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AGENCY COSTS, COLLATERAL, AND BUSINESS FLUCTUATIONS

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ABSTRACT

Bad economic times are typically associated with a high incidence of financial distress, e.g., insolvency and bankruptcy. This paper studies the role of changes in borrower solvency in the initiation and propagation of the business cycle. We first develop a model of the process of financing real investment projects under asymmetric information, extending work by Robert Townsend. A major conclusion here is that when the entrepreneurs who borrow to finance projects are more solvent (have more "collateral"), the deadweight agency costs of investment finance are lower. This model of investment finance is then embedded in a dynamic macroeconomic setting. We show that, first, since reductions in collateral in bad times increase the agency costs of borrowing, which in turn depress the demand for investment, the presence of these financial factors will tend to amplify swings in real output. Second, we find that autonomous factors which affect the collateral of borrowers (as in a "debt-deflation") can actually initiate cycles in output.

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1. Introduction

Bad economic times are typically associated with a high incidence of financial distress, as measured in increased defaults, bankruptcies, and business failures, and in the general deterioration of firm balance sheets. One might argue that these financial problems are merely passive reflections of adverse developments on the real side of the economy and therefore do not require special attention. But such a view is, in our opinion, unsupportable: First, it ignores the central allocative role played by the financial system in modern capitalist economies. Second, as a matter of theory, it appears impossible to explain why we have institutions such as noncontingent debt and bankruptcy (as opposed to Arrow-Debreu contingent contracts) without invoking asymmetric information or similar factors; such considerations having been admitted, however, there is a strong presumption that they will constrain the set of equilibria attainable by the economy. Thus, financial factors should indeed "matter," i.e., affect real outcomes.

This paper develops a theoretical model in which financial conditions are not merely a "sideshow" to real activity but play a role in both the initiation and propagation of economic cycles. The basic argument is as follows: Under the usual assumption that individuals who organize physical investment projects ("insiders") have more information about their projects than do those from who they borrow to finance these projects ("outsiders"), the best feasible financial arrangements will typically entail some deadweight losses ("agency costs") relative to the first-best, perfect information equilibrium. Further, in standard models the result obtains that, the greater the quantity of unencumbered wealth ("collateral") that insiders can bring to the project, the lower will be the expected agency costs involved in financing the investment.

Thus, periods of financial distress, when borrowers have less unencumbered collateral, are also times of relatively high agency costs in investment.

At the macroeconomic level, the proposition that insider collateral and agency costs are inversely related has two significant implications. First, since insider collateral is likely to be procyclical (borrowers are more solvent during good times), there will be a decline in agency costs during booms and a rise during recessions. This will affect the cyclical pattern of investment and, therefore, the dynamics of the cycle itself. Second, shocks to insider collateral which occur independently of aggregate output will be an initiating source of real fluctuations. A striking example of this second implication is the "debt-deflation," first analyzed by Irving Fisher (1933): During a debt-deflation, because of an unanticipated fall in the price level (or, possibly, a fall in the relative price of borrowers' collateral, e.g., farmland), there is a decline in the quantity of insider collateral relative to debt obligations. This has the effect of making those individuals in the economy with the most direct access to investment projects suddenly uncreditworthy (i.e., the agency costs associated with lending to them are high). The resulting fall in investment has negative effects on both aggregate demand and aggregate supply.[1] A preliminary analysis of debt-deflation is given below as an illustration of the effects of a shock to borrower collateral.

The plan of the paper is as follows: Section 2 below studies the process of financing physical investment in a static, one-period model with asymmetric information. Here we build on the "costly state verification" model of Townsend (1979) (see also Gale and Hellwig (1985)). Townsend's approach has the virtue of simplicity: The only agency problem in his model is that insiders, who can directly observe the realized return to their projects, may mis-report this return to the outside investors. To keep the insiders honest and to make lending possible in equilibrium, the outsiders must commit themselves to "auditing" the insiders (that is, to determining the true return, at a fixed real cost) whenever the insiders announce an unusually unfavorable outcome. In this framework it is easy to demonstrate the basic result, that the expected agency costs (here, the costs of auditing) vary inversely with the collateral brought to projects by borrowers.[2] We also extend Townsend's work in two ways important for the subsequent macroeconomic analysis: first, by allowing for endogenous determination of the identities of insiders and outsiders and of the ratio of inside to outside finance; second, by explicitly considering the effects of variations in collateral.

Section 3 embeds the model of capital finance in a dynamic macroeconomic setting, specifically, in a stochastic variant of the Diamond (1965) overlapping generations model. We demonstrate that the counter-cyclicality of agency costs in capital finance induces a greater volatility in investment and output than would otherwise exist. We also show how the financial sector can be an independent source of shocks to the real economy; in particular, we find that a debt-deflation can be thought of as a downward movement in the IS curve. Conclusions are offered in Section 4.

Some recent interesting work related to ours has been done by Farmer [1984, 1985] and Williamson [1985]. Each of these papers, like ours, motivates a link between financial factors and macroeconomic behavior by postulating agency problems in loan markets; in particular, Williamson also uses the costly state verification approach. Our paper differs from the earlier work, however, in many significant details. One of these is our emphasis on changing levels of collateral as a factor in cyclical fluctuations. Another difference is the existence of a source of intrinsic dynamics (operating through the presence of physical capital) in our model, which permits an explicit consideration of the interaction of financial shocks and the propagation mechanism. Neither of these factors is present in the earlier analyses. (Other differences from the earlier literature are discussed below.)

It is also worth mentioning the relation of this work to our earlier paper, Bernanke-Gertler [forthcoming]. That paper, which also studied the macroeconomic role of financial factors, emphasized the macroeconomic importance of the quantity of collateral held by borrowers (in that case, borrowers were identified with banks). When borrowers in general are solvent, the deadweight costs of debt finance are less, and more resources can be devoted to productive investment. The present research yields a similar message, but in this case in a setting where (unlike the earlier paper), the collateral of borrowers is endogenous in the model.

2. A Model of Financial Structure and Investment

As a prelude to a more general macroeconomic analysis, this section develops a partial equilibrium, one-period model of the process by which investment projects are financed. As noted in the introduction, the analysis here is an application of the ideas in Townsend [1979]; see also Gale and Hellwig [1985] and Williamson [1985]. We emphasize that our interest is not in developing the richest possible analysis of the financial process, but in obtaining a simple model suitable for subsequent analysis in a macroeconomic setting.

We study an economy in which there are two goods, a capital good and an output good. Both goods are homogeneous and divisible. The output good can be transformed into the capital good by an investment technology. The investment technology itself comes in discrete, nondivisible units, called "projects".

Each project requires as input exactly one unit of the output good, which, we will assume, exceeds the quantity of resources available to any single individual. In addition, there is associated with each project a (variable) "setup cost", measured in units of the output good. The setup cost is meant to reflect the resources devoted to locating a viable project and becoming familiar with the details of its operation. The setup cost also plays an important role in the informational structure of our model, and will be discussed further momentarily.

Each individual investment project is to be thought of as a draw from an infinite pool of potential projects. Projects look the same ex ante, but differ in the quantity of capital they produce ex post. Denote the physical yield of given project by the random variable $\tilde{k}$, where $\tilde{k}$ is distributed continuously over the project pool. Let H(k) be the cumulative distribution function and $[0,\kappa]$ be the support of $\tilde{k}$. Without loss of generality (this simply defines the units in which capital is measured), normalize the average physical return over the population of projects to equal one. That is,

$$\int_0^\kappa k dH = 1 \tag{2.1}$$

For the purposes of conducting a preliminary static analysis, we also temporarily impose three further assumptions: 1) The economy contains a continuum of length one of identical, risk-neutral agents. 2) Each agent is endowed with S units of savings, in the form of the output good. 3) Each agent's objective is to invest his savings in such a way as to maximize the expected quantity of the capital good that is owned after all project returns are realized. (In the next section, output and savings will be endogenous, and agents' portfolio objectives will be derived from a utility maximization problem.)

In order to motivate a role for financial structure (that is, to render the Modigliani-Miller theorem inapplicable), we must depart in some way from the assumptions of complete information and perfect markets. To do this in a simple way, we adopt the assumption of Townsend [1979] that the actual (ex post) returns to each investment project are costlessly observable only by the entrepreneur(s) ("insiders") who operate that particular project. Other agents in the economy ("outsiders") can learn the realized returns of a project only through a public auditing technology, which absorbs γ units of the capital good when operated. As a number of earlier papers have shown, this ex post informational asymmetry leads naturally to an optimal financial contract between insiders and outsiders that looks like a standard debt contract, with the insiders as residual claimants.

