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FORWARD AND SPOT EXCHANGE RATES IN A MULTI-CURRENCY WORLD

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ABSTRACT

Separate literatures study violations of uncovered interest parity (UIP) using regression-based and portfolio-based methods. We propose a decomposition of these violations into a cross-currency, a between-time-and-currency, and a cross-time component that allows us to analytically relate regression-based and portfolio-based facts, and to estimate the joint restrictions they place on models of currency returns. Subject to standard assumptions on investors' information sets, we find that the forward premium puzzle (FPP) and the "dollar trade" anomaly are intimately linked: both are driven almost exclusively by the cross-time component. By contrast, the "carry trade" anomaly is driven largely by cross-sectional violations of UIP. The simplest model the data do not reject features a cross-sectional asymmetry that makes some currencies pay permanently higher expected returns than others, and larger time series variation in expected returns on the US dollar than on other currencies. Importantly, conventional estimates of the FPP are not directly informative about expected returns, because they do not correct for uncertainty about future mean interest rates. Once we correct for this uncertainty, we never reject the null that investors expect high-interest-rate currencies to depreciate, not appreciate.

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The forward premium puzzle and the carry trade anomaly are two major stylized facts in international economics reflecting failures of uncovered interest parity. The forward premium puzzle is a fact about a regression coefficient, whereas the carry trade anomaly describes a profitable trading strategy. In this paper, we introduce a series of decompositions that allows us to show analytically how regression- and portfolio-based facts relate to each other, to test whether they are empirically distinct, and to estimate the joint restrictions they place on models of currency returns and exchange rates.

The forward premium puzzle arises in a bilateral regression of currency returns on forward premia (Fama, 1984):

$$rx_{i,t+1} = \alpha_i + \beta_i^{fpp}(f_{it} - s_{it}) + \varepsilon_{i,t+1}, \quad (1)$$

where f_{it} is the log one-period forward rate of currency i , s_{it} is the log spot rate, and $rx_{i,t+1} = f_{it} - s_{i,t+1}$ is the log excess return on currency i between time t and $t + 1$. Under covered interest parity, the forward premium, $f_{it} - s_{it}$, is equal to the interest differential between the two currencies, so that we can think of the currency return simply as the interest differential plus the rate of appreciation of the foreign currency. Although estimates of β_i^{fpp} tend to be noisy, the literature finds $\beta_i^{fpp} > 0$ for most currencies. A pooled specification that constrains all β_i^{fpp} to be identical across currencies yields point estimates significantly larger than zero and often larger than one.¹ This fact, the forward premium puzzle (FPP), has drawn a lot of interest from theorists because it suggests that “high-interest-rate currencies appreciate.” In a rational model, $\beta_i^{fpp} > 1$ requires that the risk premium on a currency must be negatively correlated with its expected rate of depreciation and be so volatile that it plays a role in determining expected changes in bilateral exchange rates.² These implications are often collectively referred to as the “Fama conditions” (Backus, Foresi, and Telmer, 2001).

The carry trade anomaly arises when sorting currencies into portfolios. It refers to the fact that lending in currencies that have high interest rates while borrowing in currencies that have low interest rates is a profitable trading strategy. The same is true for the somewhat less well-known “dollar trade” anomaly, a profitable trading strategy whereby investors go long all foreign currencies when the world average interest rate is high relative to the US interest rate, and short all foreign currencies when it is low.

The literature has often loosely connected these anomalies, for example, by attributing the

¹The same relationship is often estimated using the change in the spot exchange rate as the dependent variable, in which case, the coefficient estimate is $1 - \beta_i^{fpp}$. An equivalent way of stating the FPP is thus that $1 - \beta_i^{fpp} < 1$.

²Throughout the paper, we follow the convention in the literature and refer to conditional expected returns as “risk premia.” However, this terminology need not be taken literally. Our analysis is silent on whether currency returns are driven by risk premia, institutional frictions, or other limits to arbitrage. See Burnside et al. (2011) and Lustig et al. (2011) for a discussion.

carry trade anomaly to the FPP.³ In this paper, we propose a decomposition that produces an exact mapping between the three anomalies. We decompose the unconditional covariance of expected currency returns (“risk premia”) with forward premia into a cross-currency, a between-time-and-currency, and a cross-time component. Subject to a standard assumption on what investors know at the time of portfolio formation, each of the three components can be written either as the expected return to a linear trading strategy or as a function of a slope coefficient from a regression, similar to (1), that relates variation in expected currency returns to variation in forward premia in the corresponding dimension. These regression coefficients in turn have a clear economic interpretation: in a rational model, they correspond to the elasticity of currency risk premia with respect to forward premia in each of the three dimensions. We can thus write the systematic variation driving the carry trade, the dollar trade, as well as a number of other yet un-named trading strategies, as regression coefficients, test their statistical significance, and link them to parameters in a generic model of currency returns. Similarly, we can show that the FPP corresponds to a specific (also as-yet unnamed) trading strategy that involves going long a currency when its interest rates exceeds its own long-run mean and going short otherwise.

We first show analytically that the expected return on the carry trade is the sum of the cross-currency and the between-time-and-currency component of the unconditional covariance of currency returns with forward premia, whereas the FPP consists of the sum of the between-time-and-currency and the cross-time components. The expected return on the dollar trade equals the cross-time component. All three anomalies thus load on different dimensions of the failure of uncovered interest parity (UIP).

Using a wide range of plausible assumptions on investors’ information sets, we then estimate the elasticity of risk premia with respect to forward premia in each of the three dimensions. Our results show that 44%-100% of the systematic variation driving the carry trade is in the cross section (the cross-currency variation in α_i in (1)) rather than the time series: Currencies that have persistently higher forward premia (interest rates) pay significantly higher expected returns than currencies with persistently lower forward premia. Some of our specifications also show statistically significant variation in the cross-time dimension: expected returns on the US dollar appear to fluctuate with its average forward premium against all other currencies in the sample. This cross-time variation accounts for 100% of the dollar trade anomaly and it also explains 64%-100% of the variation that generates the FPP. By contrast, the contribution of the the between-time-and-currency component to all three anomalies is small. We usually cannot reject the null that currency risk premia are inelastic with respect to variation in forward premia in the between-time-and-currency dimension.

³Some examples include Brunnermeier et al. (2009), Verdelhan (2010), Ilut (2012), and Bacchetta and Van Wincoop (2010).

These results imply that the FPP, that is, the fact that $\beta_i^{fpp} > 0$, has no statistically significant effect on the returns to the carry trade. In this sense, the carry trade and the FPP may require distinct theoretical explanations: explaining the carry trade primarily requires explaining permanent or highly persistent differences in interest rates across currencies that are partially, but not fully, reversed by predictable movements in exchange rates. (High-interest-rate currencies depreciate, but not enough to reverse the higher returns resulting from the interest rate differential.) By contrast, explaining the FPP primarily requires explaining the dollar trade anomaly, that is, why the US dollar on average does not depreciate proportionately when its interest rate is high relative to all other currencies in the world.

The reason we find only a weak link between expected returns on the carry trade and the FPP is that the FPP itself is less informative about expected returns and risk premia than some of the previous literature may have suggested: regressions like (1) teach us about the elasticity of realized, but not necessarily the elasticity of expected returns. When using portfolios to estimate expected returns on trading strategies, we naturally require that all information used in the formation of the portfolio is available *ex ante*. Similarly, when we use regressions to estimate the elasticity of behavior (demanding a risk premium) with respect to some right-hand-side variable, this variable must be known at time t . By contrast, regressions with currency fixed effects (the α_i in (1)) do not correct for the fact that the sample mean of each currency’s forward premium is unknown to investors *ex ante*, and are thus appropriately interpreted as estimating the elasticity of realized, but not expected, returns.

This distinction is important. We show analytically that the elasticity of realized returns reflected in the FPP is always larger than the elasticity of expected returns if investors do not have perfect foresight about the future mean interest rates absorbed in the α_i . In particular, we find that the pooled version of (1) that constrains all β_i^{fpp} to be equal across currencies and uses currency fixed effects (α_i) produces coefficients larger than one primarily because future interest rates are hard to predict, and not because investors expect high interest rate currencies to appreciate. For example, in our standard specification, the weighted average of β_i^{fpp} is 1.81 (s.e.=0.53), whereas our preferred estimate for the elasticity of expected returns is only half that number (0.86, s.e.=0.34). This distinction has important theoretical implications because an elasticity of expected returns smaller than one does not require a systematic association between variation in risk premia and expected depreciations and thus potentially eliminates a long-standing puzzle in the literature on the FPP and the “Fama conditions:” investors generally expect currencies with high interest rates to depreciate and not appreciate.

Having estimated the elasticity of risk premia with respect to forward premia in each of our dimensions, we then use the variance-covariance matrix of our estimates to identify the restrictions these different violations of UIP jointly place on models of currency returns. We find that the simplest model that our regression-based analysis does not reject features positive

elasticities of risk premia with respect to forward premia in the cross-currency and cross-time dimensions, but not necessarily in the between-time-and-currency dimension. In addition, we cannot reject the hypothesis that all three elasticities are smaller than one, such that the model need not generate a correlation between expected changes in exchange rates and risk premia in any of the three dimensions.

Another interesting implication of this analysis is that the model with the best fit to the data features a higher elasticity of risk premia in the cross-time dimension than in the between-time-and-currency dimension, suggesting that the stochastic properties of the US dollar (the base currency in our analysis) may be systematically different from that of the average currency in our sample. We generalize our decomposition to show how results would differ had we chosen a different base currency, and find that the elasticity of the risk premium on the US dollar indeed appears large relative to that of other currencies: The US dollar appears to be one of a small number of currencies that pays significantly higher expected returns when its interest rate is high relative to its own currency-specific average and to the world average interest rate at the time. Based on this decomposition, we derive a simple test of the hypothesis that the elasticity of the risk premium on the US dollar is identical to that of an average country in our sample. However, we narrowly fail to reject this hypothesis.

The main substantive conclusion from our analysis is that currency risk premia may be simpler objects than previously thought. First, the most statistically significant violations of UIP are in the cross section and appear to be highly persistent over time. Second, the FPP, a long-standing puzzle in the literature, arises partially due to the fact that future mean interest rates are difficult to predict. Once we make reasonable corrections for this fact, we cannot reject the null that currency risk premia are uncorrelated with expected changes in exchange rates, neither for the US dollar nor for the other currencies in our sample. Third, there is some evidence that the US dollar is special and that, in particular, the dollar trade anomaly and the FPP are very closely related phenomena.

We make four caveats to this interpretation. First, any inference on the elasticity of risk premia requires taking a stand on the precision of investors' expectations. Although our results remain stable across a wide range of conventional approaches, we cannot exclude the possibility that richer forecasting models might produce different results. Second, our methodology does not allow us to distinguish between permanent and highly persistent differences in expected returns across currencies, and we make no claims to that effect. Third, the fact that we do not find statistically reliable evidence of a non-zero elasticity of risk premia with respect to forward premia in the between-time-and-currency dimension does not mean it does not exist. Fourth, non-linearities may exist in the functional form linking risk premia to forward premia that are not picked up by our linear (regression-based) approach.

Two largely separate literatures have described violations of UIP using regression-based

and portfolio-based methods.⁴ We contribute to this literature by providing a simple approach to reconcile the results from these two literatures and estimate the restrictions they jointly place on models of currency returns.

A large body of theoretical work studies the FPP in models with two ex-ante symmetric countries.⁵ Our analysis relates to this literature in three ways. First, it clarifies that these models are unlikely to explain the carry trade anomaly, unless they generate large and persistent cross-sectional differences in currency risk premia. Second, some influential quantitative applications of these models may be calibrated to an overstated version of the FPP because they do not correct for uncertainty about future interest rates. Third, the focus on generating a negative covariance between currency risk premia and expected depreciations in these models may be less relevant empirically than previously thought.

Papers that offer explicit models of either permanent or highly persistent asymmetries in currency risk premia include [Martin \(2012\)](#), [Hassan \(2013\)](#), [Maggiori \(2017\)](#), [Richmond \(2016\)](#), and [Ready, Roussanov, and Ward \(2017\)](#).⁶ Another strand of the literature has connected persistent currency risk premia with shocks that are themselves persistent, as in [Engel and West \(2005\)](#), [Colacito and Croce \(2011, 2013\)](#), [Gourio, Siemer, and Verdelhan \(2013\)](#), and [Colacito et al. \(2017\)](#).

Our work builds on papers that use portfolio-based analysis to study the cross section of multilateral currency returns ([Menkhoff et al., 2012, 2017](#); [Koiijen et al., 2018](#)). Most closely related is the work by [Lustig, Roussanov, and Verdelhan \(2011, 2014\)](#), who already document that a large part of carry trade returns result from cross-sectional violations of UIP and identify risk factors that explain the carry trade and the dollar trade. Our contribution is to relate these findings to established (regression-based) puzzles in the literature, and to translate them into restrictions on linear models of currency risk premia.

The remainder of this paper is structured as follows: Section 1 describes the data. Section 2 decomposes violations of UIP into trading strategies based on cross-currency, between-time-and-currency, and cross-time variation in forward premia. Section 3 maps the expected returns on each of the three trading strategies to regression coefficients and discusses the theoretical implications of these estimates. Section 4 concludes.

⁴See [Tyron \(1979\)](#), [Hansen and Hodrick \(1980\)](#), [Bilson \(1981\)](#), [Meese and Rogoff \(1983\)](#), [Backus et al. \(1993\)](#), [Evans and Lewis \(1995\)](#), [Bekaert \(1996\)](#), [Bansal \(1997\)](#), [Bansal and Dahlquist \(2000\)](#), [Chinn \(2006\)](#), [Graveline \(2006\)](#), [Burnside et al. \(2006\)](#), [Lustig and Verdelhan \(2007\)](#), [Brunnermeier et al. \(2009\)](#), [Jurek \(2014\)](#), [Corte et al. \(2009\)](#), [Bansal and Shaliastovich \(2010\)](#), [Burnside et al. \(2011\)](#), and [Sarno et al. \(2012\)](#). [Hodrick \(1987\)](#), [Froot and Thaler \(1990\)](#), [Engel \(1996\)](#), [Lewis \(2011\)](#), and [Engel \(2014\)](#) provide surveys.

⁵Examples include [Backus et al. \(2001\)](#), [Gourinchas and Tornell \(2004\)](#), [Alvarez et al. \(2009\)](#), [Verdelhan \(2010\)](#), [Burnside et al. \(2009\)](#), [Heyerdahl-Larsen \(2014\)](#), [Evans and Lyons \(2006\)](#), [Yu \(2013\)](#), [Bacchetta et al. \(2010\)](#), and [Ilut \(2012\)](#).

⁶Also see [Caballero et al. \(2008\)](#), [Govillot et al. \(2010\)](#), [Berg and Mark \(2015\)](#), [Farhi and Gabaix \(2016\)](#), [Hassan et al. \(2016\)](#), [Zhang \(2018\)](#), and [Wiradinata \(2018\)](#).

1 Data

Throughout the main text, we use monthly observations of US dollar-based spot and forward exchange rates at the 1-, 6- and 12-month horizon. All rates are from Thomson Reuters Financial Datastream. The data range from October 1983 to June 2010. For robustness checks, we also use all UK pound-based data from the same source as well as forward premia calculated using covered interest parity from interbank interest rate data, which are available for longer time horizons for some currencies. Our dataset nests the data used in recent studies on currency returns, including [Lustig et al. \(2011\)](#) and [Burnside et al. \(2011\)](#). In additional robustness checks, we replicate our findings using only the subset of data used in these studies.

Many of the decompositions we perform require balanced samples. However, currencies enter and exit the sample frequently, the most important example of which is the euro and the currencies it replaced. We deal with this issue in two ways. In our baseline sample (“1 Rebalance”), we use the largest fully balanced sample we can construct from our data by selecting the 15 currencies with the longest coverage (the currencies of Australia, Canada, Denmark, Hong Kong, Japan, Kuwait, Malaysia, New Zealand, Norway, Saudi Arabia, Singapore, South Africa, Sweden, Switzerland, and the UK from December 1990 to June 2010). In addition, we construct three alternative samples that allow for entry of currencies at 3, 6, and 12 dates during the sample period, where we chose the entry dates to maximize coverage. The “3 Rebalance” sample allows entry in December of 1989, 1997, and 2004 and covers 30 currencies. The “6 Rebalance” sample allows entry in December of 1989, 1993, 1997, 2001, 2004, and 2007 and covers 36 currencies. Our largest sample, “12 Rebalance,” allows entry in June 1986, and in June of every second year thereafter through June 2008, and covers 39 currencies. In between each of these dates, all samples are balanced except for a small number of observations removed by our data-cleaning procedure (see [Appendix A](#)). Currencies enter each of the samples if their forward and spot exchange rate data are available for at least four years prior to the rebalancing date (the reason for this prior data requirement will become apparent below).⁷

Throughout the main text, we take the perspective of a US investor and calculate all returns in US dollars. In [section 3.3](#), we discuss how our results change when we use different base currencies. [Appendix A](#) lists the coverage of individual currencies and describes our data-selection and -cleaning process in detail.

⁷The only exception we make to this rule is for the first set of currencies entering the 12 Rebalance sample, which become available in October 1983.

2 Portfolio-based Decomposition of Violations of UIP

We begin by showing that the FPP, the carry trade, and the dollar trade can be thought of as three trading strategies that capitalize on different violations of UIP. To this end, we first introduce the carry trade and derive the trading strategy corresponding to the FPP. We then use our decomposition to see how the two phenomena relate to each other and estimate their relative contributions to overall violations of UIP in the data.

2.1 The Carry Trade and the Forward Premium Trade

Consider a version of the carry trade in which, at the beginning of each month during an investment period, $t = 1, \dots, T$, we form a portfolio of all available foreign currencies, $i = 1, \dots, N$, weighted by the difference of their forward premia ($fp_{it} \equiv f_{it} - s_{it}$) to the average forward premium of all currencies at the time ($\overline{fp}_t \equiv \sum_i \frac{1}{N} fp_{it}$). Under covered interest parity, a currency's forward premium is equal to its interest rate differential with the US dollar, so that the portfolio is long currencies that have a higher interest rate than the average of all currencies at time t and short currencies that have a lower than average interest rate. We can write the return on this portfolio as

$$\sum_{i,t} [rx_{i,t+1} (fp_{it} - \overline{fp}_t)], \quad (2)$$

where, for convenience, we denote the double-sum over i and t as $\sum_{i,t}$:

$$\sum_{i,t} x_{i,t} \equiv (\sum_{i=1}^N \sum_{t=1}^T x_{i,t}). \quad (3)$$

More generally, we maintain the convention of denoting means with an overline and by omitting the corresponding subscripts throughout the paper:

$$\overline{x}_i \equiv \frac{1}{T} \sum_{t=1}^T x_{it} \quad \overline{x}_t \equiv \frac{1}{N} \sum_{i=1}^N x_{it} \quad \overline{x} \equiv \frac{1}{NT} \sum_{t=1}^T \sum_{i=1}^N x_{it}, \quad x = fp, rx. \quad (4)$$

We implement the carry trade (2) using linear portfolio weights ($fp_{it} - \overline{fp}_t$), because they allow us to relate portfolio returns directly to coefficients in linear regressions (Pedersen, 2015) and to parameters in a generic model of currency returns (as we will see below). Note however, that our results would be very similar if we sorted currencies into a number of bins and then analyzed the returns on a strategy that is long the bin with the highest-interest-rate currencies and short the bin with the lowest-interest-rate currencies, as is customary in the literature.⁸

⁸Such sorts can be thought of as non-parametric regressions (Cochrane, 2011). Appendix Table 1 shows that the Sharpe ratio on our “linear” version of the carry trade is between 80 and 105% of that of a long-short strategy using five bins as in Lustig et al. (2011). The table also shows mean returns and Sharpe ratios on the

As with this alternative formulation, the carry trade portfolio is “zero-cost” (its weights sum to zero, $\sum_i (fp_{it} - \overline{fp}_t) = 0$) and its return is neutral with respect to the dollar, that is, it is independent of the bilateral exchange rate of the US dollar against any other currencies.⁹

Table 1 shows the annualized mean return on the carry trade portfolio in our 1 Rebalance sample. Consistent with earlier research, we find that the carry trade is highly profitable and yields a mean annualized net return of 4.95% with a Sharpe ratio of 0.54. However, the table also shows that currencies which the carry trade is long (i.e., currencies with high interest rates) on average *depreciate* relative to currencies with low interest rates. Our carry trade portfolio loses 2.15 percentage points of annualized returns due to this depreciation.

As we show below, this pattern holds across a wide range of plausible variations: currencies with high interest rates thus tend to depreciate, not appreciate.¹⁰ An obvious question is then why the FPP appears to suggest the opposite. The answer is in the currency-specific intercepts, α_i , in Fama’s regression (1), reproduced here for convenience:

$$rx_{i,t+1} = \alpha_i + \beta_i^{fpp} fp_{it} + \varepsilon_{i,t+1}. \quad (1)$$

We tend to find that $\beta_i^{fpp} > 1$ in regressions in which currency fixed effects absorb the currency-specific mean forward premium ($\overline{fp}_i = \sum_{t=1}^T \frac{1}{T} fp_{it}$). If we wanted to trade on the correlation in the data that drives the FPP, we would thus have to buy currencies that have a higher forward premium (interest rate differential to the US dollar) than they usually do (Cochrane, 2001; Bekaert and Hodrick, 2008). Such a strategy, we call it the “forward premium trade,” weights each currency with the deviation of its current forward premium from its currency-specific mean. We can write the return on the forward premium trade as $\sum_{i,t} [rx_{i,t+1} (fp_{it} - \overline{fp}_i)]$.

The carry trade (2) thus exploits a correlation between currency returns and forward premia conditional on time fixed effects (\overline{fp}_t), whereas the FPP describes a correlation conditional on currency fixed effects (\overline{fp}_i). Figure 1 illustrates the difference between the carry trade and the forward premium trade for the case in which a US investor considers investing in two foreign currencies. The left panel plots the forward premium of the New Zealand dollar and the Japanese yen over time. Throughout the sample period, the forward premium of the former is always higher than the forward premium of the latter, reflecting the fact that New Zealand has consistently higher interest rates than Japan. The carry trade is always long New Zealand dollars and always short Japanese yen. By contrast, the forward premium trade evaluates the forward premium of each currency in isolation and goes long if the forward premium is higher

equally weighted strategy in Burnside et al. (2011). However, this strategy is less comparable because it is not neutral with respect to the US dollar.

⁹See Appendix B.1 for a formal proof of this statement.

¹⁰This fact is also apparent in Table 1 of Lustig et al. (2011).

than its currency-specific mean during the investment period (shown in the right panel). As a result, the forward premium trade is not “dollar neutral” in the sense that it may be long or short both foreign currencies at any given point in time.

It is immediately apparent that implementing the forward premium trade may be more difficult in practice than implementing the carry trade, because it requires an estimate of the mean forward premium of each currency (\overline{fp}_i), which is not known before the end of the investment period. In what follows, we denote investors’ ex-ante expectation of the currency-specific and the unconditional mean forward premium as

$$\overline{fp}_i^e \equiv E_{i0} [\overline{fp}_i], \quad \overline{fp}^e \equiv E_0 [\overline{fp}].$$

The ex-ante implementable version of the forward premium trade (which we show below is the version that is relevant for estimating elasticities of risk premia with respect to forward premia) has a mean return of

$$\sum_{i,t} \left[rx_{i,t+1} \left(fp_{it} - \overline{fp}_i^e \right) \right]. \quad (5)$$

2.2 Portfolio-based Decomposition

Having recast the FPP as a trading strategy, we can now ask how it relates to the carry trade. The expected returns on both portfolios load on different violations of UIP, that is, different components of the unconditional covariance between currency returns and forward premia. To show this result, we can decompose the unconditional covariance into the sum of the expected returns on three trading strategies plus a constant term. Adding and subtracting \overline{fp}_t , \overline{fp}_i^e , and \overline{fp}^e in the second bracket and re-arranging yields

$$\begin{aligned} &= \underbrace{\sum_{i,t} \left[rx_{i,t+1} \left(\overline{fp}_i^e - \overline{fp}^e \right) \right]}_{\text{Static Trade}} + \underbrace{\sum_{i,t} \left[rx_{i,t+1} \left(fp_{it} - \overline{fp}_t - \left(\overline{fp}_i^e - \overline{fp}^e \right) \right) \right]}_{\text{Dynamic Trade}} + \underbrace{\sum_{i,t} \left[rx_{i,t+1} \left(\overline{fp}_t - \overline{fp}^e \right) \right]}_{\text{Dollar Trade}} \\ &\quad + \underbrace{\sum_{i,t} \left[\overline{rx} \left(\overline{fp}^e - \overline{fp} \right) \right]}_{\text{Constant}}, \end{aligned} \quad (6)$$

where \overline{rx} again refers to the mean currency return across currencies and time periods.

The “static trade” trades on the cross-currency variation in forward premia. It is long currencies that are expected to have a high forward premium on average and short those that are expected to have a low forward premium. We may think of it as a version of the carry trade in which we do not update portfolio weights. We weight currencies once (at $t = 0$), based on our expectation of the currencies’ future mean level of interest rates, and do not

change the portfolio until the end of the investment period, T . The “dynamic trade” trades on the between-time-and-currency variation in forward premia. It is long currencies that have high forward premia relative to the average forward premium of all currencies at the time and relative to their currency-specific mean forward premium. We may think of the mean return on the dynamic trade as the incremental benefit of re-weighting the carry trade portfolio every period. Finally, the “dollar trade” trades on the cross-time variation in the average forward premium of all currencies against the US dollar. It goes long all foreign currencies when the average forward premium of all currencies against the US dollar is high relative to its unconditional mean and goes short all foreign currencies when it is low. This trading strategy was recently described by [Lustig et al. \(2014\)](#). We follow their naming convention here.

Upon inspection, the carry trade (2) is simply the sum of the static and dynamic trades,

$$\underbrace{\sum_{i,t} [rx_{i,t+1} (fp_{it} - \overline{fp}_t)]}_{\text{Carry Trade}} = \underbrace{\sum_{i,t} [rx_{i,t+1} (\overline{fp}_i^e - \overline{fp}^e)]}_{\text{Static Trade}} + \underbrace{\sum_{i,t} [rx_{i,t+1} (fp_{it} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e))]}_{\text{Dynamic Trade}},$$

whereas the forward premium trade (5) is the sum of the dynamic and the dollar trades:

$$\underbrace{\sum_{i,t} [rx_{i,t+1} (fp_{it} - \overline{fp}_i^e)]}_{\text{Forward Premium Trade}} = \underbrace{\sum_{i,t} [rx_{i,t+1} (fp_{it} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e))]}_{\text{Dynamic Trade}} + \underbrace{\sum_{i,t} [rx_{i,t+1} (\overline{fp}_t - \overline{fp}^e)]}_{\text{Dollar Trade}}.$$

The common element between the carry trade and the forward premium trade is the dynamic trade, that is, the between-time-and-currency part of the unconditional covariance between expected currency returns and forward premia. By contrast, the cross-currency component is unique to the carry trade and the cross-time component is unique to the forward premium trade. The question of whether the carry trade and the forward premium trade are related in the data thus reduces to estimating the relative contribution of the dynamic trade. On the other hand, the dollar trade is by construction independent of the carry trade.

2.3 Estimation

Estimating the expected return on each of the three trading strategies requires a model that specifies how investors form expectations given the available data. We begin by assuming that we (the econometricians) know how investors form beliefs and have access to the same data so that we can infer their true expectations, \overline{fp}_i^e :

$$\widehat{\overline{fp}_i^e} = \overline{fp}_i^e, \tag{A1}$$

where \widehat{fp}_i^e is our estimate of investors' expectation of \overline{fp}_i . In particular, we begin with the conventional assumption in the portfolio-based literature that investors simply expect \overline{fp}_i to be equal to the mean of fp_{it} across all available data prior to the investment period. However, once we re-write our decomposition in regression form in section 3, we will be able to show that the economic interpretation of our results is more general and holds under a wide range of more sophisticated models of investor beliefs that also allow for the possibility that we might estimate \overline{fp}_i^e with error.

Table 2 lists the mean returns and Sharpe ratios of the three strategies, as well as the mean returns and Sharpe ratios of the carry trade and the forward premium trade. All returns are again annualized and normalized by dividing with \overline{fp} to facilitate comparison. Columns 1-4 on the top left give the results for our 1 Rebalance sample, where we use all available data prior to December 1994 to estimate \overline{fp}_i^e and \overline{fp}^e .

Column 1 shows the results for one-month forwards, without taking into account bid-ask spreads. The mean annualized return on the static trade is 3.46% with a Sharpe ratio of 0.39. It thus contributes 70% of carry trade returns. By contrast, the dynamic trade contributes 30%, with an annualized return of 1.50% and a Sharpe ratio of 0.24. Although the forward premium trade is not commonly known as a trading strategy in foreign exchange markets, it yields similar returns to the carry trade, with a mean annualized return of 4.04% and a Sharpe ratio of 0.27. The dollar trade contributes 63% to this overall return and has a Sharpe ratio of 0.25, with the dynamic trade contributing the remaining 37%.

