NBER WORKING PAPER SERIES

EXPORT SUPPLY AND IMPORT DEMAND FUNCTIONS: A PRODUCTION THEORY APPROACH

W. Erwin Diewert

Catherine J. Morrison

Working Paper No. 2011

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 1986

Research support from the National Science Foundation, Grant SES-8420937, is gratefully acknowledged. The research reported here is part of the NBER's research programs in International Studies and Productivity. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Working Paper #2011
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ABSTRACT

In this paper we theoretically and empirically model import demand and export supply behavior of firms for the U.S. economy from 1967-1982. A producer theoretic approach based on duality theory is used to derive econometric systems of producer supply and demand functions that are consistent with profit maximizing behavior. This system is then empirically implemented and the resulting estimates used to construct a full set of supply and demand elasticities characterizing import demand and export supply functions as well as domestic output supply and labor demand. These elasticities are in turn used to derive devaluation elasticities and some estimates of the equilibrium real exchange rate that would cause the U.S. trade surplus to reach zero.

W. Erwin Diewert
Department of Economics
University of British Columbia
Vancouver, B.C., Canada
V6T 1Y2

Catherine J. Morrison
Department of Economics
Tufts University
Medford, Massachusetts
02155

I. Introduction

In this paper, we utilize a producer theory approach to the generation of export supply and import demand functions for the U.S. economy for the period 1967 to 1982. Our approach uses an economy wide GNP function[1] or restricted profit function with exports appearing as outputs and imports appearing as inputs. This approach to modeling trade functions using an integrated production theory framework was implemented by Kohli [1978] for the Canadian economy.[2]

Traditional general equilibrium models of trading economies assume that internationally traded goods can enter the consumer sector directly and hence traditional empirical approaches to the estimation of import demand and export supply functions inevitably involve modeling the household sector. However, in Kohli's approach, imports are regarded as intermediate inputs into the production sector and exports are regarded as separate nondomestic outputs produced by the private production sector as a whole.[3] Treating imported goods as intermediate goods seems reasonable since: (i) many imports are intermediates and (ii) even imported goods that are delivered to the household sector generally have domestic transportation, wholesaling and retailing inputs added to them. The advantage of the Kohli approach over the traditional approach is that we need only model the private production sector of the economy and we can ignore the difficult problems involved in modeling the consumer sector.

Duality theory may be used to derive econometrically convenient systems of producer supply and demand functions that are consistent with profit maximizing behavior.[4] However, often the estimated profit functions do not satisfy the appropriate theoretical curvature conditions. Thus in this study, we shall use an adaptation to the profit function context of a functional form due to Diewert and Wales [1985]. This functional form allows one to impse the correct curvature conditions in a global manner without destroying the flexibility of the functional form.

A brief outline of the paper follows. In Section 2, we outline the production theory approach to the generation of systems of export supply and import demand functions. In Section 3, we introduce our functional form for the profit function and we explain how the correct curvature conditions can be imposed. Empirical results are also presented in this section. In Section 4, we table various elasticities based on our estimated profit function. In Section 5, we derive some devaluation elasticities and some estimates of the equilibrium real exchange rate by year that would drive the U.S. merchandise trade surplus or deficit to zero for the years 1967 to 1982. For example, for the year 1982, we estimate that a devaluation of 28% would be required to eliminate the U.S. balance of trade deficit for that year. Section 6 concludes and our data are described and tabled in a data Appendix.

Our model of producer behavior is based on short run competitive profit maximization, holding capital fixed in each period. Thus we do not model the capital accumulation process. Moreover, we assume that producers regard wage rates as fixed and can hire any amount of labor at the going wage rate. Due to the existence of substantial amounts of unemployment during the period, we feel that this assumption is more realistic than the polar case where labor input into the private production sector is held fixed and the wage rate becomes an endogenous variable. In subsequent work, we plan to consider alternative treatments of labor.

2. A Production Theory Approach to Export Supply and Import Demand Functions

We assume that domestic and international demand and supply decisions are made by profit maximizing firms operating under perfect competition in all commodity and factor markets. The firms face vectors of domestic and international prices and a vector of domestic primary factor stocks and make decisions about domestic and international input demand and production. The technology is assumed to be represented by a production possibilities set, from which a profit function can be derived.

Specifically, let the production possibilities set for the private sector of a country in period t be a set $S_t \equiv \{(x,y,K^t)\}$ where $x=(x_1,\ldots,x_N)$ is an N dimensional vector of domestic net outputs (if $x_n<0$, then good n is an input), $y \equiv (y_1,\ldots,y_M)$ is an M dimensional vector of net exports (if $y_m<0$, then good m is an imported good that is being utilized by the private sector) and $K^t \equiv K^t{}_1,\ldots,K^t{}_J)$ is a nonnegative J dimensional vector of capital stocks utilized by the private sector in period t. Let $p^t \equiv p^t{}_1,\ldots,p^t{}_N)$ be the positive vector of domestic prices faced by domestic producers in period t and let $w^t=(w^t{}_1,\ldots,w^t{}_M)$ be the positive vector of international prices that producers face in period t. Then we can define the country's net revenue or restricted profit function (see Gorman [1968], McFadden [1966], or Diewert [1974]) by:

$$\Pi^t(p^t,w^t,K^t) \equiv \max_{x,y} \{p^t \cdot x + w^t \cdot y : (x,y,K^t) \in S^t\} \quad . \tag{1}$$

From Hotelling's Lemma, the economy's observed net output vector $x^t = (x^t{}_1,\ldots,x^t{}_N)$ and observed net export vector $(y^t{}_1,\ldots y^t{}_M)$ are equal to the vectors of derivatives of the restricted profit function with respect to the components of p^t and w^t, respectively (if the derivatives exist); i.e.,

