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# GROWTH, IMPORT DEPENDENCE AND WAR

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## ABSTRACT

Existing theories of pre-emptive war typically predict that the leading country may choose to launch a war on a follower who is catching up, since the follower cannot credibly commit to not use their increased power in the future. But it was Japan who launched a war against the West in 1941, not the West that pre-emptively attacked Japan. Similarly, many have argued that trade makes war less likely, yet World War I erupted at a time of unprecedented globalization. This paper develops a theoretical model of the relationship between trade and war which can help to explain both these observations. Dependence on strategic imports can lead follower nations to launch pre-emptive wars when they are potentially subject to blockade.

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# 1 Introduction

This paper develops a model of trade and war that speaks to two distinct literatures. The first is the literature on whether or not trade helps reduce the likelihood of warfare. The argument that it does so sits uneasily with the observation that World War I erupted at a time of unprecedented globalization. The second is the literature on war between established and rising powers. A typical prediction is that the established power (or leader) may launch a pre-emptive war against the rising power (or follower), since the latter cannot credibly commit to not use their increased power in the future. And yet it was Japan who attacked the West in 1941, not vice versa.

Our model can help to resolve both apparent paradoxes. We show that import dependence can lead a follower country to lauch pre-emptive wars against the leader if two conditions hold. First, import dependence must increase over time. Second, the country must be vulnerable to blockade in the event of war. The model can be regarded as a formalization of arguments about trade and war made by some realist scholars in the international relations literature.

Ours is a model of hegemonic war, and hegemonic wars are too infrequent for our arguments to be testable econometrically. We therefore provide a brief historical narrative in which we show how our model can help to make sense of three historical episodes: Anglo-German rivalry prior to World War I; Hitler's expansionist ambitions, and his decision to attack the Soviet Union in 1941; and Japan's decision to attack the West later in the same year. Our model formalizes some of the arguments made about these three episodes by prominent historians: Avner Offer's book on Anglo-German rivalry (Offer 1989), Adam Tooze's book on the Nazi German war economy (Tooze 2006), and Michael Barnhart's book on Japan's "preparation for total war"(Barnhart 1987).

We are sure that none of these historians would argue that the mechanism that we describe here "explains" any of these three conflicts in some monocausal way. Lest there be any misunderstanding on the subject, we do not make such a claim either: the origins of the first and second world wars were much too complicated to be "explained" by this or any other formal model. Our model has just two players, but there were many players involved in these conflicts (and so a country like Germany could be a follower relative to the UK, but a leader relative to Russia).

It assumes that conflict is motivated by just one cause (a "pie"which both players are struggling to obtain), but international rivalries in the 1910s and 1930s were multi-dimensional. It assumes that countries can be modelled as unitary actors, but internal divisions were important in Wilhelmine Germany, Imperial Japan and elsewhere. And it assumes rationality, even though many important actors in these three episodes were motivated by sentiments such as honour and dignity, or by racial or religious prejudice, or were over-optimistic about their chances in a war, or under-estimated their opponents.

Nevertheless, we hope to convince the reader that the mechanism described by our model was one factor among many at work during these three episodes, and that trade dependence can sometimes make war more rather than less likely. We should not expect an economic model to be able to explain on its own something as complicated as the outbreak of a world war, but this does not mean that it has nothing to tell us about the past, or that it cannot provide us with lessons that may be useful in the future.

## 1.1 Trade and war

The optimistic, liberal argument that international trade promotes peace is ancient but controversial (see e.g. Barbieri 1996, Rowe 2005, McDonald and Sweeney 2007, Martin, Mayer and Thoenig 2008 and Harrison and Wolf 2012). One objection is that trade can make countries dependent on others, and therefore vulnerable, in the context of an anarchic world in which countries have fundamentally different interests. In the words of John Mearsheimer, "states will struggle to escape the vulnerability that interdependence creates, in order to bolster their national security. States that depend on others for critical economic supplies will fear cutoff or blackmail in time of crisis or war; they may try to extend political control to the source of supply, giving rise to conflict with the source or with its other customers" (Mearsheimer 1990, p. 45). There is a critical difference between international and domestic trade, argues Kenneth Waltz: regions within a country "are free to specialize because they have no reason to fear the increased interdependence that goes with specialization", whereas in an anarchic world, states may fear specialization on the grounds that their potential competitors may gain more than they do, or because trade makes them "dependent on others through cooperative endeavors and exchanges of goods and services" (Waltz 2006, pp. 104, 106; see also Gilpin 1981, p. 220).

There is also a large literature on hegemonic wars between rising challengers and dominant powers (Gilpin 1981). Our paper develops a model of trade and hegemonic warfare, in the tradition of recent papers on "rationalist explanations for war" (Fearon 1995, Powell 2006). These start from the premise that wars are costly, and that rational unitary states in dispute with each other should be able to bargain their way to compromises that leave both better off (in probabilistic terms) than they would be in the event that war breaks out. Powell (2006) argues that wars can nevertheless arise as a result of commitment problems. He does so in the context of models in which a pie has to be divided between countries in a setting where (1) countries cannot pre-commit to particular divisions of the pie in the future; (2) countries have the option to launch a war to "lock in" an expected share of future flows; (3) wars are costly, in that they reduce the overall size of the pie; and (4) the distribution of power, which affects how much of the pie countries can lock in, exogenously changes over time (p. 181). For example, consider the case in which a follower exogenously catches up on a leader (Fearon 1995). The follower has an incentive to forestall a pre-emptive war by the leader, by promising the leader a sufficiently big slice of the pie in the future. Since it cannot pre-commit to this, and has an incentive to use its greater power in the future to secure a greater share of the pie, the leader may chose to launch a pre-emptive war in order to lock in a higher share of the spoils while it still has the chance.

In our model, we find that it is the follower who may declare war on the leader. International trade, and the opportunities and vulnerabilities which it implies, are central to establishing this otherwise counter-intuitive result. Central to our analysis is the assumption that the follower needs to import raw materials from the rest of the world.

We model the link between growth and changes in the distribution of power in a context in which the follower becomes increasingly dependent on imported raw materials. We assume that the leader, as befits the hegemon, can control the follower's access to imported raw materials, either because it controls the sources of supply (via formal or informal empire), or because it controls world shipping lanes and can mount a blockade of the follower. We show that if dependence on imported raw materials increases over time, the follower can become militarily weaker and not stronger, even if it is growing more rapidly, and can therefore have an incentive to start a pre-emptive war. International trade can thus be crucial in determining the likelihood of war.

While we borrow our basic theoretical mechanism from the existing literature (Powell 2006), our application of these ideas is novel. Furthermore, in our setup both the leader and the follower also care about consumption, allowing us to endogenise the share of their GDP that countries wish to devote to their armed forces. The paper closest in spirit to ours is Copeland (1996), who constructs a similar argument in which pessimistic expectations of future trade levels can lead trade-dependent countries to declare war. Our contribution is different from his, in that we provide a formal theoretical analysis, which he does not. This means among other things that we can endogenously figure out where these trade expectations come from. We also tell a story in which the processes of catch-up and structural change, and the strategic nature of trade, play central roles.[1]

# 2 Model description

We consider a world with two industrial countries, $L$ and $F$ (for "Leader" and "Follower"), and a third resource-rich country $C$. In each country, there is a large number of agents, allowing markets to be competitive. There is an infinite number of periods, indexed by $t = 1, ..., \infty$.

Agents everywhere care about consumption of a final good, $z$. In addition, in $L$ and $F$, agents also care about consumption of a "pie", $p$, which we may interpret as a range of contested issues

[1]There is a growing literature on the relationship between trade and war. Glick and Taylor (2010) estimate the impact of war on trade flows, and find that it is large. Acemoglu et al. (2012) present a dynamic model of resource trade and war, focussing on how, in the presence of an inelastic demand for resources, progressive depletion may increase the value of a resource-rich region, thus increasing the incentives for a resource-scarce country to invade the country the region belongs to (and thus appropriate the resource). They study how different market structures in the natural resource industry - perfectly competitive, or monopolistically controlled by the government of the resource-rich country - may be associated with different probabilities of war. While the main focus of their paper is on wars between resource-rich and resource-scarce countries, ours is on wars between resource-scarce industrialized countries. Caselli et al. (2015) find that war between pairs of countries is more likely when at least one country has natural resources, and when these are located near borders. Finally, a series of papers by Stergios Skaperdas and co-authors (see Garfinkel et al. 2012 for a good overview) study the pattern and welfare implications of trade in a context in which two countries may fight over a contested region. The focus of these papers is different from our own: they present static models of the impact of trade (between the two countries and the rest of the world) on the incentives for the two countries to arm and go to war over the contested region. Ours is a dynamic model of trade between the two countries and the rest of the world, where the dynamics of relative power and trade dependence determine the likelihood of war.

that the two countries must settle. Preferences in $L$ and $F$ are described by period $t$ utility

$$u_t^J = z_t^J + p_t^J,$$

where $z_t^J$ and $p_t^J$ denote, respectively, consumption of the final good and the pie by the representative agent in country $J \in \{L, F\}$. The present discounted value of utility in $J$ is

$$U_t^J = Z_t^J + P_t^J, \tag{1}$$

where $Z_t^J = \sum_{s=t}^{\infty} \delta^{s-t} z_s^J$, $P_t^J = \sum_{s=t}^{\infty} \delta^{s-t} p_s^J$, and $\delta < 1$ is the discount factor. Period $t$ and present discounted value utility in $C$ are similarly equal to $z_t^C$ and $Z_t^C$. However, $C$ does not make any strategic decisions in our model: it is $L$ and $F$ who compete for the pie, and whose decisions determine whether or not there will be a war.[2]

In both $L$ and $F$, social planners maximise eq. (1). The essential tradeoff they face is that resources can be allocated either to the production of the final consumption good, $z$, or to the production of an *army*. Armies are not valuable per se, but are useful in securing a greater share of the pie. The planners thus face a trade off between the consumption of $z$ and of $p$. In this paper, we develop a model of the strategic interaction of the two planners over an infinite number of discrete periods, as they attempt to maximise eq. (1). In each period, the planners first simultaneously set the size of their armies. Next, they decide how to share the pie (by going to war, or through peaceful negotiations). Finally, given the planners' arming and war decisions, production of the final good takes place, the pie is allocated, and consumption is realised.

We begin by describing how the final good and army are produced, as well as the two economies' endowments (Section 2.1); the way in which the pie is divided (2.2); the exact timing of actions (2.3); and the equilibrium concept used in the paper (2.4). The equilibrium of the game is then characterised in Sections 3 and 4. For brevity, we use "$L$" and "$F$" as a short-hand for "$L$'s social planner" and "$F$'s social planner".

We use the following notation. As above, a lower case latin letter, e.g. $x_t^J$, denotes the value

[2] Allowing $C$ to make strategic decisions could be allowed for in an extension. For example, $C$ could be allowed to manipulate the price of raw materials, and thus influence the likelihood of war.

of a variable $x$ in country $J \in \{L, F\}$ in period $t$, while $X_t^J = \sum_{s=t}^{\infty} \delta^{s-t} x_s^J$, denotes the present discounted value of that variable from period $t$ to infinity. Sums of variables across the two countries lose the superscript, e.g. $x_t = x_t^L + x_t^F$. Finally, greek letters denote parameters, with lower and upper cases having the same meanings as above.

## 2.1 Economic environment

The final good is competitively produced using a non-traded "industrial input" $y$ and "raw materials" $x$. The industrial input can be interpreted as all productive inputs (capital, labour, land) that need to be combined with raw materials to produce GDP.[3] Production of one unit of $z$ requires exactly one unit of each input. Then, if $\left(y_t^J\right)_z$ and $\left(x_t^J\right)_z$ units of the two inputs are allocated to the production of $z$ in country $J$ in period $t$, national production of $z$ is

$$\min\left[\left(y_t^J\right)_z, \left(x_t^J\right)_z\right].$$

The industrial input is not produced but something with which economies are endowed. Raw material supplies are also given by endowments. Endowments evolve over time, following an exogenous growth process described below. We choose $z$ as the numéraire. All owners of endowments are small enough to be price takers.

We interpret country $L$ as an industrial leader that, by the beginning of period 1, has completed its process of structural transformation, and whose economy grows at a constant, steady-state rate in all sectors. In contrast, $F$ is a follower that is still undergoing structural transformation in period 1, and only reaches steady state in period 2. By "structural transformation" we mean that $F$ is undergoing catch-up growth, and reallocating resources from the primary sector to the industrial sector. Its industrial inputs, then, initially grow faster than in steady state, while its raw materials sector (here modelled simply as an endowment of raw materials available in every period) grows more slowly (and possibly at a negative rate). Finally, we assume that, in all periods, $F$ is scarce in raw materials. To capture all this, we assume the following endowments

[3] It may also include raw materials that exist in abundant supply domestically, in which case $x$ would represent raw materials that need to be at least partially imported.

of the two inputs in $L$ and $F$:

$$\begin{aligned} y_1^L &= 1 \quad & y_2^L &= \gamma \\ x_1^L &= \rho \quad & x_2^L &= \rho\gamma \\ y_1^F &= \underline{\alpha} \quad & y_2^F &= \overline{\alpha}\gamma \\ x_1^F &= \overline{\beta} \quad & x_2^F &= \underline{\beta}\gamma, \end{aligned} \tag{2}$$

where $\gamma \geq 1$ is steady state growth, $\rho \geq 0$ captures the availability of raw materials in $L$, $0 < \underline{\alpha} \leq \overline{\alpha}$ and $\overline{\beta} \geq \underline{\beta} \geq 0$ capture structural transformation in $F$, and we assume $\underline{\alpha} \geq \overline{\beta}$ to make sure that $F$ is scarce in raw materials.[4] After period 2, all endowments grow at a constant rate $\gamma$. Note that we have normalised the initial size of $L$'s economy to 1, while $F$'s economy can have any initial size ($\underline{\alpha}$ unconstrained). As for $C$, we interpret it as a peripheral country that is abundant in raw materials. It is a large economy, relative to both $L$ and $F$. It produces both the final consumption good and raw materials, and the world relative price of raw materials in terms of the consumption good, $\eta \in [0, 1)$, is determined there. There are no transportation costs, and $L$ and $F$ can exchange unlimited quantities of raw materials for the final consumption good, or vice versa, in $C$'s markets at this fixed relative price.

All goods are tradable internationally, except for $y$ which is non-tradable. This implies that, given its scarce domestic supply, $F$ will import raw materials, and export the final good in return. If $\rho \in [0, 1)$, $L$ will have similar trade patterns, and both industrial countries will import raw materials from $C$. If $\rho > 1$, $L$ is abundant in raw materials, and exports these in exchange for imports of $z$. $F$ will import from either $C$ or $L$ (or both). Given that raw materials cost $\eta$, perfect competition in production of the final good and the fact that $z$ is the numéraire together imply that the equilibrium price of $y$ will be $1 - \eta$.[5]

In $L$ and $F$, the planner can divert resources from production of the final good to production

[4]If we interpret this environment through the lens of the Solow model, steady state growth is driven by capital accumulation, technological progress, and population growth, and faster growth during catching up is driven by faster capital accumulation in this phase. As for growth in the endowment of raw materials, steady state growth could be driven by a combination of technological progress and an exogenous process of discovery/depletion, while slower growth during catching up could be driven by a transfer of resources into industry.

[5]An increase in $\eta$ has two effects on the economy of an importing country: it increases the cost of imports, reducing national income; and it redistributes income from owners of the industrial input to owners of raw materials.

of an army, $a$. While the army does not increase utility directly, it may do so indirectly by increasing the portion of the pie that a country is able to obtain. If $\left(y_t^J\right)_a$ and $\left(x_t^J\right)_a$ units of the two inputs are allocated to the production of $a$, then an army of size

$$a_t^J = \frac{1}{c_t^J} \min\left[\left(y_t^J\right)_a, \left(x_t^J\right)_a\right]$$

is produced.[6] We assume

$$c_t^J = 1/y_t^J : \tag{3}$$

a more advanced country has a lower cost of producing an army, relative to the final good, perhaps because of a superior technology. For example, a more advanced country could have a technology that yields a more powerful army for given military expenditure. We denote military expenditure by $m_t^J = c_t^J a_t^J$.

## 2.2 Political environment

Our model follows closely the model of pre-emptive war in Powell (2006). In every period, there is a pie that the two countries must partition. The pie has size $\pi_1 > 0$ in period 1, and grows at a constant rate $\gamma$ in all periods after that.[7] The partition of the pie can be done in two ways. On the one hand, in every period $t$ in which there has been no previous war (thus, at least in period 1), the two countries may try to negotiate a *peaceful partition* of the pie involving $J$ getting a share $s_t^J$. Alternatively, they may go to *war*. This is won by $J$ with probability

$$q_t^J = \frac{a_t^J}{a_t^L + a_t^F},$$

[6]The planner could either appropriate the inputs directly, and produce the army by itself, or impose a lump sum tax on final good consumption, and use it to purchase the army from the private sector. Both interpretations, as well as a mixture of the two, are consistent with our model.

[7]The pie may represent a range of contested issues that $L$ and $F$ must settle. These could be non-economic issues, such as the division of territory that matters purely for matters of prestige, or issues that arise because of ideological concerns. Or they could be economic issues, such as the division of territories with an economic value.

and by $-J$ with reciprocal probability $1 - q_t^J$. The war gives the winner the entire current and all future pies. However, war also costs a share $\kappa \in [0, 1)$ of the present discounted value of all pies. In summary, war implies that the present discounted value of consumption of the pie, $P_t^J$, will be equal to $q_t^J \Pi_t (1 - \kappa)$, while peaceful partition implies that $p_t^J$ will be given by $s_t^J \pi_t$, with $P_{t+1}^J$ remaining to be determined in subsequent periods.

Negotiations to reach a peaceful partition work as follows. First, $L$ decides whether to enter negotiations, or to immediately start a war. In the former case, it offers $F$ a share $s_t^F$ of the current pie (so that a share $s_t^L = 1 - s_t^F$ would remain for itself). Given this offer, $F$ decides whether to accept, or to reject and start a war. If it accepts, the pie is peacefully partitioned, and the two countries move on to the next period.[8]

Note that, while war allocates the entire future stream of pies, negotiatiors cannot commit to the sharing of future pies. This *lack of commitment* is the key friction in the model, which may lead to a welfare-reducing war occurring in equilibrium.[9] To see why, suppose that a country expects to become weaker over time. Then, it knows that, unless it secures the future pies by winning a war today, it will get little of them as a result of future negotiations or conflict. Since lack of commitment prevents today's negotiators from alleviating this country's concerns, it may start a war even if, in principle, there is an overall surplus that the two parties could share. It turns out that, in equilibrium, such an inefficient war can only occur in period 1, the period in which structural transformation leads to a shift in relative power between $L$ and $F$.

As with the production of the final good, producing an army relies on raw materials, which may have to be imported. This potential dependence of the army on international trade makes it important to specify the effect of war on the two countries' capacity to trade. In this paper, we consider two alternative cases. The first is a symmetric case in which war does not affect the capacity of either country to trade. In this case, dependence on imported raw materials does not matter for relative military power. The second case is an asymmetric one, in which $L$

[8]Note that this structure of negotiations allocates all of the bargaining power to $L$. We have assumed this extreme distribution of bargaining power just for simplicity: to relax this assumption would not qualitatively change our results.

[9]As we will see, war can also be "welfare-increasing" in this model since it implies that no future military expenditures will be undertaken. Since our interest is in welfare-reducing war, we will rule this possibility out in Section 3.5 and subsequently.

may blockade $F$ in times of war, but not the other way around. This involves both $F$'s trade with $L$ (if any), and $F$'s trade with $C$. We refer to this second case as "$L$ having the capacity to blockade". It is easy to show that, if $L$ has the capacity to blockade, it always uses it in times of war. Intuitively, the disruption of $F$'s trade does not carry a direct economic cost for $L$, since it can still trade with $C$. On the other hand, as clarified below, to blockade $F$ can reduce the latter country's probability of winning the war. Thus, $L$ having the capacity to blockade is synonymous with $L$ blockading $F$ in times of war.[10] We believe this second case is an important one, since hegemonic countries may develop a naval superiority that allows them to control trade routes in case of conflict. In this second case, $F$'s dependence on imported raw materials may have important consequences for relative military power.[11] We define a state variable $\mathcal{B}$, which is one if and only if $L$ has the capacity to blockade.

An indicator variable $w_t^J$ is one if and only if country $J$ starts a war in period $t$. Then, $w_t \equiv w_t^L + w_t^F$ is one if and only if a war occurs in period $t$. We also define a state variable $\mathcal{W}_t^J$, which is one if and only if $J$ has *won* a war in some previous period $T < t$. Then, $\mathcal{W}_t \equiv \mathcal{W}_t^L + \mathcal{W}_t^F$ is a state variable indicating whether or not war has already occurred in period $t$.

## 2.3 Timing

Each period $t$ can be divided into three sub-periods, during which the following events take place:

t.1 $L$ and $F$ simultaneously set $a_t^L$ and $a_t^F$.

[10] We do not consider the possibility that $L$ uses the capacity to blockade in times of peace. An obvious justification for this assumption is that a blockade could, in itself, be regarded as an act of war.

[11] The capacity to blockade could be thought of as arising in two ways. It could arise in the context of a world in which $C$ remains independent, but in which $L$ gains control over the trade routes linking $C$ to its industrial rival. In this interpretation, the key determinant of the capacity to blockade is the relative size of the countries' navies: $L$ will have the largest navy, and will then have the ability to blockade $F$ (but not vice-versa). The capacity to blockade could also arise in a world in which $L$ gained colonial control over $C$. Colonial control would give $L$ the power to deprive its rival of the ability to import raw materials, which is what a blockade means in the context of our model. We think that the first interpretation is more consistent with the structure of our model. Our assumption is that the capacity to blockade is indivisible, and is therefore affected by war in a way that it cannot be by peaceful negotiations between the two countries. If $L$'s capacity to blockade originated from the control of colonial empires, it would be quite hard to argue for its indivisibility, since colonial empires can be divided in many different ways. In contrast, negotiations over naval power are much more discontinuous in nature - a navy is either dominant, or it is not - and so it is possible that the *expected* impact of war on naval power cannot be obtained through peaceful negotiations.

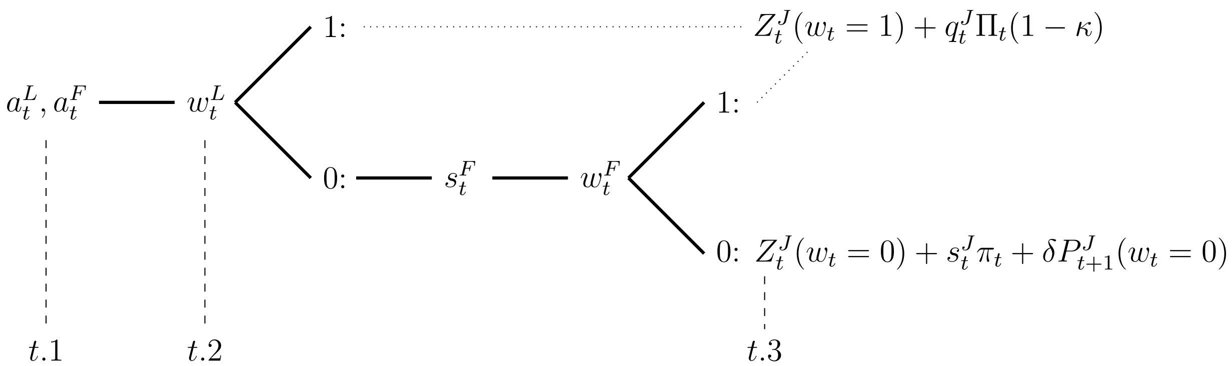
t.2 If there has been a war at some $T < t$, the winner gets the entire period $t$ pie. If there hasn't been a war, $L$ can either make an offer $s_t^F$ on how to share the period $t$ pie, or start a war ($w_t^L = 1$). If it makes an offer, $F$ may either accept, in which case the pie is peacefully partitioned, or reject the offer and start a war ($w_t^F = 1$).

t.3 Trade and production take place (if someone has started a war, i.e. $w_t = 1$, and $L$ has the capacity to blockade, i.e. $\mathcal{B} = 1$, then $F$ cannot trade). After production has taken place, if someone has started a war, it now occurs. Finally, consumption takes place.