Columns 2-4 replicate the same decomposition but take into account bid-ask spreads in forward and spot exchange markets.¹¹ Column 2 again uses one-month forward contracts, column 3 uses 6-month contracts, and column 4 uses 12-month contracts. Once we take into account bid-ask spreads, the mean returns on all trading strategies fall.¹² In the case of the dynamic trade, the mean return in column 2 actually turns negative. However, the same basic pattern persists across all columns: the static trade accounts for 70%-121% of the mean returns on the carry trade, and the dollar trade accounts for 63%-124% of the mean returns on the forward premium trade.^{13 14}

¹¹We calculate returns net of transaction costs for each currency i as $rx_{i,t+1}^{net} = I[w_{it} \geq 0](f_{it}^{bid} - s_{i,t+1}^{ask}) + (1 - I[w_{it} \geq 0])(f_{it}^{ask} - s_{i,t+1}^{bid})$, where w_{it} is the portfolio weight of currency i at time t , and I is an indicator function that is one if $w_{it} \geq 0$ and zero otherwise.

¹²Transaction costs in currency markets are thus of the same order of magnitude as the mean returns on the dynamic trade. See Burnside et al. (2006) for a discussion. However, bid-ask spreads reported on Datastream may be larger than the effective inter-dealer market spreads; see Lyons (2001) and Gilmore and Hayashi (2011).

¹³The mean returns on the three underlying trades no longer add up to the mean returns on the carry trade and the forward premium trade when we take into account bid-ask spreads. We thus calculate the percentage contribution of static (dollar) trade by dividing its mean return with the maximum of zero and the sum of the mean returns on the static (dollar) and dynamic trades.

¹⁴In a similar comparison, Lustig et al. (2011) attribute a somewhat smaller share of the static (unconditional) component in carry trade returns (53% in their standard specification). The reason for this apparent

The only potentially sensitive assumption we make in performing this decomposition is that investors expect \overline{fp}_i to be equal to the mean of fp_{it} prior to 1995. To show that our results do not depend on this particular base period (and the resulting selection of currencies in our 1 Rebalance sample), the remaining panels and columns repeat the same exercise using the 3, 6, and 12 Rebalance samples. In each case, we again assume that investors use all available data before each cutoff date to update their expectations. For example, in the 3 Rebalance sample (allowing entry of new currencies into the sample in December of 1989, 1997, and 2004), we calculate \overline{fp}_i^e for the period 1990-1997 as the mean of fp_{it} for each currency prior to 1990, for the period 1998-2004 as the mean of fp_{it} prior to 1998, and so on. In this sense, we allow investors to update their expectations and rebalance their portfolios at three dates for the 3 Rebalance sample and at six and twelve dates for the 6 Rebalance and 12 Rebalance samples, respectively.

The results remain broadly the same across the different samples, where the static trade on average contributes 85.7% of the mean returns to the carry trade, and the dollar trade on average contributes 81.3% of the mean returns on the forward premium trade. In addition, the Sharpe ratio on the dynamic trade appears economically small or even negative in all calculations that take into account the bid-ask spread (they range from -0.14 to 0.19). Whereas the carry trade delivers an economically significant Sharpe ratio in all samples (ranging from 0.12 to 0.44 net of transaction costs), the forward premium trade tends to deliver somewhat lower Sharpe ratios (ranging from 0.00 to 0.27), particularly in the samples that allow more rebalances. Appendix Table 3 shows that these patterns also hold when we exclude currencies with pegged exchange rates, use an extended sample of interest rate data, or use a wide range of alternative samples of exchange rate data used in other studies. We argue below that these patterns also continue to hold when we relax (A1). However, this additional step first requires clarifying the relationship between portfolio returns and regression coefficients.

Our main conclusion from Table 2 is that the dollar trade accounts for the majority of returns to the forward premium trade and the static trade accounts for the majority of returns to the carry trade. By contrast, the dynamic trade, the common element between the carry trade and the forward premium trade, contributes an economically small share to the returns on the two strategies. In this sense, the FPP and the dollar trade anomaly appear intimately linked, while the carry trade anomaly appears largely independent of the other two phenomena.

discrepancy is that in their exercise, they allow the carry trade to use up to 36 currencies, whereas the unconditional carry trade uses only 18 currencies. By contrast, our decomposition requires that we restrict all five trading strategies to use the same set of currencies. These differences in implementation arise because their decomposition views portfolios as the primitive (regardless of the number of their constituents), whereas our decomposition focuses on currencies $i = 1, \dots, N$ as the object of interest. See Appendix Table 2 for a detailed comparison between the two approaches.

3 Decomposition in Regression Form

Expected returns may vary across currencies, between-time-and-currency, and across time. Each of these dimensions corresponds to one of the three basic trading strategies outlined above. To test whether the variation of expected returns in each of these dimensions is statistically significant and to understand the restrictions that the results in the previous section place on models of currency returns, it is useful to rewrite (6) in terms of regression coefficients. Manipulating the expected return on the static trade (the first term on the right-hand side of (6)) yields

$$\begin{aligned} \sum_{i,t} \left[rx_{i,t+1} \left(\overline{fp}_i^e - \overline{fp}^e \right) \right] &= \sum_{i,t} \left[(rx_{i,t+1} - \overline{rx}_{t+1}) \left(\overline{fp}_i^e - \overline{fp}^e \right) \right] + \underbrace{\sum_{i,t} \left[\overline{rx}_{t+1} \left(\overline{fp}_i^e - \overline{fp}^e \right) \right]}_{=0} \\ &= \hat{\beta}^{stat} \sum_{i,t} \left(\overline{fp}_i^e - \overline{fp}^e \right)^2. \end{aligned}$$

We get the first equality by adding and subtracting \overline{rx}_{t+1} . The second equality follows from the fact that $\sum_i (\overline{fp}_i^e - \overline{fp}^e)$ is zero and does not vary across t . The third equality follows from rewriting the covariance as an OLS regression coefficient where $\hat{\beta}^{stat} = \sum_{i,t} \left[(rx_{i,t+1} - \overline{rx}_{t+1}) \left(\overline{fp}_i^e - \overline{fp}^e \right) \right] / \sum_{i,t} \left(\overline{fp}_i^e - \overline{fp}^e \right)^2$ is an estimate of the slope coefficient from the specification

$$rx_{i,t+1} - \overline{rx}_{t+1} = \beta^{stat} \left(\overline{fp}_i^e - \overline{fp}^e \right) + \epsilon_{i,t+1}^{stat}. \quad (7)$$

Appendix C.1 shows that similarly rewriting the second and third terms in (6) yields

$$\begin{aligned} &\sum_{i,t} \left[(rx_{i,t+1} - \overline{rx}) (fp_{it} - \overline{fp}) \right] \\ &= \\ &\underbrace{\hat{\beta}^{stat} \sum_{i,t} \left(\overline{fp}_i^e - \overline{fp}^e \right)^2}_{\text{Static Trade}} + \underbrace{\hat{\beta}^{dyn} \sum_{i,t} \left(fp_{i,t} - \overline{fp}_t - \left(\overline{fp}_i^e - \overline{fp}^e \right) \right)^2 + \hat{\alpha}^{dyn}}_{\text{Dynamic Trade}} + \underbrace{\hat{\beta}^{dol} \sum_{i,t} \left(\overline{fp}_t - \overline{fp}^e \right)^2 + \hat{\alpha}^{dol} - \hat{\alpha}^{dol}}_{\text{Dollar Trade}}, \end{aligned} \quad (8)$$

where $\hat{\beta}^{dyn}$ and $\hat{\beta}^{dol}$ are again OLS estimates of slope coefficients from pooled regressions of currency returns on the variation in forward premia in the relevant dimension:

$$rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx}) = \beta^{dyn} \left[(fp_{it} - \overline{fp}_t) - \left(\overline{fp}_i^e - \overline{fp}^e \right) \right] + \epsilon_{i,t+1}^{dyn}, \quad (9)$$

$$rx_{i,t+1} - \overline{rx} = \gamma + \beta^{dol} \left(\overline{fp}_t - \overline{fp}^e \right) + \epsilon_{i,t+1}^{dol}. \quad (10)$$

Because the right hand side variables in these regressions depend on investors' ex-ante expectations of future mean forward premia, \overline{fp}_i^e , the three error terms $\epsilon_{i,t+1}^{stat}$, $\epsilon_{i,t+1}^{dyn}$, and $\epsilon_{i,t+1}^{dol}$

naturally capture any errors investors may make in these forecasts. These forecast errors induce a structure in the error terms which is key to our empirical finding that investors do not appear to expect high-interest rate currencies to appreciate. We discuss it in detail below (see Appendix C.2 for a formal derivation).¹⁵

Similarly, the residuals $\hat{\alpha}^{dyn} = \sum_{i,t} [\bar{r}x_i(\bar{f}p_i - \bar{f}p - (\bar{f}p_i^e - \bar{f}p^e))]$ and $\hat{\alpha}^{dol} = \sum_{i,t} [\bar{r}x_i(\bar{f}p - \bar{f}p^e)]$ in (8) measure the covariance of currency returns with these forecast errors. By contrast, the three slope coefficients, β^{stat} , β^{dyn} and β^{dol} determine the systematic part of the mean returns calculated in Table 2. They have a simple economic interpretation. To make this interpretation transparent for the most standard class of models, we henceforth use the language of a frictionless rational model, referring to conditional expected currency returns as “currency risk premia.”¹⁶

Definition 1 *The risk premium on currency i at time t is a rational investor’s expectation of the log return on the currency, given that all currencies’ forward premia prior to and including period t are known, $E_{it}[rx_{i,t+1}]$.*

Consider a model where forward premia evolve according to some ergodic, covariance stationary process and currencies are priced by a representative rational investor who has rational expectations of future mean forward premia, $\{\bar{f}p_i^e\}_{i=1}^N$, and demands compensation for holding the static, dynamic, and dollar trade portfolios during the investment period as specified in (7), (9), and (10).¹⁷ Taken together, these three conditions imply a simple model of currency returns: Averaging (7) across t and (10) across i , and then adding the three equations yields

$$rx_{i,t+1} = \gamma + \beta^{stat} (\bar{f}p_i^e - \bar{f}p^e) + \beta^{dyn} [(fp_{it} - \bar{f}p_t) - (\bar{f}p_i^e - \bar{f}p^e)] + \beta^{dol} (\bar{f}p_t - \bar{f}p^e) + \bar{\epsilon}_i^{stat} + \epsilon_{i,t+1}^{dyn} + \bar{\epsilon}_{t+1}^{dol}. \quad (11)$$

In this simple model, the slope coefficients β^{stat} , β^{dyn} , and β^{dol} measure the elasticity of currency risk premia with respect to forward premia in the cross-currency, between-time-and-currency, and cross-time dimension, respectively. They link behavior at time t (demanding a risk premium between t and some future time period) to information investors can condition on at time t (perceived variation in forward premia). In this sense, the three elasticities are behavioral parameters in any model of currency returns, regardless of whether we think of (11)

¹⁵This correlation structure is also the reason why it is more convenient to estimate each coefficient separately using (7), (9), and (10). We show in section 3.1.2 how to conduct a joint estimation.

¹⁶In this paper, forward premia are the only drivers of risk premia. However, our decomposition can be easily generalized to account for additional drivers, as recently demonstrated by Menkhoff et al. (2017).

¹⁷More formally, the representative rational investor demands risk premia so that the error terms in (7), (9), and (10) are mean zero, covariance stationary, asymptotically orthogonal, and unconditionally uncorrelated with the right hand side variable in each of the three equations, $E_0[\bar{\epsilon}_i^{stat}(\bar{f}p_i^e - \bar{f}p^e)] = E_0[\epsilon_{i,t+1}^{dyn}((fp_{it} - \bar{f}p_t) - (\bar{f}p_i^e - \bar{f}p^e))] = E_0[\bar{\epsilon}_{t+1}^{dol}(\bar{f}p_t - \bar{f}p^e)] = 0$. Rationality also implies that the investor’s forecasts are such that $E_0[\bar{f}p_i^e(\bar{f}p_i^e - \bar{f}p^e)] = 0$.

as a generic affine model or as a first-order approximation to a non-linear model of currency returns.¹⁸

Proposition 1 *The slope coefficients β^{stat} , β^{dyn} , and β^{dol} measure the elasticity of currency risk premia with respect to forward premia in the cross-currency, between-time-and-currency, and the cross-time dimension*

$$\beta^{stat} = \frac{\text{cov}(E_{it}[\bar{r}\bar{x}_i], \bar{f}p_i^e)}{\text{var}(\bar{f}p_i^e)}, \quad \beta^{dyn} = \frac{\text{cov}(E_{it}[rx_{i,t+1} - \bar{r}\bar{x}_i], (fp_{it} - \bar{f}p_t) - (\bar{f}p_i^e - \bar{f}p^e))}{\text{var}((fp_{it} - \bar{f}p_t) - (\bar{f}p_i^e - \bar{f}p^e))}, \quad \beta^{dol} = \frac{\text{cov}(E_t[\bar{r}\bar{x}_{t+1}], \bar{f}p_t)}{\text{var}(\bar{f}p_t)}. \quad 19$$

Under assumption (A1), ordinary least squares estimates of (7), (9), and (10) yield unbiased estimates of these elasticities.

Proof. By the properties of linear regression, we can write β^{stat} as

$$\begin{aligned} \beta^{stat} &= E_0 \left[(rx_{i,t+1} - \bar{r}\bar{x}_{t+1}) (\bar{f}p_i^e - \bar{f}p^e) \right] \text{var}(\bar{f}p_i^e)^{-1} = E_0 \left[E_{it} \left\{ (rx_{i,t+1} - \bar{r}\bar{x}_{t+1}) (\bar{f}p_i^e - \bar{f}p^e) \right\} \right] \text{var}(\bar{f}p_i^e)^{-1} \\ &= E_0 \left[E_{it} \left\{ (rx_{i,t+1} - \bar{r}\bar{x}_{t+1}) \right\} (\bar{f}p_i^e - \bar{f}p^e) \right] \text{var}(\bar{f}p_i^e)^{-1} = \text{cov}(E_{it}[\bar{r}\bar{x}_i], \bar{f}p_i^e) \text{var}(\bar{f}p_i^e)^{-1}. \end{aligned}$$

The second equality applies the law of iterated expectations. The third equality uses the fact that $\bar{f}p_i^e$ and $\bar{f}p^e$ are known at time t . The proofs for β^{dyn} and β^{dol} are analogous. The second statement follows directly from the properties of OLS. ■

Which of these elasticities is statistically distinguishable from zero? Columns 1-4 of Table 3 estimate the specifications (7), (9), and (10) using our 1 Rebalance sample. As in section 2, we use assumption (A1), calculating $\bar{f}p_i^e$ as the mean of fp_{it} across all available data prior to December 1994. The standard errors for β^{stat} and β^{dol} are clustered by currency and time, respectively, whereas the standard errors for β^{dyn} are Newey-West with 12, 18, and 24 lags for the 1-, 6-, and 12-month horizons, respectively (correcting for autocorrelation in the error term within each currency). Where appropriate, we use the Murphy and Topel (1985) procedure to adjust all standard errors for the estimated regressors $\bar{f}p_i^e$ and $\bar{f}p^e$ (see Appendix C.4 for details).²⁰

The specifications in column 1 use monthly forward contracts and show a highly statistically significant estimate for β^{stat} of 0.47 (s.e.=0.08). The estimate of β^{dyn} is about the same size

¹⁸In keeping with the portfolio-based decomposition above, this model defines the returns on the three strategies relative to a single investment period $t = 1, \dots, T$ (where $\bar{f}p_i^e = E_{i0}[\bar{f}p_i]$). However, it is easy to generalize this approach to allow for overlapping investment periods, where a new investment period begins at each t and investors continuously update their expectations of future mean forward premia. Because this more general model requires more notation but offers the same economic insights we relegate it to Appendix C.3.

¹⁹Unless otherwise indicated all variances and covariances condition on the information available to the investor at $t = 0$ for the investment period starting at t : $\text{cov}(X_{it}, Y_{it}) = E_0[(X_{it} - E_0[X_{it}])(Y_{it} - E_0[Y_{it}])]$, $t = 1, \dots, T, i = 1, \dots, N$.

²⁰Where the original Murphy and Topel (1985) application assumes i.i.d. errors, we generalize their approach to use the appropriate assumption about error structure for each application, so that our approach is overall internally consistent.

0.44 (s.e.=0.25) but statistically distinguishable from zero only at the 10% level, as is the much larger estimate for β^{dol} (3.11, s.e.=1.60).

How do these elasticities map into the behavior that drives the carry trade and the FPP? Although, model (11) allows the elasticity of risk premia with respect to forward premia to differ in each of the three dimensions, the logic of Proposition 1 applies even if we constrain some of these elasticities to be equal to each other. For example, we could impose $\beta^{dyn} = \beta^{dol}$, so that our model compactly summarizes the FPP in a single coefficient

$$rx_{i,t+1} = \gamma + \beta^{stat} (\overline{fp}_i^e - \overline{fp}^e) + \beta^{fpp} (fp_{it} - \overline{fp}^e) + \bar{\epsilon}_i^{stat} + \epsilon_{i,t+1}^{fpp} \quad (12)$$

where β^{fpp} is the elasticity of risk premia with respect to forward premia in the time series dimension, which can be estimated using the regression

$$rx_{i,t+1} - \bar{r}x_i = \beta^{fpp} (fp_{it} - \overline{fp}_i^e) + \epsilon_{i,t+1}^{fpp}. \quad (13)$$

At the same time, this regression provides an estimate (and a standard error) for the systematic variation in currency risk premia driving the forward premium trade. The corresponding regression for the carry trade takes the form

$$rx_{i,t+1} - \bar{r}x_{t+1} = \beta^{ct} (fp_{it} - \overline{fp}_t) + \epsilon_{i,t+1}^{ct}, \quad (14)$$

where again the correct procedure is to regress the variation in currency returns in the relevant dimension on the portfolio weights used to implement the trading strategy.

Table 3 shows estimates of these elasticities. Again focusing on the simplest specification in column 1 using our 1 Rebalance sample, we find that the coefficients in both regressions are positive and statistically significant. Importantly however, the estimate of β^{fpp} (0.86, s.e.=0.34) is smaller than one, and smaller than we might have expected given a focus in the existing literature on the idea that investors expect currencies with high interest rates to appreciate. We discuss this finding in detail below. The estimate for β^{ct} is 0.68 (s.e.=0.27).²¹

Having represented the carry trade anomaly as a regression coefficient, it is easy to show that the carry trade and the FPP are linked by the elasticity of risk premia with respect to between-time-and-currency variation in forward premia: coefficients β^{ct} and β^{fpp} are linear combinations of β^{stat} and β^{dyn} , and β^{dyn} and β^{dol} , respectively (shown formally in Appendix C.5). The common element is β^{dyn} . Using these relationships, column 1 of Table 3 reports the partial R^2 of the static trade in the carry trade regression (62%, s.e.=23%) and the partial

²¹In both regressions, we use Newey-West standard errors with the appropriate number of lags, following the convention outlined above. In addition, we also adjust standard errors for β^{fpp} for estimated regressors \overline{fp}_i^e as above.

R^2 of the dollar trade in the forward premium trade regression (90%, s.e.=23%).²² Based on these results, we cannot reject the null hypotheses that *all* of the variation driving the carry trade originates from the static trade and *all* the variation driving the FPP originates from the dollar trade.

The remaining columns report variations of the same estimates, showing that our results are similar when we adjust for transaction costs, use forward contracts of longer maturity, include different countries in the sample, and use different time horizons for estimating \overline{fp}_i^e . The structure of the table is identical to Table 2: columns 2-4 use returns adjusted for the bid-ask spread and forward contracts at the 1-, 6-, and 12-month horizon. The remaining columns and panels repeat the same estimations using our 3, 6, and 12 Rebalance samples, where, as in Table 2, we again calculate \overline{fp}_i^e as the mean of fp_{it} for each currency using all available data prior to each cutoff date.

The pattern that emerges from these variations is similar to the results in column 1. In all samples, the coefficient on the static trade is a precisely estimated number between zero and one, and this coefficient usually explains about two thirds of the variation in risk premia driving the identification of β^{ct} . We thus always reject the null that currency risk premia do not vary with unconditional differences in forward premia across currencies. The coefficient on the dollar trade is imprecisely estimated and statistically distinguishable from zero at the 5% level in only one out of 16 specifications. Point estimates range from -0.23 to 3.72. We thus rarely reject the null that no covariance exists between risk premia and forward premia in the cross-time dimension. However, the dollar trade always explains more than half, often more than 90% of the variation driving the identification of β^{fpp} . The coefficient estimates of β^{fpp} itself are almost always smaller than one and are statistically distinguishable from zero in 7 out of our 16 specifications. Consistent with our portfolio-based results, the effect of the dynamic trade on both the carry trade and the FPP is statistically indistinguishable from zero. We reject the null that $\beta^{dyn} = 0$ in only one of our 16 specifications.

Appendix Table 4 shows that the results from Table 3 also hold across a wide range of alternative samples used in other studies and when using interest rate data to infer missing data on historical forward premia. In addition, Appendix Table 5 shows results using the same 15 currencies across rebalances, excluding developing countries from the sample, and adjusting for expected inflation.²³ All of these variations again yield very similar results. Focusing only

²² We calculate the partial R^2 as $\frac{ESS^d}{ESS^d + ESS^{dyn}}$, $d \in \{stat, dol\}$, where ESS^{dyn} refers to the explained sum of squares in specification (9) and ESS^{stat}, ESS^{dol} refer to the explained sum of squares in specifications (7) and (10), respectively. The standard errors for these objects reported in the text are calculated by bootstrapping across sample years. Because this simple bootstrap does not account for uncertainty in the estimation of forecast errors ($\overline{fp}_i^e - \overline{fp}_i$), we also repeat the same exercise in Appendix Table 6, which uses a block-bootstrap that also randomizes over the pre-sample. In this specification we again cannot reject both null hypotheses.

²³Following standard practice in the literature, this calculation assumes that investors have rational expect-

on developed economies accentuates the role of the dollar trade, yielding larger point estimates for β^{dol} and an increased partial R^2 for the dollar trade in accounting for the FPP.

As an additional robustness check, we use our 12 Rebalance sample to block-bootstrap standard errors, shown in Appendix Table 6. In this procedure, we treat each of the 12 two-year periods in between re-balancing dates as one block and draw 100,000 random samples with replacement from this set of histories. The table shows that this procedure produces somewhat wider standard errors for some of our estimates. However, the basic pattern is identical to the one in Table 3.²⁴ Finally, Appendix C.6 shows that our estimates are unlikely to be significantly affected by Stambaugh bias.

The following three subsections now apply these results of our decomposition to make progress on three inter-related issues. First, we assess analytically how our assumptions about investors' expectations (A1) affect our results, and how they interfere with the long-standing interpretation of the FPP that high-interest-rate currencies appreciate. Second, we use the standard errors around the elasticities estimated above to decide more generally which kinds of models we can reject based on regressions of currency returns on forward premia. Third, we test formally whether our finding that β^{dol} is instrumental in generating the FPP points to a potentially special role of the US dollar relative to other currencies.

3.1 Decomposition for General Models of Investor Expectations

A remaining concern with our estimates of β^{stat} , β^{dyn} , and β^{fpp} (as well as our estimates of the expected returns on the corresponding trading strategies) is that they require explicit estimates of \overline{fp}_i^e as inputs and are thus contingent on a specific model of how investors form expectations (A1). Although we have performed a number of variations in estimating these inputs, our model of investor expectations may be misspecified or we might be estimating their expectations with error. To address these difficulties, we first show how investors' ability to predict \overline{fp}_i affects the elasticities β^{stat} , β^{dyn} , and β^{fpp} in general. We then develop a simple alternative approach to estimating elasticities of risk premia that does not require specifying investor expectations of \overline{fp}_i , but only their precision. In a last step, we then discuss how recasting the FPP in this way qualifies its implications for models of exchange rate determination.

3.1.1 Relation to Fixed Effects Estimator

To see how assumptions on investor expectations affect the results of our decomposition, note that the only difference between our specifications (7), (9), and (13) and standard fixed effects

tations, that is, we proxy for expected inflation with future realized inflation.

²⁴In this specification, bootstrapping also appropriately accounts for uncertainty in the estimation of forecast errors $\overline{fp}_i^e - \overline{fp}_i$, particularly when computing standard errors for %ESS.

estimators is that they replace \overline{fp}_i and \overline{fp} with expectations \overline{fp}_i^e and \overline{fp}^e . That is, had we been interested in characterizing the elasticity of *realized* rather than expected returns, we would have simply used currency fixed effects instead of \overline{fp}_i^e in our regressions. Denoting the slope coefficients of the corresponding fixed effects specifications as β_{FE}^{stat} , β_{FE}^{dyn} , and β_{FE}^{fpp} we can show the following result:

Proposition 2 *The elasticities of expected returns with respect to forward premia β^{dyn} and β^{fpp} are smaller in absolute terms than the elasticities of realized returns with respect to forward premia β_{FE}^{dyn} and β_{FE}^{fpp} . The difference between elasticities of expected and realized returns depends only on the relative precision of investors' expectations of \overline{fp}_i ,*

$$\beta^{dyn} = \beta_{FE}^{dyn} \left(1 + \frac{\text{var}(\overline{fp}_i - \overline{fp}_i^e)}{\text{var}(fp_{it} - \overline{fp}_t - (\overline{fp}_i - \overline{fp}))} \right)^{-1}, \quad (15)$$

and

$$\beta^{fpp} = \beta_{FE}^{fpp} \left(1 + \frac{\text{var}(\overline{fp}_i - \overline{fp}_i^e)}{\text{var}(fp_{it} - \overline{fp}_i)} \right)^{-1}. \quad (16)$$

In addition, β^{stat} can be written as

$$\beta^{stat} = \beta_{FE}^{stat} \frac{\text{var}(\overline{fp}_i)}{\text{var}(\overline{fp}_i^e)} + \frac{E_0 \left[\overline{rx}_i (\overline{fp}_i - \overline{fp} - (\overline{fp}_i^e - \overline{fp}^e)) \right]}{\text{var}(\overline{fp}_i^e)}. \quad (17)$$

Proof. See Appendix C.7. ■

The difference between the elasticities of expected and realized returns thus depends only on the statistical properties of the forecast error $(\overline{fp}_i - \overline{fp}_i^e)$. If investors have perfect foresight about future average forward premia, the two concepts are identical ($\text{var}(\overline{fp}_i - \overline{fp}_i^e) = 0$). However, if they do not, $|\beta^{dyn}| < |\beta_{FE}^{dyn}|$ and $|\beta^{fpp}| < |\beta_{FE}^{fpp}|$. In this sense, the elasticity of realized returns is an upper bound for the elasticity of expected returns.

The intuition for this result is akin to attenuation bias, but in reverse. Consider, for example, equation (13). Classic attenuation bias arises when the right hand side variable in a linear regression is measured with error, resulting in a downward bias of the coefficient. Here, the opposite occurs: Any errors investors make when predicting \overline{fp}_i naturally generate variation in the mean of $\epsilon_{i,t+1}^{fpp}$ across currencies. (That is, $\sum_t \epsilon_{i,t+1}^{fpp}$ must be positive on average for currencies for which \overline{fp}_i was higher than expected and vice versa — see Appendix C.2 for a formal derivation.) A fixed effects estimator removes all of these forecast errors from the right hand side variable (in this sense measuring it with too little error), and assigns this variation to

the slope. For a given elasticity of expected returns, β_{FE}^{dyn} , and β_{FE}^{fpp} thus mechanically increase when there is uncertainty about future mean interest rate differentials.

Similarly, the elasticity of realized returns with respect to cross-sectional variation in forward premia, β_{FE}^{stat} , may differ from the elasticity of expected returns, although (in any given sample) the sign of the difference is ambiguous. By contrast, no distinction exists between the elasticity of realized and expected returns in the cross-time dimension. In that dimension, the fact that investors need to estimate \overline{fp} ex ante has no bearing on the estimate of the covariance of risk premia with forward premia, because $cov(E_t[\overline{rx}_{t+1}], \overline{fp}_t) = cov(E_t[\overline{rx}_{t+1}], \overline{fp}_t - \overline{fp}^e) = cov(E_t[\overline{rx}_{t+1}], \overline{fp}_t - \overline{fp})$, so that $\beta^{dol} = \beta_{FE}^{dol}$. (Equation (10) has a constant that absorbs any errors in predicting \overline{fp} .)

Table 4 compares estimates of elasticities of realized and expected returns (where the latter are from columns 1 and 5 of Table 3). All specifications use one-month forwards and exclude bid-ask spreads. The table shows that the difference in coefficients is considerable, particularly for β^{dyn} and β^{fpp} . For example, in our 1 Rebalance sample, the estimate of β_{FE}^{dyn} is 1.13 (s.e.=0.45) and highly statistically significant, whereas our estimate of β^{dyn} is 60% smaller and statistically insignificant (0.44, s.e.=0.25). Similarly, β_{FE}^{fpp} is 1.81 (s.e.=0.53), whereas β^{fpp} is less than half the size and smaller than one (0.86, s.e.=0.34). Taking account of the fact that investors may not have perfect foresight about future mean forward premia thus makes the difference between point estimates above and below one. Before we discuss the theoretical implications of this finding, we first probe its generality and explore how it might vary under other reasonable assumptions about investors' ability to forecast \overline{fp}_i .

3.1.2 Alternative Estimates

Equations (15) and (16) suggest that we could have calculated estimates for β^{dyn} and β^{fpp} identical to those in Table 3 without taking a stand on \overline{fp}_i^e , simply by using our pre-sample to calculate directly the variance (rather than the realizations) of forecast errors across currencies, and then backing out the elasticities of expected returns from the elasticities of realized returns. That is, estimating β^{dyn} and β^{fpp} does not require taking a stand on what investors expect, but only on the *precision* of these expectations. In this sense, assumption (A1) is not a necessary condition for our estimates in Tables 3 to be consistent. Instead, it is sufficient if the variance of forecast errors implied by our model of investor expectations converges in probability to the true variance of investors' forecast errors.

$$\sum_i \frac{1}{N-1} \left(\overline{fp}_i - \widehat{\overline{fp}_i^e} \right)^2 \xrightarrow{p} var \left(\overline{fp}_i - \overline{fp}_i^e \right). \quad (\text{A2})$$

In other words, any method for backing out investor expectations will do, as long as it gives us the right idea of how well investors can predict \overline{fp}_i .