(2) $$x^t = \nabla_p \Pi^t(p^t, w^t, K^t) \quad \text{and}$$

(3) $$y^t = \nabla_w \Pi^t(p^t, w^t, K^t) \qquad \text{for } t=1,\ldots,T,$$

where ∇ is the vector differential operator. Thus $\nabla_p \Pi^t(p^t,w^t,K^t)$ is the vector of first order derivatives of Π with respect to the components of p. Note that these net output and input vectors are explicitly based on short run decisions since they are conditional on fixed stock levels of certain inputs. The extent of fixity -- and thus the flexibility of the country to act in the short run -- will depend on how many stocks are held fixed (e.g., recognizing only the fixity of capital plant and equipment, or including also labor stocks to characterize labor hoarding, or assuming a given domestic output level which generates the negative of a cost function).

Assuming that only capital stocks are fixed, the system of equations (2) represents the economy's short run domestic output supply and labor (and possibly other domestic variable input) demand functions while the system of equations (3) contains the economy's short run export supply and import demand functions. If we assume a functional form for Π^t, and append errors to (2) and (3), then the unknown parameters which characterize Π^t (and therefore the production possibilities set S^t by duality theory) may be econometrically estimated from (2) and (3), given a time series of observations on x^t, y^t, p^t, w^t, and K^t.

3. The Normalized Quadratic Profit Function

The approach explained in the previous section for modeling export supply and import demand functions has been implemented by Kohli [1978] for Canada using the translog functional form. However, a problem with the translog functional form is that the estimated profit function frequently fails to satisfy the appropriate theoretical curvative conditions. A procedure due to Jorgenson and Fraumeni [1981] (see also Jorgenson [1984]) may be used to impose the appropriate curvature conditions, but Diewert and Wales [1985] have shown that this procedure destroys the flexibility of the translog functional form. We therefore will implement the model described in section 2 by using a special case of the Biquadratic Restricted Profit Function defined by Diewert [1985;89]. For the case of only one capital good and a constant returns to scale technology, this profit function in period t, $\Pi^t(p,w,K)$ say, is defined as follows:

$$(4) \qquad \Pi^t(p,w,K^t)/K^t \equiv [p^T,w^T]a + [p^T,w^T]b\tau + (1/2)[p_2,\ldots,p_N;w^T]B[p_2,\ldots,p_N^T;w^T]^T/p_1$$

where $K^t>0$ denotes the quantity of capital utilized during year t, τ^t is a scalar indicator of technical progress in time period t, $p \equiv [p_1,p_2,\ldots,p_N]^T$ and $w \equiv [w_1,\ldots,w_M]^T$ are N and M dimensional (column) vectors of domestic and international prices respectively, $a \equiv [a_1,\ldots,a_{N+M}]^T$ and $b \equiv [b_1,\ldots,b_{N+M}]^T$ are N+M dimensional column vectors of unknown parameters and $B \equiv [b_{ij}]$ is an N-1+M by N-1+M symmetric matrix of unknown parameters.

Note that the right hand side of (4) defines a unit profit function: it defines the maximum amount of net revenue that the economy can produce, given one unit of capital and given that all producers face the domestic prices p and the international prices w. Note that our definition of Π^t has imposed a constant returns to scale property on the technology: if we change the capital input by the scalar $\lambda>0$, then the economy's net revenue will change by the same proportional factor λ; i.e., $\Pi^t(p,w,\lambda K^t) = \lambda\Pi^t(p,w,K^t)$. The function becomes more complex, of course, if more fixed inputs are considered.

The profit function defined by (4) is a straightforward adaptation of the Generalized McFadden cost function defined in Diewert and Wales [1985]. Adapting their proofs, it can be shown that the Π^t defined by (4) is a flexible functional form for a constant returns to scale technology. It can be seen the Π^t also is linearly homogeneous in prices, although it will be convex in prices if and only if the symmetric matrix B is positive semidefinite.

For our empirical work, we utilize the data for the U.S. economy for the period 1967-1982 which was developed by Morrison and Diewert [1985]. We have two domestic goods (i.e., N=2): x_1 is the quantity of domestic sales and x_2 is (minus) labor input.[5] We have three classes of internationally traded goods: y_1 is (minus) the quantity of import demand excluding petroleum products, y_2 is (minus) petroleum imports, and y_3 is exports. The data are more fully described in the data Appendix.

From (2) and (3), differentiating Π^t with respect to the prices p_1, p_2, w_1, w_2, w_3 yields: (i) the domestic supply function, (ii) (minus) the labor demand function, (iii) (minus) the nonpetroleum import demand function, (iv) (minus) the petroleum import demand function and (v) the export supply function for the U.S. economy in period t. We divided these net supply functions by the capital utilized in the period (this made the assumption of homoskedastic errors more plausible) and appended the stochastic disturbance $u^t{}_i$ to equation i in period t. The resulting system of estimating equations is given by

$$(5) \qquad x_1^t/K^t = a_1 + b_1\tau^t - (1/2)b_{11}(p_2^t/p_1^t)^2 - (1/2)b_{22}(w_1^t/p_1^t)^2$$

$$- (1/2)b_{33}(w_2^t/p_1^t)^2 - (1/2)b_{44}(w_3^t/p_1^t)^2 - b_{12}p_2^t w_1^t/(p_1^t)^2$$

$$- b_{13}p_2^t w_2^t/(p_1^t)^2 - b_{14}p_2^t w_3^t/(p_1^t)^2 - b_{23}w_1^t w_2^t/(p_1^t)^2$$

