NBER WORKING PAPER SERIES

OPTIMAL MONETARY POLICY IN AN OPEN ECONOMY

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Working Paper No. 2018

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 1986

Earlier versions of this paper were presented to seminars at the University of Warwick and the Australian National University. The authors wish to thank the participants at these seminars for their helpful comments. In particular, comments made by Andrew Hughes-Hallett, Ngo Van Long, and Marcus Miller are gratefully acknowledged. The research reported here is part of the NBER's research program in International Studies. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

This paper analyzes the optimal intertemporal tradeoff between inflation and output in an open economy under perfect foresight. The announcement of the optimal plan may, or may not, generate an initial jump in the exchange rate. That depends upon the real adjustment costs, which such unanticipated changes impose on the economy. In the case that such jumps occur, the question of time consistency of the optimal policy arises. A time consistent solution is obtained provided: (i) the policy maker is not too myopic; (ii) the adjustment costs associated with the jump in the exchange rate are of an appropriate form. The optimal monetary rule is derived and properties of this rule, as well as the overall optimal adjustment of the economy are discussed.

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1. INTRODUCTION

The tradeoff between unemployment and inflation has been a dominant issue in macroeconomic policy for many years now. The question of the optimal choice has been investigated by a number of authors. The earliest studies, conducted in the mid 1960's were purely static; see, e.g., Lipsey (1965) and Brechling (1968). Subsequently, the analysis was extended to a dynamic context, where the source of the dynamics is inflationary expectations, which are assumed to follow some gradual evolutionary process, such as an adaptive scheme; see, e.g., Phelps (1967, 1974), Turnovsky (1981). These authors derive an optimal path along which the inflation rate adjusts gradually towards some steady state equilibrium, while the unemployment rate converges slowly towards its natural rate level. More recently, Stemp and Turnovsky (1984) have shown that if instead, inflationary expectations satisfy perfect foresight, the economy can jump instantaneously to a zero rate of inflation. However, there is a tradeoff between an initial once-and-for-all jump in the price level and the subsequent gradual adjustment of the unemployment rate towards its natural rate level.

In this paper we analyze the issue of the unemployment (or equivalently output)-inflation tradeoff in an open economy in which expectations satisfy perfect foresight.[1/] For the model we analyze, we find that even under this assumption, the optimal paths followed by output and inflation generally involve the gradual adjustment of both variables towards their respective steady state equilibria. There is therefore a dynamic intertemporal tradeoff between them. The nature of the tradeoff depends critically upon the parameters characterizing the economy, as well as the preferences of the policy maker. These include in particular the rate of time discount and the costs associated with an initial real structural adjustment in the economy, necessary to attain the optimal path. The implied optimal monetary policy along this path can be described in terms of monetary rules, which can be expressed in various equivalent ways. One particularly convenient form specifies the level of the real money stock, relative to its equilibrium, in terms of the deviation of the current real exchange rate from its long-run equilibrium level. Rules of this kind are similar to those which appear in the recent literature on exchange market intervention; see, e.g., Boyer (1978), Cox (1980), Turnovsky (1983), and the papers in Bhandari (1985).

An important, and widely discussed, aspect of optimal policy determination under rational expectations concerns the question of the time consistency of the optimal policy; see, e.g., Kydland and Prescott (1977), Turnovsky and Brock (1980). This issue is addressed in our analysis.[2/] We show that, provided the policy maker is not too myopic, the time consistency or otherwise of the opimal solutions depends crucially upon the nature of the adjustment costs associated with unanticipated changes in the exchange rate, resulting from a policy change.

2. A DYNAMIC MACRO MODEL OF A SMALL OPEN ECONOMY

Consider a small open economy operating under a regime of perfectly flexible exchange rates. The economy is assumed to be specialized in the production of a single (composite) commodity, part of which is consumed domestically, the remainder of which is exported.

In exchange, the economy imports from abroad, taking the foreign currency price of imports as given. We also assume that there exists a perfect worldwide capital market with domestic and foreign securities being regarded as perfect substitutes. For notational simplicity, the model is expressed relative to a fixed steady state, so that all variables can be interpreted in deviation form. It can therefore be summarized as follows:

$$Y = d_1(E - P) - d_2(I - \dot{P}) \qquad d_1 > 0,\ d_2 > 0 \tag{1a}$$

$$M - P = \alpha_1 Y - \alpha_2 I \qquad \alpha_1 > 0,\ \alpha_2 > 0 \tag{1b}$$

$$I = \dot{E}\ , \text{ except at points where } E \text{ jumps} \tag{1c}$$

$$\dot{P} = \gamma Y \qquad \gamma > 0 \tag{1d}$$

$$C = \delta P + (1 - \delta) E \tag{1e}$$

where

Y = real output, measured as a deviation from its natural rate level,

P = price of domestic output, expressed in logarithms,

C = consumer price index, expressed in logarithms,

E = exchange rate (measured in terms of units of foreign currency per unit of domestic currency), measured in logarithms,

I = nominal interest rate,

M = nominal money supply, expressed in logarithms.

This model will be immediately recognized as being a standard Dornbusch (1976) model, so that our description can be brief. Equation

(1a) describes the reduced form for the domestic goods market, where the demand for domestic output depends positively upon the relative price of domestic to foreign goods (where for convenience the foreign currency price of the latter are set to unity) and negatively upon the real interest rate. Money market equilibrium is specified by (1b) and is standard. Uncovered interest parity is described by (1c), where again for notational convenience we have set the foreign nominal interest rate to zero. Equation (1d) defines the rate of price adjustment in the domestic economy in terms of a simple Phillips curve. Finally, (1e) describes the domestic CPI (which we assume to be relevant to the policy maker's objective function, discussed below), to be a weighted average of the price of domestic good and the domestic price of the imported good. Note that the real interest rate in (1a) and the nominal money stock in (1b) are deflated by the price of domestic output. Little is changed, apart from additional notational complications, if the deflator is in terms of the domestic CPI.

Equations (1a)-(1e) describe the basic dynamic structure of the economy, which is seen to be extremely simple. In addition, we should note that the model embodies the familiar features of the Dornbusch model, namely sluggish goods prices, so that at any point of time P is predetermined by the past, while the exchange rate is forward looking, allowing it to undergo endogenous jumps as new information impinges on the economy. The policy maker is assumed to control the economy by appropriate choice of the nominal money stock. In addition, the announcement of a new (optimal) policy at the beginning of the planning horizon, time 0 say, leads to an initial jump in the exchange rate, which also plays an important role in determining the subsequent path of the economy.

The most critical aspect of the model concerns the formulation of the objective function. Specifically, we assume that the policy maker's objective is to

$$\text{Minimize} \quad k|E(0) - E_0|^q + \int_0^\infty [aY^2 + (1-a)\dot{C}^2]e^{-\beta t}dt \qquad (2)$$

This cost function has two components. The second is a standard quadratic loss function. It asserts that a zero rate of inflation of the CPI (i.e., a stable CPI), together with a full employment level of output ($Y = 0$), are regarded as globally optimal. The policy maker seeks to minimize the discounted intertemporal deviations about these targets, over the planning horizon, which we take to be infinite. The parameter a ($0 \leq a \leq 1$) reflects the relative importance attached to inflation and output in the intertemporal objective. As a increases, the policy maker is concerned increasingly with unemployment (output); as a decreases, the objective is weighted more heavily towards inflation. The parameter β, ($0 \leq \beta < \infty$) which measures the rate of time preference, reflects the degree of myopia of the policy maker; the larger β, the more myopic he is.