Corollary 1 *Under assumption (A2), ordinary least squares estimates of (7), (9) and (13) are consistent.*

Proof. See Appendix C.8 ■

While this corollary bolsters our confidence in the results presented so far, it also suggests using more sophisticated methods for estimating the variance of forecast errors. Figure 2 plots estimates of β^{dyn} and β^{fpp} from our 1 Rebalance sample over the variance of the forecast error. When this variance is zero (perfect foresight) we are back to our estimates of β_{FE}^{dyn} and β_{FE}^{fpp} from column 1 of Table 4 (the intercepts). The black romboids show the now familiar estimates of β^{dyn} and β^{fpp} we obtained using the pre-sample means to measure investors' expectations (column 2 of Table 4). The variance of forecast errors implied in these estimates is large compared to the between-time-and-currency variation in forward premia ($var(fp_{it} - \overline{fp}_t - (\overline{fp}_i - \overline{fp}))$) (left panel) and the time-series variation in forward premia ($var(fp_{it} - \overline{fp}_i)$), so that the difference between the estimated elasticities of expected and realized returns is quite sizable.

Would more sophisticated models of investor expectations imply smaller forecast errors? The hollow circles in the two graphs mark alternative estimates we obtain from estimating an autoregressive process,

$$fp_{it} = \alpha_i + \rho_{1,i}fp_{i,t-1} + \rho_2\overline{fp}_{t-1} + \epsilon_{it}^f, \quad (18)$$

in the pre-sample data for each currency and then calculating the implied variance of the forecast error in a sample with length $T = 186$ months under the assumption that the estimated coefficients α_i , $\rho_{1,i}$, and ρ_2 (and the standard deviations of ϵ_{it}^f) are known to investors and characterize the true process governing the evolution of fp_{it} . This calculation results in almost identical adjustments, returning an estimate of 0.47 (s.e.=0.26) for β^{dyn} and an estimate of 0.89 (s.e.=0.36) for β^{fpp} . When we repeat our calculation while imposing the same autocorrelation coefficients $\rho_{1,i} = \rho_1$ for all currencies and $\rho_2 = 0$ in (18), we obtain tighter standard errors but also a larger adjustment to both coefficients (marked with a triangle).

Of course we cannot exclude the possibility that other approaches to estimating the precision of investors' forecasts might yield different results. Appendix Table 7 lists a range of such variations. Perhaps the least conservative of these variations, using a GARCH model on the full sample (rather than the pre-sample), yields estimates of 0.45 (s.e.=0.29) and 1.01 (s.e.=0.39) for β^{dyn} and β^{fpp} , respectively. With all of these different approaches, our conclusions from Table 3 thus continue to hold: β^{dyn} is never statistically distinguishable from zero, whereas β^{fpp} is usually smaller than one and statistically significantly different from zero in some specifications.

We draw three main conclusions from this analysis. First, the elasticities of realized and expected returns are inherently different objects, akin to the difference between mean returns on trading strategies that are implementable using the information available at the time and trading strategies that rely on hindsight. Second, while we can estimate elasticities of realized returns using fixed effects estimators, estimating elasticities of expected returns requires adjusting for errors in investors' forecasts of \overline{fp}_i . Third, a wide range of reasonable adjustments for these errors returns results similar to those in Table 3, where we consistently obtain point estimates for β^{dyn} and β^{fpp} smaller than one, and the former are usually not statistically distinguishable from zero.

3.1.3 Currencies with High Risk Premia Need not Appreciate

Why is it important that β^{dyn} and β^{fpp} are below one? Most immediately, estimates below one imply that investors do not, in fact, expect high-interest-rate currencies to appreciate. Instead the finding that $\hat{\beta}_{FE}^{fpp} > 1$ arises when we look at the data with hindsight, conditioning on information not available ex-ante.

To illustrate this point, consider a Monte Carlo exercise where exchange rates, conditional on investors' information sets, are unpredictable such that $\beta^{fpp} = 1$. For concreteness, also suppose that forward premia are governed by the process (18) with $\rho_2 = 0$, and parameters for $\rho_{1,i}$ and α_i , as well as the variance of errors ϵ_{it}^f , equal to our estimates from the pre-sample. Then a fixed effects regression on a sample with 186 months yields on average an estimate of $\hat{\beta}_{FE}^{fpp}$ well above one: 1.45 (with a 90% confidence interval for the ratio $\beta_{FE}^{fpp}/\beta^{fpp}$ ranging from 1.08 to 2.32).²⁵ Thus, even if we lived in a world where exchange rates are unpredictable, ($\beta^{fpp} = 1$) a regression of realized returns on forward premia with country fixed effects would show an elasticity of realized returns with respect to forward premia far greater than one, simply because investors predict forward premia with error.²⁶

The fact that investors, conditional on the information available at the time, may in fact expect high-interest-rate currencies to depreciate rather than appreciate in turn has important implications for the kind of models we might want to write to understand the interplay between risk premia and exchange rates: a β^{fpp} smaller than one obviates the long-standing challenge in the theoretical literature to find a reason why a representative currency's risk premium

²⁵Coefficient and confidence interval are calculated by using the process for forward premia (18) to simulate 1,000 artificial series of forward premia for each currency included in our 1 Rebalance sample, and then using (16) to calculate $\hat{\beta}_{FE}^{fpp} = 1 + \widehat{var}(fp_i - \widehat{fp}_i^e) / \widehat{var}(fp_{it} - \overline{fp}_i)$ for each series. See Appendix Table 8 for details.

²⁶Appendix Table 8 also shows variations of the same Monte Carlo exercise, illustrating how the size of the difference between elasticities of expected and realized returns varies across different samples. Across specifications, we find that it increases with the heterogeneity of the countries included in the sample and with the conditional variance of mean forward premia. That is, the harder it is to predict mean forward premia for a given process for forward premia and investment horizon, the larger is $\hat{\beta}_{FE}^{fpp}$.

should be negatively correlated with expected depreciations. Following the argument in Fama (1984), we can write the elasticities of risk premia with respect to forward premia β^{stat} , β^{dyn} , β^{dol} , and their linear combinations β^{ct} and β^{fpp} in the following form:²⁷

$$\beta^{fpp} = \frac{\text{var}(E_{it}[rx_{i,t+1}] - E_{i0}[\overline{rx}_i]) + \text{cov}(E_{it}[rx_{i,t+1}] - E_{i0}[\overline{rx}_i], E_{it}[\Delta s_{i,t+1}] - E_{i0}[\overline{\Delta s}_i])}{\text{var}(E_{it}[rx_{i,t+1}] - E_{i0}[\overline{rx}_i]) + \text{var}(E_{it}[\Delta s_{i,t+1}] - E_{i0}[\overline{\Delta s}_i]) + 2\text{cov}(E_{it}[rx_{i,t+1}] - E_{i0}[\overline{rx}_i], E_{it}[\Delta s_{i,t+1}] - E_{i0}[\overline{\Delta s}_i])}. \quad (19)$$

This fraction can be larger than one only if the covariance term is negative, that is, only if a negative covariance exists between risk premia and expected depreciations. However, as long as β^{fpp} is between zero and one, Fama's analysis has no implications for this covariance. Any number between zero and one may simply result from the fact that both risk premia and expected changes in exchange rates vary over time ($\text{var}(E_{it}[rx_{i,t+1}] - E_{i0}[\overline{rx}_i])$, $\text{var}(E_{it}[\Delta s_{i,t+1}] - E_{i0}[\overline{\Delta s}_i]) > 0$).

Similarly, estimates between zero and one for β^{stat} , β^{dyn} , β^{dol} , and β^{ct} have no implications for the covariance of currency risk premia and expected changes in exchange rates in the relevant dimension. Figure 3 summarizes the implications of our estimates in Table 3 for the covariance of currency risk premia with expected appreciations. It shows all estimates of β^{stat} , β^{dyn} , and β^{dol} from the table and highlights the median estimate of each of the coefficients. (Appendix Figure 2 shows the same overview for β^{fpp} and β^{ct} .)

None of our point estimates for β^{stat} and β^{dyn} are larger than one. In fact, we can reject the hypothesis that either of the two coefficients is larger than one in all but one specification. The data thus provide little evidence that risk premia and expected appreciations are correlated in the cross-currency and the between-time-and-currency dimensions.

In fact, the only potential evidence in favor of a negative covariance between currency risk premia and expected depreciations comes from the cross-time dimension. There, a number of point estimates are above one. However, the standard errors are so large that we reject the hypothesis that $\beta^{dol} > 0$ in only one specification and *never* reject the hypothesis that $\beta^{dol} < 1$. Overall, our multilateral regressions of currency returns on forward premia thus offer little evidence of a non-zero covariance of currency risk premia with expected changes in exchange rates,²⁸ with the possible exception of the cross-time dimension.

3.2 Implications for Models of Currency Returns

Beyond the relationship between risk premia and exchange rates, another advantage of representing all three anomalies (the FPP, the carry trade, and the dollar trade) in the form of

²⁷To get an expression of this form, replace $E_{it}[rx_{i,t+1}] = fp_{it} - E_{it}\Delta s_{i,t+1}$ in the expressions given in Proposition 1. See Appendix C.9 for a detailed derivation.

²⁸This conclusion is also consistent with Sarno and Schmeling (2014), who find that currency risk premia are only weakly related to exchange rates.

regression coefficients is that we can now use the variance-covariance matrix of our estimated elasticities of risk premia with respect to forward premia from Table 3 to estimate the restrictions that these facts jointly place on models of currency returns. Our generic affine model of currency returns (11) has three parameters. A theorist wishing to focus her energy on the most salient features of the data may want to begin with the null hypothesis that each of these parameters is equal to zero and include them if and only if they significantly improve the model's fit to the data. Based on the results from Table 3, she might thus start with the simplest model the data do not clearly reject $\{\beta^{stat} > 0, \beta^{dyn} = 0, \beta^{dol} = 0\}$. This model explains returns on the carry trade as the result of static, unconditional, differences in risk premia across currencies.

Although this model explains most of the significant coefficients shown in Table 3, discarding the mean returns to the forward premium trade, and thus the FPP itself, as a statistical fluke may not be satisfactory. Columns 1-5, 7, and 8 of the 1 Rebalance and 3 Rebalance samples, show significantly positive returns to the forward premium trade. Although neither β^{dyn} nor β^{dol} are by themselves usually statistically distinguishable from zero, their convex combination (β^{fpp}) is statistically significant in these seven specifications. We might thus want to relax our model by adding an additional parameter that can explain this pattern. The three simplest options to extend the model are $\{\beta^{dyn} > 0, \beta^{dol} = 0\}$, $\{\beta^{dyn} = 0, \beta^{dol} > 0\}$, and $\{\beta^{dyn} = \beta^{dol} = \beta^{fpp} > 0\}$.

Table 5 performs χ^2 difference tests, asking which of the three extensions is best able to explain the mean returns on the forward premium trade observed in the data under the assumption that the coefficient estimates of β^{fpp} , β^{dyn} , and β^{dol} are normally distributed (see Appendix C.10 for details). The two columns in the table use the coefficient estimates and standard errors from columns 1 and 5 of the 1 Rebalance and the 3 Rebalance samples in Table 3, respectively. (Because the linear relationship between the three coefficients holds only in the absence of transaction costs, these specifications are the only two of relevance.) In both cases, we cannot reject $\beta^{dyn} = 0$ or $\beta^{dyn} = \beta^{dol}$, whereas we can reject $\beta^{dol} = 0$ at the 5% level. The two simplest models that can explain all the statistically significant correlations in Table 3 are thus $\{\beta^{stat} > 0, \beta^{dyn} = 0, \beta^{dol} > 0\}$ and $\{\beta^{stat} > 0, \beta^{dyn} = \beta^{dol} = \beta^{fpp} > 0\}$.²⁹

The conclusion from this exercise is that the data strongly reject models in which $\beta^{stat} = 0$ and, to the extent that the FPP is a robust fact in the data, also reject models in which $\beta^{dol} = 0$. A parsimonious affine model of currency returns thus need only allow for variation in currency risk premia in the cross-currency and cross-time dimensions. Any assumptions about β^{dyn} do not significantly affect the model's ability to fit the data.³⁰

²⁹This finding continues to apply when we focus on a sample of developed economies or after adjusting returns for US inflation.

³⁰Given these results one might be tempted to go a step further and impose $\beta^{stat} = \beta^{dyn} = \beta^{dol}$. Indeed, we

This finding suggests that the statistically significant violations of UIP may be fundamentally linked to asymmetries across countries: the carry trade anomaly requires static or highly persistent asymmetries in risk premia across currencies, while the FPP and the dollar trade anomaly may arise due to an especially high elasticity of the risk premium on the US dollar.

3.2.1 Example: No-arbitrage Model of Currency Returns

To illustrate how these findings restrict a specific model of currency returns, we apply our decomposition to a simplified version of the no-arbitrage model by [Lustig et al. \(2011\)](#). Their model is particularly well-suited for our purposes because it explicitly models both the cross-section and the time series of the returns on N currencies, rather than focusing only on one currency pair or on a single cross-section. For simplicity, the model exogenously specifies a stochastic discount factor (SDF) for each currency, without tracing the innovations to this SDF to fundamental shocks to productivity or demand (as would be common in the macroeconomic literature). In a world with complete markets and one representative agent per country and currency, we might think of each SDF simply as representing the growth rate of the marginal utility of consumption in a given country.

The logarithm of each country’s SDF is

$$-m_{i,t+1} = \alpha + \chi z_{it} + \sqrt{\gamma} z_{it} u_{i,t+1} + \tau z_t^w - \sqrt{\delta_i z_t^w} u_{w,t+1}, \quad (20)$$

where $u_{i,t+1}$ is a currency-specific shock and $u_{w,t+1}$ is a “global” shock that is common to all currencies. Both shocks are i.i.d. and follow a standard normal distribution. The $N + 1$ state variables, $\{z_{it}\}$ and z_t^w , evolve according to some stationary process over time and modulate the SDFs’ exposures to the two shocks. For example, if z_{it} is low, currency i has little currency-specific risk, while a high z_t^w means that global risk is high, in the sense that all currencies are highly exposed to the global shock. Following our convention above, we denote the time-zero expectation of the state variables as z^e and z^{we} , respectively. The change in the real exchange rate between two currencies, measured in country i goods per home country good is then simply $m_{h,t+1} - m_{i,t+1}$, where we index the home country with h .

While earlier work on this class of model focused on the restrictions on (20) needed to generate the FPP, the major innovation of [Lustig et al. \(2011\)](#)’s work is that they allow some currencies to be permanently be riskier than others, that is, they allow currencies to differ in

reject this hypothesis only in our 6 Rebalance sample (again using the specifications in columns 1 and 5 of Table 3). Appendix Table 9 shows estimates of this model using the specification $rx_{i,t+1} - \bar{r}\bar{x} = \beta (fp_{it} - \bar{fp}^e) + \epsilon_{i,t+1}$. 15 out of 16 estimates return values larger than zero but less than one, suggesting that such a constrained model would again be very simple: currencies with high interest rates depreciate, but not enough to reverse the higher returns resulting from the interest rate differential.

their loading on the global shock (δ_i).³¹ It is this feature that allows the model to generate cross-sectional differences in interest rates and expected currency returns.

Although some authors have argued that the US dollar may have a particularly large exposure to global shocks, we will also assume that δ_h is equal to the mean of δ_i across all currencies as doing so greatly simplifies the exposition.³² For simplification, we also assume that the number of countries is large such that at any time $\sum_i z_{it} = z^e$, and that the $\{\delta_i\}$ are normally distributed across countries with variance σ_δ^2 .

In this framework, [Lustig et al. \(2011\)](#) derive the forward premium and expected returns on currency i as

$$fp_{it} = \frac{1}{2} (z_t^w (\delta_h - \delta_i) + (\gamma - 2\chi) (z_{ht} - z_{it}))$$

and

$$E_t [rx_{i,t+1}] = \frac{1}{2} (z_t^w (\delta_h - \delta_i) + \gamma (z_{ht} - z_{it})),$$

respectively, implying that currencies that have a large loading on the global shock pay lower returns in equilibrium (because they tend to appreciate in “high-marginal-utility” states). If $\chi = 0$, expected returns are equal to forward premia, that is, investors pocket on average the interest differential and exchange rates are in random walk.

Performing our decomposition we can show

$$\beta^{stat} = 1, \quad \beta^{dyn} = \frac{\gamma(\gamma-2\chi)+\sigma_\delta^2}{(\gamma-2\chi)^2+\sigma_\delta^2}, \quad \text{and} \quad \beta^{dol} = \frac{\gamma}{\gamma-2\chi}.$$

For comparison, our baseline estimates and 95% confidence intervals for these coefficients from column 1 of [Table 3](#) are 0.47 [0.31; 0.63], 0.44 [-0.05; 0.93], and 3.11 [-0.03; 6.25], respectively.

What do these estimates teach us about this model? First, and most importantly, the model generates permanent differences in interest rates across countries ($\beta^{stat} > 0$). However, in the data, these differences are partially reversed by predictable depreciations ($\beta^{stat} < 1$). This simple version of the model cannot match these predictable depreciations because χ affects depreciations in the time-series, but not the cross-section. In the time-series, we can generate any value for β^{dol} smaller or greater than one by adjusting χ . We get $1 > \beta^{dol} > 0$ by setting $\chi < 0$, so that the dollar depreciates relative to all other currencies when it has high interest rates. By contrast, we get the dollar to appreciate in these states by setting $\chi > 0$. However, this simple version of the model does not allow the dollar’s stochastic behavior to differ from

³¹The later work cited above has then argued that such differential loadings arise in microfounded macroeconomic models as a result of differences in country size, trade centrality, or endowments of commodities.

³²The model considered here corresponds to the setup in section 4.1 of [Lustig et al. \(2011\)](#). In [Appendix C.11](#) we show the same results for a more general version of the model where we drop this assumption and also allow for variation over time in a country’s exposure to a second global shock as in [Lustig et al. \(2014\)](#). Moreover, we obtain identical results when we add the inflation process specified in the extended version of the model in [Lustig et al. \(2011\)](#) and solve for elasticities of expected real (rather than nominal) returns.

that of other currencies, so that there is a tight link between β^{dyn} and β^{dol} : both coefficients depend on the same parameters and are larger than one if and only if $\chi > 0$. As a result, the model cannot, for example, simultaneously match the point estimates for both coefficients in column 1 of Table 3. In the model, the only difference between the two coefficients is the addition of σ_{ξ}^2 in the numerator and the denominator of β^{dyn} , reflecting that time-series variation z_t^w yields an additional motive for re-weighting currencies as higher and lower global risk expands and contracts the size of permanent interest rate differentials.

To break the tight link between these two coefficients, and to reflect the dominant role of the dollar trade in generating the FPP, the model would therefore also have to allow for some kind of asymmetry between the dollar and other currencies, for example by introducing an additional state variable or assigning different parameters.

Finally, the model also naturally implies that $\beta_{FE}^{dyn} > \beta^{dyn}$ and $\beta_{FE}^{fpp} > \beta^{fpp}$, because investors do not have perfect foresight about \overline{fp}_i over a finite investment horizon:

$$\overline{fp}_i - \overline{fp}_i^e = \frac{1}{2} ((\gamma - 2\chi)(\bar{z}_h - \bar{z}_i) + (\delta_h - \delta_i)(\bar{z}^w - z^e)).$$

The difference between estimates of the elasticities of expected and realized returns can then be used to discipline the size of forecast errors relative to the variance of forward premia over time.

3.3 Is the US Dollar Special?

The important role of β^{dol} in accounting for both the dollar trade and the FPP suggests that the returns on the dollar might behave differently from the returns to other currencies. To address this question it is useful to first generalize our model of currency returns to allow for heterogeneous elasticities of risk premia with respect to forward premia across currencies.

3.3.1 Allowing for Heterogeneous Elasticities Across Currencies

Consider a generalized version of (9)

$$rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx}) = \alpha_i^{dyn} + \beta_i^{dyn} \left[(fp_{jt} - \overline{fp}_t) - (\overline{fp}_j^e - \overline{fp}^e) \right] + \tilde{\epsilon}_{i,t+1}^{dyn}, \quad (21)$$

where β_i^{dyn} can now differ across currencies. In combination with (7) and (10), this specification implies a generalized version of our model of currency returns, nesting (11) and (12):

$$rx_{i,t+1} = \gamma + \beta^{stat} (\overline{fp}_i^e - \overline{fp}^e) + \beta_i^{dyn} \left[(fp_{jt} - \overline{fp}_t) - (\overline{fp}_j^e - \overline{fp}^e) \right] + \beta^{dol} (\overline{fp}_t - \overline{fp}^e) + \tilde{\epsilon}_{i,t+1}, \quad (22)$$

where $\tilde{\epsilon}_{i,t+1} = \alpha_i^{dyn} + \bar{\epsilon}_i^{stat} + \tilde{\epsilon}_{i,t+1}^{dyn} + \bar{\epsilon}_t^{dol}$. Following the same steps as the proof of Proposition 1 we can again show that currency-specific coefficients β_i^{dyn} are measures of the elasticity of the risk premium on currency i with respect to deviations of currency i 's forward premium from its currency- and time-specific mean. In addition, Appendix C.12 shows we can re-write the decomposition in (8) as

$$\begin{aligned} & \sum_{i,t} (rx_{i,t+1} fp_{it}) \\ & = \\ & \underbrace{\hat{\beta}^{stat} \sum_{i,t} (\bar{fp}_i^e - \bar{fp}^e)^2}_{\text{Static Trade}} + \underbrace{\sum_i \hat{\beta}_i^{dyn} \sum_t (fp_{it} - \bar{fp}_t - (\bar{fp}_i - \bar{fp}))^2 + \hat{\alpha}^{dyn}}_{\text{Dynamic Trade}} + \underbrace{\hat{\beta}^{dol} \sum_{i,t} (\bar{fp}_t - \bar{fp}^e)^2 + \hat{\alpha}^{dol} - \hat{\alpha}^{dol}}_{\text{Dollar Trade}}. \end{aligned} \quad (23)$$

From comparing equations (8) and (23), it follows immediately that

$$\hat{\beta}^{dyn} \sum_{i,t} (fp_{it} - \bar{fp}_t - (\bar{fp}_i^e - \bar{fp}^e))^2 = \sum_i \hat{\beta}_i^{dyn} \sum_t (fp_{it} - \bar{fp}_t - (\bar{fp}_i - \bar{fp}))^2, \quad (24)$$

so that models (11) and (22) predict identical expected returns on the static, dynamic, dollar, carry, and forward premium trades. In other words, allowing for heterogeneous elasticities of risk premia with respect to forward premia across currencies does not change the model's ability to account for the returns on these trading strategies or the FPP. Instead, the purpose of extending the model in this way is merely to detect whether the dynamic behavior of specific currencies is significantly different from that of others.

Table 6 shows estimates of (21). To save space, we show only the coefficients using one-month forwards, without taking into account bid-ask spreads. The table shows we cannot reject the null that $\beta_i^{dyn} = 0$ for most currencies. In fact, looking across columns, we do not appear to robustly reject this null for any currency, with the possible exception of the Indian rupee, the Austrian schilling, and the Belgian franc. Although we remain open to the possibility that risk premia of these, and potentially a few other, currencies may co-move with deviations of forward premia from their time- and currency specific mean, the evidence does not appear overwhelming.

3.3.2 Changing the Base Currency

What do these results imply about the role of the US dollar? Throughout the paper, we account for returns in terms of US dollars. Asking whether the dollar is special is thus equivalent to asking whether our results would be significantly different if we had chosen a different base currency. Given a large enough sample of currencies, our estimates of the returns on the static and the dynamic trades, as well as our estimates of β^{stat} and β^{dyn} , would not change at all, as both strategies are neutral with respect to the base currency—implying that their returns are

not affected by the choice of base currency. However, our estimates of β^{dol} might be different, because the dollar trade is not neutral with respect to the returns on the dollar.³³

We now generalize our analysis to allow for an arbitrary choice of base currency. To this end, denote the elasticity of risk premia with respect to forward premia in the cross-time dimension from the perspective of an investor using currency i as base currency as β^i , $i = dol, aus, \dots, yen$.

Proposition 3 *In the limit in which the number of currencies tends to infinity ($N \rightarrow \infty$), the elasticity of the risk premium on any base currency j with respect to the average forward premium on all other foreign currencies equals the elasticity of currency i 's risk premium against the US dollar with respect to deviations of its forward premium against the US dollar from its time- and currency-specific mean,*

$$\beta^i = \beta_i^{dyn}.$$

Proof. See Appendix C.14 ■

If the number of currencies is large, the coefficients in Table 6 are thus identical to the coefficients we would estimate on the “base currency trade” (i.e., the equivalent of the dollar trade but using currency i as the base currency) of each of the other currencies in the sample. For example, had we chosen to account for all returns in terms of Japanese yen, our estimates of β^{stat} and β^{dyn} would (in a large sample of currencies) be identical to those in Table 3, but our estimate of β^{yen} would be equal to $\beta_{yen}^{dyn} = 0.55$ in column 1 of Table 6.

One can show that the elasticity of realized returns in the between-time-and-currency dimension, β_{FE}^{dyn} , is a weighted average of the β_i^{dyn} .³⁴ Thus, the null hypothesis that $\beta^{dol} = \beta_{FE}^{dyn}$ provides a test of whether the elasticity of the risk premium on the US dollar is significantly different from elasticity of the average currency in the sample. Table 7 shows we cannot reject this hypothesis in any of our samples (with a p-value of 0.17 in our 1 Rebalance sample).³⁵

However, given our results regarding the prominent role of the dollar trade as a driver of the FPP in Tables 3 and 5, our overall results are at least consistent with the notion that the risk-premium on the US dollar might have dynamics that are systematically different from those of other countries.³⁶ Indeed, Table 6 suggests that this property may be shared with a small number of other currencies.

³³See Appendix C.13 for a formal proof of these statements.

³⁴To see this, substitute the (sample equivalent) of (15) into the left hand side of (24) and simplify to get $\hat{\beta}_{FE}^{dyn} = \sum_i \omega_i \hat{\beta}_i^{dyn}$, where $\omega_i = \sum_t (fp_{it} - \bar{fp}_t - (\bar{fp}_i - \bar{fp}))^2 / \sum_i \sum_t (fp_{it} - \bar{fp}_t - (\bar{fp}_i - \bar{fp}))^2$.

³⁵When we restrict our sample of currencies in the 1 Rebalance sample to only developed economies as in Appendix Table 5, column 3, the p-value on the same test drops to 0.09, and it drops to 0.11 if we adjust for inflation (as in column 4 of Appendix Table 5).

³⁶For other evidence on the special role of the US dollar, see, for example, Gourinchas and Rey (2007), Lustig et al. (2014), and Maggiori (2017).

3.4 The Currency-specific FPP

Before concluding, we revisit the FPP in the way it is traditionally framed in the literature and discuss how its conventional interpretation should be modified in light of our analysis. Many papers on international currency returns feature a table showing a list of estimates of β_i^{fpp} from Fama's bilateral regression (1). Table 8 replicates this list using our data. Consistent with the literature, the coefficients β_i^{fpp} exhibit wide variation. Some are significantly positive, others are significantly negative, most are statistically indistinguishable from zero, but the average across point estimate is above one.

We have argued that this fact should *not* be taken as evidence that $\beta^{fpp} > 1$. That is, it does not mean that a representative currency has an elasticity of risk premia with respect to forward premia above one or that investors in general expect high-interest-rate currencies to appreciate. The reason is that a weighted average of β_i^{fpp} yields β_{FE}^{fpp} but not β^{fpp} ,

$$\sum_i \frac{1}{N} \frac{\text{var}_i(fp_{it})}{\sum_i \frac{1}{N} \text{var}_i(fp_{it})} \beta_i^{fpp} = \beta_{FE}^{fpp} > \beta^{fpp}. \quad (25)$$

(See Appendix C.15 for a formal proof.)³⁷ Mentally averaging across currency-specific estimates in Table 8 thus yields a consistent estimate of the elasticity of realized but not expected returns, and is thus by itself not useful to discipline the dynamics of risk premia. In this sense, tables like our Table 8 make the FPP look worse than it is, because they do not correct for investors' information sets. Once we make reasonable corrections for what investors know, we consistently obtain point estimates for β^{fpp} that are smaller than one, calling into question the usual interpretation of the FPP that requires a negative covariance between currency risk premia and forward premia.