The earlier literature has generally assumed that the division of the population into outside lenders and entrepreneurs is exogenous, and that the resources brought to each project by insiders are fixed. Here we propose to allow the determination of who becomes insiders and the quantity of insider equity to be endogenous. Thus we will allow projects to be operated by insider coalitions of arbitrary size n, where n will be a choice variable.[3] A coalition of n members that wishes to operate a given investment project must pay (in addition to the basic input cost of one unit of output) a setup cost of c(n) units of output. We assume the function c(.) to be twice continuously differentiable (recall that agents are represented by a continuum), with $c(0)>0$, $c'>0$, and $c''>0$. As mentioned above, incurring this setup cost permits the members of the insider coalition to locate and operate the project; very importantly, incurring this cost is also assumed to allow each insider to observe (privately) the actual return to the project. Thus an informational asymmetry exists (in the absence of public auditing) between insiders and potential outside investors.

Several comments are in order on the assumptions that we have made about the function $c(n)$. First, note that we are here strictly more general than the previous literature, which typically takes $c(1)=0$, $c(n)=\infty$, for $n>1$; i.e., it has usually been assumed that there is exactly one insider (whose identity is exogenously determined), and that no further insiders may be added. Second, $c'(n)$ must reasonably be positive, since if not, then every individual in the economy will (in equilibrium) become an insider in every project. Finally, unless $c''(n)>0$, as assumed, the equilibrium will involve only corner solutions with all lending done by insiders; see (2.10) below. Intuitively, the assumption that $c''(n)>0$ is the assumption that the marginal cost of insider finance is increasing. In a more general model, this might reflect risk-aversion or the agency costs of equity dilution. In the present model, a possible rationale is that the costs of coalition management are proportional to the number of bilateral relationships among insiders, which increases geometrically with n.

Optimal financial contracting. We now examine the optimal financial arrangements in this economy for the case where the decision to verify project outcomes via the public auditing technology is assumed to be a non-stochastic function of the returns announced by the insiders.[4]

Consider first the (traditional) case where n, the number of insiders, is fixed. If per capita savings are small relative to the required project input, the wealth of insiders may well be insufficient to finance the project; i.e., it may be that $nS < 1 + c(n)$. It is thus necessary to borrow from outsiders, in the amount of $1 + c(n) - nS$. The literature has shown (see, e.g., Williamson [1985]) that the form of the optimal contract between the insiders and outsiders is always representable as follows: There exists some return x, called the "no-default payment". When the insiders realize a return $k \geq x$, they are not audited and pay a return of x to outsiders. When $k < x$, the insiders are forced (by the non-negativity constraint) to "declare bankruptcy" (that is, accept auditing) and to forfeit all of the returns to the project to the outsiders. This arrangement is, of course, naturally interpretable as a debt contract with a probability of default.

The intuition underlying the form of the optimal contract is straightforward. With no stochastic auditing, all possible announcements by the insiders induce either the outcome "audit" or the outcome "no audit". The payment made in the no-auditing states must effectively be constant, since the insiders never have an incentive to announce a no-auditing state which has an associated payment higher than the minimum among no-auditing states; this minimum payment corresponds to the no-default payment x.

The reason that outsiders must receive all of the returns when there is auditing (which occurs when $k < x$) is as follows: With risk-neutral borrowers and lenders, a necessary condition for optimal contracting is that (subject to incentive compatibility constraints and the requirement that each lender expects a competitive return), expected auditing costs be minimized. If outside lenders receive less than the maximum possible return when there is auditing, then they must be compensated by receiving more when there is no auditing (i.e., the no-default payment x is higher). But raising the no-default payment makes the probability of auditing higher than the minimum feasible level, which cannot be optimal. Thus, outsiders receive all returns when there is auditing.

Note that, if auditing results were private information to the auditor, then a role would arise for zero-profit intermediaries to channel all funds between savers and projects. These intermediaries would internalize all auditing costs and, by holding perfectly diversified portfolios, could eliminate the need to be monitored by depositors (see Diamond (1984) and Williamson (1985)). For our purposes here, it suffices to assume that auditing results are public information. We also assume that outside lenders are able to commit in advance (say, by legal devices such as escrow accounts) to sharing the costs of auditing when bankruptcy occurs. These assumptions eliminate some motivations for intermediation that have become standard in the literature. However, in our setting individual outside investors will still have an incentive to diversify, in order to avoid the potential problem that the return to a given project might be so low as not to cover the auditing costs; for the economy as a whole, this constraint may be safely assumed never to bind, as profits from successful projects may be used to fund the auditing of unsuccessful ones. (Again, see Diamond [1984] and Williamson [1985].)

It should also be stressed that, while we describe γ as an "auditing cost", there are simple reformulations that would allow us to think of it as a cost of bankruptcy more generally (e.g., loss in productivity due to change in management or to unmonitored actions taken by insiders; see Gale and Hellwig [1985] or Farmer [1984]). These additional costs may be quite important, but since including them would not change our analysis qualitatively, we avoid unnecessary complication by not pursuing these here.

<u>Endogenizing the proportions of inside and outside finance</u>. We now allow for endogenous determination of the size of insider coalitions as a result of profit maximization and optimal financial contracting. Imposition of a zero-profit condition (which follows from the assumption of free entry into

"insidership") then allows us to find, for this one-period economy with exogenous savings and valuation of capital, the following variables: the no-default payment; the safe interest rate; the ratio of inside to outside finance; the default probability; and the number of projects in the economy. Let:

q = the value of next-period capital, relative to the output good; E(q) is the expected value of q at the time of investment

s = per capita contributions by insiders to the project, in terms of the output good

r = the safe rate of interest, in terms of the output good

Q(k) = total payments to insiders, as a function of realized return; measured in terms of the capital good

Given the results of the previous section, we can characterize the optimal contract with a variable number of insiders as the solution to the following programming problem (which may be thought of as the profit maximization problem of the "firm" that brings together investors and undertakes the project):

$$\max E(q)\left[\int_{x}^{K} (k - x)dH - \int_{0}^{K} Q(k)dH\right] \tag{2.2}$$

subject to

$$0 \leq s \leq S \tag{2.3}$$

$$\max[(k - x), 0] \geq Q(k) \geq 0 \tag{2.4}$$

$$E(q)[\int_0^K Q(k)dH] \geq rns \tag{2.5}$$

$$E(q)[\int_0^x (k - \gamma)dH + \int_x^K xdH] \geq r\,[1 + c(n) - ns] \tag{2.6}$$

where the maximization is with respect to s (the contribution of each insider), n (the number of insiders), x (the promised return if there is no default), and Q(k) (the payment schedule for insiders). The safe rate of return r will be determined endogenously below but is assumed to be taken as parametric by the framers of individual financial contracts.

Constraint (2.3) states that per capita insider contributions cannot exceed per capita savings. (Contributions can be less than savings; in principle, for example, an individual could be an insider in more than one project). (2.4) restricts total payments to insiders, in terms of the capital good, to be between zero and what remains after debt-holders are paid in each state. (2.5) and (2.6) require that each insider and outsider receives an expected return equal to the opportunity cost of his funds, as measured by the safe rate of return r. (Note that in taking expectations in (2.5) and (2.6) we use the independence of the aggregate valuation of capital and the return to individual projects.) (2.5) and (2.6) also implicitly impose the constraint that the sum of insider and outsider contributions must equal 1 + c(n).

We indicate the solution process for (2.2) in several steps. First, note that, if (2.2) is maximized, constraints (2.5) and (2.6) must always bind (that is, all agents must earn exactly their opportunity cost). We may thus use (2.5) to substitute out the second integral in (2.2). Next, we note that, for any n, s=S at the optimum; insiders should devote all of their savings to one project. (Actually, it is a matter of indifference whether insiders put their resources directly into their project or act as outside lenders on another project, as long as all of their wealth is available as collateral to back outsider loans to their own project; that is, in this risk-neutral world, it is never desirable to limit liability, except as is required by the non-negativity constraint on consumption.) The result that s=S follows from the principle that the probability of bankruptcy should be minimized[5]; the higher s, the lower the value of x (and thus the lower the bankruptcy probability, given by H(x)) which will allow (2.6) to hold. Intuitively, it is always preferable to use inside funds when they are available at the same opportunity cost as outside funds, since increased outside obligations also increase the risk of bankruptcy.

The observations of the paragraph above allow us to simplify the optimization problem to

$$\max_{n,x} E(q)\left[\int_x^K (k - x)dH\right] - rnS \tag{2.7}$$

subject to (2.6). (It will be clear that (2.4) and (2.5) will always hold in equilibrium.)

The first order necessary condition for the number of inside lenders n is:

$$c'(n) = h(x)S\gamma \tag{2.8}$$

where

$$h(x) = dH(x)/[1 - H(x)] \tag{2.9}$$

is the "hazard rate" associated with the distribution function H. We assume, conventionally, that $h'(x) > 0$, which together with $c''(n) > 0$, is sufficient to ensure that (2.8) defines a maximum.

Equation (2.8) is a relation defining the quantity of inside investors per project, given (i) the no-default payment to outsiders, x, (ii) per capita saving S, and (iii) the auditing cost γ. A rise in either S or γ increases the expected marginal benefit from adding another insider, and thus induces a rise in n. Similarly, an increase in the no-default payment x (or, equivalently, the bankruptcy probability) raises n. Thus, we may define the following implicit function for n:

$$n = n(x, S, \gamma) \qquad (2.10)$$

with $\partial n/\partial x > 0$ (given $h' > 0$), $\partial n/\partial S > 0$, $\partial n/\partial \gamma > 0$.

