NBER WORKING PAPER SERIES

INVESTMENT, TOBIN'S Q, AND MULTIPLE CAPITAL INPUTS

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Working Paper No. 2033

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
October 1986

This paper is a revised version of "On The Inappropriateness Of An Empirical Q Investment Equation," and has benefited from the comments and advice offered by D. Cox, R. Ehrenberg, R. Eisner, R. Gordon, C. Hakkio, F. Hayashi, N. Jenkinson, N. Kiefer, T. Mroz, L. Muus, G. Silverstein, K. Troske, L. Vincent, and participants at numerous seminars and the NBER Summer Institute. Financial support from the Social Science Research Council, the Hoover Institution, and the National Science Foundation under Grant No. SES-8309073 is gratefully acknowledged. Remaining errors, omissions, and conclusions are the sole responsibility of the author. The research reported here is part of the NBER's research program in Economic Fluctuations. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #2033
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ABSTRACT

Despite their solid theoretical basis, models of business investment based on Tobin's Q theory have recorded a generally disappointing empirical performance. This paper examines one possible source of misspecification. When the firm's technology is expanded to include two or more capital inputs, the investment equation following from maximizing behavior includes Q as well as a series of additional explanatory variables. The importance of these omitted variables is assessed, and the econometric evidence is mixed, as the Multi-Capital Q model clearly dominates the Conventional specification but empirical problems remain. In addition, the implications of the parameter estimates from the Conventional and Multi-Capital models for tax policy are noted.

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INVESTMENT, TOBIN'S Q, AND MULTIPLE CAPITAL INPUTS

I. INTRODUCTION

Quantifying the parameters characterizing the capital accumulation process has been the subject of economic research for a number of years. A substantial amount of our knowledge of this process is based on estimates of the "Neoclassical" investment model pioneered by Dale Jorgenson (1963). While this econometric specification is capable of explaining a substantial amount of the variation in investment expenditures, it has been criticized for a lack of careful consideration of dynamics arising from expectations (Lucas, 1976) and intertemporal aspects of the technology (Nerlove, 1972).

The Q theory approach to investment behavior, introduced by Keynes (1964) and revitalized by James Tobin (1969), offers possible solutions to these two shortcomings. In this model, a forward-looking firm faced with costs in adjusting its capital stock will have its investment expenditures determined by marginal Q, the ratio of the discounted future revenues from an additional unit of capital to the net-of-tax purchase price. Whenever this ratio differs from unity, the firm has an incentive to alter its capital stock, but its actions are tempered by the adjustment cost technology. Since marginal Q is unobservable, empirical models have utilized average Q, defined as the ratio of the value of the firm, as evaluated in financial markets, to the replacement cost of its existing capital stock. By relying on financial market data, which in principle incorporates expectations of future variables relevant to the investment decision, Q models provide a direct role for expectations in the econometric specification.

The problem of unobservable expectations has received a great deal of attention in the applied econometrics literature, and an alternative approach uses time-invariant, forecasting equations to calculate marginal Q. This class of solutions includes the two-step procedure of Abel and Blanchard (1984), the maximum likelihood estimator of Hansen and Sargent (1980), and the Euler equation technique of Hansen and Singleton (1982); in the latter case, the forecasting equations are determined by the choice of instruments. These methods arose in response to the critique of Lucas (1976), who argued that "any change in policy will systematically alter the structure of econometric models" (p. 41). Under this view, changes in policy are identified as changes in the parameters governing policy outcomes, and these parameters will generally affect forecasts of economic variables. Only if one maintains that the sample period contains no changes in policy or non-policy factors affecting the stochastic environment will the forecasting solutions to the unobservable expectations problem be valid. A significant advantage of the Q approach is that estimation can proceed even if the sample period is characterized by an unstable stochastic environonment.

However, the usefulness of Q theory is called into question by its generally disappointing empirical performance.[1] A number of the problems associated with empirical Q models are evident in the

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[1] See Chirinko (1986, Section V.B.3) for a review of the econometric results from Q models, and Clark (1979) for simulations comparing Q to other investment models.

following investment equation estimated with aggregate data for equipment, structures, and inventories,

$$I(t) / K(t) = \underset{(.021)}{.072} + \underset{(.005)}{.013}\, \Omega(t) / K(t) + \underset{(.190)}{.368}\, I(t-1) / K(t)\,, \tag{1}$$

$\overline{R}^2 = .256$ $\quad$ $m = 3.477$ $\quad$ Res Sum Sq = .5020

Sample Period: 1950-1978

where I(t) and K(t) are the investment flow and capital stock, respectively, for the three capital goods and $\Omega(t)$ is a variable closely related to average Q (to be discussed below). The coefficient of .013 is smaller than that obtained in the Q study of Summers (1981, p. 101), who found that investment responded unreasonably slowly to variations in $\Omega(t)$. His simulations indicated that, twenty years after an unexpected change in the economic environment, the capital stock would have moved only three-fourths of the way to its ultimate, steady-state value.[2] Contrary to the theory, the lagged dependent

[2] When constrained by a geometric lag distribution, the econometric estimates of Ciccolo (1975) imply that the mean lag of the adjustment to a change in the long-run capital stock is seven years (Hall, 1977, p. 89).

variable proves an important determinant of investment.[3] Furthermore, the $\bar{R}^2$ is rather low and, as measured by the m-statistic (cf., fn. 11), serial correlation in the residuals indicates that the model is misspecified. A problem thus facing the applied econometrician is that conventional Q theory does not suggest what modifications or additional explanatory variables may be useful in attenuating these deficiencies.

An examination of the investment data for equipment, structures, and inventories suggests a possible cause of this disappointing empirical performance. In Table I, the serial correlation patterns for the three capital inputs differ widely, indicating that the capital homogeneity assumption implicit in the conventional Q model may be inappropriate. Under the assumption that adjustment costs increase with the durability of the capital asset, these patterns are consistent with the adjustment cost technology underlying the Q model. Thus, the central idea to be exploited in this study is that conventional formulations of empirical Q models are misspecified by failing to recognize the possibility that the value of the firm depends on two or more capital inputs with differing adjustment cost technologies. That the relationship

[3] Fischer (1983) has shown that lagged Q should be an important determinant of investment spending. There are two objections to his result. First, he enters lags into the maximization problem by assuming that adjustment costs depend on both current and lagged investment, but it is not at all clear what phenomena are being captured by this formulation. Second, the investment schedule that follows from his model is quite different from those actually used in econometric work, and thus the relation betweeen his result and empirically significant lagged variables is not apparent.

between Q and investment is sensitive to the number of capital inputs has been noted previously (Chirinko, 1982b; Wildasin, 1984). In this paper, we investigate the specification error due to these omitted variables, show that a structural model can be preserved under a broader specification of the technology, and provide econometric evidence to evaluate the Multi-Capital Q model.