That said, our interpretation of these results is predicated on using data from multiple countries to learn about the stochastic properties of a representative currency's risk premium. If instead we were interested in modeling the dynamics of each individual currency's risk premium, respecting for example the fact that the risk premium on the Danish crown may have systematically different dynamics than the Japanese yen, we may reinterpret the coefficients in Table 8 as country-by-country results: because (1) includes a currency-specific intercept that absorbs any expectational errors, $\overline{fp}_i - \overline{fp}_i^e$, we can rewrite (1) as

$$rx_{i,t+1} - \overline{rx}_i = \alpha_i + \beta_i^{fpp} \left(fp_{it} - \overline{fp}_i^e \right) + \tilde{\xi}_{i,t+1}^{fpp}, \quad (26)$$

where $\alpha_i = \beta_i^{fpp}(\overline{fp}_i^e - \overline{fp}_i)$. Following the logic in proposition 1, we may interpret the

³⁷In keeping with our notational convention, $\text{var}_i(fp_{it})$ refers to the variance for a given currency i , as opposed to $\text{var}(fp_{it})$, which refers to the corresponding variance across i and t .

coefficients β_i^{fpp} as consistent estimates of the *currency-specific* elasticity of risk premia with respect to forward premia corresponding to the model:

$$rx_{i,t+1} - \bar{r}\bar{x} = \beta^{stat} (\overline{fp}_i^e - \overline{fp}^e) + \beta_i^{fpp} (fp_{it} - \overline{fp}_i^e) + \alpha_i + \bar{\epsilon}_i^{stat} + \tilde{\epsilon}_{i,t+1}^{fpp}. \quad (27)$$

However, this interpretation seems somewhat unappealing for three reasons: The first is its sheer complexity. For example, such a model would have to explain why the elasticities of Kuwait and South Africa have opposing signs and why Canada has a significantly larger elasticity than Japan, but about the same elasticity as Denmark. The second reason is that the model (27) ignores that the base currency may itself have a risk premium that fluctuates over time (the dollar trade), thus confounding β_i^{dyn} and β^{dol} . In this sense, the model (22), while similarly complex, may be preferable because it features only one coefficient (β^{dol}) rather than N coefficients (all β_i^{fpp}) that are specific to the base-currency chosen for the analysis. In fact, the results in Table 6 show substantially fewer significant coefficients than those in Table 8, demonstrating that estimates of β_i^{fpp} tend to be particularly high on average when using the dollar as base currency. Including β^{dol} in the model (22) thus accounts for most of the variation in currency risk premia that drives the FPP, consistent with our results above.

Finally, skeptics might argue that any evidence on heterogeneity in the dynamic behavior of risk premia across currencies would be more convincing if it were associated with a profitable trading strategy. We have shown that heterogeneity in β_i^{fpp} is not responsible for the carry trade or any of the other well-known trading strategies we have considered. In fact, among the large number of papers on the FPP, we are unaware of any systematic evidence of a strategy that would generate excess returns based on the fact that β_i^{fpp} appears to be larger for some currencies than for others. To probe this possibility, we use pre-sample data to estimate β_i^{fpp} and \overline{fp}_i^e for each currency in our 1 Rebalance sample and calculate the Sharpe ratio of each “bilateral forward premium trade,” where we weight each currency with $fp_{it} - \overline{fp}_i^e$. Appendix Figure 1 plots the ex-ante estimates $\hat{\beta}_i^{fpp}$ over the Sharpe ratio for each currency. Consistent with our prior findings, we find no relationship between the two. If anything, the slope shown is negative, meaning that the “bilateral Forward Premium Trade” generates *less attractive* returns for currencies that have a high $\hat{\beta}_i^{fpp}$.

4 Conclusion

A large empirical literature studies the forward premium puzzle, the carry trade, the dollar trade, and other violations of uncovered interest parity. However, the relationship between these anomalies and their implications for theoretical work have often remained unclear because some anomalies are identified in regression-based, others in portfolio-based analyses. As a

result, theoretical work often only loosely connects these anomalies, for example, by attributing the (portfolio-based) carry trade anomaly to the (regression-based) forward premium puzzle. In this paper, we introduced a decomposition of violations of uncovered interest parity into a cross-currency, a between-time-and-currency, and a cross-time component. Subject to a standard assumption on investor expectations, each component can be written as the expected return to a trading strategy or as a function of a slope coefficient in a regression that measures the elasticity of currency risk premia with respect to forward premia. This decomposition allowed us to show analytically how regression- and portfolio-based facts relate to each other, to test whether they are empirically distinct, and to estimate the joint restrictions they place on models of currency returns and exchange rates.

Our analysis produced four main insights. First, the cross-time component accounts for all of the systematic variation driving the dollar trade anomaly and most of the variation driving the forward premium puzzle. The two anomalies are thus intimately linked. By contrast, the cross-currency component accounts for most of the systematic variation driving the carry trade. Explaining the carry trade thus primarily requires explaining permanent or highly persistent differences in interest rates across currencies that are partially, but not fully, reversed by predictable movements in exchange rates. By contrast, explaining the forward premium puzzle primarily requires explaining cross-time variation in the expected return on the US dollar against all other currencies.

Second, having translated all three anomalies into regression coefficients with standard errors, we are able to estimate the joint restrictions they place on models of currency returns. We find the simplest model that the data do not reject features positive elasticities of risk premia with respect to forward premia in the cross-currency and cross-time dimensions, but not necessarily in the between-time-and-currency dimension. The three anomalies are thus best explained in a model with two kinds of asymmetries: a highly persistent asymmetry that makes some currencies have persistently higher interest rates and returns than others, and an asymmetry in the dynamic response of currency returns to variation in forward premia between the US dollar and other currencies. We also never reject the hypothesis that all three elasticities are smaller than one, such that high-interest-rate currencies need not systematically appreciate in any of the three dimensions.

Third, although the data seem to favor a special role of the US dollar, we (narrowly) cannot reject the hypothesis that the elasticity of the risk premium on the US dollar is identical to that of an average country. Nevertheless, the US dollar appears to be one of a small number of currencies that pay significantly higher expected returns when their interest rates are high relative to their currency-specific average and to the world average interest rate at the time.

Fourth, the tight correspondence between portfolio returns and regression coefficients in our decomposition reveals an important distinction between the elasticity of expected and

realized returns, akin to the difference between mean returns on trading strategies that are implementable, using only information available in real time, and trading strategies that rely on hindsight. Standard quantifications of the forward premium puzzle, using a pooled regression of currency returns on forward premia with currency fixed effects (or, equivalently, averaging the coefficients from currency-by-currency regressions), condition on information only available with hindsight, and are thus informative only about the dynamics of realized, but not expected returns. Once we correct for this fact, the forward premium puzzle is significantly diminished—to the point that it does not require a systematic association between currency risk premia and expected depreciations, thus potentially resolving a long-standing puzzle in the theoretical literature.

In sum, we hope our results may help guide future theoretical work, having synthesized and clarified the joint implications of three well-established anomalies in currency markets. However, we stress that our synthesis between regression- and portfolio-based facts also has natural limitations: it is confined to representing trading strategies in a linear form, it faces well-known limitations when attempting to detect time-series variation in expected returns, and our quantitative results may change if investors' expectations of future mean forward premia are significantly more precise than our simple forecasting models suggest. Nevertheless, we believe the simplicity of our approach may prove useful for distilling the theoretical implications of portfolio-based analysis in other areas of empirical asset pricing.

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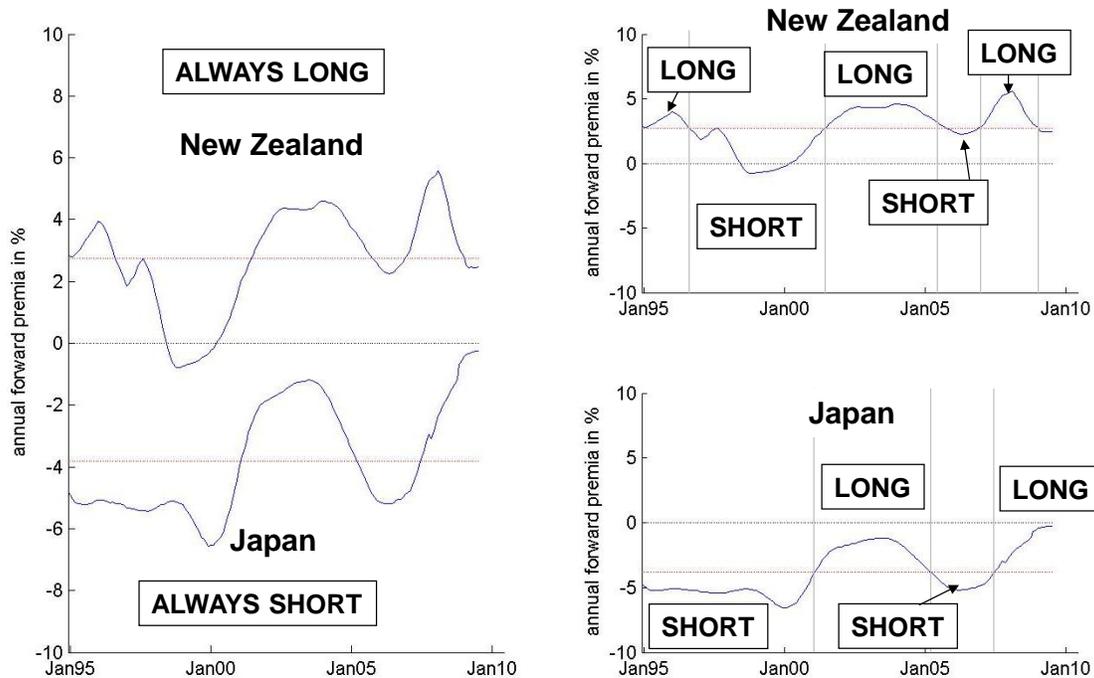


Figure 1: **Carry Trade vs. Forward Premium Trade**

Forward premia of the New Zealand dollar and Japanese yen against the US dollar 1995-2010. Left panel: Carry Trade uses $fp_{it} - \overline{fp}_t$ as portfolio weights, always long the New Zealand dollar, always short the Japanese yen; Right panel: Forward Premium Trade uses $fp_{it} - \overline{fp}_i$ as portfolio weights, goes long when a currency's forward premium exceeds its currency-specific mean. The plot cumulates monthly forward premia to the annual frequency according to $fp_{i,t} = \sum_{m=1}^{12} fp_{i,t+m}$.

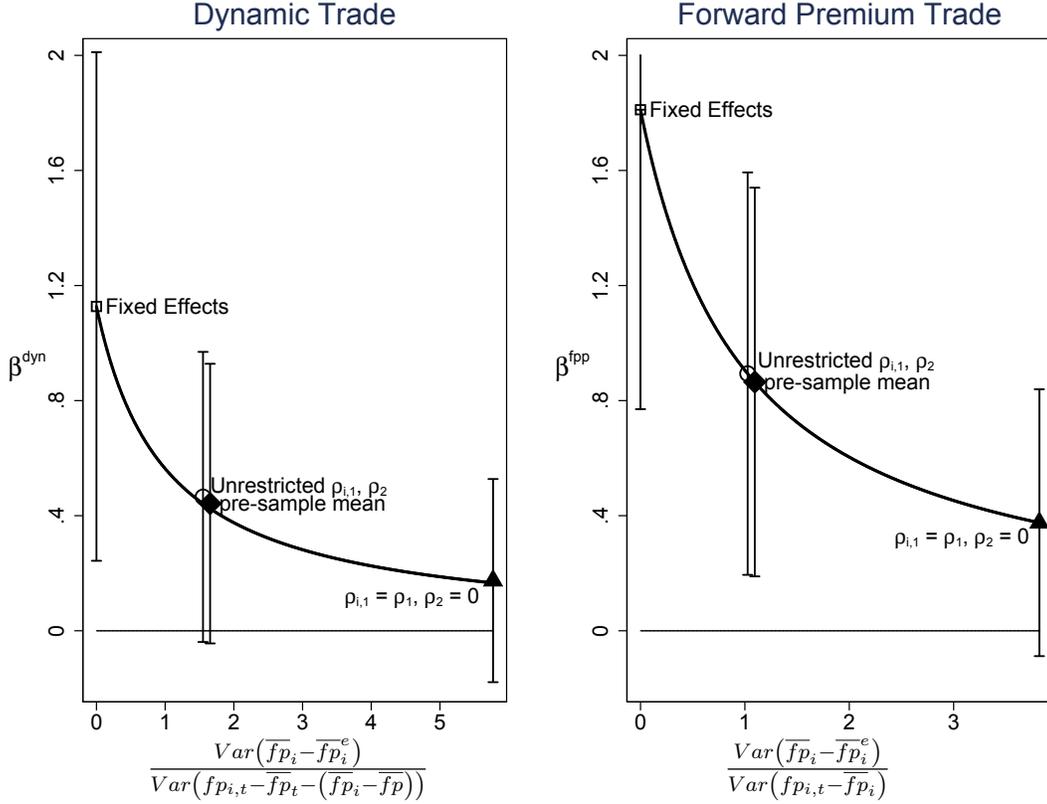


Figure 2: **Alternative Estimates of the Elasticity of Risk Premia**

Estimates of β^{dyn} and β^{dol} as a function of the estimate of β_{FE}^{dyn} and β_{FE}^{dol} from column 1 of Table 4 and the variance of the forecast error $var(\bar{fp}_i - \bar{fp}_i^e)$ as given in equations (15) and (16). Rhomboids mark the estimates from our standard specification in column 1 of Table 3. Circles mark the point estimates we obtain from estimating equation (18) over the pre-sample and then calculating the implied variance of the forecast error in a sample with length $T = 186$, months under the assumption that the estimated autocorrelation coefficients $\{\rho_{1,i}\}$ and ρ_2 , and the standard deviations of ϵ_{it}^f characterize the true process governing the evolution of fp_{it} . Triangles mark results of the same calculation when imposing $\rho_{1,i} = \rho_1$ for all currencies. Minor discrepancies between the estimate shown and the one implied by the function are due to small departures from a fully balanced sample due to our data-cleaning algorithm. Standard error bands show the 95% confidence interval.

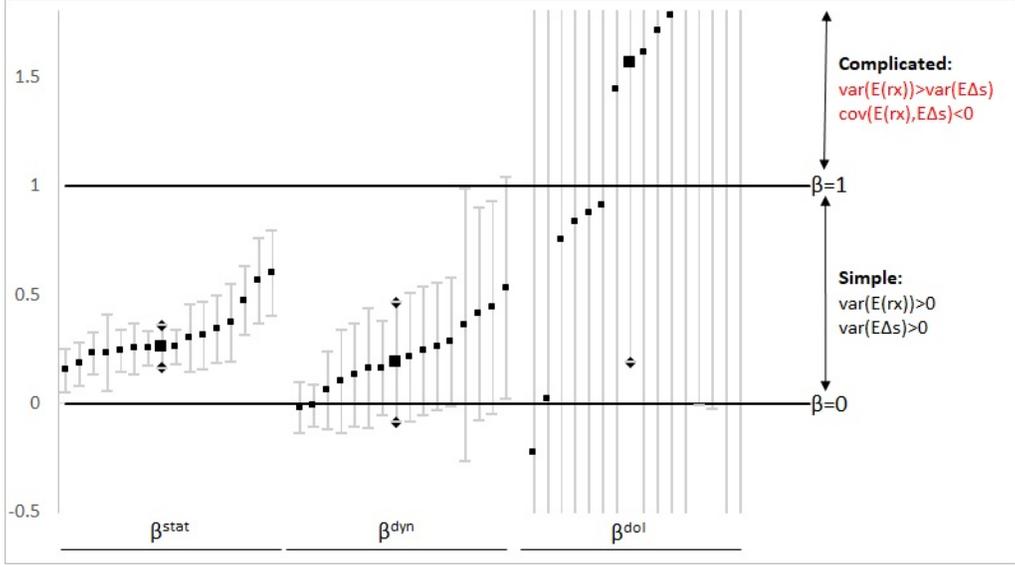


Figure 3: **Summary of Estimates of the Elasticity of Risk Premia with Respect to Forward Premia across Samples**

The figure plots all estimates and 95% confidence intervals of β^{stat} , β^{dyn} , and β^{dol} from Table 3. Small squares show point estimates, and large squares identify the median estimate for each elasticity across samples/horizons. The right-hand-side axis summarizes the implications of the estimates for linear models of currency risk premia. Appendix Figure 2 shows the same overview for β^{fpp} and β^{ct} .

Table 1: Mean Annualized Return to the Carry Trade

Carry Trade return, $\sum_{i,t} [rx_{i,t+1} (fp_{it} - \overline{fp}_t)]$	4.95
of which forward premium	7.11
of which appreciation	-2.15
<i>Sharpe Ratio</i>	0.54

Note: Mean annualized return and Sharpe Ratio of the carry trade calculated by standardizing the expression in (2) with the unconditional mean forward premium in the sample, \overline{fp} . The second and third lines give the part of the mean annualized carry trade return attributable to the forward premium ($\sum_{i,t} [fp_{it} (fp_{it} - \overline{fp}_t)]$) and appreciation ($\sum_{i,t} [(s_{it} - s_{i,t+1}) (fp_{it} - \overline{fp}_t)]$), respectively. One-month forward and spot exchange rates from the 1 Rebalance sample ranging from 12/1994 to 6/2010.

Table 2: Mean Returns on Five Trading Strategies

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sample	1 Rebalance				3 Rebalance			
Horizon (months)	1	1	6	12	1	1	6	12
Static Trade								
$\sum_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]$	3.46	1.36	3.58	3.82	3.09	0.33	2.55	2.53
Sharpe Ratio	0.39	0.15	0.32	0.32	0.37	0.04	0.24	0.22
Dynamic Trade								
$\sum_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e))]$	1.50	-0.24	0.33	1.20	1.42	-0.85	-0.12	0.45
Sharpe Ratio	0.24	-0.04	0.05	0.19	0.20	-0.12	-0.02	0.07
Dollar Trade								
$\sum_{i,t}[rx_{i,t+1}(\overline{fp}_t - \overline{fp}^e)]$	2.55	1.24	2.52	3.18	1.90	0.26	2.20	2.36
Sharpe Ratio	0.25	0.12	0.26	0.27	0.15	0.02	0.17	0.18
Carry Trade								
$\sum_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_t)]$	4.95	2.81	4.25	5.24	4.50	1.99	2.95	3.35
Sharpe Ratio	0.54	0.31	0.34	0.44	0.54	0.23	0.26	0.29
% Static Trade	70%	121%	92%	76%	69%	.	105%	85%
Forward Premium Trade								
$\sum_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_i^e)]$	4.04	1.77	3.03	4.51	3.31	0.28	2.26	2.94
Sharpe Ratio	0.27	0.12	0.20	0.27	0.18	0.02	0.12	0.16
% Dollar Trade	63%	124%	88%	73%	57%	.	106%	84%
Sample	6 Rebalance				12 Rebalance			
Static Trade								
$\sum_{i,t}[rx_{i,t+1}(\overline{fp}_i - \overline{fp}^e)]$	2.42	-0.38	1.96	1.96	3.81	0.22	2.92	2.87
Sharpe Ratio	0.29	-0.05	0.20	0.21	0.46	0.03	0.30	0.29
Dynamic Trade								
$\sum_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e))]$	1.85	-0.48	0.34	-0.08	1.65	-0.89	0.41	0.19
Sharpe Ratio	0.26	-0.05	0.04	-0.00	0.26	-0.14	0.06	0.01
Dollar Trade								
$\sum_{i,t}[rx_{i,t+1}(\overline{fp}_t - \overline{fp}^e)]$	2.09	0.23	2.39	3.64	1.88	-0.18	1.15	2.13
Sharpe Ratio	0.16	0.02	0.18	0.19	0.14	-0.01	0.09	0.13
Carry Trade								
$\sum_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_t)]$	4.28	1.66	2.81	2.23	5.45	2.19	3.95	3.45
Sharpe Ratio	0.50	0.19	0.25	0.12	0.69	0.28	0.40	0.22
% Static Trade	57%	.	85%	104%	70%	.	88%	94%
FP Trade								
$\sum_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_i^e)]$	3.95	0.74	2.92	3.71	3.53	-0.01	1.78	2.44
Sharpe Ratio	0.21	0.04	0.15	0.17	0.20	-0.00	0.10	0.12
% Dollar Trade	53%	.	88%	102%	53%	.	74%	92%
Bid-Ask Spreads	No	Yes	Yes	Yes	No	Yes	Yes	Yes

Note: Mean returns and Sharpe ratios on the Static, Dynamic, Dollar, Carry, and Forward Premium Trades defined in equations (2), (5), and (6) calculated using 1-, 6-, and 12-month currency forward contracts against the US dollar. All returns are annualized and divided by \overline{fp} estimated in the 1 Rebalance sample post 12/1994 to facilitate comparison. The table also reports the percentage contribution of Static (Dollar) Trade to the mean returns on the Carry (Forward Premium) Trade, calculated by dividing its mean return by the maximum of zero and the sum of the mean returns on the Static (Dollar) and Dynamic Trades. $\sum_{i,t}$ denotes the double-sum over i and t . See Appendix A for details.

Table 3: Estimates of the Elasticity of Risk Premia with respect to Forward Premia

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sample	1 Rebalance				3 Rebalance			
Horizon (months)	1	1	6	12	1	1	6	12
Static T: β^{stat}	0.47*** (0.08)	0.37*** (0.09)	0.56*** (0.10)	0.60*** (0.10)	0.26*** (0.05)	0.18*** (0.05)	0.26*** (0.04)	0.25*** (0.06)
Dynamic T: β^{dyn}	0.44* (0.25)	0.41* (0.25)	0.36 (0.32)	0.53** (0.26)	0.28* (0.15)	0.24 (0.15)	0.21 (0.15)	0.26* (0.15)
Dollar T: β^{dol}	3.11* (1.60)	3.09* (1.58)	3.21 (1.96)	3.72* (2.16)	0.91 (1.18)	0.83 (1.18)	1.44 (1.22)	1.78 (1.20)
Carry Trade: β^{ct}	0.68** (0.27)	0.55** (0.26)	0.62** (0.29)	0.71*** (0.26)	0.57*** (0.19)	0.45** (0.18)	0.42** (0.21)	0.43** (0.19)
% ESS Static T	62	54	79	66	56	44	72	62
Forward Premium T: β^{fpp}	0.86** (0.34)	0.83** (0.34)	0.85** (0.42)	1.09*** (0.40)	0.41** (0.20)	0.37* (0.20)	0.48** (0.21)	0.60*** (0.21)
% ESS Dollar T	90	91	94	91	75	76	93	93
N	2706	2706	2631	2541	4494	4494	4374	4230
Sample	6 Rebalance				12 Rebalance			
Static T: β^{stat}	0.23*** (0.05)	0.15*** (0.05)	0.25*** (0.04)	0.24*** (0.05)	0.34*** (0.08)	0.23*** (0.09)	0.31*** (0.08)	0.30*** (0.08)
Dynamic T: β^{dyn}	0.19 (0.14)	0.16 (0.14)	0.10 (0.12)	-0.02 (0.06)	0.16 (0.11)	0.13 (0.12)	0.06 (0.09)	-0.01 (0.05)
Dollar T: β^{dol}	0.87 (2.59)	0.75 (2.60)	1.83 (2.14)	1.56** (0.70)	1.71 (2.26)	1.61 (2.27)	0.02 (2.04)	-0.23 (1.35)
Carry Trade: β^{ct}	0.56*** (0.18)	0.45*** (0.17)	0.45** (0.19)	0.11 (0.14)	0.67*** (0.16)	0.52*** (0.16)	0.57*** (0.16)	0.22 (0.17)
% ESS Static T	70	58	92	99	90	86	99	100
Forward Premium T: β^{fpp}	0.24 (0.19)	0.20 (0.19)	0.22 (0.17)	0.08 (0.08)	0.30* (0.16)	0.26* (0.16)	0.05 (0.14)	-0.03 (0.05)
% ESS Dollar T	62	64	96	100	92	94	1	95
N	4842	4842	4712	4556	6019	6019	5874	5626
Bid-Ask Spreads	No	Yes	Yes	Yes	No	Yes	Yes	Yes

Note: Estimates of the elasticity of currency risk premia with respect to forward premia in the cross-currency (β^{stat}), between-time-and-currency (β^{dyn}), and cross-time dimension (β^{dol}) using specifications (7), (9), and (10), respectively. The table also shows the slope coefficients from specifications (14) and (13) and the partial R^2 , calculated as $\frac{ESS^d}{ESS^d + ESS^{dyn}}$, $d \in \{stat, dol\}$, where ESS^{dyn} refers to the explained sum of squares in specification (9) and ESS^{stat} , ESS^{dol} refer to the explained sum of squares in specifications (7) and (10), respectively. Standard errors are in parentheses. Standard errors for β^{stat} and β^{dol} are clustered by currency and time, respectively, whereas the standard errors for β^{dyn} , β^{ct} , and β^{fpp} are Newey-West with 12, 18, and 24 lags for the 1-, 6-, and 12-month horizons, respectively. Additionally, β^{dol} is also adjusted at the 6-, and 12-month horizon using Newey-West as per Driscoll and Kraay (1998). Where appropriate, we use the Murphy and Topel (1985) procedure to adjust all standard errors for the estimated regressors \overline{fp}_i^e and \overline{fp}^e (see Appendix C.4 for details). Asterisks denote statistical significance at the 1 (***) , 5 (**) and 10% (*) level.

Table 4: Elasticities of Realized vs. Expected Returns

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sample	1 Rebalance		3 Rebalance		6 Rebalance		12 Rebalance	
	β_{FE}^{stat}	β^{stat}	β_{FE}^{stat}	β^{stat}	β_{FE}^{stat}	β^{stat}	β_{FE}^{stat}	β^{stat}
Static Trade	0.53*** (0.13)	0.47*** (0.08)	0.43*** (0.09)	0.26*** (0.05)	0.50*** (0.11)	0.23*** (0.05)	0.65*** (0.12)	0.34*** (0.08)
	β_{FE}^{dyn}	β^{dyn}	β_{FE}^{dyn}	β^{dyn}	β_{FE}^{dyn}	β^{dyn}	β_{FE}^{dyn}	β^{dyn}
Dynamic Trade	1.13** (0.45)	0.44* (0.25)	0.83*** (0.32)	0.28* (0.15)	0.71** (0.34)	0.19 (0.14)	0.74** (0.33)	0.16 (0.11)
	β_{FE}^{fpp}	β^{fpp}	β_{FE}^{fpp}	β^{fpp}	β_{FE}^{fpp}	β^{fpp}	β_{FE}^{fpp}	β^{fpp}
F.P. Trade	1.81*** (0.53)	0.86** (0.34)	0.89*** (0.32)	0.41** (0.20)	0.77* (0.41)	0.24 (0.19)	1.04*** (0.37)	0.30* (0.16)
	<i>FE</i>		<i>FE</i>		<i>FE</i>		<i>FE</i>	
% ESS Static T	39	62	36	56	79	70	79	90
% ESS Dollar T	80	90	51	75	71	62	71	92

Note: This table compares estimates of the elasticity of realized returns with respect to forward premia, β_{FE}^{stat} , β_{FE}^{dyn} , and β_{FE}^{fpp} (the slope coefficients from regressions with currency fixed effects corresponding to (7), (9), and (13)) with estimates of the elasticity of risk premia with respect to forward premia from columns 1 and 5 in Table 3. All specifications use one-month forwards and exclude bid-ask spreads. The columns marked *FE* in the bottom panel of the table show the partial R^2 of the static trade in the carry trade regression and the partial R^2 of the dollar trade in the forward premium trade regression calculated as $\frac{ESS^d}{ESS^d + ESS^{dyn}}$, $d \in \{stat, dol\}$, where ESS^{stat} and ESS^{dyn} refer to the explained sum of squares in the equivalent of specifications (7) and (9) estimated using currency fixed effects and ESS^{dol} refers to the explained sum of squares in specification (10). The unlabeled columns show the partial R^2 calculated using the corresponding specifications without currency fixed effects from columns 1 and 5 of Table 3 for comparison. Asterisks denote statistical significance at the 1 (***) , 5 (**) and 10% (*) level.

Table 5: χ^2 Difference Tests

	(1)	(2)
Sample	1 Rebalance	3 Rebalance
<i>Null Hypothesis</i>	<i>p-values</i>	
$\beta^{dyn} = 0$	0.16	0.30
$\beta^{dol} = 0$	0.02	0.04
$\beta^{dol} = \beta^{dyn}$	0.11	0.15

Note: χ^2 difference tests of the ability of restricted linear models of currency returns to explain the returns on the forward premium trade documented in columns 1 and 5 of Table 2. *p-values* are calculated under the assumption that the coefficient estimates of β^{fpp} , β^{dyn} , and β^{dol} in columns 1 and 5 of Table 3 are normally distributed. See Appendix C.10 for details.

Table 6: Currency-specific Elasticities of Risk Premia with Respect to Forward Premia

Sample	(1) 1 Rebalance	(2) 3 Rebalance	(3) 6 Rebalance	(4) 12 Rebalance
Australia	1.03	0.23	-0.26	-0.33
Austria			4.29***	4.40***
Belgium		2.59**	2.95**	3.79***
Canada	1.20	1.45	1.31	2.84
Czech Rep.		-0.76	2.68	7.30***
Denmark	1.91	0.69	0.56	0.33
ECU			-0.50	-1.25***
Euro		2.75	4.29	2.04
France		0.90	0.82	0.15
Germany		2.08	2.16	3.64*
Hong Kong	1.66	1.12	0.20	0.62
Hungary		6.06**	8.69*	6.27***
Iceland				-5.93**
India		3.66***	3.44***	3.59***
Indonesia				2.67***
Ireland			1.24	1.18**
Italy		-1.31	-1.43	-0.27
Japan	0.55	0.80	-0.72	-0.27
Korea			-1.76	-1.05
Kuwait	1.33	1.59	0.44	0.96
Malaysia	-1.64	-2.44	-2.17	-2.46**
Mexico		0.91	0.76	1.96
Netherlands		2.39*	2.50	3.88*
New Zealand	-0.84	-0.19	-1.77	-2.09
Norway	0.55	-0.69	-0.84	-0.95
Philippines		1.03	0.25	1.00
Poland		-3.08	-1.61	5.43***
Saudi Arabia	2.72*	2.40	1.40	3.43**
Sweden	3.08***	-0.09	0.16	0.18
Singapore	1.25*	0.09	0.27	0.11
Slovak Rep.				21.76***
Spain			1.61	-2.22**
Switzerland	1.59	2.90	3.03	4.50*
Taiwan		0.70	1.00	0.07
Thailand		1.55	1.63*	1.75*
Turkey			-0.27	2.18
UAE		1.21	3.77***	3.21**
United Kingdom	2.86	2.52***	2.83**	-0.58
South Africa	2.34**	2.27**	2.95***	0.92

Note: Currency-specific elasticities of risk premia with respect to forward premia, β_i^{dyn} , estimated using (21). Asterisks denote statistical significance at the 1 (***), 5 (**), and 10% (*) level. Standard errors (not shown) are Newey-West using 12 lags. 1-month forward contracts used throughout.