As noted above, with a forward looking exchange rate, the change in monetary policy at time zero will cause the exchange rate to undergo an initial unanticipated discontinuous change from its previously inherited level E_0 say, at that time. Given that the price of output is predetermined at any instant, this change in the nominal exchange rate translates into an instantaneous unanticipated jump in the relative price E - P, which in turn causes an unanticipated jump in output. These jumps in real magnitudes impose real (structural) adjustment costs on the economy and these need to be taken into account in assessing the overall benefits of the optimal stabilization policy to the economy.

The initial adjustment costs are specified to be proportional to the absolute value of the jump, raised to the arbitrary power q.[3/] This is a more general functional form than the more usual quadratic function appearing in the second component in (2), but the choice is deliberate. As will be shown below, the time consistency or otherwise of the optimal policy depends critically upon q. Specifically, if $q \leq 1$, the solution is time consistent; for $q > 1$ it is time inconsistent. Thus the quadratic function ($q = 2$) is time inconsistent, while if the initial costs are evaluated in terms of the absolute jumps ($q = 1$), time consistency is obtained.

The magnitude of the initial jump in the exchange rate will be shown to depend upon the size of the coefficient k. In the limiting case when $k = 0$, we will see that the optimal monetary policy will move the economy instantaneously to steady state. The dynamic time path therefore degenerates. Thus, a well-defined optimal intertemporal adjustment path is obtained by, in effect, balancing off the initial costs associated with implementing the policy, with the subsequent improvement in the performance of the economy. This procedure for generating a gradual adjustment process is not new. In his well known survey article on distributed lags, Griliches (1967) demonstrated how distributed lag investment functions could be derived by balancing off the costs of an initial adjustment in the capital stock, with the costs of being away from some target level. Our approach here is essentially analogous.

3. THE OPTIMAL MONETARY POLICY PROBLEM

The formal monetary stabilization problem is to choose the nominal money stock M(t) so as to minimize the loss function specified in (2) subject to the constraints on the economy given by (1). In order to solve this problem it is convenient to define:

$m \equiv M - P =$ real money stock (in logarithms),

$s \equiv E - P =$ relative price of foreign to domestic goods (in logarithms); i.e., the real exchange rate.

We can then use equations (1) to express the variables Y, $\dot{C}$, and $\dot{s}$, in terms of m and s as follows:

$$Y = \phi_1 m + \phi_2 s \tag{3a}$$

$$\dot{C} = \eta_1 m + \eta_2 s \tag{3b}$$

$$\dot{s} = \theta_1 m + \theta_2 s \tag{3c}$$

where

$$\phi_1 \equiv \frac{d_2}{D} > 0 \; ; \; \phi_2 \equiv \frac{\alpha_2 d_1}{D} > 0$$

$$\eta_1 \equiv \frac{\delta\gamma d_2 - (1-\delta)(1-d_2\gamma)}{D} \; ; \; \eta_2 \equiv \frac{d_1[\delta\gamma\alpha_2 + (1-\delta)\alpha_1]}{D} > 0$$

$$\theta_1 \equiv -\frac{1}{D} < 0 \; ; \; \theta_2 = \frac{d_1(\alpha_1 - \alpha_2\gamma)}{D}$$

$$D \equiv \alpha_2(1-d_2\gamma) + \alpha_1 d_2$$

We shall assume $1 > d_2\gamma$, thereby ensuring $D > 0$. This imposes an upper bound on the speed of price adjustment γ consistent with having a downward sloped IS curve in Y - I space.

Given the assumption that price of output moves gradually, it is reasonable to assume that P(t) is observed at time t. Thus by continuously accommodating the nominal money stock (the monetary instrument) to the predetermined price level, the monetary authorities can in effect control the real money stock m. In fact, it is analytically convenient to treat m rather than M as being the monetary control variable. Secondly, we assume that the monetary authority observes the nominal exchange rate instantaneously. Thus the relative price s(t) is observable to the policy maker at time t, and in fact the optimal monetary policies will be obtained as feedback solutions in terms of s.[4/] Thus the formal optimal stabilization problem is

$$\underset{m}{\text{Minimize}} \quad k|s(0) - s_0|^q + \frac{1}{2}\int_0^\infty [aY^2 + (1-a)\dot{C}^2]e^{-\beta t}dt \quad (4)$$

subject to (3a)-(3c).

Since the function describing the adjustment costs associated with the initial jump in the real exchange rate, i.e., $k|s(0) - s_0|^q$, is generally nondifferentiable at the point $s(0) = s_0$, the optimization problem specified by equations (3a)-(3c), (4) can most easily be solved by decomposing it into the following two subproblems:

Problem 1:

$$\text{Find } L_1 = \text{Min } k(s(0) - s_0)^q + \frac{1}{2}\int_0^\infty [aY^2 + (1-a)C'^2]e^{-\beta t}dt \quad (5a)$$

subject to equations (3a)-(3c) and $s(0) \geq s_0$

Problem 2:

$$\text{Find } L_2 = \text{Min } k(s_0 - s(0))^q + \frac{1}{2}\int_0^\infty [aY^2 + (1-a)\dot{C}^2]e^{-\beta t}dt \qquad (5b)$$

subject to equations (3a)-(3c) and $s_0 \geq s(0)$

The solution to the original problem is then a solution for which $s(0) \geq s_0$ is $L_1 \leq L_2$ or a solution for which $s_0 \geq s(0)$ if $L_2 \leq L_1$.

4. DETERMINATION OF OPTIMALITY CONDITIONS

To solve the optimization problem specified by both Problems 1 and 2, we first write down the Hamiltonian function:

$$H \equiv \frac{1}{2} e^{-\beta t}[a[\phi_1 m + \phi_2 s]^2 + (1-a)[\eta_1 m + \eta_2 s]^2]$$
$$+ \mu e^{-\beta t}[\dot{s} - \theta_1 m - \theta_2 s] \qquad (6)$$

where $\mu e^{-\beta t}$ is the discounted Lagrange multiplier associated with the dynamic equation (3c). The Euler equations with respect to m and s are then given by

$$\psi_{11}m + \psi_{12}s - \theta_1\mu = 0 \qquad (7a)$$

$$\psi_{12}m + \psi_{22}s + (\beta-\theta_2)\mu = \dot{\mu} \qquad (7b)$$

where

$$\psi_{ij} \equiv a\phi_i\phi_j + (1-a)\eta_i\eta_j \qquad i = 1, 2, \ j = 1, 2$$

These two equations, in conjunction with equations (3a)-(3c), then define the dynamic time paths for the optimal solutions for m and s, and hence for Y and $\dot{C}$.

Note further that the long-run solution is obtained by setting $\dot{s} = \dot{\mu} = 0$ in (7a), (7b). From this substitution we immediately see that in long-run equilibrium $m = s = 0$. Consequently, the long-run dynamic time path of the economy converges to $Y = \dot{C} = 0$.