The theoretical development of the Multi-Capital Q model is contained in Section II, which relates investment expenditures, (unobserved) marginal Q, and (observed) average Q. We give particular attention to the way in which debt finance affects the definition of average Q. Section III considers specification issues relating to the investment model. In Section IV, the Conventional and Multi-Capital Q models are estimated with a variety of econometric techniques. The simulated responses of investment expenditures and capital accumulation to an unanticipated change in the economic environment are calculated for both models in Section V. A summary and conclusions are provided in Section VI.

II. THE MULTI-CAPITAL Q MODEL - THEORY

The representative firm chooses variable and quasi-fixed inputs to maximize the present discounted value of its cash flow (V(0)) over an infinite horizon,

$$V(0) = \int_0^\infty \exp(-\int_0^t \rho(s)\, ds)\ \{R(t) - C(t) - T(t) - F(t)\}\, dt\ , \qquad (2)$$

where $\rho(s)$ is the discounting factor at time s and R(t), C(t), T(t), and F(t) represent revenues, operating costs, taxes, and net financing costs, respectively, at time t. Revenues (R(t)) are generated by selling a single product in a competitive market at an exogenous price (p(t)). The production technology ($\Phi[t]$) depends on labor (L(t)) and J distinctive types of capital ($K_j(t)$, $j \varepsilon J$), and is characterized by constant returns to scale with respect to all inputs and by Inada conditions,

$$R(t) = p(t)\, \Phi[L(t), K_j(t), \forall j \varepsilon J]\ . \qquad (3)$$

Operating costs (C(t)) arise from purchasing inputs and adjusting the capital stocks. The firm purchases labor services at price w(t) and new capital ($I_j(t)$) at prices $v_j(t)$ in perfectly competitive factor markets. To capture the quasi-fixed nature of capital, we assume that the firm incurs adjustment costs when incorporating new capital goods into the production process. These internal costs can be viewed as the movement of real resources from producing output toward installing capital goods, are valued in terms of the opportunity cost of foregone output, and increase at an increasing rate as investment exceeds replacement needs, defined as the product of an exponential depreciation rate (δ_j) and the existing capital stock. These properties apply to the adjustment cost functions ($\Gamma_j[I_j(t),K_j(t)], \forall j \varepsilon J$) that are homogeneous of degree one in both arguments,

$$C(t) = w(t)\, L(t) + \sum_{j \varepsilon J} v_j(t)\, I_j(t) + p(t) \sum_{j \varepsilon J} \Gamma_j[I_j(t),K_j(t)] \,, \tag{4}$$

An income tax is assessed at rate $\tau(t)$ against the firm's revenues less labor and adjustment costs, and is reduced by tax credits ($k_j(t)$) extended on the purchase of investment goods. The effective cost of the jth capital good is further lowered by tax depreciation allowances granted against current taxes but based on both current and past investments,

$$\tau(t) \int_{-\infty}^{t} D_j(t\text{-}s,s)\;\; v_j(s)\;\; I_j(s)\, ds \,, \tag{5}$$

where $D_j(t\text{-}s,s)$ are tax depreciation allowances per dollar of the jth investment made t-s periods ago according to the tax code in effect in period s. When embedded in the firm's discounted cash flow expression (2), equation (5) can be rearranged to isolate those factors that depend on current decisions and those that are predetermined at time t,

$$v_j(t)\ (k_j(t)+z_j(t))\ I_j(t)\ +\ A_j(t,0)\ , \tag{6a}$$

$$z_j(t) = \int_t^{\infty} \exp(-\int_t^{s} \rho(u)\,du)\ \tau(s)\ D_j(s-t,t)\ ds\ , \tag{6b}$$

$$A_j(t,0) = \tau(t) \int_{-\infty}^{0} D_j(t-s,s)\ v_j(s)\ I_j(s)\ ds\ . \tag{6c}$$

$$t \varepsilon [0,\infty)$$

The expression for $z_j(t)$ represents the present discounted value of current and future tax depreciation allowances flowing from a dollar of investment in period t, and (6c) represents the total value of tax depreciation allowances claimed at time t on the jth capital asset purchased before time 0. Total taxes (T(t)) paid by the firm, excluding those directly associated with debt finance, are as follows,

$$T(t) = \tau(t)\ \{p(t)\ \Phi[L(t),K_j(t),\forall j \varepsilon J]\ -\ w(t)\ L(t)\ -\ p(t) \sum_{j \varepsilon J} \Gamma_j[I_j(t),K_j(t)]\}$$

$$-\ \sum_{j \varepsilon J}\ v_j(t)\ (k_j(t)+z_j(t))\ I_j(t)\ -\ A_j(t,0) \tag{7}$$

Net financing costs (F(t)) for the jth type of capital are introduced into the model by augmenting the firm's cash flow to reflect the acquisition and retirement of debt and net-of-tax interest payments.[4] A firm that finances a proportion (b(s)) of its net-of-tax investment with debt will have its cash flow incremented by

$$X_j(s) \;=\; b(s)\,(1-k_j(s)-z_j(s))\;v_j(s)\;I_j(s)\;. \tag{8a}$$

Debt policy is exogenous to this model, and debt is retired at an exponential rate (η). In period t, the cash flow devoted to retirements equals the amount of debt issued at time s ($X_j(s)$) multiplied by the "survival" factor (exp-η(t-s)) and the retirement rate (η), and summed from time t backward,

$$\int_{-\infty}^{t} \eta\; X_j(s)\;\exp(-\eta(t-s))\;ds\;. \tag{8b}$$

Interest payments at time t are the product of the amount of debt issued at time s surviving at time t and the interest rate (i(s)) prevailing at time s, summed from time t backward. In recognition of the deductibility of interest payments against income taxes, they are multiplied by one less the tax rate at time t,

$$(1-\tau(t))\int_{-\infty}^{t} i(s)\;X_j(s)\;\exp(-\eta(t-s))\;ds\;. \tag{8c}$$

[4] This formulation of debt finance follows from joint work with Stephen King (1983).

