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CONSTRAINED DISCRETION AND CENTRAL BANK TRANSPARENCY

Francesco Bianchi
Leonardo Melosi

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ABSTRACT

We develop and estimate a general equilibrium model in which monetary policy can deviate from active inflation stabilization and agents face uncertainty about the nature of these deviations. When observing a deviation, agents conduct Bayesian learning to infer its likely duration. Under constrained discretion, only short deviations occur: Agents are confident about a prompt return to the active regime, macroeconomic uncertainty is low, welfare is high. However, if a deviation persists, agents' beliefs start drifting, uncertainty accelerates, and welfare declines. If the duration of the deviations is announced, uncertainty follows a reverse path. When estimated to match past U.S. experience, our model suggests that transparency lowers uncertainty and increases welfare.

Francesco Bianchi
Department of Economics
Duke University
213 Social Sciences Building
Box 90097
Durham, NC 27708
and CEPR
and also NBER
francesco.bianchi@duke.edu

Leonardo Melosi
Federal Reserve Bank of Chicago
230 S. LaSalle street
Chicago, IL 60604
lmelosi@frbchi.org

1 Introduction

The last two decades have witnessed two major breakthroughs in the practice of central banking worldwide. First, most central banks have adopted a monetary policy framework that Bernanke and Mishkin (1997) have termed *constrained discretion*. Bernanke (2003) explains that under constrained discretion, the central bank retains some flexibility in de-emphasizing inflation stabilization so as to pursue alternative short-run objectives such as unemployment stabilization. However, such flexibility is constrained to the extent that the central bank should maintain a strong reputation for keeping inflation and inflation expectations firmly under control. Second, many countries have taken remarkable steps to make their central bank more transparent (Bernanke et al., 1999 and Mishkin 2001).[1]

As a result of these changes, some key questions lie at the heart of modern monetary policy making. First, for how long can a central bank de-emphasize inflation stabilization before the private sector starts fearing a return to a period of high and volatile inflation as in '70s? Second, does transparency play an essential role for effective monetary policy making? In other words, should a central bank be explicit about the future course of monetary policy? The recent financial crisis has triggered a prolonged period of accommodative monetary policy that some members of the Federal Open Market Committee fear could lead to a disanchoring of inflation expectations.[2] As a result, the research questions outlined above are at the center of the policy debate.

In order to address them, we develop and estimate a model in which the anti-inflationary stance of the central bank can change over time and agents face uncertainty about the nature of deviations from active inflation stabilization. When monetary policy alternates between prolonged periods of active inflation stabilization, *active regime*, and short periods during which the emphasis on inflation stabilization is reduced, *short-lasting passive regime*, the model captures the monetary approach described as constrained discretion. However, the central bank can also engage in a prolonged deviation from the active regime and move to a *long-lasting passive regime*. Agents in the model are fully rational and able to infer if monetary policy is active or not. However, when the passive rule prevails, they are uncertain about the nature of the observed deviation. In other words, agents are not sure if the central bank is engaging in a short or long-lasting deviation from the active regime. The central bank

[1] Since May 1999, the Federal Open Market Committee (FOMC) has included explicit language about the likely future policy stance in its official statements, as documented in Rudebusch and Williams (2008). Industrialized countries such Canada, Spain, Sweden, and the United Kingdom have publicly announced a target range for inflation and also introduced a wide variety of instruments for communicating with the public. These include regular release of macroeconomic forecasts, discussions of the policy responses to keep inflation on target, and prompt releases of minutes.

[2] As an example see Plosser (2012).

can then follow two possible communication strategies: *Transparency* and *no transparency*. Under no transparency, the nature of the deviation is not revealed. Under transparency, the duration of *every* deviation is announced.

Under no transparency, when passive monetary policy prevails, agents conduct Bayesian learning in order to infer the likely duration of the deviation from active monetary policy. Given that the behavior of the monetary authority is unchanged across the two passive regimes, the only way for *rational* agents to learn about the nature of the deviation consists of keeping track of the number of consecutive deviations. As agents observe more and more realizations of the passive rule, they become increasingly convinced that the long-lasting passive regime is occurring. As a result, the more the central bank deviates from active inflation stabilization, the more agents become discouraged about a quick return to the active regime. We then solve the model keeping track of the joint evolution of policy makers' behavior and agents' beliefs using the methods developed in Bianchi and Melosi (2014b). The latter methods are based on the idea of expanding the number of regimes to take into account the learning mechanism. Once a regime is defined in terms of both policy makers' behavior and agents' beliefs, the model can be solved using any of the methods developed for perfect information Markov-switching models. The resulting solution implies that the model dynamics evolve over time in response to the evolution of policy makers' behavior *and* agents' beliefs.

The ability of generating smooth changes in agents' beliefs in response to central bank actions makes the model an ideal laboratory to study the macroeconomic and welfare implications of constrained discretion. In the model, social welfare is shown to be a function of agents uncertainty about future inflation and future output gaps. Both of these measures of uncertainty keep increasing as agents observe more and more deviations from the active policy and update their beliefs about the duration of the passive policy. In standard models, monetary policy affects agents' welfare by influencing the unconditional variances of the endogenous variables. In our nonlinear setting, policy actions exert dynamic effects on uncertainty. Therefore, welfare evolves over time in response to the short-run fluctuations of uncertainty. To our knowledge, this feature is new in the literature and allows us to study changes in the *macroeconomic risk* due to policy actions and communication strategies and the associated welfare implications.

We measure uncertainty taking into account agents' beliefs about the evolution of monetary policy. As long as the number of deviations from the active regime is low, the increase in uncertainty is very modest and in line with the levels implied by the active regime. This is because agents regard the early deviations as temporary. However, as the number of deviations increases and fairly optimistic agents become fairly pessimistic, uncertainty starts increasing and eventually converges to the values implied by the long-lasting passive regime. As a result, for each horizon, our measure of uncertainty is now higher than its long run value. This is because agents take into account that while in the short run a prolonged period of passive monetary policy will prevail, in the long run the economy will surely visit the active regime again. Therefore, an important result arises: Deviations from the active regime that last only a few periods have no disruptive consequences on welfare because they do not have a large impact on agents' uncertainty regarding future monetary policy. Instead, if a central bank deviates for a prolonged period of time, the disanchoring of agents' uncertainty occurs, causing sizeable welfare losses.

The model under the assumption of no transparency is fitted to U.S. data. In line with previous contributions, we identify prolonged deviations from active monetary policy in the '60s and the '70s. However, we also find that the Federal Reserve has recurrently engaged in short-lasting passive policies since the early '80s, supporting the view that constrained discretion has been the predominant approach to U.S. monetary policy in the last three decades. In the analysis, we abstract from the reasons why the Federal Reserve has engaged in such deviations. In fact, we consider such recurrent deviations as a *given* of our analysis. The objective of this paper is to use the estimated model to evaluate how quickly agents' beliefs respond to policy makers' behavior and announcements, what this implies for the evolution of uncertainty and welfare, and what the potential gains are from reducing the uncertainty about the conduct of monetary policy.

The paper introduces a practical definition of reputation: a central bank has strong reputation if it is less likely to engage in long-lasting deviations from active policies. It is worth pointing out that the proposed definition of central bank reputation is not only reflected in the in sample frequency of regime changes, but it also manifests itself affecting agents' beliefs and, consequently, the general equilibrium properties of the macroeconomy. Therefore, the proposed definition of central bank reputation has the advantage of being measurable in the data, while at the same time being in line with the seminal contributions of Kydland and Prescott (1977), Barro and Gordon (1983), and Gali and Gertler (2007).

The Federal Reserve is found to benefit from strong reputation. Based on the estimates, pessimism and hence agents' uncertainty about future inflation change very sluggishly in response to deviations from active monetary policy. In fact, uncertainty is found to stay anchored and move only very gradually as the Federal Reserve deviates from active monetary policy. This finding has the important implication that the Federal Reserve can conduct passive policies for up fairly large number of years before the disanchoring of inflation expectations and an overall increase in macroeconomic uncertainty occur.

While this result implies that the Federal Reserve can successfully implement constrained discretion even without transparency, our findings suggest that increasing transparency would improve welfare. The estimated model suggests that the welfare gains from transparency range between 0.67% to 6.63%. A transparent central bank systematically announces the duration of any deviation from the active regime beforehand. The implications of such a communication strategy vary based on the nature of the deviation. When the central bank engages in a short lasting deviation, announcing its duration immediately removes the *fear of the '70s*: When agents are not informed about the exact nature of an observed deviation, whenever a short deviation occurs, *ex-ante* agents cannot rule out the possibility of a long-lasting deviation of the kind that characterized the early part of the sample. As a result, *ex-post*, agents turn out to have overstated the persistence of the observed deviation. How large this effect is depends on the central bank reputation, captured by the conditional probability of engaging in a long lasting deviation.

When instead a deviation is in fact long lasting, the model allows us to highlight an important trade-off associated with transparency. First, in the short run being transparent reduces welfare because agents are told that passive monetary policy will prevail for a while and thereby future shocks are expected to have more dramatic inflationary/deflationary consequences. It follows that, if the duration of the announced deviation is long enough, over the early periods uncertainty is higher than when no announcement is made. This short-run effect on welfare arises because the central bank publicly commits to a policy that de-emphasizes inflation stabilization for the announced number of future periods. Agents understand that such a commitment to follow the announced policy course limits the central bank's ability of countering the inflationary consequences of future shocks that might occur during the implementation of the announced policy. Therefore, the announcement leads to a higher macroeconomic risk and associated detrimental effects on welfare. Second, as time goes by, agents know that the prolonged period of passive monetary policy is coming to an end. This leads to a reduction in the level of uncertainty at every horizon with an associated improvement in welfare. Notice, that this is exactly the opposite of what occurs when no announcement is made: Agents, in this case, become more and more discouraged about the possibility of moving to the active regime and uncertainty increases. To our knowledge, this is the first paper that studies this critical trade-off and its welfare implications through the lens of an estimated DSGE model.

This paper makes three main methodological contributions to the existing literature. First, we estimate a microfounded general equilibrium model with changes in policy makers' behavior and Bayesian learning. To the best of our knowledge, this is the first paper that *estimates* a DSGE model with Markov-switching parameters and Bayesian learning.[3] Second,

[3] The learning mechanism implies that agents' beliefs are not invariant to the duration of a certain policy.

we show how to model systematic and recurrent policy makers' announcements in a general equilibrium framework. In light of the recent development of forward guidance, we believe that this contribution should be of independent interest. Finally, we show how to characterize and compute social welfare in a Markov switching DSGE model with Bayesian learning and announcements. In doing so, we combine the methods developed by Bianchi (2013a) to measure uncertainty in MS-DSGE models with the solution methods for MS-DSGE models with learning developed by Bianchi and Melosi (2014b) and the solution methods for MS-DSGE models with announcements developed in this paper.

Our modeling framework goes beyond the assumption of *anticipated utility* that is often used in the learning literature.[4] Such an assumption implies that agents forecast future events assuming that their beliefs will never change in the future. Instead, agents in our models *know that they do not know*. Therefore, when forming expectations, they take into account that their beliefs will evolve according to what they will observe in the future. In our context, it is possible to go beyond the anticipated utility assumption because we can keep track of the joint evolution of policymakers' behavior and agents' beliefs. Using anticipated utility would break the link between the observed policy path and the *future policy course*. This link is key for the dynamics of uncertainty. To understand why, consider a prolonged sequence of deviations from the active regime. This would have two effects. First, monetary policy is less active in stabilizing inflation. Second, agents become more pessimistic about a return to the active regime. Both effects are reflected in the model solution with important consequences for the expected impact of future shocks and, consequently, the evolution of uncertainty and welfare.

This paper is part of a broader research agenda that aims to model the evolution of agents' beliefs in general equilibrium models. In Bianchi and Melosi (2014a), we study a model in which the current policy makers' behavior influences agents' beliefs about the way debt will be stabilized. In Bianchi and Melosi (2013), we develop methods to study the evolution of agents' beliefs in general equilibrium models. Unlike those two papers, in this paper we conduct a full estimation of a DSGE model with parameter instability and information frictions. We use the results to assess how anchored inflation expectations and uncertainty are in the U.S. economy and to investigate the welfare implications of forward-looking communication by the Federal Reserve.

Eusepi and Preston (2010) study monetary policy communication in a model where

Therefore, the model captures a very intuitive idea: Agents in the late '70s were arguably more pessimistic about a return to the active regime with respect to the early '70s. This feature was not present in previous contributions such as Bianchi (2013b) and Davig and Doh (2008).

[4] For some prominent examples see Marcet and Sargent (1989b,a) Cho, Williams, and Sargent (2002), and Evans and Honkapohja (2001, 2003).

agents face uncertainty about the value of model parameters. Unlike Eusepi and Preston (2010), agents in our model are not bounded rational, they only have incomplete information. Cogley, Matthes, and Sbordone (2011) address the problem of a newly-appointed central bank governor who inherits a high average inflation rate from the past and wants to disinflate. In their model, agents conduct Bayesian learning over the coefficients that characterize the conduct of monetary policy, but they are bounded rational to the extent that use anticipated utility to form expectations. In our model, regime changes are recurrent, agents learn about the regime in place as opposed to the Taylor rule parameters, and expectations reflect the possibility of changes in beliefs and policy makers' behavior. Finally, the tractability of our approach allows us to conduct a full estimation.

Schorfheide (2005) considers an economy in which agents use Bayesian learning to infer changes in a Markov-switching inflation target. In that paper agents solve a filtering problem to disentangle a persistent component from a transitory component. The learning mechanism is treated as external to the model, implying that the model needs to be solved in every period in order to reflect the change in agents' beliefs regarding the persistent and transitory components. Consequently, when agents form expectations they do not take into account how their beliefs will respond to future observations. On the contrary, in this paper agents form expectations by knowing that they do not know. Furthermore, the method developed in Schorfheide (2005) cannot be immediately extended to models in which agents learn about changes in the stochastic properties of the model parameters.

The paper is then related to a growing literature that models parameter instability to capture changes in the evolution of the macroeconomy. This consists of three branches: Davig and Leeper (2007), Farmer, Waggoner, and Zha (2009), and Foerster, Rubio-Ramirez, Waggoner, and Zha (2011) develop solution methods for Markov-switching rational expectations models, Justiniano and Primiceri (2008), Benati and Surico (2009), Bianchi (2013b), Bianchi and Ilut (2013), Davig and Doh (2008), and Fernandez-Villaverde and Rubio-Ramirez (2008) introduce parameter instability in estimated dynamic equilibrium models, while Sims and Zha (2006), Primiceri (2005), Cogley and Sargent (2005), and Boivin and Giannoni (2006) work with structural VARs. Finally, our work is also linked to papers that study the impact of monetary policy decisions on inflation expectations, such as Nimark (2008), Mankiw, Reis, and Wolfers (2004), Del Negro and Eusepi (2010), and Melosi (2014a and 2014b).

This paper is organized as follows. Section 2 introduces the baseline model. In Section 3, we show how to solve the model under no transparency and transparency. In Section 4, the model under the assumption of no transparency is fitted to U.S. data. In Section 5 we use the estimated model to assess the welfare implications of introducing transparency. In Section 6 we study some extensions and assess the robustness of our results. Section 7 concludes.

