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ABSTRACT

The price-rent ratio is highly volatile and predicts future returns for commercial real estate. Price-rent variations in commercial real estate also tend to comove with investment and output. We develop a general equilibrium model that explicitly introduces a rental market and incorporates collateral constraints on production as a key ingredient. Our estimation identifies discount-rate shocks as the most important factor in (1) driving price-rent variations, (2) producing the long-horizon predictability of real estate returns, and (3) linking the dynamics in commercial real estate to those in the production sector.

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I. Introduction

The rise and fall of real estate prices in the past decades and the 2008 financial crisis triggered by the collapse of real estate prices have generated a great deal of research on the impact of real estate prices on the macroeconomy. Most research has focused on consumers' behavior and the residential real estate market. When we study firms' investment dynamics, it is often the commercial real estate market that becomes relevant. In a recent paper, Chaney, Sraer, and Thesmar (2012) provide micro evidence that links the commercial real estate price to investment. They estimate that a \$1 increase in a representative U.S. firm's value of real estate raises its investment by \$0.06. At the aggregate level, however, the link between commercial real estate prices and investment dynamics has been largely unexplored.

In this paper we develop a medium-size dynamic stochastic general equilibrium (DSGE) model and show that this model is capable of *reproducing quantitatively* a variety of stylized facts about the commercial real estate price and the business cycle if one incorporates two key ingredients: shocks to households' discount rate and collateral constraints on firms' production. We confront our model with financial and macroeconomic time series and estimate the model using the Bayesian method to account for the following facts:

(1) *Volatility*: Commercial real estate price fluctuates much more than rent and output do. Over the past 20 years, while the volatility (measured by the standard deviation of quarterly changes) is 1.245% for rent and 0.697% for output, the volatility of real estate prices is 4.171%.

(2) *Long-horizon predictability*: Price-rent variation predicts long-horizon real estate returns.[1] Simple ordinary least squares (OLS) regressions of real estate returns at different horizons on the rent-price ratio (the valuation ratio) show that the slope coefficients are significantly positive and they increase with the horizon and that the fit measure, R^2, also increases with the horizon. Figure 1 plots the contemporaneous price-rent ratio against real estate returns of the subsequent five years to capture the key fact: high prices relative to rents have preceded many years of low returns and low prices have preceded high returns.

(3) *Comovement*: The price-rent ratio tends to comove with output as demonstrated by Figure 2. The contemporaneous correlation between output and price-rent ratio is 0.83 for log level and 0.48 for year-over-year growth.[2]

How to reproduce all these macro-finance facts within one structural framework has been a central but challenging task in the macro-finance literature. Our model builds on the DSGE

[1]See Ghysels, Plazzi, Torous, and Valkanov (2012) for an extensive survey of this literature.

[2]When output and the price-rent ratio are log-linearly detrended, the correlation is 0.47. When these series are hp-filtered, the correlation is 0.63.

literature with a combination of two distinctive features: we introduce a rental market of commercial real estate and assume that firms face collateral constraints when financing working capital. Without modeling the rental market explicitly, the existing macroeconomic models (Iacoviello, 2005; Iacoviello and Neri, 2010; Liu, Wang, and Zha, 2013; Liu, Miao, and Zha, 2016, for example) show that real estate price and rent move in comparable magnitude so that there is little price-rent variation in comparison to the data. But price-rent variation is central to both the long-horizon predictability of real estate returns (Cochrane, 2011; Ghysels, Plazzi, Torous, and Valkanov, 2013) and the business cycle (Figure 2).

By controlling for an array of commonly used shocks such as technology and labor supply shocks, we find that shocks to the discount rate are the key to generating the data dynamics that quantitatively reproduce stylized facts (1)-(3). The key intuition is that the rental price of commercial real estate is determined by the marginal product of the real estate property in firms' production, but the real estate price is a forward looking variable, equal to the discounted present value of future rents. Shocks to such a discount are intertemporal and affect future appreciations of the price; they do not directly affect the current rent.

The discount-rate shock is a parsimonious way of modeling the variation in discount rates stressed by Hansen and Jagannathan (1991), Campbell and Ammer (1993) and Cochrane (2011) and can sometimes be interpreted as a sentiment shock as in Barberis, Shleifer, and Vishny (1998) and Dumas, Kurshev, and Uppal (2009). In the macroeconomic literature (Smets and Wouters, 2003; Primiceri, Schaumburg, and Tambalotti, 2006; Galí, 2015, for example), the discount-rate shock is called a "preference" shock to capture shifts in aggregate demand; its asset pricing implications were first discussed by Albuquerque, Eichenbaum, Luo, and Rebelo (2016), who construct a general equilibrium model of an endowment economy to show that discount-rate shocks can generate the observed risk premium and weak correlation between consumption growth and stock returns. In their model, however, these shocks do not affect macroeconomic movements or generate the long-horizon predictability of excess stock returns.

We follow Albuquerque, Eichenbaum, Luo, and Rebelo (2016) by introducing discount-rate shocks and extend their model to a production economy with an emphasis on the implications of commercial real estate prices on investment and the business cycle. One key contribution of our paper is to show that in our production economy model, discount-rate shocks can generate price-rent fluctuations over the business cycle. Because firms' collateral assets provide the collateral (liquidity) premium, discount-rate shocks can also generate the long-horizon predictability of excess real estate returns. These discoveries are important for several reasons. First, the long-horizon predictability is prevalent in the asset pricing literature in general and the real estate literature in particular. Which shock in general equilibrium models can generate the data dynamics reproducing the predictability result has been an open and challenging issue. Second, the discount-rate shock in our model is an macroeconomic shock commonly used in the literature. What is new in our paper is the finding that the collateral channel for firms' production, as documented by Chaney, Sraer, and Thesmar (2012), is essential for reproducing facts (1)-(3). Third, our estimation shows that the discount-rate shock is an empirically important source for the linkage between the real estate market and the business cycle. Moreover, the quantitative importance of the real estate value for investment at the macro level is consistent with the findings of Chaney, Sraer, and Thesmar (2012) at the micro level.

The existing general equilibrium models with real estate markets typically fail to generate price-rent variation.[3] One exception is Favilukis, Ludvigson, and Nieuwerburgh (2017), who study a calibrated two-sector overlapping-generations model of housing and non-housing production in which heterogeneous households face limited opportunities to insure against aggregate and idiosyncratic risks. They examine how a relaxation of financing constraints leads to a large boom in house prices. Another exception is Kaplan, Mitman, and Violante (2017), who study house prices and their impact on household consumption by incorporating several macroeconomic shocks. They introduce a rental market in residential real estate and find that shocks to agent beliefs about future demand of real estate are the main driver of price-rent variation. Their belief shocks are similar to our discount-rate shocks in the sense that both types of shocks affect the expectations of future house price appreciation.

Our emphasis is different: we develop a general equilibrium model with a production economy that is tractable to be estimated against the data and focus on some key *asset pricing* aspects of the general equilibrium model. We find that discount-rate shocks are the only macroeconomic shocks driving the large movements of commercial real estate price *relative to rent*. In our model, moreover, price-rent variation generated by discount-rate shocks is central to the dynamic interactions between the commercial real estate price and the business cycle. Traditional business-cycle shocks, such as shocks to technology and labor supply, cannot explain price-rent movements that are quantitatively comparable to the observed time series.

Although price-rent variation is necessary for the long-horizon predictability of real estate returns, it is by no means sufficient. We show that in response to discount-rate shocks, firms' real estate values used as collateral play a key role in generating both the long-horizon predictability and the high volatility of real estate prices observed in the data. The rise

[3]See Campbell, Davis, Gallin, and Martin (2009); Piazzesi and Schneider (2009); Kiyotaki, Michaelides, and Nikolov (2011); Caplin and Leahy (2011); Burnside, Eichenbaum, and Rebelo (2011); Pintus and Wen (2013); Head, Lloyd-Ellis, and Sun (2014); Justiniano, Primiceri, and Tambalotti (2017) for models of housing. This literature focuses on residential housing but does not reproduce all the three facts (1)-(3) simultaneously in the dynamic general equilibrium framework.

of real estate prices relaxes a firm's collateral constraint and thus facilitates its production, while the rental price determined by the marginal product of real estate property does not move much. This result underlies the comovement between price-rent ratio and output. In the decomposition of real estate price, our estimation reveals that collateral values contribute to one third of the volatility of real estate prices while rents contribute very little.

Our estimated discount-rate shock itself exhibits an extremely small volatility (0.067%), but this small persistent shock contributes to not only all of the observed large price-rent fluctuation (3.91%) but also 48% of the observed investment fluctuation (1.68%) and a quarter of the observed output fluctuation (0.70%). Although the discount-rate shock connects the dynamics in the real estate market to those in the production sector, it is the model's internal transmission mechanism that amplifies this small shock into the large volatilities of price-rent ratio, investment, and output. Thus, our findings demonstrate the importance of incorporating discount-rate shocks and firms' collateral values in an otherwise standard DSGE model, in order to account for various asset pricing facts and their relationship to the business cycle.

The rest of the paper is organized as follows. In Section II we construct a medium-size general equilibrium model with a production economy. In Section III we estimate the model against several U.S. time series, report the estimated results, analyze the impulse responses, and discuss the linkage between price-rent dynamics and aggregate fluctuations. Section IV presents the empirical results of the predictability of real estate returns and the volatilities of real estate price, investment, and output. Section V studies a simplified version of the model to gain intuition into our empirical findings by deriving the closed-form theoretical results. Section VI concludes the paper. Detailed derivations, proofs, and estimation procedures are provided in appendices.

II. The model

We study an economy with a representative household, a continuum of intermediate-goods producers, and a continuum of heterogeneous final-goods firms. The representative household maximizes its utility and accumulates physical capital. There are a variety of intermediate goods and each good is produced by a continuum of identical competitive producers. The heterogeneous final-goods firms are indexed by idiosyncratic productivity shocks. They trade commercial real estate properties among themselves and rent out real estate properties to intermediate-goods producers. Financial frictions occur in the final-goods sector; firms in this sector borrow against their real estate value to finance working capital.

II.1. **Households.** The representative household maximizes the expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \Theta_t \beta^t \left[\log\left(C_t - \gamma C_{t-1}\right) - \psi_t \frac{N_t^{1+\nu}}{1+\nu} \right],$$

where C_t and N_t represent consumption and labor supply. The parameters $\beta \in (0,1)$ and $\gamma \in (0,1)$ represent the subjective discount rate and habit formation. The variables $\theta_t \equiv \Theta_t/\Theta_{t-1}$ and ψ_t are exogenous shocks to the discount rate and labor supply and follow the AR(1) processes as

$$\log \theta_t = (1-\rho_\theta) \log \theta + \rho_\theta \log \theta_{t-1} + \sigma_\theta \varepsilon_{\theta,t}, \tag{1}$$

$$\log \psi_t = (1-\rho_\psi) \log \psi + \rho_\psi \log \psi_{t-1} + \sigma_\psi \varepsilon_{\psi,t}, \tag{2}$$

where $\varepsilon_{\theta,t}$ and $\varepsilon_{\psi,t}$ are iid standard normal random variables. Albuquerque, Eichenbaum, Luo, and Rebelo (2016) introduce discount-rate shocks like ours as demand shocks in their endowment economy to study asset pricing implications.[4] In our model with a production economy, we will test the importance of discount-rate shocks in linking price-rent variation to the business cycle.

The household chooses consumption C_t, investment I_t, the capital utilization rate u_t, and government bonds B_{t+1} subject to intertemporal budget constraint

$$C_t + \frac{I_t}{Z_t} + \frac{B_{t+1}}{R_{ft}} \leq w_t N_t + R_{kt}\left(u_t K_t\right) + D_t + B_t,$$

where K_t, w_t, D_t, R_{kt}, and R_{ft} represent respectively capital, wage, dividend income, the rental price of capital, and the risk-free interest rate. The variable Z_t represents an aggregate investment-specific technology shock that has both permanent and transitory components (Greenwood, Hercowitz, and Krusell, 1997; Krusell, Ohanian, Ríos-Rull, and Violante, 2000):

$$Z_t = Z_t^p v_{zt}, \; Z_t^p = Z_{t-1}^p g_{zt},$$

$$\log g_{zt} = (1-\rho_z) \log g_z + \rho_z \log(g_{z,t-1}) + \sigma_z \varepsilon_{zt}, \tag{3}$$

$$\log v_{zt} = \rho_{v_z} \log v_{z,t-1} + \sigma_{v_z} \varepsilon_{v_z,t}, \tag{4}$$

where $\varepsilon_{z,t}$ and $\varepsilon_{v_z,t}$ are iid standard normal random variables.

Investment is subject to quadratic adjustment costs (Christiano, Eichenbaum, and Evans, 2005). Capital evolves according to the law of motion

$$K_{t+1} = (1-\delta_t) K_t + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2\right] I_t,$$

where $\delta_t \equiv \delta(u_t)$ is the capital depreciation rate in period t, g_I denotes the steady state growth rate of investment, and Ω is the investment adjustment cost parameter.

[4]Preference shocks used by Galí (2015) and other macroeconomic models relate to the log level of Θ_t. The shock process of $\log \theta_t$ relates to the discount rate β itself. We call it a discount-rate shock.

II.2. **Intermediate-goods producers.** There is a continuum of intermediate goods. Each intermediate good $j \in [0,1]$ is produced by a continuum of identical competitive producers of measure unity. The representative producer owns a constant-returns-to-scale technology to produce good j by hiring labor $N_t(j)$, renting real estate property $H_t(j)$ from final-goods firms, and renting capital $K_t(j)$ from the household. The producer's decision problem is

$$\max_{N_t(j), H_t(j), K_t(j)} P_{Xt}(j) X_t(j) - w_t N_t(j) - R_{ct} H_t(j) - R_{kt} K_t(j),$$

where $X_t(j) \equiv A_t \left[K_t^{1-\phi}(j) H_t^{\phi}(j) \right]^{\alpha} N_t^{1-\alpha}(j)$, $P_{Xt}(j)$ represents the competitive price of good j, and R_{ct} is the rental price of commercial real estate. The aggregate neutral technology shock A_t consists of permanent and transitory components (Aguiar and Gopinath, 2007):

$$A_t = A_t^p \nu_{a,t}, \;\; A_t^p = A_{t-1}^p g_{at},$$

$$\log g_{at} = (1-\rho_a) \log g_a + \rho_a \log(g_{a,t-1}) + \sigma_a \varepsilon_{at}, \tag{5}$$

$$\log \nu_{a,t} = \rho_{v_a} \log \nu_{a,t-1} + \sigma_{v_a} \varepsilon_{v_{a,t}}, \tag{6}$$

where ε_{at} and $\varepsilon_{v_{a,t}}$ are iid standard normal random variables.

II.3. **Final-goods firms.** There is a continuum of heterogeneous competitive firms. Each firm $i \in [0,1]$ combines intermediate goods $x_t^i(j)$ to produce final consumption goods with the standard aggregation technology

$$y_t^i = a_t^i \exp\left(\int_0^1 \log x_t^i(j) dj \right), \tag{7}$$

where a_t^i represents an idiosyncratic productivity shock drawn independently and identically from a fixed distribution with pdf $f(a)$ and cdf $F(a)$ on the $(0,\infty)$ support. Firm i purchases intermediate good j at the price $P_{Xt}(j)$. The total spending on working capital is $\int_0^1 P_{Xt}(j) x_t^i(j) dj$. The firm finances its working capital with the standard collateral constraint as in Jermann and Quadrini (2012) and Buera and Moll (2013),

$$\int_0^1 P_{Xt}(j) x_t^i(j) dj \leq \lambda p_t h_t^i, \tag{8}$$

where $0 < \lambda < 1$, h_t^i is commercial real estate held by firm i, and p_t is the real estate price. We do not introduce an exogenous shock to λ because this shock cannot generate the observed variation and persistence of real estate price as shown by Liu, Wang, and Zha (2013) and Kaplan, Mitman, and Violante (2017). The collateral constraint is not always binding; whether a particular firm's collateral constraint binds depends on the realization of its individual productivity.

Firm i trades real estate properties and rents some of them to the producers. The firm's income comes from profits and rents; its flow-of-funds constraint is given by

$$d_t^i + p_t(h_{t+1}^i - h_t^i) = y_t^i - \int_0^1 P_{Xt}(j) x_t^i(j) dj + R_{ct} h_t^i, \tag{9}$$

where d_t^i denotes dividend and the initial condition h_0^i is given. Subject to (7), (8), and (9), the firm's objective is to maximize the discounted present value of future dividends

$$\max_{x_t^i(j), h_{t+1}^i \geq 0} E_0 \sum_{t=0}^{\infty} \frac{\beta^t \Lambda_t}{\Lambda_0} d_t^i,$$

where the marginal utility of consumption is

$$\Lambda_t = \frac{\Theta_t}{C_t - \gamma C_{t-1}} - \beta\gamma E_t \frac{\Theta_{t+1}}{C_{t+1} - \gamma C_t}$$

and $\beta^t \Lambda_t / \Lambda_0$ is the household's stochastic discount factor (SDF).