Finding the optimal coalition size effectively completes the solution of the firm's problem, since, given n, the safe rate r, and the expected value of capital E(q), the constraint (2.6) determines the no-default payment x. Also determined is the ratio of outside to inside finance, which decreases in n, and the probability of bankruptcy H(x), which increases in x.

Equilibrium with free entry into inside lending. Although the safe rate r is exogenous to the individual firm, it is determined in market equilibrium. To show this, we begin by imposing the condition that firms make zero profits in the competitive equilibrium. Setting the maximized value of (2.7) equal to zero yields

$$E(q)\int_x^K (k - x)dH = rnS \qquad (2.11)$$

Competition ensures that the expected value of the project yield conditional on solvency must equal the expected total earnings of inside lenders.

Alternatively, combining constraint (2.6) with (2.11), we find that the zero-profit condition can be expressed as

$$E(q)\mu = r \tag{2.12}$$

where

$$\mu = \frac{1 - \gamma H(x)}{1 + c(n)} \tag{2.13}$$

Equation (2.12) is an arbitrage condition which states that in the competitive equilibrium, the expected value of the project yield net of default costs, per unit of saving invested, must equal the riskless interest rate. The quantity μ is the expected amount of capital produced per unit of the output good invested, given the probability of bankruptcy $H(x)$, the auditing cost γ, and the number of insiders n. (It is worth noting that the competitive allocation in this model is the same as would arise from a social planning problem in which μ is maximized subject to (2.6).) Since the expression (2.13) reflects the influence of financial factors on the physical efficiency of the investment process (note that μ would be constant in a perfect-information world), we will refer to μ as "financial efficiency". This concept will be discussed further below.

There are several implications of (2.12) which contrast with the previous literature. First, increases in the riskless rate of interest are not necessarily associated with greater bankruptcy risk, since increases in r could reflect a greater expected return to investment (e.g., due to a rise in $E(q)$). This differs from Farmer [1984, 1985], in which there is a monotonic relation between the riskless rate and default risk. Thus, unlike Farmer's, our formulation allows the possibility of the combination of a procyclical (or acyclical) real interest rate and countercyclical default risk, which (according to our reading of the evidence) seems more plausible. Second, the fact that the expected returns to inside and outside lending are equal in equilibrium implies that individuals will be indifferent to what role they play. As a result, (2.12) also implies that there will be no "credit rationing" in our model of the sort that occurs when the identities of borrowers and lenders are exogenously given (see, e.g., Williamson [1985]).

Given S, γ, and E(q), and given that n is an implicit function of x, equation (2.14) and the constraint that outside lenders receive the safe rate of return in expectation (equation (2.6), holding with equality) jointly determine x and r.

Figure 1 illustrates this outcome in (r,x) space. The (aa) and ($\ell\ell$) curves portray combinations of r and x which satisfy (2.12) and (2.6), respectively. (It may be helpful to interpret x simply as a measure of bankruptcy risk, since the probability of bankruptcy H(x) is monotone increasing in x.)

The (aa) curve slopes downward. (The appendix gives the algebra.) A rise in the safe rate r implies that, in the competitive equilibrium, the expected return to an investment project must rise. Since E(q) is given, financial efficiency μ must increase. This is possible only with a decline in the total obligation in solvent states to outsiders x. A fall in x lowers the bankruptcy probability, which increases the expected project return both because the expected bankruptcy cost declines, and because the firm lowers the proportion of inside finance (refer to (2.10)), which reduces the project setup cost c(n).

The ($\ell\ell$) curve slopes upward, assuming that

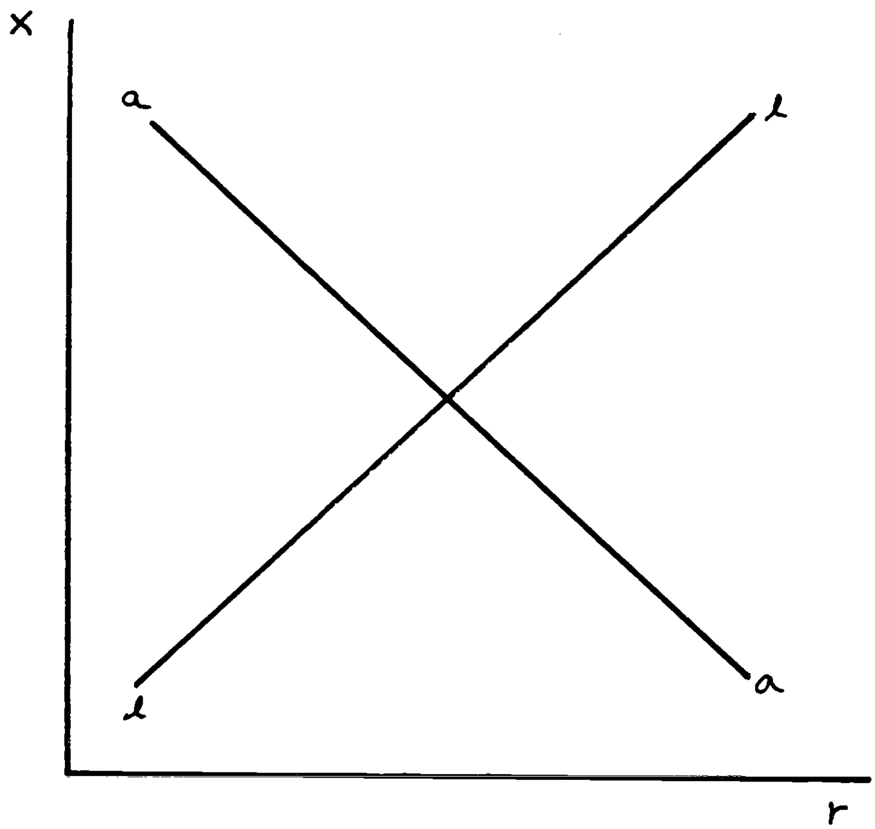

Figure 1

Equilibrium in the static model

$$\gamma h(x) < 1 \tag{2.14}$$

which we impose. (A sufficient condition for (2.14) to hold is that the auditing cost be less than the project's input cost.) An increase in r raises the opportunity cost to outside lenders; hence, the expected return to the firm's debt must rise in order to satisfy the constraint (2.7). A rise in x will accomplish this; that is, an increase in the total promised payment to outsiders conditional on solvency will raise the expected debt return, assuming that the associated rise in the expected bankruptcy cost is not too large (which (2.14) ensures).

Given the equilibrium value of x, it is straightforward to calculate the economy-wide default probability $H(x)$, the number of insiders per project $n = n(x,S,\gamma)$, financial efficiency μ, the ratio of outside to inside finance (given by $(1+c(n)-nS)/(nS)$), the risky rate of return (equal to $x/(1+c(n)-nS)$), etc. We may also calculate the number of projects undertaken in equilibrium, m, as

$$m = S/(1 + c(n)) \tag{2.15}$$

(2.15) uses the assumption that the population line is of length one, so that aggregate savings are S. We ignore the technical point that m must be an integer, which is of vanishing importance to the economy as S and therefore m become large.

<u>Comparative statics</u>. Using Figure 1, it is possible to perform some interesting comparative statics exercises, which we now briefly consider. Details are in the appendix.

(1) A rise in per capita saving S lowers x and raises r. Both the (aa) and $\ell\ell$ curves shift down, with the movement of the ($\ell\ell$) curve dominating. Intuitively, a rise in S increases the resources each insider investor can bring to the project; it also increases the marginal gain from adding an inside investor. The firm thus has the incentive to lower the ratio of outside to inside finance which, in turn, lowers x. The expected rate of return to each project (and thus, in equilibrium, the safe rate of return) rises also for two reasons: First, with lower x the bankruptcy probability falls. Second, as each insider brings more funds to the project, the marginal setup cost per unit of insider funds is lower.

The next section exploits the result that higher income and thus higher savings, through their salutary effect on insider equity, can in and of themselves raise returns and lower bankruptcy risk in the economy; this provides a mechanism by which output shocks can persist. Note also that an unanticipated redistribution of wealth from insiders to outsiders, which takes place after the identities of insiders have been determined but before financial contracts are drawn, has the same qualitative effect on x and r in this model as a general fall in S. This is consistent with Fisher's [1933] view that an unanticipated deflation that expropriates the debtor class to the benefit of creditors may have adverse effects on risk, return, and investment in the economy. We provide a more extensive analysis of "debt-deflation" in Section 3 below.