2 The Model

The model is a prototypical three-equation New-Keynesian model (Clarida, Gali, and Gertler, 2000 and Woodford, 2003), which has been used for empirical studies (Lubik and Schorfheide, 2004). We make two main departures from this standard framework. First, we assume that households and firms have incomplete information, in a sense to be made clear shortly. Second, we assume parameter instability in the monetary policy rule.

Households: The representative household maximizes

$$E\left[\sum_{t=0}^{\infty}\beta^t G_t\left((1-\sigma)^{-1}C_t^{1-\sigma}-(1+\psi)^{-1}N_t^{1+\psi}\right)|\mathcal{F}_0\right],$$

where C_t is composite consumption and N_t are hours worked in period t. The parameter $\beta \in (0,1)$ is the discount factor, the parameter $\psi \geq 0$ is the inverse of the Frisch elasticity of labor supply. $E[\cdot|\mathcal{F}_0]$ is the expectation operator conditioned on information of private agents available at time 0. The information set $\mathcal{F}_t$ contains the history of all model variables and volatility regimes ξ_t^v but not the history of policy regimes ξ_t^p that, as we shall show, determine the parameter value of the central bank's reaction function. G_t is an exogenous process affecting the discount factor of households and is assumed to follow a stationary first-order autoregressive process:

$$\ln G_t = \left(1-\rho_g\right)\ln G + \rho_g \ln G_{t-1} + \sigma_{g,\xi_t^v}\eta_{gt},\ \eta_{gt} \sim N(0,1). \tag{1}$$

where η_{gt} is an i.i.d. Gaussian shock and σ_{g,ξ_t^v} is determined by the exogenous variable ξ_t^v, which is assumed to follow a discrete Markov-switching process. As it is common in the literature we assume $G = 1$ implying that the discount factor in steady state is given by β. We refer to η_{gt} as preference shock.

Composite consumption in period t is given by the Dixit-Stiglitz aggregator

$$C_t = \left(\int_0^1 C_{it}^{1-1/\varepsilon_t} di\right)^{\frac{\varepsilon_t}{\varepsilon_t - 1}},$$

where C_{it} is consumption of a differentiated good i in period t and $\varepsilon_t > 1$ determines the elasticity of substitution between consumption goods. The elasticity of substitution is determined by the following exogenous process:

$$\ln M_t = (1-\rho_m)\ln M + \rho_m \ln M_{t-1} + \sigma_{m,\xi_t^v}\eta_{mt},\ \eta_{mt} \sim N(0,1) \tag{2}$$

where $M_t = (\varepsilon_t - 1)^{-1}$ and η_{mt} is referred to as price markup shock. Analogously to the preference shocks, the standard deviation of the markup shock $\eta_{m,t}$ is determined by the discrete Markov-switching process ξ_t^v.

The flow budget constraint of the representative household in period t reads

$$P_t C_t + B_t = R_{t-1} B_{t-1} + W_t N_t + D_t - T_t,$$

where P_t is the price level in period t, B_{t-1} is the stock of one-period nominal government bonds held by the household between period $t-1$ and period t, R_{t-1} is the gross nominal interest rate on those bonds, W_t is the nominal wage rate, D_t are nominal aggregate profits, and T_t are nominal lump-sum taxes in period t. The price level is given by

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon_t} di \right)^{1/(1-\varepsilon_t)}. \tag{3}$$

In every period t, the representative household chooses a consumption vector, labor supply, and bond holdings subject to the sequence of the flow budget constraints and a no-Ponzi-scheme condition. The representative household takes as given the nominal interest rate, the nominal wage rate, nominal aggregate profits, nominal lump-sum taxes, and the prices of all consumption goods.

Firms: There is a continuum of monopolistically competitive firms of mass one. Firms are indexed by i. Firm i supplies a differentiated good i. Firms face Calvo-type nominal rigidities and the probability of re-optimizing prices in any given period is given by $1-\theta$ independent across firms. Those firms that are not allowed to re-optimize index their prices to the steady-state inflation rate Π_*. Those firms that are allowed to re-optimize their price choose their price $P_t^*(i)$ so as to maximize:

$$\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} \left(\Pi_*^k P_t^*(i) Y_{t+k}(i) - W_{t+k} N_{t+k}(i) \right) | \mathcal{F}_t \right]$$

where $Q_{t,t+k}$ is the stochastic discount factor measuring the time t utility of one unit of consumption good available at time $t+k$, $N_t(i)$ is amount of labor hired, and $Y_t(i)$ is the amount of differentiated good produced by firm i. Firms are endowed with an identical technology of production:

$$Y_t(i) = Z_t N_t(i)^{1-\alpha}.$$

The variable Z_t captures exogenous shifts of the marginal costs of production and is assumed to follow a stationary first-order autoregressive process:

$$\ln Z_t = (1-\rho_z) \ln Z + \rho_z \ln Z_{t-1} + \sigma_{z,\xi_t^v} \eta_{zt}, \;\; \eta_{zt} \sim N(0,1). \tag{4}$$

We refer to the innovations η_{zt} as technology shocks. Again, the Markov-switching process ξ_t^v determines the volatility regime for the technology shock. Re-optimizing firms face a sequence of demand constraints:

$$Y_{t+k}(i) = \left(\Pi_*^k P_t^*(i) / P_{t+k}\right)^{-\varepsilon_t} Y_{t+k}$$

Policy Makers: There is a monetary authority and a fiscal authority. The flow budget constraint of the fiscal authority in period t reads

$$T_t + B_t = R_{t-1} B_{t-1}.$$

The fiscal authority has to finance maturing government bonds. The fiscal authority can collect lump-sum taxes or issue new government bonds. We assume that the fiscal authority follows a Ricardian fiscal policy. The monetary authority sets the nominal interest rate R_t according to the Taylor rule

$$R_t = R_{t-1}^{\rho_{r,\xi_t^p}} \left[\left(\frac{\Pi_t}{\Pi_*}\right)^{\phi_{\pi,\xi_t^p}} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_{y,\xi_t^p}} \right]^{1-\rho_{r,\xi_t^p}} \exp\left(\sigma_{r,\xi_t^v} \eta_{rt}\right), \ \eta_{rt} \sim N(0,1) \qquad (5)$$

where $\Pi_t = (P_t / P_{t-1})$ is inflation and Y_t is aggregate output in period t, and Y_t^* is the potential output. The variable η_{rt} captures non-systematic exogenous deviations of the nominal interest rate R_t from the rule. The standard deviation of the monetary shock is assumed to depend on the volatility regime ξ_t^v that follows a discrete Markov process. The variable ξ_t^p is the policy regime that determines the policy coefficients of the rule reflecting the emphasis of the central bank on inflation stabilization relative to output gap stabilization in any period t.

2.1 Volatility and Policy Regimes

The standard deviations of the preference shocks, the markup shocks, the technology shocks, and the monetary shocks are determined by the volatility of regime ξ_t^v. The volatility regime follows a discrete Markov process and can assume two values: High and Low. The low volatility regime is characterized by standard deviations that are strictly smaller than those associated with the high volatility regime. Transition matrix that governs the evolution of the two volatility regimes ξ_t^v is the following:

$$\mathcal{P}_v = \begin{bmatrix} p_H & 1-p_H \\ 1-p_L & p_L \end{bmatrix}$$

where p_H (p_L) captures the probability of staying in the high (low) volatility regime. Unlike the policy regimes ξ_t^p, the realizations of the volatilities regimes ξ_t^v are perfectly observed by the agents (i.e., $\xi_t^v \in \mathcal{F}_t$, any t).

We model changes in the central bank's emphasis on inflation and output gap stabilization by introducing a Markov-switching process ξ_t^p with three regimes that evolve according to the matrix:

$$\mathcal{P}_p = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ 1-p_{22} & p_{22} & 0 \\ 1-p_{33} & 0 & p_{33} \end{bmatrix} \tag{6}$$

The realized regime determines the monetary policy parameters of the central bank's reaction function. In symbols, for $j \in \{1,2,3\}$:

$$\left(\rho_R\left(\xi_t^p = j\right), \phi_\pi\left(\xi_t^p = j\right), \phi_y\left(\xi_t^p = j\right)\right) = \begin{bmatrix} \left(\rho_R^A, \phi_\pi^A, \phi_y^A\right), \text{ if } j = 1 \\ \left(\rho_R^P, \phi_\pi^P, \phi_y^P\right), \text{ if } j = 2 \\ \left(\rho_R^P, \phi_\pi^P, \phi_y^P\right), \text{ if } j = 3 \end{bmatrix} \tag{7}$$

Under Regime 1, the *active regime*, the central bank's main goal is to stabilize inflation and the Taylor principle is satisfied $\phi_\pi\left(\xi_t^p = 1\right) = \phi_\pi^A \geq 1$. Under Regime 2, the *short-lasting passive regime*, the central bank de-emphasizes inflation stabilization by deviating from the Taylor principle $\phi_\pi^P < 1$, but only for short periods of time (on average). The *same* parameter combination also characterizes Regime 3, the *long-lasting passive regime.* However, under Regime 3 deviations are generally more prolonged. In other words, Regime 2 is less persistent than Regime 3: $p_{22} < p_{33}$. Summarizing, the two passive regimes do not differ in terms of the response to inflation ϕ_π^P and the output gap ϕ_y^P, but only in terms of their relative persistence.

The three policy regimes are meant to capture the recurrent changes in the Federal Reserve's attitude toward inflation and output stabilization in the postwar period. A number of empirical works (Clarida, Gali, and Gertler, 2000, Lubik and Schorfheide, 2004, Bianchi, 2013) have documented that the Federal Reserve de-emphasized inflation stabilization for *prolonged* periods of time in the 1970s. Furthermore, as argued by Bernanke (2003), while the Federal Reserve has been mostly focused on actively stabilizing inflation and inflation expectations starting from the early 1980, it has also occasionally engaged in *short-lasting* policies whose objective was not stabilizing inflation in the short run. This monetary policy approach has been dubbed *constrained discretion.* We introduce this three-regime structure so as to give the model enough flexibility to explain both the long-lasting passive monetary policy of the 1970s as well as the recurrent and short-lasting passive policies of post-1970s.

The probabilities p_{11}, p_{12}, p_{22} govern the evolution of monetary policy when the central bank follows constrained discretion. The larger p_{12} is vis-a-vis to p_{11}, the more frequent the short-lasting deviations are. The larger p_{22} is, the more persistent the short-lasting deviations are. The probability p_{13} controls how likely it is that constrained discretion is abandoned in favor of a prolonged deviation from the active regime. The ratio $p_{12}/(1-p_{11})$ captures the relative probability of a short-lasting deviation conditional on having deviated to passive regimes and can be interpreted as a measure of central bank's *reputation.* This is because this composite parameter controls how likely it is that the central bank will abandon constrained discretion the moment it starts deviating from the active regime. When $p_{12}/(1-p_{11})$ is close to unity, agents expect that the central bank will refrain from engaging in 1970s-style long-lasting passive policies that - as we shall show - are invariably associated with heightened inflation instability. As it will become clear later on, central bank reputation has deep implications for the general equilibrium properties of the macroeconomy. Therefore, the parameters of the transition matrix do not only affect the frequency with which the different regimes are observed, but also the law of motion of the economy across the different regimes. This is because agents are fully rational and form expectations taking into account the possibility of regime changes, implying that their beliefs matter for the way shocks propagate through the economy. Therefore, the proposed definition of central bank reputation has the important advantage of being measurable, even over a relatively short period of time.

2.2 Communication Strategies

In the model, regime changes do not affect the steady state, but only the way the economy propagates around it. We then log-linearize the model around the steady-state equilibrium. We then obtain:[5]

$$y_t = \mathbb{E}(y_{t+1}|\mathcal{F}_t) - \sigma^{-1}[i_t - \mathbb{E}_t(\pi_{t+1}|\mathcal{F}_t)] + g_t \tag{8}$$

$$\pi_t = \beta\mathbb{E}_t(\pi_{t+1}|\mathcal{F}_t) + \kappa(y_t - z_t) + m_t \tag{9}$$

$$r_t = \rho_{R,\xi_t^p} r_{t-1} + \left(1 - \rho_{R,\xi_t^p}\right)\left[\phi_{\pi,\xi_t^p}\pi_t + \phi_{y,\xi_t^p}(y_t - z_t)\right] + \sigma_{r,\xi_t^v}\eta_{r,t} \tag{10}$$

$$g_t = \rho_g g_{t-1} + \sigma_{g,\xi_t^v}\eta_{gt} \tag{11}$$

$$z_t = \rho_z z_{t-1} + \sigma_{z,\xi_t^v}\eta_{zt} \tag{12}$$

$$m_t = \rho_m m_{t-1} + \sigma_{m,\xi_t^v}\eta_{m,t} \tag{13}$$

where lowercase variables denote log-deviations of uppercase variables from their steady state equilibrium and $\kappa \equiv \frac{\theta}{(1-\beta\theta)(1-\theta)}\frac{\sigma(1-\alpha)+\psi+\alpha}{1-\alpha+\alpha\varepsilon}$ is the slope of the Phillips curve. The model can then be solved under different assumptions on what the central bank communicates to

[5]Following Lubik and Schorfheide (2004) we rescale the preference process G_t.

agents about the future monetary policy course. Central bank communication affects agents' information set $\mathcal{F}_t$. We consider two cases: *no transparency* and *transparency*.

If the central bank is not transparent, it never announces the duration of passive policies. We call this approach *no transparency*. We make a minimal departure from the assumption of perfect information assuming that agents can observe the history of all the endogenous variables, the history of the structural shocks as well as the history of the volatility regimes ξ_t^v but not the policy regimes ξ_t^p. It should be noted that agents are always able to infer if monetary policy is currently active or passive. However, when monetary policy is passive, agents cannot immediately figure out whether the short-lasting Regime 2 or the long-lasting Regime 3 is in place. To see why, recall that the two passive regimes are observationally equivalent to agents, given that ϕ_π^p and ϕ_y^p are the same across the two regimes. Therefore, agents conduct Bayesian learning in order to infer which one of the two regimes is in place. In the next section we will discuss how agents' beliefs evolve as agents observe more and more deviations from the active regime.[6]

Under *transparency* all the information held by the central bank is communicated to agents. We assume that the central bank knows for how long it will be deviating from active monetary policy. Therefore, a transparent central bank announces the duration of passive policies, revealing to agents exactly *when* monetary policy will switch back to the active regime. It is important to emphasize that agents form their beliefs by taking into account that the central bank will *systematically* announce the duration of passive policy. We assume that central bank's announcements are truthful and are believed as such by rational agents. In Section 7, we will consider the case in which the central bank has much less information about the duration of its policy course and can only announce the likely duration of the passive policies; that is, the type of passive regime (i.e., $\xi_t^p \in \{2, 3\}$) that the central bank will carry out. This case corresponds to a form of transparency in which the central bank communicates only the likely duration rather than the actual duration of the passive policy.