II.4. **Equilibrium.** The markets clear in real estate, government bond, and intermediate-goods sectors:

$$\int_0^1 h_t^i di = \int_0^1 H_t(j) dj = 1,\ B_t = 0,\ \int_0^1 x_t^i(j) di = X_t(j) = A_t \left[K_t^{1-\phi}(j)\, H_t^{\phi}(j) \right]^{\alpha} N_t^{1-\alpha}(j).$$

Since the equilibrium is symmetric across intermediate-goods producers, we have

$$P_{Xt}(j) = P_{Xt},\ H_t(j) = H_t,\ N_t(j) = N_t,\ K_t(j) = u_t K_t,$$
$$X_t(j) = X_t = A_t \left[(u_t K_t)^{1-\phi} H_t^{\phi} \right]^{\alpha} N_t^{1-\alpha}$$

for all j. The household's dividend income and aggregate output are

$$D_t = \int_0^1 d_t^i di \text{ and } Y_t = \int_0^1 y_t^i di.$$

The competitive equilibrium consists of price sequences $\{w_t, R_{ct}, R_{kt}, p_t, R_{ft}, P_{Xt}\}_{t=0}^{\infty}$ and allocation sequences $\{C_t, I_t, u_t, N_t, Y_t, B_{t+1}, K_{t+1}, X_t, D_t\}_{t=0}^{\infty}$ such that (a) given the prices, the allocations solve the optimizing problems for households, intermediate-goods producers, and final-goods firms and (b) all markets clear.

III. Estimation and analysis

III.1. **Data and estimation.** We take the Bayesian approach and estimate the log-linearized version of the model presented in Section II. The model has six commonly used macroeconomic shocks represented by AR(1) processes (1), (2), (3), (4), (5), and (6). It is estimated against a number of key U.S. time series over the period from 1995Q2 to 2017Q2:[5] the price-rent ratio in commercial real estate, the quality-adjusted relative price of investment, real

[5]The repeated sales price of commercial real estate is available from 1996Q2 until present. We allow four lags in estimation. Therefore, the sample including four lags begins in 1995Q2.

per capita consumption, real per capita investment (in consumption units), and per capita hours worked.[6] Since our model features long-run growth, we detrend our model to make it stationary. We use $\tilde{x}_t$ to denote the detrended variable of x_t and use $\hat{x}_t \equiv \log \tilde{x}_t - \log \tilde{x}$ to denote the log deviation from the steady-state value $\tilde{x}$. The detailed description of data and estimation method are provided in Appendices A and B.[7]

There are five structural parameters to be estimated: the inverse Frisch elasticity ν, the collateral elasticity χ, the steady-state elasticity of capacity utilization δ''/δ', the habit formation γ, and the investment adjustment cost Ω. In particular, the collateral elasticity parameter χ measures a percentage change in the endogenous total factor productivity (TFP) in response to a one-percent change in the collateral value relative to output (see Appendix C for interpretations). The other structural parameters are either calibrated or indirectly estimated by solving the steady state (see Appendix B and Supplemental Appendix G for details).

The five directly estimated parameters are reported in Table 1, along with 90% probability intervals. The posterior probability intervals indicate that all these structural parameters are tightly estimated. The estimated inverse Frisch elasticity of labor supply is 0.34, consistent with a range of values discussed in the literature (Keane and Rogerson, 2011). The collateral elasticity is tightly estimated around 0.045. According to Appendix C, we calculate that aggregate investment through the endogenous TFP increases by 0.2% for a 1% increase in the collateral value (relative to output).[8]

The steady state elasticity of capacity utilization δ''/δ' is 0.85, in line with a range of values reported in the literature (Jaimovich and Rebelo, 2009). The small value means that an increase in the marginal cost is small when capacity increases, which implies that capacity is responsive to shocks. The estimated habit formation γ and capital-adjustment cost Ω are very small. Hence, these factors are not important in driving the dynamics of consumption and investment.

Table 2 reports the estimated persistence and standard-deviation parameters of exogenous shock processes. Among all shocks, the discount-rate shock is the most persistent. But its estimated standard deviation is considerably smaller than those of all other shocks except the stationary investment-specific shock. The probability intervals for the estimated standard

[6]In Chaney, Sraer, and Thesmar (2012), commercial real estate prices are approximated by house prices because these two series are highly correlated (the correction is over 0.90). All our empirical results hold for house prices as well; see our previous working paper (Miao, Wang, and Zha, 2014). In this paper, we construct the time series of commercial real estate price ourselves.

[7]The complete equilibrium system, the detrended stationary equilibrium system, the steady state, and the log-linearized equilibrium system are presented in Supplemental Appendices E, F, G, and H.

[8]This estimated elasticity is in line with the observed evidence during the Great Recession period when investment dropped by 11% while commercial real estate price (relative to output) dropped by 39%.

deviation of the discount-rate shock are particularly tight. Such a small standard deviation implies that any large effects on real estate price and aggregate variables must come from the model's internal propagation mechanism, which is discussed in Section III.3.

III.2. **Impulse responses of discount-rate and technology shocks.** Our focus in this paper is on the financial and real impacts of discount-rate shocks *after* controlling for all other common shocks studied in the literature. Among other common shocks, the permanent neutral technology shock is most important in driving the business cycle. We thus compare the estimated dynamic responses to a discount-rate shock with those to a technology shock in Figures 3 and 4. Although the discount-rate shock process is assumed to be of AR(1) and the estimated capital-adjustment cost is extremely small, the discount-rate shock generates a sizable hump-shaped response of investment in magnitude comparable to the dynamic response to a technology shock. In the following subsection (Section III.3), we explain the model's propagation mechanism for generating such a hump shape.

The dynamic responses of labor hours to discount-rate and technology shocks also have a similar magnitude, although the initial two-quarter response of hours to a technology shock is negative due to the wealth effect of a permanent nature of this shock. Large differences show up in the dynamic responses of real estate price, rent, and consumption. To a technology shock, the dynamic responses of these three variables are comparable in magnitude, but the comparability breaks down for the responses to a discount-rate shock (Figure 4). The dynamic response of real estate price to a discount-rate shock is considerably larger than the response to a technology shock, while the dynamic response of rent to a discount-rate shock is much smaller than the response to a technology shock. As a result, the price-rent ratio responds more to a discount-rate shock than to a technology shock by orders of magnitude. It can be seen from Figures 3 and 4 that the price-rent ratio comoves with investment and therefore output in response to a discount-rate shock, as we observe in the data (Figure 2).

The dynamic response of aggregate output to a discount-rate shock is about one third of the response to a technology shock (top row of Figure 3). This difference is because the consumption response to a discount-rate shock is much smaller than to a technology shock. Although the consumption response to a discount-rate shock is negative initially for the first year, it rises subsequently for most of the five-year horizon. Without financial frictions, by contrast, the consumption response to a discount-rate shock is negative for almost the entire forecasting period of five years while the responses of investment and hours remain positive (solid lines in Figure 5). Moreover, the response of real estate price from the model without financial frictions is significantly smaller than the response from the benchmark model. Thus, the two ingredients commonly used in the modern macroeconomic literature, credit constraint and endogenous TFP, are essential to mitigating the opposite movements between consumption and investment responses as well as to generating a large volatility of real estate price.

III.3. **Propagation mechanism.** A tractable feature of our heterogeneous model is that one can obtain a closed-form solution to the aggregation problem. The closed-form solution is essential to make our estimation and empirical analysis feasible. In Supplemental Appendix E we list all the equilibrium equations for solving and estimating the model. In this subsection we emphasize the key equilibrium dynamics and highlight the role of financial frictions in the transmission mechanism.

Denote the average cost of intermediate goods by

$$a_t^* \equiv \exp\left[\int_0^1 \log P_{Xt}(j)dj\right] = P_{Xt}. \tag{10}$$

The following two key propositions establish the close link between asset prices and the production economy.

Proposition 1. The optimal output for firm i is given by

$$y_t^i = \begin{cases} \lambda \frac{a_t^i}{a_t^*} p_t h_t^i & \text{if } a_t^i \geq a_t^* \\ 0 & \text{otherwise} \end{cases},$$

where the average cost a_t^* and aggregate output Y_t are determined jointly by the two simultaneous equations:

$$\lambda \frac{p_t}{a_t^*} \int_{a_t^*}^{\infty} a f(a) da = Y_t, \tag{11}$$

and

$$Y_t = A_t \left(u_t K_t\right)^{\alpha(1-\phi)} H_t^{\alpha\phi} N_t^{1-\alpha} \left[\frac{1}{1-F(a_t^*)} \int_{a_t^*}^{\infty} a f(a) da\right], \tag{12}$$

where the term in square brackets is the endogenously determined TFP.

Proof. See Appendix D.1. □

Proposition 2. The asset pricing equation is

$$p_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[R_{ct+1} + p_{t+1} + \lambda p_{t+1} \int_{a_{t+1}^*}^{\infty} \frac{a - a_{t+1}^*}{a_{t+1}^*} f(a) da\right] \tag{13}$$

and the rental price of real estate is determined by

$$R_{ct} = \alpha\phi a_t^* A_t \left(u_t K_t\right)^{\alpha(1-\phi)} H_t^{\alpha\phi-1} N_t^{1-\alpha}. \tag{14}$$

Proof. See Appendix D.2. □

Proposition 1 states that the average cost of intermediate goods, a_t^*, is also a threshold productivity level, above which productive firms choose to produce. For a given value of Y_t, equation (11) describes the relationship between the threshold productivity a_t^* and the real estate price p_t. This relationship is represented by an upward sloping curve on the (a_t^*, p_t) graph (see the bottom panel of Figure 6). Since equation (11) is derived from the collateral constraint, we call this relationship the collateral constraint curve.

The asset pricing equation (13) in Proposition 2 departs from the standard one in that the SDF and rent are not the only factors moving the real estate price. In addition to the future rent, the future collateral premium represented by

$$\int_{a_{t+1}^*}^{\infty} \frac{a - a_{t+1}^*}{a_{t+1}^*} f(a) da$$

also influences the real estate price. For productive firms ($a_{t+1}^i \geq a_{t+1}^*$), the collateral premium reflects the average profit generated by a one-dollar credit. Our estimation shows that the collateral premium, not the rent, accounts for at least one third of the volatility in the real estate price.

Equation (14) shows that the discount-rate shock does not affect the current rent R_{ct} directly. It has an indirect effect through its impact on other variables such as N_t and a_t^*. On the other hand, the discount-rate shock has a direct effect on the expected appreciation of future prices through its impact on the SDF ($\beta \Lambda_{t+1}/\Lambda_t$) as in equation (13). Consequently, the discount-rate shock has the potential to explain the high volatility of price-rent ratios.

The relationship between a_t^* and p_t represented by the asset pricing equation is negative, holding everything else fixed. An increase in the current threshold productivity level a_t^* raises the future threshold productivity level a_{t+1}^*. As a_{t+1}^* rises, the future collateral premium falls. Since the real estate price is a forward looking variable, the current price falls as well. Thus, the asset pricing curve representing (13) is downward sloping on the (a_t^*, p_t) plane. The two curves, collateral constraint and asset pricing, determine a_t^* and p_t jointly in the financial market as plotted in the bottom panel of Figure 6.

To make transparent the connection between the real estate market and the production economy, one should note that the real wage and labor hours are jointly determined by the labor supply equation

$$\frac{\Lambda_t}{\Theta_t} w_t = \psi_t N_t^{\nu}$$

and the labor demand equation

$$(1-\alpha) Y_t = \frac{\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da}{1 - F(a_t^*)} w_t N_t.$$

Using these two equations to eliminate w_t, we obtain the equilibrium equation that determines labor hours:

$$N_t^{1+\nu} = \frac{1 - F(a_t^*)}{\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da} \frac{(1 - \alpha) Y_t \frac{\Lambda_t}{\Theta_t}}{\psi_t}. \tag{15}$$

The top panel of Figure 6 plots two curves on the (N_t, Y_t) graph, with the convex curve representing the production equation (12) and the concave curve representing the labor-market equation (15).

A discount-rate shock affects both real and financial sectors simultaneously. Figure 6 illustrates the transmission mechanism of this shock. Suppose that the initial equilibrium is Point A at the steady state. According to equation (13), a positive shock to discount rate delivers a direct impact on the real estate price through the SDF ($\beta\Lambda_{t+1}/\Lambda_t$), shifting the asset pricing curve upward and raising the threshold productivity. As

$$TFP_t \equiv \frac{1}{1 - F(a_t^*)} \int_{a_t^*}^{\infty} af(a)da$$

is driven by the response of a_t^* to the discount-rate shock, a rise of threshold productivity increases aggregate output through the endogenous TFP and thus demand for investment and credit to finance working capital. An increase of aggregate output shifts the collateral constraint curve upward according to equation (11). The direct effect of the discount-rate shock on asset prices dominates the indirect effect on aggregate output so that the net effect on the threshold productivity is positive (bottom panel of Figure 6). The equilibrium moves from Point A to Point B on impact, with an increase of both threshold productivity and real estate price.

As an increase of threshold productivity raises aggregate output on impact and shifts the production curve upward, it simultaneously shifts the labor-market curve upward so long as the endogenous TFP relative to the average cost a_t^*

$$\frac{1}{1 - F(a_t^*)} \int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a)\, da$$

increases with a_t^* and its impact on consumption (and its marginal utility Λ_t) is relatively small.

With capital accumulation, it is optimal for households to postpone consumption for investment. Thus, the hump-shaped response of investment propels a further increase of aggregate output and thus shifts the asset pricing curve further in subsequent periods. As a result of higher investment and output, the collateral constraint curve moves up further, generating an even higher real estate price. As long as the discount-rate shock is persistent as in our estimation, both the asset pricing curve and the collateral constraint curve continue to shift upward, moving the equilibrium from Point B to Point C (bottom panel of Figure 6), with a persistent increase in the real estate price. In equilibrium, however, the threshold productivity level a_t^* does not have to move much as shown in the figure.

At the same time, a higher level of investment continues to shift the production curve and the labor-market curve upward, moving the equilibrium from Point B to Point C (top panel of Figure 6) and generating even higher output. The ripple effect through such interactions between the financial setor and the production sector is the key feature of this propagation mechanism.

IV. Predictability and volatility

The preceding section documents the dynamic responses of key financial and economic variables to a discount-rate shock and explains the propagation mechanism behind the linkage between financial and production sectors. In the real estate sector, a key fact is that the price-rent ratio has a long-horizon predictability of real estate returns (Ghysels, Plazzi, Torous, and Valkanov, 2013). The observed linkage between real estate and production sectors is the comovements between price-rent ratio, investment, and output. These comovements imply that the volatility of price-rent ratio and the volatilities of investment and output are not isolated. From the lense of our general equilibrium model, the discount-rate shock is the only shock that relates price-rent variation to the volatilities of investment and output. An important question is how much of the observed predictability and volatility can be explained by the dynamics generated by discount-rate shocks.

IV.1. **Empirical results of predictability.** Ghysels, Plazzi, Torous, and Valkanov (2013) argue that the price-rent ratio should have power in predicting real estate returns and propose to run the predictive regression

$$r_{t\to t+k} = \alpha_0 + \alpha_1 \log\left(R_{ct}/p_t\right) + \varepsilon_{t,k},$$

where the real estate return from t to $t+k$ is defined as $r_{t\to t+k} = \log\left(p_{t+k}/p_t\right)$ and the slope coefficient α_1 is positive.[9] Table 3 reports the regression results of the slope coefficient α_1 and the fit measure R^2 from both actual and model-simulated data at different horizons ($k = 8, 12, 16, 20, 24$). Using the posterior mode estimates of model parameters, we simulate a sample of time series of model variables with only discount-rate shocks for 88 periods (the same length as the actual sample size). For each simulated sample, we run the previous predictive regression. We repeat the simulation 100,000 times and compute the median

[9]Following Ghysels, Plazzi, Torous, and Valkanov (2013) and the real estate literature, we use returns without rent in our empirical estimation. Since the time series data on real estate price and rent are available only in index format, we are unable to construct returns with rent. But in the simulated data from our model, we can calculate returns with rent as well as excess returns and our predictability results do not change much. See Section V.3 for further disccusions.

values of α_1 and R^2 as well as the corresponding 90% probability intervals. As one can see, the 90% probability interval of R^2 contains the regresson estimate from the actual data at each horizon. Conversely, the 90% confidence interval of α_1 from the actual data contains the median value of α_1 from the simulated data at each horizon. Both α_1 and R^2 increase with the forecasting horizon k.[10] Overall, the model results are consistent with the data, especially for the model's ability to predict long-horizon real estate returns by the price-rent ratio.

The samples generated by other shocks in our model do not have such a predictive power. Table 4 reports the regression results from the samples simulated by growth technology shocks. At each horizon ($k = 8, 12, 16, 20, 24$), the median estimate of α_1 is negative and outside the 90% probability interval implied by the actual data. The median values of R^2 are all less than 0.05 and their 90% probability intervals implied by the simulated data do not contain the value of R^2 calculated from the actual data except for the short horizon of 8 quarters. Thus, the samples generated by technology shocks do not have a power in predicting future real estate returns.[11] The failure to predict applies to the samples generated by other shocks as well. A comparison of Tables 3 and 4 indicates clearly the essential role of discount-rate shocks in predicting the long-horizon returns of real estate by price-rent variation.

