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ABSTRACT

Collateral constraints widely used in models of financial crises feature a pecuniary externality: Agents do not internalize how borrowing decisions taken in "good times" affect collateral prices during a crisis. We show that agents in a competitive equilibrium borrow more than a financial regulator who internalizes this externality. We also find, however, that under commitment the regulator's plans are time-inconsistent, and hence focus on studying optimal, time-consistent policy without commitment. This policy features a state-contingent macroprudential debt tax that is strictly positive at date t if a crisis has positive probability at t + 1. Quantitatively, this policy reduces sharply the frequency and magnitude of crises, removes fat tails from the distribution of returns, and increases social welfare. In contrast, constant debt taxes are ineffective and can be welfare-reducing, while an optimized "macroprudential Taylor rule" is effective but less so than the optimal policy.

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1 Introduction

Following the 2008 Global Financial Crisis, the realization that credit booms are infrequent but perilous events that often end in similar crises (see, for example, Mendoza and Terrones (2008) and Reinhart and Rogoff (2009)) has resulted in a strong push for a new "macroprudential" form of financial regulation. The objective of this new regulation is to adopt a macroeconomic perspective of credit dynamics, with a view to defusing credit booms in their early stages as a prudential measure to prevent them from turning into crises (see for example Borio, 2003 and Bernanke, 2010). Efforts to move financial regulation in this direction, however, have moved faster and further than our understanding of how financial policies influence the transmission mechanism driving financial crises, particularly in the context of quantitative models that can be used to design and evaluate these policies.

This paper aims to fill this gap by answering three key questions: First, can credit frictions affecting individual borrowers produce strong financial amplification effects that result in macroeconomic crises? Second, if the answer to the first question is yes, what is the optimal design of macroprudential policy, particularly when commitment and credibility are issues at stake? Third, how effective is this policy at affecting private borrowing incentives in a prudential manner, reducing the magnitude and frequency of crises, and improving social welfare?

This paper provides answers to these questions derived from the theoretical and quantitative analysis of a dynamic stochastic general equilibrium model with a collateral constraint linking borrowing capacity to the market value of collateral assets. We start by developing a normative theory of the optimal macroprudential policy with and without commitment. Then we calibrate the model to data from industrial countries, and solve it numerically to show that, in the absence of macroprudential policy, the model embodies a strong financial amplification mechanism that produces financial crises. Then we solve for the optimal, time-consistent macroprudential policy of a regulator who cannot commit to future policies, and compute a state-contingent schedule of debt taxes that supports the optimal allocations as a competitive equilibrium. We evaluate the effectiveness of this policy for reducing the probability and magnitude of crises and increasing social welfare, and compare it with the effectivenss of simpler policy rules.

The collateral constraint is occasionally-binding and limits total debt (one-period debt plus within-period working capital) to a fraction of the market value of physical assets that can be posted as collateral, which are in fixed supply. This constraint is the engine of the model's financial amplification mechanism. When the constraint binds, Irving Fisher's classic debt-deflation effect is set in motion: Agents fire-sale assets to meet their obligations forcing price declines that tighten further the constraint and trigger further asset fire-sales. The result is a financial crisis driven by a nonlinear feedback loop between asset fire sales and borrowing capacity.

Focusing on financial frictions models with collateral constraints is important because of the prevalence of secured lending worldwide. The relevance of collateral in residential mortgage markets is self evident, but in addition, evidence cited by Gan (2007) shows that roughly 70 percent of all commercial and industrial loans are secured with collateral in the United States, the United Kingdom and Germany, and that real estate is a dominant form of collateral for firm financing in these three countries and in 58 emerging economies. In line with this evidence, Chaney et al. (2012) found that movements in U.S. local real estate prices are statistically significant for explaining cross-sectional variations in U.S. corporate investment. Moreover, there is also evidence showing that a sizable share of working capital financing requires collateral, and that it plays an important role in the drop in economic activity during financial crises. The Federal Reserve's 2013 *Survey of Terms of Business Lending* shows that 40 percent of commercial and industrial loans with less than a year of maturity used collateral. Amiti and Weinstein (2011) provide empirical evidence showing that trade credit is a key determinant of firm-level exports during financial crises.

The normative theory we study highlights a pecuniary externality similar to those used in the related literature on credit booms and macroprudential policy (e.g. Lorenzoni, 2008; Korinek, 2009; Bianchi, 2011; Stein, 2012): Individual agents facing a collateral constraint taking prices as given do not internalize how their borrowing decisions in "good times" affect collateral prices, and hence aggregate borrowing capacity, in "bad times" in which the collateral constraint binds. This creates a market failure that results in equilibria that can be improved upon by a financial regulator who faces the same credit constraint but internalizes the externality.

In our setup, this pecuniary externality implies that private agents fail to internalize the Fisherian debt-deflation effect that crashes asset prices and causes a crisis when the constraint binds. Moreover, when this happens production plans are also affected, because working capital loans pay for a fraction of the cost of inputs, and these loans are also subject to the collateral constraint. This results in a sudden increase in effective factor costs and a fall in output when the constraint binds. In turn, this affects expected dividend streams and therefore asset prices, and introduces an additional vehicle for the pecuniary externality to operate.

We study the optimal policy problem of a financial regulator who chooses the level of credit to maximize private utility subject to resource, collateral and implementability constraints. This regulator internalizes the pecuniary externality and cannot commit to future policies. The inability to commit is modeled explicitly by solving for optimal time-consistent macroprudential policy as a Markov perfect equilibrium, in which the effects of the regulator's optimal plans on future regulator's plans are taken into account. We followed this approach because we show that, under commitment, the regulator promises lower future consumption to prop up asset prices when the collateral constraint binds, but reneging is optimal ex post. Hence, in the absence of effective commitment devices, the optimal macroprudential policy under commitment is not credible.

We provide theoretical results showing that the regulator can decentralize its equilibrium allocations as a competitive equilibrium with optimal state-contingent debt taxes. A key element of these taxes is what we label a *macroprudential debt tax*, which is levied in good times when collateral constraints do not bind at date t but can bind with positive probability at $t+1$, and we show that this tax is always positive. When the constraint binds at t, the optimal taxes include two other components, which combined can be positive or negative: One captures the regulator's "ex post" incentives to influence asset prices to prop up credit when collateral constraints are already binding, and the other captures its incentives to influence the optimal plans of future regulators due to the lack of commitment.

The quantitative results show that the optimal policy reduces sharply the frequency and severity of financial crises. The probability of crises is 4 percent in the unregulated decentralized equilibrium v. close to zero in the equilibrium attained by the regulator. When a crisis occurs, asset prices drop 43.7 percent and the equity premium rises to 4.8 percent in the former, v. 5.4 and 0.7 percent respectively in the latter. Without regulation, the output drop is about 28 percent larger and the distribution of asset returns features an endogenous "fat tail". In terms of welfare, the optimal policy yields a sizable average gain of 1/3rd of a percent computed as the standard Lucas-style compensating variation in consumption that equates expected lifetime utility with and without policy. The optimal macroprudential debt tax is about 3.6 percent on average, fluctuates roughly half as much as GDP and has a correlation of 0.7 with leverage.

We also evaluate the effectiveness of policy rules simpler than the optimal policy. Fixed debt taxes are ineffective at best, and at worst they can be welfare-reducing. In contrast, a "macroprudential Taylor rule" that makes the tax an isoleastic function of the debt position relative to a target performs better. Optimizing the elasticity of this rule to maximize the average welfare gain, we construct a welfare-increasing rule that is effective at reducing the probability and magnitude of crises, albeit less so than the optimal policy.

This paper contributes to the growing quantitative macro-finance literature by developing a non-linear quantitative framework suitable for the normative analysis of macroprudential policy. Most of this literature, including this article, follows in the steps of the work on financial accelerators initiated by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997).[1] In particular, we follow Mendoza (2010) in analyzing the non-linear dynamics of an occasionally binding collateral constraint. He showed that a constraint of this kind produces financial crises that match the key features of observed crises, but his work abstracted from normative issues, which are the main focus of this study.

There are also quantitative studies of pecuniary externalities due to collateral constraints. In particular, Bianchi (2011) studies the effects of a debt tax in a setting in which the borrowing capacity is linked to the relative price of nontradable goods to tradable goods. Benigno et al. (2013) show in a similar setup that there can be a role for ex-post policies to reallocate labor from the non-tradables sector to the tradables sector, and show how this reduces precautionary savings. This paper differs from these studies in that it focuses on assets as collateral, and on asset prices as a key factor driving debt dynamics and the pecuniary externality, instead of the

[1]See, for example, Jermann and Quadrini (2012), Perri and Quadrini (2011), Khan and Thomas (2013), Bigio (2015), Boissay, Collard, and Smets (2015), Arellano and Kehoe (2012).

relative price of nontradable goods. This is important because private debt contracts commonly use assets as collateral, and also because the forward-looking nature of asset prices introduces effects that are absent otherwise. In particular, expectations of future crises affect the discount rates applied to future dividends and distort asset prices even in periods of financial tranquility. This also drives the time consistency issues that we tackle in this study and that were absent from previous work. Our model also differs in that we introduce working capital financing subject to the collateral constraint, which implies that the asset fire-sales also affect adversely production, factor allocations and dividend rates.

This paper is also related to the work of Jeanne and Korinek (2010), who studied a model in which assets serve as collateral. In their model, however, aggregate, not individual, assets are collateral for private borrowing, output follows an exogenous Markov-switching process, and debt is limited to the sum of a fraction of the value of collateral plus an exogenous constant. In addition, in their setup the planner faces asset prices that are predetermined in states in which the collateral constraint binds, while we study a time-consistent Markov perfect equilibrium in which the planner internalizes how borrowing choices made when the constraint binds affect prices contemporaneously via changes in current consumption and in the optimal plans of future regulators. Moreover, we can also prove that the optimal macroprudential tax is positive, while Jeanne and Korinek obtain a tax that depends on equilibrium objects with a potentially ambiguous sign.[2] The two studies also differ sharply in their quantitative implications. In their calibration, the constant term in the credit limit is much larger than the fraction of the value of assets that serve as collateral, and the probability of crises equals the exogenous probability of a low-output regime. As a result, debt taxes cannot affect the frequency of crises and have small effects on their magnitude. In contrast, in our model both the probability of crises and output dynamics are endogenous, and the optimal policy reduces sharply the incidence and magnitude of crises.

Our analysis is also related to other studies on inefficient borrowing and its policy implications. In particular, Schmitt-Grohé and Uribe (2014) and Farhi and Werning (2012) examine the use of prudential capital controls as a tool for smoothing aggregate demand in the presence of nominal rigidities and a fixed exchange rate regime. In earlier work, Uribe (2006) examined an economy with an aggregate borrowing limit and compared the borrowing decisions with those of an economy where the borrowing limit applies to individual agents.The literature on participation constraints in credit markets initiated by Kehoe and Levine (1993) has also studied inefficiencies that result from endogenous borrowing limits (e.g. Jeske, 2006 and Wright, 2006).

The rest of the paper is organized as follows: Section 2 presents the theoretical analysis. Section 3 conducts the quantitative analysis. Section 4 provides conclusions. In addition, an extended Appendix provides further details on various aspects of the theoretical and quantitative analyses.

[2] Compare in particular condition (14) in this paper v. (19) in Jeanne and Korinek (2010), and the optimal tax results in Prop. 1 and eq. (17) here v. eq. (21) in their paper. Korinek and Mendoza (2014) explain why their treatment of the planner's problem imposes time-consistency by construction (see p. 325).

2 A Model of Financial Crises & Macroprudential Policy

In this Section we study a small-open-economy model of financial crises driven by an occasionally binding collateral constraint. We characterize first a decentralized competitive equilibrium (DE) without regulation, following an approach similar to Mendoza (2010), in which a representative firm-household (the "agent") makes both production and consumption-savings plans for simplicity.[3] Then we analyze the optimal policy problem of a constrained-efficient social planner (SP) who is unable to commit to future policies, and demonstrate that the SP's allocations can be supported as a competitive equilibrium with state-contingent debt taxes. Finally, we compare the results with those obtained under commitment.

2.1 Firm-Household Optimization Problem

The representative agent has an infinite life horizon and preferences given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t - G(h_t)) \tag{1}$$

In this expression, $\mathbb{E}(\cdot)$ is the expectations operator, β is the subjective discount factor, c_t is consumption, and h_t is labor supply (we follow the standard convention of using lowercase letters for individual variables and uppercase letters for aggregate variables). The utility function $u(\cdot)$ is a standard concave, twice-continuously differentiable function that satisfies the Inada condition. The argument of $u(\cdot)$ is the composite commodity $c_t - G(h_t)$ defined by Greenwood et al. (1988). $G(h)$ is a convex, strictly increasing and continuously differentiable function that measures the disutility of labor. This formulation of preferences removes the wealth effect on labor supply, which prevents a counterfactual increase in labor supply during crises.

The agent combines physical assets, intermediate goods (v_t), and labor (h_t) to produce final goods using a production technology such that $y = z_t F(k_t, h_t, v_t)$, where F is a twice-continuously differentiable, concave production function andz_t is a productivity shock. This shock has compact support and follows a finite-state, stationary Markov process. Intermediate goods are traded in competitive world markets at a constant exogenous price p_v in terms of domestic final goods (i.e. p_v can be interpreted as the terms of trade taken as given by the small open economy, and is also the marginal rate of transformation between final goods and intermediate goods). The profits of the agent are given by $z_t F(k_t, h_t, v_t) - p_v v_t$.