Equations (8a-c) can be embedded in the firm's discounted cash flow expression (2), and rearranged to obtain the following expression for the firm's net financing costs,

$$F(t) = \sum_{j \varepsilon J} v_j(t)\ (1-k_j(t)-z_j(t))\ \psi(t)\ I_j(t)\ -\ B_j(t,0)\ , \tag{9a}$$

$$\psi(t) = b(t)\{\{\int_t^{\infty} \exp(-\int_t^{s} (\rho(u)+\eta)\ du)\ [(1-\tau(s))\ i(t) + \eta]\ ds\} - 1.0\}, \tag{9b}$$

$$B_j(t,0) = (1-\tau(t)) \int_{-\infty}^{0} i(s)\ X_j(s)\ \exp(-\eta(t-s))\ ds$$

$$+\ \eta \int_{-\infty}^{0} X_j(s)\ \exp(-\eta(t-s))\ ds. \qquad t\varepsilon[0,\infty) \tag{9c}$$

Equation (9b) represents the potential subsidy for the purchase of investment goods provided by debt finance. If financial policy eliminates arbitrage opportunities between the net-of-tax cost of borrowing and the capitalization rate on debt, then $\psi(t)$ becomes unity, and the potential advantage of debt financing, arising from the tax deductability of interest payments, would be eliminated. Equation (9c) represents the value of the net-of-tax interest and retirement payments at time t on debt acquired prior to time 0 and, when discounted by $\rho(t)$ and integrated from zero to infinity, becomes the market value of the firm's debt at time 0 ($B^*(0)$). This interpretation holds for any time paths of the variables, but becomes apparent when we assume $i(s)$, $\tau(t)$, and $\rho(t)$, are constant from time zero onward,[5]

$$B^*(0) = [((1-\tau(0))\, i(0) + \eta) \,/\, (\rho(0) + \eta)] \;\; FV(0) \,, \tag{10a}$$

$$FV(0) = \sum_{j \varepsilon J} \int_{-\infty}^{0} X_j(s) \, \exp(\eta s) \, ds \,, \tag{10b}$$

where $FV(0)$ is the face value of debt remaining at time 0. If the costs of debt and equity are equated through financial policy, then the term in braces in (10a) is unity, and the face and market values of debt are identical.

To complete our specification of the cash flow problem facing the firm, we assume that movements in the capital stocks are governed by the following transition equations,

$$K_j(t) = I_j(t) - \delta_j \, K_j(t) \,. \qquad \forall j \varepsilon J \tag{11}$$

The firm is assumed to maximize (2) constrained by (3), (4), (7), (9), and (11) and, in order to analyze the optimal choices of labor and capital inputs, we construct the following current-value Hamiltonian,

[5] The general expression for the market value of debt is as follows,

$$B^*(0) = FV(0) \int_{0}^{\infty} \exp\Big(- \int_{0}^{t} (\rho(u)+\eta)\, du\Big) \Big\{ \int_{-\infty}^{0} \omega(s)[(1-\tau(t))i(s)+\eta] \, ds\Big\} \, dt,$$

where $\omega(s)$ is the percentage of the face value of debt issued in period s.

$$H[L(t),\ I_j(t),\ K_j(t),\ \mu_j(t),\ \forall j \varepsilon J] \ = \ \exp(-\int_0^t \rho(s)\ ds) \tag{12}$$

$$\{(1-\tau(t))\ \{p(t)\ \Phi[L(t),K_j(t),\forall j\varepsilon J] \ - \ w(t)\ L(t) \ - \ p(t) \sum_{j\varepsilon J} \Gamma_j[I_j(t),K_j(t)]\}$$

$$- \sum_{j\varepsilon J} \tilde{v}_j(t)\ I_j(t) \ + \ \mu_j(t)\ (I_j(t) - \delta_j\ K_j(t)) \ + \ A_j(t,0) \ - \ B_j(t,0)\}\ ,$$

$$\tilde{v}_j(t) \ = \ v_j(t)\ (1-k_j(t)-z_j(t))\ (1+\psi(t)) \qquad t\varepsilon[0,\infty)$$

where (12) is written in terms of the control ($L(t)$, $I_j(t)$), state ($K_j(t)$), and current-value co-state ($\mu_j(t)$) variables, and $\tilde{v}_j(t)$ is the purchase price of new capital adjusted for the effects of taxes and financing.[6] Note that $A_j(t,0)$ and $B_j(t,0)$ are predetermined in period t, and thus do not affect the firm's profit-maximizing decisions. Necessary conditions for the maximization of (12) are obtained by applying

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[6] In regard to debt finance, our formulation differs in two respects from that found in the Q models of Summers (1981) and Poterba and Summers (1983). First, our derivation permits the marginal and average levels of debt finance to vary, and thus average Q (or Ω in the current notation) will reflect the time variation in the market value of debt. Second, the specification of the purchase price of new capital allows for the possibility that financial policy may not eliminate the subsidy associated with debt finance. When the percentage of debt finance is constant (as has been assumed in previous studies), it is unlikely that this subsidy will be zero, especially when nominal interest payments are tax deductible.

Pontryagin's Maximum Principle and, for purposes of the present analysis, we consider the following conditions pertaining to labor and the jth type of capital,

$$\Phi_L[t] = w(t) / p(t) , \tag{13a}$$

$$\mu_j(t) = \int_t^{\infty} \exp(- \int_t^{s} (\rho(u)+\delta) \, du) \, (1-\tau(t)) \, p(t)) \, \{\Phi_{Kj}[s] - \Gamma_{Kj}[s]\} \, ds , \tag{13b}$$

$$\mu_j(t) = \tilde{v}_j(t) + (1-\tau(t)) \, p(t)) \, \Gamma_{Ij}[t] . \tag{13c}$$

$$\forall j \varepsilon J$$

Equation (13a) is the familiar marginal productivity condition for a variable factor of production. The marginal benefit of an additional unit of capital is defined in (13b) as the sum of current and future marginal products weighted by the rates of discount and depreciation. Along the optimal path, this marginal benefit must equal the total marginal costs of acquiring capital. These involve the sum of purchase and marginal adjustment costs, and the requisite equality is given by (13c).