IV.2. **Empirical results of volatility.** Using the same samples simulated with discount-rate shocks as in the preceding section, one can compute the volatilities of investment, consumption, output, rental price, real estate price, and price-rent ratio. Table 5 reports these volatilities for the simulated data against the actual data. For the model-simulated data, the table reports the median values of volatilities with 90% probability intervals (the last three columns). Judging by the median values, one can see that although discount-rate shocks account for only 11% of the observed volatility for consumption and 7% for rent, these shocks explain 48% of the observed volatility for investment, 25% for output, 94% for real estate price, and 100% for price-rent ratio. The volatility of discount-rate shocks is by itself diminutive, with the median value of 0.00067. Through the model's propagation mechanism discussed in Section III.3, however, these shocks are capable of generating the dynamic data that link the volatilities in the financial sector to those in the production sector.[12]

[10] In Section V we use a simplified version of our general equilibrium model to obtain a closed-form solution to the first-order approximation. This solution enables us to provide a formal proof of this empirical result.

[11] A formal proof is given for the simplified model discussed in Section V.

[12] Since the interest rate is determined by $1 = \beta R_{ft} E_t \frac{\Lambda_{t+1}}{\Lambda_t}$, the volatility of the interest rate R_f has a magnitude similar to the volatility of the discount rate. Indeed, its median estimate is 0.0005 with the tight 90% probability interval [0.00046, 0.0006]. The relatively small fluctuation of R_f is the reason why our model is capable of generating the long-horizon predictability of *excess* returns, as discussed in Section V.3.

In the counterfactual economy with no financial frictions, by contrast, discount-rate shocks can explain only 63% of price-rent variation or the volatility of real estate price as shown in Table 6. To see how financial frictions play such an important role in the real estate sector, we establish the following proposition by log-linearizing equation (13).

Proposition 3. Define $\eta = \frac{a^* f(a^*)}{1-F(a^*)}$. Under a suitable normalization of distribution f, the real estate price can be decomposed into three components:

$$\hat{p}_t = \hat{p}_{1t} + \hat{p}_{2t} + \hat{p}_{3t},$$

where

$$\hat{p}_{1t} = E_t\left(\hat{\theta}_{t+1} + \hat{\Lambda}_{t+1} - \hat{\Lambda}_t\right) + \beta E_t \hat{p}_{1t+1},$$

$$\hat{p}_{2t} = \frac{\beta \tilde{R}_c}{\tilde{p}} E_t \hat{R}_{ct+1} + \beta E_t \hat{p}_{2t+1},$$

$$\hat{p}_{3t} = \lambda(1-\beta) E_t\left[\hat{p}_{t+1} - \frac{(1+\mu)(\mu-\chi)}{\mu^2}\left(\hat{p}_{t+1} - \hat{Y}_{t+1}\right)\right] + \beta E_t \hat{p}_{3t+1}.$$

Proof. The decomposition uses a log-linearized method similar to the Campbell and Shiller (1988) decomposition. The detailed proof is provided in Appendix D.3. □

The first component $\hat{p}_{1t}$ is the contribution from the stochastic discount factor, which is mainly driven by discount-rate variation. The second component $\hat{p}_{2t}$ reflects the contribution from rent. The third component $\hat{p}_{3t}$ represents the contribution from the collateral premium.

Using the simulated data with discount-rate shocks alone, we calculate $std(\hat{p}_t) = 12.91\%$, $std(\hat{p}_{1t}) = 8.20\%$, $std(\hat{p}_{2t}) = 0.34\%$, and $std(\hat{p}_{3t}) = 4.40\%$, where std stands for standard deviation. In this case,

$$std(\hat{p}_t) \approx std(\hat{p}_{1t}) + std(\hat{p}_{2t}) + std(\hat{p}_{3t}).$$

The estimated volatility contribution from the collateral premium is about one third:[13]

$$\frac{std(\hat{p}_{3t})}{std(\hat{p}_t)} = \frac{4.4}{12.91} = 0.34.$$

This estimation result reveals that the collateral premium is an important force in driving the fluctuation of real estate price. The absence of the contribution from the collateral premium, therefore, is the main reason for the only 63% explanation of the volatility in the real estate sector as reported in Table 6 for the counterfactual economy without financial frictions.

[13]We perform a similar exercise for the growth rate of real estate price from our simulated data. The corresponding volatilities are $std(\Delta\hat{p}_t) = 3.9\%$, $std(\Delta\hat{p}_{1t}) = 2.46\%$, $std(\Delta\hat{p}_{2t}) = 0.10\%$, and $std(\Delta\hat{p}_{3t}) = 1.36\%$. Thus, the volatility contribution from the collateral premium is also about one third: $\frac{std(\Delta\hat{p}_{3t})}{std(\Delta\hat{p}_t)} = \frac{1.360}{3.910} = 0.348$.

V. Understanding amplification and predictability

To understand how discount-rate and technology shocks play disparate roles in predicting real estate returns and in amplifying the fluctuations of real estate variables relative to the volatility of output, we simplify our model by fixing the supply of labor at $N_t = 1$ and removing investment from the original model. This simplification enables us to solve for a closed-form solution to the log-linearized model and thus gain insight into the empirical results we have obtained in this paper.

V.1. **The simplified model.** In the simplified stationary model we focus on two exogenous shocks: the technology shock A_t and the discount-rate shock θ_t. The household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \Theta_t u\left(C_t\right)$$

subject to $C_t = w_t + D_t$, where $u\left(C_t\right) = \log C_t$. Hence, $\Lambda_t = \frac{\Theta_t}{C_t}$. The intermediate-goods producer's problem is

$$\max_{N_t(j), H_t(j)} \; P_{Xt}(j) X_t\left(j\right) - w_t N_t(j) - R_{ct} H_t(j),$$

subject to $X_t\left(j\right) \equiv A_t H_t^{\alpha}(j) N_t^{1-\alpha}\left(j\right)$. The problem for final-goods firms remains the same as in the original model. With this simplification, we have the following closed-form results.

Proposition 4. The log-linearized solutions for $\hat{a}_t^*$, $\hat{Y}_t$, $\hat{p}_t$, and $\hat{R}_{ct}$ are

$$\hat{a}_t^* = \frac{\pi}{1+\eta} \hat{\theta}_t, \tag{16}$$

$$\hat{Y}_t = \hat{C}_t = \hat{A}_t + \frac{\eta\mu}{1+\mu} \hat{a}_t^*, \tag{17}$$

$$\hat{p}_t = \hat{A}_t + \left[1+\eta\right] \hat{a}_t^*, \tag{18}$$

$$\hat{R}_{ct} = \hat{A}_t + \hat{a}_t^*, \tag{19}$$

where

$$\pi = \chi \frac{1+\eta}{\eta} \frac{1+\mu}{\mu} \frac{\rho_\theta}{1-\rho_\theta \kappa},$$

$$\kappa = 1 - (1-\beta)(1-\lambda) - \chi(1-\beta)(1-\lambda)\left(1 - \frac{1+\mu}{\mu\eta}\right) - \lambda(1-\beta) \frac{\chi}{\eta} \frac{(1+\mu)^2}{\mu^2}.$$

Proof. See Appendix D.4. □

V.2. **Theoretical results of volatility.** The volatility derivation follows directly from Proposition 4. For the data generated by technology shocks, we have

$$\frac{std(\hat{p}_t)}{std(\hat{R}_{ct})} = 1, \frac{std(\hat{p}_t - \hat{R}_{ct})}{std(\hat{Y}_t)} = 0.$$

The real estate price and the rental price fluctuate in the same magnitude so that technology shocks are unable to produce price-rent variation. For the time series generated by discount-rate shocks, by contrast, we have

$$\frac{std(\hat{p}_t)}{std(\hat{R}_{ct})} = 1 + \eta, \ \frac{std(\hat{p}_t - \hat{R}_{ct})}{std(\hat{Y}_t)} = \eta.$$

Clearly, the real estate price is always more volatile than the rental price and as long as $\eta > 1$, which is the case in our estimation, the price-rent ratio is more volatile than output. The intuition is that, unlike technology shocks, discount-rate shocks are intertemporal shocks that do not influence the current rent or output directly. They have a smaller indirect effect on rent through endogenous TFP as revealed by the term $\hat{a}_t^*$, but a larger effect on the future appreciation of real estate price.

V.3. **Theoretical results of predictability.** To gain economic intuition behind our empirical predictability results, we begin with the standard asset pricing equation

$$E_t M_{t+1} R_{it+1} = 1,$$

where R_{it+1} denotes the one-period return of any asset i and the pricing kernel, denoted by M_{t+1}, is

$$M_{t+1} = \frac{\theta_{t+1} \beta u'(C_{t+1})}{u'(C_t)}.$$

Because the asset pricing equation also holds for the risk-free interest rate

$$R_{ft} E_t M_{t+1} = 1, \tag{20}$$

the log-linearized solution leads to

$$E_t \hat{R}_{it+1} - \hat{R}_{ft} = 0.$$

For standard models, therefore, there is no equity premium and hence no predictability.[14]

The asset pricing equation in our model is different as it involves the collateral premium for the first-order solution. For the risk-free interest rate, equation (20) continues to hold. But for the real estate return, we rewrite the asset pricing equation represented by (13) as

$$E_t M_{t+1} \left(R_{t+1}^{re} + L_{t+1} \right) = 1, \tag{21}$$

where R_{t+1}^{re} denotes the real estate return

$$R_{t+1}^{re} \equiv \frac{p_{t+1} + R_{ct+1}}{p_t}$$

[14] When taking into account second-order Jensen term, Albuquerque, Eichenbaum, Luo, and Rebelo (2016) show that the equity premium remains constant in their baseline model even when the valuation ratio fluctuates with discount-rate shocks. This result implies no predictability for excess returns. The reason is simple: discount-rate shocks drive returns and risk-free rates in the same magnitude and hence cancel out both terms.

and L_{t+1} denotes the collateral (liquidity) premium

$$L_{t+1} \equiv \lambda \frac{p_{t+1}}{p_t} \int_{a_{t+1}^*}^{\infty} (\frac{a}{a_{t+1}^*} - 1) f(a) da.$$

We now use the closed-form solution to show that predictability is related to the collateral premium. Log-linearizing equation (21) leads to

$$\frac{R^{re}}{R^{re} + L} E_t \hat{R}_{t+1}^{re} + \frac{L}{R^{re} + L} E_t \hat{L}_{t+1} + E_t \hat{M}_{t+1} = 0.$$

Log-linearizing equation (20) gives us

$$\hat{R}_{ft} + E_t \hat{M}_{t+1} = 0.$$

It follows from the preceding two equations that

$$E_t \hat{R}_{t+1}^{re} - \hat{R}_{ft} = \frac{L}{R^{re}} \left(\hat{R}_{ft} - E_t \hat{L}_{t+1} \right).$$

One can see from Proposition 4 that $\hat{\theta}_{t+1}$ has a direct impact on a_{t+1}^* and hence on $\hat{L}_{t+1}$. Since the stochastic process for $\hat{\theta}_t$ is very persistent, the discount-rate shock $\hat{\theta}_t$ moves the excess return of real estate directly by influencing both R_{ft} and $E_t \hat{L}_{t+1}$. While this result is intuitive, it is challenging to turn this intuition into the theoretical result of predictability. Specifically, if one runs the OLS regression of excess return $\hat{R}_{t+k}^{re} - \hat{R}_{ft}$ on the valuation ratio $\hat{v}_t \equiv \hat{R}_{ct} - \hat{p}_t$, is the coefficient of the valuation ratio positive? For our model, the answer is affirmative and the following proposition establishes this predictability result.

Proposition 5. For the time series generated by discount-rate shocks, the regression coefficient

$$\alpha_1 \equiv E \left[\left(\hat{R}_{t+k}^{re} - \hat{R}_{ft} \right) \mid \hat{v}_t \right]$$

is positive for $k \geq 1$ with the parameter values obtained by our estimation.

Proof. See Appendix D.5. □

We use the simulated data generated by discount-rate shocks with our estimated parameter values and run the OLS regression of $\hat{R}_{t+k}^{re} - \hat{R}_{ft}$ on the valuation ratio $\hat{v}_t \equiv \hat{R}_{ct} - \hat{p}_t$ for different horizons k. The results have a pattern remarkably similar to what is reported in Table 3 (thus, not reported to save the space). Since the rent and price time series in actual data are available only in terms of index, the real estate return is often computed without rent in the real estate literature (Ghysels, Plazzi, Torous, and Valkanov, 2013). For the theoretical underpinning of the empirical results pertaining to the real estate return without rent as reported in Table 3, we have the following proposition.

Proposition 6. Denote the k-period return of real estate (without rent) by $\hat{r}_{t+k} \equiv \hat{p}_{t+k} - \hat{p}_t$. For the time series generated by discount-rate shocks, the following two results hold:

- $\alpha_1 \equiv E \left[\hat{r}_{t+k} \mid \hat{v}_t \right] = \left(1 - \rho_\theta^k \right) \frac{\eta}{1+\eta} \hat{v}_t$,

- $R^2_{r,v} = \frac{1}{2}\left(1 - \rho_\theta^k\right)$, where $R^2_{r,v}$ is the R^2 measure for the OLS regression of $\hat{r}_{t+k}$ on $\hat{v}_t$.

For the time series generated by technology shocks, however, $\alpha_1 \equiv E\left[\hat{r}_{t+k} \mid \hat{v}_t\right] = 0$.

Proof. See Appendix D.6. □

According to Proposition 6, there is no predictability of real estate returns from the time series driven by technology shocks. For discount-rate shocks, the opposite is true. The regression coefficient α_1 increases with the forecast horizon but is always less than one; the fit measure R^2 increases with the forecast horizon to around 0.5 for long forecast horizons. These closed-form results are in line with our empirical results obtained from the more realistic benchmark model (Table 3).

In summary, the simplified model illustrates the special role played by discount-rate shocks in generating the data dynamics that feature the two distinct asset pricing properties observed in actual data: predictability and volatility. To be sure, the simplified model excludes investment and is incapable of explaining the relationship between price-rent variation and the business cycle (fact (3) discussed in the introduction). But the insight gained from the closed-form results provides a theoretical explanation of the empirical results we have obtained in this paper.

VI. Conclusion

We argue to imbed households' discount-rate shocks and firms' collateral constraints in the dynamic general equilibrium framework. This addition keeps the model tractable to be estimated and at the same time substantially improves the model's performance in accounting for (i) the large volatility of price-rent ratio relative to investment and output, (ii) the long-horizon predictability of real estate returns, and (iii) the dynamic relationship between price-rent variations and the business cycle. For dynamic general equilibrium models, the long-horizon predictability has proven difficult to obtain. We find that the collateral premium is the most important factor in generating the predictability result as well as the comovements among financial and economic variables. Fluctuations of discount-rate shocks are extremely small in magnitude but the model's internal propagation mechanism translates these small shocks into the large volatilities of price-rent ratio, investment, and output.

Because the 2008 financial crisis was triggered by the collapse of real estate prices and the sharp fall of investment, this paper focuses on commercial real estate and its relation to investment and inevitably abstracts from other dimensions that merit further study in the future. One such dimension is to include mortgage markets for households. Another dimension is to extend the model to explain asset pricing facts in the stock market. We hope that the mechanism and insight developed in this paper lays the groundwork for extending the model along these and other important dimensions.

Table 1. Posterior estimates of structural parameters

		Posterior estimates		
Parameter	Representation	Mode	Low	High
ν	Inv Frisch elasticity	0.343	0.088	1.100
χ	Collateral elasticity	0.045	0.044	0.045
δ''/δ'	Capacity utilization	0.850	0.676	1.243
γ	Habit formation	0.558	0.480	0.634
Ω	Capital adjustment	0.245	0.164	0.386

Note: "Low" and "High" denote the bounds of the 90% probability interval for each parameter.

Table 2. Posterior estimates of shock parameters

		Posterior estimates		
Parameter	Representation	Mode	Low	High
ρ_z	Permanent investment tech	0.0941	0.0274	0.2765
ρ_{ν_z}	Stationary investment tech	0.0000	0.0114	0.4779
ρ_a	Permanent neutral tech	0.5664	0.4294	0.7403
ρ_{ν_a}	Stationary neutral tech	0.8211	0.7560	0.8835
ρ_θ	Discount rate	0.9994	0.9986	0.9997
ρ_ψ	Labor supply	0.9941	0.9838	0.9967
σ_z	Permanent investment tech	0.0053	0.0044	0.0059
σ_{ν_z}	Stationary investment tech	0.0001	0.00007	0.0019
σ_a	Permanent neutral tech	0.0027	0.0019	0.0038
σ_{ν_a}	Stationary neutral tech	0.0087	0.0078	0.0108
σ_θ	Discount rate	0.0002	0.00018	0.0003
σ_ψ	Labor supply	0.0080	0.0065	0.0124

Note: "Low" and "High" denote the bounds of the 90% probability interval for each parameter.

TABLE 3. Prediction of real estate returns by the price-rent ratio at different horizons with model results based on simulated data from estimated discount-rate shocks

Predictive regression: $r_{t\to t+k} = \alpha_0 + \alpha_1 \log(R_{ct}/p_t) + \varepsilon_{t+k}$						
Horizon	Data (α_1)	Model (α_1)	Data (R^2)	Model (R^2)		
Quarter (year)		Median		Median	Low	High
8 (2)	0.20 (0.07, 0.33)	0.37	0.08	0.20	0.04	0.39
12 (3)	0.37 (0.20, 0.54)	0.55	0.15	0.30	0.07	0.52
16 (4)	0.58 (0.39, 0.78)	0.70	0.26	0.38	0.09	0.62
20 (5)	0.77 (0.58, 0.96)	0.82	0.40	0.42	0.11	0.69
24 (6)	0.82 (0.65, 1.00)	0.89	0.50	0.51	0.13	0.74

Note: The table reports the OLS estimates of α_1 and R^2 from both actual data and model-simulated data. The numbers in parentheses in the column headed by Data (α_1) represent the 90% confidence interval of the estimated coefficient. The real estate return from t to $t+k$ is defined as $r_{t\to t+k} = \log(p_{t+k}/p_t)$. "Low" and "High" denote the bounds of the 90% probability interval of the simulated data from the model.