$$- b_{24}w_1^t w_3^t/(p_1^t)^2 - b_{34}w_2^t w_3^t/(p_1^t)^2 + u_1^t \ ;$$

$$(6) \qquad x_2^t/K^t = a_2 + b_2\tau^t + b_{11}p_2^t/p_1^t + b_{12}w_1^t/p_1^t + b_{13}w_2^t/p_1^t + b_{14}w_3^t/p_1^t + u_2^t$$

$$(7) \qquad y_1^t/K^t = a_3 + b_3\tau^t + b_{12}p_2^t/p_1^t + b_{22}w_1^t/p_1^t + b_{23}w_2^t/p_1^t + b_{24}w_3^t/p_1^t + u_3^t$$

$$(8) \qquad y_2^t/K^t = a_4 + b_4\tau^t + b_{13}p_2^t/p_1^t + b_{23}w_1^t/p_1^t + b_{33}w_2^t/p_1^t + b_{34}w_3^t/p_1^t + u_4^t$$

$$(9) \qquad y_3^t/K^t = a_5 + b_5\tau^t + b_{14}p_2^t/p_1^t + b_{24}w_1^t/p_1^t + b_{34}w_2^t/p_1^t + b_{44}w_3^t/p_1^t + u_5^t$$

There are five a_i, five b_i and ten b_{ij} parameters that must be estimated given that the cross equation symmetry constraints on the b_{ij} in equations (5)-(9) have been imposed. To estimate this system, we first assume that the error vectors $j^t \equiv [u^t{}_1, \ldots u^t{}_5]^T$ are independently distributed with a multivariate normal distribution with zero means and covariance matrix Ω. We then were able to obtain the maximum likelihood estimates for a,b and B using the iterative Zellner SYSTEM command in version 5 of SHAZAM (see White [1978]) because the system is linear. RESTRICT statements were used to impose the cross equation symmetry restrictions. The algorithm took 65 iterations to converge. The log of the likelihood function after one iteration was 210.25 and after 65 iterations was 260.04. The equation R-squares between the observed and predicted values were: .24, .80, .45, .25, and .93. These R-squares are relatively low because of our division of the quantities on the left hand side of (5)-(9) by capital to eliminate trends due to the growth in the economy. Our parameter estimates may be found in Table 1 below.

Unfortunately, this unconstrained specification resulted in a violation of the required curvature properties for the technology. Testing for this violation requires verifying that the matrix of second order partial derivatives of Π with respect to prices, $\nabla^2_{zz}\pi(z^t,K^t)$ is positive semidefinite. Since the price by definition is positive and the functional form is basically a quadratic, the convexity conditions are satisfied globally if and if the estimated B matrix is positive semidefinite, which can be determined by checking whether the eigenvalues are all positive. For our estimated unconstrained model, one of these eigenvalues turned out to be negative.

Since the estimated B matrix did not satisfy the required properties we imposed positive semi-definiteness on B by a reparametrization due to Wiley, Schmidt and Bramble [1973] and discussed and extended in Diewert [1985] and Diewert and Wales [1985]. The process requires replacing the components of B by the corresponding elements of a 4 by 4 lower triangular matrix C, where

10) $B=CC^T; \quad C \equiv [c_{ij}]; \quad i,j = 1,2,3,4; \quad c_{ij}=0 \text{ for } j>i$.

Due to the symmetry restrictions imposed on the b_{ij} parameters, this reparameterization does not change the total number of parameters to be estimated. However the equations become substantially more complicated since the b_{ij} parameters, which enter linearly in (5)-(9) given the restrictions on the model, must be replaced by

11) $b_{ij} = \Sigma_{k=1}^{4} \; c_{ik}c_{jk} \quad \text{for } i,j=1,\ldots,4$.

Using equations (10) or (11), the b_{ij}'s appearing in (5)-(9) were replaced by these functions of the c_{ij}'s, and the resulting parameters a_i, b_i, and C were estimated with the nonlinear regression command in SHAZAM using the Davisdon=Fletcher-Powell algorithm which required 345 iterations to converge. At the final stage, the log of the likelihood function was 257.8 and the equation by equation R-squares between the observed and predicted values were: .23, .79, .37, .25 and .91. Thus the likelihood decreased negligibly going from the B unconstrained case to the B constrained case and there was little loss of fit.

Once estimates for the elements of the C matrix have been found, estimates for the elements of the B matrix can be obtained using matrix equation (10), or, equivalently, equations (11). These estimates for the b_{ij} parameters along with the corresponding estimates for the a_i and b_i's are reported in the last column of Table 1. Standard errors are not reported since standard asymptotic theory is unfortunately not applicable when parameters are subject to inequality constraints.[6] As the reader can see from Table 1, in most cases the constrained parameter estimates are of the same sign and order of magnitude as the corresponding unconstrained case.

In the following sections, we use only the constrained estimates in order to form a system of U.S. net domestic supply functions by differentiating our estimated profit function with respect to domestic prices (recall (2)) and a system of U.S. net export supply functions by differentiating with respect to international prices (recall (3)).

4. Trade Elasticities

We differentiate $\Pi^t(p^t{}_1, p^t{}_2, w^t{}_1, w^t{}_2, w^t{}_3, K^t)$ with respect to p_1 and p_2 to obtain a domestic supply function $x^t{}_1 = \partial\Pi^t/\partial p_1$ and (minus) a labor demand function $x^t{}_2 = \partial\Pi^t/\partial p_2$. Similarly, differentiation of Π^t with respect to w_1, w_2, and w_3 yields $y^t{}_1 = \partial\Pi^t/\partial w_1$ = (minus) a nonpetroleum import demand function, $y^t{}_2 = \partial\Pi^t/\partial w_2$ = (minus) a petroleum products import demand function and $y^t{}_3 = \partial\Pi^t/\partial w_3$ = an export supply function.