3 Beliefs Dynamics and Model Solution

Different communication strategies imply different dynamics of beliefs and hence different solution methods. Let us first discuss how to solve the model in which the central bank is

[6] It might be argued that the central bank could try to signal the kind of deviation perturbing the Taylor rule parameters across the two rules. For example, $\phi_\pi (s_t = 3) = \phi_\pi (s_t = 2) + \xi$ for $\xi \neq 0$ and small. However, the point of the paper is exactly to capture agents' uncertainty about the duration of passive policies. Therefore, the model would be extended to allow for a total of four passive regimes: a long-lasting Regime 4 in which $\phi_\pi = \phi_\pi (s_t = 2)$ and $p_{44} > p_{22}$ and a short-lasting Regime 5 in which $\phi_\pi = \phi_\pi (s_t = 3)$ and $p_{55} < p_{33}$.

not transparent. Since agents know the history of endogenous variables and shocks, they can exactly infer the policy mix that is in place at each point in time. However, while the active regime is fully revealing, when the passive regime is prevailing, agents do not know whether the central bank is engaging in a short-lasting deviation or a long-lasting one. Agents have to learn the nature of the deviation in order to form expectations over the endogenous variables of the economy.

To solve the model under no transparency we use the methods developed in Bianchi and Melosi (2014b). We briefly report the main features of this solution method so as to make this paper self-contained. Denote the number of consecutive deviations from the active regime at time t as $\tau_t \in \{0, 1, ...\}$, where $\tau_t = 0$ means that monetary policy is active at time t. Conditional on having observed $\tau_t \geq 1$ consecutive deviations from the active regime at time t, agents believe that the central bank will keep deviating in the next period $t+1$ with the following probability:[7]

$$prob\left\{\tau_{t+1} \neq 0 | \tau_t \neq 0\right\} = \frac{p_{22}\left(p_{12}/p_{13}\right)\left(p_{22}/p_{33}\right)^{\tau_t - 1} + p_{33}}{\left(p_{12}/p_{13}\right)\left(p_{22}/p_{33}\right)^{\tau_t - 1} + 1}. \tag{14}$$

Equation (14) makes it clear that $prob\left\{\tau_{t+1} \neq 0 | \tau_t \neq 0\right\} = prob\left\{\tau_{t+1} \neq 0 | \mathcal{F}_t\right\}$ as τ_t is a sufficient statistic for the probability of being in the passive regime next period. Furthermore, this equation captures the dynamics of agents' beliefs about observing yet another period of passive policy in the next period, which is the key state variable we use to solve the model under no transparency. It should be observed that this equation has a number of properties that are quite insightful to the key mechanism of the model at hand. It is useful to observe that the probability of observing yet another period of passive policy in the next period is a weighted average of the probabilities p_{22} and p_{33} with weights that vary with the number of consecutive periods of passive policy τ_t. When agents observe the central bank deviating from the active regime for the first time ($\tau_t = 1$), the weights for the probabilities p_{22} and p_{33} are $p_{12}/\left(1 - p_{11}\right)$ and $p_{13}/\left(1 - p_{11}\right)$, respectively. These weights can be interpreted as agents' *priors* about which passive regime is in place when the first deviation is observed. As more and more periods of passive policy are observed ($\tau_t \uparrow$), the weight assigned to the short-lasting passive Regime 2 *monotonically* decreases due to the fact that $p_{33} > p_{22}$. Consequently, as the first period of passive policy is observed, agents' beliefs about observing a passive policy in the next period are at their lower bound. Furthermore, as the central bank keeps on deviating, agents get increasingly convinced that the economy has entered a long-lasting deviation, given that under this policy regime long deviations are more likely.

[7] This result can be derived by applying the Bayes' theorem and then combining the resulting probabilities with the transition matrix H. The proof is straightforward and is shown in Bianchi and Melosi (2014b).

Importantly, how low is the lower bound for the probability of observing yet another period of passive policy will depend on the level of the central bank's reputation. High reputation makes the weight $p_{12}/(1-p_{11})$ close to one, implying that the probability of observing a second consecutive period of passive policy will be very close to p_{22}, the value associated with a short lasting deviation. When reputation is high, it is very unlikely that the central bank engages in a long-lasting passive policy. Therefore, as the first period of passive policy is observed, agents are quite confident to have entered the short-lasting passive regime (Regime 2). If the central bank keeps deviating from the active regime, agents will eventually become convinced of being in the long-lasting passive regime (Regime 3) regardless of the level of the central bank's reputation, $p_{12}/(1-p_{11})$.[8] After a sufficiently long-lasting passive policy, the probability of observing an additional deviation in the next period degenerates to the persistence of the long-lasting Regime 3. Formally: $\lim_{\tau_t \to \infty} prob\{\tau_{t+1} \neq 0 | \tau_t \neq 0\} = p_{33}$. Hence, p_{33} is the *upper bound* for the probability that agents attach to staying in the passive regime next period. It follows that rational agents cannot get more convinced to observe yet another passive policy in the next period than when they are sure to be in the long-lasting Regime 3. More formally, for each $e > 0$, there exists an integer τ^* such that:

$$p_{33} - prob\{\tau_{t+1} \neq 0 | \tau_t = \tau^*\} < e, \tag{15}$$

with the important result that for any $\tau_t > \tau^*$, agents' beliefs can be effectively approximated using the properties of the long-lasting passive regime (Regime 3).

Endowed with these results, we can solve the model under no transparency by expanding the number of regimes in order to take into account the evolution of agents' beliefs. Now each regime is characterized by central bank's behavior and the *number of observed consecutive deviations from the active policy at any time t* $\tau_t \in \{0, 1, ..., \tau^*\}$. The mapping to the parameter values of the policy rule is as follows:

$$\left(\rho_r(\tau_t = j), \phi_\pi(\tau_t = j), \phi_y(\tau_t = j)\right) = \begin{bmatrix} \left(\rho_r^A, \phi_\pi^A, \phi_y^A\right), \text{ if } j = 0 \\ \left(\rho_r^P, \phi_\pi^P, \phi_y^P\right), \text{ if } 1 \leq j < \tau^* \end{bmatrix} \tag{16}$$

The transition matrix for this new set of regimes $\tau_t \in \{0, 1, ..., \tau^*\}$ can be derived by equation (14) as shown in Appendix A.

Endowed with these results regarding the dynamics of agents' beliefs, one can recast the

[8] We abstract from the extreme case in which the central bank's reputation is such that $p_{12}/(1-p_{11}) = 1$. In this case, agents' beliefs will not evolve at all as the central bank deviates Another limit case is when the central bank's reputation is at its lowest; that is, $p_{12}/(1-p_{11}) = 0$. In this case, agents know that any passive policy is surely of the long-lasting type and do not update their beliefs during the implementation of the passive policy. We do not consider these two extreme cases in this paper.

Markov-switching DSGE model under no transparency and learning as a Markov-switching Rational Expectations model. Now regimes are defined in terms of the observed consecutive duration of the passive regimes, τ_t, which, unlike the primitive set of policy regime $\xi_t^p \in \{1, 2, 3\}$, belongs to the agents' information set $\mathcal{F}_t$. This result allows us to solve this model by applying any of the methods developed to solve Markov-switching rational expectations models, such as Davig and Leeper (2007), Farmer, Waggoner, and Zha (2009), and Foerster, Rubio-Ramirez, Waggoner, and Zha (2011). We use Farmer, Waggoner, and Zha (2009) to solve the model with learning once the policy regimes are redefined as described above.

It is worth emphasizing that this way of recasting the learning process allows us to tractably model the behavior of agents that *know that they do not know*. In other words, agents are aware of the fact that their beliefs will change in the future according to what they observe in the economy. This represents a substantial difference with the anticipated utility approach, in which agents form expectations without taking into account that their beliefs about the economy will change over time. Furthermore, our approach differs from the one traditionally used in the learning literature in which agents form expectations according to a reduced form law of motion that is updated recursively (for example, using discounted least squares regressions). The advantage of adaptive learning is the extreme flexibility given that, at least in principle, no restrictions need to be imposed on the type of parameter instability characterizing the model. However, such flexibility does not come without a cost, given that agents are not really aware of the model they live in, but only of the implied law of motion. Instead, in this paper, agents fully understand the model and they are aware of the trade-offs that characterize it. However, they are uncertain about the central bank' future behavior, and this uncertainty has important consequences for the law of motion of the economy.

When the central bank is *transparent*, the exact duration of every deviation from active policy is truthfully announced. In this model the *number of announced deviations from the active policy yet to be carried out* τ_t^a is a sufficient statistic that captures the dynamics of beliefs. Hence, we redefine the set of policy regimes in terms of this variable with the following mapping to the parameter values of the policy rule:

$$\left(\rho_r\left(\tau_t^a=j\right), \phi_\pi\left(\tau_t^a=j\right), \phi_y\left(\tau_t^a=j\right)\right)=\left[\begin{array}{c}\left(\rho_r^A, \phi_\pi^A, \phi_y^A\right), \text{ if } j=0 \\ \left(\rho_r^P, \phi_\pi^P, \phi_y^P\right), \text{ if } 1 \leq j<\tau_*^a\end{array}\right] \quad (17)$$

where τ_*^a is a large number at which we truncate the redefined set of regimes.[9] The regimes $\tau_t^a \in \{0, 1, ..., \tau_*^a\}$ governed by the $(\tau_*^a+1) \times (\tau_*^a+1)$ transition matrix $\widetilde{\mathcal{P}}^A = [\widetilde{p}_A^{1\prime}, \widetilde{p}_A^{2\prime}]'$, where $\widetilde{p}_A^1$ is a $1 \times (\tau_*^a+1)$ vector whose $j-th$ element $\widetilde{p}_A(j)$ is p_{11} if $j=1$ and $p_{12}p_{22}^{j-2}p_{21}+p_{13}p_{33}^{j-2}p_{31}$

[9] Since $p_{33}<1$, it can be easily show that the higher the truncation regime τ_a^*, the lower the probability that the realized duration is larger than τ_a^*, the lower the approximation error.

(the probability that the realized passive policy will last exactly $j-1$ consecutive periods) for any $2 \leq j \leq \tau_*^a + 1$. The $\tau_*^a \times (\tau_*^a + 1)$ matrix $\widetilde{p}_A^2$ is defined as $\left[\mathbf{I}_{\tau_*^a}, \mathbf{0}_{\tau_*^a \times 1}\right]$, where $\mathbf{I}_{\tau_*^a}$ is a $\tau_*^a \times \tau_*^a$ identity matrix and $\mathbf{0}_{\tau_*^a \times 1}$ is $\tau_*^a \times 1$ column vector of zeros. Note that regimes are ordered from the smallest number of deviations (zero, the active policy) to the largest one (τ_*^a).

Similarly to the case of no transparency, we have recasted the Markov-switching DSGE model under transparency as a Markov-switching Rational Expectations model, in which the regimes are redefined in terms of the number of *announced deviations* from the active regimes *yet to be carried out*, τ_t^a, which, unlike the policy regime ξ_t^p, belongs to the agents' information set $\mathcal{F}_t$. This result allows us to solve the model under transparency by applying any of the methods developed to solve Markov-switching rational expectations models.

4 Empirical Analysis

In order to put discipline on the parameter values, the model under the assumption of no transparency is fitted to US data. We believe that the model with a non-transparent central bank is the better suited to capture the Federal Reserve communication strategy in our sample that ranges from mid-1950s to prior the great recession. We then use the results to quantify the Federal Reserve reputation and the potential gains from making the Federal Reserve more transparent.

This section is organized as follows. Section 4.1 briefly deals with the Bayesian estimation of the model. In Section 4.2 we show the evolution of agents' beliefs about future monetary policy, which is key to understand the welfare implications of transparency.

4.1 Data and Estimation

For observables, we use three series of U.S. quarterly data: the (HP filtered) real GDP per capita, the annualized quarterly inflation (GDP deflator), and the Federal Funds Rate (FFR). The sample spans from 1954:III to 2008:I. Table 1 reports the prior and the posterior distribution of model parameters.[10] To keep the dimensionality of the state space tractable, we measure the output gap using the HP-filtered GDP. For a detailed discussion of the estimation strategy see Bianchi (2013b). We shut down the process for the technology z_t as its parameters cannot be identified. The parameter rr^* denotes the steady-state equilibrium real interest rate. The parameter σ_π is the standard deviation of the measurement error associated with inflation.

[10]The convergence statistics of the Gibbs sampler are reported in Appendix B.

	Posterior				Prior		
Name	Mode	Mean	5%	95%	Type	Mean	Std.
ϕ_{π}^{A}	1.6033	1.6332	1.2896	2.0583	N	1.8	0.3
ϕ_{y}^{A}	0.2450	0.2763	0.0934	0.6042	G	0.25	0.15
ρ_{R}^{A}	0.6253	0.6899	0.5384	0.8916	B	0.7	0.15
ϕ_{π}^{P}	0.7456	0.7587	0.4926	1.0537	N	0.8	0.3
ϕ_{y}^{P}	0.3706	0.4194	0.2056	0.6867	G	0.25	0.15
ρ_{R}^{P}	0.8725	0.8549	0.7269	0.9107	B	0.7	0.15
p_{11}	0.7887	0.7825	0.6254	0.9251	B	0.85	0.1
p_{22}/p_{33}	0.9132	0.8326	0.6529	0.9438	B	0.75	0.1
p_{33}	0.9538	0.9401	0.8862	0.9779	B	0.9	0.05
$p_{12}/(1-p_{11})$	0.9186	0.8773	0.7738	0.9595	B	0.9	0.05
σ^{-1}	4.5927	4.6988	3.0468	6.7451	G	3	1
κ	0.0185	0.0173	0.0079	0.0290	G	0.3	0.2
ρ_g	0.8335	0.8312	0.7862	0.8755	B	0.8	0.1
ρ_μ	0.9462	0.9288	0.8909	0.9617	B	0.7	0.15
$100 \cdot rr^*$	0.4230	0.4359	0.3404	0.5330	G	0.6	0.3
100π	0.5151	0.5142	0.4693	0.5582	G	0.5	0.025
$100\sigma_{R}^{H}$	0.3029	0.3228	0.2613	0.3973	IG	0.31	0.4
$100\sigma_{g}^{H}$	0.2942	0.3128	0.2364	0.3971	IG	0.38	0.4
$100\sigma_{\mu}^{H}$	1.3700	2.2249	1.1936	3.7360	IG	1	0.8
$100\sigma_{R}^{L}$	0.0714	0.0771	0.0654	0.0908	IG	0.31	0.4
$100\sigma_{g}^{L}$	0.1349	0.1420	0.1127	0.1757	IG	0.38	0.4
$100\sigma_{\mu}^{L}$	0.4798	0.7567	0.4383	1.2573	IG	1	0.8
p_H	0.9086	0.8856	0.7951	0.9515	Dir	0.8333	0.1034
p_L	0.9646	0.9545	0.9198	0.9796	Dir	0.8333	0.1034
$100\sigma_\pi$	0.2772	0.2741	0.2369	0.3131	IG	0.15	0.3

Table 1: Posterior modes, means, and 90% error bands of the model parameters. Type N, G, B, IG stand for Normal, Gamma, Beta, Inversed Gamma density, respectively. Dir stands for the Dirichelet distribution

At the posterior mode, the passive policy implies a higher output-gap coefficient ϕ_y than that implied by the active policy. The probability of being in the short-lasting passive regime conditional on having switched to passive policies, $p_{12}/(1-p_{11})$, plays a critical rule in the model. As noticed in Section 2, this parameter value relates to the strength of the Federal Reserve *reputation* to refrain from long-lasting deviations. This parameter is found to be fairly close to one, confirming that the Federal Reserve has strong reputation. This number means that as agents observe a deviation from the active regime, they expect that the Federal Reserve is conducting a short-lasting passive policy with probability 0.92.

