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INEFFICIENTLY LOW SCREENING WITH WALRASIAN MARKETS

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ABSTRACT

Financial intermediaries devote resources to finding and screening borrowers before lending capital. By retaining only sufficiently good matches, informed lenders exacerbate adverse selection problems for others lending in the same market. Failure to internalize this implies that informed lenders are too selective in the matches they retain. The resulting under-use of capital pushes the cost of capital down, decreasing the benefit of being informed rather than uninformed and prompting a reallocation of resources from screening to matching. Compared to the constrained efficient allocation, the decentralized equilibrium has too little screening, too little informed credit, and too much uninformed credit.

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1 Introduction

In many models with asymmetric information about asset values, screening by a potential buyer lowers the average quality of assets available to others. It could be a consumer buying a used car, a firm hiring a new worker, or a bank financing a new firm. Potential buyers do not internalize the negative externality that their screening imparts, so the standard intuition is that decentralized screening will be inefficiently high.

I revisit the efficiency properties of screening in decentralized markets, motivated by a simple yet fundamentally important distinction: adverse selection is imparted not by the act of screening but by the retention decision of a buyer who has successfully screened. I demonstrate that screening and retention can behave quite differently, and should thus be decoupled, when the worst asset that would be profitable to retain upon successful screening is endogenous. Failure to internalize the adverse selection problem makes informed buyers too selective in the asset qualities they retain. This implies an under-utilization of capital that lowers funding costs and leads buyers to choose to become informed less often. Decentralized screening may therefore be inefficiently low, not inefficiently high.

The setting for my analysis is a model of financial intermediation. There is a continuum of heterogeneous borrowers who need capital to produce. Borrowers differ in production ability and have private information about their types. An equal mass of ex ante identical lenders has access to capital and intermediates it by hiring workers to perform two activities: matching and screening. Matching activities include the creation and marketing of products to attract new business, while screening activities target the information gap between borrowers and lenders. The probability that each activity succeeds is increasing in the amount of labor devoted to it, but labor is costly because workers must be paid a wage to forgo leisure. The wage and the cost of capital for lenders are each determined in Walrasian markets.

The intermediation process involves several decisions by the lender, namely how many workers to hire, how to allocate these workers between matching and screening, and whether to provide capital when the matching activity succeeds. If only matching succeeds, then the lender chooses the probability of providing capital knowing only the distribution from which the borrower was drawn (uninformed retention strategy). If both matching and screening succeed, then the lender decides whether to provide capital based on the borrower's type (informed retention strategy).

The optimal strategy of an informed lender is to retain all types above a chosen threshold. Informed lenders do not internalize that a higher threshold worsens the distribution of available borrowers, hence the informed retention strategy is too selective (i.e., the lowest type retained is too high) relative to a social planner who faces the same technologies and constraints. Uninformed retention, on the other hand, improves the available distribution by forgoing a future informed match where only borrowers above the threshold would be retained. Failure to internalize this improvement implies that the retention probability of an uninformed lender is also too low relative to the constrained efficient planner.

The under-retention of borrowers by all lenders implies an under-use of capital. In a Walrasian market, the price of capital would fall to encourage more retention. For the market mechanism to implement the constrained efficient retention strategies, however, the distributional externalities from informed and uninformed retention would have to be of equal strength. I show that they are not. Informed lenders are the direct source of adverse selection in the distribution of available borrowers, so the negative distributional externality imparted by the informed retention threshold is stronger than the positive distributional externality imparted by uninformed retention. As a result, the under-use of capital by informed lenders pushes the price of capital below what would be needed to correct the under-use of capital by uninformed lenders. The Walrasian mechanism then implements a decentralized equilibrium with over-retention by uninformed lenders and under-retention by informed ones.

The probability of uninformed retention is naturally bounded from above by one. I show that both the planner and the decentralized lenders are constrained by this upper bound when the expected duration of borrower projects is sufficiently short. In other words, both would like to retain more uninformed matches than they form, making the margin of adjustment the allocation of labor across intermediation activities. Moving the marginal unit of labor from screening to matching increases uninformed matches relative to informed ones in this region of the parameter space. The distributional externality from the matching activity is then positive and, all else constant, the fraction of labor devoted to matching will be inefficiently high for the same reason that uninformed retention was inefficiently high in the earlier discussion: the negative distributional externality imparted by the informed retention threshold is stronger than the positive distributional externality imparted by the matching activity, pushing the price of capital below the price that would correct the latter.

Next, I show that the total amount of labor hired by an individual unmatched lender is approximately efficient when (i) labor is inelastically supplied by workers and (ii) borrower projects are expected to be of sufficiently short duration. It then follows from the explanations above that too much labor will be devoted to matching while too little labor will be devoted to screening in the decentralized equilibrium. An extension to elastic labor supply can deliver under-investment in both matching and screening, but, on aggregate, there is still too much uninformed credit relative to the planner's solution and too little credit overall.

Related Literature The prediction of inefficiently low screening contrasts with a large body of literature. Broecker (1990) provides an early study of screening externalities in a model where banks attract borrowers via Bertrand competition and operate a noisy screening technology at zero cost. Several papers have since allowed for costly screening (e.g., Cao and Shi (2001), Hauswald and Marquez (2006), Direr (2008), Gehrig and Stenbacka (2011), Fishman and Parker (2015)) and, whenever screening is found to be inefficient in these papers, the conclusion is that screening is inefficiently high.[1] In models with separating contracts à la Rothschild and Stiglitz (1976) rather than screening technologies, the analog to inefficiently high screening would be a separating equilibrium when the planner wants pooling. Dell'Ariccia and Marquez (2006) model screening via separating contracts and find that any inefficiency involves the decentralized economy settling on a separating equilibrium.

[1] The dynamic model of Fishman et al (2020) also features inefficiently high screening. Under certain parameters, Gehrig and Stenbacka (2011) find cycles with delayed screening but even this does not culminate in insufficient information production unless firms are assumed to die in the interim.

In a very different environment, Guerrieri et al (2010) also find that inefficiencies tend to be in the direction of too much separation (i.e., partial pooling would pareto dominate).[2]

The run-up to the 2007-09 financial crisis seems at odds with inefficiently high screening by banks. If anything, information acquisition was too low and originate-to-distribute models with securitization have become a popular destination for answers. My results, derived in an originate-to-hold environment, suggest that securitization is not a necessary condition for too little screening, even if one can make the case that it was sufficient in this particular crisis.[3] In principle, environments like Grossman and Stiglitz (1980) and Verrecchia (1982) could also be used to study whether too much or too little information is acquired without appealing to securitization. Agents learn about a common fundamental and prices are at least partly revealing of total knowledge. In my model, each bank is learning about a different idiosyncratic borrower, not about an aggregate state. Therefore, inefficiency is not driven by a failure to internalize that others benefit from information acquisition through partial revelation of this information by prices. The price of capital does not play this role here.[4]

Instead, the role of prices in my model relates more to a growing literature on pecuniary externalities. Dávila and Korinek (2018) distinguish between two types of pecuniary externalities: collateral externalities that arise because of price-dependent financial constraints and distributive externalities that arise because of incomplete insurance markets. In my model, the feedback between lender decisions through the price of capital is more similar to a distributive pecuniary externality, not to be confused with the distributional externalities discussed earlier which were non-pecuniary and involved changing the composition of the

[2]There is also a flavor of too much separation in Fuchs and Skrzypacz (2015), as policies that disincentivize the use of trading delays to signal quality achieve efficiency gains.

[3]The negative effect of securitization on screening is shown empirically in Keys et al (2010) and Purnanandam (2011) but loan sales must be incentive compatible (e.g., Gorton and Pennacchi (1995)) so it is a separate theoretical question whether access to a securitization technology implies too little screening relative to the second-best. To this point, Vanasco (2017) allows cash flows from potentially screened assets to be securitized and finds that the direction of inefficiency can go either way.

[4]Recently, Colombo et al (2014), Llosa and Venkateswaran (2017), and Mackowiak and Wiederholt (2018) have considered learning about a common fundamental in coordination environments rather than environments with partially revealing prices. They find cases where information acquisition is inefficiently low, but, once again, the mechanisms and applications are very different from mine because of the nature of the information being acquired.

borrower pool. My results involve a pecuniary externality because two markets for capital (one for lenders that are informed and one for lenders that are uninformed) would allow the Walrasian economy to achieve constrained efficiency. Specifically, the distributional externality from each decision would be priced in a separate market, eliminating the feedback that exists when there is only one market for capital.

A classic result in the literature on pecuniary externalities is that excessive borrowing can arise in equilibrium because firms do not internalize that more leverage will require more fire sales should a negative shock hit (e.g., Lorenzoni (2008)). There are no fire sales in my model and the aggregate effects are such that uninformed credit is excessive but total credit can be inefficiently low because of under-retention of borrowers by informed lenders.

The rest of the paper proceeds as follows. Sections 2 and 3 study an environment without any Walrasian markets. Section 4 then introduces the market for capital while Section 5 introduces the market for labor. Section 6 concludes. All proofs are in Appendix A.

2 Baseline Model

Time is discrete. There are two groups of agents of equal mass, firms and lenders. All agents are infinitely-lived, risk neutral, and have discount factor $\beta \in (0, 1)$.

There is a continuum of firm types, denoted by ω and distributed uniformly over the unit interval. Each firm has private information about its type. Each firm also has a production project that requires one unit of capital input. Time to project completion is an i.i.d geometric random variable with parameter $\mu \in (0, 1]$. A completed (mature) project generates $y(\omega)$ units of output, where $y'(\cdot) > 0$.

Capital is endowed to a unit mass of ex ante identical lenders. Firms do not have their own capital and cannot store project output. Lenders do not have access to the same production projects as firms. Instead, lenders can intermediate capital to firms, as will be described in Section 2.1, or invest in a simple technology that yields g' units of output per unit of capital invested. Section 4 will replace the simple technology with an interbank market where lenders can trade capital with each other at a Walrasian price. Until then, each lender has one unit of capital, there is no trade between lenders, and g' is a constant satisfying:

Assumption 1 $y(0) < \frac{g'}{\mu} < y(1)$

Under Assumption 1, the simple technology is more productive than the project of the worst firm but less productive than the project of the best firm and thus not always socially preferable to intermediation.

2.1 Intermediation Technologies: Matching and Screening

Intermediation of capital by lenders to firms (borrowers) involves two technologies. The first is matching: lenders can create and/or advertise standardized financial products to match firms with capital. Formally, I model a one-to-one matching technology that is only available to unmatched lenders. The mass of unmatched lenders equals the mass of unmatched firms each period and a lender's matching probability depends only on his own matching effort.

The second intermediation technology is screening: a matched lender can investigate the quality of his match to determine whether he wants to retain the match and extend capital. Lenders cannot (costlessly) commit to actions that will dissuade certain firms from making themselves available to match. For now, I will also assume that retained matches are exogenously dissolved once the underlying project matures. Combined with the fact that all loans involve exactly one unit of capital, there are not enough instruments to offer separating contracts in lieu of screening.[5] Relationship lending, discussed in Appendix B, will provide an alternative means of learning but only after some time has elapsed.

Although lenders may want to undertake both matching and screening, it is either too costly or too time-consuming to make each activity succeed with certainty. I introduce a resource constraint to capture this. In particular, each lender is endowed with $z \in (0, \infty)$

[5] Abstracting from separating contracts is less stark than may initially seem. Separation is not free. The lender has to forgo some rents to ensure incentive compatibility for all borrower types. See also the fixed costs incurred per contract in Livshits et al (2016). In a different environment with non-exclusive contracts, Attar et al (2011) show that separation may not even be feasible.

units of non-transferable effort in every period that he is unmatched. For the moment, z is a parameter. In Section 5, I will model z as labor and endogenize the unmatched lender's decision of how much labor to hire when the wage is determined in a Walrasian market.[6]

A lender who allocates $\pi \in [0, z]$ units of his effort to matching gets a borrower with probability $p(\pi)$ and discovers that borrower's type with probability $p(z-\pi)$ immediately thereafter. The function $p(\cdot)$ satisfies $p(0) = 0$, $p(\infty) = 1$, $p'(\cdot) > 0$, and $p''(\cdot) < 0$. Also:

Assumption 2 $\frac{p'(z-\pi)}{1-p(z-\pi)} < \frac{p'(\pi)}{p(\pi)} - \frac{p''(\pi)}{p'(\pi)} - \frac{p''(z-\pi)}{p'(z-\pi)}$ *for any* $\pi \in (0, z)$

With $p'(\cdot) > 0$ and $p''(\cdot) < 0$, a stricter version of Assumption 2 is $p''(\cdot) \leq -\frac{p'(\cdot)^2}{1-p(\cdot)}$, interpretable as follows: if $p(\cdot)$ increases rapidly and/or approaches one, it picks up enough curvature to slow down. This ensures that lenders face economically meaningful tradeoffs when allocating finite resources and will be sufficient for uniqueness of equilibrium later on.

2.2 Retention Decisions and Project Completion

Consider a lender who is unmatched at the beginning of date t. He first chooses his matching effort π_t. With probability $1-p(\pi_t)$, he does not attract a borrower, in which case he operates the simple technology for one period and is unmatched at the beginning of date $t+1$. With probability $p(\pi_t)$, he does attract a borrower and exerts screening effort $z - \pi_t$. Successful screening occurs with probability $p(z-\pi_t)$ and means that the lender's information set contains the borrower's true type (i.e., the lender is "informed"). Unsuccessful screening occurs with probability $1 - p(z - \pi_t)$ and means that the lender's information set only contains the distribution from which the match was drawn (i.e., the lender is "uninformed"). Denote by $\psi_t(\cdot)$ the distribution from which matches are drawn at date t.

[6] Costly screening can be motivated as in the literature that followed Broecker (1990). Costly matching can be motivated by non-price competition for borrowers (e.g., Heider and Inderst (2012)) or search frictions (e.g., Becsi et al (2013)). I have in mind the former. My matching technology is set up so that no externalities are imparted through the ratio of borrowers to lenders in the market, which would be the key variable in a model of random search (e.g., Hosios (1990), Yashiv (2007)). While some search models with heterogeneous agents have also been used to study market composition (e.g., Shimer and Smith (2001)), they abstract from asymmetric information and are hence silent on whether screening is too high or too low.

Conditional on his information set, a newly matched lender must decide whether to retain the borrower he just attracted or whether to let him go, operate the simple technology, and try for another borrower in $t+1$. Let $I_t(\omega) \in [0,1]$ denote the probability that a newly matched lender retains a borrower whose type he knows to be ω. This is the informed retention strategy. The uninformed retention strategy, defined as the probability that a newly matched lender retains a borrower whose type he does not know, is denoted by $\alpha_t \in [0,1]$.

Once retention decisions have been made, newly matched and retained borrowers undertake production. At the end of date t, each matched borrower (new or continuing) discovers if his project has matured. The output from a mature project is also observed by the lender who financed it (but not by other lenders) and split with the borrower according to a constant fraction, which, to reduce notation, is just set to one in favor of the lender. In the baseline model, each lender eats the output he receives, whether from a mature project or the simple technology, and starts the next period with a new one-unit endowment of capital. This assumption will be relaxed in Section 4 (i.e., aggregate capital will be endogenous and allocated among lenders via the interbank market).

2.3 Quantity and Quality of Available Borrowers

I will focus on symmetric equilibria where all lenders choose the same effort allocation π_t, the same informed retention strategy $I_t(\omega)$, and the same uninformed retention strategy α_t.

Fraction $p(\pi_t)[1-p(z-\pi_t)]$ of unmatched type ω firms are drawn into uninformed matches at the beginning of date t and retained with probability α_t while fraction $p(\pi_t)\,p(z-\pi_t)$ are drawn into informed matches and retained with probability $I_t(\omega)$. Let $n_{t-1}(\omega)$ denote the fraction of type ω firms that are in matches after retention decisions at date $t-1$. Fraction μ of these matches mature at the end of $t-1$, leaving fraction $1-(1-\mu)\,n_{t-1}(\omega)$ of type ω firms unmatched at the beginning of date t. The law of motion for $n_t(\omega)$ is then:

$$n_t(\omega) = (1-\mu)\,n_{t-1}(\omega) + p(\pi_t)\left[[1-p(z-\pi_t)]\,\alpha_t + p(z-\pi_t)\,I_t(\omega)\right]\left[1-(1-\mu)\,n_{t-1}(\omega)\right] \tag{1}$$

In steady state, $n_t(\omega) = n_{t-1}(\omega)$ for each ω and thus the steady state fraction of type ω firms that receive financing is:

$$n(\omega) = \frac{p(\pi)[[1 - p(z - \pi)]\alpha + p(z - \pi) I(\omega)]}{\mu + (1 - \mu) p(\pi)[[1 - p(z - \pi)]\alpha + p(z - \pi) I(\omega)]} \tag{2}$$

Unless otherwise indicated, I focus on steady states and drop time subscripts.

The relative likelihood that an unmatched firm is of type ω is:

$$\psi(\omega) = \frac{1 - (1 - \mu) n(\omega)}{A} \tag{3}$$

where:

$$A = 1 - (1 - \mu) \int_0^1 n(\omega) d\omega \tag{4}$$

is the mass of open matches at the beginning of each period. The quality distribution of available matches is summarized by $\psi(\cdot)$.

2.4 Lender Value Functions

Let $J(\omega)$ denote the value of a match with type ω, conditional on retention. With probability μ, the match matures, giving the lender a one-period return of $y(\omega)$ and a continuation value of βU, where U denotes the value of an unmatched lender. With probability $1-\mu$, the match does not mature and is carried over to the next period, giving the lender a one-period return of zero and a continuation value of $\beta J(\omega)$. Therefore:

$$J(\omega) = \mu [y(\omega) + \beta U] + \beta (1 - \mu) J(\omega) \tag{5}$$

Consider now U. An unmatched lender can achieve a one-period return of g' and a continuation value of βU by investing his capital in the simple technology. If he were to instead provide the capital to a type ω firm, his net return would be $J(\omega) - g' - \beta U$. Of course, the lender only finds a firm with probability $p(\pi)$, the firm is only type ω with probability density $\psi(\omega)$, and type ω is only retained with probability $[1-p(z-\pi)]\alpha+p(z-\pi)I(\omega)$. Integrating over firm types, the value of an unmatched lender is thus:

$$U = g' + \beta U + p(\pi)\int_0^1 [[1-p(z-\pi)]\alpha + p(z-\pi)I(\omega)][J(\omega)-g'-\beta U]\psi(\omega)\,d\omega \quad (6)$$

A decentralized lender chooses his effort allocation $\pi \in [0,z]$ and informed and uninformed retention strategies $I(\cdot) \in [0,1]$ and $\alpha \in [0,1]$ to maximize U taking as given the distribution $\psi(\cdot)$. The dimensionality of the problem is simplified by the following lemma, which reduces informed retention to a reservation strategy around a scalar $\xi \in [0,1]$:

Lemma 1 *There is a unique $\xi \in [0,1]$ such that successfully screened matches are retained if and only if $\omega \geq \xi$. That is, $I(\omega)=0$ for $\omega<\xi$ and $I(\omega)=1$ for $\omega \geq \xi$.*

Using the recursive structure of the value functions, the problem can be written as:

Lemma 2 *Define the expected one-period net return from intermediating an open match:*

$$\begin{aligned}\Gamma(\pi,\xi,\alpha,\psi(\cdot),g') \equiv\ & p(\pi)[1-p(z-\pi)]\alpha\int_0^1[\mu y(\omega)-g']\psi(\omega)\,d\omega \\ & +p(\pi)p(z-\pi)\int_\xi^1[\mu y(\omega)-g']\psi(\omega)\,d\omega\end{aligned}$$

and the recursive discount rate:

$$D(\pi,\xi,\alpha,\psi(\cdot)|\beta) \equiv 1+\frac{\beta(1-\mu)}{1-\beta(1-\mu)}p(\pi)\left[[1-p(z-\pi)]\alpha+p(z-\pi)\int_\xi^1\psi(\omega)\,d\omega\right]$$

Lenders in the decentralized economy solve:

$$\max_{\pi\in[0,z],\xi\in[0,1],\alpha\in[0,1]}\left\{\left(D\left(\pi,\xi,\alpha,\overline{\psi}(\cdot)|\beta\right)\right)^{-1}\times\Gamma\left(\pi,\xi,\alpha,\overline{\psi}(\cdot),g'\right)\right\}$$

where the notation $\overline{\psi}(\cdot)$ means $\psi(\cdot)$ is being taken as given.

A decentralized equilibrium is a triple (π^*,ξ^*,α^*) that solves the fixed point problem implicit in Lemma 2. Specifically, for any triple (π_0,ξ_0,α_0), equation (3) defines a distribution $\psi_0(\cdot)$.

Taking as given this distribution, the first order conditions for the maximization problem in Lemma 2 deliver a triple $(\pi_1, \xi_1, \alpha_1) \equiv \mathcal{T}(\pi_0, \xi_0, \alpha_0)$. A decentralized equilibrium then solves $(\pi^*, \xi^*, \alpha^*) = \mathcal{T}(\pi^*, \xi^*, \alpha^*)$.

2.5 Constrained Efficiency Benchmark

Consider a social planner who holds the economy's unit endowment of capital and must allocate it to achieve production. The planner puts equal weight on all types and everyone is risk neutral. Hence, welfare is measured by the total present discounted value of output:

$$\mathcal{W} = \frac{1}{1-\beta}\left[g' + \int_0^1 [\mu y(\omega) - g'] \, n(\omega) \, d\omega\right] \tag{7}$$

With one unit of capital, the planner can operate the simple technology and generate g' units of output at the end of the period. If he were to instead give the capital to a type ω firm, $y(\omega)$ units of output would be generated at the end of the period with probability μ. The planner is subject to the same constraints as lenders in the decentralized economy, meaning he can only allocate capital to firms by dividing effort z between the two intermediation technologies described in Section 2.1. The fraction of type ω firms that receive capital is therefore $n(\omega)$, with $n(\omega)$ in the welfare function given by equation (2).

The following lemma writes the planner's problem in a form more easily comparable to the problem solved by decentralized lenders in Lemma 2:

Lemma 3 *With g' constant, the planner solves:*

$$\max_{\pi \in [0,z], \xi \in [0,1], \alpha \in [0,1]} \{A \times \Gamma(\pi, \xi, \alpha, \psi(\cdot), g')\}$$

where A, as defined in equation (4), is equivalent to the limit:

$$A = \lim_{\beta \to 1} (D(\pi, \xi, \alpha, \psi(\cdot) | \beta))^{-1} \tag{8}$$

A constrained efficient allocation is a triple $\left(\widehat{\pi}, \widehat{\xi}, \widehat{\alpha}\right)$ that solves the maximization problem in Lemma 3, taking into account that $\psi(\cdot)$ is endogenously determined as per equation (3).

The first difference between the problem of a decentralized lender in Lemma 2 and the planner's problem in Lemma 3 is that the planner takes into account how his decisions affect the distribution of available borrowers $\psi(\cdot)$. Decentralized lenders take this distribution as given and hence fail to internalize the *distributional externality* that they impart on each other. The quality of available borrowers affects the attractiveness of intermediation relative to the simple technology so distributional externalities can change how other lenders intermediate. Failure to internalize this can have real effects.

The second difference is that the planner internalizes how his decisions affect the total mass of open matches available to firms, i.e., $(D(\cdot|1))^{-1}$ in Lemma 3 rather than just $(D(\cdot|\beta))^{-1}$ in Lemma 2. By the recursive nature of his problem, a decentralized lender internalizes how his decisions affect his future availability, but, unless $\beta = 1$, this is not the same as internalizing how the current availability of other lenders is affected.[7] I will call the difference between the decentralized and planning problems arising from $\beta < 1$ an *extensive externality*. This externality would be moot if there was free entry of unmatched lenders because the expected net return $\Gamma(\cdot)$ would be driven to zero in both Lemmas 2 and 3, making it irrelevant what exactly multiplies $\Gamma(\cdot)$. The extensive externality thus appears in my model because barriers to free entry allow lenders to make positive profit.

3 Inefficiencies in the Baseline Model

This section explores how the extensive and distributional externalities affect the decentralized equilibrium relative to the constrained efficient allocation. The focus will be on $\mu \in (0, 1)$, as intertemporal preservation of matches is necessary for inefficiency:

Proposition 1 *If $\mu = 1$, then the decentralized equilibrium is constrained efficient.*

[7] See the decentralized bargaining model of Elliott and Nava (2019) for other examples where an inefficiency disappears as the discount factor converges to 1.

The proof of Proposition 1 is straightforward. With $\mu = 1$, any matches formed at the beginning of the period break by the end of the period. Every firm is therefore available at the beginning of every period, regardless of the choices of π, ξ, and α, so there are no extensive or distributional externalities stemming from these choices.

The results that follow are organized around the uninformed retention strategy α. Section 3.1 considers the case where both the planner and the decentralized lenders choose $\alpha = 1$. Section 3.2 then shows what happens when the constraint $\alpha \leq 1$ is slack.

3.1 Full Retention of Uninformed Matches

Suppose both the planner and the decentralized lenders choose $\alpha = 1$. The goal here is to build intuition for the rest of the paper so existence and uniqueness of solutions are taken as given. These properties will be formally established for the Walrasian model of Section 4.

3.1.1 Extensive Externalities

To isolate only the extensive externalities, we can consider a version of the model where the distribution $\psi(\cdot)$ is exogenous because available borrowers draw new types from the uniform (population) distribution at the beginning of every period:

Proposition 2 *Fix $\psi(\cdot) = 1$. For any $\beta < 1$, the decentralized solution has (i) π inefficiently high and (ii) ξ inefficiently low.*

Proposition 2 indicates that the extensive externality from π is negative. An increase in π implies a substitution of effort from screening to matching, generating more uninformed matches. The total mass of open matches then falls because uninformed matches are always retained ($\alpha = 1$). The extensive externality from ξ is, in contrast, positive. An increase in ξ implies that informed lending is more selective, leading to more open matches. The planner therefore chooses a lower value of π and a higher value of ξ than the decentralized lenders when the distribution of available borrowers is exogenous.

3.1.2 Distributional Externalities

Return now to the full model which also has $\psi(\cdot)$ endogenous. Since informed retention always follows a reservation strategy, the distribution of available borrowers involves only two values: ψ_L which will denote the density for any $\omega < \xi$ and ψ_H which denotes the density for any $\omega \geq \xi$, where $\psi_L \xi + \psi_H (1 - \xi) = 1$.

As shown next, the distributional externality from ξ is negative and reverses the prediction of ξ inefficiently low in Proposition 2:

Proposition 3 *With $\psi(\cdot)$ endogenous, $\xi^* = \widehat{\xi}$ for $\beta = 0$ and $\xi^* > \widehat{\xi}$ for any $\beta > 0$.*

The extensive externalities discussed in Section 2.5 are strongest at $\beta = 0$. Proposition 3 indicates that the positive extensive externality from ξ is exactly offset by a negative distributional externality when $\beta = 0$. As β increases, the extensive externality becomes muted while the distributional one is unchanged, leading to ξ^* inefficiently high.

The negative distributional externality from ξ is driven by the effect of ξ on ψ_H. As ξ increases, informed lenders hold out for a smaller set of borrower types, making any one of those types less available in the steady state (i.e., ψ_H is lower). On the flip side, the range of types fully available to uninformed lenders expands, which means that the probability of drawing any one type from this range also falls (i.e., ψ_L is lower). For an unmatched lender, both the very best and the very worst types are now less likely to be drawn. The fact that intermediation is most valuable with high types drives the direction of the distributional externality in Proposition 3. Specifically, informed lenders are too selective because they do not internalize that the returns to intermediation for other lenders fall when the probability of drawing high types falls.

Consider next the distributional externality from π. Only informed lenders can change the distribution of available borrowers, and they do so by returning borrowers below type ξ to the available pool. Unmatched lenders have no borrowers to return and uninformed lenders do not know enough about their borrowers to condition retention decisions on type.

Thus, the allocation of effort between matching and screening affects the distribution $\psi(\cdot)$ by affecting the probability that an unmatched lender becomes an informed lender.

Proposition 4 *Consider $\psi(\cdot)$ endogenous. If $\beta = 0$ so that $\xi^* = \widehat{\xi}$, then $\pi^* > \widehat{\pi}$ and, by continuity, there is a $\overline{\beta} \in (0,1]$ such that $\pi^* > \widehat{\pi}$ for any $\beta \leq \overline{\beta}$.*

To understand Proposition 4, it is important to recognize that the distributional externality from π is non-monotone. In more detail, the probability that an unmatched lender forms an informed match is $p(\pi)\, p(z-\pi)$, which is maximized at $\pi = \frac{z}{2}$ given the concavity of $p(\cdot)$. Increasing π within the range $\pi \in \left(0, \frac{z}{2}\right)$ increases the formation of informed matches and worsens the available distribution, while increasing π within the range $\pi \in \left(\frac{z}{2}, z\right)$ decreases the formation of informed matches and improves the available distribution. I will later establish that $\pi \in \left(\frac{z}{2}, z\right)$ when $\alpha = 1$ is optimal.[8] Thus, the distributional externality from π is positive and the baseline model will only have $\pi^* > \widehat{\pi}$ if the negative extensive externality from π is sufficiently strong, which, according to Proposition 4, it proves to be at low β.

3.2 Partial Retention of Uninformed Matches

Consider now the case where uninformed retention is unconstrained by $\alpha \leq 1$. The following proposition shows that π is constrained efficient. Any inefficiencies only appear in the retention decisions ξ and α:

Proposition 5 *For parameters where both the planner and the decentralized lenders choose $\alpha < 1$, the decentralized π is constrained efficient. In contrast, the efficiency properties of the decentralized ξ are still as in Proposition 3. If $\beta = 0$ so that $\xi^* = \widehat{\xi}$, then $\alpha^* < \widehat{\alpha}$ for parameters where both the planner and the decentralized lenders choose $\alpha \in (0,1)$ and, by continuity, there is a $\overline{\overline{\beta}} \in (0,1]$ such that $\alpha^* < \widehat{\alpha}$ for any $\beta \leq \overline{\overline{\beta}}$.*

[8] See specifically the proof of Proposition 10. Although Proposition 10 applies to the Walrasian model, the part of the proof that shows $\pi > \frac{z}{2}$ in any decentralized equilibrium where $\alpha = 1$ is optimal does not rely on the market clearing condition and is easy to rederive for the baseline model.

The extensive externality from α is negative. Intuitively, the total mass of open matches falls when uninformed lenders retain a higher fraction of the matches they form. The distributional externality from α is instead positive. Retaining an uninformed match rather than holding out for an informed one frees up some type $\omega \geq \xi$ borrowers in the steady state. Higher α thus increases ψ_H and decreases ψ_L, improving the available distribution. Holding the informed retention threshold ξ at its constrained efficient value, Proposition 5 says that the distributional externality from α dominates the extensive one, leading the planner to choose a higher value of α than the decentralized lenders.

Comparing the results in Section 3.1 to Proposition 5 reveals that π becomes the margin of adjustment when both the planner and the decentralized lenders are constrained by $\alpha \leq 1$. With this constraint binding, both want to retain more uninformed matches than they form, so they increase π to form more such matches. However, with ξ at its constrained efficient value, Proposition 4 says that the extensive externality from π dominates the distributional one, hence the planner will increase π less aggressively than the decentralized lenders.

