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ABSTRACT

This paper studies the maturity and stream of payments of sovereign debt. Using Bloomberg bond data for eleven emerging economies, we document that countries react to crises by issuing debt with shortened maturity but back-load payment schedules. To account for this pattern, we develop a sovereign default model with an endogenous choice of debt maturity and payment schedule. During recessions, the country prefers its payments to be more back-loaded—delaying relatively larger payments—to smooth consumption. However, such a back-loaded schedule is expensive given that later payments carry higher default risk. To reduce borrowing costs, the country optimally shortens maturity. When calibrated to the Brazilian data, the model can rationalize the observed patterns of maturity and payment schedule, as an optimal trade-off between consumption smoothing and endogenous borrowing cost.

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1 Introduction

At least since the work of Rodrik and Velasco (1999) on the maturity of emerging market debt, international economists have been puzzled by emerging economies' heavy issuance of short-term debt during crises. Short-term debt is particularly subject to roll-over risk, which is harmful to consumption smoothing. We argue that this is less puzzling than one might think. Countries also adjust the stream of promised payments to be more back-loaded, i.e., relatively larger payments are scheduled closer to maturity, while the smaller payments are due sooner. This allows the sovereign to mitigate the downsides of short-term borrowing.

In this paper we introduce a parsimonious measure, the average growth rate of the scheduled payments, to capture the timing and relative size of coupons and principals of sovereign debt. A higher growth rate implies a more back-loaded schedule. We document that countries react to recessions by increasing payment growth and by shortening maturity. During recessions, countries prefer to delay relatively larger payments to smooth consumption. However, a schedule with such a high payment growth is expensive, given that later payments carry higher default risk. To reduce borrowing costs, the country optimally shortens the time to maturity. This choice reflects the tension between present debt burden and lack of enforcement in the future.

To understand how an emerging economy chooses the maturity and, more important, the growth rate of scheduled payments for external debt, we explore the individual bond data of eleven emerging markets from the Bloomberg Professional service using panel IV methods. We report two major findings on sovereign debt issuance. First, the payment growth rate is higher when output is low and spread is high. This implies that promised payments are more back-loaded during downturns. Second, the maturity is shorter during such episodes, consistent with the evidence presented by Arellano and Ramanarayanan (2012) and Broner, Lorenzoni, and Schmukler (2013).

Our model extends the standard sovereign default framework of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008) by introducing a flexible choice of payment schedule. A small, open economy can issue only state-uncontingent bonds in the international financial markets. Its government can choose to default over its bond, subject to a punishment of output loss and temporary exclusion from international markets. We depart from the literature and allow the government to issue bonds with different maturities and schedules. For example, the government may issue a T-period, back-loaded (front-loaded) long-term bond. Before the bond matures, the government makes periodic payments that increase (decrease) over time.

The payment schedule and maturity of sovereign debt are determined by the interplay of two incentives: smoothing consumption and reducing default risk. To smooth consumption, the sovereign would like to align payments with future output, i.e., larger payments ought to to be scheduled in periods with higher expected output. Thus, a more back-loaded schedule is preferable during economic downturns, since the government can repay the bulk of its obligation in the future, when the economy is expected to recover. Therefore, under the consumption-smoothing incentive, the growth rate of payments and current output should be negatively correlated.

The government must also take into consideration its default risk when making choices over payment schedules, since high default risk leads to high borrowing cost. A more back-loaded bond is particularly expensive during downturns. The reason is that such a contract specifies that most payments be made in the distant future, which subjects lenders to large losses if the government defaults in the meantime. To reduce borrowing cost while enjoying the consumption-smoothing benefit of back-loaded contracts, the government chooses a shorter maturity in economic downturns. Contracts with shorter maturity allow lenders to receive their investment returns sooner. Lenders therefore bear less default risk and offer a higher bond price.

We calibrate the model to match key moments for the Brazilian economy. Our model generates volatilities of consumption and trade balance similar to the data. The model replicates key features of sovereign debt. The median maturity is about nine years in the model and ten years in the data. The median growth rate of payment is five percent in the data and six percent in the model, which implies that, on average, countries issue back-loaded bonds.

Most important, our model matches the cyclical behavior of issuance well. When the spread increases above its mean, maturity shortens from seven to about three years, while the payment growth rate increases from 3.4% to roughly eight percent. By looking across quartiles of spread or GDP we find that the cyclical properties of issuance in both model and data are similar and fairly monotonic.

This paper makes two contributions. Empirically, we construct a parsimonious measure of payment schedule and document the role of back-loading for consumption smoothing during downturns. Most studies in the literature, such as those of Broner, Lorenzoni, and Schmukler (2013) and Arellano and Ramanarayanan (2012), address this margin by focusing on the portfolio composition of both short and long debt.

Theoretically, we model the endogenous choice of payment schedule and maturity. The literature often restricts borrowing either to a one-period bond or to exogenous payment schedules. A new line of work studying long-term sovereign debt as in Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009) uses perpetuity bonds to avoid the curse of dimensionality. Such perpetuity bonds are restricted to a front-loaded payment schedule[1], opposite those covered in the data. Another line of work studies maturity for fiscal policy purposes, using zero-coupon bonds, e.g. Lustig, Sleet, and Yeltekin (2008).

[1]For example, in Arellano and Ramanarayanan (2012), one unit of the perpetuity bond promises payments $\left\{1, \delta, \delta^2, \dots\right\}$ and so forth, forever. This requires the gross growth rate δ to be bounded above, to keep the state space bounded.

2 Empirical Analysis

This section documents how the maturity and payment schedule of new issuances vary with underlying fundamentals, using bond-level data. Our key finding is that during financial distress the sovereign shortens maturity and schedules payments to be more back-loaded, i.e., they promise smaller payments in the near future and larger payments later.

We study a sample of eleven emerging market sovereigns[2]: Argentina, Brazil, Mexico, Russia, Colombia, Uruguay, Venezuela, Hungary, Poland, Turkey, and South Africa. Using the *Bloomberg Professional* database, we extract information on the schedule of coupons and principal of external debt. We focus on foreign-currency denominated bonds and exclude bonds with special features (e.g., collateralized) and those with guarantees from international financial institutions, e.g., the IMF.

We construct promised cash flows from coupons and principal payments. Since countries issue debt denominated in various currencies, we need to convert these flows to real US dollars using exchange rates provided by the IMF and the CPI series from the Bureau of Labor Statistics. LIBOR rates from EconStats.com are used whenever a bond specifies its coupon rate relative to such a reference rate. We document key facts about these bond-level issuance data, in connection with GDP and the spread series provided by Broner, Lorenzoni, and Schmukler (2013).[3] Appendix A contains further information on the data used.

2.1 Payment Schedule, Maturity, and Duration

We start by defining key concepts. We characterize bonds using three measures: maturity T, Macaulay duration D, and the growth rate of payments δ. Consider a sovereign country i in period t. Let $c_t^i(s)$ denote the cash flow—in real US dollar terms—promised by the portfolio issued at period t to be paid $s \in \{1, 2, \dots, N_t^i\}$ periods later. N_t^i refers to the number of periods until the last payment is scheduled. Let n be the number of periods in a year.

Whenever multiple bonds are issued during a given time period, e.g. in the same week, we sum over the cross-section of promised cash flows, at each future period, resulting in a single stream of payments $c_t^i(s)$, as if the country had issued a single bond making all the payments scheduled by the actual bonds issued. Such constructed streams are assigned a maturity T_t^i (measured in terms of years since the issue date) given by the average maturity of the actually issued bonds, weighted by each bond's real principal value. We label the promised cash-flow profile $\{c_t^i(s)\}_{s=1}^{N_t}$ as *payment schedule*. To compute the annualized growth rate of payment δ_t^i, we regress the promised cash flows

[2]This is the same set of countries considered in Broner, Lorenzoni, and Schmukler (2013).