The agent's budget constraint is:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t} = q_t k_t + b_t + [z_t F(k_t, h_t, v_t) - p_v v_t] \tag{2}$$

[3] We show in Section B of the Appendix that the competitive equilibrium is the same if we separate the optimization problems of households and firms (assuming a frictionless equity market).

where b_t and k_t are holdings of one-period non-state-contingent foreign bonds and domestic physical assets respectively, q_t is the market price of assets, and R_t is the world-determined gross real interest rate also taken as given by the small open economy.[4] Since assets are in fixed unit supply, the market-clearing condition in the asset market is simply $k_t = 1$. R_t is stochastic and, like the productivity shocks, it follows a finite-state, stationary Markov process with compact support.

The assumption that the economy is small and open relative to world financial market fits well the advanced economies we targeted to calibrate the model in Section 3. Even in the United States, interest rates have become increasingly dependent on external factors as a result of financial globalization. Mendoza and Quadrini (2010) use data from the *Flow of Funds* of the Federal Reserve to show that about 1/2 of the surge in net credit in the U.S. economy since the mid 1980s was financed by foreign capital inflows, and by 2010 more than half of the stock of treasury bills was owned by foreign agents. Still, Section H of the Appendix reports results showing how our quantitative findings vary if we replace the exogenous R_t process with an inverse supply-of-funds curve, which allows the real interest rate to increase as debt rises. Assuming that R_t follows a standard stationary process turns out to be conservative, because in the pre-2008-crisis boom years real interest rates displayed a protracted decline, and introducing this drop strengthens our results by enhancing the overborrowing effect of the pecuniary externality.

The firm-household also faces a working capital constraint that requires a fraction $\theta \leq 1$ of the cost of inputs $p_v v_t$ to be paid in advance of production using foreign credit. This credit is a within-period loan which effectively carries a zero interest rate. In contrast, in the conventional working capital setup the marginal cost of inputs carries a financing cost determined by R_t and thus responds to interest rate shocks (e.g. Uribe and Yue, 2006). Our formulation isolates the effect of working capital due to the need to provide collateral for these funds, as explained below, which is present even without the effect of R_t on marginal factor costs.

The agent faces a collateral constraint that limits total debt, including both intertemporal debt and working capital loans, not to exceed a fraction κ_t of the market value of beginning-of-period asset holdings (i.e. κ_t imposes a ceiling on the leverage ratio):

$$-\frac{b_{t+1}}{R_t} + \theta p_v v_t \leq \kappa_t q_t k_t \tag{3}$$

We show in Section A.5 of the Appendix that this collateral constraint can be derived as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents lenders from collecting more than a fraction κ_t of the value of the assets owned by a defaulting debtor. Note also that, while bonds and working capital are explicitly modeled as credit from abroad, this credit could also be provided by a domestic financial system that has unrestricted access to world capital markets and faces the same enforcement friction.

[4] An equivalent formulation is to assume deep-pockets, risk-neutral lenders that discount utility at rate $\beta^* = 1/R$. They are unaffected by domestic financial policies because their return on savings remains the same.

The model allows for shocks to κ_t, which can be viewed as financial shocks that lead creditors to adjust collateral requirements on borrowers (e.g. Jermann and Quadrini, 2012 and Boz and Mendoza, 2014). It is important to note, however, that neither the nature of the financial amplification mechanism nor the normative arguments we develop later rely on κ_t being stochastic. In fact, models with constant κ have been shown to be able to produce crises dynamics with realistic features in response to productivity shocks of standard magnitudes (see Mendoza (2010)).

The agent maximizes (1) subject to (2) and (3) taking prices as given. This maximization problem yields the following optimality conditions for each date $t = 0, ..., \infty$:

$$z_t F_h(k_t, h_t, v_t) = G'(h_t) \tag{4}$$

$$z_t F_v(k_t, h_t, v_t) = p_v(1 + \theta\mu_t/u'(t)) \tag{5}$$

$$u'(t) = \beta R_t \mathbb{E}_t\left[u'(t+1)\right] + \mu_t \tag{6}$$

$$q_t u'(t) = \beta \mathbb{E}_t\left[u'(t+1)\left(z_{t+1}F_k(k_{t+1}, h_{t+1}, v_{t+1}) + q_{t+1}\right) + \kappa_{t+1}\mu_{t+1}q_{t+1}\right] \tag{7}$$

where $\mu_t \geq 0$ is the Lagrange multiplier on the collateral constraint and $u'(t)$ denotes $u'(c_t - G(h_t))$.

Condition (4) is the labor-market optimality condition equating the marginal disutility of labor supply with the marginal productivity of labor demand, which is also the wage rate. Condition (5) is a similar condition setting the demand for intermediate goods by equating their marginal productivity with their marginal cost. The latter includes the financing cost $\theta\mu_t/u'(t)$, which is incurred only when the collateral constraint binds.

The last two optimality conditions are the Euler equations for bonds and assets respectively. When the collateral constraint binds, condition (6) implies that the marginal benefit of borrowing to increase c_t exceeds the expected marginal cost by an amount equal to the shadow price of relaxing the credit constraint (i.e. the agent faces an effective real interest rate higher than R_t). Condition (7) equates the marginal cost of an extra unit of assets with its marginal benefit. When the collateral constraint binds, the fact that assets serve as collateral increases the marginal benefit of buying assets by $\beta\mathbb{E}_t\kappa_{t+1}\mu_{t+1}q_{t+1}$.

Following Mendoza and Smith (2006), the interaction between the collateral constraint and asset prices can be illustrated by studying how the constraint alters the standard conditions for asset pricing and excess returns. First, using the definition of asset returns ($R^q_{t+1} \equiv \frac{z_{t+1}F_k(t+1)+q_{t+1}}{q_t}$) and iterating forward on (7) we can express the pricing condition as the expected present value of dividends (the marginal product of capital) discounted with R^q_{t+1} :

$$q_t = \mathbb{E}_t \sum_{j=0}^{\infty} \left(\prod_{i=0}^{j} \mathbb{E}_{t+i} R^q_{t+1+i} \right)^{-1} z_{t+j+1}F_k(t+j+1), \tag{8}$$

Second, combining (6) and (7) and the definition of R^q_{t+1}, the expected excess return on assets relative to bonds (i.e. the equity premium, $R^{ep}_t \equiv \mathbb{E}_t(R^q_{t+1} - R_t)$) can be decomposed into a liquidity premium, a collateral effect, and a risk premium as follows:

$$R^{ep}_t = \underbrace{\frac{\mu_t}{u'(t)\mathbb{E}_t m_{t+1}}}_{\text{Liquidity Premium}} - \underbrace{\frac{\mathbb{E}_t\left(\phi_{t+1} m_{t+1}\right)}{\mathbb{E}_t m_{t+1}}}_{\text{Collateral Effect}} - \underbrace{\frac{cov_t(m_{t+1}, R^q_{t+1})}{\mathbb{E}_t\left[m_{t+1}\right]}}_{\text{Risk Premium}} \tag{9}$$

where $m_{t,t+1} \equiv \frac{\beta^j u'(c_{t+1})}{u'(c_t)}$ is the one-period ahead stochastic discount factor, and $\phi_{t+1} \equiv \kappa_{t+1} \frac{\mu_{t+1}}{u'(c_t)} \frac{q_{t+1}}{q_t}$.

The liquidity premium increases R^{ep}_t when the constraint binds, with an effect proportional to μ_t. The collateral effect pushes R^{ep}_t in the opposite direction, because buying more assets at date t improves borrowing capacity at $t+1$ if the constraint can bind then.[5] The effect of the risk premium depends on how the covariance between the stochastic discount factor and the return on assets changes. The expectation of a binding collateral constraint rises the premium, because it makes the covariance "more negative" as it makes it harder to smooth consumption, but if the constraint is already binding the covariance rises as the constraint tigthens, reducing the risk premium. If the liquidity premium dominates, conditions (8)and (9) imply that a binding collateral constraint exerts pressure to fire-sell assets, raises excess returns and lowers asset prices.

The above mechanism is at the core of the model's pecuniary externality: higher *individual* debt leads to more frequent fire sales, driving excess returns up and asset prices down, which in turn reduces the *aggregate* borrowing capacity of the economy. In addition, because of the efficiency loss induced by the diminished access to working capital financing when the collateral constraint binds, the stream of dividends is also distorted. Moreover, because expected returns rise whenever the collateral constraint is expected to bind at any future date, condition (8) also implies that asset prices at t are affected by collateral constraints not just when the constraints binds at t, but whenever it is expected to bind at any future date. Hence, expectations about future excess returns (i.e. future liquidity and risk premia, and future collateral effects) and dividends feed back into current asset prices. This interaction will play an important role in the normative analysis.

The assumption that assets are not traded internationally is not innocuous. If assets are traded by foreign investors in frictionless markets, asset prices are not affected by a domestic collateral constraint, because they are priced discounting at the world's risk-free rate (see Mendoza and Smith, 2006). But if investors face trading costs or other frictions, prices respond and our findings about the optimal policy to tackle the pecuniary externality still hold.[6]

[5] A similar effect is present when k_{t+1} serves as collateral instead of k_t, but its timing changes. In this case, the marginal benefit of holding more assets as collateral shows up as the term $-\mu_t \kappa_t$ in R^{ep}_t (see Mendoza and Smith, 2006 and Bianchi and Mendoza, 2010)

[6] The optimal policy would be more complex, because the planner would have incentives to alter prices to extract monopolistic rents from foreigners.

2.2 Unregulated Decentralized Competitive Equilibrium

We define and solve for the DE in recursive form. We denote as s the triplet of date-t realizations of shocks $s = \{z_t, \kappa_t, R_t\}$ and separate individual bond holdings under the agent's control, b, from the economy's aggregate bond position, B, on which prices depend. Hence, the state variables for the agent's problem are the individual states (b, k) and the aggregate states (B, s). Aggregate capital is not a state variable because it is in fixed supply. In addition, in order to form expectations of future prices, the agent needs a "perceived" law of motion $B' = \Gamma(B, s)$ governing the evolution of the economy's bond position, and a conjectured asset pricing function $q(B, s)$.

The agent's recursive optimization problem is:

$$V(b,k,B,s) = \max_{b',k',c,h,v} u(c - G(h)) + \beta \mathbb{E}_{s'|s} V(b',k',B',s') \tag{10}$$

$$\begin{aligned} s.t. \quad q(B,s)k' + c + \frac{b'}{R} &= q(B,s)k + b + [zF(k,h,v) - p_v v] \\ -\frac{b'}{R} + \theta p_v v &\leq \kappa q(B,s)k \\ B' &= \Gamma(B,s) \end{aligned}$$

The solution to this problem is characterized by the decision rules $\hat{b}(b,k,B,s)$, $\hat{k}(b,k,B,s)$, $\hat{c}(b,k,B,s)$, $\hat{v}(b,k,B,s)$ and $\hat{h}(b,k,B,s)$. The decision rule for bond holdings induces an "actual" law of motion for aggregate bonds, which is given by $\hat{b}(B,1,B,s)$, and the recursive form of (8) induces an "actual" pricing function $\hat{q}(B,s)$.

Definition (Recursive Competitive Equilibrium). *A recursive competitive equilibrium is defined by an asset pricing function $q(B,s)$, a perceived law of motion for aggregate bond holdings $\Gamma(B,s)$, and decision rules $\hat{b}'(b,k,B,s), \hat{k}'(b,k,B,s), \hat{c}(b,k,B,s), \hat{h}(b,k,B,s), \hat{v}(b,k,B,s)$ with associated value function $V(b,k,B,s)$ such that:*

1. $\left\{\hat{b}(b,k,B,s), \hat{k}(b,k,B,s), \hat{c}(b,k,B,s), \hat{h}(b,k,B,s), \hat{v}(b,k,B,s), \hat{\mu}(b,k,B,s)\right\}$ *and* $V(b,k,B,s)$ *solve the agents' recursive optimization problem, taking as given* $q(B,s)$ *and* $\Gamma(B,s)$.

2. *The market for assets clear* $\hat{k}(B,1,B,s) = 1$

3. *The resource constraint holds:* $\frac{\hat{b}'(B,1,B,s)}{R} + \hat{c}(B,1,B,s) = zF(1, \hat{h}(B,1,B,s), \hat{v}(B',1,B,s')) + B - p_v \hat{v}(b,1,B,s)$

4. *The perceived law of motion for aggregate bonds and perceived asset pricing function are consistent with the actual law of motion and actual pricing function respectively:* $\Gamma(B,s) = \hat{b}(B,1,B,s)$ *and* $q(B,s) = \hat{q}(B,s)$.

2.3 Time-Consistent Planner's Problem

Comparing competitive equilibria with and without credit constraints, private agents borrow *less* in the former, because the constraints limit the amount they can borrow, and also because they build precautionary savings to self-insure against the risk of the sharp consumption adjustments caused by the constraints. In contrast, in the normative analysis that follows we study constrained-efficient allocations chosen by a planner or regulator who also faces the collateral constraint. We show that the DE with collateral constraints displays overborrowing relative to the SP's borrowing decisions when the collateral constraint does not bind. Hence, the DE with collateral constraints features *underborrowing* relative to the DE without collateral constraints but *overborrowing* relative to the SP equilibrium with the constraints.