Notice that the externalities associated with α here and π in Section 3.1 have similar signs (negative extensive, positive distributional) but the relative strengths differ. In particular, the amount of matching effort π has a stronger effect on the quantity of open matches A than on the quality distribution $\psi(\cdot)$, whereas the probability of uninformed retention α has a stronger effect on the quality distribution $\psi(\cdot)$ than on the quantity A.

4 Inefficiencies with a Walrasian Interbank Market

The analysis so far has assumed that the cost of intermediating capital is forgone investment in an alternative technology. I now analyze what happens when lenders instead face a cost of capital that is determined in a Walrasian market.

4.1 Capital Market Clearing

Set $g' = 0$ to eliminate the simple technology and replace Assumption 1 with:

Assumption 3 $y(0) < 1 < y(1)$ *and* $\int_0^1 y(\omega)\, d\omega < 1$

This ensures that not all projects are worth their capital input and that the accumulation of capital will be bounded.

Next, introduce into the baseline environment a Walrasian interbank market for capital with market clearing price R. The price is quoted so that R is the present discounted value of a lender's gross cost of funds. Lenders who do not have enough capital to finance their matches borrow from the interbank market. For all other lenders, interbank trade is the opportunity cost of proceeding with a match. The interbank market allows us to abstract from the distribution of capital across lenders and focus instead on the aggregate capital stock, which is now endogenously determined.

The price R adjusts until the equilibrium values of π, ξ, and α clear the capital market. In steady state, the capital market clears when the demand for capital from newly formed matches equals the supply of capital generated by newly maturing matches. In any given period, fraction $n(\omega)$ of type ω firms are in matches and fraction μ of these matches mature. The supply of capital generated by newly maturing matches is therefore $\mu \int_0^1 y(\omega)\, n(\omega)\, d\omega$.[9] On the demand side, the fraction of type ω firms in newly formed matches is given by the second term on the right-hand side of equation (1). We can see from equation (1) that this term is simply $\mu n(\omega)$ in steady state. Since firm projects each require one unit of capital, the demand for capital from newly formed matches is $\mu \int_0^1 n(\omega)\, d\omega$. The market clearing condition is therefore:

$$\mu \int_0^1 [1 - y(\omega)]\, n(\omega)\, d\omega = 0 \tag{9}$$

with $n(\cdot)$ as defined in (2). Equation (9) implicitly assumes that lenders return all the proceeds from maturing matches back to the interbank market. This is not necessary for the results. One could instead let lenders eat a small fraction $\varepsilon > 0$ so that only $(1-\varepsilon)\, y(\cdot)$ is

[9] I will keep the assumption of the baseline model that the output from a matured project is paid in full to the lender. Assuming instead that the lender only gets an exogenous fraction $\kappa \in (0,1)$ of the output while the borrower consumes the rest delivers exactly the same results, so setting $\kappa = 1$ eliminates the extra parameter without changing the insights. In a previous version of the paper, I showed that the main results are also robust to allowing κ to be endogenously chosen by the lender.

returned. Larger ε would mean a smaller capital stock and thus require a higher R to clear the interbank market.

4.2 Decentralized Lenders

The price R is taken as given by each individual lender, as is the distribution $\psi(\cdot)$. The value of an unmatched lender is now:

$$U = \beta U + p(\pi) \int_0^1 [[1 - p(z - \pi)] \alpha + p(z - \pi) I(\omega)] [J(\omega) - R - \beta U] \psi(\omega) d\omega \qquad (10)$$

where $J(\omega)$ is still as per equation (5). It is straightforward to show that informed retention still follows a reservation strategy and that, by the recursive structure of the value functions, lenders in the decentralized economy solve:

$$\max_{\pi \in [0,z], \xi \in [0,1], \alpha \in [0,1]} \left\{ \left(D\left(\pi, \xi, \alpha, \overline{\psi}(\cdot) | \beta\right) \right)^{-1} \times \Gamma\left(\pi, \xi, \alpha, \overline{\psi}(\cdot), \widetilde{R}\right) \right\}$$

where $\widetilde{R} \equiv [1 - \beta(1 - \mu)] R$. This is the objective function in Lemma 2, but with $\widetilde{R}$ in place of g'. The decentralized problem is thus as before, conditional on the determination of R. Parallel to Section 3, I will focus first on $\alpha = 1$. Then, in Section 4.5, I consider $\alpha < 1$ and characterize the optimal choice of α.

Proposition 6 *Define:*

$$S(\pi, \xi, \psi(\cdot) | \beta) \equiv \frac{\mu p(\pi)}{D(\pi, \xi, 1, \psi(\cdot) | \beta)} \left[\int_0^1 y(\omega) \psi(\omega) d\omega - p(z - \pi) \int_0^\xi y(\omega) \psi(\omega) d\omega \right]$$

and:

$$V(\pi, \xi, \psi(\cdot) | \beta) \equiv \frac{p(\pi)}{D(\pi, \xi, 1, \psi(\cdot) | \beta)} \left[1 - p(z - \pi) \int_0^\xi \psi(\omega) d\omega \right]$$

With $\alpha = 1$, the first order conditions for the decentralized problem are:

$$S'_i - V'_i \widetilde{R} = 0 \text{ for } i \in \{\pi, \xi\} \qquad (11)$$

where S_i' and V_i' denote the partial derivatives of $S(\cdot|\beta)$ and $V(\cdot|\beta)$ with respect to i. A decentralized equilibrium is then a triple $\{\pi^*, \xi^*, R^*\}$ satisfying (11) and market clearing as per (9). Under Assumption 3 and $p(z)$ sufficiently high, there exists an equilibrium with $\pi^* \in (0, z)$ and $\xi^* \in (0, 1)$ and, under Assumption 2, this equilibrium is unique.

The condition on $p(z)$ in Proposition 6 is just a lower bound on the endowment of intermediation resources z. It prevents the resource constraint from being so tight that lenders cannot pursue enough intermediation to clear the interbank market.

4.3 Planner's Problem

Return to the welfare function in equation (7), setting $g' = 0$. The planner now faces an aggregate feasibility constraint equivalent to (9). The Lagrange multiplier on this constraint, λ, is the shadow price of capital in the planner's problem while R is the market price of capital in the decentralized equilibrium. The planner's Lagrangian is then:

$$\mathcal{L} = \frac{\mu}{1-\beta} \int_0^1 \left[y(\omega) - \lambda\left[1 - y(\omega)\right]\right] n(\omega)\, d\omega$$

Proposition 7 *With $\alpha = 1$, the planner's first order conditions are:*

$$S_i' - V_i' r + X_i = 0 \text{ for } i \in \{\pi, \xi\} \tag{12}$$

where:

$$r \equiv \frac{\mu}{1+\lambda}\left[\lambda + \frac{(1-\beta)(1-A)}{1-\beta(1-\mu)}\right]$$

and:

$$X_i \equiv \left[S_{\psi_L}' - V_{\psi_L}' r\right]\frac{\partial \psi_L}{\partial i} + \left[S_{\psi_H}' - V_{\psi_H}' r\right]\frac{\partial \psi_H}{\partial i}$$

The constrained efficient allocation is then a triple $\left(\widehat{\pi}, \widehat{\xi}, \widehat{\lambda}\right)$ satisfying (12) and the aggregate feasibility constraint in (9). If μ is not too small, then $\left(\widehat{\pi}, \widehat{\xi}, \widehat{\lambda}\right)$ is unique.

The condition on μ in Proposition 7 is sufficient, not necessary. We will see in Section 4.5 that the optimality of $\alpha = 1$ for both the planner and the decentralized lenders requires μ above some threshold, so the condition in Proposition 7 is not restrictive.

4.4 Full Retention of Uninformed Matches

We can now compare the decentralized solution to the constrained efficient allocation to see whether the predictions of the Walrasian model under the restriction of $\alpha = 1$ differ from those of the baseline model in Section 3.1.

4.4.1 Results with Exogenous Distribution

It follows immediately from equations (11) and (12) that the decentralized equilibrium of the Walrasian model is constrained efficient in the absence of distributional externalities. Mathematically, if $\psi(\cdot)$ is exogenously reset every period as in Proposition 2, then $X_i = 0$ for $i \in \{\pi, \xi\}$ and a decentralized price of $\widetilde{R} = r$ implements the planner's solution.

Proposition 2 delivered a decentralized equilibrium with too much matching relative to screening (π inefficiently high) and too much informed retention (ξ inefficiently low). Thus, too much capital was tied up in existing matches because of the extensive externalities from π and ξ. With a Walrasian market for capital, any over-use of capital will cause the price of capital to rise, which will then prompt lenders to screen more (lower π) and be more selective in who they finance upon successful screening (higher ξ). A Walrasian market therefore prices in the extensive effects seen earlier, guiding the decentralized equilibrium to the constrained efficient allocation when there are no distributional externalities.

The key here is that the extensive externalities from π and ξ have similar effects on the price of capital, enabling one market clearing price to correct them both. The relationship lending extension in Appendix B provides an example where this is not the case – in fact, the extensive externalities from π and ξ have opposite effects on the price of capital once relationship lending is introduced – so, even without distributional externalities, the Walrasian market does not correct the extensive effects.

4.4.2 Results with Endogenous Distribution

Return to $\psi(\cdot)$ endogenously determined as per equation (3). Then $X_i \neq 0$ and it is clear from equations (11) and (12) that a decentralized price, specifically $\widetilde{R} = r - \frac{X_\xi}{V'_\xi}$, implements the planner's solution if and only if $X_\xi = \frac{V'_\xi}{V'_\pi} X_\pi$. In other words, there would have to be a specific proportionality between the distributional externalities from π and ξ, as measured by X_ξ and X_π, otherwise the decentralized equilibrium with market clearing will not be constrained efficient. This proportionality does not hold in general:

Proposition 8 *Fix $\alpha = 1$. For $\mu \in (0,1)$, the decentralized equilibrium with market clearing involves $\pi^* > \widehat{\pi}$ and $\xi^* > \widehat{\xi}$. If ξ were fixed at $\widehat{\xi}$, there would be an equilibrium price R_π that implements $\widehat{\pi}$. If π were fixed at $\widehat{\pi}$, an equilibrium price $R_\xi < R_\pi$ would implement $\widehat{\xi}$.*

Note that Proposition 8 holds for any discount factor β. This includes $\beta = 1$, which, from Section 2.5, eliminates the extensive externalities. Two implications from Proposition 8 are then immediate. First, the Walrasian market fails to price in the distributional externalities, even absent any extensive effects. Second, because the Walrasian market did not fail to price in the extensive effects absent any distributional externalities (Subsection 4.4.1), the intuition for Proposition 8 must lie in the distributional externalities.

Recall the negative distributional externality from ξ, which also delivered too little informed retention ($\xi^* > \widehat{\xi}$) in Proposition 3. Too little capital is thus used in matches with firms, so, with a Walrasian interbank market for capital, the price of capital will fall. Informed lenders will then become less selective in who they retain (lower ξ), helping to correct the inefficiency in ξ. Simultaneously though, a fall in the price of capital will prompt unmatched lenders to reallocate effort from screening to matching (higher π) and, unless the distributional externality from π discussed after Proposition 4 is sufficiently positive, the interbank price that corrects the inefficiency in ξ will "over-correct" the inefficiency in π. Proposition 8 says that the distributional externality from π is never sufficiently positive, so the decentralized equilibrium in the Walrasian model with $\alpha = 1$ settles on both π and ξ inefficiently high.[10] In a very different setting, Colombo et al (2014) discuss the acquisition of information separately from its use. Employing that classification here, the use of information (as captured by ξ) imparts a stronger effect on the distribution of available borrowers than the acquisition of information (as captured by $z - \pi$).

A corollary of Proposition 8 is that two prices are necessary to absorb the disproportionality between the distributional externalities from ξ and π: one price that deals with the effect from ξ and another that deals with the effect from π. Alternatively, the policy-maker would need an instrument that changes the strength with which R affects π relative to ξ. Appendix C explores this further using a matching tax.

The aggregate implications of Proposition 8 are summarized next:

Proposition 9 *In the environment of Proposition 8: (i) uninformed lending is too high; (ii) informed lending is too low; (iii) total lending is too low.*

The first two parts of Proposition 9 are intuitive given that unmatched lenders over-do matching relative to screening while informed lenders are too selective in the types they retain. The third part then reveals that the composition of informed versus uninformed lending results in an inefficiently small credit market overall.[11]

4.5 Optimality of Uninformed Retention

The previous section assumed that uninformed matches are always retained by the planner and the decentralized lenders. I now remove this assumption and allow the probability of accepting an uninformed match, $\alpha \in [0, 1]$, to be endogenously chosen:

[10] As π^* rises above $\widehat{\pi}$, too many uninformed matches are formed and, with $\alpha = 1$, retained. This leads to an over-use of capital which prevents the price of capital from falling by enough to fully correct the under-use stemming from ξ. For this reason, there is still $\xi^* > \widehat{\xi}$.

[11] Both screening and total credit being inefficiently low contrasts with Dell'Ariccia and Marquez (2006). In Ruckes (2004), screening and total credit can both fall as economic prospects worsen, but that is a comparative static result with respect to the aggregate state, not a comparison between the decentralized equilibrium and the constrained efficient allocation for any given state.

Proposition 10 *Suppose $p\left(\frac{z}{2}\right)$ is not too low, which is effectively a lower bound on z and qualitatively similar to the condition in Proposition 6. There exists a $\overline{\mu} \in (0,1)$ such that $\alpha^* = \widehat{\alpha} = 1$ if and only if $\mu \geq \overline{\mu}$.*

The results of Section 4.4 therefore apply whenever the exogenous match separation rate μ is sufficiently high.

Figure 1 illustrates what happens for the entire range of separation rates. Details on the construction of Figure 1 are presented in Appendix D. To simplify the exposition, I assume that project output is linear in firm type, specifically $y(\omega) = \theta\omega$. The lowest value of μ for which lenders in the decentralized economy optimally choose $\alpha = j$ is denoted by μ_j, where $j \in \{0, 1\}$. The analogous thresholds for the planner are denoted by $\widehat{\mu}_j$. Appendix D establishes $\mu_0 < \widehat{\mu}_0 < \widehat{\mu}_1$ and $\mu_0 < \mu_1 < \widehat{\mu}_1$ but the position of $\widehat{\mu}_0$ relative to μ_1 depends on parameters. For completeness, Figure 1 illustrates both possibilities, with the left panel drawn for parameters where $\widehat{\mu}_0 < \mu_1$ and the right panel drawn for parameters where $\widehat{\mu}_0 > \mu_1$. Both panels deliver the same messages about the directions of any inefficiencies.

For low values of μ, the top row of Figure 1 shows that both the planner and the lenders in the decentralized economy optimally reject uninformed matches ($\alpha^* = \widehat{\alpha} = 0$). The quality of an uninformed match is not discovered until it breaks so the opportunity cost of unknowingly being in a bad match is high when the separation rate is low.

Notice from the second and third rows of Figure 1 that π and ξ are also constrained efficient for values of μ yielding $\alpha^* = \widehat{\alpha} = 0$. When uninformed matches are always rejected ($\alpha = 0$), the model is isomorphic to one where firm types are public information and there exists a single intermediation technology that delivers a randomly drawn match with probability $\widetilde{p}(\pi) \equiv p(\pi)\, p(z - \pi)$. In this environment, it would be optimal for both the planner and the decentralized lenders to maximize the contact rate with firms, $\widetilde{p}(\pi)$, given an endowment of intermediation resources z which need not be fully exhausted. It would also be the case that the quantity of open matches A and the quality distribution $\psi(\cdot)$ depend on π only through $\widetilde{p}(\pi)$, hence the marginal effect of π on A and $\psi(\cdot)$ is zero when $\widetilde{p}(\pi)$

is maximized. The decentralized choice of π is therefore constrained efficient under $\alpha = 0$. With both π and α constrained efficient, the Walrasian interbank market can just price in the net externality from ξ, guiding ξ to its constrained efficient value.[12]

For intermediate values of μ, the decentralized ξ in Figure 1 becomes inefficiently high. This is as in Proposition 8. However, instead of π also being inefficiently high, it is now α that is inefficiently high. Unmatched lenders get the allocation of resources between matching and screening correct but are too willing to accept uninformed matches relative to the planner. Although the inefficiency shows up in α (the probability of accepting an uninformed match) rather than π (the effort allocation that determines the probability of forming an uninformed match), the flavor of the problem is similar to before: decentralized lenders fund too many uninformed matches. It is then straightforward to show that Proposition 9 still holds.[13]

The result on α here contrasts with the baseline model, where the decentralized α was too low (Proposition 5) because of a positive distributional externality. Now, the decentralized α is too high, reflecting the combination of the distributional externalities and the Walrasian market. All else constant, ξ inefficiently high causes the price of capital to fall. This prompts both informed and uninformed lenders to be less selective, decreasing ξ and increasing α. The negative distributional externality from ξ is stronger than the positive distributional externality from α and hence the decentralized α is pushed above its constrained efficient value. Based on this intuition, we should see a similar reversal in the direction of inefficiency of α in the baseline model if the simple technology that served as an alternative to intermediation is changed from a linear technology to one that exhibits sufficiently strong decreasing returns to scale.[14] Appendix E confirms this.

[12] In the baseline model, where there is no market to price in this net externality, ξ would still be inefficiently high because the negative distributional externality from ξ dominates the positive extensive one (Section 3).

[13] To sketch the proof, total capital K is proportional to welfare (thus the decentralized K is too low) and uninformed lending K_N is proportional to αA when $\pi = \frac{z}{2}$, where A is decreasing in K. With α too high and K too low, it follows that K_N is too high and informed lending K_I is too low.

[14] All else constant, ξ inefficiently high means that too much capital is invested in the simple technology, depressing its marginal return when there are decreasing returns to scale. This is similar to a decrease in the price of capital in the Walrasian model.

As μ continues to increase, Figure 1 shows that π also becomes inefficiently high. Eventually, both the planner and the decentralized lenders settle on accepting all uninformed matches ($\alpha^* = \widehat{\alpha} = 1$ as in Proposition 10) with $\pi^* > \widehat{\pi}$ and $\xi^* > \widehat{\xi}$ as per Proposition 8.

5 Endogenous Resource Constraint

The analysis so far has assumed that intermediation resources z are exogenous. I now extend the Walrasian model of Section 4 to consider what happens when z is endogenously chosen by unmatched lenders. In particular, instead of being endowed with z units of effort, unmatched lenders now choose an amount of labor z to hire, along with choosing how to allocate this labor between matching and screening.

Denote by $\overline{L}$ the total amount of labor available in the economy. For the moment, labor is inelastically supplied at wage W each period, where W is an equilibrium object. There are two changes to the analysis. First, the value of an unmatched lender becomes:

$$U = -Wz + \beta U + p(\pi) \int_0^1 \left[\left[1 - p(z-\pi)\right]\alpha + p(z-\pi) I(\omega)\right]\left[J(\omega) - R - \beta U\right]\psi(\omega)\, d\omega \tag{13}$$

Second, the newly introduced labor market has to clear:

$$Az = \overline{L} \tag{14}$$

The left-hand side of (14) is the aggregate demand for labor by unmatched lenders while the right-hand side is aggregate labor supply. To avoid unnecessarily complicating the analysis, workers deposit all of their labor income back into the banking system so that capital market clearing is still given by equation (9).

The rest of the details are collected in the proof of the following proposition:

Proposition 11 *Suppose $\overline{L}$ is not too low. Then there is a $\overline{\overline{\mu}} < 1$ above which the decentralized equilibrium has more matching and less screening than the constrained efficient allocation, even when the amount of intermediation resources z is endogenized.*

Allowing z to be endogenously chosen introduces the possibility of an individual lender devoting more resources π to the matching technology without also devoting fewer resources $z-\pi$ to screening. The key insight from Proposition 11 is that under-investment in screening, as found in Proposition 8, is robust to this possibility.

Figure 2 presents a concrete example. As in Figure 1, project output is given by $y(\omega) = \theta\omega$. The intermediation technologies are characterized by $p(x) = 1 - \exp(-\upsilon x)$, which satisfies the curvature assumptions imposed earlier. Labor supply is normalized to $\overline{L} = 1$ and the figure is drawn for $\theta = 1.75$ and $\upsilon = 2.5$. Figure 2 plots the decentralized equilibrium and the constrained efficient allocation for all values of μ where $\alpha^* = \widehat{\alpha} = 1$ is optimal. The exception is the top right panel, which zooms into the top left panel for a subset of μ.

Notice from Figure 2 that the decentralized acquisition of intermediation resources is approximately efficient (i.e., $z^* \approx \widehat{z}$). Appendix F presents some derivations that support this result. Specifically, I use a second-order Taylor approximation around the constrained efficient allocation to show that any deviations in ξ and π in the neighborhood of the planner's $\widehat{\xi}$ and $\widehat{\pi}$ will have only a second-order effect on z. In other words, ξ and π can be inefficient without a large inefficiency in z. Appendix F also shows that all of the following are approximately efficient when z is approximately efficient: the total mass of open matches A, the total amount of credit K, and total welfare $\mathcal{W}$. It is still the case that uninformed credit is inefficiently high and informed credit is inefficiently low, but the overall welfare loss is small when labor is inelastically supplied in a Walrasian market.

Appendix G outlines what happens if labor supply is instead endogenous and elastic to the wage. With more intermediation resources, capital can be better allocated to borrowers. This generates more output and allows more loans to be made, relaxing the market clearing constraint on capital. Workers do not internalize this when choosing how much labor to supply to the intermediaries. The decentralized labor supply is then too low relative to the constrained efficient allocation, as is the decentralized z.[15] There is still under-investment

[15] The decentralized z would also be too low if z were effort exerted directly by the banker at some increasing disutility. The reason would again be failure to internalize that intermediation resources relax the capital market constraint. See Appendix G for further discussion.

in screening and now there can also be under-investment in matching (i.e., both π and $z - \pi$ are too low). Informed lenders are still too selective in who they retain. Uninformed credit is again too high but the approximate efficiency result on total credit disappears. Instead, there is too little credit overall and the welfare losses are more substantial.

6 Conclusion

A standard intuition from models with asymmetric information about asset values is that screening imparts adverse selection on others buying in the same market. Buyers fail to internalize this, so screening is inefficiently high. However, adverse selection is technically imparted by the retention decision of a buyer who has successfully screened, not by the act of screening itself. I show that this distinction is critical when the cutoff between profitable and unprofitable assets is endogenously determined. In the context of banking, failure to internalize the adverse selection problem means that informed lenders (i.e., those who have successfully screened borrowers) are too selective in the types they retain relative to a social planner who faces the same technologies and constraints. This implies an under-use of capital. The price of capital then falls to encourage the retention of more borrower types. As this happens, the benefit of being informed rather than uninformed also falls, prompting a reallocation of intermediation effort from screening to matching. Screening is inefficiently low in the decentralized equilibrium, not inefficiently high. While I have illustrated these forces using lenders who find and screen borrowers, a similar insight could also apply to firms who engage in R&D by finding and screening ideas.

References

Attar, A., T. Mariotti, and F. Salanié. 2011. "Nonexclusive Competition in the Market for Lemons." *Econometrica*, 79(6): 1869-1918.

Becsi, Z., V. Li, and P. Wang. 2013. "Credit Mismatch and Breakdown." *European Economic Review*, 59(1): 109-125.

Broecker, T. 1990. "Credit-Worthiness Tests and Interbank Competition." *Econometrica*, 58(2): 429-452.

Cao, M. and S. Shi. 2001. "Screening, Bidding, and the Loan Market Tightness." *European Finance Review*, 5(1-2): 21-61.

Colombo, L., G. Femminis, and A. Pavan. 2014. "Information Acquisition and Welfare." *Review of Economic Studies*, 81(4): 1438-1483.

Dávila, E. and A. Korinek. 2018. "Pecuniary Externalities in Economies with Financial Frictions." *Review of Economic Studies*, 85(1): 352-395.

Dell'Ariccia, G. and R. Marquez. 2006. "Lending Booms and Lending Standards." *Journal of Finance*, 61(5): 2511-2546.

Direr, A. 2008. "Multiple Equilibria in Markets with Screening." *Journal of Money, Credit and Banking*, 40(4): 791-798.

Elliott, M. and F. Nava. 2019. "Decentralized Bargaining in Matching Markets: Efficient Stationary Equilibria and the Core." *Theoretical Economics*, 14(1): 211-251.

Fishman, M. and J. Parker. 2015. "Valuation, Adverse Selection, and Market Collapses." *Review of Financial Studies*, 28(9): 2575-2607.

Fishman, M., J. Parker, and L. Straub. 2020. "A Dynamic Theory of Lending Standards." Mimeo.

Fuchs, W. and A. Skrzypacz. 2015. "Government Interventions in a Dynamic Market with Adverse Selection." *Journal of Economic Theory*, 158(A): 371-406.

Gehrig, T. and R. Stenbacka. 2011. "Decentralized Screening: Coordination Failure, Multiple Equilibria, and Cycles." *Journal of Financial Stability*, 7(2): 60-69.

Gorton, G. and G. Pennacchi. 1995. "Banks and Loan Sales: Marketing Non-marketable Assets." *Journal of Monetary Economics*, 35(3): 389-411.

Grossman, S. and J. Stiglitz. 1980. "On the Impossibility of Informationally Efficient Markets." *American Economic Review*, 70(3): 393-408.

Guerrieri, V., R. Shimer, and R. Wright. 2010. "Adverse Selection in Competitive Search Equilibrium." *Econometrica*, 78(6): 1823-1862.

Hauswald, R. and R. Marquez. 2006. "Competition and Strategic Information Acquisition in Credit Markets." *Review of Financial Studies*, 19(3): 967-1000.

Heider, F. and R. Inderst. 2012. "Loan Prospecting." *Review of Financial Studies*, 25(8): 2381-2415.

Hosios, A. 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment." *Review of Economic Studies*, 57(2): 279-298.

Keys, B., T. Mukherjee, A. Seru, and V. Vig. 2010. "Did Securitization Lead to Lax Screening? Evidence from Subprime Loans." *Quarterly Journal of Economics*, 125(1): 307-362.

Livshits, I., J. MacGee, and M. Tertilt. 2016. "The Democratization of Credit and the Rise in Consumer Bankruptcies." *Review of Economic Studies*, 83(4): 1673–1710.

Llosa, L. and V. Venkateswaran. 2017. "Efficiency with Endogenous Information Choice." Mimeo.

Lorenzoni, G. 2008. "Inefficient Credit Booms." *Review of Economic Studies*, 75(3): 809-833.

Mackowiak, B. and M. Wiederholt. 2018. "Lack of Preparation for Rare Events." *Journal of Monetary Economics*, 100(1): 35-47.

Purnanandam, A. 2011. "Originate-to-Distribute Model and the Subprime Mortgage Crisis." *Review of Financial Studies*, 24(6): 1881-1915.

Rothschild, M. and J. Stiglitz. 1976. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." *Quarterly Journal of Economics*, 90(4): 629-649.

Ruckes, M. 2004. "Bank Competition and Credit Standards." *Review of Financial Studies*, 17(4): 1073-1102.

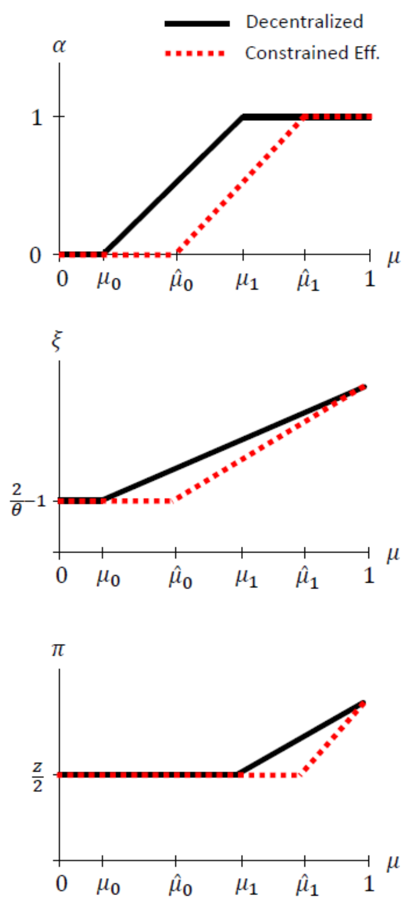
Shimer, R. and L. Smith. 2001. "Matching, Search, and Heterogeneity." *Advances in Macroeconomics*, 1(1): Article 5.

Vanasco, V. 2017. "The Downside of Asset Screening for Market Liquidity." *Journal of Finance*, 72(5): 1937-1982.

Verrecchia, R. 1982. "Information Acquisition in a Noisy Rational Expectations Economy." *Econometrica*, 50(6): 1415-1430

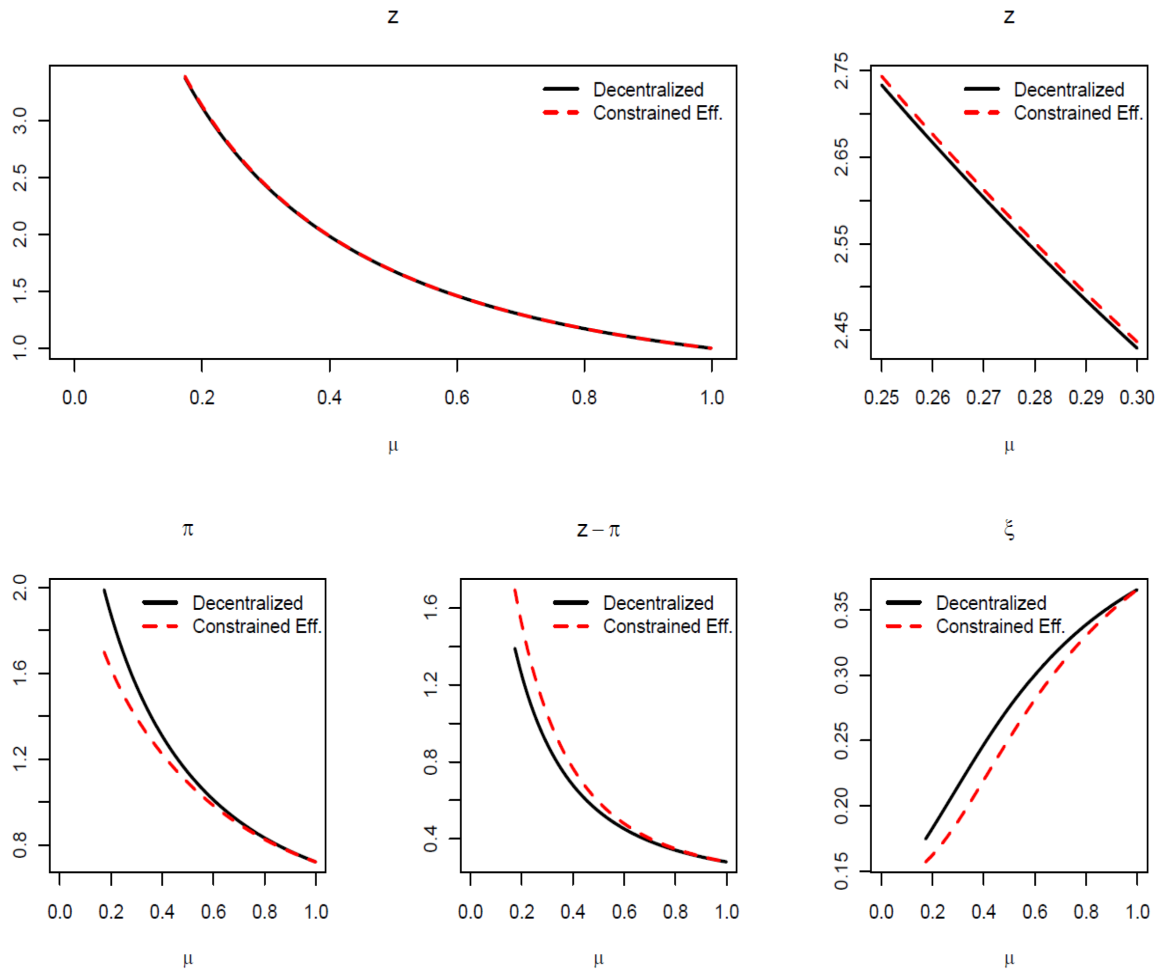
Yashiv, E. 2007. "Labor Search and Matching in Macroeconomics." *European Economic Review*, 51(8): 1859-1895.