(2) An increase in the auditing cost γ has an ambiguous effect on x, but definitely lowers r. (The dominant factor is a shift of the (aa) curve to the left; the movement of the $\ell\ell$ curve is ambiguous.) In the former case, the offsetting forces are as follows: The rise in γ lowers the expected return to risky debt, ceteris paribus, which implies that outsiders must be compensated

with a higher return in solvent states; this tends to raise x. On the other hand, the larger bankruptcy cost induces the firm to lower the proportion of outside finance; this depresses x. The safe interest rate declines unambiguously because the rise in the auditing cost lowers the expected project return.

(3) A rise in E(q), the expected value of capital, increases r proportionately, the safe rate simply responding to movements in the expected value of the project return. The rise in E(q) has no effect on x, which is the no-default payment measured in capital goods, though it does affect the value of this payment measured in the output good, E(q)x. Essential to the result that x does not change is that, in this partial equilibrium setting, the change in r does not in turn affect other variables, such as the level of savings S.

3. Dynamic General Equilibrium with Agency Costs of Investment Finance

In this section we embed the static model of capital formation with inside and outside finance into a generic real business cycle model, i.e., a simple stochastic neoclassical growth model. This framework allows us to illustrate starkly the role of financial factors, since in standard versions of the real business cycle model (e.g. Kydland and Prescott [1982], Long and Plosser [1983]), the assumption of perfect markets implies that financial structure is irrelevant. (But see King and Plosser [1984] for an early attempt to bring financial considerations into a version of this model.) To develop intuition about the relation of this to more traditional approaches, we also derive the IS curve for our model and use this device to characterize the real effects of financial factors.

For tractability, the particular neoclassical growth model that we use is a stochastic version of Diamond's [1965] overlapping generations framework, in which agents are assumed to live for two periods. A technical problem with having longer-lived agents is that differential success of individual investors over time would force us to keep track of an ever-changing wealth distribution, which in turn would complicate the analysis of the division of the population between inside and outside lenders. Issues of dynamic coalition formation and reputation would also arise, which, while important, are beyond the scope of the present paper. We believe, however, that the qualitative predictions of our analysis would survive in alternative formalizations.

Technology. As in the previous section there are two goods, an output good and a capital good. The output good is now taken to be produced by a constant returns technology using capital and labor. Let y be the quantity of the output good per unit of labor input, k the amount of capital per unit of labor input, and $\tilde{\theta}$ a random productivity shock. We assume that production of the output good in each period is governed by:

$$y_t = \tilde{\theta}_t(1/\sigma)k_t^{\sigma} \qquad 0 < \sigma < 1 \tag{3.1}$$

where the random variable $\tilde{\theta}_t$ is i.i.d., is distributed continuously over a finite and non-negative support, and has a mean equal to $\bar{\theta}$. Subscripts denote the time period.

In each period, output is divided between consumption, C_t, and input for producing new capital goods, S_{kt}, where both variables are in units per worker. Thus

$$y_t = C_t + S_{kt} \tag{3.2}$$

The economy's "savings for capital" S_{kt} are assumed to be transformed into the next period's capital stock k_{t+1} by projects financed by inside and outside lending, as in the previous section. Assuming further that (1) capital is revolving (i.e., it depreciates in one period) and (2) the number of workers is normalized to one, the capital stock evolves as follows:

$$k_{t+1} = \sum_{i=1}^{m_t} [k_t(i) - \gamma I_t(i, Z)] \tag{3.3}$$

where m_t, the number of projects undertaken in t, is given (as above) by

$$m_t = S_{kt}/(1 + c(n_t)) \tag{3.4}$$

and again we neglect the fact that m_t must be an integer. The notation $k_t(i)$ stands for the realized outcome of project i; γ, recall, is the auditing cost; and $I_t(i,Z_t)$ is an indicator function that takes on the value one if $k_t(i) \in Z_t$, where Z_t is defined as the set of outcomes that induces auditing in period t, and is zero otherwise.

The expected production of capital goods can be written as

$$E(k_{t+1}) = \mu_t S_{kt} \tag{3.5}$$

where μ_t is the "financial efficiency" concept defined by (2.13) above.

<u>Consumers</u>. There are overlapping generations of two-period lived identical individuals. The population of each generation is a continuum with length normalized to equal one. Let $C_t(1)$ and S_t be consumption and saving, respectively, by a young person at t, $C_t(2)$ consumption by an old person, τ_t lump-sum taxes (transfers, if negative) levied on the young, and B_t the

quantity of one-period discount government bonds acquired by the typical young person. The young are assumed to acquire capital from the old at price q_t for use in producing current output. Then each young person at time t faces the following budget constraint:

$$C_t(1) = y_t - \tau_t - q_t k_t - S_t \tag{3.6}$$

where y_t obeys (3.1). Savings S_t can take the form either of lending to capital-producing projects or the purchase of government discount bonds:

$$S_t = S_{kt} + (1/r_t)B_t \tag{3.7}$$

where $1/r_t$ is the price of a discount bond acquired at t. Finally, expected consumption in the second period of life is[6]

$$E\{C_{t+1}(2)\} = E(q_{t+1})\mu_t S_{kt} + B_t \tag{3.8}$$

Each individual has the following utility function:

$$E\{U(C_t(1),\ C_{t+1}(2))\} \equiv \ell n C_t(1) + \beta \ell n E\{C_{t+1}(2)\} \tag{3.9}$$

(3.9) imposes risk neutrality, which permits us to use the analysis of optimal contracting from the previous section. Selden [1978] provides motivation for this formulation of the utility function, which was also used by Farmer [1984]; alternatively, we could have followed Williamson [1985] and allowed utility to be additively linear in second period consumption. Each individual has a fixed endowment of one unit of labor, which is supplied inelastically during the first period of life. At t, each young consumer chooses the vector $\{C_t(1), S_t, S_{kt}, B_t, E\{C_{t+1}(2)\}\}$ to maximize (3.9), subject to (3.1), (3.6), (3.7), and (3.8).

The first-order necessary conditions are:

$$q_t = \theta_t k_t^{\sigma-1} \tag{3.10}$$

$$S_t = \Gamma y_{dt} \tag{3.11}$$

$$E(q_{t+1})\mu_t = r_t \tag{3.12}$$

where

$$\Gamma = [\beta/(1 + \beta)][1 - \sigma] \tag{3.13}$$

and

$$y_{dt} \equiv y_t - \tau_t \tag{3.14}$$

Equation (3.10) defines the young consumer's demand for capital, for use in current production. Because of the constant returns technology, it also has the interpretation of a market demand curve. (3.11) characterizes both individual and aggregate saving behavior; conveniently, saving is just proportional to disposable income. (3.12) states that the expected returns from lending to investment projects and buying government bonds must be equal; note that this is just the equation for the (aa) curve (2.12), described in the previous section.

Government. The government does not consume resources, but only issues one-period discount bonds B_t and levies taxes τ. In each period the quantity of bonds redeemed must equal the discounted value of new bonds issued plus net taxes:

$$B_{t-1} = (1/r_t)B_t + \tau_t \tag{3.15}$$

We make the process followed by the quantity of outstanding bonds stochastic by assuming that taxes evolve randomly:

$$\tau_t = B_{t-1} - \tilde{z}_t \, B_{t-1}{}^{1-u}/r_t \tag{3.16}$$

where $\tilde{z}$ is a random variable with finite support and positive mean $\bar{z}$, and $0<u<1$. (3.16) implies that the quantity of bonds follows an autonomous and stationary first-order process:

$$B_t = \tilde{z}_t B_{t-1}{}^{1-u} \tag{3.17}$$

Equilibrium. To analyze the stochastic equilibrium of this model, we begin with the momentary (within-period) equilibrium, turning subsequently to the dynamics.

In studying the momentary equilibrium we take as given the inherited capital stock k_t, the current realization of the productivity shock θ_t, and the current and lagged realizations of the random quantity of government bonds, B_t and B_{t-1} (see equations (3.15)-(3.17)). Equilibrium is then described by the eight equations (E.1) through (E.8), collected for convenience in Table 1. The eight endogenous variables whose values are determined as the solution to these

Table 1. Momentary equilibrium

(E.1) $$y_t = \theta(1/\sigma)k_t^{\sigma}$$

(E.2) $$E(q_{t+1})\mu_t = r_t \qquad \text{(aa)}$$

(E.3) $$E(q_{t+1})[\int_o^{x_t}(k - \gamma)dH + \int_{x_t}^{K} x_t dH] = r_t[1 + c(n_t) - n_t S_t] \qquad (\ell\ell)$$

(E.4) $$\mu_t = (1 - \gamma H(x_t))/(1 + c(n_t))$$

(E.5) $$n_t = n_t(x_t, S_t, \gamma)$$

(E.6) $$S_t = \Gamma(y_t - B_{t-1} + (1/r_t)B_t)$$

(E.7) $$S_{kt} = \Gamma y_t - (1 - \Gamma)(1/r_t)B_t - \Gamma B_{t-1}$$

(E.8) $$E(q_{t+1}) = \bar{\theta}(\mu_t S_{kt})^{\sigma-1}$$

equations are: output y_t; the safe rate r_t; the risky payoff x_t (equivalently, the rate of bankruptcy $H(x_t)$); financial efficiency μ_t; the number of insiders per project n_t; total savings S_t; savings devoted to capital projects S_{kt}; and the expected price of capital $E(q_{t+1})$.

