MICRO DATA AND MACRO TECHNOLOGY

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ABSTRACT

We develop a framework to estimate the aggregate capital-labor elasticity of substitution by aggregating
the actions of individual plants, and use it to assess the decline in labor's share of income in the US
manufacturing sector. The aggregate elasticity reflects substitution within plants and reallocation across
plants; the extent of heterogeneity in capital intensities determines their relative importance. We use
micro data on the cross-section of plants to build up to the aggregate elasticity at a point in time. Our
approach places no assumptions on the evolution of technology, so we can separately identify shifts
in technology and changes in response to factor prices. We find that the aggregate elasticity for the
US manufacturing sector has been stable since 1970 at about 0.7. Mechanisms that work solely through
factor prices cannot account for the labor share's decline. Finally, the aggregate elasticity is substantially
higher in less-developed countries.

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A data appendix is available at http://www.nber.org/data-appendix/w20452

1   Introduction

Over the last several decades, labor's share of income in the US manufacturing sector has fallen by more than 15 percentage points.[1] A variety of mechanisms have been proposed to explain declining labor shares; these can be separated into two categories. Some reduce the labor share solely by altering factor prices. For example, Piketty (2014) maintained that declining labor shares resulted from increased capital accumulation, and Karabarbounis and Neiman (2014) argued that they stem from falling capital prices. Other mechanisms, such as automation and offshoring, would be viewed through the lens of an aggregate production function as a change in technology.[2]

As Hicks (1932) first pointed out, the crucial factor in assessing the relevance of these mechanisms is the aggregate capital-labor elasticity of substitution, which governs how aggregate factor shares respond to changing factor prices. More generally, as a central feature of aggregate technology, the elasticity helps answer a wide variety of economic questions. These include, for example, the welfare impact of corporate tax changes, the impact of international interest rate differentials on output per worker, the speed of convergence to a steady state, and how trade barriers shape patterns of specialization.[3]

Unfortunately, obtaining the elasticity is difficult; Diamond et al. (1978) proved that the elasticity cannot be identified from time series data on output, inputs, and marginal products alone. Instead, identification requires factor price movements that are independent of the bias of technical change. Economists have thus explored two different approaches to estimating the elasticity.

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[1]The overall labor share has declined by roughly 8 percentage points over the same time period. While the manufacturing sector is relatively small, representing 13 percent of value added and 8 percent of employment in 2010 (24 percent and 27 percent respectively in 1970), it accounts for between 30 percent and 45 percent of decline in the overall labor share, depending on whether the counterfactual freezes the manufacturing labor share at its 1970 level or both manufacturing's labor share and its share of value added at its 1970 level.

[2]Formally, changes in factor compensation can be decomposed into the response to shifts in factor prices (holding technology fixed) and changes due to shifts in technology (holding factor prices fixed). We call the latter the "bias of technical change," and we use the term loosely to include everything other than factor prices, including changes in production possibilities or in composition due to changing tastes or trade. Changes in the supply of capital or labor (as in Piketty (2014)) would alter relative factor prices but leave technology unchanged.

[3]See Hall and Jorgenson (1967), Mankiw (1995), and Dornbusch et al. (1980).

The first approach uses the aggregate time series and places strong parametric assumptions on the aggregate production function and bias of technical change for identification. The most common assumptions are that there has been no bias or a constant bias over time. Leon-Ledesma et al. (2010) demonstrated that, even under these assumptions, it is difficult to obtain the true aggregate elasticity. Not surprisingly, the estimates found in this literature range widely from 0.5 to 1.5, hindering inference about the decline in the labor share.[4]

The second approach uses micro production data with more plausibly exogenous variation in factor prices, and yields the micro capital-labor elasticity of substitution. Houthakker (1955), however, famously showed that the micro and macro elasticities can be very different; an economy of Leontief micro units can have a Cobb–Douglas aggregate production function.[5] Given Houthakker's result, it is unclear whether the micro elasticity can help answer the many questions that hinge on the aggregate elasticity.

In this paper, we show how the aggregate elasticity of substitution can be recovered from the plant-level elasticity. Building on Sato (1975), we show that the aggregate elasticity is a convex combination of the plant-level elasticity of substitution and the elasticity of demand.[6] In response to a wage increase, plants substitute towards capital. In addition, capital-intensive plants gain market share from labor-intensive plants. The degree of heterogeneity in capital intensities determines the relative importance of within-plant substitution and reallocation. For example, when all plants produce with the same capital intensity, there is no reallocation of resources across plants.

Using this framework, we build the aggregate capital-labor elasticity from its individual components. We estimate micro production and demand parameters. Since Levhari (1968), it has been well known that Houthakker's result of a unitary elasticity of substitu-

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[4] Although Berndt (1976) found a unitary elasticity of substitution in the US time series assuming neutral technical change, Antras (2004) and Klump et al. (2007) subsequently found estimates from 0.5 to 0.9 allowing for biased technical change. Karabarbounis and Neiman (2014) estimate an aggregate elasticity of 1.25 using cross-country panel variation in capital prices. Piketty (2014) estimates an aggregate elasticity between 1.3 and 1.6. Herrendorf et al. (2014) estimate an elasticity of 0.84 for the US economy as a whole and 0.80 for the manufacturing sector. Alvarez-Cuadrado et al. (2014) estimate an elasticity of 0.78 for the manufacturing sector.

[5] Houthakker assumed that factor-augmenting productivities follow independent Pareto distributions. The connection between Pareto distributions and a Cobb-Douglas aggregate production function is also emphasized in Jones (2005), Lagos (2006), and Luttmer (2012).

[6] Sato (1975) showed this for a two-good economy. See also Miyagiwa and Papageorgiou (2007).

tion is sensitive to the distribution of capital intensities. Rather than making distributional assumptions, we directly measure the empirical distribution using cross-sectional micro data.

Thus, given the set of plants that existed at a point in time, we estimate the aggregate elasticity of substitution *at that time.* Our strategy allows both the aggregate elasticity and the bias of technical change to vary freely over time, opening up a new set of questions. Because our identification does not impose strong parametric assumptions on the time path of the bias, our approach is well suited for measuring how the bias has varied over time and how it has contributed to the decline in labor's share of income. We can also examine how the aggregate elasticity has changed over time and whether it varies across countries at different stages of development.

We first estimate the aggregate elasticity for the US manufacturing sector using the US Census of Manufactures. In 1987, our benchmark year, we find an average plant-level elasticity of substitution of roughly one-half. Given the heterogeneity in capital shares and our estimates of other parameters, the aggregate elasticity in 1987 was 0.71.[7] Reallocation thus accounts for roughly one-third of substitution. Despite large structural changes in manufacturing over the past forty years,[8] the aggregate elasticity has been stable. We find the elasticity has risen slightly from 0.67 in 1972 to 0.75 in 2007.

We then use this estimate to decompose the decline in labor's share of income in the manufacturing sector since 1970. We have several findings. First, since our estimated aggregate elasticity is below one, mechanisms such as increased capital accumulation or declining capital prices would raise the labor share. Second, the contribution of factor prices to the labor share exhibits little variation over time, and thus does not match the accelerating decline of the labor share. Rather, the acceleration is accounted for by changes in the pace of the bias of technical change. Third, wages grew more slowly before 1970; a counterfactual exercise that keeps wage growth at its pre-1970 pace indicates that this can explain about one sixth of the decline in the labor share. Finally, a shift in the composition of industries accounts for part of the acceleration in the labor share's decline since 2000. These findings suggest

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[7]Our estimate is a long-run elasticity of substitution between capital and labor, owing to a proper interpretation of our estimates of plant-level parameters. It is thus an upper bound on the short run elasticity.

[8]Elsby et al. (2013) have emphasized the shift in industry composition in the context of labor's share of income.

the decline in the labor share stems from factors that affect technology, broadly defined, including automation and offshoring, rather than mechanisms that work solely through factor prices.

Finally, our approach allows us to examine how the shape of technology varies across countries at various stages of development; policies or frictions that lead to more variation in capital shares raise the aggregate elasticity. Using their respective manufacturing censuses, we find an average manufacturing sector elasticity of 0.84 for Chile, 0.84 for Colombia, and 1.11 for India. These differences are quantitatively important; the response of output per worker to a change in the interest rate is over fifty percent larger in India than in the US, as is the welfare cost of capital taxation. They also imply that a decline in capital prices decreases the labor share in India but increases it in the US.

Our work complements the broad literature that examines how changes in factor prices and technology have affected the distribution of income. Krusell et al. (2000) studied how a capital-skill complementarity and a declining price of capital can change factor shares and raise the skill premium. Acemoglu (2002), Acemoglu (2003), and Acemoglu (2010) showed how factor prices and the aggregate capital-labor elasticity of substitution can determine the direction of technical change. Burstein et al. (2014) studied how changes in technology and supplies of various factors alter factor compensation. Autor et al. (2003) and Autor et al. (2013) studied how changes in technology and trade affect factor income.

The remainder of the paper is organized as follows. In Section 2, we present our theoretical analysis of the aggregation problem, while in Section 3 we estimate the aggregate elasticity for manufacturing. Section 4 examines the robustness of our estimates. In Section 5, we use our estimates of the aggregate elasticity to assess the changes in the labor share. In Section 6, we examine how the aggregate elasticity varies across countries and discuss policy implications. Finally, in Section 7 we conclude.

2 Theory

This section characterizes the aggregate elasticity of substitution between capital and labor in terms of production and demand elasticities of individual plants. We begin with a simplified

environment in which we describe the basic mechanism and intuition. We proceed to enrich the model with sufficient detail to take the model to the data by incorporating materials and allowing for heterogeneity across industries.

A number of features—adjustment costs, an extensive margin, non-constant returns to scale, imperfect pass-through, and varying production and demand elasticities across plants in the same industry—are omitted from our benchmark model. We postpone a discussion of these until Section 4.

2.1   Simple Example

Consider a large set of plants I whose production functions share a common, constant elasticity of substitution between capital and labor, σ. A plant produces output Y_i from capital K_i and labor L_i using the following CES production function:

$$Y_i = \left[(A_i K_i)^{\frac{\sigma-1}{\sigma}} + (B_i L_i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

Productivity differences among plants are factor augmenting: A_i is i's capital-augmenting productivity and B_i i's labor-augmenting productivity.

Consumers have Dixit–Stiglitz preference across goods, consuming the bundle $Y = \left(\sum_{i \in I} D_i^{\frac{1}{\varepsilon}} Y_i^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$. Plants are monopolistically competitive, so each plant faces an isoelastic demand curve with a common elasticity of demand $\varepsilon > 1$.

Among these plants, aggregate demand for capital and labor are defined as $K \equiv \sum_{i \in I} K_i$ and $L \equiv \sum_{i \in I} L_i$ respectively. We define the aggregate elasticity of substitution, σ^{agg}, to be the partial equilibrium response of the aggregate capital-labor ratio, K/L, to a change in relative factor prices, w/r:[9]

$$\sigma^{agg} \equiv \frac{\mathrm{d}\ln K/L}{\mathrm{d}\ln w/r} \tag{2}$$

We neither impose nor derive a parametric form for an aggregate production function. Given the allocation of capital and labor, σ^{agg} simply summarizes, to a first order, how a change in factor prices would affect the aggregate capital-labor ratio.

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[9]Since production and demand are homogeneous of degree one, a change in total spending would not alter the aggregate capital-labor ratio. We address non-homothetic environments in Web Appendix B.6.

Let $\alpha_i \equiv \frac{rK_i}{rK_i + wL_i}$ and $\alpha \equiv \frac{rK}{rK + wL}$ denote the cost shares of capital for plant i and in aggregate. The plant-level and industry-level elasticities of substitution are closely related to the changes in these capital shares:

$$\sigma - 1 = \frac{\mathrm{d}\ln rK_i/wL_i}{\mathrm{d}\ln w/r} = \frac{\mathrm{d}\ln \alpha_i/(1-\alpha_i)}{\mathrm{d}\ln w/r} = \frac{1}{\alpha_i(1-\alpha_i)}\frac{\mathrm{d}\alpha_i}{\mathrm{d}\ln w/r} \tag{3}$$

$$\sigma^{agg} - 1 = \frac{\mathrm{d}\ln rK/wL}{\mathrm{d}\ln w/r} = \frac{\mathrm{d}\ln \alpha/(1-\alpha)}{\mathrm{d}\ln w/r} = \frac{1}{\alpha(1-\alpha)}\frac{\mathrm{d}\alpha}{\mathrm{d}\ln w/r} \tag{4}$$

The aggregate cost share of capital can be expressed as an average of plant capital shares, weighted by size:

$$\alpha = \sum_{i \in I} \alpha_i \theta_i \tag{5}$$

where $\theta_i \equiv \frac{rK_i + wL_i}{rK + wL}$ denotes plant i's expenditure on capital and labor as a fraction of the aggregate expenditure. To find the aggregate elasticity of substitution, we can simply differentiate equation (5):

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\ln w/r} = \sum_{i \in I}\frac{\mathrm{d}\alpha_i}{\mathrm{d}\ln w/r}\theta_i + \sum_{i \in I}\alpha_i \frac{\mathrm{d}\theta_i}{\mathrm{d}\ln w/r}$$

Using equations 3 and 4, this can be written as

$$\sigma^{agg} - 1 = \frac{1}{\alpha(1-\alpha)}\sum_{i \in I}\alpha_i(1-\alpha_i)(\sigma-1)\theta_i + \frac{1}{\alpha(1-\alpha)}\sum_{i \in I}\alpha_i\theta_i\frac{\mathrm{d}\ln\theta_i}{\mathrm{d}\ln w/r} \tag{6}$$

The first term on the right hand side is a substitution effect that captures the change in factor intensity holding fixed each plant's size, θ_i. σ measures how much an individual plant changes its mix of capital and labor in response to changes in factor prices. The second term is a reallocation effect that captures how plants' sizes change with relative factor prices. By Shephard's Lemma, a plant's cost share of capital α_i measures how relative factor prices affect its marginal cost. When wages rise, plants that use capital more intensively gain a relative cost advantage. Consumers respond to the subsequent changes in relative prices by shifting consumption toward the capital intensive goods. This reallocation effect is larger

when demand is more elastic, because customers respond more to changing relative prices. Formally, the change in i's relative expenditure on capital and labor is

$$\frac{\mathrm{d}\ln\theta_i}{\mathrm{d}\ln w/r} \quad = \quad (\varepsilon-1)(\alpha_i-\alpha) \tag{7}$$

After some manipulation (see Appendix A for details), we can show that the industry elasticity of substitution is a convex combination of the micro elasticity of substitution and elasticity of demand:

$$\sigma^{agg} \quad = \quad (1-\chi)\sigma + \chi\varepsilon \tag{8}$$

where $\chi \equiv \sum_{i\in I} \frac{(\alpha_i-\alpha)^2}{\alpha(1-\alpha)}\theta_i$.

The first term, $(1-\chi)\sigma$, measures substitution between capital and labor within plants. The second term, $\chi\varepsilon$, captures reallocation between capital- and labor-intensive plants.

We call χ the heterogeneity index. It is proportional to the cost-weighted variance of capital shares and lies between zero and one.[10] When each plant produces at the same capital intensity, χ is zero and there is no reallocation across plants. Each plant's marginal cost responds to factor price changes in the same way, so relative output prices are unchanged. In contrast, if some plants produce using only capital while all others produce using only labor, all factor substitution is across plants and χ is one. When there is little variation in capital intensities, within-plant substitution is more important than reallocation.

2.2 Baseline Model

This section describes the baseline model we will use in our empirical implementation. The baseline model extends the previous analysis by allowing for heterogeneity across industries and using a production structure in which plants use materials in addition to capital and labor.