Figure 1:
Walrasian Model with Endogenous Uninformed Retention Strategy (α)



Notes: This figure is drawn for $y(\omega) = \theta\omega$, where $\theta \in (1, 2)$ to satisfy Assumption 3. Consider $z > 2p^{-1}\left(\frac{4(2-\theta)}{4-\theta}\right) \equiv z_a(\theta)$ with $\theta > \frac{4}{3}$ so that $\widehat{\mu}_1 < 1$. If $\theta \in \left(\frac{4}{3}, \theta_0\right)$ where $\theta_0 \approx 1.6274$, the right panel applies for all $z > z_a(\theta)$. Otherwise, if $\theta \in (\theta_0, 2)$, there exists a $z_b(\theta) > z_a(\theta)$ such that the left panel applies for $z \in (z_a(\theta), z_b(\theta))$ while the right panel applies for $z > z_b(\theta)$.

Figure 2:
Walrasian Model with Endogenous Intermediation Resources (z)



Notes: This figure is drawn for $y(\omega) = 1.75\omega$ and $p(x) = 1 - \exp(-2.5x)$ with $\overline{L} = 1$. The results are plotted for values of μ where both the planner and the decentralized lenders optimally choose $\alpha = 1$. The top right panel zooms into the top left panel for a subset of μ.

Online Appendix for

"Inefficiently Low Screening with Walrasian Markets"

Kinda Hachem

UVA Darden and NBER

April 2020

Appendix A – Proofs

Proof of Lemma 1

The lender chooses $\pi \in [0, z]$, $\alpha \in [0, 1]$, and $I(\omega) \in [0, 1]$ for each $\omega \in [0, 1]$ to maximize U. By standard contraction mapping arguments, there is a unique U satisfying the recursion in equation (6) with $J(\omega)$ as per (5). The first order condition for $I(\omega)$ is just:

$$\upsilon_1(\omega) - \upsilon_0(\omega) \overset{sign}{=} J(\omega) - g' - \beta U$$

where $\upsilon_0(\omega) \geq 0$ and $\upsilon_1(\omega) \geq 0$ are Lagrange multipliers on $I(\omega) \geq 0$ and $I(\omega) \leq 1$ respectively. Complementary slackness holds as usual. Defining:

$$\xi \equiv \arg \min_{\omega \in [0,1]} |J(\omega) - g' - \beta U|$$

and using $J'(\cdot) > 0$, which follows from the assumption of $y'(\cdot) > 0$, completes the proof. ■

Proof of Lemma 2

Rearrange (5) to isolate $J(\omega)$. Substitute into (6) then rearrange (6) to isolate U. This gives:

$$U = \frac{1}{1-\beta} \frac{g' + p(\pi) \int_0^1 [[1 - p(z-\pi)]\alpha + p(z-\pi) I(\omega)] \left[\frac{\mu y(\omega)}{1-\beta(1-\mu)} - g'\right] \psi(\omega) d\omega}{1 + \frac{\beta(1-\mu)}{1-\beta(1-\mu)} p(\pi) \left[[1 - p(z-\pi)]\alpha + p(z-\pi) \int_0^1 I(\omega) \psi(\omega) d\omega\right]}$$

Multiply both sides by $1 - \beta(1-\mu)$ then rewrite as:

$$U = \frac{1}{1-\beta} \left[g' + \frac{\frac{1}{1-\beta(1-\mu)} p(\pi) \int_0^1 [[1 - p(z-\pi)]\alpha + p(z-\pi) I(\omega)] [\mu y(\omega) - g'] \psi(\omega) d\omega}{1 + \frac{\beta(1-\mu)}{1-\beta(1-\mu)} p(\pi) \left[[1 - p(z-\pi)]\alpha + p(z-\pi) \int_0^1 I(\omega) \psi(\omega) d\omega\right]}\right] \tag{A.1}$$

Invoking Lemma 1 completes the proof. ■

Proof of Lemma 3

The first step is to show that the planner's informed retention strategy, $I(\cdot)$, can also be reduced to a scalar ξ. Use equation (2) to substitute $n(\omega)$ out of the welfare function (7):

$$\mathcal{W} = \frac{1}{1-\beta} \left[g' + \int_0^1 [\mu y(\omega) - g'] \frac{p(\pi) [[1 - p(z-\pi)]\alpha + p(z-\pi) I(\omega)]}{\mu + (1-\mu) p(\pi) [[1 - p(z-\pi)]\alpha + p(z-\pi) I(\omega)]} d\omega\right] \tag{A.2}$$

Taking derivatives with respect to $I(\omega)$ gives:

$$\frac{\partial \mathcal{W}}{\partial I(\omega)} \stackrel{sign}{=} \mu y(\omega) - g'$$

In words, the planner sets $I(\omega) = 1$ if $y(\omega) > \frac{g'}{\mu}$ and $I(\omega) = 0$ if $y(\omega) < \frac{g'}{\mu}$. He is indifferent between any $I(\omega) \in [0,1]$ for $y(\omega) = \frac{g'}{\mu}$. Given $y'(\cdot) > 0$, this is a reservation strategy of the form defined in Lemma 1, but with ξ such that $y(\xi) = \frac{g'}{\mu}$.

The second step is to show that the planner's objective function can be rewritten as $A \times \Gamma(\pi, \xi, \alpha, \psi(\cdot), g')$. In steady state, equations (1) and (3) imply:

$$\mu n(\omega) = p(\pi)\left[\left[1 - p(z-\pi)\right]\alpha + p(z-\pi) I(\omega)\right]\psi(\omega) A \tag{A.3}$$

Using (A.3) to substitute $n(\omega)$ out of the welfare function (7):

$$\mathcal{W} = \frac{1}{1-\beta}\left[g' + \frac{1}{\mu} A \int_0^1 p(\pi)\left[\left[1-p(z-\pi)\right]\alpha + p(z-\pi) I(\omega)\right]\left[\mu y(\omega) - g'\right]\psi(\omega)\, d\omega\right]$$

The reservation strategy result above then implies:

$$\mathcal{W} \propto \mu g' + A \times \Gamma(\pi, \xi, \alpha, \psi(\cdot), g')$$

so the planner does indeed maximize $A \times \Gamma(\pi, \xi, \alpha, \psi(\cdot), g')$.

The last step is to show that A as defined in equation (4) satisfies (8). Using (A.3) to substitute $n(\omega)$ out of (4) gives:

$$A = 1 - \frac{1-\mu}{\mu} A p(\pi)\left[\left[1 - p(z-\pi)\right]\alpha + p(z-\pi)\int_0^1 I(\omega)\psi(\omega)\, d\omega\right]$$

Rearrange to isolate A and use the reservation strategy result above to get:

$$A = \frac{1}{1 + \frac{1-\mu}{\mu} p(\pi)\left[\left[1-p(z-\pi)\right]\alpha + p(z-\pi)\int_\xi^1 \psi(\omega)\, d\omega\right]}$$

This is just $D(\pi, \xi, \alpha, \psi(\cdot)|1)^{-1}$, completing the proof. ■

Proof of Proposition 1

Notice that $\mu = 1$ in equation (3) implies $\psi(\cdot) = 1$. Also note from the definition of $D(\cdot|\beta)$ in Lemma 2 that $\mu = 1$ implies $D(\cdot|\beta) = 1$ for any $\beta \in [0,1]$. Therefore, both the planner and the decentralized lenders are choosing $\pi \in [0,z]$, $\xi \in [0,1]$, and $\alpha \in [0,1]$ to maximize $\Gamma(\pi, \xi, \alpha, 1, g')$. This will give them the same first order conditions, completing the proof. ■

Proof of Proposition 2

Since lenders in the decentralized economy take as given the distribution of available firms, we can solve the problem of an unmatched lender for a general distribution $\psi(\cdot)$ then impose $\psi(\cdot) = 1$ to see what happens under the assumptions of Proposition 2.

The decentralized choice of π satisfies $\frac{\partial U}{\partial \pi} = 0$, where U is as defined in equation (6) with $\alpha = 1$. This yields:

$$\int_0^1 \left[1 - p(z-\pi)(1 - I(\omega)) + \frac{p(\pi) p'(z-\pi)}{p'(\pi)} (1 - I(\omega))\right] [J(\omega) - g' - \beta U] \psi(\omega) d\omega = 0$$

which, using Lemma 1, can be rewritten as:

$$\left[1 - p(z-\pi) + \frac{p(\pi) p'(z-\pi)}{p'(\pi)}\right] \int_0^{\xi} [J(\omega) - g' - \beta U] \psi(\omega) d\omega + \int_{\xi}^1 [J(\omega) - g' - \beta U] \psi(\omega) d\omega = 0 \tag{A.4}$$

We also know from the proof of Lemma 1 that the decentralized choice of ξ satisfies:

$$J(\xi) - g' - \beta U = 0 \tag{A.5}$$

Using (5) to substitute out $J(\cdot)$, we can rewrite (A.4) as:

$$\left[1 - p(z-\pi) + \frac{p(\pi) p'(z-\pi)}{p'(\pi)}\right] \int_0^{\xi} [y(\omega) - y(\xi)] \psi(\omega) d\omega + \int_{\xi}^1 [y(\omega) - y(\xi)] \psi(\omega) d\omega = 0 \tag{A.6}$$

and (A.5) as:

$$y(\xi) = \frac{g'}{\mu} + \frac{\beta(1-\mu)}{\mu} [(1-\beta) U - g'] \tag{A.7}$$

Next, use (A.1) to write:

$$U = \frac{1}{1-\beta} \left[g' + \frac{\frac{p(\pi)}{1-\beta(1-\mu)} \left[[1 - p(z-\pi)] \int_0^{\xi} [\mu y(\omega) - g'] \psi(\omega) d\omega + \int_{\xi}^1 [\mu y(\omega) - g'] \psi(\omega) d\omega\right]}{1 + \frac{\beta(1-\mu)}{1-\beta(1-\mu)} p(\pi) \left[1 - p(z-\pi) \int_0^{\xi} \psi(\omega) d\omega\right]} \right] \tag{A.8}$$

when $\alpha = 1$. Substituting (A.8) into (A.7) and rearranging, we can simplify (A.7) to:

$$y(\xi) = \frac{g'}{\mu} + \frac{\beta(1-\mu) p(\pi)}{1-\beta(1-\mu)} \left[[1 - p(z-\pi)] \int_0^{\xi} [y(\omega) - y(\xi)] \psi(\omega) d\omega + \int_{\xi}^1 [y(\omega) - y(\xi)] \psi(\omega) d\omega\right] \tag{A.9}$$

The decentralized equilibrium solves equations (A.6) and (A.9), conditional on the specification of the distribution $\psi(\cdot)$. Proposition 2 sets $\psi(\cdot) = 1$ so equation (A.6) becomes:

$$\left[1 - p(z-\pi) + \frac{p(\pi) p'(z-\pi)}{p'(\pi)}\right] \int_0^{\xi} [y(\omega) - y(\xi)] d\omega + \int_{\xi}^1 [y(\omega) - y(\xi)] d\omega = 0 \tag{A.10}$$

and equation (A.9) becomes:

$$y(\xi) = \frac{g'}{\mu} + \frac{\beta(1-\mu)p(\pi)}{1-\beta(1-\mu)}\left[[1-p(z-\pi)]\int_0^{\xi}[y(\omega)-y(\xi)]\,d\omega + \int_{\xi}^{1}[y(\omega)-y(\xi)]\,d\omega\right] \tag{A.11}$$

Consider now the planner. We cannot use $y(\xi) = \frac{g'}{\mu}$ as derived in the proof of Lemma 3 because that derivation used $n(\cdot)$ based on $\psi(\cdot)$ endogenous. With $\psi(\cdot) = 1$, we know from the statements of Lemmas 2 and 3 that the planner's problem is the same as the problem of the decentralized lenders evaluated at $\beta = 1$. We can thus characterize the planner's solution as an intersection between equations (A.10) and (A.11) when the latter is evaluated at $\beta = 1$. The decentralized equilibrium is instead an intersection between (A.10) and (A.11) when the latter is evaluated at the actual β.

It will help to visualize these equations in a two-dimensional space with π on the horizontal axis and ξ on the vertical axis. Since $p(\cdot)$ is concave, (A.10) defines a negative relationship between π and ξ. Differentiating (A.11):

$$\frac{d\xi}{d\pi} = \frac{\left[1-p(z-\pi)+\frac{p(\pi)p'(z-\pi)}{p'(\pi)}\right]\int_0^{\xi}[y(\omega)-y(\xi)]\,d\omega + \int_{\xi}^{1}[y(\omega)-y(\xi)]\,d\omega}{\left[\frac{1-\beta(1-\mu)}{\beta(1-\mu)} + p(\pi)[1-p(z-\pi)\,\xi]\right]\frac{y'(\xi)}{p'(\pi)}}$$

Equation (A.11) thus achieves a critical point at any intersection with (A.10). Taking second derivatives reveals that any critical point of (A.11) is a maximum. Now, for any given π, differentiate (A.11) to get $\frac{\partial\xi}{\partial\beta} \overset{sign}{=} y(\xi) - \frac{g'}{\mu}$. This is zero at $\pi = 0$ and positive otherwise, meaning that (A.11) evaluated at $\beta = 1$ lies above (A.11) evaluated at $\beta < 1$ when plotted in π-ξ space. It must then be the case that the planner's solution involves higher ξ and lower π than the decentralized equilibrium. ■

Proof of Proposition 3

The proof of Lemma 3 established that the planner's choice of ξ solves:

$$y\left(\widehat{\xi}\right) = \frac{g'}{\mu} \tag{A.12}$$

when $\psi(\cdot)$ is endogenous. For the decentralized solution, return to the proof of Lemma 2, specifically the derivations conditional on a general distribution $\psi(\cdot)$. Substitute (A.6) into (A.9) to write:

$$y(\xi^*) = \frac{g'}{\mu} + \frac{\beta(1-\mu)}{1-\beta(1-\mu)}\frac{p^2(\pi^*)\,p'(z-\pi^*)}{p'(\pi^*)}\int_0^{\xi^*}[y(\xi^*)-y(\omega)]\,\psi^*(\omega)\,d\omega \tag{A.13}$$

at the decentralized equilibrium. With some abuse of notation, $\psi^*(\cdot)$ is used here to denote the distribution $\psi(\cdot)$ evaluated at the equilibrium values of π and ξ when $\alpha = 1$. This distribution will be formally derived in the proof of Proposition 4. For now, it suffices to note that $\mu > 0$ implies some availability for each type so $\psi(\omega) > 0$ for all ω. Therefore, $\pi^* > 0$ in (A.13) implies $y(\xi^*) > \frac{g'}{\mu}$ and, with $y'(\cdot) > 0$, we can conclude $\xi^* > \widehat{\xi}$. ■

Proof of Proposition 4

Using the notation defined at the beginning of Subsection 3.1.2, $\psi(\omega) = \psi_L$ for any $\omega < \xi$ and $\psi(\omega) = \psi_H$ for any $\omega \geq \xi$. With $\alpha = 1$, evaluating equations (2) and (3) at $I(\omega) = 0$ yields:

$$\psi_L = \frac{\mu/A}{\mu + (1-\mu)\, p(\pi)\, [1 - p(z-\pi)]}$$

while evaluation at $I(\omega) = 1$ yields:

$$\psi_H = \frac{\mu/A}{\mu + (1-\mu)\, p(\pi)}$$

where, from equation (4), we can get:

$$A = \frac{\mu}{\mu + (1-\mu)\, p(\pi)} \left(1 + \frac{(1-\mu)\, p(\pi)\, p(z-\pi)\, \xi}{\mu + (1-\mu)\, p(\pi)\, [1 - p(z-\pi)]}\right) \tag{A.14}$$

The decentralized equilibrium satisfies equations (A.6) and (A.13) with $\psi(\omega)$ as just derived. For ease of reference, define:

$$f_1(\pi) \equiv 1 - p(z-\pi) + \frac{p(\pi)\, p'(z-\pi)}{p'(\pi)}$$

and:

$$f_2(\pi) \equiv \frac{\psi_H}{\psi_L} = \frac{\mu + (1-\mu)\, p(\pi)\, [1 - p(z-\pi)]}{\mu + (1-\mu)\, p(\pi)}$$

so that (A.6) can be expressed as:

$$\frac{f_1(\pi^*)}{f_2(\pi^*)} = \frac{\int_{\xi^*}^{1} [y(\omega) - y(\xi^*)]\, d\omega}{\int_0^{\xi^*} [y(\xi^*) - y(\omega)]\, d\omega} \tag{A.15}$$

at the equilibrium (π^*, ξ^*).

We know from the proofs of Lemma 3 and Proposition 3 that the constrained efficient choice of ξ satisfies equation (A.12) so it just remains to get the planner's first order condition for π. Return to the welfare function as written in (A.2) and take the derivative with respect to π to get:

$$\frac{\partial \mathcal{W}}{\partial \pi} \propto p'(\pi) \int_0^1 \left[y(\omega) - \frac{g'}{\mu}\right] \frac{1 - p(z-\pi)(1 - I(\omega)) + \frac{p(\pi)p'(z-\pi)}{p'(\pi)}(1 - I(\omega))}{[\mu + (1-\mu)\, p(\pi)\, [1 - p(z-\pi)(1 - I(\omega))]]^2} d\omega$$

when $\alpha = 1$. Setting to zero and using the reservation strategy result for $I(\cdot)$ in Lemma 3 yields:

$$\frac{f_1(\widehat{\pi})}{f_2^2(\widehat{\pi})}=\frac{\int_{\widehat{\xi}}^1\left[y(\omega)-y\left(\widehat{\xi}\right)\right]d\omega}{\int_0^{\widehat{\xi}}\left[y\left(\widehat{\xi}\right)-y(\omega)\right]d\omega} \tag{A.16}$$

where I have also used (A.12) to substitute out $\frac{g'}{\mu}$.

Assumption 2 is sufficient for $\frac{f_1(\pi)}{f_2(\pi)}$ to be increasing in π. Specifically, $\frac{d}{d\pi}\frac{f_1(\pi)}{f_2(\pi)}>0$ requires:

$$\left[2-\frac{p(\pi)}{p'(\pi)}\left(\frac{p''(z-\pi)}{p'(z-\pi)}+\frac{p''(\pi)}{p'(\pi)}\right)\right]\frac{p'(z-\pi)}{p(z-\pi)}>\frac{(1-\mu)f_1(\pi)\left[\frac{p(\pi)p'(z-\pi)}{p(z-\pi)}-\frac{\mu p'(\pi)}{\mu+(1-\mu)p(\pi)}\right]}{\mu+(1-\mu)p(\pi)[1-p(z-\pi)]}$$

The right-hand side of the above inequality is maximized at $\mu=0$ so a sufficient condition can be found by imposing $\mu=0$ and rearranging to get Assumption 2.

Therefore, under Assumption 2, equation (A.15) defines a negative relationship between π and ξ. For a given value of ξ, it also yields a higher value of π than equation (A.16). As in the proof of Proposition 2, it will help to visualize this in a two-dimensional space. Imagine now a plot with ξ on the horizontal axis and π on the vertical axis. Equation (A.15) maps a downward-sloping curve that lies above the curve mapped out by (A.16).

Next compare equations (A.12) and (A.13). The planner's ξ is independent of π so (A.12) is just a vertical line on the plot. If $\beta=0$, then equation (A.13) overlaps this vertical line so we have $\xi^*=\widehat{\xi}$ and $\pi^*>\widehat{\pi}$. If $\beta>0$, then equation (A.13) intersects the vertical line defined by (A.12) at $\pi=0$ but lies to the right for any $\pi>0$. By continuity, $\pi^*>\widehat{\pi}$ extends to any β below some positive threshold. ■

Proof of Proposition 5

Start with the planner. Taking derivatives of the welfare function in (A.2) with respect to α and π, we get:

$$\frac{\partial\mathcal{W}}{\partial\alpha}\overset{sign}{=}\int_0^1\frac{\mu y(\omega)-g'}{\left[\mu+(1-\mu)p(\pi)\left[[1-p(z-\pi)]\alpha+p(z-\pi)I(\omega)\right]\right]^2}d\omega$$

and:

$$\frac{\partial\mathcal{W}}{\partial\pi}\overset{sign}{=}\int_0^1\frac{[\mu y(\omega)-g']\left[\left(1-p(z-\pi)+\frac{p(\pi)p'(z-\pi)}{p'(\pi)}\right)\alpha+\left(p(z-\pi)-\frac{p(\pi)p'(z-\pi)}{p'(\pi)}\right)I(\omega)\right]}{\left[\mu+(1-\mu)p(\pi)\left[[1-p(z-\pi)]\alpha+p(z-\pi)I(\omega)\right]\right]^2}d\omega$$

The proposition considers parameters where $\widehat{\alpha}<1$, implying either $\widehat{\alpha}=0$ or $\widehat{\alpha}$ solving $\frac{\partial\mathcal{W}}{\partial\alpha}=0$. In both cases, this delivers:

$$\frac{\partial\mathcal{W}}{\partial\pi}\overset{sign}{=}\left(p(z-\pi)-\frac{p(\pi)p'(z-\pi)}{p'(\pi)}\right)\int_0^1\frac{[\mu y(\omega)-g']I(\omega)}{\left[\mu+(1-\mu)p(\pi)\left[[1-p(z-\pi)]\alpha+p(z-\pi)I(\omega)\right]\right]^2}d\omega$$

Invoking the reservation strategy result from the proof of Lemma 3, the solution to $\frac{\partial \mathcal{W}}{\partial \pi} = 0$ is just $\widehat{\pi} = \frac{z}{2}$. The solution to $\frac{\partial \mathcal{W}}{\partial \alpha} = 0$ is then:

$$\left(1 + \frac{(1-\mu)\, p^2\left(\frac{z}{2}\right)}{\mu + (1-\mu)\, p\left(\frac{z}{2}\right)\left[1 - p\left(\frac{z}{2}\right)\right]\widehat{\alpha}}\right)^2 \int_0^{\widehat{\xi}} \left[y(\omega) - y\left(\widehat{\xi}\right)\right] d\omega + \int_{\widehat{\xi}}^1 \left[y(\omega) - y\left(\widehat{\xi}\right)\right] d\omega = 0 \tag{A.17}$$

with $y\left(\widehat{\xi}\right) = \frac{g'}{\mu}$.

Consider now the decentralized lenders. Notice:

$$\frac{\partial \psi_L}{\partial \alpha} = -\frac{(1-\mu)^2\, p^2(\pi)\, p(z-\pi)\left[1 - p(z-\pi)\right](1-\xi)}{\left[\mu + (1-\mu)\, p(\pi)\left[\left[1 - p(z-\pi)\right]\alpha + p(z-\pi)\,\xi\right]\right]^2} < 0$$

and:

$$\frac{\partial \psi_H}{\partial \alpha} = -\frac{\xi}{1-\xi}\frac{\partial \psi_L}{\partial \alpha} > 0$$

when $\xi \in (0,1)$. As with the other choice variables, this distributional externality (in addition to the extensive externality) is not internalized by the decentralized lenders when choosing α. Taking derivatives of the unmatched value U in (A.1) with respect to α and π, we get:

$$\frac{\partial U}{\partial \alpha} \overset{sign}{=} \int_0^1 \left[\frac{\mu\left[y(\omega) + \beta U\right]}{1 - \beta(1-\mu)} - g' - \beta U\right]\psi(\omega)\, d\omega$$

and:

$$\frac{\partial U}{\partial \pi} \overset{sign}{=} \int_0^1 \left[\alpha + \frac{\left(p(z-\pi) - \frac{p(\pi)p'(z-\pi)}{p'(\pi)}\right) I(\omega)}{1 - p(z-\pi) + \frac{p(\pi)p'(z-\pi)}{p'(\pi)}}\right]\left[\frac{\mu\left[y(\omega) + \beta U\right]}{1 - \beta(1-\mu)} - g' - \beta U\right]\psi(\omega)\, d\omega$$

The proposition considers parameters where $\alpha^* < 1$, implying either $\alpha^* = 0$ or α^* solving $\frac{\partial U}{\partial \alpha} = 0$. In both cases, this delivers:

$$\frac{\partial U}{\partial \pi} \overset{sign}{=} \left(p(z-\pi) - \frac{p(\pi)\, p'(z-\pi)}{p'(\pi)}\right)\int_0^1 I(\omega)\left[\frac{\mu\left[y(\omega) + \beta U\right]}{1 - \beta(1-\mu)} - g' - \beta U\right]\psi(\omega)\, d\omega$$

Invoking the reservation strategy result from the proof of Lemma 1, the solution to $\frac{\partial U}{\partial \pi} = 0$ is just $\pi^* = \frac{z}{2}$, which is constrained efficient given $\widehat{\pi} = \frac{z}{2}$ as derived earlier. The solution to $\frac{\partial U}{\partial \alpha} = 0$ is then:

$$\left(1 + \frac{(1-\mu)\, p^2\left(\frac{z}{2}\right)}{\mu + (1-\mu)\, p\left(\frac{z}{2}\right)\left[1 - p\left(\frac{z}{2}\right)\right]\alpha^*}\right)\int_0^{\xi^*} \left[y(\omega) - y(\xi^*)\right] d\omega + \int_{\xi^*}^1 \left[y(\omega) - y(\xi^*)\right] d\omega = 0 \tag{A.18}$$

with:

$$y\left(\xi^*\right) = \frac{g'}{\mu} + \frac{\beta\left(1-\mu\right)p^2\left(\frac{z}{2}\right)}{1-\beta\left(1-\mu\right)} \frac{1}{1+\frac{(1-\mu)p^2\left(\frac{z}{2}\right)\xi^*}{\mu+(1-\mu)p\left(\frac{z}{2}\right)\left[1-p\left(\frac{z}{2}\right)\right]\alpha^*}} \int_{\xi^*}^{1}\left[y\left(\omega\right)-y\left(\xi^*\right)\right]d\omega \tag{A.19}$$

If $\beta = 0$, then $y\left(\xi^*\right) = \frac{g'}{\mu}$ and hence $\xi^* = \widehat{\xi}$. If $\beta > 0$, then $y\left(\xi^*\right) > \frac{g'}{\mu}$ and hence $\xi^* > \widehat{\xi}$. To compare α^* and $\widehat{\alpha}$, imagine a plot with ξ on the horizontal axis and α on the vertical axis. Equation (A.17) maps an upward-sloping curve that lies above the upward-sloping curve mapped out by (A.18). The planner's ξ is independent of α so $y\left(\widehat{\xi}\right) = \frac{g'}{\mu}$ is just a vertical line on the plot. If $\beta = 0$, then equation (A.19) overlaps this vertical line so we have $\alpha^* < \widehat{\alpha}$. If $\beta > 0$, then equation (A.19) intersects this vertical line at $\alpha = 0$ but lies to the right for any $\alpha > 0$. By continuity, $\alpha^* < \widehat{\alpha}$ extends to any β below some positive threshold. ■

Proof of Proposition 6

Start by deriving the first order conditions in (11). The lender chooses π and ξ to maximize $\frac{\Gamma\left(\pi,\xi,1,\overline{\psi}(\cdot),\widetilde{R}\right)}{D\left(\pi,\xi,1,\overline{\psi}(\cdot)|\beta\right)}$, where $\Gamma\left(\cdot\right)$ and $D\left(\cdot|\beta\right)$ are as defined in Lemma 2. Expand $\Gamma\left(\pi,\xi,1,\overline{\psi}\left(\cdot\right),\widetilde{R}\right)$ to write:

$$\begin{aligned}\Gamma\left(\pi,\xi,1,\overline{\psi}\left(\cdot\right),\widetilde{R}\right) &= \mu p\left(\pi\right)\left[\int_0^1 y\left(\omega\right)\overline{\psi}\left(\omega\right)d\omega - p\left(z-\pi\right)\int_0^{\xi} y\left(\omega\right)\overline{\psi}\left(\omega\right)d\omega\right] \\ &\quad -p\left(\pi\right)\left[1-p\left(z-\pi\right)\int_0^{\xi}\overline{\psi}\left(\omega\right)d\omega\right]\widetilde{R}\end{aligned}$$

The lender's objective function can then be expressed as:

$$\frac{\Gamma\left(\pi,\xi,1,\overline{\psi}\left(\cdot\right),\widetilde{R}\right)}{D\left(\pi,\xi,1,\overline{\psi}\left(\cdot\right)|\beta\right)} = S\left(\pi,\xi,\overline{\psi}\left(\cdot\right)|\beta\right) - V\left(\pi,\xi,\overline{\psi}\left(\cdot\right)|\beta\right)\widetilde{R}$$

where $S\left(\cdot|\beta\right)$ and $V\left(\cdot|\beta\right)$ are as defined in Proposition 6. Since lenders in the decentralized economy take as given a distribution $\overline{\psi}\left(\cdot\right)$, their first order conditions are simply $S_i' - V_i'\widetilde{R} = 0$ for $i \in \{\pi, \xi\}$.

Now move to existence and uniqueness. Use $n\left(\cdot\right)$ as per equation (2), with $\alpha = 1$ as well as $I\left(\omega\right) = 1$ if $\omega \geq \xi$ and $I\left(\omega\right) = 0$ otherwise, to rewrite the market clearing condition in equation (9) as:

$$\frac{1-p\left(z-\pi\right)}{f_2\left(\pi\right)} = \frac{\int_{\xi}^{1}\left[y\left(\omega\right)-1\right]d\omega}{\int_0^{\xi}\left[1-y\left(\omega\right)\right]d\omega} \tag{A.20}$$

where $f_2\left(\pi\right)$ is as defined in the proof of Proposition 4.