It is of some interest to consider the various supply and demand elasticities with respect to prices.[7] In Table 2 below, we list the domestic supply elasticities $Ex_1p_i = (p^t{}_i/x^t{}_1)(\partial x^t{}_1/\partial p_i)$, i=1,2, and $Ex_1w_j = (w^t{}_j/x^t{}_1)\cdot(\partial x^t{}_1/\partial w_j)$, j=1,2,3 for alternate years. In tables 3-6 we list the price elasticities for the labor demand, nonpetroleum import demand, petroleum import demand and export supply functions, respectively.

From Table 2, it can be seen that in 1982, a 1% increase in the domestic price level or the price of nonpetroleum imports would increase domestic supply (holding capital and other prices fixed) by about .8% and .01% respectively while a 1% increase in the wage rate, the price of exports or the price of petroleum imports would decrease domestic supply by about .7%, .1% and .1% respectively. It is interesting to note that the one sign change appearing over time in these tables is for petroleum imports; until 1973 a 1% increase in the price of petroleum imports corresponded to an increase in domestic supply and later it caused a decrease in supply. This has important implications about the impact of the OPEC price shocks on domestic production.

In the cost function context, two inputs are Hicks [1946] -Allen [1938;504] substitutes (complements) if the cross partial derivative of the cost function with respect to the prices of the two goods is positive (nonpositive). In the profit function case, we define two distinct goods to be <u>substitutes</u> (complements) if the cross partial derivative of the profit function with respect to the two prices is <u>negative</u> (nonnegative). Hicks [1946;311-312] shows that in the two input case, the two inputs must be substitutes while in the three input case, at least two pairs of inputs must be substitutes. Hicks [1946;321] and Diewert [1974;143-145] show more generally that in the case of only two variable goods, the two goods must be substitutes while in the three variable good case, at least two pairs of goods must be substitutes. Since the elasticities in Table 2 are second order derivatives of the profit function with respect to p_1 and one of the other prices times the other price divided by $x^t{}_1$, it can be seen that domestic sales and exports are substitutes, which we might expect. Domestic sales and labor are substitutes while sales with either of the two types of imports are complementary pairs of goods.

From Table 3, it can be seen that in 1982, a 1% increase in the domestic price level, the price of imports or the price of petroleum imports would increase the demand for labor by about 1.1%, .1%, and .02% respectively, while a 1% increase in the wage rate or in the price of exports would decrease labor demand by about 1.2% and .1% respectively. It appears that labor is quite highly substitutable with domestic sales, and slightly substitutable with both nonpetroleum and petroleum imports. Labor is complementary with exports. The relatively large own price elasticity of demand for labor means that there would appear to be a rather high wage-unemployment tradeoff; i.e., moderation in wage demands would lead to relatively large employment increases. Conversely, the small elasticity with respect to petroleum imports implies that there is very little tradeoff between use of energy and labor in production.

Turning to Table 4, in 1982 a 1% increase in the wage rate or the price of petroleum imports would increase the demand for nonpetroleum imports by about 1% and .09% respectively, while a 1% increase in the domestic price level, the price of nonpetroleum imports or the price of exports would decrease the demand for nonpetroleum imports by about .2%, .7%, and .2%. Note that the own price elasticity of demand for nonpetroleum imports was close to -1 for much of the period. It can be seen that nonpetroleum imports are substitutable with labor (as might be expected) and petroleum imports, and are complementary with domestic sales and exports. Note that the substitutability with labor is particularly strong.

From Table 5, in 1982 a 1% increase in the wage rate or in the price of nonpetroleum imports would increase the demand for petroleum imports by about .3% and .2% respectively, while a 1% increase in the domestic price level, the price of petroleum imports or the price of exports would decrease the demand for petroleum imports by about .25%, .6%, and .1% respectively. It can be seen that petroleum imports are substitutable with labor and nonpetroleum imports, and are complementary with domestic sales and exports. Note that although all elasticities are small, the complementarity with sales is relatively strong in the first part of the sample, decreases, and then rises again approximately to its initial value, whereas the substitutability with labor is larger at the beginning of the period (Ey_2p_2 = .414 in 1967) and diminishes consistently over time (Ey_2p_2 = .965 in 1982). Conversely, the own elasticity begins very small (Ey_2w_2 = -.182 in 1967) and increases over time, with a noticeable jump post 1973 and a peak in 1980 of -.894 about the time of the second OPEC shock.

Finally, turning to Table 6, we see that in 1982 a 1% increase in the wage rate, in the price of nonpetroleum imports, in the price of petroleum imports or in the price of exports would increase the supply of exports by about .8%, .2%, .07%, and .3% respectively while a 1% increase in the price of domestic sales (holding other prices and capital input constant as usual) would decrease the supply of exports by about 1.4%. Thus exports are strongly substitutable with domestic sales, are quite strongly complementary with labor and are weakly complementary with the two classes of imports. The own price elasticity of export supply stayed fairly constant between .32 and .375 over the sample period.

Although these elasticities are interesting in their own right, it may be even more important to consider their combined implications for questions that we may ask about international trade. One very interesting question which may be posed, for example, is how large a change in the real exchange rate would be required to create a zero trade surplus. We turn now to an illustration of this application of the above elasticities.