## 2.4 Definition of equilibrium

We focus on Markov-perfect Subgame Perfect Nash Equilibra (SPNE). Then, in each period, all relevant information about the previous history is summarised by the state variable $\mathcal{W}_t$, which specifies whether or not war has already occurred. That is to say, given $\mathcal{W}_t$, equilibrium strategies must prescribe optimal actions for each possible action played in all previous sub-periods only.

In period $t$, in any node such that $\mathcal{W}_t = 0$, the structure of the game is as follows




where the expressions at the end of each branch denote payoffs, and $Z_t^J(w_t = 1)$ and $Z_t^J(w_t = 0)$ are the present discounted values of consumption with and without war in period $t$. If there is war, each player gets its expected share of the pie arising from war, $q_t^J \Pi_t(1-\kappa)$. If there is no war, each player gets its peacefully negotiated share of the pie this period, plus the present discounted value of its share of the pie in the following period (which will depend among other things on whether there is a war in the following or subsequent periods). Equilibrium strategies must prescribe actions

$$\left(a_t^L\right)^*, \left[w_t^L(a_t^L, a_t^F)\right]^*, \left[s_t^F(a_t^L, a_t^F)\right]^* = \arg\max \left\{ w_t \left[ Z_t^L(w_t = 1) + q_t^L \Pi_t (1-\kappa) \right] + \right.$$
$$\left. + (1-w_t) \left[ Z_t^L(w_t = 0) + (1 - s_t^F)\pi_t + \delta P_{t+1}^L(w_t = 0) \right] \right\} \tag{4}$$

$$\left(a_t^F\right)^*, \left[w_t^F(a_t^L, a_t^F, s_t^F)\right]^* = \arg\max \left\{ w_t \left[ Z_t^F(w_t = 1) + q_t^F P_t (1-\kappa) \right] + \right.$$
$$\left. + (1-w_t) \left[ Z_t^F(w_t = 0) + s_t^F \pi_t + \delta P_{t+1}^F(w_t = 0) \right] \right\}. \tag{5}$$

The difference between these two expressions reflects the fact that only $L$ can offer a peaceful partition of the pie, $s_t^F$; $F$ has to take this as given. In any node such that $\mathcal{W}_t = 1$, equilibrium strategies must prescribe actions

$$\left(a_t^J\right)^* = \arg\max \left\{ Z_t^J + \mathcal{W}_t^J \Pi_t (1-\kappa) \right\}. \tag{6}$$

It is easy to anticipate that, in this second case, in which the allocation of all the pies has already been determined (it is given by the second expression on the right hand side of (6)), and the sole concern is to maximise $Z_t^J$, arming will always be set equal to zero in equilibrium.

To simplify the identification of a unique SPNE, we focus on a subset of equilibria which we call "balanced growth path SPNEs". These are defined by

**Definition 1.** *A balanced growth path SPNE is a SPNE in which, in $t \geq 2$ where war has not yet occurred, if war does not occur, then $m_{t+1}^J = \gamma m_t^J$ for $J \in \{L, F\}$.*

By focusing on balanced-growth path SPNEs, we impose the requirement that, from period 2 onwards and until there is a war (if ever), military expenditures grow at rate $\gamma$. This is a reasonable restriction given that, from period 2 onwards, all relevant parameters for the arming decisions are scaled up by a factor $\gamma$ in every period. Our strategy is consistent with the standard approach in the growth literature, which is to focus on balanced growth paths only.

# 3 Preliminary results

In every period in which war has not yet occurred, the planners first allocate resources between producing the consumption good and the army (these decisions we will henceforth refer to as arming decisions). These decisions determine their bargaining power in negotiations, and whether or not they decide to go to war. Given arming decisions (taken in sub-period t.1) and given the occurrence or non-occurrence of war (decided in sub-period t.2), agents optimally trade, produce, and consume (in sub-period t.3). To solve the game, we need to derive optimal arming and war decisions in every period. This is complicated, since it depends on dynamic calculations about future behaviour in both countries. We therefore proceed in steps, as follows.

In Section 3.1, we begin by deriving equilibrium consumption, determined in sub-period t.3, given arming decisions and given the occurrence or non-occurrence of war (determined in the previous two sub-periods). Next, we turn to optimal war decisions, given arming decisions in the previous sub-period. Finally, we consider optimal arming decisions.

In order to decide whether to go to war or not, planners need to compare payoffs with and without war. These payoffs depend both on consumption and on the share of the pie. Payoffs with war are relatively easy to find, since: the war determines who will get the pie in all subsequent periods; the war means that there will be no future wars, and that optimal arming will be zero in subsequent periods; and we have already determined optimal consumption given arming and war decisions in Section 3.1. We present these payoffs, for given arming decisions in the previous sub-period, in Section 3.2. Payoffs without war are harder to calculate, as they depend on future arming and war decisions, which are themselves part of the equilibrium to be determined. We proceed in several steps: the remainder of Section 3 establishes some essential preliminary findings, before our main results are established in Section 4.[12]

In a first step, in Section 3.3, we derive optimal arming in a period when there is war. This arming decision is important for two reasons. First, it allows us to completely specify payoffs if there is a war, given the results in Section 3.2. Second, it turns out that arming decisions with peace from period 2 onwards are the same as if there were war.[13] Having derived this

[12] A formal proof of our results is contained in Online Appendix B.

[13] This will be established in Section 3.6.

optimal arming decision, we can now track the evolution of relative power over time (Section 3.4). In a third step, in Section 3.5, we impose restrictions on the parameter space to rule out an uninteresting case in which war must always occur. This case arises because of the channel identified by Garfinkel and Skaperdas (2000): by going to war now, countries can save on future military expenditure. This channel implies that, if the cost of war $\kappa$ is low enough, the *effective* cost of war may be negative (that is, war may be welfare increasing), in which case war must occur. In order to focus on our own channel, this possibility will be ruled out by assuming $\kappa \geq \hat{\kappa}$, where $\hat{\kappa}$ is a threshold cost of war between zero and one. In a fourth step, in Section 3.6, we present a lemma (Lemma 3) showing that, in the remaining parameter space, if there is no war in period 1, war will never occur, but that (as previously mentioned) countries will continue to arm in every period as if there were war.

Lemma 3 gives us everything we need to calculate payoffs without war in period 1, given period 1 arming decisions, while Sections 3.1-3.3 give us everything we need to calculate payoffs with war in period 1, again taking that period's arming decisions as given. In Section 4, therefore, we can finally turn to optimal war decisions in period 1, by comparing the payoffs with and without war (taking arming decisions in period 1 as given). Finally, we complete the derivation of the equilibrium by calculating optimal arming decisions in period 1. Because our goal is to show that war *may* occur, to simplify the analysis we focus on the case in which the effective cost of war is close to zero ($\kappa$ is close to $\hat{\kappa}$).

## 3.1 Equilibrium consumption given arming and war decisions

The payoffs of both $L$ and $F$ depend in part on their consumption of the final good. It is therefore useful to begin by showing how equilibrium consumption in $t.3$ depends on war and arming decisions, taken in $t.2$ and $t.1$ respectively.

Suppose that, at time $t$, war has not yet occurred ($\mathcal{W}_t = 0$). If no war is started in $t.2$ ($w_t = 0$), international trade is not disrupted. Given abundance of $x$ in the world as a whole, endowments of $y$ must be fully utilised in equilibrium. Then, in $F$, $y_t^F - x_t^F$ units of raw materials must be imported at a price of $\eta$. Balanced trade requires that $F$ export $i_t^F$ units of the final good, where $i_t^F$, the net consumption cost of imports, is given by

$$i_t^F = \eta \left( y_t^F - x_t^F \right).$$

At least $i_t^F$ of the final good must be produced, and arming decisions, $a_t^F$, must have taken this into account: it must be the case that $c_t^F a_t^F \leq y_t^F - i_t^F$.[14] Given arming decisions, resource utilisation by the army is

$$\begin{aligned}
(y_t^F)_a &= c_t^F a_t^F \\
(x_t^F)_a &= c_t^F a_t^F,
\end{aligned}$$

and resource utilisation in the production of the final good,

$$\begin{aligned}
(y_t^F)_z &= y_t^F - c_t^F a_t^F \\
(x_t^F)_z &= y_t^F - c_t^F a_t^F.
\end{aligned}$$

It follows that consumption of the final good is

$$z_t^F = y_t^F - i_t^F - c_t^F a_t^F.$$

Turning now to $L$, there are two cases. If $y_t^L > x_t^L$, then $y_t^L - x_t^L$ units of raw materials must be imported, at a cost $\eta \left( y_t^L - x_t^L \right)$. If $y_t^L \leq x_t^L$, instead, $L$ is abundant in raw materials, and will export $x_t^L - y_t^L$ units to either $F$ or $C$. This raises $L$'s consumption of the final good by $\eta \left( x_t^L - y_t^L \right)$. It follows that the net consumption cost to $L$ of its trade in raw materials is

$$i_t^L = \eta \left( y_t^L - x_t^L \right), \tag{7}$$

[14] Any attempt to produce $\epsilon$ more units of army than $y_t^F - i_t^F$ would *reduce* the overall size of the army, as it would reduce by $\epsilon/\eta - \epsilon \geq 0$ the units of raw materials available to the army.

and that $L$'s consumption of the final good is

$$z_t^L = y_t^L - i_t^L - c_t^L a_t^L. \tag{8}$$

Suppose next that a war has been started in $t.2$ ($w_t = 1$). If $\mathcal{B} = 0$ ($L$ does not have the capacity to blockade), the war does not lead to any trade disruption. Then, consumption by both countries is still as above. If $\mathcal{B} = 1$, $F$'s imports are constrained to be zero. Given arming decisions, resource utilisation by the army is as above, while in the production of the final good we have

$$(y_t^F)_z = x_t^F - c_t^F a_t^F$$
$$(x_t^F)_z = x_t^F - c_t^F a_t^F.$$

Note that, if $\mathcal{B} = 1$, the maximum amount of raw materials available to $F$'s army in times of war is $x_t^F$. But since the army is only actually used in times of war, arming decisions must have taken this constraint into account: it must be the case that $c_t^F a_t^F \leq x_t^F$. This implies that, in this simple model, the loss of trade associated with a blockade only hits production of the final good. In other words, the blockade only constrains the *planned* size of the army, rather than the extent to which this plan is implemented.[15] Because blockades therefore hit production and consumption of the final good, it is easy to derive an expression for the gains from trade for $F$, which are the same as $F$'s consumption loss in case of a blockade:[16] they are

$$g_t^F = (1-\eta)\left(y_t^F - x_t^F\right). \tag{9}$$

[15]If $c_t^F a_t^F \leq x_t^F$, there is always enough domestic raw materials to implement the planned size of the army. Note that the army may still rely on imports if there is no war (the model does not pin down the allocation of imported products), in which case war would result in a reallocation of domestic raw materials to military use.

[16]Gains from trade are easy to derive in this case, because the planned size of the army can be implemented even if imports are forced to be zero. It is then easy to quantify the loss of welfare associated with the suppression of foreign trade: it is simply equal to the loss of consumption of the final good. If instead the planned size of the army relied on imports, as might for example be the case if raw materials and industrial inputs were substitutes in the production of armies, their loss would result not only in a lower consumption of the final good, but also in a smaller army. The effect of the latter would depend on the political equilibrium, making the gains from trade much harder to calculate.

Putting together the various cases considered so far, $F$'s consumption of the final good can be written as

$$z_t^F = y_t^F - i_t^F - \mathcal{B} w_t g_t^F - c_t^F a_t^F. \tag{10}$$

Turning now to $L$, even if there is a blockade it can still trade with $C$. It follows that the net consumption cost of $L$'s trade in raw materials is still $i_t^L$, and $L$'s consumption of $z$ can still be written as in (8).

Finally, consider the case in which war has already occurred ($\mathcal{W}_t = 1$). Because war and blockades cannot occur anymore, endowments of $y$ must be fully utilised in period $t$. Consumption of the final good can still be written as in eq. (8) and (10), with $w_t = 0$. Thus, these two expressions denote equilibrium consumption in all cases.

## 3.2 Payoffs with war

Suppose again that war has not yet occurred in period $t$. We now derive payoffs if a country starts a war, taking arming decisions in the previous sub-period as given. To begin with, note that war may occur at most once. Then, if a war occurs in one period, we would expect that arming will be set to zero in all subsequent periods. This intuition is confirmed by

**Lemma 1.** *In a SPNE, in any $t > 1$, if war has already occurred, $\left(a_s^L\right)^* = 0$ and $\left(a_s^F\right)^* = 0$ for $s \geq t$.*

*Proof.* Take any $t > 1$, and suppose $\mathcal{W}_t = 1$. Substituting (8) and (10) into (6), we see that, in a SPNE, arming decisions must satisfy

$$\left(a_t^J\right)^* = \arg\max \left[ Y_t^J - I_t^J - \left( c_t^J a_t^J + \sum_{s=t+1}^{\infty} \delta^{s-t} c_s^J a_s^J \right) + \mathcal{W}_t^J \Pi(1-\kappa) \right].$$

Since $a_t^J$ only enters the maximands negatively, the solution is clearly $\left(a_t^L\right)^* = \left(a_t^F\right)^* = 0$. □

We are now ready to derive the payoffs if a country starts a war. Let these be denoted by

$V_t^J\left(w_t = 1|a_t^L, a_t^F\right)$. Using (8) and (10), together with Lemma 1, we can find $Z_t^J\left(w_t = 1\right)$. Then,

$$V_t^L\left(w_t = 1|a_t^L, a_t^F\right) = y_t^L - i_t^L - c_t^L a_t^L + \delta\left(Y_{t+1}^L - I_{t+1}^L\right) + \frac{a_t^L}{a_t^L + a_t^F}\Pi_t(1-\kappa) \tag{11}$$

$$V_t^F\left(w_t = 1|a_t^L, a_t^F\right) = y_t^F - i_t^F - \mathcal{B}g_t^F - c_t^F a_t^F + \delta\left(Y_{t+1}^F - I_{t+1}^F\right) + \frac{a_t^F}{a_t^L + a_t^F}\Pi_t(1-\kappa). \tag{12}$$

As discussed above, war implies destruction (the size of $\Pi_t$ is decreased by $\kappa$) and possibly trade disruption (if $\mathcal{B} = 1$, $F$'s consumption is decreased by $g_t^F$). However, it also gives the winner control of the contested resource for the rest of time ($\Pi_t$), and creates a peaceful world in which no further consumption is sacrificed to wasteful arming (no $c_s^J a_s^J$ is subtracted from payoffs in any period $s > t$).

Note that eq. (11) and (12) give payoffs from war, taking arming decisions in the previous sub-period as given. We still have to derive payoffs from peace, taking arming decisions as given; and derive optimal arming decisions. It is however convenient to first derive optimal arming decisions in a period when there is war.

## 3.3 Optimal arming in a period when there is war

Consider any period $t$ such that, in sub-period $t.2$, one country starts a war. If the occurrence of war was exogenously given, then arming decisions in $t.1$ would be extremely simple: anticipating the exogenous coming of war, countries would, in equilibrium, choose $a_t^L$ and $a_t^F$ that simultaneously maximise (11) and (12), subject to the relevant constraints. In reality, of course, the occurrence of war in $t.2$ is endogenous to arming decisions in $t.1$. However, we show in Online Appendix B that if, in a SPNE, war occurs in period $t$, then optimal arming levels must be precisely those that simultaneously maximise (11) and (12). Indeed, those arming levels turn out to be selected even in periods when there is peace.[17] In what follows we therefore derive those arming levels.

When selecting $a_t^L$ and $a_t^F$ that simultaneously maximise (11) and (12), countries face the two types of constraints discussed in Section 3.1. First, armies cannot use more than the available

[17]Section 3.6 shows that this is the case from period 2 onwards, while Section 4 shows that, in the case that we focus on, this is also true in period 1.

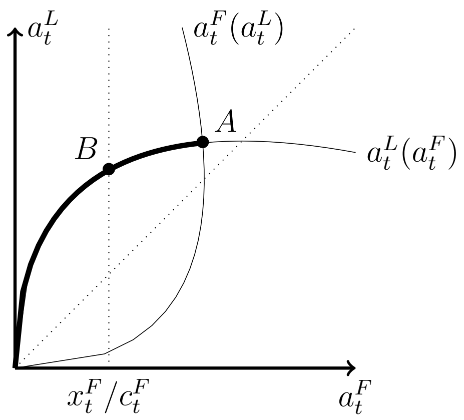



Figure 1: Best response functions, case $c_t^L < c_t^F$.

endowment of the industrial input, $c_t^J a_t^J \leq y_t^J - i_t^J$. Second, if $\mathcal{B} = 1$, $F$'s army cannot use more than the domestic endowment of raw materials, $c_t^F a_t^F \leq x_t^F$. To simplify, we assume that the former constraint is not binding.[18] That allows us to consider the impact of the second constraint.

Suppose first that not even the second constraint is binding. We are then looking for $a_t^L$ and $a_t^F$ that simultaneously maximise (11) and (12), subject to no constraint. Set $\partial V_t^J \left(w_t = 1 | a_t^L, a_t^F\right) / \partial a_t^J = 0$ for $J \in \{L, F\}$, then solve for $a_t^J$ as a function of $a_t^{-J}$. This yields the best response functions

$$a_t^L(a_t^F) = \left[\frac{\Pi_t(1-\kappa)}{c_t^L} a_t^F\right]^{\frac{1}{2}} - a_t^F$$
$$a_t^F(a_t^L) = \left[\frac{\Pi_t(1-\kappa)}{c_t^F} a_t^L\right]^{\frac{1}{2}} - a_t^L,$$

which are plotted in Figure 1 (drawn for the case $c_t^L < c_t^F$). Solving them together yields the unconstrained optimum,

$$\left(a_t^L\right)^{w,u} = \Pi_t(1-\kappa)\frac{c_t^F}{(c_t^L + c_t^F)^2} \tag{13}$$
$$\left(a_t^F\right)^{w,u} = \Pi_t(1-\kappa)\frac{c_t^L}{(c_t^L + c_t^F)^2}, \tag{14}$$

[18] This and all other assumptions will be satisfied in the numerical examples that we consider in Online Appendix C.

which is represented by point $A$ in the figure. Investment in arming increases in both countries if the net pie becomes bigger, or if the cost of arming falls proportionately everywhere. In addition, the country with the lower cost of arming ($L$ in the case of Figure 1) invests relatively more. At the unconstrained optimum, relative military power can be written as $q_t^F = c_t^L/(c_t^L + c_t^F)$, or

$$\left(q_t^F\right)^{w,u} = \frac{y_t^F}{y_t^L + y_t^F}. \tag{15}$$

In words, $F$ is relatively more powerful, the larger is its economy relative to $L$'s.

Next, suppose that the constraint $c_t^F a_t^F \leq x_t^F$ is binding: in other words, $\mathcal{B} = 1$, and $c_t^F \left(a_t^F\right)^{w,u} > x_t^F$. Optimal arming is then

$$\left(a_t^L\right)^{w,c} = \left[\frac{\Pi_t(1-\kappa)}{c_t^L} \frac{x_t^F}{c_t^F}\right]^{\frac{1}{2}} - \frac{x_t^F}{c_t^F} \tag{16}$$

$$\left(a_t^F\right)^{w,c} = \frac{x_t^F}{c_t^F}, \tag{17}$$

and is represented by point $B$ in the figure.[19] At the constrained optimum, relative military power can be written as $\left(q_t^F\right)^{w,c} = \left[\left(c_t^L/\Pi_t(1-\kappa)\right)\left(x_t^F/c_t^F\right)\right]^{\frac{1}{2}}$, or

$$\left(q_t^F\right)^{w,c} = \left(\frac{x_t^F}{\Pi_t(1-\kappa)} \frac{y_t^F}{y_t^L}\right)^{\frac{1}{2}}. \tag{18}$$

In words, $F$'s relative power is now constrained by domestic availability of raw materials, $x_t^F$. It is still increasing in the relative development of $F$'s economy, however, as this determines how efficient $F$'s army is in using available raw materials.

In what follows, we assume that, if $L$ has the capacity to blockade ($\mathcal{B} = 1$), $F$ is always constrained by its domestic endowment of raw materials ($c_t^F \left(a_t^F\right)^{w,u} > x_t^F$). Then, optimal

[19] To see that $B$ is a Nash equilibrium, note that $L$ is on its best response function; as for $F$, it is at its maximum allowed investment level, and below its unconstrained optimum: given that, as it is easy to show, $V_t^F\left(w_t = 1 | a_t^L, a_t^F\right)$ is strictly concave in $a_t^F$, this must be $F$'s best response. A similar logic can be used in the case $c_t^L \geq c_t^F$.

arming levels are

$$\left(a_t^J\right)^w = \begin{cases} \left(a_t^J\right)^{w,u} & \text{if} \quad \mathcal{B} = 0 \\ \left(a_t^J\right)^{w,c} & \text{if} \quad \mathcal{B} = 1 \end{cases}, \tag{19}$$

with $\left(q_t^J\right)^w$ and $\left(m_t^J\right)^w$ similarly defined.[20]

Let $V_t^J\left(w_t = 1\right)$ denote payoffs in a period where countries go to war. Putting together the results of this and the previous section, we can write

$$V_t^L\left(w_t = 1\right) \equiv V_t^L\left(w_t = 1 | \left(a_t^L\right)^w, \left(a_t^F\right)^w\right) \tag{20}$$

$$V_t^F\left(w_t = 1\right) \equiv V_t^F\left(w_t = 1 | \left(a_t^L\right)^w, \left(a_t^F\right)^w\right). \tag{21}$$

## 3.4 Evolution of relative military power

How does relative military power, $\left(q_t^F\right)^w$ evolve over time? Suppose first that $\mathcal{B} = 0$. The evolution of $\left(q_t^F\right)^w$ can be found by writing the RHS of (15) as a function of parameters in each period, as we do in Appendix A. It is represented in the left-hand panel of Figure 2. $F$'s relative power increases between period 1 and period 2, as $F$ catches up on $L$. It remains constant from period 2 onwards, when both countries grow at steady state rate.

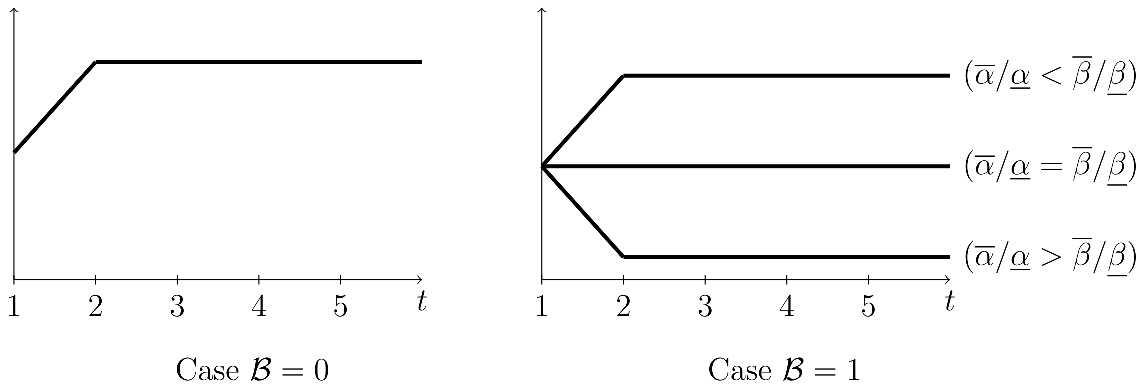



Figure 2: Evolution of $\left(q_t^F\right)^w$ over time.