Figure 1 shows the estimated probabilities of the active policy regime (upper panel)[11] and the estimated probabilities of the high volatility regimes (lower panel). In line with previous studies, it emerges that the 1970s and the early 1980s were periods of high volatility. While

[11] As discussed in Section 3, we estimate the model after we have redefined the set of regimes as the number of consecutive deviations from the active policy τ_t. Therefore, we cannot tease out the evolution of the probability of the short-lasting passive regime and the long-lasting passive regime.

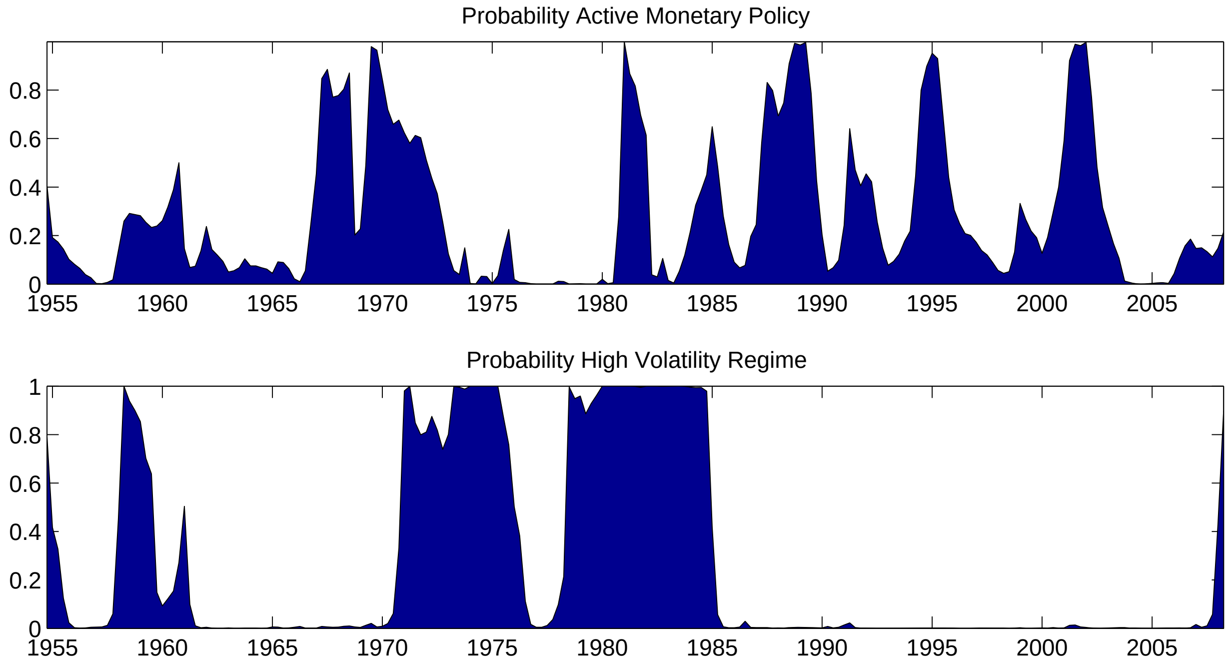

Figure 1: Estimated probability of the active monetary policy regime and the high volatility regime.

the 1970s were characterized by a fairly long-lasting passive policy, the Federal Reserve has alternated periods of active policy to periods of rather short-lasting passive policy in the post-1970s. The most recent approach to monetary policy closely resembles the idea of constrained discretion discussed by Bernanke (2003); that is, a monetary policy approach whose main objective is to stabilize inflation through active policies but the central bank may sometimes de-emphasize inflation stabilization for rather short periods of time.

4.2 Beliefs Dynamics

Monetary policy decisions on stabilizing inflation and communication strategies critically affect the social welfare and the macroeconomic equilibrium by influencing agents' pessimism about future monetary policy. In this paper, we will use the word *pessimism* to precisely mean agents' expectations about the duration of an observed passive policy. A high level of pessimism means that agents expect an observed passive policy to last for relatively long; that is, close to the expected duration of the long-lasting passive Regime $(1 - p_{33})^{-1}$. While expecting a longer lasting deviation from the active regime is not necessarily welfare decreasing, we will show that expecting a prolonged period of passive policy impairs social welfare in the estimated model.

We measure pessimism by computing the expected number of consecutive periods of

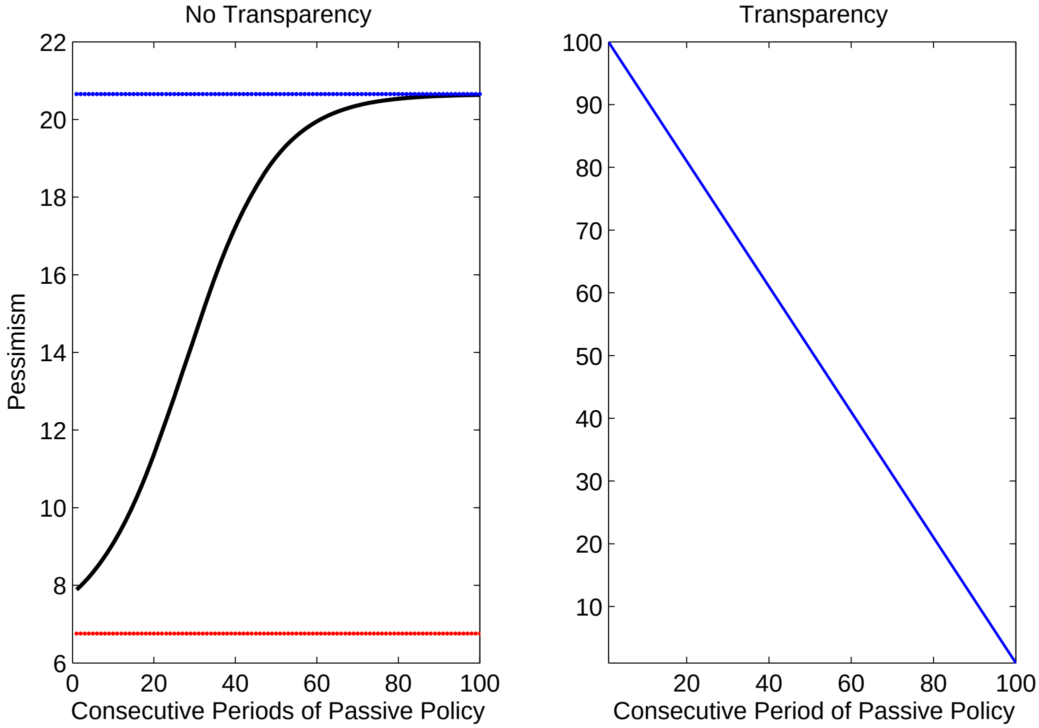

Figure 2: Pessimism on the vertical axis is measured as the number of expected consecutive deviations ahead. On the left plot the two horizontal lines denote the smallest lower bound $(1-p_{22})^{-1}$ and upper bound of pessimism $(1-p_{33})^{-1}$. These statistics are computed at the posterior mode.

passive monetary policy. Let us assume that the central bank decides to engage in passive policies lasting one hundred consecutive periods. While a policy of such a long duration is clearly quite implausible for the U.S., this example is illustrative of how transparency affects pessimism relative to no transparency. Figure 2 reports the evolution of pessimism under no transparency (left graph) and under transparency (right graph) at the posterior mode.[12] The two horizontal lines mark the smallest lower bound and upper bound for pessimism. The former is given by the expected duration of the short-lasting passive Regime $(1-p_{22})^{-1}$. The smallest lower bound is attained at the first period of passive policy only if the conditional probability of a long lasting deviation is zero: $p_{12}/(1-p_{11}) = 1$. The left graph shows that the intercept of the solid line is quite close to the bottom dashed line, implying that agents's

[12]Under no transparency, the pessimism after having observed τ_t consecutive deviations from the active policy is computed as follows:

$$prob\{\xi_t^p = 2|\tau_t\}\frac{1}{1-p_{22}} + [1 - prob\{\xi_t^p = 2|\tau_t\}]\frac{1}{1-p_{33}}$$

where $prob\{\xi_t^p = 2|\tau_t\} = p_{12}p_{22}^{\tau_t - 1}/\left(p_{12}p_{22}^{\tau_t-1} + p_{13}p_{33}^{\tau_t-1}\right)$ is the probability of being in Regime 2 conditional on having observed τ_t consecutive periods of passive regime.

mostly expect that the Federal Reserve is engaging in a short-lasting deviation as the first period of passive policy is observed. This result is due to the fact that the Federal Reserve's reputation is estimated to be fairly high ($p_{12}/(1-p_{11})=0.92$).

The upper bound for pessimism is given by the expected duration of the long-lasting passive policy $(1-p_{33})^{-1}$ and is attained only after a very large number of consecutive deviations from the active regime. Such a gradual increase in pessimism suggests that the Federal Reserve can enjoy a great deal of leeway in deviating from active monetary policy in order to stabilize alternative short-lasting objectives. This result is again due to the strong reputation of the Federal Reserve. If the reputation coefficient $p_{12}/(1-p_{11})$ were close to zero, then the expected number of consecutive deviations would experience a larger jump and, hence, the convergence to the upper bound would be much faster than what shown in Figure 2.

As shown in the right graph, pessimism follows an inverse path under transparency. Unlike the case of no transparency, agents' pessimism is very high at the initial stage of the deviation from the passive policy but it decreases as the time goes by. This result comes from assuming that agents are fully rational and the announcement is truthful. As the one hundred periods of passive monetary policy are announced ($t=0$), an immediate rise in pessimism occurs. As the number of periods of passive policy yet to be carried out decreases, agents' pessimism declines accordingly. At the end of the policy ($t=100$), pessimism reaches its lowest level, with agents expecting to return to the active regime with probability one in the following period. It should be noted that at the end of the announced deviation transparency allows the central bank to lower agents' pessimism below the smallest lower bound attainable under no transparency: This result emerges because the central bank is able to inform agents about the exact period in which the passive policy will be terminated. This assumption will be relaxed in Section 6.2.

To sum up, Figure 2 allows us to isolate two important effects of transparency on agents' pessimism about future monetary policy: (i) *transparency raises pessimism at the beginning of the policy*; (ii) *transparency anchors down pessimism at the end of the policy.* As we shall show, these two effects play a critical role for the welfare implications of transparency.

5 Welfare Implications of Transparency

In this section, we use the model to assess the welfare implications of introducing transparency. Before proceeding, it is worth emphasizing that the regime changes considered in this paper do not affect the steady state, but only the way the economy fluctuates around the steady state. The period welfare function can then be obtained by taking a log-quadratic approximation of the representative household's utility function around the deterministic steady state:[13]

$$\mathbb{W}_i\left(s_t\left(i\right), \xi_t^v; \theta\right) = -\sum_{h=1}^{\infty} \beta^h \left[\left(\varepsilon/\lambda\right) var_i\left(\pi_{t+h}|s_t\left(i\right), \xi_t^v\right) + \left[\sigma + \frac{\psi+\alpha}{1-\alpha}\right] var_i\left(y_{t+h}|s_t\left(i\right), \xi_t^v\right)\right], \tag{18}$$

where $i \in \{N, T\}$, $\lambda \equiv \kappa\left[\sigma + \frac{\psi+\alpha}{1-\alpha}\right]^{-1}$, $var_i\left(\cdot\right)$ stands for the stochastic variance associated with agents' forecasts of inflation, and the output gap at horizon h, and , θ is the vector of model parameter. The subscript i refers to the communication strategy: $i = N$ stands for the case of no transparency, while $i = T$ denotes transparency. Finally, $s_t\left(i\right)$ denotes the policy regime: $s_t\left(i = N\right) = \tau_t$ and $s_t\left(i = T\right) = \tau_t^a$. Recall from Section 3, the policy regime s_t is the observed duration of passive policy for the case of no transparency and the number of periods of announced passive policy yet to be carried out in the case of transparency. The steady-state demand elasticity $\varepsilon = \left(1+\mu\right)/\mu$, the Frisch labor supply elasticity parameter ψ, and the capital labor share α are not identifiable. We set this parameter equal to 6, which Rotemberg and Woodford (1999) argue to deliver plausible markups for the U.S. economy. Following Rios-Rull, Schorfheide, Fuentes-Albero, Kryshko, and Santaeulalia-Llopis (2012) we calibrate the (inverse) Frisch labor supply elasticity parameter ψ to 0.5. The capital income share α is set equal to 0.3.

The output gap enters the welfare function because it reflects the difference between the marginal rate of substitution and the marginal product of labor, which is a measure of the economy's aggregate inefficiency (Woodford, 2003, Steinsson (2003), and Gali, 2008). Inflation deviations from its steady-state level reduce welfare by raising price dispersion. The elasticity of substitution between two differentiated goods ε raises the weight of inflation fluctuations relative to the output gap because it amplifies the welfare losses associated with any given price dispersion. Nominal rigidities, whose size is inversely related to the slope of the New Keynesian Phillips curve κ, raise the degree of price dispersion resulting from any given deviation from the steady-state inflation rate.

Equation (18) makes it explicit that social welfare depends on agents' *uncertainty* about future inflation and the future output gap. It should be noted that agents' uncertainty in any given period captures the *macroeconomic risk* associated with the observed policy regime and communication strategy, $s_t\left(i\right)$. Unlike standard New Keynesian models with fixed parameters where welfare is merely a function of the unconditional variance of inflation and the output gap, our model allows to study the *dynamic* effects of policy actions and

[13] The derivation closely follows Woodford (2003), Gali (2008), Coibion, Gorodnichenko, and Wieland (2012). Furthermore, we assume that the inefficiency generated by the market power are removed by the suitable choice of subsidies so that the steady-state equilibrium is efficient. A detailed derivation of the welfare function is in Appendix C.

forward-looking communication on welfare. Furthermore, the learning mechanism plays an important role in our welfare analysis by linking the concept of reputation, which can be directly measured in the data, to the central bank's ability of controlling the dynamics of the macroeconomic risk associated with policy actions. This last point will be the focus of the next session.