In addition, the optimal solution must satisfy the following transversality condition as $t \to \infty$

$$\lim_{t\to\infty} s\mu e^{-\beta t} = 0 \tag{8}$$

Furthermore, the fact that $s(0)$ is endogenously determined imposes the following constraints on the initial time point 0, see, e.g., Kamien and Schwartz (1971):

<u>For Problem 1</u>:

$$\text{If } s(0) > s_0 \text{ then } \mu(0) = kq(s(0) - s_0)^{q-1} \tag{9a}$$

$$\text{If } s(0) = s_0 \text{ then } \mu(0) \leqq kq(s(0) - s_0)^{q-1} \tag{9b}$$

<u>For Problem 2</u>:

$$\text{If } s_0 > s(0) \text{ then } \mu(0) = -kq(s_0 - s(0))^{q-1} \tag{10a}$$

$$\text{If } s_0 = s(0) \text{ then } \mu(0) \geqq -kq(s_0 - s(0))^{q-1} \tag{10b}$$

5. TRANSITIONAL DYNAMICS

The main item of interest is the transitional dynamics implied by the optimal solutions (7a), (7b), together with (3a)-(3c). We begin by solving (7a) for m, namely,

$$m = \frac{\theta_1\mu - \psi_{12}s}{\psi_{11}} \tag{11}$$

Next, substituting for m from (11) into (7b) and (3c) the dynamic solution reduces to the following pair of equations in μ and s

$$\begin{pmatrix} \dot{\mu} \\ \dot{s} \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} \mu \\ s \end{pmatrix} \tag{12}$$

where

$$h_{11} = \beta + \frac{\psi_{12}\theta_1 - \psi_{11}\theta_2}{\psi_{11}} \qquad h_{12} = \frac{\psi_{11}\psi_{22} - \psi_{12}^2}{\psi_{11}} > 0$$

$$h_{21} = \frac{\theta_1^2}{\psi_{11}} > 0 \qquad h_{22} = \frac{\theta_2\psi_{11} - \theta_1\psi_{12}}{\psi_{11}}$$

The eigenvalues of this system satisfy the quadratic equation

$$\lambda^2 - (h_{11} + h_{22})\lambda + h_{11}h_{22} - h_{12}h_{21} = 0$$

i.e.,

$$\lambda^2 - \beta\lambda + K = 0 \tag{13}$$

where

$$K \equiv h_{11}h_{22} - h_{12}h_{21}$$

$$= \beta h_{22} - (h_{22}^2 + h_{12}h_{21})$$

Hence,

$$K \lesssim 0, \text{ if either } h_{22} < 0$$

$$\text{or if } h_{22} > 0, \text{ and } \beta < (h_{22}^2 + h_{12}h_{21})/h_{22} \tag{14a}$$

$$K > 0, \text{ if } h_{22} > 0 \text{ and } \beta > (h_{22}^2 + h_{12}h_{21})/h_{22} \tag{14b}$$

The roots λ_1, λ_2, or (13) are given by

$$\lambda_1, \lambda_2 = \frac{1}{2}[\beta \pm \sqrt{\beta^2 - 4K}] \tag{15}$$

In particular, λ_1, λ_2 are always real and if we denote the larger by λ_1, then

$$\lambda_1 \gtreqqless \max[h_{11}, h_{22}] > \beta/2 > 0 \tag{16a}$$

$$\lambda_2 \lesseqqgtr \min[h_{11}, h_{22}] < \beta/2 \tag{16b}$$

Also, $\lambda_2 \gtrless 0$ according as $K \gtrless 0$. Hence it follows from (14a, 14b) that

$$\lambda_2 < 0, \text{ if } 0 < \beta < \beta^* \tag{17a}$$

$$\lambda_2 > 0, \text{ if } \beta > \beta^* \tag{17b}$$

where

$$\beta^* = \begin{cases} \infty, \text{ if } h_{22} < 0 \\ (h_{22}^2 + h_{12}h_{21})/h_{22}, \text{ if } h_{22} > 0 \end{cases}$$

In other words, the optimal solution will have two unstable roots if the policy maker is sufficiently myopic (i.e., β is sufficiently large); otherwise, the optimal solution will have one stable and one unstable root. The critical level of myopia, β^*, depends upon the parameters in the economy.

The solution to the dynamic system (12) can be expressed in the form

$$\begin{pmatrix} \mu \\ s \end{pmatrix} = \begin{pmatrix} h_{12} & h_{12} \\ \lambda_1 - h_{11} & \lambda_2 - h_{11} \end{pmatrix} \begin{pmatrix} n_1 e^{\lambda_1 t} \\ n_2 e^{\lambda_2 t} \end{pmatrix} \tag{18}$$

where n_1 and n_2 are arbitrary constants. Given (16a), (16b), in order for the transversality conditions (8) to hold, we require $n_1 = 0$ and hence the solution becomes

$$\hat{\mu}(t) = \mu(0)e^{\lambda_2 t} \tag{19a}$$

$$\hat{s}(t) = \left(\frac{\lambda_2 - h_{11}}{h_{12}}\right)\hat{\mu}(t) \tag{19b}$$

where $\mu(0)$ satisfies the initial conditions given by equations (9), (10) and $\hat{\mu}$, $\hat{s}$ denote optimal values.

It will be recalled from the previous discussion that if the policy maker is sufficiently myopic (i.e., if β is sufficiently large) then $0 < \lambda_2 < \beta/2$, and the path derived from the optimality conditions is unstable. This is a plausible outcome, because when the policy maker is myopic much more weight is assigned to initial values along the optimal path. In the extreme case when $\beta > \beta^*$, the policy maker is so unconcerned with these later values that it is of little consequence for the loss function that the optimal path diverges.

When the optimal path diverges, then given that an optimal path has been chosen at time 0, say, it will always be optimal to revise the initial path at some future time $t = t_0$ say. This property of the optimal solution, which is known as time inconsistency, is clearly undesirable. In fact it can be argued that if all solutions to the optimization problem are time inconsistent, then no truly optimal solution will exist.[5/]

Throughout the rest of this paper we shall restrict analysis to the case where the optimal path converges (i.e., $\lambda_2 < 0$). As will be shown in the next section, for a large range of cases, such a path is consistent with a time-path that would be chosen at any time in the future, and hence is time consistent.

Equations (19a), (19b) describe the time path of the real exchange rate, s, and the costate variable, μ, along the optimal path that will be followed after an initial jump in the exchange rate. Combining (19b) with (11), and noting the definitions of h_{ij}, the real money stock along the optimal path is related to the real exchange rate by the simple feedback rule

$$\hat{m}(t) = \left(\frac{\lambda_2-\theta_2}{\theta_1}\right)\hat{s}(t) \qquad t > 0 \tag{20}$$

Rules of the form (20) characterize much of the current discussion of exchange market intervention; see Boyer (1978), Cox (1980), Turnovsky (1983) and papers in Bhandari (1985). Since $\theta_1 < 0$, and assuming the more plausible case where $\lambda_2 < 0$, $\theta_2 > 0$, we see that the coefficient of $\hat{s}$ in (20) is positive. This means that following the initial jump in the exchange rate, it will be optimal for the policy maker to respond to a depreciating real exchange rate by increasing the real money supply; i.e., to "lean with the wind." Note that by using the definitions of m and s, the rule can be expressed equivalently in nominal terms as

$$\hat{M}(t) = \left(\frac{\lambda_2-\theta_2}{\theta_1}\right)\hat{E}(t) + \left(\frac{\theta_1+\theta_2-\lambda_2}{\theta_1}\right)\hat{P}(t) \tag{20'}$$