We formulate the SP's problem in a manner similar to the "primal approach" to optimal policy analysis, with the planner choosing allocations subject to resource, implementability and collateral constraints. In particular, the SP chooses b_{t+1} on behalf of the representative firm-household subject to those constraints, but lacking the ability to commit to future policies. Since asset prices remain market-determined, the private agent's Euler equation for assets enters in the SP's problem as an implementability constraint. The planner thus does not set asset prices, but it does internalize how its borrowing decisions affect them.

The optimization problem of the private agent changes because b_{t+1} is no longer a choice variable, and this in turn has two implications (see Section A.1 of the Appendix for the complete formulation of the agent's optimization problem in the constrained-efficient equilibrium). First, the agent now takes as given a transfer T_t, which matches the resources added or subtracted by the SP's bond choices (the SP's budget constraint is $T_t = b_t - \frac{b_{t+1}}{R_t}$). Second, the private agent's problem no longer has an Euler equation for bonds, but the agent still faces the working capital constraint, and hence the optimality conditions for v_t and k_{t+1} are still (5) and (7).

Following Klein et al. (2008) and Klein et al. (2007), we focus on Markov-stationary policy rules that are expressed as functions of the payoff-relevant state variables (b, s). Since the SP cannot commit to future policy rules, it chooses its policy rules at any given period taking as given the policy rules that represent future SP's decisions, and a Markov-perfect equilibrium is characterized by a fixed point in these policy rules. At this fixed point, the policy rules of future planners that the current planner takes as given to solve its optimization problem match those that the current planner finds optimal to choose. Hence, the planner does not have the incentive to deviate from other planner's policy rules, thereby making these rules time-consistent.

Let $\mathcal{B}(b, s)$ be the policy rule for bond holdings of future planners that the SP takes as given, and $\{\mathcal{C}(b, s), \mathcal{H}(b, s), \mathcal{V}(b, s), \boldsymbol{\mu}(b, s), \mathcal{Q}(b, s)\}$ the associated recursive functions that return the values of the corresponding variables under that policy rule. Given these functions, the optimization problem of the private agents yields a standard Euler equation for assets (see equation (A.3) of the Appendix), which becomes the SP's asset pricing implementability constraint that can be rewritten recursively as follows:

$$q = \frac{\beta \mathbb{E}_{s'|s} \left\{ u'(\mathcal{C}(b',s') - G'(\mathcal{H}(b',s')))(\mathcal{Q}(b',s') + z' F_k(1, \mathcal{H}(b',s'), \mathbf{v}(b',s')) \ + \kappa' \boldsymbol{\mu}(b',s') \mathcal{Q}(b',s') \right\}}{u'(c - G(h))} \tag{11}$$

This expression shows that the b' choice of the planner affects asset prices directly, since it affects date-t marginal utility (the denominator of the stochastic discount factor). In addition, the choice of b' affects asset prices indirectly by affecting the bond holdings chosen by future planner's, along with their associated future allocations and prices.

As noted earlier, the SP maximizes the utility of the representative firm-household subject to the resource, collateral and implementability constraints. In addition, the planner faces as constraints the optimality conditions for labor and intermediate goods, and the Khun-Tucker conditions associated with the collateral constraint in the DE. We show in Section A.3 of the Appendix, however, that these additional constraints are not binding and can thus be ignored. Hence, taking again as given $\{\mathcal{B}(b,s), \mathcal{C}(b,s), \mathcal{H}(b,s), \mathcal{V}(b,s), \boldsymbol{\mu}(b,s), \mathcal{Q}(b,s)\}$ the SP's optimization problem can be represented in recursive form as follows:

$$\begin{aligned}
\mathcal{V}(b,s) &= \max_{c,b',q,h,v} u(c - G(h)) + \beta \mathbb{E}_{s'|s} \mathcal{V}(b',s') \qquad (12) \\
c + \frac{b'}{R} &= b + zF(1,h,v) - p^v v \\
\frac{b'}{R} - \theta p^v v &\geq -\kappa q \\
q u'(c - G(h)) &= \beta \mathbb{E}_{s'|s} [u'(\mathcal{C}(b',s') - G(\mathcal{H}(b',s')))(\mathcal{Q}(b',s') + z' F_k(1, \mathcal{H}(b',s'), \mathbf{v}(b',s'))) \\
&\quad + \kappa' \boldsymbol{\mu}(b',s') \mathcal{Q}(b',s')]
\end{aligned}$$

The economy's resource constraint has the multiplier $\lambda \geq 0$. The collateral constraint has the multiplier $\mu^* \geq 0$), which differs from μ because the private and social values from relaxing the collateral constraint differ. The asset pricing implementability constraint has the multiplier $\xi \geq 0$. As mentioned earlier, this constraint requires the planner to choose allocations such that q satisfies the pricing condition from the private asset market.

Given the definition of the recursive planner's problem, it is straightforward to define the constrained-efficient equilibrium:

Definition. *The recursive constrained-efficient equilibrium is defined by the policy function $b'(b,s)$ with associated decision rules $c(b,s)$, $h(b,s)$, $v(b,s)$, $\mu(b,s)$, pricing function $q(b,s)$ and value function $\mathcal{V}(b,s)$, and the conjectured function characterizing the decision rule of future planners $\mathcal{B}(b,s)$ and the associated decision rules $\mathcal{C}(b,s)$, $\mathcal{H}(b,s)$, $\mathbf{v}(b,s)$, $\boldsymbol{\mu}(b,s)$ and asset prices $\mathcal{Q}(b,s)$, such that these conditions hold:*[7]

[7] Note that there is a decision rule $\mu(b,s)$ even tough μ does not appear in the SP's problem, because as we show in Section A.1 of the Appendix, constraint (5) does not bind and this yields $\mu_t = (\mu_t^*/\lambda_t)u'(t)$.

1. *Planner's optimization:* $\mathcal{V}(b,s)$ *and the functions* $b'(b,s)$,$c(b,s)$, $h(b,s)$, $v(b,s)$, $\mu(b,s)$, *and* $q(b,s)$ *solve the Bellman equation defined in Problem (12) given* $\mathcal{B}(b,s)$, $\mathcal{C}(b,s)$, $\mathcal{H}(b,s)$, $\mathbf{v}(b,s)$, $\boldsymbol{\mu}(b,s)$ *and* $\mathcal{Q}(b,s)$.

2. *Time consistency (Markov stationarity): The conjectured policy rules that represent optimal choices of future planners match the corresponding recursive functions that represent optimal plans of the current regulator:* $b'(b,s) = \mathcal{B}(b,s)$, $c(b,s) = \mathcal{C}(b,s)$, $h(b,s) = \mathcal{H}(b,s)$, $v(b,s) = \mathbf{v}(b,s)$, $\mu(b,s) = \boldsymbol{\mu}(b,s)$, $q(b,s) = \mathcal{Q}(b,s)$.

2.4 Comparison of Equilibria and Optimal Policy

The SP and DE solutions differ in two key respects: First, private agent's fail to internalize how borrowing choices made at date t affect asset prices at date $t+1$ in states in which the collateral constraint binds. Second, they also do not take into account that when the collateral constraint binds already at t, date-t asset prices can be pushed up to enhance borrowing capacity by changing current borrowing choices or by affecting the decisions of future regulators. We characterize these differences by comparing the optimality conditions for consumption, bonds, and asset prices across the two environments.

The SP's optimality conditions re-written in sequential form are the following:[8]

$$c_t :: \qquad \lambda_t = u'(t) - \xi_t u''(t) q_t \tag{13}$$

$$b_{t+1} :: \qquad u'(t) = \beta R_t \mathbb{E}_t \left\{ u'(t+1) - \xi_{t+1} u''(t+1) \mathcal{Q}_{t+1} + \xi_t \Omega_{t+1} \right\} + \xi_t u''(t) q_t + \mu_t^* \tag{14}$$

$$q_t :: \qquad \xi_t = \frac{\kappa_t \mu_t^*}{u'(t)} \tag{15}$$

where Ω_{t+1} collects all the terms with derivatives that capture the effects of the planner's choice of b_{t+1} on q_t via effects on the actions of future planners in the right-hand-side of the implementability constraint.[9] Ω_{t+1} is composed of three terms. The first, captures how an extra unit of b_{t+1} affects future consumption and labor disutility, and thus affects the discounting of future asset returns (i.e. future marginal utility) that applies when determining q_t. In our quantitative work, this term is always negative, since $c_{t+1} - g(h_{t+1})$ rises with b_{t+1} and $u'' < 0$. The second term includes the effects by which higher b_{t+1} alters q_t by affecting asset prices and dividends at $t+1$. Numerically, asset prices tend to be increasing in bond holdings, and so this second term is usually positive. The third term captures how changes in b_{t+1} affect the tightness of the collateral constraint at $t+1$,

[8]These expressions are obtained by assuming that the policy and value functions are differentiable, and then applying the standard Envelope theorem to the first-order conditions of the planner's problem.

[9]In recursive form, $\Omega' = \frac{1}{R}\big[u''(\mathcal{C}(b',s') - G(\mathcal{H}(b',s')))\{\mathcal{Q}(b',s') + z'F_k(1,\mathcal{H}(b',s'),\mathbf{v}(b,s)(b',s'))\}\cdot \{\mathcal{C}_b(b',s') - G'(\mathcal{H}(b',s'))\mathcal{H}_b(b',s')\} + u'(\mathcal{C}(b',s') - G'(\mathcal{H}(b',s')))\{\mathcal{Q}_b(b',s') + z'[F_{kh}(1,\mathcal{H}(b',s'),\mathbf{v}(b',s'))\mathcal{H}_b(b',s') + F_{kv}(1,\mathcal{H}(b',s'),\mathbf{v}(b',s'))\mathbf{v}_b(b',s')]\} + \kappa'[\mu_b(b',s')\mathcal{Q}(b',s') + \mu(b',s')\mathcal{Q}_b(b',s')]\big]$

thereby affecting the value of collateral and asset prices at t. This third effect is negative. These three effects imply that the sign of Ω_{t+1} is ambiguous, but numerically we find that $\Omega_{t+1} < 0$, implying that the planner has higher incentives to borrow at the margin when the constraint binds.

Next we compare the optimality conditions of the SP and DE. Compare first the condition for c_t. The planner's condition is eq. (13), while the corresponding condition in the DE takes the standard form $\lambda_t = u'(t)$. Thus, the shadow value of wealth for the private agent is simply the marginal utility of current consumption, while the social shadow value of wealth adds the amount by which an increase in c_t reduces marginal utility and relaxes the implementability constraint.[10] Moreover, condition (15) shows that the social benefit from relaxing the implementability constraint is positive at date t if and only if the collateral constraint binds for the social planner at t, i.e., $\mu^* > 0 \leftrightarrow \xi_t > 0$. These two conditions together show that, when the collateral constraint binds, the marginal social benefit of wealth of an extra unit of c_t considers how the extra consumption raises equilibrium asset prices, which in turn relaxes the collateral constraint (i.e. combining (15) and (13) we obtain $-u''(t)q_t\frac{\kappa_t\mu_t^*}{u'(t)}$). If the collateral constraint does not bind, $\mu_t^* = \xi_t = 0$ and the shadow values of wealth in the DE and SP coincide.

Compare next the SP's Generalized Euler equation for bonds (14) with the corresponding Euler equation in the DE. This comparison highlights the two main properties that distinguish the DE and SP outcomes:

(1) *Effects via* q_{t+1}: Condition (14) indicates that the differences identified above in the private and social marginal utilities of wealth, which are differences in marginal benefits of bond holdings "ex post" when the collateral constraint binds, induce differences "ex ante," when the constraint is not binding. In particular, if $\mu_t = 0$, the marginal cost of increasing debt at date t in the DE is the standard term $\beta R_t \mathbb{E}_t u'(t+1)$. In contrast, the second term in the right-hand-side of (14) shows that the marginal social cost of borrowing is higher, because the SP internalizes the effect by which the larger debt at t reduces borrowing ability at $t+1$ if the credit constraint binds then. We can use again (15) to make this evident, by rewriting the second term in the right-hand-side of (14) as $-u''(t+1)q_{t+1}\frac{\kappa_{t+1}\mu_{t+1}^*}{u'(t+1)}$, which is positive for $\mu_{t+1}^* > 0$. Intuitively, since the planner values more consumption when the constraint binds ex-post, it borrows less ex-ante (i.e. there is overborrowing in the DE relative to the SP).

(2) *Effects via* q_t: The two Euler equations for bonds also differ in that condition (14) includes effects that reflect the SP's ability to induce changes in current asset prices when the constraint binds at t (i.e. $\mu_t > 0$). There are two effects of this kind: First, the term $\xi_t u''(t)q_t$ shows that, when $\mu_t > 0$, the SP internalizes that increasing c_t raises q_t and provides more borrowing capacity. This effect, when present, reduces the social marginal benefit of savings. Second, since the planner cannot commit to future policies, it takes into account how future planners respond to changes in its debt choice (which is a state variable of the next-period's planner). As explained above,

[10] Note that $-\xi_t u''(t)q_t > 0$ because $u''(\cdot) < 0$ and $\xi_t > 0$, as condition (15) implies. Hence, $\lambda_t > u'(t)$.

the derivatives of the future decision rule and pricing function with respect to b_{t+1} are included in Ω_{t+1} and are only relevant when $\mu_t > 0$, otherwise they vanish. Since Ω_{t+1} has an ambiguous sign, this effect can either increase or reduce the social marginal benefit of savings.