[3]The spreads are at a weekly frequency and measured by the differences in the (annualized) yield-to-maturity relative to equivalent U.S. (or German) bonds. Their yield curve estimates deliver spread for bonds of the maturities either up to three years, between six and nine years, or over twelve years.

over the number of years elapsed since the issue date t,

$$\log c_t^i(s) = \text{constant} + \delta_t^i \frac{s}{n} + \epsilon_t^i(s) \tag{1}$$

where $\epsilon_t^i(s)$ is an error term reflecting deviations of the actual schedule from a perfect exponential sequence. Table 1 reports country-level, average R-squared statistics for these regressions.

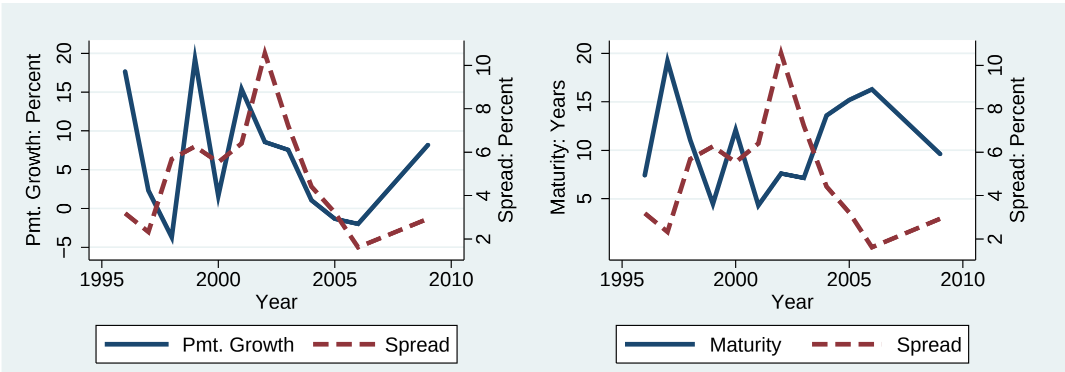
Duration $D_t^i(j)$ measures the average length of time to payment. It is given by

$$D_t^i = \sum_{s=1}^{N_t^i} \frac{c_t^i(s)\, R^{-s/n}}{\sum_{s=1}^{N_t^i}\{c_t^i(s)\, R^{-s/n}\}}\, \frac{s}{n}, \tag{2}$$

where R denotes the gross annual, real, risk-free rate, which we fix at 3.2 percent, following Arellano and Ramanarayanan (2012). Thus, D_t^i represents the risk-free version of the Macaulay duration and is referred to simply as the *duration.* This is the measure commonly studied in the literature. Although this measure reflects both maturity and the payment schedule, it obscures their independent roles for issuance choice, as documented in our analysis.

Table 1 reports the summary statistics for the eleven countries in our sample. The average payment growth for weekly new issuances is 19%, while the average maturity is about nine years, and average duration is six years. We are interested in how emerging markets vary issuance characteristics with the business cycle, as reflected in the nine-year interest rate spread. We use the series from the yield curve estimation of Broner, Lorenzoni, and Schmukler (2013). For all countries except Russia, the maturity is negatively correlated with the spread, with correlations ranging from -0.05 to -0.48. Payments are back-loaded when the interest rate is high, with a positive correlation for most countries. Uruguay is an outlier in that it has issued a substantial number of zero-coupon bonds.

Figure 1: Maturity, Payment Growth, and Spread (Brazil)



Note: Pmt. growth denotes the growth rate of payments for annual new issuances.

We illustrate the dynamics of payment growth and maturity in relation to the spread for the case of Brazil, in Figure 1. The growth rate of scheduled payments co-moves with the spread, with a correlation of 0.51. On the other hand, the maturity has a negative correlation of -0.48.

2.2 Regression Analysis

Given the suggestive correlations reported above, we undertake a more systematic analysis of the data by employing the specification introduced by Broner, Lorenzoni, and Schmukler (2013). We regress our measures on the interest rate spread, controlling for country fixed-effects, country-specific time trends and the other controls used in Broner et al.[4] We also report two-stage least-squares estimates, using their identification strategy of instrumenting for sovereign spreads with the Credit Suisse First Boston (CSFB) High Yield Index. This index measures the spread on high-yield debt securities issued by the US corporate sector. Conditions in the US corporate debt market are a demand-side factor for sovereign debt markets via the portfolio problem faced by investors.

Table 2 reports our estimates. In all specifications, financial conditions are statistically significant determinants of issuance choice, with a positive coefficient in maturity regressions and negative in payment growth regressions. Throughout, results for duration are similar to those for maturity. The OLS and IV coefficients share the same sign, the IV coefficients are more precisely estimated, and the OLS coefficients are systematically closer to zero. This points to a potential attenuation bias induced by mismeasurement of spreads or the omission of a variable negatively correlated with spreads.

The magnitudes of these results are also economically significant. For every one percentage point increase in the spread, emerging markets will raise the growth rate of payments by eight to nine percentage points, back-load the schedule, and reduce maturity by about 1.3 years. Since spreads are quite volatile, for example, varying between two and ten percent for Brazil, our findings imply substantial variation in these debt characteristics over the cycle.

Constructed cash flows and their estimated payment growth δ require an explicit choice of issuance frequency. To minimize any bias induced by time aggregation and for consistency with the Broner, Lorenzoni, and Schmukler (2013) methodology, we use a weekly issuance frequency for our empirical section. This means we aggregate all bonds issued within one week into a composite bond that schedules all the payments of the individual bonds. In Appendix C we document that our empirical results are robust to a choice of yearly frequency for issuance.

Our analysis highlights two main results. First, the maturity of bonds shortens during crisis periods. This is consistent with the existing work on maturity choice of emerging markets. To the best of own knowledge, our second finding is new to the literature: sovereigns also adjust payment schedules in response to crises by issuing more back-loaded bonds.

[4] All variables are six-month moving averages (measured by using 26-week rolling window). All independent variables are logged and then demeaned for a given country, where spread variables are log-spreads $100*log(1+spread)$. For more details, see section 4 in Broner et al.

Table 1: Summary Statistics: All Countries

Country	Num. Bonds		Issuance						
	All Bonds	Zero Coupon Bonds	Maturity (T): Years	Payment Growth (δ): Percent	Duration (D): Years	Spread (r): Percent	Corr (T, r)	Corr (δ, r)	$R-$sq
Argentina	321	131	4.90	22.75	4.21	8.92	-0.05	-0.01	0.38
Brazil	78	0	11.43	11.39	6.98	4.44	-0.48	0.51	0.23
Colombia	107	0	7.56	30.08	5.44	3.98	-0.44	0.43	0.39
Hungary	29	0	8.71	14.98	7.20	0.94	-0.35	0.07	0.22
Mexico	108	1	9.71	15.16	6.72	2.31	-0.25	0.22	0.22
Poland	63	0	12.53	11.88	8.86	0.66	-0.28	0.35	0.18
Russia	22	0	9.94	14.23	6.90	5.85	0.32	0.13	0.21
South Africa	25	0	9.87	14.09	7.22	1.96	-0.06	0.25	0.22
Turkey	200	23	6.27	25.61	5.23	4.20	-0.21	-0.05	0.29
Uruguay	572	483	2.64	41.42	2.73	4.64	-0.20	-0.47	0.72
Venezuela	50	0	10.82	10.70	7.04	3.94	-0.11	0.20	0.26
Avg.	143	58	8.58	19.30	6.23	3.80	-0.19	0.15	0.30

Note: this table provides characteristics of weekly issues of bonds. $R-$sq refers to the $R-$squared from the estimation of the payment growth (δ), Equation 1. Pmt. Growth (δ) refers to the annual growth—in percentage points—in the payment schedule, and Spread (r) the percentage nine-year interest rate spread. The row labeled "Avg." refers to the simple averages across the eleven countries. Sample periods are: July 1993 - May 2003 for Argentina, July 1994 - June 2009 for Brazil, January 1993 - June 2009 for Colombia, February 1990 - June 2009 for Hungary, January 1991 - June 2009 for Mexico, October 1994 - June 2009 for Poland, January 1993 - June 2009 for Russia, October 1991 - June 2009 for South Africa, January 1990 - June 2009 for Turkey, January 1993 - May 2003 for Uruguay, and July 1991 - June 2009 for Venezuela.