In a similar fashion we can express the optimal time paths followed by output and inflation in terms of the time path followed by the real exchange rate. These are derived by substituting (20) and (19b) into (3a), (3b), respectively, to yield[6/]

$$\hat{Y}(t) = \left(\frac{\phi_1\lambda_2+\theta_1\phi_2-\theta_2\phi_1}{\theta_1}\right)\hat{s}(t) \equiv \omega_1\hat{s}(t) \tag{21a}$$

$$t > 0$$

$$\dot{\hat{C}}(t) = \left(\frac{\eta_1\lambda_2+\theta_1\eta_2-\theta_2\eta_1}{\theta_1}\right)\hat{s}(t) \equiv \omega_2\hat{s}(t) \tag{21b}$$

Recalling the definitions of ϕ_i, θ_i, we find $\theta_1\phi_2 - \theta_2\phi_1 < 0$. Thus for $\lambda_2 < 0$, the coefficient $\omega_1 > 0$. That is, along the optimal path domestic output increases with the real exchange rate. On the other hand, ω_2 in (21b) is ambiguous. Differentiating (1e) with respect to t and noting the definition of s, we may express (21b) as

$$\begin{aligned}\dot{\hat{C}} &= \dot{\hat{P}} + (1-\delta)\dot{\hat{s}} \\ &= [\gamma\omega_1 + (1-\delta)\lambda_2]\hat{s}\end{aligned} \tag{21b'}$$

enabling us to see the source of the ambiguity. On the one hand, $\dot{C}$ responds positively through the inflation of domestic goods to domestic output and hence to the relative price s; on the other hand, it is negatively related through changes in $\dot{s}$ to s. The net effect depends upon which of these two effects dominate.

Combining equations (21a), (21b), we observe that the optimal paths for output and the rate of inflation of the CPI lie along the straight line

$$\dot{\hat{C}}(t) = \left(\frac{\eta_1\lambda_2+\theta_1\eta_2-\theta_2\eta_1}{\phi_1\lambda_2+\theta_1\phi_2-\theta_2\phi_1}\right)\hat{Y}(t) \equiv \frac{\omega_2}{\omega_1}\hat{Y}(t) \tag{22}$$

It is clear from the above observations that this locus may be either positively or negatively sloped depending upon whether $\omega_2 \gtrless 0$. Finally, substituting (20) into (3c) and noting (21a), (21b), we see that along the optimal path all variables converge at the same rate λ_2 to their respective steady states.

6. ENDOGENOUS INITIAL JUMPS IN EXCHANGE RATE

Having determined the dynamic time path of the economy following the initial jump in the exchange rate, all that remains to determine the solution for the optimal time path is to determine the initial condition for the exchange rate, namely, s(0), itself.

By combining the transversality conditions at time 0, given by equations (9) and (10), together with the optimality conditions (19a), (19b), and the comparison of the loss functions for Problems 1 and 2, we find that the initial conditions for s(0) for the optimization problem are given by

$$s(0) = -\left(\frac{h_{11}-\lambda_2}{h_{12}}\right)kq(s(0)-s_0)^{q-1} \text{ , if } s(0) > s_0 \tag{23a}$$

$$-\left(\frac{h_{11}-\lambda_2}{h_{12}}\right)kq(s(0)-s_0)^{q-1} \leqq s(0) \leqq \left(\frac{h_{11}-\lambda_2}{h_{12}}\right)kq(s_0-s(0))^{q-1} \text{ , if } s(0) = s_0 \tag{23b}$$

$$s(0) = \left(\frac{h_{11}-\lambda_2}{h_{12}}\right)kq(s_0-s(0))^{q-1} \text{ , if } s_0 > s(0) \tag{23c}$$

The nature of the endogenous jumps in the exchange rate depends crucially upon the form of the adjustment costs as represented in (2'). Specifically, the time consistency of the initial jump depends upon the magnitude of q, while the magnitude of the initial jump depends upon the inherited real exchange rate s_0, together with the cost parameter k. It is convenient to consider the three cases $q = 1$, $0 < q < 1$, $q > 1$ separately.

Case 1: $q = 1$

In this case conditions (23a)-(23c) reduce to

$$s(0) = -\left(\frac{h_{11}-\lambda_2}{h_{12}}\right)k \ , \text{ if } s(0) > s_0 \tag{24a}$$

$$-\left(\frac{h_{11}-\lambda_2}{h_{12}}\right)k \leqq s(0) \leqq \left(\frac{h_{11}-\lambda_2}{h_{12}}\right)k \ , \text{ if } s(0) = s_0 \tag{24b}$$

$$s(0) = \left(\frac{h_{11}-\lambda_2}{h_{12}}\right)k \ , \text{ if } s_0 > s(0) \tag{24c}$$

It can be deduced from these three equations that if the inherited real exchange rate s_0 lies in a specific closed interval bounded by

$$-\left(\frac{h_{11}-\lambda_2}{h_{12}}\right)k \leqq s_0 \leqq \left(\frac{h_{11}-\lambda_2}{h_{12}}\right)k \tag{25}$$

then it will be optimal for no initial jump in the real exchange rate s to occur.[7/] Since any initial jump in the real exchange rate is achieved through a jump in the nominal rate (P moves continuously throughout) this means that if (25) holds, it will be optimal for no jump in the nominal exchange rate to occur. However, if s_0 lies outside this interval, then the nominal exchange rate will jump so that the real exchange rate s moves instantaneously to the nearest boundary of this closed interval.

The boundaries of the closed interval defined in (25) depend critically upon k, the magnitude of the cost associated with the initial jump. If $k \to \infty$, then it is clear that it is never optimal to have an initial jump in the exchange rate E. On the other hand, if $k \to 0$, then the closed interval (25) collapses to 0, and it will be optimal for the real exchange rate to always jump instantaneously to its steady state equilibrium value, (which we have chosen to be zero). Thirdly, the boundaries of the closed interval are symmetric about the steady-state equilibrium level $\tilde{s} = 0$.

Once the initial condition for the optimal time path for the real exchange rate s has been determined, equations (19a), (19b) show that s will then converge monotonically towards its steady state level, whenever β is sufficiently small to ensure $\lambda_2 < 0$. Since the steady state $\tilde{s} = 0$ is always within the boundaries of the closed interval defined by (25), it will not be optimal for the exchange rate to jump again after the initial jump. In the case $q = 1$, therefore, the optimal policy will clearly be time consistent.

This example of a time consistent solution is illustrated in Figure 1a. Initially, the real exchange rate is at $s_0 < -(h_{11}-\lambda_2)k/h_{12}$. At the initial instant that optimization begins, the jump in the nominal exchange rate causes the real exchange rate to jump to $s(0) = -(h_{11}-\lambda_2)k/h_{12}$ and thereafter it begins to converge monotonically towards equilibrium (zero).

Case 2: $0 < q < 1$

In this case condition (23b) reduces to

$$-\infty < s_0 = s(0) < \infty \qquad \text{(23b')}$$

Thus if $s_0 < 0$, then either $s(0)$ satisfies (23a) or $s(0) = s_0$. As in the case $q = 1$, it may be optimal either for an initial jump in s to occur, or for no initial jump in s to occur, depending upon the inherited value of the exchange rate, as well as the adjustment cost parameters k and q, and other parameters in the economy. Note that in contrast to the case $q = 1$, where the point to which $s(0)$ jumps (when it is in fact optimal to jump) is independent of the inherited value s_0 (see (24a) or (24b)), in the present case $s(0)$ is in part determined by s_0; see (23a), (23b). It can further be shown that no subsequent jumps beyond the initial point will be optimal, although details of this are omitted.[8/] Thus for $0 < q < 1$, the solution will be time consistent and the optimal path will be of the same general form to that for $q = 1$; i.e., a possible initial jump followed by convergence towards equilibrium.