Table 7: Is the US Dollar Special?

	(1)	(2)	(3)	(4)
Sample	1 Rebalance	3 Rebalance	6 Rebalance	12 Rebalance
β^{dol}	3.11* (1.60)	0.91 (1.18)	0.87 (2.59)	1.71 (2.26)
$\beta_{FE}^{dyn} = \sum_i \omega_i \beta_i^{dyn}$	1.13** (0.45)	0.83*** (0.32)	0.71** (0.34)	0.74** (0.33)
p-val($\beta^{dol} = \sum_i \omega_i \beta_i^{dyn}$)	0.17	0.96	0.95	0.65

Note: This table compares point estimates of β^{dol} from columns 1 and 5 of Table 3 with the weighted average of estimates of β_i^{dyn} from columns 1-4 of Table 6. One can show that $\sum_i \omega_i \beta_i^{dyn} = \beta_{FE}^{dyn}$, where $\omega_i = \frac{\sum_t (fp_{it} - \overline{fp}_t - (\overline{fp}_i - \overline{fp}))^2}{\sum_i \sum_t (fp_{it} - \overline{fp}_t - (\overline{fp}_i - \overline{fp}))^2}$. To obtain the p-value of the test $\beta^{dol} = \sum_i \omega_i \beta_i^{dyn}$, we run a bivariate panel regression of $rx_{it} - \overline{rx}_i$ on both $\overline{fp}_t - \overline{fp}$ and $fp_{it} - \overline{fp}_t - (\overline{fp}_i - \overline{fp})$, and test if the two resulting coefficients (β^{dol} and β_{FE}^{dyn}) are equal. Standard errors are clustered by time. There is a small discrepancy between β^{dol} estimated from this multivariate regression and the estimates shown in the first row of this table due to few data exclusions resulting from our data-filtering procedure. Asterisks denote statistical significance at the 1 (***) , 5 (**) and 10% (*) level. See Appendix A for details.

Table 8: Traditional Bilateral Forward Premium Puzzle Regressions

	(1)	(2)	(3)	(4)
Sample	1 Rebalance	3 Rebalance	6 Rebalance	12 Rebalance
Australia	3.25*	2.15*	2.06	1.86
Austria			6.27***	0.09
Belgium			3.03	3.99*
Canada	4.36***	2.31***	4.47***	4.73***
Czech Rep.		-3.60*	-5.50	5.28***
Denmark	4.43***	1.13	0.96	1.45
ECU			1.49	-4.10***
Euro			3.63	4.38*
France			0.73	0.34
Germany			1.90	3.33
Hong Kong	1.05***	1.03***	1.06***	1.14***
Hungary		2.34	8.04	7.40***
Iceland				0.42
Indonesia				3.97**
Ireland			4.26*	1.86**
Italy			-2.09	-2.59*
Japan	2.55***	2.88***	3.32	2.03
Korea			-2.45	-2.52
Kuwait	-1.94***	-2.08***	-2.00***	-1.78**
Malaysia	-1.96**	-1.72	-2.61*	-1.10
Mexico		-0.73	-0.37	2.01
Netherlands			2.00	1.84
New Zealand	1.10	1.26	-2.06	-1.58
Norway	1.89	-0.12	-1.07	-0.88
Philippines		0.85	3.51	2.77
Poland		-5.99***	-5.80*	3.40*
Saudi Arabia	1.36***	1.46***	1.47***	1.58***
Sweden	3.37**	0.02	-0.75	-1.25
Singapore	0.74	1.31*	1.13	2.66***
Slovak Rep.				11.47***
Spain			5.42***	-3.42*
Switzerland	3.59**	2.37**	3.57	4.58***
Taiwan		-0.05	-0.05	0.55
Thailand		0.96	1.07	2.26**
Turkey			-0.99	-0.82
UAE		1.15***	1.15***	1.19***
United Kingdom	2.66*	0.63	0.88	0.06
South Africa	2.43**	2.44*	2.65**	1.33
β_{FE}^{fpp}	1.81***	0.89***	0.77*	1.04***
β^{fpp}	0.86**	0.41**	0.24	0.30*

Note: Estimates of the currency-specific elasticity of risk premia with forward premia β_i^{fpp} using the specification $rx_{i,t+1} = \alpha_i + \beta_i^{fpp} fp_{it} + \epsilon_{it}$. Standard errors (not shown) are Newey-West using 12 lags. 1-month forward contracts used throughout. The table also reproduces for comparison the corresponding estimates of β_{FE}^{fpp} and β^{fpp} as in Table 4. Asterisks denote statistical significance at the 1 (***) , 5 (**) and 10% (*) level.

Online Appendix

to

“Forward and Spot Exchange Rates in a Multi-Currency World”

by

Tarek A. Hassan and Rui C. Mano

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A Appendix to Section 1

We use two different types of data: foreign exchange data, which comprises spot and forward rates for maturities of 1, 6, and 12 months, and interbank interest rate data, for maturities of

1 and 12 months. All data are monthly, retrieved at the last trading day of the month.

We use an algorithm to clean the foreign exchange data based on departures from Covered Interest Parity (CIP) and discrepancies between different sources of data. The algorithm is described below.

A.1 Interest Rate Data

We use two different sources for interbank interest rate data. The first is sourced from Global Financial Data (GFD). This source comprises interbank rates (mostly local LIBOR rates) for maturities 1 and 12 months. The second source is Datastream (DS) Eurocurrency rates for the 1- and 12-month maturity, which comprise a smaller cross section of currencies. Generally, these series are virtually equal to each other.

- GFD Interbank rates: mnemonics for these series are $IBccg1D$ and $IBccg12D$ for 1- and 12-month maturities, respectively. ccg is the country code for each country in GFD, which are not the official ISO currency codes.
- DS Interbank Eurocurrency rates: mnemonics for 1 and 12 months are $ECiso1M$ and $ECiso1Y$, respectively. As mentioned above, DS uses ISO codes. Check in the FX Data subsection for details.

In both cases, we did not use the series for 2, 3, and 6 months because their coverage tends to be less extensive, both in the cross-section and time-series dimension. See the data provider's websites for details on respective detailed methodology.

A.2 Spot and Forward Rates

We use data on dollar-based spot and forward exchange rates from Datastream (DS) to construct currency returns. Datastream contains four sources of these data: World Markets PLC/Reuters (WM/R), Thomson/Reuters (T/R), HSBC, and Barclays Bank PLC (BB). The most comprehensive in terms of currencies is WM/R. However, this series only begins in December 1996. T/R goes back to May 1990. Both HSBC and BB are not available for recent years but have data back to October 1983 (BB) and October 1986 (HSBC) for some currencies. All providers also offer spot exchange rates corresponding to their forward rates. The mnemonics for these series are: $dsisoSP$ for spot and $dsiso1F$, $-3F$, $-6F$, and $-1Y$ or $-YF$ for 1-, 3-, 6-, and 12-month-maturity forwards. ds corresponds to the dataset mnemonic: TD for Thomson/Reuters, BB for Barclays Bank, and MB for HSBC. WM/R has a different structure for spot and forward rates. The mnemonics for spot rates do not have a clear pattern other than some abbreviation of the currency name and the dollar sign in the end (e.g., $AUSTDO\$$ for the Australian Dollar quote). The forward rates follow the pattern given above

for the other sources with mnemonic *US*. Datastream uses the *iso* codes as country codes. To check ISO codes specified by the International Organization for Standardization (ISO), go to <http://www.oanda.com/help/currency-iso-code-country>.

The general rules for mnemonics (e.g., departures from ISO codes) have some exceptions. In addition to mid rates, bid and offer quotes are also available. To distinguish between these three, DS codes have a suffix *-Ex* where *x* is B, R, or O, respectively, for bid, mid, and offer quotes. See the data provider's website for details on respective detailed methodology.

In addition to dollar-based data, we complement our spot and forward data with pound-based data from another provider also available through DS listed as BMI. These data include one-month forward and spot rates for 14 European currencies, the US dollar, and Japanese yen from January 1976 onward. These are same as those in [Burnside et al. \(2006\)](#).

In time periods in which they overlap, the data from the different providers are very similar. We assemble a comprehensive panel of dollar-based forward premia and currency returns in three steps. First, we use forward and spot rates from the same source to construct a panel of forward premia and currency returns from each provider. (The data providers vary on the fixing time. Using a forward rate from one source with a spot from another could therefore lead to inaccuracies.) Second, we combine the panels in the following order: When available we use WM/R data, which appears to be the most recent and most accurate source. We fill in missing observations using the Thomson/Reuters, HSBC and Barclays Bank datasets in that order. In a final step, we check the consistency of the data using the following algorithm.

For observations for which we have information on a single dollar-based forward premium, we compare the forward premia to differentials in the interbank rates at the one-month horizon. If the interest rate differential in the Global Financial Data (GFD) data is within 20bps of the interest differential sourced from DS, we exclude the observation if the one-month forward premium deviates from the one-month GFD interest differential by more than 50bps (a dramatic violation of covered interest parity). By this criterion, we exclude Italy 1/1985 and 2/1985; Switzerland 2/1985; Germany 2/1985; United Kingdom 3/1985; Belgium 7/1990; and Indonesia 12/1997, 3/1998, 5/1998-7/1998, 2/2001-11/2002.

For observations for which we have information on a single forward premium, a forward premium from the pound-based data and information on interest rate differentials from one source, we again check if the one-month forward premium deviates from the interest differential by more than 50bps. If it does, we check the forward premium from the pound-based dataset. If the pound-based forward premium deviates from the interest differential by less than 50bps, we exclude this observation. By this criterion, we exclude Austria 1/1990-2/1990; Spain 9/1987, 5/1988; Ireland 11/1986, 11/1987, 1/1989, 1/1991, 9/1992-11/1992, 1/1993; Belgium 2/1985; and Norway 2/1985.

For observations for which we have information on the forward premium from multiple dollar-based sources and information on interest differentials from one source, we again check

if the 1-month forward premium deviates from the interest differential by more than 50bps. If it does we check the forward premium from the alternative sources. If the forward premia from one other source deviates from the interest differential by less than 50bps we substitute this observation. By this criterion we replace Norway 5/1988, Sweden 5/1988, Malaysia 12/1993, and Belgium 10/1987 and 5/1988 with data from BB; and Iceland 2/2009 and Thailand 12/2006, 11/2008 with data from TD.

For observations for which we have information on the forward premium from multiple dollar-based sources and information on interest differentials from both GFD and DS, we check if the interest rate differential in the GFD data is within 20bps of the interest differential sourced from DS. If so, we check if the one-month forward premium deviates from one of the interest differentials by more than 50bps. If it does, we check the forward premium from the alternative sources. If the forward premium from one other source deviates from the interest differential by less than 50bps we substitute this observation. By this criterion, we replace Switzerland 1/1989, Germany 5/1988, France 1/1989, Italy 5/1988, Netherlands 5/1988, United Kingdom 1/1989 with data from BB; and Singapore 10/1997 and Thailand 10/2003 with data from TD.

Following [Lustig et al. \(2011\)](#), we drop South Africa 8/1985 and Turkey before 11/2001 due to large covered interest parity departures we could not verify. Finally, we drop Malaysia 8/1998-6/2005 and Indonesia 1/2003-5/2007 because forward rates are zero.

Our “1 Rebalance,” “3 Rebalance,” “6 Rebalance,” and “12 Rebalance” samples are built with the dollar-based data after applying the above algorithm and exclusions.

In addition, we look at four alternative samples: “1 Rebalance (no fixed),” “LRV,” “4 Rebalances (CIP),” and “BER.” “1 Rebalance (no fixed)” is the same as “1 Rebalance,” excluding Saudi Arabia riyal and Hong Kong dollar. “LRV” is the same as “1 Rebalance” but instead of using our data cleaning algorithm, we use the notes provided in p.8 of [Lustig et al. \(2011\)](#) to approximate as best as we can the dataset used there. “4 Rebalance (CIP)” is a sample with four rebalances at 6/1983, 12/1989, 12/1997, and 12/2004 where we extended our dollar-based data with both pound-based data and interest rate differentials. Finally, “BER” uses the same pound-based data as [Burnside et al. \(2006\)](#) with the same rebalancing periods as “4 Rebalance (CIP).”

B Appendix to Section 2

A number of the proofs below make use of the following identities for all $x_{it}, y_{it} = fp_{it}, rx_{i,t+1}$

$$\sum_{i,t} [\bar{x}_t y_{it}] = \sum_{i,t} [\bar{x}_t \bar{y}_t],$$

$$\sum_{i,t} [\bar{x} y_{it}] = \sum_{i,t} [\bar{x} \bar{y}_i] = \sum_{i,t} [\bar{x} \bar{y}_t] = \sum_{i,t} [\bar{x} \bar{y}],$$

and

$$\sum_{i,t} [\bar{x}_i y_{it}] = \sum_{i,t} [\bar{x}_i \bar{y}_i].$$

To see that these identities must hold consider for example the first statement above. Using the definitions (3) and (4), we can then show

$$\begin{aligned} \sum_{i,t} [\bar{x}_t y_{it}] &= \sum_i \sum_t \bar{x}_t y_{it} = \sum_t \bar{x}_t (\sum_i y_{it}) \\ &= \sum_t \bar{x}_t (N \bar{y}_t) = \sum_i \sum_t \bar{x}_t \bar{y}_t \\ &= \sum_{i,t} [\bar{x}_t \bar{y}_t]. \end{aligned}$$

The remaining statements follow using the same steps.

B.1 The Carry Trade is neutral with respect to the US dollar

To see this result formally, note that the return on an equally weighted portfolio of all foreign currencies relative to the US dollar is $\bar{r}\bar{x}_{t+1} = \sum_i \frac{1}{N} r x_{i,t+1}$ from (4). In addition, we have that

$$\sum_{i,t} [\bar{r}\bar{x}_{t+1} (f p_{it} - \bar{f} p_t)] = 0,$$

such that

$$\sum_{i,t} [(r x_{i,t+1} - \bar{r}\bar{x}_{t+1}) (f p_{it} - \bar{f} p_t)] = \sum_{i,t} [r x_{i,t+1} (f p_{it} - \bar{f} p_t)]. \quad (28)$$

The returns to the carry trade are thus independent of the returns on the US dollar.

C Appendix to Section 3

C.1 Detailed derivation of (8)

Re-writing the second term on the right-hand side of (6) yields

$$\begin{aligned} \sum_{i,t} [r x_{i,t+1} (f p_{it} - \bar{f} p_t - (\bar{f} p_i^e - \bar{f} p^e))] &= \sum_{i,t} [(r x_{i,t+1} - \bar{r}\bar{x}_{t+1} - (\bar{r}\bar{x}_i - \bar{r}\bar{x})) (f p_{it} - \bar{f} p_t - (\bar{f} p_i^e - \bar{f} p^e))] \\ &\quad + \sum_{i,t} [(\bar{r}\bar{x}_{t+1} + (\bar{r}\bar{x}_i - \bar{r}\bar{x})) (f p_{it} - \bar{f} p_t - (\bar{f} p_i^e - \bar{f} p^e))] \\ &= \sum_{i,t} [(r x_{i,t+1} - \bar{r}\bar{x}_{t+1} - (\bar{r}\bar{x}_i - \bar{r}\bar{x})) (f p_{it} - \bar{f} p_t - (\bar{f} p_i^e - \bar{f} p^e))] \\ &\quad + \sum_{i,t} [\bar{r}\bar{x}_i (f p_{it} - \bar{f} p_t - (\bar{f} p_i^e - \bar{f} p^e))] \\ &= \hat{\beta}^{dyn} \sum_{i,t} ((f p_{i,t} - \bar{f} p_t) - (\bar{f} p_i^e - \bar{f} p^e))^2 + \\ &\quad + \sum_{i,t} [\bar{r}\bar{x}_i (f p_i - \bar{f} p - (\bar{f} p_i^e - \bar{f} p^e))]. \end{aligned}$$

We again get the first equality from adding and subtracting $\bar{r}\bar{x}_{t+1} + (\bar{r}\bar{x}_i - \bar{r}\bar{x})$. The second equality again follows from the fact that $\sum_{i,t} (f p_{it} - \bar{f} p_t - (\bar{f} p_i^e - \bar{f} p^e)) = 0$ and does not

vary across t . The third equality then follows from re-writing the first term as an OLS regression coefficient where

$$\hat{\beta}^{dyn} = \sum_{i,t} \left[(rx_{i,t+1} - \bar{rx}_{t+1} - (\bar{rx}_i - \bar{rx})) (fp_{it} - \bar{fp}_t - (\bar{fp}_i^e - \bar{fp}^e)) \right] / \sum_{i,t} \left((fp_{i,t} - \bar{fp}_t) - (\bar{fp}_i^e - \bar{fp}^e) \right)^2$$

is the OLS estimate of the slope coefficient in (9).

Similarly, we can rewrite the third term on the right-hand side of (6) as

$$\begin{aligned} \sum_{i,t} \left[rx_{i,t+1} (\bar{fp}_t - \bar{fp}^e) \right] &= \sum_{i,t} \left[(\bar{rx}_{it} - \bar{rx}_i) (\bar{fp}_t - \bar{fp}^e) \right] + \sum_{i,t} \left[\bar{rx}_i (\bar{fp}_t - \bar{fp}^e) \right] \\ &= \sum_{i,t} \left[(rx_{it} - \bar{rx}_i) (\bar{fp}_t - \bar{fp}^e) \right] + \sum_{i,t} \left[\bar{rx}_i (\bar{fp}_t - \bar{fp}^e) \right] \\ &= \hat{\beta}^{dol} \sum_{i,t} (\bar{fp}_t - \bar{fp}^e)^2 + \sum_{i,t} \left[\bar{rx} (\bar{fp} - \bar{fp}^e) \right], \end{aligned}$$

where $\hat{\beta}^{dol}$ the OLS estimate of the slope coefficient in (10).

C.2 Structure of Error Terms

Some readers might find it useful to first read section 3.1 of the main text before reading this this appendix.

Note that the only difference between our specifications (7), (9), and (10) and standard fixed effects estimators is that they replace \bar{fp}_i and \bar{fp} with expectations \bar{fp}_i^e and \bar{fp}^e . That is, had we been interested in characterizing the elasticity of *realized* rather than expected returns, we would have simply used currency fixed effects instead of \bar{fp}_i^e in our regressions. Denoting the slope coefficients of the corresponding fixed effects specifications as β_{FE}^{stat} , β_{FE}^{dyn} , and β_{FE}^{dol} , we can write realized currency returns without loss of generality as

$$rx_{i,t+1} = \gamma + \beta_{FE}^{stat} (\bar{fp}_i - \bar{fp}) + \beta_{FE}^{dyn} (fp_{i,t} - \bar{fp}_t - (\bar{fp}_i - \bar{fp})) + \beta_{FE}^{dol} (\bar{fp}_t - \bar{fp}) + \vartheta_{i,t+1}, \quad (29)$$

where $\vartheta_{i,t+1}$ is mean-zero and orthogonal to $(\bar{fp}_i - \bar{fp})$, $(\bar{fp}_i^e - \bar{fp}^e)$, $(fp_{i,t} - \bar{fp}_t - (\bar{fp}_i - \bar{fp}))$, and $(\bar{fp}_t - \bar{fp})$. It follows directly that

$$\bar{rx}_i = \beta_{FE}^{stat} (\bar{fp}_i - \bar{fp}) + \bar{\vartheta}_i, \quad \bar{rx}_{t+1} = \beta_{FE}^{dol} (\bar{fp}_t - \bar{fp}) + \bar{\vartheta}_{t+1}, \quad \text{and} \quad \bar{rx} = \gamma + \bar{\vartheta}. \quad (30)$$

We can then write variation of returns in the between-time-and-currency dimension as

$$rx_{i,t+1} - \bar{rx}_i - \bar{rx}_{t+1} + \bar{rx} = \beta_{FE}^{dyn} (fp_{i,t} - \bar{fp}_t - (\bar{fp}_i - \bar{fp})) + (\vartheta_{i,t+1} - \bar{\vartheta}_i - \bar{\vartheta}_{t+1} + \bar{\vartheta}).$$

Adding and subtracting terms reveals the structure of the error term, $\epsilon_{i,t+1}^{dyn}$, in (9)

$$rx_{i,t+1} - \bar{r}x_i - \bar{r}x_{t+1} + \bar{r}x = \beta^{dyn} (fp_{i,t} - \bar{f}p_t - (\bar{f}p_i^e - \bar{f}p^e)) + \underbrace{(\beta_{FE}^{dyn} - \beta^{dyn}) (fp_{i,t} - \bar{f}p_t - (\bar{f}p_i - \bar{f}p)) - \beta^{dyn} ((\bar{f}p_i - \bar{f}p) - (\bar{f}p_i^e - \bar{f}p^e)) + (\vartheta_{i,t+1} - \bar{\vartheta}_i - \bar{\vartheta}_{t+1} + \bar{\vartheta})}_{\epsilon_{i,t+1}^{dyn}}$$

The first two terms in the bracket reflect errors investors make when predicting $\bar{f}p_i$: the first term depends only on the variance of the forecast error ($var(\bar{f}p_i^e - \bar{f}p_i)$). It is zero if and only if investors have perfect foresight, $\bar{f}p_i^e = \bar{f}p_i$, as in that case $\beta_{FE}^{dyn} = \beta^{dyn}$ (see Proposition 2 for details). The second term clearly depends on the forecast error itself.

We can now verify that $\epsilon_{i,t+1}^{dyn}$ is uncorrelated with the regressor, $E_0[\epsilon_{i,t+1}^{dyn}((\bar{f}p_i - \bar{f}p) - (\bar{f}p_i^e - \bar{f}p^e) - \bar{f}p^e)] = 0$. Noting that $E_0[(fp_{i,t} - \bar{f}p_t - (\bar{f}p_i^e - \bar{f}p^e))(\vartheta_{i,t+1} - \bar{\vartheta}_i - \bar{\vartheta}_{t+1} + \bar{\vartheta})] = 0$, plugging in the structure shown above, and re-arranging yields

$$E_0 \left[(\beta_{FE}^{dyn} - \beta^{dyn}) (fp_{i,t} - \bar{f}p_t - (\bar{f}p_i - \bar{f}p)) (fp_{i,t} - \bar{f}p_t - (\bar{f}p_i^e - \bar{f}p^e)) \right] = E_0 \left[\beta^{dyn} ((\bar{f}p_i - \bar{f}p) - (\bar{f}p_i^e - \bar{f}p^e)) (fp_{i,t} - \bar{f}p_t - (\bar{f}p_i^e - \bar{f}p^e)) \right].$$

Adding and subtracting $(\bar{f}p_i - \bar{f}p)$ in the last round bracket on both sides of the equation yields

$$\begin{aligned} & (\beta_{FE}^{dyn} - \beta^{dyn}) E_0 \left[(fp_{i,t} - \bar{f}p_t - (\bar{f}p_i - \bar{f}p))^2 \right] + E_0 \left[(fp_{i,t} - \bar{f}p_t - (\bar{f}p_i - \bar{f}p)) \underbrace{((\bar{f}p_i - \bar{f}p) - (\bar{f}p_i^e - \bar{f}p^e))}_{=0} \right] \\ = & \beta^{dyn} E_0 \left[\underbrace{((\bar{f}p_i - \bar{f}p) - (\bar{f}p_i^e - \bar{f}p^e))}_{=0} (fp_{i,t} - \bar{f}p_t - (\bar{f}p_i - \bar{f}p)) \right] + \beta^{dyn} E_0 \left[((\bar{f}p_i - \bar{f}p) - (\bar{f}p_i^e - \bar{f}p^e))^2 \right]. \end{aligned}$$

Re-arranging these expressions yields

$$\beta_{FE}^{dyn} = \beta^{dyn} \left(1 + \frac{E_0 \left[((\bar{f}p_i - \bar{f}p) - (\bar{f}p_i^e - \bar{f}p^e))^2 \right]}{E_0 \left[(fp_{i,t} - \bar{f}p_t - (\bar{f}p_i - \bar{f}p))^2 \right]} \right),$$

which is true, as shown in (15).

Similarly, from (30) we can write variation of returns in the cross-currency dimension as

$$\bar{r}x_i - \bar{r}x = \beta_{FE}^{stat} (\bar{f}p_i - \bar{f}p) + \bar{\vartheta}_i - \bar{\vartheta}. \quad (31)$$

Adding and subtracting terms again yields the structure of the error term, $\bar{\epsilon}_i^{stat}$,

$$\bar{r}x_i - \bar{r}x = \beta^{stat} \left(\overline{fp}_i^e - \overline{fp}^e \right) + \underbrace{(\beta_{FE}^{stat} - \beta^{stat}) (\overline{fp}_i - \overline{fp}) + \beta^{stat} \left((\overline{fp}_i - \overline{fp}) - (\overline{fp}_i^e - \overline{fp}^e) \right)}_{\bar{\epsilon}_i^{stat}} + \bar{\vartheta}_i - \bar{\vartheta}, \quad (32)$$

where again the first term can be shown to depend on the variance of the forecast error (refer again to Proposition 2).

Following the same steps, we can again verify that $E_0 \left[\bar{\epsilon}_i^{stat} \left(\overline{fp}_i^e - \overline{fp}^e \right) \right] = 0$: Noting that $E_0 \left[(\bar{\vartheta}_i - \bar{\vartheta}) \left(\overline{fp}_i^e - \overline{fp}^e \right) \right] = 0$, plugging in the structure in (32), and re-arranging yields

$$\begin{aligned} & E_0 \left[(\beta_{FE}^{stat} - \beta^{stat}) (\overline{fp}_i - \overline{fp}) \left(\overline{fp}_i^e - \overline{fp}^e \right) \right] \\ &= E_0 \left[\beta^{stat} \left(\left(\overline{fp}_i^e - \overline{fp}^e \right) - (\overline{fp}_i - \overline{fp}) \right) \left(\overline{fp}_i^e - \overline{fp}^e \right) \right]. \end{aligned}$$

Adding and subtracting $(\overline{fp}_i - \overline{fp})$ in the last round bracket on both sides of the equation and eliminating terms yields

$$\begin{aligned} & \beta_{FE}^{stat} E_0 \left[\left((\overline{fp}_i - \overline{fp})^2 \right) \right] - \underbrace{\beta_{FE}^{stat} E_0 \left[\left(\left(\overline{fp}_i^e - \overline{fp}^e \right) - (\overline{fp}_i - \overline{fp}) \right)^2 \right]}_{=-E_0 \left[(\bar{r}x_i - \bar{r}x) (\overline{fp}_i^e - \overline{fp}^e - (\overline{fp}_i - \overline{fp})) \right]} \\ &= \beta^{stat} E_0 \left[\left(\overline{fp}_i^e - \overline{fp}^e \right)^2 \right], \end{aligned}$$

For the substitution of the second term on the right hand side note that using (31) we can write

$$\begin{aligned} & E_0 \left[(\bar{r}x_i - \bar{r}x) \left(\overline{fp}_i^e - \overline{fp}^e - (\overline{fp}_i - \overline{fp}) \right) \right] \\ &= E_0 \left[(\beta_{FE}^{stat} (\overline{fp}_i - \overline{fp}) + \bar{\vartheta}_i - \bar{\vartheta}) \left(\overline{fp}_i^e - \overline{fp}^e - (\overline{fp}_i - \overline{fp}) \right) \right]. \end{aligned}$$

Again noting that $\bar{\vartheta}_i$ is orthogonal to forward premia, and adding and subtracting $(\overline{fp}_i^e - \overline{fp}^e)$ yields

$$\begin{aligned} E_0 \left[(\bar{r}x_i - \bar{r}x) \left(\overline{fp}_i^e - \overline{fp}^e - (\overline{fp}_i - \overline{fp}) \right) \right] &= -\beta_{FE}^{stat} E_0 \left[\left(\left(\overline{fp}_i^e - \overline{fp}^e \right) - (\overline{fp}_i - \overline{fp}) \right)^2 \right] \\ &\quad + \underbrace{\beta_{FE}^{stat} E_0 \left[\left(\overline{fp}_i^e - \overline{fp}^e \right) \left(\overline{fp}_i^e - \overline{fp}^e - (\overline{fp}_i - \overline{fp}) \right) \right]}_{=0 \text{ by rationality}} \end{aligned}$$

Substituting this expression as indicated above and simplifying yields (17).

Finally, we have

$$\overline{rx}_{t+1} - \overline{rx} = \beta_{FE}^{dol} (\overline{fp}_t - \overline{fp}) + \overline{\vartheta}_{t+1} - \overline{\vartheta}. \quad (33)$$

Because the specification (10) contains a constant term we have that $\beta_{FE}^{dol} = \beta^{dol}$ (again see Proposition 2 for details). It suffices to add and subtract terms to show

$$\overline{rx}_{t+1} - \overline{rx} = \underbrace{\beta^{dol} (\overline{fp}^e - \overline{fp})}_{\gamma} + \beta^{dol} (\overline{fp}_t - \overline{fp}^e) + \underbrace{\overline{\vartheta}_{t+1} - \overline{\vartheta}}_{\overline{\epsilon}_{t+1}^{dol}}, \quad (34)$$

where $E_0 \left[\overline{\epsilon}_{t+1}^{dol} (\overline{fp}_t - \overline{fp}^e) \right] = 0$ by construction.