Table 2: Regression: Payment Growth, Maturity, and Duration

	Dependent Variable: Payment Growth Rate (δ)					
	OLS	IV	OLS	IV	OLS	IV
3-y Spread	0.97	8.89***				
	[2.15]	[1.52]				
9-y Spread			1.74	7.96***		
			[3.11]	[1.40]		
12-y Spread					2.50	8.07***
					[2.33]	[1.41]
Controls	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.16	0.15	0.16	0.15	0.16	0.15
	Dependent Variable: Maturity (T)					
3-y Spread	-0.01	-1.37***				
	[0.04]	[0.17]				
9-y Spread			-0.31***	-1.23***		
			[0.08]	[0.16]		
12-y Spread					-0.13**	-1.24***
					[0.06]	[0.16]
Controls	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.38	0.40	0.39	0.40	0.38	0.40
	Dependent Variable: Duration (D)					
3-y Spread	3e-3	-0.78***				
	[0.03]	[0.08]				
9-y Spread			-0.16***	-0.70***		
			[0.05]	[0.07]		
12-y Spread					-0.06	-0.71***
					[0.04]	[0.07]
Controls	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.42	0.45	0.43	0.45	0.42	0.45
First Stage						
CSFB HYI		2.71***		3.03***		2.99***
		[0.21]		[0.12]		[0.14]

Note: The number of observations is 4515. This table reports OLS and 2SLS (IV) regressions of payments growth, maturity, and duration on spreads, controlling for country fixed-effects, country-specific time trends, the real exchange rate, terms of trade, and an investment grade dummy. For the IV regressions, spread variables are instrumented by the Credit Suisse First Boston High Yield Index (CSFB HYI). Standard errors are robust to heteroskedasticity and within country serial correlation. ** significant at 5%; *** at 1%.

3 Model

We study optimal maturity and payment schedule of sovereign debt in a small, open economy model with default. A benevolent government borrows from a continuum of competitive lenders by issuing uncontingent debt with a flexible choice of maturity and payment schedule. The debt contract has limited enforcement, in that payments are state-uncontingent and the sovereign government has the option to default.

3.1 Technology, preferences, and international contracts

The economy receives a stochastic endowment y, which follows a first-order Markov process. The government is benevolent, and its objective is to maximize the utility of the representative consumer given by,

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where c_t denotes consumption in period t, $0 < \beta < 1$ the discount factor, and $u(\cdot)$ the period utility function, satisfying the usual Inada conditions. Each period, the government may borrow abroad by issuing a bond contract and decides whether to default on the outstanding debt. All the proceeds of the government are transferred as a lump sum to the representative consumer. We assume the government has access to enough policy instruments[5] to be capable of perfectly control the overall national level of borrowing, thus avoiding any issues related to private sector over-borrowing, as discussed in Jeske (2005).

While in *good credit standing*, the government has the option to default on its debt. Following the sovereign default literature, we assume that after default, the debt is written off and the government switches to *bad credit standing*. The government is then subject to output losses and temporary exclusion from international financial markets. With probability ϕ, international lenders forgive a government in bad credit standing and resume lending to it.[6] Given default risk, lenders charge bond prices that compensate them for expected losses.

A bond contract specifies a maturity T and a payment schedule given by the growth rate of payments δ, the number of units issued b, and a bond price q. For such a contract, conditional on not defaulting, the government repays $(1+\delta)^{-\tau}$ with $0 \leq \tau \leq T$ periods to maturity. When δ is negative, the payments shrink over time (front-loaded).[7] When δ equals zero, the contract is "flat" as the payments are constant over T periods. When δ is positive, the payments grow over time (back-loaded). The contract also nests the zero coupon bond, when we let δ go to infinity.

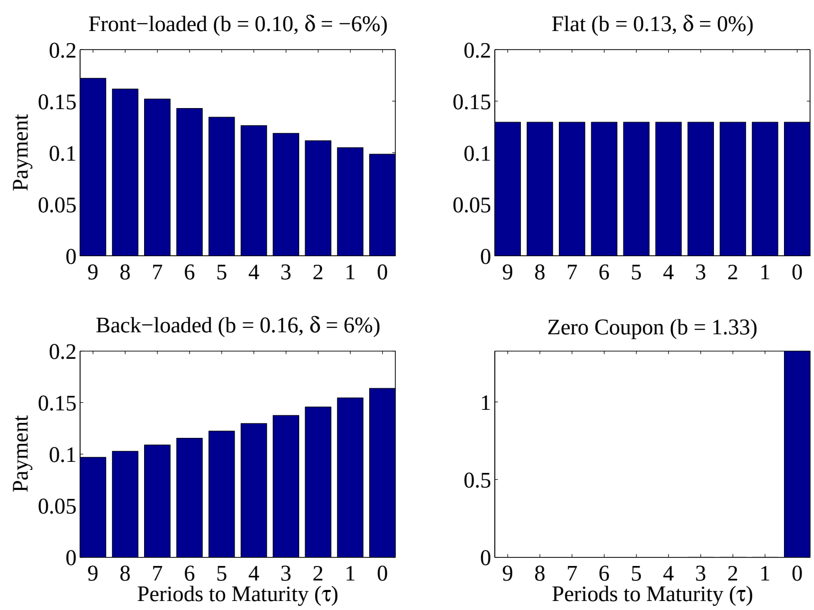
[5] For example, capital control policies, as studied by Kehoe and Perri (2004), Wright (2006), Kim and Zhang (2012), or Na, Schmitt-Grohé, Uribe, and Yue (2014).

[6] Our model abstracts from renegotiation. Yue (2010), D'Erasmo (2008), and Benjamin and Wright (2009) study debt renegotiation explicitly. Quantitatively, the predictions of such models in terms of standard business-cycle statistics of emerging economics are similar as that in Arellano (2008), without renegotiation.

[7] This is the case covered by the perpetuity bond in Arellano and Ramanarayanan (2012).

Figure 2 shows examples of schedules for different cases of δ, for ten-year bonds. To make contracts comparable, we pick the number of bond units issued b to finance one unit of consumption for all cases, using the risk-free bond price. With a more back-loaded schedule, the number of units issued b has to be larger due to discounting.

Figure 2: Payment schedule



Note: Payment schedules for bond contracts with different δ, against periods to maturity τ. The number of units issued b is picked so that all bonds finance 1 unit of consumption at the respective risk-free bond price.

To mitigate the curse of dimensionality implicit in using richer descriptions of debt contracts, we assume that the government can only hold one type of bond at a time. If the government wants to change its payment schedule, it has to buy back the outstanding debt before it can issue a new contract. Under this assumption, the state of a government with good credit standing is $z = (T, \delta, b, y)$, including its income shock y and the outstanding units b, with remaining maturity T and growth rate of payments δ.

3.2 Equilibrium

The government's problem The government in good credit standing chooses whether to default d, with $d = 1$ denoting default:

$$V(z) = \max_{d \in \{0,1\}} \left\{ d\, V^d(y) + (1-d)\, V^n(z) \right\} \tag{3}$$

where V^d and V^n are the defaulting and repaying values respectively.