The equations of Table 1 are interpreted as follows:

(E.1) is the production function (3.1), given the realized value of θ_t.

(E.2) states that, given risk-neutrality, the expected return to capital investment must equal the safe rate r_t available on bonds. This is again the (aa) curve (2.12), derived in Section 2.

(E.3) is the condition that outside lenders to capital projects must receive the safe rate of return in expectation. This equation was called the ($\ell\ell$) curve in Section 2; see (2.6).

(E.4) defines "financial efficiency", as in (2.13).

(E.5) gives the optimal number of inside lenders per project (equation 2.12); equivalently we could have written out the first-order condition (2.8) explicitly.

(E.6) and (E.7) are identities defining S_t and S_{kt}. They follow from (3.7), (3.11), (3.14), and (3.15).

(E.8) is an approximate expression for the expected price of capital $E(q_{t+1})$. It is derived in several steps. First, we take expectations of both sides of (3.10) to obtain

$$E(q_{t+1}) = \bar{\theta}E(k_{t+1}{}^{\sigma-1}) \tag{3.18}$$

Next, since k_{t+1} is a sum of independent random variables, it can be shown by using the law of large numbers that

$$\lim_{m_t \to \infty} E(k_{t+1}{}^{\sigma-1})/E(k_{t+1})^{\sigma-1} = 1 \tag{3.19}$$

where $m_t = S_{kt}/(1+c(n_t))$ is the number of investment projects. Thus, if the number of projects is large (i.e., aggregate savings is large relative to the project size), then it is reasonable to use the following approximation of (3.18):

$$E(q_{t+1}) = \bar{\theta}E(k_{t+1})^{\sigma-1} \tag{3.20}$$

Together with (3.5), (3.20) implies (E.8).[7]

Given k_t and θ_t, the production function (E.1) alone is sufficient to determine output y_t. Thus financial factors do not affect output in momentary equilibrium, because aggregate supply is inelastic. We will show, however, that these factors do affect the process of capital formation, thereby having an impact on future output. Further, it is possible to characterize the effects of financial factors on an "IS curve"; this exercise provides insight into the implications for settings with alternative formulations of aggregate supply.

To solve for the allocation in momentary equilibrium, the model, consider first the determination of x_t and μ_t. Note that equations (E.2) through (E.5) form the identical model analyzed in Section 2, given $E(q_{t+1})$ and S_t. The comparative statics exercises done with that model showed that x_t does not depend on $E(q_{t+1})$. x_t does however depend (negatively) on per capita savings S_t; since S_t in turn is proportional to disposable income y_{dt}, we may write

$$x_t = x(y_{dt}) \tag{3.21}$$

where $x' < 0$. Similarly, within the subsystem (E.2)-(E.5) it can be shown[8] that financial efficiency μ depends positively on S_t. We therefore also have

$$\mu_t = \mu(y_{dt}) \tag{3.22}$$

where $\mu' > 0$. Thus increased disposable income, because it raises the quantity of collateral that insiders can bring to projects, lowers the rate of bankruptcy $H(x)$ and raises the efficiency of the investment process.

Let us now obtain the "IS curve" for this model. Substitute (E.8) and (3.22) into (E.2) to obtain

$$r_t = \bar{\theta}\mu(y_{dt})^{\sigma} S_{kt}^{\ \sigma-1} \tag{3.23}$$

(3.23) gives the combinations of interest rates and output such that savings is equal to the quantity of input used in investment projects. The production function (E.1) and (3.23) together determine output y_t and the safe interest rate r_t in momentary equilibrium. (See Figure 2.) Determination of the rest of the variables in the system then follows from simple substitutions. In particular, x_t, μ_t, S_t, and S_{kt} follow directly from (3.22), (3.23), (E.6), and (E.7) respectively. n_t and $E(q_{t+1})$ are then obtained from (E.5) and (E.8).

In the case with no government, the quantity of bonds and net taxes are both always zero; we then have simply $y_{dt} = y_t$ and $S_{kt} = S_t = \Gamma y_t$. The IS curve then reduces to

$$r_t = \bar{\theta}\mu(y_t)^{\sigma}(\Gamma y_t)^{\sigma-1} \tag{3.24}$$

The slope of the IS curve in this case is given by

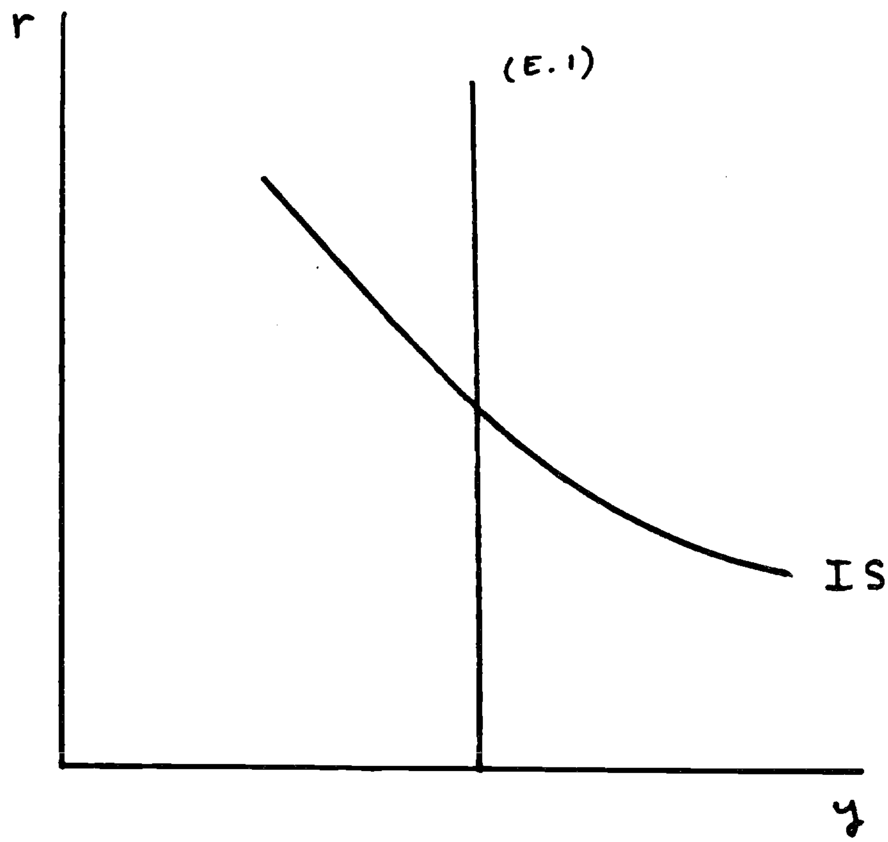

Figure 2

Momentary equilibrium

$$\left.\frac{\partial r}{\partial y}\right|_{IS} = (\sigma - 1 + \sigma\eta)(r_t/y_t) \tag{3.25}$$

where $\eta > 0$ is equal to the elasticity of financial efficiency μ with respect to disposable income. Note that, if we neglect η, the IS curve slopes down (since $\sigma < 1$), and for the conventional reason: An increase in income leads to more saving, which can only be absorbed into investment if the interest rate falls. However, the presence of η makes the IS curve flatter than in the usual case, and could in principle make it upward sloping. The reason is that, at higher levels of income, increased insider equity lowers the agency costs of financing projects and thus increases the demand for investment funds. (Formally, the effect is analogous to including income in the investment demand equation in the derivation of the standard IS curve.) If the costs of finance are very sensitive to the level of insider equity (η large) and the elasticity of output with respect to capital is not too far below one, then increasing output may raise the demand for investment funds faster than it raises the supply of savings; in this case, the IS curve slopes up.

When the government sector is reinstated in the model, the slope of the IS curve is given by the more complicated expression

$$\left.\frac{\partial r}{\partial y}\right|_{IS} = \frac{[(\sigma - 1)\Gamma(y_{dt}/S_{kt}) + \sigma\eta](r_t/y_{dt})}{(1 + v_t)} \tag{3.26}$$

where

$$v_t = [(1 - \sigma)(1 - \Gamma)(y_{dt}/S_{kt}) + \sigma\eta]B_t/(r_t y_{dt}) \tag{3.27}$$

and $v_t > 0$ if $B_t > 0$. The IS curve is now even flatter, compared to the case without government debt. One can identify two effects. The first, captured by the first term in brackets in the expression for v_t, is the conventional Diamond [1965] effect: A fall in r increases the price of newly issued government bonds, which diverts saving from productive investment; a higher level of income than before is needed to generate enough savings to equate investment and the savings devoted to investment. The second effect, represented by the second term in brackets in (3.27) is a financial effect: A fall in interest rates acts as a capital gain for the young, since it reduces the taxes they must pay in order to re-finance the government debt (see 3.15). As above, higher income for the young increases insider collateral, lowers bankruptcy risk, and increases the demand for investment funds at any value of the safe interest rate; thus income must increase even more to generate enough savings to satisfy this investment demand.