Next, simplify the decentralized first order conditions. Going through the algebra, we find that $S_{\xi}' - V_{\xi}'\widetilde{R} = 0$ reduces to:

$$y(\xi) = \frac{\widetilde{R}}{\mu} + \frac{\beta(1-\mu)p(\pi)}{1-\beta(1-\mu)} \left[\begin{array}{l} [1-p(z-\pi)] \int_0^{\xi} [y(\omega) - y(\xi)] \psi(\omega) d\omega \\ + \int_{\xi}^{1} [y(\omega) - y(\xi)] \psi(\omega) d\omega \end{array} \right] \tag{A.21}$$

Using (A.21) to substitute out $\widetilde{R}$, the condition $S'_\pi - V'_\pi \widetilde{R} = 0$ simplifies to (A.15) from the baseline model. Therefore, with a Walrasian interbank market and $\alpha = 1$, the decentralized equilibrium is a pair (π^*, ξ^*) that solves equations (A.15) and (A.20). The price $R^* \equiv \frac{\widetilde{R}^*}{1-\beta(1-\mu)}$ can then be recovered from (A.21).

Let $\pi_l(\xi)$ and $\pi_k(\xi)$ denote the functions implicitly defined by (A.15) and (A.20) respectively. Proving existence of equilibrium thus amounts to proving the existence of a pair (π^*, ξ^*) such that $\pi^* = \pi_l(\xi^*) = \pi_k(\xi^*)$.

Begin with equation (A.20). Define $\overline{\xi}$ such that $y(\overline{\xi}) \equiv 1$. Assumption 3 and $y'(\cdot) > 0$ imply that $\overline{\xi}$ exists uniquely and is interior. Differentiating (A.20) reveals $\pi'_k(\overline{\xi}) = 0$ and $\pi'_k(\xi)\left[y(\xi) - y(\overline{\xi})\right] < 0$ for $\xi \neq \overline{\xi}$. A necessary condition for $\pi^* > 0$ is $\pi_k(\overline{\xi}) > 0$. To ensure $\pi_k(\overline{\xi}) > 0$, we need $p(z) > 1 - \frac{\int_{\overline{\xi}}^1 [y(\omega)-1] d\omega}{\int_0^{\overline{\xi}} [1-y(\omega)] d\omega}$ or, equivalently, $p(z)$ sufficiently high.

The properties of $\pi_k(\cdot)$ just established also imply existence of unique points $\xi_{k,1} \in (0, \overline{\xi})$ and $\xi_{k,2} \in (\overline{\xi}, 1)$ defined by $\pi_k(\xi_{k,1}) \equiv 0$ and $\pi_k(\xi_{k,2}) \equiv 0$. The restriction to $\xi_{k,1} > 0$ and $\xi_{k,2} < 1$ reflects the fact that $\pi_k(\cdot)$ is not defined at $\xi = 0$ or $\xi = 1$ under Assumption 3 and $p(z) < 1$.

Turn to equation (A.15). When evaluated at $\xi = \overline{\xi}$, the right-hand sides of (A.15) and (A.20) are the same so $p(z)$ sufficiently high also ensures $\pi_l(\overline{\xi}) > 0$.

We can show $\pi_l(\overline{\xi}) < \pi_k(\overline{\xi})$ in two steps. First, the left-hand side of (A.20) is increasing in π. Second, the left-hand side of (A.15) equals the left-hand side of (A.20) plus a function of π. This function is zero if $\pi = 0$ and positive otherwise. Therefore, $\pi_l(\overline{\xi}) < \pi_k(\overline{\xi})$.

The following lemma completes the existence proof by finding a point $\xi \in [\xi_{k,1}, \overline{\xi})$ such that $\pi_l(\xi) > \pi_k(\xi)$:

Lemma A.1 *If $\pi_l(\xi_{k,1})$ exists, then $\pi_l(\xi_{k,1}) > \pi_k(\xi_{k,1})$. If $\pi_l(\xi_{k,1})$ does not exist, then there is a point $\xi_z \in (\xi_{k,1}, \overline{\xi})$ such that $\pi_l(\xi_z) = z > \pi_k(\xi_z)$.*

Proof. Equation (A.20) and the definition of $\xi_{k,1}$ yield:

$$1 - p(z) = \frac{\int_{\xi_{k,1}}^1 [y(\omega) - 1] d\omega}{\int_0^{\xi_{k,1}} [1 - y(\omega)] d\omega} < \frac{\int_{\xi_{k,1}}^1 \left[y(\omega) - y(\xi_{k,1})\right] d\omega}{\int_0^{\xi_{k,1}} \left[y(\xi_{k,1}) - y(\omega)\right] d\omega}$$

where the inequality follows from $y(\xi_{k,1}) < y(\overline{\xi}) \equiv 1$. Return to equation (A.15). If $\pi_l(\xi_{k,1})$ exists, then the above inequality implies $\pi_l(\xi_{k,1}) > 0 \equiv \pi_k(\xi_{k,1})$. If $\pi_l(\xi_{k,1})$ does not exist, then it must be the case that:

$$\frac{\int_{\xi_{k,1}}^1 \left[y(\omega) - y(\xi_{k,1})\right] d\omega}{\int_0^{\xi_{k,1}} \left[y(\xi_{k,1}) - y(\omega)\right] d\omega} > 1 + \frac{p(z)\, p'(0)}{p'(z)}$$

With $\frac{d}{dx}\left(\frac{\int_x^1 [y(\omega)-y(x)]d\omega}{\int_0^x [y(x)-y(\omega)]d\omega}\right) < 0$, we can thus look for a point $\xi_z > \xi_{k,1}$ satisfying $\pi_l(\xi_z) = z$. Substituting $\pi = z$ into equation (A.20) returns $\int_0^1 y(\omega)\, d\omega = 1$. This violates Assumption 3 so we can conclude $\pi_k(\cdot) < z$ and thus $\pi_l(\bar{\xi}) < z$. Therefore, if $\pi_l(\xi_{k,1})$ does not exist, there is a point $\xi_z \in (\xi_{k,1}, \bar{\xi})$ such that $\pi_l(\xi_z) = z > \pi_k(\xi_z)$. □

We have now shown existence of an equilibrium (π^*, ξ^*) with $\xi^* \in (\xi_{k,1}, \bar{\xi}) \subset (0,1)$ and $\pi^* = \pi_k(\xi^*) \in (0, z)$. Consider next uniqueness. Under Assumption 2, the left-hand side of (A.15) is increasing in π. It is also straightforward to see that the right-hand side of (A.15) is decreasing in ξ. Therefore, $\pi_l'(\cdot) < 0$. We already know $\pi_k'(\xi) > 0$ for any $\xi \in (\xi_{k,1}, \bar{\xi})$ so, to conclude uniqueness, we just need to show that all equilibria satisfy $\xi^* \in (\xi_{k,1}, \bar{\xi})$. We can do this by rearranging equations (A.15) and (A.20) to isolate $\int_{\xi^*}^1 y(\omega)\, d\omega$ in each then equating to get:

$$1 - y(\xi^*) = \frac{\frac{p(\pi^*)p'(z-\pi^*)}{p'(\pi^*)}\left[\mu + (1-\mu)\, p(\pi^*)\right]\int_0^{\xi^*}\left[y(\xi^*) - y(\omega)\right] d\omega}{\mu\left[1 - p(z-\pi^*)\,\xi^*\right] + (1-\mu)\, p(\pi^*)\left[1 - p(z-\pi^*)\right]} > 0$$

Invoking $y(\bar{\xi}) \equiv 1$ and $y'(\cdot) > 0$ establishes the desired result. ■

Proof of Proposition 7

Start by deriving the first order conditions in (12). Use equation (A.3) with $\alpha = 1$ to rewrite the planner's Lagrangian as:

$$\mathcal{L} = \frac{1+\lambda}{1-\beta} A \int_0^1 \left[y(\omega) - \frac{\lambda}{1+\lambda}\right] p(\pi)\left[1 - p(z-\pi)(1 - I(\omega))\right]\psi(\omega)\, d\omega$$

Following the proof of Lemma 3, it can be shown that the planner still follows a reservation strategy for informed retention. We can then simplify the planner's Lagrangian to:

$$\mathcal{L} = \frac{1+\lambda}{1-\beta}\frac{1}{\mu}\left[S(\pi, \xi, \psi(\cdot)|1) - \frac{\mu\lambda}{1+\lambda} V(\pi, \xi, \psi(\cdot)|1)\right]$$

where $S(\cdot|\beta)$ and $V(\cdot|\beta)$ are as defined in Proposition 6. Notice from these definitions:

$$S(\cdot|1) = S(\cdot|\beta)\frac{D(\cdot|\beta)}{D(\cdot|1)}$$

and:

$$V(\cdot|1) = V(\cdot|\beta)\frac{D(\cdot|\beta)}{D(\cdot|1)}$$

with $D(\cdot|\beta)$ and $D(\cdot|1)$ evaluated at $\alpha = 1$. The planner's Lagrangian is therefore:

$$\mathcal{L} = \frac{1+\lambda}{1-\beta}\frac{1}{\mu}\left[S(\pi, \xi, \psi(\cdot)|\beta) - \frac{\mu\lambda}{1+\lambda} V(\pi, \xi, \psi(\cdot)|\beta)\right]\frac{D(\pi, \xi, 1, \psi(\cdot)|\beta)}{D(\pi, \xi, 1, \psi(\cdot)|1)}$$

and his first order condition with respect to $i \in \{\pi, \xi\}$ is:

$$
\begin{aligned}
0 &= S_i'(\cdot|\beta) + S_\psi'(\cdot|\beta)\frac{\partial\psi}{\partial i} - \frac{\mu\lambda}{1+\lambda}\left[V_i'(\cdot|\beta) + V_\psi'(\cdot|\beta)\frac{\partial\psi}{\partial i}\right] \\
&\quad + \left[S(\cdot|\beta) - \frac{\mu\lambda}{1+\lambda}V(\cdot|\beta)\right]\left[\frac{D_i'(\cdot|\beta) + D_\psi'(\cdot|\beta)\frac{\partial\psi}{\partial i}}{D(\cdot|\beta)} - \frac{D_i'(\cdot|1) + D_\psi'(\cdot|1)\frac{\partial\psi}{\partial i}}{D(\cdot|1)}\right]
\end{aligned}
$$

where $S_\psi'(\cdot|\beta)\frac{\partial\psi}{\partial i}$ is short-hand for $S_{\psi_L}'(\cdot|\beta)\frac{\partial\psi_L}{\partial i} + S_{\psi_H}'(\cdot|\beta)\frac{\partial\psi_H}{\partial i}$. Similarly for $V_\psi'(\cdot|\beta)\frac{\partial\psi}{\partial i}$ and $D_\psi'(\cdot|\beta)\frac{\partial\psi}{\partial i}$.

From the definition of $D(\cdot|\beta)$ in Lemma 2:

$$
\frac{1-\beta(1-\mu)}{\beta\mu}\frac{D(\cdot|\beta)}{D(\cdot|1)} - 1 = \frac{1-\beta}{\beta\mu}\frac{1}{D(\cdot|1)}
$$

and:

$$
D_i'(\cdot|1) = \frac{1-\beta(1-\mu)}{\beta\mu}D_i'(\cdot|\beta)
$$

and:

$$
D_i'(\cdot|\beta) = \frac{\beta(1-\mu)}{1-\beta(1-\mu)}D^2(\cdot|\beta)V_i'(\cdot|\beta)
$$

for $i \in \{\pi, \xi, \psi\}$. Also note that the definitions of $V(\cdot|\beta)$ and $D(\cdot|\beta)$ together with (8) imply:

$$
V(\cdot|1) = \frac{\mu}{1-\mu}(1-A)
$$

We can then simplify the planner's first order conditions for $i \in \{\pi, \xi\}$ to:

$$
\begin{aligned}
& S_i'(\cdot|\beta) + S_\psi'(\cdot|\beta)\frac{\partial\psi}{\partial i} \\
= \;& \left[\frac{\mu\lambda}{1+\lambda} + \frac{(1-\beta)(1-A)}{1-\beta(1-\mu)}\left(\frac{S(\cdot|\beta)}{V(\cdot|\beta)} - \frac{\mu\lambda}{1+\lambda}\right)\right]\left[V_i'(\cdot|\beta) + V_\psi'(\cdot|\beta)\frac{\partial\psi}{\partial i}\right]
\end{aligned}
$$

Now use (A.3) with $\alpha = 1$ and the reservation strategy result for informed retention to write the aggregate feasibility constraint in equation (9) as:

$$
p(\pi)\left[\int_0^1 y(\omega)\psi(\omega)\,d\omega - p(z-\pi)\int_0^\xi y(\omega)\psi(\omega)\,d\omega\right] = p(\pi)\left[1 - p(z-\pi)\int_0^\xi \psi(\omega)\,d\omega\right]
$$

or, equivalently:

$$
S(\cdot|\beta) = \mu V(\cdot|\beta)
$$

This allows us to further simplify the planner's first order conditions for $i \in \{\pi, \xi\}$ to:

$$S_i'(\cdot|\beta) - rV_i'(\cdot|\beta) + \left[S_\psi'(\cdot|\beta) - rV_\psi'(\cdot|\beta)\right]\frac{\partial\psi}{\partial i} = 0$$

where r is as defined in the statement of Proposition 7.

Now move to uniqueness. Setting $g' = 0$, the welfare function in equation (7) is:

$$\mathcal{W} = \frac{\mu}{1-\beta}\int_0^1 y(\omega)\, n(\omega)\, d\omega \tag{A.22}$$

Use $n(\cdot)$ as per equation (2), with $\alpha = 1$ as well as $I(\omega) = 1$ if $\omega \geq \xi$ and $I(\omega) = 0$ otherwise, to rewrite:

$$\mathcal{W} = \frac{\mu}{1-\beta}\frac{p(\pi)}{\mu + (1-\mu)\, p(\pi)}\left[\frac{1 - p(z-\pi)}{f_2(\pi)}\int_0^\xi y(\omega)\, d\omega + \int_\xi^1 y(\omega)\, d\omega\right] \tag{A.23}$$

Any constrained efficient allocation must satisfy the aggregate feasibility constraint (9) which, as shown in the proof of Proposition 6, can also be expressed as (A.20). Recall from the same proof that $\pi_k(\xi)$ denotes the function implicitly defined by (A.20).

Equation (A.23) with π evaluated at $\pi_k(\xi)$ defines a function:

$$\mathcal{W}_k(\xi) \equiv \frac{\mu}{1-\beta}\frac{p(\pi_k(\xi))}{\mu + (1-\mu)\, p(\pi_k(\xi))}\frac{\xi\int_0^1 y(\omega)\, d\omega - \int_0^\xi y(\omega)\, d\omega}{\int_0^\xi [1 - y(\omega)]\, d\omega}$$

By definition, $\mathcal{W}_k'\left(\widehat{\xi}\right) = 0$ for any $\widehat{\xi}$ that is part of a constrained efficient allocation when $\alpha = 1$.

The first step in the uniqueness proof is to establish $\mathcal{W}_k''\left(\widehat{\xi}\right) < 0$ for any $\widehat{\xi}$ such that $\mathcal{W}_k'\left(\widehat{\xi}\right) = 0$. Taking derivatives:

$$\begin{aligned}\mathcal{W}_k'(\xi) &= \left[\frac{\mu\frac{p'(\pi_k(\xi))}{p(\pi_k(\xi))}}{\mu + (1-\mu)\, p(\pi_k(\xi))}\pi_k'(\xi) - \frac{1 - y(\xi)}{\int_0^\xi [1 - y(\omega)]\, d\omega}\right]\mathcal{W}_k(\xi) \\ &\quad + \frac{\mu}{1-\beta}\frac{p(\pi_k(\xi))}{\mu + (1-\mu)\, p(\pi_k(\xi))}\frac{\int_0^1 y(\omega)\, d\omega - y(\xi)}{\int_0^\xi [1 - y(\omega)]\, d\omega}\end{aligned}$$

It is straightforward to show that $\mathcal{W}_k'\left(\widehat{\xi}\right) = 0$ rearranges to:

$$\frac{\mu\frac{p'\left(\pi_k\left(\widehat{\xi}\right)\right)}{p\left(\pi_k\left(\widehat{\xi}\right)\right)}}{\mu + (1-\mu)\, p\left(\pi_k\left(\widehat{\xi}\right)\right)}\pi_k'\left(\widehat{\xi}\right) = \frac{1 - \int_0^1 y(\omega)\, d\omega}{\widehat{\xi}\int_0^1 y(\omega)\, d\omega - \int_0^{\widehat{\xi}} y(\omega)\, d\omega}\frac{\int_0^{\widehat{\xi}}\left[y\left(\widehat{\xi}\right) - y(\omega)\right] d\omega}{\int_0^{\widehat{\xi}} [1 - y(\omega)]\, d\omega} \tag{A.24}$$

where differentiation of (A.20) implies:

$$\pi_k'(\xi) = \frac{1}{\frac{p'(z-\pi)}{p(z-\pi)[1-p(z-\pi)]} + \frac{(1-\mu)p'(\pi)}{\mu+(1-\mu)p(\pi)}} \frac{1-y(\xi)}{\int_\xi^1 [y(\omega)-1]\, d\omega} \tag{A.25}$$

At $\widehat{\xi}$ such that $\mathcal{W}_k'\left(\widehat{\xi}\right) = 0$, the second derivative of $\mathcal{W}_k(\cdot)$ satisfies:

$$\begin{aligned}
\frac{\mathcal{W}_k''\left(\widehat{\xi}\right)}{\mathcal{W}_k\left(\widehat{\xi}\right)} &= \frac{\mu\frac{p'(\pi)}{p(\pi)}}{\mu+(1-\mu)p(\pi)}\pi_k''\left(\widehat{\xi}\right) \\
&+ \frac{\mu\frac{p'(\pi)}{p(\pi)}}{\mu+(1-\mu)p(\pi)}\left[\frac{p''(\pi)}{p'(\pi)} - \frac{p'(\pi)}{p(\pi)} - \frac{(1-\mu)p'(\pi)}{\mu+(1-\mu)p(\pi)}\right]\left(\pi_k'\left(\widehat{\xi}\right)\right)^2 \\
&- \frac{\mu\frac{p'(\pi)}{p(\pi)}}{\mu+(1-\mu)p(\pi)}\left[\frac{\mu\frac{p'(\pi)}{p(\pi)}}{\mu+(1-\mu)p(\pi)}\pi_k'\left(\widehat{\xi}\right) - \frac{2\left[1-y\left(\widehat{\xi}\right)\right]}{\int_0^{\widehat{\xi}}[1-y(\omega)]\, d\omega}\right]\pi_k'\left(\widehat{\xi}\right) \\
&- \frac{y'\left(\widehat{\xi}\right)\left[1-\int_0^1 y(\omega)\, d\omega\right]}{\left[\int_0^1 y(\omega)\, d\omega - \frac{1}{\widehat{\xi}}\int_0^{\widehat{\xi}} y(\omega)\, d\omega\right]\int_0^{\widehat{\xi}}[1-y(\omega)]\, d\omega}
\end{aligned}$$

where π is evaluated at $\pi_k\left(\widehat{\xi}\right)$ and differentiation of (A.25) implies:

$$\begin{aligned}
\pi_k''(\xi) &= \frac{\frac{p'(z-\pi)}{p(z-\pi)[1-p(z-\pi)]}}{\frac{p'(z-\pi)}{p(z-\pi)[1-p(z-\pi)]} + \frac{(1-\mu)p'(\pi)}{\mu+(1-\mu)p(\pi)}}\left[\frac{p''(z-\pi)}{p'(z-\pi)} - \frac{p'(z-\pi)}{p(z-\pi)} + \frac{p'(z-\pi)}{1-p(z-\pi)}\right](\pi_k'(\xi))^2 \\
&+ \frac{\frac{(1-\mu)p'(\pi)}{\mu+(1-\mu)p(\pi)}}{\frac{p'(z-\pi)}{p(z-\pi)[1-p(z-\pi)]} + \frac{(1-\mu)p'(\pi)}{\mu+(1-\mu)p(\pi)}}\left[\frac{(1-\mu)p'(\pi)}{\mu+(1-\mu)p(\pi)} - \frac{p''(\pi)}{p'(\pi)}\right](\pi_k'(\xi))^2 \\
&- \frac{1}{\frac{p'(z-\pi)}{p(z-\pi)[1-p(z-\pi)]} + \frac{(1-\mu)p'(\pi)}{\mu+(1-\mu)p(\pi)}}\left[\frac{y'(\xi)}{\int_\xi^1 [y(\omega)-1]\, d\omega} + \left(\frac{1-y(\xi)}{\int_\xi^1 [y(\omega)-1]\, d\omega}\right)^2\right]
\end{aligned}$$

Going through the algebra, we find:

$$\begin{aligned}
&\mathcal{W}_k''\left(\widehat{\xi}\right) \overset{sign}{=} \\
&\left[\begin{array}{l}
\frac{p''(\pi)}{p'(\pi)} + \frac{p''(z-\pi)}{p'(z-\pi)} + \frac{p'(z-\pi)}{1-p(z-\pi)} - \frac{p'(\pi)}{p(\pi)} \\
-\frac{2\mu(1-\mu)}{\mu+(1-\mu)p(\pi)[1-p(z-\pi)]}\frac{p'(\pi)p(z-\pi)}{\mu+(1-\mu)p(\pi)}\left(1+\frac{p'(\pi)[1-p(z-\pi)]}{p(\pi)p'(z-\pi)}\right) \\
-\frac{\mu-(1-\mu)p(\pi)[1-p(z-\pi)]}{\mu+(1-\mu)p(\pi)[1-p(z-\pi)]}\left(\frac{p'(\pi)}{p(\pi)} + \frac{p'(z-\pi)}{1-p(z-\pi)}\right)
\end{array}\right]\frac{p'(z-\pi)\left(\pi_k'\left(\widehat{\xi}\right)\right)^2}{p(z-\pi)[1-p(z-\pi)]} \\
&- \frac{\frac{p(\pi)}{\mu}\left[1-\mu+\frac{p'(z-\pi)[\mu+(1-\mu)p(\pi)]}{p'(\pi)p(z-\pi)[1-p(z-\pi)]}\right]\left[1-\int_0^1 y(\omega)\, d\omega\right]y'\left(\widehat{\xi}\right)}{\left[\int_0^1 y(\omega)\, d\omega - \frac{1}{\widehat{\xi}}\int_0^{\widehat{\xi}} y(\omega)\, d\omega\right]\int_0^{\widehat{\xi}}[1-y(\omega)]\, d\omega} - \frac{y'\left(\widehat{\xi}\right)}{\int_{\widehat{\xi}}^1 [y(\omega)-1]\, d\omega}
\end{aligned}$$

where π is evaluated at $\pi_k\left(\widehat{\xi}\right)$.

With $y'(\cdot) > 0$ and Assumptions 2 and 3, a sufficient condition for $\mathcal{W}_k''\left(\widehat{\xi}\right) < 0$ is $\mu \geq \frac{p(z)}{1+p(z)}$ or, qualitatively, μ not too small.

The second step in the uniqueness proof is to establish $\pi_k'\left(\widehat{\xi}\right) > 0$. This follows immediately from (A.24) under Assumption 3 and $y'(\cdot) > 0$. ■

Proof of Proposition 8

Recall from the proof of Proposition 6 that the equilibrium pair (π^*, ξ^*) solves equations (A.15) and (A.20). The constrained efficient pair $\left(\widehat{\pi}, \widehat{\xi}\right)$ must also satisfy (A.20), in addition to (A.24) from the proof of Proposition 7. Combine (A.24) with the expression for $\pi_k'(\xi)$ in (A.25) to get:

$$\frac{f_1(\widehat{\pi})}{f_2^2(\widehat{\pi})} = \frac{1-p(z-\widehat{\pi})}{f_2(\widehat{\pi})}\left[1 + \frac{\left(1-\frac{1-p(z-\widehat{\pi})}{f_2(\widehat{\pi})}\right)\left[1-y\left(\widehat{\xi}\right)\right]}{1-\int_0^1 y(\omega)\,d\omega}\frac{\widehat{\xi}\int_0^1 y(\omega)\,d\omega - \int_0^{\widehat{\xi}} y(\omega)\,d\omega}{\int_0^{\widehat{\xi}}\left[y\left(\widehat{\xi}\right)-y(\omega)\right]d\omega}\frac{\int_0^{\widehat{\xi}}[1-y(\omega)]\,d\omega}{\int_{\widehat{\xi}}^1[y(\omega)-1]\,d\omega}\right]$$

where $\widehat{\pi} \equiv \pi_k\left(\widehat{\xi}\right)$. Now use (A.20) to rewrite as:

$$\frac{f_1(\widehat{\pi})}{f_2^2(\widehat{\pi})} = \frac{\int_{\widehat{\xi}}^1[y(\omega)-1]\,d\omega}{\int_0^{\widehat{\xi}}[1-y(\omega)]\,d\omega} + \frac{1-y\left(\widehat{\xi}\right)}{\int_0^{\widehat{\xi}}[1-y(\omega)]\,d\omega}\frac{\widehat{\xi}\int_0^1 y(\omega)\,d\omega - \int_0^{\widehat{\xi}} y(\omega)\,d\omega}{\int_0^{\widehat{\xi}}\left[y\left(\widehat{\xi}\right)-y(\omega)\right]d\omega}$$

which simplifies to (A.16) from the baseline model. Therefore, the constrained efficient allocation is now a pair $\left(\widehat{\pi}, \widehat{\xi}\right)$ that solves (A.16) and (A.20).

As in the proof of Proposition 6, use $\pi_l(\xi)$ and $\pi_k(\xi)$ to denote the functions implicitly defined by (A.15) and (A.20) respectively. The decentralized market achieves (π^*, ξ^*) such that $\pi^* = \pi_l(\xi^*) = \pi_k(\xi^*)$. Letting $\pi_e(\xi)$ denote the function implicitly defined by (A.16), the planner achieves $\left(\widehat{\pi}, \widehat{\xi}\right)$ such that $\widehat{\pi} = \pi_e\left(\widehat{\xi}\right) = \pi_k\left(\widehat{\xi}\right)$. The proof of Proposition 6 established $\pi_k'(\xi^*) > 0$ and $\pi_l'(\cdot) < 0$. The proof of Proposition 7 also established $\pi_k'\left(\widehat{\xi}\right) > 0$. Therefore, to show $\widehat{\xi} < \xi^*$ and $\widehat{\pi} < \pi^*$, it will be enough to show $\pi_e\left(\widehat{\xi}\right) < \pi_l\left(\widehat{\xi}\right)$.

With $f_1(\pi)$ and $f_2(\pi)$ as defined in the proof of Proposition 4, (A.15) and (A.16) yield:

$$\frac{f_1\left(\pi_l\left(\widehat{\xi}\right)\right)}{f_2\left(\pi_l\left(\widehat{\xi}\right)\right)} = \frac{\int_{\widehat{\xi}}^1\left[y(\omega)-y\left(\widehat{\xi}\right)\right]d\omega}{\int_0^{\widehat{\xi}}\left[y\left(\widehat{\xi}\right)-y(\omega)\right]d\omega} = \frac{f_1\left(\pi_e\left(\widehat{\xi}\right)\right)}{f_2\left(\pi_e\left(\widehat{\xi}\right)\right)^2} > \frac{f_1\left(\pi_e\left(\widehat{\xi}\right)\right)}{f_2\left(\pi_e\left(\widehat{\xi}\right)\right)}$$

where the inequality follows from $f_2(\cdot) \in (0,1)$. The proof of Proposition 4 established the sufficiency of Assumption 2 for $\frac{d}{d\pi}\frac{f_1(\pi)}{f_2(\pi)} > 0$ so we can now conclude $\pi_l\left(\widehat{\xi}\right) > \pi_e\left(\widehat{\xi}\right)$.

It remains to characterize R_π and R_ξ as defined in the statement of Proposition 8. If ξ is fixed at $\widehat{\xi}$, then (A.20) implies $\pi^* = \widehat{\pi}$. The interbank price that implements $\widehat{\pi}$ as an equilibrium can be obtained by evaluating the decentralized first order condition for π, namely $S'_\pi - V'_\pi \widetilde{R} = 0$, at $\xi = \widehat{\xi}$ and $\pi = \widehat{\pi}$. Calling this price $\widetilde{R}_\pi$, we get:

$$R_\pi \equiv \frac{\widetilde{R}_\pi}{1-\beta(1-\mu)} = \frac{\mu D(\cdot|\beta)}{1-\beta(1-\mu)} \left[\frac{\frac{f_1(\widehat{\pi})}{f_2(\widehat{\pi})} \int_0^{\widehat{\xi}} y(\omega)\, d\omega + \int_{\widehat{\xi}}^1 y(\omega)\, d\omega}{1-\widehat{\xi} + \frac{f_1(\widehat{\pi})}{f_2(\widehat{\pi})}\widehat{\xi}} - \frac{\beta(1-\mu)}{\mu} \frac{S(\cdot|\beta)}{1-\beta(1-\mu)} \right]$$

Similarly, if π is fixed at $\widehat{\pi}$, then (A.20) implies $\xi^* = \widehat{\xi}$. The interbank price that implements $\widehat{\xi}$ as an equilibrium can be obtained by evaluating the decentralized first order condition for ξ, namely $S'_\xi - V'_\xi \widetilde{R} = 0$, at $\xi = \widehat{\xi}$ and $\pi = \widehat{\pi}$. Calling this price $\widetilde{R}_\xi$, we get:

$$R_\xi \equiv \frac{\widetilde{R}_\xi}{1-\beta(1-\mu)} = \frac{\mu D(\cdot|\beta)}{1-\beta(1-\mu)} \left[y\left(\widehat{\xi}\right) - \frac{\beta(1-\mu)}{\mu} \frac{S(\cdot|\beta)}{1-\beta(1-\mu)} \right]$$

For $R_\xi < R_\pi$, we need:

$$\frac{f_1(\widehat{\pi})}{f_2(\widehat{\pi})} \int_0^{\widehat{\xi}} \left[y(\omega) - y\left(\widehat{\xi}\right) \right] d\omega + \int_{\widehat{\xi}}^1 \left[y(\omega) - y\left(\widehat{\xi}\right) \right] d\omega > 0$$

which is true by (A.16) and $f_2(\cdot) \in (0,1)$. ■

Proof of Proposition 9

Total lending is defined as $K \equiv \int_0^1 n(\omega)\, d\omega$. It is immediate from (A.22) that welfare is proportional to K at any pair (π, ξ) satisfying equation (9). Specifically, $\mathcal{W} = \frac{\mu}{1-\beta} K$ or, equivalently, $K = \frac{1-\beta}{\mu} \mathcal{W}$. By market clearing, the decentralized equilibrium satisfies (9) which, in the planner's problem, is the aggregate feasibility constraint. The decentralized equilibrium is therefore a feasible allocation. The planner does not choose it (see Proposition 8) and the planner's solution is unique (see Proposition 7), implying $\mathcal{W}^* < \widehat{\mathcal{W}}$ and hence $K^* < \widehat{K}$.