Case 3: $q > 1$

In this case inequalities (23b) reduce to

$$s_0 = s(0) = 0 \tag{23b''}$$

If $|s_0| > 0$, then (23a), (23c) together imply[9/]

$$|s_0| > |s(0)| > 0 \tag{26a}$$

But then if after the initial jump, the policy maker immediately reoptimizes, choosing the initial condition after the second optimization to be given by $s(0+)$, then

$$|s_0| > |s(0)| > |s(0+)| > 0 \tag{26b}$$

Hence $s(0+) \neq s(0)$, unless $s_0 = 0$. Thus, when $q > 1$, the recomputed path is in general discontinuous and the optimal solution is in general time inconsistent.

An example of this time inconsistent case is given in Figure 1b. At the beginning of the optimization period, the real exchange rate s jumps from s_0 to $s(0)$. If the policy maker reoptimizes at time t_0, then it will be optimal for the exchange rate to jump again at that time.

An intuitive explanation of these results can be seen by noting that the marginal cost of a jump in the exchange rate is given by

$$kq|s(0) - s_0|^{q-1}$$

When $0 < q < 1$, the marginal cost of a jump decreases with the magnitude of the jump. Thus it is "cheaper" to have one large jump than two small ones. On the other hand, when $q > 1$, the marginal cost of a jump increases with the magnitude of the jump. In this case two smaller jumps are less costly than a single large one.[10/]

7. SOLUTION FOR OPTIMAL TIME PATH

We are now in a position to construct the optimal time paths for output and inflation. This involves a consideration of the optimal transitional paths given in (19a,19b), (20), and (21a,21b), as well as the endogenous initial jumps in the exchange rate described in Section 6. Throughout this section we shall restrict discussion to time consistent solutions. We shall also consider the case $q = 1$, for which $s(0)$ when it jumps, does so to a point independent of s_0. The case $q < 1$ is essentially similar, the only difference being that $s(0)$ is a function of s_0, as seen from (23a).

We assume that initially the economy starts from a point having an arbitrary real money stock m_0 and real exchange rate s_0. Corresponding to these values, (3a), (3b) imply initial values of output and inflation, namely,

$$Y_0 = \phi_1 m_0 + \phi_2 s_0$$

$$\dot{C}_0 = \eta_1 m_0 + \eta_2 s_0$$

For the sake of being concrete we shall assume that m_0, s_0, the inherited values, have been chosen such that $Y_0 < 0$, $\dot{C}_0 > 0$; i.e., the economy begins in a state of stagflation.

Typical optimal paths for inflation and output are illustrated in Figures 2A, 2B. In both these figures, the locus XX' defines the optimal adjustment path for output and CPI inflation described by equation (22). It has been drawn on the assumption that the domestic price inflation responds slowly to output changes (i.e., γ is small), so that the coefficient ω_2 in (21b) is negative. This implies that XX' has a negative slope. Given our choice of units, the steady state of the economy is at the point where this line passes through the origin.

The appropriate initial points on the line XX' lie within the closed interval defined by AB. In effect, this locus is the analogue of the closed set derived from (23b) in Y-$\dot{C}$ space. Within this set no initial jump in E (or s) occurs. If the economy starts outside this set it must move towards the nearer endpoint A or B. The coordinates of these points are obtained by substituting for $\mu(0)$ from the initial conditions (9), (10) into (19) and (21), at time t = 0.

The loci Z_1Z_1', Z_2Z_2' (as well as WS, W'S') have slopes given by

$$\left(\frac{d\dot{C}}{dY}\right)_{ZZ'} = \frac{\eta_1}{\phi_1} \tag{27}$$

where the partial derivatives ϕ_1, η_1, are obtained from (3a), (3b). Basically, these derivatives define the slopes of the lines along which inflation and output will move instantaneously in response to an initial change in the money supply, without taking account of the jump in E which such a change may or may not induce (depending upon adjustment costs). Since $\phi_1 > 0$, its slope depends upon whether $\eta_1 \gtrless 0$.

It is seen from its definition that η_1 has two effects. On the one hand, an increase in m will raise output, thereby increasing the rate of inflation of domestic goods and hence that of the overall CPI. On the other hand, the monetary expansion will tend to lower the domestic interest rate and hence the rate of exchange depreciation, thereby lowering the rate of inflation of the CPI. We take the case where the latter effect dominates, so that the Z_iZ_i' lines are negatively sloped.[11/]

Figure 2 illustrates two cases. The first is where the initial output and inflation rate lie at a point such as V which lies outside the area defined by Z_1Z_1' and Z_2Z_2'. The second is where it is initially at W, which lies within the area. We shall consider these in turn.

Suppose that the economy is initially at V, with $Y_0 < 0$ and a positive rate of inflation $\dot{C}_0 > 0$. The initial adjustment involves moving the system instantaneously to A on the optimal locus XX'. This jump implies an instantaneous increase in real output ($dY(0) > 0$), accompanied by an instantaneous reduction in the rate of inflation ($d\dot{C}(0) < 0$). To see the adjustments involved, we consider equations (3a), (3b). Letting $dX(0) \equiv X(0) - X_0$, we have

$$dY(0) = \phi_1 dm(0) + \phi_2 ds(0)$$

$$d\dot{C}(0) = \eta_1 dm(0) + \eta_2 ds(0)$$

Solving these equations for dm(0), ds(0), yields

$$dm(0) = \frac{\eta_2 dY(0) - \phi_2 d\dot{C}(0)}{\phi_1\eta_2 - \phi_2\eta_1} \tag{28a}$$

$$ds(0) = \frac{-\eta_1 dY(0) + \phi_1 d\dot{C}(0)}{\phi_1\eta_2 - \phi_2\eta_1} \tag{28b}$$

From the definitions of ϕ_i, η_i, we can readily show $\phi_1\eta_2 - \phi_2\eta_1 > 0$. It then follows that with $dY(0) > 0$, $d\dot{C}(0) < 0$, $\phi_2 > 0$, $\eta_2 > 0$,

$$dm(0) > 0 \tag{29a}$$

Also, since V lies to the left of Z_1Z_1', we have

$$\frac{d\dot{C}(0)}{dY(0)} > \left(\frac{d\dot{C}}{dY}\right)_{ZZ'} = \frac{\eta_1}{\phi_1}$$

implying

$$\phi_1 d\dot{C}(0) - \eta_1 dY(0) > 0$$

so that

$$ds(0) > 0 \tag{29b}$$

Thus the movement from V to A is achieved by an initial monetary expansion. This gives rise to a depreciation of the real exchange rate, which with the price of domestic output sluggish, is brought about by a depreciation of the nominal rate. The expressions for the initial changes in s(0) and m(0) are obtained from (24) and (20), namely,

$$ds(0) = \left(\frac{\lambda_2 - h_{11}}{h_{12}}\right)k - s_0 > 0 \tag{30a}$$

$$dm(0) = \left(\frac{\lambda_2 - \theta_2}{\theta_1}\right)\left(\frac{\lambda_2 - h_{11}}{h_{12}}\right)k - m_0 > 0 \tag{30b}$$

It follows further from (30) that

$$s_0 < s(0) < 0 \tag{30a'}$$

$$m_0 < m(0) < 0 \tag{30b'}$$

Consequently, the original stagflation illustrated by V is the result of a low real money supply, coupled with a value of the real exchange rate which is overvalued compared with its long-run equilibrium value.