Dynamics. Period-to-period dynamics in the model are created by the presence of capital. The law of motion governing output is of the form

$$y_{t+1} = y'(y_t, \hat{k}_t(i), \tilde{\theta}_{t+1}; \gamma) \qquad (3.28)$$

where the function y' is given by

$$y'(\cdot) = \tilde{\theta}_{t+1}(1/\sigma)(\sum_{i=1}^{m_t}[k_t(i) - \gamma I_t(i, Z)])^{\sigma} \qquad (3.29)$$

In (3.29), recall that m_t is the number of projects undertaken in t, and is an increasing function of y_t; $\hat{k}_t(i)$ is the vector of realizations of the projects undertaken in t; and I is the indicator function which takes the value one for projects which are audited.

A useful approximation[9] to (3.29) is

$$y_{t+1} = \tilde{\theta}_{t+1}(1/\sigma)(\mu_t S_{kt})^{\sigma} \tag{3.30}$$

which in the case without government debt reduces to

$$y_{t+1} = \tilde{\theta}_{t+1}(1/\sigma)(\mu_t \Gamma y_t)^{\sigma} \tag{3.31}$$

How does the presence of financial factors affect output dynamics following an innovation to θ? An indicator of the sensitivity of this first order system to a temporary productivity shock is simply the derivative of expected output with respect to current output. From (3.30) it follows that

$$\partial y_{t+1}/\partial y_t = \sigma(\eta_{yd} + \rho)y_{t+1}/y_t \tag{3.32}$$

where $\eta_{yd} > 0$ is the elasticity of financial efficiency μ_t with respect to disposable income y_{dt}, and $\rho > 0$ is the elasticity of savings for capital formation S_{kt} with respect to output y_t. In the model without government debt, $\rho=1$.

In the conventional real business cycle model, $\eta_{yd} = 0$. (3.32) thus suggests that the financial factors we have introduced into the model magnify the persistence of a shock to productivity. The mechanism is as follows: Higher productivity increases income and per capita savings, which raises both the number of insiders per project and the amount of equity each insider can bring. This lowers the risk of bankruptcy and increases the efficiency of the investment process (μ increases). Both the higher level of investment (due to last assumption is meant to reflect the real-world fact that non-entrepreneurs cannot be transformed into entrepreneurs without some lapse of time.

The equations describing momentary equilibrium after the redistribution are listed as (D.1) through (D.9) (D for "debt-deflation") in Table 2. We describe them briefly:

(D.1) is the production function.

(D.2) is the (aa) curve. Because the number of investment projects may be decreased but not increased after the redistribution, the (aa) curve here is in principle an inequality; the return to an investment project could be greater than the return to bonds if the number of projects selected before the surprise redistribution turned out to be a binding constraint. It can be shown[14], however, that for $\varepsilon < 1$ (D.2) must hold with equality.

(D.3) is the ($\ell\ell$) curve, which states that outside lending and holding bonds must pay the same return. Note that, on the right hand side of (D.3), per capita savings S_t has been replaced by S_t^i, where S_t^i denotes the per capita savings of insiders after the redistribution ($S_t^i < S_t$). The replacement of S_t with S_t^i implies that a greater share of each project must be financed by the funds of outsiders.

(D.4) restates the definition of financial efficiency. According to (D.5), for the purposes of comparative statics exercises with respect to the redistribution parameter ε, the coalition size n can be treated as fixed. (D.6) and (D.7) re-state the definitions of per capita savings S_t and per capita "savings for investment" S_{kt}. These definitions remain valid despite the redistribution because total gross income is unchanged and because savings is a fixed proportion of disposable income. (Note that the savings of the average outsider will exceed S_t, however.)

Table 2. Momentary equilibrium with debt-deflation

(D.1) $$y_t = \theta_t(1/\sigma)k_t^{\sigma}$$

(D.2) $$E(q_{t+1})\mu_t = r_t \qquad \text{(aa)'}$$

(D.3) $$E(q_{t+1})[\int_o^{x_t}(k - \gamma)dH + \int_{x_t}^{\kappa} x_t dH] = r_t[1 + c(n_t) - n_t S_t^i] \qquad (\ell\ell)'$$

(D.4) $$\mu_t = (1 - \gamma H(x_t))/(1 + c(n_t))$$

(D.5) $$n_t = \bar{n}$$

(D.6) $$S_t = \Gamma(y_t - B_{t-1} + (1/r_t)B_t)$$

(D.7) $$S_{kt} = \Gamma y_t - (1 - \Gamma)(1/r_t)B_t - \Gamma B_{t-1}$$

(D.8) $$S_t^i = \Gamma(\varepsilon y_t + 1/r_t B_t - B_{t-1})$$

(D.9) $$E(q_{t+1}) = \bar{\theta}(\mu_t S_{kt})^{\sigma-1}$$

higher savings) and the greater efficiency of investment raise the level of the capital stock, which propagates the output shock over time.[10]

Realistically, this financial effect may be interpreted as acting through corporate liquidity and profits. In good times, when firms are flush with inside funds, outside funds for project finance also become relatively easy to obtain; the increased capital formation that follows from this enhances the general prosperity, both through an aggregate supply effect (as in our formal model) and, presumably, through an aggregate demand effect as well. Conversely, in bad times, low levels of insider collateral and high bankruptcy risk make lenders more wary, which reduces investment and reinforces the fall in output.

In the model with government debt, there is an additional, slightly more subtle effect by which the presence of financial factors increases the sensitivity of y_{t+1} to y_t. In the standard model, a rise in output lowers interest rates, which raises the price of government debt and diverts savings away from productive investment. The same effect occurs in our model here, except that, because financial factors make the IS curve flatter (or possibly positively sloped), the diversion of resources away from capital is unambiguously smaller. That is, the parameter ρ in (3.32) is larger here than in the standard case. Thus, once again, financial factors tend to act in the direction of increasing the persistence of the response to real shocks.

Financial shocks. We have shown above that financial factors can affect the propagation over time of real (productivity) shocks to output. We now consider briefly the effects of shocks originating in the financial sector (including government finance). We look at (1) innovations in government debt, (2) shocks to the auditing cost, and (3) "debt-deflation".

(1) An innovation in current government debt B_t (which is equivalent to a change in lump-sum taxes; see (3.15)-(3.17)) is a shock to the IS curve. Noting that (for fixed r) $\partial y_{dt}/\partial B_t = 1/r_t$ and $\partial S_{kt}/\partial B_t = -(1 - \Gamma)/r_t$, we can write

$$\left.\frac{\partial y_t}{\partial B_t}\right|_{IS} = \frac{[1 - \sigma)\Gamma(y_{dt}/S_{kt}) + \sigma\eta]}{[(1 - \sigma)\Gamma(y_{dt}/S_{kt}) - \sigma\eta]r_t} \tag{3.36}$$

The derivative in (3.36) is positive, so (as usual) a positive shock to bonds (equivalently, a cut in taxes) shifts the IS curve up and to the right (assuming that the condition for a downward sloping IS curve is met).[11] In the absence of financial effects (η=0), the expression in (3.36) is simply $1/r_t$, and the macroeconomic impact of the debt shock is the same as in Diamond [1965]; specifically, some savings are diverted away from productive investment to bonds, so that future capital and output falls. More precisely, it can be shown that the increase in B_t unequivocally raises net bond wealth B_t/r_t, which raises disposable income and current consumption, and reduces investment.

If $\eta > 0$ (that is, financial efficiency responds strictly positively to increased income), the effect of the debt shock on the IS curve is unambiguously increased, so that interest rates rise more than when financial factors are absent. Higher disposable income raises financial efficiency and thus actual capital formation, offsetting to some degree the negative effects of the tax cut on capital.

We stress that it is for technical reasons only (specifically, the difficulty in calculating the dynamic stochastic equilibrium) that our present model does not allow an IS shift to affect current output. In principle, one could imagine replacing the inelastic labor supply of our model with an intertemporal substitution mechanism (so that output supplied increases with the safe interest rate), or a Keynesian LM curve and sticky nominal wages. In one of these more general models, we would expect to find that a tax cut raises current output (as IS shifts up), at the same time that it crowds out future output. In this case, while the financial factors discussed in this paper would tend to offset the tendency of a tax cut to crowd out capital in the long run, in the short run they would act, because financial efficiency and therefore investment increase as income increases, to reinforce the positive effect of the tax cut on output.

(2) An increase in the auditing cost γ (which could reflect either a deterioration in the auditing technology or perhaps institutional changes affecting the cost of bankruptcy) shifts the IS curve down and left. The derivative is

$$\left.\frac{\partial y_t}{\partial \gamma}\right|_{IS} = \frac{\sigma\delta y_{dt}}{\gamma[(1 - \sigma)\Gamma(y_{dt}/S_{kt}) - \sigma\eta]} \tag{3.37}$$

where $\delta < 0$ is the elasticity of financial efficiency μ with respect to γ. (3.37) is negative if the IS curve itself is downward sloping. Higher auditing costs lower expected project returns and hence depress the safe rate of return and the flow of investment.