Notice a key difference between the q_t and q_{t+1} effects: The latter is only relevant when the constraint has a positive probability of becoming binding at $t+1$, while the former are only relevant when the constraint is binding at t. Existing studies of macroprudential policy focus mainly on the effects via q_{t+1}, but the above discussion suggests that the effects operating via q_t should also be part of the analysis.

We show now that the SP's equilibrium can be decentralized with a state-contingent tax on debt τ_t.[11] The price of bonds becomes $1/[R_t(1+\tau_t)]$ in the budget constraint of the private agent in the regulated competitive equilibrium, and there is also a lump-sum transfer T_t rebating tax revenue.[12] The agents' Euler equation for bonds becomes:

$$u'(t) = \beta R_t(1+\tau_t)\mathbb{E}_t u'(t+1) + \mu_t \tag{16}$$

Analyzing the SP's optimality conditions together with those of the regulated and unregulated DE leads to the following proposition:

Proposition 1 (Decentralization with Debt Taxes)**.** *The constrained-efficient equilibrium can be decentralized with a state-contingent tax on debt with tax revenue rebated as a lump-sum transfer and the tax rate set to satisfy:*

$$1+\tau_t = \frac{1}{\mathbb{E}_t u'(t+1)}\mathbb{E}_t\left[u'(t+1) - \xi_{t+1}u''(t+1)\mathcal{Q}_{t+1} + \xi_t\Omega_{t+1}\right] + \frac{1}{\beta R_t \mathbb{E}_t u'(t+1)}\left[\xi_t u''(t) q_t\right]$$

where the arguments of the functions have been shorthanded as dates to keep the expression simple. Proof: See Appendix A.2 .

The optimal tax schedule has two components that match the q_t and q_{t+1} effects on the social marginal benefit of savings identified in the SP's Euler equation. First, matching the pecuniary externality via q_{t+1}, we have a component denoted the *macroprudential debt tax*, τ^{MP}, which is a tax levied only when the collateral constraint is not binding at t but may bind with positive probability at $t+1$. Thus, this tax hampers credit growth in good times to lower the risk of future financial instability. Using (15), the macroprudential debt tax reduces to:

$$\tau_t^{MP} = \left(\frac{-\mathbb{E}_t[\xi_{t+1}u''(t+1)\mathcal{Q}(t+1)]}{\mathbb{E}_t[u'(t+1)]}\right) \tag{17}$$

[11] Following Bianchi (2011), it is also possible to decentralize the planner's problem using measures targeted directly to financial intermediaries, such as capital requirements, reserve requirements or loan-to-value ratios.

[12] The tax can also be expressed as a tax on the income generated by borrowing, so that the post-tax price would be $(1-\tau_t^R)(1/R_t)$. The two treatments are equivalent if we set $\tau_t^R = \tau_t/(1+\tau_t)$.

We can also demonstrate that this tax is non-negative. It is zero whenever the constraint is not expected to bind at $t+1$, but otherwise it is strictly positive, since $u' > 0, u'' < 0$ and $\xi \geq 0$. Thus, the tax is strictly positive whenever there is a positive probability that the collateral constraint (or equivalently the implementability constraint, given condition (15)) can become binding at $t+1$.

The second component of the optimal debt tax is formed by the two terms that match the effects operating through q_t, and hence are only present if the collateral constraint binds at t. Since the term $\xi_t u''(t) q_t$ is negative, it pushes for a debt subsidy, but since the term with Ω_{t+1} has an ambiguous sign, the combined effect also has an ambiguous sign and thus the second component of the tax can be positive or negative.

The above optimal policy analysis modeled the SP as choosing allocations and bonds directly subject to an implementability constraint, and showing that those allocations can be decentralized using debt taxes. In Section A.3 of the Appendix, we demonstrate that the same outcome can be obtained if we model instead the planner as choosing directly optimal debt taxes under discretion facing allocations and prices that are competitive equilibria. In particular, we show that this approach yields the same allocations and the same taxes. In addition, we also study in the Appendix a case in which debt taxes are restricted to be positive. This is interesting because the optimal τ_t we derived could be negative, which would require introducing other forms of taxation to finance subsidies, particularly lump-sum taxes. Our results show that the optimal macroprudential debt tax τ_t^{MP} has the same form as the one we derived here.

While it was possible to characterize theoretically the differences in the optimality conditions of the DE and SP, the optimal debt tax and the sign of the macroprudential debt tax, comparing the levels of debt and asset prices in the two equilibria is only possible via numerical simulation. Still, we can develop some intuition using elements of this analysis.

Borrowing decisions and asset prices are related, both when the collateral constraint binds and when it does not. When it binds, it is obvious that higher asset prices support higher debt. When it does not bind, expectations of higher asset prices reduce the need to build precautionary savings and lead to higher borrowing, since collateral constraints are expected to be more relaxed. Hence, understanding differences in asset prices is key for understanding differences in debt choices across the DE and SP. In turn, given the asset pricing condition, differences in expected asset returns are key for understanding how prices differ, and these differences can be characterized analytically.

Expected returns in the DE are characterized by the condition we derived for the equity premium (eq. (9)). The planner's excess returns are given by the following expression, which follows from applying the same treatment to the SP's optimality conditions as we did in the DE:

$$R_t^{ep} = \underbrace{\frac{\mu_t + \xi_t u''(t) q_t + \beta R_t \mathbb{E}_t \xi_t \Omega_{t+1}}{u'(t) \mathbb{E}_t m_{t+1}}}_{\text{Liquidity Premium}} - \underbrace{\frac{\mathbb{E}_t \left(\phi_{t+1} m_{t+1}\right)}{\mathbb{E}_t m_{t+1}}}_{\text{Collateral Premium}} - \underbrace{\frac{cov_t(m_{t+1}, R_{t+1}^q)}{\mathbb{E}_t m_{t+1}}}_{\text{Risk Premium}} - \underbrace{\frac{\beta R_t \mathbb{E}_t \left(\xi_{t+1} u''(t+1) \mathcal{Q}_{t+1}\right)}{u'(t) \mathbb{E}_t m_{t+1}}}_{\text{Externality Premium}} \tag{18}$$

Excess returns for the SP differ from the DE in two respects. First, they carry an "externality premium," because the SP internalizes the q_{t+1} effects of borrowing decisions. In fact, simplifying further this premium yields $R_t \tau_t^{MP}$, which is intuitive because the macroprudential tax rateis equal to the magnitude of the wedge the q_{t+1} effect drives into the SP's Euler equation for bonds relative to the DE. Second, the SP's liquidity premium includes two terms absent from the liquidity premium in the DE, which are related to the SP's effects on q_t when the constraint binds at t. As noted before, the first of these terms is negative, which lowers the return on assets, and the second term has an ambiguous sign. In addition to these first-order effects via the externality and liquidity premia, there are also second-order effects operating via endogenous changes in all four premia in the SP's excess returns, since the SP has a stronger precautionary-savings motive and supports allocations and prices that produce less risk.

The net effect of the four premia in the SP's returns can increase or decrease asset prices in the economy with regulation v. the DE. First, the externality premium pushes asset returns higher and asset prices lower, which tilts the portfolio towards bonds and away from risky assets. Second, the additional terms in the liquidity premium can push returns higher or lower, since their combined value has an ambiguous sign. Third, the second-order effects via changes in precautionary savings and risk can have ambiguous effects too, since higher demand for bonds weakens demand for assets, lowering their price, but lower risk premia reduce expected returns, increasing asset prices.

Quantitatively, under our baseline calibration, expected returns are generally higher, asset prices lower and debt smaller for the SP than the DE, and particularly so in the good-times regions of the state space in which the macroprudential tax is used. In contrast, during financial crises (which become very infrequent under the optimal policy) returns are significantly lower, prices higher and debt higher for the SP than the DE (see Section I of the Appendix for a detailed comparison of the quantitative asset pricing features of both economies). The lack of commitment is important for these results too. Under commitment, as we describe below, the planner considers how borrowing at any date t affects asset prices in previous periods, which creates a force to sustain higher asset prices even when the constraint does not bind.

2.5 Time Inconsistency under Commitment

We focused on studying optimal policy without commitment because we found that the problem under commitment yields time-inconsistent optimal plans.[13] A comprehensive analysis of this issue is beyond the scope of this paper, but we provide here the argument that shows why optimal policy under commitment is time-inconsistent.

The planner chooses at date 0 policy rules in a once-and-for-all fashion (see Section D of the Appendix for a detailed description of the planner's optimization problem under commitment and

[13] This time-inconsistency problem does not arise in Lorenzoni (2008)'s classic model of fire sales because in his model the asset price is determined by a static condition linking relative productivity of households and entrepreneurs, rather than expectations about future marginal utility. Similarly, in Bianchi (2011), borrowing capacity is determined by a static price of non-tradable goods.

a numerical example). In contrast with the problem without commitment, we found that in this case we do need to carry as constraints the optimality conditions of factor allocations and the Khun-Tucker conditions associated with the collateral constraint in the DE, because it cannot be guaranteed that they are always nonbinding.

The first-order conditions for consumption, bond holdings and asset prices under commitment in sequential form are the following:[14]

$$c_t :: \qquad \lambda_t = u'(t) - \xi_t u''(t) q_t + \xi_{t-1} u''(t)(q_t + z_t F_k(t) + \kappa_t \mu_t q_t) \tag{19}$$

$$b_{t+1} :: \qquad \lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t^* + \mu_t \nu_t \tag{20}$$

$$q_t :: \qquad \xi_t = \xi_{t-1}(1 + \kappa_t \mu_t) + \frac{\kappa_t(\mu_t \nu_t + \mu_t^*)}{u'(t)} \tag{21}$$

The time-inconsistency problem is evident from the presence of the lagged multipliers in the first and third conditions.[15] According to (19), the planner internalizes how an increase in consumption at time t helps relax the borrowing constraint at time t and makes it tighter at $t-1$. As (21) shows, this implies that the Lagrange multiplier on the implementability constraint ξ_t follows a positive, non-decreasing sequence, which increases every time the constraint binds. Intuitively, when the constraint binds at t, the planner promises lower future consumption so as to prop up asset prices and borrowing capacity at t, but ex post when $t+1$ arrives it is sub-optimal to keep this promise. In line with this intuition, we found in the numerical example of the Appendix that the planner with commitment supports higher asset prices and higher debt than in the DE (which is the opposite of what we found vis-a-vis the SP without commitment).

Section D of the Appendix also shows that state-contingent debt taxes can still be be used to decentralize the solutions of the problem under commitment as a competitive equilibrium, except that again this is a non-credible policy because of the time-inconsistency of the planner's optimal plans. The macroprudential component of this tax has the same form as in the problem without commitment only if the collateral constraint has *never* been binding up to date t and is expected to bind with some probability at $t+1$. Otherwise, even if it does not bind at t, the optimal macroprudential taxes differ with and without commitment.

[14] The problem has seven Lagrange multipliers, but in these first-order conditions only four appear. λ_t, μ_t^*, ξ_t, which are assigned to the same constraints as in the problem without commitment, and ν_t which is the multiplier assigned to the constraint that requires the complementary slackness condition of the DE to hold.

[15] It should be understood that time $t-1$ variables include the history up to time $t-1$ and time t variables represent the history up to time $t-1$ in addition to a time t exogenous disturbance.

3 Quantitative Analysis

This Section studies the model's quantitative implications by conducting numerical simulations for a baseline calibration. The first part describes the calibration and the rest discusses the results.

3.1 Calibration

We calibrate the model to annual frequency using OECD data between 1984 and 2012.[16] For some variables (e.g. housing wealth and working capital), we used only U.S. data because of data availability limitations.

The functional forms for preferences and technology are the following:

$$u(c - G(h)) = \frac{\left(c - \chi \frac{h^{1+\omega}}{1+\omega}\right)^{1-\sigma} - 1}{1-\sigma} \quad \omega > 0, \sigma > 1$$
$$F(k, h, v) = e^{z} k^{\alpha_k} v^{\alpha_v} h^{\alpha_h}, \qquad \alpha_k, \alpha_v, \alpha_h \geq 0 \quad \alpha_k + \alpha_v + \alpha_v \leq 1$$

TFP and R follow independent AR(1) processes.[17] TFP shocks follow an AR(1) process: $z_t = \bar{z} + \rho_z z_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0, \sigma_\varepsilon)$ and $\bar{z}$ normalized so that mean output equals one. The AR(1) process for the logged gross real interest rate is: $ln(R_t) = (1 - \rho_R)\bar{R} + \rho_R ln(R_{t-1}) + \varsigma_t$ with $\varsigma_t \sim N(0, \sigma_\varsigma)$. These shocks are discretized using Tauchen's quadrature method with 3 realizations for each shock. κ follows a standard two-state, regime-switching Markov process with $\{\kappa^L < \kappa^H\}$, where κ^H represents a normal credit regime and κ^L is a regime with unusually tight credit conditions, in the sense that switches from κ^H to κ^L are infrequent and the mean duration of the κ^L regime is low. For simplicity, this process is assumed to be independent from the Markov processes of z and R. The continuation transition probabilities are denoted $P_{L,L}$ and $P_{H,H}$ for κ^L and κ^H respectively, and the long-run probabilities are given by $P^L = P_{H,L}/(P_{L,H} + P_{H,L})$ and $P^H = P_{L,H}/(P_{L,H} + P_{H,L})$. The mean durations are $1/P_{L,H}$ and $1/P_{H,L}$ for κ^L and κ^H respectively.