5. Devaluation and the Balance of Trade

Let e be an exchange rate and define the balance of trade function in period t, $B^t(e)$, as the value of exports minus the value of imports expressed in world prices.[8] Since net export supply functions can be obtained by differentiating the profit function with respect to the international prices w_j (recall (3), Hotelling's Lemma), we may define $B^t(e)$ as follows:

$$(12) \qquad B^t(e) \equiv \textstyle\sum_{j=1}^{3} w_j^t \partial\Pi^t(ep_1^t, ep_2^t, w_1^t, w_2^t, w_3^t, K^t)/\partial w_j \quad .$$

Since e is a hypothetical real exchange rate which expresses the price of nontraded U.S. domestic goods relative to internally traded goods, a decline in e implies a devaluation of the U.S. dollar.[9] If we define Π^t by (4) using our constrained parameter estimates, it can be seen that $B^t(1)$ corresponds to the estimated trade balance for the U.S. for the year t; $B^t(1)>0$ (<0) indicates that the estimated trade balance is a surplus (deficit). The $B^t(1)$ are tabled in Table 7 below.

Differentiate (12) with respect to e and evaluate the resulting derivatives at e=1:

$$(13) \qquad B^{t'}(1) = \sum_{j=1}^{3} w_j^t \sum_{i=1}^{2} p_i^t \, \partial^2 \Pi^t(p^t, w^t, K^t)/\partial w_j \partial p_i$$

where $y^t{}_j(p,w,K)$ is the jth net export supply function for the U.S. economy. Since $B^t(1)\neq 0$ in our sample, we can convert $B^{t'}(1)$ into an elasticity by multiplying it by $1/B^t(1)$:

$$(14) \qquad B^{t'}(1)/B^t(1) = \sum_{j=1}^{3} w_j^t y_j^t \sum_{i=1}^{2} (p_i^t/y_j^t)(\partial y_j^t(p^t, w^t, K^t)/\partial p_i)/B^t(1)$$

$$= \sum_{j=1}^{3} \sum_{i=1}^{2} s_j^t E y_j p_i^t$$

where $s_j = \sum_j w^t{}_j y^t{}_j)/B^t(1)$ for j=1,2,3. Note that $\sum^3{}_{j=1} s^t{}_j = 1$ and $Ey_j p^t{}_i$ is the period t price elasticity of y_j with respect to p_i (see Tables 4, 5, and 6 for a partial listing for these elasticities). Equations (12) and (14) enable us to compute $B^{t'}$.

A linear approximation to $B^t(e)$ is:

$$(15) \qquad B^t(e) \cong B^t(1) + B^{t'}(1)(e-1) \quad .$$

From (15), it can be seen that a one cent appreciation in the real exchange rate in period t will change the balance of trade net surplus by approximately $.01B^{t'}(1)$ billions of dollars (all quantities are measured in billions of 1972 dollars and all prices are set equal to 1 in 1972). These numbers are tabled in Table 7. It can be seen that these <u>trade balance impacts</u> are all negative as we might expect;[10] a devaluation of domestic as compared to traded goods always improves the U.S. balance of trade. Note that the effectiveness of a hypothetical devaluation improves over the sample period [11]: a one cent devaluation in the late sixties would tend to improve the trade balance by only .5 billion dollars while the same devaluation in 1981 would improve the balance by about 4 billion dollars.

If we take the right hand side of (15) and set it equal to zero, we may solve for an approximate equilibrium real exchange rate that would induce a zero trade surplus in each year. The resulting series e^t may also be found in Table 7. Note that $e^{1982} = .72$, so that an approximate 28% devaluation would be required to eliminate the estimated 1982 trade deficit. However, in addition to the crudeness of the approximation (15), the reader should note that our model assumes that domestic prices are "sticky" in the short run so that the relative price of output and labor remain unchanged. In the long run, domestic demand conditions may alter these prices and hence our equilibrium exchange rate may be only a short run equilibrium. In addition, moving to an analysis of long run adjustment is much more complex because the dynamics of investment and different capital vintages should be incorporated in order to model lags associated with real world devaluations.

Note, finally, that the model of real exchange rate devaluation or appreciation developed here can be adapted to yield a producer oriented theory of purchasing power parities[12]: an equilibrium PPP price vector for a country in period t could be defined to be the price vector $e^t p^t$ where the scalar exchange rate e^t) solves the following equation:

$$(16) \quad w^t \cdot \nabla \Pi_w^t(e^t p^t, w^t, K^t) = T$$

where w^t is the vector of international prices that all countries face, $\nabla_w \Pi^t$ is the country's net export vector and T denotes the long run equilibrium level of "transfers" (i.e., remittances abroad, net tourist expenditures abroad, net international debt service,[13] etc.) that the production sector of the country should make in period t. If labor were treated as a fixed good so that it appears in the "capital" vector K^t, and there were only one domestic good, then the scalar $e^t p^t$ would be the country's equilibrium PPP. This formulation is useful because it shows that a country's equilibrium PPP will not generally be unity even if technology sets are identical across countries. The equilibrium PPP depends on: (i) the country's long run level of net transfers T, (ii) the country's technological capabilities (i.e., the production set S^t or its dual Π^t), (iii) the world price vector w^t, and (iv) the country's endowments of labor skills, capital stocks and natural resources that are embodied in the vector K^t. Changes in any of these variables will generally change the country's equilibrium PPP.