In Figure 2A we have also drawn the locus VV', the slope of which is

$$\left(\frac{d\dot{C}}{dY}\right)_{VV'} = \frac{\eta_2}{\phi_2} \tag{31}$$

This represents the relative effects of an exchange rate change on the rate of inflation and level of output, with the nominal (and real) money supply held constant. Thus the instantaneous jump from V to A can be decomposed into a jump due to the change in the exchange rate (along VV'), coupled with a jump due to the monetary expansion (along V'A), both of which occur simultaneously at the beginning of the optimization.

Letting $(d\dot{C}/dY)_{XX'}$ denote the slope of the inflation-output tradeoff given by XX', we can show by substitution that

$$\left(\frac{d\dot{C}}{dY}\right)_{VV'} > \left(\frac{d\dot{C}}{dY}\right)_{XX'} > \left(\frac{d\dot{C}}{dY}\right)_{ZZ'} \tag{32}$$

Figures 2A and 2B are drawn with the respective slopes satisfying (32).

Despite the initial depreciation of the exchange rate associated with the move from V to A, the domestic currency remains overvalued relative to its long-run equilibrium. Further devaluation is required. With $s(0) < 0$, the monetary authorities set $m(0) < 0$, in accordance with the optimal rule (20). As a result, the domestic exchange rate immediately continues to depreciate; i.e., $\dot{s}(0) = \lambda_2 s(0) > 0$. This causes output to continue increasing and with $\omega_2 < 0$, for CPI inflation to continue decreasing. Along the optimal path, the real exchange s continues to depreciate, although at a decreasing rate, as steady state is approached. This adjustment is mirrored by the money supply. Likewise, output and inflation continue to follow the optimal path XX' until the steady-state equilibrium $Y = \dot{\tilde{C}} = 0$ is reached.

It is also possible to consider the case where the domestic price inflation effect in (21b') dominates, so that the coefficient $\omega_2 > 0$. In this case XX' is positively sloped. The exposition remains similar to the above except that after the jump in the exchange rate, the economy will be driven to a level of deflation ($\dot{C}(0) < 0$), as well as unemployment. In this case the effect of the monetary expansion along the optimal path on output will have a greater effect on domestic prices than before. The added demand now induces inflationary pressure leading to a gradual moderation in the deflation initially generated.

We turn now to the second case illustrated in Figure 2b, where the economy starts out at W which lies within the area bounded by Z_1Z_1' and Z_2Z_2'. In this case it will be optimal for no initial jump in the exchange rate to occur and the optimal path will be reached by means of an initial expansion in the money supply alone. In this case the move is along WS which involves an increase in output accompanied by a decrease in inflation. Thereafter, output and inflation follow XX' as before. Note that it is possible for the increase in output resulting from the initial monetary expansion to take the economy beyond the steady state level 0. In this case, the economy will reduce output and depending upon the magnitude of ω_2, inflate or deflate along the optimal path XX'. This is illustrated by the move along W'S' from the initial point W' in Figure 2b.

Finally, Figure 3 illustrates the implied time paths for the levels of CPI and output (drawn for $\omega_2 < 0$). The relationship between these paths and those in Figure 2 is self-explanatory.

8. FURTHER DISCUSSION

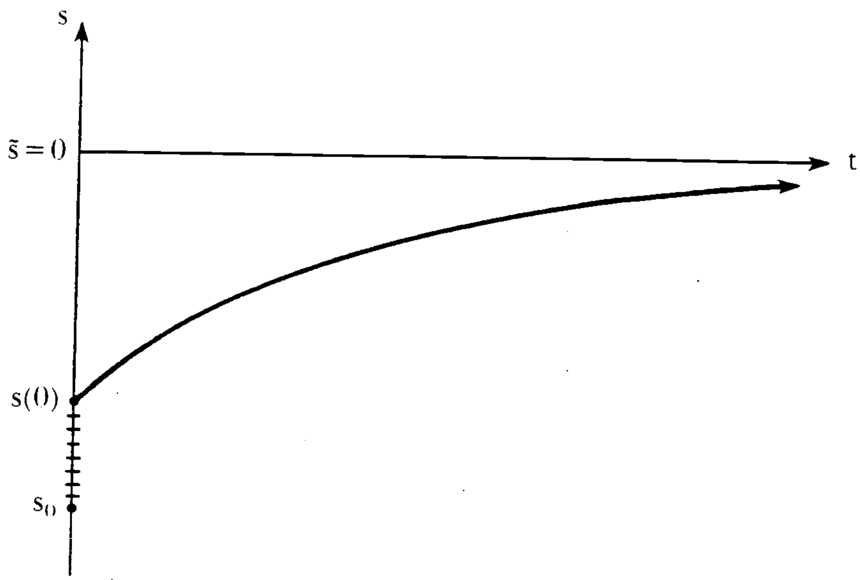
In this paper we have analyzed the optimal intertemporal tradeoff between inflation and output in an open economy which is characterized by perfect foresight in the formation of exchange rate expectations. The announcement of the optimal plan, may or may not, generate an initial unanticipated jump in the exchange rate. That depends upon the real adjustment costs, which such unanticipated changes impose on the economy. In the case that such jumps do occur, the question of time consistency of the optimal path arises. We have obtained a time consistent solution, provided: (i) the policy maker is not too myopic; (ii) the adjustment costs associated with the unanticipated jump in the exchange rate are of an appropriate, but reasonable, form.

The optimal monetary rule can be expressed in feedback form with the real money supply being adjusted to the real exchange rate. In most cases the relationship is a positive one, implying a "leaning with the wind" policy. The corresponding adjustments in output and inflation can also be expressed in terms of the real exchange rate. Accordingly, along the optimal path the relationship between output and inflation is a linear one. This may be either positively or negatively sloped, depending upon whether the CPI inflation rate is positively or negatively related to the real exchange rate.

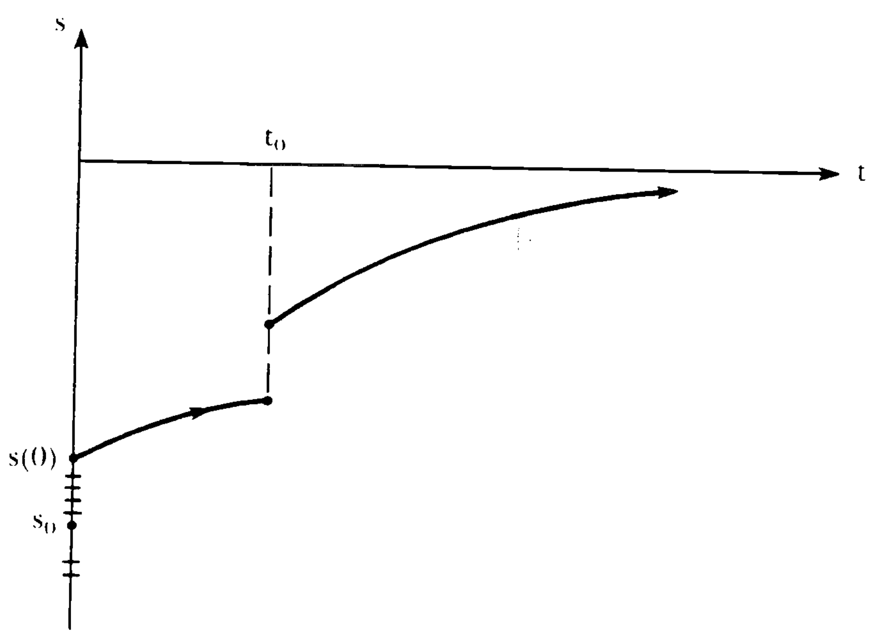
We have analyzed in some detail the case where the economy starts from stagflation; output is below full employment equilibrium, while the inflation rate is positive. In the case that it is optimal for the exchange rate to undergo an initial jump, the optimal monetary policy calls for an initial expansion, thereby generating an initial increase in output and in the <u>level</u> of the CPI, followed by an instantaneous reduction in its rate of inflation. Thereafter, as the economy moves along the optimal path, the monetary growth remains positive, but declining, so that output and inflation converge to their respective equilibria. If the costs associated with the initial jump in the exchange rate are sufficiently high, there will be no initial jump, and the optimal output-inflation tradeoff path is reached by means of a change in the money supply alone. The exchange rate then always moves continuously.