The calibration proceeds in two steps. First, a subset of parameter values are set using direct empirical evidence or standard values from the literature. Second, given these parameter values, the remaining six parameters are simultaneously determined by solving the model to target jointly six moments from the data.

In the first step, we set the parameters of the R process, the values of the two κ regimes, and the values of $\{\sigma, \omega, \alpha_h, \theta, \alpha_\nu, \chi\}$. To calibrate the interest-rate process, we follow the standard approach in the international macro literature of measuring the world real interest rate using

[16] We include all 34 OECD countries for simplicity. The cross-country averages of national accounts ratios and time-series moments used to calibrate the model change only slightly excluding the 9 OECD emerging economies. The data were gathered from *OECD National Accounts Statistics* and the United Nations *UNdata*.

[17] This assumption is in line with the observation that the Basu-Fernald U.S. Solow residual estimates are uncorrelated with the U.S. real interest rate on 90-day Tbills.

Table 1: Calibration

Parameters set independently	Value	Source/Target
Risk aversion	$\sigma = 1.$	Standard value
Share of inputs in gross output	$\alpha_v = 0.45$	Cross country average OECD
Share of labor in gross output	$\alpha_h = 0.352$	OECD GDP Labor share = 0.64
Labor disutility coefficient	$\chi = 0.352$	Normalization to yield average $h = 1$
Frisch elasticity	$1/\omega = 2$	Keane and Rogerson (2012)
Working capital coefficient	$\theta = 0.16$	U.S. Working capital/GDP ratio=0.133
Tight credit regime	$\kappa^L = 0.75$	U.S. post-crisis LTV ratios
Normal credit regime	$\kappa^H = 0.90$	U.S. pre-crisis LTV ratios
Interest rate	$\bar{R} = 1.1\%, \rho_R = 0.68$	U.S. 90-day T-Bills
	$\sigma_R = 1.86\%$	
Parameters set by simulation	**Value**	**Target**
TFP shock	$\rho_z = 0.78, \sigma_z = 0.01$	OECD average for std. and autoc. of GDP
Share of assets in gross output	$\alpha_k = 0.008$	Value of collateral matches total credit
Discount factor	$\beta = 0.95$	$NFA = -25$ percent
Transition prob. κ^H to κ^L	$P_{H,L} = 0.1$	4 crises every 100 years (See Appendix E.2)
Transition prob. κ^L to κ^L	$P_{L,L} = 0.$	1 year duration of crises (See Appendix E.2)

the annualized ex-post real return on 90-day U.S. T-bills. This yields $\bar{R} = 1.01, \rho_R = 0.68$ and $\sigma_\varsigma = 1.38$ percent.

The values of the credit regimes are set to $\kappa^L = 0.75$ and $\kappa^H = 0.9$. These values are consistent with evidence on loan-to-value (LTV) ratios for both households and firms in the United States and abroad during the financial crisis and prior to the crisis. Demyanyk and Hemert (2011) and Duca and Murphy (2011) show that U.S. mortgage LTV ratios peaked at about 0.9 in the run-up to the crisis, hence we set $\kappa^H = 0.9$. Favilukis et al. (2010) note that there was a significant drop in LTV ratios during the crisis, down to maximum values in the 0.75-0.8 range. These LTV ratios are also in the range of cross-country estimates reported by Nguyen and Qian (2012). They report LTVs for both firms and households ranging between 0.72 and 0.9 based on firm-level survey data from the *World Bank Enterprise Survey*.

The CRRA coefficients is set to $\sigma = 1$, which is commonly used in open-economy DSGE models. The Frisch elasticity of labor supply ($1/\omega$) is set equal to 2, in the range of estimates typically used in Macro models (see Keane and Rogerson, 2012). The parameter χ is set so that mean hours are equal to 1, which requires $\chi = \alpha_h$ (with α_h calibrated as described below).

Using national accounts data for all OECD members, we obtained a GDP-weighted average of the ratio of total intermediate goods to gross output of about 0.45 in the 1980-2012 period. Hence we set $\alpha_v = 0.45$. The share of labor in gross output is then set so that it yields the standard OECD labor share in GDP of 0.64 (see Stockman and Tesar, 1995). This implies $\alpha_h = 0.64^*(1-\alpha_v) = 0.352$.

The value of θ is set to be consistent with an empirical estimate of working capital financing based on cross-sectional U.S. data for 2013. In particular, we measure working capital as the sum of trade credit liabilities of nonfinancial businesses from the *Flow of Funds* dataset, plus the total of commercial and industrial loans extended by commercial banks with maturity of less than one year, from the *Survey of Terms of Business Lending*. This yields an estimate of 13.3 percent of GDP for total working capital financing in 2013. Hence, since total working capital as a share of GDP in the model is given by $\theta p_v v/(F(k,h,v) - p_v v)$, and the ratio of total intermediate goods to GDP in U.S. data for 2013 was 0.8, it follows that θ=0.133/.8=0.16.

The second stage of the calibration sets the values of $\{\rho_z, \sigma_z, \alpha_k, \beta, P_{L,L} P_{H,L},\}$. The values of these six parameters are set jointly so that the DE solution matches the corresponding six target moments from the data listed in Table 1.

The values of ρ_z and σ_z are targeted to match the average autocorrelation and standard deviation of the linearly-detrended cyclical component of output across all OECD countries in the 1984-2010 period. The average standard deviation is 0.05 and the average autocorrelation is 0.76. Matching these moments requires setting $\rho_z = 0.78$ and $\sigma_z = 0.01$.

The target for setting the value of β is an estimate of the net foreign asset position (NFA) as a share of GDP that excludes the government sector (since government is not included in the model).[18] This estimate was constructed using U.S. data from the *Flow of Funds* dataset for 2013. We did not target the time-series average because the U.S. NFA-GDP ratio has displayed a marked downward trend since the early 1980s due to the Global Imbalances phenomenon. Since the *Flow of Funds* provide a breakdown of domestic v. foreign financing only in terms of the overall funding for the total domestic nonfinancial sectors, which includes the government, we compute first the fraction of the net credit liabilities of the domestic nonfinancial sectors financed by the rest of the world (0.2), and then apply this fraction to the total net credit liabilities of the *private* domestic nonfinancial sectors (-1.21), which yields a private NFA position of $0.2 \times (-1.21) = -0.249$ as a share of GDP. The model's decentralized equilibrium yields an unconditional mean of b as share of GDP that matches this ratio with $\beta = 0.95$.

In setting the share of capital in value added, we cannot follow the standard approach of setting it to the observed share of about 1/3rd, because this estimate includes capital income accrued to the entire capital stock, while the model considers only capital that is in fixed supply. Instead, we set the capital share so that the value of assets usable as collateral can support levels of leverage comparable with those observed in the data (i.e. the values in the interval defined by κ^L and κ^H). In particular, we set α_k so that the collateral constraint in the κ^L regime holds with equality when

[18] We also control for the absence of government purchases by deducting a time- and state-invariant amount of autonomous expenditures in the resource and budget constraints, calibrated to match the 16 percent average share of government expenditures in GDP in U.S. data over the 1984-2012 period.

evaluated at the unconditional averages of asset prices, debt and working capital. That is, we adjust α_k until this condition holds: $E[q_t] = \frac{-E[b_{t+1}+\theta p_v v_t]}{\kappa^L}$. Given the NFA target of -24.9 percent of GDP, the working capital estimate of 13.3 percent of GDP, and the value of κ^L, the condition holds when $\alpha_k = 0.008$.[19]

Finally, we calibrate the transition probabilities of the credit regime switching process so as to match the frequency and duration of financial crises in the data. To construct estimates of these two statistics, we applied the methodology proposed by Forbes and Warnock (2012) to identify the timing and duration of sharp changes in financial conditions. A financial crisis is defined as an event in which linearly-detrended current account is above two-standard deviations from its mean. Since the current account is the overall measure of financing of the economy vis-a-vis the rest of the world, this unusually large current accounts represent unusually large drops in foreign financing. The starting (ending) dates of the events are set in the year within the previous (following) two years in which the current account first rose (fell) above (below) one standard deviation. Using the data for all OECD countries over the 1984-2012 period, we obtained financial crises with a frequency of 4 percent and a mean duration of 1 year. The model calibrated with $P_{L,L} = 0$ and $P_{HH} = 0.9$, and applying the same criteria to define financial crises and their duration, yields financial crises with a frequency of 3.8 percent and a mean duration of 1 year.

The model is solved using a global, nonlinear solution algorithm taking into account the occasionally binding, stochastic credit constraint. The DE solution is obtained using a time iteration algorithm. In the SP's problem, we use a nested fixed-point algorithm: In the inner loop, we solve for policy functions and value functions using value function iteration, given future policies. In the outer loop, we update future policies given the solution to the Bellman equation, which ensures Markov stationarity. Further details are provided in Section J of the Appendix.

3.2 Financial Crises Dynamics

In order to analyze the model's ability to generate financial crises, and the effectiveness of the optimal policy at reducing the frequency and severity of crises, we conduct an event analysis of model-simulated data for the DE and SP economies. We examine averages across financial crises events in a long time-series simulation, defining crises in the same way as in the data.[20]

The first important result of this event analysis is that the time-consistent macroprudential policy reduces significantly the frequency of crises. The model was calibrated so that the DE matches the 4-percent crises frequency observed in the data. Under the same calibration, the

[19]This value is similar to what would be obtained if we impose the same average price of the baseline calibration in a deterministic steady state in which the constraint is not binding. In this case, the steady-state pricing condition implies $q = \alpha_k(1-\beta)$, and using the same average price and the same value of β as in the baseline calibration would imply $\alpha_k = 0.012$. Risk and binding borrowing constraints alters the implied value of α_k.

[20]In Section F of the Appendix we follow a different approach and examine instead the DE's predicted time-series dynamics for the global financial crisis using a window spanning the 2000-2009 period, and compare these dynamics with U.S. and European data and with what the event would have looked like under the optimal policy.

frequency of crises in the SP is only 0.02 percent. Thus, financial crises become extremely rare under the optimal policy .[21]

The ability of the DE to generate financial crises and the effect of the optimal policy on their severity are illustrated by constructing event windows with the simulated data comparing the DE and SP. The results are presented in Figure 1, which shows nine-year event windows for total credit (bonds plus working capital) as a share of GDP, asset prices, output, and consumption, as well as windows that show the evolution of the exogenous shocks.

We construct comparable event windows for the two economies following this procedure: First, we simulate the DE for 100,000 periods and identify financial crises using the event-study methodology we borrowed from the empirical literature described earlier. Second, we construct nine-year event windows centered at the crisis year, denoted date t, by computing averages for each variable across the cross section of crisis events at each date. This produces the DE dynamics plotted as the red, continuous lines in Figure 1. Third, we take the initial bond position at $t-5$ of the DE and the sequences of shocks the DE went through in the nine-year window, and pass them through the policy functions of the SP. Finally, we average in each date the cross-sectional sample of the SP to generate the averages shown as the blue, dashed line in Figure 1.

Panel (a) of Figure 1 shows that the pecuniary externality results in significant overborrowing in the DE in the periods before the crisis. At $t-5$ both DE and SP start from the same credit-GDP ratio by construction. But starting at $t-4$, and for the rest of the pre-crisis years, credit under the SP is roughly 3 percentage points of GDP below the DE average. In contrast, credit in the DE rises in the years before the crises, peaking at about 38 percent of GDP. As a result of this overborrowing, the DE builds up more leverage and experiences a larger collapse in credit when a financial crisis hits. Credit falls amost 18 percentage points of GDP in the DE between $t-1$ and t, v. 1.5 percentage points in the SP, and although it rises at a fast pace after the crisis, four years later it remains below its long-run average. Note, however, that by then the DE is again generating more credit than the SP.[22]

Asset prices (panel (b)), output (panel (c)) and consumption (panel (d)) also fall more sharply in the DE than the SP. The declines in consumption and asset prices are particularly larger (-26 v. -8 percent for consumption and -43.7 v. -5.4 percent for asset prices). The asset price collapse plays an important role in explaining the more pronounced decline in credit in the DE, because it reflects the full impact of the Fisherian deflation. Output falls almost 2 percentage points more in the DE than in the SP, because of the higher shadow price of inputs produced by the tighter binding constraint on access to working capital.

Panel (e) shows that financial crises are preceded largely by regimes with κ^H and coincide with

[21] We identify financial crises for the SP using the credit thresholds of the DE in levels. Re-computing the thresholds using the standard deviation of credit in the SP, which is smaller, the frequency of crises rises slightly but remains much lower than in the DE. Our results are also largely robust to alternative crises definitions.

[22] The model produces large credit drops partly because all intertemporal credit is in the form of one-period bonds, whereas loans in the data have on average a longer maturity.