6. Conclusion

We have shown how a consistent framework for simultaneously modeling a country's domestic supply, labor demand, import demand and export supply functions can be obtained using producer theory. We followed in the footsteps of Kohli [1978] and others, except that we adapted to the profit function context some of the recent work by Diewert and Wales [1985] on flexible functional forms that satisfy curvature conditions globally. Use of the producer-oriented framework to model export supply and import demand functions is much simpler than traditional general equilibrium approaches since we do not have to model the consumer side of the model and the associated aggregation over consumers problem. However, consideration of only one "building block" of the full general equilibrium model, as with any applied production theory analysis, requires partial equilibrium assumptions such as exogeneity of domestic prices rather than allowing these prices to be affected by changes in international prices. Our partial equilibrium framework requires assumptions about conditional distributions which may not be true and thus our estimates may be subject to some simultaneous equations bias.

Additional problems and deficiencies arise from the simplifications incorporated in our model including: (i) a high level of aggregation, (ii) the assumption of constant returns to scale, (iii) the assumption of price taking behavior (in particular, possible induced changes in foreign prices are ignored and possible monopolistic behavior is assumed away), (iv) the neglect of natural resource stocks, (v) the use of the assumption of homogeneous capital across time periods (in particular, we have not allowed for possible adjustment costs involved in installing new capital equipment), and (vi) avoidance of consideration of the various taxes, tariffs, subsidies and quotas that impact the allocation of resources within the private production sector of the U.S. economy. We have shown, however, that we can effectively deal with many interesting issues within the framework of this simple model. Although it is possible to extend the analysis to include, say, imperfect competition and nonconstant returns to scale, these extensions cause additional aggregation and computation problems that, at least for a first attempt, are best avoided. Finally, it would be useful to push our data base forward through 1985, so that we could attempt to model the very recent experience of the U.S. economy.[14] These extensions and developments will be the focus of future research.

7. Data Appendix

The data required to calculate our indexes include price and quantity information on national output, capital and labor inputs, exports and imports. We have developed the output, import and export data for 1968-82 from the National Income and Product Accounts (U.S. Department of Commerce [1981],[1982],[1983]), and have used real capital stock data constructed by the Bureau of Labor Statistics (U.S. Department of Labor [1983]) and real labor data updated from Jorgenson and Fraumeni [1981], since these series closely approximate our theoretically ideal indexes.

More specifically, we have calculated the value of output ($P^t{}_Y Y^t$) as gross domestic business product including tenant occupied housing output, property taxes, and Federal subsidies to businesses, but excluding Federal, State and Local indirect taxes and owner occupied housing. The corresponding price index ($P^t{}_Y$), was computed by cumulating the Business Gross Domestic Product Chain Price index.

The value of merchandise exports and imports were determined by adding the durable and nondurable export and import values, respectively, reported in the National Accounts. Tariff revenues were added to the value of imports. For 1967-82, value and price data for nine different types of exports and ten types of imports were available, which were used to compute chain price indexes. An aggregate export price index $P^t{}_X$ was calculated as a translog (or Divisia in SHAZAM terminology) price index in the nine types of exports and the corresponding quantity series X^t was determined implicitly. Translog or Tornqvist indexes are defined in Diewert [1978]. An aggregate nonpetroleum import price index $P^t{}_M$ was calculated as a translog or divisia price index in the nine types of nonpetroleum imports and the corresponding quantity series

M^t was determined implicitly. The price index $P^t{}_O$ and the value series for petroleum imports were taken directly from the National Accounts and the corresponding quantity series O_t was determined by division.

Using the values of imports and exports, $P^t{}_M M^t$, $P^t{}_O O^t$ and $P^t{}_X X^t$, tax adjusted gross domestic private business sales to domestic purchasers, or absorption, was calculated as $P^t{}_S S^t = P^t{}_Y Y^t - P^t{}_X X^t + P^t{}_M M^t + P^t{}_O O^t$. The corresponding price ($P^t{}_S$) determined by cumulating the gross domestic purchases chain price index from the National Accounts, and the constant dollar quantity S^t was calculated by division.

For our labor quantity series L^t, we used the series constructed by D. Jorgenson and B. Fraumeni, whieh is conveniently tabled by the U.S. Department of Labor [1983;77]. Our total private labor compensation series, $P^t{}_L L^t$, was taken from the same publication. The price of labor, $P^t{}_L$, was determined by division.[15]]

For our capital services quantity series K^t we used the private business sector (excluding government enterprises) constant dollar capital serivces input tabled by the U.S. Department of Labor [1983;77]. In order to ensure that the value of privately produced outputs equals the value of privately utilized inputs, we determined the price of capital services $P^t{}_K$ residually, i.e., $P_K \equiv (P^t{}_Y Y^t - P^t{}_L L^t)/K^t$.

Changing now to the notation used in the main text of our paper, we set $x^t \equiv S^t$, $p^t \equiv P^t{}_S$; $x^t{}_2 \equiv -L^t$, $p^t{}_2 \equiv P^t{}_L$; $y^t{}_1 \equiv -M^t$, $w^t{}_1 \equiv P^t{}_M$; $y^t{}_2 \equiv -O^t$, $w^t{}_2 \equiv P^t{}_O$; $y^t{}_3 \equiv X^t$, $w^t{}_3 \equiv P^t{}_X$. The price series are tabled below in Table 8 while the quantity series are tabled in Table 9.

Finally, we calculated a translog price index using $p^t{}_1$, $w^t{}_1$, $w^t{}_2$, $w^t{}_3$ as prices and $x^t{}_1$, $y^t{}_1$, $y^t{}_2$, $y^t{}_3$ as the corresponding quantity weights. Call the resulting price and net output series PA^t and QA^t. We calculated a translog quantity index using K^t and L^t as quantities and $P^t{}_K$ and $P^t{}_L$ as the corresponding price weights. Call the resulting input quantity and price series QB^t and PB^t. We took τ^t to be QA^t/QB^t, a translog productivity index. For a theoretical justification of this index, see Theorem 1 in Diewert and Morrison [1986]. The series τ^t may be found in Table 9 below.