If it defaults, the government gets its debt written off but receives a lower endowment $h(y) \leq y$. With probability ϕ, a government in bad credit standing will return to market, without any debt. The defaulting value satisfies

$$V^d(y) = u[h(y)] + \beta\,\mathbf{E}\left\{(1-\phi)\,V^d(y') + \phi\,V(0,0,0,y')\right\}. \tag{4}$$

If it repays, the government can continue the current contract, with value V^c, or issue new debt and receive value V^r. We use $x = 0$ to denote continuing the current contract and $x = 1$ to denote issuing new debt. Specifically, the problem under no default is given by

$$V^n(z) = \max_{x \in \{0,1\}} \{x\,V^r(z) + (1-x)\,V^c(z)\} \tag{5}$$

where the value when continuing to service outstanding debt is

$$V^c(z) = u\left[y - \frac{b}{(1+\delta)^T}\right] + \beta\,\mathbf{E}\,V(T-1,\delta,b,y'), \tag{6}$$

and the value when choosing a new bond is

$$\begin{aligned} V^r(z) &= \max_{T',\delta',b'} \{u(c) + \beta\,\mathbf{E}\,V(T',\delta',b',y')\} \\ \text{s.t. } c &= y - \frac{b}{(1+\delta)^T} + q(T',\delta',b',y)\,b' - q^{\text{rf}}(T-1,\delta)\,b. \end{aligned} \tag{7}$$

If it chooses to issue, the government must retire outstanding obligations, at the risk-free bond price q^{rf}. The proceeds from the sale of the new bond are $q(T',\delta',b',y)\,b'$, where the bond price schedule for new issuance, q, reflects future default risk and thus depends on the current endowment level y and the payment structure.

We assume that when buying back old bonds, the government faces a cost given by the risk-free bond price q^{rf}, the upper limit for the secondary-market price. This high cost is consistent with evidence on expensive buybacks discussed in Bulow and Rogoff (1988) and proxies for issuance costs in a reduced form. Here we abstract from issues of debt dilution, as studied by the recent literature on long-term sovereign debt, e.g., Hatchondo, Martinez, and Sosa-Padilla (2014) and Sanchez, Sapriza, and Yurdagul (2014). We conduct sensitivity analysis with respect to alternative buyback costs, allowing for dilution, in section 4.4.

International financial intermediaries Lenders[8] are risk neutral, competitive, and face a constant world interest rate r. The bond price schedule must guarantee that lenders break even in expectation. For a bond with remaining maturity T' and growth rate δ', its risk-free price is defined

[8] We assume that lenders have deep pockets and thus can unilaterally satisfy the country's loan demand. This rules out self-fulfilling crises due to lenders' failure to coordinate, as in Cole and Kehoe (2000).

recursively as

$$q^{\mathrm{rf}}\left(T', \delta'\right) = \begin{cases} \dfrac{1}{1+r}\left[\dfrac{1}{(1+\delta)^{T'}} + q^{\mathrm{rf}}\left(T'-1, \delta\right)\right] & \text{for } T' \geq 1 \\ \dfrac{1}{1+r} & \text{for } T' = 0. \end{cases} \tag{8}$$

With default risk, lenders charge a higher interest rate to compensate for losses in the default event. For $T' \geq 1$, the bond price is therefore given by

$$\begin{aligned} q\left(T', \delta', b', y\right) = {} & \frac{1}{1+r}\mathbf{E}\left\{\left(1 - d\left(T', \delta', b', y'\right)\right) \times \right. \\ & \left. \left[\frac{1}{(1+\delta)^{T'}} + \left(1 - x\left(T', \delta', b', y'\right)\right) q\left(T'-1, \delta', b', y'\right) + x\left(T', \delta', b', y'\right) q^{\mathrm{rf}}\left(T'-1, \delta'\right)\right]\right\}, \end{aligned} \tag{9}$$

and for $T' = 0$ the bond price reduces to the usual one-period bond case

$$q\left(0, \delta', b', y\right) = \frac{1}{1+r}\mathbf{E}\left\{1 - d\left(0, \delta', b', y'\right)\right\}. \tag{10}$$

The risky bond price reflects expected payments to lenders. If the government repays next period, lenders receive a payment of $(1+\delta)^{-T'}$ per unit outstanding. The repaying government may choose to restructure its debt $x' = 1$ and so repurchase its outstanding debt at the risk-free bond price q^{rf}. Note that maturity T' and payment schedule δ' affect the risky bond price in two ways: on one hand, conditional on no default, they matter for expected discounted payment and thus the risk-free component of q, the corresponding q^{rf}. On the other hand, both maturity and payment schedule matter for future default decisions and thus the default premium priced into q.

Definition of equilibrium The equilibrium consists of policy functions T', δ', b', d', x', value functions V, V^d, V^n, V^c, the bond price schedule q, and the risk-free schedule q^{rf}, such that, given the world interest rate r,

(a) policies and values satisfy the government's problem (3-7), given the bond prices, and

(b) lenders charge break-even bond prices (9) consistent with government policies, and the risk-free bond price schedule is given by (8).

4 Quantitative Analysis

We calibrate the model for the Brazilian economy over the period from 1996 to 2009 and study its implications for standard business cycle statistics and, most important, for the maturity and payment schedule of sovereign debt. We discuss the incentives faced by a country when designing its bond issuance. Finally, we conduct sensitivity analysis related to the cost of retiring outstanding debt and alternative shock specifications.

4.1 Calibration

We calibrate the parameter values of the model to match key moments in the yearly Brazilian data. The per-period utility function $u(c)$ exhibits a constant coefficient of relative risk aversion, σ,

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}. \tag{11}$$

The economy is subject to two, independent shocks: an endowment shock and a sudden stop shock. The endowment of this economy follows an AR(1) process

$$\log(y_t) = \rho \log(y_{t-1}) + \eta\, \varepsilon_t, \tag{12}$$

where the idiosyncratic shock ε_t follows the standard Normal distribution. Every period, with a constant probability p^{ss}, the country enters a *sudden stop state*, in which endowment is reduced and the country can only lower its debt burden. While in this state, the country has a constant probability p^{ret} of recovering in the next period.[9]

Following Arellano and Ramanarayanan (2012), the output of a country with a bad credit standing $h(y)$ is given by

$$h(y) = \min \{\{y,\ (1 - \lambda_d)\, \mathbf{E}\, y\}\} \tag{13}$$

where $\mathbf{E}\, y$ is the unconditional mean of y and $\lambda_d \in [0, 1]$ captures the default penalty. During sudden stop, the endowment is capped by $(1 - \lambda_s)\, \mathbf{E}\, y$.

To compare model and data, we define the *yield to maturity* as the constant interest rate $\hat{r}$ such that the present value of payments computed using this interest rate is equal to the market price of the bond, i.e., $\hat{r}$ is implicitly defined by

$$q\left(T', \delta', b', y\right) = \sum_{\tau=T'}^{0} \exp\left[-\hat{r} \times \left(T' + 1 - \tau\right)\right] \frac{1}{\left(1 + \delta'\right)^{\tau}}. \tag{14}$$

The *spread* is the difference between the yield to maturity $\hat{r}$ and the risk-free rate r:

$$\text{spread}\left(T', \delta', b', y\right) \equiv \hat{r}\left(T', \delta', b', y\right) - r. \tag{15}$$

Table 3 presents the calibrated parameter values. The risk-aversion parameter σ is set to two as is standard in the literature. The risk-free interest rate is set to 3.2 percent to target the average annual yield to maturity for US government bonds. The persistence and volatility of the AR(1) output process are taken from Arellano and Ramanarayanan (2012), who calibrate these two parameters to the HP-filtered Brazilian GDP. They pick $\rho = 0.9$ and compute the standard deviation $\eta = 0.017$. The probability of a defaulting country regaining access to the international financial market ϕ is

[9] For a version of the model with an explicit sudden stop state, see Appendix B.