Let N be the set of industries and I_n be the set of plants in industry n. We assume that each plant's production function has a nested CES structure.

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[10]A simple proof: $\sum_{i\in I}(\alpha_i-\alpha)^2\theta_i = \sum_{i\in I}\alpha_i^2\theta_i - \alpha^2 \leq \sum_{i\in I}\alpha_i\theta_i - \alpha^2 = \alpha - \alpha^2 = \alpha(1-\alpha)$. It follows that $\chi=1$ if and only if each plant uses only capital or only labor (i.e., for each i, $\alpha_i \in \{0,1\}$).

Assumption 1 *Plant i in industry n produces with the production function*

$$F_{ni}\left(K_{ni}, L_{ni}, M_{ni}\right) = \left(\left[\left(A_{ni}K_{ni}\right)^{\frac{\sigma_n-1}{\sigma_n}} + \left(B_{ni}L_{ni}\right)^{\frac{\sigma_n-1}{\sigma_n}}\right]^{\frac{\sigma_n}{\sigma_n-1}\frac{\zeta_n-1}{\zeta_n}} + \left(C_{ni}M_{ni}\right)^{\frac{\zeta_n-1}{\zeta_n}}\right)^{\frac{\zeta_n}{\zeta_n-1}} \tag{9}$$

so its elasticity of substitution between capital and labor is σ_n. Further, i's elasticity of substitution between materials and its capital-labor bundle is ζ_n.

We also assume that demand has a nested structure with a constant elasticity at each level of aggregation. Such a structure is consistent with a representative consumer whose preferences exhibit constant elasticities of substitution across industries and across varieties within each industry:

$$Y \equiv \left[\sum_{n \in N} D_n^{\frac{1}{\eta}} Y_n^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \qquad Y_n \equiv \left(\sum_{i \in I_n} D_{ni}^{\frac{1}{\varepsilon_n}} Y_{ni}^{\frac{\varepsilon_n-1}{\varepsilon_n}}\right)^{\frac{\varepsilon_n}{\varepsilon_n-1}} \tag{10}$$

This demand structure implies that each plant in industry n faces a demand curve with constant elasticity ε_n. Letting q be the price of materials, each plant maximizes profit

$$\max_{P_{ni}, Y_{ni}, K_{ni}, L_{ni}, M_{ni}} P_{ni}Y_{ni} - rK_{ni} - wL_{ni} - qM_{ni}$$

subject to the technological constraint $Y_{ni} = F_{ni}\left(K_{ni}, L_{ni}, M_{ni}\right)$ and the demand curve $Y_{ni} = Y_n(P_{ni}/P_n)^{-\varepsilon_n}$, where $P_n \equiv \left(\sum_{i \in I_n} D_{ni}P_{ni}^{1-\varepsilon_n}\right)^{\frac{1}{1-\varepsilon_n}}$ is the price index for industry n.

The industry-level elasticity of substitution between capital and labor for industry n measures the response of the industry's capital-labor ratio to a change in relative factor prices:

$$\sigma_n^N \equiv \frac{\mathrm{d}\ln K_n/L_n}{\mathrm{d}\ln w/r}$$

The derivation of this industry elasticity of substitution follows Section 2.1 up to equation (6). As before, $\alpha_{ni} = \frac{rK_{ni}}{rK_{ni}+wL_{ni}}$ is plant i's capital share of non-materials cost and $\theta_{ni} = \frac{rK_{ni}+wL_{ni}}{rK_n+wL_n}$ plant i's share of industry n's expenditure on capital and labor. We will show that reallocation depends on plants' expenditures on materials. We denote plant i's materials share of its total cost as $s_{ni}^M \equiv \frac{qM_{ni}}{rK_{ni}+wL_{ni}+qM_{ni}}$. Because producers of intermediate inputs use capital and labor, changes in r and w would impact the price of materials. We

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define $\alpha^M \equiv \frac{\mathrm{d}\ln q/w}{\mathrm{d}\ln r/w}$ to be the capital content of materials.

Proposition 1 *Under Assumption 1, the industry elasticity of substitution is:*

$$\sigma_n^N = (1 - \chi_n)\sigma_n + \chi_n \left[(1 - \bar{s}_n^M)\varepsilon_n + \bar{s}_n^M \zeta_n\right]$$

where $\chi_n = \sum_{i \in I_n} \frac{(\alpha_{ni} - \alpha_n)^2}{\alpha_n(1 - \alpha_n)}\theta_{ni}$ *and* $\bar{s}_n^M = \frac{\sum_{i \in I_n}(\alpha_{ni} - \alpha_n)(\alpha_{ni} - \alpha^M)\theta_{ni}s_{ni}^M}{\sum_{i \in I_n}(\alpha_{ni} - \alpha_n)(\alpha_{ni} - \alpha^M)\theta_{ni}}$

The proofs of all propositions are contained in Appendix A.

Relative to equation (8), the demand elasticity is replaced by a convex combination of the elasticity of demand, ε_n, and the elasticity of substitution between materials and the capital-labor bundle, ζ_n. This composite term measures the change in i's share of its industry's expenditure on capital and labor, θ_{ni}. Intuitively, a plant's expenditure on capital and labor can fall because its overall scale declines or because it substitutes towards materials. The cost share of materials determines the relative importance of each. As materials shares approach zero, all shifts in composition are due to changes in scale, and Proposition 1 reduces to equation (8). In contrast, as a plant's materials share approaches one, changes in its cost of capital and labor have a negligible impact on its marginal cost, and hence a negligible impact on its sales. Rather, the change in its expenditure on capital and labor is determined by substitution between materials and the capital-labor bundle.

The aggregate elasticity parallels the industry elasticity; aggregate capital-labor substitution consists of substitution within industries and reallocation across industries. Proposition 2 shows that expression for the aggregate elasticity parallels the expressions for the industry elasticity in Proposition 1 with plant and industry variables replaced by industry and aggregate variables respectively.

Proposition 2 *The aggregate elasticity between capital and labor,* $\sigma^{agg} = \frac{\mathrm{d}\ln K/L}{\mathrm{d}\ln w/r}$*, is:*

$$\sigma^{agg} = (1 - \chi^{agg})\bar{\sigma}^N + \chi^{agg}\left[(1 - \bar{s}^M)\eta + \bar{s}^M\bar{\zeta}^N\right] \tag{11}$$

where $\chi^{agg} \equiv \sum_{n \in N} \frac{(\alpha_n - \alpha)^2}{\alpha(1 - \alpha)}\theta_n$, $\bar{s}^M \equiv \sum_{n \in N} \frac{(\alpha_n - \alpha)(\alpha_n - \alpha^M)\theta_n}{\sum_{n' \in N}(\alpha_{n'} - \alpha)(\alpha_{n'} - \alpha^M)\theta_{n'}}s_n^M$,
$\bar{\sigma}^N \equiv \sum_{n \in N} \frac{\alpha_n(1 - \alpha_n)\theta_n}{\sum_{n' \in N}\alpha_{n'}(1 - \alpha_{n'})\theta_n}\sigma_n^N$, *and* $\bar{\zeta}^N \equiv \sum_{n \in N} \frac{(\alpha_n - \alpha)(\alpha_n - \alpha^M)\theta_n s_n^M}{\sum_{n' \in N}(\alpha_{n'} - \alpha)(\alpha_{n'} - \alpha^M)\theta_{n'}s_{n'}^M}\zeta_n^N$.

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Substitution within industries depends on $\bar{\sigma}^N$, a weighted average of the industry elasticities of substitution defined in Proposition 1. $\bar{\zeta}^N$ is similarly a weighted average of industry level elasticities of substitution between materials and non-materials (we relegate the expression ζ_n^N to Appendix A). χ^{agg} is the cross-industry heterogeneity index and is proportional to the cost-weighted variance of industry capital shares.

3 US Aggregate Elasticity of Substitution

The methodology developed in the previous section shows how to recover the aggregate capital-labor elasticity from micro parameters and the distribution of plant expenditures. We now use plant-level data on US manufacturing plants to estimate all of the micro components of the aggregate elasticity. We then assemble these components to estimate the aggregate capital-labor elasticity of substitution for the US manufacturing sector and examine its behavior over time.

3.1 Data

The two main sources of micro data on manufacturing plants are the US Census of Manufactures and Annual Survey of Manufactures (ASM). The Census of Manufactures is a census of all manufacturing plants conducted every five years. It contains more than 180,000 plants per year.[11] The Annual Survey of Manufactures tracks about 50,000 plants over five year panel rotations between Census years, and includes the largest plants with certainty.

In this study, we primarily use factor shares measured at the plant level. For the Census samples, we measure capital by the end year book value of capital, deflated using an industry specific current cost to historic cost deflator. The ASM has the capital and investment history required to construct perpetual inventory measures of capital. Thus, we create perpetual inventory measures of capital, accounting for retirement data from 1973 to 1987 as in Caballero et al. (1995) and using NIPA investment deflators for each industry-capital type. Capital costs consist of the total stock of structures and equipment capital multiplied

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[11]This excludes small Administrative Record plants with fewer than five employees, for whom the Census only tracks payroll and employment. We omit these in line with the rest of the literature using manufacturing Census data.

by the appropriate rental rate, using a Jorgensonian user cost of capital based upon an external real rate of return of 3.5 percent as in Harper et al. (1989). For labor costs, both surveys contain total salaries and wages at the plant level, but the ASM subsample includes data on supplemental labor costs including benefits as well as payroll and other taxes. For details about data construction, see Web Appendix C.

The Census of Manufactures, unlike the ASM subsample, only contains capital data beginning in 1987. Further, industry definitions change from SIC to NAICS in 1997. Given all of these considerations, we take the following approach to estimating the aggregate elasticity.

We use the full 1987 Census of Manufactures to estimate the micro elasticities and examine their robustness. We then use the ASM in each year for the relevant information on the composition of plants – the heterogeneity indices and materials shares – because we extend the analysis from 1972 to 2007. So, for example, to compute the aggregate elasticity of substitution in 1977, we combine estimates of micro parameters from the 1987 Census of Manufactures with information from the 1977 ASM.

Throughout, we use a plant's total cost of labor as a measure of its labor input. We view employees of different skill as supplying different quantities of efficiency units of labor, so using the wage bill controls for differences in skill. In Section 4.4 we show that our methodology is valid even if wages per efficiency unit of labor vary across plants.

3.2 Micro Heterogeneity

The extent of heterogeneity in capital intensities, as measured by the heterogeneity index, determines the relative importance of within-plant substitution and reallocation. Figure 1a depicts these indices for each industry in 1987. Across industries, the indices average 0.1 and are all less than 0.2. Similarly, Figure 1b shows how the average heterogeneity index evolves over time. While heterogeneity indices are rising, they remain relatively small. Industry capital shares exhibit even less variation; the cross-industry heterogeneity index χ^{agg} is 0.05.[12]

Given this level of heterogeneity, the plant-level elasticity of substitution between capital

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[12]It may seem surprising that these heterogeneity indices are so small. Note, however, that the numerator of the heterogeneity index is a *cost-weighted* variance of capital shares, which would likely be smaller than an unweighted variance. In addition, since capital shares are less than one, their variance is smaller than their standard deviation.
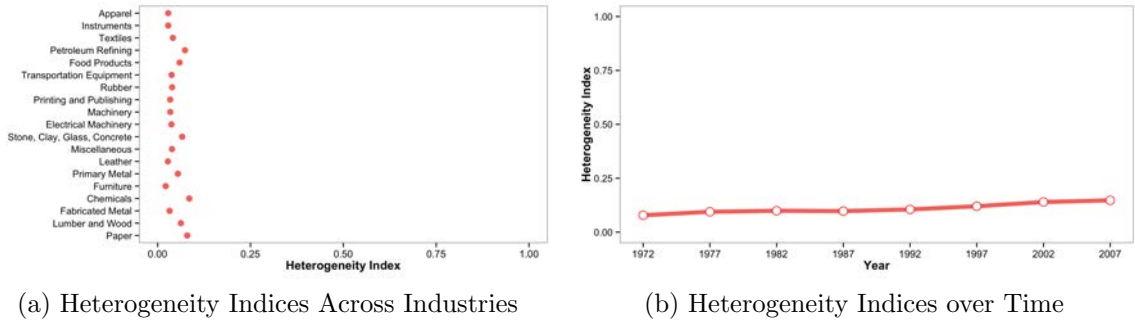
(a) Heterogeneity Indices Across Industries   (b) Heterogeneity Indices over Time

Figure 1 Heterogeneity Indices

Note: The left figure displays the heterogeneity index, χ_n, in 1987 for each industry. The right figure displays the average heterogeneity index over time.

and labor is a primary determinant of the aggregate elasticity. Therefore, we begin with a thorough investigation of this elasticity.

3.3   Plant Level Elasticity of Substitution

We obtain the plant-level elasticity of substitution from Raval (2014). We describe the methodology and estimates in detail in order to explain how we map the theory to the data, and then compare these estimates to others from the literature.

Given cost minimization, the relationship between relative expenditures on capital and labor rK_{ni}/wL_{ni} and relative factor prices w/r identifies this elasticity. We exploit wage differences across local areas in the US in order to identify the micro elasticity of substitution between capital and labor. Because these wage differences are persistent, with a correlation between 1990 and 2000 of 0.90, they identify plants' long-run response to a permanent change in factor prices. We run the regression:

$$\log \frac{rK_{ni}}{wL_{ni}} = (\sigma_n - 1) \log w_{ni}^{MSA} + CONTROLS + \epsilon_{ni}$$

where w_{ni}^{MSA} is the wage for the MSA in which the plant is located. The regression only uses plants in a single year; the implicit assumption is that capital is mobile so all plants face the same cost of capital. The local wage should reflect the cost of an efficiency unit of labor in an MSA, the wages are estimated controlling for observable measures of skill. To obtain the

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MSA wages, we first use data from the Population Censuses to estimate a residual wage for each person after controlling for education, experience, and demographics. We then average this residual within the MSA.[13] All regressions control for industry fixed effects, as well as plant age and multi-unit status.

This specification has several attractive properties. First, the dependent variable uses a plant's wage bill rather than a count of employees. If employees supply efficiency units of labor, using the wage bill automatically controls for differences in skill across workers and, more importantly, across plants. Second, plants may find it costly to adjust capital or labor. Deviations of a plant's capital or labor from static cost minimization due to adjustment costs would be in the residual, but should be orthogonal to the MSA wage. Third, the MSA wage and plant wage bills are calculated using different data sources, so we avoid division bias from measurement error in the wage in the dependent and independent variables.

Using all manufacturing plants, Raval (2014) estimates a plant level elasticity of substitution close to one-half in both 1987 and 1997. In this paper, we allow plant elasticities of substitution to vary by two digit SIC industry and so run separate regressions for each industry. Figure 2 displays the estimates by industry along with 95 percent confidence intervals.[14] Most of the estimates range between 0.4 and 0.7. The remainder of this section addresses a number of potential issues with our plant level elasticity estimates.

Endogeneity

We estimated the plant level elasticity of substitution using cross-sectional wage differences across US locations. A natural question is whether these wage differences are exogenous to non-neutral productivity differences. For example, to the extent that higher MSA wages were caused by higher labor-augmenting productivity or unobserved skills, our estimate of the elasticity of substitution would be biased towards one as $\sigma - 1$ would be attenuated.

To address such endogeneity problems, we use a version of Bartik (1991)'s instrument for labor demand, which is based on the premise that MSAs differ in their industrial composition. When an industry expands nationwide, MSAs more heavily exposed to that industry experience larger increases in labor demand. Thus given each MSA's initial industrial com-

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[13]For details about how we construct this wage, see Web Appendix C.2.
[14]We list these estimates in Web Appendix D.1.