FOOTNOTES

[1]This concept can be traced to Samuelson [1953].

[2]See Woodland [1982; 3690375] for a summary of Kohli's approach and references to related empirical literature. M. Denny and D. Burgess used this approach in their unpublished Ph.D. theses; see Denny [1972] and Burgess [1974].

[3]Of course, some of these export goods could be highly or even perfectly substitutable with the corresponding domestic outputs.

[4]See Diewert [1974] [1982], Fuss and McFadden [1978] and Jorgenson [1984] for references to the rapidly growing literature that utilizes duality theory.

[5]This series was taken from Jorgenson and Fraumeni [1981].

[6]See Gourieroux, Holly and Monfort [1982]. Replacing B by CC^t is equivalent to estimating B subject to the usual determinantal inequality conditions for positive semidefiniteness. One of these inequalities turns out to be binding in our case and this is what leads to a failure in the usual asymptotic theory.

[7]All supply and demand elasticities with respect to the capital input are unity and this is due to our maintained hypothesis of constant returns to scale. We assumed a constant returns to scale technology in order to avoid conceptual problems that can occur when aggregating over producers.

[8]We ignore tariffs, subsidies and quotas in what follows.

[9]See Dornbusch [1980] for further discussion of the real exchange rate concept.

[10]If there were only one domestic variable good, it can be shown using the properties of profit functions that $B^{t'}(1)$ must be nonpositive; see Diewert [1974;145].

[11]This reflects the fact that the trade elasticities tabled in the previous section tend to become larger in magnitude over the sample period.

[12]See Dornbusch [1985] for a nice exposition of purchasing power parity theories.

[13]Net capital movements abroad should be excluded.

[14]The main obstacle to accomplishing this task is the unavailability of a sensible measure for U.S. real labor input into the private production sector. The Bureau of Labor Statistics [1983] for historical reasons uses unweighted manhours as its official measure of labor input. The conceptually more desirable wage rate weighted series developed by Jorgenson and Fraumeni [1981] which we utilized is only available through 1982.

[15]We wish to thank Mike Harper at BLS and Barbara Fraumeni for their help in providing updated series.

TABLE 1. PARAMETER ESTIMATES FOR THE UNCONSTRAINED AND CONSTRAINED MODELS

COEF. SYMBOL	UNCONSTRAINED ESTIMATES	STANDARD ERROR	T RATIO	CONSTRAINED ESTIMATES
a_1	3.7639	.281	13.39	3.8242
a_2	-3.3242	.264	-12.57	-3.4414
a_3	- .23701	.051	- 4.61	- .23835
a_4	- .047639	.013	- 3.57	- .050116
a_5	- .079179	.062	- 1.27	- .061502
b_1	.31215	.282	1.11	.29409
b_2	- .63288	.167	- 3.79	- .60521
b_3	.11210	.030	3.77	.12770
b_4	.036548	.012	2.96	.037821
b_5	.001142	.030	.038	- .025091
b_{11}	2.0682	.269	7.70	2.191161
b_{22}	.065458	.030	2.19	.114062
b_{33}	.001921	.0008	2.43	.002099
b_{44}	- .046765	.043	- 1.08	.050206
b_{12}	- .19818	.045	- 4.38	- .006116
b_{13}	- .008982	.011	- .81	- .057495
b_{14}	.18578	.067	2.83	.556407
b_{23}	- .004425	.002	- 2.63	- .003128
b_{24}	.095383	.031	1.94	.035664
b_{34}	.005532	.003	3.08	.002245

TABLE 2. DOMESTIC SUPPLY ELASTICITIES

YEAR	Ex_1p_1	Ex_1p_2	Ex_1w_1	Ex_1w_2	Ex_1w_3
1967	.615	-.550	.011	.0012	-.077
1969	.714	-.651	.016	.0013	-.081
1971	.752	-.688	.017	.0014	-.082
1973	.843	-.761	.016	.0018	-.099
1975	.883	-.753	.006	-.0007	-.135
1977	.852	-.738	.009	-.0006	-.122
1979	.943	-.809	.008	-.003	-.140
1981	.875	-.732	.010	-.017	-.136
1982	.803	-.690	.013	-.011	-.115

TABLE 3. LABOR DEMAND ELASTICITIES

YEAR	Ex_2p_1	Ex_2p_2	Ex_2w_1	Ex_2w_2	Ex_2w_3
1967	.894	- .911	.091	.0029	-.076
1969	1.018	-1.036	.094	.0028	-.079
1971	1.080	-1.102	.100	.0030	-.081
1973	1.155	-1.186	.120	.0037	-.093
1975	1.150	-1.191	.145	.0110	-.115
1977	1.151	-1.195	.141	.0111	-.108
1979	1.235	-1.287	.156	.0143	-.119
1981	1.162	-1.214	.148	.0214	-.118
1982	1.146	-1.191	.135	.0183	-.108

TABLE 4. NONPETROLEUM IMPORT DEMAND ELASTICITIES

YEAR	Ey_1p_1	Ey_1p_2	Ey_1w_1	Ey_1w_2	Ey_1w_3
1967	-.259	1.349	- .834	.025	-.282
1969	-.334	1.293	- .732	.021	-.248
1971	-.324	1.214	- .685	.019	-.224
1973	-.266	1.355	- .850	.025	-.265
1975	-.087	1.456	-1.098	.080	-.351
1977	-.128	1.310	- .958	.072	-.296
1979	-.106	1.342	-1.014	.089	-.311
1981	-.121	1.158	- .878	.121	-.281
1982	-.165	1.023	- .720	.093	-.231