Table 3: Benchmark Parameter Values

		Value	Target/Source
Parameters calibrated independently			
σ	Risk-aversion	2.0	Standard value
r	Risk-free rate	3.2%	Arellano and Ramanarayanan (2012)
ρ	Shock persistence	0.9	Arellano and Ramanarayanan (2012)
η	Shock volatility	0.017	Arellano and Ramanarayanan (2012)
ϕ	Prob. of return to market	0.17	Benjamin and Wright (2009)
p^{ss}	Prob. of sudden stop (s.s.)	0.10	Bianchi, Hatchondo, and Martinez (2012)
p^{ret}	Prob. of s.s. recovery	0.75	Bianchi, Hatchondo, and Martinez (2012)
Parameters calibrated jointly			
β	Discount factor	0.88	Jointly: Mean of 9y and 3y spreads, median maturity, and the debt service to GDP ratio.
λ_d	Output loss due to default	5.0%	
λ_s	Output loss due to s.s.	-0.5%	
$\overline{T}$	Max. maturity	15	

Note: this table provides the benchmark parameter values used in calibrating the model.

set to 0.17, following Arellano and Ramanarayanan (2012). The annual probability of sudden stop p^{ss} and recovery p^{ret} are chosen to be 0.10 and 0.75, consistent with the quarterly values used by Bianchi, Hatchondo, and Martinez (2012). The four remaining parameters, the discount factor β, the output loss parameters λ_d and λ_s, together with the maximum maturity $\overline{T}$ are chosen jointly, to match the average three-year and nine-year spreads, median maturity, and the debt service to GDP ratio.

Table 4 compares the baseline model (column 2) and data (column 1) statistics for Brazil. The model matches the targeted moments well. For both data and model, we focus on new issuance.[10] The median maturity is 10.3 years in the data and nine years in the model. The model also replicates payment growth and key business cycle features of emerging markets well. It predicts a six percent growth rate of payments, consistent with the data, where the median growth rate of payments is 4.9%, implying a back-loaded payment schedule for new issuance. It generates excess volatility of spreads relative to the data. Consumption is more volatile than output, as documented by Neumeyer and Perri (2005). The volatility of consumption is 1.1 times that of output in both the model and the data. The model produces a volatile trade balance (normalized by GDP), 55 percent in the model and 36 percent in the data. In Brazil, the spreads for all maturities are countercyclical. The correlations are -0.49, -0.57, and -0.52 for three-year, nine-year, and twelve-year with GDP, respectively. Table 4 reports the average of these correlations, -0.53. This correlation is also negative

[10]Following the sovereign default literature, for computational reasons, we restrict the sovereign to hold only one asset at a time. It then must be the case that this period's issuance will be next period's stock. In contrast, in the data the stock at any one time is the accumulation of many issuances, at various moments in the past. Faced with a choice between targeting stocks and matching flows (issuance), we follow the literature and study issuance, e.g., Arellano and Ramanarayanan (2012) and Broner, Lorenzoni, and Schmukler (2013).

Table 4: Key Statistics: Data vs. Model

	Data	Baseline	No SS
Targeted Moments			
Mean 9-y Spread	4.4%	3.7%	3.8%
Mean 3-y Spread	4.5%	5.2%	2.3%
Debt Service / GDP	5.3%	4.5%	5.7%
Median Maturity	10.3	9.0	10.0
Other Moments			
Median Payment Growth	4.9%	6.0%	3.0%
Std 9-y Spread	2.7%	4.2%	4.3%
Std 3-y Spread	4.0%	5.9%	3.9%
Std C / Std Y	110%	113%	113%
Std NX/Y / Std Y	36%	55%	56%
Corr B/Y, Y	-0.87	-0.23	0.26
Corr Spread, Y	-0.53	-0.34	-0.33

Note: "No SS" refers to the calibrated model without sudden stop shocks. Std denotes standard deviation and Corr correlation. C is consumption, Y is GDP, NX is net export, B is total debt.

in the model, -0.34.

In the data the debt-to-GDP ratio is strongly countercyclical, with a correlation with GDP of -0.87. To replicate this behavior, we need to include a sudden stop shock that creates an additional precautionary saving motive, discouraging excessive borrowing during good times when spreads are low. Eliminating the sudden stop increases the correlation of debt-to-GDP with GDP to 0.26 but leaves other moments relatively unchanged, as shown in the third column of Table 4.

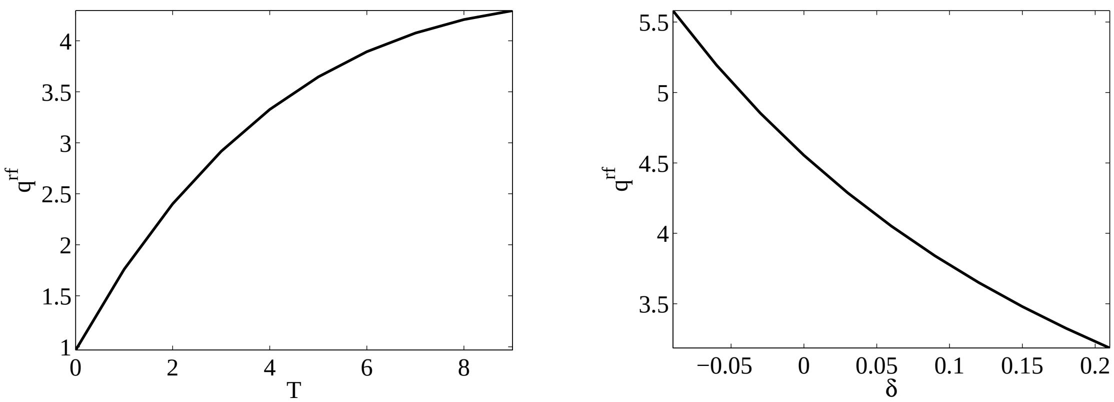
4.2 Bond Price Schedule

The choice of optimal contracts depends on government's preferences and the bond price schedule it faces. This schedule depends on future governments' default incentives, which are determined by two channels: lack of commitment and debt burden. Contracts which make eventual default more tempting for the government (lack of commitment) or which require higher payments (debt burden), will carry higher default risk, lower prices and therefore be less attractive for debt finance.

The bond price reflects the lender's opportunity cost, the equivalently structured risk-free bond price q^{rf}. This price varies with T' and δ', due to the changes they induce in the size and number of payments. All other contract characteristics constant, longer maturity implies more payments and thus a higher risk-free bond price. See Figure 3(a). A high δ' is associated with back-loaded payments, which are subject to compounded discounting and thus have lower present value, resulting in a lower risk-free price. See Figure 3(b).