Residuals in specifications (7) and (10)

Note that in our empirical applications of this model we use the full panel of data to estimate β^{stat} and β^{dol} . To derive the structure of the error term in the specifications (7) and (10), note that from (29) and (34) we have

$$rx_{i,t+1} - \overline{rx}_{t+1} = \beta^{stat} (\overline{fp}_i^e - \overline{fp}^e) + \beta^{dyn} (fp_{i,t} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e)) + \epsilon_i^{stat} + \epsilon_{t+1}^{dyn}.$$

Further, substituting $\beta^{dyn} (fp_{i,t} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e)) + \epsilon_{t+1}^{dyn} = rx_{i,t+1} - \overline{rx}_i - \overline{rx}_{t+1} + \overline{rx}$ yields

$$\epsilon_{i,t+1}^{stat} = \overline{\epsilon}_i^{stat} + rx_{i,t+1} - \overline{rx}_i - \overline{rx}_{t+1} + \overline{rx},$$

where again $E_0 \left[\epsilon_{i,t+1}^{stat} (\overline{fp}_i^e - \overline{fp}^e) \right] = E_0 \left[\overline{\epsilon}_i^{stat} (\overline{fp}_i^e - \overline{fp}^e) \right] = 0$, and $\epsilon_{i,t+1}^{stat}$ is the error term from (7) and $\sum_t \epsilon_{i,t+1}^{stat} = \overline{\epsilon}_i^{stat}$.

Similarly, from (29) and (30), and (34) we have

$$rx_{i,t+1} - \overline{rx} = \gamma + \beta^{dol} (\overline{fp}_t - \overline{fp}^e) + \underbrace{\beta_{FE}^{stat} (\overline{fp}_i - \overline{fp}) + \beta_{FE}^{dyn} (fp_{i,t} - \overline{fp}_t - (\overline{fp}_i - \overline{fp}))}_{\overline{\epsilon}_{i,t+1}^{dol}} + \vartheta_{i,t+1} - \overline{\vartheta}.$$

Because $(\overline{fp}_i - \overline{fp})$ and $(fp_{i,t} - \overline{fp}_t - (\overline{fp}_i - \overline{fp}))$ are by construction orthogonal to $(\overline{fp}_t - \overline{fp}^e)$, we again have $E_0 \left[\overline{\epsilon}_{i,t+1}^{dol} (\overline{fp}_t - \overline{fp}^e) \right] = E_0 \left[\epsilon_t^{dol} (\overline{fp}_t - \overline{fp}^e) \right] = 0$, where $\overline{\epsilon}_{i,t+1}^{dol}$ is the error term from (10) and $\sum_i \overline{\epsilon}_{i,t+1}^{dol} = \overline{\epsilon}_t^{dol}$.

C.3 Generalized Model: Overlapping Investment Periods

Some readers might find it useful to first read section 3.1 of the main text before reading this this appendix.

Consider an extension of the model in Section 3 where currencies are priced by a sequence of rational marginal investors. The marginal investor at each point in time τ has an investment horizon of T periods. There is an infinite number of investment periods. Denote the population mean forward premium of currency i as \overline{fp}_i , whereas the mean in each investment period is denoted as \overline{fp}_i^τ , and τ is the beginning of the investment period, $\tau+1, \dots, \tau+T$. As before, each marginal investor observes each currency's entire history of forward premia and has rational expectations of mean forward premia $\left\{ \overline{fp}_i^{\tau,e} \right\}_{\tau,i}$, where $\overline{fp}_i^{\tau,e} \equiv E_{i\tau} \left[\overline{fp}_i^\tau \right]$. In particular, rational expectations again imply that $E_\tau \left[\overline{fp}_i^{\tau,e} \left(\overline{fp}_i^\tau - \overline{fp}_i^{\tau,e} \right) \right] = 0$ for all τ . In addition to covariance stationarity of forward premia, we now also require that the precision of investors' forecasts of future mean forward premia is invariant across investment periods. That is, the variance ratio $var_\tau \left(fp_{i,t} - \overline{fp}_i^\tau \right) / var_\tau \left(fp_{i,t} - \overline{fp}_i^{\tau,e} \right)$ is well-defined and constant for all τ . To save space we re-derive the main results of the paper only for the model corresponding to (12). The extension to the three-dimensional case is straight-forward.

Each marginal investor demands compensation for holding the static and forward premium trade portfolios as specified in (7) and (13), so that during the investment period beginning in τ ,

$$rx_{i,t+1} = \gamma + \beta^{stat} \left(\overline{fp}_i^{\tau,e} - \overline{fp}_i^{\tau,e} \right) + \beta^{fpp} \left(fp_{i,t} - \overline{fp}_i^{\tau,e} \right) + \bar{\epsilon}_i^{stat,\tau} + \epsilon_{i,t+1}^{fpp,\tau}, \quad t = \tau+1, \dots, \tau+T, \quad (35)$$

and $E_\tau \left[\bar{\epsilon}_i^{stat,\tau} \left(\overline{fp}_i^{\tau,e} - \overline{fp}_i^{\tau,e} \right) \right] = E_\tau \left[\epsilon_{i,t+1}^{fpp,\tau} \left(fp_{i,t} - \overline{fp}_i^{\tau,e} \right) \right] = 0$.

The proofs of Propositions 1 and 2 are as follows: For any given τ we have from (13),

$$\begin{aligned} \beta^{fpp} &= \frac{E_\tau \left[(rx_{i,t+1} - \bar{rx}_{i,\tau}) \left\{ fp_{i,t} - \overline{fp}_i^{\tau,e} \right\} \right]}{var_\tau \left(fp_{i,t} - \overline{fp}_i^{\tau,e} \right)}, \\ &= \frac{E_\tau \left[E_{it} \left[(rx_{i,t+1} - \bar{rx}_{i,\tau}) \right] \left\{ fp_{i,t} - \overline{fp}_i^{\tau,e} \right\} \right]}{var_\tau \left(fp_{i,t} - \overline{fp}_i^{\tau,e} \right)} \end{aligned}$$

for all $t = \tau+1, \dots, \tau+T$. Similarly, we have from (7),

$$\begin{aligned} \beta^{stat} &= \frac{E_\tau \left[(\bar{rx}_{i,\tau} - \bar{rx}_\tau) \left(\overline{fp}_i^{\tau,e} - \overline{fp}_\tau^e \right) \right]}{var_\tau \left(\overline{fp}_i^{\tau,e} - \overline{fp}_\tau^e \right)}, \\ &= \frac{E_\tau \left[E_{it} \left[\bar{rx}_{i,\tau} - \bar{rx}_\tau \right] \left(\overline{fp}_i^{\tau,e} - \overline{fp}_\tau^e \right) \right]}{var_\tau \left(\overline{fp}_i^{\tau,e} - \overline{fp}_\tau^e \right)} \end{aligned}$$

for all $t = \tau+1, \dots, \tau+T$.

We can then follow the usual steps from Proposition 2 to show that

$$\beta^{fpp} = \left(\frac{E_\tau \left[(rx_{i,t+1} - \bar{rx}_i^\tau) (fp_{i,t} - \bar{fp}_i^\tau) \right]}{\underbrace{\text{var}_\tau (fp_{i,t} - \bar{fp}_i^\tau)}_{=\beta_{FE}^{fpp,\tau}}} + \frac{E_\tau \left[(rx_{i,t+1} - \bar{rx}_i^\tau) (\bar{fp}_{i,\tau} - \bar{fp}_i^{e,\tau}) \right]}{\underbrace{\text{var}_\tau (fp_{i,t} - \bar{fp}_i^\tau)}_{=0}} \right) \frac{\text{var}_\tau (fp_{i,t} - \bar{fp}_i^\tau)}{\text{var}_\tau (fp_{i,t} - \bar{fp}_i^{e,\tau})}, t = \tau + 1, \dots, \tau + T$$

$$\beta^{fpp} = \beta_{FE}^{fpp,\tau} \frac{\text{var}_\tau (fp_{i,t} - \bar{fp}_i^\tau)}{\text{var}_\tau (fp_{i,t} - \bar{fp}_i^{e,\tau})}, t = \tau + 1, \dots, \tau + T.$$

which also demonstrates that $\beta_{FE}^{fpp} = \beta_{FE}^{fpp}$ for all τ , as the variance ratio on the right hand side is time-invariant by assumption. Similarly,

$$\beta^{stat} = \left(\frac{E_\tau \left[\bar{rx}_i^\tau ((\bar{fp}_i^\tau - \bar{fp}^\tau)) \right]}{\underbrace{\text{var}_\tau (\bar{fp}_i^\tau - \bar{fp}^\tau)}_{\beta_{FE}^{stat}}} + \frac{E_\tau \left[\bar{rx}_i^\tau (\bar{fp}_i^{e,\tau} - \bar{fp}^{e,\tau} - (\bar{fp}_i^\tau - \bar{fp}^\tau)) \right]}{\text{var}_\tau (\bar{fp}_i^\tau - \bar{fp}^\tau)} \right) \frac{\text{var}_\tau (\bar{fp}_i^\tau - \bar{fp}^\tau)}{\text{var}_\tau (\bar{fp}_i^{e,\tau} - \bar{fp}^{e,\tau})}, t = \tau + 1, \dots, \tau + T$$

which again also demonstrates that $\beta_{FE}^{stat} = \beta_{FE}^{stat}$ for all τ , due to covariance stationarity (all the variance ratios on the right hand side are time-invariant by assumption).

This result also implies that we can write the data generating process for realized returns without loss of generality as

$$rx_{i,t+1} = \gamma + \beta_{FE}^{stat} (\bar{fp}_i - \bar{fp}) + \beta_{FE}^{fpp} (fp_{i,t} - \bar{fp}_i) + \vartheta_{i,t+1}, \quad (36)$$

where $\vartheta_{i,t+1}$ is orthogonal to $(\bar{fp}_i - \bar{fp})$, $(fp_{i,t} - \bar{fp}_i)$, and $(fp_{i,t} - \bar{fp}_i^{e,\tau})$. Moreover, the coefficients β_{FE}^{stat} and β_{FE}^{fpp} are stable over time as shown above. This implies that for each investment period beginning at $t = \tau$ we can also write

$$rx_{i,t+1} = \gamma + \beta_{FE}^{stat} (\bar{fp}_i^\tau - \bar{fp}^\tau) + \beta_{FE}^{fpp} (fp_{i,t} - \bar{fp}_i^\tau) + \zeta_{i,t+1}^\tau, \quad t = \tau + 1, \dots, \tau + T, \quad (37)$$

where $\zeta_{i,t+1}^\tau = (\beta_{FE}^{stat} - \beta_{FE}^{fpp}) (\bar{fp}_i - \bar{fp}_i^\tau) + \beta_{FE}^{stat} (\bar{fp}^\tau - \bar{fp}) + \vartheta_{i,t+1}$.

It follows directly that for a given investment period beginning at $t = \tau$:

$$\bar{rx}_i^\tau = \gamma + \beta_{FE}^{stat} (\bar{fp}_i^\tau - \bar{fp}^\tau) + \bar{\zeta}_i^\tau. \quad (38)$$

We can then write variation of returns in the time series dimension for any investment period τ as

$$rx_{i,t+1} - \bar{rx}_i^\tau = \beta_{FE}^{fpp} (fp_{i,t} - \bar{fp}_{i,\tau}) + \zeta_{i,t+1}^\tau - \bar{\zeta}_i^\tau.$$

Adding and subtracting terms yields the structure of the error term, $\epsilon_{i,t+1}^{fpp,\tau}$, in (13)

$$\begin{aligned} rx_{i,t+1} - \bar{r}\bar{x}_i^\tau &= \beta^{fpp} \left(fp_{i,t} - \bar{fp}_i^{e,\tau} \right) \\ &\quad + \underbrace{\left(\beta_{FE}^{fpp} - \beta^{fpp} \right) \left(fp_{i,t} - \bar{fp}_i^\tau \right) - \beta^{fpp} \left(\bar{fp}_{i,t} - \bar{fp}_i^{e,\tau} \right)}_{\epsilon_{i,t+1}^{fpp,\tau}} + \zeta_{i,t+1}^\tau - \bar{\zeta}_i^\tau, \end{aligned}$$

where $\zeta_{i,t+1}^\tau - \bar{\zeta}_i^\tau = \vartheta_{i,t+1} - \bar{\vartheta}_i^\tau$.

To verify that, $E_\tau \left[\epsilon_{i,t+1}^{fpp,\tau} \left(fp_{i,t} - \bar{fp}_i^{e,\tau} \right) \right] = 0$, recall that $E_\tau \left[\left(fp_{i,t} - \bar{fp}_i^{e,\tau} \right) \left(\vartheta_{i,t+1} - \bar{\vartheta}_i^\tau \right) \right] = 0$. Plugging in the remainder of the the error term from the expression above and re-arranging yields

$$E_\tau \left[\left(\beta_{FE}^{fpp} - \beta^{fpp} \right) \left(fp_{i,t} - \bar{fp}_i^\tau \right) \left(fp_{i,t} - \bar{fp}_i^{e,\tau} \right) \right] = E_\tau \left[\beta^{fpp} \left(\bar{fp}_i - \bar{fp}_i^{e,\tau} \right)^2 \right]$$

Adding and subtracting \bar{fp}_i^τ in the last round bracket on the left hand side of the equation yields

$$\begin{aligned} &\left(\beta_{FE}^{fpp} - \beta^{fpp} \right) E_\tau \left[\left(fp_{i,t} - \bar{fp}_i^\tau \right)^2 \right] + \underbrace{\left(\beta_{FE}^{fpp} - \beta^{fpp} \right) E_\tau \left[\left(\bar{fp}_{i,\tau} - \bar{fp}_i^{e,\tau} \right) \left(fp_{i,t} - \bar{fp}_i^\tau \right) \right]}_{=0} \\ &= \beta^{fpp} E_\tau \left[\left(\bar{fp}_{i,\tau} - \bar{fp}_i^{e,\tau} \right)^2 \right] \end{aligned}$$

Re-arranging these expressions yields

$$\beta_{FE}^{fpp} = \beta^{fpp} \left(1 + \frac{E_\tau \left[\left(\bar{fp}_i^\tau - \bar{fp}_i^{e,\tau} \right)^2 \right]}{E_\tau \left[\left(fp_{i,t} - \bar{fp}_i^\tau \right)^2 \right]} \right)$$

which is true, as shown in (25).

The proof showing that $E_0 \left[\epsilon_i^{stat} \left(\bar{fp}_i^e - \bar{fp}^e \right) \right] = 0$ is analogous to the one shown in Appendix C.2.

C.4 Choice of Standard Errors

Standard errors for estimates of β^{stat} are clustered by currency because the panel is composed of repeated values in the time-series dimension. Similarly, standard errors for estimates of β^{dol} are clustered by time.

Standard errors for estimates of β^{dyn} are corrected for both heteroscedasticity and serial correlation using a Newey-West adjustment (Bartlett kernel) with a 12-month lag. For horizons

larger than one month, we must additionally take into account the fact that returns overlap. Therefore, for the 6- and 12-month horizons, the standard errors of estimates of β^{dyn} are corrected for serial correlation at 12- and 24-month lags. Throughout, we calculate standard errors for β_i^{dyn} , β_i^{fpp} , and β^{fpp} in the same way as those for β^{dyn} . In the case of β^{dol} we use a Newey-West adjustment in addition to clustering by time using the [Driscoll and Kraay \(1998\)](#) procedure at the 6- and 12-month horizons.

Finally, an additional adjustment to the standard errors for estimates of β^{stat} , β^{dyn} , and β^{fpp} is made following [Murphy and Topel \(1985\)](#) to account for the fact that the inputs $\{\overline{fp}_i^e\}$ are estimated in the pre-sample.

In addition, to check the robustness of our results to sample variance in the estimation of average forward premia, we bootstrap standard errors across blocks of rebalances in [Table 6](#). We choose the 12 Rebalance sample as our population and run our regressions on bootstrapped draws with replacement from those original 12 blocks of data. Standard errors presented are for 100,000 draws.

C.5 Details on the coefficients β^{ct} and β^{fpp}

Equation (13) defines $\beta^{fpp} = \frac{E_0[(rx_{i,t+1} - \overline{rx}_i)(fp_{it} - \overline{fp}_i^e)]}{var(fp_{it} - \overline{fp}_i^e)}$. Multiply by $var(fp_{it} - \overline{fp}_i^e)$, add and subtract $(\overline{fp}_t - \overline{fp}^e)$ from the term that multiplies $(rx_{i,t+1} - \overline{rx}_i)$ inside the expectation, and reorganize to get

$$\beta^{fpp} var(fp_{it} - \overline{fp}_i^e) = E_0[(rx_{i,t+1} - \overline{rx}_i)(fp_{it} - \overline{fp}_i^e - (\overline{fp}_t - \overline{fp}^e))] + E_0[(rx_{i,t+1} - \overline{rx}_i)(\overline{fp}_t - \overline{fp}^e)].$$

Adding and subtracting $\overline{rx}_{t+1} - \overline{rx}$ to the returns term in the first expectation above,

$$\begin{aligned} \beta^{fpp} var(fp_{it} - \overline{fp}_i^e) &= E_0[(rx_{i,t+1} - \overline{rx}_i - (\overline{rx}_{t+1} - \overline{rx})) (fp_{it} - \overline{fp}_i^e - (\overline{fp}_t - \overline{fp}^e))] + \\ &+ E_0[(\overline{rx}_{t+1} - \overline{rx})(fp_{it} - \overline{fp}_i^e - (\overline{fp}_t - \overline{fp}^e))] + E_0[(rx_{i,t+1} - \overline{rx}_i)(\overline{fp}_t - \overline{fp}^e)]. \end{aligned}$$

Note that the first term equals $\beta^{dyn} var(fp_{it} - \overline{fp}_i^e - (\overline{fp}_t - \overline{fp}^e))$, as defined in equation (9). Gathering terms yields

$$\begin{aligned} \beta^{fpp} var(fp_{it} - \overline{fp}_i^e) &= \beta^{dyn} var(fp_{it} - \overline{fp}_i^e - (\overline{fp}_t - \overline{fp}^e)) + \\ &E_0[(\overline{rx}_{t+1} - \overline{rx})(fp_{it} - \overline{fp}_i^e)] + E_0[(rx_{i,t+1} - \overline{rx}_i - (\overline{rx}_{t+1} - \overline{rx}))(\overline{fp}_t - \overline{fp}^e)]. \end{aligned}$$

The last term is equal to zero since $(\overline{fp}_t - \overline{fp}^e)$ do not vary across i , and $\sum_i (rx_{i,t+1} - \overline{rx}_i) / N = \overline{rx}_{t+1} - \overline{rx}$. Additionally, the second term simplifies to $E_0[(\overline{rx}_{t+1} - \overline{rx})(\overline{fp}_t - \overline{fp}^e)]$, because

$(\overline{rx}_{t+1} - \overline{rx})$ do not vary across i . Using the definition of β^{dol} from equation (10),

$$\beta^{fpp} \text{var} \left(fp_{it} - \overline{fp}_i^e \right) = \beta^{dyn} \text{var} \left(fp_{it} - \overline{fp}_i^e - \left(\overline{fp}_t - \overline{fp}^e \right) \right) + \beta^{dol} \text{var} \left(\overline{fp}_t - \overline{fp}^e \right).$$

Finally, because

$$\begin{aligned} \text{var} \left(fp_{it} - \overline{fp}_i^e \right) &= \text{var} \left(fp_{it} - \overline{fp}_t + \overline{fp}^e - \overline{fp}_i^e + \overline{fp}_t - \overline{fp}^e \right) \\ &= \text{var} \left(fp_{it} - \overline{fp}_t + \overline{fp}^e - \overline{fp}_i^e \right) + \text{var} \left(\overline{fp}_t - \overline{fp}^e \right) \\ &\quad + \underbrace{2\text{cov} \left(fp_{it} - \overline{fp}_t + \overline{fp}^e - \overline{fp}_i^e, \overline{fp}_t - \overline{fp}^e \right)}_{=0}, \end{aligned}$$

one arrives at

$$\begin{aligned} \beta^{fpp} &= \frac{\text{var} \left(fp_{it} - \overline{fp}_i^e - \left(\overline{fp}_t - \overline{fp}^e \right) \right)}{\text{var} \left(fp_{it} - \overline{fp}_t + \overline{fp}^e - \overline{fp}_i^e \right) + \text{var} \left(\overline{fp}_t - \overline{fp}^e \right)} \beta^{dyn} \\ &\quad + \frac{\text{var} \left(\overline{fp}_t - \overline{fp}^e \right)}{\text{var} \left(fp_{it} - \overline{fp}_t + \overline{fp}^e - \overline{fp}_i^e \right) + \text{var} \left(\overline{fp}_t - \overline{fp}^e \right)} \beta^{dol}, \end{aligned}$$

demonstrating that β^{fpp} is a linear combination of β^{dyn} and β^{dol} .

Take the definition of β^{ct} as in equation (14):

$$\beta^{ct} = E_0 \left[(rx_{i,t+1} - \overline{rx}_{t+1}) (fp_{it} - \overline{fp}_t) \right] \left[\text{var} (fp_{it} - \overline{fp}_t) \right]^{-1}.$$

Add and subtract $(\overline{rx}_i - \overline{rx})$ in the expectation term:

$$E_0 \left[(rx_{i,t+1} - \overline{rx}_{t+1}) (fp_{it} - \overline{fp}_t) \right] = E_0 \left[(rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx})) (fp_{it} - \overline{fp}_t) + (\overline{rx}_i - \overline{rx}) (fp_{it} - \overline{fp}_t) \right].$$

Note that $E_0 \left[(\overline{rx}_i - \overline{rx}) (fp_{it} - \overline{fp}_t) \right] = \beta_{FE}^{stat} \text{var} (\overline{fp}_i - \overline{fp})$ as defined in C.7. Moreover, from (17), we have that

$$\beta_{FE}^{stat} \text{var} (\overline{fp}_i - \overline{fp}) = \beta^{stat} \text{var} \left(\overline{fp}_i^e - \overline{fp}^e \right) + E_0 \left[(\overline{rx}_i - \overline{rx}) \left(\overline{fp}_i^e - \overline{fp}^e - (\overline{fp}_i - \overline{fp}) \right) \right],$$

which means

$$\begin{aligned} E_0 \left[(rx_{i,t+1} - \overline{rx}_{t+1}) (fp_{it} - \overline{fp}_t) \right] &= E_0 \left[(rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx})) (fp_{it} - \overline{fp}_t) \right] + \\ &\quad \beta^{stat} \text{var} \left(\overline{fp}_i^e - \overline{fp}^e \right) + E_0 \left[(\overline{rx}_i - \overline{rx}) \left(\overline{fp}_i^e - \overline{fp}^e - (\overline{fp}_i - \overline{fp}) \right) \right]. \end{aligned}$$

Add and subtract $\left(\overline{fp}_i^e - \overline{fp}^e\right)$ from the forward premia to get

$$E_0 \left[(rx_{i,t+1} - \overline{rx}_{t+1}) (fp_{it} - \overline{fp}_t) \right] = E_0 \left[(rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx})) (fp_{it} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e) + (\overline{fp}_i^e - \overline{fp}^e)) \right] \\ + \beta^{stat} var \left(\overline{fp}_i^e - \overline{fp}^e \right) + E_0 \left[(\overline{rx}_i - \overline{rx}) \left(\overline{fp}_i^e - \overline{fp}^e - (\overline{fp}_i - \overline{fp}) \right) \right].$$

From equation (9) we know that β^{dyn} is such that

$$E_0 \left[(rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx})) (fp_{it} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e)) \right] = \beta^{dyn} var \left(fp_{it} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e) \right),$$

which means

$$E_0 \left[(rx_{i,t+1} - \overline{rx}_{t+1}) (fp_{it} - \overline{fp}_t) \right] = E_0 \left[(rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx})) (\overline{fp}_i^e - \overline{fp}^e) \right] \\ + \beta^{dyn} var \left(fp_{it} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e) \right) + \beta^{stat} var \left(\overline{fp}_i^e - \overline{fp}^e \right) \\ + E_0 \left[(\overline{rx}_i - \overline{rx}) \left(\overline{fp}_i^e - \overline{fp}^e - (\overline{fp}_i - \overline{fp}) \right) \right].$$

Note that $E_0 \left[(rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx})) (\overline{fp}_i^e - \overline{fp}^e) \right] = 0$ and let

$\alpha^{dyn} = E_0 \left[(\overline{rx}_i - \overline{rx}) \left(\overline{fp}_i^e - \overline{fp}^e - (\overline{fp}_i - \overline{fp}) \right) \right]$. Collect terms to get

$$\beta^{ct} = \frac{\alpha^{dyn}}{var \left(fp_{it} - \overline{fp}_t \right)} + \beta^{dyn} \frac{var \left(fp_{it} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e) \right)}{var \left(fp_{it} - \overline{fp}_t \right)} + \beta^{stat} \frac{var \left(\overline{fp}_i^e - \overline{fp}^e \right)}{var \left(fp_{it} - \overline{fp}_t \right)}.$$

C.6 Possibility of Stambaugh Bias

Because forward premia are persistent, it is possible that our estimates may be affected by Stambaugh bias. Consider the following autoregressive system for each currency i :

$$rx_{i,t+1} = \alpha + \beta_i^{fpp} fp_{i,t} + u_{i,t+1} \quad (39)$$

$$fp_{i,t+1} = \gamma + \phi fp_{i,t} + v_{i,t+1}, \quad (40)$$

where the error terms u_{t+1} and v_{t+1} are normally-distributed with mean zero, and a potentially currency-specific covariance matrix Σ such that:

$$\Sigma = \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \cdot & \sigma_v^2 \end{bmatrix}.$$

Stambaugh (1999) shows that in this environment,

$$E[\hat{\beta}_i^{fpp} - \beta_i^{fpp}] = \frac{\sigma_{uv}}{\sigma_v^2} E[\hat{\phi} - \phi] \quad (41)$$

for each currency i .

One can show that, for a given precision of investors' expectations of future mean forward premia (\overline{fp}_i), the overall elasticity of risk-premia with respect to forward premia, β^{fpp} , is a linear function of these country-specific regressions coefficients β_i^{fpp} (see Proposition 2 and equation (25) in the main text).

Appendix Table 10 uses data from our 1 Rebalance sample and the formulas above to estimate $\frac{\sigma_{uv}}{\sigma_v^2}$ and the size of the bias for each individual currency under the (possibly extreme) assumption that $\phi = 0.99$ (see Panel A). The Table shows two main results: First, the estimated biases for some currencies are positive and negative for others, so that the aggregation of all of these biases has a negligible effect on our overall estimate of β^{fpp} (shown in Panel B). Taking the bounds for all of the estimated currency-specific Stambaugh biases at face value implies that our estimates of β^{fpp} should be only 0.01, lower than its true value (corresponding to 1.1% of the size of the estimated coefficient in our main specification). Similarly, conducting the Stambaugh calculation directly for the mean return across currencies (\overline{rx}_{t+1}) and the mean forward premium (\overline{fp}_t), yields an estimated bias in the estimate of β^{dol} of 0.07 (or 2.1% of the size of the estimated coefficient). Second, consistent with the existing literature (Moon and Velasco, 2017), we also find that the estimated Stambaugh bias for the currency-specific coefficients (β_i^{fpp}) is also modest for most currencies. Possible exceptions are the the New Zealand dollar and the Singapore dollar, where the bias may reach up to -27% and +19% of the point estimate, respectively.

C.7 Detailed proof of Proposition 2

From (9) we have that

$$\beta^{dyn} = \frac{E_0 \left[E_{it} (rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx})) \left\{ fp_{it} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e) \right\} \right]}{\text{var} \left(fp_{it} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e) \right)}.$$

Taking iterated expectations, adding and subtracting $(\overline{fp}_i - \overline{fp})$ in the curly brackets, and multiplying and dividing with $\text{var} (fp_{it} - \overline{fp}_t - (\overline{fp}_i - \overline{fp}))$ yields

$$\beta^{dyn} = \left(\beta_{FE}^{dyn} + \frac{E_0 \left((rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx})) \left[(\overline{fp}_i - \overline{fp}) - (\overline{fp}_i^e - \overline{fp}^e) \right] \right)}{\text{var} (fp_{it} - \overline{fp}_t - (\overline{fp}_i - \overline{fp}))} \right) \frac{\text{var} (fp_{it} - \overline{fp}_t - (\overline{fp}_i - \overline{fp}))}{\text{var} (fp_{it} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e))},$$

where $\beta_{FE}^{dyn} = \frac{E_0((rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx})) [fp_{it} - \overline{fp}_t - (\overline{fp}_i - \overline{fp})])}{\text{var}(fp_{it} - \overline{fp}_t - (\overline{fp}_i - \overline{fp}))}$ is the slope coefficient from the fixed effects specification $rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx}) = \beta_{FE}^{dyn} ((fp_{it} - \overline{fp}_t) - (\overline{fp}_i - \overline{fp})) + \epsilon_{i,t+1}$.