If $\mathcal{B} = 1$, the evolution of $\left(q_t^F\right)^w$ can be found by writing the RHS of (18) as a function

[20]In Appendix A, we write $\left(a_t^J\right)^w$, $\left(q_t^J\right)^w$ and $\left(m_t^J\right)^w$ as functions of parameters. In steady state, $\left(a_t^J\right)^w$ grows at a rate $2\gamma$, $\left(m_t^J\right)^w$ at a rate $\gamma$ (so that it is a constant share of GDP), and $\left(q_t^F\right)^w$ is constant.

of parameters, and is represented in the figure's right-hand side panel. There are now three cases, depending on how $F$'s speed of catching up compares with the speed with which, during structural transformation, it becomes more dependent on imported raw materials. If $F$ catches up faster than it becomes more import-dependent ($\overline{\alpha}/\underline{\alpha} > \overline{\beta}/\underline{\beta}$), then its relative power increases between periods 1 and 2. However, at lower speeds of catching up ($\overline{\alpha}/\underline{\alpha} < \overline{\beta}/\underline{\beta}$), $F$'s relative power decreases. Intuitively, even though $F$ becomes more efficient at arming, its increased dependence on imported raw materials that are subject to blockade has a stronger, negative effect on its capacity to arm. In the knife edge case in which $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta}$ $F$'s relative power remains constant between periods 1 and 2.[21]

### 3.5 Ruling out welfare-increasing war

There are two distinct reasons why war may occur in this model. The first is the one highlighted by Garfinkel and Skaperdas (2000). As these authors pointed out, a desirable feature of war is that, by permanently allocating the pie to the winner, it removes the need to arm in future periods. In contrast, so long as there is peace, there is a pie that needs to be allocated in every period, and this forces countries to arm so as to strengthen their position in negotiations. Because arming is costly, this effect makes war more attractive for both countries. Indeed, if it is strong enough, war becomes welfare increasing, as the joint payoff of the two countries is higher with war than without. When this is the case, negotiations can never succeed, since the maximum that one country is willing to offer is less than the minimum that the other is willing to accept. War must then always occur.

In this paper, we want to focus on a second channel, in which war may occur as the result of trade-related shifts in relative power. We therefore want to rule out the case in which, because

[21] Allowing raw materials and the industrial input to be substitutes in production would imply that, when blockaded, $F$ could choose more $y$-intensive ways of producing both the final good and the army. As mentioned above (footnote 16), this would make it possible for it to choose an army that in peacetime would be produced with more raw materials than allowed for by domestic endowments alone. The possibility of substituting between the two inputs would alleviate the effect of a blockade on relative power, but would not fully eliminate it (provided that raw materials and the industrial input are not perfect substitutes) given that $F$ would still have to choose a sub-optimal vector of inputs. Because, in $F$, raw materials become relatively scarcer between period 1 and period 2, this country would find it increasingly hard to make up for lost imports by substituting $y$ for $x$: we would then find that, for some parameter values, $F$ would still become weaker between periods 1 and 2, which is what the central result of this paper hinges on.

of a high future cost of arming, war must always occur. As it turns out, this can be done by ruling out very low values of $\kappa$, the exogenous cost of war. When $\kappa$ is low, war is likely to be welfare increasing for two reasons. On the one hand, a low $\kappa$ means that the war has limited destructive effects. On the other hand, this implies that, in negotiations, the outside option of going to war is valuable: in turn, this induces countries to invest a lot in arming until there is war, in order to strengthen their position in negotiations. Indeed, given

**Assumption 1.** $\frac{\delta\gamma}{1-\delta\gamma}2\underline{\beta} > \mathcal{B}\left(1-\eta\right)\left(\overline{\alpha}-\underline{\beta}\right)$,

(an assumption that we further comment on in footnote 22) the following lemma establishes that war must occur immediately if $\kappa$ is sufficiently low, for any value of the parameters:

**Lemma 2.** *There exists $\hat{\kappa} \in (0,1)$ such that, if $\kappa < \hat{\kappa}$, in the unique balanced growth path SPNE, war is welfare increasing, and always occurs in period 1.*

*Proof.* In Online Appendix B. □

The threshold $\hat{\kappa}$ is such that, in period 1, the "effective" cost of war - that is, the cost of war net of savings related to the future cost of arming - is zero if $\kappa = \hat{\kappa}$. Intuitively, the fact that there is now a benefit from going to war pushes up the zero cost-of-war threshold, relative to a model with no costly arming where the threshold would be at $\kappa = 0$.[22]

To focus on our own channel, we impose

**Assumption 2.** $\kappa \geq \hat{\kappa}$.

Given Assumption 2, the rest of the paper focuses on the case in which, in equilibrium, war is welfare decreasing, or its effective cost is positive. As we show below, war will then only occur in the presence of shifts in relative power. Since such shifts will only occur between periods 1 and 2, this will imply that war can only occur in period 1.[23]

[22] If $\mathcal{B} = 1$, war has an additional cost, due to the fact that trade disruption occurs immediately, as opposed to at some future date. If this cost is large, the effective cost of war is negative for $\kappa = 0$, and the threshold $\hat{\kappa}$ is negative. The role of Assumption 1 is to rule out this case, by requiring that the discount rate be high (and that therefore the cost from anticipating trade disruption be low). It is desirable for the model to feature $\hat{\kappa} > 0$, since, as further explained below, this allows us to consider the case in which the effective cost of war is close to zero ($\kappa \to \hat{\kappa}$ from above).

[23] Assumption 2 implies that the future cost of arming will not on its own eliminate the bargaining range. On the other hand, in period 1, when war remains possible because of shifts in relative power, the future cost of arming will still be a determinant of the size of the bargaining range, as we will see below (see eq. 28).

## 3.6 Subgame starting in period 2

Before proceeding further, we introduce

**Assumption 3.** $\underline{\beta}\overline{\alpha} \geq \mathcal{B}(1-\eta)\left(\overline{\alpha}-\underline{\beta}\right)$.

Assumption 3 only poses a restriction on parameters if $\mathcal{B}=1$. It requires $F$'s economy to be large, relative to its gains from trade. The assumption is needed in order to ensure the existence of a balanced growth path SPNE of the game.[24]

The following lemma describes the SPNE of the subgame starting in period 2. For conciseness, we only present the equilibrium *path*. The full description of the equilibrium is presented in Online Appendix B.

**Lemma 3.** *Suppose war does not occur in period 1. In the unique balanced growth path SPNE of the subgame that starts in period 2, war never occurs. For all $t \geq 2$,*

- $a_t^L = \left(a_t^L\right)^w$ *and* $a_t^F = \left(a_t^F\right)^w$.
- *Negotiators agree on an allocation of the pie that leaves $F$ exactly as well off as with war.*

*Proof.* In Online Appendix B. □

If war is avoided in period 1, it does not occur anymore. Intuitively, negotiations can only fail if one country expects to become relatively weaker over time: the impossibility to commit to a future sharing of the pie can lead to a situation in which any offer is not good enough for this country. However, from period 2 onwards, the economies of $L$ and $F$ grow at the same rate. Then, their arming technology gets better at the same rate, and no shift in the balance of power is expected (see Figure 2). This is enough to ensure that the minimum share $F$ must be offered is less than the entire current pie, and that the maximum share $L$ is willing to offer is greater than zero. Negotiations must then succeed in every period.

[24] As stated in Lemma 3 below, in the unique equilibrium, $F$'s payoff is equal to its payoff from going to war, and $F$ arms so as to maximise this payoff. By requiring that the trade cost of war be moderate, Assumption 3 ensures that this payoff be positive: if it were negative, $F$ would choose not to arm, and the equilibrium would collapse. Full details are provided in the proof to Lemma 7 in Online Appendix B (which is referred to by the proof to Lemma 3)

Note that arming decisions with peace are the same as if there were war. This is because, in equilibrium, countries receive a payoff which is equal to their outside options, plus a share (which is one for $L$, and zero for $F$) of the surplus from not going to war. Since the outside options are payoffs with war, and neither the surplus nor the way it is shared depend on current arming levels,[25] countries arm so as to maximise their payoffs with war. Given that equilibrium arming is $\left(a_t^J\right)^w$, military expenditure is in both countries a constant share of GDP.

Also, note that $F$ is offered (and accepts) a share of the pie such that the entire surplus from not going to war is captured by $L$ in every period. Intuitively, by moving first, $L$ can offer $F$ the minimum it requires for not starting a war, and keep the rest of the surplus for itself.

# 4 Equilibrium

We finally turn to optimal decisions in period 1. We proceed in two steps. First we determine whether or not the two countries will go to war, taken arming decisions in period 1 as given. Next, we determine optimal arming decisions, allowing us to solve for the full equilibrium of the model, and to determine whether there will be war or not.

Payoffs with war were derived in (11)-(12) and Section 3.3. Payoffs without war, given period 1 arming decisions, can be derived using Lemma 3. According to the lemma, if war does not occur in period 1, it does not occur anymore. Furthermore, in period 2, $F$'s payoff is driven down to its payoff from going to war. It follows that $F$'s payoff with no war is equal to its current payoff in period 1, plus the discounted value of its payoff from going to war in period 2. Next, note that, if $F$ is driven down to its war-time payoff, then $L$'s payoff in period 2 must be equal to its payoff from going to war, plus the entire surplus from permanently avoiding war from period

[25] As shown in the proof to Lemma 3, and to Lemma 7 in Online Appendix B, the surplus from not going to war depends on future, not current arming levels. As for the way it is shared, provided that the bargaining range is entirely included in the interval $[0, 1]$ (which is ensured by Assumption 3), this only depends on the structure of negotiations. So, we could have assumed a different structure (for example, we could have assumed that countries bargain à la Nash), and the first point of Lemma 3 would be unchanged.

2 onwards. Payoffs without war can then be written as

$$V_1^L\left(w_1=0|a_1^L,a_1^F\right)=y_1^L-i_1^L-c_1^La_1^L+s_1^L\pi_1+\delta\left[V_2^L\left(w_2=1\right)+\kappa\Pi_2-\frac{\delta\left(m_3\right)^w}{1-\delta\gamma}+\mathcal{B}g_2^F\right] \quad (22)$$

$$V_1^F\left(w_1=0|a_1^L,a_1^F\right)=y_1^F-i_1^F-c_1^Fa_1^F+s_1^F\pi_1+\delta V_2^F\left(w_2=1\right). \quad (23)$$

The surplus from avoiding war from period 2 onwards is made up of the last three terms in eq. (22). First, there is a positive term, $\kappa\Pi_2$, which captures the fact that the destruction associated with war is permanently avoided. Second, a negative term captures the fact that, contrary to what would happen if there was a war in period 2, countries must pay for military expenditures not only in period 2 (which cost is included in $V_2^F\left(w_2=1\right)$), but also in all subsequent periods. Considering military expenditure in both countries, this carries a combined cost $\left(m_3\right)^w$ in the next period (3), which then grows (in discounted value terms) at a constant rate $\delta\gamma$ in subsequent periods. Finally, a further benefit of never going to war is that the trade disruption implied by war is avoided. This is captured by the term $\mathcal{B}g_2^F$. Although such trade costs are born by $F$, a higher trade cost of war actually increases $L$'s payoff in equilibrium, since it makes $F$'s outside option less attractive and thus weakens its position in negotiations.

We can now proceed using backward induction. Suppose $L$'s negotiators have offered $F$ a share $s_1^F$ of the pie. When does $F$ accept? To answer this question, let $\underline{s}_1^F\left(a_1^L,a_1^F\right)$ be the share that leaves $F$ indifferent between accepting or not. Clearly, then, $\underline{s}_1^F\left(a_1^L,a_1^F\right)$ is also the minimum share that $F$ is willing to accept. It is given by

$$\begin{aligned}\underline{s}_1^F\left(a_1^L,a_1^F\right)&=\arg_{s_F^1}\left\{V_1^F\left(w_1=0|a_1^L,a_1^F\right)=V_1^F\left(w_1=1|a_1^L,a_1^F\right)\right\}\\&=\arg_{s_1^F}\left\{y_1^F-i_1^L-c_1^Fa_1^F+s_1^F\pi_1+\delta V_2^F\left(w_2=1\right)=V_1^F\left(w_1=1|a_1^L,a_1^F\right)\right\}.\end{aligned}$$

Using (12) and re-arranging, the threshold can be re-written as

$$\begin{aligned}\underline{s}_1^F\left(a_1^L,a_1^F\right)=&\frac{a_1^F}{a_1^F+a_1^L}(1-\kappa)-\left[\left(q_2^F\right)^w-\frac{a_1^F}{a_1^F+a_1^L}\right]\frac{\delta\Pi_2}{\pi_1}(1-\kappa)\\&+\frac{\delta\left(m_2^F\right)^w}{\pi_1}+\mathcal{B}\frac{\delta g_2^F-g_1^F}{\pi_1}.\end{aligned} \quad (24)$$

Eq. (24) is an important equation that we comment on in detail below. Before doing that, however, we also derive the share that leaves $L$ indifferent between making an offer that gets accepted and starting a war. This is given by $\arg_{s_F^1}\left\{V_1^L\left(w_1=0|a_1^L,a_1^F\right)=V_1^L\left(w_1=1|a_1^L,a_1^F\right)\right\}$, or

$$\arg_{s_1^L}\left\{y_1^L-i_1^L-c_1^La_1^L+s_1^L\pi_1+\delta\left[V_2^L\left(w_2=1\right)+\kappa\Pi_2-\frac{\delta\left(m_3\right)^w}{1-\delta\gamma}+\mathcal{B}g_2^F\right]=V_1^L\left(w_1=1|a_1^L,a_1^F\right)\right\},$$

which, using (11) and re-arranging, can be written as

$$\underline{s}_1^F\left(a_1^L,a_1^F\right)+\frac{1}{\pi_1}\overbrace{\left[\kappa\Pi_1-\frac{\delta(m_2)^w}{1-\delta\gamma}+\mathcal{B}g_1^F\right]}^{\equiv K_1}. \tag{25}$$

This is the maximum share that $L$ is willing to offer (provided it expects its offer to be accepted).

The difference between the minimum share that $F$ is willing to accept and the maximum share that $L$ is willing to offer must be equal to the surplus from striking an agreement. Given that, if countries strike an agreement in period 1, war will never occur (Lemma 3), this surplus must be equal to the surplus from permanently avoiding war from *period 1* onwards. Indeed, this is what we see in equation (25) (note that, compared to the expression for the previously discussed surplus from avoiding war from *period 2* onwards, all subscripts are one period earlier). Again, the surplus from permanently avoiding war is made up of the gain from avoiding destruction and trade disruption in perpetuity, but there is also a cost due to the fact that both countries must continue to arm in all subsequent periods. We denote this gain by $K_1$ in what follows. As explained in Section 3.5, the threshold $\hat{\kappa}$ is such that $K_1=0$ if and only if $\kappa=\hat{\kappa}$.

Because $K_1\geq 0$ in our range of parameters, war is welfare reducing. Then, one might expect that negotiators should be able to avoid war. This however does not need to be the case. *Lack of commitment* explains this inefficiency: since negotiators cannot commit to future agreements, they may be unable to offer enough to a country whose war prospects are better today than tomorrow. To gain some intuition, consider equation (24). If $F$ expects to become weaker over time $(\left(q_2^F\right)^w<a_1^F/(a_1^L+a_1^F))$, or if the cost of arming for one more period is very high ($\left(m_2^F\right)^w$ is high), or if the trade cost of war increases over time ($g_1^F<\delta g_2^F$), then $F$ may require more than the entire current pie to be induced not to start a war ($\underline{s}_1^F\left(a_1^L, a_1^F\right) > 1$). Since negotiators cannot allocate future pies, they cannot avoid war. Similarly, from (25), if $L$ expects to become weaker over time, or expects that the future cost of arming will be high, not even the possibility of keeping the entire pie for itself will be enough to prevent it from going to war.[26]

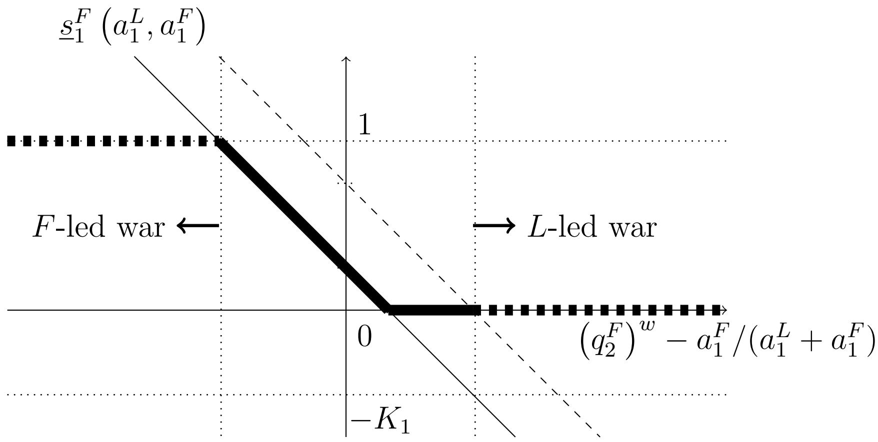



Figure 3: Welfare reducing war.

Figure 3 plots the minimum share that $F$ must be offered (the solid downward sloping line) alongside the maximum share that $L$ is willing to offer (the dashed downward sloping line), as a function of the rise in $F$'s military power. The solid thick line represents the outcome of successful negotiations (i.e. negotiations that avoid war). If $\underline{s}_1^F\left(a_1^L, a_1^F\right) \in [0, 1]$, the best $L$ can do is to offer exactly $\underline{s}_1^F\left(a_1^L, a_1^F\right)$: by definition, that is both sufficient to avoid war, and the cheapest way to do so. But what if $\underline{s}_1^F\left(a_1^L, a_1^F\right) > 1$, or $\underline{s}_1^F\left(a_1^L, a_1^F\right) < 0$? In the first case, the best $L$ can do is to offer 1. This, however, is not sufficient to avoid war, which must then occur. In the second case, the best $L$ can do is to offer 0. This is more than what $L$ would ideally like to offer, but it is sufficient to avoid war, and the cheapest *feasible* way to do so. But does $L$ want to make such an offer? Clearly, it does so if $\underline{s}_1^F\left(a_1^L, a_1^F\right) \geq -K_1$, since then the maximum that $L$ is willing to offer (eq. 25) is more than 0. Otherwise, this country prefers to start a war than to offer anything.

We next introduce the following:

[26] As anticipated in Section 3.5, the future cost of arming is a determinant of the size of the bargaining range: a higher $\left(m_2^F\right)^w$ increases the minimum that $F$ must be offered, while a higher $\left(m_2^L\right)^w$ decreases the maximum that $L$ is willing to offer.

**Definition 2.** *A $J$-led war is a war that takes place when there exists a peaceful partition that would induce $-J$ to prefer peace to war, but $J$ prefers war to such a partition.*

Applying this definition to the case just discussed, it is evident that there is a $F$-led war in period 1 if and only if $\underline{s}_1^F\left(a_1^L,a_1^F\right) > 1$, and there is an $L$-led war if and only if $\underline{s}_1^F\left(a_1^L,a_1^F\right) < -K_1$.[27]

Having found how war decisions depend on $\underline{s}_1^F\left(a_1^L,a_1^F\right)$, we move back one sub-period and examine the arming decisions that determine this threshold. These must simultaneously satisfy

$$\left(a_1^L\right)^* = \arg \max \; V_1^L\left(a_1^L,a_1^F\right) \tag{26}$$

$$\left(a_1^F\right)^* = \arg \max_{c_1^F a_1^F \le x_1^F} V_1^F\left(a_1^L,a_1^F\right), \tag{27}$$

where

$$V_1^L\left(a_1^L,a_1^F\right) = V_1^L\left(w_1=1|a_1^L,a_1^F\right) + \begin{cases} 0 & \text{if } \underline{s}_1^F\left(a_1^L,a_1^F\right) > 1 \\ K_1 & \text{if } \underline{s}_1^F\left(a_1^L,a_1^F\right) \in [0,1] \\ K_1 + \underline{s}_1^F\left(a_1^L,a_1^F\right)\pi_1 & \text{if } \underline{s}_1^F\left(a_1^L,a_1^F\right) \in [-K_1,0) \\ 0 & \text{if } \underline{s}_1^F\left(a_1^L,a_1^F\right) < -K_1 \end{cases}$$

$$V_1^F\left(a_1^L,a_1^F\right) = V_1^F\left(w_1=1|a_1^L,a_1^F\right) + \begin{cases} 0 & \text{if } \underline{s}_1^F\left(a_1^L,a_1^F\right) > 1 \\ 0 & \text{if } \underline{s}_1^F\left(a_1^L,a_1^F\right) \in [0,1] \\ -\underline{s}_1^F\left(a_1^L,a_1^F\right)\pi_1 & \text{if } \underline{s}_1^F\left(a_1^L,a_1^F\right) \in [-K_1,0) \\ 0 & \text{if } \underline{s}_1^F\left(a_1^L,a_1^F\right) < -K_1 \end{cases}.$$

The above maximands have an intuitive interpretation. If arming decisions are such that $\underline{s}_1^F\left(a_1^L,a_1^F\right)$ is either greater than 1 or smaller than $-K_1$, then a war occurs, and both countries obtain their payoffs with war. If this minimum share lies between zero and one, then war is

[27] We prefer to characterise a war based on Definition 2, and not based on who starts the war, because the latter approach depends on the specific tie-breaking rule used, while the former approach does not. When deriving the full equilibrium in Online Appendix B, we show that, under reasonable tie-breaking rules, if there is a $J$-led war, this is always started by $J$. These rules can be summarised as follows: given equal wartime and peacetime payoffs, a country prefers not to start a war. Note that, had we assumed the opposite, both an $L$-led war and an $F$-led war would be started by $L$, the country who moves first. More details are provided in the proofs to Propositions 1 and 2.

avoided, and $L$ can offer the minimum that $F$ is willing to accept. It follows that $F$ is driven down to its war payoff, while $L$ reaps the entire gain from not going to war, $K_1$. Finally, if the minimum share is just below zero, then war is avoided, but $L$ must offer more than the minimum. Relative to the previous case, $F$'s payoff must be higher, and $L$'s payoff lower (note that the term $\underline{s}_1^F\left(a_1^L, a_1^F\right)\pi_1$ is negative in this range).

What armies will countries have in equilibrium, and will this lead to war? To provide a general answer to this question would require maximising the expressions in (26) and (27), two complicated functions of $a_1^L$ and $a_1^F$. Here, we adopt a simpler approach: instead of looking at the entire range $\kappa \in [\hat{\kappa}, 1]$, we focus on the case in which $\kappa$ is close enough to $\hat{\kappa}$. This is enough for our purposes: to show that a welfare reducing war *can* occur, it is enough to show that it can occur if its effective cost is small enough (but still positive).[28] This approach simplifies the problem in (26) and (27), since the $V_1^J\left(a_1^L, a_1^F\right)$ for both countries converge to $V_1^J\left(w_1 = 1 | a_1^L, a_1^F\right)$. The solution is $\left(a_1^L\right)^* = \left(a_1^L\right)^w$ and $\left(a_1^F\right)^* = \left(a_1^F\right)^w$, implying that the minimum share $F$ must be offered converges to

$$\underbrace{\left[\left(q_1^F\right)^w - \delta\gamma\left(q_2^F\right)^w\right]\frac{\Pi_1}{\pi_1}(1-\hat{\kappa})}_{\text{Shift in relative power}} \qquad + \overbrace{\frac{\delta\left(m_2^F\right)^w}{\pi_1}}^{\text{Additional military expenditure}} \qquad \underbrace{+\mathcal{B}\frac{\delta g_2^F - g_1^F}{\pi_1}}_{\text{Increase in the trade cost of war}} \quad . \tag{28}$$

From the previous discussion, we know that an $L$-led war occurs if and only if the above share is smaller than $-K_1$, and an $F$-led war occurs if and only if it is greater than one. However, for $\kappa$ close enough to $\hat{\kappa}$, $K_1$ is close to zero. So, what we have to check is whether or not the above share is smaller than 0 or greater than 1. All have we to do, then, is to write $\left(q_1^F\right)^w$, $\left(q_2^F\right)^w$, $\left(m_2^F\right)^w$, $g_1^F$ and $g_2^F$ as functions of the fundamental parameters of the model, and examine the value of the resulting expression. We separately consider the case in which $L$ does not have the capacity to blockade ($\mathcal{B} = 0$), and the case when it does ($\mathcal{B} = 1$). Again, we only report here the (period 1) equilibrium path, while the full description of the equilibrium is contained in the proofs.