To isolate the consequences of default risk, Figure 4 plots the market bond price schedule

Figure 3: Risk-Free Bond Price q^{rf}



(a) Maturity (b) Payment Growth

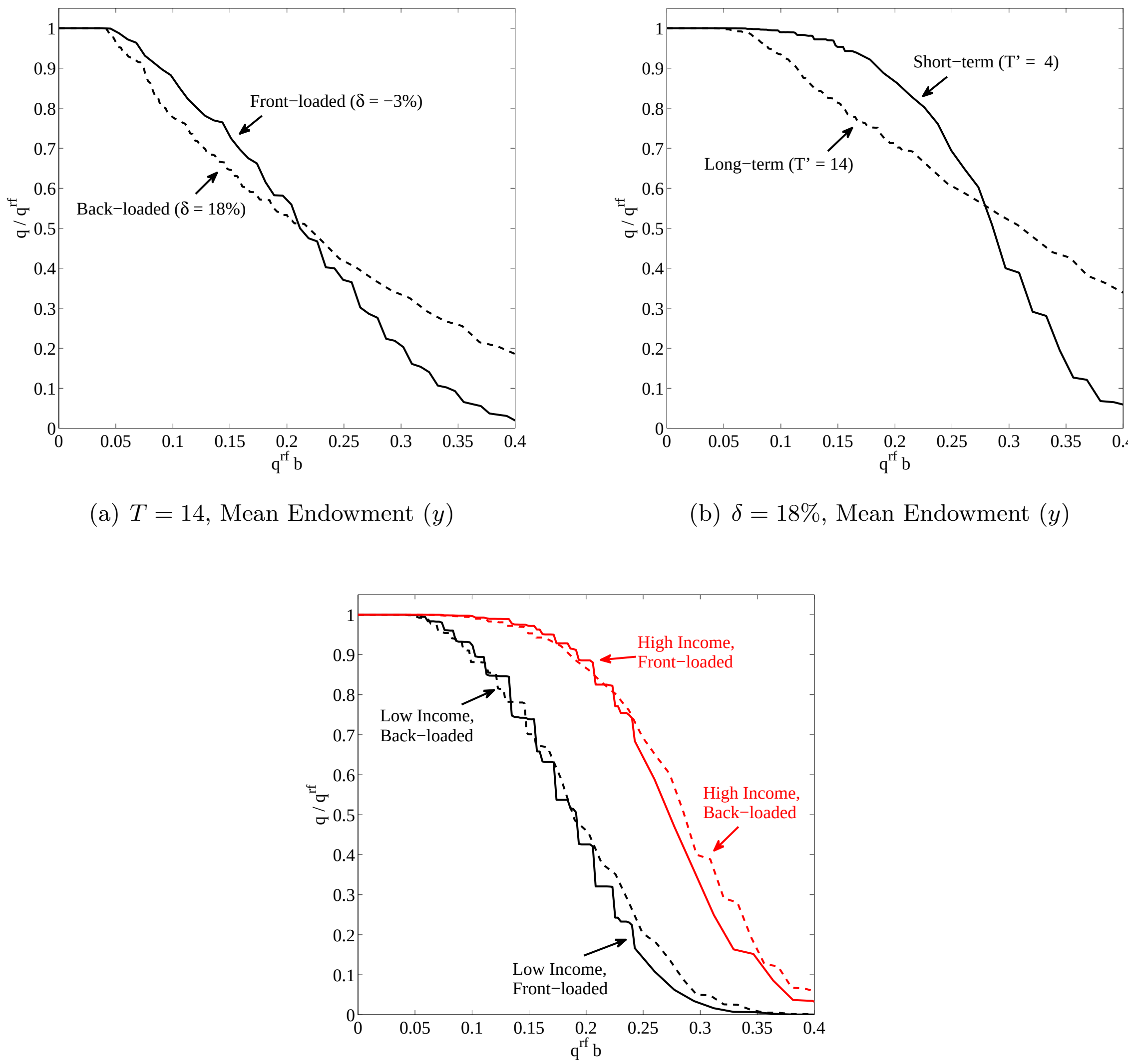
$q(T', \delta', b', y)$ relative to the risk-free bond price $q^{\text{rf}}(T', \delta')$ as a function of $q^{\text{rf}}(T', \delta')b$. We normalize the number of units b with q^{rf} to facilitate comparisons of debt values across different contracts. For any given T' and δ', issuing more units means a higher debt burden and thus higher risk of default and a lower bond price.

Figure 4(a) compares the bond price across growth rates of payments, $\delta = -3\%$ versus $\delta = 18\%$, for a fixed $T = 14$ and mean endowment. Consider an increase in δ, i.e., a more back-loaded contract. In the absence of commitment, distant promises are less credible, leading to higher default incentives and a lower bond price. On the other hand, more back-loading induces smaller payments in the near future and larger payments later. Overall, due to compound discounting, this implies a lower debt burden, lower default risk, and a higher bond price. For back-loaded contracts, the debt burden effect dominates for high levels of debt and the government gets better bond prices.

Figure 4(b) compares the bond price across maturity choices, $T = 4$ versus $T = 14$, for a fixed $\delta = 18\%$ and mean endowment. When extending the maturity, there is a greater lack of commitment and the bond price is lower. At the same time, debt burden in any one period is decreased, reducing default incentives. Overall, when less is borrowed, the price schedule is higher for short-term debt.

When maturity is short, the bond price schedule becomes insensitive to the choice of δ, as shown in Figure 4(c). This is because the two channels are fairly balanced. This suggests that when countries shorten maturity during recessions, they are likely to smooth consumption by back-loading payments. Figure 4(c) also illustrates the role of income in determining bond prices: higher income implies fewer incentives to default and thus the country can borrow more cheaply.

Figure 4: Bond Price Schedule



(a) $T = 14$, Mean Endowment (y)

(b) $\delta = 18\%$, Mean Endowment (y)

(c) $T = 4$

4.3 Maturity and Payment Schedule

We now turn our focus to understanding how maturity and payment structure of issuance vary with the business cycle. We use the spread and output as our preferred cyclical indicators. Table 5 reports key statistics for Brazil and their model counterparts. In the data, during normal times when the spread is below its historic mean, the growth rate of payments is about five percent, with a maturity of roughly 14 years and a duration of nine years. During periods of financial stress, when the spread is above average, payments become more back-loaded, with a growth rate of 7.1%, maturity shortens to about nine years, and duration is reduced to six years. These patterns are consistent with the findings in Section 2 where we studied a broader set of countries.

Our model matches the observed cyclicality of maturity, payment growth, and duration well. When the spread increases above its mean, the payment growth rate increases from 3.4% to roughly eight percent, while maturity shortens from seven to about three years, and the duration decreases from five to three years. Even though the model generates a lower maturity and duration, on average, relative to the data, it successfully matches the magnitude and sign of their changes: in both data and model maturity decreases by about four years, when spread increases above its mean. By looking across quartiles of the spread we find that the cyclical properties of issuance are fairly monotonic, and similar between model and data. However, the model has an excessively low maturity when spread is below its 25th percentile. In the model, countries face a low spread either when income is high or when they have close to no debt. In the model we find that countries shocked with a low endowment realization, while carrying close to no debt, choose not to default but face low spreads. Even so, due to the low endowment, they choose to borrow short-term, as our mechanism predicts.

Using GDP as a cyclical indicator, we get a similar message. When GDP is below trend, the country shortens maturity from 13 to nine years but back-loads the payment from 3.2 percent to 8.4 percent. Duration follows the dynamics of maturity. In particular, it also shortens by 2.5 years.

The payment schedule and maturity of sovereign debt are determined by the interplay of two incentives: (i) smoothing consumption, and (ii) lowering the borrowing cost by reducing default risk. To smooth consumption, the sovereign would like to align payments with future output, i.e. scheduling larger payments for periods with higher expected output. Given the mean-reverting nature of the output process considered, the growth rate of output decreases with the current output. Thus, a more back-loaded schedule is preferable during economic downturns since the government can repay the bulk of its obligation in the future, when the economy is expected to recover. Under the consumption-smoothing incentive, the growth rate of payments and current output should be negatively correlated.

The government also takes into consideration the borrowing cost it faces when making choices over payment schedules. During downturns, when income is low, the range of debt levels for which back-loaded contracts offer better bond prices shrinks, as Figure 4(b) shows. This makes the sovereign more likely to face a tighter bond price if it were to choose a more back-loaded contract. To reduce the borrowing cost, while enjoying the consumption-smoothing benefit of more back-loaded contracts, the government chooses a shorter maturity in downturns to mitigate its lack of commitment. Moreover, for short maturities, the differences in bond price schedules for different payment growth rates are small, as Figure 4(c) shows.