Consider next uninformed lending, $K_N \equiv \int_0^1 \phi(\omega)\, d\omega$, where $\phi(\omega)$ is the fraction of type ω firms that receive uninformed financing when $\alpha = 1$. In steady state:

$$\mu \phi(\omega) = p(\pi)[1 - p(z-\pi)][1 - (1-\mu) n(\omega)]$$

which, using (3) and $\int_0^1 \psi(\omega)\, d\omega = 1$, implies:

$$K_N = p(\pi)[1 - p(z-\pi)] \frac{A}{\mu}$$

Substituting in A as per (A.14), we can write:

$$K_N = \frac{p(\pi)[1-p(z-\pi)]}{\mu + (1-\mu) p(\pi)} \left(1 + \frac{(1-\mu) p(\pi) p(z-\pi) \xi}{\mu + (1-\mu) p(\pi)[1 - p(z-\pi)]} \right)$$

Taking derivatives, we find $\frac{\partial K_N}{\partial \xi} > 0$ and $\frac{\partial K_N}{\partial \pi} > 0$. Therefore, $K_N^* > \widehat{K}_N$ is implied by $\pi^* > \widehat{\pi}$ and $\xi^* > \widehat{\xi}$, which were shown in Proposition 8.

Informed lending, denoted by K_I, must satisfy $K_I + K_N = K$ so $K_I^* < \widehat{K}_I$ follows immediately from $K_N^* > \widehat{K}_N$ and $K^* < \widehat{K}$. ■

Proof of Proposition 10

It will be verified below that both the planner and the decentralized lenders still follow reservation strategies for informed retention when α is endogenous. Evaluating (2) and (3) at $I(\omega) = 0$ yields:

$$\psi_L = \frac{\mu/A}{\mu + (1-\mu)\, p(\pi)\, [1 - p(z-\pi)]\, \alpha}$$

while evaluation at $I(\omega) = 1$ yields:

$$\psi_H = \frac{\mu/A}{\mu + (1-\mu)\, p(\pi)\, [\alpha + (1-\alpha)\, p(z-\pi)]}$$

so we can define the following replacement for $f_2(\pi)$ in the proof of Proposition 4:

$$\widetilde{f}_2(\pi, \alpha) \equiv \frac{\mu + (1-\mu)\, p(\pi)\, [1 - p(z-\pi)]\, \alpha}{\mu + (1-\mu)\, p(\pi)\, [\alpha + (1-\alpha)\, p(z-\pi)]}$$

Notice $\frac{\partial}{\partial \alpha} \widetilde{f}_2(\pi, \alpha) > 0$ and $\widetilde{f}_2(\pi, 1) = f_2(\pi)$.

Now use $n(\cdot)$ as per equation (2), with $I(\omega) = 1$ if $\omega \geq \xi$ and $I(\omega) = 0$ otherwise, to rewrite (9) as:

$$\frac{\alpha}{\alpha + (1-\alpha)\, p(z-\pi)} \frac{1 - p(z-\pi)}{\widetilde{f}_2(\pi, \alpha)} = \frac{\int_\xi^1 [y(\omega) - 1]\, d\omega}{\int_0^\xi [1 - y(\omega)]\, d\omega} \tag{A.26}$$

Setting $\alpha = 1$ in (A.26) would return (A.20).

Decentralized Equilibrium An unmatched lender in the decentralized economy chooses π, $I(\cdot)$, and α to maximize U as defined in equation (10). It is straightforward to show that the first order condition for $I(\omega)$ again yields a reservation strategy with a threshold ξ such that $J(\xi) = R + \beta U$. The first order conditions for π and α then simplify to:

$$\alpha \frac{f_1(\pi)}{\widetilde{f}_2(\pi, \alpha)} \int_0^\xi [y(\omega) - y(\xi)]\, d\omega + [1 - (1-\alpha)\, f_1(\pi)] \int_\xi^1 [y(\omega) - y(\xi)]\, d\omega = 0 \tag{A.27}$$

and:

$$\gamma_1 - \gamma_0 \overset{sign}{=} \int_0^\xi [y(\omega) - y(\xi)]\, d\omega + \widetilde{f}_2(\pi, \alpha) \int_\xi^1 [y(\omega) - y(\xi)]\, d\omega \tag{A.28}$$

where $\gamma_0 \geq 0$ and $\gamma_1 \geq 0$ are Lagrange multipliers on $\alpha \geq 0$ and $\alpha \leq 1$ respectively. The decentralized equilibrium involves π, ξ, and α solving (A.26), (A.27), and (A.28).

Equation (A.28) holds with complementary slackness. Therefore, to support an equilibrium with $\alpha = 1$, we need:

$$\int_0^{\xi^*} [y(\omega) - y(\xi^*)]\, d\omega + f_2(\pi^*) \int_{\xi^*}^1 [y(\omega) - y(\xi^*)]\, d\omega \geq 0 \tag{A.29}$$

where π^* and ξ^* solve (A.15) and (A.20). Notice that (A.15) is just (A.27) evaluated at $\alpha = 1$ while (A.20) is just (A.26) evaluated at $\alpha = 1$. Substituting (A.15) into (A.29), the desired inequality simplifies to $\frac{p'(\pi^*)}{p(\pi^*)} \leq \frac{p'(z-\pi^*)}{p(z-\pi^*)}$. Given that $p(\cdot)$ is increasing and concave, this just means that we need $\pi^* \geq \frac{z}{2}$.

I will now show existence of a threshold separation rate, $\mu_1 \in (0,1)$, such that (A.15) and (A.20) yield $\pi^* > \frac{z}{2}$ if and only if $\mu > \mu_1$.

The first step is to show $\frac{d\pi^*}{d\mu} > 0$. Let $h(\xi)$ and $k(\xi)$ denote the right-hand sides of (A.15) and (A.20) respectively. Also write $f_2(\pi,\mu)$ to make explicit that $f_2(\cdot)$ in (A.15) and (A.20) depends on μ. Differentiate equations (A.15) and (A.20) then combine to get:

$$\frac{d\pi^*}{d\mu} = \frac{\left[\frac{h'(\xi^*)}{h(\xi^*)} - \frac{k'(\xi^*)}{k(\xi^*)}\right] \frac{f'_{2\mu}(\pi^*,\mu)}{f_2(\pi^*,\mu)}}{\frac{h'(\xi^*)}{h(\xi^*)}\left[\frac{p'(z-\pi^*)}{1-p(z-\pi^*)} - \frac{f'_{2\pi}(\pi^*,\mu)}{f_2(\pi^*,\mu)}\right] - \frac{k'(\xi^*)}{k(\xi^*)}\left[\frac{f'_1(\pi^*)}{f_1(\pi^*)} - \frac{f'_{2\pi}(\pi^*,\mu)}{f_2(\pi^*,\mu)}\right]}$$

where $f'_{2\mu}(\pi,\mu)$ is short-hand for $\frac{\partial}{\partial\mu} f_2(\pi,\mu)$. It is straightforward to show $h'(\xi) < 0$, $f'_{2\pi}(\pi,\mu) > 0$, and $\frac{p'(z-\pi)}{1-p(z-\pi)} > \frac{f'_{2\pi}(\pi,\mu)}{f_2(\pi,\mu)}$. We also know $y(\xi^*) < 1$ from the proof of Proposition 6 so it is easy to show $k'(\xi^*) > 0$. Finally, $\frac{f'_1(\pi)}{f_1(\pi)} > \frac{f'_{2\pi}(\pi,\mu)}{f_2(\pi,\mu)}$ follows from Assumption 2. We can now conclude $\frac{d\pi^*}{d\mu} > 0$.

The second step is to ensure $\pi^* > \frac{z}{2}$ at $\mu = 1$. Equations (A.15) and (A.20) reduce to $f_1(\pi) = h(\xi)$ and $1 - p(z-\pi) = k(\xi)$ respectively at $\mu = 1$. We already know from the proof of Proposition 6 that, on a two-dimensional graph with ξ on the horizontal axis and π on the vertical, (A.15) is downward-sloping while (A.20) is upward-sloping for any $\xi < \overline{\xi}$. We also know that any equilibrium involves $\xi < \overline{\xi}$. Define ξ_x such that $h(\xi_x) \equiv f_1\left(\frac{z}{2}\right) = 1$ or, equivalently, $y(\xi_x) \equiv \int_0^1 y(\omega)\, d\omega$. With $y'(\cdot) > 0$ and Assumption 3, $\xi_x < \overline{\xi}$. For $\pi^* > \frac{z}{2}$, we just need $k(\xi_x) > 1 - p\left(\frac{z}{2}\right)$ or, equivalently, $p\left(\frac{z}{2}\right) > \frac{1-\int_0^1 y(\omega)d\omega}{\int_0^{\xi_x}[1-y(\omega)]d\omega}$. Therefore, $p\left(\frac{z}{2}\right)$ sufficiently high ensures $\pi^* > \frac{z}{2}$ at $\mu = 1$.

We can now conclude that there is a $\mu_1 < 1$ at which $\pi^* = \frac{z}{2}$. Using (A.15) and (A.20):

$$\mu_1 \equiv \left[1 + \frac{1}{p\left(\frac{z}{2}\right)} \frac{\int_0^{\xi_1} [1 - y(\omega)]\, d\omega}{\int_0^{\xi_1} [y(\xi_1) - y(\omega)]\, d\omega} \frac{\int_0^1 y(\omega)\, d\omega - y(\xi_1)}{1 - \int_0^1 y(\omega)\, d\omega}\right]^{-1}$$

with ξ_1 implicitly defined by:

$$\frac{\int_0^{\xi_1} [y(\xi_1) - y(\omega)]\, d\omega}{\int_0^{\xi_1} [1 - y(\omega)]\, d\omega} \frac{\int_{\xi_1}^1 [y(\omega) - 1]\, d\omega}{\int_{\xi_1}^1 [y(\omega) - y(\xi_1)]\, d\omega} \equiv 1 - p\left(\frac{z}{2}\right)$$

To confirm $\mu_1 > 0$, note that (A.15) tells us $h(\xi_1) = \frac{f_1\left(\frac{z}{2}\right)}{f_2\left(\frac{z}{2},\mu_1\right)} = \frac{1}{f_2\left(\frac{z}{2},\mu_1\right)} > 1$ or, equivalently, $y(\xi_1) < \int_0^1 y(\omega)\, d\omega$.

Constrained Efficient Allocation The planner chooses π, $I(\cdot)$, and α to maximize welfare as defined in (A.22) subject to the aggregate feasibility constraint (9). His Lagrangian can be expressed as:

$$\mathcal{L} = \int_0^1 y(\omega)\, n(\omega)\, d\omega + \lambda \int_0^1 [y(\omega) - 1]\, n(\omega)\, d\omega + \gamma_0 \alpha + \gamma_1 (1-\alpha)$$

where $\lambda \geq 0$ is the Lagrange multiplier on (9), $\gamma_0 \geq 0$ and $\gamma_1 \geq 0$ are Lagrange multipliers on $\alpha \geq 0$ and $\alpha \leq 1$ respectively, and $n(\cdot)$ is as defined in (2). It is straightforward to show that the first order condition for $I(\omega)$ yields a reservation strategy with a threshold ξ such that $y(\xi) = \frac{\lambda}{1+\lambda}$. The first order conditions for π and α then simplify to:

$$\alpha \frac{f_1(\pi)}{\widetilde{f}_2^2(\pi,\alpha)} \int_0^\xi [y(\omega) - y(\xi)]\, d\omega + [1 - (1-\alpha) f_1(\pi)] \int_\xi^1 [y(\omega) - y(\xi)]\, d\omega = 0 \tag{A.30}$$

and:

$$\gamma_1 - \gamma_0 \overset{sign}{=} \int_0^\xi [y(\omega) - y(\xi)]\, d\omega + \widetilde{f}_2^2(\pi,\alpha) \int_\xi^1 [y(\omega) - y(\xi)]\, d\omega \tag{A.31}$$

The constrained efficient allocation involves π, ξ, and α solving (A.26), (A.30), and (A.31).

Equation (A.31) holds with complementary slackness. Therefore, to support a constrained efficient allocation with $\alpha = 1$, we need:

$$\int_0^{\widehat{\xi}} \left[y(\omega) - y\left(\widehat{\xi}\right)\right] d\omega + f_2^2(\widehat{\pi}) \int_{\widehat{\xi}}^1 \left[y(\omega) - y\left(\widehat{\xi}\right)\right] d\omega \geq 0 \tag{A.32}$$

where $\widehat{\pi}$ and $\widehat{\xi}$ solve (A.16) and (A.20). Notice that (A.16) is just (A.30) evaluated at $\alpha = 1$ and, as above, (A.20) is just (A.26) evaluated at $\alpha = 1$. Substituting (A.16) into (A.32), the desired inequality reduces to $\widehat{\pi} \geq \frac{z}{2}$.

I will now show existence of a threshold separation rate, $\widehat{\mu}_1 \in (0,1)$, such that (A.16) and (A.20) yield $\widehat{\pi} > \frac{z}{2}$ if and only if $\mu > \widehat{\mu}_1$. The proof goes through the same steps as the one for the decentralized equilibrium.

The first step is to show $\frac{d\widehat{\pi}}{d\mu} > 0$. Differentiate (A.16) and (A.20) then combine to get:

$$\frac{d\widehat{\pi}}{d\mu} = \frac{\left[\frac{h'\left(\widehat{\xi}\right)}{h\left(\widehat{\xi}\right)} - 2\frac{k'\left(\widehat{\xi}\right)}{k\left(\widehat{\xi}\right)}\right] \frac{f'_{2\mu}(\widehat{\pi},\mu)}{f_2(\widehat{\pi},\mu)}}{\frac{h'\left(\widehat{\xi}\right)}{h\left(\widehat{\xi}\right)} \left[\frac{p'(z-\widehat{\pi})}{1-p(z-\widehat{\pi})} - \frac{f'_{2\pi}(\widehat{\pi},\mu)}{f_2(\widehat{\pi},\mu)}\right] - \frac{k'\left(\widehat{\xi}\right)}{k\left(\widehat{\xi}\right)} \left[\frac{f'_1(\widehat{\pi})}{f_1(\widehat{\pi})} - 2\frac{f'_{2\pi}(\widehat{\pi},\mu)}{f_2(\widehat{\pi},\mu)}\right]} \tag{A.33}$$

We know $\widehat{\xi} < \xi^*$ from Proposition 8 so $y\left(\widehat{\xi}\right) < 1$ and $k'\left(\widehat{\xi}\right) > 0$. A sufficient condition for $\frac{d\widehat{\pi}}{d\mu} > 0$ is then $\frac{f'_1(\pi)}{f_1(\pi)} \geq 2\frac{f'_{2\pi}(\pi,\mu)}{f_2(\pi,\mu)}$ or, equivalently:

$$\frac{\frac{p'(\pi)}{p(\pi)} - \frac{1}{2}\left[\frac{p''(\pi)}{p'(\pi)} + \frac{p''(z-\pi)}{p'(z-\pi)}\right]}{\frac{p'(\pi)}{p(\pi)} + \frac{p'(z-\pi)}{1-p(z-\pi)}} \geq \frac{(1-\mu)\, p(\pi)\, [1 - p(z-\pi)]}{\mu + (1-\mu)\, p(\pi)\, [1-p(z-\pi)]} \left[1 - \frac{\mu \frac{p'(\pi)}{p(\pi)} \frac{p(z-\pi)}{p'(z-\pi)}}{\mu + (1-\mu)\, p(\pi)}\right]$$

Assumption 2 will guarantee this if the right-hand side is less than $\frac{1}{2}$. Note that the right-hand side is indeed less than $\frac{1}{2}$ for any $\mu \geq \frac{p(z)}{1+p(z)}$. Therefore, $\frac{d\widehat{\pi}}{d\mu} > 0$ for μ sufficiently high. Since the numerator in (A.33) is strictly negative, $\frac{d\widehat{\pi}}{d\mu} < 0$ for lower μ would require a discontinuity which is ruled out with well-behaved functional forms.

The second step is to ensure $\widehat{\pi} > \frac{z}{2}$ at $\mu = 1$. Equations (A.16) and (A.20) reduce to $f_1(\pi) = h(\xi)$ and $1 - p(z - \pi) = k(\xi)$ respectively at $\mu = 1$. These are exactly the same equations as the decentralized equilibrium when $\mu = 1$ so $\pi^* > \frac{z}{2}$ at $\mu = 1$ also implies $\widehat{\pi} > \frac{z}{2}$ at $\mu = 1$.

We can now conclude that there is a $\widehat{\mu}_1 < 1$ at which $\widehat{\pi} = \frac{z}{2}$. Using (A.16) and (A.20):

$$\widehat{\mu}_1 \equiv \left[1 + \frac{1}{p\left(\frac{z}{2}\right)} \frac{\int_0^{\widehat{\xi}_1} [1 - y(\omega)]\, d\omega}{1 - \int_0^1 y(\omega)\, d\omega} \left(\sqrt{\frac{\int_{\widehat{\xi}_1}^1 \left[y(\omega) - y\left(\widehat{\xi}_1\right)\right] d\omega}{\int_0^{\widehat{\xi}_1} \left[y\left(\widehat{\xi}_1\right) - y(\omega)\right] d\omega}} - 1 \right)\right]^{-1}$$

with $\widehat{\xi}_1$ implicitly defined by:

$$\frac{\int_{\widehat{\xi}_1}^1 [y(\omega) - 1]\, d\omega}{\int_0^{\widehat{\xi}_1} [1 - y(\omega)]\, d\omega} \sqrt{\frac{\int_0^{\widehat{\xi}_1} \left[y\left(\widehat{\xi}_1\right) - y(\omega)\right] d\omega}{\int_{\widehat{\xi}_1}^1 \left[y(\omega) - y\left(\widehat{\xi}_1\right)\right] d\omega}} \equiv 1 - p\left(\frac{z}{2}\right)$$

To confirm $\widehat{\mu}_1 > 0$, note that (A.16) tells us $h\left(\widehat{\xi}_1\right) = \frac{f_1\left(\frac{z}{2}\right)}{f_2^2\left(\frac{z}{2}, \widehat{\mu}_1\right)} = \frac{1}{f_2^2\left(\frac{z}{2}, \widehat{\mu}_1\right)} > 1$.

Defining $\overline{\mu} \equiv \max\{\mu_1, \widehat{\mu}_1\}$ completes the proof. ■

Proof of Proposition 11

The planner chooses π, $I(\cdot)$, α, and z to maximize welfare as defined in (A.22) subject to the aggregate feasibility constraints for capital (9) and labor (14). Informed retention will still be characterized by a reservation strategy around a threshold ξ. Using A as per (8) with $\psi(\cdot)$ as per the proof of Proposition 10, we can rewrite (14) as:

$$\frac{\mu z}{\mu + (1-\mu)\, p(\pi)\left[\alpha + (1-\alpha)\, p(z-\pi)\right]} \left[1 + \frac{(1-\mu)\, p(\pi)\, p(z-\pi)\, \xi}{\mu + (1-\mu)\, p(\pi)\left[1 - p(z-\pi)\right]\alpha}\right] = \overline{L} \tag{A.34}$$

The constrained efficient allocation boils down to a quadruple $\{\pi, \xi, \alpha, z\}$ solving (A.26), (A.30), (A.31), and (A.34), where (A.31) holds with complementary slackness.[16] The Lagrange multipliers on (9) and (14) can then be recovered from the first order conditions for ξ and z.

Consider now the decentralized problem. An unmatched lender in the decentralized economy maximizes U as defined in (13) with $J(\omega)$ as per (5), taking as given the distribution $\psi(\cdot)$. Informed retention will again be characterized by a reservation strategy. The

[16]Both $f_1(\cdot)$ as defined in the proof of Proposition 4 and $\widetilde{f}_2(\cdot)$ as defined in the proof of Proposition 10 depend on z so, for the purposes of the current proof, any equations that depend on $f_1(\cdot)$ and/or $\widetilde{f}_2(\cdot)$ should be understood to depend on $f_1(\pi, z)$ and/or $\widetilde{f}_2(\pi, \alpha, z)$.

decentralized first order conditions for ξ and z are:

$$\frac{\mu\left[y\left(\xi\right)+\beta U\right]}{1-\beta\left(1-\mu\right)}-R-\beta U=0 \tag{A.35}$$

and:

$$W=\frac{\mu p\left(\pi\right)p'\left(z-\pi\right)}{1-\beta\left(1-\mu\right)}\left[\alpha\int_{0}^{\xi}\left[y\left(\xi\right)-y\left(\omega\right)\right]\psi\left(\omega\right)d\omega+\left(1-\alpha\right)\int_{\xi}^{1}\left[y\left(\omega\right)-y\left(\xi\right)\right]\psi\left(\omega\right)d\omega\right] \tag{A.36}$$

respectively. The conditions for π and α then reduce to (A.27) and (A.28), where (A.28) holds with complementary slackness. Capital market clearing is still given by (A.26). Labor market clearing is given by (A.34). The decentralized equilibrium is therefore a quadruple $\{\pi,\xi,\alpha,z\}$ solving (A.26), (A.27), (A.28), and (A.34). The prices R and W can then be recovered from (A.35) and (A.36).

Assume $\alpha=1$ then validate by verifying $\pi>\frac{z}{2}$ for both the decentralized lenders and the planner. Equations (A.26) and (A.34) are common to the lenders and the planner. With $\alpha=1$, we can express (A.26) as $\frac{1-p(z-\pi)}{f_2(\pi,z,\mu)}=k\left(\xi\right)$, where $f_2\left(\cdot\right)$ is as defined in the proof of Proposition 4, except writing $f_2\left(\pi,z,\mu\right)$ instead of $f_2\left(\pi\right)$ to make explicit the dependence on z and μ, and $k\left(\xi\right)$ is as defined in the proof of Proposition 10. We can also express (A.34) as $\mu z=Q\left(\pi,z,\xi,\mu\right)$ when $\alpha=1$, where:

$$Q\left(\pi,z,\xi,\mu\right)\equiv\frac{\left[\mu+\left(1-\mu\right)p\left(\pi\right)\right]\overline{L}}{1+\frac{(1-\mu)p(\pi)p(z-\pi)\xi}{\mu+(1-\mu)p(\pi)[1-p(z-\pi)]}}$$

The third equation for the decentralized lenders is (A.27) which can be expressed as $\frac{f_1(\pi,z)}{f_2(\pi,z,\mu)}=h\left(\xi\right)$ when $\alpha=1$, where $f_1\left(\cdot\right)$ is as defined in the proof of Proposition 4, except writing $f_1\left(\pi,z\right)$ instead of $f_1\left(\pi\right)$ to make explicit the dependence on z, and $h\left(\xi\right)$ is as defined in the proof of Proposition 10. The planner's third equation is (A.30) which can be expressed as $\frac{f_1(\pi,z)}{f_2^2(\pi,z,\mu)}=h\left(\xi\right)$ when $\alpha=1$. Notice that (A.27) and (A.30) are equivalent at $\mu=1$. Therefore, the decentralized z, π, and ξ are all constrained efficient at $\mu=1$.

Start by differentiating (A.26) and (A.34) under the assumption of $\alpha=1$ to get:

$$-\frac{p'\left(z-\pi\right)}{f_2\left(\pi,z,\mu\right)}\frac{dz}{d\mu}+\frac{p'\left(z-\pi\right)}{f_2\left(\pi,z,\mu\right)}\frac{d\pi}{d\mu}-\frac{1-p\left(z-\pi\right)}{f_2^2\left(\pi,z,\mu\right)}\left[f'_{2\pi}\frac{d\pi}{d\mu}+f'_{2z}\frac{dz}{d\mu}+f'_{2\mu}\right]=k'\left(\xi\right)\frac{d\xi}{d\mu} \tag{A.37}$$

and:

$$z+\mu\frac{dz}{d\mu}=Q'_{\pi}\frac{d\pi}{d\mu}+Q'_{z}\frac{dz}{d\mu}+Q'_{\xi}\frac{d\xi}{d\mu}+Q'_{\mu} \tag{A.38}$$

Also differentiate (A.27) and (A.30) under the same assumption to get:

$$\frac{f'_{1\pi}}{f_2\left(\pi,z,\mu\right)}\frac{d\pi}{d\mu}+\frac{f'_{1z}}{f_2\left(\pi,z,\mu\right)}\frac{dz}{d\mu}-\frac{f_1\left(\pi,z\right)}{f_2^2\left(\pi,z,\mu\right)}\left[f'_{2\pi}\frac{d\pi}{d\mu}+f'_{2z}\frac{dz}{d\mu}+f'_{2\mu}\right]=h'\left(\xi\right)\frac{d\xi}{d\mu} \tag{A.39}$$

and:

$$\frac{f'_{1\pi}}{f_2^2(\pi,z,\mu)}\frac{d\pi}{d\mu}+\frac{f'_{1z}}{f_2^2(\pi,z,\mu)}\frac{dz}{d\mu}-2\frac{f_1(\pi,z)}{f_2^3(\pi,z,\mu)}\left[f'_{2\pi}\frac{d\pi}{d\mu}+f'_{2z}\frac{dz}{d\mu}+f'_{2\mu}\right]=h'(\xi)\frac{d\xi}{d\mu}\quad\text{(A.40)}$$

Evaluate (A.37), (A.38), and (A.39) at $\mu = 1$ then combine to isolate:

$$\left.\frac{dz^*}{d\mu}\right|_{\mu=1}=Q'_\mu - z$$

and:

$$\left.\frac{d\pi^*}{d\mu}\right|_{\mu=1}=\frac{h'(\xi)\left[1-p(z-\pi)\right]-k'(\xi)f_1(\pi,z)}{h'(\xi)p'(z-\pi)-k'(\xi)f'_{1\pi}}f'_{2\mu}+\frac{h'(\xi)p'(z-\pi)+k'(\xi)f'_{1z}}{h'(\xi)p'(z-\pi)-k'(\xi)f'_{1\pi}}\left(Q'_\mu - z\right)$$

and:

$$\left.\frac{d\xi^*}{d\mu}\right|_{\mu=1}=\frac{\left[1-p(z-\pi)\right]f'_{1\pi}-f_1(\pi,z)p'(z-\pi)}{h'(\xi)p'(z-\pi)-k'(\xi)f'_{1\pi}}f'_{2\mu}+\frac{p'(z-\pi)\left[f'_{1\pi}+f'_{1z}\right]}{h'(\xi)p'(z-\pi)-k'(\xi)f'_{1\pi}}\left(Q'_\mu - z\right)$$

These derivatives tell us how the decentralized equilibrium changes if we move slightly below $\mu = 1$.

Now evaluate (A.40) at $\mu = 1$ then combine with (A.37) and (A.38), also evaluated at $\mu = 1$, to isolate:

$$\left.\frac{d\widehat{z}}{d\mu}\right|_{\mu=1}=Q'_\mu - z$$

and:

$$\left.\frac{d\widehat{\pi}}{d\mu}\right|_{\mu=1}=\frac{h'(\xi)\left[1-p(z-\pi)\right]-2k'(\xi)f_1(\pi,z)}{h'(\xi)p'(z-\pi)-k'(\xi)f'_{1\pi}}f'_{2\mu}+\frac{h'(\xi)p'(z-\pi)+k'(\xi)f'_{1z}}{h'(\xi)p'(z-\pi)-k'(\xi)f'_{1\pi}}\left(Q'_\mu - z\right)$$

and:

$$\left.\frac{d\widehat{\xi}}{d\mu}\right|_{\mu=1}=\frac{\left[1-p(z-\pi)\right]f'_{1\pi}-2f_1(\pi,z)p'(z-\pi)}{h'(\xi)p'(z-\pi)-k'(\xi)f'_{1\pi}}f'_{2\mu}+\frac{p'(z-\pi)\left[f'_{1\pi}+f'_{1z}\right]}{h'(\xi)p'(z-\pi)-k'(\xi)f'_{1\pi}}\left(Q'_\mu - z\right)$$

These derivatives tell us how the constrained efficient allocation changes if we move slightly below $\mu = 1$.

The two sets of derivatives ($\left.\frac{di^*}{d\mu}\right|_{\mu=1}$ and $\left.\frac{d\widehat{i}}{d\mu}\right|_{\mu=1}$ for $i \in \{z, \pi, \xi\}$) are evaluated at the same values of z, π, and ξ since the decentralized equilibrium is constrained efficient at $\mu = 1$. It is easy to see:

$$\left.\frac{dz^*}{d\mu}\right|_{\mu=1} = \left.\frac{d\widehat{z}}{d\mu}\right|_{\mu=1}$$

We can also show:

$$\left.\frac{d\pi^*}{d\mu}\right|_{\mu=1} - \left.\frac{d\widehat{\pi}}{d\mu}\right|_{\mu=1} = \frac{k'(\xi)\, f_1(\pi, z)}{h'(\xi)\, p'(z-\pi) - k'(\xi)\, f'_{1\pi}} f'_{2\mu} < 0$$

where the sign follows from $h'(\cdot) < 0$, $f'_{2\mu} > 0$, the curvature of $p(\cdot)$, and the result from the proof of Proposition 6 that any equilibrium has $y(\xi) < 1$ and hence $k'(\xi) > 0$. If $\left.\frac{d\widehat{\pi}}{d\mu}\right|_{\mu=1} \leq 0$, then the decentralized π increases more than the planner's π as we move below $\mu = 1$. If $\left.\frac{d\widehat{\pi}}{d\mu}\right|_{\mu=1} > 0$, then the planner's π decreases as we move below $\mu = 1$ while the decentralized π either decreases by less or increases. Combined with $\left.\frac{dz^*}{d\mu}\right|_{\mu=1} = \left.\frac{d\widehat{z}}{d\mu}\right|_{\mu=1}$, this means that the decentralized equilibrium has π inefficiently high and $z - \pi$ inefficiently low as we move below $\mu = 1$.

To complete the proof, we just need to confirm $\alpha = 1$ (or, equivalently, $\pi > \frac{z}{2}$) at $\mu = 1$. A continuity argument can then be invoked to conclude $\pi > \frac{z}{2}$ (and thus $\alpha = 1$) slightly below $\mu = 1$. Note that $\mu = 1$ reduces (A.34) to $z = \overline{L}$, pinning down z independently of π and ξ. With $\overline{L}$ not too low, $p\left(\frac{z}{2}\right)$ is not too low and we can follow the proof of Proposition 10 to conclude that both the planner and the decentralized lenders choose $\pi > \frac{z}{2}$ at $\mu = 1$. ■

Appendix B – Relationship Lending Extension

In models of relationship lending, lenders acquire information about their borrowers over repeated interactions and use that information in subsequent financing.[17] The analysis in the main text abstracts from relationship lending by dissolving matches after one interaction of average duration $\frac{1}{\mu}$. This appendix explores how, if at all, the allocation of resources between matching and screening is affected by an ability to resolve information frictions over time through relationship lending. For brevity, I will focus on the case of $\alpha = 1$. The relevant comparison is therefore to Sections 3.1 and 4.4, depending on whether or not there is a Walrasian market.

To proceed, I fix the time to project completion at one period and re-interpret μ as an exogenous rate of match separation. Relationship lending is introduced by assuming that uninformed matches last at most one period. Specifically, an uninformed lender whose match does not exogenously separate after one period learns his borrower's type, and hence becomes an informed lender, by virtue of having interacted with the borrower over the course of the first period.