Now note that the second term in the round brackets is equal to zero and write

$$\beta^{dyn} = \beta_{FE}^{dyn} \frac{\text{var} \left(fp_{it} - \overline{fp}_t - (\overline{fp}_i - \overline{fp}) \right)}{\text{var} \left(fp_{it} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e) \right)}. \quad (42)$$

Finally, replace

$$\begin{aligned} \text{var} \left(fp_{it} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e) \right) &= \text{var} \left(fp_{it} - \overline{fp}_t - (\overline{fp}_i - \overline{fp}) + (\overline{fp}_i - \overline{fp}) - (\overline{fp}_i^e - \overline{fp}^e) \right) \\ &= \text{var} \left(fp_{it} - \overline{fp}_t - (\overline{fp}_i - \overline{fp}) \right) + \text{var} \left(\overline{fp}_i - \overline{fp}^e \right) \end{aligned}$$

and cancel terms to get (15).

From (13) we have that

$$\beta^{fpp} = E_0 \left[E_{it} (rx_{i,t+1} - \overline{rx}_i) \left\{ fp_{it} - \overline{fp}_i^e \right\} \right] \text{var} \left(fp_{it} - \overline{fp}_i^e \right)^{-1}.$$

Taking iterated expectations, adding and subtracting \overline{fp}_i in the curly brackets, and multiplying and dividing with $\text{var} \left(fp_{it} - \overline{fp}_i \right)$ yields

$$\beta^{fpp} = \left(\beta_{FE}^{fpp} + \frac{E_0 \left((rx_{i,t+1} - \overline{rx}_i) \left[\overline{fp}_i - \overline{fp}_i^e \right] \right)}{\text{var} \left(fp_{it} - \overline{fp}_i \right)} \right) \frac{\text{var} \left(fp_{it} - \overline{fp}_i \right)}{\text{var} \left(fp_{it} - \overline{fp}_i^e \right)},$$

where $\beta_{FE}^{fpp} = E_0 \left((rx_{i,t+1} - \overline{rx}_i) \left[fp_{it} - \overline{fp}_i \right] \right) \text{var} \left(fp_{it} - \overline{fp}_i \right)^{-1}$ is the slope coefficient from the fixed effects specification $rx_{i,t+1} - \overline{rx}_i = \beta_{FE}^{fpp} \left(fp_{it} - \overline{fp}_i \right) + \epsilon_{i,t+1}^{fpp}$. The second term in the round brackets is equal to zero and so

$$\beta^{fpp} = \beta_{FE}^{fpp} \frac{\text{var} \left(fp_{it} - \overline{fp}_i \right)}{\text{var} \left(fp_{it} - \overline{fp}_i^e \right)},$$

which leads to equation (16).

From (7) we have that

$$\beta^{stat} = E_0 \left[E_{it} \left((\overline{rx}_i - \overline{rx}) \right) \left\{ \overline{fp}_i^e - \overline{fp}^e \right\} \right] \text{var} \left(\overline{fp}_i^e - \overline{fp}^e \right)^{-1}$$

Taking iterated expectations, adding and subtracting $(\overline{fp}_i - \overline{fp})$ in the curly brackets, and

multiplying and dividing with $\text{var}(\overline{fp}_i - \overline{fp})$ yields:

$$\beta^{stat} = \left(\beta_{FE}^{stat} + \frac{E_0 \left((\overline{rx}_i - \overline{rx}) \left[(\overline{fp}_i^e - \overline{fp}^e) - (\overline{fp}_i - \overline{fp}) \right] \right)}{\text{var}(\overline{fp}_i - \overline{fp})} \right) \frac{\text{var}(\overline{fp}_i - \overline{fp})}{\text{var}(\overline{fp}_i^e - \overline{fp}^e)},$$

where $\beta_{FE}^{stat} = E_0 [(\overline{rx}_i - \overline{rx}) \{ \overline{fp}_i - \overline{fp} \}] \text{var}(\overline{fp}_i - \overline{fp})^{-1}$. Because \overline{fp} and \overline{fp}^e are constants, one can disregard them when measuring $\text{var}(\cdot)$, yielding (17).

C.8 Proof of Corollary 1

Consider (13):

$$rx_{i,t+1} - \overline{rx}_i = \beta^{fpp} (fp_{it} - \overline{fp}_i^e) + \epsilon_{i,t+1}^{fpp}.$$

Under the assumption that we observe \overline{fp}_i^e without error (A1) we have

$$E[\hat{\beta}^{fpp}] = \beta^{fpp},$$

where

$$\hat{\beta}^{fpp} = \frac{\sum_{i,t} (rx_{i,t+1} - \overline{rx}_i) (fp_{it} - \overline{fp}_i^e)}{\sum_{i,t} (fp_{it} - \overline{fp}_i^e)^2}.$$

However, in reality we may observe \overline{fp}_i^e with error and instead run the regression

$$rx_{i,t+1} - \overline{rx}_i = \tilde{\beta}^{fpp} (fp_{it} - \widehat{\overline{fp}_i^e}) + \nu_{i,t+1}^{fpp},$$

where \widehat{fp}_i^e is our estimate of investors' expectation of \overline{fp}_i . The OLS estimate of $\tilde{\beta}^{fpp}$ is then given as

$$\begin{aligned}
& \sum_{i,t} \left[(rx_{i,t+1} - \overline{rx}_i) \{ fp_{it} - \widehat{fp}_i^e \} \right] \left(\sum_{i,t} (fp_{it} - \widehat{fp}_i^e)^2 \right)^{-1} \\
&= \left[\sum_{i,t} (rx_{i,t+1} - \overline{rx}_i) (fp_{it} - \overline{fp}_i^e) + \underbrace{\sum_{i,t} (rx_{i,t+1} - \overline{rx}_i) (\overline{fp}_i^e - \widehat{fp}_i^e)}_{=0} \right] \left(\sum_{i,t} (fp_{it} - \widehat{fp}_i^e)^2 \right)^{-1} \\
&= \frac{\sum_{i,t} (rx_{i,t+1} - \overline{rx}_i) (fp_{it} - \overline{fp}_i^e) \sum_{i,t} (fp_{it} - \overline{fp}_i^e)^2}{\sum_{i,t} (fp_{it} - \overline{fp}_i^e)^2 \sum_{i,t} (fp_{it} - \widehat{fp}_i^e)^2} \\
&= \hat{\beta}^{fpp} \frac{\sum_{i,t} (fp_{it} - \overline{fp}_i^e)^2}{\sum_{i,t} (fp_{it} - \widehat{fp}_i^e)^2}
\end{aligned}$$

where $\hat{\beta}^{fpp}$ is the OLS estimate we would have obtained if $\widehat{fp}_i^e = \overline{fp}_i^e$.

Finally, under assumption (A2) and using that $\epsilon_{i,t+1}^{fpp}$ is asymptotically orthogonal to $(fp_{it} - \widehat{fp}_i^e)$:

$$\begin{aligned}
& \text{plim}_{N \rightarrow \infty} \left(\frac{\sum_{i,t} (fp_{it} - \overline{fp}_i^e)^2}{\sum_{i,t} (fp_{it} - \widehat{fp}_i^e)^2} \hat{\beta}^{fpp} \right) = \left(\frac{\text{plim}_{N \rightarrow \infty} \sum_{i,t} (fp_{it} - \overline{fp}_i^e)^2}{\text{plim}_{N \rightarrow \infty} \sum_{i,t} (fp_{it} - \widehat{fp}_i^e)^2} \text{plim}_{N \rightarrow \infty} \hat{\beta}^{fpp} \right) \\
& \frac{\text{plim}_{N \rightarrow \infty} \sum_{i,t} \left((fp_{it} - \overline{fp}_i^e)^2 + (\overline{fp}_i^e - \widehat{fp}_i^e)^2 \right)}{\text{plim}_{N \rightarrow \infty} \sum_{i,t} \left((fp_{it} - \overline{fp}_i^e)^2 + (\overline{fp}_i^e - \widehat{fp}_i^e)^2 \right)} \beta^{fpp} = \frac{\text{plim}_{N \rightarrow \infty} \left(\sum_{i,t} (fp_{it} - \overline{fp}_i^e)^2 \right) + \text{var}(\overline{fp}_i^e - \widehat{fp}_i^e)}{\text{plim}_{N \rightarrow \infty} \left(\sum_{i,t} (fp_{it} - \overline{fp}_i^e)^2 \right) + \text{var}(\overline{fp}_i^e - \widehat{fp}_i^e)} \beta^{fpp} \\
& = \beta^{fpp},
\end{aligned}$$

concluding the proof.

C.9 Deriving the ‘‘Fama Conditions’’

We have already seen that the slope coefficient in (13) can be written as

$$\beta^{fpp} = \frac{E_0 \left[(rx_{i,t+1} - \overline{rx}_i) \{ fp_{it} - E_{i0}[\overline{fp}_i] \} \right]}{\text{var}(fp_{i,t} - E_{i0}[\overline{fp}_i])}, \quad (43)$$

The next step is to observe that, without loss of generality,

$$fp_{it} = E_{it}[rx_{i,t+1}] + E_{it}[\Delta s_{i,t+1}],$$

and, as a result,

$$E_{i0} [\overline{fp}_i] = E_{i0} [\overline{rx}_i] + E_{i0} [\overline{\Delta s}_i].$$

Substituting these two expressions in the curly brackets on the right hand side of (43) yields

$$\beta^{fpp} = \frac{E_0 [(rx_{i,t+1} - \overline{rx}_i) \{ (E_{it} [rx_{i,t+1}] + E_{it} [\Delta s_{i,t+1}]) - (E_{i0} [\overline{rx}_i] + E_{i0} [\overline{\Delta s}_i]) \}]]}{var ((E_{it} [rx_{i,t+1}] + E_{it} [\Delta s_{i,t+1}]) - (E_{i0} [\overline{rx}_i] + E_{i0} [\overline{\Delta s}_i]))}.$$

To draw any conclusions about the properties of expected returns (or currency risk premia) we must now take expectations conditional on i and t in the square brackets and re-arrange to get

$$\beta^{fpp} = \frac{E_0 [(E_{it} [rx_{i,t+1}] - E_{i0} [\overline{rx}_i]) (E_{it} [rx_{i,t+1}] - E_{i0} [\overline{rx}_i]) + E_0 [(E_{it} [rx_{i,t+1}] - E_{i0} [\overline{rx}_i]) (E_{it} [\Delta s_{i,t+1}] - E_{i0} [\overline{\Delta s}_i])]]}{var ((E_{it} [rx_{i,t+1}] - E_{i0} [\overline{rx}_i]) + (E_{it} [\Delta s_{i,t+1}] - E_{i0} [\overline{\Delta s}_i]))}.$$

Note however, that the next step, transforming the first term into a variance, and the second term into a covariance requires an additional assumption that is made implicitly in Fama (1984) – that investors do not update their expectations of the mean returns on a given currency during the investment period

$$E_{it} [\overline{rx}_i] = E_{i0} [\overline{rx}_i]. \quad (\text{F1})$$

Once we impose this condition, we can write

$$\beta^{fpp} = \frac{var (E_{it} [rx_{i,t+1}] - E_{i0} [\overline{rx}_i]) + cov (E_{it} [rx_{i,t+1}] - E_{i0} [\overline{rx}_i], E_{it} [\Delta s_{i,t+1}] - E_{i0} [\overline{\Delta s}_i])}{var (E_{it} [rx_{i,t+1}] - E_{i0} [\overline{rx}_i]) + var (E_{it} [\Delta s_{i,t+1}] - E_{i0} [\overline{\Delta s}_i]) + 2cov (E_{it} [rx_{i,t+1}] - E_{i0} [\overline{rx}_i], E_{it} [\Delta s_{i,t+1}] - E_{i0} [\overline{\Delta s}_i])}.$$

which is the basis both for our and for Fama's claim that

$$cov (E_{it} [rx_{i,t+1}] - E_{i0} [\overline{rx}_i], E_{it} [\Delta s_{i,t+1}] - E_{i0} [\overline{\Delta s}_i]) < 0 \text{ if } \beta^{fpp} > 1,$$

where Fama (1984), in addition, imposes

$$E_{i0} [\overline{fp}_i] = \overline{fp}_i. \quad (\text{F2})$$

so that, in his application, $\beta^{fpp} = \beta_{FE}^{fpp}$.³⁸

The derivations for β^{stat} , β^{dyn} , β^{dol} , and β^{ct} are analogous.

³⁸While one can argue that (F2) is irrelevant when attempting to estimate the elasticity of risk premia with respect to forward premia for only one currency pair ($N = 1$) rather than a "representative" currency (β_i^{fpp} rather than β^{fpp} in our notation), (F1) is needed regardless of which of the two objects we consider.

C.10 Details on χ^2 difference tests

This section gives analytical details for the construction of the χ^2 difference test statistics used to calculate the p-values in Table 5. These are distance metric statistics (Newey and West, 1987) where we focus solely on variance coming from estimation uncertainty about β^{dol} and β^{dyn} .

The objective of these tests is to determine which aspect of behavior can best explain the expected returns on the forward premium trade, given that the hypothesis tests in Table 3 were inconclusive, in the sense that neither β^{dol} nor β^{dyn} are statistically distinguishable from zero. However, for the forward premium trade to arise, at least one of these parameters must be non-zero. To this end, we take as given the returns on the trading strategy, as well as the parts explained by the constants (expectational errors), and ask if our ability to explain the returns to the trading strategy significantly changes under the null that $\beta^{dyn} = 0$, $\beta^{dol} = 0$ or $\beta^{dol} = \beta^{dyn}$.

For the hypothesis that $\beta^{dyn} = 0$, we calculate

$$X^r = \frac{\left(\sum_{it} rx_{it} (fp_{it} - \bar{fp}_i^e) - \hat{\alpha}_{dyn} - \hat{\alpha}_{dol} - \hat{\beta}^{dol} \sum_{it} \left[(\bar{fp}_t - \bar{fp}^e) - (\bar{fp} - \bar{fp}^e) \right] \right)^2}{\widehat{Var} \left(\hat{\beta}^{dol} \sum_{it} \left[(\bar{fp}_t - \bar{fp}^e) - (\bar{fp} - \bar{fp}^e) \right] \right)^2},$$

where the denominator is the variance of the returns to the forward premium trade under the null. Similarly, for $\beta^{dol} = 0$,

$$X^r = \frac{\left(\sum_{it} rx_{it} (fp_{it} - \bar{fp}_i^e) - \hat{\alpha}_{dyn} - \hat{\alpha}_{dol} - \hat{\beta}^{dyn} \sum_{it} \left[(fp_{it} - \bar{fp}_t) - (\bar{fp}_i^e - \bar{fp}^e) \right] \right)^2}{\widehat{Var} \left(\hat{\beta}^{dyn} \sum_{it} \left[(fp_{it} - \bar{fp}_t) - (\bar{fp}_i^e - \bar{fp}^e) \right] \right)^2},$$

and for $\beta^{dol} = \beta^{dyn}$, we estimate the restricted common coefficient $\hat{\beta}_r$ such that:

$$X^r = \frac{\left(\sum_{it} rx_{it} (fp_{it} - \bar{fp}_i^e) - \hat{\alpha}_{dyn} - \hat{\alpha}_{dol} - \hat{\beta}_r \sum_{it} \left[(fp_{it} - \bar{fp}_t) - (\bar{fp}_i^e - \bar{fp}^e) \right] - \hat{\beta}_r \sum_{it} \left[(\bar{fp}_t - \bar{fp}^e) - (\bar{fp} - \bar{fp}^e) \right] \right)^2}{\widehat{Var} \left(\hat{\beta}_r \sum_{it} \left[(fp_{it} - \bar{fp}_t) - (\bar{fp}_i^e - \bar{fp}^e) \right] + \hat{\beta}_r \sum_{it} \left[(\bar{fp}_t - \bar{fp}^e) - (\bar{fp} - \bar{fp}^e) \right] \right)^2},$$

where in each case

$$X^r - X^u \sim \chi_1.$$

with

$$X^u = \frac{\left(\sum_{it} rx_{it} (fp_{it} - \bar{fp}_i^e) - \hat{\alpha}_{dyn} - \hat{\alpha}_{dol} - \hat{\beta}^{dyn} \sum_{it} \left[(fp_{it} - \bar{fp}_t) - (\bar{fp}_i^e - \bar{fp}^e) \right] - \hat{\beta}^{dol} \sum_{it} \left[(\bar{fp}_t - \bar{fp}^e) - (\bar{fp} - \bar{fp}^e) \right] \right)^2}{\widehat{Var} \left(\hat{\beta}^{dyn} \sum_{it} \left[(fp_{it} - \bar{fp}_t) - (\bar{fp}_i^e - \bar{fp}^e) \right] + \hat{\beta}^{dol} \sum_{it} \left[(\bar{fp}_t - \bar{fp}^e) - (\bar{fp} - \bar{fp}^e) \right] \right)^2}.$$

C.11 Generalized No-arbitrage Model

The no-arbitrage model in [Lustig et al. \(2014\)](#) is more general, allowing δ_h to differ from the mean exposure across countries and allowing for time-variation in heterogeneous exposures to an additional global shock. The SDF is

$$-m_{i,t+1} = \alpha + \chi z_{it} + \sqrt{\gamma z_{it}} u_{i,t+1} + \tau z_t^w - \sqrt{\delta_i z_t^w} u_{w,t+1} + \sqrt{\kappa z_{it}} u_{t+1}^g,$$

where u_{t+1}^g again follows a standard normal distribution. These generalizations do not substantially change our conclusions. Forward premia and expected returns are

$$fp_{it} = \frac{1}{2} (z_t^w (\delta_h - \delta_i) + (\gamma + \kappa - 2\chi) (z_{ht} - z_{it}))$$

and

$$E_t [rx_{i,t+1}] = \frac{1}{2} (z_t^w (\delta_h - \delta_i) + (\gamma + \kappa) (z_{ht} - z_{it})).$$

Performing our decomposition yields

$$\beta^{stat} = 1, \quad \beta^{dyn} = \frac{(\gamma + \kappa)(\gamma + \kappa - 2\chi) + \sigma_\delta^2}{(\gamma + \kappa - 2\chi)^2 + \sigma_\delta^2}, \quad \text{and} \quad \beta^{dol} = \frac{-2\delta_h \bar{\delta} + \bar{\delta}^2 + (\gamma + \kappa)(\gamma + \kappa - 2\chi) + \delta_h^2}{-2\delta_h \bar{\delta} + \bar{\delta}^2 + (\gamma + \kappa - 2\chi)^2 + \delta_h^2}.$$

C.12 Derivation of (23)

In Section [C.1](#), we derived equation (8):

$$\begin{aligned} & \sum_{i,t} [(rx_{i,t+1} - \bar{r}\bar{x}) (fp_{it} - \bar{f}p)] \\ & = \\ & \underbrace{\hat{\beta}^{stat} \sum_{i,t} (\bar{f}p_i^e - \bar{f}p^e)^2}_{\text{Static Trade}} + \underbrace{\hat{\beta}^{dyn} \sum_{i,t} (fp_{i,t} - \bar{f}p_t - (\bar{f}p_i^e - \bar{f}p^e))^2 + \hat{\alpha}^{dyn}}_{\text{Dynamic Trade}} + \underbrace{\hat{\beta}^{dol} \sum_{i,t} (\bar{f}p_t - \bar{f}p^e)^2 + \hat{\alpha}^{dol} - \hat{\alpha}^{dol}}_{\text{Dollar Trade}}, \end{aligned}$$

Use $\hat{\beta}^{dyn} \sum_{i,t} (fp_{i,t} - \bar{f}p_t - (\bar{f}p_i^e - \bar{f}p^e))^2 = \hat{\beta}_{FE}^{dyn} \sum_{i,t} (fp_{i,t} - \bar{f}p_t - (\bar{f}p_i - \bar{f}p))^2$ (as shown in [Appendix C.7](#), equation (42)), together with the definition of β_{FE}^{dyn} ,

$$\hat{\beta}_{FE}^{dyn} \sum_{i,t} (fp_{i,t} - \bar{f}p_t - (\bar{f}p_i - \bar{f}p))^2 = \sum_{i,t} [(rx_{it} - \bar{r}\bar{x}_{t+1} - (\bar{r}\bar{x}_i - \bar{r}\bar{x})) (fp_{it} - \bar{f}p_t - (\bar{f}p_i - \bar{f}p))].$$

We can then write

$$\begin{aligned} \hat{\beta}^{dyn} \sum_{i,t} (fp_{i,t} - \bar{f}p_t - (\bar{f}p_i^e - \bar{f}p^e))^2 & = \sum_i \sum_t [(rx_{it} - \bar{r}\bar{x}_{t+1} - (\bar{r}\bar{x}_i - \bar{r}\bar{x})) (fp_{it} - \bar{f}p_t - (\bar{f}p_i - \bar{f}p))] \\ & = \sum_i \hat{\beta}_i^{dyn} \sum_t (fp_{it} - \bar{f}p_t - (\bar{f}p_i - \bar{f}p))^2 \end{aligned}$$

Substituting into equation (8) leads to (23).

C.13 Appendix to Section 3.3.2

Denote by $fp_{i,t}^j$ the forward premium of currency i against currency j at time t . If $j = USD$, we simply write $fp_{i,t}$ as before. For any two currencies, i and j , it must be true by convertibility (existence of triangular trades) that:

$$\begin{aligned} fp_{i,t}^j &= fp_{i,t} - fp_{j,t} \\ rx_{i,t+1}^j &= rx_{i,t+1} - rx_{j,t+1}. \end{aligned} \quad (44)$$

Taking means over time of the equations in (44) one gets:

$$\begin{aligned} \overline{fp}_i^j &= \overline{fp}_i - \overline{fp}_j \\ \overline{rx}_i^j &= \overline{rx}_i - \overline{rx}_j \end{aligned} \quad (45)$$

Take the mean over currencies of equation (44) to get

$$\begin{aligned} \frac{\sum_{i \neq j} fp_{i,t}^j}{N} &= \frac{\sum_{i \neq j} fp_{i,t}}{N} - fp_{j,t} \\ \overline{fp}_t^j &= \frac{\sum_i fp_{i,t}}{N} - fp_{j,t} \left(1 + \frac{1}{N}\right) \\ \overline{fp}_t^j &= \overline{fp}_t - fp_{j,t} \left(\frac{N+1}{N}\right). \end{aligned}$$

If $N \rightarrow \infty$,

$$\begin{aligned} \overline{fp}_t^j &= \overline{fp}_t - fp_{j,t} \\ \overline{rx}_{t+1}^j &= \overline{rx}_{t+1} - rx_{j,t+1}, \end{aligned} \quad (46)$$

where we followed the same steps for excess returns.

Finally, take means over currencies j in equation (45):

$$\begin{aligned} \frac{\sum_{i \neq j} \overline{fp}_i^j}{N} &= \frac{\sum_{i \neq j} \overline{fp}_i}{N} - \overline{fp}_j \\ \overline{fp}^j &= \frac{\sum_i \overline{fp}_i}{N} - \overline{fp}_j \left(1 + \frac{1}{N}\right) \\ \overline{fp}^j &= \overline{fp} - \overline{fp}_j \left(1 + \frac{1}{N}\right). \end{aligned}$$

Again, if $N \rightarrow \infty$,

$$\begin{aligned}\overline{fp}^j &= \overline{fp} - \overline{fp}_j \\ \overline{rx}^j &= \overline{rx} - \overline{rx}_j,\end{aligned}\tag{47}$$

where we used the same steps for excess returns as for forward premia.

Claim 2 *If $N \rightarrow \infty$, β^{stat} and β^{dyn} are independent of the choice of base currency.*

Proof. *By the definition of β^{stat} in equation (7), where the US dollar is the base currency,*

$$\beta^{stat} = cov\left(\overline{rx}_i - \overline{rx}, \overline{fp}_i^e - \overline{fp}^e\right) \left[var\left(\overline{fp}_i^e - \overline{fp}^e\right) \right]^{-1}.$$

Using (44)-(47) we can write $\overline{rx}_i^j - \overline{rx}^j = \overline{rx}_i - \overline{rx}_j - (\overline{rx} - \overline{rx}_j) = \overline{rx}_i - \overline{rx}$. Taking conditional expectations in these equations also yields $\overline{fp}_i^{e,j} - \overline{fp}^{e,j} = \overline{fp}_i^e - \overline{fp}^e$. Thus,

$$\beta^{stat} = cov\left(\overline{rx}_i^j - \overline{rx}^j, \overline{fp}_i^{e,j} - \overline{fp}^{e,j}\right) \left[var\left(\overline{fp}_i^{e,j} - \overline{fp}^{e,j}\right) \right]^{-1}$$

for any base currency j other than the US dollar as well.

By the definition of β^{dyn} in equation (9), where the US dollar is the base currency,

$$\beta^{dyn} = cov\left(rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx}), fp_{i,t} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e)\right) \left[var\left(fp_{i,t} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e)\right) \right]^{-1}.$$

Using (44)-(47) we can again write

$$\begin{aligned}rx_{i,t+1}^j - \overline{rx}_{t+1}^j - (\overline{rx}_i^j - \overline{rx}^j) &= (rx_{i,t+1} - rx_{j,t+1}) - (\overline{rx}_{t+1} - rx_{j,t+1}) - (\overline{rx}_i - \overline{rx}_j - (\overline{rx} - \overline{rx}_j)) = \\ &= rx_{i,t+1} - \overline{rx}_{t+1} - (\overline{rx}_i - \overline{rx}),\end{aligned}$$

and similarly for forward premia by taking the conditional expectations operator through equations (45), (46), and (47). Thus,

$$\beta^{dyn} = cov\left(rx_{i,t+1}^j - \overline{rx}_{t+1}^j - (\overline{rx}_i^j - \overline{rx}^j), fp_{i,t}^j - \overline{fp}_t^j - (\overline{fp}_i^{e,j} - \overline{fp}^{e,j})\right) \left[var\left(fp_{i,t}^j - \overline{fp}_t^j - (\overline{fp}_i^{e,j} - \overline{fp}^{e,j})\right) \right]^{-1}$$

for any base currency j other than the US dollar as well. ■

C.14 Proof of Proposition 3

First, we generalize our notation to account for returns in units of different currencies. Denote by $fp_{i,t}^j$ the forward premium of currency i against currency j at time t , where for the US

dollar, we maintain $fp_{i,t}^{dol} = fp_{i,t}$. By convertibility, we have

$$fp_{i,t}^j = fp_{i,t} - fp_{j,t}, \quad \Delta s_{i,t+1}^j = \Delta s_{i,t+1} - \Delta s_{j,t+1}, \quad \text{and thus } rx_{i,t+1}^j = rx_{i,t+1} - rx_{j,t+1},$$

where we again use the convention that $\Delta s_{i,t+1}^j$ and $rx_{i,t+1}^j$ refer to values in terms of currency j . If $N \rightarrow \infty$, we can also write $\overline{fp}_t^j = \overline{fp}_t - fp_{j,t}$ and consequently, $\overline{fp}^{e,j} = \overline{fp}^e - \overline{fp}_j^e$ (see Appendix C.13 for a formal derivation). We can then write $\overline{fp}_t^j - \overline{fp}^{e,j} = -\left(fp_{j,t} - \overline{fp}_j^e - \left(\overline{fp}_t - \overline{fp}^e\right)\right)$ and, analogously, $\overline{rx}_{t+1}^j - \overline{rx}^j = -\left(rx_{j,t+1} - \overline{rx}_j - \left(\overline{rx}_{t+1} - \overline{rx}\right)\right)$.

Using these identities, we can then show that, for a given base currency j ,

$$\begin{aligned} E_0 \left[\left(\overline{rx}_{t+1}^j - \overline{rx}^j \right) \left(\overline{fp}_t^j - \overline{fp}^{e,j} \right) \right] &= E_{j0} \left[\left(\overline{rx}_{t+1}^j - \overline{rx}^j \right) \left(\overline{fp}_t^j - \overline{fp}^{e,j} \right) \right] \\ &= E_{j0} \left[\left(rx_{j,t+1} - \overline{rx}_j - \left(\overline{rx}_{t+1} - \overline{rx} \right) \right) \left(fp_{j,t} - \overline{fp}_j^e - \left(\overline{fp}_t - \overline{fp}^e \right) \right) \right]. \end{aligned}$$

By definition, the left-hand side of this equation is equal to $cov \left(\overline{rx}_{t+1}^j - \overline{rx}^j, \overline{fp}_t^j - \overline{fp}^{e,j} \right) = \beta^j var \left(\overline{fp}_t^j \right)$. Similarly, the right-hand side can be replaced with $cov_j \left(rx_{j,t+1} - \overline{rx}_{t+1}, fp_{j,t} - \overline{fp}_t \right) = \beta_j^{dyn} var_j \left(fp_{j,t} - \overline{fp}_t \right) = \beta_j^{dyn} var \left(\overline{fp}_t^j \right)$, where the last equality again uses the identities above. It follows that $\beta^j = \beta_j^{dyn}$.