Figure 2 Plant Elasticity of Substitution by Industry, 1987
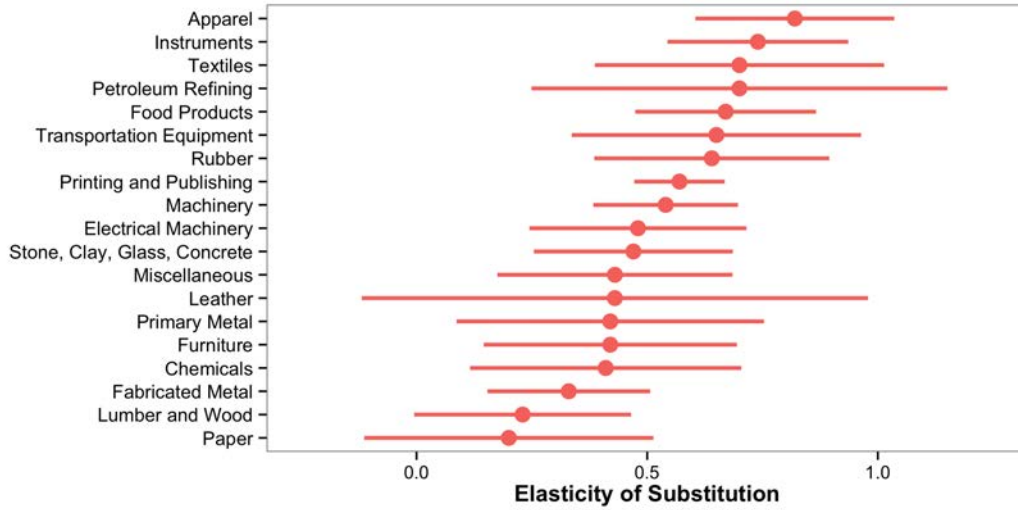
Note: For each industry, this graph plots the plant level elasticity of substitution between capital and labor as estimated in Raval (2014), together with the 95 percent confidence interval for each estimate. Standard errors are clustered at the MSA-industry level.

position, we can construct the change in each MSA's labor demand due to the change in each industry's nationwide employment. We restrict the instrument to non-manufacturing industries only.[15]

Table I contains estimates of the elasticity of substitution using these instruments in the third column.[16] The first two columns contain our baseline least squares estimates. The first column contains the average plant-level elasticity when we estimate separate elasticities for each two-digit SIC industry.[17] The second column estimates a common elasticity across all

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[15]Formally, the instrument is constructed as follows: Let $g_n(t) = \frac{1}{10}\ln(L_n(t)/L_n(t-10))$ be the national growth rate of industry n, and let $\omega_{j,n}(t)$ be the share of MSA j's employment working in industry n. The instrument is the interaction between initial MSA employment shares and 10 year national employment growth rates: $Z_j(t) = \sum_{n \in N^S} \omega_{j,n}(t-10)g_n(t)$, where N^S is the set of non-manufacturing four-digit SIC industries.

[16]Although all other specifications use wages from data on workers from the Population Censuses, here we use wages based on data on average payroll to employment across all establishments in an MSA from the Longitudinal Business Database. The Population Censuses are only conducted every 10 years in different years from the Economic Censuses, so the wages from the Population Censuses do not match the year of the Economic Census. For most of our specifications, this mismatch is not a problem because our wage variation is highly persistent. This mismatch becomes a problem if we want to use short run variation in wages from labor demand shocks. While the wages from establishment data do not control for differences in individual worker characteristics, the labor demand instrument should be orthogonal to the measurement error in wages. Because the SIC industry definitions changed from 1972 SIC basis to 1987 SIC basis in 1987, for 1987 we use the 1976-1986 instrument.

[17]This average, along with other averages across industries, is a weighted average where the weight on industry n is $\frac{\alpha_n(1-\alpha_n)\theta_n}{\sum_{n \in N} \alpha_n(1-\alpha_n)\theta_n}$ as in Proposition 2.

industries in manufacturing. In both years, the IV estimates are close to one-half, ranging from 0.49 to 0.52, and are similar to the baseline least squares estimates.

Table I Robustness Checks for Plant Capital-Labor Substitution Elasticity

	(1) Separate OLS	(2) Single OLS	(3) Bartik Instrument	(4) Equipment Capital	(5) Firm FE	(6) State FE
1987	0.52	0.52 *(0.04)*	0.49 *(0.05)*	0.53 *(0.03)*	0.49 *(0.05)*	0.48 *(0.05)*
1997	0.52	0.46 *(0.03)*	0.52 *(0.08)*		0.48 *(0.08)*	0.46 *(0.04)*

Note: Standard errors are in parentheses. The table contains five specifications. The first specification estimates a separate plant elasticity of substitution for each industry, and then averages them using the cross industry weights used for aggregation. The second specification estimates a single common elasticity of substitution for the entire manufacturing sector. The third specification estimates the elasticity of substitution in an IV framework using Bartik labor demand instruments. The fourth specification estimates the plant elasticity of substitution through least squares, defining capital to only include equipment capital. The fifth specification estimates the plant elasticity of substitution through least squares but includes firm fixed effects, excluding all single plant firms. The sixth specification estimates the plant elasticity of substitution through least squares but includes state fixed effects.

All regressions include industry fixed effects, age fixed effects, and a multi-unit status indicator. Wages used are the average log wage for the MSA. In the IV specification, the wage is computed as payroll/number of employees at the establishment level; in all other cases, the wage is computed as wage and salary income over total number of hours worked adjusted for differences in worker characteristics. Standard errors are clustered at the industry-MSA level.

Rental Rate of Capital

Our estimate of the micro elasticity of substitution would be biased if rental rates vary systematically with local wages. Three reasons might cause these rental rates to vary with wages. First, the cost of some kinds of capital, such as structures, may reflect local wages. To examine this, we estimate the elasticity of substitution between labor and equipment capital, which is more plausibly mobile across locations.[18] Second, the cost of capital could vary through differences in lending rates from banks in different locations, or from differences in firm creditworthiness or access to capital markets. To control for these differences, we add firm fixed effects for plants that belong to multi-unit firms. Third, individual states could have different capital taxes or investment subsidies; we control for these through state fixed effects. Table I reports the estimates from these specifications in columns (4)-(6); in all cases, the estimates are quite close to those from the baseline specification.

Extensive Margin

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[18]Because the Census does not separate equipment and structures capital in 1997, we only estimate this specification for 1987.

In the baseline model presented in Section 2, plants could substitute across inputs and change size. However, the set of existing plants remained fixed, so there was no extensive margin of adjustment. Would a higher wage cause entering firms to choose more capital intensive technologies? If so, the aggregate elasticity of substitution should account for that shift.

Here we provide an example of a putty-clay model in which there is adjustment along both the intensive and extensive margins. We will show that while we must interpret our estimated micro elasticity of substitution differently, our estimate of the macro elasticity is unchanged.

Consider an environment in which plant i has some core characteristics $\{\mathcal{A}_i, \mathcal{B}_i\}$. Upon entering, it can choose a technology $\{A_i, B_i\}$ from the menu

$$\left[(A_i/\mathcal{A}_i)^{1-\sigma^{ext}} + (B_i/\mathcal{B}_i)^{1-\sigma^{ext}} \right]^{\frac{1}{1-\sigma^{ext}}} \leq 1$$

Once it has selected its technology, it cannot change to an alternative technology. After entering, it produces using the production function

$$Y_i = \left[(A_i K_i)^{\frac{\sigma^{int}-1}{\sigma^{int}}} + (B_i L_i)^{\frac{\sigma^{int}-1}{\sigma^{int}}} \right]^{\frac{\sigma^{int}}{\sigma^{int}-1}} \tag{12}$$

σ^{int} thus represents a short run elasticity. Once the plant has entered, it cannot switch $\{A_i, B_i\}$, so σ^{int} is the response of i's capital-labor ratio to relative factor prices:

$$\frac{K_i}{L_i} = \left(\frac{A_i}{B_i} \right)^{\sigma^{int}-1} \left(\frac{r}{w} \right)^{-\sigma^{int}}$$

A shift in factor prices would also alter entering plants' choices of technologies. Given factor prices, i's choice of technologies will satisfy $\left(\frac{A_i K_i}{B_i L_i} \right)^{\frac{\sigma^{int}-1}{\sigma^{int}}} = \left(\frac{A_i/\mathcal{A}_i}{B_i/\mathcal{B}_i} \right)^{\sigma^{ext}-1}$. Along with i's choice of capital and labor, this implies that, after entry, the entering plant's capital-labor ratio will be

$$\frac{K_i}{L_i} = \left(\frac{\mathcal{A}_i}{\mathcal{B}_i} \right)^{\sigma^{total}-1} \left(\frac{r}{w} \right)^{-\sigma^{total}}$$

where σ^{total} is defined to satisfy

$$\frac{1}{\sigma^{total} - 1} = \frac{1}{\sigma^{int} - 1} + \frac{1}{\sigma^{ext} - 1}$$

σ^{total} represents a long run elasticity of substitution. If, for example, the wage was high and remained high, all entering plants would choose more capital intensive technologies, and σ^{total} would capture the resulting shifts in capital-labor ratios.

If the true model contains both an intensive and extensive margin, then how should we interpret our estimates of the micro elasticity? Our estimation strategy uses cross-sectional differences; we compare capital-labor ratios across locations with different wages. These differences in capital-labor ratios come from some combination of the intensive and extensive margins. Because geographic wage differences are extremely persistent, our estimated micro elasticity corresponds to σ^{total}; in high-wage locations, past entering cohorts would have selected technologies that reflected the higher cost of labor.

With our methodology we are unable to distinguish between the intensive and extensive margins of adjustment. Fortunately, doing so is not necessary to build up to a long-run aggregate elasticity.[19]

Sorting

Our estimates do not account for the possibility that plants select locations in response to factor prices. To see why this might matter, consider the following extreme example: Suppose plants cannot adjust their factor usage but can move freely. Then we would expect to find more labor intensive plants in locations with lower wages. A national increase in the wage would not, however, change any plant's factor usage. Thus, to the extent that this channel is important, our estimated elasticity will overstate the true elasticity.

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[19]Houthakker (1955) also features an extensive margin. We do not have a general result for that type of environment, but we can show that under his assumptions our methodology would deliver an estimate of the aggregate elasticity of substitution of one. The argument that our estimated micro elasticity captures both the intensive and extensive margins of adjustment is the same, but the mapping from Houthakker's model to our parameter estimates is more opaque. In that model, even though individual plants have Leontief production functions, one can show that the equilibrium distribution of capital shares (α_i) in the cross section is independent of factor prices. Thus we would estimate a unit plant-level elasticity of substitution. Houthakker also assumed that each plant's capacity was constrained by a fixed factor. This, in combination with his distributional assumptions, implies that the average revenue-cost ratio in that model is infinite. Thus we would estimate a unit demand elasticity (see Section 3.4). Thus, for any heterogeneity index, our estimation strategy would yield a Cobb-Douglas aggregate production function.

Plants ability to sort across locations likely varies by industry. We would expect industries in which plants are more mobile to be more clustered in particular areas. This could depend, for example, on how easily goods can be shipped to other locations.

Raval (2014) addressed this by looking at a set of ten large four-digit industries located in almost all MSAs and states. These are industries for which we would expect sorting across locations to be least important. The leading example of this is ready-mixed concrete; because concrete cannot be shipped very far, concrete plants exist in every locality. Elasticities for these industries are similar to the estimates for all industries in our baseline, with average elasticities of 0.49 for 1987 and 0.61 for 1997.

Alternative Estimates

Our estimation strategy uses cross-sectional variation to estimate a long-run plant-level elasticity. Chirinko (2008) surveyed the existing literature estimating short- and long-run elasticities of substitution at various levels of aggregation. This literature typically uses variation in the user cost of capital over time stemming from changes in tax-laws or the price of capital that differentially affect asset types.

Chirinko et al. (2011) and Barnes et al. (2008) provided estimates that are conceptually closest to ours, as they used long-run movements in the user cost of capital to identify the long-run micro elasticity for US and UK public firms respectively. Their approach estimates the elasticity using the capital first order condition and allows for biased technical change at the industry level. Because they use variation within firms over time, their estimates are not biased by sorting across locations. In addition, because their estimated elasticities contain only the intensive margin of adjustment, we would expect them to be slightly lower than ours. Each estimated a micro elasticity of substitution of 0.4. We find it comforting that two approaches that use very different sources of variation yield similar estimates.

3.4   Aggregation

We now estimate the remaining plant-level production and demand parameters. We then use them to aggregate to the manufacturing-level elasticity of substitution.

The reallocation effect depends upon both the plant elasticity of substitution between materials and non-materials, ζ, and the elasticity of demand, ε.

To identify ζ, we use the same cross-area variation in the wage. Across MSAs, the local wage varies but the prices of capital and materials are fixed. Cost minimization implies that ζ measures the response of relative expenditures of materials and non-materials to changes in their respective prices: $1 - \zeta = \frac{d \ln[(rK_{ni}+wL_{ni})/qM_{ni}]}{d \ln(\lambda_{ni}/q)}$, where λ_{ni} is the cost index of i's capital-labor bundle. Holding fixed the prices of materials and capital, a change in the local wage would increase these relative prices by $d \ln \lambda_{ni}/q = (1 - \alpha_{ni}) d \ln w_{ni}^{MSA}$. To a first-order approximation, the response of $(rK_{ni} + wL_{ni})/qM_{ni}$ to the local wage would be $(1 - \zeta)(1 - \alpha_{ni})$. We therefore estimate ζ using the regression

$$\log \frac{rK_{ni} + wL_{ni}}{qM_{ni}} = (1 - \zeta)(1 - \alpha_{ni})(\log w_{ni}^{MSA}) + CONTROLS + \epsilon_{ni}$$

Table II contains these estimates for 1987 and 1997. Because we use the full Census for each estimate, our estimate of ζ is common across industries. The first column contains our baseline estimate. This implicitly assumes a national market for materials. The second column adjusts the regression to account for the fact that some materials are sourced locally; an increase in the local wage would raise the cost of such locally sourced materials. See Web Appendix D.3 for details. These estimates are a bit lower than one; in our subsequent calculations, we use the 1987 MSA level estimate of 0.90.[20]

Table II Elasticities of Substitution between Materials and Non-Materials for the Manufacturing Sector

	No Local Content	Local Content
1987	0.90 *(0.06)*	0.87 *(0.07)*
1997	0.67 *(0.04)*	0.63 *(0.05)*
N	$\approx 140,000$	

Note: Standard errors are in parentheses. All regressions include industry fixed effects, age fixed effects and a multi-unit status indicator, and have standard errors clustered at the two digit industry-MSA level. Specifications with local content are as described in Web Appendix D.3.

Within industries, the demand elasticity tells us how much consumers substitute across plants when relative prices change. We estimate the elasticity of demand using the impli-

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[20] Atalay (2014) pursued a complementary approach using differences in materials prices across plants in the US Census of Manufactures and finds an estimate of 0.65, within the range of Table II. See Appendix F of his paper. This differs from his main estimate of this elasticity which uses shorter-run industry-level variation and includes non-manufacturing industries, and so may not reflect the long-run, plant-level elasticity for manufacturing required for our model.

cations of profit maximization; optimal price setting behavior implies that the markup over marginal cost is equal to $\frac{\varepsilon}{\varepsilon-1}$. Thus, we invert the average markup across plants in an industry to obtain the elasticity of demand. The assumption of constant returns to scale implies that each plant's markup is equal to its ratio of revenue to cost.[21] Figure 3 displays the elasticities of demand for each manufacturing industry in 1987.[22] Across industries, elasticities of demand vary between three and five.