III. THE MULTI-CAPITAL Q MODEL - SPECIFICATION

The optimizing conditions associated with the maximization of (12) form the basis of the Multi-Capital Q model of investment. By dividing both sides of (13c) by $\tilde{v}_j(t)$, we derive the positive relationship between investment, as embedded in the adjustment cost function, and marginal Q, the ratio of the shadow price for the jth capital good to its net purchase price.[7] Since this equation contains an unobservable variable, it is of little immediate use in applied work. For a firm that uses only equity finance and one capital input, Hayashi (1982a) has developed the conditions under which the unobserved shadow price of capital can be related to financial market data. These conditions include that the production and adjustment cost technologies exhibit constant returns to scale, the firm participates in perfectly competitive output and factor markets, capital depreciates exponentially, and the discounted values of the capital stocks are constrained by transversality conditions.[8] Modifying Hayashi's proof to incorporate multiple capital goods and debt finance, we obtain the following relationship between shadow prices and financial data,

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[7] This relationship has been noted by, among others, Abel (1979), Mussa (1977), Sargent (1979), and Yoshikawa (1980).

[8] The transversality condition is written as follows,

$$\lim_{T\to\infty} \exp\left(-\int_0^T \rho(s)\,ds\right) \mu_j(T)\ K_j(T) = 0 . \qquad \forall j \varepsilon J$$

$$V(0) = \sum_{j \varepsilon J} \mu_j(0)\ K_j(0) + A^*(0) - B^*(0)\ , \tag{14a}$$

$$A^*(0) = \int_0^\infty \exp(-\int_0^t \rho(s)\, ds) \sum_{j \varepsilon J} A_j(t,0)\ dt, \tag{14b}$$

$$B^*(0) = \int_0^\infty \exp(-\int_0^t \rho(s)\, ds) \sum_{j \varepsilon J} B_j(t,0)\ dt, \tag{14c}$$

The intuition behind this results is rather straightforward. The assumption of competitive markets ensures that, after period 0, the firm is unable to earn any profits, and the equity value of the firm is determined by the quasi-rents from the stocks available at the beginning of the planning period. The most important of these is fixed capital and, under constant returns to scale, the marginal and average return to capital are identical. Thus, the value of the jth capital good is the product of its shadow price and the existing stock. Assuming momentarily that the firm uses only one capital good and that A*(0) and B*(0) are zero, we see that (14a) delivers the basic relationship between marginal and average Q, the latter defined as the ratio of the financial value of the firm to its replacement cost,

$$\text{average } Q \equiv V(0)\ /\ \tilde{v}(0)\ K(0) = \mu(0)\ /\ \tilde{v}(0) \equiv \text{marginal } Q. \tag{15}$$

More generally, the equity value depends on the other capital goods and their shadow prices, plus the discounted value of tax depreciation allowances due to past investments ($A^*(0)$), less the discounted liability due to past financing committments ($B^*(0)$).

To obtain an investment equation amenable to testing, we need to impose some assumptions about the intertemporal technology, and choose the following parameterization of the adjustment cost functions,

$$\Gamma_j[I_j(t),K_j(t)] = (\gamma_j/2)\ [I_j(t)/K_j(t) - \delta_j]^2\ K_j(t)\,. \qquad \forall j \varepsilon J \tag{16}$$

As noted in the introduction, past studies have found that lagged variables have been significant in Q investment equations, and lagged dependent variables are included in the general econometric specification to represent any dynamics not fully captured by (16). While these lagged variables compromise the structural interpretation of the investment equations, they are included in the general specification because their statistical significance serves as a test for misspecification in the expanded model. Furthermore, it would appear preferable to account explicitly for the dynamics rather than doing so implicitly through a GLS correction. Combining (13c), (14), and (16), we obtain the following system of investment equations for the J capital goods,

$$I_j(t) = \delta_j K_j + (1/\gamma_j)\,\Omega(t) - (\gamma_k/\gamma_j)\, I_k(t)$$

$$+ (\gamma_k \cdot \delta_k/\gamma_j)\, K_k(t) + \lambda_j\, I_j(t\text{-}1) + \varepsilon_j(t), \qquad \forall j,k \varepsilon J,\ k \neq j \tag{17a}$$

$$\Omega(t) = \{V(t) + B^*(t) - A^*(t) - \sum_{j \varepsilon J} \tilde{v}_j(t)\, K_j(t)\} \;/\; ((1-\tau(t))\, p(t), \tag{17b}$$

where the k subscript represents all but the jth capital good and (17) corresponds to the case where J=2. A white-noise error term ($\varepsilon_j(t)$) reflects non-systematic variations in $I_j(t)$ and approximation errors that have arisen in the development of the model,[9] and λ_j is the parameter for the lagged dependent variable.

Equations (17) highlight that, in the Multi-Capital Q model, market value data will not prove to be a "sufficient statistic" for signaling investment expenditures for any particular capital input. In (17b), $\Omega(t)$ is defined as the difference between the financial value of the firm (adjusted for tax depreciation) and the replacement cost of the capital stocks all deflated by the net-of-tax output price. (Note that the same $\Omega(t)$ enters all J investment equations and, if adjustment costs were valued by the price of investment goods

[9] An additive error term can be derived from the theoretical model if we assume that technology shocks affect either the production function (Hayashi, 1982b) or the adjustment cost function (Poterba and Summers, 1983). The maximization problem has not been formulated with these additional assumptions because neither result in any restrictions on the estimation procedure, other than to warn of the omnipresent possibility of correlation between error terms and endogenous regressors (i.e., $I_k(t)$).

rather than output, $\Omega(t)$ would be replaced by average Q less unity (cf., (15)).) In (17a), a given value of $\Omega(t)$ indicates the profitable level of investment activity for the entire firm. The own capital stock influences gross investment positively due to replacement needs. The regressors subscripted by k affect $I_j(t)$ through adjustment costs. The higher $I_k(t)$, the greater the adjustment costs for the kth capital good and, hence, the fewer resources available for investment in the jth capital good. Since a larger capital stock lowers adjustment costs, $K_k(t)$ has a positive effect on investment expenditures. The failure to include these latter variables in econometric equations implies that conventional formulations of the Q model may be misspecified. The stronger the association between $\Omega(t)$ and investment expenditures for the kth capital good, the greater the upward bias in γ_j.[10] The econometric results presented in the next section will allow us to assess the extent of this bias and the degree to which the Multi-Capital model attenuates the other empirical problems associated with Q theory.

[10] Alternatively, it has been noted that the large estimated values of γ_j are an inevitable outcome of relating volatile financial market data to the less variable investment series (Shapiro, 1986, p. 531). Insofar as the flow variables are strongly, positively correlated with $\Omega(t)$, the estimated γ_j's from (17), ceteris paribus, should fall.