(3) A "debt-deflation" is an unanticipated deflation that redistributes wealth from borrowers to creditors. According to the logic of our approach, such a redistribution should have real effects because it reduces the collateral held by the "borrowing class" and thus increases the agency costs of subsequent loans.

Our formal model does not incorporate the feature that individuals currently borrowing in order to invest are also relatively likely to have taken out loans previously. We thus attempt to capture the debt-deflation phenomenon in a stylized way, as follows: We assume that in each period t, there is an instant, after production is realized, in which potential investment projects are selected and insider coalitions are formed; after this instant, no additional projects can be considered until the next period. At a subsequent instant within t, agents decide on portfolios, financial contracts are drawn up, and resources are committed to the projects. Now let us imagine that, between the moment of coalition formation and project selection and the time at which resources are committed, there is a totally unanticipated and one-time redistribution of (before-tax) income from those agents who had just joined investment coalitions to those who had not; specifically, those who had joined coalitions have their income reduced to a fraction ε, $0 < \varepsilon < 1$, of its previous level. (This is supposed to capture the idea that under a debt-deflation, those who are currently in a position to undertake investment projects experience an unanticipated deterioration of collateral.[12]) After this redistribution, agents have the following options: Those who had joined coalitions and suffered the loss of income may continue with their projects if they wish; however, their coalition sizes are fixed at the level initially determined[13] and the financial contracts they draw up with outsiders must, naturally, reflect their straitened financial circumstances. Alternatively, those who had joined coalitions may abandon their investment projects without further penalty (i.e., they do not lose their setup cost) and become outside lenders to other projects or holders of government bonds. (Insiders who abandon their projects do not regain the income they lost in the redistribution.) Individuals who did not become insiders originally may hold bonds or become outside lenders, as before; importantly, though, they may not start new investment projects as insiders (within the current period). This

(D.8) defines S_t^i, the average savings of individuals who initially joined investment coalitions. The parameter ε, recall, is the redistribution parameter which is applied to gross income. Per capita taxes, $B_{t-1} - (1/r_t)B_t$, are lump-sum and are thus not affected by the redistribution.

To analyze this system, use (D.2), (D.7), and (D.9) to write an expanded version of the (aa) curve as

$$\text{(aa)'} \qquad \bar{\theta}\mu_t^{\sigma}[\Gamma y_t - (1 - \Gamma)(1/r_t)B_t - \Gamma B_{t-1}]^{\sigma-1} = r_t \tag{3.38}$$

Note that, given y_t, B_t, B_{t-1}, and the dependence of μ_t only on x_t, (3.38) is a (negative) relation between r_t and x_t only. Similarly, expand the $\ell\ell$ curve, using (D.3), (D.7), (D.8), and (D.9), to read

$$\bar{\theta}[\mu_t(\Gamma y_t - (1 - \Gamma)(1/r_t)B_t - \Gamma B_{t-1})]^{\sigma-1}[\int_o^{x_t}(k - \gamma)dH + \int_{x_t}^{\kappa} x_t dH] =$$

$$(\ell\ell)' \qquad r_t[1 + c(n_t) - n_t\Gamma(\varepsilon Y_t + 1/r_t B_t - B_{t-1})] \tag{3.39}$$

which again is a relationship (here positive) between x_t and r_t.

Let us see how a fall in ε (say, from $\varepsilon = 1$ to $\varepsilon < 1$) affects the (aa)' and $(\ell\ell)$' curves. First, ε does not appear in (aa)', so this curve does not move. However, a fall in ε can be shown to shift the $(\ell\ell)$' curve up and to the left in (r,x) - space. (See Figure 3.) Intuitively, a fall in ε reduces insider capital, so that more outside finance is required per project; thus, at any given safe rate of return r_t, the total non-default payment to be made to outsiders, x_t, must rise. Alternatively, at any given safe rate, as the amount of insider capital declines, the risk of bankruptcy must rise.

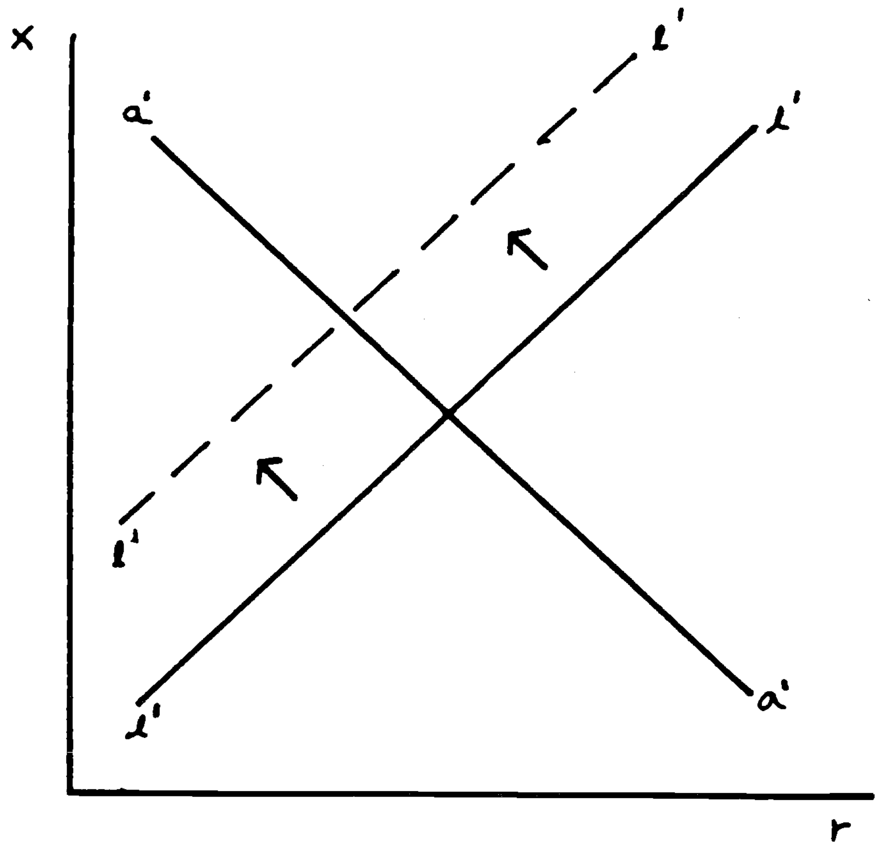

Figure 3

Debt-deflation

The IS curve in this example is the solution of (3.38) and (3.39) for r_t in terms of y_t. From Figure 3 we see that a borrower-to-creditor redistribution ($\varepsilon < 1$) causes r_t to fall for any y_t, i.e., the IS curve shifts down. We may conclude that a debt-deflation lowers safe interest rates (i.e., raises the price of safe assets); this may be interpreted as the result of the "flight to quality" which is common during periods of financial crisis. Debt-deflation also raises the rate of bankruptcy, lowers financial efficiency, and (from (D.6) and (D.7)) raises the total savings of the young while lowering the amount of savings devoted to capital formation. Lower savings for capital formation plus lower financial efficiency together imply less capital and thus less output in the future. Because of our model's assumption of inelastic aggregate supply, y_t is not affected by the debt-deflation; but, as noted above, in any model in which IS shifts affect output contemporaneously, we would expect to see debt-deflation reduce current output as well. Thus, what appears to be a pure redistribution, because of its effect on insider collateral, can have a variety of real effects.

4. Conclusion

We have constructed a simple neoclassical model of intrinsic business cycle dynamics in which financial factors play an important role. A key point is that borrower solvency (here defined as the amount of collateral available to secure outside loans) is inversely related to the agency costs of undertaking physical investments. Increased solvency in good times thus expands investment demand, which in turn tends to magnify swings in output. Further, financial disturbances (such as debt-deflations) which affect insiders' collateral or the costs of bankruptcy may have real effects. These general conclusions would survive, we suspect, in models with characterizations of the agency problem and the macroeconomic setting that are richer than the simple approaches used here.

A question that does deserve investigation, though, is the relevance of this analysis to an economy that is dominated by large, publicly held firms. The financial model used here probably works best as a description of smaller, closely held entities, for which "collateral" is easily identifiable as the personal stake of the owner/managers; how to map this model into a situation where ownership is diffused and is divorced from management is less clear. In particular, our conclusions would be substantially weakened if most firms in the economy could quickly raise funds through outside equity issues.

One response to this is to note that closely-held firms are in fact a large part of all contemporary capitalist economies. Beyond this, as has been argued by Greenwald, Stiglitz, and Weiss (1984), it should be pointed out that agency problems also impinge on publicly held firms, probably restricting their ability to issue outside equity. For example, equity issuance may dilute managerial incentives (by lowering the debt-equity ratio) or send a bad signal to outside investors. A useful direction for further research would be to extend this present approach to encompass an agency model of the publicly held firm. Beyond increasing the applicability of the model, such an extension might also deliver testable implications about differences in the cyclical behavior of publicly and privately owned companies.