C.15 Derivation of (25)

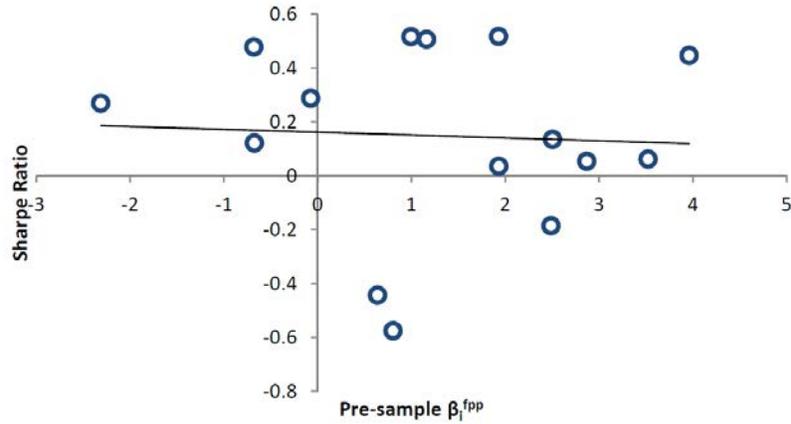
By the properties of OLS we have $\hat{\beta}_{FE}^{fpp} = \frac{\sum_{i,t} [(rx_{it} - \overline{rx}_i) (fp_{it} - \overline{fp}_i)]}{\sum_{i,t} (fp_{it} - \overline{fp}_i)^2}$. In addition (1) introduced β_i^{fpp} , where $\hat{\beta}_i^{fpp}$ can be written as $\hat{\beta}_i^{fpp} = \sum_t [(rx_{i,t+1} - \overline{rx}_i) (fp_{it} - \overline{fp}_i)] / \sum_t (fp_{it} - \overline{fp}_i)^2$, because a currency-specific constant is in the regression. We can then write

$$\begin{aligned} \hat{\beta}_{FE}^{fpp} &= \sum_{i,t} [(rx_{it} - \overline{rx}_i) (fp_{it} - \overline{fp}_i)] / \sum_{i,t} (fp_{it} - \overline{fp}_i)^2 \\ &= \sum_i \sum_t [(rx_{it} - \overline{rx}_i) (fp_{it} - \overline{fp}_i)] / \sum_i \sum_t (fp_{it} - \overline{fp}_i)^2. \end{aligned}$$

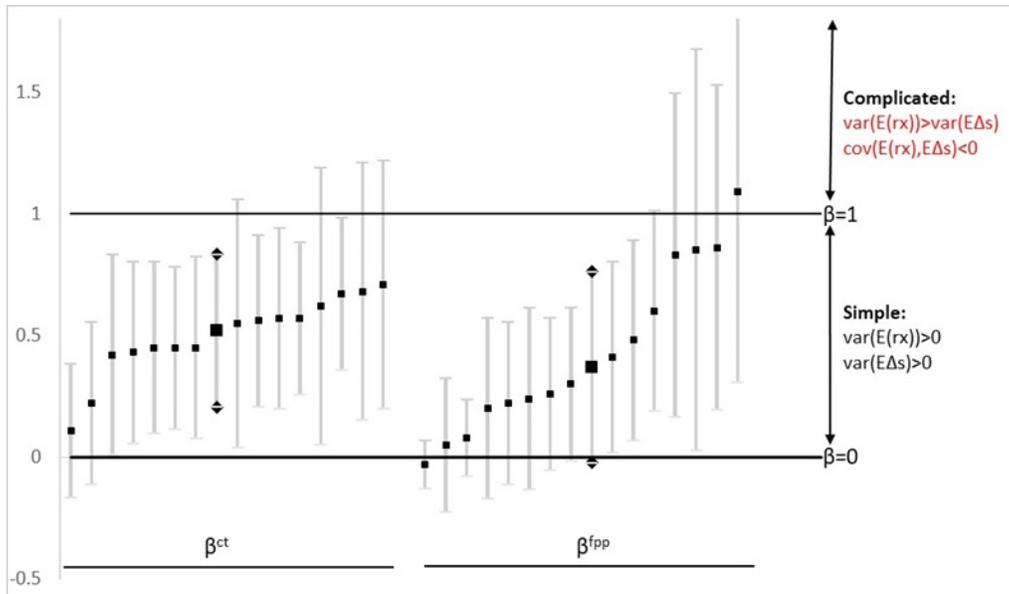
After dividing and multiplying each term inside the summation by the currency-level variance of forward premium, $\sum_t (fp_{it} - \overline{fp}_i)^2$, one gets

$$\begin{aligned} \hat{\beta}_{FE}^{fpp} &= \sum_i \left(\frac{\sum_t [(rx_{it} - \overline{rx}_i) (fp_{it} - \overline{fp}_i)]}{\sum_t (fp_{it} - \overline{fp}_i)^2} \sum_t (fp_{it} - \overline{fp}_i)^2 \right) / \sum_i \sum_t (fp_{it} - \overline{fp}_i)^2 \\ &= \sum_i \left(\hat{\beta}_i^{fpp} \frac{\sum_t (fp_{it} - \overline{fp}_i)^2}{\sum_{i,t} (fp_{it} - \overline{fp}_i)^2} \right), \end{aligned}$$

which yields (25).



Appendix Figure 1: Sharpe Ratio on bilateral forward premium trade for each currency in 1 Rebalance sample plotted over that currency's β_i^{fpp} estimated in the pre-sample. 1-month forward contracts used throughout.



Appendix Figure 2: **Summary of Estimates of the Elasticity of Risk Premia with Respect to Forward Premia across Samples and Horizons**

The figure plots all estimates and 95% confidence intervals of β^{fpp} and β^{ct} from Table 3. Small squares show point estimates, and large squares identify the median estimate for each elasticity across samples/horizons. The right-hand-side axis summarizes the implications of the estimates for linear models of currency risk premia.

Appendix Table 1: Implementing the Carry Trade Using Alternative Weighting Schemes

	1 Rebalance			3 Rebalance			6 Rebalance			12 Rebalance		
Expected Return	4.95	6.43	2.73	4.50	4.60	3.11	4.28	4.60	2.97	5.45	5.29	2.88
Sharpe Ratio	0.54	0.66	0.80	0.54	0.53	0.69	0.50	0.55	0.67	0.69	0.66	0.63
max \$ short	0	0	-0.60	0	0	-0.42	0	0	-0.23	0	0	-0.17
max \$ long	0	0	0.71	0	0	0.75	0	0	0.69	0	0	0.72
Linear weights	Yes			Yes			Yes			Yes		
HML		Yes			Yes			Yes			Yes	
Equally weighted			Yes			Yes			Yes			Yes

Note: Mean returns and Sharpe ratios achieved by three different implementations of the carry trade across our four main samples. (1) “Linear weights”: weight each currency by the difference between its forward premium and the average forward premium across currencies at the time as in equation (2); (2) “HML”: separate currencies into five portfolios and go long the currencies in the last portfolio (highest forward premia) and short the currencies on the first portfolio (lowest forward premia) as described in [Lustig et al. \(2011\)](#); (3) “Equally weighted”: go long all currencies whose forward premium is larger than zero and short all other currencies, normalizing total investment to \$1 as described in [Burnside et al. \(2011\)](#).

Appendix Table 2: Comparing Carry Trade and Static Trade

	(1)	(2)	(3)	(4)	(5)	(6)
Panel I: All Countries						
Static Trade						
Expected Return	4.90	4.90	4.90	5.36	4.51	3.46
Sharpe Ratio	0.46	0.46	0.46	0.47	0.42	0.39
Carry Trade						
Expected Return	10.15	7.18	7.05	6.94	6.43	4.95
Sharpe Ratio	1.13	0.64	0.63	0.64	0.66	0.54
Ratio Static/Carry	48%	68%	70%	77%	70%	70%
Max total curr.	36	18	18	13	15	15
Max curr. short	6	3	3	2	3	0
Max curr. long	6	3	3	3	3	0
Currencies added and subtracted relative to 1 Rebalance sample	- kwd sar + bef dem frf frf itl nlg	- kwd sar + bef dem frf frf itl nlg	- kwd sar + bef dem frf frf itl nlg	- kwd sar - kwd sar	=	=
Same # curr. in Static & Carry T.	No	Yes	Yes	Yes	Yes	Yes
Prtf. Construction	HML	HML	HML	HML	HML	linear weights
Time Period	1/95-12/09	1/95-12/09	1/95-12/09	1/95-12/09	1/95-6/10	1/95-6/10
Data Source	LRV	LRV	LRV	LRV	Hassan-Mano	Hassan-Mano

	(1)	(2)	(3)	(4)	(5)	(6)
Panel II: Developed						
Static Trade						
Expected Return	4.23	4.23	4.83	5.60	4.51	3.46
Sharpe Ratio	0.46	0.46	0.47	0.41	0.42	0.39
Carry Trade						
Expected Return	6.75	5.33	6.72	6.06	6.43	4.95
Sharpe Ratio	0.64	0.45	0.50	0.43	0.66	0.54
Ratio Static/Carry	63%	79%	72%	92%	70%	70%
Max total curr.	14	14	14	8	15	15
Max curr. short	2	2	2	1	3	0
Max curr. long	4	4	3	2	3	0
Currencies added and subtracted relative to 1 Rebalance sample	- hkd kwd myr sar sgd zar + bef dem frf itl nlg	- hkd kwd myr sar sgd zar + bef dem frf itl nlg	- hkd kwd myr sar sgd zar + bef dem frf itl nlg	- hkd kwd myr sar sgd zar + bef dem frf itl nlg	- hkd kwd myr sar sgd zar + bef dem frf itl nlg	=
Same # curr. in Static & Carry T.	No	Yes	Yes	Yes	Yes	Yes
Prtf. Construction	HML	HML	HML	HML	HML	linear weights
Time Period	1/95-12/09	1/95-12/09	1/95-12/09	1/95-12/09	1/95-6/10	1/95-6/10
Data Source	LRV	LRV	LRV	LRV	Hassan-Mano	Hassan-Mano

Note: This table compares our decomposition of carry trade from Table 2 to a similar exercise in Lustig et al. (2011) (LRV). It shows that the procedure in LRV attributes a lower percentage to the static trade predominantly due to the inclusion of additional currencies in the carry trade relative to the static trade. Column (6) of Panel I replicates our results from Table 2. Column (1) replicates closely the results from Table 2 in LRV. The remaining columns show step by step the differences in the two procedures. Panel I uses the full sample of currencies. Panel II uses only 15 developed countries' currencies: Australia, Belgium, Canada, Denmark, Germany, Euro, France, Italy, Japan, Netherlands, New Zealand, Norway, Sweden, Switzerland, and the UK as in LRV. Data come from two sources: "LRV" was downloaded on 6/12/2014 from <http://web.mit.edu/adrienv/www/Data.html>; "Hassan-Mano" denotes the data used throughout this paper. The Static Trade uses only currencies that were available prior to December 1994 in all columns. The Carry Trade is either not constrained to use the same currencies as the Static Trade (1) or constrained to do so (2)-(6). (3) uses the same data as (2) but assigns currencies to portfolios to minimize the difference in the number of countries in all portfolios. (4) uses the same data and portfolio allocation as (3) but excludes euro-zone currencies, which we excluded to get a balanced sample. Column (5) uses our 1 Rebalance sample with 15 currencies, which differs slightly from the sample used in LRV on three dimensions: (1) it goes beyond Dec09 to Jun10; (2) it extends the time coverage for some currencies; and (3) uses a different filtering algorithm for cleaning the data. Column (6) uses the same data as (5), but weights all currencies linearly by their forward premia to construct the static and carry trades, rather than building the HML portfolio. This is Table 1 in this paper.

Appendix Table 3: Currency Portfolios Using Alternative Samples

	(1)	(2)	(3)	(4)	(5)	(6)
Sample	1 Rebalance (no fixed)				LRV	
Horizon (months)	1	1	6	12	1	1
Static Trade						
$\sum_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]$	3.36	1.38	3.64	3.97	4.10	1.96
Sharpe Ratio	0.44	0.18	0.37	0.38	0.47	0.22
Dynamic Trade						
$\sum_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e))]$	1.05	-0.62	-0.25	0.50	1.02	-0.76
Sharpe Ratio	0.18	-0.11	-0.04	0.09	0.16	-0.12
Dollar Trade						
$\sum_{i,t}[rx_{i,t+1}(\overline{fp}_t - \overline{fp}^e)]$	2.86	1.37	3.01	3.82	2.42	1.09
Sharpe Ratio	0.24	0.12	0.26	0.27	0.24	0.11
Carry Trade						
$\sum_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_t)]$	4.41	2.31	3.70	4.68	5.12	2.93
Sharpe Ratio	0.50	0.26	0.32	0.41	0.55	0.32
% Static Trade	76%	182%	107%	89%	80%	163%
Forward Premium Trade						
$\sum_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_i^e)]$	3.90	1.65	2.97	4.48	3.44	1.10
Sharpe Ratio	0.27	0.11	0.20	0.26	0.23	0.07
% Dollar Trade	73%	183%	109%	88%	70%	330%
Sample	4 Rebalance (CIP)				BER	
Static Trade						
$\sum_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]$	4.87	0.19		2.97	5.11	-6.78
Sharpe Ratio	0.53	0.02		0.21	0.49	-0.63
Dynamic Trade						
$\sum_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_t - (\overline{fp}_i^e - \overline{fp}^e))]$	0.93	-2.34		0.11	1.12	-5.94
Sharpe Ratio	0.12	-0.31		0.01	0.21	-1.09
Dollar Trade						
$\sum_{i,t}[rx_{i,t+1}(\overline{fp}_t - \overline{fp}^e)]$	4.51	2.55		4.26	6.30	1.54
Sharpe Ratio	0.31	0.17		0.26	0.26	0.06
Carry Trade						
$\sum_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_t)]$	5.80	2.08		3.62	6.23	-2.78
Sharpe Ratio	0.71	0.25		0.24	0.63	-0.28
% Static Trade	84%	.		99%	82%	.
Forward Premium Trade						
$\sum_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_i^e)]$	5.44	1.57		4.54	7.42	-1.36
Sharpe Ratio	0.27	0.08		0.22	0.30	-0.05
% Dollar Trade	83%	747%		99%	85%	.
Bid-Ask Spreads	No	Yes	Yes	Yes	No	Yes

Note: This table replicates all calculations in Table 2 using alternative data samples. Columns 1-4 of the top panel use the 1 Rebalance sample but drop currencies that have a fixed official exchange rate with respect to the US dollar. Columns 5 and 6 of the top and bottom panels use samples that are as close as possible to the samples used in Lustig et al. (2011) and Burnside et al. (2006). Columns 1-4 of the bottom panel use an extended sample using all available US dollar- and UK pound-based forward data as well as forward rates imputed using interest rate data. See Appendix A for details.

Appendix Table 4: Estimates of the Elasticity of Risk Premia with respect to Forward Premia Using Alternative Samples

	1 Rebalance (no fixed)				LRV	
	(1)	(2)	(3)	(4)	(5)	(6)
Horizon (months)	1	1	6	12	1	1
Static T: β^{stat}	0.52*** (0.08)	0.44*** (0.08)	0.63*** (0.10)	0.66*** (0.10)	0.57*** (0.09)	0.45*** (0.10)
Dynamic T: β^{dyn}	0.41 (0.28)	0.38 (0.28)	0.28 (0.36)	0.46 (0.29)	0.43* (0.25)	0.40 (0.25)
Dollar T: β^{dol}	3.12* (1.61)	3.11** (1.57)	3.28 (2.25)	3.80* (2.24)	3.32** (1.59)	3.23* (1.82)
Carry Trade: β^{ct}	0.63** (0.26)	0.50* (0.26)	0.56** (0.28)	0.65** (0.25)	0.69** (0.27)	0.56** (0.26)
% ESS Static T	71	68	90	78	73	65
Forward Premium T: β^{fpp}	0.96** (0.40)	0.92** (0.40)	0.95* (0.50)	1.22*** (0.47)	0.88** (0.35)	0.84** (0.35)
% ESS Dollar T	94	95	98	95	92	93
N	2334	2334	2269	2191	2616	2616
	4 Rebalance (CIP)				BER	
Static T: β^{stat}	0.21*** (0.06)	0.13*** (0.03)		0.24*** (0.06)	0.26*** (0.03)	0.19 (0.14)
Dynamic T: β^{dyn}	0.18* (0.11)	0.15 (0.11)		0.20 (0.12)	0.38*** (0.15)	0.19* (0.11)
Dollar T: β^{dol}	1.83 (1.19)	1.72 (1.20)		2.06* (1.10)	1.31 (1.32)	1.46 (1.25)
Carry Trade: β^{ct}	0.57*** (0.16)	0.39** (0.17)		0.36** (0.17)	0.67*** (0.18)	0.38** (0.18)
% ESS Static T	69	55		74	53	68
Forward Premium T: β^{fpp}	0.42*** (0.15)	0.38*** (0.15)		0.63*** (0.18)	0.74*** (0.25)	0.60*** (0.20)
% ESS Dollar T	94	95		96	88	96
N	5558	5533		5231	3161	3997
Bid-Ask Spreads	No	Yes	Yes	Yes	No	Yes

Note: This table replicates all calculations in Table 3 using alternative data samples. Columns 1-4 of the top panel use the 1 Rebalance sample but drop currencies that have a fixed official exchange rate with respect to the US dollar. Columns 5 and 6 of the top and bottom panels use samples that are as close as possible to the samples used in [Lustig et al. \(2011\)](#) and [Burnside et al. \(2006\)](#). Columns 1-4 of the bottom panel use an extended sample using all available US dollar- and UK pound-based forward data as well as forward rates imputed using interest rate data. Asterisks denote statistical significance at the 1 (***), 5 (**), and 10% (*) level. See Appendix A for details.

Appendix Table 5: Estimates of the Elasticity of Risk Premia with respect to Forward Premia calculated using a fixed set of currencies, focusing on developed countries, and after adjusting for US inflation

	<i>Baseline Results</i>	<i>Fixed set of currencies</i>	<i>Developed Countries</i>	<i>Inflation adjusted</i>	<i>Baseline Results</i>	<i>Fixed set of currencies</i>	<i>Developed Countries</i>	<i>Inflation adjusted</i>
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sample	1 Rebalance				3 Rebalance			
Static T: β^{stat}	0.47*** (0.08)	0.47*** (0.08)	0.52*** (0.10)	0.47*** (0.08)	0.26*** (0.05)	0.29*** (0.04)	0.25*** (0.05)	0.26*** (0.05)
Dynamic T: β^{dyn}	0.44* (0.25)	0.44* (0.25)	0.40 (0.35)	0.44* (0.25)	0.28* (0.15)	0.27 (0.19)	0.21 (0.18)	0.27* (0.15)
Dollar T: β^{dol}	3.11* (1.60)	3.11* (1.60)	3.82** (1.52)	3.46** (1.62)	0.91 (1.18)	1.70 (1.08)	1.23 (1.14)	0.90 (1.18)
Carry Trade: β^{ct}	0.68** (0.27)	0.68** (0.27)	0.90** (0.40)	0.68** (0.27)	0.57*** (0.19)	0.51** (0.22)	0.63** (0.29)	0.56*** (0.19)
% ESS Static T	62	62	85	62	56	63	68	57
Forward Premium T: β^{fpp}	0.86** (0.34)	0.86** (0.34)	1.85*** (0.44)	0.92*** (0.35)	0.41** (0.20)	0.58** (0.25)	0.55** (0.24)	0.41** (0.20)
% ESS Dollar T	90	90	99	92	75	91	95	75
N	2706	2706	1674	2706	4494	3407	2759	4428
Sample	6 Rebalance				12 Rebalance			
Static T: β^{stat}	0.23*** (0.05)	0.26*** (0.03)	0.22*** (0.05)	0.22*** (0.05)	0.34*** (0.08)	0.38*** (0.04)	0.18** (0.09)	0.33*** (0.09)
Dynamic T: β^{dyn}	0.19 (0.14)	0.17 (0.19)	0.12 (0.20)	0.18 (0.14)	0.16 (0.11)	0.08 (0.15)	0.05 (0.15)	0.15 (0.12)
Dollar T: β^{dol}	0.87 (2.59)	1.85 (2.94)	1.18 (2.87)	1.29 (2.65)	1.71 (2.26)	2.55 (1.75)	1.15 (2.27)	1.85 (2.28)
Carry Trade: β^{ct}	0.56*** (0.18)	0.52** (0.22)	0.63** (0.29)	0.54*** (0.17)	0.67*** (0.16)	0.68*** (0.18)	0.66*** (0.25)	0.65*** (0.16)
% ESS Static T	70	79	83	71	90	98	96	91
Forward Premium T: β^{fpp}	0.24 (0.19)	0.28 (0.26)	0.26 (0.29)	0.27 (0.20)	0.30* (0.16)	0.35* (0.20)	0.26 (0.25)	0.30* (0.16)
% ESS Dollar T	62	90	97	81	92	99	99	94
N	4842	3503	2759	4728	6019	4128	3191	5899

Note: This table replicates calculations without bid-ask spreads in Table 3 using alternative assumptions and samples. Columns 1 and 5 replicate the results from Table 3. Columns 2 and 6 use the same countries included in the 1 Rebalance sample, while still updating expectations of $\overline{f p_i^c}$ at 3, 6, and 12 dates (but not allowing entry of currencies as in Table 3). Columns 3 and 7 include only developed countries (see caption of Appendix Table 2). Columns 4 and 8 subtract the lead of 1-year U.S. inflation from returns.

Appendix Table 6: Bootstrapped Standard Errors for 12 Rebalance Sample

	(1)	(2)	(3)	(4)
Horizon (months)	1	1	6	12
Static T: β^{stat}	0.34** (0.15)	0.23 (0.15)	0.31*** (0.12)	0.30*** (0.11)
Dynamic T: β^{dyn}	0.16 (0.11)	0.13 (0.12)	0.06 (0.14)	-0.01 (0.11)
Dollar T: β^{dol}	1.71 (2.36)	1.61 (2.40)	0.02 (2.88)	-0.23 (1.76)
Carry Trade: β^{ct}	0.67*** (0.24)	0.52** (0.25)	0.57** (0.23)	0.22 (0.27)
% ESS Static T	90*** (15)	86*** (24)	99*** (18)	100*** (15)
Forward Premium T: β^{fpp}	0.30 (0.27)	0.26 (0.28)	0.05 (0.36)	-0.03 (0.24)
% ESS Dollar T	92*** (24)	94*** (23)	1 (22)	95*** (23)
Bid-Ask Spreads	No	Yes	Yes	Yes

Note: This tables uses our 12 Rebalance sample to block-bootstrap standard errors corresponding to columns 5-8 of Table 3. In this procedure, we treat each of the 12 two-year periods in between re-balancing dates as one block and draw 100,000 random samples with replacement from this set of histories. Using this approach to constructing standard errors, the table also reports standard errors for % ESS Static T and % ESS Dollar T.

Appendix Table 7: Alternative Estimates of β^{dyn} and β^{fpp} using Different Approaches to Estimating the Variance of Forecast Errors, $var(\overline{fp}_i - \overline{fp}_i^e)$

Restrictions on (18)	β^{dyn}	β^{fpp}
$\rho_{1,i} = \rho_1, \rho_2 = 0$	0.17 (0.18)	0.38 (0.24)
$\rho_2 = 0$	0.45* (0.25)	0.88** (0.35)
unrestricted	0.47* (0.26)	0.89** (0.36)
unrestricted GARCH(1,1)	0.45 (0.28)	1.01** (0.39)

Note: Estimates of β^{dyn} and β^{fpp} calculated by combining (15) and (16) with different approaches to estimating the variance of forecast errors, $var(\overline{fp}_i - \overline{fp}_i^e)$. The estimates in the top panel are calculated by estimating (18) in the pre-sample data for each currency and then calculating the implied variance of the forecast error in a sample with length $T = 186$ months. Bottom panel: The GARCH(1,1) model relaxes the assumption that innovations to forward premia in (18), ϵ_{it}^f , are distributed with constant variance and assumes instead $\epsilon_{it}^f \sim \mathcal{N}(0, \sigma_{i,t}^2)$ where $\sigma_{i,t}^2 = b_i^0 + b_i^1(\epsilon_{i,t-1}^f)^2 + b_i^2\sigma_{i,t-1}^2$. If the estimated coefficients in the GARCH imply nonstationary dynamics, we revert to an AR(1) specification. In case the estimated $\rho_{1,i}$ coefficient is larger than 1, we set it to 0.999. Asterisks denote statistical significance at the 1 (***) , 5 (**) and 10% (*) level. See section 3.1.2 of the main text for details.

Appendix Table 8: Estimates of β_{FE}^{fpp} in Monte Carlo Simulations where Exchange Rates are Unpredictable ($\beta^{fpp} = 1$)

Process for f_{pit}	$\beta_{FE}^{fpp}/\beta^{fpp}$		
	JPY, AUD and NZD	Developed Countries	1 Rebalance Sample
common AR(1)	1.09 [1.01, 1.25]	1.10 [1.05, 1.17]	1.12 [1.06, 1.19]
AR(1)	1.10 [1.01, 1.27]	1.13 [1.05, 1.24]	1.45 [1.08, 2.32]
GARCH(1,1)	1.29 [1.02, 1.9]	1.21 [1.08, 1.42]	1.92 [1.15, 3.33]

Note: This table shows the average $\hat{\beta}_{FE}^{fpp}$ and 90% confidence intervals obtained from imposing $\beta^{fpp} = 1$ and simulating 1,000 time paths of forward premia for each of the currencies in our sample. Three models were used to generate these time paths: (i) “common AR(1)” as in equation (18) where $\rho_{i,1} = \rho_1$ and $\rho_2 = 0$; (ii) “country-specific AR(1)” as in equation (18) where $\rho_2 = 0$; and (iii) “GARCH(1,1)” as described in the note of Appendix Table 7. Three different samples of currencies were used: (i) “JPY, AUD and NZD” — a sample restricted to the Japanese yen, Australian dollar and New Zealand dollar; (ii) “Developed countries” using only developed countries from our 1 Rebalance sample (see caption of Appendix Table 2); and (iii) our 1 Rebalance sample. Parameters are estimated in each of these (full) samples and then, for given parameters, simulated for an artificial sample of 180-months.

Appendix Table 9: Estimates of the Elasticity of Risk Premia with respect to Forward Premia Using Constrained Model

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Sample	1 Rebalance				3 Rebalance			
Horizon (months)	1	1	6	12	1	1	6	12
Overall Beta: β	0.97*** (0.35)	0.85** (0.34)	0.92** (0.39)	1.05*** (0.37)	0.60*** (0.22)	0.49** (0.22)	0.58** (0.24)	0.62*** (0.24)
N	2706	2706	2631	2541	4494	4494	4374	4230
Sample	6 Rebalance				12 Rebalance			
Overall Beta: β	0.59*** (0.21)	0.48** (0.21)	0.57*** (0.22)	0.26 (0.17)	0.70*** (0.19)	0.57*** (0.19)	0.66*** (0.19)	0.37* (0.20)
N	4842	4842	4712	4556	6019	6019	5874	5644
Bid-Ask Spreads	No	Yes	Yes	Yes	No	Yes	Yes	Yes

Note: Estimates of the elasticity of currency risk premia with respect to forward premia for a constrained model in which $\beta^{stat} = \beta^{dyn} = \beta^{dol}$ using the specification

$$rx_{i,t+1} - rx = \alpha + \beta (fp_{it} - \overline{fp}^e) + \epsilon_{i,t+1}.$$

Standard errors are in parentheses. Asterisks denote statistical significance at the 1 (***) , 5 (**) and 10% (*) level.

Appendix Table 10: Bounds for Stambaugh Bias in Estimates of β_i^{fpp} , β^{fpp} , and β^{dol} Calculated using Kendall's Approximation

Panel A		Stambaugh Bias in $\hat{\beta}_i^{fpp}$		
<i>Currency</i>	$\frac{\sigma_{uv}}{\sigma_v^2}$	$E[\hat{\phi} - \phi]$	$E[\hat{\beta}_i^{fpp} - \beta_i^{fpp}]$	$\hat{\beta}_i^{fpp}$
AUD	15.9	-0.02	-0.34	3.25
CAD	-12.7	-0.02	0.27	4.36
CHF	6.6	-0.02	-0.14	3.59
DKK	3.1	-0.02	-0.07	4.43
HKD	0.1	-0.02	0.00	1.05
JPY	6.1	-0.02	-0.13	2.55
KWD	-1.2	-0.02	0.03	-1.94
MYR	3.0	-0.02	-0.06	-1.96
NOK	-0.5	-0.02	0.01	1.89
NZD	13.8	-0.02	-0.30	1.10
SAR	0.0	-0.02	0.00	1.36
SEK	10.6	-0.02	-0.23	3.37
SGD	-6.6	-0.02	0.14	0.74
UK	-1.1	-0.02	0.02	2.66
ZAR	-8.8	-0.02	0.19	2.43

Panel B		Stambaugh Bias in $\hat{\beta}^{fpp}$ and $\hat{\beta}^{dol}$		
<i>Coefficient</i>	$\frac{\sigma_{uv}}{\sigma_v^2}$	$E[\hat{\phi} - \phi]$	$E[\hat{\beta} - \beta]$	$\hat{\beta}$
β^{fpp}	n/a	-0.02	-0.01	0.86
β^{dol}	-3.4	-0.02	0.07	3.27

Note: Panel A shows estimates of $\frac{\sigma_{uv}}{\sigma_v^2}$ for each currency in our 1 Rebalance sample estimated using (48) and (49); the bias in the autoregressive coefficient in (49) calculated under the assumption that $\phi = 0.99$ and using and Kendall's approximation, $E[\hat{\phi} - \phi] = -\frac{1+3\phi}{T}$; the resulting bias in estimates of β_i^{fpp} calculated using (50); and the point estimate of β_i^{fpp} for each currency. The first line in Panel B shows the Stambaugh bias in β^{fpp} implied by these numbers, calculated using the expressions given in Proposition 2 and (25), under the additional assumption that the variance of the forecast error of $\bar{f}p_i$ is equal to that estimated for our 1 Rebalance sample, $\frac{\text{var}(\bar{f}p_i - \bar{f}p_i^e)}{\text{var}(fp_{it} - \bar{f}p_i)} = 1.08$. The second line in Panel B also shows an estimate for the Stambaugh bias in β^{dol} , calculated in the same way as the numbers in Panel A, but using $\bar{r}x_{t+1}$ and $\bar{f}p_t$.