The overall scale elasticity is $\bar{s}_n^M \zeta + (1 - \bar{s}_n^M)\varepsilon_n$. $\bar{s}_n^M$ is an average of materials shares, which are high in manufacturing; the average across industries in 1987 is 0.54. Figure 3 contains our estimates of the scale elasticities; they average 2.26 across industries.

Figure 3 Elasticity of Demand and Scale Elasticity by Industry, 1987
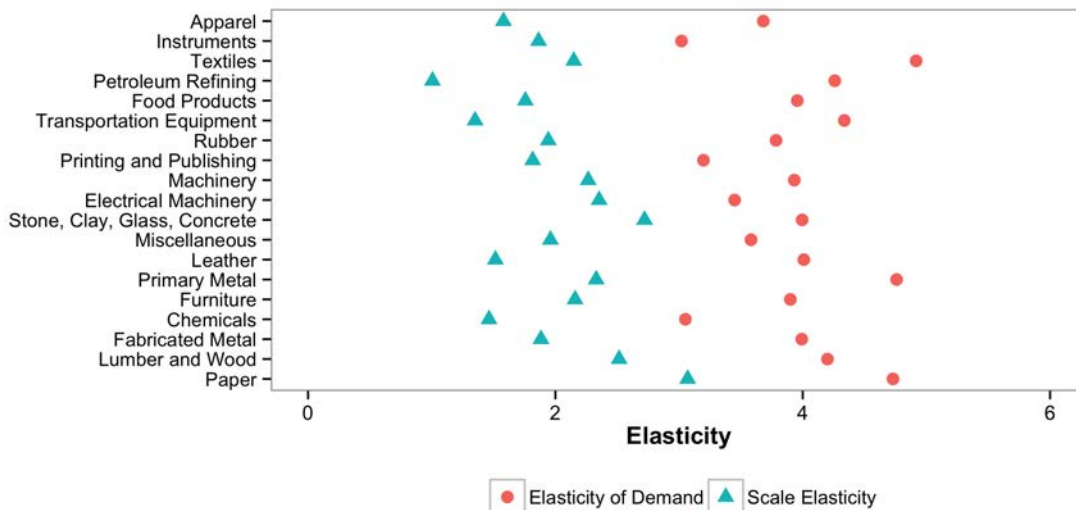

Note: For each industry, this graph plots both the elasticity of demand, estimated from revenue-cost ratios, and the scale elasticity $\bar{s}_n^M \zeta + (1 - \bar{s}_n^M)\varepsilon_n$.

To aggregate across industries, we need one more elasticity, the cross industry elasticity of demand η. We estimate this elasticity using industry-level panel data by regressing quantity on price, using average cost as an instrument for supply. Web Appendix D.4 contains the details of this analysis. Across specifications, we find estimates centered around one. We thus set η to one. As we would expect, the cross-industry demand elasticity η is much lower than the plant-level demand elasticities; varieties in the same industries are better

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[21]We relax the assumption of constant returns to scale in Section 4.3, and explore alternative estimation strategies for the demand elasticity in Section 4.1.

[22]We list these estimates in Web Appendix D.2.

(a) Plant and Industry Elasticities

(b) Aggregate Elasticity Across Time
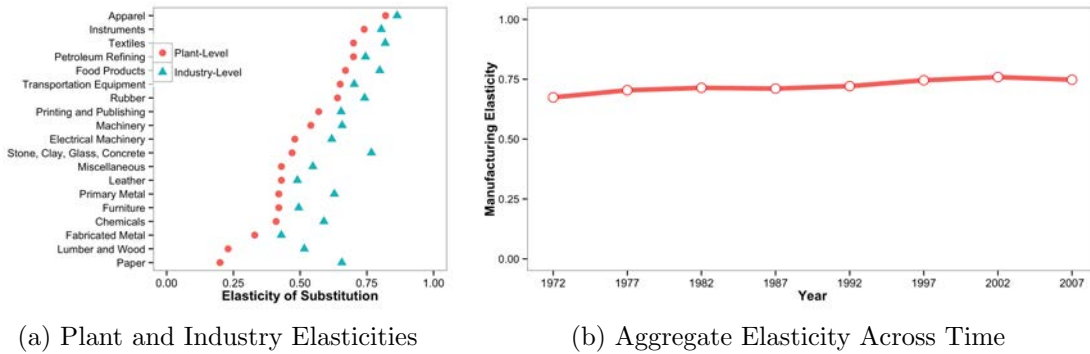
Figure 4 Elasticities of Substitution and Aggregation

Note: The left figure displays the plant level elasticity and industry level elasticity of substitution for each industry. The right figure displays the manufacturing level elasticity of substitution for each Census year from 1972-2007.

substitutes than varieties in other industries.

We can now combine the substitution and reallocation effects to estimate the industry and manufacturing sector level elasticities of substitution. In Figure 4a, we depict the plant level and industry level elasticities of substitution. Because the heterogeneity indices tend to be small, the industry-level elasticities of substitution are only moderately higher than the plant-level elasticities. The average industry elasticity is 0.69 and the overall manufacturing level elasticity of substitution is 0.71.[23] Within-plant substitution accounts for 69 percent of industry substitution and 63 percent of overall substitution for manufacturing.

The manufacturing sector has evolved over time. Our methodology allows us to examine how much changes in the composition of plants caused the aggregate elasticity of substitution to vary over time.[24] Figure 4b depicts the aggregate elasticity of substitution from 1972 to 2007. The aggregate elasticity has risen slightly from 0.67 to 0.75. In Web Appendix D.5, we examine the robustness of this approach by using estimates of the micro production and demand elasticities based on the 1997 Census of Manufactures. The resulting time path of aggregate elasticities is virtually identical to those that use the 1987 micro estimates. This result provides support for models that assume a stable elasticity of substitution over time.

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[23]For our methodology, uncertainty about specification is likely more important than statistical imprecision. That being said, the standard error of our estimate of the aggregate capital-labor elasticity is 0.03. We arrive at this by using the standard errors from our micro level elasticity estimates.

[24]After 1997, industry definitions switch from two digit SIC basis to three digit NAICS basis. We assign SIC plant elasticities to the equivalent NAICS industries.

4   Robustness

In this section, we examine the robustness and sensitivity of our estimates to alternative modeling assumptions. We first examine our estimates of the micro elasticities of demand. We also discuss measurement error and changes in the specification of the economic environment, including imperfect pass-through, returns to scale, adjustment costs, and other misallocation frictions.

4.1   Elasticity of Demand

For several homogeneous products, the US Census of Manufactures collects both price and physical quantity data. We can thus estimate the elasticity of demand by regressing quantity on price, instrumenting for price using average cost. This approach is similar to Foster et al. (2008). This strategy implies an average industry-level capital-labor elasticity of substitution of 0.52 among these industries, close to the estimate of 0.54 using our baseline strategy.[25] The trade literature finds estimates in the same range using within industry variation across imported varieties to identify the elasticity of demand. For example, Imbs and Mejean (2011) find a median elasticity of 4.1 across manufacturing industries.

Web Appendix B.5 generalizes the analysis to a homothetic demand system with arbitrary demand elasticities and imperfect pass-through of marginal cost. In that environment, the formula for the industry elasticity in Proposition 1 is unchanged except the elasticity of demand ε_n is replaced by a weighted average of the quantity $b_{ni}\varepsilon_{ni}$; ε_{ni} is i's local demand elasticity and b_{ni} is i's local rate of relative pass-through (the elasticity of its price to a change in marginal cost). Note that under Dixit-Stiglitz preferences, $b_{ni} = 1$ and $\varepsilon_{ni} = \varepsilon_n$ for each i. Here, however, if a plant passes through only three-quarters of a marginal cost increase, then the subsequent change in scale would be three-quarters as large. Given a pass-through rate of three-quarters, our estimate of the 1987 aggregate elasticity of substitution would be

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[25]Foster et al. (2008) instrument for price using plant-level TFP. We cannot use their estimates directly because they assume plants produce using homogeneous Cobb-Douglas production functions. Because we maintain the assumption of constant returns to scale, the appropriate analogue to plant-level TFP is average cost. Directly using the demand elasticities of Foster et al. (2008) would yield an average industry-level elasticity of substitution of 0.54.

0.67.

4.2 Non-CES Production Functions

In our baseline model, we assumed that within an industry, each plant produced using a nested CES production function with common elasticities. Here we relax the strong functional form assumption. While we maintain that each plant's production function has constant returns to scale, we impose no further structure. In particular, we do not assume substitution elasticities are constant, that inputs are separable, or that technological differences across plants are factor augmenting. Rather, we relate the industry-level elasticity of substitution to local elasticities of individual plants.

For plant i, we define the local elasticities σ_{ni} and ζ_{ni} to satisfy $\sigma_{ni} - 1 \equiv \frac{d \ln r K_{ni}/w L_{ni}}{d \ln w/r}$ and $\zeta_{ni} - 1 \equiv \frac{1}{\alpha^M - \alpha_{ni}} \frac{d \ln \frac{q M_{ni}}{r K_{ni} + w L_{ni}}}{d \ln w/r}$.[26] In Appendix A we derive an expression for the industry elasticity of substitution in this environment. The resulting expression is identical to Proposition 1 except the plant elasticities of substitution are replaced with weighted averages of the plants' local elasticities, $\bar{\sigma}_n \equiv \sum_{i \in I_n} \frac{\alpha_{ni}(1 - \alpha_{ni})\theta_{ni}}{\sum_{i' \in I_n} \alpha_{ni'}(1 - \alpha_{ni'})\theta_{ni'}} \sigma_{ni}$ and $\bar{\zeta}_n \equiv \sum_{i \in I_n} \frac{(\alpha_{ni} - \alpha_n)(\alpha_{ni} - \alpha^M)s_{ni}^M}{\sum_{i' \in I_n}(\alpha_{ni'} - \alpha_n)(\alpha_{ni'} - \alpha^M)s_{i'}^M} \zeta_{ni}$:

$$\sigma_n^N = (1 - \chi_n)\bar{\sigma}_n + \chi_n \left[(1 - \bar{s}_n)\varepsilon_n + \bar{s}_n^M \bar{\zeta}_n \right]$$

4.3 Returns to Scale

Our baseline estimation assumed that each plant produced using a production function with constant returns to scale. Alternatively, we can assume that plant i produces using the production function:

$$Y_{ni} = F_{ni}(K_{ni}, L_{ni}, M_{ni}) = G_{ni}(K_{ni}, L_{ni}, M_{ni})^\gamma$$

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[26]If i's production function takes a nested CES form as in Assumption 1, σ_{ni} and ζ_{ni} would equal σ_n and ζ_n respectively. The definition of σ_{ni} is straightforward but ζ_{ni} is more subtle; see Appendix A for details. Here, however, these elasticities are not parameters of a production function. Instead, they are defined locally in terms of derivatives of i's production function evaluated at the cost-minimizing input bundle. Exact expressions for σ_{ni} and ζ_{ni} are given in Web Appendix B.1.

where G_{ni} has constant returns to scale and $\gamma < \frac{\varepsilon_n}{\varepsilon_n - 1}$. Relative to the baseline, two things change, as shown in Web Appendix B.3. First, the industry elasticity of substitution becomes

$$\sigma_n^N = (1 - \chi_n)\bar{\sigma}_n + \chi_n \left[\bar{s}_n^M \bar{\zeta}_n + (1 - \bar{s}_n^M) x_n \right]$$

where x_n is defined to satisfy $\frac{x_n}{x_n - 1} = \frac{1}{\gamma} \frac{\varepsilon_n}{\varepsilon_n - 1}$. Thus the scale elasticity is a composite of two parameters, the elasticity of demand and the returns to scale. When the wage falls, the amount a labor-intensive plant would expand depends on both.

Second, when we divide a plant's revenue by total cost, we no longer recover the markup. Instead, we get

$$\frac{P_{ni} Y_{ni}}{r K_{ni} + w L_{ni} + q M_{ni}} = \frac{1}{\gamma} \frac{\varepsilon_n}{\varepsilon_n - 1} = \frac{x_n}{x_n - 1}$$

Fortunately, this means that the procedure we used in the baseline delivers the correct aggregate elasticity of substitution even if we mis-specify the returns to scale. To see this, when we assumed constant returns to scale, we found the elasticity of demand by computing $\frac{\frac{P_{ni} Y_{ni}}{r K_{ni} + w L_{ni} + q M_{ni}}}{\frac{P_{ni} Y_{ni}}{r K_{ni} + w L_{ni} + q M_{ni}} - 1}$. With alternative returns to scale, this would no longer give the elasticity of demand, ε_n; rather, it gives the correct scale elasticity, x_n.[27]

4.4 Adjustment Costs and Distortions

Section 2 showed that the relative importance of within-plant substitution and reallocation depends upon the variation in cost shares of capital. Implicit in that environment was that this variation came from non-neutral differences in technology. On the other hand, some of this heterogeneity may be due to adjustment costs or other distortions as in the recent misallocation literature; see Banerjee and Duflo (2005), Restuccia and Rogerson (2008), Hsieh and Klenow (2009). A natural question arises: What are the implications for the aggregate elasticity of substitution if these differences come from distortions?

Consider an alternative environment in which plants pay idiosyncratic prices for their

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[27]This specification of returns to scale rules out some features such as fixed costs of production. In an environment in which production functions exhibited such features, x_n would be replaced by a weighted average of individual plants' local scale elasticities. For details see Web Appendix B.6, which also derives an aggregate elasticity when production functions are non-homothetic.

inputs.[28] We are interested in how changes in factor prices impact the relative compensation of capital and labor. To be precise, suppose plant i pays factor prices $r_{ni} = T_{Kni}r$ and $w_{ni} = T_{Lni}w$, and we define the industry elasticity of substitution in relation to the change in relative factor compensation in response to a change in w/r (holding fixed the idiosyncratic components of factor prices, $\{T_{Kni}, T_{Lni}\}_{i \in I_n}$) so that it satisfies[29]

$$\sigma_n^N - 1 = \frac{\mathrm{d}\ln\left(\sum_{i \in I_n} r_{ni} K_{ni} / \sum_{i \in I_n} w_{ni} L_{ni}\right)}{\mathrm{d}\ln w/r}$$

As shown in Web Appendix B.4, it turns out that the expression for the industry elasticity of substitution is exactly the same as in Proposition 1, provided that we define $\alpha_i = \frac{r_i K_i}{r_i K_i + w_i L_i}$ and $\theta_i = \frac{r_{ni} K_{ni} + w_{ni} L_{ni}}{\sum_{i' \in I_n} r_{ni'} K_{ni'} + w_{ni'} L_{ni'}}$. Thus, as long as expenditures are measured correctly, no modifications are necessary.

Alternatively, a plant's shadow value of an input may differ from its marginal expenditure on that input. This could happen if the input is fixed in the short run or if use of that input is constrained by something other than prices. In the presence of such "unpaid" wedges, our expression for the industry elasticity would change slightly.

If we observed these unpaid wedges, we could compute the industry elasticity. However, measuring these wedges presents a challenge. Differences in plants' cost shares of capital could reflect differences in unpaid wedges or differences in technology; data on revenue and input expenditures alone are not sufficient to distinguish between the two.

To get a sense of how big of an issue these unpaid wedges might represent, we consider the following thought experiment. Suppose all variation in cost shares of capital were due to "unpaid" wedges. In that case, the aggregate elasticity of substitution for the manufacturing sector in 1987 would be 0.71, the same as our baseline, while in 1997 the aggregate elasticity would be 0.91 rather than 0.75 in the baseline (see Web Appendix B.4 for details). This suggests that misallocation would not substantially alter our analysis.