Start by redefining $n(\omega)$, the fraction of type ω firms financed each period. The only difference relative to the law of motion in (1) is that lenders whose matches are not exogenously separated can make the retention decision again. Mathematically:

$$n_t(\omega) = I_t(\omega)(1-\mu)\, n_{t-1}(\omega) + p(\pi_t)\left[1 - p(z-\pi_t) + p(z-\pi_t)\, I_t(\omega)\right]\left[1 - (1-\mu)\, n_{t-1}(\omega)\right] \tag{B.1}$$

which, in steady state, becomes:

$$n(\omega) = \frac{p(\pi)\left[1 - p(z-\pi) + p(z-\pi)\, I(\omega)\right]}{1 - (1-\mu)\, I(\omega) + (1-\mu)\, p(\pi)\left[1 - p(z-\pi) + p(z-\pi)\, I(\omega)\right]} \tag{B.2}$$

The mass of available matches, A, and the distribution of available borrowers, $\psi(\cdot)$, are still given by equations (4) and (3) respectively but using $n(\cdot)$ as per (B.2).

The rest of this appendix establishes two main results. Section B.1 shows that relationship lending changes the direction of the extensive externality from π. Section B.2 then shows that the distributional externalities together with the Walrasian market for capital recover the main insight from Proposition 8 (i.e., both π and ξ inefficiently high in the decentralized equilibrium) for parameters where the extensive externality is not too strong.

[17] Some empirical evidence for the idea that relationship lending involves information acquisition comes from Lummer and McConnell (1989). They find that stock prices (i) react positively when a firm announces the renewal of an existing lending agreement with its bank and (ii) do not react in a statistically significant way to announcements of new lending agreements. They interpret this as evidence that banks accumulate inside information about their customers over time. A common view in the banking literature is that a relationship lender acquires new information by observing the same borrower over time or across products in interactions afforded by the relationship, not by actively re-screening the borrower in every period of the relationship. See Boot (2000) as well as Hachem (2011), Cohen et al (2019), and the references therein.

B.1 Baseline Model

The value to a lender of accepting a match with a type ω firm is now given by:

$$J(\omega) = y(\omega) + \beta\mu U + \beta(1-\mu)\left[I(\omega)J(\omega) + (1-I(\omega))(g' + \beta U)\right] \quad \text{(B.3)}$$

The firm delivers $y(\omega)$ at the end of the period. With probability μ, the match is exogenously broken and the lender enters next period unmatched. With probability $1-\mu$, the match is not exogenously broken and the lender can decide whether to accept the borrower again or endogenously separate and enter the next period unmatched. The value of an unmatched lender, U, is still given by equation (6) but with $J(\cdot)$ as per (B.3).

Use equations (6) and (B.3) to isolate:

$$U = \frac{1}{1-\beta}\left[g' + \frac{p(\pi)\int_0^1 \frac{1-p(z-\pi)(1-I(\omega))}{1-\beta(1-\mu)I(\omega)}\left[y(\omega)-g'\right]\psi(\omega)\,d\omega}{1+\beta(1-\mu)p(\pi)\int_0^1 \frac{1-p(z-\pi)(1-I(\omega))}{1-\beta(1-\mu)I(\omega)}\psi(\omega)\,d\omega}\right] \quad \text{(B.4)}$$

Decentralized lenders choose $\pi \in [0,z]$ and $I(\cdot) \in [0,1]$ to maximize U as defined in (B.4), taking as given $\psi(\cdot)$. As in Lemma 1, informed retention will be characterized by a reservation strategy around some cutoff ξ. To facilitate comparison to Lemmas 2 and 3, define:

$$\widetilde{\Gamma}(\pi,\xi,\psi(\cdot),g'|\beta) \equiv p(\pi)\left[\left[1-p(z-\pi)\right]\int_0^\xi \left[y(\omega)-g'\right]\psi(\omega)\,d\omega + \frac{\int_\xi^1\left[y(\omega)-g'\right]\psi(\omega)\,d\omega}{1-\beta(1-\mu)}\right]$$

and:

$$\widetilde{D}(\pi,\xi,\psi(\cdot)|\beta) \equiv 1+\beta(1-\mu)p(\pi)\left[\left[1-p(z-\pi)\right]\int_0^\xi \psi(\omega)\,d\omega + \frac{\int_\xi^1 \psi(\omega)\,d\omega}{1-\beta(1-\mu)}\right]$$

Maximization of U as per (B.4) by the decentralized lenders amounts to choosing $\pi \in [0,z]$ and $\xi \in [0,1]$ to maximize $\frac{\widetilde{\Gamma}(\pi,\xi,\psi(\cdot),g'|\beta)}{\widetilde{D}(\pi,\xi,\psi(\cdot)|\beta)}$, taking as given $\psi(\cdot)$.

Consider now the constrained efficiency benchmark. The welfare function is:

$$\mathcal{W} = \frac{1}{1-\beta}\left[g' + \int_0^1 \left[y(\omega)-g'\right]n(\omega)\,d\omega\right]$$

with $n(\cdot)$ as per (B.2). In steady state, equations (B.1) and (3) imply:

$$n(\omega) = \frac{p(\pi)\left[1-p(z-\pi)(1-I(\omega))\right]\psi(\omega)A}{1-(1-\mu)I(\omega)} \quad \text{(B.5)}$$

Substituting (B.5) into (4) and rearranging to isolate A yields:

$$A = \frac{1}{1+(1-\mu)p(\pi)\int_0^1 \frac{1-p(z-\pi)(1-I(\omega))}{1-(1-\mu)I(\omega)}\psi(\omega)\,d\omega} \quad \text{(B.6)}$$

We can now substitute (B.5) with A as per (B.6) into the welfare function to write:

$$\mathcal{W}=\frac{1}{1-\beta}\left[g'+\frac{p(\pi)\int_0^1\frac{1-p(z-\pi)(1-I(\omega))}{1-(1-\mu)I(\omega)}\left[y(\omega)-g'\right]\psi(\omega)\,d\omega}{1+(1-\mu)\,p(\pi)\int_0^1\frac{1-p(z-\pi)(1-I(\omega))}{1-(1-\mu)I(\omega)}\psi(\omega)\,d\omega}\right] \tag{B.7}$$

The planner chooses $\pi\in[0,z]$ and $I(\cdot)\in[0,1]$ to maximize $\mathcal{W}$ as defined in (B.7). This amounts to choosing $\pi\in[0,z]$ and $\xi\in[0,1]$ to maximize $A\times\widetilde{\Gamma}(\pi,\xi,\psi(\cdot),g'|1)$, where $A=\lim_{\beta\to1}\frac{1}{\widetilde{D}(\pi,\xi,\psi(\cdot)|\beta)}$.

Compare the decentralized and planning problems. There are still distributional externalities because, unlike the planner, decentralized lenders take $\psi(\cdot)$ as given. There are also still extensive externalities, as the decentralized objective function only coincides with the planner's objective if there is no intertemporal discounting. However, the fact that β now appears in $\widetilde{\Gamma}(\cdot)$, not just $\widetilde{D}(\cdot)$, means that $\beta\to1$ does more than make decentralized lenders internalize their full effect on A. In other words, with relationship lending, the extensive externality is about more than just the mass of available matches.

Proposition B.1 below shows that this difference changes the direction of the extensive externality from π relative to the baseline model without relationship lending. Specifically, comparing Proposition B.1 to Propositions 2, 3, and 4 in the main text, we see that the decentralized π is now inefficiently low as a result of the extensive effect. Higher π still decreases the mass of available matches A so the reversal in Proposition B.1 must be explained by the presence of β in $\widetilde{\Gamma}(\cdot)$. The intuition lies in the fact that relationship lending generates informed matches from previously uninformed ones, mitigating the tradeoff between matching and screening in the initial allocation of intermediation resources. By the recursive nature of his problem, a decentralized lender takes into account that choosing higher π increases the probability he forms an uninformed match today and becomes informed tomorrow. But this is discounted at rate β and is therefore not the same as internalizing how his decisions affect the stock of previously uninformed matches available to become informed in a symmetric steady state. The latter is what the planner internalizes, leading the planner to choose a higher value of π than the decentralized lenders.

Proposition B.1 *If $\psi(\cdot)$ is exogenously reset every period, then the decentralized ξ and π in the baseline model with relationship lending are both inefficiently low. If $\psi(\cdot)$ is endogenous, then the decentralized ξ is too high but, for β low, the decentralized π is still too low.*

Proof. Start with $\psi(\cdot)$ endogenous. The planner's first order conditions reduce to:

$$y(\xi)=g'$$

and:

$$\frac{1-p(z-\pi)+\frac{p(\pi)p'(z-\pi)}{p'(\pi)}}{\mu\left[1+\frac{(1-\mu)[1-p(\pi)p(z-\pi)]}{\mu+(1-\mu)p(\pi)}\right]^2}=\frac{\int_\xi^1\left[y(\omega)-y(\xi)\right]d\omega}{\int_0^\xi\left[y(\xi)-y(\omega)\right]d\omega} \tag{B.8}$$

In contrast, the decentralized first order conditions reduce to:

$$y(\xi)=g'+\frac{\beta(1-\mu)\,p(\pi)^2\,\frac{p'(z-\pi)}{p'(\pi)}\left[\mu+(1-\mu)\,p(\pi)\right]\int_0^\xi\left[y(\xi)-y(\omega)\right]d\omega}{\mu+(1-\mu)\,p(\pi)\left[\mu+(1-\mu)\,\xi-\mu(1-\xi)\,p(z-\pi)\right]}$$

and:

$$\frac{1-p(z-\pi)+\frac{p(\pi)p'(z-\pi)}{p'(\pi)}}{\mu\left[1+\frac{(1-\mu)[1-p(\pi)p(z-\pi)]}{\mu+(1-\mu)p(\pi)}\right]}=\frac{1}{1-\beta(1-\mu)}\frac{\int_{\xi}^{1}[y(\omega)-y(\xi)]\,d\omega}{\int_{0}^{\xi}[y(\xi)-y(\omega)]\,d\omega} \tag{B.9}$$

If $\beta>0$, then $\xi^*>\widehat{\xi}$, where stars denote the solution to the decentralized system and hats denote the solution to the planner's system. If $\beta=0$, then $\xi^*=\widehat{\xi}$ so $\pi^*<\widehat{\pi}$. By continuity, $\pi^*<\widehat{\pi}$ for β low.

The rest of the proof considers what happens when $\psi(\cdot)$ is exogenously reset every period. In this case, $\psi(\cdot)=1$ and the decentralized first order conditions simplify to:

$$\left[1-p(z-\pi)+\frac{p(\pi)p'(z-\pi)}{p'(\pi)}\right]\int_0^{\xi}[y(\omega)-y(\xi)]\,d\omega+\frac{\int_{\xi}^{1}[y(\omega)-y(\xi)]\,d\omega}{1-\beta(1-\mu)}=0 \tag{B.10}$$

and:

$$y(\xi)=g'+\beta(1-\mu)p(\pi)\left[[1-p(z-\pi)]\int_0^{\xi}[y(\omega)-y(\xi)]\,d\omega+\frac{\int_{\xi}^{1}[y(\omega)-y(\xi)]\,d\omega}{1-\beta(1-\mu)}\right] \tag{B.11}$$

Going through the algebra, we then find that the planner's first order conditions amount to (B.10) and (B.11) evaluated at $\beta=1$. The discount factor β directly affects the decentralized choice of ξ in equation (B.11) because the lender compares the value of keeping the borrower today to the value he could get tomorrow if unmatched. We saw the same thing in the model without relationship lending (e.g., equation (A.11)). Now, however, β also directly affects the decentralized choice of π in equation (B.10) because relationship lending gives the lender an alternative way to learn tomorrow. We did not have this in the model without relationship lending (e.g., equation (A.10)).

Fully differentiate (B.11) to get:

$$\frac{d\xi}{d\beta}=\frac{\frac{(1-\mu)p(\pi)}{1-\beta(1-\mu)}\left[\beta(1-\mu)[1-p(z-\pi)]+\frac{p(\pi)p'(z-\pi)}{p'(\pi)}\right]}{1+\beta(1-\mu)p(\pi)\left[[1-p(z-\pi)]\xi+\frac{1-\xi}{1-\beta(1-\mu)}\right]}\frac{1}{y'(\xi)}\int_0^{\xi}[y(\xi)-y(\omega)]\,d\omega$$

where I have used (B.10) to simplify terms. Clearly, $\frac{d\xi}{d\beta}>0$ so we can conclude $\xi^*<\widehat{\xi}$.

Now use (B.10) to rewrite (B.11) as:

$$y(\xi)=g'+\frac{p(\pi)^2p'(z-\pi)}{p'(\pi)}\left[\int_0^{\xi}[y(\xi)-y(\omega)]\,d\omega-\frac{\int_{\xi}^{1}[y(\omega)-y(\xi)]\,d\omega}{1-p(z-\pi)+\frac{p(\pi)p'(z-\pi)}{p'(\pi)}}\right] \tag{B.12}$$

Equation (B.12) implicitly defines a function $\pi_x(\xi)$ which is independent of β. In other words, $\pi_x(\xi)$ is the same for the planner and the decentralized lenders. Letting $\pi_a(\xi|\beta)$ denote the function implicitly defined by (B.10), the decentralized equilibrium is an intersection between

$\pi_x(\xi)$ and $\pi_a(\xi|\beta)$ while the constrained efficient allocation is an intersection between $\pi_x(\xi)$ and $\pi_a(\xi|1)$. Differentiating (B.12) yields:

$$\pi_x'(\xi) = \frac{\left[1 - \frac{p(\pi)^2 p'(z-\pi)}{p'(\pi)}\left[\xi + \frac{1-\xi}{1-p(z-\pi)+\frac{p(\pi)p'(z-\pi)}{p'(\pi)}}\right]\right] y'(\xi)}{\left[2\frac{p'(\pi)}{p(\pi)} - \frac{p''(\pi)}{p'(\pi)} - \frac{p''(z-\pi)}{p'(z-\pi)}\right]\left[y(\xi) - g' + p(\pi)\left[\frac{\frac{p(\pi)p'(z-\pi)}{p'(\pi)}}{1-p(z-\pi)+\frac{p(\pi)p'(z-\pi)}{p'(\pi)}}\right]^2 \int_\xi^1 [y(\omega) - y(\xi)]\, d\omega\right]}$$

where all instances of π on the right-hand side are evaluated at $\pi_x(\xi)$. Combining (B.12) with $\pi_x'(\xi) = 0$, we find that any critical point of $\pi_x(\cdot)$ is a solution to (B.12) and:

$$1 - p(z-\pi) + \frac{p(\pi)\, p'(z-\pi)}{p'(\pi)} = \frac{\int_\xi^1 [y(\omega) - g']\, d\omega}{\int_0^\xi [g' - y(\omega)]\, d\omega} \tag{B.13}$$

Notice that (B.13) is equivalent to (B.10) when $\beta = 0$ and $y(\xi) = g'$. In other words, the function implicitly defined by (B.13) uniquely intersects $\pi_a(\xi|0)$ at $\xi = \xi_a \equiv y^{-1}(g')$. Also notice that $\xi = \xi_a$ is the unique solution to $\pi_x(\xi) = \pi_a(\xi|0)$. Therefore, $\pi_x(\xi)$ has a critical point at $\xi = \xi_a$. Moreover, $\pi_x''(\xi_a) \overset{sign}{=} \frac{p'(\pi)}{p(\pi)} - \frac{p'(z-\pi)}{p(z-\pi)}$ which is positive if and only if $\pi < \frac{z}{2}$. Since the relevant π is $\pi_x(\xi_a)$ and we know $\pi_x(\xi_a) = \pi_a(\xi_a|0)$, it follows that $\pi_x(\cdot)$ achieves a minimum at $\xi = \xi_a$ if and only if $\pi_a(\xi_a|0) < \frac{z}{2}$. Returning to (B.10), we can show that $\pi_a(\xi_a|0) < \frac{z}{2}$ is equivalent to $g' > \int_0^1 y(\omega)\, d\omega$.

The next step is to show that $\xi = \xi_a$ is the unique critical point of $\pi_x(\xi)$. To do this, note that the derivative of the right-hand side of (B.13) with respect to ξ has the same sign as $\left[\int_0^1 y(\omega)\, d\omega - g'\right][y(\xi) - g']$. In other words, the function implicitly defined by (B.13) has a unique critical point at $\xi = \xi_a$ and, with $g' > \int_0^1 y(\omega)\, d\omega$, this critical point is a maximum. Therefore, (B.12) achieves a minimum at $\xi = \xi_a$ while (B.13) achieves a maximum so the only intersection between (B.12) and (B.13) is indeed $\xi = \xi_a$.

We have now shown that $g' > \int_0^1 y(\omega)\, d\omega$ implies $\pi_x(\xi)$ convex with a unique critical point that lies on $\pi_a(\cdot|0)$. This is also the only intersection between $\pi_x(\cdot)$ and $\pi_a(\cdot|0)$. It is straightforward to show that higher β shifts $\pi_a(\cdot|\beta)$ away from the origin (in two-dimensional space with ξ on the horizontal axis and π on the vertical) while leaving $\pi_x(\cdot)$ unchanged. Since $\pi_a(\cdot|\beta)$ is downward sloping, we can now conclude that $\pi^* < \widehat{\pi}$. ■

B.2 Walrasian Model

The capital market clearing condition, which is also the aggregate feasibility condition in the planner's problem, is still given by (9) but with $n(\cdot)$ as per (B.5).[18] Formally:

$$[1 - p(z-\pi)] \int_0^\xi [1 - y(\omega)]\, \psi(\omega)\, d\omega + \frac{1}{\mu} \int_\xi^1 [1 - y(\omega)]\, \psi(\omega)\, d\omega = 0 \tag{B.14}$$

[18] Technically, the left-hand side of (9) is no longer multiplied by μ but this is moot since the right-hand side is zero.

Proposition B.2 below summarizes the results of the Walrasian model with relationship lending. The first part says that the decentralized equilibrium is not constrained efficient in the absence of distributional externalities. We know from Proposition B.1 that the extensive externalities result in both π and ξ inefficiently low when there is relationship lending. With ξ too low, there is an over-use of capital so the price of capital will rise in a Walrasian market. However, with π too low, there is an under-use of capital so the price of capital will fall in a Walrasian market. These two forces counteract each other, making it possible to have a market clearing equilibrium with both ξ and π too low. This is in contrast to the model without relationship lending (see specifically Subsection 4.4.1) where the Walrasian market priced in the extensive effects and delivered constrained efficiency in the absence of distributional externalities. The difference is that the extensive externalities resulted in π too high and ξ too low without relationship lending. Accordingly, there was an unambiguous over-use of capital which made capital more expensive, pushing π down and ξ up until the constrained efficient allocation was reached. When β is high, the extensive externalities are muted so the direction of inefficiency is driven by the distributional externalities. The second part of Proposition B.2 shows that the results of the Walrasian model without relationship lending still hold in this case. In particular, relationship lending does not change the finding that distributional externalities lead to both π and ξ inefficiently high when there is a Walrasian market for capital. The reasons are similar to those in the discussion of Proposition 8.

Proposition B.2 *Set $g' = 0$ and introduce a Walrasian market for capital. If $\psi(\cdot)$ is exogenously reset every period, then the decentralized π and ξ in the Walrasian model with relationship lending are both inefficiently low. If $\psi(\cdot)$ is endogenous, then there exists a unique $\mathcal{B} \in (0,1)$ such that the decentralized π and ξ are both: (i) inefficiently low if $\beta < \mathcal{B}$; (ii) constrained efficient if $\beta = \mathcal{B}$; (iii) inefficiently high if $\beta > \mathcal{B}$.*

Proof. Consider first the exogenously reset distribution (i.e., $\psi(\cdot) = 1$). The decentralized first order conditions still combine to deliver (B.10), while the planner's first order conditions still combine to deliver (B.10) evaluated at $\beta = 1$. Drawn in two dimensions, with ξ on the horizontal axis and π on the vertical axis, (B.10) is a downward-sloping curve which shifts away from the origin as β increases.

With $\psi(\cdot) = 1$, equation (B.14) simplifies to:

$$1 - p(z - \pi) = \frac{1}{\mu} \frac{\int_\xi^1 [1 - y(\omega)]\, d\omega}{\int_0^\xi [y(\omega) - 1]\, d\omega} \tag{B.15}$$

This defines an upward-sloping curve until $\xi = \overline{\xi} \equiv y^{-1}(1)$. Therefore, to show that the planner chooses both ξ and π higher than the decentralized equilibrium, it will be enough to show that the planner's ξ satisfies $y(\xi) < 1$. The first order condition for the planner's informed retention strategy delivers:

$$\begin{aligned} \frac{\mu\lambda}{1+\mu\lambda} \quad = \quad & \left[1 + (1-\mu)\, p(\pi) \left[1 - p(z-\pi)\,\xi + \tfrac{(1-\mu)(1-\xi)}{\mu}\right]\right] y(\xi) \\ & - (1-\mu)\, p(\pi) \left[[1 - p(z-\pi)] \int_0^\xi y(\omega)\, d\omega + \tfrac{1}{\mu} \int_\xi^1 y(\omega)\, d\omega\right] \end{aligned} \tag{B.16}$$

where λ denotes the Lagrange multiplier on (B.15) in the planning problem. Combining (B.15) and (B.16):

$$[1-y(\xi)]\left[1+(1-\mu)p(\pi)\left[[1-p(z-\pi)]\xi+\tfrac{1-\xi}{\mu}\right]\right]=\frac{1}{1+\mu\lambda}$$

The planner's solution thus satisfies $y(\xi)<1$, completing the proof for the case of $\psi(\cdot)=1$.

Now consider $\psi(\cdot)$ endogenous. The decentralized first order conditions still combine to deliver (B.9) while the planner's first order conditions still combine to deliver (B.8). Using $\psi(\cdot)$ as per (3) with $n(\cdot)$ as per (B.2), equation (B.14) becomes:

$$\frac{1-p(z-\pi)}{1+\frac{(1-\mu)[1-p(\pi)p(z-\pi)]}{\mu+(1-\mu)p(\pi)}}=\frac{\int_{\xi}^{1}[1-y(\omega)]\,d\omega}{\int_{0}^{\xi}[y(\omega)-1]\,d\omega} \tag{B.17}$$

Let $\widetilde{\pi}_e(\xi)$, $\widetilde{\pi}_l(\xi)$, and $\widetilde{\pi}_k(\xi)$ denote the functions implicitly defined by equations (B.8), (B.9), and (B.17) respectively. The decentralized equilibrium (π^*,ξ^*) satisfies $\pi^*=\widetilde{\pi}_k(\xi^*)=\widetilde{\pi}_l(\xi^*)$ while the constrained efficient allocation $\left(\widehat{\pi},\widehat{\xi}\right)$ satisfies $\widehat{\pi}=\widetilde{\pi}_k\left(\widehat{\xi}\right)=\widetilde{\pi}_e\left(\widehat{\xi}\right)$.

Define:

$$\mathcal{B}\equiv\frac{1-p(\widehat{\pi})p(z-\widehat{\pi})}{1+(1-\mu)p(\widehat{\pi})[1-p(z-\widehat{\pi})]}\in(0,1)$$

Notice that equations (B.8) and (B.17) are independent of β so $\widehat{\pi}$ is also independent of β and $\mathcal{B}$ is explicitly defined. With $\beta=\mathcal{B}$ in equation (B.9), the planner's allocation solves the system of equations that defines the decentralized equilibrium for any $\mu\in(0,1)$ so we can conclude $(\pi^*,\xi^*)=\left(\widehat{\pi},\widehat{\xi}\right)$.[19] Turning now to $\beta\neq\mathcal{B}$, the following lemma will be useful:

Lemma B.1 *$\widetilde{\pi}_l'(\cdot)<0$, $\widetilde{\pi}_k'(\xi^*)>0$, and $\widetilde{\pi}_k'\left(\widehat{\xi}\right)>0$*

Proof. Define $f_3(\pi)\equiv 1+\frac{(1-\mu)[1-p(\pi)p(z-\pi)]}{\mu+(1-\mu)p(\pi)}$ and $h(\xi)\equiv\frac{\int_{\xi}^{1}[y(\omega)-y(\xi)]d\omega}{\int_{0}^{\xi}[y(\xi)-y(\omega)]d\omega}$. With $f_1(\pi)$ as per the proof of Proposition 4, $\widetilde{\pi}_l(\cdot)$ solves $\frac{f_1(\widetilde{\pi}_l(\xi))}{f_3(\widetilde{\pi}_l(\xi))}=\frac{\mu h(\xi)}{1-\beta(1-\mu)}$. Some algebra reveals that Assumption 2 is sufficient for $\frac{d}{d\pi}\frac{f_1(\pi)}{f_3(\pi)}>0$ so $h'(\xi)<0$ implies $\widetilde{\pi}_l'(\cdot)<0$. To establish $\widetilde{\pi}_k'(\xi^*)>0$, rewrite equations (B.9) and (B.17) to isolate $\int_{\xi^*}^{1}y(\omega)\,d\omega$ then equate. Rearrange the equated expression to isolate $1-y(\xi^*)$. The result implies $1-y(\xi^*)>0$ which, by differentiation of equation (B.17) and Assumption 3, means $\widetilde{\pi}_k'(\xi^*)>0$. Finally, the first order condition for the planner's informed retention strategy can be used to conclude $y\left(\widehat{\xi}\right)<1$ so $\widetilde{\pi}_k'\left(\widehat{\xi}\right)>0$ is also true. □

Given Lemma B.1, showing $(\pi^*,\xi^*)\ll\left(\widehat{\pi},\widehat{\xi}\right)$ amounts to showing $\widetilde{\pi}_l\left(\widehat{\xi}\right)<\widetilde{\pi}_e\left(\widehat{\xi}\right)$. Similarly, showing $(\pi^*,\xi^*)\gg\left(\widehat{\pi},\widehat{\xi}\right)$ amounts to showing $\widetilde{\pi}_l\left(\widehat{\xi}\right)>\widetilde{\pi}_e\left(\widehat{\xi}\right)$. With $f_1(\pi)$, $f_3(\pi)$,

[19] Existence of equilibrium and the sufficiency of Assumption 2 for uniqueness of this equilibrium is proven similarly to Proposition 6. Also, with $\mu=1$, equations (B.8) and (B.9) are identical so the decentralized equilibrium is constrained efficient for any β.

and $h(\xi)$ as defined in the proof of Lemma B.1, $\widetilde{\pi}_l(\cdot)$ and $\widetilde{\pi}_e(\cdot)$ solve $\frac{f_1(\widetilde{\pi}_l(\xi))}{f_3(\widetilde{\pi}_l(\xi))} = \frac{\mu h(\xi)}{1-\beta(1-\mu)}$ and $\frac{f_1(\widetilde{\pi}_e(\xi))}{f_3(\widetilde{\pi}_e(\xi))} = \mu f_3(\widetilde{\pi}_e(\xi)) h(\xi)$ respectively. If $\beta < \mathcal{B}$, then $\frac{1}{1-\beta(1-\mu)} < f_3(\widehat{\pi}) = f_3\left(\widetilde{\pi}_e\left(\widehat{\xi}\right)\right)$ and, therefore, $\frac{f_1\left(\widetilde{\pi}_l\left(\widehat{\xi}\right)\right)}{f_3\left(\widetilde{\pi}_l\left(\widehat{\xi}\right)\right)} < \mu f_3\left(\widetilde{\pi}_e\left(\widehat{\xi}\right)\right) h\left(\widehat{\xi}\right) = \frac{f_1\left(\widetilde{\pi}_e\left(\widehat{\xi}\right)\right)}{f_3\left(\widetilde{\pi}_e\left(\widehat{\xi}\right)\right)}$. We know $\frac{d}{d\pi}\frac{f_1(\pi)}{f_3(\pi)} > 0$ from the proof of Lemma B.1 so $\frac{f_1\left(\widetilde{\pi}_l\left(\widehat{\xi}\right)\right)}{f_3\left(\widetilde{\pi}_l\left(\widehat{\xi}\right)\right)} < \frac{f_1\left(\widetilde{\pi}_e\left(\widehat{\xi}\right)\right)}{f_3\left(\widetilde{\pi}_e\left(\widehat{\xi}\right)\right)}$ implies $\widetilde{\pi}_l\left(\widehat{\xi}\right) < \widetilde{\pi}_e\left(\widehat{\xi}\right)$. In other words, $(\pi^*, \xi^*) \ll \left(\widehat{\pi}, \widehat{\xi}\right)$ if $\beta < \mathcal{B}$. In an analogous manner, $\beta > \mathcal{B}$ yields $\frac{f_1\left(\widetilde{\pi}_l\left(\widehat{\xi}\right)\right)}{f_3\left(\widetilde{\pi}_l\left(\widehat{\xi}\right)\right)} > \frac{f_1\left(\widetilde{\pi}_e\left(\widehat{\xi}\right)\right)}{f_3\left(\widetilde{\pi}_e\left(\widehat{\xi}\right)\right)}$ so $\widetilde{\pi}_l\left(\widehat{\xi}\right) > \widetilde{\pi}_e\left(\widehat{\xi}\right)$ and thus $(\pi^*, \xi^*) \gg \left(\widehat{\pi}, \widehat{\xi}\right)$. ■

References

Boot, A. 2000. "Relationship Banking: What Do We Know?" *Journal of Financial Intermediation*, 9(1): 7-25.

Cohen, J., K. Hachem, and G. Richardson. 2019. "Relationship Lending and the Great Depression." *Review of Economics and Statistics*, forthcoming.

Hachem, K. 2011. "Relationship Lending and the Transmission of Monetary Policy." *Journal of Monetary Economics*, 58(6-8): 590-600.

Lummer, S. and J. McConnell. 1989. "Further Evidence on the Bank Lending Process and the Capital Market Response to Bank Loan Agreements." *Journal of Financial Economics*, 25(1): 99-122.

Appendix C – Corrective Taxation

Proposition 8 suggests that constrained efficiency cannot be achieved by simply changing the level of the interbank rate R. The problem is that the interbank rate in the decentralized equilibrium was simultaneously too high and too low: it was too high from the perspective of achieving the constrained efficient ξ but too low from the perspective of achieving the constrained efficient π.

One way to address this involves changing the strength with which R affects π relative to ξ. Imagine a government that can costlessly observe the resource allocation decision of lenders and tax any resources devoted to matching. Starting from the decentralized pair (π^*, ξ^*), the direct effect of introducing such a tax is to decrease π, freeing up capital and pushing R down. As the price of capital falls, ξ decreases and π increases. However, π does not increase by as much as it would have absent the tax since taxation of matching activities makes such activities less attractive. Since unmatched lenders rely on matching activities to find potential borrowers, the tax also decreases the attractiveness of being unmatched, putting additional downward pressure on ξ.