[28] As already mentioned, $K_1 = 0$ if and only if $\kappa = \hat{\kappa}$. Furthermore, since $(m_2)^w$ is decreasing in $\kappa$, $K_1$ is increasing in $\kappa$.

Suppose $L$ does not have the capacity to blockade. The results we obtain are reported in

**Proposition 1.** *Suppose $\kappa \to \hat{\kappa}$. Then, $\left(a_1^L\right)^* = \left(a_1^L\right)^w$ and $\left(a_1^F\right)^* = \left(a_1^F\right)^w$. If $L$ does not have the capacity to blockade, there cannot be a $F$-led war. There is an $L$-led war if and only if*

$$\frac{\underline{\alpha}}{1+\underline{\alpha}} - \delta\gamma \left(\frac{\overline{\alpha}}{1+\overline{\alpha}}\right)^2 < 0. \tag{29}$$

*Proof.* In Online Appendix B. □

If $L$ does not have the capacity to blockade, $F$'s military power increases as it catches up to the leader (see Figure 2). Then, the term $\left(q_1^F\right)^w - \delta\gamma\left(q_2^F\right)^w$ in eq. (28) can be negative. If this *shift in relative power* is large enough, this may make $\underline{s}_t^F\left(a_1^L, a_1^F\right)$ negative, leading to an $L$-led war. The condition for this to happen is presented in condition (29). The expression is true if $F$ catches up fast enough, that is if $\overline{\alpha}$ is large relative to $\underline{\alpha}$. Proposition 1 is simply the well-known result that an industrial leader may find it optimal to start a pre-emptive war against a catching-up follower. Intuitively, catching up will make the follower more powerful in the future ($q_2^F > q_1^F$), and the follower cannot commit not to use this augmented power against the leader. In these circumstances, $L$ may want to start a pre-emptive war so as to defeat the follower before it is too late. In Online Appendix C, we present two vectors of parameters which satisfy all assumptions of the paper, and such that, if $\mathcal{B} = 0$, there is, respectively, no war and a $L$-led war in period 1.

If $L$ does have the capacity to blockade, we can obtain the results reported in

**Proposition 2.** *Suppose $\kappa \to \hat{\kappa}$. Then, $\left(a_1^L\right)^* = \left(a_1^L\right)^w$ and $\left(a_1^F\right)^* = \left(a_1^F\right)^w$. If $L$ has the capacity to blockade, there can be both an $F$-led war and an $L$-led war. There is an $F$-led war if and only if*

$$\left[\left(\overline{\beta}\underline{\alpha}\right)^{\frac{1}{2}} - \delta\gamma\left(\underline{\beta}\overline{\alpha}\right)^{\frac{1}{2}}\right]\frac{\left[\Pi_1(1-\hat{\kappa})\right]^{\frac{1}{2}}}{\pi_1} + \frac{\delta\gamma\underline{\beta}}{\pi_1} + (1-\eta)\frac{\delta\gamma\left(\overline{\alpha}-\underline{\beta}\right) - \left(\underline{\alpha}-\overline{\beta}\right)}{\pi_1} > 1, \tag{30}$$

*while there is an $L$-led war if and only if the LHS of the above inequality is less than zero.*

*Proof.* In Online Appendix B. □

If $L$ has the capacity to blockade, an $F$-led war is now possible. Two channels make it potentially attractive for $F$ to start a war. First, there is the *shift in relative power* channel, which is captured by the first term in (28), or on the LHS of (30). From Figure 2, we know that $F$ may now become *weaker* over time $(\left(q_1^F\right)^w > \left(q_2^F\right)^w)$: that happens when $F$'s catching up is not fast enough to make up for its increased dependence on imported raw materials, $\overline{\alpha}/\underline{\alpha} < \overline{\beta}/\underline{\beta}$. In this case, the first term in (28), or on the LHS of (30), is large and positive: this may imply that the minimum share that $F$ is willing to accept is greater than one, making a $F$-led war unavoidable. Intuitively, if faced with a large enough decline in its relative power, $F$ may find it optimal to go to war immediately. The second channel is the *increase in the trade cost of war* channel, which is captured by the third term in (28), or on the LHS of (30). Intuitively, if dependence on imported raw materials grows fast (so that $\overline{\alpha} - \underline{\beta}$ is much greater than $\underline{\alpha} - \overline{\beta}$), $F$ faces a much higher trade cost of war in period 2 than in period 1. Anticipating that this will make it weaker over time, by a logic similar to that of a decline in relative power, $F$ may then decide to start a war immediately. It turns out that, even if $F$'s relative power is constant or increasing (which, from Figure 2, is the case if $\overline{\alpha}/\underline{\alpha} \geq \overline{\beta}/\underline{\beta}$), this second channel may still make an $F$-led war unavoidable.[29]

On the other hand, an $L$-led war may still occur: this may happen when $\overline{\alpha}/\underline{\alpha}$ is much larger than $\overline{\beta}/\underline{\beta}$. Intuitively, $F$'s prodigious economic growth makes its military technology increasingly sophisticated, and increasingly good at using scarce domestic raw materials. Then, $F$'s relative power increases fast between period 1 and period 2, making the first term in (28), or on the LHS of (30), negative. If this effect is large enough, (28), or the LHS of (30), may then be negative, making a $L$-led war unavoidable.

These results are summarised by the following:

**Corollary 1.** *If $\overline{\alpha}/\underline{\alpha} \leq \overline{\beta}/\underline{\beta}$, there is either peace or a $F$-led war. If $\overline{\alpha}/\underline{\alpha} > \overline{\beta}/\underline{\beta}$, there can be a*

[29] As explained in footnote 23, the third channel contained in eq. (28), the *additional military expenditure* channel, cannot, in itself, be a cause of war. This can be seen by setting $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta} = 1$, which "shuts down" both the shift in relative power channel and the increase in the trade cost of war channel: as shown in the proof to Corollary 1, there can never be a war in this case. However, the additional military expenditure channel still contributes to determining the size of the bargaining range, and may therefore matter in conjunction with the other two channels.

*$L$-led war, peace or a $F$-led war.*

*Proof.* In Online Appendix B. $\square$

In Online Appendix C, we present three vectors of parameters which satisfy all assumptions of the paper, and such that, if $\mathcal{B} = 1$, there is, respectively, no war, an $F$-led war, and an $L$-led war in period 1.

# 5 Extensions

So far, we have assumed that, if $L$ has the capacity to blockade, countries must take this initial condition as given. In this section, we briefly consider two ways in which this assumption can be relaxed. We begin by looking at a case in which $F$ can attack $C$, and thus conquer enough raw materials to become immune to a blockade. We then consider the possibility that $L$ may surrender the capacity to blockade.

## 5.1 Conquest of $C$

Suppose that, before any other event takes place, $F$ may attack $C$. We assume that, if $F$ is indifferent between attacking or not, it does not attack. For simplicity, we also assume that an attack costs nothing (or very little), and is always successful. As a result of a successful attack, $F$ annexes a portion of $C$'s territory producing no less than $\overline{\alpha} - \underline{\beta}$ in raw materials in period 1, and $\gamma^{t-1}$ times that amount in period $t > 1$. In order to focus on our security of supply channel, we assume that even if $F$ conquers a portion of $C$, it must still pay for any raw materials it imports from there, at the original price $\eta$. Thus, if $\mathcal{B} = 0$, $F$ will never attack $C$, since it gains nothing by doing so. However, as soon as conquered resources become part of $F$'s endowment, they become non-blockadable by $L$. This can give the follower a strategic incentive to attack.

Suppose then that $\mathcal{B} = 1$. The choice to attack or not is equivalent to a choice between playing the baseline game when $\mathcal{B} = 0$, or playing it when $\mathcal{B} = 1$. Because $F$ receives its wartime payoff in both cases, it will attack if and only if its wartime payoff is higher in the former case, than in the latter. In terms of Figure 1, it will attack if and only if its wartime payoff is higher at point A than at point B. It is possible to show that, if $F$ is equal in size (in terms of its endowment of $y$) or larger than $L$, or if it is smaller, but is severely constrained in its arming decisions, then its wartime payoff is higher at point A than at point B. In either case, $F$ attacks $C$. Intuitively, by attacking, $F$ can increase its chances of winning a war against $L$, as well as reduce its trade costs from such a war. This benefits $F$ both if the war actually occurs, and if it does not, since it increases its bargaining power in negotiations.[30]

What does $F$'s attack on $C$ actually imply for bilateral relations between $L$ and $F$? Comparing Propositions 1 and 2, we see that the model allows for a rich set of cases. If $F$ does not attack, and $L$'s capacity to blockade remains intact, there can an $F$-led war, no war, or an $L$-led war (Proposition 2). If it does attack, there may either be no war or an $L$-led war (Proposition 1). The actual impact of an attack on $C$ thus depends on economic fundamentals. Three possibilities seem of particular interest. First, if $F$'s dependence on imported raw materials is growing fast, but its economy is not catching up too rapidly on $L$'s economy, then the attack will stave off the $F$-led war that would otherwise have occurred. Second, if catch-up growth is rapid, and $F$'s import dependence is not growing rapidly, then the attack may trigger an $L$-led war, when otherwise there would have been peace. And third, if $F$ is growing rapidly *and* becoming much more import-dependent, then the attack will transform what would otherwise have been an $F$-led war into an $L$-led war.

Finally, because we have assumed that an attack makes $F$ fully self-sufficient, it can never be the case that, after an attack, there is an $F$-led war. However, it is easy to imagine a more general case in which $F$ would first attack $C$, and then attack $L$. All is required for this is for $F$ to be capable of conquering only a portion of $C$, so that the raw materials that it can grab from $C$ are not enough to make it fully self-sufficient. In this case its dependence on imported raw materials may still grow fast enough during structural transformation for it to attack the leader. However, it may well make sense for $F$ to first attack $C$, so as to increase the probability that it

[30]Perhaps surprisingly, $F$'s wartime payoff is not always higher at point A than at point B. $F$'s arming decision at point $A$ is optimal *given $L$'s arming decision.* This does not rule out the possibility that if *both* countries choose different arming levels (e.g. those at point B), $F$'s payoff might be higher. It turns out that, if $F$ is smaller than $L$, and B is not too far from A, then $F$'s wartime payoff can be higher at point B. This is because, in this case, at point A $F$ arms "too much" against a powerful opponent. Interestingly, then, the model admits a case in which $F$ chooses not to attack $C$ in order to avoid an escalation in arming levels.

will defeat the leader.[31]

## 5.2 Surrendering the capacity to blockade

We again focus on the case $\mathcal{B} = 1$. Suppose that, before any other event takes place, $L$ may decide to surrender the capacity to blockade, with immediate effect. This is again a choice between playing the baseline game when $\mathcal{B} = 1$, or playing it when $\mathcal{B} = 0$. The effect on bilateral relations between $L$ and $F$ will depend on economic fundamentals, as in the previous section. When does $L$ surrender to capacity to blockade? To answer this question, note that $L$'s payoff is equal to its wartime payoff if there is a war, and to its wartime payoff plus the surplus $K_1$ if there is no war. It is possible to show that $L$'s wartime payoff is always higher for $\mathcal{B} = 1$ than for $\mathcal{B} = 0$, because its chances of winning the war are higher in the former case.[32] Then, of the three examples considered in the previous section, $L$ will only consider surrendering the capacity to blockade in the first, when $F$'s import dependence is growing fast but its economy is not growing too rapidly. In this case surrendering the capacity to blockade can stave off an $F$-led war: $L$ would then have to trade off the fall in its wartime payoff against its ability to reap the surplus $K_1$ from not going to war. In both other cases, the fact that $F$'s economy is growing fast implies that, if $L$ surrenders the capacity to blockade, it will then have to start a war against $F$: this cannot possibly be an optimal choice for $L$.

# 6 A brief historical discussion

Our model predicts that war can arise when a rising power finds its import dependence growing more rapidly than it is converging on the industrial leader. In this case, its relative military strength will be declining over time, and the follower may have an incentive to strike before it is too late. Indeed, the rising cost to consumers of wartime blockades may give the follower an

[31] Conquering a portion of $C$ will move the equilibrium along the leader's best response function in Figure 1 from B in the direction of A. As in the case when the follower is capable of conquering all of $C$, and the equilibrium moves all the way to A, for many parameter values shifting the equilibrium in this way will increase the follower's payoff from war.

[32] That is to say, contrary to what we saw for $F$, points A and B in Figure 1 are unambiguously ranked in $L$'s preferences: B always dominates A.

incentive to go to war, even in circumstances when its relative power is not declining. The model also predicts that the follower may attack resource-rich peripheral areas, in an attempt to become more self-sufficient, or entirely self-sufficient, in raw materials. It may do so prior to launching an attack on the leader. It may even do so in circumstances when it knows that this will provoke an attack upon it by the leader, when otherwise the two countries would not have gone to war. If the follower is not only becoming rapidly more import-dependent, but is also converging rapidly on the leader, then conquering the country supplying raw materials transforms what would have been a follower-led war into a leader-led war. In this case, while it is the leader who decides to go to war against the follower, the root cause of the war remains the follower's incentive to fight, arising from its import-dependence.

There is a substantial body of historical literature which suggests that this trade-dependence mechanism was at work in the first half of the twentieth century, and that concerns over the supply of imported raw materials was an important motivating factor at various points in time for both German and Japanese military planners. In the words of Azar Gat, "the quest for self-sufficiency in strategic war materials became a cause as well as an effect of the drive for empire, most notably in the German and Japanese cases towards and during the Second World War" (Gat 2006, p. 556). This seems especially obvious in the Japanese case.

To repeat: the world is much more complicated than the simple structure envisaged in our model, or any other, and we do not argue that our mechanism can "explain" the Second World War in some monocausal way. However, our model provides useful insights into the origins of this war, especially in the Pacific. It is much less useful in understanding the origins of the First World War, which lie elsewhere, but does provide insights into the Anglo-German naval rivalry which preceded it, and which helps explain Britain's decision to join the war once it had started. We therefore provide a very brief account of the build-ups to the Second World War in Asia, the Second World War in Europe, and the First World War. In each case, we indicate how the mechanisms identified by our model are relevant in understanding the episode in question, as well as some of the ways (certainly not all) in which reality was more complex than allowed for in the theoretical discussion above.

## 6.1 World War II in Asia

Japan's industrial output had been growing more rapidly than American output since 1890 (Bénétrix et al. 2015). Between 1920 and 1938, Japan's industrial output grew at an average of 6.7 per cent per annum, much higher than the growth rates recorded in the USA (1.2 per cent, although that reflected the severity of the Great Depression) and UK (3 per cent) over the same period. Rapid growth meant an increase in Japan's relative military power, already dramatically displayed during the Russo-Japanese war of 1904-5. This was a case where $\overline{\alpha}/\underline{\alpha}$ was unambiguously high.

However, $\overline{\beta}/\underline{\beta}$ was also very high in interwar Japan. Japan was endowed with very few natural resources, and rapid growth meant greater dependence on trade: by the eve of the war Japan was producing "only 16.7 per cent of her total iron ore consumption, 62.2 per cent of her steel consumption, 40.6 per cent of her aluminium consumption, 20.2 per cent of her crude oil consumption, and 31.3 per cent of her salt consumption". Japan was completely reliant on imports for such strategic minerals as nickel and bauxite (Milward 1977, pp. 31-2). The United States was a major supplier of several crucial materials to Japan, including oil, scrap iron, and raw cotton (Liberman 1996, p. 169); it supplied Japan with two thirds of her oil in 1936 (Millward, *op. cit.*). On the other hand, if Japan managed to seize control over not only Manchuria and China, but Southeast Asia as well, then planners estimated that she would be self-sufficient in the major strategic commodities, aside from nickel (*ibid*).

A group of "total war" military officers became convinced that Japan would only be secure if it was self-sufficient. "War hereafter would be protracted...and nations had to be able to supply themselves during wartime with adequate quantities of raw materials and manufactured goods. Reliance on other countries for the materiel of war was a sure path to defeat... The need for security became, slowly, an impulse for empire, and it led directly to the Pacific War" (Barnhart 1987, p. 9). And so Japan invaded Manchuria in 1931, China in 1937, French Indochina in 1940, and South East Asia more generally in 1941, the latter invasion implying direct confrontation with the Western powers.

Unlike what we assume in the extension to our simple model (Section 5.1), conquering China was far from costless, and increased the need for imported raw materials from the West (Yasuba

1996). It also increased Western suspicion of Japan and aid to China. The US response confirmed in the minds of Japanese planners that their basic assumption, that a reliance on trade was dangerous for national security, was correct. In July 1940 the President was empowered to ban the export of strategic commodities, and soon the US had banned the export of scrap iron and steel, aviation fuel and other commodities. While in the short run Japan could live with this, having stockpiled American raw materials since 1937, the ban on oil exports which came in July 1941 was a different matter, and was seen as a *de facto* declaration of war.[33] The fact that critical raw materials were now in short supply became an argument, not for restraint, but for an immediate all-out war (Ferguson 2007), since it implied that $q^F_{t+1} < q^F_t$.

This case seems the one that best fits our model. Japan was growing relatively rapidly, and becoming more dependent on imported raw materials, just as is true of the follower country in our model. The European imperial powers and the United States possessed colonies which produced vital raw materials, or (as in the case of US oil) produced those raw materials domestically. This gave them an ability to blockade which was used by the United States in the run-up to war. Japan's invasions of Manchuria, China and Southeast Asia corresponded to invasions first of $C$, and then of $L$, in our model, a possibility discussed in Section 5.1. They were motivated by a desire for economic and strategic self-sufficiency, to be formalised via the creation of a Greater East Asia Co-Prosperity Sphere. This would have deprived the Western powers of the ability to blockade Japan. But trying to achieve such self-sufficiency was a high risk gamble, since attacking Southeast Asia required launching an attack on the Western powers, despite Japan's economic and military inferiority relative to America.

## 6.2 World War II in Europe

Again and again, Hitler returned in his speeches and writings to the need for secure supplies of both food and raw materials. The key was the Soviet Union. As early as 1931 he told a Party member that "Europe needs the grain, meat, the wood, the coal, the iron, and the oil from Russia in order to be able to survive" (Overy 2009, p. 51), and shortly before the war began he

[33] As is well known, Roosevelt had not envisaged the oil embargo as being a complete one, but the State Department officials who implemented the embargo ensured that it became one (Iriye 1987, p. 150).

told a Swiss diplomat that "I need the Ukraine, so that no one will starve us out as they did in the last war" (Hildebrand 1973, p. 88).

In a speech to the heads of the armed forces of November 1937, Hitler stated that:

"There was a pronounced military weakness in those States which depended for their existence on foreign trade. As our foreign trade was carried on over the sea routes dominated by Britain, it was more a question of security of transport than one of foreign exchange, which revealed in time of war the full weakness of our food situation. The only remedy...lay in the acquisition of greater living space...areas producing raw materials can be more usefully sought in Europe, in immediate proximity to the Reich, than overseas..." [34]

Germany was extremely or entirely dependent on imports for its supplies of such strategically vital raw materials as bauxite, chromium, copper, iron, lead, nickel, oil, rubber, and zink (Volkmann 1990, p. 246). The 1934 New Plan and 1936 Four Year Plan therefore tried to promote import substitution: in terms of our model, increasing $\underline{\beta}$. The annexations of Austria and Czechoslovakia in 1938 and 1939 provided the Reich with lignite, coal, and iron ore, as well as heavy industry (Overy 2002, pp. 197, 227), and Germany also tried to increase its economic hold over the resources of Hungary, Bulgaria and Romania via a series of bilateral deals. Dominating Poland was "necessary, in order to guarantee the supply of agricultural products and coal for Germany"(Overy 2002, p. 222). However, the ultimate prize, Russian resources, were still essential in order to make the Nazi empire blockade-proof (Kaiser 1980, pp. 277-9; Volkmann 1990, p. 258; Hildebrand 1973, p. 92). The conclusion of the Nazi-Soviet pact was thus crucial for Hitler, who could now invade Poland confident that even if Britain and France intervened, "We need not be afraid of a blockade. The East will supply us with grain, cattle, coal, lead and zinc."And indeed, in 1940 the USSR supplied Germany with 74 per cent of its phosphates imports, 67 per cent of its imported asbestos, and 34 per cent of its oil (Tooze 2006, p. 321). Ultimately, however, Hitler's aim was to grab these resources, so as to be able to rival the Anglo-American powers, rather than to buy them from the Communist enemy. It is in that light that his decision to invade the Soviet Union in 1941 needs to be understood.

[34] According to notes taken at the meeting, the so-called Hossbach memorandum, available at http:/germanhistorydocs.ghi-dc.org/pdf/eng/English50.pdf.

There was nothing rational about Hitler's racial theories and rabid nationalism. However, his desire for *Lebensraum* is quite consistent with our model. *Vis à vis* the Western nations, the Nazi state was a rising power. However, its dependence on trade left it vulnerable to blockade by sea. One obvious solution was to attack Eastern Europe, which corresponded to $C$ in our model. Indeed, as Section 5.1 argues, it might even have made sense to attack Poland in 1939, despite the fact that this risked war with France and Britain. And in the long run, conquering Russia was the only way to achieve complete self-sufficiency in raw materials.

## 6.3 Anglo-German naval rivalry and World War I

Our model does not explain the origins of World War I, which as every schoolchild knows lie in a dispute between Austria-Hungary and Serbia. Our model does, however, have something to say about Britain's decision to enter the war, as well as about the Anglo-German naval rivalry which preceded it.

The British economy had been heavily dependent on international trade from the time of the Industrial Revolution (Clark et al. 2015), and naval supremacy was therefore a strategic imperative for that country. By the late 19th century, industrialization and structural change were making Germany increasingly dependent on imports of food and raw materials as well. 74 per cent of these imports were arriving by sea, either directly or indirectly (Offer 1989, p. 335), implying that they were potentially vulnerable to blockade by the British.

According to Avner Offer (1989), a key factor underlying Anglo-German naval rivalry was the fact that *both* Germany and Britain were increasingly dependent on overseas imports of food and raw materials. "The economies of both Britain and Germany came to depend on hundreds of merchant ships that entered their ports every month. Overseas resources, the security of the sea lanes and the economics of blockade affected the war plans of the great powers and influenced their decision to embark on war" (Offer 1989, p. 1).

In 1898 Germany embarked on a naval buildup whose aim was to achieve naval parity with Britain, not globally, but locally (that is to say, in the waters between the two countries). But this strategy completely underestimated the importance of preserving naval hegemony in British eyes: it was essential both for the security of the Empire, and of Britain herself. The result was a naval arms race which Britain eventually won, but which in the process helped to shift British strategic thinking in an anti-German, rather than a pro-German, direction. As Sir Edward Grey, the British Foreign Secretary, told the Canadian Prime Minister, in 1912, "There are practically no limits to the ambitions which might be indulged by Germany, or to the brilliant prospects open to her in every quarter of the globe, if the British navy were out of the way. The combination of the strongest Navy with that of the strongest Army would afford wider possibilities of influence and action than have yet been possessed by any Empire in Modern Times" (Steiner 1977, p. 42). As Section 5.2 suggests, abandoning the capacity to blockade a rival that was growing as rapidly as Germany was unthinkable to the British.

The failure to make any headway in challenging Britain's naval superiority prompted some in Germany to argue for a strategy of German continental dominance, based on a European economic bloc with Germany at its centre (Strachan 2001, pp. 46-7). This was also unacceptable to Britain, since it would have granted Germany access to Atlantic ports, weakening or eliminating Britain's capacity to blockade her. As Grey said in 1911, if a European power achieved continental hegemony Britain would permanently lose its control of the sea, which would in turn mean its separation from the Dominions and the end of the Empire (Howard 1972, pp. 51-52). Paradoxically, Britain's traditional maritime orientation meant that it was *more* likely that she would intervene in a war in which France risked being destroyed by Germany.