To assess the desirability of transparency, we compute the model predicted welfare gains/losses from transparency as follows:

$$\Delta\mathbb{W}^e(\theta) = p_T^*(\tau_t^a, \xi_t^v)' \cdot \mathbb{W}_T(\tau_t^a, \xi_t^v; \theta) - p_N^*(\tau_t, \xi_t^v)' \cdot \mathbb{W}_N(\tau_t, \xi_t^v; \theta) \tag{19}$$

where $p_T^*(\tau^a, \xi_t^v)$ stands for the vector of the ergodic joint probabilities of a passive policy of *announced* duration τ_t^a and volatility regime ξ_t^v. $p_N^*(\tau_t, \xi_t^v)$ stands for the vector of ergodic joint probabilities of a passive policy of *observed* duration τ_t and volatility regime ξ_t^v are realized. It is important to emphasize that welfare gains from transparency are not conditioned on a particular shock or policy path. Instead, the welfare gain is measured by the unconditional long-run change in welfare that arises if the central bank systematically announces the duration of any deviation from active monetary policy.

Uncertainty about the future output gap plays only a minor role for social welfare since the estimated value of the slope of the Phillips curve κ is very small (see Table 1) and standard calibrations for the elasticity of substitution ε range from 6 to 10. Such a flat Phillips curve is a standard finding when DSGE models are estimated using U.S. data. Therefore, welfare turns out to be tightly related to agents' uncertainty about future inflation, which, as we shall show, depends on the time-varying level of pessimism about observing a future switch to active monetary policy. If the central bank has lower reputation, agents take into account that long-lasting deviations from the active regime are more frequent and potentially more persistent. Consequently, agents expect more drastic inflationary or deflationary consequences from future shocks and thereby they become more uncertain about future inflation as the central bank engages in passive policies. As a result, social welfare deteriorates faster than in the case of strong reputation. As shown in Section 4.2, the level of pessimism responds to central bank behavior, namely the frequency and duration of deviations from active policy and the communication strategy.

Section 5.1 outlines how uncertainty evolves as the central bank conducts passive policies of different duration and under different communication strategies. In Section 5.2, we use the model to assess the welfare implications of increasing central bank transparency.

5.1 Evolution of Uncertainty

We have shown that agents' uncertainty about future inflation crucially affects social welfare in the estimated model. In this section, we will show how uncertainty is tightly linked to agents' pessimism about observing active monetary policy in the future. As shown in Section 4.2, transparency has two effects on pessimism: (i) pessimism rises at the beginning of the policy (henceforth, the *short-run effect of transparency on pessimism*); (ii) pessimism is anchored down at the end of the policy (henceforth, the *anchoring effect of transparency on pessimism*). As we shall show, these two effects play a critical role for the welfare implications of enhancing the transparency of the central bank.

To illustrate how uncertainty responds to pessimism, we consider the case in which the Federal Reserve conducts a forty-quarter-long deviation from active monetary policy.[14] Figure 3 illustrates the evolution of uncertainty about inflation at different horizons under no transparency, left panel, and under transparency, right panel. At each point in time, the evolution of agents' uncertainty is measured by the h-period ahead standard deviation of inflation at different horizons re-scaled by the standard deviation conditional on monetary policy being currently active and given the communication strategy:

$$sd_i\left(\pi_{t+h}|\tau_t\,,\xi_t^v\right) = 100\left[\sqrt{var_i\left(\pi_{t+h}|s_t\left(i\right),\xi_t^v\right)} - \sqrt{var_i\left(\pi_{t+h}|s_t\left(i\right)=0,\xi_t^v\right)}\right]$$

where $i \in \{N, T\}$ and $\xi_t^v \in \{L, H\}$.[15] We analytically compute the conditional standard deviations taking into account regime uncertainty using the methods described in Bianchi (2013a).

It should be noted that the normalizing factor $var_i\left(\pi_{t+h}|s_t\left(i\right)=0,\xi_t^v\right)$ is not the same across communication strategies $i \in \{N, T\}$, implying that the left and right panels are not directly comparable. This is because transparency determines an overall reduction in uncertainty that manifests itself also under the active regime, even if under the active regime no announcement is made. A transparent central bank enjoys lower uncertainty even when monetary policy is active because agents understand that should a passive policy of any durations be implemented in the future, the central bank will announce its duration before-

[14] The analysis is conducted for an economy at the steady-state and hence without conditioning on a particular shock. The exercise is only conditioned on the policy path and intends to facilitate the exposition of the welfare implications of transparency in the next section.

[15] The graphs plot the results for h from 1 to 60: At horizon $h = 0$, uncertainty is zero as agents observe current inflation. We rescale the uncertainty using the volatility conditional on being in the active regime at horizon $h = 0$ so as to purge the effect of heteroskedasticity on agents' uncertainty from the graph. In the interest of space, we report only the dynamics of uncertainty conditional on being in the low volatility regime while the passive policy is carried out. The dynamics of uncertainty conditional on being in the high volatility regime follows a very similar pattern and is available upon request.

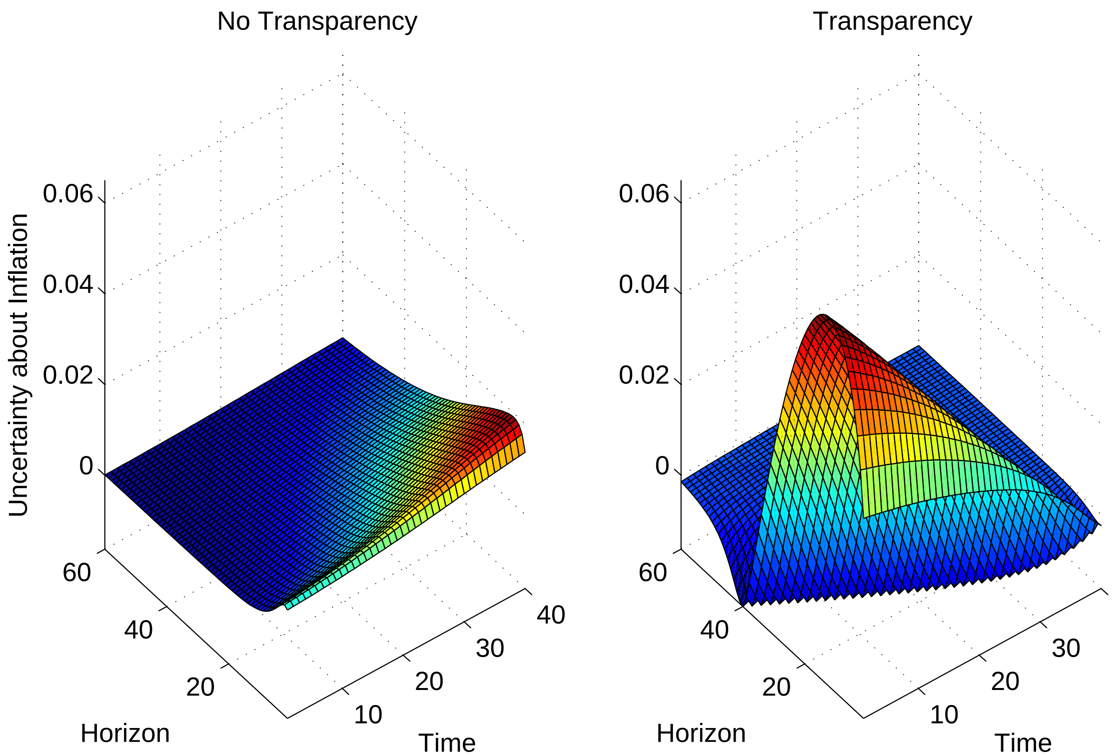

Figure 3: Evolution of uncertainty about inflation at different horizons (h) over forty periods of passive policy (time) under low volatility (i.e., $\xi_t^v = L$). The vertical axis reports the standard deviations (rescaled for the standard deviation associated with currently being in the active regime) in percentage points at the posterior mode.

hand. As it will soon become clear, such a communication strategy is effective in reducing uncertainty by removing the fear of a long lasting deviation for the frequent short lasting deviations and creating an anchoring effect for the sporadic long lasting deviations. However, Figure 3 is illustrative of the dynamics of uncertainty about future inflation as a prolonged passive policy is carried out. When we will compute the welfare implications of transparency, we will not normalize our measure of uncertainty.

As shown in the left graph, when the central bank does not announce its policy course beforehand, uncertainty about future inflation is relatively low at the beginning of the policy because agents interpret the first deviations from active policy as short lasting. This result is driven by the high reputation of the Federal Reserve, implying that agents attach 92% probability of being in the short-lasting passive regime as the first period deviation from active policy is observed. As more and more periods of passive policy are observed, agents get progressively more persuaded that the observed deviation may have a long-lasting nature and uncertainty about future inflation *gradually* takes off. Uncertainty rises because expecting a longer spell of passive policies raises concerns about the central bank's ability of controlling the inflationary consequences of future unanticipated shocks. Note that the increase in uncertainty occurs at every horizon because agents expect passive monetary policy to prevail for many periods ahead and thereby anticipate that the inflationary/deflationary consequences of future shocks will be more severe. It is worth emphasizing that the pattern of agents' uncertainty over time mimics the evolution of pessimism depicted in Figure 2. Summarizing, under no transparency, following a prolonged deviation from the active regime uncertainty starts low and then gradually accelerates. Since higher uncertainty maps into higher welfare loss, the progressive *disanchoring* of uncertainty about future inflation is a reason of concern for a non-transparent central bank.

The right graph illustrates the dynamics of uncertainty about future inflation in the case of transparency. Upon announcement agents become suddenly more uncertain about future inflation because of the *short-run effect of transparency on pessimism*. This is captured by the pronounced hump-shaped dynamics of short- and medium-horizon uncertainty. The announcement commits the central bank to follow a passive policy for the next forty periods, causing agents to expect more dramatic inflationary/deflationary consequences from all those disturbances that will materialize during the implementation of the announced policy path. The rise in uncertainty upon the announcement is clearly welfare decreasing and captures the main reason why the central bank may be reluctant to explicitly communicate their future policy course.

Compared to the case of no transparency, short-horizon uncertainty is larger at the beginning of the policy. However, at this early stage of the passive policy, uncertainty about forty-quarter-ahead inflation appears to be smaller in the case of transparency. This result is due to the *anchoring effect of transparency on pessimism*. While agents know monetary policy will be passive for forty quarters, they also know there will be a switch to the active regime thereafter. Announcing the timing of the return to active monetary policy determines a fall in uncertainty in correspondence of the horizons that coincide with announced date (40 quarters in this numerical example). In the graph, such a decline in uncertainty shows up as a *valley* in the surface representing the level of uncertainty. As we shall show, this feature of transparency has the effect of raising social welfare by systematically anchoring agents' uncertainty at the end of prolonged deviations from the active regime.

While under no transparency uncertainty *increases* across all horizons as the policy is implemented, under transparency uncertainty *decreases* across all horizons because agents are aware that the end of the prolonged period of passive monetary policy is approaching. This depends on the *anchoring effect of transparency on pessimism*. Furthermore, announcing the duration of passive policies triggers a fall in uncertainty at every horizon because it eliminates policy uncertainty for the duration of the announced policy. Even though this effect is not easy to observe in Figure 3, this is certainly an additional welfare-increasing effect. Finally, under the active regime uncertainty about future inflation is found to be lower at every horizon under transparency. This result tends to raise the social welfare associated with transparency. Note that this finding does not hold for all parameter values and hence is due to the estimated parameters for the U.S. In the next section, we will show that this last fact plays a critical role for the welfare comparison between transparency and no transparency.

For the sake of brevity, we do not discuss the evolution of uncertainty about the output gap. As mentioned before, since the estimated value of the slope of the Phillips curve κ is very small when compared to the elasticity of substitution between goods ε, uncertainty about future output plays a negligible role in our welfare analysis.

It is also important to point out that the variance about future inflation conditional on monetary policy to be current active is lower under transparency than that under no transparency at all horizons. In symbols, $var_N\left(\pi_{t+h}|\tau_t = 0, \xi_t^v\right) > var_T\left(\pi_{t+h}|\tau_t^a = 0, \xi_t^v\right)$.This point is important and will be deserve more attention in the next sections.

5.2 Welfare Gains from Transparency

This section derives the welfare gains from enhancing central bank transparency in our model estimated to the U.S. economy. In Section 5.2.1, we conduct a numerical exercise to illustrate the dynamics of the welfare gains/losses from transparency during the implementation of a passive policy. In Section 5.2.2, we compute and discuss the model predicted welfare gains from transparency.

5.2.1 A Numerical Example

For the sake of illustrating the dynamics of welfare, let us consider a passive policy of duration 40 quarters.[16] Figure 4 shows the dynamics of welfare $\mathbb{W}_i\left(s_t\left(i\right), \xi_t^v; \theta\right)$, defined in equation (18), over time as this policy is implemented under the two communication schemes: no transparency $i = N$ and transparency $i = T$. We make this computation conditional on being in the high volatility regime (left graph) and in the low volatility regime (right graph) at time t. It should be observed that under the high volatility regime the welfare under transparency (red solid line) is lower than the welfare under no transparency (blue dashed line) at an early stage of the passive policy. Under the low volatility regime the welfare associated with transparency is always higher than that associated with no transparency

[16] This is a numerical example and is made for the sake of illustrating the evolution of welfare. We pick a fairly prolonged deviation from the active regime so as to make these dynamics more visible in the graphs. Such a long-lasting passive policy has a low probability of occuring based on our estimates.

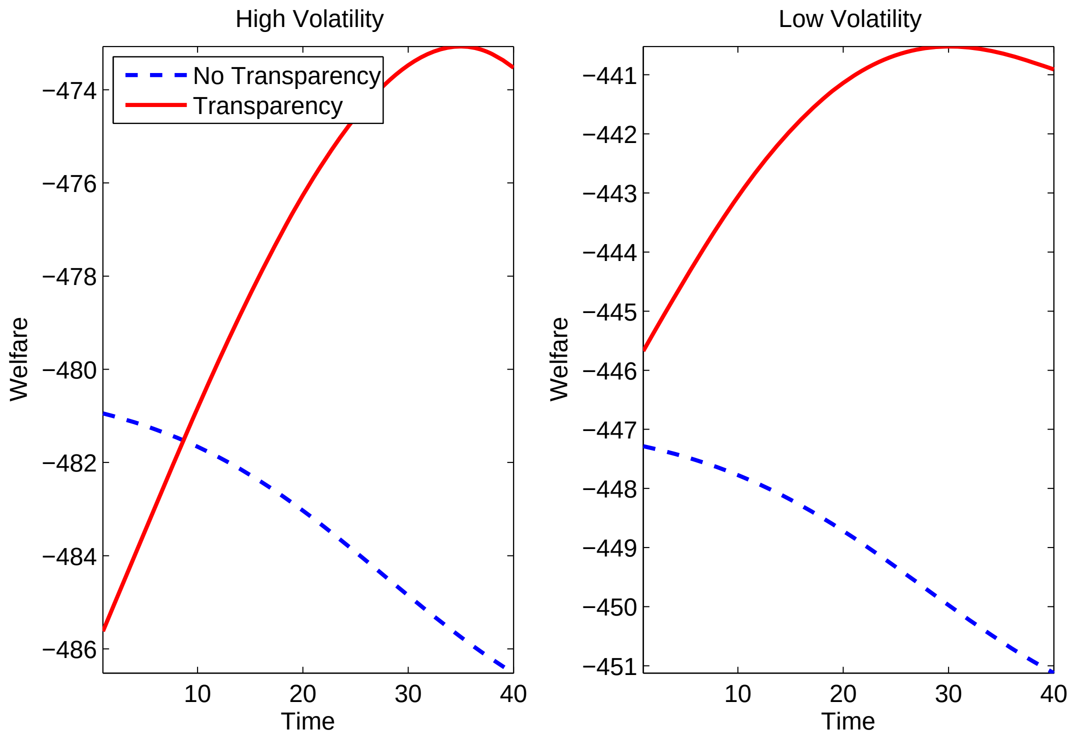

Figure 4: Evolution of welfare $\mathbb{W}_i\left(s_t\left(i\right), \xi_t^v; \theta\right)$ defined in equation (18) as a passive policy of duration 40 quarters is implemented under no transparency ($i = N$), the blue dashed line, and under transparency ($i = T$), the red solid line. Parameter values are set at their posterior mode.

at any time. However, larger gains from transparency, measured by the vertical distance between the two lines, are reaped at the end of the passive policy. As discussed earlier, when the announcement is made, agents become suddenly more pessimistic and hence being transparent lowers welfare compared to no transparency at the beginning of the policy.