The following proposition formalizes the above discussion:

Proposition C.1 *Consider a policy which taxes π at a per-unit rate τ then transfers all the tax revenues lump-sum to the interbank market. There is a $\tau > 0$ that implements the constrained efficient allocation $\left(\widehat{\pi}, \widehat{\xi}\right)$ as a decentralized equilibrium when $\alpha = 1$.*

Proof. The proposed tax changes the value of an unmatched lender from (10) to:

$$U = \beta U + p(\pi) \int_0^1 [1 - p(z - \pi)(1 - I(\omega))] [J(\omega) - R - \beta U] \psi(\omega) d\omega - \tau\pi$$

where $J(\omega)$ is still as per equation (5). Informed retention still follows a reservation strategy with:

$$y(\xi) = \frac{\widetilde{R}}{\mu} + \frac{\beta(1-\mu)}{\mu}(1-\beta) U$$

and the lender's first order condition for π simplifies to:

$$\frac{f_1(\pi)}{f_2(\pi)} \int_0^{\xi} [y(\omega) - y(\xi)] d\omega + \int_{\xi}^1 [y(\omega) - y(\xi)] d\omega - \frac{1 - \beta(1-\mu)}{\mu p'(\pi) \psi_H} \tau = 0 \tag{C.1}$$

The equilibrium now involves a pair (π^*, ξ^*) satisfying equations (C.1) and (A.20). The constrained efficient allocation $\left(\widehat{\pi}, \widehat{\xi}\right)$ still solves equations (A.16) and (A.20). Define:

$$\widehat{\tau} \equiv \frac{\mu}{1 - \beta(1-\mu)} \frac{(1-\mu) p'(\widehat{\pi})}{\mu + (1-\mu) p(\widehat{\pi})} \frac{p(\widehat{\pi}) p(z - \widehat{\pi})}{1 + \frac{(1-\mu) p(\widehat{\pi}) p(z - \widehat{\pi}) \widehat{\xi}}{\mu + (1-\mu) p(\widehat{\pi}) [1 - p(z - \widehat{\pi})]}} \int_{\widehat{\xi}}^1 \left[y(\omega) - y\left(\widehat{\xi}\right)\right] d\omega$$

If $\tau = \widehat{\tau}$, then $\left(\widehat{\pi}, \widehat{\xi}\right)$ satisfies equation (C.1), completing the proof. Note $\widehat{\tau} > 0$, meaning that the resources allocated to matching are taxed. ■

The lump-sum transfer of tax revenues ensures that capital market clearing is still given by equation (9), which is the equation on which (A.20) is based. Specifically, the government takes the tax out of the average profits of lenders. All else constant, this would reduce the amount of capital remitted to the interbank market by the lenders. The lump-sum transfer offsets this exactly to preserve (9).

The tax in Proposition C.1 was predicated on costless observation of π by the government. This may not be possible if π is literal effort, in which case the value of Proposition C.1 is simply pedagogical in that it reinforces the nature of the externalities. If, in contrast, the resources that lenders allocate between matching and screening take the form of physical labor rather than effort, then a tax on π may be possible.

Appendix D – Construction of Figure 1

Decentralized Equilibrium From the proof of Proposition 10, there is a decentralized equilibrium with $\alpha^* = 1$ if and only if $\mu \geq \mu_1$. Using $y(\omega) = \theta\omega$ and the expression for μ_1 in the aforementioned proof, we can write:

$$\mu_1 = \left[1 + \frac{(2-\theta\xi_1)(1-2\xi_1)}{(2-\theta)\, p\left(\frac{z}{2}\right)\xi_1}\right]^{-1} \tag{D.1}$$

where ξ_1 solves:

$$\frac{2(1-\theta\xi_1)}{(2-\theta\xi_1)(1-\xi_1)} = p\left(\frac{z}{2}\right) \tag{D.2}$$

Equation (D.2) is a quadratic in ξ_1 with one positive root:

$$\xi_1 = \frac{\sqrt{4\left[1-p\left(\frac{z}{2}\right)\right] + \left(\frac{2}{\theta}-1\right)^2 p^2\left(\frac{z}{2}\right)} - 2 + \left(1+\frac{2}{\theta}\right)p\left(\frac{z}{2}\right)}{2p\left(\frac{z}{2}\right)} \tag{D.3}$$

We can then get $\mu_1 \in (0,1)$ with:

$$2 > \theta > \underline{\theta} \equiv \frac{4\left[2-p\left(\frac{z}{2}\right)\right]}{4-p\left(\frac{z}{2}\right)} \tag{D.4}$$

Recall that the proof of Proposition 10 assumed $p\left(\frac{z}{2}\right) > \frac{1-\int_0^1 y(\omega)d\omega}{\int_0^{\xi_x}[1-y(\omega)]d\omega}$, where ξ_x was defined by $y(\xi_x) \equiv \int_0^1 y(\omega)\, d\omega$. Under $y(\omega) = \theta\omega$, this assumption simplifies to $p\left(\frac{z}{2}\right) > \frac{4(2-\theta)}{4-\theta}$ or, equivalently, $\theta > \underline{\theta}$. The condition $\theta > \underline{\theta}$ is necessary and sufficient for (D.3) to deliver $\xi_1 < \frac{1}{2}$ which, together with $\theta < 2$, implies $\mu_1 \in (0,1)$ in (D.1). Note that $\int_0^1 y(\omega)\, d\omega < 1$ in Assumption 3 is equivalent to $\theta < 2$ when $y(\omega) = \theta\omega$.

From the proof of Proposition 10, we know $\frac{d\pi^*}{d\mu} > 0$ when $\alpha^* = 1$. It is straightforward to also show $\frac{d\xi^*}{d\mu} > 0$ when $\alpha^* = 1$.

Consider now an equilibrium with $\alpha^* = 0$. Equations (A.26) and (A.27) reduce to $\pi^* = \frac{z}{2}$ and $\xi^* = \frac{2}{\theta} - 1$. Using (A.28), we then need to confirm $\frac{1}{\mu} \geq 1 + \frac{\theta(3\theta-4)}{(2-\theta)^2 p^2\left(\frac{z}{2}\right)}$. Let μ_0 be the μ at which this holds with equality:

$$\mu_0 \equiv \left[1 + \frac{\theta(3\theta-4)}{(2-\theta)^2 p^2\left(\frac{z}{2}\right)}\right]^{-1}$$

There is a decentralized equilibrium with $\alpha^* = 0$ if and only if $\mu \leq \mu_0$. Notice that $\theta > \frac{4}{3}$ ensures $\mu_0 \in (0,1)$. Also notice that $\theta > \frac{4}{3}$ is ensured by $\theta > \underline{\theta}$. We can now compare μ_0 and μ_1. The condition for $\mu_0 < \mu_1$ is:

$$\frac{3\theta-4}{(2-\theta)\, p\left(\frac{z}{2}\right)} \frac{\theta\xi_1}{2-\theta\xi_1} > 1 - 2\xi_1 \tag{D.5}$$

To simplify, rewrite (D.2) as:

$$\frac{\theta\xi_1}{2-\theta\xi_1} = 1-(1-\xi_1)\,p\left(\tfrac{z}{2}\right)$$

Substitute this into the left-hand side of (D.5), rearrange to isolate ξ_1, then substitute ξ_1 as per (D.3). After some algebra, we get that $\mu_0 < \mu_1$ just requires:

$$2\left[2-p\left(\frac{z}{2}\right)\right]\left(\frac{2}{\theta}-1\right)^2 < 1$$

This is true for any $\theta \in (\underline{\theta}, 2)$ so $\mu_0 < \mu_1$ follows.

Finally, consider an equilibrium with $\alpha^* \in (0,1)$. This requires $\gamma_0 = \gamma_1 = 0$ in (A.28). Combining equations (A.26), (A.27), and (A.28) then yields $\pi^* = \frac{z}{2}$ and:

$$\alpha^* = \frac{1}{1-p\left(\frac{z}{2}\right)}\left[p\left(\frac{z}{2}\right)\frac{\xi^{*2}}{1-2\xi^*} - \frac{\mu}{(1-\mu)\,p\left(\frac{z}{2}\right)}\right] \tag{D.6}$$

where ξ^* solves:

$$\frac{(2-\theta)(1-\xi^*)\,\xi^*}{(1-\theta\xi^*)(1-2\xi^*)} = \frac{2\mu}{(1-\mu)\,p^2\left(\frac{z}{2}\right)} \tag{D.7}$$

The decentralized equilibrium with $\alpha^* \in (0,1)$ prevails if and only if $\mu \in (\mu_0, \mu_1)$. Notice:

$$\frac{d\xi^*}{d\mu} = \frac{2\,(1-\theta\xi^*)^2\,(1-2\xi^*)^2}{(1-\mu)^2\,p^2\left(\frac{z}{2}\right)(2-\theta)\left[1-2\xi^*+(2-\theta)\,\xi^{*2}\right]} > 0$$

Going through the algebra, we can also get:

$$\frac{d\alpha^*}{d\mu} = \frac{\theta^2\,(1-2\xi^*)^2\left[\frac{1}{\theta}\left(1-\frac{2}{\theta}\right)+\frac{2}{\theta}\xi^*-\xi^{*2}\right]}{(1-\mu)^2\,p\left(\frac{z}{2}\right)\left[1-p\left(\frac{z}{2}\right)\right](2-\theta)\left[1-2\xi^*+(2-\theta)\,\xi^{*2}\right]}$$

Therefore, the condition for $\frac{d\alpha^*}{d\mu} > 0$ is:

$$\xi^{*2} - \frac{2}{\theta}\xi^* + \frac{1}{\theta}\left(\frac{2}{\theta}-1\right) < 0 \tag{D.8}$$

Expand (D.7) to isolate ξ^{*2} then substitute into (D.8) to rewrite (D.8) as:

$$-\left(2-\theta+\frac{2\theta\mu}{(1-\mu)\,p^2\left(\frac{z}{2}\right)}\right)\xi^* < \frac{3\theta-4}{2-\theta}\,\frac{2\mu}{(1-\mu)\,p\left(\frac{z}{2}\right)^2} - \frac{2-\theta}{\theta}$$

The left-hand side is negative while the right-hand side is increasing in μ. Therefore, the right-hand side being positive at $\mu = \mu_0$ will be sufficient for $\frac{d\alpha^*}{d\mu} > 0$. It is straightforward to show that this sufficient condition is true.

Constrained Efficient Allocation From the proof of Proposition 10, the planner chooses $\widehat{\alpha} = 1$ if and only if $\mu \geq \widehat{\mu}_1$. Using $y(\omega) = \theta\omega$ and the expression for $\widehat{\mu}_1$ in the aforementioned proof, we can write:

$$\widehat{\mu}_1 = \left[1 + \frac{\frac{\theta}{2-p\left(\frac{z}{2}\right)}\left(1 - \frac{4}{\theta} + \frac{2}{2-p\left(\frac{z}{2}\right)}\right)}{(2-\theta)\, p\left(\frac{z}{2}\right)} \right]^{-1}$$

Three comments are in order. First, the bounds on θ that ensure $\mu_1 \in (0,1)$ also ensure $\widehat{\mu}_1 \in (0,1)$. Second, it is straightforward to show $\frac{d\widehat{\xi}}{d\mu} > 0$ when $\widehat{\alpha} = 1$ (recall that $\frac{d\widehat{\pi}}{d\mu} > 0$ when $\widehat{\alpha} = 1$ was already shown in the proof of Proposition 10). Third, $\widehat{\mu}_1 > \mu_1$.

The proof of $\widehat{\mu}_1 > \mu_1$ proceeds by contradiction. Suppose $\widehat{\mu}_1 \leq \mu_1$. From the proof of Proposition 10, we know $\widehat{\pi} = \frac{z}{2}$ with $\widehat{\alpha} = 1$ at $\mu = \widehat{\mu}_1$ and $\pi^* = \frac{z}{2}$ with $\alpha^* = 1$ at $\mu = \mu_1$. We also know $\frac{d\widehat{\pi}}{d\mu} > 0$ whenever $\widehat{\alpha} = 1$. Therefore, $\widehat{\mu}_1 \leq \mu_1$ implies $\widehat{\pi} \geq \frac{z}{2} = \pi^*$ at $\mu = \mu_1$. However, Proposition 8 established $\widehat{\pi} < \pi^*$ for any $\mu \in (0,1)$ where both the decentralized equilibrium and the planner's solution have $\alpha = 1$. With $\widehat{\mu}_1 \leq \mu_1$, both have $\alpha = 1$ at $\mu = \mu_1$ so $\widehat{\pi} \geq \pi^*$ cannot be true. In other words, $\widehat{\mu}_1 \leq \mu_1$ leads to a contradiction so it must be the case that $\widehat{\mu}_1 > \mu_1$.

Consider now a constrained efficient allocation with $\widehat{\alpha} = 0$. Equations (A.26) and (A.30) reduce to $\widehat{\pi} = \frac{z}{2}$ and $\widehat{\xi} = \frac{2}{\theta} - 1$. Using (A.31), we must then confirm $\frac{1}{\mu} \geq 1 + \frac{3\theta - 4}{(2-\theta)p^2\left(\frac{z}{2}\right)}$. Let $\widehat{\mu}_0$ be the μ at which this holds with equality:

$$\widehat{\mu}_0 \equiv \left[1 + \frac{3\theta - 4}{(2-\theta)\, p^2\left(\frac{z}{2}\right)} \right]^{-1}$$

Notice that $\frac{4}{3} < \theta < 2$ ensures $\widehat{\mu}_0 \in (0,1)$. It is then easy to see $\widehat{\mu}_0 > \mu_0$. Also notice that $\widehat{\mu}_0 < \widehat{\mu}_1$ reduces to:

$$4\theta - 6 + (2-\theta)\, p\left(\tfrac{z}{2}\right) < \frac{3\theta - 4}{p\left(\frac{z}{2}\right)}$$

The left-hand side is increasing in $p\left(\frac{z}{2}\right)$. Evaluating it as $p\left(\frac{z}{2}\right) \to 1$ yields $3\theta - 4$ so the inequality is true for any $p\left(\frac{z}{2}\right) \in (0,1)$. We can thus conclude $\widehat{\mu}_0 < \widehat{\mu}_1$, where the planner chooses $\widehat{\alpha} = 0$ if and only if $\mu \leq \widehat{\mu}_0$.

Finally, consider a constrained efficient allocation with $\widehat{\alpha} \in (0,1)$. This requires $\gamma_0 = \gamma_1 = 0$ in (A.31). Combining equations (A.26), (A.30), and (A.31) then yields $\widehat{\pi} = \frac{z}{2}$ and:

$$\widehat{\alpha} = \frac{1}{1 - p\left(\frac{z}{2}\right)} \left[p\left(\frac{z}{2}\right) \frac{\widehat{\xi}}{1 - 2\widehat{\xi}} - \frac{\mu}{(1-\mu)\, p\left(\frac{z}{2}\right)} \right] \tag{D.9}$$

where $\widehat{\xi}$ solves:

$$\frac{2 - \theta}{\left(1 - 2\widehat{\xi}\right)\left(4 - \theta - 2\theta\widehat{\xi}\right)} = \frac{\mu}{(1-\mu)\, p^2\left(\frac{z}{2}\right)} \tag{D.10}$$

There are two possible solutions for $\widehat{\xi}$. However, we need $\widehat{\xi} < \frac{1}{2}$ otherwise $\widehat{\alpha}$ cannot be positive. The only valid solution is therefore:

$$\widehat{\xi} = \frac{1}{\theta} - \sqrt{\frac{2-\theta}{4\theta}\left[\frac{2-\theta}{\theta} + \left(\frac{1}{\mu} - 1\right) p^2 \left(\frac{z}{2}\right)\right]}$$

It is easy to see $\frac{d\widehat{\xi}}{d\mu} > 0$. We can also use (D.9) and (D.10) to show:

$$\frac{d\widehat{\alpha}}{d\mu} = \frac{1}{8(1-\mu)^2 p\left(\frac{z}{2}\right)\left[1 - p\left(\frac{z}{2}\right)\right]} \frac{\theta^2}{2-\theta} \frac{\left(1 - 2\widehat{\xi}\right)^2}{1 - \theta\widehat{\xi}} > 0$$

Note that the planner chooses $\widehat{\alpha} \in (0, 1)$ if and only if $\mu \in (\widehat{\mu}_0, \widehat{\mu}_1)$.

Lemma D.1 *If $\widehat{\mu}_0 < \mu_1$, then $\widehat{\alpha} < \alpha^*$ and $\widehat{\xi} < \xi^*$ for $\mu \in (\widehat{\mu}_0, \mu_1)$.*

Proof. Using (D.6) and (D.9), we see that $\widehat{\alpha} < \alpha^*$ amounts to $\frac{\xi^{*2}}{1-2\xi^*} > \frac{\widehat{\xi}}{1-2\widehat{\xi}}$. A necessary condition is $\xi^* > \widehat{\xi}$ so establishing $\widehat{\alpha} < \alpha^*$ will also establish $\xi^* > \widehat{\xi}$.

Recall $\widehat{\mu}_0 > \mu_0$ and $\widehat{\mu}_1 > \mu_1$. Also recall $\frac{d\alpha^*}{d\mu} > 0$ for $\mu \in (\mu_0, \mu_1)$. Therefore, $\alpha^* > 0 = \widehat{\alpha}$ at $\mu = \widehat{\mu}_0$ and $\alpha^* = 1 > \widehat{\alpha}$ at $\mu = \mu_1$.

The rest proceeds by contradiction. In particular, suppose there is a $\widetilde{\mu} \in (\widehat{\mu}_0, \mu_1)$ such that $\widehat{\alpha} > \alpha^*$. Then there is a $\widetilde{\mu}_x \in (\widehat{\mu}_0, \widetilde{\mu})$ such that $\widehat{\alpha} = \alpha^*$ and $\frac{d\widehat{\alpha}}{d\mu} > \frac{d\alpha^*}{d\mu}$. There must also be a $\widetilde{\mu}_y \in (\widetilde{\mu}, \mu_1)$ such that $\widehat{\alpha} = \alpha^*$ and $\frac{d\widehat{\alpha}}{d\mu} < \frac{d\alpha^*}{d\mu}$. Rewrite $\frac{d\widehat{\alpha}}{d\mu} < \frac{d\alpha^*}{d\mu}$ as:

$$\frac{1}{8}\frac{\left(1-2\widehat{\xi}\right)^2}{1-\theta\widehat{\xi}} < \frac{(1-2\xi^*)^2\left[\frac{1}{\theta}\left(1-\frac{2}{\theta}\right) + \frac{2}{\theta}\xi^* - \xi^{*2}\right]}{1 - 2\xi^* + (2-\theta)\xi^{*2}} \tag{D.11}$$

Now use $\frac{\xi^{*2}}{1-2\xi^*} = \frac{\widehat{\xi}}{1-2\widehat{\xi}}$ (which comes from $\widehat{\alpha} = \alpha^*$) to rewrite (D.11) in terms of only ξ^*:

$$T(\xi^*) \equiv 8\left(1 - 2\xi^* + 2\xi^{*2}\right)\left[\frac{1}{\theta}\left(1 - \frac{2}{\theta}\right) + \frac{2}{\theta}\xi^* - \xi^{*2}\right] - 1 > 0 \tag{D.12}$$

We can prove a contradiction here. The first step is to show $T\left(\frac{1}{2}\right) < 0$. The second step is to show $T'(\xi^*) > 0$ for any $\xi^* \in \left(0, \frac{1}{2}\right)$. We can restrict attention to $\xi^* < \frac{1}{2}$ since this is necessary for $\alpha^* > 0$. Taking first derivatives:

$$T'(\xi^*) = \left(\frac{4}{\theta}\right)^2 \left[2 - \left(4 + 2\theta + \theta^2\right)\xi^* + 3\theta(2+\theta)\xi^{*2} - 4\theta^2\xi^{*3}\right]$$

Now take second and third derivatives to get:

$$T''(\xi^*) = -\left(\frac{4}{\theta}\right)^2 \left[4 + 2\theta + \theta^2 - 6\theta(2+\theta)\xi^* + 12\theta^2\xi^{*2}\right]$$

and:

$$T'''(\xi^*) = \frac{96}{\theta}\left[2(1-\theta\xi^*) + \theta(1-2\xi^*)\right] > 0$$

respectively. We can then evaluate:

$$T''\left(\frac{1}{2}\right) = -\left(\frac{4}{\theta}(2-\theta)\right)^2 < 0$$

and:

$$T'\left(\frac{1}{2}\right) = \frac{4}{\theta}(2-\theta) > 0$$

and:

$$T\left(\frac{1}{2}\right) = -2\left(\frac{2}{\theta}-1\right)^2 < 0$$

With $T'''(\xi^*) > 0$ and $T''\left(\frac{1}{2}\right) < 0$, we can conclude that $T'(\xi^*)$ is decreasing in ξ^*. With $T'\left(\frac{1}{2}\right) > 0$, we can then conclude that $T(\xi^*)$ is increasing in ξ^*. Together with $T\left(\frac{1}{2}\right) < 0$, this rules out the existence of a $\xi^* \in \left(0, \frac{1}{2}\right)$ satisfying $T(\xi^*) > 0$ so (D.12) cannot hold and, hence, there cannot exist a $\widetilde{\mu} \in (\widehat{\mu}_0, \mu_1)$ such that $\widehat{\alpha} > \alpha^*$. □

The left panel of Figure 1 is drawn for $\widehat{\mu}_0 < \mu_1$. If instead $\widehat{\mu}_0 > \mu_1$, then the decentralized equilibrium reaches $\alpha^* = 1$ while the planner is still at $\widehat{\alpha} = 0$. This is illustrated in the right panel of Figure 1.

The last step is to reduce $\widehat{\mu}_0 < \mu_1$ to a condition on parameters. Using the expressions for $\widehat{\mu}_0$ and μ_1 derived above, $\widehat{\mu}_0 < \mu_1$ is equivalent to:

$$\theta p\left(\frac{z}{2}\right)\xi_1^2 - \left[5\theta - 4 + 2p\left(\frac{z}{2}\right)\right]\xi_1 + 2 < 0 \tag{D.13}$$

where ξ_1 solves (D.2). Note that we can expand (D.2) to get:

$$\theta p\left(\frac{z}{2}\right)\xi_1^2 - \left[2p\left(\frac{z}{2}\right) + \theta p\left(\frac{z}{2}\right) - 2\theta\right]\xi_1 - 2\left[1 - p\left(\frac{z}{2}\right)\right] = 0$$

which helps simplify (D.13) to:

$$\xi_1 > \frac{2\left[2 - p\left(\frac{z}{2}\right)\right]}{7\theta - \theta p\left(\frac{z}{2}\right) - 4} \tag{D.14}$$

Now use equation (D.3) to substitute out ξ_1. After some algebra, (D.14) can be expressed as $\widetilde{T}\left(p\left(\frac{z}{2}\right)|\theta\right) > 0$, where:

$$\widetilde{T}\left(p\left(\frac{z}{2}\right)|\theta\right) \equiv \left(\theta^2 - \frac{7\theta}{2} + 2\right)p^2\left(\frac{z}{2}\right) - \left(\frac{\theta^3}{4} + \frac{31\theta^2}{4} - 17\theta + 8\right)p\left(\frac{z}{2}\right) + \frac{21\theta^2}{4} - 10\theta + 4$$

It is easy to show $\widetilde{T}(0|\theta) > 0$, $\widetilde{T}(1|\theta) < 0$, and $\widetilde{T}''(\cdot|\theta) < 0$ for any $\theta \in \left(\frac{4}{3}, 2\right)$. This implies existence of a unique $\overline{p}(\theta) \in (0,1)$ such that $\widetilde{T}(\overline{p}(\theta)|\theta) = 0$ and $\widetilde{T}\left(p\left(\frac{z}{2}\right)|\theta\right) > 0$ if and only if $p\left(\frac{z}{2}\right) < \overline{p}(\theta)$. Accordingly, $\widehat{\mu}_0 < \mu_1$ if $p\left(\frac{z}{2}\right) < \overline{p}(\theta)$ while $\widehat{\mu}_0 > \mu_1$ if $p\left(\frac{z}{2}\right) > \overline{p}(\theta)$.

Recall from (D.4) that the analysis imposes $\theta \in (\underline{\theta}, 2) \subset \left(\frac{4}{3}, 2\right)$, where $\theta > \underline{\theta}$ is equivalent to $p\left(\frac{z}{2}\right) > \frac{4(2-\theta)}{4-\theta}$. Therefore, for the left panel of Figure 1 to be relevant, we need $\frac{4(2-\theta)}{4-\theta} < \overline{p}(\theta)$. Going through the algebra, $\overline{p}(\theta)$ is given by:

$$\overline{\rho}(\theta) = \frac{\frac{\theta^3}{4} + \frac{31\theta^2}{4} - 17\theta + 8 - \sqrt{\left(\frac{\theta^3}{4} + \frac{31\theta^2}{4} - 17\theta + 8\right)^2 - 4\left(\theta^2 - \frac{7\theta}{2} + 2\right)\left(\frac{21\theta^2}{4} - 10\theta + 4\right)}}{2\theta^2 - 7\theta + 4}$$

and showing $\frac{4(2-\theta)}{4-\theta} < \overline{\rho}(\theta)$ amounts to showing:

$$8\theta^7 + 2\theta^6 - 681\theta^5 + 3732\theta^4 - 8320\theta^3 + 9088\theta^2 - 4864\theta + 1024 > 0$$

This inequality is satisfied by any $\theta \in (\theta_0, 2)$, where $\theta_0 \approx 1.6274$.

Define $z_a(\theta)$ and $z_b(\theta)$ such that $p\left(\frac{z_a(\theta)}{2}\right) \equiv \frac{4(2-\theta)}{4-\theta}$ and $p\left(\frac{z_b(\theta)}{2}\right) \equiv \overline{\rho}(\theta)$. If $\theta \in (\theta_0, 2)$, then $z_a(\theta) < z_b(\theta)$. The left panel in Figure 1 applies for any $z \in (z_a(\theta), z_b(\theta))$ while the right panel applies for any $z > z_b(\theta)$. If instead $\theta \in \left(\frac{4}{3}, \theta_0\right)$, then $z_a(\theta) > z_b(\theta)$. The left panel in Figure 1 does not apply while the right panel applies for any $z > z_a(\theta)$.

Appendix E – Baseline with Non-Linear Alternative

Return to the baseline model of Sections 2 and 3. Normalizing the aggregate stock of capital in the economy to one, the total amount of capital invested in the simple technology is:

$$K_a \equiv 1 - \int_0^1 n(\omega)\, d\omega$$

where $\int_0^1 n(\omega)\, d\omega$ with $n(\cdot)$ as per (2) represents the total amount of capital in intermediated projects. Total output from the simple technology is then $g(K_a)$, where $g(0) = 0$ and $g'(\cdot) > 0$. The analysis in Sections 2 and 3 assumed $g(\cdot)$ linear, that is, $g(K_a) = g'K_a$ for some constant $g' > 0$. This appendix considers what happens when $g(\cdot)$ is non-linear.

The welfare function that the planner maximizes is now:

$$\mathcal{W} = \frac{1}{1-\beta}\left[g(K_a) + \mu \int_0^1 y(\omega)\, n(\omega)\, d\omega\right]$$

Notice that this delivers the welfare function in (7) if $g(K_a) = g'K_a$. The planner's first order condition for $I(\cdot)$ still delivers a reservation strategy. Specifically, he sets $I(\omega) = 0$ for $\omega < \xi$ and $I(\omega) = 1$ for $\omega \geq \xi$, where ξ is implicitly defined by:

$$\mu y(\xi) = g'(K_a) \tag{E.1}$$

This is similar to equation (A.12) in the proof of Proposition 3, except that $g'(K_a)$ is no longer a constant. The planner's first order conditions for π and α then simplify to (A.30) and (A.31) respectively.

Lenders in the decentralized economy take as given the marginal return $g'(K_a)$. Therefore, the problem of a decentralized lender is still as in Lemma 2.

E.1 Full Retention of Uninformed Matches

First consider $\alpha = 1$ for both the planner and the decentralized lenders. The relevant comparison is to Section 3.1.

With $\alpha = 1$, the planner's first order condition for π reduces to equation (A.16). The decentralized solution is characterized by equations (A.13) and (A.15), where g' in (A.13) is evaluated at $g'(K_a)$. On a plot with ξ on the horizontal axis and π on the vertical axis, we know from the proof of Proposition 4 that (A.15) maps a downward-sloping curve that lies above the curve mapped out by (A.16). Accordingly, it only remains to determine the relative positions of the curves mapped out by (A.13) and (E.1).

Using $n(\cdot)$ as per (2) with $\alpha = 1$, we can write:

$$K_a = 1 - \frac{p(\pi)}{\mu + (1-\mu)\, p(\pi)}\left[1 - \frac{\mu p(z-\pi)\,\xi}{\mu + (1-\mu)\, p(\pi)\left[1 - p(z-\pi)\right]}\right]$$

It is easy to see that K_a is decreasing in π and increasing in ξ. Intuitively, more capital is available for the simple technology when unmatched lenders are less keen on matching with firms and/or informed lenders are more selective in the firms they retain.

Suppose the simple technology exhibits diminishing marginal returns, $g''(\cdot) < 0$. This implies $\frac{\partial g'}{\partial \xi} < 0$ and $\frac{\partial g'}{\partial \pi} > 0$. It then follows immediately from $y'(\cdot) > 0$ that (E.1) maps an upward-sloping curve when graphed with ξ on the horizontal axis and π on the vertical. Notice that (A.13) collapses to (E.1) if $\beta = 0$, in which case (A.15) above (A.16) on this graph implies that both π and ξ are too high in the decentralized equilibrium relative to the constrained efficient allocation. If instead $\beta > 0$, then $g'(K_a)$ must be lower in (A.13) than in (E.1) for the same value of ξ. Accordingly, π must be lower in (A.13) than in (E.1) for the same value of ξ, implying that (E.1) lies above the curve mapped out by (A.13). This means that the decentralized ξ is inefficiently high for any $\beta > 0$, while a continuity argument establishes that the decentralized π is inefficiently high for any β below some positive threshold. The only difference relative to Propositions 3 and 4 is that the decentralized ξ is now inefficiently high even at $\beta = 0$; all other results are qualitatively the same.

E.2 Partial Retention of Uninformed Matches

Now consider parameters where both the planner and the decentralized lenders choose $\alpha < 1$. The relevant comparison is to Section 3.2.

With $\alpha < 1$, the planner's first order conditions reduce to $\pi = \frac{z}{2}$ and (A.17) as in the proof of Proposition 5, along with (E.1) evaluated at:

$$K_a = 1 - \frac{p\left(\frac{z}{2}\right)\left[1-p\left(\frac{z}{2}\right)\right]\alpha\xi}{\mu + (1-\mu)\,p\left(\frac{z}{2}\right)\left[1-p\left(\frac{z}{2}\right)\right]\alpha} - \frac{p\left(\frac{z}{2}\right)\left[\alpha + (1-\alpha)\,p\left(\frac{z}{2}\right)\right](1-\xi)}{\mu + (1-\mu)\,p\left(\frac{z}{2}\right)\left[\alpha + (1-\alpha)\,p\left(\frac{z}{2}\right)\right]} \tag{E.2}$$

The decentralized solution is characterized by $\pi = \frac{z}{2}$, (A.18), and (A.19) as in the proof of Proposition 5, where g' in (A.19) is evaluated at $g'(K_a)$.