Table 5: Payment Growth, Maturity, and Duration: Cyclical Properties

Spread	Below Mean	Above Mean	Δ	Q1	Q2	Q3	Q4
Payment Growth (δ, %)							
Data	5.0	7.1	2.1	0.2	6.7	4.6	12.7
Baseline	3.4	7.9	4.5	2.5	5.6	7.6	8.1
No SS	1.2	3.6	2.4	0.7	2.0	5.9	3.4
P. Dilution	6.8	9.4	2.6	6.6	7.0	8.6	9.6
Maturity (T, Years)							
Data	13.5	8.6	-4.9	17.7	12.9	10.3	5.9
Baseline	7.0	2.8	-4.2	6.0	11.8	6.7	3.5
No SS	8.2	5.3	-2.9	7.1	11.4	7.8	5.0
P. Dilution	5.9	2.8	-3.1	4.2	12.2	4.4	3.1
Duration (D, Years)							
Data	9.0	6.3	-2.7	11.1	8.5	7.4	4.7
Baseline	5.2	3.0	-2.2	4.4	8.8	5.8	3.5
No SS	5.5	4.0	-1.5	4.6	7.8	6.3	3.8
P. Dilution	5.0	3.0	-2.0	3.8	9.5	4.2	3.2
GDP							
Payment Growth (δ, %)							
Data	8.4	3.2	-5.2	8.5	11.0	-1.3	10.0
Baseline	8.5	1.8	-6.7	8.8	8.4	6.7	0.1
No SS	4.5	-2.1	-6.6	3.8	6.4	4.1	-5.0
P. Dilution	9.1	6.2	-2.9	10.5	8.8	8.2	4.9
Maturity (T, Years)							
Data	8.9	13.2	4.3	8.6	8.0	13.1	13.3
Baseline	3.3	11.4	8.1	1.1	10.0	10.8	11.7
No SS	5.6	10.7	5.1	3.8	11.2	11.8	10.2
P. Dilution	2.2	11.9	9.7	0.4	8.5	11.4	12.3
Duration (D, Years)							
Data	6.4	8.9	2.5	6.1	6.0	9.2	8.6
Baseline	3.4	7.3	3.9	1.7	8.3	8.2	7.0
No SS	4.5	6.5	2.0	3.0	8.8	8.7	5.4
P. Dilution	2.6	8.8	6.2	1.3	7.1	9.1	8.7

Note: All "Data" values are based on annual issuance and refer to means conditional either on spread or GDP. Payment growth (δ, %) is percentage growth in the daily payment schedule. "No SS" and "P. Dilution" refer to the model without the sudden stop shock and the one with a partial dilution buyback price respectively.

4.4 Sensitivity Analysis

In this section, we conduct sensitivity analysis for the case without sudden stop shock and alternative bond buyback prices.

We first recalibrate the model assuming no sudden stop shock. In particular, the median maturity is calibrated to be 10 years as in the baseline model. The third column of Table 4 shows that the model without sudden stop shocks generates similar volatilities of spread and net export as in the baseline. The sudden stop shock, however, matters for characteristics of debt issuance. The sovereign issues fewer back-loaded bonds, the median payment growth is reduced to three percent, compared to six percent in the baseline model. This is mainly driven by borrowing choices of high-income states, under which the sovereign tends to issue relatively front-loaded bonds if it borrows. Without sudden stop shock, high-income states have fewer precautionary motives and thus borrow more. We therefore observe lower median payment growth and positively correlated debt-to-GDP and GDP.

Though having a low payment growth, the model without sudden stop still produces countercyclical payment growth and procyclical maturity issuance. When the interest rate spread becomes high (i.e., GDP is below trend), the payment growth rate of new issuance increases by 2.4% points, and the maturity shortens by three years, as Table 5 shows. These findings are robust to the use of GDP as our conditioning variable.

We now consider an alternative specification of bond buyback price. In our main analysis we used the risk-free bond price q^{rf} to retire outstanding debt, thus abstracting from any issues raised by long-term debt dilution. First, we consider the "full dilution" case with buyback at the competitive, secondary market price. This price results from valuing outstanding debt using the default probabilities implied by new issuance. The logic is that if the government retires all but a measure zero of outstanding bonds, these bonds' remaining payments would be subjected to the same default risk as the newly issued bond. This makes the buyback price a function of both current state variables (T, δ, y) and issuance characteristics (T', δ', b'). The full dilution bond price q^{fd} is given by

$$\begin{aligned} q^{\mathrm{fd}}\left(T, \delta, y, T', \delta', b'\right) = & \frac{1}{1+r}\,\mathbf{E}\left(1 - d'\left(T', \delta', b', y'\right)\right)\Big\{(1+\delta)^{-T} \\ & + x\left(T', \delta', b', y'\right)\; q^{\mathrm{fd}}\left(T-1, \delta, y', T'', \delta'', b''\right) \\ & + \left(1 - x\left(T', \delta', b', y'\right)\right)\; q^{\mathrm{fd}}\left(T-1, \delta, y', T'-1, \delta', b'\right)\Big\} \end{aligned} \tag{16}$$

where $\langle T'', \delta'', b'' \rangle$ are the optimal choices in state $\langle T', \delta', b', y' \rangle$, conditional on restructuring. Consistent with Sanchez, Sapriza, and Yurdagul (2014), we find that, under full dilution, short-term debt strictly dominates and only one period bonds are issued in the ergodic distribution of the model. This is clearly inconsistent with the data. Sanchez, Sapriza, and Yurdagul (2014) show that with the introduction of sudden stop shocks, a higher level of risk aversion, or a debt restructuring procedure can revert this extreme result.

Given the lack of variation in optimal maturity under full dilution, we study a hybrid case, labeled "partial dilution," in which the buyback price is an average of the risk-free price and the full dilution price. The partial dilution price is given by

$$q^{\text{pd}}\left(T, \delta, y, T', \delta', b'\right) = \frac{1}{2}\left[q^{\text{rf}}\left(T, \delta\right) + q^{\text{fd}}\ \left(T, \delta, y, T', \delta', b'\right)\right]. \tag{17}$$

For our numerical results, we keep maximum maturity $\overline{T} = 15$ as in the baseline, and recalibrate other parameters. The partial dilution model can deliver cyclicality results in line with our baseline and the data. However, on average, this produces shorter maturity and higher payment growth on average, relative to baseline. The overall effect of dilution is to shorten maturity and increase back-loading of payments. This level effect leaves intact our key findings, in terms of the magnitude of changes in issuance characteristics, with respect to spread or GDP. See Table 5.

5 Conclusion

In this paper we address the outstanding puzzle of short-term borrowing by emerging markets during crises. We go beyond the previous characterization of debt in terms of duration and instead consider two complementary measures: payment growth rate and maturity. In our model, as in the data, countries in crisis issue bonds with back-loaded payments and shorter maturity. This renders the choice of maturity less of a puzzle, given that such a schedule helps with risk-sharing during downturns.

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A Data Appendix

A.1 Exchange Rate, U.S. CPI, and LIBOR

Sovereigns often schedule payments over the course of 20 or 30 years in the future since the issue date. In order to evaluate such promised payments in terms of real U.S. dollars, several assumptions are necessary:

- Exchange Rate: Under the assumption that foreign exchange rates are Martingales, the expected future exchange rate is equal to the current value.
- U.S. CPI: For the U.S. CPI, we assume perfect-foresight because the U.S. CPI is quite stable.
- LIBOR: When the coupon rate is expressed as a spread over the LIBOR rate, e.g., the floating coupon-rate bond, we take as our benchmark the perfect-foresight case in measuring the LIBOR rates in the future.