IV. ECONOMETRIC RESULTS

The Multi-Capital Q model was estimated with data for nonfinancial corporations over the period 1950-1978 for three capital goods - equipment, structures, and inventories. The length of the sample and the number of capital goods were determined by data availability, and detailed information concerning data sources is provided in the Glossary. For each capital input, (17a) was scaled by $K_j(t)$ to eliminate the possible spurious correlation arising from the trends in the flow and stock variables (Granger and Newbold, 1974). Since no estimation technique dominates under even modest violations of maintained assumptions, results are reported for a variety of estimators.

Ordinary Least Squares estimates of the Conventional (CV) and Multi-Capital (MC) models are presented in Table II, columns 1-3 and 4-6, respectively. For the CV model, the bulk of the explanatory power of the Q model can be traced to the structures equation. As assessed by the m-statistic,[11] the residuals are serially correlated and, contrary to a strict interpretation of Q theory, the lagged dependent variables in the equipment and structures equations are significant. The results for the MC model are mixed. The $\overline{R}^2$'s are improved substantially, and serially correlated residuals are absent.

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[11] The m-statistic is distributed t under the null hypothesis of no serial correlation. The m-statistic is calculated in a regression of the residuals from an equation with a lagged dependent variable (e.g., (17a)) on all of the explanatory variables in the initial regression plus the lagged residual; m is the t-statistic on the lagged residual. Performing a t-test on this coefficient is asymptotically equivalent to the Durbin h-test. However, it can be calculated for all possible values of the estimated parameters, and has performed better than the h-statistic in Monte Carlo experiments (Harvey, 1981, p. 276).

However, the only significant coefficient on $\Omega(t)$ appears in the structures equation, which also contains a significant lagged dependent variable. While a number of the additional flow and stock variables are significant, most are of the wrong sign. The CV specification is nested within the MC and, for each of the three investment equations, the zero restrictions on the four additional regressors ($I_k(t)$, $K_k(t)$, $\forall k,j \varepsilon J$, $k \neq j$) are rejected at the 1% level.

Key to our development of the Multi-Capital Q model is the assumption that the adjustment cost parameters differ among the capital inputs, and two tests suggest that the null hypothesis -- $\gamma_j = \gamma_k$, $\forall j,k \varepsilon J$, $k \neq j$ -- is rejected by the data. First, the rejections of the CV model reported above are strong evidence in favor of the MC specification.[12] Second, we can interpret the aggregate equation (1) as a restricted case of the separate MC equations (17) and, regardless of which capital good served as the dependent variable, the restricted model was always rejected at the 1% level.[13]

Estimates for the MC model may be biased by correlations between error terms and the investment flows (the capital stocks

[12] These results hold whether or not a lagged dependent variable is included as a regressor.

[13] To ensure that (1) is nested within (17a), the separate MC equations have all been scaled by the aggregate capital stock instead of $K_j(t)$ and the constant term in the restricted model is interpreted as a weighted average of the depreciation rates for the separate capital goods. The tests were conducted for each of the capital goods four different ways: with or without a lagged dependent variable, and for two (equipment and structures) or three (equipment, structures, and inventory) capital goods. This aggregate Q investment equation stands in contrast to Wildasin's (1984) Proposition II, which is based on an aggregate investment equation specified as the ratio of nominal variables.

and $\Omega(t)$, dated at the beginning of the period, are predetermined), and Table III contains two sets of estimates that avoid this potential problem. Reduced form estimates are contained in the first three columns, and all of the coefficients on $\Omega(t)$ are significant. An implication of the adjustment cost technology is that these coefficients should be lowest for structures (representing the largest adjustment costs) and highest for inventories, a prediction borne-out by the estimates. Instrumental variables (IV) regressions for the MC model are presented in columns 4-6 of Table III and, relative to OLS, these estimates remain essentially unchanged.[14]

By recognizing the contemporaneous correlation between the ε_j's we can enhance the efficiency of the coefficient estimates, and can conduct an additional test of the specification. Three-Stage Least Squares (3SLS) estimates are presented in Table IV. For the MC model, all of the coefficients on $\Omega(t)$ are significant at the 1% level, though the coefficient in the equipment equation is negative. Coefficients on the lagged dependent variables and tests for serially correlated residuals are both insignificant. The system of equations characterizing the MC model has 12 regressors not contained in the CV model, and 11 of these are statistically significant, 4 of which are of the correct sign. Not surprisingly, a comparison of the CV and MC models based on the differences in their Criteria, distributed $\chi^2(12)$,

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[14] The instrument list included all of the predetermined variables in the three equation system plus $k_{eq}(t)$, $z_{eq}(t)$, $\tau(t)$. The R^2's for the first-stage regressions were no lower than .78. When the instrument list was reduced to $k_{eq}(t)$, $z_{eq}(t)$, $\tau(t)$, and all predetermined variables in a given equation, the parameter estimates were largely unaffected.

rejects the restrictions defining the Conventional Q model at the 1% level. Further support for the MC model is obtained by comparing the IV and 3SLS estimates with the test proposed by Hausman (1978), Durbin (1954), and Wu (1973). If misspecification is present in the system, the 3SLS estimates will differ substantially from the corresponding IV estimates. For the three equation system, the HDW test statistic is distributed $\chi^2(21)$, and equals 6.372 ($P > .999$), indicating that the null hypothesis of no misspecification can not be rejected.[15]

To gain additional insight into the MC model, we examine the role of inventory capital, which has been excluded in a number of previous Q studies. Table V contains estimates of equation systems for equipment and structures, and the results change little for the CV model (columns 1-2) or for the MC model when the inventory terms remain as regressors (columns 3-4). These inventory terms are removed in the regressions reported in columns 5-6, and the coefficients on the lagged dependent variables increase sharply. Since inventories play an important role in buffering the firm against shocks, failing to include these terms may lead to misspecified dynamics captured in an ad hoc manner by lagged variables in conventional Q models. Formally, a comparison of the Criteria for the

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[15] This statistic is based on the IV estimates in Table III, columns 4-6, and the 3SLS estimates in Table IV, columns 4-6. The test statistic was computed with the 3SLS residual covariance matrix, and can be interpreted as the Lagrange multiplier version of the HDW test. In comparing the IV and 3SLS estimates, note that the standard errors of the estimated coefficients from the single equation estimators (Tables II and III) are adjusted by the degrees of freedom, whereas the the estimated coefficients from the systems estimators (Tables IV and V) are adjusted by the sample size.

models in Table V rejects the hypothesis that inventories can be excluded. Complementary evidence is provided by the HDW test statistics of 8.261 ($P > .875$) for columns 3-4 and 13.721 ($P > .186$) for columns 5-6. While both tests are below conventional significance levels, the substantial increase in the latter P value confirms the importance of the inventory variables.