If $\beta = 0$, then (A.19) simplifies to (E.1). This implied $\xi^* = \widehat{\xi}$ in Proposition 5 since g' was a constant. Comparison of (A.17) and (A.18) then implied $\alpha^* < \widehat{\alpha}$. Now that we are considering a simple technology with decreasing returns to scale, g' depends on ξ and α so we can no longer follow the same reasoning. To this point, let $\alpha_c(\xi)$ denote the function implicitly defined by (E.1) with K_a as per (E.2). Differentiating yields:

$$\alpha_c'(\xi) = \frac{\frac{p^2\left(\frac{z}{2}\right)}{\left[\mu+(1-\mu)p\left(\frac{z}{2}\right)\left[1-p\left(\frac{z}{2}\right)\right]\alpha\right]\left[\mu+(1-\mu)p\left(\frac{z}{2}\right)\left[\alpha+(1-\alpha)p\left(\frac{z}{2}\right)\right]\right]} - \frac{y'(\xi)}{g''(K_a)}}{\frac{p\left(\frac{z}{2}\right)\left[1-p\left(\frac{z}{2}\right)\right]\xi}{\left[\mu+(1-\mu)p\left(\frac{z}{2}\right)\left[1-p\left(\frac{z}{2}\right)\right]\alpha\right]^2} + \frac{p\left(\frac{z}{2}\right)\left[1-p\left(\frac{z}{2}\right)\right](1-\xi)}{\left[\mu+(1-\mu)p\left(\frac{z}{2}\right)\left[\alpha+(1-\alpha)p\left(\frac{z}{2}\right)\right]\right]^2}} > 0$$

so, at $\beta = 0$, we have $\alpha^* < \widehat{\alpha}$ if and only if $\xi^* < \widehat{\xi}$.

Equations (A.17) and (A.18) implicitly define functions that I will denote by $\alpha_p(\xi)$ and $\alpha_d(\xi)$ respectively. The constrained efficient allocation solves $\widehat{\alpha} = \alpha_p\left(\widehat{\xi}\right) = \alpha_c\left(\widehat{\xi}\right)$ while the decentralized equilibrium solves $\alpha^* = \alpha_d(\xi^*) = \alpha_c(\xi^*)$ when $\beta = 0$. On a plot with ξ on the horizontal axis and α on the vertical axis, we know from the proof of Proposition 5 that (A.17) maps an upward-sloping curve that lies above the upward-sloping curve mapped out by (A.18). Therefore, showing that $\alpha_c(\xi)$ is less steep than $\alpha_d(\xi)$ at any point where these two functions intersect will be sufficient to show $\alpha^* > \widehat{\alpha}$ and $\xi^* > \widehat{\xi}$ when $\beta = 0$. Note that this would constitute a reversal of the result on α in Proposition 5.

For $\alpha_c'(\xi) < \alpha_d'(\xi)$ when $\alpha_c(\xi) = \alpha_d(\xi)$, we need:

$$(1-\mu)^2 p^2\left(\frac{z}{2}\right)\left[\frac{p^2\left(\frac{z}{2}\right)}{\left[\mu+(1-\mu)\,p\left(\frac{z}{2}\right)\left[1-p\left(\frac{z}{2}\right)\right]\alpha\right]^2}\frac{\int_0^\xi\left[y(\xi)-y(\omega)\right]d\omega}{\int_\xi^1\left[y(\omega)-y(\xi)\right]d\omega}-\frac{y'(\xi)}{g''(K_a)}\right]$$
$$<\frac{y'(\xi)}{\int_0^\xi\left[y(\xi)-y(\omega)\right]d\omega}\left(1-\xi+\frac{\int_\xi^1\left[y(\omega)-y(\xi)\right]d\omega}{\int_0^\xi\left[y(\xi)-y(\omega)\right]d\omega}\xi\right)\left[\xi+\left(\frac{\int_0^\xi\left[y(\xi)-y(\omega)\right]d\omega}{\int_\xi^1\left[y(\omega)-y(\xi)\right]d\omega}\right)^2(1-\xi)\right]$$

To fix ideas, consider $y(\omega)=\theta\omega$ as in the construction of Figure 1. The condition for $\alpha_c'(\xi)<\alpha_d'(\xi)$ when $\alpha_c(\xi)=\alpha_d(\xi)$ simplifies to:

$$-g''\left(\frac{\mu}{(1-\mu)^2 p^2\left(\frac{z}{2}\right)}\frac{1-2\xi}{\xi(1-\xi)}-\frac{\mu}{1-\mu}\right)>\theta(1-\mu)^2 p^2\left(\frac{z}{2}\right)\frac{\xi^2(1-\xi)^2}{1-2\xi(1-\xi)}\tag{E.3}$$

where ξ solves:

$$\mu\theta\xi=g'\left(\frac{\mu}{(1-\mu)^2 p^2\left(\frac{z}{2}\right)}\frac{1-2\xi}{\xi(1-\xi)}-\frac{\mu}{1-\mu}\right)$$

and α is then given by:

$$\alpha=\frac{1}{p\left(\frac{z}{2}\right)\left[1-p\left(\frac{z}{2}\right)\right]}\left[p^2\left(\frac{z}{2}\right)\frac{\xi^2}{1-2\xi}-\frac{\mu}{1-\mu}\right]\tag{E.4}$$

Notice from (E.4) that $\xi\in\left(0,\frac{1}{2}\right)$ is necessary for α to be well-defined. We can also show:

$$\frac{\partial}{\partial\xi}\left(\frac{\xi^2(1-\xi)^2}{1-2\xi(1-\xi)}\right)=\frac{2\xi(1-\xi)(1-2\xi)}{1-2\xi(1-\xi)}\left(1+\frac{\xi(1-\xi)}{1-2\xi(1-\xi)}\right)>0$$

for $\xi\in\left(0,\frac{1}{2}\right)$. Therefore, a sufficient condition for (E.3) can be found by evaluating the right-hand side of (E.3) at $\xi=\frac{1}{2}$. Defining:

$$x\equiv\frac{\mu}{(1-\mu)^2 p^2\left(\frac{z}{2}\right)}\frac{1-2\xi}{\xi(1-\xi)}-\frac{\mu}{1-\mu}\equiv\Delta(\xi)$$

we can rewrite this sufficient condition more compactly as:

$$g''(x^*)<-\frac{\theta}{8}(1-\mu)^2 p^2\left(\frac{z}{2}\right)$$

where x^* solves:

$$g'(x^*)=\mu\theta\Delta^{-1}(x^*)$$

This is just a statement about $g(\cdot)$ being sufficiently concave. In other words, for parameters where both the planner and the decentralized lenders choose $\alpha<1$, the baseline model delivers $\alpha^*>\widehat{\alpha}$ at $\beta=0$ if the simple technology is assumed to exhibit sufficiently strong decreasing returns to scale.

Appendix F – Analytical Supplement to Figure 2

Impose $\alpha = 1$ for both the planner and the decentralized lenders (recall that Figure 2 is plotted for values of μ where this is indeed optimal). Use the functional forms in the main text to write the aggregate capital condition (A.26) as:

$$\frac{\exp(\upsilon z) - \exp(\upsilon \pi)}{\exp(\upsilon \pi) - 1 + \mu} = \frac{2 - \theta}{\mu [\theta(1 + \xi) - 2](1 - \xi)} \tag{F.1}$$

Also use (F.1) along with $\overline{L} = 1$ and the functional forms to write the aggregate labor condition (A.34) in terms of only z and ξ:

$$z = \frac{\exp(\upsilon z) - 1 + \mu}{\mu \exp(\upsilon z) + (1 - \mu)(2 - \theta)\left[\frac{1}{\theta(1+\xi)-2} + \frac{\exp(\upsilon z)}{2-\theta\xi}\right]} \equiv \Upsilon(z, \xi) \tag{F.2}$$

Next, use (F.1) and the functional forms to write the planner's remaining equation, (A.30), as:

$$\exp(\upsilon \widehat{z}) = \left(\frac{2 - \theta\widehat{\xi}}{\theta\left(1 + \widehat{\xi}\right) - 2}\right)^2 \tag{F.3}$$

Similarly, use (F.1) and the functional forms to write the decentralized lenders' remaining equation, (A.27), as:

$$\exp(\upsilon z^*) = \left(\frac{2 - \theta\xi^*}{\theta(1 + \xi^*) - 2}\right)^2 + \frac{2(1 - \mu)(2 - \theta)(1 - \theta\xi^*)}{\mu[\theta(1 + \xi^*) - 2]^2(1 - \xi^*)\xi^*} \tag{F.4}$$

Return now to (F.2). Equation (F.2) defines an implicit function $z(\xi)$ which is common to both the decentralized and planner solutions. Taking a second order Taylor expansion around the planner's solution, we can write:

$$z(\xi) \approx z\left(\widehat{\xi}\right) + z'\left(\widehat{\xi}\right)\left(\xi - \widehat{\xi}\right) + \frac{1}{2}z''\left(\widehat{\xi}\right)\left(\xi - \widehat{\xi}\right)^2$$

where:

$$z'(\xi) = \frac{\Upsilon'_\xi}{1 - \Upsilon'_z}$$

and:

$$z''(\xi) = \frac{\Upsilon''_{\xi\xi}}{1 - \Upsilon'_z} + \frac{2\Upsilon''_{z\xi} + \Upsilon''_{zz} z'(\xi)}{1 - \Upsilon'_z} z'(\xi)$$

Restrict $\theta < 2$ as in Appendix D. With $\Upsilon(\cdot)$ as defined in (F.2), we can show:

$$\Upsilon'_\xi \overset{sign}{=} \left(\frac{2 - \theta\xi}{\theta(1 + \xi) - 2}\right)^2 - \exp(\upsilon z)$$

Equation (F.3) then implies $\Upsilon'_{\xi} = 0$ at the planner's solution, reducing the Taylor approximation to:

$$z^* - \widehat{z} \approx \frac{1}{2} z'' \left(\widehat{\xi}\right) \left(\xi^* - \widehat{\xi}\right)^2 \tag{F.5}$$

where $z^* \equiv z(\xi^*)$ and $\widehat{z} \equiv z\left(\widehat{\xi}\right)$.

Going through the relevant derivatives, we get:

$$z''\left(\widehat{\xi}\right) \equiv -\frac{2\frac{(1-\mu)(2-\theta)}{\theta}\left[1+\exp\left(\frac{\upsilon\widehat{z}}{2}\right)\right]^4 \exp\left(-\frac{\upsilon\widehat{z}}{2}\right)\widehat{z}^2}{\exp(\upsilon\widehat{z}) - (1-\mu)\left[1+\upsilon\widehat{z}+\upsilon\frac{2-\theta}{\theta}\left[1+\exp\left(\frac{\upsilon\widehat{z}}{2}\right)\right]\widehat{z}^2\right]}$$

where $\widehat{z}$ solves:

$$\widehat{z} = \frac{\exp(\upsilon\widehat{z}) - 1 + \mu}{\mu\exp(\upsilon\widehat{z}) + \frac{(1-\mu)(2-\theta)}{\theta}\left[1+\exp\left(\frac{\upsilon\widehat{z}}{2}\right)\right]^2} \tag{F.6}$$

The characterization of $\widehat{z}$ in (F.6) comes from rearranging (F.3) to isolate $\widehat{\xi}$ then substituting into (F.2) and simplifying. It is straightforward to show that (F.6) implies a strictly positive denominator in the expression for $z''\left(\widehat{\xi}\right)$; a sufficient condition is just:

$$\exp(\upsilon\widehat{z})\left[1+\exp\left(\frac{\upsilon\widehat{z}}{2}\right) - \upsilon\widehat{z}\right] > 1 + (1+\upsilon\widehat{z})\exp\left(\frac{\upsilon\widehat{z}}{2}\right)$$

which is true by properties of the exponential function for any $\upsilon\widehat{z} > 0$. With $z''\left(\widehat{\xi}\right)$ well behaved, the difference between z^* and $\widehat{z}$ in (F.5) is small.

A corollary of z approximately efficient is that all of the following are also approximately efficient: the total mass of available matches A, the total amount of credit K, and total welfare $\mathcal{W}$. The result on A comes from (14). The result on $K \equiv \int_0^1 n(\omega)\, d\omega$ then comes from the definition of A in (4). The result on $\mathcal{W}$ comes from (9) and the result on K. However, it is still the case that uninformed credit is too high and informed credit is too low. From the proof of Proposition 9, uninformed credit is $K_N = p(\pi)\left[1 - p(z-\pi)\right]\frac{A}{\mu}$. We know from Proposition 8 that the decentralized π is inefficiently high when the planner and the decentralized economy are assumed to have the same z. The same ideas apply here with z approximately efficient so, with A also approximately efficient, K_N is inefficiently high. Informed credit is just $K_I = K - K_N$ so, with K approximately efficient and K_N inefficiently high, K_I is inefficiently low.

Appendix G – Elastic Labor Supply

Suppose there are workers who solve a simple utility maximization problem to determine labor supply. By supplying L units of labor, a worker earns WL at disutility $\frac{1}{2\ell}L^2$. This implies the labor supply function $L^* = \ell W$. Labor market clearing then changes from (14) to:

$$Az = \ell W \tag{G.1}$$

The rest of the equations for the decentralized equilibrium follow the proof of Proposition 11. Specifically, capital market clearing is still given by (A.26) and the decentralized first order conditions still deliver (A.27), (A.28), (A.35), and (A.36), where (A.28) holds with complementary slackness.[20]

G.1 Constrained Inefficiency

The planner's problem is summarized by the following Lagrangian:

$$\mathcal{L} = \mu \int_0^1 y(\omega)\, n(\omega)\, d\omega + \lambda_1 \int_0^1 [y(\omega) - 1]\, n(\omega)\, d\omega + \gamma_0 \alpha + \gamma_1 (1-\alpha) - \frac{1}{2\ell}L^2 + \lambda_2 [L - Az]$$

with $n(\cdot)$ as defined in (2) and A as defined in (4). This is similar to the Lagrangian in the proof of Proposition 10, but also taking into account the disutility of labor, $\frac{1}{2\ell}L^2$, and the aggregate feasibility constraint for labor, $Az = L$, which has Lagrange multiplier $\lambda_2 \geq 0$.[21]

The planner's first order condition for L delivers $\widehat{L} = \ell\lambda_2$ so the aggregate feasibility constraint for labor changes from (14) to:

$$Az = \ell\lambda_2 \tag{G.2}$$

The planner's first order condition for $I(\omega)$ yields a reservation strategy with threshold ξ defined by:

$$y(\xi) = \frac{\lambda_1 - (1-\mu)\lambda_2 z}{\mu + \lambda_1} \tag{G.3}$$

The aggregate feasibility constraint for capital is still given by (A.26) and the planner's first order conditions for π and α still deliver (A.30) and (A.31), where (A.31) holds with complementary slackness. Finally, the planner's first order condition for z is:

[20] Any equations referenced in this appendix that depend on $f_1(\cdot)$ and/or $\widetilde{f}_2(\cdot)$ as defined in the proofs of Propositions 4 and 10 respectively should be understood to depend on $f_1(\pi, z)$ and/or $\widetilde{f}_2(\pi, \alpha, z)$.

[21] The first term in the Lagrangian here is scaled by μ since output is produced at the end of a match while disutility of labor is incurred every period. In the Lagrangian in the proof of Proposition 10, all terms other than the first term were multiplied by Lagrange multipliers so it did not matter whether the first term was also scaled by μ.

$$\frac{\lambda_2}{\mu+\lambda_1} = \frac{p(\pi)\, p'(z-\pi)\left[\alpha \int_0^{\xi} [y(\xi)-y(\omega)]\, d\omega + (1-\alpha)\, \widetilde{f}_2^2(\pi,\alpha,z) \int_{\xi}^1 [y(\omega)-y(\xi)]\, d\omega\right]}{[\mu+(1-\mu)\, p(\pi)\, [1-p(z-\pi)]\, \alpha]\left[\xi + \widetilde{f}_2(\pi,\alpha,z)(1-\xi)\right]} \tag{G.4}$$

which completes the characterization.

Consider $y(\omega) = 1.75\omega$ and $p(x) = 1 - \exp(-2.5x)$ as in Figure 2. For the remaining parameters, I set $\beta = 0.95$ and consider different values of ℓ. For each ℓ, Figure G.1 compares the decentralized equilibrium with the constrained efficient allocation for all values of μ where $\alpha^* = \widehat{\alpha} = 1$ is optimal and the decentralized equilibrium has $R > 0$ with $\beta = 0.95$.[22] A red marker at the coordinates (μ, ℓ) means that the equilibrium value of the indicated variable is inefficiently high at this combination of μ and ℓ. A blue marker means that the equilibrium value is inefficiently low. The decentralized choice of z tends to be too low relative to the constrained efficient allocation, with $z - \pi$ too low and π also typically too low. The decentralized choice of ξ is too high, total credit K is too low, and uninformed credit K_N tends to be too high. The corollary that allowed us to conclude approximately similar welfare for the decentralized and planning solutions in the case of inelastic labor supply (see the discussion of Figure 2 in Section 5) no longer applies. When ℓ is very low, the decentralized K is so low relative to the planner's K that there is a small region of the parameter space with K_N also too low. However, even in that region, I verify that the ratio of K_N to K is too high.

G.2 Intuition and Discussion

Notice from Figure G.1 that the decentralized equilibrium differs from the planner's solution even at $\mu = 1$. This was not the case with fixed labor supply in the proof of Proposition 11. Labor supply in the decentralized equilibrium is now $L = \ell W$, with W still given by equation (A.36). Substituting $\mu = 1$ into (A.36) delivers:

$$W = p(\pi)\, p'(z-\pi)\left[\alpha \int_0^{\xi} [y(\xi)-y(\omega)]\, d\omega + (1-\alpha) \int_0^1 [y(\omega)-y(\xi)]\, d\omega\right] \equiv F(z,\pi,\xi,\alpha)$$

The decentralized labor supply is therefore:

$$L^* = \ell F(z,\pi,\xi,\alpha) \tag{G.5}$$

Labor supply in the planner's solution is $L = \ell\lambda_2$. Substituting $\mu = 1$ into (G.4) delivers:

[22] With labor supply fixed at $\overline{L}$, the decentralized equilibrium in the proof of Proposition 11 was characterized by four equations that pinned down π, ξ, α, and z independently of β and two equations that pinned down R and W as functions of β. It is easy to see from (A.35) that $\beta = 0$ always supports $R > 0$. Figure 2 could thus be generated without specifying β then finding the highest β consistent with $R > 0$ for all the plotted values of μ. Given the other parameters, any $\beta \leq 0.58$ would support $R > 0$ for all plotted μ (i.e., $\mu \geq 0.17$) in Figure 2. As β increases, the lowest μ consistent with $R > 0$ increases so $\beta = 0.95$ would support $R > 0$ for $\mu \geq 0.39$ in Figure 2. Now, however, W enters the labor market clearing condition because the supply of labor is elastic to the wage. Therefore, the decentralized π, ξ, α, and z cannot be determined without first specifying β and $R > 0$ must then be verified without β as a "free" parameter.

$$\frac{\lambda_2}{1+\lambda_1} = F(z,\pi,\xi,\alpha)$$

where we recall that λ_1 is the Lagrange multiplier on the aggregate feasibility constraint for capital. The constrained efficient labor supply is therefore:

$$\widehat{L} = (1+\lambda_1)\,\ell F(z,\pi,\xi,\alpha) \tag{G.6}$$

The expressions for L^* and $\widehat{L}$ in equations (G.5) and (G.6) are the same if and only if $\lambda_1 = 0$. With $\lambda_1 > 0$, the aggregate feasibility constraint on capital is binding and, at the wage that prevails in the decentralized equilibrium, the planner would make workers supply more labor than they actually do because intermediation resources relax the capital market constraint. Decentralized workers fail to internalize this effect when choosing how much labor to supply, regardless of the value of μ.

This intuition for the inefficiency in z does not depend on workers being separate agents from the intermediaries. To see this explicitly, eliminate the labor market and suppose z is effort exerted by each unmatched lender at some disutility $c(z)$. The value of an unmatched lender changes from (13) to:

$$U = -c(z) + \beta U + p(\pi)\int_0^1 \left[\left[1-p(z-\pi)\right]\alpha + p(z-\pi)\,I(\omega)\right]\left[J(\omega) - R - \beta U\right]\psi(\omega)\,d\omega$$

and the planner's Lagrangian is:

$$\mathcal{L} = \mu\int_0^1 y(\omega)\,n(\omega)\,d\omega + \lambda_1\int_0^1 \left[y(\omega)-1\right]n(\omega)\,d\omega + \gamma_0\alpha + \gamma_1(1-\alpha) - Ac(z)$$

The decentralized equilibrium still involves (A.26), (A.27), (A.28), and (A.35) but now the combination of (G.1) and (A.36) is replaced by:

$$c'(z) = \frac{\mu p(\pi)\,p'(z-\pi)}{1-\beta(1-\mu)}\left[\alpha\int_0^{\xi}\left[y(\xi)-y(\omega)\right]\psi(\omega)\,d\omega + (1-\alpha)\int_{\xi}^1\left[y(\omega)-y(\xi)\right]\psi(\omega)\,d\omega\right] \tag{G.7}$$

Similarly, the constrained efficient allocation still involves (A.26), (A.30), and (A.31) but now the combination of (G.2) and (G.4) is replaced by:

$$\frac{c'(z)}{\mu+\lambda_1} = \frac{p(\pi)\,p'(z-\pi)\left[\alpha\int_0^{\xi}\left[y(\xi)-y(\omega)\right]d\omega + (1-\alpha)\,\widetilde{f}_2^2(\pi,\alpha,z)\int_{\xi}^1\left[y(\omega)-y(\xi)\right]d\omega\right]}{\left[\mu+(1-\mu)\,p(\pi)\left[1-p(z-\pi)\right]\alpha\right]\left[\xi+\widetilde{f}_2(\pi,\alpha,z)(1-\xi)\right]} \tag{G.8}$$

with:

$$y(\xi) = \frac{\lambda_1 - (1-\mu)\,c(z)}{\mu+\lambda_1} \tag{G.9}$$

instead of (G.3). Consider $c(z) = \frac{1}{2\ell}z^2$. At $\mu = 1$, equation (G.7) replicates the combination of (G.1) and (A.36) while equations (G.8) and (G.9) replicate the combination of (G.2), (G.3), and (G.4). Intuitively, it does not matter whether the lender incurs a disutility to create z or whether he pays a worker who incurs it. In both cases, individual agents fail to internalize that more intermediation resources relax the capital market constraint.

G.3 Additional Comparative Statics

Consider $y(\omega) = \theta\omega$ and the intermediation technologies:

$$p_m(\pi) = 1 - \exp(-\eta\pi)$$

for matching and:

$$p_s(z - \pi) = 1 - \exp(-\upsilon(z - \pi))$$

for screening. The analysis so far has restricted $\eta = \upsilon$. Here, I allow η to differ from υ to get some additional comparative statics for the decentralized equilibrium of the model with endogenous labor supply.

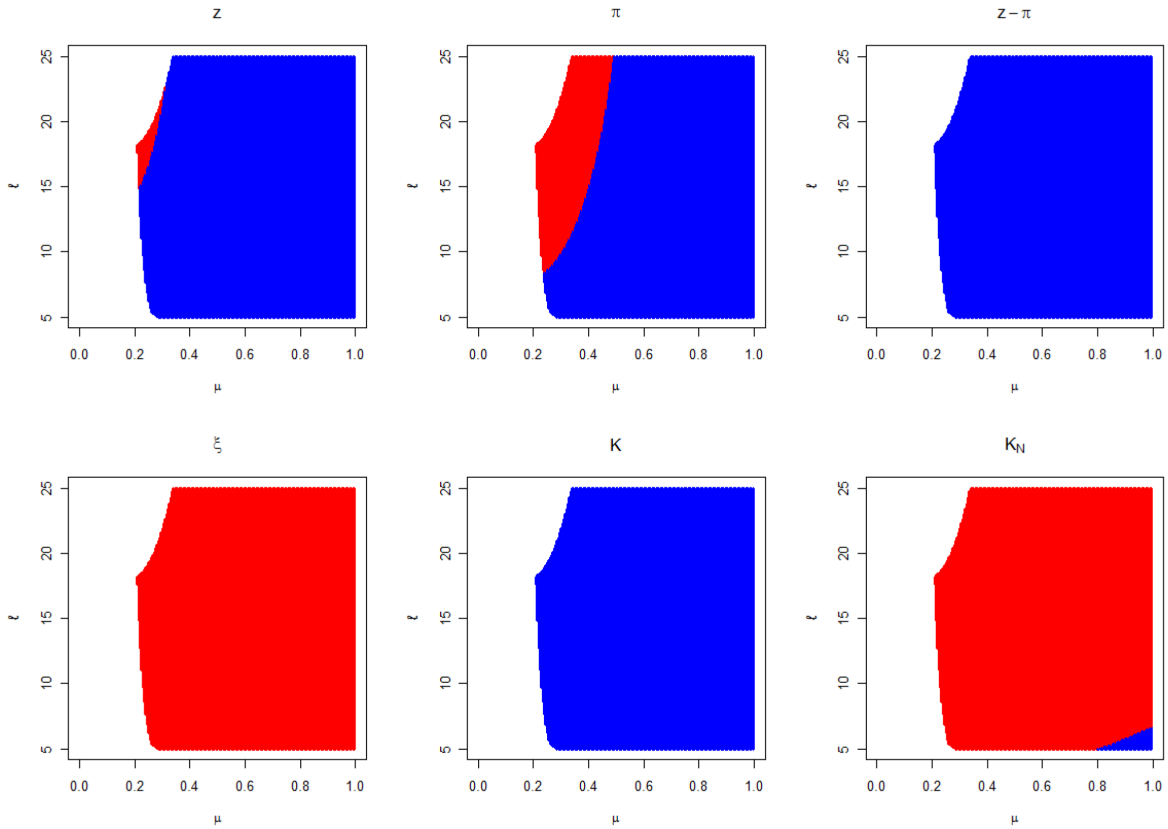
The left column of Figure G.2 shows that an increase in μ (or equivalently a decrease in the average match duration $\frac{1}{\mu}$) leads to a decrease in total credit K, an increase in uninformed credit K_N, and thus an unambiguous increase in the ratio $\frac{K_N}{K}$. The middle column of Figure G.2 shows that a decrease in η leads to the same effects: lower K, higher K_N, and higher $\frac{K_N}{K}$. Lower η means that more matching resources π are needed to achieve a given matching probability. In other words, matching becomes harder as η decreases so lower values of η could capture an increase in competitive pressure (e.g., because of exogenous reductions in barriers to entry in banking). The right column of Figure G.2 shows that an increase in the aggregate productivity parameter θ increases both K and K_N, with the ratio $\frac{K_N}{K}$ again rising.

Loutskina and Strahan (2011) show that the share of mortgages originated by lenders with little to no private information about their borrowers increased over the period 1992 to 2006. Increases in $\frac{K_N}{K}$ are thus an empirically relevant phenomenon and taking the model to data could be a fruitful extension for future work.

References

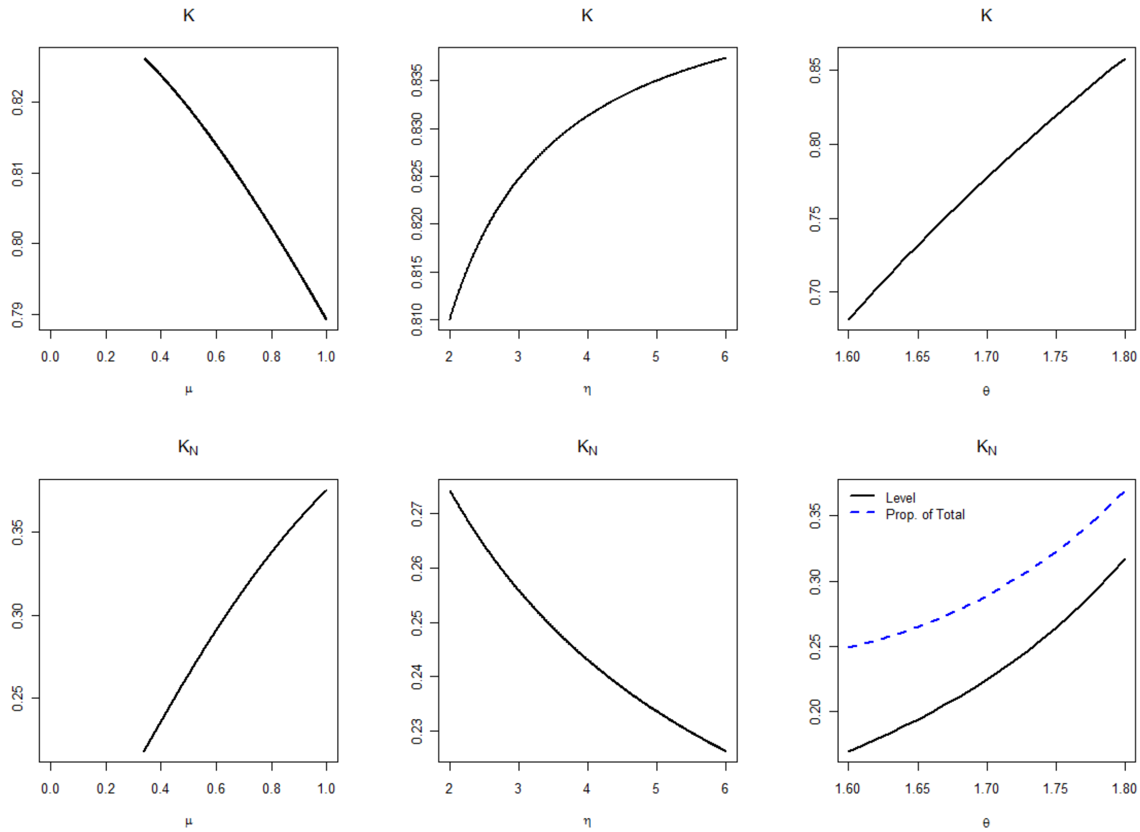
Loutskina, E. and P. Strahan. 2011. "Informed and Uninformed Investment in Housing: The Downside of Diversification." *Review of Financial Studies*, 24(5): 1447-1480.

Figure G.1:
Walrasian Model with Endogenous z and Elastic Labor Supply



Notes: This figure is drawn for $y(\omega) = 1.75\omega$ and $p(x) = 1 - \exp(-2.5x)$ with $\beta = 0.95$. A red (blue) marker at the coordinates (μ, ℓ) means that the variable indicated above the plot is higher (lower) in the decentralized equilibrium than in the constrained efficient allocation at this combination of μ and ℓ. Markers are only plotted for combinations of μ and ℓ where both the planner and the decentralized lenders optimally choose $\alpha = 1$ and the decentralized equilibrium has $R > 0$. A separate plot for the ratio of K_N to K is omitted for brevity; for all plotted markers, this ratio is higher in the decentralized equilibrium than in the constrained efficient allocation.

Figure G.2:
Additional Results for Decentralized Equilibrium with Elastic Labor Supply



Notes: This figure is drawn for $\upsilon = 2.5$, $\beta = 0.95$, and $\ell = 25$. The left column uses $\theta = 1.75$ and $\eta = 2.5$ and varies μ. The middle column uses $\theta = 1.75$ and $\mu = 0.5$ and varies η. The right column uses $\eta = 2.5$ and $\mu = 0.5$ and varies θ.