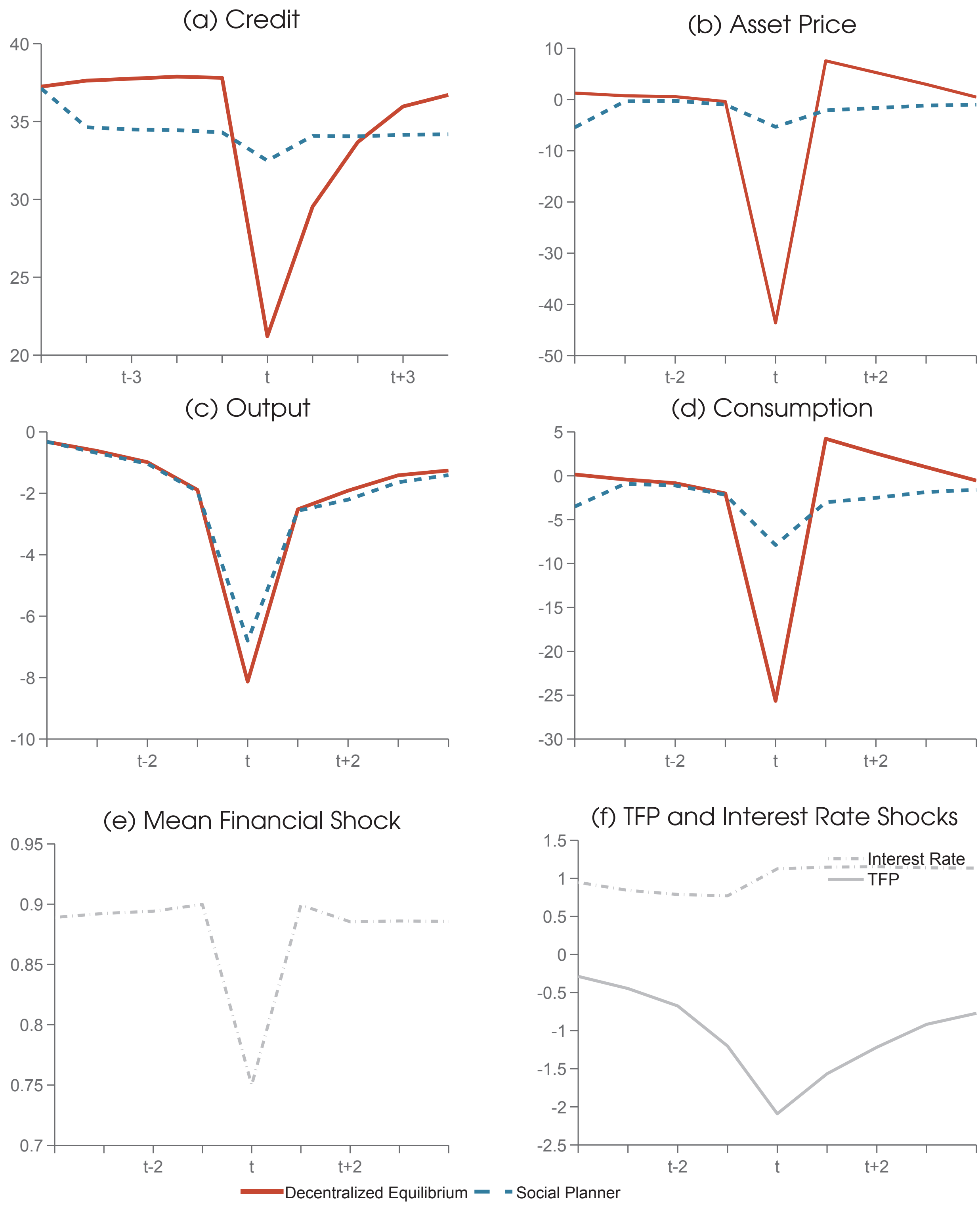

Figure 1: Comparison of Crises Dynamics

Note: Panel (a) shows the credit-GDP ratio in percent. Panels (b),(c),(d),(f) are plotted as percent differences relative to unconditional averages.

Table 2: Summary Statistics

	Decentralized Equilibrium	Social Planner
Crisis statistics		
Probability of crisis	4.0	0.02
Asset Price Drop	-43.7	-5.4
Equity Premium	4.8	0.7
Mean tax and welfare gains		
Macroprudential Debt Tax		3.6
Welfare Gains		0.30

Note: The price drop and equity premium are averages conditional on financial crises events. The debt tax and welfare gains are unconditional averages over the model's limiting distribution of bonds and shocks. The welfare gains are measured as compensating variations in consumption that equate expected lifetime utility for the DE and SP at each point in the state space.

regime switches to κ^L. Panel (f) shows that TFP is declining on average before financial crises, and reaches a through of about 2 percent below the mean when a crisis hits, and after that it recovers. The real interest rate falls slightly on average before financial crises, then rises about 50 basis points when crises occur and remains stable in the years after. Thus, financial crises in the DE are associated *on average* with adverse TFP, interest rate and financial shocks. Note, however, that the model also generates crises with positive TFP shocks when leverage is sufficiently high and an adverse financial shock hits.

Summing up, this event analysis delivers two main results: First, financial amplification driven by the Fisherian mechanism is strong in the model, producing financial crises with deep recessions.[23] Second, the pecuniary externality is quantitatively large, resulting in an optimal policy that is very effective at reducing the magnitude and frequency of crises.

Table 2 shows additional statistics that summarize the effectiveness of the optimal policy. In addition to the reductions in the probability of crises and the asset price collapse during a crisis documented above, this Table shows that the excess return on assets averages 4.8 percent during financial crises in the DE v. 0.8 percent in the SP. About half of the large excess return in the DE is due to the collateral effect identified in equation (9), and the rest is due to the liquidity and risk premia. The average debt tax over the entire state space is 3.6 percent, and the welfare gain of the optimal policy is 30 basis points in terms of the standard compensating variation in consumption (see subsection 3.4 for an explanation of how the welfare gains are meaured).

[23] Mendoza (2010) shows that this holds also in models with capital accumulation.

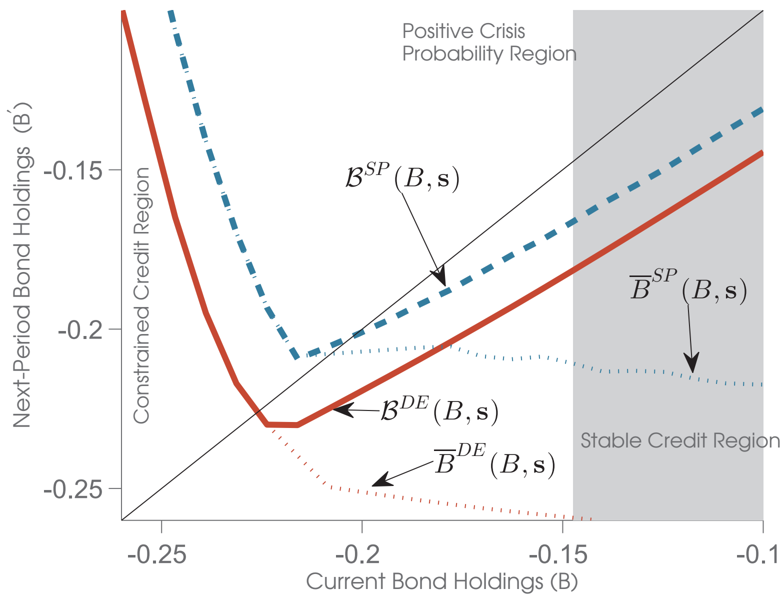

Figure 2: Bond Decision Rules and Borrowing Limits.

3.3 Borrowing Decisions and Amplification

The manner in which the optimal policy reduces amplification and tackles the pecuniary externality can be illustrated by comparing borrowing decisions across the DE and SP. Figure 2 shows the decision rules for bonds $\mathcal{B}^{DE}(B, s)$ and $\mathcal{B}^{SP}(B, s)$ as the red-continuous and blue-dashed curves respectively. Bond choices for $t+1$ (B') are shown in the vertical axis as functions of bond holdings at t in the horizontal axis (B), for values of the shocks set to κ^L, high R and average TFP. The Figure also shows the debt limits of each economy ($\overline{\mathcal{B}}^{DE}(B, s)$ and $\overline{\mathcal{B}}^{SP}(B, s)$).[24]

The bond decision rules are divided into three regions: The leftmost region is the *constrained credit region*, which is defined by the values of B that represent sufficiently high initial debt (low B) such that the collateral constraint already binds for the SP. The center region is the *positive crisis probability region*. This region is characterized by financial instability, in the sense that the constraint is not binding at t, but values of B' chosen by private agents in the DE are low enough so that for some values of the shocks at $t + 1$ the collateral constraint binds. As shown in Section 2, this is the region in which the regulator uses the macroprudential debt tax. At equilibrium, the long-run probability of observing states in this region region is almost 94 percent in the SP solution. Hence, while crises are near-zero probability events under the optimal policy, macroprudential debt taxes are used nearly all the time. Finally, the rightmost region is the *stable credit region*, where B is high enough so that both the constraint is not binding at t and the probability of hitting it next period is zero for both DE and SP.

The V-shaped bond decision rules are a feature of financial frictions models that incorporate a strong Fisherian deflation mechanism. This is in contrast with standard Bewley-style incomplete-

[24]These limits are defined as $-\kappa q^{DE}(B, s) + p_v \hat{v}^{DE}(B, 1, B, s)$ and $-\kappa q^{SP}(B, s) + p_v \hat{v}^{SP}(B, s)$

markets models of heterogeneous agents and RBC models of the small open economy, both of which produce monotonically-increasing decision rules. The point at which the decision rules switch slope corresponds to the value of B at which the collateral constraint is marginally binding in each economy (i.e. it holds with equality but the choice of debt is exactly the same debt allowed by the credit constraint). To the right of this point, the collateral constraint does not bind and the decision rules are upward sloping. To the left of this point, the decision rules are sharply decreasing in B, because a reduction in B results in a sharp fall in asset prices caused by the Fisherian deflation mechanism, which tightens the borrowing constraint, thus increasing B'.[25] In line with these results, the decision rules lie above their corresponding borrowing limits to the right of the values of B at which the constraint becomes binding, and to the left the decision rules must be equal to their corresponding borrowing limits.

A second, and more important, feature of the bond decision rules from the perspective of the normative analysis we are conducting, is that the SP's decision rule is uniformly higher than in the DE for all values of B (i.e. there is "overborrowing" in the DE relative to the SP in all three regions). Recall, however, that as we explained in Section 2, prices and bond choices can be higher or lower in the SP than in the DE. Indeed we found that with other parameterizations the two decision rules can be closer, and there can even be instances in which the DE chooses higher B'.

The differences in bond choices across DE and SP may seem small, but they lead to large differences in prices and allocations when a crisis occurs. The non-linear financial amplification dynamics that make this possible, and the SP's ability to weaken them, are illustrated in Figure 3. This Figure shows the decision rules for bonds over the interval $-0.25 \leq B \leq -0.17$ for two different triples of s. The ones labeled *positive shock* are for κ^H, and the ones labeled *negative shock* are for κ^L, using for both average TFP and a high value of R. The ray from the origin is the stationary choice (45-degree) line, where $B' = B$.

Figure 3 can be used to visualize the dynamics of financial amplification in the DE, and compare them with the SP via the following experiment: Assume both DE and SP start in a hypothetical first period with bonds at point O, which is the intersection of DE's bond decision rule under positive shocks with the 45-degree line. Starting at this point, agents in the DE choose bond holdings D such that $D = O$, since $B' = B$. Hence, DE ends the period with the same amount of bonds it started with. Assume then that the second period arrives and the realization of κ is κ^L. The DE starts at point D but now the collateral constraint becomes binding, and the Fisherian deflation of asset prices forces a sharp, nonlinear upward adjustment of the bond position such that B' increases to point D', reducing debt from about -0.245 to about -0.185. Compare this with what happens in the SP case. Starting at O, the planner's decision rule increases bond holdings (lowers debt) to about -0.22, to end the first period at point P. Hence, SP ends with debt slightly below what agents in the DE chose (i.e. -0.22 v. -0.245 respectively). The second

[25] In Bianchi (2011), this non-monotonicity arises from how consumption affects the marginal rate of substitution between tradables and non-tradables and thus the relative price of nontradables.

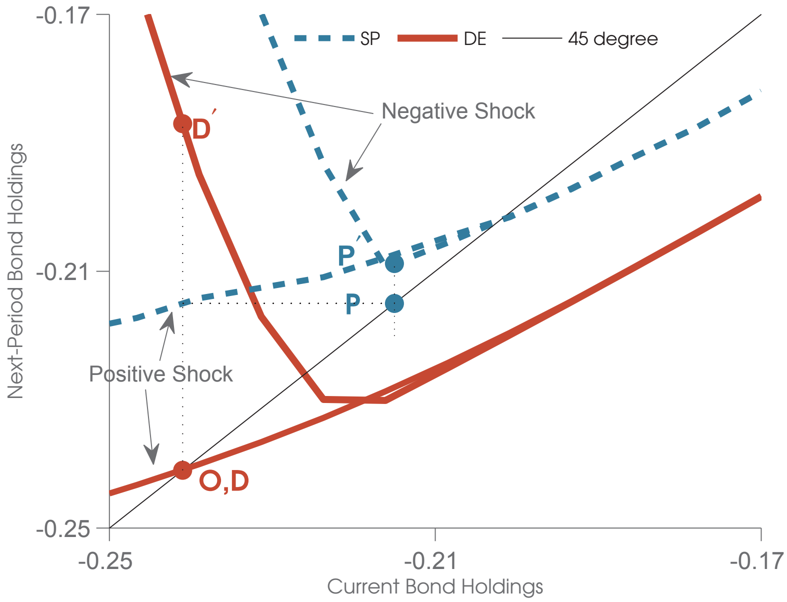

Figure 3: Amplification Dynamics in Response to Adverse Shocks

period arrives, but now the correction in debt triggered by the binding collateral constraint is small, as the choice of B' rises from P to P'. Hence, the slightly smaller initial debt of the SP v. the DE in the second period results in a sharply smaller upward adjustment in B' for the planner (about 50 basis points in percent of GDP v. roughly 700). This is the mechanism that produces the SP's significantly smaller financial crises shown in the event analysis.