V. SIMULATIONS

To evaluate further the Multi-Capital Q model, we examine in this section the response of investment expenditures to changes in V(t) (hence $\Omega(t)$) in both the CV and MC models. We assume that V(t) increases unexpectedly by \$1, perhaps due to a lump sum tax rebate,[16] and use the estimated parameters to calculate the increments to investment and the capital stocks.[17] These calculations will be only approximate because we are not constraining the solution to be consistent with the eventual steady-state (i.e., remain on the stable manifold). A comparison of our method to that used by Summers (1981, Table 6) indicates that the understatement of the accumulated capital stock is small: the bias is approximately 0% during the first five years, 1% in the tenth year, and 13% in the fiftieth year.

The simulation results are reported in the Tables at the bottom three rows for each equation, and the response of investment is quite slow. Ten years after the \$1 shock, increments to the accumulated sum of the three capital stocks range between \$.19 and \$.37. These results do not vary systematically with the CV or MC specifications, and are quite consistent with the value of \$.32 from the equation

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[16] These simulation results are comparable to and consistent with the evidence from other econometric investment models discussed in Chirinko (1986).

[17] Since the lagged dependent variables are not part of the formal Q model and lead to explosive behavior, they have been excluded in the simulations. The system of investment equations (17) was augmented by the capital accumulation equations (11) with geometric depreciation rates of .13, .04, and 0.0 for equipment, structures, and inventory, respectively. When the three capital inputs are aggregated, the depreciation rate was set to .061.

based on aggregate capital (1). A result not apparent in the aggregate equation is that inventories are accumulated at a disproportionately fast rate in the conventional model. For the sample period, the inventory stock is 25% of the aggregate, but the simulated changes in inventories are between 51-56% (CV models) and 19-32% (MC models). In sum, one finds that the modifications to the Q model undertaken in this paper do little to attenuate the unreasonably sluggish response of investment to variations in $\Omega(t)$.

VI. SUMMARY AND CONCLUSIONS

The poor empirical performance of Q models has led us to reexamine the conventional framework. By recognizing the possibility that the value of the firm depends on two or more capital inputs whose adjustment cost technologies may differ, we have derived an econometric specification that nests the Conventional model as a special case. Econometric evidence was generated with equipment, structures, and inventories considered as capital inputs, and the Conventional model was rejected in favor of the Multi-Capital specification. However, for a number of the variants of the Multi-Capital model, lagged variables proved significant, serial correlation in the residuals was important, and a number of the regressors had incorrect signs. Furthermore, as has been the case in previous work with Q models, the response of the capital stock to unexpected changes in the economic environment remained unreasonably slow.

The results reported in this paper thus provide substantial support for the Multi-Capital model as a useful extension to the Q framework, but indicate that further work remains. In particular, inventories were shown to play a key role in the modeling of fixed investment expenditures within the Q framework. One possible direction would be to modify the technology in order to recognize a buffer stock role for inventory investment but retain a traditional adjustment cost approach for equipment and structures.[18] Alternatively, the dynamics of fixed capital accumulation may be too complex to be captured adequately by the adjustment cost technology. A more satisfactory model of capital accumulation may thus have to be based on an intertemporal technology that recognizes explicitly delivery, expenditure, and gestation lags and the possibility of non-exponential capital decay.

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[18] Given the availability of suitable data, the MC framework could be extended to other capital inputs such as research and development, human capital, or advertising/goodwill. However, in the event that the data associated with these types of capital are plagued by substantial measurement error, their inclusion may compromise the other coefficient estimates (cf., Fisher, 1980, p. 158).

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GLOSSARY

SOURCES:

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LEGEND:

BOP - beginning of the period.
C - column.
FLO - flow over the period.
L - line.
MOP - middle of the period.
T - table.

DEFINITIONS:

A* - Present discounted value of tax depreciation allowances from assets purchased prior to period t, BOP: CS.

b - Leverage ratio: B* / (B* + V).

B* - Current dollar market value of debt issued by nonfinancial corporate business, BOP: (NETINT * (MIP / INT)) / i).

DIV - Current dollar dividends for nonfinancial corporate business, BOP: NIPA, T1.13, L31.

DSP_c - Standard & Poor's dividend-common stock price ratio, BOP: S&P, p. 127.

DSP_p - Standard & Poor's dividend-preferred stock price ratio, BOP: S&P, p. 118.

i - Moody's nominal interest rate on corporate Aaa bonds, BOP: NIPA, S-16, and BS.

I_j - Constant dollar investment for nonfinancial corporate business (j = equipment, structures, inventories), FLO: BEAU.

INT - Current dollar net interest paid by nonfinancial corporate business, FLO for the previous period: NIPA, T1.13, L35.

k_j - Rate of investment credit (j = equipment, structures): CS, p. 54.

K_j - Constant dollar replacement value of the capital stock for nonfinancial corporate business (j = equipment, structures, inventories), BOP: BEAU.

$K_j\$$ - Current dollar replacement value of the capital stock for nonfinancial corporate business (j = equipment, structures, inventories), BOP: BAL, T705, L4.

MIP - Current dollar net monetary interest paid by nonfinancial corporate business, FLO for the previous period: NIPA, T8.7, L7 less L25.

NETINT - Current dollar net interest paid by nonfinancial corporate business, BOP: NIPA, T1.13, L35.

NFA - Current dollar noninterest bearing net financial assets of nonfinancial corporate business, BOP: BAL T705, L9, L16, L17, L18, less L37, L38, L39.

η - Rate of debt retirement: .1087, which implies that 90% of a debt issue would be retired after 20 years.

p - Implicit price deflator for gross domestic purchases, BOP: NIPA, T7.3, L15.

ρ - Firms' nominal rate of discount, BOP: (w_{div} DSP_c + $(1-w_{div})$ DSP_p) + LAG4(SP/SP), where the latter variable equals the mean of the percentage change in SP over the previous four periods.

SP - Standard & Poor's composite stock price index, MOP: CEA, TB-90.

τ - Rate of federal taxation of corporate income: ST, T2, C6.

τ_j - Tax credits and deductions on capital services (j = equipment, structures): $(1 - k_j - \tau z_j)$.

v_j - Price index for investment expenditures on asset j: calculated implicitly with $K_j\$$.