TABLE 5. PETROLEUM IMPORT DEMAND ELASTICITIES

YEAR	Ey_2p_1	Ey_2p_2	Ey_2w_1	Ey_2w_2	Ey_2w_3
1967	-.286	.414	.245	-.182	-.190
1969	-.315	.424	.230	-.161	-.179
1971	-.264	.347	.188	-.130	-.141
1973	-.259	.354	.212	-.155	-.152
1975	.033	.352	.254	-.453	-.186
1977	.027	.310	.217	-.400	-.154
1979	.107	.323	.234	-.499	-.165
1981	.422	.335	.243	-.822	-.178
1982	.253	.264	.177	-.563	-.131

TABLE 6. EXPORT SUPPLY ELASTICITIES

YEAR	Ey_3p_1	Ey_3p_2	Ey_3w_1	Ey_3w_2	Ey_3w_3
1967	-1.626	.988	.246	.017	.375
1969	-1.596	1.003	.229	.015	.349
1971	-1.565	.992	.226	.015	.332
1973	-1.508	.929	.235	.016	.329
1975	-1.425	.795	.242	.040	.348
1977	-1.430	.814	.240	.042	.334
1979	-1.397	.782	.238	.048	.329
1981	-1.385	.752	.230	.073	.331
1982	-1.405	.790	.224	.067	.324

TABLE 7. IMPACT ON THE TRADE BALANCE OF A ONE CENT INCREASE

YEAR t	ESTIMATED TRADE BALANCE $B^t(1)$	IMPACT ON TRADE BALANCE $.01B^{t'}(1)$	EQUILIBRIUM EXCHANGE RATE e^t
1967	1.16	- .49	1.023
1968	.37	- .51	1.007
1969	- .30	- .55	.995
1970	- 1.80	- .62	.971
1971	- 4.96	- .66	.925
1972	- 7.59	- .74	.897
1973	.45	-1.01	1.004
1974	16.42	-1.68	1.098
1975	10.86	-1.90	1.057
1976	- 5.25	-1.86	.972
1977	- 9.93	-2.08	.952
1978	-12.04	-2.36	.949
1979	- 9.53	-2.95	.968
1980	-25.28	-3.82	.934
1981	-51.16	-4.04	.873
1982	-104.05	-3.72	.720

TABLE 8. PRICE SERIES

YEAR	p^t_1	p^t_2	w^t_1	w^t_2	w^1_3	P^t_K
1967	0.80182	0.72204	0.78489	0.871	0.84947	0.92252
1968	0.82588	0.77277	0.79758	0.865	0.86270	0.95383
1969	0.86056	0.82668	0.82298	0.858	0.89104	0.94054
1970	0.90359	0.88226	0.88232	0.880	0.93788	0.90078
1971	0.95239	0.93318	0.92638	0.959	0.96959	0.95611
1972	1.00000	1.00000	1.00000	1.000	1.00000	1.00000
1973	1.04900	1.06135	1.17053	1.277	1.16592	1.09631
1974	1.10873	1.15432	1.48154	4.197	1.48958	1.08115
1975	1.22404	1.22917	1.63038	4.334	1.66677	1.21930
1976	1.33298	1.32006	1.65766	4.599	1.71765	1.32665
1977	1.40762	1.40568	1.80812	4.971	1.78929	1.48218
1978	1.49912	1.53092	1.98778	4.981	1.90020	1.57942
1979	1.61455	1.66222	2.20819	7.034	2.16876	1.64533
1980	1.76632	1.78329	2.50159	11.554	2.40338	1.70086
1981	1.95356	1.93094	2.57416	12.971	2.63230	1.93726
1982	2.12937	2.06104	2.55045	12.064	2.62128	1.86782

TABLE 9. QUANTITY SERIES, billions of 1972 dollars

YEAR	x^t_1	x^t_2	y^t_1	y^t_2	y^t_3	K^t	τ^t
1967	771.141	-543.092	-31.556	-2.4006	36.095	254.516	1.08920
1968	827.861	-556.789	-38.374	-2.7560	38.968	266.369	1.09250
1969	857.460	-576.440	-40.286	-3.0874	40.863	278.533	1.08921
1970	843.920	-570.485	-41.851	-3.3261	45.277	290.697	1.04660
1971	866.445	-573.463	-45.245	-3.8060	44.652	300.990	1.00943
1972	913.824	-595.496	-51.124	-4.6500	49.353	311.907	1.00000
1973	973.382	-625.866	-53.038	-6.5896	61.205	326.255	1.00142
1974	992.963	-630.035	-51.805	-6.3400	65.936	340.914	0.98079
1975	995.917	-611.574	-43.473	-6.2337	63.973	350.272	0.87702
1976	991.136	-635.394	-53.597	-7.5175	66.597	356.822	0.83067
1977	1072.649	-666.360	-58.176	-9.0490	66.911	366.179	0.82180
1978	1142.027	-705.067	-66.492	-8.4946	74.145	379.279	0.81093
1979	1178.245	-736.629	-67.450	-8.5985	82.617	393.627	0.78878
1980	1156.462	-743.179	-65.758	-6.8720	91.496	407.663	0.74589
1981	1180.657	-759.257	-70.896	-5.9825	88.060	419.203	0.687631
1982	1113.010	-738.415	-71.869	-5.0729	79.770	435.110	0.625447

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