An important aspect of our analysis concerns the specification of the Phillips curve, (1d), in which the price of output adjusts to deviations in output alone. If, on the other hand, the price adjusts fully to changes in the CPI, as the result of some wage indexation scheme, say, our results are changed somewhat. In particular, the stable loci XX' in Figs 2A, 2B become horizontal, coinciding with the Y axis, implying that inflation can be reduced instantaneously to zero. At the same time, output still adjusts slowly, reflecting the initial adjustment costs.[12/]

A further interesting aspect of our results is the ability to generate time-consistent solutions. These are obtained by imposing costs on the initial jumps in the exchange rate and assuming these to be from a specified class of functions. Such costs are introduced on the grounds that unanticipated changes in the exchange rate impose real disturbances and hence real adjustment costs on the economy. The feature of the specification which generates time consistency is the fact that the marginal cost of a jump in the exchange rate decreases with the size of the jump, thus making it optimal to make an initial large jump and discouraging any subsequent jumps. By choosing a general class of adjustment cost functions, we have demonstrated that the phenomenon of time consistency is not simply the result of choosing a particular arbitrary cost function. Furthermore, the procedure used here has general applicability and may prove to be useful in other stabilization contexts.

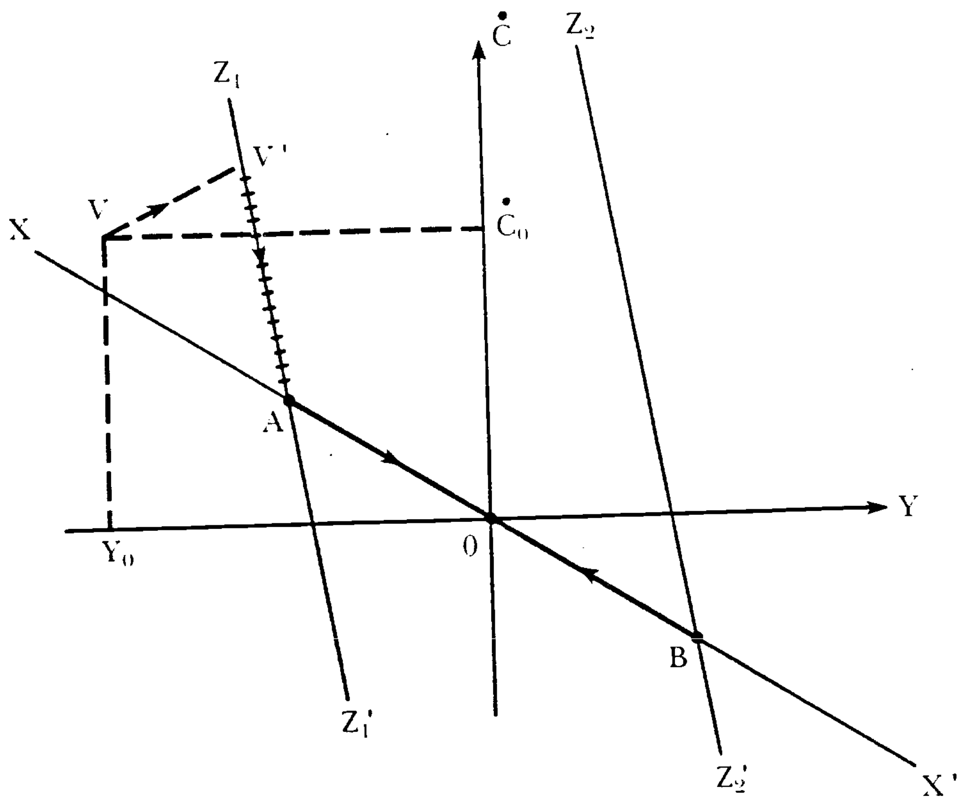


A. Time-consistent Solution ($q \leq 1$)

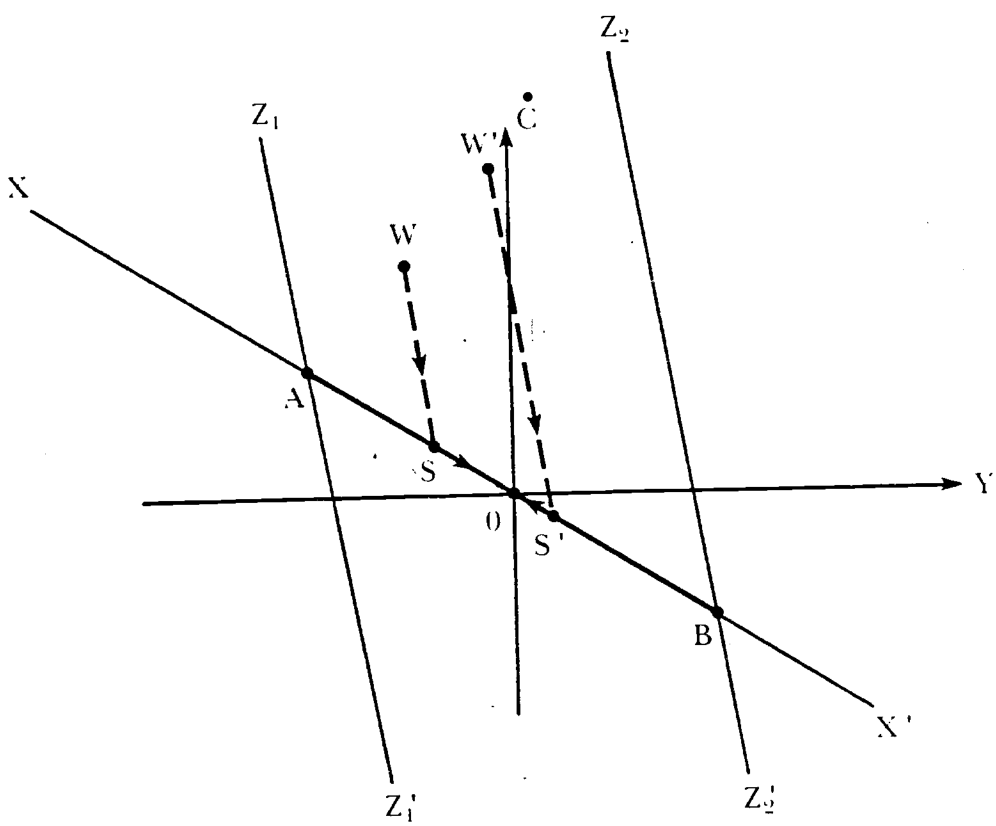


B. Time-inconsistent Solution ($q > 1$)

Figure 1. Optimal Time Paths for Real Exchange Rate

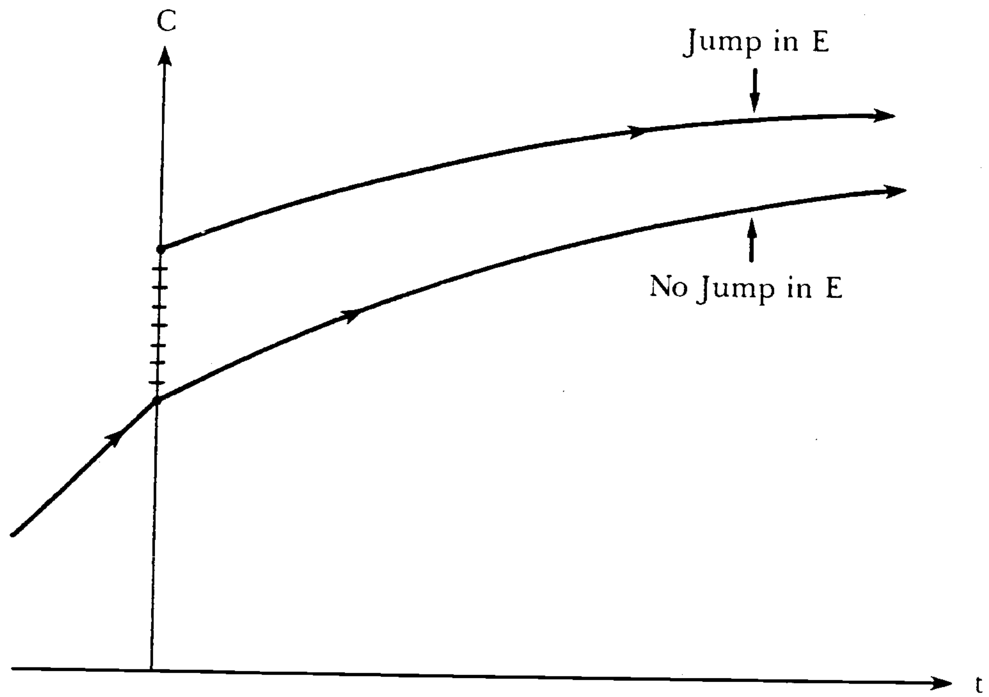


A. Jump in Exchange Rate

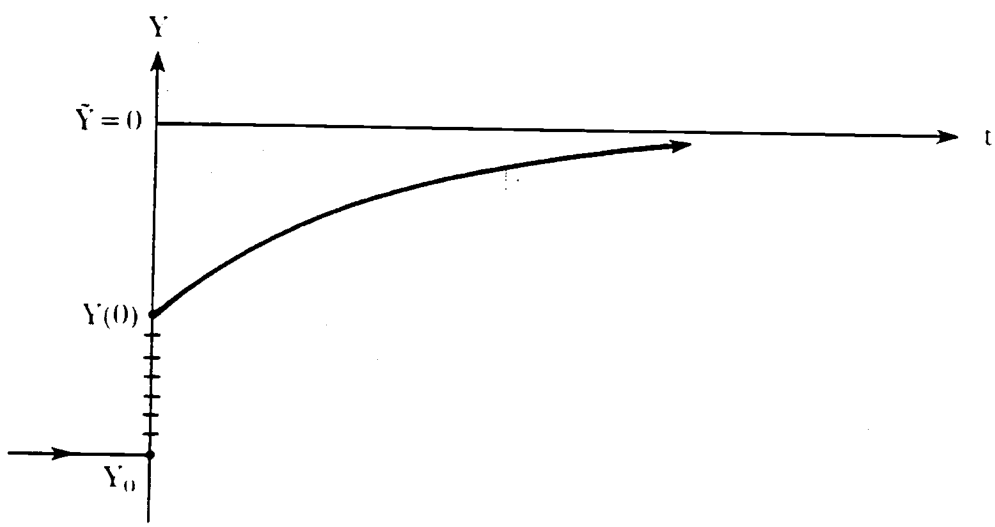


B. No Jump in Exchange Rate

Figure 2. Optimal Output-Inflation Tradeoff



A. CPI



B. Output

Figure 3. Time Paths for CPI and Output

FOOTNOTES

1/ For some simulations of optimal monetary policies in an open economy see Driffill (1982).

2/ The issue of time consistency within the context of linear-quadratic dynamic control models, with specific reference to problems of international macro policy, is also discussed by Miller and Salmon (1984).

3/ The reason for specifying the costs in terms of the absolute value is simply to ensure that they are always nonnegative for all $q > 0$.