V - Current dollar market value of equity for nonfinancial corporate business, BOP: $DIV / (w_{div}\ DSP_c + (1\text{-}w_{div})\ DSP_p)$.

Ω - Difference between the value of the firm evaluated on financial markets and the net-of-tax replacement value of its assets, BOP: $[(V + B^*) - (\tau_e K_e\$ + \tau_s K_s\$ + (1\text{-}\tau)K_i\$ + NFA)] / (1\text{-}\tau)\ p$.

w_{div} - Percentage of dividends paid on common stock: VF, p. 358, fn. 11, extended for the current study.

ψ - Subsidy for the purchase of investment goods provided by debt finance, BOP: $b\{[((1\text{-}\tau)i + \eta) / (\rho + \eta)] - 1.0\}$.

z_j - Present discounted value of current and future tax depreciation allowances per dollar of investment in period t (j = equipment, structures): CS, Appendix E.

TABLE I

SERIAL CORRELATION COEFFICIENTS

EQUIPMENT, STRUCTURES, INVENTORIES*

Lag Length	Equipment	Structures	Inventories
	(1)	(2)	(3)
1	.728	.896	.263
2	.425	.753	-.200
3	.374	.658	-.189

* Sample period 1953-1978. All series scaled by their own capital stocks. Critical values are .496 and .388 at the 1% and 5% levels, respectively.

TABLE II

CONVENTIONAL AND MULTI-CAPITAL Q MODELS

ORDINARY LEAST SQUARES ESTIMATES*

Variable or Statistic	Conventional			Multi-Capital		
	EQ (1)	ST (2)	IN (3)	EQ (4)	ST (5)	IN (6)
$\Omega(t)$	.0041	.0029[a]	.0051	-.0012	.0025[a]	.0031
	(.0020)	(.0010)	(.0035)	(.0018)	(.0008)	(.0027)
$I_{eq}(t)$	——	——	——	——	.2706[a]	1.3900[a]
	——	——	——	——	(.1000)	(.1744)
$K_{eq}(t)$	.0449	——	——	.0692	-.0795[a]	-.0304
	(.0312)	——	——	(.0545)	(.0266)	(.1064)
$I_{st}(t)$	——	——	——	.6410	——	-.8944
	——	——	——	(.3114)	——	(.4812)
$K_{st}(t)$	——	.0254	——	-.1567[a]	.0942[a]	.2485[a]
	——	(.0128)	——	(.0452)	(.0197)	(.0631)
$I_{in}(t)$	——	——	——	.5096[a]	-.0913	——
	——	——	——	(.0671)	(.0680)	——
$K_{in}(t)$	——	——	.0372[a]	.2625[a]	-.0140	-.5282[a]
	——	——	(.0105)	(.0978)	(.0556)	(.1609)
$I_{eq}(t-1)$	.7820[a]	——	——	.0868	——	——
	(.1818)	——	——	(.1832)	——	——
$I_{st}(t-1)$	——	.7398[a]	——	——	.4314[a]	——
	——	(.1382)	——	——	(.1451)	——
$I_{in}(t-1)$	——	——	.1558	——	——	.1490
	——	——	(.1941)	——	——	(.1419)
$\bar{R}^2$	.4324	.7154	.0331	.8259	.8527	.7275
m	1.8572	2.7819	3.9303	1.2659	.2230	-.2731
Res. Sum. Sq.	.5878	.0793	3.3425	.1526	.0347	.7970
Criterion	82.155	111.20	56.954	101.71	123.18	77.741
Simulations ($dK_i/d\Omega$)						
second year	.0115	.0082	.0144	.0089	.0082	.0137
fifth year	.0395	.0311	.0593	.0327	.0329	.0341
tenth year	.0694	.0649	.1414	.0634	.0724	.0550

*Estimates based on equation (17). The dependent variable is indicated by the column heading. All explanatory variables are scaled by the capital stock for the dependent variable; thus, for a given equation, the corresponding capital stock represents the constant term. Sample period 1950-1978. Standard errors in parentheses. Residual Sum of Squares multiplied by 10^{-2}.

[a] Significant at the 1% level. [b] Significant at the 5% level.

TABLE III

MULTI-CAPITAL Q MODELS

REDUCED FORM AND INSTRUMENTAL VARIABLES ESTIMATES*

Variable or Statistic	Reduced Form EQ (1)	Reduced Form ST (2)	Reduced Form IN (3)	Instrumental Variables EQ (4)	Instrumental Variables ST (5)	Instrumental Variables IN (6)
$\Omega(t)$	.0082[a]	.0037[a]	.0119[a]	-.0022	.0029[a]	.0047
	(.0021)	(.0009)	(.0034)	(.0026)	(.0009)	(.0033)
$I_{eq}(t)$	——	——	——	——	.3226[b]	1.3813[a]
	——	——	——	——	(.1449)	(.1975)
$K_{eq}(t)$	-.1631	-.1048[a]	-.1936	.0849	-.0864[a]	-.0794
	(.0946)	(.0403)	(.1526)	(.0719)	(.0281)	(.1188)
$I_{st}(t)$	——	——	——	.7910	——	-1.2534
	——	——	——	(.4994)	——	(.6126)
$K_{st}(t)$	.2023[a]	.1008[a]	.4729[a]	-.1832[a]	.1083[a]	.2887[a]
	(.0697)	(.0297)	(.1118)	(.0716)	(.0242)	(.0752)
$I_{in}(t)$	——	——	——	.5863[a]	-.1731	——
	——	——	——	(.0902)	(.1005)	——
$K_{in}(t)$	.1128	.0718	-.4170	.2767[a]	-.0247	-.4665[a]
	(.1577)	(.0653)	(.2574)	(.1020)	(.0692)	(.1812)
$I_{eq}(t-1)$	1.2828[a]	.1408	1.7194[a]	.0019	——	——
	(.3308)	(.1388)	(.5285)	(.2230)	——	——
$I_{st}(t-1)$	-1.7448[a]	.2318	-2.9245[a]	——	.3532[b]	——
	(.4894)	(,2048)	(.7812)	——	(.1573)	——
$I_{in}(t-1)$	-.0655	-.0225	.0620	——	——	.1491
	(.1406)	(.0604)	(.2261)	——	——	(.1444)
$\bar{R}^2$	.5911	.7968	.3753	——	——	——
m	.6759	2.4944	-.3128	.3913	-.3978	-.3189
Res. Sum. Sq.	.3583	.0479	1.8274	.1628	.0381	.8199
Criterion	89.334	118.51	65.709	10.334	8.9431	8.5567
Simulations ($dK_i/d\Omega$)						
second year	.0231	.0104	.0335	.0108	.0086	.0173
fifth year	.0796	.0407	.0704	.0459	.0369	.0482
tenth year	.1315	.0779	.0884	.1105	.0825	.0902

*Estimates based on equation (17). The dependent variable is indicated by the column heading. All explanatory variables are scaled by the capital stock for the dependent variable; thus, for a given equation, the corresponding capital stock represents the constant term. Sample period 1950-1978. Standard errors in parentheses. Residual Sum of Squares multiplied by 10^{-2}. Criterion for IV divided by the variance of the regression error.