# 7 Conclusions

This paper has developed a model of the links between growth, trade and military power in which a follower country may choose to launch a pre-emptive attack on a leader, despite the fact that it is growing more rapidly. Faster growth may not translate into greater future military strength if it is accompanied by increased dependence on imported raw materials, and the leader has the capacity to blockade; since the leader cannot pre-commit to not use this capacity in the future, the follower may choose to launch a pre-emptive war.

In our view, this mechanism is most likely to have been at work during the post-Industrial Revolution period. Rapid industrial growth involved profound structural change, and it is this structural change which made rising powers potentially vulnerable to blockade. We would not therefore expect to see our mechanism at work during the 18th century or earlier, even though there were of course many wars involving the English/British, Dutch and French trying to establish naval superiority over each other.

But neither was it the case that rapid industrial growth and structural change *necessarily* led to war from the 19th century onwards. There are several historical examples of countries rapidly catching up on established leaders, without their attacking either the leader or adjacent sources of raw materials. Sometimes this was because these rising powers were impossible to blockade. Thus, a classic example of a follower country catching up on and overtaking a leader, without provoking a war, is the United States' ascent relative to Britain. Our mechanism could not have been at work in this instance, since the United States was a vast continental economy abundant in raw materials, and impossible to blockade. Nor did Russia, or the Soviet Union, launch preemptive wars against Germany in 1914, 1939 or 1941. Again, our mechanism would not have been expected to work in this instance, since Russia was another vast, resource-abundant country that was impossible to blockade.

On the other hand, neither did the USSR attack the West after 1945 (or vice versa), despite the fact that the former was growing more rapidly than the latter until the 1970s, and that Russia was importing food by the end of this period. Nuclear weapons are one obvious reason why the peace was kept on this occasion. Nor has China's rise over the past three decades provoked an attack on its trading partners, despite the fact that it is becoming increasingly import-dependent.

Several historians have noted that there was a circularity to some of the strategic and military logics driving nations to war in the 1930s. In the case of Germany, David Kaiser (1980, p. 282) wrote that "Having insisted upon rearmament for the sake of conquest, he (Hitler) found himself in a situation where conquest was the only means of continuing rearmament. His belief that Germany must conquer a self-sufficient economic empire, rather than rely upon world trade, had become a self-fulfilling prophecy." In the case of Japan, Hatano and Asada (1989, pp. 399-400) comment that Japanese military thinking during this period "was characterised by peculiarly circular reasoning: to prepare for hostilities with the Anglo-American powers, Japan would have to march into Indochina to obtain raw materials; the United States would counter by imposing an economic embargo; this in turn would compel Japan to seize the Dutch East Indies to secure essential oil, a step that would lead to hostilities with the United States." Ralph Hawtrey (1952, p. 72) wrote that "the principal cause of war is war itself", in that "the aim for which war is judged worth while is most often something which itself affects military power."

As Kaiser noted, the danger with circular logics is that they can become self-fulfilling. Standard political economy considerations imply that it would be difficult if not impossible to unwind today's globalization, on which the Chinese economy depends: production is so fragmented, and the Chinese and Western economies so inter-dependent, that a move away from free trade would be impossibly costly, not just in the aggregate, but for large corporations that wield considerable political as well as economic power. This paper sounds a cautionary note (although one hopes that the costs of war have now become so enormous as to make it unthinkable): if strategic considerations were ever allowed to gain an upper hand, globalization would become more fragile, and the world would become a much more dangerous place.

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# Appendix A: key variables as functions of parameters

$$\left(a_1^L\right)^{w,u} = \frac{\pi_1}{1-\delta\gamma}(1-\kappa)\frac{\frac{1}{\underline{\alpha}}}{\left(1+\frac{1}{\underline{\alpha}}\right)^2} = \frac{\pi_1}{1-\delta\gamma}(1-\kappa)\frac{\underline{\alpha}}{(1+\underline{\alpha})^2} \tag{31}$$

$$\left.\left(a_t^L\right)^{w,u}\right|_{t>1} = \frac{\gamma^{t-1}\pi_1}{1-\delta\gamma}(1-\kappa)\frac{\overline{\alpha}\gamma^{t-1}}{(1+\overline{\alpha})^2} \tag{32}$$

$$\left(a_1^F\right)^{w,u} = \frac{\pi_1}{1-\delta\gamma}(1-\kappa)\frac{(\underline{\alpha})^2}{(1+\underline{\alpha})^2} \tag{33}$$

$$\left.\left(a_t^F\right)^{w,u}\right|_{t>1} = \frac{\gamma^{t-1}\pi_1}{1-\delta\gamma}(1-\kappa)\frac{(\overline{\alpha})^2\,\gamma^{t-1}}{(1+\overline{\alpha})^2} \tag{34}$$

$$\left(m_1^L\right)^{w,u} = \left(m_1^F\right)^{w,u} = \frac{\pi_1}{1-\delta\gamma}(1-\kappa)\frac{\underline{\alpha}}{(1+\underline{\alpha})^2} \tag{35}$$

$$\left.\left(m_t^L\right)^{w,u}\right|_{t>1} = \left.\left(m_t^F\right)^{w,u}\right|_{t>1} = \frac{\gamma^{t-1}\pi_1}{1-\delta\gamma}(1-\kappa)\frac{\overline{\alpha}}{(1+\overline{\alpha})^2} \tag{36}$$

$$\left(q_1^F\right)^{w,u} = \frac{\underline{\alpha}}{1+\underline{\alpha}} \tag{37}$$

$$\left.\left(q_t^F\right)^{w,u}\right|_{t>1} = \frac{\overline{\alpha}}{1+\overline{\alpha}} \tag{38}$$

$$\left(a_1^L\right)^{w,c} = \left[\frac{\pi_1}{1-\delta\gamma}\left(1-\kappa\right)\overline{\beta}\underline{\alpha}\right]^{\frac{1}{2}} - \overline{\beta}\underline{\alpha} \tag{39}$$

$$\left.\left(a_t^L\right)^{w,c}\right|_{t>1} = \left[\frac{\gamma^{t-1}\pi_1}{1-\delta\gamma}\left(1-\kappa\right)\gamma^{t-1}\underline{\beta}\overline{\alpha}\gamma^{2(t-1)}\right]^{\frac{1}{2}} - \underline{\beta}\overline{\alpha}\gamma^{2(t-1)} \tag{40}$$

$$\left(a_1^F\right)^{w,c} = \frac{\overline{\beta}}{\frac{1}{\underline{\alpha}}} = \overline{\beta}\underline{\alpha} \tag{41}$$

$$\left.\left(a_t^F\right)^{w,c}\right|_{t>1} = \frac{\underline{\beta}\gamma^{t-1}}{\frac{1}{\overline{\alpha}\gamma^{t-1}}} = \underline{\beta}\overline{\alpha}\gamma^{2(t-1)} \tag{42}$$

$$\left(m_1^L\right)^{w,c} = \left[\frac{\pi_1}{1-\delta\gamma}\left(1-\kappa\right)\overline{\beta}\underline{\alpha}\right]^{\frac{1}{2}} - \overline{\beta}\underline{\alpha} \tag{43}$$

$$\left.\left(m_t^L\right)^{w,c}\right|_{t>1} = \left[\frac{\pi_1}{1-\delta\gamma}\left(1-\kappa\right)\underline{\beta}\overline{\alpha}\gamma^{2(t-1)}\right]^{\frac{1}{2}} - \underline{\beta}\overline{\alpha}\gamma^{t-1} \tag{44}$$

$$\left(m_1^F\right)^{w,c} = \overline{\beta} \tag{45}$$

$$\left.\left(m_t^F\right)^{w,c}\right|_{t>1} = \underline{\beta}\gamma^{t-1} \tag{46}$$

$$\left(q_1^F\right)^{w,c} = \left[\frac{1-\delta\gamma}{\pi_1(1-\kappa)}\overline{\beta}\underline{\alpha}\right]^{\frac{1}{2}} \tag{47}$$

$$\left.\left(q_t^F\right)^{w,c}\right|_{t>1} = \left[\frac{1-\delta\gamma}{\pi_1(1-\kappa)}\underline{\beta}\overline{\alpha}\right]^{\frac{1}{2}} \tag{48}$$

# Online Appendix B: proofs

In this section, we first provide a set of additional results (Section B.1), which we then build on to provide proofs (Section B.2) to all results in the main text.

Section B.1 contains four additional lemmas, Lemmas 4-7. Lemma 4 establishes that arming levels $\left(a_t^L\right)^w$ and $\left(a_t^L\right)^w$ must be chosen in a period when war occurs. Lemma 5 defines the gain from delaying war by one period, $k_t$; it establishes that such a gain is constant from period 1 onwards (if $\mathcal{B}=0$) or from period 2 onwards (if $\mathcal{B}=1$); and it derives a threshold $\underline{\kappa} \in (0,1)$ such that this constant value of $k_t$ is negative if and only if $\kappa < \underline{\kappa}$. Finally, Lemmas 6 and 7 describe the full balanced growth path SPNE of the subgame that starts in period 2, for the case $\kappa < \underline{\kappa}$ and $\kappa \geq \underline{\kappa}$ respectively. Results in this section use the following tie-breaking rules:

***Tie-breaking rule 1****: If $J \in \{L, F\}$ is indifferent between an action which leads to war with $-J$, and an action which does not, it chooses the latter.*

***Tie-breaking rule 2****: Between an action that leads to $J$ starting a war, and an action that leads to $-J$ starting a war, $J \in \{L, F\}$ prefers the latter.*

## B.1 Additional results

**Lemma 4.** *If, in the SPNE, war occurs in period $t$, then it must be the case that $a_t^L = \left(a_t^L\right)^w$ and $a_t^F = \left(a_t^F\right)^w$.*

*Proof.* Suppose that, in equilibrium, war occurs in $t$. Let $(\mathbf{A}_t)^w$ be the set such that, in the equilibrium under consideration, $\left[a_t^L, a_t^F\right] \in (\mathbf{A}_t)^w$ implies $w_t = 1$. Let $\left[\hat{a}_t^L, \hat{a}_t^F\right]$ denote equilibrium arming levels. It must be that $\hat{a}_t^L$ lies on $L$'s best response function, $\hat{a}_t^L = a_t^L\left(\hat{a}_t^F\right)$. Suppose otherwise. Then, $L$'s payoff should be at least as high with arming levels $\left[\hat{a}_t^L, \hat{a}_t^F\right]$ as with arming levels $\left[a_t^L\left(\hat{a}_t^F\right), \hat{a}_t^F\right]$. But if $\left[a_t^L\left(\hat{a}_t^F\right), \hat{a}_t^F\right] \in (\mathbf{A}_t)^w$, that contradicts the definition of $a_t^L\left(\hat{a}_t^F\right)$. If $\left[a_t^L\left(\hat{a}_t^F\right), \hat{a}_t^F\right] \notin (\mathbf{A})^w$, that again contradicts the definition of $a_t^L\left(\hat{a}_t^F\right)$, since $L$'s payoff must be at least as high if there is peace with arming levels $\left[a_t^L\left(\hat{a}_t^F\right), \hat{a}_t^F\right]$ as if there is war with those same arming levels (given that $L$ can always set $w_t^L = 1$) and, by the definition of $a_t^L\left(\hat{a}_t^F\right)$, $L$'s payoff in the latter case must be higher than if there is war with arming levels $\left[\hat{a}_t^L, \hat{a}_t^F\right]$. Similarly, it can be shown that it must be that $\hat{a}_t^F = a_t^F\left(\hat{a}_t^L\right)$. Together, $\hat{a}_t^L = a_t^L\left(\hat{a}_t^F\right)$ and $\hat{a}_t^F = a_t^F\left(\hat{a}_t^L\right)$

imply $\left[\hat{a}_t^L, \hat{a}_t^F\right] = \left[\left(a_t^L\right)^w, \left(a_t^F\right)^w\right]$. □

**Lemma 5.** *Let $k_t$ be the joint welfare gain from delaying war by one period in period $t$ (expressed as a share of period $t$ pie). Such a gain can be written as*

$$k_t \equiv \kappa - \frac{\delta (m_{t+1})^w}{\pi_t} + \mathcal{B} \frac{g_t^F - \delta g_{t+1}^F}{\pi_t}, \tag{49}$$

*If $\mathcal{B} = 0$, such a gain is constant from period 1 onwards, while if $\mathcal{B} = 1$ it is lower in period 1 than in period 2, and constant from period 2 onwards. In either case, let $k$ be the expression denoting this constant value of $k_t$. There exists $\underline{\kappa} \in (0, 1)$ such that $k < 0$ if $\kappa < \underline{\kappa}$, $k = 0$ if $\kappa = \underline{\kappa}$, and $k > 0$ if $\kappa > \underline{\kappa}$.*

*Proof.* By Lemma 4, if war occurs in period $t+1$, it must be that $a_{t+1}^L = \left(a_{t+1}^L\right)^w$ and $a_{t+1}^F = \left(a_{t+1}^F\right)^w$. By definition, $k_t$ can then be written as $1/\pi_t$ times

$$\begin{aligned} & y_t^L - i_t^L - c_t^L a_t^L + s_t^L \pi_t + \delta V_{t+1}^L \left(w_{t+1} = 1\right) - V_t^L \left(w_t = 1 | a_t^L, a_t^F\right) + \\ & + y_t^F - i_t^F - c_t^F a_t^F + s_t^F \pi_t + \delta V_{t+1}^F \left(w_{t+1} = 1\right) - V_t^F \left(w_t = 1 | a_t^L, a_t^F\right), \end{aligned}$$

which, using (11)-(12) and (20)-(21) and re-arranging, can be written as in (49). If $\mathcal{B} = 0$, $k_t$ is constant over time, since, as evident from (44) and (46), $(m_{t+1})^w$ grows at a rate $\gamma$ like $\pi_t$. If $\mathcal{B} = 1$, the term $k_t$ is constant from period 2 onwards, since $g_t^F - \delta g_{t+1}^F$ can then be written as $(1 - \delta\gamma) g_t^F$, which also grows at a rate $\gamma$. However, $k_1 < k_2$, since $g_t^F$ grows faster than $\gamma$ between period 1 and 2. From (44) and (46), we see that $(m_{t+1})^w = \left(m_{t+1}^L\right)^w + \left(m_{t+1}^F\right)^w$ is continuously decreasing in $\kappa$. It follows that $k$ is continuously increasing in $\kappa$. All we need to show is that there exists $\underline{\kappa} \in (0, 1)$ such that $k = 0$ if $\kappa = \underline{\kappa}$. If $\mathcal{B} = 0$, $\underline{\kappa}$ is easy to derive explicitly. Using (35) and (36),

$$\begin{aligned} \arg_\kappa \{k = 0\} &= \arg_\kappa \left\{ \kappa - \frac{\delta \left(m_{t+1}\right)^w}{\pi_t} = 0 \right\} = \arg_\kappa \left\{ \kappa - \frac{\delta\gamma}{1 - \delta\gamma} (1 - \kappa) \frac{2\overline{\alpha}}{(1 + \overline{\alpha})^2} = 0 \right\} \\ &= \frac{\frac{\delta\gamma}{1-\delta\gamma} \frac{2\overline{\alpha}}{(1+\overline{\alpha})^2}}{1 + \frac{\delta\gamma}{1-\delta\gamma} \frac{2\overline{\alpha}}{(1+\overline{\alpha})^2}} \in (0, 1). \end{aligned} \tag{50}$$

If $\mathcal{B} = 1$, using (43)-(46), we can write:

$$k = \kappa - \frac{\delta\gamma\left\{\left[\frac{\pi_1}{1-\delta\gamma}(1-\kappa)\underline{\beta}\overline{\alpha}\right]^{\frac{1}{2}} - \underline{\beta}\overline{\alpha} + \underline{\beta}\right\}}{\pi_1} + \frac{(1-\delta\gamma)(1-\eta)\left(\overline{\alpha}-\underline{\beta}\right)}{\pi_1}. \tag{51}$$

Because $k$ is continuously increasing in $\kappa$, all we need to show is $k < 0$ for $\kappa = 0$, and $k > 0$ for $\kappa \to 1$. If $\kappa = 0$, condition $k < 0$ can be written as

$$-\frac{\delta\gamma}{1-\delta\gamma}\left[\left(\frac{\pi_1}{1-\delta\gamma}\underline{\beta}\overline{\alpha}\right)^{\frac{1}{2}} - \underline{\beta}\overline{\alpha} + \underline{\beta}\right] + (1-\eta)\left(\overline{\alpha}-\underline{\beta}\right) < 0. \tag{52}$$

By assumption, it is $x_t^F \leq \left(m_t^F\right)^{w,u}$. Using (36) and (2), it is easy to see that this implies $\pi_1/(1-\delta\gamma)\,\overline{\alpha} > \underline{\beta}(1+\overline{\alpha})^2$. Then, a sufficient condition for (81) to hold is

$$-\frac{\delta\gamma}{1-\delta\gamma}\left[\left(\underline{\beta}^2(1+\overline{\alpha})^2\right)^{\frac{1}{2}} - \underline{\beta}\overline{\alpha} + \underline{\beta}\right] + (1-\eta)\left(\overline{\alpha}-\underline{\beta}\right) < 0$$

$$\frac{2\delta\gamma}{1-\delta\gamma}\underline{\beta} > (1-\eta)\left(\overline{\alpha}-\underline{\beta}\right),$$

which is the same as Assumption 1. If $\kappa \to 1$, condition $k > 0$ converges to

$$1 + \frac{\underline{\beta}(\overline{\alpha}-1)}{\pi_1} + \frac{(1-\delta\gamma)(1-\eta)\left(\overline{\alpha}-\underline{\beta}\right)}{\pi_1} > 0. \tag{53}$$

Again, using (36) and (2), it is easy to see that the assumption $x_t^F \leq \left(m_t^F\right)^{w,u}$ implies $\underline{\beta}/\pi_1 < (1-\kappa)/(1-\delta\gamma) * \overline{\alpha}/(1-\overline{\alpha})^2$. Then, $\kappa \to 1$ implies $\overline{\beta}/\pi_1 \to 0$ (or, in other terms, $\delta\,(m_{t+1})^w/\pi_t \to 0$). It follows that (53) must hold true. $\square$

**Lemma 6.** *Suppose war does not occur in period 1. If $\kappa < \underline{\kappa}$, where $\underline{\kappa}$ was defined in Lemma 5, the unique balanced growth path SPNE of the subgame starting in period 2 is as follows. In*

*$t \geq 2$, if war has not yet occurred, actions are*

$$\left[w_t^F\left(a_t^L, a_t^F, s_t^F\right)\right]^* = \begin{cases} 0 & \text{if} \quad s_t^F \geq \underline{s}_t^F\left(a_t^L, a_t^F\right) \\ 1 & \text{if} \quad s_t^F < \underline{s}_t^F\left(a_t^L, a_t^F\right) \end{cases} \tag{54}$$

$$\left[s_t^F(a_t^L, a_t^F)\right]^* = \begin{cases} s & \text{if} \quad \underline{s}_t^F\left(a_t^L, a_t^F\right) > 1 \\ s' & \text{if} \quad \underline{s}_t^F\left(a_t^L, a_t^F\right) \in (0, 1] \\ 0 & \text{if} \quad \underline{s}_t^F\left(a_t^L, a_t^F\right) \leq 0 \end{cases} \tag{55}$$

$$\left[w_t^L(a_t^L, a_t^F)\right]^* = \begin{cases} 0 & \text{if} \quad \underline{s}_t^F\left(a_t^L, a_t^F\right) > 0 \\ 1 & \text{if} \quad \underline{s}_t^F\left(a_t^L, a_t^F\right) \leq 0 \end{cases} \tag{56}$$

$$(a_t^L)^* = \left(a_t^L\right)^w$$
$$(a_t^F)^* = \left(a_t^F\right)^w,$$

*where*

$$\underline{s}_t^F\left(a_t^L, a_t^F\right) \equiv \arg\left\{y_t^F - i_t^F - c_t^F a_t^F + s_t^F \pi_t + \delta V_{t+1}^F\left(w_{t+1} = 1\right) = V_t^F\left(w_t = 1 | a_t^L, a_t^F\right)\right\},$$

*and $\left(a_t^J\right)^w$ were defined in (19), and $s \in [0, 1]$, and $s' \in \left[0, \underline{s}_t^F\left(a_t^L, a_t^F\right)\right)$. In $t > 2$, if war has already occurred, actions are $\left(a_t^L\right)^* = \left(a_t^F\right)^* = 0$.*

*Proof.* ***Preliminaries.*** First, note that the term $k$, defined in Lemma 5, is the joint welfare gain (expressed as a share of the current pie) from delaying war by one period. As shown in that lemma, given $\kappa < \underline{\kappa}$, we have $k < 0$. Next, note that, in $t \geq 2$, if war has not yet occurred, the actions in (54)-(56) imply $w_t = 1$ (one country starts a war).[35] The proof proceed in three steps, and various sub-steps. ***Step 1.*** **The proposed outcome is a balanced growth path SPNE of the subgame starting in period 2**. Step 1.1. In $t > 2$, if war has already occurred, by Lemma 1, $\left(a_t^L\right)^*$ and $\left(a_t^F\right)^*$ are optimal actions. Step 1.2. In $t \geq 2$, if war has not yet occurred, if $w_t = 0$, then $w_{t+1} = 1$. It follows that $\underline{s}_t^F\left(a_t^L, a_t^F\right)$ is the share that leaves $F$ indifferent between starting a war in $t$, or not. Then, $\left[w_t^F\left(a_t^L, a_t^F, \pi_t^F\right)\right]^*$ follows from backward induction, and from tie-breaking rule 1. Next, note that the share that leaves $L$ indifferent between offering that

[35] If $\underline{s}_t^F\left(a_t^L, a_t^F\right) \leq 0$ it is $L$ that starts a war, otherwise it is $F$.

share (and being accepted) and starting a war,

$$\arg\left\{y_t^L - i_t^L - c_t^L a_t^L + s_t^L \pi_t + \delta V_{t+1}^L\left(w_{t+1}=1\right) = V_t^L\left(w_t = 1|a_t^L, a_t^F\right)\right\}, \tag{57}$$

can, substituting (11) into (57) and re-arranging, be written as $\underline{s}_t^F\left(a_t^L, a_t^F\right) + k$. Since $k < 0$, it follows that the share that leaves $L$ indifferent is lower than $\underline{s}_t^F\left(a_t^L, a_t^F\right)$. Then, $\left[s_t^F\left(a_t^L, a_t^F\right)\right]^*$ follows from backward induction, and $\left[w_t^L\left(a_t^L, a_t^F\right)\right]^*$ from backward induction and from tie-breaking rule 2. Finally, given $\left[w_t^F\left(a_t^L, a_t^F, \pi_t^F\right)\right]^*$, $\left[s_t^F\left(a_t^L, a_t^F\right)\right]^*$ and $\left[w_t^L\left(a_t^L, a_t^F\right)\right]^*$, it is $w_t = 1$ for any $a_t^L$ and $a_t^F$. Then, $(a_t^L)^*$ and $(a_t^F)^*$ follow from backward induction. ***Step 2 (intermediate result).*** **In $t \geq 2$, if war has not yet occurred, in any balanced growth path SPNE of the subgame starting in period $t$, war must occur immediately.** Take any balanced growth path SPNE of the subgame starting in period $t$, and denote equilibrium actions by $\hat{}$. Step 2.1. In the SPNE under consideration, war cannot happen in $T > t$. Suppose it did. By the logic of Step 1.2 (replacing $t$ with $T-1$), actions in $T-1$ would have to be as in (54)-(56). By the Preliminaries, that would imply $w_{T-1} = 1$: a contradiction. Step 2.2 (intermediate step). Suppose that, in the SPNE under consideration, war never happened. For $s \geq t$, let

$$\hat{k} \equiv \kappa - \frac{\delta \hat{m}_{s+1}}{\pi_s} + \mathcal{B}\frac{(1-\delta\gamma)\, g_s^F}{\pi_s},$$

which, given $\hat{m}_{s+1} = \gamma \hat{m}_s$ in a balanced growth path SPNE, and given $\pi_s = \gamma \pi_{s-1}$ and $g_s^F = \gamma g_{s-1}^F$, does not depend on $s$. Then, the joint surplus from not going to war in period $s$ can be written as $\sum_{v=s}^{\infty} \delta^{v-s} \hat{k} \pi_v = \hat{k} \Pi_s$. Let $\hat{e}_{s+1}^J \in [0,1]$ be the share of such surplus appropriated by $J$ in the subgame starting in period $s+1$, and let

$$\underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F) \equiv \arg\left\{y_s^F - i_s^F - c_s^F a_s^F + s_s^F \pi_s + \delta\left[V_{s+1}^F(w_{s+1}=1|\hat{a}_{s+1}^L, \hat{a}_{s+1}^F) + \hat{e}_{s+1}^F k \Pi_{s+1}\right] = V_s^F\left(w_s = 1|a_s^L, a_s^F\right)\right\}, \tag{58}$$

be the share that leaves $F$ indifferent between starting a war in period $s$, or not. Then, backward induction and tie-breaking rule 1 require

$$\hat{w}_s^F\left(a_s^L, a_s^F, s_s^F\right) = \begin{cases} 0 & \text{if } \; s_s^F \geq \underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F) \\ 1 & \text{if } \; s_s^F < \underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F) \end{cases} . \tag{59}$$