TABLE 4. Prediction of real estate returns by the price-rent ratio at different horizons with model results based on simulated data from estimated technology shocks

Predictive regression: $r_{t\to t+k} = \alpha_0 + \alpha_1 \log(R_{ct}/p_t) + \varepsilon_{t+k}$						
Horizon	Data (α_1)	Model (α_1)	Data (R^2)	Model (R^2)		
Quarter (year)		Median		Median	Low	High
8 (2)	0.20 (0.07, 0.33)	-0.26	0.08	0.02	0.00	0.09
12 (3)	0.37 (0.20, 0.54)	-0.34	0.15	0.03	0.00	0.10
16 (4)	0.58 (0.39, 0.78)	-0.41	0.26	0.03	0.00	0.12
20 (5)	0.77 (0.58, 0.96)	-0.46	0.40	0.03	0.00	0.13
24 (6)	0.82 (0.65, 1.00)	-0.48	0.50	0.04	0.01	0.14

Note: The table reports the OLS estimates of α_1 and R^2 from both actual data and model-simulated data. The numbers in parentheses in the column headed by Data (α_1) represent the 90% confidence interval of the estimated coefficient. The real estate return from t to $t+k$ is defined as $r_{t\to t+k} = \log(p_{t+k}/p_t)$. "Low" and "High" denote the bounds of the 90% probability interval of the simulated data from the model.

TABLE 5. Volatilities explained by discount-rate shocks (%)

Description	Volatility	Data	Model			
			% Explained	Median	Low	High
Investment	$std\,(\Delta \log I_t)$	1.679	48.1	0.808	0.734	0.884
Output	$std\,(\Delta \log Y_t)$	0.697	25.1	0.175	0.159	0.191
Consumption	$std\,(\Delta \log C_t)$	0.444	11.9	0.053	0.045	0.061
Rental price	$std\,(\Delta \log R_{ct})$	1.245	6.7	0.084	0.077	0.091
Real estate price	$std\,(\Delta \log p_t)$	4.171	93.7	3.910	3.611	4.193
Price-rent ratio	$std\,(\Delta \log(p_t/R_{ct}))$	3.909	100	3.923	3.625	4.211
Discount rates	$std\,(\log \theta_t)$			0.00067	0.00046	0.001

Note: "Low" and "High" denote the bounds of the 68% probability interval of the simulated data from the model.

TABLE 6. Real estate volatilities explained by discount-rate shocks (%) in the model with no financial frictions

Description	Volatility	Data	Model			
			% Explained	Median	Low	High
Real estate price	$std\,(\Delta \log p_t)$	4.171	62.5	2.607	2.409	2.796
Price-rent	$std\,(\Delta \log(p_t/R_{ct}))$	3.909	63.2	2.472	2.285	2.656

Note: "DR" stands for discount rate. "Low" and "High" denote the bounds of the 90% probability interval of the samples generated by discount-rate shocks in the model without financial frictions.

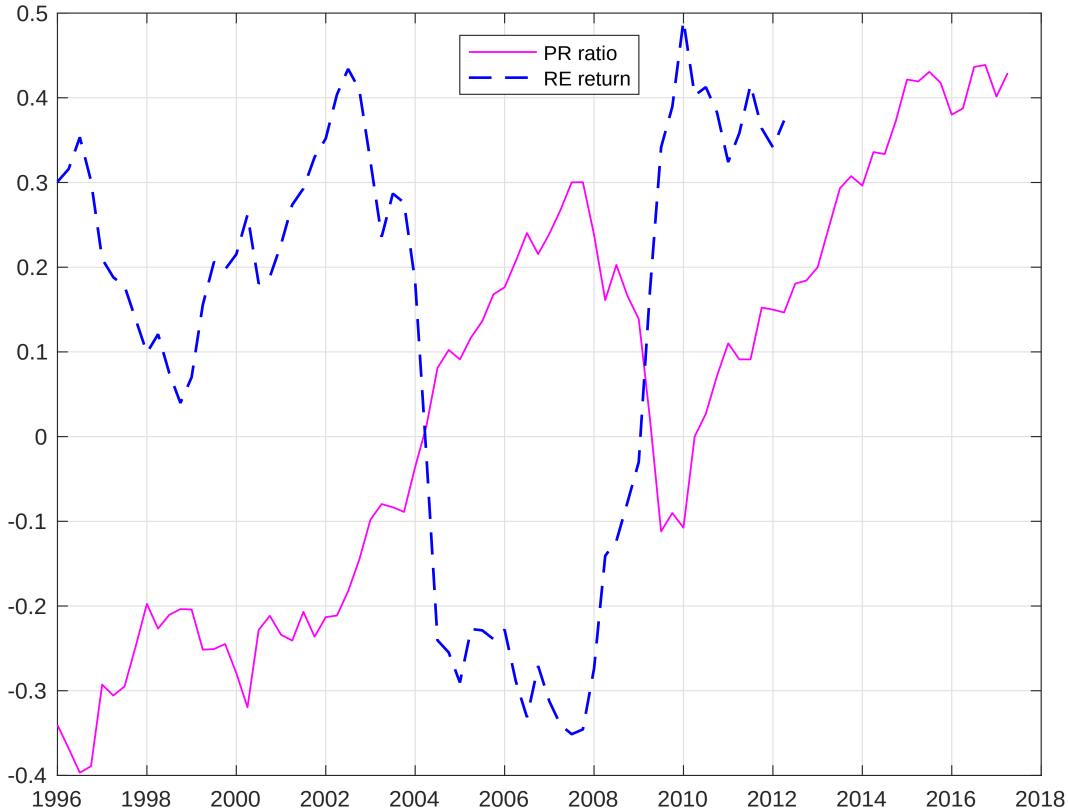

FIGURE 1. "PR" stands for the price-rent ratio and "RE" stands for real estate. The PR ratio is in log value and scaled to make it visually comparable with the returns. The real estate return is over the next five-year horizon. The price-rent ratio is in log at the current time.

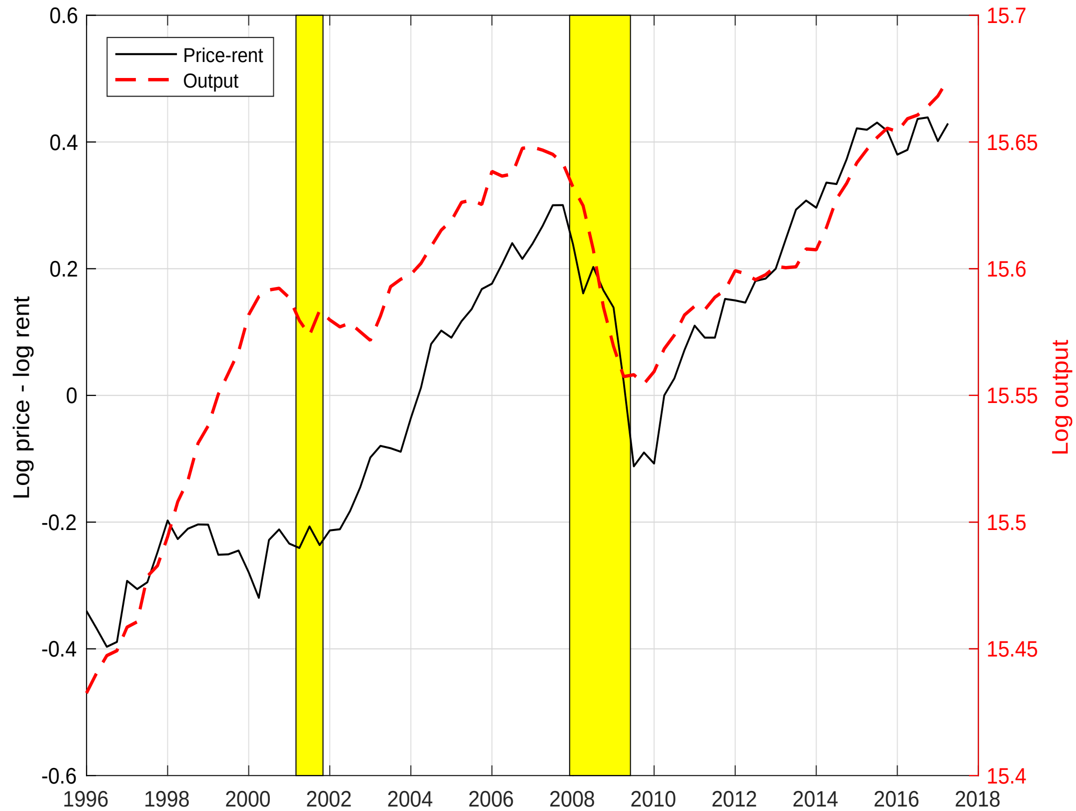

FIGURE 2. The time series of the log price-rent ratio in the U.S. real estate sector (the left scale) and the time series of log output in the U.S. economy (the right scale).

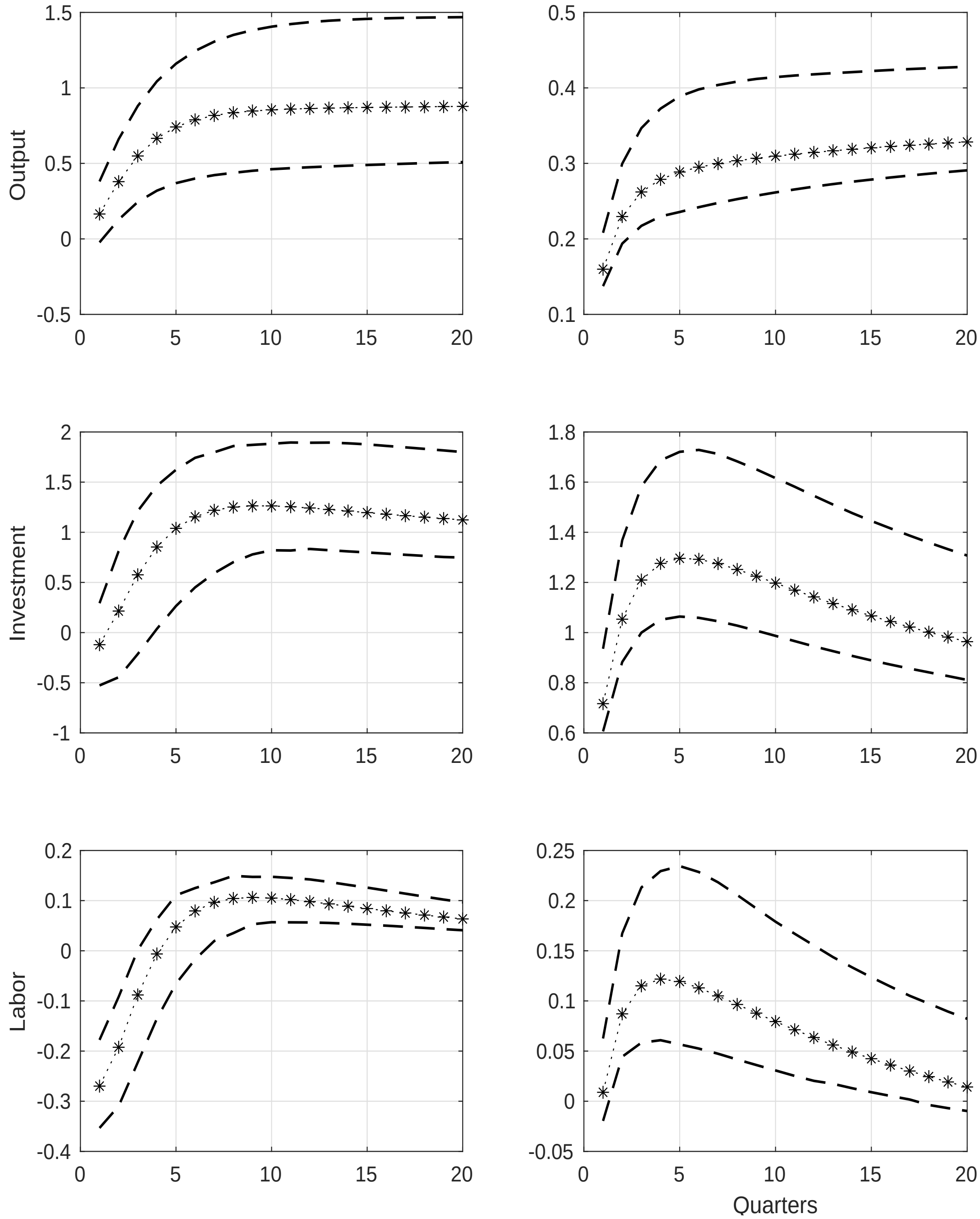

FIGURE 3. Impulse responses (%) to a one-standard-deviation shock to neutral technology growth (left panel) and to discount rates (right panel). The starred line represents the estimated response. The dashed lines represent the 0.90 probability error bands.

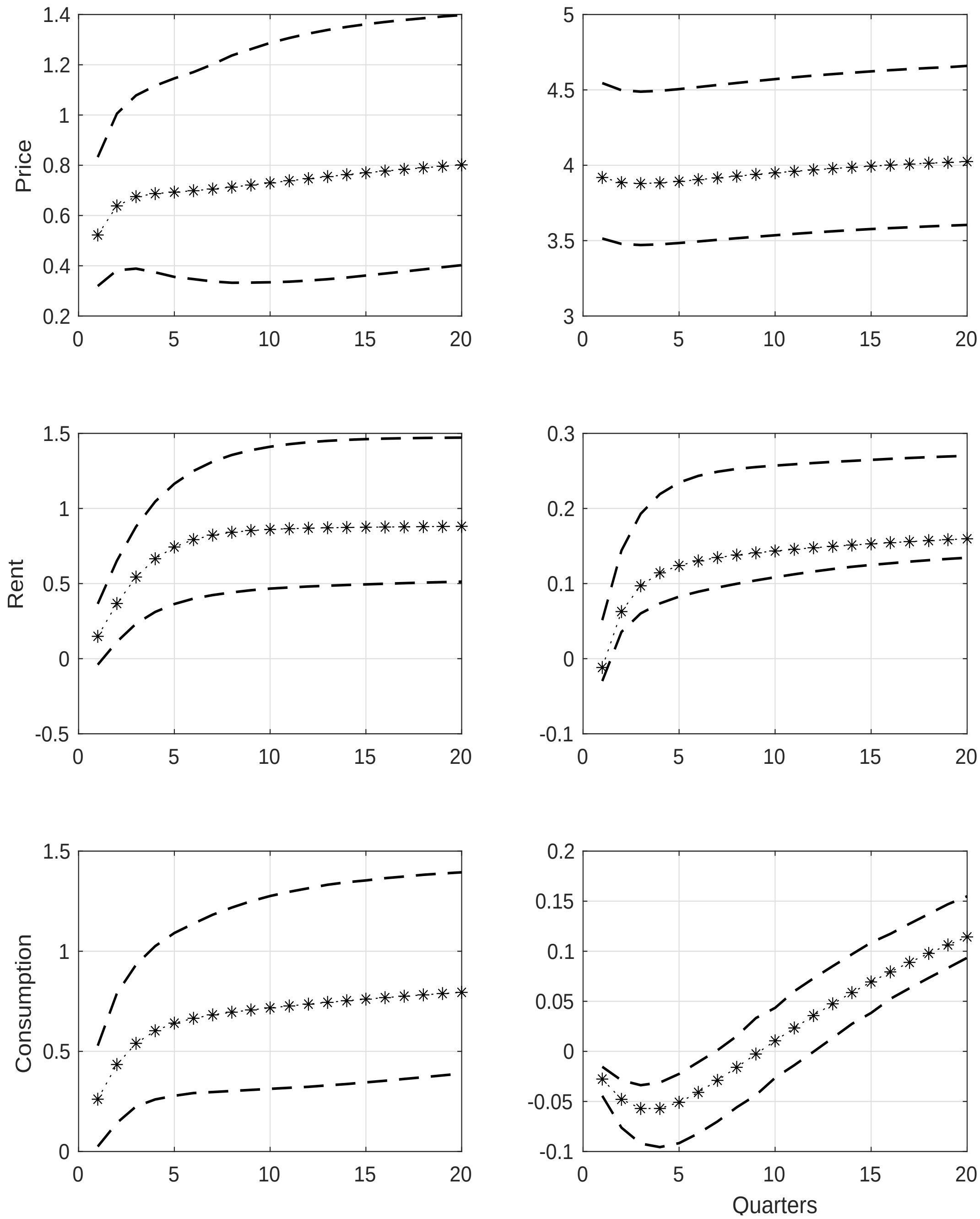

Figure 4. Impulse responses (%) to a one-standard-deviation shock to neutral technology growth (left panel) and to discount rates (right panel). The starred line represents the estimated response. The dashed lines represent the 0.90 probability error bands.