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[28]In fact, our identification of the plant-level elasticity of substitution relies on plants (in different locations) facing different wages.

[29]In this environment the mapping between changes in the capital-labor ratio and changes in factor compensation is fuzzy; generically, $\frac{\mathrm{d}\ln K_n/L_n}{\mathrm{d}\ln w/r} - 1 \neq \frac{\mathrm{d}\ln \sum_{i \in I_n} r_{ni} K_{ni} / \sum_{i \in I_n} w_{ni} L_{ni}}{\mathrm{d}\ln w/r}$.

4.5 Measurement Error

The measured heterogeneity indices may be affected by measurement error. Two types of measurement error might play a role. First, if the costs of capital or labor are misreported or constructed incorrectly, we would likely overstate the heterogeneity indices as they would include both true heterogeneity and measurement error. In the extreme case in which all measured heterogeneity reflects mismeasurement, then the true aggregate elasticity would equal the plant-level elasticity. A second type of measurement error would work in the opposite direction. As described by White et al. (2012), data for some plants in the Census are imputed, reducing the measured dispersion of productivity. This means that our measured heterogeneity indices might understate the true heterogeneity in capital intensities.

A reason to think measurement error might not play a big role is that both types of measurement discussed above are much more important for small plants than for large plants, and the heterogeneity indices are weighted by expenditure.

5 The Decline of the Labor Share

Figure 5 depicts the labor share of income in the US for the manufacturing sector and for the aggregate economy in the post-war period.[30] The labor share for manufacturing has fallen since 1970, from about 0.73 to 0.55 in 2011. The steepest decline has been since 2000; the labor share fell from roughly 0.65 to 0.55 in one decade. The labor share has fallen for the overall economy as well, though not by as much, falling from 0.70 in 1970 to 0.62 in 2011.

Labor's share of income could change in response to factor prices or for other reasons which, through the lens of an aggregate production function, would be viewed as biased technical change. The aggregate elasticity determines the impact of changes in factor prices;

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[30]The labor share for manufacturing comes from the BLS Multifactor Productivity Series, while the aggregate labor share comes from Fernald (2012); both are based on data from the National Accounts. As Gomme and Rupert (2004), Krueger (1999), and Elsby et al. (2013) point out, the major issue with calculating the labor share is whether proprietors' income accrues to labor or capital. Both series assume that the share of labor for proprietors is the same as for corporations. For the manufacturing sector, proprietors' income represented 1.4 percent of income on average since 1970. Industry definitions change over this period from SIC basis to NAICS basis; for consistency, we include NAICS Publishing Industries in manufacturing because Publishing was included in manufacturing on a SIC basis.

Figure 5 Labor Share over Time
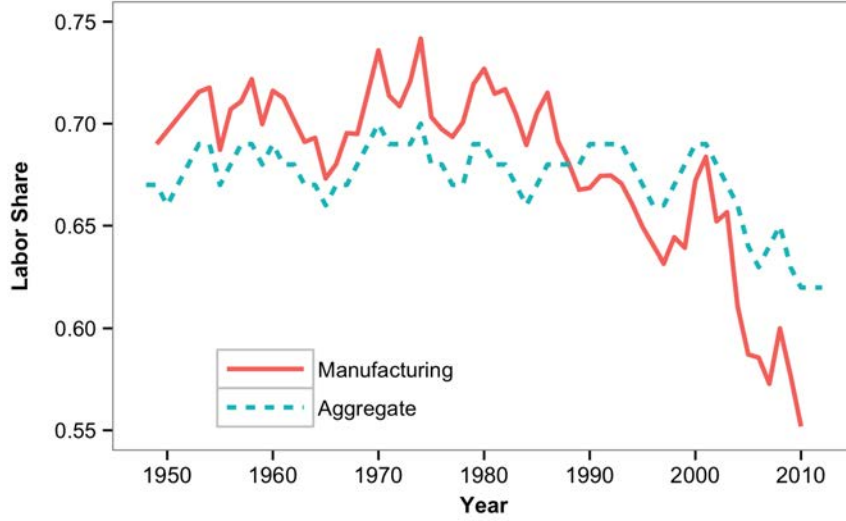
Note: The solid line is the labor share for manufacturing based on data from the BLS Multifactor Productivity series. The dashed line is the labor share for the overall economy from Fernald (2012).

we ascribe the residual to the bias of technical change. We now measure the bias of technical change, and examine how it has varied over time.[31]

Formally, let $s^{v,L}$ denote labor's share of value added. Then the change in the labor share can be decomposed into two terms

$$\mathrm{d}s^{v,L} = \frac{\partial s^{v,L}}{\partial \ln w/r} \mathrm{d}\ln w/r + \left(\mathrm{d}s^{v,L} - \frac{\partial s^{v,L}}{\partial \ln w/r} \mathrm{d}\ln w/r \right)$$

The first measures the contribution of changes in factor prices. The second measures a residual that we label the "bias of technical change".

To execute this decomposition, we need measures of factor prices. We base our rental prices on NIPA deflators for equipment and structures, and wages on average compensation per hour worked adjusted for changes in worker quality over time.[32] Our measures of value

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[31]See Web Appendix D.6 for the details of this calculation. Total income can be decomposed into payments to labor, payments to fixed capital, and a residual that we label "profit". Any return to intangible capital or entrepreneurship would be included in this category. Consistent with Section 2, we assume changes in factor prices do not alter the "profit" share of gross output. See Web Appendix D.7 for a comparison to alternative econometric methods using only aggregate data.

[32]For rental prices, we develop a Tornqvist index for the rental price to account for changing shares of two digit industries and different types of capital over time. For wages, we use BLS data on total compensation and hours for each industry, correcting for labor quality using indices from Jorgenson et al. (2013). Jorgenson et al. (2013) measures labor quality as the deviation of total hours from a Tornqvist index of hours across many different cells that represent workers with different amounts of human capital, as in Jorgenson et al.

added and input expenditures are based on NIPA data. Our estimates of expenditures on fixed capital combine NIPA data on equipment and structures capital with our rental prices.[33] Finally, we allow the aggregate elasticity to vary across time, linearly interpolating between the Census years in which we estimated the elasticity.

Table III displays the annualized change in the labor share and each of its components before and after the year 2000. This type of decomposition cannot provide a full explanation of the decline in the labor share. Nevertheless, any explanation should be consistent with the patterns that we depict.

We have several findings. First, an estimated aggregate elasticity less than one implies that two mechanisms that have been put forward – an acceleration in investment specific technical change as in Karabarbounis and Neiman (2014) or increased capital accumulation as in Piketty (2014) – would have raised rather than lowered labor's share of income. The declining cost of capital relative to labor raised the labor share by a total of 3.2 percentage points over the 1970-2010 period.[34] Second, the contribution of changes in factor prices has been relatively constant over the past 40 years, at 0.08 percentage points per year in the 1970-1999 period and 0.07 percentage points per year in the 2000-2010 period. This means that other explanations that work through factor prices, such as a decline in labor supplied by prime-aged males or changes in benefits, would have trouble matching the timing of the decline in the labor share.

Instead, there has been an acceleration in the bias of technical change over the past 40 years; the labor share has decreased about half a percentage point faster in the period since

(2005).

[33]The labor share from NIPA is based on firm level data, rather than production level data, and so would include non-manufacturing establishments of a manufacturing firm, such as a firm's headquarters. In 1987, the total wages and salaries from the production data (from the NBER-CES Productivity database) was 88.2 percent of the wages and salaries from NIPA. While our aggregate elasticities are estimated using production data, we apply these elasticities to labor shares from NIPA. This is our preferred estimate. See Web Appendix D.6.2 for details and alternative measures.

[34]Our rental prices are based upon official NIPA deflators for equipment and structures capital. However, Gordon (1990) has argued that the NIPA deflators underestimate the actual fall in equipment prices over time. In Web Appendix D.6.3 we use an alternative rental price series for equipment capital that Cummins and Violante (2002) developed by extending the work of Gordon (1990). Their series extends to 1999, so we compare our baseline to these rental prices during the 1970-1999 period. Using the Cummins and Violante (2002) equipment prices implies that the wage to rental price ratio has increased by 3.8 percent per year, instead of 2.0 percent per year with the NIPA deflators. This change increases the contribution of factor prices to the labor share between 1970 and 1999 by 0.05 percentage points per year, or about 1.9 percentage points over the period.

2000, and this is accounted for by the bias of technical change. The cumulative decrease in the labor share due to the bias is almost the same in the 1970-1999 and 2000-2010 periods at about 10.6 and 9.4 percentage points respectively. The bias may stem from different sources, including automation, IT investment, the decline of unions, or offshoring.

Table III Contributions to Labor Share Change

| | Annual Contribution | | | Cumulative Contribution | | |
Period	Labor Share	Factor Prices	Bias	Labor Share	Factor Prices	Bias
1970-1999	-0.27	0.08	-0.35	-8.12	2.50	-10.61
2000-2010	-0.79	0.07	-0.85	-8.64	0.71	-9.35

Note: The factor price and bias contributions are as defined in the text. Annual Contributions are in percentage points per year and Cumulative Contributions are in percentage points.

From 1970-2011, annual growth in real wages in the manufacturing sector was 1.9 percentage points lower than 1954-1969. How much of the decline in the labor share can be explained by the slowdown in wage growth? If wage growth had continued to grow at the same robust pace, how much higher would the labor share be?

We can use our estimated aggregate elasticity to assess this counterfactual.[35] We find that if wages grew faster, the manufacturing labor share in 2010 would be 0.58, compared to its actual value of 0.55. Thus the slowdown in wages can explain about one-sixth of the total decline.

5.1 Industry Decomposition

What drives the bias of technical change? One possibility is that industries have become more capital intensive. Alternatively, the composition of industries within manufacturing could have shifted. Changes in trade barriers or the cost of outsourcing, for example, could change the aggregate capital intensity by shifting industry composition.

Figure 6 shows that industries that were more labor intensive tended to shrink relative to capital intensive industries. Thus shifts in composition likely played a role in the labor

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[35]Note that this counterfactual involves extrapolating our local estimate of the aggregate elasticity of substitution.

Figure 6 Shift in Industry Composition
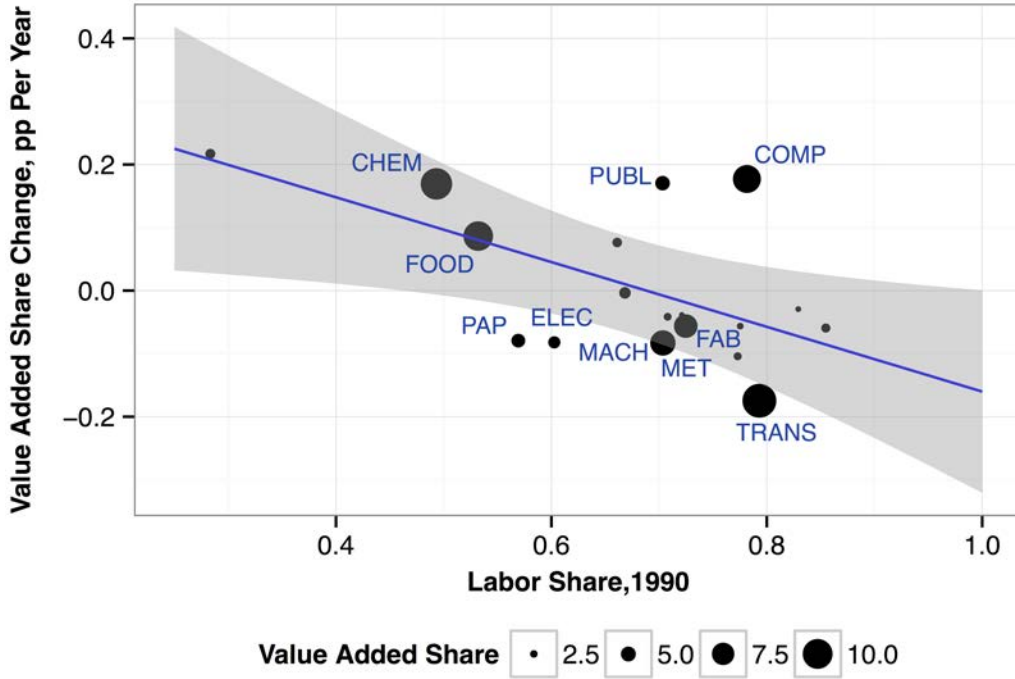
**Note:** The figure depicts each industry's change in value added share in between 1990 and 2010 and its labor share of income in 1990. The regression line depicts the linear regression of the change in value added share on the 1990 labor share, weighting each industry by its 1990 share of value added. All industries with a value added share above 4 percent are labeled; the NAICS industries represented are: CHEM Chemical Products, COMP Computer and Electronic Products, ELEC Electrical Equipment, Appliances, and Components, FAB Fabricated Metal Products, FOOD Food and Beverage and Tobacco Products, MACH Machinery, MET Primary Metals, PAP Paper Products, PUBL Publishing Industries, and TRANS Transportation Equipment.

share's decline.

We can go further and decompose the sources of aggregate bias between biased technical change within industries and compositional change across industries. Formally, let $s_n^{v,L}$ denote labor's share of industry n's value added, and let v_n denote industry n's share of total value added. Since $s^{v,L} = \sum_n v_n s_n^{v,L}$, the change in labor's share of income can be decomposed into within- and between-industry components

$$\mathrm{d}s^{v,L} = \sum_n v_n \mathrm{d}s_n^{v,L} + \sum_n \left(s_n^{v,L} - s^{v,L}\right) \mathrm{d}v_n$$

(a) Annual Change   (b) Cumulative Change
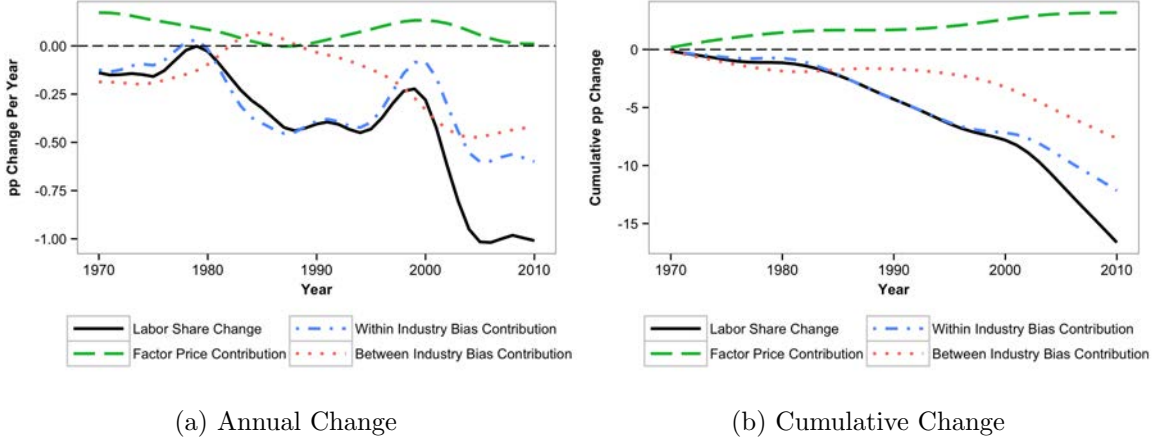
Figure 7 Change in Labor Share Components over Time

Note: All changes are in percentage points. Each contribution to the labor share change has been smoothed using a Hodrick-Prescott filter. The contributions from factor prices, within-industry bias, and between-industry bias are as defined in the text.