Next, note that the share that leaves $L$ indifferent between offering that share (and being accepted) and starting a war,

$$\arg\left\{y_s^L - i_s^L - c_s^L a_s^L + s_s^L \pi_s + \delta\left[V_{s+1}^L\left(w_{s+1} = 1\right) + \hat{e}_{s+1}^L \hat{k}\Pi_s\right] = V_s^L\left(w_s = 1 | a_s^L, a_s^F\right)\right\}, \tag{60}$$

can, substituting (11) into (60) and re-arranging, be written as $\underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F) + \hat{k}\Pi_s$. Step 2.3. It cannot be the case that, in the SPNE under consideration, war never happens, and $a_s^J = \left(a_s^J\right)^w$ for $J \in \{L, F\}$. Suppose this was the case. It would then be the case that $\hat{k} = k < 0$. Then, the share that leaves $L$ indifferent, $\underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F) + k\Pi_s$, would be less than $\underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F)$, given which

$$\hat{s}_s^F(a_s^L, a_s^F) = \begin{cases} s & \text{if } \; \underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F) > 1 \\ s' & \text{if } \; \underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F) \in (0, 1] \\ 0 & \text{if } \; \underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F) \leq 0 \end{cases} \qquad \hat{w}_t^L(a_t^L, a_t^F) = \begin{cases} 0 & \text{if } \; \underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F) > 0 \\ 1 & \text{if } \; \underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F) \leq 0 \end{cases},$$

where $s \in [0, 1]$ and $s' \in \left[0, \underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F)\right]$, would follow, respectively, from backward induction, and from backward induction and tie-breaking rule 2. But given $\hat{w}_s^F\left(a_s^L, a_s^F, s_s^F\right)$, $\hat{s}_s^F(a_s^L, a_s^F)$ and $\hat{w}_t^L(a_t^L, a_t^F)$, it would be the case that $w_s = 1$ for any $a_s^L$ and $a_s^F$, a contradiction. Step 2.4. It cannot be the case that, in the SPNE under consideration, war never happens, and $a_s^J \neq \left(a_s^J\right)^w$ for at least one $J \in \{L, F\}$. Suppose this was the case. It cannot be that $\hat{k} < 0$, or, by the logic of Step 2.3, we would have $w_s = 1$, a contradiction. Given $\hat{k} \geq 0$, the share that leaves $L$ indifferent, $\underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F) + \hat{k}\Pi_s$, is at least as high as $\underline{\hat{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F)$. Then,

$$\hat{s}^F_s(a^L_s,a^F_s)=\begin{cases} s & \text{if} \quad \hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})>1 \\ \underline{s}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1}) & \text{if} \quad \hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})\in[0,1] \\ 0 & \text{if} \quad \hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})<0 \end{cases} \qquad \hat{w}^L_s(a^L_s,a^F_s)=\begin{cases} 0 & \text{if} \quad \hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})\geq -\frac{\hat{k}\Pi_s}{\pi_s} \\ 1 & \text{if} \quad \hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})< -\frac{\hat{k}\Pi_s}{\pi_s} \end{cases}, \tag{61}$$

follow, respectively, from backward induction and tie-breaking rule 1, and from backward induction and tie-breaking rule 2. Also,

$$\hat{a}^L_s=\arg\max\quad V^L_s\left(w_s=1|a^L_s,a^F_s\right)+\begin{cases} 0 & \text{if} \quad \hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})>1 \\ \hat{k}\Pi_s & \text{if} \quad \hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})\in(0,1] \\ \hat{k}\Pi_s+\hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})\pi_s & \text{if} \quad \hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})\in\left[-\frac{\hat{k}\Pi_s}{\pi_s},0\right] \\ 0 & \text{if} \quad \hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})<-\frac{\hat{k}\Pi_s}{\pi_s} \end{cases} \tag{62}$$

$$\hat{a}^F_s=\arg\max_{c^F_s a^F_s\leq x^F_s}\quad V^F_s(w_s=1|a^L_s,a^F_s)+\begin{cases} 0 & \text{if} \quad \hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})>1 \\ 0 & \text{if} \quad \hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})\in(0,1] \\ -\hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})\pi_s & \text{if} \quad \underline{s}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})\in\left[-\frac{\hat{k}\Pi_s}{\pi_s},0\right] \\ 0 & \text{if} \quad \hat{\underline{s}}^F_s(a^L_s,a^F_s,\hat{e}^F_{s+1})<-\frac{\hat{k}\Pi_s}{\pi_s} \end{cases} \tag{63}$$

follow from backward induction. Given (59) and (61), for war not to occur in period $s$, it must be that $\hat{\underline{s}}^F_s(\hat{a}^L_s,\hat{a}^F_s,\hat{e}^F_{s+1})\in\left[-\frac{\hat{k}\Pi_s}{\pi_s},1\right]$. However, that cannot be the case. First, it cannot be the case that $\hat{\underline{s}}^F_s(\hat{a}^L_s,\hat{a}^F_s,\hat{e}^F_{s+1})\in\left[-\frac{\hat{k}\Pi_s}{\pi_s},0\right]$. To see this, note that, substituting (12) into (58) and re-arranging, $\hat{\underline{s}}^F_s(\hat{a}^L_s,\hat{a}^F_s,\hat{e}^F_{s+1})$ can be written as

$$\hat{\underline{s}}^F_s(\hat{a}^L_s,\hat{a}^F_s,\hat{e}^F_{s+1})=\hat{q}^F_s\frac{\Pi_s}{\pi_s}(1-\kappa)-\hat{q}^F_{s+1}\frac{\delta\Pi_{s+1}}{\pi_s}(1-\kappa)+\frac{\delta\hat{m}^F_{s+1}}{\pi_s}-\mathcal{B}\frac{(1-\delta\gamma)\,g^F_s}{\pi_s}-\frac{\delta\hat{e}^F_{s+1}k\Pi_{s+1}}{\pi_s}. \tag{64}$$

Then, substituting in (11)-(12) and (64), and re-arranging, payoffs in the case under consideration can be written as

$$V_s^L\left(w_s=1|\hat{a}_s^L,\hat{a}_s^F\right)+\hat{k}\Pi_s+\underline{\hat{s}}_s^F(\hat{a}_s^L,\hat{a}_s^F,\hat{e}_{s+1}^F)\pi_s=d_s^L(\hat{e}_{s+1}^F)-c_s^L\hat{a}_s^L \tag{65}$$

$$V_s^F\left(w_s=1|\hat{a}_s^L,\hat{a}_s^F\right)-\underline{\hat{s}}_s^F(\hat{a}_s^L,\hat{a}_s^F,\hat{e}_{s+1}^F)\pi_s=d_s^F\left(\hat{e}_{s+1}^F\right)-c_s^F\hat{a}_s^F, \tag{66}$$

where the $d_s^J\left(\hat{e}_{s+1}^F\right)$ are expressions which do not depend on $a_t^J$. It follows that at least one country could obtain a higher payoff by decreasing arming by $\epsilon$.[36] Next, it cannot be the case that $\underline{\hat{s}}_s^F(\hat{a}_s^L,\hat{a}_s^F,\hat{e}_{s+1}^F)\in(0,1)$. In that case, payoffs would be $V_s^L\left(w_s=1|a_s^L,a_s^F\right)+\hat{k}\Pi_s$ and $V_s^F\left(w_s=1|a_s^L,a_s^F\right)$ (with $\hat{k}\Pi_s$ an expression that does not depend on $a_t^J$). Since $\hat{a}_s^J\neq\left(a_s^J\right)^w$ for at least one of $J\in\{L,F\}$, given concavity of $V_s^J(w_s=1|a_s^L,a_s^F)$ in $a_s^J$, at least one of $J\in\{L,F\}$ could obtain a higher payoff by either increasing or decreasing arming by $\epsilon$. Finally, it cannot be the case that $\underline{s}_s^F(\hat{a}_s^L,\hat{a}_s^F,\hat{e}_{s+1}^F)=1$. To see this, note that, in a balanced growth path SPNE, it must be that $\hat{q}_s^F=\hat{q}_{s+1}^F$. Then, (64) can be written as

$$\begin{aligned}
&\hat{q}_s^F(1-\kappa)+\frac{\delta\hat{m}_{s+1}^F}{\pi_s}-\mathcal{B}\frac{(1-\delta\gamma)\,g_t^F}{\pi_s}-\frac{\delta\hat{e}_{s+1}^F\hat{k}\Pi_s}{\pi_s}=\\
&\underbrace{\hat{q}_s^F(1-\kappa)+\kappa}_{<1}-\underbrace{\left[\kappa-\frac{\delta\hat{m}_{s+1}^F}{\pi_s}+\mathcal{B}\frac{(1-\delta\gamma)\,g_t^F}{\pi_s}\right]}_{>\hat{k}\geq 0}-\underbrace{\frac{\delta\hat{e}_{s+1}^F\hat{k}\Pi_s}{\pi_s}}_{\geq 0}<1.
\end{aligned}$$

Since it cannot be the case that $\underline{\hat{s}}_s^F(\hat{a}_s^L,\hat{a}_s^F,\hat{e}_{s+1}^F)\in\left[-\frac{\hat{k}\Pi_s}{\pi_s},1\right]$, a war must occur in $s$, a contradiction. Step 2.5. In summary, we have shown that, in any balanced growth path of the subgame starting in period $t$, war cannot happen in $T>t$ (Step 2.1), nor can it be the case that war never occurs (Step 2.2). It follows that war must occur immediately. ***Step 3.*** **The proposed SPNE is the unique balanced growth path SPNE of the subgame starting in period 2.** In $t>2$, if war has already occurred, by Lemma 1, $\left(a_t^L\right)^*$ and $\left(a_t^F\right)^*$ are the unique optimal actions. Next, take any $t\geq 2$, and suppose that war has not yet occurred. If $w_t=0$, by Step 2, it is the case that $w_{t+1}=1$. By the logic of Step 1.2, then, $\left[w_t^F\left(a_t^L,a_t^F,s_t^F\right)\right]^*$, $\left[s_t^F\left(a_t^L,a_t^F\right)\right]^*$, $\left[w_t^L\left(a_t^L,a_t^F\right)\right]^*$, $(a_t^L)^*$ and $(a_t^F)^*$ are uniquely pinned down by backward induction,

[36] If $\underline{\hat{s}}_s^F(\hat{a}_s^L,\hat{a}_s^F,\hat{e}_{s+1}^F)=-\frac{\hat{k}\Pi_s}{\pi_s}$, it is $L$ who can obtain a higher payoff by decreasing arming (with $\underline{\hat{s}}_s^F(\hat{a}_s^L,\hat{a}_s^F,\hat{e}_{s+1}^F)$ remaining in the assumed range); if $\underline{\hat{s}}_s^F(\hat{a}_s^L,\hat{a}_s^F,\hat{e}_{s+1}^F)=0$, it is $F$; if $\underline{\hat{s}}_s^F(\hat{a}_s^L,\hat{a}_s^F,\hat{e}_{s+1}^F)\in\left(-\frac{\hat{k}\Pi_s}{\pi_s},0\right)$, it is both.

and by tie-breaking rules 1 and 2. $\square$

**Lemma 7.** *Suppose war does not occur in period 1. If $\kappa \geq \underline{\kappa}$, where $\underline{\kappa}$ was defined in Lemma 5, the unique balanced growth path SPNE of the subgame that starts in period 2 is as follows. In $t \geq 2$, if war has not yet occurred, actions are*

$$\left[w_t^F\left(a_t^L, a_t^F, s_t^F\right)\right]^* = \begin{cases} 0 & \textit{if} \;\; s_t^F \geq \underline{s}_t^F\left(a_t^L, a_t^F\right) \\ 1 & \textit{if} \;\; s_t^F < \underline{s}_t^F\left(a_t^L, a_t^F\right) \end{cases} \tag{67}$$

$$\left[s_t^F(a_t^L, a_t^F)\right]^* = \begin{cases} s & \textit{if} \;\; \underline{s}_t^F\left(a_t^L, a_t^F\right) > 1 \\ \underline{s}_t^F\left(a_t^L, a_t^F\right) & \textit{if} \;\; \underline{s}_t^F\left(a_t^L, a_t^F\right) \in [0,1] \\ 0 & \textit{if} \;\; \underline{s}_t^F\left(a_t^L, a_t^F\right) < 0 \end{cases} \tag{68}$$

$$\left[w_t^L(a_t^L, a_t^F)\right]^* = \begin{cases} 0 & \textit{if} \;\; \underline{s}_t^F\left(a_t^L, a_t^F\right) \geq -k\Pi_t/\pi_t \\ 1 & \textit{if} \;\; \underline{s}_t^F\left(a_t^L, a_t^F\right) < -k\Pi_t/\pi_t \end{cases} \tag{69}$$

$$(a_t^L)^* = \left(a_t^L\right)^w \tag{70}$$

$$(a_t^F)^* = \left(a_t^F\right)^w, \tag{71}$$

*where $k$ was defined in Lemma 5, and*

$$\underline{s}_t^F\left(a_t^L, a_t^F\right) \equiv \arg\left\{y_t^F - i_t^F - c_t^F a_t^F + s_t^F \pi_t + \delta V_{t+1}^F\left(w_{t+1} = 1\right) = V_t^F\left(w_t = 1 | a_t^L, a_t^F\right)\right\}, \tag{72}$$

*and $\left(a_t^J\right)^w$ were defined in (19), and $s \in [0,1]$. In $t > 2$, if war has already occurred, actions are $\left(a_t^L\right)^* = \left(a_t^F\right)^* = 0$.*

*Proof.* ***Preliminaries.*** First, note that the term $k$, defined in Lemma 5, is the joint welfare gain (expressed as a share of the current pie) from delaying war by one period. As shown in that lemma, given $\kappa \geq \underline{\kappa}$, we have $k \geq 0$. Next, note that in $t \geq 2$, if war has not yet occurred, the actions in (67)-(71) imply $w_t = 0$. They also imply that $F$ receives payoff $V_t^F\left(w_t = 1\right)$, and $L$ receives payoff $V_t^L\left(w_t = 1\right) + k\Pi_t$. To see this, note first that the actions in (67)-(69) imply $w_t = 0$ iff $\underline{s}_t^F\left(\left(a_t^L\right)^w, \left(a_t^F\right)^w\right) \in [-k\Pi_t/\pi_t, 1]$, and they imply that $F$ receives payoff $V_t^F\left(w_t = 1\right)$ iff $\underline{s}_t^F\left(\left(a_t^L\right)^w, \left(a_t^F\right)^w\right) \in [0,1]$. But $\underline{s}_t^F\left(\left(a_t^L\right)^w, \left(a_t^F\right)^w\right) \in (0,1)$. To see the latter point, note that, substituting (12) into (72), $\underline{s}_t^F\left(\left(a_t^L\right)^w,\left(a_t^F\right)^w\right)$ can be written as

$$\begin{aligned}\underline{s}_t^F\left(\left(a_t^L\right)^w,\left(a_t^F\right)^w\right) &= \left(q_t^F\right)^w\frac{\Pi_t}{\pi_t}(1-\kappa)-\left(q_{t+1}^F\right)^w\frac{\delta\Pi_{t+1}}{\pi_t}(1-\kappa)+\frac{\delta\left(m_{t+1}^F\right)^w}{\pi_t}-\mathcal{B}\frac{(1-\delta\gamma)\,g_t^F}{\pi_t}\\ &= \left(q_t^F\right)^w(1-\kappa)+\frac{\delta\left(m_{t+1}^F\right)^w}{\pi_t}-\mathcal{B}\frac{(1-\delta\gamma)\,g_t^F}{\pi_t},\end{aligned} \tag{73}$$

where we have used the fact that $\left(q_t^F\right)^w=\left(q_{t+1}^F\right)^w$. On the one hand, $\underline{s}_t^F\left(\left(a_t^L\right)^w,\left(a_t^F\right)^w\right)<1$, since (73) can be re-written as

$$\underbrace{\left(q_t^F\right)^w(1-\kappa)+\kappa}_{\in(0,1)}-\underbrace{\left[\kappa-\frac{\delta\left(m_{t+1}^F\right)^w}{\pi_t}+\mathcal{B}\frac{(1-\delta\gamma)\,g_t^F}{\pi_t}\right]}_{>k\geq 0}<1.$$

On the other hand, under Assumption 3, $\underline{s}_t^F\left(\left(a_t^L\right)^w,\left(a_t^F\right)^w\right)>0$. To see this, note that, using (73) and the fact that $\left(m_{t+1}^F\right)^w=\gamma\left(m_t^F\right)^w$, such inequality can be re-written as

$$\left(q_t^F\right)^w\frac{\pi_t}{1-\delta\gamma}(1-\kappa)+\frac{\delta\gamma\left(m_t^F\right)^w}{1-\delta\gamma}\geq\mathcal{B}g_t^F. \tag{74}$$

A sufficient condition for (74) to hold can be obtained by subtracting $\left(m_t^F\right)^w>0$ from the LHS and re-arranging,

$$\left(q_t^F\right)^w\frac{\pi_t}{1-\delta\gamma}(1-\kappa)-\left(m_t^F\right)^w\geq\mathcal{B}g_t^F. \tag{75}$$

The LHS of (75) is $F$'s gain from war (gross of any trade cost) if $a_t^L=\left(a_t^L\right)^w$ and $a_t^F=\left(a_t^F\right)^w$. Because $a_t^F=\left(a_t^F\right)^w>0$ maximises such gain, and not $a_t^F=0$ (the latter being a level of arming at which the LHS is zero), the LHS must be positive.[37] If $\mathcal{B}=0$, then, (75) always holds. If $\mathcal{B}=1$, substituting in (48) and (46), (75) can be written as

$$\left[\underline{\beta}\overline{\alpha}\frac{\pi_1}{1-\delta\gamma}(1-\kappa)\right]^{\frac{1}{2}}-\underline{\beta}\geq(1-\eta)\left(\overline{\alpha}-\underline{\beta}\right). \tag{76}$$

[37] To verify, if $\mathcal{B}=0$, plug in $\left(q_t^F\right)^w=c_t^L/(c_t^L+c_t^F)$ and $\left(m_t^F\right)^w=\Pi_t(1-\kappa)\,c_t^L/\left(c_t^L+c_t^F\right)^2$. The LHS can then be written as $\left(\frac{c_t^L}{c_t^L+c_t^F}\right)^2\frac{\pi_t}{1-\delta\gamma}(1-\kappa)>0$. If $\mathcal{B}=1$, as shown below, the LHS can be written as

By assumption, it is $x_t^F \leq \left(m_t^F\right)^{w,u}$. Using (36) and (2), it is easy to see that this implies $\overline{\alpha}\pi_1 / (1-\delta\gamma)(1-\kappa) > \underline{\beta}(1+\overline{\alpha})^2$. Then, a sufficient condition for (76) to hold is $\underline{\beta}(1+\overline{\alpha}) - \underline{\beta} = \underline{\beta}\overline{\alpha} \geq (1-\eta)\left(\overline{\alpha} - \underline{\beta}\right)$, that is Assumption 3. We have shown that, in $t \geq 2$, if war has not yet occurred, $w_t = 0$. This implies that it is also true that $w_s = 0$ for $s > t$ (war is forever delayed). Given that, as shown, $F$ receives payoff $V_t^F(w_t = 1)$ in period $t$, $L$ must then receive payoff $V_t^L(w_t = 1) + k\Pi_t$. The rest of the proof proceed in four steps, and in various sub-steps. ***Step 1.*** **The proposed outcome is a balanced growth path SPNE of the subgame starting in period 2.** Step 1.1. In $t > 2$, if war has already occurred, by Lemma 1, $(a_t^L)^*$ and $(a_t^F)^*$ are optimal actions. Step 1.2. Take $t \geq 2$, and suppose war has not yet occurred. If $w_t = 0$, by the Preliminaries, in period $t+1$, $F$ and $L$ receive payoffs $V_{t+1}^F(w_{t+1} = 1)$ and $V_{t+1}^L(w_{t+1} = 1) + k\Pi_{t+1}$. It follows that $\underline{s}_t^F\left(a_t^L, a_t^F\right)$ is the share that leaves $F$ indifferent between starting a war in $t$, or not. Then, $\left[w_t^F\left(a_t^L, a_t^F, \pi_t^F\right)\right]^*$ follows from backward induction, and from tie-breaking rule 1. Next, note that the share that leaves $L$ indifferent between offering that share (and being accepted) and starting a war,

$$\arg\left\{y_t^L - i_t^L - c_t^L a_t^L + s_t^L \pi_t + \delta\left[V_{t+1}^L(w_{t+1} = 1) + k\Pi_{t+1}\right] = V_t^L\left(w_t = 1 | a_t^L, a_t^F\right)\right\}, \tag{77}$$

can, substituting (11) into (77) and re-arranging, be written as $\underline{s}_t^F\left(a_t^L, a_t^F\right) + k\Pi_t$. It follows that the share that leaves $L$ indifferent is at least as high as $\underline{s}_t^F\left(a_t^L, a_t^F\right)$. Then, $\left[s_t^F\left(a_t^L, a_t^F\right)\right]^*$ follows from backward induction and from tie-breaking rule 1, and $\left[w_t^L\left(a_t^L, a_t^F\right)\right]^*$ from backward induction and from tie-breaking rule 2. Finally, given $\left[w_t^F\left(a_t^L, a_t^F, \pi_t^F\right)\right]^*$, $\left[s_t^F\left(a_t^L, a_t^F\right)\right]^*$ and

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$\left[\frac{c_t^L}{c_t^F}\frac{\pi_t}{1-\delta\gamma}(1-\kappa)x_t^F\right]^{\frac{1}{2}} - x_t^F$. This must be positive, or else

$$\frac{c_t^L}{c_t^F}\Pi_t(1-\kappa) < x_t^F$$

$$\frac{c_t^L c_t^F}{\left(c_t^L + c_t^F\right)^2}\Pi_t(1-\kappa) < x_t^F\left(1 + \frac{c_t^L}{c_t^F}\right),$$

which is in contradiction with the assumption that $\left(m_t^F\right)^{w,u} > x_t^F$, or $c_t^L c_t^F / \left(c_t^L + c_t^F\right)^2 \Pi_t(1-\kappa) > x_t^F$.