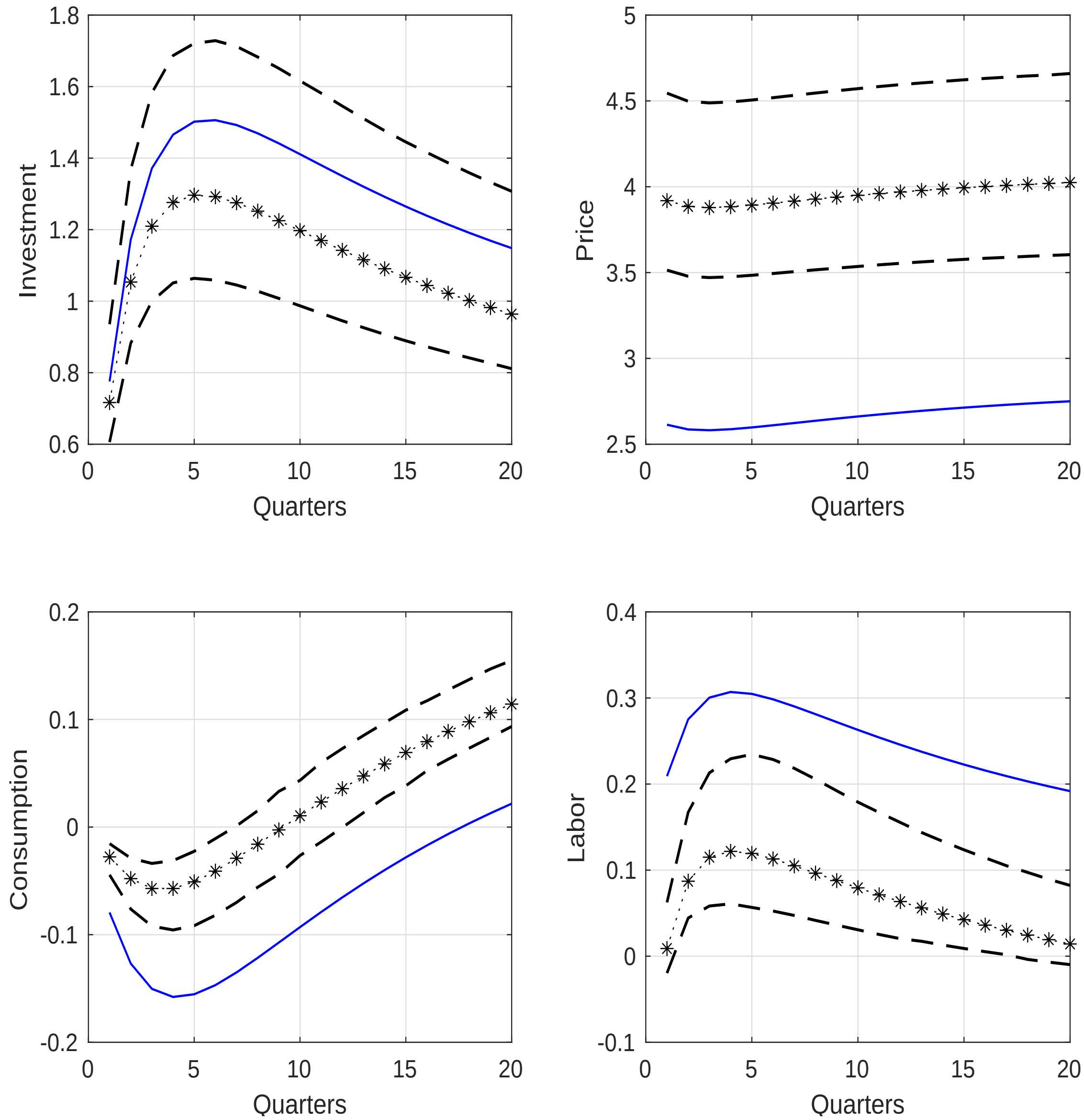

FIGURE 5. Impulse responses (%) to a one-standard-deviation shock to discount rates. The starred line represents the estimated response. The dashed lines represent the 0.90 probability error bands. The solid line represents the counterfactual response for an economy without financial frictions.

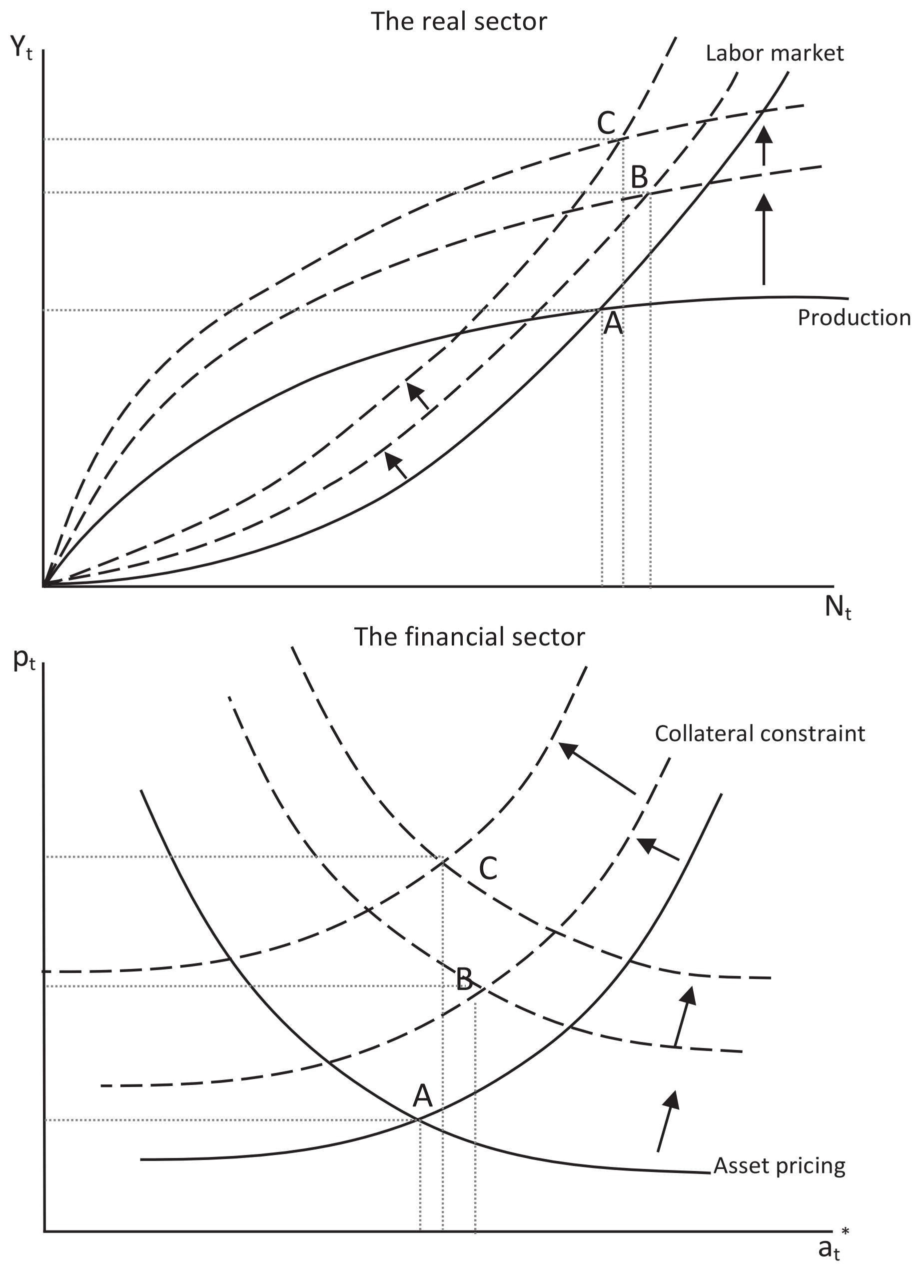

FIGURE 6. An illustration of the propagation mechanism: the production and labor-market equations are (12) and (15) and the collateral constraint and asset pricing equations are (11) and (13).

Appendix A. Data

All the quarterly time series used in this paper were constructed by Patrick Higgins at the Federal Reserve Bank of Atlanta, some of which were collected directly from the Haver Analytics Database (Haver for short). In this section, we describe the details of data construction.

The model estimation is based on six U.S. aggregate time series: the real price of commercial real estate (p_t^{Data}), the real rental price (R_{ct}^{Data}), the quality-adjusted relative price of investment ($(1/Z_t)^{\text{Data}}$), real per capita consumption (C_t^{Data}), real per capita investment (I_t^{Data}), and per capita total hours (H_t^{Data}). All variables except hours and relative price of investment are deflated by the price of nondurable consumption goods and non-housing services.

These series are constructed as follows:

- $p_t^{\text{Data}} = \frac{\text{pCommRE}}{\text{PriceNonDurPlusServExHous}}$.
- $R_{ct}^{\text{Data}} = \frac{\text{TortoTotalRent}}{\text{PriceNonDurPlusServExHous}}$.
- $(1/Z_t)^{\text{Data}} = \frac{\text{GordonPriceCDplusES}}{\text{PriceNonDurPlusServExHous}}$.
- $C_t^{\text{Data}} = \frac{(\text{NomConsNHSplusND})/\text{PriceNonDurPlusServExHous}}{\text{POPSMOOTH\_USECON}}$.
- $I_t^{\text{Data}} = \frac{(\text{CDX\_USNA + nominveqipp})/\text{PriceNonDurPlusServExHous}}{\text{POPSMOOTH\_USECON}}$.
- $H_t^{\text{Data}} = \frac{\text{AggHours}}{\text{POPSMOOTH\_USECON}}$.

Sources for the constructed data, along with the Haver keys (all capitalized letters) to the data, are described below.

pCommRE: Commercial real-estate price index. The construction of this series is based on the series named as “FL075035503” from the Flow of Funds Accounts database provided by the Board of Governors of the Federal Reserve System.[15] Note that the price index through 1996Q1 is *not based on repeated sales* but instead relies on a weighted-average of three appraisal-based commercial property price series (per square foot): retail property, office property, and warehouse/industrial property. These series come from National Real Estate Investor (NREI). The weights applied to the NREI are not revised and are calculated using annual data from the Survey of Current Business. From 1996Q2 on, the commercial property price index is the Costar Commercial Repeat Sales Index published by “National Real Estate Investor.”

TortoTotalRent: Rental price index for commercial real estate. Tornqvist aggregate of Torto Wheaton Research Index for rental prices of retail properties, Torto Wheaton Research Index for rental prices of office properties (commercial excluding retail), and

[15]See the Federal Reserve Board of Governors’ website `http://www.federalreserve.gov/apps/fof/SeriesAnalyzer.aspx?s=FI075035503&t=`. Unpublished tables for level time series are available at `http://www.federalreserve.gov/datadownload/Choose.aspx?rel=Z.1`

Torto Wheaton Research Index for rental prices of industrial properties. Detailed description of the series is available at `http://www.cohenasset.com/pdfs/Torto%20Wheaton%20Research%20Methodology.pdf`. The data, downloaded from the CBRE Econometrics Advisors website, were constructed by the Torto Wheaton Research (TWR) hedonic approach (Wheaton and Torto, 1994) and (Malpezzi, 2002, Chapter 5).

PriceNonDurPlusServExHous: Consumer price index. Price deflator of non-durable consumption and non-housing services, constructed by Tornqvist aggregation of price deflator of non-durable consumption and non-housing related services (2009=100).

GordonPriceCDplusES: Price of investment goods. Quality-adjusted price index for consumer durable goods, equipment investment, and intellectual property products investment. This is a weighted index from a number of individual price series within this category. For each individual price series from 1947 to 1983, we use Gordon (1990)'s quality-adjusted price index. Following Cummins and Violante (2002), we estimate an econometric model of Gordon's price series as a function of time trend and several macroeconomic indicators in the National Income and Product Account (NIPA), including the current and lagged values of the corresponding NIPA price series. The estimated coefficients are then used to extrapolate the quality-adjusted price index for each individual price series for the sample from 1984 to 2008. These constructed price series are annual. We use Denton (1971)'s method to interpolate these annual series at quarterly frequency. We then use the Tornqvist procedure to construct the quality-adjusted price index from the interpolated individual quarterly price series.

NomConsNHSplusND: Nominal personal consumption expenditures. Nominal nondurable goods and non-housing services (SAAR, billion of dollars). It is computed as CNX_USNA + CSX_USNA - CSRUX_USNA, where CNX_USNA is nominal non-durable goods consumption (SAAR, million of dollars), CSX_USNA is nominal service consumption (SAAR, million of dollars), and CSRUX_USNA is nominal housing and utilities consumption (SAAR, million of dollars).

POPSMOOTH_USECON: Population. Smoothed civilian noninstitutional population with ages 16 years and over (thousands). This series is smoothed by eliminating breaks in population from 10-year censuses and post-2000 American Community Surveys using the "error of closure" method. This fairly simple method is used by the Census Bureau to get a smooth monthly population series and reduce the unusual influence of drastic demographic changes.[16]

[16]The detailed explanation can be found at `http://www.census.gov/popest/archives/methodology/intercensal_nat_meth.html`.

CDX_USNA: Consumer durable goods expenditures. Nominal personal consumption expenditures: durable goods (SAAR, million of dollars).

nominveqipp: Nominal equipment and intellectual property products investment (SAAR, million of dollars).

AggHours: Total hours in the non-farm business (NFB) sector. It is calculated as (Average hours per workers in NFB sector) times (Total civilian employment from Household Survey). The series is normalized to one at 1948Q1.

Appendix B. Estimation procedure

We apply the Bayesian methodology to the estimation of the log-linearized medium-scale structural model, using our own C/C++ code. The advantage of using our own code instead of using Dynare is the flexibility and accuracy we have for finding the posterior mode. We generate over a half million draws from the prior as a starting point for our optimization routine and select the estimated parameters that give the highest posterior probability density. The optimization routine is a combination of NPSOL software package and the csminwel routine provided by Christopher A. Sims.

In estimation, we use the log-linearized equilibrium conditions, reported in Appendix E, to form the posterior probability function fit to the six quarterly U.S. time series from 1995Q2 to 2017Q2: the price-rent ratio in commercial real estate, the quality-adjusted relative price of investment, real per capita consumption, real per capita investment (in consumption units), and per capita hours worked. Excluding the four lags, the sample for estimation begins with 1996Q2 when the repeated-sales price of commercial real estate became available.

We fix the values of certain parameters as an effective way to sharpen the identification of some key parameters in the model. The capital share $\alpha(1-\phi)$ is set at 0.33, consistent with the average capital income share. The share of land in production is estimated at $\phi = 0.07$ by solving the steady state (see Appendix G). The growth rate of aggregate investment-specific technology, $g_z = 1.01$, is consistent with the average growth rate of the inverse relative price of investment goods. The growth rate of aggregate output, $g_\gamma = 1.003$, is consistent with the average common growth rate of consumption and investment. The interest rate R_f is set at 1.01. The steady state capacity utilization u is set at 1. The steady-state labor supply as a fraction of the total time is normalized at $N = 0.3$. To solve the steady state, we impose three additional restrictions to be consistent with the data: 1) the capital-output ratio is 1.125 at annual frequency; 2) the investment-capital ratio is 0.22 at annual frequency; and 3) the rental-income-to-output ratio is 0.1.[17]

[17]The output data used in our model is a sum of personal consumption expenditures and private domestic investment. Consumption is the private expenditures on nondurable goods and nonhousing services.

We estimate five structural parameters as well as all the persistence and volatility parameters that govern exogenous shock processes. The five structural parameters are the inverse Frisch elasticity of labor supply ν, the collateral elasticity χ, the elasticity of capacity utilization $\delta''(1)/\delta'(1)$, the habit formation γ, and the investment-adjustment cost Ω. The remaining parameters are then obtained from the steady state relationships that satisfy the aforementioned data ratio restrictions. These parameters are: the capital depreciation rate ($\delta = 0.0437$), the subjective discount factor ($\beta = 0.993$), the collateral elasticity ($\chi = 0.045$), the capacity utilization rate ($\delta'(1) = 0.0638$), and the labor disutility ($\psi = 4.027$).

For the estimated parameters, we specify a prior that covers a wide range of values that are economically plausible (Table 7). The prior for ν, χ, γ, or Ω has a distribution with the shape hyperparameter $a = 1$. This hyperparameter value is specified to allow a *positive* probability density at the zero value. The implied 90% prior probability bounds are consistent with the values considered in the literature. The prior distribution for $\delta''(1)/\delta'(1)$ is designed to cover the range consistent with Jaimovich and Rebelo (2009).

The prior for the persistence parameters of exogenous shock processes follows the beta distribution with the 90% probability interval between 0.01 and 0.45. Such a prior favors stationarity. The prior for the standard deviations of shock processes follows the inverse gamma distribution with the 90% probability interval between 0.0001 and 2.0. The standard deviation prior specification is far more diffuse than what is used in the literature.

Appendix C. Collateral elasticity

In this appendix we derive the expression for the collateral elasticity χ and interpret its economic meaning. Log-linearizing the endogenous TFP

$$TFP_t = \frac{1}{1 - F(a_t^*)} \int_{a_t^*}^{\infty} a f(a) da \tag{A1}$$

yields

$$\widehat{TFP}_t = \frac{a^* f(a^*)}{1 - F(a^*)} \hat{a}_t^* - \frac{(a^*)^2 f(a^*)}{\int_{a^*}^{\infty} a f(a) da} \hat{a}_t^*.$$

Define

$$\eta = \frac{a^* f(a^*)}{1 - F(a^*)}.$$

From equations (10) and (12) we deduce the markup as

$$\mu_t = \frac{Y_t}{P_{Xt} X_t} - 1 = \frac{\int_{a_t^*}^{\infty} a f(a) da}{a_t^* (1 - F(a_t^*))} - 1.$$

Investment is the private expenditures on consumer durable goods and fixed investment in equipment and intellectual property. Accordingly, we measure capital stock using the annual stocks of equipment, intellectual products, and consumer durable goods.