However, transparency lowers pessimism as the passive policy is implemented because agents expect less and less periods of passive policy ahead. Therefore, welfare generally increases as the passive policy is implemented. In contrast welfare is downward sloping under no transparency. When the central bank does not communicate the duration of passive policies, agents' pessimism gradually unfold, progressively lowering welfare.

5.2.2 Model Predicted Welfare Gains from Transparency

To assess the welfare gains from transparency we use formula (19), which combines the welfare associated with the augmented policy regimes (τ_t for the case of no transparency and τ_t^a for the case of transparency) and their ergodic probabilities. To facilitate the comparison, we redefine the regimes under transparency τ_t^a in terms of observed periods of passive policy

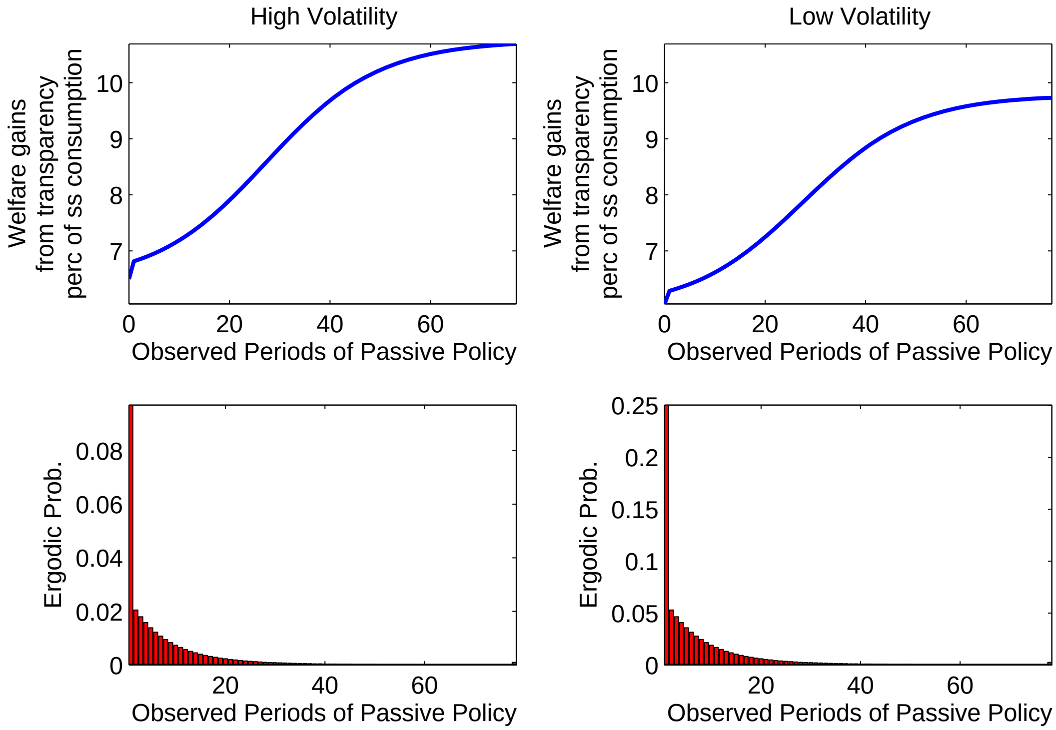

Figure 5: The upper graphs report the dynamics of the welfare gains from transparency as a function of the observed periods of passive policy (τ_t). The lower graph reports the ergodic probability of observing the periods of passive policy on the x-axis (τ_t). Parameter values are set at their posterior mode.

τ_t and recompute welfare under transparency associated with these new set of regimes as shown in Appendix D.

The upper panels of Figure 5 shows the welfare gains from being transparent associated with having observed passive policy for τ_t periods. Once again, we consider the case in which the economy is initially under the high and low volatility regimes. The lower panels report the ergodic probabilities associated with these events. It should be observed that the welfare gains from transparency grow fast for passive policies of short duration and then slow down as their duration increases. This negative second-order derivative is explained by the fact that announcing deviations of longer and longer durations progressively strengthens the *short-run effect of transparency on pessimism* which raises the risk of macroeconomic instability, as shown in Figure 3. In fact, for extremely long-lasting passive policy welfare gains from transparency start declining. However, the lower graphs show that such long-lasting deviations have virtually zero probability to occur in the US economy and hence do not significantly influence the computation of the model predicted welfare gains from transparency based on equation (19). It should be observed that welfare gains from transparency are positive for all the durations of passive policies reported on the x-axis under both low volatility and high volatility. This result implies that the anchoring effects due to transparency dominate its short-run effects. In other words, transparency is welfare improving because it allows the central bank to effectively sweep away the fear of a return to the 1970s-type of passive policies.

Furthermore, it is worth emphasizing that the upper plots capture the welfare gains from *systematically* announcing the duration of passive policies. This explains why when the central bank conducts an active policy ($\tau_t = 0$), the welfare gains from transparency are *not* zero. They are, in fact, positive capturing the welfare gains from expecting that the central bank will systematically and truthfully announce the duration of any future passive policy. This result is the mirror image of what discussed in the previous section: Transparency implies that under the active regime uncertainty is lower at every horizon because agents anticipate the future conduct of monetary policy.

The overall welfare gain is obtained combining the welfare gains conditional on having observed a specific number of deviations, with their corresponding ergodic probabilities. The plausible durations of passive policies in the U.S. are shown in the lower graphs, which report the ergodic probabilities of observing a policy regime of a given duration conditional on currently being in the high volatility regime and in the low volatility regime. Quite interestingly, in the case of low volatility the ergodic probability appears to be less skewed to the right than when the macroeconomic volatility is high. In both cases, the distribution attributes large probability mass to active policies and fairly small probability mass of passive policies of duration longer than twenty quarters.

To sum up, Figure 5 shows that welfare gains from transparency are positive for passive policies of plausible durations for the U.S. under both volatility regimes, implying that the *model predicted welfare gains from transparency* computed in equation (19) are positive based on our estimates. More precisely, the model's predicted welfare gains from transparency amount to roughly 6.63% of steady-state consumption for the U.S. economy.

6 Robustness and Extensions

In this section we conduct a robustness exercise and consider an extension of the benchmark communication strategy. In Section 6.1 we investigate whether transparency is welfare increasing for passive policies of every plausible duration. In Section 6.2, we relax the assumption that the central bank knows exactly the realized duration of the ongoing passive policy and investigate whether transparency would still deliver welfare improvements. Specifically, we study the effects of a central bank that announces the likely duration of passive policies revealing which type of passive regime is in place as opposed to announcing the exact

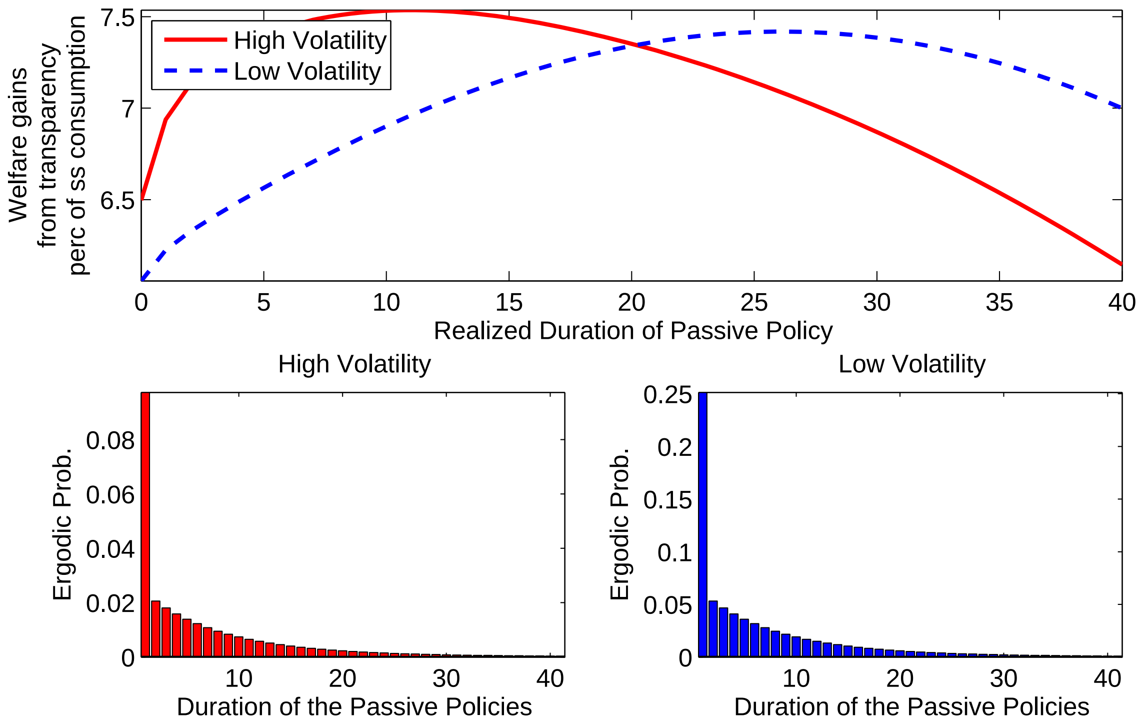

Figure 6: Upper graph: Average per-period welfare gains from transparency associated with policy of a given duration. Lower graphs: the ergodic distribution of the duration of passive policies. A passive policy with duration of zero period on the x-axis corresponds to an active policy. Parameter values are set at their posterior mode.

duration of the deviation.

6.1 Short-Run Benefits from Transparency

In the previous section we showed that embracing transparency would raise the social welfare compared to no transparency. The computation of expected welfare gains from transparency is obtained using the ergodic distribution of the policy regimes and hence captures the *long-run* gains. Furthermore, it should be noted that these long-run welfare gains have been computed under the assumption that agents understand that the central bank will *systematically* announce the duration of *every* passive policy. The fact that welfare gains from transparency are positive in the active regime (when no announcement is actually made) suggests that this systematic feature of the central bank's communication policy contributes to raise the welfare gains from transparency. A transparent central bank enjoys higher welfare when monetary policy is active because agents understand that should a passive policy of any durations be implemented in the future, the central bank will announce its duration beforehand. However, it remains to be seen if the central bank is better off following transparency for any possible duration of the deviation. In other words, are there deviations for which the central bank would rather be not transparent?

We find that the gains from transparency occur for every plausible duration of the passive policy. To see this, the upper plot of Figure 6 shows the dynamics of the average per-period welfare gains from transparency associated with passive policy of various durations, while the lower plots in Figure 6 report the corresponding ergodic probabilities. The important result is that welfare gains are positive for all plausible durations of the passive policies. This finding suggests that the central bank is better off by announcing passive policies of *every* plausible duration. Quite interestingly, the upper graph suggests that the central bank is better off even if it has to announce passive of policy of fairly long duration. This is an important result that implies that the overall reduction of uncertainty that occurs thanks to transparency overcomes any short run loss associated with announcing a prolonged deviation fro active inflation stabilization.

It should be noted the difference between the welfare gains from transparency in Figure 5 and those of Figure 6. Figure 5 reports the welfare gains from having announced the duration of the ongoing passive policy when τ_t deviations out of the announced $\tau_t^a \geq \tau_t$ have been observed. Figure 6 shows the average per-period welfare gains, should the Federal Reserve decide to announce *a passive policy of a certain duration.* The latter measure evaluates the average of welfare gains from transparency across periods of policy implementation whereas the former measure coupled with the ergodic probability distribution of the observed durations of policies (τ_t) captures the expected benefit from being transparent over the *long run.*

6.2 Limited Information

We have modeled transparency as a communication strategy in which the central bank shares all the information about the policy regime to households and firms. Since we assume that the central bank knows the exact duration of its passive policies, transparency implies that such information is shared with the public. In this section, we relax the assumption that the central bank knows the exact duration of passive policies. Rather, we assume that the central bank knows only the *expected duration* of the deviations from the active regime; that is, the bank perfectly knows only if the passive policy is short-lasting, Regime 2, or long-lasting, Regime 3. Now, under transparency the central bank truthfully announces the *expected duration* of passive regimes to households and firms. It should be noted that since the central bank truthfully tells the type of passive regime that is realized to agents, the model boils down to a MS DSGE model with perfect information given that now the history

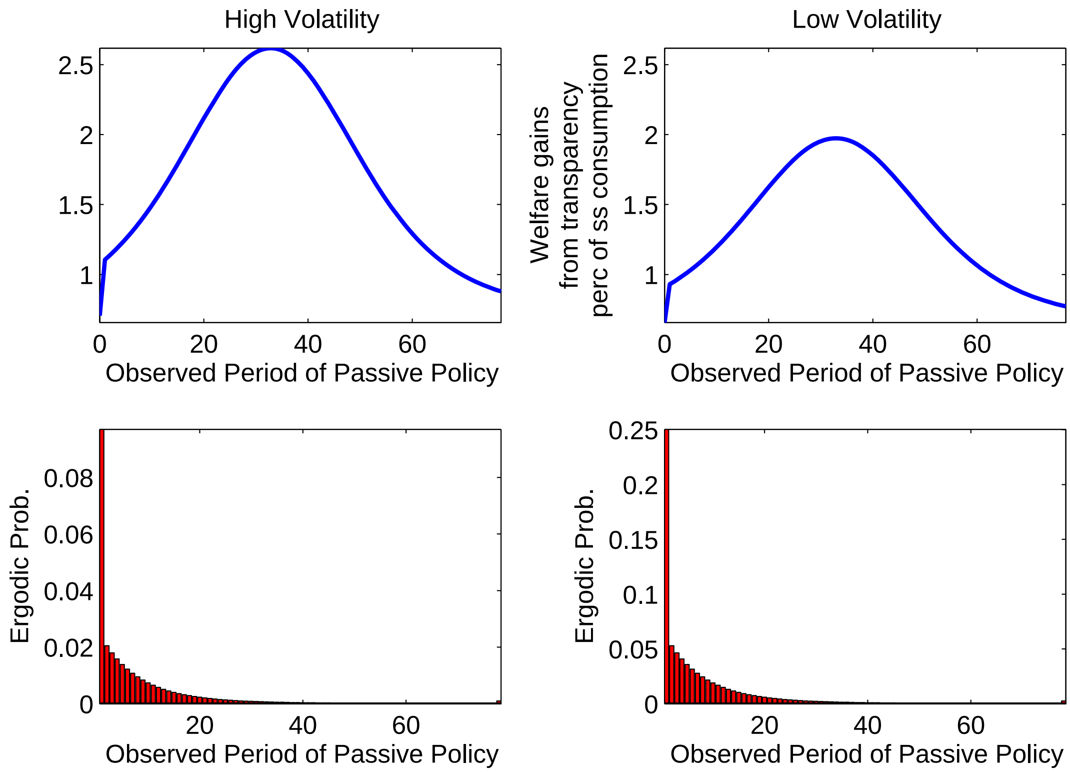

Figure 7: The upper graphs report the dynamics of the welfare gains from transparency as a function of the observed periods of passive policy (τ_t). The lower graph reports the ergodic probability of observing the periods of passive policy on the x-axis (τ_t). Parameter values are set at their posterior mode.

of policy regimes $\xi_t^p \in \{1, 2, 3\}$ belongs to the agents' information set, $\mathcal{F}_t$.