The differences in borrowing decisions and asset prices of the DE and SP are also reflected in two important differences in the cumulative long-run distributions of realized asset returns (see Figure 4). First, the SP shows more mass at higher returns, which partly reflects that the externality premium identified in equation (18) is large, or, since this premium can be expressed as $R_t\tau_t^{MP}$, it can also be viewed as an implication of the macroprudential debt tax. Second, the distribution for the DE displays a fat left tail, which corresponds to states in which negative shocks hit when agents have a relatively high level of debt. Intuitively, the standard effect of negative shocks reducing expected dividends and putting downward pressure on asset returns is amplified by the effect of asset fire-sales that occur if the collateral constraint binds.

The fat tail of the distribution of asset returns in the DE, and its substantial effects on the risk premium due to the associated time-varying risk of financial crises, are important results because they are an endogenous equilibrium outcome resulting from the non-linear asset pricing dynamics when the debt-deflation mechanism is at work. Fat tails in asset returns are also highlighted in the recent literature on asset pricing and "rare disasters" but this literature generally treats financial disasters as resulting from exogenous stochastic processes. More details on the asset pricing implications and the implications of macroprudential policy are reported in Appendix I.

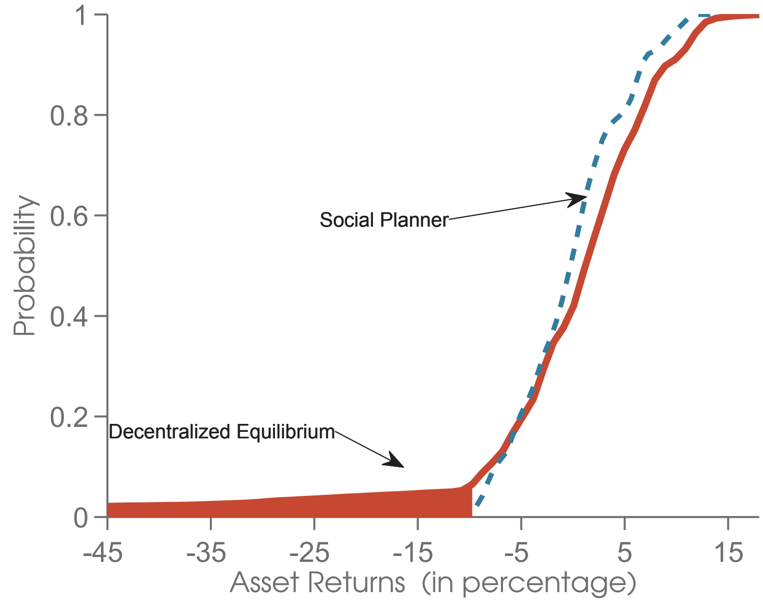

Figure 4: Ergodic distribution of Asset returns

3.4 Macroprudential Debt Tax & Welfare Effects

We now study the quantitative features of the macroprudential debt tax (τ^{MP}) and its welfare implications. Panel (a) of Figure 5 shows the tax schedule as a function of B for average TFP, high R and κ^H. Panel (b) shows the event-analysis evolution of the tax around financial crises.

Panel (a) shows that the tax is zero when the value of B at date t is such that the economy is in the stable credit region, because the probability of hitting the collateral constraint at $t+1$ is also zero. When B is low enough to be in the positive-crisis-probability region the constraint is still not binding at t but it can bind at $t+1$, and so the tax is positive. In this region, the tax is higher at lower B (i.e, it is increasing in current debt), because this makes it more likely that the constraint will become binding at $t+1$, and that if it does it will be more binding than at higher values of B. The tax can be as high as 13 percent when debt is about 30 percent.

Panel (b) shows that the tax is positive and rising in the four years prior to the financial crisis. Recall from Figure 1 that these are also the years in which credit and leverage rise in the unregulated DE. Hence, the policy is taxing debt to reduce the overborrowing that occurs in the good times, so as to mitigate the magnitude of a financial crisis when it occurs. The tax peaks at about 12 percent in the year just before the crisis. When the crisis hits the tax is zero, because at this point $\mu_t > 0$ and the probability of $\mu_{t+1} > 0$ is zero, and hence the prudential aspect of the policy vanishes. The tax increases slightly the year after the crisis and then rises rapidly to reach about 7 percent at $t+4$.

As noted earlier, the long-run average of the macroprudential debt tax is 3.6 percent. In addition, it has a standard deviation that is roughly half the standard deviation of GDP, and a correlation of 0.7 with the leverage ratio. This is consistent with the prudential rationale behind

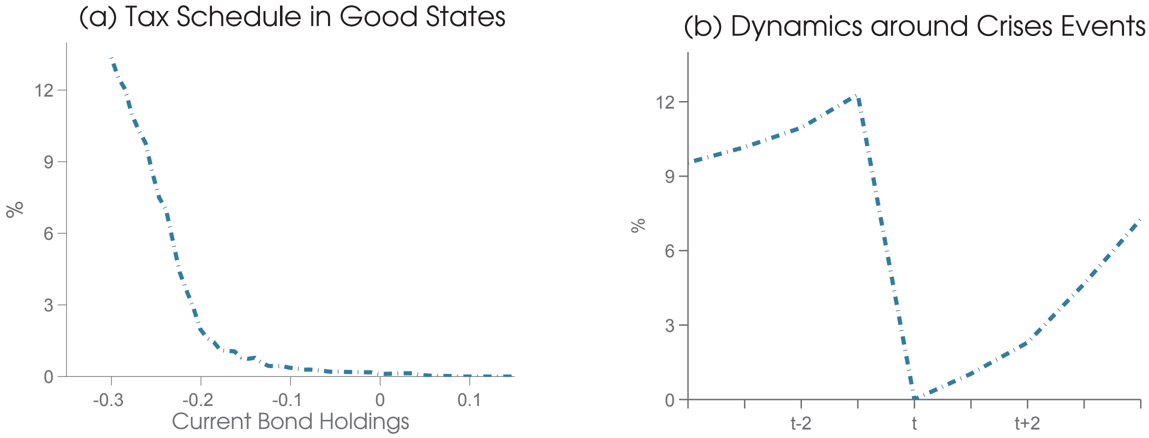

Figure 5: Optimal Macroprudential Tax

the tax: The tax is high when leverage is building up and low when the economy is deleveraging. Note, however, that since leverage itself is negatively correlated with GDP, the tax also has a negative GDP correlation. Finally, the tax also has a positive correlation with "credit conditions" as reflected in κ_t, again in line with arguments often used to favor macroprudential policy.

Jeanne and Korinek (2010) also computed macroprudential debt taxes to correct a similar pecuniary externality, but found that they have much weaker effects on financial crises than in our setup: The asset price drop is reduced from 12.3 to 10.3 percent, compared with 43.6 to 5.4 percent in our analysis. In their model, the credit constraint is determined by the aggregate level of assets $\bar{K}$ and a constant term ψ (i.e. their constraint is $\frac{b_{t+1}}{R} \geq -\kappa q_t \bar{K} - \psi$), with parameters calibrated to $\kappa = 0.046$, $\psi = 3.07$ and $q_t \bar{K} = 4.8$. This implies that the effects of the credit constraint are driven mainly by ψ, and only 7 percent of the borrowing ability depends on the value of assets (0.07=0.046*4.8/(0.046*4.8+3.07)). As a result, the Fisherian deflation effect and the pecuniary externality are both weak, and thus macroprudential policy cannot be very effective. Moreover, since they model output as an exogenous, regime-switching Markov process, such that the probability of a crisis equals the exogenous probability of the low-output regime, macroprudential policy cannot affect the probability of crises either.

We study the welfare implications of the optimal policy by calculating welfare effects as compensating consumption variations for each initial state (B, s) that equalize expected utility across the DE and SP. Formally, for a given initial state (B, s) at date 0, the welfare effect of the optimal policy is computed as the value of $\gamma(B, s)$ that satisfies this condition:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{DE}(1+\gamma) - G(h_t^{DE})) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{SP} - G(h_t^{SP})) \quad (22)$$

The mean welfare gain reported earlier is the average $\gamma(B, s)$ computed with the DE's ergodic distribution. In addition, we analyze below the variation of the welfare gains across (B, s) pairs and in the dynamics around crises.

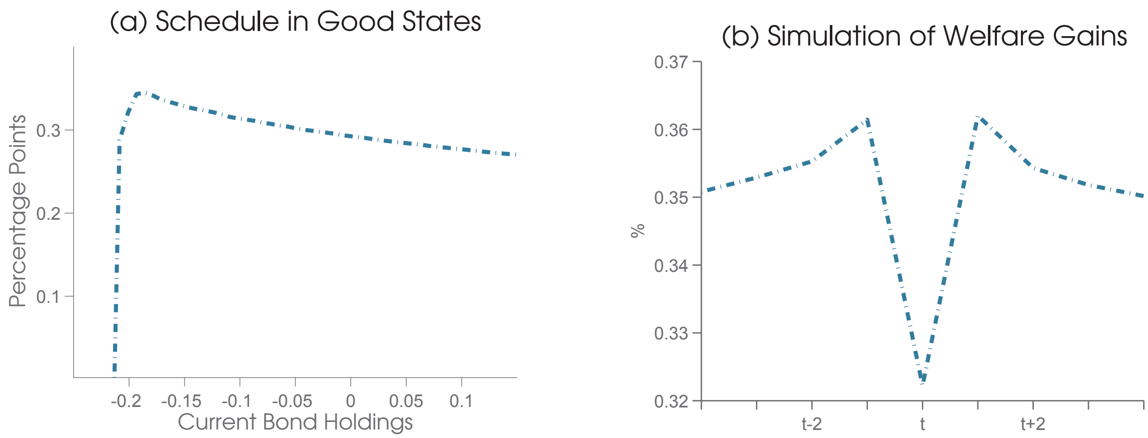

Figure 6: Welfare Gains of the Optimal Policy

The welfare gains of the optimal policy arise from two sources. First, the reduced variability of consumption in the SP v. the DE, due to the fact that the credit constraint binds more often in the DE, and when it binds it induces a larger adjustment in asset prices and consumption. Second, the efficiency loss in production that occurs in the DE due to the effect of the credit friction on working capital and factor allocations. Again, since the collateral constraint binds more often in the DE than in the SP, there is a larger efficiency loss in the former.

The welfare gains of the optimal policy are illustrated in Figure 6. Panel (a) shows the state-contingent schedule of welfare gains as a function of B for the same "good" state with average TFP, high R and κ^H as in Figure 5. Panel (b) shows the event-analysis dynamics of the welfare gains around financial crises events. In looking at these results, keep in mind that the values of γ are generally small because the model is in the class of stationary-consumption, representative-agent models with CRRA preferences that produce small welfare effects due to consumption variability (see Lucas, 1987). Moreover, although the efficiency loss in production at work when the collateral constraint binds can be relatively large and add to the welfare effects, this is a low-probability event because of the low probability of hitting the constraint.

The schedule of welfare gains as a function of B in panel (a) is bell-shaped in the region with a positive probability of crisis at $t+1$. It rises sharply as B rises from -0.25, peaking at about a 0.35 welfare gain when $B = -0.18$, and then falls gradually. The welfare gains continue to fall gradually, and almost linearly, as B moves into the stable credit region, and reaches 0.26 percent when $B = 0.15$. This pattern is due to the differences in the optimal plans of the regulator vis-a-vis private agents in the DE. In the region where a crisis is possible at $t+1$, the SP's allocations projected as of date t for the future differ sharply from those of the DE, because of the latter's higher magnitude and frequency of crises, and this generally enlarges the welfare gains of the optimal policy. Notice that, since the regulator's allocations involve more savings and less current consumption, there are welfare losses in terms of current utility for the regulator, but these are far outweighed by less vulnerability to sharp decreases in future consumption during financial crises. As the level of debt falls and the economy enters the stable credit region, financial crises are unlikely at $t+1$, or are likely much further into the future, and thus the welfare gains of the policy (or the costs of the externality) decrease.

Panel (b) shows that the welfare gains of the optimal policy rise in the years before the crisis to a peak around 0.36 percent at $t-1$. When the crisis hits the welfare gain drops to near 0.32 percent, because by then the crisis has arrived and the prudential aspect of the optimal policy is less valuable, but right after the crisis the welfare gain increases sharply. As noted before, the unconditional average welfare gain computed using the DE's ergodic distribution is about 0.3 percent. These welfare gains may seem small, but they are much higher than the welfare gains of eliminating business cycles obtained with the same CRRA coefficient of $\sigma = 1$ in calibrations to U.S. data (e.g. Lucas (1987) estimated a gain of only 0.0005).