TABLE 7. Prior distributions of structural and shock parameters

Parameter	Distribution	a	b	Low	High
ν	Gamma(a,b)	1.0	3.0	0.017	1.000
χ	Gamma(a,b)	1.0	30	0.0017	0.100
δ''/δ'	Gamma(a,b)	4.6	17	0.100	0.500
γ	Beta(a,b)	1.0	2.0	0.026	0.776
Ω	Gamma(a,b)	1.0	0.5	0.100	6.000
ρ_z	Beta(a,b)	1.0	5.0	0.010	0.450
ρ_{ν_z}	Beta(a,b)	1.0	5.0	0.010	0.450
ρ_a	Beta(a,b)	1.0	5.0	0.010	0.450
ρ_{ν_a}	Beta(a,b)	1.0	5.0	0.010	0.450
ρ_θ	Beta(a,b)	1.0	5.0	0.010	0.450
ρ_ξ	Beta(a,b)	1.0	5.0	0.010	0.450
ρ_ψ	Beta(a,b)	1.0	5.0	0.010	0.450
σ_z	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_{ν_z}	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_a	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_{ν_a}	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_θ	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_ξ	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_ψ	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000

Note: "Low" and "High" denote the bounds of the 90% probability interval for each parameter.

The steady-state markup is

$$\mu = \frac{\int_{a^*}^{\infty} af(a)da}{a^*(1-F(a^*))} - 1.$$

With the definitions of η and μ, we have

$$\widehat{TFP}_t = \frac{a^* f(a^*)}{1-F(a^*)}\frac{\mu}{1+\mu}\hat{a}_t^* = \frac{\eta\mu}{1+\mu}\hat{a}_t^*,$$

Log-linearizing the stationary version of equation (11) gives us

$$\hat{a}_t^* = \frac{1+\mu}{1+\eta+\mu}\left(\hat{p}_t - \hat{Y}_t\right). \quad \text{(A2)}$$

It follows that

$$\widehat{TFP}_t = \frac{\eta\mu}{1+\eta+\mu}(\hat{p}_t - \hat{Y}_t).$$

We define the collateral elasticity as

$$\chi = \frac{\eta\mu}{1+\eta+\mu}.$$

From the previous expression, one can see that χ measures a percentage change of endogenous TFP in response to a one-percent change in the collateral value (relative to output). From the log-linearized version of the aggregate output equation

$$\hat{Y}_t = \alpha(1-\phi)(\hat{u}_t + \hat{K}_t) + \\ (1-\alpha)\hat{N}_t + \frac{\eta\mu}{1+\eta+\mu}\left(\hat{p}_t - \hat{Y}_t\right) - \frac{\alpha(1-\phi)}{1-\alpha(1-\phi)}\left(\hat{g}_{zt} + \hat{g}_{vzt} + \hat{g}_{at} + \hat{g}_{vat}\right),$$

one can see that it also measures a percentage change in output, holding everything else constant. From the log-linearized version of the resource constraint equation

$$\frac{\tilde{C}}{\tilde{Y}}\hat{C}_t + \frac{\tilde{I}}{\tilde{Y}}\hat{I}_t = \hat{Y}_t,$$

it follows that $\chi\tilde{Y}/\tilde{I}$ measures a percentage change in aggregate investment in response to a one-percent change in the collateral value.

Appendix D. Proposition proofs

D.1. **Proof of Proposition 1.** We rewrite firm i's decision problem as the Bellman equation

$$V_t(h_t^i, a_t^i) = \max_{x_t^i(j), h_{t+1}^i \geq 0} d_t^i + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}(h_{t+1}^i, a_{t+1}^i) \tag{A3}$$

subject to (7), (8), and (9).

To solve the firm's decision problem, we first derive the unit cost of production. Define the total cost of producing y_{it} as

$$\Phi(y_t^i, a_t^i) \equiv \min_{x_t^i(j)} \int P_{Xt}(j) x_t^i(j) dj$$

subject to $a_t^i \exp\left(\int_0^1 \log x_t^i(j) dj\right) \geq y_t^i$. Cost minimization implies that

$$\Phi(y_t^i, a_t^i) = y_t^i \frac{a_t^*}{a_t^i}, \tag{A4}$$

where the average cost a_t^* is given by equation (10) and the demand for each $x_t^i(j)$ satisfies

$$P_{Xt}(j) x_t^i(j) = a_t^* \exp\left(\int_0^1 \log x_t^i(j) dj\right). \tag{A5}$$

Using the cost function in (A4), we rewrite firm i's budget constraint as

$$d_t^i + p_t(h_{t+1}^i - h_t^i) \leq y_t^i - y_t^i \frac{a_t^*}{a_t^i} + R_{ct} h_t^i. \tag{A6}$$

Conjecture the value function in the form of

$$V_t\left(h_t^i, a_t^i\right) = v_t\left(a_t^i\right) h_t^i, \tag{A7}$$

where $v_t\left(a_t^i\right)$ satisfies the first-order condition

$$\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} v_{t+1}(a_{t+1}^i) = p_t. \tag{A8}$$

Equation (A8) is an equilibrium restriction on the real estate price. If $p_t > \beta E_t\left[v_{t+1}\left(a_{t+1}^i\right)\Lambda_{t+1}/\Lambda_t\right]$, firm i would prefer to sell all real estate so that $h_{t+1}^i = 0$. All other firms would not hold real estate because the preceding inequality holds for any i as a_t^i is an iid process. This would violate the market clearing condition for the real estate market. If $p_t < \beta E_t\left[v_{t+1}\left(a_{t+1}^i\right)\Lambda_{t+1}/\Lambda_t\right]$, all firms would prefer to own real estate as much as possible, which again violates the market clearing condition.

We rewrite the credit constraint (8) as

$$y_t^i \frac{a_t^*}{a_t^i} \le \lambda p_t h_t^i. \tag{A9}$$

Substituting equations (A6) and (A7) into equation (A3), we rewrite the firm's problem as

$$v_t\left(a_t^i\right) h_t^i = \max_{y_t^i, h_{t+1}^i} y_t^i \left(1 - \frac{a_t^*}{a_t^i}\right) + R_{ct} h_t^i - p_t(h_{t+1}^i - h_t^i) + p_t h_{t+1}^i, \tag{A10}$$

subject to (A9). The optimal solution to (A10) is

$$y_t^i = \begin{cases} \lambda \frac{a_t^i}{a_t^*} p_t h_t^i & \text{if } a_t^i \ge a_t^* \\ 0 & \text{otherwise} \end{cases}. \tag{A11}$$

Aggregating individual firms' output in (A11) gives

$$Y_t = \int_0^1 y_t^i di = \lambda \int_{a_t^i \ge a_t^*}^{\infty} \frac{a_t^i}{a_t^*} p_t da_t^i \int_0^1 h_t^i di = \lambda \frac{p_t}{a_t^*} \int_{a_t^*}^{\infty} a f(a) da. \tag{A12}$$

From equations (7), (8), and (A5) one can see that the total production cost is given by

$$P_{Xt} X_t = \int_0^1 \int_0^1 P_{Xt} x_t^i(j) di dj = \int_{a_t^i \ge a_t^*}^{\infty} \frac{a_t^*}{a_t^i} y_t^i di = \lambda p_t \left[1 - F\left(a_t^*\right)\right].$$

Using $P_{Xt} = a_t^*$ and $X_t(j) = X_t = A_t\left[\left(u_t K_t\right)^{1-\phi} H_t^{\phi}\right]^{\alpha} N_t^{1-\alpha}$, we derive

$$\lambda p_t = \frac{a_t^* A_t \left[\left(u_t K_t\right)^{1-\phi} H_t^{\phi}\right]^{\alpha} N_t^{1-\alpha}}{1 - F(a_t^*)}. \tag{A13}$$

Combining this equation and (A12) gives the aggregate production function

$$Y_t = A_t \left[\left(u_t K_t\right)^{1-\phi} H_t^{\phi}\right]^{\alpha} N_t^{1-\alpha} \frac{\int_{a_t^*}^{\infty} a f(a) da}{1 - F(a_t^*)}.$$

D.2. **Proof of Proposition 2.** Substituting equation (A11) into the Bellman equation (A10) and matching the coefficients, we obtain

$$v_t\left(a_t^i\right) = \begin{cases} \left(\frac{a_t^i}{a_t^*} - 1\right)\lambda p_t + R_{ct} + p_t & \text{if } a_t^i \geq a_t^* \\ R_{ct} + p_t & \text{otherwise} \end{cases}.$$

Substituting the above expression into (A8) gives the asset pricing equation

$$p_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left[R_{ct+1} + p_{t+1} + \lambda p_{t+1}\int_{a_{t+1}^*}^{\infty} \frac{a - a_{t+1}^*}{a_{t+1}^*} f(a)da\right].$$

The first order condition for intermediate goods producers' optimal problem with respect to real estate gives

$$\alpha\phi P_{Xt}(j) A_t K_t\left(j\right)^{\alpha(1-\phi)} N_t\left(j\right)^{(1-\alpha)} H_t^{\alpha\phi-1}(j) = R_{ct}.$$

Given the symmetric equilibrium in the intermediate goods sector and the market clear conditions, the previous equation becomes

$$R_{ct} = \alpha\phi a_t^* A_t\left(u_t K_t\right)^{\alpha(1-\phi)} H_t^{\alpha\phi-1} N_t^{1-\alpha}. \tag{A14}$$

D.3. **Proof of Proposition 3.** In Supplemental Appendix G, we derive the log-linearized equilibrium system. One of the log-linearized equations pertains to the real estate price, which involves the term

$$\int_{a^*}^{\infty} \frac{a - a^*}{a^*} f(a)da.$$

Since the data does not pin down the distribution f or a^*, we normalize the distribution such that

$$\int_{a^*}^{\infty} \frac{a - a^*}{a^*} f(a)da = \frac{1}{\beta} - 1. \tag{A15}$$

Given this normalization, we log-linearize the stationary version of equation (13) as

$$\hat{p}_t + \hat{\Lambda}_t = E_t\left(\hat{\theta}_{t+1} + \hat{\Lambda}_{t+1}\right) + \frac{\beta(\tilde{R}_c/\tilde{Y})}{\tilde{p}/\tilde{Y}} E_t\hat{R}_{ct+1} + \beta E_t\hat{p}_{t+1} +$$
$$\frac{(1-\beta)(\lambda\tilde{p}/\tilde{Y})}{\tilde{p}/\tilde{Y}} E_t\left[\hat{p}_{t+1} - \frac{1+\mu}{\mu}\frac{1+\mu}{1+\eta+\mu}\left(\hat{p}_{t+1} - \hat{Y}_{t+1}\right)\right].$$

We rewrite the previous equation as

$$\begin{aligned}\hat{p}_t &= E_t\left(\hat{\theta}_{t+1} + \hat{\Lambda}_{t+1} - \hat{\Lambda}_t\right) + \frac{\beta(\tilde{R}_c/\tilde{Y})}{\tilde{p}/\tilde{Y}} E_t\hat{R}_{ct+1} \\ &\quad + \lambda(1-\beta)E_t\left[\hat{p}_{t+1} - \frac{1+\mu}{\mu}\frac{1+\mu}{1+\eta+\mu}\left(\hat{p}_{t+1} - \hat{Y}_{t+1}\right)\right] + \beta E_t\hat{p}_{t+1}.\end{aligned}$$

Let

$$\hat{p}_t = \hat{p}_{1t} + \hat{p}_{2t} + \hat{p}_{3t},$$

where

$$\begin{aligned}
\hat{p}_{1t} &= E_t\left(\hat{\theta}_{t+1}+\hat{\Lambda}_{t+1}-\hat{\Lambda}_t\right)+\beta E_t\hat{p}_{1t+1},\\
\hat{p}_{2t} &= \frac{\beta\tilde{R}_c}{\tilde{p}}E_t\hat{R}_{ct+1}+\beta E_t\hat{p}_{2t+1},\\
\hat{p}_{3t} &= \lambda(1-\beta)E_t\left[\hat{p}_{t+1}-\frac{1+\mu}{\mu}\frac{1+\mu}{1+\eta+\mu}\left(\hat{p}_{t+1}-\hat{Y}_{t+1}\right)\right]+\beta E_t\hat{p}_{3t+1}.
\end{aligned}$$

Substituting $\eta=\chi(1+\mu)/(\mu-\chi)$ into the last equation, we complete the proof of the proposition.

D.4. **Proof of Proposition 4.** For our simple model, equation (A12) can be simplified as

$$Y_t=C_t=\frac{\lambda p_t}{a_t^*}\int_{a_t^*}^{\infty}af(a)da.$$

Since the collateral constraint (8) binds for firms with $a_t^i\geq a_t^*$, aggregating this constraint in a symmetric equilibrium yields

$$P_{Xt}X_t=[1-F(a_t^*)]\lambda p_t.$$

Since $P_{Xt}=a_t^*$ and $X_t=A_tN_t^{1-\alpha}=A_t$, we have

$$P_{Xt}X_t=[1-F(a_t^*)]\lambda p_t=a_t^*A_t.$$

This equation implies that

$$\lambda p_t=\frac{a_t^*A_t}{1-F(a_t^*)}.$$

Substituting this equation into the preceding equation for Y_t yields

$$Y_t=C_t=A_t\frac{\int_{a_t^*}^{\infty}af(a)da}{1-F(a_t^*)}. \tag{A16}$$

The log-linearized version of the previous equation is

$$\hat{Y}_t=\hat{C}_t=\hat{A}_t+\frac{\eta\mu}{1+\mu}\hat{a}_t^*.$$

Similarly, the simplified version of equation (A13) is

$$\lambda p_t=\frac{a_t^*A_t}{[1-F(a_t^*)]}.$$

Log-linearizing the previous equation gives

$$\hat{p}_t=\hat{A}_t+[1+\eta]\,\hat{a}_t^*. \tag{A17}$$

The simplified version of equation (A14) is

$$R_{ct}=\alpha\phi a_t^*A_t,$$

whose log-linearized version is

$$\hat{R}_{ct}=\hat{A}_t+\hat{a}_t^*.$$

The asset pricing equation, represented by (13), can be simplified to

$$p_t \frac{1}{C_t} = \beta E_t \frac{\theta_{t+1}}{C_{t+1}} \left[R_{ct+1} + p_{t+1} + \lambda p_{t+1} \int_{a^*_{t+1}}^{\infty} \frac{a - a^*_{t+1}}{a^*_{t+1}} f(a) da \right].$$

Log-linearizing the preceding equation and then combining the log-linearized equation with equation (A17) gives us

$$\begin{aligned} \frac{\eta+1+\mu}{1+\mu} \hat{a}^*_t &= \rho_\theta \hat{\theta}_t + [1-(1-\beta)(1-\lambda)] \frac{\eta+1+\mu}{1+\mu} E_t \hat{a}^*_{t+1} \\ &\quad + (1-\beta)(1-\lambda) E_t (1 - \frac{\eta\mu}{1+\mu}) \hat{a}^*_{t+1} \\ &\quad - \frac{(1-\beta)(1+\mu)\lambda}{\mu} E_t \hat{a}^*_{t+1}, \end{aligned}$$

or equivalently

$$\begin{aligned} \hat{a}^*_t &= \rho_\theta \hat{\theta}_t \frac{1+\mu}{\eta+1+\mu} + [1-(1-\beta)(1-\lambda)] E_t \hat{a}^*_{t+1} \\ &\quad + (1-\beta)(1-\lambda) E_t \frac{1+\mu-\eta\mu}{\eta+1+\mu} \hat{a}^*_{t+1} \\ &\quad - \frac{(1-\beta)(1+\mu)\lambda}{\mu} \frac{1+\mu}{\eta+1+\mu} E_t \hat{a}^*_{t+1}. \end{aligned}$$

Given the AR(1) process of a^*_t, we have

$$\begin{aligned} \hat{a}^*_t &= \rho_\theta \frac{1+\mu}{\eta+1+\mu} \frac{1}{1-\rho_\theta \kappa} \hat{\theta}_t \\ &= \frac{\pi}{1+\eta} \hat{\theta}_t, \end{aligned}$$

where κ and π are defined in Proposition 4. Note that $\kappa < 1$ for our estimated parameter values.