4/ We ignore informational asymmetries, i.e. the possibility that E may be observed before P or even H.

5/ This result follows because if the optimal solution is time inconsistent, there is no optimal time period between endogenous jumps.

6/ Combining (20) with (21a) or (21b) it is also possible to express the optimal monetary rule in terms of Y or $\dot{C}$. However, these forms are less appealing since information on current exchange rate is generally more readily available than information on current output or inflation.

7/ This can be established by the following simple argument. Suppose s_0 lies in the closed interval (25), and s_0 jumps to $s(0) > s_0$, say. Then from (24a), $s(0) = -(h_{11}-\lambda_2)k/h_{12}$. Hence it follows that $-(h_{11}-\lambda_2)k/h_{12} > s_0$, inconsistent with s_0 lying in the closed interval (25). The case where s_0 jumps to $s(0) < s_0$ can be argued similarly.

8/ The detailed arguments are available from the authors on request.

9/ Inequality (26a) can be established by contradiction. Suppose $s_0 > 0$. Then either $s(0) > s_0 > 0$, or $s(0) < s_0$. If $s(0) > s_0$, then (24a) implies $s(0) < 0$ and hence $s(0) < s_0$, which is a contradiction. Thus $s(0) < s_0$, in which case (23c) implies $s_0 > s(0) > 0$ implying (26a). The case where $s_0 < 0$ can be reasoned similarly.

10/ Equivalently, these conclusions follow from the mathematical property that for $a > 0$, $b > 0$,

$$(a + b)^q > a^q + b^q \qquad q > 1$$

$$(a + b)^q \leq a^q + b^q \qquad q \leq 1$$

11/ It is important to emphasize that $\eta_1 < 0$ does not necessarily mean that a monetary expansion will necessarily lead to a lower CPI inflation rate. This is because it measures only a partial effect and does not take account of any induced jump in the exchange rate. In fact, one can show that an increase in m alone will (in the absence of adjustment costs) lead to a devaluation of the exchange rate. This devaluation in the exchange rate tends to increase the domestic inflation rate $\dot{P}$, but as is known from the Dornbusch model, this can lead to a decline in $\dot{E}$, the rate of depreciation of the nominal exchange rate. Since $\dot{C} = \delta\dot{P} + (1-\delta)\dot{E}$, the overall effect on the CPI inflation rate is in general ambiguous.

12/ We are grateful to a referee for drawing this to our attention. Actually, a previous version of this paper analyzes the case of an expectations-augmented Phillips curve in detail.

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