The change in the overall labor can thus be decomposed into three terms:

$$
\begin{aligned}
\mathrm{d}s^{v,L} &= \frac{\partial s^{v,L}}{\partial \ln w/r}\mathrm{d}\ln w/r + \sum_n v_n\left(\mathrm{d}s_n^{v,L} - \frac{\partial s_n^{v,L}}{\partial \ln w/r}\mathrm{d}\ln w/r\right) \\
&\quad + \sum_n \left(s_n^{v,L} - s^{v,L}\right)\left(\mathrm{d}v_n - \frac{\partial v_n}{\partial \ln w/r}\mathrm{d}\ln w/r\right)
\end{aligned}
\tag{13}
$$

The first term measures the contribution of factor price changes. The second term measures within-industry contributions to the bias of technical change. It is the weighted average of the changes in industries' labor shares dues to technical change. The third term measures the between-industry contribution to the bias of technical change. It is the covariance between industry growth due to technical change and its labor share of income. If the covariance is positive, labor-intensive industries have grown relative to capital-intensive industries, which raised labor's share of value added.

Figure 7 shows the contribution of the within-industry bias and between-industry bias as in equation (13). Figure 7a shows the annualized rate of change while Figure 7b shows the cumulative change.[36]

This decomposition points to the importance of both technical change within industries

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[36]We smooth each contribution by applying a Hodrick-Prescott filter; the change in the labor share is then the sum of all the components. We use a smoothing parameter of 60.

and compositional changes in the decline of the labor share of manufacturing. The within-industry bias contribution has been large since the 1980s; overall, it accounts for about 12.7 percentage points of the decline in the labor share, of which about 4.4 percentage points were in the 2000s. The between-industry bias was low for much of the sample period, but rose in the 2000s. Overall, it accounts for about 7.2 percentage points of the decline in the labor share, of which 4.9 percentage points were in the 2000s. The increase the importance of compositional change in the 2000s may point to multiple factors behind the decline in the labor share.

Finally, we can examine how changes within industries are related to changes across industries. Figure 8 plots each industry's annual change in value added against the annual change in its labor share. There was a clear shift in this relationship between the first and second halves of the sample period. After 1990, the industries that grew more were those with the largest decline in labor share; before 1990, the two were much less correlated. This pattern is inconsistent with a simple story in which the labor-intensive segment of an industry moves abroad; it is consistent with automation or a more complex offshoring story in which both labor intensive tasks move abroad and production using capital intensive tasks expands. The largest industries that have both expanded and become more capital intensive are Computer Equipment and Chemicals. These industries include R&D intensive companies such as Apple and Pfizer and may suggest the increasing importance of intangible capital.

6 Cross–Country Elasticities

How does the aggregate capital-labor elasticity vary across countries? Production technologies may differ across countries for a number of reasons. Researchers have generally found greater variation in capital intensity and productivity in less developed countries; see Hsieh and Klenow (2009), Bartlesman et al. (2013). This may stem from resource misallocation. Alternatively, if lower wages in less developed countries reduce adoption of new capital intensive technologies, as in Acemoglu and Zilibotti (2001), developing countries would operate a mix of old and new technologies. In our framework, greater microeconomic heterogeneity implies a higher aggregate capital-labor elasticity of substitution. We now examine how the

(a) 1970-1990	(b) 1990-2010
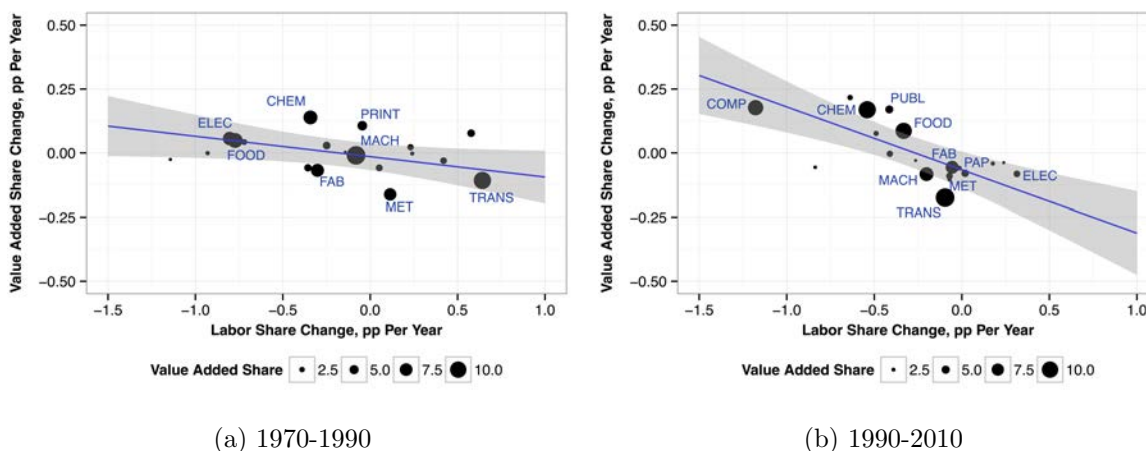
Figure 8 Changes in Value Added Shares and Changes in Labor Shares

Note: All changes are annualized and are in percentage points. Each regression line depicts the linear regression of the change in value added share on the change in labor share, weighting each industry by its share of value added in the initial year. All industries with a value added share above 4 percent are labeled. In the 1970-1990 figure, the SIC industries represented are: CHEM Chemical Products, ELEC Electronic and Other Electric Equipment, FAB Fabricated Metal Products, FOOD Food and Kindred Products, MACH Industrial Machinery and Equipment, MET Primary Metals, PRINT Printing and Publishing, and TRANS Transportation Equipment. In the 1990-2010 figure, the NAICS industries represented are: CHEM Chemical Products, COMP Computer and Electronic Products, ELEC Electrical Equipment, Appliances, and Components, FAB Fabricated Metal Products, FOOD Food and Beverage and Tobacco Products, MACH Machinery, MET Primary Metals, PAP Paper Products, PUBL Publishing Industries, and TRANS Transportation Equipment.

greater microeconomic heterogeneity found in less-developed countries affects the aggregate elasticity.

6.1 Estimates

We examine the aggregate elasticity for three less developed countries – Chile, Colombia, and India – using micro datasets on production for each country. Web Appendix C.7 contains the details of each dataset; on average, there are roughly 5,000 plants per year for Chile from 1986–1996, 7,000 for Colombia from 1981–1991, and 30,000 for India from 2000–2003. In computing the aggregate elasticity, we allow the composition of plants to change across countries but fix production and demand elasticities at their US 1987 values for matching industries. Figure 9 depicts these estimates; we find an average aggregate manufacturing elasticity of 0.84 in Chile, 0.84 in Colombia, and 1.11 in India, all of which are higher than the US 1987 value of 0.70.

33

Why are aggregate elasticities higher in the less developed countries? Figure 9 depicts the average plant level and industry level elasticities; most of the difference across countries is due to larger industry level elasticities. For India and Chile, greater heterogeneity in capital intensities accounts for roughly 70 percent of the difference in aggregate elasticities.[37] The higher elasticity for Colombia is due to a combination of multiple factors, including a different industrial composition, more heterogeneity, and lower materials shares.

Figure 9 Manufacturing Elasticity of Substitution Across Countries
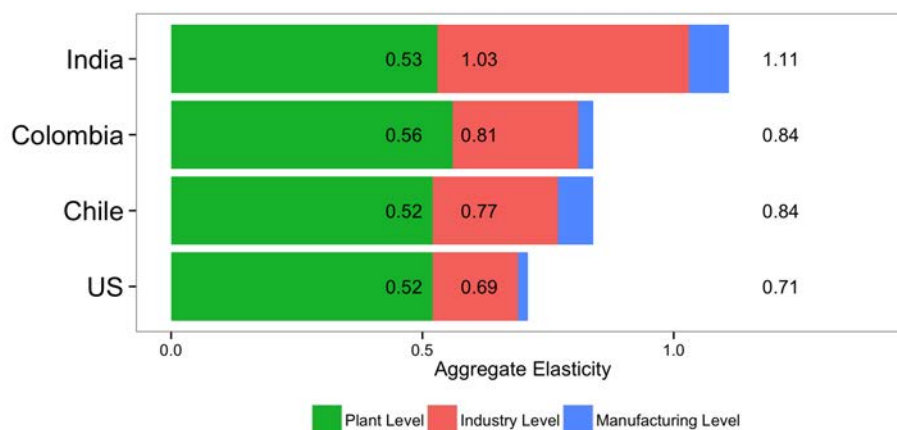
Note: This figure records the average plant level, industry level, and manufacturing level elasticities of substitution. The elasticities for each country is an average over the relevant time period, which is 1986-1996 for Chile, 1981-1991 for Colombia, 2000-2003 for India, and 1987 for the US.

6.2 Policy Implications

These cross-country differences in elasticities can imply large differences in outcomes. We examine two potential policy changes that would affect output per worker through a change in the capital rental rate. The first policy change lowers foreign interest rates to the US real interest rate; the second sets corporate taxes to zero. Our approach allows us to examine the effect of each policy change without assuming that each country shares the same aggregate technology.

The capital rental rate is composed of the real interest rate R, depreciation rate δ, and

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[37] Across industries, the average heterogeneity index in Chile is 0.15, 50 percent higher than the US 1987 value, and 0.30 in India, three times the US value.

effective corporate tax rate τ:

$$r = \frac{R + \delta}{1 - \tau}$$

Table IV contains these rental rates; see Web Appendix C.7 for the details of the construction of each variable. The real interest rate is higher than the US for all three developing countries, with an interest rate differential of 1.8 percentage points for Chile, 1.9 percentage points for India, and 5.3 percentage points for Colombia. Higher real interest rates imply that all three countries have higher capital rental rates than the US; the Chilean capital rental rate is 9 percent higher than the US rate, the Indian rate 17 percent higher, and the Colombian rate 51 percent higher. Capital tax rates are roughly similar for all countries.

Table IV Cross Country Differences in Real Interest Rates, Effective Corporate Taxes, and Capital Rental Rates

	Chile	Colombia	India	US
Real Interest Rate	5.9%	9.5%	6.0%	4.2%
Effective Corporate Tax Rate	15.1%	24.3%	20.3%	18.2%
Capital Rental Rate	18.1%	25.0%	19.4%	16.6%

Note: This table records the average real interest rate from 1992–2011 using the private sector lending rate from the IMF International Financial Statistics adjusted for inflation using the change in the GDP deflator, the one year effective corporate tax rate from Djankov et al. (2010), and the capital rental rate given the average real interest rate, corporate tax rate, and a depreciation rate of 9.46 percent.

Table V displays the change in output per worker from both policy changes, as well as the change in output per worker if all countries shared the US aggregate elasticity. The elasticity of output per worker to relative factor prices is proportional to the aggregate elasticity:

$$\frac{\mathrm{d}\ln Y/L}{\mathrm{d}\ln r/w} = \alpha \sigma^{agg}$$

where α is the capital share.[38]

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[38]Two notes: First, because the elasticity that we estimate is local, counterfactual predictions for non-local changes in rental rates involves extrapolating from our local elasticity. Second, we hold the wage fixed in these experiments; the change in wage induced from changes in rental rates depends upon labor supply as well as demand. Our estimates are a lower bound on the full effects of removing interest rate differentials, while the wage change for the capital tax change depends upon whether the revenue loss is compensated for by other tax changes.

If all interest rates fell to the US interest rate, output per worker would increase by 5.5 percent in Chile, 12.4 percent in Colombia, and 8.5 percent in India. These effects are exacerbated because the aggregate elasticity of substitution is larger in these countries than in the US. For example, the impact of the interest rate differential in India is more than fifty percent larger than it would be if India had the same elasticity of substitution as the US.

The story is similar with corporate tax rates. Reducing the corporate tax rate to zero would raise output per worker by 3.4 percent for the US, 7.3 percent for Chile, 10.5 percent for Colombia, and 14.8 percent for India; with the US aggregate elasticity, this change becomes 6.2 percent for Chile, 8.9 percent for Colombia, and 9.5 percent for India.[39] Differences in aggregate technology across countries have large effects on the outcome of these policy changes.

Table V Change in Output per Worker from Policy Changes Affecting the Capital Rental Rate

Policy Change	Chile	Colombia	India	US
Equalize to US Interest Rate	5.5%	12.4%	8.5%	0%
Equalize to US Interest Rate, using US Elasticities	4.6%	10.5%	5.4%	0%
Zero Corporate Tax	7.3%	10.5%	14.8%	3.4%
Zero Corporate Tax, using US Elasticities	6.2%	8.9%	9.5%	3.4%

Note: This table records the change in output per worker from two policy experiments- equalizing all interest rates to the US interest rate and setting the corporate tax rate to zero. For each experiment, we examine the change in output per worker using the country elasticity and using the US 1987 elasticity.

7 Conclusion

This paper has developed a new approach to estimate the aggregate elasticity of substitution between capital and labor by building up from micro structural parameters and the cross-sectional distribution of plant-level expenditures. Our approach has several advantages. We can estimate the aggregate elasticity at each point in time, which means we can separately examine the bias of technical change. In addition, with cross-sectional micro data, we can

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[39] We cannot interpret the change in output per worker from these policy changes as a gain in welfare because that comparison contrasts two different steady states. However, Chamley (1981) found in a full general equilibrium model that the welfare cost of capital taxation is proportional to the elasticity of substitution.

examine the robustness of our model and identification strategy.

We then applied our methodology to data on plants in the US manufacturing sector. While we estimated an average plant-level elasticity of roughly 0.5, our estimates indicated that the aggregate elasticity of substitution between capital and labor has been close to 0.7 from 1972 to 2007. Thus roughly 63 percent of aggregate factor substitution happened within plants.

We then measured the contributions of factor prices and two components of the bias of technology to the fall of the labor share in manufacturing since 1970. To do this, we used our estimates of aggregate elasticities and the history of factor prices. We found that the bias of technical change within industries has steadily decreased the labor share, accounting for most of the shift. There has been a sharp drop in the labor share since 2000 that coincides with a shift in the composition of industries. We found that mechanisms that work through factor prices alone cannot account for the decline in the labor share.

Our work indicates that explaining the decline in the labor share requires understanding the determinants of technical change, broadly defined, including automation, IT investment, offshoring, or shifts in composition driven by changing tastes or trade. Our decomposition suggests that there may be more than one force at work. While we have applied our methodology to the change in the US labor share, we believe it can be useful in understanding the evolution of skill premia and inequality.

Finally, our approach allowed us to estimate how the aggregate elasticity varies across time and countries, and to understand the underlying reasons for such variation. In particular, greater heterogeneity in capital intensities implies a higher aggregate elasticity of substitution. Elasticities in all three of the developing countries we examined were higher than the US, with an average manufacturing sector elasticity of 0.84 for Chile, 0.84 for Colombia, and 1.11 for India. Our estimates can help illuminate how and why policies have differential effects across countries.

References

Acemoglu, Daron, "Directed Technical Change," *Review of Economic Studies*, 2002, *69* (4), 781–809.

_ , "Labor- And Capital-Augmenting Technical Change," *Journal of the European Economic Association*, 2003, *1* (1), 1–37.

_ , "When Does Labor Scarcity Encourage Innovation?," *Journal of Political Economy*, 2010, *118* (6), 1037–1078.

_ **and Fabrizio Zilibotti**, "Productivity Differences," *Quarterly Journal of Economics*, 2001, *116* (2), 563–606.

Alvarez-Cuadrado, Francisco, Ngo Van Long, and Markus Poschke, "Capital-Labor Substitution, Structural Change and the Labor Income Share," Technical Report, CESifo Working Paper 2014.

Antras, Pol, "Is the US Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution," *Contributions to Macroeconomics*, 2004, *4* (1).