The upper graphs of Figure 7 show the welfare gains from transparency associated with observing different durations of passive policies under the two volatility regimes. The lower graphs report the ergodic probability of observing passive policies of different durations where the one with zero duration corresponds to active policy.[17] The important result that emerges from this graph is that welfare gains from transparency are always positive for policies of any plausible duration and any volatility regime.

Quite interestingly, the pattern of the welfare gains from transparency is qualitatively similar to the one depicted in Figure 5 reporting the welfare gains from transparency when the central bank has full information. The main difference is the duration of passive policy above which the welfare gains from transparency starts declining. The turning point occurs at shorter durations in the case of limited information of the central bank. The welfare gains

[17] Computing the upper graphs requires to transform the primitive regimes, $\xi_t^p \in \{1, 2, 3\}$, into the set of regimes used for the case of no transparency that are defined in terms of the observed durations of passive policies τ_t. The details of this transformation are provided in Appendix D.

from transparency are reduced because while the central bank is still able to remove the fear of a long lasting deviation from the active regime whenever a short lasting deviation occurs, the anchoring effect deriving from exactly announcing when the active regime will be again in place is lost.

Welfare gains from transparency are smaller than in the case of full information by the central bank. In fact, the model predicted welfare gains from transparency amounts to 0.67% of steady-state consumption. Thus, our analysis suggests that the welfare gains from transparency are positive and are quantified to range from 0.67% to 6.63% depending on the degree of information of the central bank.

7 Concluding Remarks

In the model, the central bank alternates active policies aimed to stabilize inflation and passive policies that de-emphasize inflation stabilization. Agents observe when monetary policy becomes passive but they face uncertainty regarding its nature. Importantly, when passive policies are observed, they cannot rule out the possibility that a persistent sequence of deviations is in fact a return to the kind of monetary policy that characterized the 1970s. Instead, they have to keep track of the number of deviations to learn if monetary policy entered a short-lasting or a long-lasting period of passive monetary policy. The longer the deviation from the active policy is, the more pessimistic about the evolution of future monetary policy agents become. This implies that as the central bank keeps deviating, uncertainty increases and welfare deteriorates.

We develop a general equilibrium model in which the central bank can deviate from active inflation stabilization. Agents observe when monetary policy becomes passive but they face uncertainty regarding its nature. Importantly, when passive policies are observed, they cannot rule out the possibility that a persistent sequence of deviations is in fact a return to the kind of monetary policy that characterized the 1970s. Instead, they have to keep track of the number of deviations to learn if monetary policy entered a short-lasting or a long-lasting period of passive monetary policy. The longer the deviation from the active policy is, the more pessimistic about the evolution of future monetary policy agents become. This implies that as the central bank keeps deviating, uncertainty increases and welfare deteriorates.

When the model is fitted to U.S. data, we find that the Federal Reserve benefits from strong reputation. As a result, the Federal Reserve can deviate for a fairly prolonged period of time from active monetary policy before losing control over agents' uncertainty about future inflation. Nevertheless, increasing the transparency of the Federal Reserve would improve welfare by anchoring agents' pessimism when facing exceptionally prolonged periods of passive monetary policy and removing the fear of the '70s for the frequent short lasting deviations.

In the model, agents learn only the persistence of passive policies, while the active regime is fully revealing. This implies that agents' expectations are completely revised as soon as the central bank returns to the active regime. In Bianchi and Melosi (2014b) we develop a more general methodology that could be used to study a model in which agents have to learn about the likely duration of both passive and active policies. This extension implies that central bank reputation varies over time. While this feature is very interesting, it would make the task of solving the model computationally challenging, preventing us from estimating the model. We regard estimation as an important ingredient of the paper because the proposed framework is new in the literature, with the result that the parameters controlling central bank reputation cannot be borrowed from previous contributions. Furthermore, we believe that this extension is unlikely to affect the main conclusions of the paper. This is because announcing the return to a long lasting period of active monetary policy would still have the effect of anchoring agents' pessimism and uncertainty.

A nice feature of the paper is to introduce a convenient way to model gradual changes in beliefs about future policy decisions and macroeconomic outcomes. In the parsimonious setting studied in this paper, we have shown that waves of agents' pessimism or optimism about future policy actions play a central role in shaping the response of macroeconomic variables and households' welfare to macroeconomic shocks in forward-looking rational expectations models. Expanding the analysis to state-of-the-art monetary DSGE models such as Christiano, Eichenbaum, and Evans, 2005 and Smets and Wouters, 2007 would be of great interest, but quite challenging from a computational point of view.

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A Solving the Model with No Transparency

It is very important to emphasize that the evolution of agents' beliefs about the future conduct of monetary policies plays a critical role in the Markov-switching model with learning. In fact, three policy regimes ξ_t^p are not a sufficient statistic for the dynamics of the endogenous variables in the model with learning. Instead, agents expect different dynamics for next period's endogenous variables depending on their beliefs about a return to the active regime.

To account for agents learning we expand the number of regimes and redefine them as a combination between central bank' behaviors and agents' beliefs. Bianchi and Melosi (2014b) show that the Markov-switching model with learning described previously can be recast in terms of an expanded set of $(\tau_t^* + 1) > 3$ new regimes, where $\tau_t^* > 0$ is defined by the condition (15). These new set of regimes constitute a sufficient statistics for the endogenous variables in the model as they capture the evolution of agents' beliefs about observing a switch to the active regime in the next period. The $\tau^* + 1$ regimes are given by

$$\left[\left(\xi_t^p = 1, \tau_t = 0\right), \left(\xi_t^p \neq 1, \tau_t = 1\right), \left(\xi_t^p \neq 1, \tau_t = 2\right), ..., \left(\xi_t^p \neq 1, \tau_t = \tau^*\right)\right],$$

and the transition matrix $\widetilde{P}_p$ is defined using equation (14); that is,

$$\widetilde{P}_p = \begin{bmatrix} p_{11} & p_{12}+p_{13} & 0 & \dots & 0 & 0 \\ 1-\frac{p_{12}p_{22}+p_{13}p_{33}}{p_{12}+p_{13}} & 0 & \frac{p_{12}p_{22}+p_{13}p_{33}}{p_{12}+p_{13}} & \dots & 0 & 0 \\ 1-\frac{p_{12}p_{22}^2+p_{13}p_{33}^2}{p_{12}p_{22}+p_{13}p_{33}} & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1-\frac{p_{22}(p_{12}/p_{13})(p_{22}/p_{33})^{\tau^*-2}+p_{33}}{(p_{12}/p_{13})(p_{22}/p_{33})^{\tau^*-2}+1} & 0 & 0 & 0 & 0 & \frac{p_{22}(p_{12}/p_{13})(p_{22}/p_{33})^{\tau^*-2}+p_{33}}{(p_{12}/p_{13})(p_{22}/p_{33})^{\tau^*-2}+1} \\ 1-\frac{p_{22}(p_{12}/p_{13})(p_{22}/p_{33})^{\tau^*-1}+p_{33}}{(p_{12}/p_{13})(p_{22}/p_{33})^{\tau^*-1}+1} & 0 & 0 & 0 & 0 & \frac{p_{22}(p_{12}/p_{13})(p_{22}/p_{33})^{\tau^*-1}+p_{33}}{(p_{12}/p_{13})(p_{22}/p_{33})^{\tau^*-1}+1} \end{bmatrix}.$$

B Convergence

Table 2 reports results based on the Brooks-Gelman-Rubin potential reduction scale factor using within and between variances based on the four multiple chains used in the paper. The five chains consist of $540,000$ draws each, the first $40,000$ draws are dropped, and of the remaining draws 1 every $1,000$ draws is saved. The numbers are very close to 1 and therefore well below the 1.2 benchmark value used as an upper bound for convergence.

Potential Scale Reduction Factor

Parameter	PSRF	Parameter	PSRF	Parameter	PSRF	Parameter	PSRF
ϕ_π^A	1.00	H_{11}^{mp}	1.01	τ	1.01	σ_R^H	1.00
ϕ_y^A	1.00	H_{22}^{mp}/H_{33}^{mp}	1.00	κ	1.00	σ_g^H	1.00
ρ_R^A	1.00	H_{33}^{mp}	1.00	ρ_g	1.00	σ_m^H	1.00
ϕ_π^P	1.00	$H_{12}^{mp}/(1-H_{11}^{mp})$	1.00	ρ_m	1.02	σ_R^L	1.00
ϕ_y^P	1.05	H_{11}^{σ}	1.00	r	1.00	σ_g^L	1.00
ρ_R^P	1.00	H_{22}^{σ}	1.00	π	1.00	σ_m^L	1.01
						σ_π	1.00

Table 2: The table reports the Gelman-Rubin Potential Scale Reduction Factor (PSRF) for four chains of 540,000 draws each (1 every 1000 is stored). Values below 1.2 are regarded as indicative of convergence.

C Welfare Function

It is worth emphasizing that the regime changes considered in this paper do not affect the steady state, but only the way the economy fluctuates around the steady state. Therefore, we are going to follow the literature and we derive a second order approximation of the representative household's utility function around the steady state. The representative household's utility function are described by the following function:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi}$$

Define the $\widehat{x}_t = \ln \frac{X_t}{X}$ as the log-deviation of the generic variable X_t from its own steady-state value. The utility function above can be equivalently written as

$$U(C_t, N_t) = \frac{C^{1-\sigma} e^{(1-\sigma)\hat{c}_t}}{1-\sigma} - \frac{N^{1+\psi} e^{(1+\psi)\hat{n}_t}}{1+\psi}$$

The second order Taylor expansion around the steady-state equilibrium yields

$$\begin{aligned} U(Y_t, N_t) &\simeq C^{1-\sigma}\hat{c}_t - N^{1+\psi}\hat{n}_t + \frac{1}{2}C^{1-\sigma}(1-\sigma)\hat{c}_t^2 - \frac{1}{2}N^{1+\psi}(1+\psi)\hat{n}_t^2 \\ &\simeq Y^{1-\sigma}\left(\hat{y}_t + \frac{1}{2}(1-\sigma)\hat{y}_t^2\right) - N^{1+\psi}\left(\hat{n}_t + \frac{1}{2}(1+\psi)\hat{n}_t^2\right) \end{aligned} \quad (20)$$

where in the second line we use the market clearing condition $\hat{c}_t = \hat{y}_t$.

Using the production function we can write

$$N_t(i)^{1-\alpha} = \frac{Y_t(i)}{Z_t}$$

Using the fact that the demand for the variety produced by firm i is $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon_t} Y_t$ we obtain

$$N_t(i) = \left(\frac{Y_t}{Z_t}\right)^{\frac{1}{1-\alpha}} \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon_t}{1-\alpha}}$$

Integrating both sides of the above equation across firms i, we obtain

$$N_t = \left(\frac{Y_t}{Z_t}\right)^{\frac{1}{1-\alpha}} \int \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon_t}{1-\alpha}} di$$

where we use the fact that the labor market clearing requires that $N_t = \int N_t(i)\, di$. The above equation can be log-linearized around a symmetric steady state to get the following:

$$(1-\alpha)\hat{n}_t = (\hat{y}_t - z_t) + d_t \tag{21}$$

where we define $d_t \equiv (1-\alpha)\ln \int \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon_t}{1-\alpha}} di$.

Lemma 1 *In a neighborhood of a symmetric steady state, and up to a second-order approximation,* $d_t \simeq \frac{\varepsilon}{2\Theta} var_i \ln P_t(i)$, *where* $\Theta \equiv \left[\frac{1+\alpha(\varepsilon-1)}{(1-\alpha)}\right]^{-1}$ *is the strategic complementarity parameter.*

Proof. Let $\hat{p}_t(i) \equiv \ln P_t(i) - \ln P_t$. Note that $\hat{p}_t(i)$ is a *stationary* variable because of the assumption of perfect indexation of price to steady-state inflation. Notice that

$$\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon_t}{1-\alpha}} = \exp\left[-\frac{\varepsilon_t}{1-\alpha}\hat{p}_t(i)\right]$$

Taking the second-order approximation of this object around the symmetric steady state:[18]

$$\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon_t}{1-\alpha}} \simeq 1 - \frac{\varepsilon}{1-\alpha}\hat{p}_t(i) + \frac{\varepsilon^2}{2(1-\alpha)^2}\hat{p}_t^2(i)$$

Integrating both sides of this equation across firms leads to

$$\int \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon_t}{1-\alpha}} di \simeq 1 - \frac{\varepsilon}{1-\alpha}\int \hat{p}_t(i)\, di + \frac{\varepsilon^2}{2(1-\alpha)^2}\int \hat{p}_t^2(i)\, di \tag{22}$$

Note that from the definition of the price level we obtain $1 = \int e^{\hat{p}_t(i)(1-\varepsilon_t)} di$. Taking the

[18]Note that since $\widehat{p}_t(i)$ is equal to zero in steady state, taking the Taylor expansion with respect to the elasticity of substitution $\hat{\varepsilon}_t \equiv \ln(\varepsilon_t/\varepsilon)$ would be immaterial. To keep the derivation notationally tractable, we do not take the Taylor expansion with respect to $\hat{\varepsilon}_t$.

second-order approximation of this expression yields

$$0 \simeq (1-\varepsilon) \int \hat{p}_t(i)\, di + \frac{(1-\varepsilon)^2}{2} \int \hat{p}_t(i)^2\, di$$

and after rearranging

$$\int \hat{p}_t(i)\, di = \frac{\varepsilon - 1}{2} \int \hat{p}_t(i)^2\, di \tag{23}$$

Substituting equation (23) in equation (22) allows us to write

$$\int \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon_t}{1-\alpha}} di = 1 - \frac{\varepsilon}{1-\alpha}\left[\frac{\varepsilon-1}{2}\int \hat{p}_t(i)^2\, di\right] + \frac{\varepsilon^2}{2(1-\alpha)^2}\int \hat{p}_t(i)^2\, di$$

After same straightforward manipulation, we obtain

$$\int \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon_t}{1-\alpha}} di = 1 + \frac{1}{2}\frac{\varepsilon}{1-\alpha}\frac{1}{\Theta}\int \hat{p}_t(i)^2\, di. \tag{24}$$

Recalling the definition of $\hat{p}_t(i)$ we can write $E_i \hat{p}_t(i)^2 \equiv \int \hat{p}_t(i)^2\, di = \int [\ln P_t(i) - \ln P_t]^2\, di$. Note also that up to a first-order approximation, the price index equation implies that $\ln P_t = E_i \ln P_t(i)$.[19] Hence, we can write $E_i \hat{p}_t(i)^2 \simeq \int (\ln P_t(i) - E_i \ln P_t(i))^2\, di = var_i(\ln P_t(i)/P_t)$.