D.5. **Proof of Proposition 5.** Denote the k-period ahead return by

$$R^{re}_{t,t+k} = \frac{p_{t+k} + R_{ct+k}}{p_t}.$$

Given normalization (A15), we have

$$\frac{R_c}{p} = (1-\lambda)(\frac{1}{\beta} - 1).$$

Hence, the log-linearized return is given by

$$\hat{R}^{re}_{t,t+k} = \frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta} \hat{R}_{ct+k} + \frac{\beta}{(1-\lambda)(1-\beta)+\beta} \hat{p}_{t+k} - \hat{p}_t. \tag{A18}$$

From equations (18) and (19) we have

$$\hat{R}_{ct} - \hat{p}_t = -\eta \hat{a}^*_t = -\varphi \hat{\theta}_t, \tag{A19}$$

where we define $\varphi = \eta\frac{1+\mu}{\eta+1+\mu}\frac{\rho_\theta}{1-\rho_\theta\kappa}$. Substituting equations (18), (19), and (A19) into the preceding equation leads to

$$\begin{aligned}\hat{R}^{re}_{t,t+k} &= \hat{p}_{t+k} - \hat{p}_t + \frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta}[\hat{R}_{ct+k} - \hat{p}_{t+k}] \\ &= (\eta+1)(\hat{a}^*_{t+k} - \hat{a}^*_t) - \frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta}\eta a^*_{t+k}.\end{aligned}$$

Then substituting equations (16) and (A19) into the previous equation leads to

$$\begin{aligned}\hat{R}^{re}_{t,t+k} &= \pi\left[\hat{\theta}_{t+k} - \hat{\theta}_t\right] + \frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta}[\hat{R}_{ct+k} - \hat{p}_{t+k}] \\ &= \pi\left[\hat{\theta}_{t+k} - \hat{\theta}_t\right] - \frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta}\varphi\hat{\theta}_{t+k},\end{aligned}$$

where $\varphi = \eta\frac{1+\mu}{\eta+1+\mu}\frac{\rho_\theta}{1-\rho_\theta\kappa} < \pi = (1+\eta)\frac{1+\mu}{\eta+1+\mu}\frac{\rho_\theta}{1-\rho_\theta\kappa}$. It follows that

$$\begin{aligned}E_t\left\{\hat{R}^{re}_{t,t+k} \mid \left(\hat{R}_{ct} - \hat{p}_t\right)\right\} &= \left[(\rho^k_\theta - 1)\pi - \rho^k_\theta\frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta}\varphi\right]\theta_t \\ &= \left[\rho^k_\theta\frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta}\varphi - (\rho^k_\theta - 1)\frac{\pi}{\varphi}\right]\left(\hat{R}_{ct} - \hat{p}_t\right).\end{aligned}$$

where the regression coefficient

$$\begin{aligned}&\rho^k_\theta\frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta} - (\rho^k_\theta - 1)\frac{\pi}{\varphi} \\ =\ &\rho^k_\theta\frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta} - (\rho^k_\theta - 1)\frac{\eta+1}{\eta} \\ >\ &0\end{aligned}$$

increases with k. The corresponding measure of fit is

$$R^2 = \frac{\left((\eta+1)(\rho^k_\theta - 1) - \frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta}\eta\rho^k_\theta\right)^2}{\left[(\eta+1) - \frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta}\eta\right]^2(1-\rho^{2k}_\theta) + \left[(\eta+1)(\rho^k_\theta - 1) - \frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta}\eta\rho^k_\theta\right]}.$$

The predictability result for real estate returns, as shown above, also holds for excess returns. One can see from equations (17) and (18) that the risk-free interest rate in response to the discount-rate shock (i.e., $\hat{A}_t = 0$) is

$$\begin{aligned}\hat{R}_{ft} &= E_t\hat{C}_{t+1} - \hat{C}_t - \rho_\theta\hat{\theta}_t \\ &= \frac{\eta\mu}{1+\mu}\frac{1+\mu}{\eta+1+\mu}\frac{1}{1-\rho_\theta\kappa}(\rho_\theta - 1)\hat{\theta}_t - \rho_\theta\hat{\theta}_t \\ &= \frac{\eta\mu}{1+\mu}(\rho_\theta - 1)\hat{a}^*_t - \rho_\theta\hat{\theta}_t.\end{aligned}$$

From equation (A18) we have

$$\begin{aligned} E_t \hat{R}^{re}_{t,t+k} &= \hat{p}_{t+k} - \hat{p}_t + \frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta} E_t[\hat{R}_{ct+k} - \hat{p}_{t+k}] \\ &= [\eta+1]\left(\rho_\theta^k - 1\right)\hat{a}_t^* - \frac{(1-\lambda)(1-\beta)}{(1-\lambda)(1-\beta)+\beta}\eta\rho_\theta^k \hat{a}_t^*. \end{aligned}$$

The excess return is then given by

$$\begin{aligned} E_t \hat{R}^{re}_{t,t+k} - \hat{R}_{ft} = & \frac{(1-\beta)(1-\lambda)+\beta(\eta+1)}{(1-\beta)(1-\lambda)+\beta}(\rho_\theta^k - \rho_\theta) a_t^* \\ & + \left[\frac{(1-\beta)\,(1+\mu)\,\lambda}{\mu} - \frac{\lambda(1-\lambda)(1-\beta)^2}{(1-\beta)(1-\lambda)+\beta}\eta\right]\rho_\theta \hat{a}_t^*. \end{aligned}$$

Substituting equation (A19) into the preceding equation leads to

$$\begin{aligned} E_t \hat{R}^{re}_{t,t+k} - \hat{R}_{ft} = & \frac{(1-\beta)(1-\lambda)+\beta(\eta+1)}{(1-\beta)(1-\lambda)+\beta}\frac{\rho_\theta - \rho_\theta^k}{\eta}\left(\hat{R}_{ct} - \hat{p}_t\right) \\ & + \left[\frac{\lambda(1-\lambda)(1-\beta)^2}{(1-\beta)(1-\lambda)+\beta} - \frac{(1-\beta)\,(1+\mu)\,\lambda}{\eta\mu}\right]\rho_\theta\left(\hat{R}_{ct} - \hat{p}_t\right). \end{aligned}$$

Note that $\rho_\theta - \rho_\theta^k \geq 0$. As long as η is sufficiently large, the term in the square brackets is positive, which is true for our estimated parameters. Hence if we run a regression of the excess return on the log value of the rent-price ratio $\hat{R}_{ct} - \hat{p}_t$, the coefficient will be positive.

D.6. **Proof of Proposition 6.** From the proof of Proposition 4, we have the following equilibrium condition:

$$\frac{p_t}{R_{ct}} = \frac{1}{\lambda\left[1 - F(a_t^*)\right]}.$$

Log-linearizing this condition delivers the first-order solutions in closed form as

$$\hat{v}_t \equiv \hat{R}_{ct} - \hat{p}_t = \hat{a}_t^* - \pi\hat{\theta}_t = -\frac{\eta\pi}{1+\eta}\hat{\theta}_t.$$

Note

$$\hat{r}_{t+k} \equiv \hat{p}_{t+k} - \hat{p}_t = \hat{A}_{t+k} - \hat{A}_t + \pi\left[\hat{\theta}_{t+k} - \hat{\theta}_t\right].$$

With discount-rate shocks, we have

$$\begin{aligned} \hat{r}_{t+k} &= \pi\left[\hat{\theta}_{t+k} - \hat{\theta}_t\right] \\ &= (\rho_\theta^k - 1)\pi\hat{\theta}_t + \pi\sigma_\theta[\varepsilon_{\theta t+k} + \rho_\theta\varepsilon_{\theta t+k-1} + \cdots + \rho_\theta^{k-1}\varepsilon_{\theta t+1}]. \end{aligned}$$

It follows that

$$\begin{aligned} E[\hat{r}_{t+k} \mid \hat{v}_t] &= E[\hat{r}_{t+k} \mid \hat{\theta}_t] \\ &= (\rho_\theta^k - 1)\pi\hat{\theta}_t \\ &= (1 - \rho_\theta^k)\frac{1+\eta}{\eta}\hat{v}_t. \end{aligned}$$

If we run a regression of $\hat{r}_{t+k}$ on the valuation ratio $\hat{v}_t$, the coefficient will be positive.

With discount-rate shocks, the correlation between $\hat{r}_{t+k}$ and $\hat{v}_t$ is

$$\rho_{r,v} = \frac{(1-\rho_\theta^k)\sqrt{\frac{1}{1-\rho_\theta^2}}}{\sqrt{\left(1-\rho_\theta^k\right)^2 \frac{1}{1-\rho_\theta^2} + \frac{1-\rho_\theta^{2k}}{1-\rho_\theta^2}}},$$

which implies

$$\begin{aligned} R_{r,v}^2 &= \frac{\left(1-\rho_\theta^k\right)^2 / 1-\rho_\theta^2}{\left(1-\rho_\theta^k\right)^2 \frac{1}{1-\rho_\theta^2} + \frac{1-\rho_\theta^{2k}}{1-\rho_\theta^2}} \\ &= \frac{\left(1-\rho_\theta^k\right)^2}{\left(1-\rho_\theta^k\right)^2 + 1 - \rho_\theta^{2k}} \\ &= \frac{\left(1-\rho_\theta^k\right)^2}{2-2\rho_\theta^k} \\ &= \frac{1}{2}(1-\rho_\theta^k). \end{aligned}$$

Thus, the fit measure R^2 increases with the forecast horizon.

With technology shocks, however, the log value of the rent-price ratio is $\hat{v}_t = \hat{R}_{ct} - \hat{p}_t = 0$. Clearly, there is no predictability for technology shocks.

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Supplementary Appendices

(Not intended for publication)

In the supplementary appendices, all labels for equations, tables, and propositions begin with S, which stands for a *supplement* to the main text.

Appendix E. Equilibrium conditions

The equilibrium for this economy is characterized by the following system of equations.

(E1) Marginal utility of consumption Λ_t :

$$\Lambda_t = \frac{\Theta_t}{C_t - \gamma C_{t-1}} - \beta\gamma E_t \frac{\Theta_{t+1}}{C_{t+1} - \gamma C_t}. \tag{S1}$$

(E2) Labor supply w_t :

$$\Lambda_t w_t = \Theta_t \psi_t N_t^{\nu}. \tag{S2}$$

(E3) Real estate rent R_{ct} :

$$R_{ct} = \frac{\alpha\phi\xi_t Y_t / H_t}{\frac{1}{1-F(a_t^*)}\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a)da}. \tag{S3}$$

(E4) Investment I_t :

$$\begin{aligned} \frac{1}{Z_t} \quad &= \quad Q_{kt}\left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2 - \Omega\left(\frac{I_t}{I_{t-1}} - g_I\right)\frac{I_t}{I_{t-1}}\right] \\ &\quad + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} Q_{kt+1}\Omega\left(\frac{I_{t+1}}{I_t} - g_I\right)\frac{I_{t+1}^2}{I_t^2}. \end{aligned} \tag{S4}$$

(E5) Marginal Tobin's Q_{kt} :

$$Q_{kt} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}(u_{t+1}R_{kt+1} + (1 - \delta(u_{t+1}))Q_{kt+1}). \tag{S5}$$

(E6) Capital utilization u_t :

$$R_{kt} = \delta'(u_t)Q_{kt}. \tag{S6}$$

(E7) Real estate price p_t :

$$p_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left[R_{ct+1} + p_{t+1} + \lambda p_{t+1}\int_{a_{t+1}^*} (\frac{a}{a_{t+1}^*} - 1)f(a)da\right]. \tag{S7}$$

(E8) Rent of capital R_{kt} :

$$R_{kt}u_t K_t = \alpha(1-\phi)\frac{Y_t}{\frac{1}{1-F(a_t^*)}\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a)da}. \tag{S8}$$

(E9) Labor demand N_t :

$$w_t N_t = (1-\alpha)\frac{Y_t}{\frac{1}{1-F(a_t^*)}\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a)da}. \tag{S9}$$

(E10) Aggregate output Y_t :

$$Y_t = A_t\left(u_t K_t\right)^{\alpha(1-\phi)} H_t^{\alpha\phi} N_t^{1-\alpha}\left[\frac{1}{1-F(a_t^*)}\int_{a_t^*}^{\infty} af(a)da\right]. \tag{S10}$$

(E11) Collateral constraint a_t^* :

$$\lambda \frac{p_t}{a_t^*} \int_{a_t^*}^{\infty} a f(a) da = Y_t. \tag{S11}$$

(E12) Aggregate capital accumulation K_t :

$$K_{t+1} = (1 - \delta(u_t))K_t + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2\right] I_t. \tag{S12}$$

(E13) Resource constraint C_t :

$$C_t + \frac{I_t}{Z_t} = Y_t. \tag{S13}$$

(E14) Interest rate R_{ft}:

$$1 = \beta R_{ft} E_t \frac{\Lambda_{t+1}}{\Lambda_t}. \tag{S14}$$

We have 14 equations for the following 14 variables:

(V1) Λ_t: Marginal utility of consumption.

(V2) w_t: Real wage.

(V3) I_t: Investment.

(V4) $Q_{k,t}$: Price of capital.

(V5) u_t: Capacity utilization rate.

(V6) p_t: Real estate price.

(V7) R_{kt}: Rental price of capital.

(V8) N_t: Total labor supply.

(V9) Y_t: Output.

(V10) a_t^*: Cutoff value for investment.

(V11) K_{t+1}: Capital.

(V12) C_t: Consumption.

(V13) R_{ct}: Rental price of real estate.

(V14) R_{ft}: Risk-free interest rate.

Appendix F. Stationary equilibrium conditions

We make the following transformations of variables:

$$\tilde{C}_t \equiv \frac{C_t}{\Gamma_t},\ \tilde{I}_t \equiv \frac{I_t}{Z_t \Gamma_t},\ \tilde{Y}_t \equiv \frac{Y_t}{\Gamma_t},\ \tilde{K}_t \equiv \frac{K_t}{\Gamma_{t-1} Z_{t-1}},$$

$$\tilde{w}_t \equiv \frac{w_t}{\Gamma_t},\ \tilde{R}_{ct} \equiv \frac{R_{ct}}{\Gamma_t},\ \tilde{p}_t \equiv \frac{p_t}{\Gamma_t}$$

$$\tilde{R}_{kt} \equiv R_{kt} Z_t,\ \tilde{Q}_{kt} \equiv Q_{kt} Z_t, \tilde{\Lambda}_t \equiv \frac{\Lambda_t}{\Theta_t} \Gamma_t,$$

where $\Gamma_t = Z_t^{\frac{\alpha(1-\phi)}{1-\alpha(1-\phi)}} A_t^{\frac{1}{1-\alpha(1-\phi)}}$. The other variables are stationary and there is no need to transform them.

Let $G_{zt} = \frac{Z_t}{Z_{t-1}}$ and $G_{at} = \frac{A_t}{A_{t-1}}$. Then

$$\begin{aligned} \log G_{zt} &= \log g_{zt} + \log g_{\nu z,t}, \\ \log G_{at} &= \log g_{at} + \log g_{\nu a,t}. \end{aligned}$$

where

$$\begin{aligned} \log g_{\nu z,t} &= \log \nu_{z,t} - \log \nu_{z,t-1}, \\ \log g_{\nu a,t} &= \log \nu_{a,t} - \log \nu_{a,t-1}. \end{aligned}$$

Denote by $g_{\gamma t} \equiv \Gamma_t / \Gamma_{t-1}$ the gross growth rate of Γ_t. We have

$$\log g_{\gamma t} = \frac{\alpha(1-\phi)}{1-\alpha(1-\phi)} \log G_{zt} + \frac{1}{1-\alpha(1-\phi)} \log G_{at}. \tag{S15}$$

Denote by g_γ the nonstochastic steady state of $g_{\gamma t}$, which satisfies

$$\log g_\gamma \equiv \frac{\alpha(1-\phi)}{1-\alpha(1-\phi)} \log g_z + \frac{1}{1-\alpha(1-\phi)} \log g_a. \tag{S16}$$

On the nonstochastic balanced growth path, investment and capital grow at the rate of $g_I \equiv g_\gamma g_z$; consumption, output, real wages, price of commercial real estate, and the rental rate of commercial property grow at the rate of g_γ; and the rental rate of capital, Tobin's marginal Q, and the relative price of investment goods decrease at the rate g_z. Below we display the corresponding equilibrium equations for the stationary variables.