FOOTNOTES

1 The debt-deflation phenomenon was probably important during the Great Depression in the U.S., at the levels of both financial intermediaries and ultimate borrowers; see Bernanke (1983) and Hamilton (1986).

2 We emphasize, however, that this proposition is quite general. For example, in his analysis of the perhaps more familiar Holmstrom (1979) principal-agent setup, in which agents' unobserved actions affect project returns, Sappington (1983) demonstrated a similar inverse relationship between the agent's wealth and the agency costs of the principal-agent relationship.

3 The purpose of allowing a variable coalition size is to introduce a second source of variation in the aggregate debt-equity ratio (the first source being changes in the per capita contributions of insiders). We think this adds an interesting dimension to the analysis, but the main qualitative results of the paper can be obtained without it, given that intertemporal fluctuations in insider resources provide a source of variation in collateral. Having this feature, however, allows us to highlight the role of collateral within the static model.

4 Allowing for at least some restricted forms of stochastic auditing yields some interesting modifications of the analysis but does not change the basic results; this is discussed in an appendix available from the authors.

5 After substitution of (2.5) (holding with equality) into (2.2) and the addition of (2.6) (also holding with equality) to (2.2), the objective function can be written (for a given n) as simply min H(x), subject to the constraints.

6 (3.8) is in fact an approximation, since it is derived using $E(q_{t+1}k_{t+1}) = E(q_{t+1})E(k_{t+1})$, which is not exactly true here (q_{t+1} and k_{t+1} are not independent). A formal justification for this approximation when the number of investment projects m is large is given in footnote 7 below.

7 Note also that $E(q_{t+1}k_{t+1}) = \bar{\theta}E\{k^{\sigma}_{t+1}\}$ is, by the LLN, approximately equal to $\bar{\theta}E\{k_{t+1}\}^{\sigma}$ for large m_t. By (3.20) this is in turn approximately equal to

$E(q_{t+1})E(k_{t+1})$, which justifies the assumption in (3.8).

8 A sketch of the proof is as follows: Divide (E.3) by (E.2) to eliminate $E(q_{t+1})$ and r_t from (E.3). (E.3), (E.4), and (E.5) now form a subsystem in $\bar{\mu}$, x, and $\bar{n}$. This subsystem is identical to the set of equations studied in Appendix B, with $E(q_{t+1})$ set equal to one and r_t set equal to

μ_t. Total differentiation therefore shows that $d\mu/dS > 0$, as it showed that $dr/dS > 0$ in the analogous system in Appendix B.

9 To see that (3.30) is a reasonable approximation, note that

$$\lim_{m_t \to \infty} \left(\sum_{i=1}^{m_t} [k(i) - \gamma I_t(i, z)]/\mu_t S_{k_t}\right)^\sigma =$$

$$\lim \left(\sum_{i=1}^{m_t} [k_t(i) - \gamma I_t(i, z)]/[m_t(1 - \gamma H(x_t))\right)^\sigma =$$

$$\lim \left(\sum_{i=1}^{m_t} [k_t(i)/m_t - \gamma I_t(i, z)/m_t]/[1 - \gamma H(x_t)]\right)^\sigma =$$

$$([1 - \gamma H(x_t)]/[1 - \gamma H(x_t)])^\sigma = 1$$

10 Though here movements in the capital stock generate all the variation in output, we emphasize that the financial effects described here will magnify the business cycle in any setting where output variability increases with the income sensitivity of investment demand.

11 If the IS curve is upward-sloping, the derivative (3.36) is negative. However, the effects of the change in bonds on interest rates and investment are the same as in the basic case. Similar observations hold for the cases analyzed below.

12 We do not explicitly consider the usual question of why financial contracts are rarely indexed to the price level. For present purposes, let us assume that there is a fixed cost of indexing, and that the ex ante probability of debt-deflation is sufficiently small so as to make considering this contingency not worthwhile.

13 This is for simplicity; it is not essential to the qualitative results.

14 Suppose not: then it must be that $S_{kt} = S^*_{kt}$, where, here and below, starred variables indicate the equilibrium values when $\varepsilon=1$ (i.e., when there is no redistribution). From (D.7), this implies $r_t = r^*_t$. From (D.4) and (D.9), given S_{kt} fixed, x_t and $E(q_{t+1})$ covary positively. Thus, from (D.3), since $S^i_t < S^*_t$, $r_t = r^*_t$, and $n_t = n^*_t$, it must be that $x_t > x^*_t$. From (D.2) and (D.9), $\bar{\theta}(\mu_t)^\sigma(S^*_{kt})^{\sigma-1} = r^*_t$ (since (D.2) holds with equality when $\varepsilon=1$). But since $x_t > x^*_t$, this implies
$\bar{\theta}\ \mu^\sigma_t(S^*_{kt})^{\sigma-1} = \bar{\theta}\ \mu^\sigma_t S^{\sigma-1}_{kt} < r^*_t = r_t$, a contradiction.

Appendix: Comparative Statics

This appendix presents some formal results for the partial equilibrium model of Section 2. We first derive the slopes of the (aa) and ($\ell\ell$) curves, and then conduct some comparative static experiments. We consider the effects on x and r of the following changes: a rise in S, a rise in γ, and a rise in E{q}.

For convenience, we restate the equations describing the (aa) and ($\ell\ell$) curves ((2.12) and (2.6), respectively.)

$$\text{(aa)}\quad E\{q\}[1 - \gamma H(x)]/[1 - c(n)] = r \qquad \text{(A.1)}$$

$$(\ell\ell)\quad E\{q\}[(k^e - \gamma)H(x) + x(1 - H(x))]/[1 + c(n) - nS] = r \qquad \text{(A.2)}$$

where

$$n = n\ (x,\ S,\ \gamma) \qquad n_1,\ n_2,\ n_3 > 0 \qquad \text{(A.3)}$$

$$k^e \equiv \int_o^x k\,dH/H(x) \qquad \text{(A.4)}$$

The (aa) and ($\ell\ell$) curves determine x and r, given (A.3), (A.4), S, γ, and E{q}.

<u>Slopes of the (aa) and($\ell\ell$) curves</u>. Differentiating both (A.1) and (A.2) with respect to x and r yields:

$$\left.\frac{\partial x}{\partial r}\right|_{aa} = -\{r[\frac{\gamma dH}{1 - \gamma H(x)} + \frac{c'n_1}{(1 + c(n))}]\}^{-1} < 0,$$

$$\left.\frac{\partial x}{\partial r}\right|_{\ell\ell} = -\{r[\frac{1 - H(x) - \gamma dH}{(k^e - \gamma)H(x) + x(1 - H(x))} + \frac{n_1(S - c')}{1 + c(n) - nS}]\}^{-1} > 0.$$

Thus, the (aa) curve slopes downward and the ($\ell\ell$) curve upward.

A rise in S. Totally differentiating (A.1) and (A.2) with respect to x, r, and S yields.

$$\frac{\partial x}{\partial S} = -[\frac{c'n_2}{1 + c(n)} + \frac{(s - c')n_2 + n}{1 + c(n) - ns}]/\Delta < 0,$$

$$\frac{\partial r}{\partial S} = [\frac{\gamma dH}{1 - \gamma H(x)} + \frac{c'n_1}{1 + c(n)}] \frac{nr}{1 + c(n) - nS} /\Delta > 0,$$

where

$$\Delta = \frac{\gamma dH}{1 - \gamma H(x)} + \frac{c'n_1}{1 + c(n)} + \frac{1-H(x) - \gamma dH}{(k^e - \gamma)H(x) + x(1 - H(x))} + \frac{n_1(S - c')}{1 + c(n) - nS} > 0.$$

A rise in S therefore lowers x and increases r.

A rise in γ. Totally differentiating (A.1) and (A.2) with respect to x, r, and γ yields:

$$\frac{\partial x}{\partial \gamma} = [\frac{nS}{1 - c(n) - ns} \cdot \frac{H(x)}{1 - \gamma H(x)} - \frac{n_3 c'}{1 + c(n)} - \frac{n_3(S - c')}{1 + c(n) - nS}]/\Delta \lesseqgtr 0.$$

$$\frac{\partial r}{\partial \gamma} = - [n_1 S + (1 + c(n)] \frac{1 - H(x)}{1 - \gamma H(x)} \cdot \frac{H(x)}{1 - \gamma H(x)} \cdot \frac{r}{(1 - c(n) - nS)} /\Delta < 0.$$

Thus, a rise in γ has an ambiguous effect on x, but definitely lowers r.

A rise in E{q}. Totally differentiating (A.1) and (A.2) with respect to x, r, and E{q} yields:

$$\frac{\partial x}{\partial E\{q\}} = 0$$

$$\frac{\partial r}{\partial E\{q\}} = r/E\{q\} > 0.$$

A rise in E{q} has no effect on x, but increases r.

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