Then we can use this result to write

$$\int \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon_t}{1-\alpha}} di = 1 + \frac{1}{2}\frac{\varepsilon}{1-\alpha}\frac{1}{\Theta} var_i(\ln P_t(i)) \tag{25}$$

Finally, combining the definition of d_t with the equation above leads to the following:

$$\begin{aligned} d_t &\equiv (1-\alpha)\ln \int \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon_t}{1-\alpha}} di \\ &\simeq \frac{\varepsilon}{2\Theta} var_i(\ln P_t(i)) \end{aligned}$$

where in the last line we use the fact that $\ln(1+n) \simeq n$ for n sufficiently small. QED. ■

Using equation (21) we can rewrite the second-order approximation of the utility function

[19] Write the price index as $1 = \int e^{\hat{p}_t(i)(1-\varepsilon)} di$. Taking the loglinearization yields $0 = E_i \hat{p}_t(i)$. Recall that $\hat{p}_t(i) \equiv \ln P_t(i) - \ln P_t$, implying $\ln P_t = E_i \ln P_t(i)$.

(20) as follows:

$$\begin{aligned} U\left(C_t, N_t\right) &\simeq y^{1-\sigma}\left(\hat{y}_t+\frac{1}{2}(1-\sigma) \hat{y}_t^2\right) \\ &-\frac{N^{1+\psi}}{1-\alpha}\left(\hat{y}_t+\frac{\varepsilon}{2 \Theta} var_i\left\{\ln P_t(i)\right\}+\frac{1}{2} \frac{1+\psi}{1-\alpha}\left(\hat{y}_t-z_t\right)^2\right) \\ &+t.i.p.+h.o.t. \end{aligned} \tag{26}$$

where $t.i.p.$ collects all terms independent of policies (e.g., z_t) and $h.o.t.$ stands for higher-order term. We can write

$$\begin{aligned} \frac{U\left(Y_t, N_t\right)}{Y^{1-\sigma}} &\simeq \left(\hat{y}_t+\frac{1}{2}(1-\sigma) \hat{y}_t^2\right) \\ &-\frac{N}{Y} \frac{N^{\psi}}{Y^{-\sigma}} \frac{1}{1-\alpha}\left(\hat{y}_t+\frac{\varepsilon}{2 \Theta} var_i\left\{\ln P_t(i)\right\}+\frac{1}{2} \frac{1+\psi}{1-\alpha}\left(\hat{y}_t-z_t\right)^2\right) \\ &+t.i.p.+h.o.t. \end{aligned} \tag{27}$$

Efficiency of the steady state implies that[20]

$$\begin{aligned} -\frac{N^{\psi}}{C^{-\sigma}} &= MPN \\ &= (1-\alpha) \frac{Y}{N} \end{aligned}$$

Substituting the last equation into equation (27) and re-arranging yield:

$$\begin{aligned} \frac{U\left(Y_t, N_t\right)}{Y^{1-\sigma}} &\simeq -\frac{1}{2}\left[\sigma+\frac{\psi+\alpha}{1-\alpha}\right] \hat{y}_t^2-\frac{\varepsilon}{2 \Theta} var_i\left\{\ln P_t(i)\right\}+\frac{1+\psi}{1-\alpha} \hat{y}_t z_t \\ &+t.i.p.+h.o.t. \end{aligned} \tag{28}$$

Note that the log-deviation of the efficient level of output from its steady-state level is given by $\hat{y}_t^e=\frac{1+\psi}{\sigma(1-\alpha)+\psi+\alpha} z_t$. Hence, we can substitute this expression for z_t and write

$$\begin{aligned} \frac{U\left(Y_t, N_t\right)}{Y^{1-\sigma}} &\simeq -\frac{\varepsilon}{2 \Theta} var_i\left\{\ln P_t(i)\right\}-\frac{1}{2}\left[\sigma+\frac{\psi+\alpha}{1-\alpha}\right]\left(\hat{y}_t^2-2 \hat{y}_t \hat{y}_t^e\right) \\ &+t.i.p.+h.o.t. \end{aligned} \tag{29}$$

[20] As standard, we assume that the inefficiency generated by the market power are removed by the suitable choice of subsidies so that the steady-state equilibrium can be regarded efficient.

We can complete the square and write

$$\frac{U(C_t, N_t)}{Y^{1-\sigma}} \simeq -\frac{\varepsilon}{2\Theta} var_i \{\ln P_t(i)\} - \frac{1}{2}\left[\sigma + \frac{\psi+\alpha}{1-\alpha}\right](\hat{y}_t - \hat{y}_t^e)^2 + t.i.p. + h.o.t. \quad (30)$$

Accordingly, a second-order approximation to the consumer's welfare function can be expressed as a fraction of steady-state consumption (and up to additive terms independent of policy) as follows[21]

$$\mathbb{W} = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\varepsilon}{\Theta} var_i \{\ln P_t(i)\} + \left[\sigma + \frac{\psi+\alpha}{1-\alpha}\right](\hat{y}_t - \hat{y}_t^e)^2\right], \quad (31)$$

where E_0 is the expectation operator conditional on the information set that agents has at time 0, $\mathcal{F}_0$.

Lemma 2 *Under Calvo pricing with non-zero inflation steady state and perfect indexation to steady state inflation* $\sum_{t=0}^{\infty} \beta^t var_i \{\ln P_t(i)\} = \frac{\theta}{(1-\beta\theta)(1-\theta)} \sum_{t=0}^{\infty} \beta^t \hat{\pi}_t^2$.

Proof. In each period the distribution of prices is given by θ times the price distribution in the previous period times the gross steady-state inflation rate *plus* an atom of height $(1-\theta)$ at the optimal reset price. Let us denote

$$\bar{P}_t \equiv E_i \ln P_t(i). \quad (32)$$

Observe that Calvo pricing implies

$$\begin{aligned} \bar{P}_t - \bar{P}_{t-1} - \ln \Pi_* &= E_i \left[\ln P_t(i) - \bar{P}_{t-1} - \ln \Pi_*\right] \\ &= \theta E_i \left[(\ln P_{t-1}(i) + \ln \Pi_*) - \bar{P}_{t-1} - \ln \Pi_*\right] + (1-\theta)\left[\ln P_t^* - \bar{P}_{t-1} - \ln \Pi_*\right] \\ &= \theta E_i \left[\ln P_{t-1}(i) - \bar{P}_{t-1}\right] + (1-\theta)\left[\ln P_t^* - \bar{P}_{t-1} - \ln \Pi_*\right] \quad (33) \end{aligned}$$

where $\ln P_t^*$ is the optimal resetting price for those firms that are allowed to re-optimize their price and Π_* is the steady-state gross inflation rate. Notice that $E_i \ln P_{t-1}(i) \equiv \bar{P}_{t-1}$ and hence $E_i \left[\ln P_{t-1}(i) - \bar{P}_{t-1}\right] = 0$. Therefore,

$$\bar{P}_t - \bar{P}_{t-1} - \ln \Pi_* = (1-\theta)\left[\ln P_t^* - \bar{P}_{t-1} - \ln \bar{\Pi}\right] \quad (34)$$

[21] Since we analyze welfare for an economy that is currently at the steady state, we set the value of the exogenous process $G_t = G = 1$.

Analogously, denote the cross-sectional variance of prices $var_i \{\ln P_t(i)\}$ as Δ_t. This can be equivalently expressed as

$$\begin{aligned}\Delta_t &= var_i \left[\ln P_t(i) - \bar{P}_{t-1} - \ln \Pi_*\right] \\ &= E_i \left[\left(\ln P_t(i) - \bar{P}_{t-1} - \ln \Pi_*\right)^2\right] - \left(E_i \ln P_t(i) - \bar{P}_{t-1} - \ln \Pi_*\right)^2 \end{aligned} \tag{35}$$

Observe that the property of the cross-sectional distribution of prices under Calvo pricing allows us to write

$$E_i \left[\left(\ln P_t(i) - \bar{P}_{t-1} - \ln \Pi_*\right)^2\right] = \theta E_i \left[\left(\ln P_{t-1}(i) - \bar{P}_{t-1}\right)^2\right] + (1-\theta)\left[\ln P_t^* - \bar{P}_{t-1} - \ln \Pi_*\right]^2 \tag{36}$$

Notice that since $E_i \ln P_{t-1}(i) - \bar{P}_{t-1} = 0$, then

$$E_i \left[\left(\ln P_{t-1}(i) - \bar{P}_{t-1}\right)^2\right] = var_i \left[\ln P_{t-1}(i)\right] \equiv \Delta_{t-1} \tag{37}$$

Also taking the square of both sided of equation (34) implies that

$$(1-\theta)\left[\ln P_t^* - \bar{P}_{t-1} - \ln \Pi_*\right]^2 = \frac{1}{1-\theta}\left(\bar{P}_t - \bar{P}_{t-1} - \ln \Pi_*\right)^2 \tag{38}$$

Using the results in equations (37) and (38) in combination with equation (36) yields

$$E_i \left[\left(\ln P_t(i) - \bar{P}_{t-1} - \ln \Pi_*\right)^2\right] = \theta \Delta_{t-1} + \frac{1}{1-\theta}\left(\bar{P}_t - \bar{P}_{t-1} - \ln \Pi_*\right)^2 \tag{39}$$

Plugging equation (39) into equation (35) and also observing that the definition (32) implies that the second term of equation (35) is equal to $\left(\bar{P}_t - \bar{P}_{t-1} - \ln \Pi_*\right)^2$ allow us to leads to

$$\begin{aligned}\Delta_t &= \theta \Delta_{t-1} + \frac{1}{1-\theta}\left(\bar{P}_t - \bar{P}_{t-1} - \ln \Pi_*\right)^2 - \left(\bar{P}_t - \bar{P}_{t-1} - \ln \Pi_*\right)^2 \\ &= \theta \Delta_{t-1} + \frac{\theta}{1-\theta}\left(\bar{P}_t - \bar{P}_{t-1} - \ln \Pi_*\right)^2 .\end{aligned}$$

Now note that up to a first-order approximation, the price index equation implies that $\ln P_t = \bar{P}_t$ and hence the last term of the recursive equation for price dispersion is the log-deviation of inflation from its steady state value Π_*, which we denote with $\hat{\pi}_t$. Finally,

$$\Delta_t = \theta \Delta_{t-1} + \frac{\theta}{1-\theta}\hat{\pi}_t^2 .$$

Integrating forward for any given (small) initial degree of price dispersion Δ_0 we obtain:

$$\Delta_t = \theta^{t+1}\Delta_0 + \sum_{s=0}^{t} \theta^{t-s}\left(\frac{\theta}{1-\theta}\right)\hat{\pi}_s^2.$$

Since the initial price dispersion Δ_0 is independent of any policy implemented in periods $t \geq 0$, one obtains

$$\sum_{t=0}^{\infty} \beta^t \Delta_t = \frac{\theta}{(1-\theta)(1-\theta\beta)} \sum_{t=0}^{\infty} \beta^t \hat{\pi}_t^2$$

QED. ■

Hence the welfare function can be rewritten as follows

$$\mathbb{W} = -\frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\varepsilon}{\lambda}\hat{\pi}_t^2 + \left[\sigma + \frac{\psi+\alpha}{1-\alpha}\right](\hat{y}_t - \hat{y}_t^e)^2\right] \tag{40}$$

where we use the definition $\lambda \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}\Theta$ and $\widehat{\widetilde{y}}_t \equiv (\hat{y}_t - \hat{y}_t^e)$ is the welfare-relevant output gap.

D Transformation of Regimes under Transparency

In Figure 5 we express welfare under transparency in terms of number of observed deviations from the active regime. This corresponds to the definition of policy regime under no transparency. This is done in order to facilitate the analysis of how the welfare gains from transparency varies with passive policies of duration τ.

Let us compute the probability that i consecutive periods of passive policy has been announced conditional on having observed τ period of passive policy:

$$\alpha(i) = \frac{p_{12}p_{22}^{i-1}p_{21} + p_{13}p_{33}^{i-1}p_{31}}{\sum_{j=\tau}^{\tau_*^a+\tau-1}\left[p_{12}p_{22}^{j-1}p_{21} + p_{13}p_{33}^{j-1}p_{31}\right]} \text{ for any } \tau \leq i \leq \tau + \tau_*^a$$

Note that the numerator captures the probability that a deviation of duration i is realized and hence announced (recall all announcements are truthful). The denominator is the probability of (announcing) a passive policy lasting τ periods or longer (up to the truncation τ^*).

The welfare associated with a policy that has been deviating for τ consecutive periods under transparency is given by

$$\widetilde{\mathbb{W}}_T(\tau, \xi^v; \theta) = \sum_{j=0}^{\tau_a^*} \alpha(j+\tau)\,\mathbb{W}_T(\tau_a = j, \xi^v; \theta) \tag{41}$$

Note the difference from $\mathbb{W}_T(\tau_a, \xi^v; \theta)$ in equation (18), which is the welfare function defined in terms of policy regimes for the case of transparency (i.e., τ_a the number of announced deviations yet to be carried out). $\mathbb{W}_T(\tau, \cdot)$ is the welfare under transparency associated with a announcing τ_a periods of passive policy. $\widetilde{\mathbb{W}}_T(\tau, \xi^v; \theta)$ is the welfare under transparency associated with having observed τ consecutive periods of passive policy. We can show that this recasting of policy regimes leads to a negligible approximation error as

$$p_N^*(\tau, \xi_t^v)' \cdot \widetilde{\mathbb{W}}_T(\tau) \approx p_T^*(\tau_a, \xi_t^v)' \cdot \mathbb{W}_T(\tau_a, \xi^v; \theta)$$

When we compute the welfare gains from transparence under limited information by the central bank in Section 6.1, we compute $\overline{\mathbb{W}}_T(\tau, \xi^v; \theta) = \alpha \mathbb{W}_p(s_t = 2) + (1 - \alpha)\mathbb{W}_p(s_t = 3)$ where $\mathbb{W}_p$ denotes the welfare under perfect information and $s_t \in \{1, 2, 3\}$ the primitive set of policy regimes and the weight α is defined as follows:

$$\alpha(\tau) = \frac{p_{12} p_{22}^{\tau-1} p_{21}}{p_{12} p_{22}^{\tau-1} p_{21} + p_{13} p_{33}^{\tau-1} p_{31}},$$

which capture the probability of being in the short-lasting passive regime conditional on having observed τ consecutive deviations from the active policy.