Atalay, Enghin, "How Important Are Sectoral Shocks?," 2014. Mimeo.

Autor, David H., David Dorn, and Gordon H. Hanson, "The China Syndrome: Local Labor Market Effects of Import Competition in the United States," *American Economic Review*, 2013, *103*, 2121–68.

_ , **Frank Levy, and Richard J. Murmane**, "The Skill Content of Recent Technological Change: An Empirical Exploration," *Quarterly Journal of Economics*, 2003, *118*, 1279–1333.

Banerjee, Abhijit and Esther Duflo, "Growth Theory Through The Lens Of Development Economics," *Handbook of Economic Growth*, 2005, *1*, 473–552.

Barnes, Sebastian N, Simon Price, and Maria Barriel, "The Elasticity of Substitution: Evidence from a UK Firm-level Data Set," 2008.

Bartik, Timothy J., *Who Benefits from State and Local Economic Development Policies?*, W. E. Upjohn Institute for Employment Research, 1991.

Bartlesman, Eric, John Haltiwanger, and Stefano Scarpetta, "Cross-Country Differences in Productivity: The Role of Allocation and Selection," Technical Report 1 2013.

Berndt, Ernst R., "Reconciling Alternative Estimates of the Elasticity of Substitution," *Review of Economics and Statistics*, 1976, *58* (1), 59–68.

Burstein, Ariel, Eduardo Morales, and Jonathan Vogel, "Accounting for Changes in Between-Group Inequality," 2014. Mimeo.

Caballero, Ricardo, Eduardo Engel, and John Haltiwanger, "Plant Level Adjustment and Aggregate Investment Dynamics," *Brookings Papers on Economic Activity*, 1995, *2*, 1–54.

Chamley, Christophe, "The Welfare Cost of Capital Income Taxation in a Growing Economy," *Journal of Political Economy*, 1981, *89* (3), 468–96.

Chirinko, Robert S., "Sigma: The Long and Short of It," *Journal of Macroeconomics*, 2008, *30*, 671–686.

_ , **Steven M. Fazzari, and Andrew P. Meyer**, "A New Approach to Estimating Production Function Parameters: The Elusive Capital-Labor Substitution Elasticity," *Journal of Business and Economic Statistics*, 2011, *29* (4), 587–594.

Cummins, Jason and Giovanni Violante, "Investment-Specific Technical Change in the US (1947-2000): Measurement and Macroeconomic Consequences," *Review of Economic Dynamics*, 2002, *5* (2), 243–284.

Diamond, Peter, Daniel McFadden, and Miguel Rodriguez, "Measurement of the Elasticity of Factor Substitution and Bias of Technical Change," in Melvyn Fuss and Daniel McFadden, eds., *Production Economics: A Dual Approach to Theory and Applications*, 1978, chapter 5.

Djankov, Simeon, Tim Ganser, Caralee McLiesh, Rita Ramalho, and Andrei Schliefer, "The Effect of Corporate Taxes on Investment and Entrepreneurship," *American Economic Journal: Macroeconomics*, 2010, *2* (3), 31–64.

Dornbusch, Rudiger, Stanley Fischer, and Paul Samuelson, "Heckscher-Ohlin Trade Theory with a Continuum of Goods," *Quarterly Journal of Economics*, 1980, *95* (2), 203–224.

Elsby, Michael W. L., Bart Hobijn, and Aysegul Sahin, "The Decline of the U.S. Labor Share," *Brookings Papers on Economic Activity*, 2013.

Fernald, John, "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity," September 2012. Mimeo.

Foster, Lucia, John Haltiwanger, and Chad Syverson, "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?," *American Economic Review*, March 2008, *98* (1), 394–425.

Gomme, Paul and Peter Rupert, "Measuring Labor's Share of Income," 2004. Policy Discussion Paper - No 7, Federal Reserve Bank of Cleveland.

Gordon, Robert J., *The Measurement of Durable Goods Prices*, University of Chicago Press, 1990.

Greenstreet, David, "Exploiting Sequential Learning to Estimate Establishment-Level Productivity Dynamics and Decision Rules," 2007. Mimeo.

Hall, Robert E. and Dale W. Jorgenson, "Tax Policy and Investment Behavior," *American Economic Review*, 1967, *57* (3), 391–414.

Harper, Michael J., Ernst R. Berndt, and David O. Wood, "Rates of Return and Capital Aggregation Using Alternative Rental Prices," in D.W. Jorgenson and R. London, eds., *Technology and Capital Formation*, Cambridge, MA: MIT Press, 1989.

Herrendorf, Berthold, Christopher Herrington, and Akos Valentinyi, "Sectoral Technology and Structural Transformation," Technical Report 2014.

Hicks, John R., *The Theory of Wages*, MacMillan and Co, 1932.

Houthakker, Hendrik, "The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis," *The Review of Economic Studies*, 1955, *23* (1), pp. 27–31.

Hsieh, Chang-Tai and Peter Klenow, "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 2009, *124* (4), 1403–1448.

Imbs, Jean and Isabelle Mejean, "Elasticity Optimism," May 2011. Mimeo.

Jones, Chad, "The Shape of Production Functions and the Direction of Technical Change," *Quarterly Journal of Economics*, 2005, *120* (2), 517–549.

Jorgenson, Dale W., Mun S. Ho, and John D. Samuels, "A Prototype Industry-level Production Account for the United States, 1947-2010," 2013. Mimeo.

_ , _ , and Kevin J. Stiroh, "Growth of U.S. Industries and Investments in Information Technology and Higher Education," in "Measuring Capital in the New Economy" NBER Chapters, National Bureau of Economic Research, Inc, October 2005, pp. 403–478.

Karabarbounis, Loukas and Brent Neiman, "The Global Decline of the Labor Share," *Quarterly Journal of Economics*, 2014, *129*, 61–103.

Klump, Rainer, Peter McAdam, and Alpo Willman, "Factor Substitution and Factor Augmenting Technical Progress in the US," *Review of Economics and Statistics*, 2007, *89* (1), 183–192.

Krueger, Alan B., "Measuring Labor's Share," *American Economic Review*, 1999, *89* (2), 45–51.

Krusell, Per, Lee Ohanian, Jose-Victor Rios-Rull, and Giovanni Violante, "Capital-skill Complementarity and Inequality: A Macroeconomic Analysis," *Econometrica*, 2000, *68* (5), 10291053.

Lagos, Ricardo, "A Model of TFP," *Review of Economic Studies*, October 2006, *73* (4), 983–1007.

Leon-Ledesma, Miguel A., Peter McAdam, and Alpo Willman, "Identifying the Elasticity of Substitution with Biased Technical Change," *The American Economic Review*, 2010, *100* (4), 1330–1357.

Levhari, David, "A Note on Houthakker's Aggregate Production Function in a Multifirm Industry," *Econometrica*, 1968, *36* (1), pp. 151–154.

Luttmer, Erzo G. J., "Growth with a Fixed Factor," 2012. Mimeo.

Mankiw, N. Gregory, "The Growth of Nations," *Brookings Papers on Economic Activity*, 1995, *1995* (1), 275–326.

Miyagiwa, Kaz and Chris Papageorgiou, "Endogenous Aggregate Easticity of Substitution," *Journal of Economic Dynamics and Control*, 2007, *31* (9), 2899–2919.

Piketty, Thomas, *Capital in the Twenty-First Century*, Belknap Press, 2014.

Raval, Devesh, "The Micro Elasticity of Substitution and Non Neutral Technology," 2014. Mimeo.

Restuccia, Diego and Richard Rogerson, "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments," *Review of Economic Dynamics*, 2008, *11* (4), 707–720.

Sato, Kazuo, *Production Functions and Aggregation*, Elsevier, 1975.

Tybout, James R. and Mark Roberts, *Industrial Evolution in Developing Countries: Industrial Evolution in Developing Countries: Micro Patterns of Turnover, Productivity and Market Structure*, Oxford University Press, 1996.

White, T Kirk, Jerome P Reiter, and Amil Petrin, "Plant-level Productivity and Imputation of Missing Data in US Census Manufacturing Data," Technical Report, National Bureau of Economic Research 2012.

Appendix

A   Proofs of Propositions

Appendix A describes the aggregate elasticity of substitution between capital and labor when each plant's production function exhibits constant returns to scale. Web Appendix B derives expressions for the aggregate elasticity under alternative assumptions. Web Appendix B.1 characterizes local elasticities of substitution and Web Appendix B.2 derives preliminary results under the assumption that plants' production functions are homothetic. The assumption of constant returns to scale is relaxed in Web Appendix B.3. Web Appendix B.4 introduces misallocation frictions. Web Appendix B.5 generalizes the demand system to allow for arbitrary elasticities of demand and imperfect pass-through. Web Appendix B.6 relaxes the assumption that production functions are homothetic.

Throughout, we use the following notation for relative factor prices:

$$\omega \equiv \frac{w}{r}$$
$$\mathfrak{q} \equiv \frac{q}{r}$$

In addition, we define $p_{ni} \equiv P_{ni}/r$ and $p_n \equiv P_n/r$ to be plant i's and industry n's prices respectively normalized by the rental rate. It will also be useful to define plant i's cost function (normalized by r) to be

$$z_{ni}(Y_{ni}, \omega, \mathfrak{q}) = \min_{K_{ni}, L_{ni}, M_{ni}} K_{ni} + \omega L_{ni} + \mathfrak{q} M_{ni} \qquad \text{subject to} \qquad F_{ni}(K_{ni}, L_{ni}, M_{ni}) \geq Y_{ni}$$

and industry n's cost as $z_n = \sum_{i \in I_n} z_{ni}$. Two results will be used repeatedly. First, Shephard's lemma implies that for each i:

$$(1 - s_{ni}^M)(1 - \alpha_{ni}) = \frac{\partial \ln z_{ni}}{\partial \ln \omega} \tag{14}$$

$$s_{ni}^M = \frac{\partial \ln z_{ni}}{\partial \ln \mathfrak{q}} \tag{15}$$

Second, since $\alpha_n = \sum_{i \in I_n} \alpha_{ni} \theta_{ni}$, then for any quantity κ_n,

$$\sum_{i \in I_n} (\alpha_{ni} - \alpha_n) \kappa_n \theta_{ni} = 0 \tag{16}$$

When production functions take the functional form in defined in Assumption 1, it is relatively straightforward to derive the industry-level elasticity of substitution. However, when we aggregate across industries, we have no justification for using any particular functional form for the industry-level production function, or how capital, labor, and materials are nested in the industry-level production function. We will therefore derive a formula for the industry-level elasticity of substitution in a way that nests nicely: the same formula used to go from plant-level to industry-level can be used to go from industry level to aggregate.

Consequently here we relax the functional form assumption on plants' production functions. While we maintain that each plant's production function has constant returns to scale, we impose

no further structure.

Assumption 1′ *Plant i in industry n uses the constant returns to scale production function $F_{ni}(K_{ni}, L_{ni}, M_{ni})$.*

The purpose of this is twofold. First, it provides guidance on the robustness of the formulas in Proposition 1. Second, we will later use the formulas here to aggregate across industries and derive an expression for the elasticity of substitution between capital and labor for the manufacturing sector as a whole.

For plant i, we define the local elasticities σ_{ni} and ζ_{ni} to satisfy

$$\sigma_{ni} - 1 = \frac{d \ln K_{ni}/\omega L_{ni}}{d \ln \omega} \tag{17}$$

$$\zeta_{ni} - 1 = \frac{1}{\alpha^M - \alpha_{ni}} \frac{d \ln \frac{\mathsf{q} M_{ni}}{K_{ni} + \omega L_{ni}}}{d \ln \omega} \tag{18}$$

If i's production function takes a nested CES form as in Assumption 1, σ_{ni} and ζ_{ni} would equal σ_n and ζ_n respectively.[40] Under Assumption 1′, they are defined locally in terms of derivatives of F_{ni} evaluated at i's cost-minimizing input bundle. Exact expressions for σ_{ni} and ζ_{ni} are given in Web Appendix B.1 of the web appendix. Proposition 1′ then expresses the industry elasticity of substitution σ_n^N. The resulting expression is identical to Proposition 1 except the plant elasticities of substitution are replaced with weighted averages of the plants' local elasticities, $\bar{\sigma}_n$ and $\bar{\zeta}_n$.

Proposition 1′ *Under Assumption 1′, the industry elasticity of substitution $\sigma_n^N = \frac{d \ln K_n/L_n}{d \ln \omega}$ is:*

$$\sigma_n^N = (1 - \chi_n)\bar{\sigma}_n + \chi_n \left[(1 - \bar{s}_n^M)\varepsilon_n + \bar{s}_n^M \bar{\zeta}_n \right] \tag{19}$$

where $\chi_n \equiv \sum_{i \in I_n} \frac{(\alpha_{ni} - \alpha_n)^2 \theta_{ni}}{\alpha_n (1 - \alpha_n)}$ and $\bar{s}_n^M \equiv \sum_{i \in I_n} \frac{(\alpha_{ni} - \alpha_n)(\alpha_{ni} - \alpha^M)}{\sum_{j \in I_n} (\alpha_{nj} - \alpha_n)(\alpha_{nj} - \alpha^M)} s_{ni}^M$ as in Proposition 1 and

$$\bar{\sigma}_n \equiv \sum_{i \in I_n} \frac{\alpha_{ni}(1 - \alpha_{ni})\theta_{ni}}{\sum_{j \in I_n} \alpha_{nj}(1 - \alpha_{nj})\theta_{nj}} \sigma_{ni}$$

$$\bar{\zeta}_n \equiv \sum_{i \in I_n} \frac{(\alpha_{ni} - \alpha_n)(\alpha_{ni} - \alpha^M) s_{ni}^M}{\sum_{j \in I_n} (\alpha_{nj} - \alpha_n)(\alpha_{nj} - \alpha^M) s_{nj}^M} \zeta_{ni}$$

Proof. As discussed in the text, the definitions of σ_{ni} and σ_n^N imply

$$\sigma_{ni} - 1 = \frac{d \ln \frac{r K_{ni}}{w L_{ni}}}{d \ln \omega} = \frac{1}{\alpha_{ni}(1 - \alpha_{ni})} \frac{d\alpha_{ni}}{d \ln \omega}, \qquad \forall i \in I_n$$

$$\sigma_n^N - 1 = \frac{d \ln \frac{r K_n}{w L_n}}{d \ln \omega} = \frac{1}{\alpha_n(1 - \alpha_n)} \frac{d\alpha_n}{d \ln \omega}$$

---

[40]The definition of σ_{ni} is straightforward but ζ_{ni} requires some elaboration. Suppose that F_{ni} takes the nested CES form of Assumption 1. Let $\lambda_{ni} \equiv \frac{1}{r} \left[(r/A_{ni})^{1-\sigma_n} + (w/B_{ni})^{1-\sigma_n} \right]^{\frac{1}{\sigma_n - 1}}$ be the marginal cost of i's capital-labor bundle. Then cost minimization implies $\frac{\mathsf{q} M_{ni}}{K_{ni} + \omega L_{ni}} = \left(\frac{\mathsf{q}/C_{ni}}{\lambda_{ni}} \right)^{1-\zeta_n}$. Since $\frac{d \ln \mathsf{q}}{d \ln \omega} = 1 - \alpha^M$ and, from Shephard's Lemma, $\frac{d \ln \lambda_{ni}}{d \ln \omega} = 1 - \alpha_{ni}$, we have that $\frac{d \ln \frac{\mathsf{q} M_{ni}}{K_{ni} + \omega L_{ni}}}{d \ln \omega} = (\zeta_n - 1)(\alpha^M - \alpha_{ni})$.