[a] Significant at the 1% level. [b] Significant at the 5% level.

TABLE IV

CONVENTIONAL AND MULTI-CAPITAL Q MODELS

THREE STAGE LEAST SQUARES ESTIMATES*

Variable or Statistic	Conventional EQ (1)	Conventional ST (2)	Conventional IN (3)	Multi-Capital EQ (4)	Multi-Capital ST (5)	Multi-Capital IN (6)
$\Omega(t)$	.0041[b]	.0023[b]	.0053	-.0042[b]	.0032[a]	.0070[a]
	(.0019)	(.0009)	(.0033)	(.0018)	(.0007)	(.0026)
$I_{eq}(t)$	—	—	—	—	.5220[a]	1.5876[a]
	—	—	—	—	(.1060)	(.1235)
$K_{eq}(t)$	.0385	—	—	.1405[b]	-.0989[a]	-.2298[a]
	(.0209)	—	—	(.0552)	(.0230)	(.0859)
$I_{st}(t)$	—	—	—	1.2157[a]	—	-1.9877[a]
	—	—	—	(.3300)	—	(.4645)
$K_{st}(t)$	—	.0086	—	-.2343[a]	.1427[a]	.3787[a]
	—	(.0105)	—	(.0423)	(.0168)	(.0568)
$I_{in}(t)$	—	—	—	.6224[a]	-.3138[a]	—
	—	—	—	(.0533)	(.0700)	—
$K_{in}(t)$	—	—	.0391[a]	.2155[a]	-.0841	-.3318[b]
	—	—	(.0087)	(.0752)	(.0547)	(.1321)
$I_{eq}(t-1)$	.8191[a]	—	—	-.0066	—	—
	(.1213)	—	—	(.0789)	—	—
$I_{st}(t-1)$	—	.9223[a]	—	—	.2089	—
	—	(.1140)	—	—	(.1118)	—
$I_{in}(t-1)$	—	—	.1064	—	—	.0007
	—	—	(.1400)	—	—	(.0491)
m	1.7795	2.0914	3.8745	.0121	-.6636	-.3819
Res. Sum. Sq.	.5888	.0846	3.3511	.1961	.0525	1.0189
Criterion		{61.364}			{21.560}	
Simulations ($dK/d\Omega$)						
second year	.0115	.0065	.0149	.0212	.0088	.0357
fifth year	.0392	.0241	.0618	.0856	.0659	.0557
tenth year	.0680	.0483	.1486	.1355[c]	.1624[c]	.0707[c]

*Estimates based on equation (17). The dependent variable is indicated by the column heading. All explanatory variables are scaled by the capital stock for the dependent variable; thus, for a given equation, the corresponding capital stock represents the constant term. Sample period 1950-1978. Standard errors in parentheses. Residual Sum of Squares multiplied by 10^{-2}.

[a] Significant at the 1% level. [b] Significant at the 5% level. [c] The insignificant coefficient set to zero.

TABLE V

CONVENTIONAL AND MULTI-CAPITAL Q MODELS

THREE STAGE LEAST SQUARES ESTIMATES*

Variable or Statistic	Conventional		Multi-Capital			
	EQ	ST	EQ	ST	EQ	ST
	(1)	(2)	(3)	(4)	(5)	(6)
$\Omega(t)$	.0041[b]	.0023[b]	-.0046[b]	.0030[a]	.0085[b]	.0024[a]
	(.0019)	(.0009)	(.0020)	(.0008)	(.0036)	(.0007)
$I_{eq}(t)$	—	—	—	.5087[a]	—	.1050[b]
	—	—	—	(.1086)	—	(.0455)
$K_{eq}(t)$	.0332	—	.1481[b]	-.0935[a]	-.0484	-.0473[a]
	(.0260)	—	(.0571)	(.0239)	(.0846)	(.0123)
$I_{st}(t)$	—	—	1.3206[a]	—	-1.0474	—
	—	—	(.3713)	—	(.7165)	—
$K_{st}(t)$	—	.0085	-.2544[a]	.1356[a]	.1395	.0658[a]
	—	(.0106)	(.0530)	(.0186)	(.1020)	(.0138)
$I_{in}(t)$	—	—	.6231[a]	-.2928[a]	—	—
	—	—	(.0760)	(.0774)	—	—
$K_{in}(t)$	—	—	.2472[a]	-.0857	—	—
	—	—	(.0860)	(.0557)	—	—
$I_{eq}(t-1)$	.8503[a]	—	-.1159	—	1.0654[a]	—
	(.1514)	—	(.1724)	—	(.2830)	—
$I_{st}(t-1)$	—	.9236[a]	—	.2605[b]	—	.5876[a]
	—	(.1151)	—	(.1273)	—	(.1033)
$I_{in}(t-1)$	—	—	—	—	—	—
	—	—	—	—	—	—
m	1.6505	2.2302	.4390	-.6971	1.4557	1.0480
Res. Sum. Sq.	.5910	.0847	.2022	.0489	.7537	.0421
Criterion	{40.631}		{20.723}		{35.883}	
Simulations ($dK_i/d\Omega$)						
second year	.0115	.0065	-.0055	.0057	.0152	.0083
fifth year	.0393	.0244	-.0278	.0239	.0518	.0303
tenth year	.0690	.0498	-.0856	.0618	.0922	.0600

*Estimates based on equation (17). The dependent variable is indicated by the column heading. All explanatory variables are scaled by the capital stock for the dependent variable; thus, for a given equation, the corresponding capital stock represents the constant term. Sample period 1950-1978. Standard errors in parentheses. Residual Sum of Squares multiplied by 10^{-2}.
[a] Significant at the 1% level. [b] Significant at the 5% level.