(SE1) Marginal utility of consumption:

$$\tilde{\Lambda}_t = \frac{1}{\tilde{C}_t - \gamma \tilde{C}_{t-1}/g_{\gamma t}} - \beta\gamma E_t \theta_{t+1} \frac{1}{\tilde{C}_{t+1} g_{\gamma t+1} - \gamma \tilde{C}_t}. \tag{S17}$$

(SE2) Labor supply:

$$\tilde{\Lambda}_t \tilde{w}_t = \psi_t N_t^\nu. \tag{S18}$$

(SE3) Real estate rent:

$$\tilde{R}_{ct} = \frac{\alpha\phi\tilde{Y}_t}{\frac{1}{1-F(a_t^*)} \int_{a_t^*}^\infty \frac{a}{a_t^*} f(a) da}. \tag{S19}$$

(SE4) Investment:

$$\begin{aligned} 1 \;\; = \;\; & \tilde{Q}_{kt} \left[1 - \frac{\Omega}{2} \left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} G_{zt} g_{\gamma t} - g_I \right)^2 - \Omega \left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} G_{zt} g_{\gamma t} - g_I \right) \frac{\tilde{I}_t}{\tilde{I}_{t-1}} G_{zt} g_{\gamma t} \right] \\ & + \beta E_t \theta_{t+1} \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \tilde{Q}_{kt+1} \Omega \left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t} g_{\gamma t+1} G_{zt+1} - g_I \right) \frac{\tilde{I}_{t+1}^2}{\tilde{I}_t^2} g_{\gamma t+1} G_{zt+1}. \end{aligned} \tag{S20}$$

(SE5) Marginal Tobin's Q:

$$\tilde{Q}_{kt} = \beta E_t \theta_{t+1} \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t} \frac{1}{g_{\gamma t+1} G_{zt+1}} [u_{t+1} \tilde{R}_{kt+1} + (1 - \delta(u_{t+1})) \tilde{Q}_{kt+1}]. \tag{S21}$$

(SE6) Capital utilization:

$$\tilde{R}_{kt} = \delta'(u_t)\tilde{Q}_{kt}. \tag{S22}$$

(SE7) Real estate price:

$$\tilde{p}_t = \beta E_t \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\theta_{t+1}\left[\tilde{R}_{ht+1} + \tilde{p}_{t+1} + \tilde{p}_{t+1}\int_{a^*_{t+1}}^{\infty}(\frac{a}{a^*_{t+1}} - 1)f(a)da\right]. \tag{S23}$$

(SE8) Rental rate of capital:

$$\tilde{R}_{kt}u_t\tilde{K}_t = \frac{\alpha(1-\phi)G_{zt}g_{\gamma t}\tilde{Y}_t}{\frac{1}{1-F(a^*_t)}\int_{a^*_t}^{\infty}\frac{a}{a^*_t}f(a)da}. \tag{S24}$$

(SE9) Labor demand:

$$\tilde{w}_t N_t = \frac{(1-\alpha)\tilde{Y}_t}{\frac{1}{1-F(a^*_t)}\int_{a^*_t}^{\infty}\frac{a}{a^*_t}f(a)da}. \tag{S25}$$

(SE10) Aggregate output:

$$\tilde{Y}_t = \frac{1}{(G_{zt}G_{at})^{\frac{\alpha(1-\phi)}{1-\alpha(1-\phi)}}}\left(u_t\tilde{K}_t\right)^{\alpha(1-\phi)} H_t^{\alpha\phi}N_t^{1-\alpha}\frac{\int_{a^*_t}^{\infty}af(a)da}{1-F(a^*_t)}. \tag{S26}$$

(SE11) Collateral constraint:

$$\lambda\frac{\tilde{p}_t}{a^*_t}\int_{a^*_t}^{\infty}af(a)da = \tilde{Y}_t. \tag{S27}$$

(SE12) Aggregate capital accumulation:

$$\tilde{K}_{t+1} = (1-\delta(u_t))\frac{\tilde{K}_t}{g_{zt}g_{\gamma t}} + \left[1 - \frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}g_{zt}g_{\gamma t} - g_I\right)^2\right]\tilde{I}_t. \tag{S28}$$

(SE13) Resource constraints:

$$\tilde{C}_t + \tilde{I}_t = \tilde{Y}_t. \tag{S29}$$

(SE14) Interest rate:

$$1 = \beta R_{ft}E_t\left[\frac{\tilde{\Lambda}_{t+1}\theta_{t+1}}{\tilde{\Lambda}_t}\frac{1}{g_{\gamma,t+1}}\right]. \tag{S30}$$

Appendix G. Solving the steady state

(SS1) β or R_f: From (S30),

$$\beta = \frac{g_\gamma}{R_f}. \tag{S31}$$

Given $(R_f)^{\text{Data}} = 1.01$, we know β.

(SS2) $\tilde{\Lambda}$: From equation (S17), we have $\tilde{\Lambda}_t = \frac{1}{\tilde{C}_t - \gamma \tilde{C}_{t-1}/g_\gamma} - \beta\gamma E_t \theta_{t+1} \frac{1}{\tilde{C}_{t+1} g_{\gamma t+1} - \gamma \tilde{C}_t}$. Thus,

$$\tilde{\Lambda} = \frac{g_\gamma - \beta\gamma}{\tilde{C}(g_{\gamma t} - \gamma)},$$

which leads to

$$\tilde{\Lambda}\tilde{Y} = \frac{g_\gamma - \beta\gamma}{(\tilde{C}/\tilde{Y})(g_\gamma - \gamma)}, \tag{S32}$$

where $\tilde{C}/\tilde{Y}$ is given in (S42). In estimation, however, once we are given $(\tilde{I}/\tilde{K})^{\text{Data}}$and $(\tilde{K}/\tilde{Y})^{\text{Data}}$, we know in effect $(\tilde{C}/\tilde{Y})^{\text{Data}}$ and $(\tilde{I}/\tilde{Y})^{\text{Data}}$. We need to verify that the model-based ratio $\tilde{C}/\tilde{Y}$ backed out from (S42) must be exactly the same as $(\tilde{C}/\tilde{Y})^{\text{Data}}$ when $(\tilde{I}/\tilde{Y})^{\text{Data}}$ is given.

(SS3) $\tilde{Q}_k$: From equation (S20),

$$1 = \tilde{Q}_k.$$

(SS4) δ or $\tilde{I}$: From equation (S28),

$$\delta = 1 - \left(1 - \frac{\tilde{I}}{\tilde{K}}\right) g_z g_\gamma.$$

Given $(\tilde{I}/\tilde{K})^{\text{Data}}$, we obtain δ.

(SS5) $\tilde{R}_k$: From equation (S21),

$$\tilde{Q}_k = \frac{\beta}{g_\gamma g_z}\left[u\tilde{R}_k + (1 - \delta(u))\tilde{Q}_k\right].$$

With $u = 1$, we have

$$\tilde{R}_k = \frac{g_\gamma g_z}{\beta} - (1 - \delta(1)). \tag{S33}$$

Once we derive $\delta(1)$ or δ in item (SS4), we can solve for $\tilde{R}_k$.

(SS6) $\delta'(1)$ or u: From equation (S22), $\delta'(1)$ is determined by

$$\delta'(1) = \tilde{R}_k,$$

This determination utilizes the normalization $u = 1$.

(SS7) μ or $\tilde{K}$: The steady-state markup is

$$\mu = \frac{\int_{a^*}^{\infty} \frac{a}{a^*} f(a)\, da}{1 - F(a^*)} - 1 > 0.$$

From equation (S24), we have

$$\tilde{R}_k \tilde{K} = \frac{\alpha(1-\phi) g_z g_\gamma \tilde{Y}}{1 + \mu},$$

which leads to

$$\mu = \alpha(1-\phi) g_z g_\gamma \frac{\tilde{Y}}{\tilde{K}} \frac{1}{\tilde{R}_k} - 1. \tag{S34}$$

Given $(\tilde{K}/\tilde{Y})^{\text{Data}}$, we can solve for μ and ϕ jointly from (S34) and (S37). Note that $\mu > 0$ must hold.

If we were to estimate μ instead, we would then determine the capital-output ratio as

$$\frac{\tilde{K}}{\tilde{Y}} = \frac{\alpha(1-\phi)g_z g_\gamma}{(1+\mu)\tilde{R}_k}. \tag{S35}$$

(SS8) a^*: Note that

$$1+\mu = \frac{\int_{a^*}^{\infty} \frac{a}{a^*} f(a)\,da}{1-F(a^*)}. \tag{S36}$$

If we have the value of μ (see below) and specify the probability density $f(a)$, we can in principle obtain a^*. In practice, we do not need $f(a)$ nor a^* for first-order dynamics and thus we do not derive either one explicitly.

(SS9) ϕ or R_c: (S19) implies that

$$R_c = \alpha\phi\frac{Y}{1+\mu}.$$

In principle, we can solve for the rent of real estate property R_c. In estimation, however, we use the relationship

$$\frac{\tilde{R}_c}{\tilde{Y}} = \frac{\alpha\phi}{1+\mu}. \tag{S37}$$

Given $\left(\tilde{R}_c/\tilde{Y}\right)^{\text{Data}}$ (we use the ratio of rental income to output because H is normalized to be 1), we can obtain μ and ϕ jointly from (S34) and (S37).

(SS10) λ or $\tilde{p}$: From equation (S23),

$$\tilde{p} = \beta\theta\left[\frac{\alpha\phi}{1+\mu}\tilde{Y} + \tilde{p} + \lambda\tilde{p}\int_{a^*}^{\infty}\left(\frac{a}{a^*}-1\right)f(a)\,da\right]. \tag{S38}$$

We normalize $\theta = 1$. For a given value of λ, we have

$$\frac{\tilde{p}}{\tilde{Y}} = \frac{\beta\alpha\phi}{(1+\mu)\left[1-\beta-\beta\lambda\int_{a^*}^{\infty}\left(\frac{a}{a^*}-1\right)f(a)\,da\right]}. \tag{S39}$$

This expression can be further simplified with normalization (A15).

(SS11) $\tilde{w}$: From equation (S25),

$$\tilde{w}N = (1-\alpha)\frac{\tilde{Y}}{1+\mu}.$$

In principle, once we normalize N and solve for Y, we can obtain w. In practice, we do not need to know w or Y for first-order dynamics and therefore we do not need to obtain either of these variables explicitly, only implicitly.

As shown in (SS12), the normalization of N enables us to back out the value of ψ (steady state disutility level). We use the following relationship to determine ψ in (SS12):

$$\frac{\tilde{w}}{\tilde{Y}} = \frac{(1-\alpha)}{N(1+\mu)}. \tag{S40}$$

(SS12) ψ or N: From equation (S18), we obtain ψ as

$$\psi = \frac{(\tilde{\Lambda}\tilde{Y})\,(\tilde{w}/\tilde{Y})}{N^{\nu}}, \tag{S41}$$

where $\tilde{\Lambda}\tilde{Y}$ is given by (S32), $\tilde{w}/\tilde{Y}$ is given by (S40), and N is normalized to, say, 1/3.

(SS13) $\tilde{Y}$: It follows from equation (S26) that

$$\tilde{Y} = \tilde{A}\tilde{K}^{\alpha(1-\phi)}N^{1-\alpha}\widetilde{\mathrm{TFP}},$$

where

$$\widetilde{\mathrm{TFP}} = \frac{1}{1-F(a^*)}\int_{a^*}^{\infty} af(a)\,da.$$

In principle, once the probability density function $f(a)$ is given and if a^* is known, we know TFP. By dividing $\tilde{K}$ on both sides and given $(\tilde{K}/\tilde{Y})^{\mathrm{Data}}$, we obtain $\tilde{K}$ and then $\tilde{Y}$.

In estimation, we do not need to solve for $\tilde{Y}$ or $\tilde{K}$ because the scale $\tilde{A}$ is arbitrary; nor do we need to know $\widetilde{\mathrm{TFP}}$ as it does not affect first-order dynamics. This part is written for completeness, even if it is never used or needed for estimation. The scale of $\tilde{A}$ or $\tilde{Y}$ is implicitly chosen such that $\tilde{I}/\tilde{Y} = (\tilde{I}/\tilde{Y})^{\mathrm{Data}}$.

(SS14) $\tilde{C}$: From equation (S29) we have

$$\frac{\tilde{C}}{\tilde{Y}} = 1 - \frac{\tilde{I}}{\tilde{Y}}. \tag{S42}$$

In principle, after we obtain $\tilde{Y}$ and $\tilde{I}$, we can obtain $\tilde{C}$. In practice, given $(\tilde{K}/\tilde{Y})^{\mathrm{Data}}$ and $(\tilde{I}/\tilde{K})^{\mathrm{Data}}$, the ratios $\tilde{I}/\tilde{Y}$ and $\tilde{C}/\tilde{Y}$ automatically match the data. First-order dynamics only need these ratios.

Appendix H. Log-linearized system

Following is the log-linearized equilibrium system.

(L1) Marginal utility of consumption:

$$\begin{aligned}\hat{\Lambda}_t\,(g_\gamma - \beta\gamma)\,(g_\gamma - \gamma) = {} & \left[-g_\gamma^2\hat{C}_t + \gamma g_\gamma\left(\hat{C}_{t-1} - \hat{g}_{\gamma t}\right)\right] \\ & - \beta\gamma E_t\left[-g_\gamma\left(\hat{C}_{t+1} + \hat{g}_{\gamma t+1}\right) + \gamma\hat{C}_t + \hat{\theta}_{t+1}(g_\gamma - \gamma)\right]. \end{aligned} \tag{S43}$$

(L2) Labor supply:

$$\hat{\Lambda}_t + \hat{w}_t = \hat{\psi}_t + \nu\hat{N}_t. \tag{S44}$$

(L3) Real estate rent:

$$\hat{R}_{ct} = \hat{Y}_t + \frac{1+\mu-\eta\mu}{1+\eta+\mu}\left(\hat{p}_t - \hat{Y}_t\right). \tag{S45}$$

(L4) Investment:

$$\begin{aligned} 0 = \hat{Q}_{kt} - \Omega\left(g_z g_\gamma\right)^2 \left[\hat{I}_t - \hat{I}_{t-1} + \hat{g}_{zt} + \hat{g}_{vzt} + \hat{g}_{\gamma t}\right] \\ + \beta\Omega\left(g_z g_\gamma\right)^2 E_t\left(\hat{I}_{t+1} - \hat{I}_t + \hat{g}_{zt+1} + \hat{g}_{\gamma t+1} + \hat{g}_{vzt+1}\right). \end{aligned} \tag{S46}$$

(L5) Marginal Tobin's Q_k:

$$\begin{aligned} \hat{Q}_{kt} + \hat{\Lambda}_t &= E_t\left[\hat{\theta}_{t+1} + \hat{\Lambda}_{t+1} - \hat{g}_{\gamma t+1} - \hat{g}_{zt+1} - \hat{g}_{vzt+1}\right] \\ &+ (1-\beta(1-\delta))E_t\left(\hat{u}_{t+1} + \hat{R}_{kt+1}\right) \\ &+ \beta(1-\delta)E_t\left[\hat{Q}_{kt+1} - \frac{\delta'(1)}{1-\delta}\hat{u}_{t+1}\right]. \end{aligned} \tag{S47}$$

(L6) Capacity utilization:

$$\hat{R}_{kt} = \frac{\delta''(1)}{\delta'(1)}\hat{u}_t + \hat{Q}_{kt}. \tag{S48}$$

(L7) Real estate price:

$$\begin{aligned} \hat{p}_t + \hat{\Lambda}_t = E_t\left(\hat{\theta}_{t+1} + \hat{\Lambda}_{t+1}\right) + \frac{\beta(\tilde{R}_h/\tilde{Y})}{\tilde{p}/\tilde{Y}} E_t\hat{R}_{ht+1} + \beta E_t\hat{p}_{t+1} + \\ \lambda(1-\beta)E_t\left[\hat{p}_{t+1} - \frac{1+\mu}{\mu}\frac{1+\mu}{1+\eta+\mu}\left(\hat{p}_{t+1} - \hat{Y}_{t+1}\right)\right]. \end{aligned} \tag{S49}$$

(L8) Rental rate of capital:

$$\hat{R}_{kt} + \hat{u}_t + \hat{K}_t = \hat{Y}_t + \hat{g}_{zt} + \hat{g}_{\gamma t} + \hat{g}_{vzt} + \frac{1+\mu-\eta\mu}{1+\eta+\mu}\left(\hat{p}_t - \hat{Y}_t\right). \tag{S50}$$

(L9) Labor demand:

$$\begin{aligned} \hat{w}_t + \hat{N}_t &= \hat{Y}_t + [1 - \frac{\eta\mu}{1+\mu}]\hat{a}^*_t \\ &= \hat{Y}_t + \frac{1+\mu-\eta\mu}{1+\eta+\mu}\left(\hat{p}_t - \hat{Y}_t\right). \end{aligned} \tag{S51}$$

(L10) Aggregate output:

$$\begin{aligned} \hat{Y}_t = \alpha(1-\phi)(\hat{u}_t + \hat{K}_t) + \\ (1-\alpha)\hat{N}_t + \frac{\eta\mu}{1+\eta+\mu}\left(\hat{p}_t - \hat{Y}_t\right) - \frac{\alpha(1-\phi)}{1-\alpha(1-\phi)}\left(\hat{g}_{zt} + \hat{g}_{vzt} + \hat{g}_{at} + \hat{g}_{vat}\right). \end{aligned} \tag{S52}$$

(L11) Collateral constraint:

$$\hat{a}^*_t = \frac{1+\mu}{1+\eta+\mu}\left(\hat{p}_t - \hat{Y}_t\right). \tag{S53}$$

(L12) Aggregate capital accumulation:

$$\hat{K}_{t+1} = \frac{(1-\delta)}{g_z g_\gamma}\hat{K}_t + \left(1 - \frac{1-\delta}{g_z g_\gamma}\right)\hat{I}_t - \frac{\delta'(1)}{g_z g_\gamma}\hat{u}_t - (1-\delta)\left[\frac{\hat{g}_{zt} + \hat{g}_{vzt}}{g_z g_\gamma} + \frac{\hat{g}_{\gamma t}}{g_z g_\gamma}\right]. \tag{S54}$$

(L13) Resource constraint:

$$\frac{\tilde{C}}{\tilde{Y}}\hat{C}_t + \frac{\tilde{I}}{\tilde{Y}}\hat{I}_t = \hat{Y}_t. \tag{S55}$$

(L14) Interest rate:

$$0 = \hat{R}_{ft} + E_t\left[\hat{\Lambda}_{t+1} + \hat{\theta}_{t+1} - \hat{\Lambda}_t - \hat{g}_{\gamma,t+1}\right],$$

which leads to

$$\hat{R}_{ft} = E_t\left[\hat{\Lambda}_t - \hat{\Lambda}_{t+1} - \hat{\theta}_{t+1} + \hat{g}_{\gamma,t+1}\right] \tag{S56}$$

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