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Since $\alpha_n = \sum_{i \in I_n} \alpha_{ni} \theta_{ni}$, we can differentiate to get

$$\sigma_n^N - 1 \quad = \quad \frac{1}{\alpha_n(1 - \alpha_n)} \left[\sum_{i \in I_n} \frac{d\alpha_{ni}}{d\ln\omega} \theta_{ni} + \sum_{i \in I_n} \alpha_{ni} \frac{d\theta_{ni}}{d\ln\omega} \right]$$

$$= \quad \frac{\sum_{i \in I_n} \alpha_{ni}(1 - \alpha_{ni})\theta_{ni}(\sigma_{ni} - 1)}{\alpha_n(1 - \alpha_n)} + \frac{\sum_{i \in I_n} \alpha_{ni} \frac{d\theta_{ni}}{d\ln\omega}}{\alpha_n(1 - \alpha_n)}$$

Using the definition of $\bar{\sigma}_n$ and $\sum_{i \in I_n} \frac{d\theta_{ni}}{d\ln\omega} = 0$, we have

$$\sigma_n^N - 1 = (\bar{\sigma}_n - 1) \sum_{i \in I_n} \frac{\alpha_{ni}(1 - \alpha_{ni})\theta_{ni}}{\alpha_n(1 - \alpha_n)} + \frac{\sum_{i \in I_n}(\alpha_{ni} - \alpha_n)\frac{d\theta_{ni}}{d\ln\omega}}{\alpha_n(1 - \alpha_n)} \tag{20}$$

Since $\chi_n = \sum_{i \in I_n} \frac{(\alpha_{ni} - \alpha_n)^2 \theta_{ni}}{\alpha_n(1 - \alpha_n)}$, one can verify that $\sum_{i \in I_n} \frac{\alpha_{ni}(1 - \alpha_{ni})\theta_{ni}}{\alpha_n(1 - \alpha_n)} = 1 - \chi_n$, which gives

$$\sigma_n^N = (1 - \chi_n)\bar{\sigma}_n + \frac{\sum_{i \in I_n}(\alpha_{ni} - \alpha_n)\frac{d\theta_{ni}}{d\ln\omega}}{\alpha_n(1 - \alpha_n)} + \chi_n \tag{21}$$

We now find an expression for $\frac{d\ln\theta_{ni}}{d\ln\omega}$. θ_{ni} can be written as

$$\theta_{ni} = \frac{rK_{ni} + wL_{ni}}{\sum_{j \in I_n} rK_{nj} + wL_{nj}} = \frac{(1 - s_{ni}^M)z_{ni}}{\sum_{j \in I_n}(1 - s_{nj}^M)z_{nj}} = \frac{(1 - s_{ni}^M)z_{ni}}{(1 - s_n^M)z_n}$$

Since $d\ln\frac{1 - s_{ni}^M}{s_{ni}^M} = \frac{1}{s_{ni}^M}d\ln(1 - s_{ni}^M)$, the definition of ζ_n implies

$$\frac{d\ln(1 - s_{ni}^M)}{d\ln\omega} = s_{ni}^M(\zeta_n - 1)(\alpha_{ni} - \alpha^M)$$

The change in i's expenditure on all inputs depends on its expenditure shares:

$$\frac{z_{ni}}{z_n} \quad = \quad \frac{rK_{ni} + wL_{ni} + qM_{ni}}{\sum_{j \in I_n} rK_{nj} + wL_{nj} + qM_{nj}} = \frac{\frac{\varepsilon_n - 1}{\varepsilon_n}P_{ni}Y_{ni}}{\sum_{j \in I_n}\frac{\varepsilon_n - 1}{\varepsilon_n}P_{nj}Y_{nj}} = \frac{\frac{\varepsilon_n - 1}{\varepsilon_n}P_{ni}^{1-\varepsilon_n}Y_n P_n^{\varepsilon_n}}{\sum_{j \in I_n}\frac{\varepsilon_n - 1}{\varepsilon_n}P_{nj}^{1-\varepsilon_n}Y_n P_n^{\varepsilon_n}}$$

$$= \quad \left(\frac{p_{ni}}{p_n}\right)^{1-\varepsilon_n}$$

The change in i's price depends on the change in its marginal cost

$$\frac{d\ln p_{ni}}{d\ln\omega} \quad = \quad \frac{d\ln\frac{\varepsilon_n}{\varepsilon_n - 1}mc_{ni}}{d\ln\omega} = \frac{d\ln mc_{ni}}{d\ln\omega} = (1 - s_{ni}^M)(1 - \alpha_{ni}) + s_{ni}^M(1 - \alpha^M) \tag{22}$$

Putting these pieces together, we have $\theta_{ni} = \frac{1 - s_{ni}^M}{1 - s_n^M}\left(\frac{p_{ni}}{p_n}\right)^{1-\varepsilon_n}$, so differentiating and using equa-

tion (16) yields

$$
\begin{aligned}
\sum_{i \in I_n}(\alpha_{ni} - \alpha_n)\theta_{ni}\frac{d\ln\theta_{ni}}{d\ln\omega} &= \sum_{i \in I_n}(\alpha_{ni} - \alpha_n)\theta_{ni}\left[\frac{d\ln 1 - s_{ni}^M}{d\ln\omega} + (1 - \varepsilon_n)\frac{d\ln p_{ni}}{d\ln\omega}\right] \\
&= \sum_{i \in I_n}(\alpha_{ni} - \alpha_n)\theta_{ni}\left\{\begin{array}{c} s_{ni}^M(\zeta_{ni} - 1)(\alpha_{ni} - \alpha^M) \\ +(1 - \varepsilon_n)\left[(1 - s_{ni}^M)(1 - \alpha_{ni}) + s_{ni}^M(1 - \alpha^M)\right]\end{array}\right\} \\
&= \sum_{i \in I_n}(\alpha_{ni} - \alpha_n)\theta_{ni}\left\{s_{ni}^M(\zeta_{ni} - \varepsilon_n)(\alpha_{ni} - \alpha^M) + (1 - \varepsilon_n)(1 - \alpha_{ni})\right\}
\end{aligned}
$$

Using the definitions of $\bar{\zeta}_n$, $\bar{s}_n^M$, and χ_n, this becomes

$$
\begin{aligned}
\sum_{i \in I_n}(\alpha_{ni} - \alpha_n)\theta_{ni}\frac{d\ln\theta_{ni}}{d\ln\omega} &= \sum_{i \in I_n}(\alpha_{ni} - \alpha_n)\theta_{ni}\left[s_{ni}^M(\bar{\zeta}_n - \varepsilon_n)(\alpha_{ni} - \alpha^M) + (1 - \varepsilon_n)(1 - \alpha_{ni})\right] \\
&= \sum_{i \in I_n}(\alpha_{ni} - \alpha_n)\theta_{ni}\left[\bar{s}_n^M(\bar{\zeta}_n - \varepsilon_n)(\alpha_{ni} - \alpha^M) + (1 - \varepsilon_n)(1 - \alpha_{ni})\right] \\
&= \sum_{i \in I_n}(\alpha_{ni} - \alpha_n)\theta_{ni}\left[\bar{s}_n^M(\bar{\zeta}_n - \varepsilon_n)(\alpha_{ni} - \alpha_n) + (1 - \varepsilon_n)(\alpha_n - \alpha_{ni})\right] \\
&= \alpha_n(1 - \alpha_n)\chi_n\left[(\bar{\zeta}_n - \varepsilon_n)\bar{s}_n^M - (1 - \varepsilon_n)\right]
\end{aligned}
$$

Finally, we can plug this back into equation (21) to get

$$
\sigma_n^N = (1 - \chi_n)\bar{\sigma}_n + \chi_n\left[\bar{s}_n^M\zeta_n + (1 - \bar{s}_n^M)\varepsilon_n\right]
$$

■

To build up to the aggregate elasticity of substitution between capital and labor, we proceed in exactly the same way as with the industry-level elasticity. Define ζ_n^N to satisfy $(\zeta_n^N - 1)(\alpha_n - \alpha^M) = \frac{d\ln\frac{1-s_n^M}{s_n^M}}{d\ln\omega}$. The claim below will give an expression for ζ_n^N in terms of plant level elasticities and choices.

Proposition 2′ *Under Assumption 1′, the aggregate elasticity between capital and labor is*

$$
\sigma^{agg} = (1 - \chi^N)\bar{\sigma}^N + \chi_N\left[\bar{s}^M\bar{\zeta}^N + (1 - \bar{s}^M)\eta\right]
$$

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where χ^N, $\bar{\sigma}^N$, $\bar{\zeta}^N$, $\bar{s}^M$, and ζ_n^N are defined as

$$\chi^N = \sum_{n \in N} \frac{(\alpha_n - \alpha)^2 \theta_n}{\alpha(1-\alpha)}$$

$$\bar{\sigma}^N = \sum_{n \in N} \frac{\alpha_n(1-\alpha_n)\theta_n}{\sum_{n' \in N} \alpha_{n'}(1-\alpha_{n'})\theta_{n'}} \sigma_n^N$$

$$\bar{\zeta}^N \equiv \frac{\sum_{n \in N}(\alpha_n - \alpha)(\alpha_n - \alpha^M)\theta_n s_n^M \zeta_n^N}{\sum_{n \in N}(\alpha_n - \alpha)(\alpha_n - \alpha^M)\theta_n s_n^M}$$

$$\bar{s}^M = \frac{\sum_{n \in N}(\alpha_n - \alpha)(\alpha_n - \alpha^M)\theta_n s_n^M}{\sum_{n \in N}(\alpha_n - \alpha)(\alpha_n - \alpha^M)\theta_n}$$

$$\zeta_n^N = \sum_{i \in I_n} \theta_{ni} \left[\zeta_{ni} \frac{s_{ni}^M(\alpha_{ni} - \alpha^M)}{s_n^M(\alpha_n - \alpha^M)} + \varepsilon_n \left\{ 1 - \frac{s_{ni}^M(\alpha_{ni} - \alpha^M)}{s_n^M(\alpha_n - \alpha^M)} \right\} \right]$$

Proof. Note that since $\frac{z_{ni}}{z_n} = \frac{P_{ni}Y_{ni}}{P_n Y_n}$, we have both $\frac{d\ln Y_n}{d\ln \omega} = \sum_{i \in I_n} \frac{z_{ni}}{z_n} \frac{d\ln Y_{ni}}{d\ln \omega}$ and $\frac{d\ln p_n}{d\ln \omega} = \sum_{i \in I_n} \frac{z_{ni}}{z_n} \frac{d\ln p_{ni}}{d\ln \omega}$. With the first, we can derive an expression for the change in cost that parallels equation (27):

$$\frac{d\ln z_n}{d\ln \omega} = \sum_{i \in I_n} \frac{z_{ni}}{z_n} \frac{d\ln z_{ni}}{d\ln \omega}$$

$$= \sum_{i \in I_n} \frac{z_{ni}}{z_n} \left[\frac{d\ln Y_{ni}}{d\ln \omega} + (1 - s_{ni}^M)(1 - \alpha_{ni}) + s_{ni}^M(1 - \alpha^M) \right]$$

$$= \frac{d\ln Y_n}{d\ln \omega} + (1 - s_n^M)(1 - \alpha_n) + s_n^M(1 - \alpha^M)$$

With the second, we can derive an expression for the change in the price level that parallels equation (22)

$$\frac{d\ln p_n}{d\ln \omega} = \sum_{i \in I_n} \frac{z_{ni}}{z_n} \frac{d\ln p_{ni}}{d\ln \omega}$$

$$= \sum_{i \in I_n} (1 - \alpha_{ni})(1 - s_{ni}^M) + s_{ni}^M(1 - \alpha^M)$$

$$= (1 - \alpha_n)(1 - s_n^M) + s_n^M(1 - \alpha^M)$$

Thus following the exact logic of Proposition 1′, we have that

$$\sigma^{agg} = (1 - \chi^N)\bar{\sigma}^N + \chi_N \left[\bar{s}^M \bar{\zeta}^N + (1 - \bar{s}^M)\eta \right]$$

It remains only to derive the expression for ζ_n^N. Begin with $1 - s_n^M = \sum_{i \in I_n}(1 - s_{ni}^M)\frac{z_{ni}}{z_n}$. Differentiating each side gives:

$$\frac{d\ln(1 - s_n^M)}{d\ln \omega} = \sum_{i \in I_n} \frac{(1 - s_{ni}^M)z_{ni}}{(1 - s_n^M)z_n} \left[\frac{d\ln(1 - s_{ni}^M)}{d\ln \omega} + \frac{d\ln z_{ni}/z_n}{d\ln \omega} \right] = \sum_{i \in I_n} \theta_{ni} \left[\frac{d\ln(1 - s_{ni}^M)}{d\ln \omega} + \frac{d\ln z_{ni}/z_n}{d\ln \omega} \right]$$

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Using $(\alpha_{ni} - \alpha^M)(\zeta_{ni} - 1) = \frac{1}{s_{ni}^M} \frac{d \ln(1 - s_{ni}^M)}{d \ln \omega}$ and $(\alpha_n - \alpha^M)(\zeta_n^N - 1) = \frac{1}{s_n} \frac{d \ln(1 - s_n^M)}{d \ln \omega}$ gives

$$s_n^M (\alpha_n - \alpha^M)(\zeta_n^N - 1) = \sum_{i \in I_n} \theta_{ni} \left[s_{ni}^M (\alpha_{ni} - \alpha^M)(\zeta_{ni} - 1) + \frac{d \ln z_{ni}/z_n}{d \ln \omega} \right]$$

Finally, we have

$$
\begin{aligned}
\frac{d \ln z_{ni}/z_n}{d \ln \omega} &= \frac{d \ln Y_{ni}}{d \ln \omega} + (1 - s_{ni}^M)(1 - \alpha_{ni}) + s_{ni}^M (1 - \alpha^M) - \frac{d \ln Y_n}{d \ln \omega} - (1 - s_n^M)(1 - \alpha_n) - s_n^M (1 - \alpha^M) \\
&= (-\varepsilon_n) \frac{d \ln p_{ni}/p_n}{d \ln \omega} + (1 - s_{ni}^M)(1 - \alpha_{ni}) + s_{ni}^M (1 - \alpha^M) - (1 - s_n^M)(1 - \alpha_n) - s_n^M (1 - \alpha^M) \\
&= (1 - \varepsilon_n) \left[(1 - s_{ni}^M)(1 - \alpha_{ni}) + s_{ni}^M (1 - \alpha^M) - (1 - s_n^M)(1 - \alpha_n) - s_n^M (1 - \alpha^M) \right] \\
&= (\varepsilon_n - 1) \left[s_n^M (\alpha_n - \alpha^M) - s_{ni}^M (\alpha_{ni} - \alpha^M) + (\alpha_{ni} - \alpha_n) \right]
\end{aligned}
$$

Plugging this in, we have

$$s_n^M (\alpha_n - \alpha^M)(\zeta_n^N - 1) = \sum_{i \in I_n} \theta_{ni} \left\{ \begin{array}{c} s_{ni}^M (\alpha_{ni} - \alpha^M)(\zeta_{ni} - 1) \\ +(\varepsilon_n - 1) \left[s_n^M (\alpha_n - \alpha^M) - s_{ni}^M (\alpha_{ni} - \alpha^M) + (\alpha_{ni} - \alpha_n) \right] \end{array} \right\}$$

Using $\sum_{i \in I_n} \theta_{ni}(1 - \varepsilon_n)(\alpha_{ni} - \alpha_n) = 0$, dividing through by $s_n^M (\alpha_n - \alpha^M)$ and subtracting 1 from each side gives the result. ∎