$\left[w_t^L\left(a_t^L, a_t^F\right)\right]^*$, it follows from backward induction that $a_t^L$ and $a_t^F$ must simultaneously solve

$$a_t^L = \arg \max \quad V_t^L(w_t = 1|a_t^L, a_t^F) + \begin{cases} 0 & \text{if } \underline{s}_t^F(a_t^L, a_t^F) > 1 \\ k\Pi_t & \text{if } \underline{s}_t^F(a_t^L, a_t^F) \in (0, 1] \\ k\Pi_t + \underline{s}_t^F(a_t^L, a_t^F)\pi_t & \text{if } \underline{s}_t^F(a_t^L, a_t^F) \in \left[-\frac{k\Pi_t}{\pi_t}, 0\right] \\ 0 & \text{if } \underline{s}_t^F(a_t^L, a_t^F) < -\frac{k\Pi_t}{\pi_t} \end{cases} \tag{78}$$

$$a_t^F = \arg \max_{c_t^F a_t^F \leq x_t^F} V_t^F(w_s = 1|a_t^L, a_t^F) + \begin{cases} 0 & \text{if } \underline{s}_t^F(a_t^L, a_t^F) > 1 \\ 0 & \text{if } \underline{s}_t^F(a_t^L, a_t^F) \in (0, 1] \\ -\underline{s}_t^F(a_t^L, a_t^F)\pi_t & \text{if } \underline{s}_t^F(a_t^L, a_t^F) \in \left[-\frac{k\Pi_t}{\pi_t}, 0\right] \\ 0 & \text{if } \underline{s}_t^F(a_t^L, a_t^F) < -\frac{k\Pi_t}{\pi_t} \end{cases}. \tag{79}$$

That $\left(a_t^L\right)^w$ solves (78) given $a_t^F = \left(a_t^F\right)^w$ follows from the following three facts. First, by definition, $\left(a_t^L\right)^w$ maximises $V_t^L\left(w_t = 1|a_t^L, \left(a_t^F\right)^w\right) + k\Pi_t$ (given that $k\Pi_t$ does not depend on $a_t^L$). Second, $\underline{s}_t^F\left(\left(a_t^L\right)^w, \left(a_t^F\right)^w\right) \in (0, 1)$. Third, $k\Pi_t + \underline{s}_t^F\left(a_t^L, \left(a_t^F\right)^w\right)\pi_t \leq k\Pi_t$ for $\underline{s}_t^F(a_t^L, \left(a_t^F\right)^w) \in \left[-\frac{k\Pi_t}{\pi_t}, 0\right]$. Does $\left(a_t^F\right)^w$ solve (79) given $a_t^L = \left(a_t^L\right)^w$? Since, by definition, $\left(a_t^F\right)^w$ maximises $V_t^F(w_t = 1|\left(a_t^L\right)^w, a_t^F)$, and since $\underline{s}_t^F(\left(a_t^L\right)^w, \left(a_t^F\right)^w) \in (0, 1)$, a necessary and sufficient condition for this is that $F$'s payoff be higher when choosing $\left(a_t^F\right)^w$, then when choosing any $a_t^F$ such that $\underline{s}_t^F(\left(a_t^L\right)^w, a_t^F) \in \left[-\frac{k\Pi_t}{\pi_t}, 0\right]$. Since, in the latter range, $F$'s payoff is $y_t^F - i_t^F - c_t^F a_t^F + \delta V_{t+1}^F(w_{t+1} = 1)$, $F$'s payoff in the latter case is at most $y_t^F - i_t^F + \delta V_{t+1}^F(w_{t+1} = 1)$. Then, a sufficient condition for $\left(a_t^F\right)^w$ to solve (79) given $a_t^L = \left(a_t^L\right)^w$ is

$$V_t^F(w_t = 1) \geq y_t^F - i_t^F + \delta V_{t+1}^F(w_{t+1} = 1). \tag{80}$$

Using (12), (80) can be re-written as

$$\begin{aligned} \left(q_t^F\right)^w \pi_t(1-\kappa) - (1-\delta\gamma)\left(m_t^F\right)^w - \mathcal{B}(1-\delta\gamma)\, g_t^F \geq 0 \\ \left(q_t^F\right)^w \frac{\pi_t}{1-\delta\gamma}(1-\kappa) - \left(m_t^F\right)^w \geq \mathcal{B} g_t^F. \end{aligned} \tag{81}$$

Condition (81) is the same as (75), which has been shown to hold under Assumption 3. Since

$\left(a_t^L\right)^w$ solves (78) given $a_t^F = \left(a_t^F\right)^w$, and $\left(a_t^F\right)^w$ solves (79) given $a_t^L = \left(a_t^L\right)^w$, $\left(a_t^L\right)^*$ and $\left(a_t^F\right)^*$ simultaneously solve (78) and (79). They then follow from backward induction. ***Step 2 (intermediate result)*** **In $t \geq 2$, if war has not yet occurred, in any balanced growth path SPNE of the subgame starting in $t$, in which war never occurs, it is the case that $a_s^L = \left(a_s^L\right)^w$ and $a_s^F = \left(a_s^F\right)^w$ for $s \geq t$, and $F$ and $L$ receive payoffs $V_s^F\left(w_s = 1\right)$ and $V_s^L\left(w_s = 1\right) + k\Pi_s$.** Take any balanced growth path SPNE of the subgame starting in period $t$, in which war never occurs, and denote equilibrium actions by $\hat{}$. Step 2.1. By Step 2.4 in the proof to Lemma 6, it must be that $\hat{a}_s^J = \left(a_s^J\right)^w$ for $J \in \{L, F\}$. Step 2.2. Given $\hat{a}_s^J = \left(a_s^J\right)^w$ for $J \in \{L, F\}$, the joint surplus from not going to war in period $s$ can be written as $k\Pi_s$. Let $e_{s+1}^J$ be the share of the surplus from not going to war appropriated by $F$ in the subgame that starts in period $s+1$, and

$$\underline{s}_s^F\left(a_s^L, a_s^F, e_{s+1}^F\right) \equiv \arg\left\{y_s^F - i_s^F - c_s^F a_s^F + s_s^F \pi_s + \delta\left[V_{s+1}^F(w_{s+1} = 1) + e_{s+1}^F k\Pi_{s+1}\right] = V_s^F\left(w_s = 1 | a_s^L, a_s^F\right)\right\},$$

that is the share that leaves $F$ indifferent between starting a war in period $s$, or not. Next, note that the share that leaves $L$ indifferent between offering that share (and being accepted) and starting a war,

$$\arg\left\{y_s^L - i_s^L - c_s^L a_s^L + s_s^L \pi_s + \delta\left[V_{s+1}^L\left(w_{s+1} = 1\right) + e_{s+1}^L k\Pi_s\right] = V_s^L\left(w_s = 1 | a_s^L, a_s^F\right)\right\}, \quad (82)$$

can, substituting (11) into (82) and re-arranging, be written as $\underline{s}_s^F\left(a_s^L, a_s^F, e_{s+1}^F\right) + k\Pi_s$. Then, backward induction and tie-breaking rules 1 and 2 require actions to be as in (59) and (61) (replacing $\hat{\underline{s}}_s^F(a_s^L, a_s^F, \hat{e}_{s+1}^F)$ with $\underline{s}_s^F\left(a_s^L, a_s^F, e_{s+1}^F\right)$ and $\hat{k}$ with $k$). Given such actions, for war not to occur in period $s$, it must be that $\underline{s}_s^F\left(\left(a_s^L\right)^w, \left(a_s^F\right)^w, \hat{e}_{s+1}^F\right) \in [-k\Pi_s/\pi_s, 1]$. However it cannot be the case that $\underline{s}_s^F\left(\left(a_s^L\right)^w, \left(a_s^F\right)^w, \hat{e}_{s+1}^F\right) \in \left[-\frac{k\Pi_s}{\pi_s}, 0\right]$, or else payoffs could be re-written as

$$V_s^L\left(w_s = 1 | \left(a_s^L\right)^w, \left(a_s^F\right)^w\right) + k\Pi_s + \underline{s}_s^F\left(\left(a_s^L\right)^w, \left(a_s^F\right)^w, e_{s+1}^F\right)\pi_s = d_s^L(e_{s+1}^F) - c_s^L\left(a_s^L\right)^w \quad (83)$$

$$V_s^F\left(w_s = 1 | \left(a_s^L\right)^w, \left(a_s^F\right)^w\right) - \underline{s}_s^F\left(\left(a_s^L\right)^w, \left(a_s^F\right)^w, e_{s+1}^F\right)\pi_s = d_s^F\left(e_{s+1}^F\right) - c_s^F\left(a_s^F\right)^w, \quad (84)$$

where the $d_s^J\left(e_{s+1}^F\right)$ are expressions which do not depend on $a_t^J$. It follows that at least one country could obtain a higher payoff by decreasing arming by $\epsilon$. Given $\underline{s}_s^F\left(\left(a_s^L\right)^w, \left(a_s^F\right)^w, e_{s+1}^F\right) \in (0,1]$, it is the case that $\hat{s}\left(\left(a_s^L\right)^w, \left(a_s^F\right)^w\right) = \underline{s}_s^F\left(\left(a_s^L\right)^w, \left(a_s^F\right)^w, e_{s+1}^F\right)$. This implies that $F$ receives payoff $V_s^F(w_s = 1)$, which in turn implies that $L$ receives payoff $V_s^F(w_s = 1) + k\Pi_s$.[38] ***Step 3 (intermediate result).*** **In $t \geq 2$, if war has not yet occurred, in any balanced growth path SPNE of the subgame starting in period $t$, war never occurs.** Take any balanced growth path SPNE of the game starting in period $t$. We now show that, in such a SPNE, in any $s \geq t$, war does not occur. In order to do that, we derive optimal actions in period $s$ for any possible equilibrium path of the subgame starting in $s+1$, and show that those actions do not lead to war. There are two possible cases. Step 3.1. First, in the equilibrium path of the subgame that starts in period $s+1$, war never occurs. By Step 2, in period $s+1$, $F$ and $L$ receive payoffs $V_{s+1}^F(w_{s+1} = 1)$ and $V_{s+1}^L(w_{s+1} = 1) + k\Pi_{s+1}$. By the logic of Step 1.2 (replacing $t$ with $s$), $\left[w_s^F\left(a_s^L, a_s^F, \pi_s^F\right)\right]^*$, $\left[s_s^F\left(a_s^L, a_s^F\right)\right]^*$, $\left[w_s^L\left(a_s^L, a_s^F\right)\right]^*$, $\left(a_s^L\right)^*$ and $\left(a_s^F\right)^*$ follow from backward induction and from tie-breaking rules 1 and 2. By the Preliminaries, it is then the case that $w_s = 0$. Step 3.2. Second, in the equilibrium path of the subgame that starts in period $s+1$, war occurs in $T \geq s+1$. By Lemma 4, $a_T^J = \left(a_T^J\right)^w$. It follows that, if $w_{T-1} = 0$, then $w_T = 1$, and, in period $T$, $F$ and $L$ receive payoffs $V_T^F(w_T = 1)$ and $V_T^L(w_T = 1)$. Consider optimal actions in period $T-1$. By the logic of Step 1.2 (replacing $t$ with $T-1$), $\left[w_{T-1}^F\left(a_{T-1}^L, a_{T-1}^F, s_{T-1}^F\right)\right]^*$ follows from backward induction, and from tie-breaking rule 1. Next, note that, the share that leaves $L$ indifferent between offering that share (and being accepted) and starting a war,

$$\arg\left\{y_{T-1}^L - i_{T-1}^L - c_{T-1}^L a_{T-1}^L + s_{T-1}^L \pi_{T-1} + \delta V_T^L(w_T = 1) = V_{T-1}^L\left(w_{T-1} = 1 | a_{T-1}^L, a_{T-1}^F\right)\right\}, \tag{85}$$

can, substituting (11) into (85) and re-arranging, be written as $\underline{s}_{T-1}^F\left(a_{T-1}^L, a_{T-1}^F\right) + k$. Again by the logic of Step 1.2 (additionally replacing $k\Pi_{T-1}$ with $k\pi_{T-1}$), $\left[s_{T-1}^F\left(a_{T-1}^L, a_{T-1}^F\right)\right]^*$, $\left[w_{T-1}^L(a_{T-1}^L, a_{T-1}^F)\right]^*$, $\left(a_{T-1}^L\right)^*$ and $\left(a_{T-1}^F\right)^*$ follow from backward induction, and from tie-breaking rule 2. By the logic of the Preliminaries (replacing $t$ with $T-1$, and $k\Pi_{T-1}$ with $k\pi_{T-1}$), these actions imply $w_{T-1} = 0$, and that $F$ and $L$ receive payoffs $V_{T-1}^F(w_{T-1} = 0)$ and $V_{T-1}^L(w_{T-1} = 0) + k$. If $T = s+1$, it is established that $w_s = 0$. If $T > s+1$, it is established that, if $w_{T-2} = 0$, then $w_{T-1} = 0$, and,

[38] It is then the case that $e_s^F = 0$. Indeed, we have shown in the Preliminaries that $\underline{s}_s^F\left(\left(a_s^L\right)^w, \left(a_s^F\right)^w, 0\right) \in (0,1)$.

in period $T-1$, $F$ and $L$ receive payoffs $V^F_{T-1}(w_{T-1}=0)$ and $V^L_{T-1}(w_{T-1}=0)+k$. Consider optimal actions in period $T-2$. By the logic of the analysis just conducted for period $T-1$ (now replacing $t$ with $T-2$, and $k\Pi_{T-2}$ with $k\pi_{T-2}+\delta k\pi_{T-1}$) it follows that $w_{T-2}=0$, and $F$ and $L$ receive payoffs $V^F_{T-2}(w_{T-2}=0)$ and $V^L_{T-2}(w_{L-2}=0)+k\pi_{T-2}+\delta k\pi_{T-1}$. If $T=s+2$, it is established that $w_s=0$. If $T>s+2$, $w_s=0$ can be established using the logic of the analysis just conducted recursively. ***Step 4.*** **The proposed SPNE is the unique balanced growth path SPNE of the subgame starting in period 2.** In $t>2$, if war has already occurred, by Lemma 1, $\left(a^L_t\right)^*$ and $\left(a^F_t\right)^*$ are the unique optimal actions. Next, take any $t\geq 2$, and suppose that war has not yet occurred. If $w_t=0$, by Step 2 and 3, in period $t+1$, $F$ and $L$ receive payoffs $V^F_{t+1}(w_{t+1}=1)$ and $V^L_{t+1}(w_{t+1}=1)+k\Pi_{t+1}$. By the logic of Step 1.2, then, $\left[w^F_t\left(a^L_t,a^F_t,s^F_t\right)\right]^*$, $\left[s^F_t\left(a^L_t,a^F_t\right)\right]^*$, $\left[w^L_t\left(a^L_t,a^F_t\right)\right]^*$, $(a^L_t)^*$ and $(a^F_t)^*$ are uniquely pinned down by backward induction, and by tie-breaking rules 1 and 2. □

## B.2 Proofs of results in the main text

**Lemma 1.** Provided in main text. ■

**Lemma 2.** We begin by summarising a few results (established earlier in this Online Appendix) that are used in the proof, and we then proceed to the proof itself.

*Summary of earlier results.* In Lemma 5, we derived the joint welfare gain from delaying war by one period (expressed as a share of the current pie); we showed that, if $\mathcal{B}=0$, this is constant from period 1 onwards, while if $\mathcal{B}=1$, it increases between period 1 and period 2, and is then constant from period 2 onwards; and, letting $k$ denote this constant value of the joint welfare gain, we showed that there exists $\underline{\kappa}\in(0,1)$ such that $k<0$ if and only if $\kappa<\underline{\kappa}$. In Lemmas 6 and 7, we showed that, in the unique balanced growth path SPNE of the subgame starting in period 2, if $\kappa<\underline{\kappa}$, war is welfare increasing (since the gain from delaying it is negative), and always occurs in period 2, while if $\kappa\geq\underline{\kappa}$, war is welfare reducing, and never occurs; and, in the latter case, $F$ and $L$ receive payoffs $V^F_2(w_2=1)$ and $V^L_2(w_2=1)+k\Pi_2$ in period 2.

*Proof.* Let

$$\underline{s}_1^F\left(a_1^L, a_1^F\right) \equiv \arg\left\{y_1^F - i_1^F - c_1^F a_1^F + s_1^F \pi_1 + \delta V_2^F\left(w_2 = 1\right) = V_1^F\left(w_1 = 1 | a_1^L, a_1^F\right)\right\},$$

that is the share that leaves $F$ indifferent between starting a war in period 1, or not (note that, by the results summarised above, $F$ always receives payoff $V_2^F\left(w_2 = 1\right)$ in period 2). By backward induction and tie-breaking rule 1, in a SPNE, it must be that

$$w_1^F\left(a_1^L, a_1^F, s_1^F\right) = \begin{cases} 0 & \text{if} \quad s_1^F \geq \underline{s}_1^F\left(a_1^L, a_1^F\right) \\ 1 & \text{if} \quad s_1^F < \underline{s}_1^F\left(a_1^L, a_1^F\right) \end{cases}. \tag{86}$$

If $\mathcal{B} = 0$, let $\hat{\kappa} \equiv \underline{\kappa}$. If $\kappa < \hat{\kappa}$, the share that leaves $L$ indifferent between offering that share (and it being accepted) and starting a war must be equal to

$$\arg\left\{y_1^L - i_1^L - c_1^L a_1^L + s_1^L \pi_1 + \delta V_2^L\left(w_2 = 1\right) = V_1^L\left(w_1 = 1 | a_1^L, a_1^F\right)\right\}, \tag{87}$$

since, by the results summarised above, there is a war in period 2. Using (11), (87) can be re-written as

$$\underline{s}_1^F\left(a_1^L, a_1^F\right) + \kappa - \frac{\delta m_2}{\pi_1} = \underline{s}_1^F\left(a_1^L, a_1^F\right) + k.$$

The term $k$ is the joint welfare gain (expressed as a share of period 1 pie) from delaying war by one period in period 1. As explained above, it is equal to $k$. By the results summarised above, since $\kappa < \underline{\kappa}$, $k < 0$. Then, war is welfare increasing in period 1. Since the share that leaves $L$ indifferent is less than $\underline{s}_1^F\left(a_1^L, a_1^F\right)$, by backward induction and tie-breaking rule 2, in a SPNE, it must be that

$$s_1^F(a_1^L, a_1^F) = \begin{cases} s & \text{if} \quad \underline{s}_1^F\left(a_1^L, a_1^F\right) > 1 \\ s' & \text{if} \quad \underline{s}_1^F\left(a_1^L, a_1^F\right) \in (0,1] \\ 0 & \text{if} \quad \underline{s}_1^F\left(a_1^L, a_1^F\right) \leq 0 \end{cases} \tag{88}$$

$$w_1^L(a_1^L, a_1^F) = \begin{cases} 0 & \text{if} \quad \underline{s}_1^F\left(a_1^L, a_1^F\right) > 0 \\ 1 & \text{if} \quad \underline{s}_1^F\left(a_1^L, a_1^F\right) \leq 0 \end{cases}, \tag{89}$$

where $s \in [0,1]$ and $s' \in \left[0, \underline{s}_1^F\left(a_1^L, a_1^F\right)\right)$. Given (86), (88) and (89), we have $w_1 = 1$.

If $\mathcal{B} = 1$, there are two cases. If $\kappa < \underline{\kappa}$, the share that leaves $L$ indifferent between offering that share (and being accepted) and starting a war, still defined as in (87), can now be re-written (using 11) as

$$\underline{s}_1^F\left(a_1^L, a_1^F\right) + \kappa - \frac{\delta m_2}{\pi_1} + \frac{g_1^F - \delta g_2^F}{\pi_1}.$$

The term

$$\begin{aligned} \kappa - \frac{\delta m_2}{\pi_1} + \frac{g_1^F - \delta g_2^F}{\pi_1} &= \kappa - \frac{\delta m_2}{\pi_1} + \frac{(1-\delta\gamma)\frac{g_2^F}{\gamma}}{\pi_1} + \frac{g_1^F - \frac{g_2^F}{\gamma}}{\pi_1} \\ &= k + \frac{g_1^F - \frac{g_2^F}{\gamma}}{\pi_1} \\ &= k + \frac{(1-\eta)\left[\underline{\alpha} - \overline{\beta} - \left(\overline{\alpha} - \underline{\beta}\right)\right]}{\pi_1} < k \end{aligned}$$

is the joint welfare gain (expressed as a share of period 1 pie) from delaying war by one period in period 1. As explained above, it is less than $k$, which by the results summarised above, is negative (since $\kappa < \underline{\kappa}$). Then, war is welfare increasing in period 1. Since the share that leaves $L$ indifferent is less than $\underline{s}_1^F\left(a_1^L, a_1^F\right)$, by backward induction and tie-breaking rule 2, in a SPNE, actions must be as in (88) and (89). Given (86), (88) and (89), $w_1 = 1$. If $\kappa \geq \underline{\kappa}$, the share that leaves $L$ indifferent between offering that share (and it being accepted) and starting a war, must now be equal to

$$\arg\left\{y_1^L - i_1^L - c_1^L a_1^L + s_1^L \pi_1 + \delta\left[V_2^L\left(w_2 = 1\right) + \delta k \Pi_2\right] = V_1^L\left(w_1 = 1 | a_1^L, a_1^F\right)\right\}, \tag{90}$$

since, by the results summarised above, if war does not occur in period 1, it does not occur anymore. Using (11), (90) can be re-written as

$$\underline{s}_1^F\left(a_1^L, a_1^F\right) + \kappa - \frac{\delta m_2}{\pi_1} + \frac{g_1^F - \delta g_2^F}{\pi_1} + \delta k \frac{\Pi_2}{\pi_1}.$$

The term

$$\begin{aligned}\kappa - \frac{\delta m_2}{\pi_1} + \frac{g_1^F - \delta g_2^F}{\pi_1} + \delta k \frac{\Pi_2}{\pi_1} &= k + \frac{(1-\eta)\left[\underline{\alpha} - \overline{\beta} - \left(\overline{\alpha} - \underline{\beta}\right)\right]}{\pi_1} + \delta k \frac{\Pi_2}{\pi_1} \\ &= \frac{k}{1-\delta\gamma} + \frac{(1-\eta)\left[\underline{\alpha} - \overline{\beta} - \left(\overline{\alpha} - \underline{\beta}\right)\right]}{\pi_1}\end{aligned} \tag{91}$$

is the joint welfare gain (expressed as a share of period 1 pie) from permanently delaying war in period 1. To determine its sign, we use another result established earlier in this Online Appendix. In the proof to Lemma 5, it was shown that $k$ is continuously increasing in $\kappa$, and increases from 0 to $1 + (1-\delta\gamma)(1-\eta)\left(\overline{\alpha} - \underline{\beta}\right)/\pi_1$ as $\kappa$ increases from $\underline{\kappa}$ to 1. It follows that the RHS of (91), which is continuously increasing in $k$, is continuously increasing from $(1-\eta)\left[\underline{\alpha} - \overline{\beta} - \left(\overline{\alpha} - \underline{\beta}\right)\right]/\pi_1 < 0$ to

$$\begin{aligned}&\frac{1 + \frac{(1-\delta\gamma)(1-\eta)\left(\overline{\alpha} - \underline{\beta}\right)}{\pi_1}}{1-\delta\gamma} + \frac{(1-\eta)\left[\underline{\alpha} - \overline{\beta} - \left(\overline{\alpha} - \underline{\beta}\right)\right]}{\pi_1} \\ &= \frac{1}{1-\delta\gamma} + \frac{(1-\eta)\left(\underline{\alpha} - \overline{\beta}\right)}{\pi_1} > 0,\end{aligned}$$

as $\kappa$ increases from $\underline{\kappa}$ to 1. Then, there exists $\hat{\kappa} \in (\underline{\kappa}, 1)$ such that, if $\kappa < \hat{\kappa}$, the RHS of (91) is negative, and war is welfare increasing in period 1. In addition, since the share that leaves $L$ indifferent is less than $\underline{s}_1^F\left(a_1^L, a_1^F\right)$, by backward induction and tie-breaking rule 2, in a SPNE, actions must be as in (88) and (89). Given (86), (88) and (89), $w_1 = 1$. ■

**Lemma 3.** The Lemma is valid under Assumption 2, $\kappa \geq \hat{\kappa}$. The threshold $\hat{\kappa}$ was defined in Lemma 2. As shown in the proof to that lemma, it is $\hat{\kappa} \geq \underline{\kappa}$, where $\underline{\kappa} \in (0,1)$ is a threshold defined earlier in this Online Appendix. Then, $\kappa \geq \hat{\kappa}$ implies $\kappa \geq \underline{\kappa}$. It follows that, if war does not occur in period 1, the unique balanced growth path SPNE of the subgame starting in period 2 is as described in Lemma 7. In such an equilibrium, as shown in the proof to that lemma (see in particular the Preliminaries), for $t \geq 2$, it is the case that $w_t = 0$, $a_t^L = \left(a_t^L\right)^w$ and $a_t^F = \left(a_t^L\right)^w$, and $F$ receives payoff $V_t^F\left(w_t = 1\right)$. ■

**Proposition 1.** If $w_1 = 0$, by Lemma 3, $F$ receives payoff $V_2^F\left(w_2 = 1\right)$ in period 2; as for $L$'s payoff in period 2, given $\kappa \to \hat{\kappa}$, it converges to $V_1^L\left(w_1 = 1\right)$. Then,

$$\left[w_1^F\left(a_1^L, a_1^F, s_1^F\right)\right]^* = \begin{cases} 0 & \text{if} \quad s_1^F \geq \underline{s}_1^F\left(a_1^L, a_1^F\right) \\ 1 & \text{if} \quad s_1^F < \underline{s}_1^F\left(a_1^L, a_1^F\right) \end{cases} \tag{92}$$

$$\left[s_1^F(a_1^L, a_1^F)\right]^* = \begin{cases} s & \text{if} \quad \underline{s}_1^F\left(a_1^L, a_1^F\right) > 1 \\ \underline{s}_1^F\left(a_1^L, a_1^F\right) & \text{if} \quad \underline{s}_1^F\left(a_1^L, a_1^F\right) \in [0,1] \\ 0 & \text{if} \quad \underline{s}_1^F\left(a_1^L, a_1^F\right) < 0 \end{cases}, \tag{93}$$

(where $s \in [0,1]$) follow from backward induction, and from tie-breaking rule 1). It also follows from backward induction that $\left[w_1^L(a_1^L, a_1^F)\right]^*$ converges to

$$\left[w_1^L(a_1^L, a_1^F)\right] = \begin{cases} 0 & \text{if} \quad \underline{s}_1^F\left(a_1^L, a_1^F\right) \geq 0 \\ 1 & \text{if} \quad \underline{s}_1^F\left(a_1^L, a_1^F\right) < 0 \end{cases}. \tag{94}$$

Finally, given $\left[w_t^F\left(a_1^L, a_1^F, s_1^F\right)\right]^*$, $\left[s_1^F(a_1^L, a_1^F)\right]^*$ and $\left[w_1^L(a_1^L, a_1^F)\right]^*$, it follows from backward induction that $(a_1^L)^*$ and $(a_1^F)^*$ converge to

$$a_t^L = \arg \max \quad V_t^L(w_t = 1|a_t^L, a_t^F) \tag{95}$$

$$a_t^F = \arg \max_{c_t^F a_t^F \leq x_t^F} \quad V_t^F(w_s = 1|a_t^L, a_t^F), \tag{96}$$

that is to $(a_1^L)^w$ and $(a_1^F)^w$. Clearly, as $\kappa$ approaches $\hat{\kappa}$, the equilibrium becomes one in which there is an $F$-led war iff $\underline{s}_1^F\left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) > 1$, no war if $\underline{s}_1^F\left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) \in [0,1]$, and an

$L$-led war iff $\underline{s}_1^F \left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) < 0$.