Note that our sample includes bonds with non-fixed coupon rate, e.g., floating and variable coupon-rate bonds, as well as the fixed coupon-rate bond. By contrast, frequently in the literature, non-fixed coupon-rate bonds are excluded from the analysis mainly for convenience rather than for economic reasons. We must address all of these cases consistently to produce a coherent picture of payment timing and size. For example, a variable coupon bond often specifies that coupon rates rise with the length of time to payments in a step-wise form; this has important implications for the growth rate of promised payments, i.e., positive growth rate of promised payments.

A.2 Sample Selection: Excluding Bonds with Special Features

We exclude from the sample bonds that are either denominated in local currencies or of special features for the reason explained in Broner, Lorenzoni, and Schmukler (2013). First, we focus on bonds denominated in foreign currencies for following reason. In many cases for emerging market economies, sovereign bonds are denominated in foreign currencies. Sovereigns do issue bonds denominated in their local currencies; in such a case, sovereigns would have an option to dilute their debt burden by adjusting the inflation rate in local currency terms, which is not the case for the bonds denominated in foreign currencies and is ruled out by the standard sovereign-default models, such as the one studied in this paper. [11] Thus, we simply focus on foreign-currency denominated bonds by excluding local-currency denominated bonds from our sample. Second, for the same reason as above, we exclude from the sample bonds with special features that are absent in our model and infrequently observed in the data: for instance, we exclude either collateralized

[11] Moreover, as discussed in Broner, Lorenzoni, and Schmukler (2013), if both foreign- and local-currency denominated bonds were included in the sample, then the regression analysis of bond characteristics would require controlling for the time-varying exchange-rate risk premium, which is difficult to measure.

bonds or bonds with the special guarantees provided by the third-party institutions such as the *IMF*, *World Bank*, and leading foreign governments/banks.

B Full Model with Sudden Stop Shock and (Partial) Dilution

We present the full model with a sudden stop shock and dilution. The state space must be extended to include $s \in \{0,1\}$ an indicator for whether the country is in a sudden stop state. Under circumstances of (partial) dilution, the buyback bond price is a function of not only issuance characteristics but also the outstanding debt structure.

B.1 Value Functions

$$V(T,\delta,b,y,s) = \max_d \left\{ V^d(y), \max_x \left\{ V^c(T,\delta,b,y,s), V^r(T,\delta,b,y,s) \right\} \right\}$$

$$V^d(y) = u\left[h_d(y)\right] + \beta\,\mathbf{E}_{y'|y}\left\{(1-\psi)V^d(y') + \psi V(0,0,0,y',0)\right\}$$

$$\begin{aligned} V^c(T,\delta,b,y,s) =& u\left[sh_s(y) + (1-s)y - (1+\delta)^{-T}\,b\right] \\ &+\beta\,\mathbf{E}_{y'|y,s'|s}\left\{\mathbb{1}_{T>0}\cdot V(T-1,\delta,b,y',s') + \mathbb{1}_{T=0}\cdot V(0,0,0,y',s')\right\} \end{aligned}$$

$$\begin{aligned} V^r(T,\delta,b,y,s) =& \max_{T',\delta',b'} u(c) + \beta\,\mathbf{E}_{y'|y,s'|s}\,V(T',\delta',b,y',s') \\ \text{s.t. } c =& s\,h_s(y) + (1-s)y - (1+\delta)^{-T}\,b \\ & - q^{\text{bb}}(T-1,\delta,y,s,T',\delta',b')\ b + q(T',\delta',b',y,s)\ b' \\ & q^{\text{bb}}(T-1,\delta,y,s,T',\delta',b')\ b \geq q(T',\delta',b',y,s)\ b' \qquad \text{if } s=1 \end{aligned}$$

B.2 Bond Prices

Risk-free bond price:

$$q^{\text{rf}}(T,\delta) = \frac{1}{R}\left\{(1+\delta)^{-T} + q^{\text{rf}}(T-1,\delta)\right\}$$

New issuance price:

$$\begin{aligned} q(T',\delta',b',y,s) =& \frac{1}{R}\,\mathbf{E}_{y'|y,s'|s}\left(1-d'(T',\delta',b',y',s')\right)\left\{(1+\delta')^{-T'}\right. \\ &+ x(T',\delta',b',y',s')\ q^{\text{bb}}(T'-1,\delta',y',s',T'',\delta'',b'') \\ &\left.+ \left(1-x(T',\delta',b',y',s')\right)\ q(T'-1,\delta',b',y',s')\right\} \end{aligned}$$

$$q^{\text{bb}}(T,\delta,y,s,T',\delta',b') = q^{\text{rf}}(T,\delta)$$

Full dilution buyback price:

$$q^{\mathrm{fd}}\left(T,\delta,y,s,T',\delta',b'\right)=\frac{1}{R}\,\mathbf{E}_{y'|y,s'|s}\left(1-d'\left(T',\delta',b',y',s'\right)\right)\Big\{(1+\delta)^{-T}$$
$$+\,x\left(T',\delta',b',y',s'\right)\,q^{\mathrm{fd}}\left(T-1,\delta,y',s',T'',\delta'',b''\right)$$
$$+\left(1-x\left(T',\delta',b',y',s'\right)\right)\,q^{\mathrm{fd}}\left(T-1,\delta,y',s',T'-1,\delta',b'\right)\Big\}$$

$\langle T'',\delta'',b''\rangle$ are the optimal choices in state $\langle T',\delta',b',y',s'\rangle$, conditional on restructuring.
Partial dilution buyback price:

$$q^{\mathrm{pd}}\left(T,\delta,y,s,T',\delta',b'\right)=\xi\,q^{\mathrm{rf}}\left(T,\delta\right)+(1-\xi)\,q^{\mathrm{fd}}\left(T,\delta,y,s,T',\delta',b'\right)$$

ξ controls the degree of dilution.

C Robustness to Issuance Frequency

In Table 6 we document the robustness of our main findings under the alternative assumption of a yearly issuance frequency, which is in line with our calibrated period length. Quantitatively, the results are largely unaltered for maturity T and duration D while our payment growth measure δ is more sensitive to time aggregation. The coefficients preserve their significance and signs.

Table 6: Regression: Payment Growth, Maturity, and Duration: Annual Issuance

	OLS	IV	OLS	IV	OLS	IV
	Dependent Variable: Payment Growth Rate (δ)					
3-y Spread	0.65	3.09***				
	[0.41]	[0.34]				
9-y Spread			2.95**	2.64**		
			[1.26]	[1.28]		
12-y Spread					2.26**	2.78*
					[1.03]	[1.48]
R^2	0.36	0.32	0.41	0.32	0.41	0.32
	Dependent Variable: Maturity (T)					
3-y Spread	-0.02	-1.64				
	[0.12]	[1.02]				
9-y Spread			-0.37	-1.41***		
			[0.25]	[0.42]		
12-y Spread					-0.17	-1.48***
					[0.19]	[0.30]
R^2	0.37	0.44	0.39	0.44	0.38	0.44
	Dependent Variable: Duration (D)					
3-y Spread	-0.01	-0.98***				
	[0.07]	[0.15]				
9-y Spread			-0.22	-0.84***		
			[0.15]	[0.19]		
12-y Spread					-0.10	-0.88***
					[0.11]	[0.15]
R^2	0.39	0.48	0.42	0.48	0.40	0.48
First Stage						
CSFB HYI		2.24***		2.62***		2.49***
		[0.75]		[0.42]		[0.50]

Note: The number of observation is 96. This table reports OLS and 2SLS (IV) regressions of the growth rate of payments and the maturity on the short- and long-term spreads, controlling for country fixed-effects, country-specific time trends, the real exchange rate, terms of trade, and an investment grade dummy. For the IV regressions, spread variables are instrumented by the Credit Suisse First Boston High Yield Index (CSFB HYI). Standard errors are robust to heteroskedasticity and within country serial correlation. ** significant at 5%; *** at 1%.