We have

$$\underline{s}_1^F \left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) = \left[\left(q_1^F\right)^w - \delta\gamma \left(q_2^F\right)^w\right] \frac{\Pi_1}{\pi_1}(1-\hat{\kappa}) + \frac{\delta \left(m_2^F\right)^w}{\pi_1}.$$

If it were the case that $(q_1^F)^w = \left(q_2^F\right)^w$, this could be written as

$$\left(q_1^F\right)^w (1-\hat{\kappa}) + \frac{\delta \left(m_2^F\right)^w}{\pi_1} = \underbrace{\left(q_1^F\right)^w (1-\hat{\kappa}) + \hat{\kappa}}_{\in(0,1)} - \underbrace{\left[\hat{\kappa} - \frac{\delta \left(m_2^F\right)^w}{\pi_1}\right]}_{>k=0} < 1.$$

Then, since $(q_1^F)^w = \frac{\underline{\alpha}}{1+\underline{\alpha}} \leq \frac{\overline{\alpha}}{1+\overline{\alpha}} = \left(q_2^F\right)^w$, it is always true that $\underline{s}_1^F \left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) < 1$. To see this, note that if we increased $\left(q_1^F\right)^w$ to the value $\frac{\overline{\alpha}}{1+\overline{\alpha}}$, $\underline{s}_1^F \left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right)$ would unambiguously increase. However, this increased value of $\underline{s}_1^F \left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right)$ would still be less than 1, since it would then be the case that $\left(q_1^F\right)^w = \left(q_2^F\right)^w$.

It follows that there cannot be an $F$-led war. Using (37), (38) and (36), condition $\underline{s}_1^F \left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) < 0$ can be written as

$$\left[\frac{\underline{\alpha}}{1+\underline{\alpha}} - \delta\gamma\frac{\overline{\alpha}}{1+\overline{\alpha}}\right] \frac{\Pi_1}{\pi_1}(1-\hat{\kappa}) + \delta\gamma\frac{\Pi_1}{\pi_1}(1-\hat{\kappa})\frac{\overline{\alpha}}{(1+\overline{\alpha})^2} < 0$$
$$\left[\frac{\underline{\alpha}}{1+\underline{\alpha}} - \delta\gamma\left(\frac{\overline{\alpha}}{1+\overline{\alpha}}\right)^2\right] \frac{\Pi_1}{\pi_1}(1-\hat{\kappa}) < 0.$$

Then, there is an $L$-led war if and only if the condition in the proposition holds. ■

**Proposition 2.** The first part of the proof is identical to that of the proof to Proposition 1. It is the case that

$$\underline{s}_1^F \left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) = \left[\left(q_1^F\right)^w - \delta\gamma \left(q_2^F\right)^w\right] \frac{\Pi_1}{\pi_1}(1-\hat{\kappa}) + \frac{\delta \left(m_2^F\right)^w}{\pi_1} - \frac{g_1^F - \delta g_2^F}{\pi_1}.$$

Using (47), (48), (46) and (9), and imposing $\underline{s}_1^F \left(a_1^L, a_1^F\right) > 1$ as a necessary and sufficient condition for an $F$-led war, and $\underline{s}_1^F \left(a_1^L, a_1^F\right) < 0$ as a necessary and sufficient condition for a $L$-led war, we obtain the two conditions in the proposition. ■

**Corollary 1.** Consider first the case $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta}$. This can be divided into two subcases, $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta} = 1$ and $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta} > 1$. If $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta} = 1$, the LHS of (30) can be written as

$$(1-\delta\gamma)\left(\overline{\beta}\underline{\alpha}\right)^{\frac{1}{2}}\frac{\left[\Pi_1(1-\hat{\kappa})\right]^{\frac{1}{2}}}{\pi_1}+\frac{\delta\gamma\underline{\beta}}{\pi_1}-(1-\eta)\frac{(1-\delta\gamma)\left(\underline{\alpha}-\overline{\beta}\right)}{\pi_1},$$

which, using (48) and (46), and (9), can be re-written as

$$\left(q_1^F\right)^w(1-\kappa)+\frac{\delta\left(m_2^F\right)^w}{\pi_1}-\frac{(1-\delta\gamma)\,g_1^F}{\pi_1}. \tag{97}$$

The last expression was shown in the Preliminaries of the Proof to Lemma 7 to lie between 0 and 1, implying that there is peace. Next consider the case $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta} > 1$. The LHS of (30) can now be written as

$$(1-\delta\gamma)\left(\overline{\beta}\underline{\alpha}\right)^{\frac{1}{2}}\frac{\left[\Pi_1(1-\hat{\kappa})\right]^{\frac{1}{2}}}{\pi_1}+\frac{\delta\gamma\underline{\beta}}{\pi_1}+(1-\eta)\frac{\delta\gamma\left(\overline{\alpha}-\underline{\beta}\right)-\left(\underline{\alpha}-\overline{\beta}\right)}{\pi_1}. \tag{98}$$

As shown in numerical example 4 in Online Appendix C, this expression can be greater than 1, implying that there can be a $F$-led war. Next, (98) is greater than zero, implying that there cannot be a $L$-led war. To see this, note that one can always simultaneously decrease $\overline{\alpha}$ and $\overline{\beta}$ (while keeping $\underline{\alpha}$) and ($\underline{\beta}$ constant) so as to obtain $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta} = 1$. Such a change unambiguously decreases (98), making it equal to (97), or to an expression which we have shown to be greater than zero. The result then follows.

Second, consider the case $\overline{\alpha}/\underline{\alpha} < \overline{\beta}/\underline{\beta}$. That there can be peace (i.e., that the RHS of 30 can lie between zero and one) follows from continuity, since: if $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta} = 1$, the LHS of (30) lies between zero and one; the case $\overline{\alpha}/\underline{\alpha} < \overline{\beta}/\underline{\beta}$ can be obtained starting from $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta} = 1$, and increasing $\overline{\beta}$ by a small amount; and the LHS of (30) is continuous in $\overline{\beta}$. Similarly, that there can be a $F$-led war (i.e. that condition 30 can hold) follows from continuity, since the case $\overline{\alpha}/\underline{\alpha} < \overline{\beta}/\underline{\beta}$ can be obtained starting from $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta} > 1$, and increasing $\overline{\beta}$ by a small amount. There cannot be a $L$-led war (i.e. the LHS of 30 cannot be negative). To see this, note that one can always decrease $\overline{\beta}$ so as to obtain $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta} \geq 1$. Such a change unambiguously decreases the LHS of (30), making it equal to either (97) or (98), or two expressions which were shown to be greater than zero. The result then follows.

Finally, consider the case $\overline{\alpha}/\underline{\alpha} > \overline{\beta}/\underline{\beta}$. Again, that there can be peace or an $F$-led war (i.e. that the LHS of 30 can take value between 0 and 1, or be greater than 1) follows from continuity, since this case can be obtained starting from $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta} = 1$ or from $\overline{\alpha}/\underline{\alpha} = \overline{\beta}/\underline{\beta} > 1$, and increasing $\overline{\alpha}$ by a small amount. There can also be a $L$-led war (i.e. the LHS of 30 can be less than 0), as shown in numerical example 5 in Online Appendix C. ■

# Online Appendix C: Numerical examples

Let $\phi = \left[\delta, \gamma, \rho, \underline{\alpha}, \overline{\alpha}, \underline{\beta}, \overline{\beta}, \pi_1, \eta\right]^{\mathbf{T}}$ denote the vector of parameters of the model (excluding $\kappa$, which is set to be equal to an endogenous threshold). In this section, we find three vectors, $\phi'$, $\phi''$ and $\phi'''$, that satisfy all relevant parametric assumptions, and such that

- If $\mathcal{B} = 0$, there is peace if $\phi = \phi'$, an $L$-led war if $\phi = \phi''$.
- If $\mathcal{B} = 1$, there is peace if $\phi = \phi'$, an $F$ led war if $\phi = \phi''$, an $L$-led war if $\phi = \phi'''$.

We begin by summarising the parametric assumptions of the paper in Section C.1, and we then provide numerical examples in Section C.2.

## C.1: Summary of parametric assumptions

The parametric assumptions of the paper can be divided in two groups: those that define the basic characteristics of the environment, and those that ensures that the endogenous variables are within the desired ranges. The former need to hold for any $\mathcal{B}$, while the latter are specific to a value of $\mathcal{B}$.

The first group of assumptions are

$$\delta < 1 \tag{99}$$

$$\gamma \geq 1 \tag{100}$$

$$\rho \geq 0 \tag{101}$$

$$0 < \underline{\alpha} \leq \overline{\alpha} \tag{102}$$

$$0 \leq \underline{\beta} \leq \overline{\beta} \tag{103}$$

$$\underline{\alpha} \geq \overline{\beta} \tag{104}$$

$$\pi_1 > 0 \tag{105}$$

$$\eta \in [0, 1)\,. \tag{106}$$

In addition, it must be that $\kappa \in [0, 1)$; however this is ensured by the fact that we set $\kappa = \hat{\kappa}$ (see below), and that, by Lemma 2, $\hat{\kappa} \in (0, 1)$.

If $\mathcal{B} = 0$, the second group of assumptions are as follows. First, it should be the case that

$$\kappa = \hat{\kappa}, \tag{107}$$

where the endogenous threshold $\hat{\kappa}$ is defined as in (50) (recall from the proof to Lemma 2 that $\hat{\kappa} = \underline{\kappa}$ in this case). Next, it should be the case that $\left(m_t^J\right)^{w,u} \leq y_t^J - i_t^J$ for $J \in \{L, F\}$ and $t \geq 1$: in both countries, the national endowment of the industrial input should be enough to produce the unconstrained optimal arming level (see Section 3.3). The next four conditions, derived using (35), (36), and (2), ensure that this is true, respectively, in $L$ in $t = 1$, in $L$ in $t \geq 2$, in $F$ in $t = 1$, and in $F$ in $t \geq 2$:

$$\frac{\pi_1}{1-\delta\gamma}\left(1-\hat{\kappa}\right)\frac{\underline{\alpha}}{\left(1+\underline{\alpha}\right)^2} \leq \left(1-\eta\right)+\eta\rho \tag{108}$$

$$\frac{\pi_1}{1-\delta\gamma}\left(1-\hat{\kappa}\right)\frac{\overline{\alpha}}{\left(1+\overline{\alpha}\right)^2} \leq \left(1-\eta\right)+\eta\rho. \tag{109}$$

$$\frac{\pi_1}{1-\delta\gamma}\left(1-\hat{\kappa}\right)\frac{\underline{\alpha}}{\left(1+\underline{\alpha}\right)^2} \leq \left(1-\eta\right)\underline{\alpha}+\eta\overline{\beta} \tag{110}$$

$$\frac{\pi_1}{1-\delta\gamma}\left(1-\hat{\kappa}\right)\frac{\overline{\alpha}}{\left(1+\overline{\alpha}\right)^2} \leq \left(1-\eta\right)\overline{\alpha}+\eta\underline{\beta}. \tag{111}$$

If $\mathcal{B} = 1$, the second group of assumptions are as follows. First, it should be the case that

$$\kappa = \hat{\kappa}, \tag{112}$$

where the endogenous threshold $\hat{\kappa}$ is now defined as the value that makes the RHS of (91) equal to zero. Next, we need to make sure that $\left(m_t^L\right)^{w,c} \leq y_t^L - i_t^L$ for $t \geq 1$: $L$'s endowment of the industrial input should always be enough to produce the constrained optimal arming level.[39] The next two conditions, derived using (43), (44), and (2), ensure that this is true, respectively, in

[39]Note that, for $F$, it is $\left(m_t^F\right)^{w,u} > \left(m_t^F\right)^{w,c}$. For $L$, however, if $\left(a_t^F\right)^{w,u} > \left(a_t^L\right)^{w,u}$, it may be $\left(a_t^L\right)^{w,c} > \left(a_t^L\right)^{w,u}$, which implies $\left(m_t^L\right)^{w,c} > \left(m_t^L\right)^{w,u}$.

$t = 1$ and $t > 1$:

$$\left[\frac{\pi_1}{1-\delta\gamma}\left(1-\hat{\kappa}\right)\overline{\beta}\underline{\alpha}\right]^{\frac{1}{2}} - \overline{\beta}\underline{\alpha} \leq (1-\eta) + \eta\rho \tag{113}$$

$$\left[\frac{\pi_1}{1-\delta\gamma}\left(1-\hat{\kappa}\right)\underline{\beta}\overline{\alpha}\gamma^{2(t-1)}\right]^{\frac{1}{2}} - \underline{\beta}\overline{\alpha}\gamma^{t-1} \leq (1-\eta) + \eta\rho. \tag{114}$$

Also, it should be the case that $x_t^F \leq \left(m_t^F\right)^{w,u}$ for $t \geq 1$: in $F$, the national endowment of raw materials should never be enough to produce the unconstrained optimal arming level (see Section 3.3). The next two conditions, derived using (35), (36) and (2), ensure that this is true for $t = 1$ and $t > 1$ respectively:

$$\overline{\beta} < \frac{\pi_1}{1-\delta\gamma}\left(1-\hat{\kappa}\right)\frac{\underline{\alpha}}{\left(1+\underline{\alpha}\right)^2} \tag{115}$$

$$\underline{\beta} < \frac{\pi_1}{1-\delta\gamma}\left(1-\hat{\kappa}\right)\frac{\overline{\alpha}}{\left(1+\overline{\alpha}\right)^2}. \tag{116}$$

Finally, one additionally needs to check that Assumptions 1 and Assumption 3 hold. This requires

$$\frac{2\delta\gamma}{1-\delta\gamma}\underline{\beta} > (1-\eta)\left(\overline{\alpha}-\underline{\beta}\right) \tag{117}$$

$$\overline{\alpha}\underline{\beta} > (1-\eta)\left(\overline{\alpha}-\underline{\beta}\right). \tag{118}$$

## C.1: Numerical examples

Consider the vectors

$$\phi' = \begin{bmatrix} \delta' = 0.900 \\ \gamma' = 1.100 \\ \rho' = 0.900 \\ \underline{\alpha}' = 4.000 \\ \overline{\alpha}' = 4.000 \\ \overline{\beta}' = 0.400 \\ \underline{\beta}' = 0.400 \\ \pi' = 0.900 \\ \eta' = 0.800 \end{bmatrix} \quad \phi'' = \begin{bmatrix} \delta' \\ \gamma' \\ \rho' \\ \underline{\alpha}'' = 2.000 \\ \overline{\alpha}'' = 6.000 \\ \overline{\beta}'' = 0.600 \\ \underline{\beta}'' = 0.200 \\ \pi' \\ \eta' \end{bmatrix} \quad \phi''' = \begin{bmatrix} \delta' \\ \gamma' \\ \rho' \\ \underline{\alpha}''' = 2.000 \\ \overline{\alpha}''' = 6.000 \\ \overline{\beta}''' = 0.400 \\ \underline{\beta}''' = 0.400 \\ \pi' \\ \eta' \end{bmatrix},$$

which clearly satisfy (99)-(106).

**Numerical example 1:** $\mathcal{B} = 0$, $\phi = \phi'$. Using (50), we find $\hat{\kappa} = 0.964$.[40] We then set $\kappa = 0.964$, so that (107) holds. Simple calculations yield

$$\begin{aligned} y_1^L - i_1^L &= 0.910 \\ y_2^L - i_2^L &= 1.012 \\ y_1^F - i_1^F &= 1.120 \\ y_2^F - i_2^F &= 1.232 \\ \left(m_1^L\right)^{w,u} = \left(m_1^F\right)^{w,u} &= 0.441 \\ \left(m_2^L\right)^{w,u} = \left(m_2^F\right)^{w,u} &= 0.485, \end{aligned}$$

given which it is easy to check that (108)-(111) also hold. It follows that $\phi'$ satisfies all relevant

[40] All derivations and calculations used in this section are available from the authors upon request.

parametric assumptions. Simple calculations also give

$$\left(q_1^F\right)^w = 0.800$$
$$\left(q_2^F\right)^w = 0.800$$
$$\underline{s}_1^F\left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) = 0.509.$$

Since $\underline{s}_1^F\left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) \in (0,1)$, there is no war in period 1.

**Numerical example 2:** $\mathcal{B} = 0$, $\phi = \phi''$. Using (50), we find $\hat{\kappa} = 0.960$. We then set $\kappa = 0.960$, so that (107) holds. Simple calculations yield

$$y_1^L - i_1^L = 0.920$$
$$y_2^L - i_2^L = 1.012$$
$$y_1^F - i_1^F = 0.880$$
$$y_2^F - i_2^F = 1.496$$
$$\left(m_1^L\right)^{w,u} = \left(m_1^F\right)^{w,u} = 0.792$$
$$\left(m_2^F\right)^{w,u} = \left(m_2^F\right)^{w,u} = 0.480,$$

given which it is easy to check that (108)-(111) also hold. It follows that $\phi''$ satisfies all relevant parametric assumptions. Simple calculations also give

$$\left(q_1^F\right)^w = 0.667$$
$$\left(q_2^F\right)^w = 0.857$$
$$\underline{s}_1^F\left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) = -0.240.$$

Since $\underline{s}_1^F\left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) < 0$, there is an $L$-led war in period 1.

**Numerical example 3:** $\mathcal{B} = 1$, $\phi = \phi'$. To find $\hat{\kappa}$, recall that this is defined as the value that makes the RHS of (91) equal to zero. This expression can be written as a quadratic in $(1-\kappa)^{\frac{1}{2}}$, which can be used to recover $\hat{\kappa}$. This gives $\hat{\kappa} = 0.970$. We then set $\kappa = 0.970$, so that (112) holds. The value of $y_t^J - i_t^J$, for $J \in \{L, F\}$ and $t \in \{1,2\}$, is the same as for $\mathcal{B} = 0$. Simple calculations yield

$$\left(m_1^L\right)^{w,c} = 0.489$$
$$\left(m_2^L\right)^{w,c} = 0.538$$
$$\left(m_1^F\right)^{w,u} = 0.436$$
$$\left(m_2^F\right)^{w,u} = 0.455,$$

given which it is easy to check that (113)-(116) also hold.[41] It is also easy to check that (117) and (118) hold. It follows that $\phi'$ satisfies all relevant parametric assumptions. Simple calculations also give

$$\left(q_1^F\right)^w = 0.766$$
$$\left(q_2^F\right)^w = 0.766$$
$$\underline{s}_1^F\left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) = 0.455.$$

Since $\underline{s}_1^F\left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) \in (0,1)$, there is no war in period 1.

**Numerical example 4:** $\mathcal{B} = 1$, $\phi = \phi''$. Proceeding as in the last case, we find $\hat{\kappa} = 0.967$. We then set $\kappa = 0.967$, so that (112) holds. The value of $y_t^J - i_t^J$, for $J \in \{L, F\}$ and $t \in \{1, 2\}$, is the same as for $\mathcal{B} = 0$. Simple calculations yield

$$\left(m_1^L\right)^{w,c} = 0.682$$
$$\left(m_2^L\right)^{w,c} = 0.750$$
$$\left(m_1^F\right)^{w,u} = 0.656$$
$$\left(m_2^F\right)^{w,u} = 0.398,$$

given which it is easy to check that (113)-(116) also hold. It is also easy to check that (117) and (118) hold. It follows that $\phi''$ satisfies all relevant parametric assumptions. Simple calculations

[41] We are not reporting $\left(m_1^F\right)^{w,c}$ and $\left(m_2^F\right)^{w,c}$ because these variables are not required to check any assumption. By construction, they are equal to $\overline{\beta}$ and $\underline{\beta}$ respectively.

also give

$$\left(q_1^F\right)^w = 0.638$$
$$\left(q_2^F\right)^w = 0.638$$
$$\underline{s}_1^F\left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) = 1.206.$$

Since $\underline{s}_1^F\left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) > 1$, there is an $F$-led war in period 1.

**Numerical example 5:** $\mathcal{B} = 1$, $\phi = \phi'''$. Proceeding as in the first case, we find $\hat{\kappa} = 0.962$. We then set $\kappa = 0.962$, so that (112) holds. Simple calculations yield

$$y_1^L - i_1^L = 0.920$$
$$y_2^L - i_2^L = 1.012$$
$$y_1^F - i_1^F = 0.729$$
$$y_2^F - i_2^F = 1.672$$
$$\left(m_1^L\right)^{w,c} = 0.861$$
$$\left(m_2^L\right)^{w,c} = 0.525$$
$$\left(m_1^F\right)^{w,u} = 0.767$$
$$\left(m_2^F\right)^{w,u} = 0.465,$$

given which it is easy to check that (113)-(116) also hold. It is also easy to check that (117) and (118) hold. It follows that $\phi'''$ satisfies all relevant parametric assumptions. Simple calculations also give

$$\left(q_1^F\right)^w = 0.482$$
$$\left(q_2^F\right)^w = 0.834$$
$$\underline{s}_1^F\left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) = -0.003.$$

Since $\underline{s}_1^F\left(\left(a_1^L\right)^w, \left(a_1^F\right)^w\right) < 0$, there is an $L$-led war in period 1.