3.5 Simple Macroprudential Rules

The state-contingent nature of the optimal policy has the usual drawback that complex state-contingent rules are difficult to implement in practice, and therefore rarely used. In particular, there is concern for the ability of regulators to track accurately financial conditions and adjust macroprudential tools optimally and in a timely fashion (e.g. Cochrane, 2013). On the other hand, if macroprudential policy is limited to the relatively simple rules that regulators typically use, the question that is often raised is whether these simpler rules are effective. In light of these concerns, we examine the effectiveness of two simple rules: First, a time- and state-invariant debt tax (a "fixed tax"). Second, in the spirit of Taylor's rule for monetary policy, a "Macroprudential Taylor Rule" that makes the tax a function of credit. The key insight of this analysis is that simple rules can still be welfare improving if they are designed carefully, otherwise they can yield outcomes that are worse than the unregulated decentralized equilibrium.

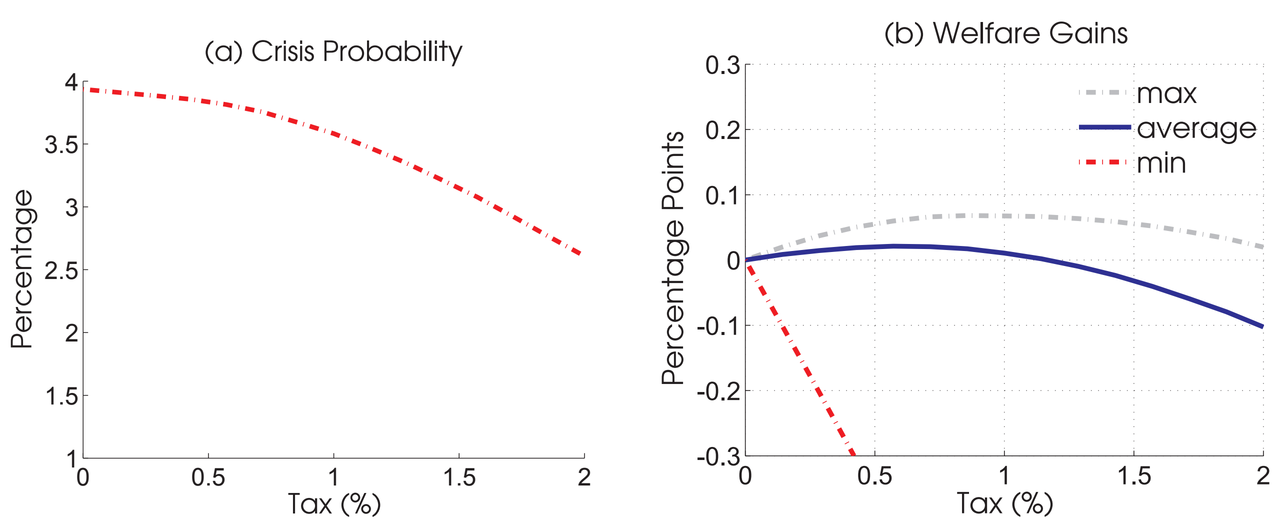

Figure 7: Effects of Fixed Debt Taxes on Probability of Crises and Welfare

Consider first the fixed tax. Figure 7 shows the effects of fixed taxes ranging from 0 to 2 percent on the long-run probability of financial crises (panel(a)) and on welfare (panel (b)). Fixed taxes reduce the likelihood of crises monotonically from 4 to 2.6 percent as the tax rises from 0 to 2 percent. This occurs because as debt is taxed more, agents build less leverage and are less vulnerable to crises, but this does not mean that they are necessarily better off. Recall in particular that the optimal τ^{MP} fluctuates roughly half as much as GDP and is positively correlated with leverage, while this rule keeps the tax constant. As a result, fixed taxes yield welfare gains when computed conditional on initial states in which the constraint is not binding, but can produce welfare losses otherwise. This is due to the negative short-run effects of debt taxes on asset prices and the tightness of the collateral constraint, which occur in turn because the increased cost of borrowing shifts demand from assets to bonds. There is also a positive, second-order effect of debt taxes on asset prices, because of a reduction in the riskiness of assets, but this effect is dominated by the first-order effect of taxes on the relative demand for bonds.

Panel (b) shows the average, maximum and minimum welfare gain for constant taxes in the 0 to 2 percent range (recall that the average welfare gain under the optimal policy is 0.3 percent). This plot illustrates two results. First, fixed taxes are always inferior to the optimal policy: The maximum (average) welfare gain of fixed taxes peaks at about 0.07 (0.03) percent with a tax of 0.6 percent, significantly smaller than the SP's average welfare gain (see Table 3). Second, some fixed taxes are welfare-reducing. Fixed taxes reduce welfare in a subset of the state space, which is reflected in the fact that the minimum welfare gain is always negative for all values of fixed taxes in panel (b), and this subset grows as the fixed tax rate rises (the largest welfare cost reaches -1.5 percent as the tax approaches 2 percent). When the subset of the state space in which fixed taxes cause welfare losses is large enough, the average welfare gain also turns negative. This occurs for fixed debt taxes above 1.2 percent.

Fixed taxes can reduce welfare because they reduce asset prices when the collateral constraint binds, making financial crises worse. This suggests that a regulator considering only fixed taxes should trade off the prudential benefit of the taxes in restraining credit growth in good times against their adverse effects in making financial crises worse. This tradeoff is reflected in the welfare-maximizing fixed tax of about 0.6 percent (see panel (b)). This tax is significantly smaller than the 3.6 percent average optimal macroprudential tax, and it achieves an average welfare gain about 1/10th of that obtained with the optimal tax (see Table 3).

Fixed taxes are also much less effective at reducing the magnitude of financial crises. As Figure 8 shows, under the welfare-maximizing fixed tax of 0.6 percent, crisis dynamics are about the same as in the unregulated DE. Adding to this result the above findings showing a small reduction in the probability of crises (from 4 to 3.6 percent) and a negligible average welfare gain (0.03 percent), we conclude that fixed debt taxes are an ineffective macroprudential policy tool. Moreover, since fixed taxes higher than 1.2 percent reduce welfare, in fact they are *at best* ineffective.

The macroprudential Taylor rule allows the tax to vary with the borrowing choice, according

Table 3: Performance of Optimal and Simple Policy Rules

	Decentralized Equilibrium	Optimal Policy	Best Taylor	Best Fixed
Welfare Gains (%)	–	0.30	0.09	0.03
Crisis Probability (%)	4.0	0.02	2.2	3.6
Drop in Asset Prices (%)	−43.7	−5.4	−36.3	−41.3
Equity Premium (%)	4.8	0.77	3.9	4.3
Tax Statistics				
Mean	–	3.6	1.0	0.6
Std relative to GDP	–	0.5	0.2	–
Correlation with Leverage	–	0.7	0.3	–

Note: Moments for optimal policy are for the macroprudential debt tax. "Mean" under Best Fixed corresponds to the welfare-maximizing fixed tax.

to the following piecewise, isoelastic function:[26] $\tau_t = \max[0, (1+\tau_0)(b_{t+1}/\bar{b})^{\eta} - 1]$, where τ_0 is a constant term, and η is the elasticity of the tax with respect to the excess of the borrowing choice b_{t+1} relative to a target $\bar{b}$. Note that, since under the baseline calibration b_{t+1} is always negative, the ratio $b_{t+1}/\bar{b}$ is positive, and hence $\eta > 0$ implies that the tax rises as debt rises above its target. The cutoff at zero rules out subsidies on debt, in line with the result that the macroprudential tax is non-negative, and allows us to avoid having to model other taxes to pay for these subsidies.

We set τ_0 to the value of the welfare-maximizing fixed tax (0.6 percent) and search numerically for an "optimal" pair $(\eta, \bar{b})$ that maximizes the average welfare gain, computing the average as before, using the ergodic distribution of the DE without regulation.[27] This procedure yields $\eta = 2$ and $\bar{b} = -0.23$, which is 200 basis points lower than the DE average, in line with the notion that the macroprudential tax aims to reign on overborrowing.

The macroprudential Taylor rule yields an average debt tax of 1 percent, higher than the best fixed tax of 0.6 percent, but lower than the mean optimal tax of 3.6 percent (see Table 3). This rule also yields taxes that fluctuate less and are less correlated with leverage than the optimal taxes. In terms of the effectiveness of this policy at affecting crises probability, magnitude of crises and welfare gains, the rule is much better than the fixed taxes, but still clearly inferior to the optimal policy. The rule yields a welfare gain of about 1/10th of a percent, a third of the gain under the optimal policy, and lowers the probability of crisis to about half of the 4 percent in the DE. Figure 8 shows that crises dynamics are less severe than in the DE and under fixed taxes,

[26] We also evaluated rules including other variables like asset prices, the interest rate, TFP, output or the leverage ratio, or conditioning on whether the collateral constraint binds, but their performance did not yield noticeable gains compared with this simpler rule.

[27] Optimizing the various formulations of the rule that we studied is computationally intensive, because for each one the model has to be solved for all combinations of values of the relevant elasticity coefficients specified in pre-determined grids.

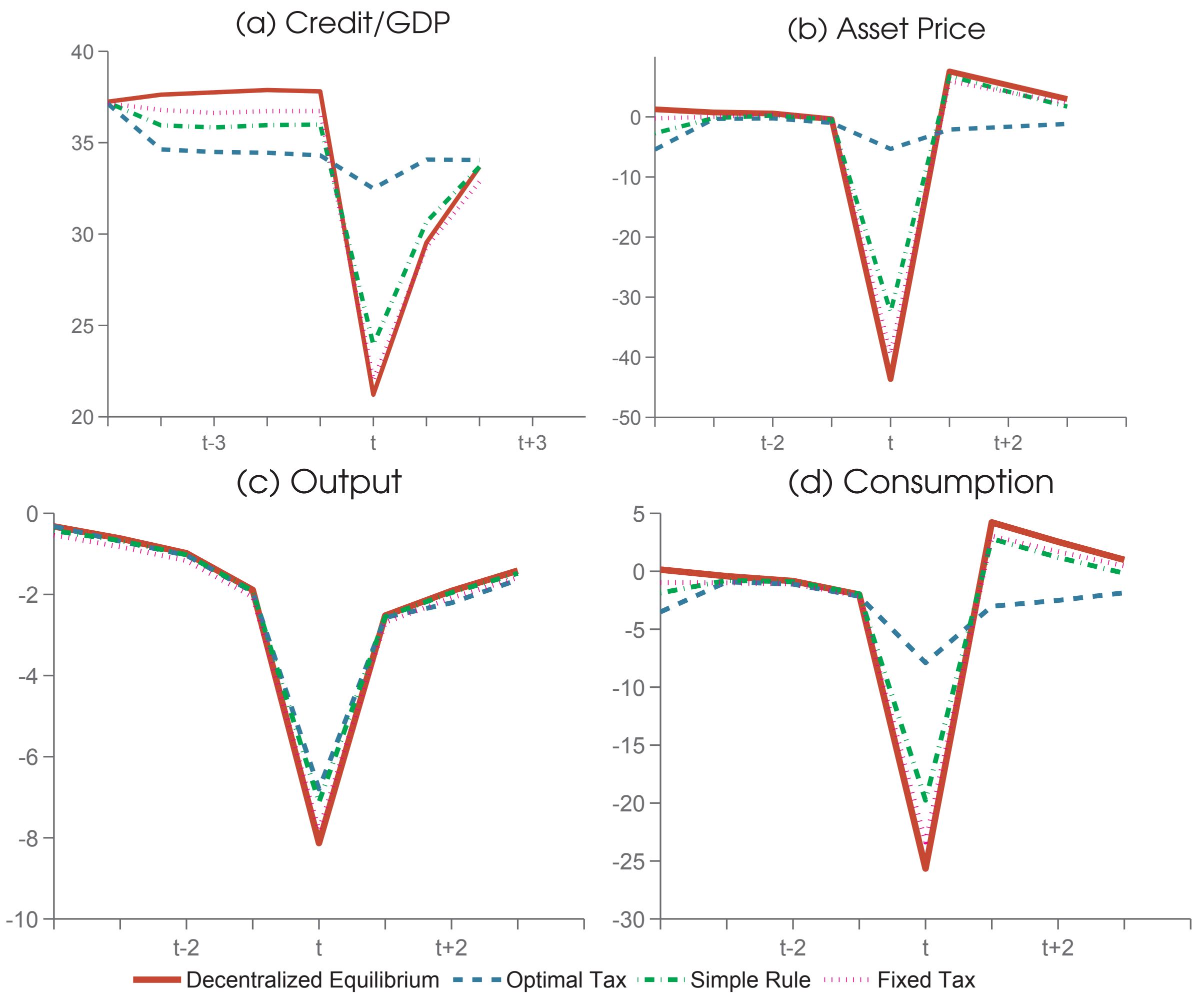

Figure 8: Event Analysis: Decentralized Equilibrium, Optimal Policy and Simple Policies

but still more severe than under the optimal policy. Recall also that these are results that hold for optimized values of the parameters of the tax rule. As with the fixed tax, one can produce outcomes that are significantly inferior for other parameter values.

An issue often discussed together with the effectiveness of simple macroprudential policy rules is whether it is feasible to construct a parsimonious statistical framework that can yield accurate "early warnings" of financial crises. We examined this issue by conducting an experiment similar to the one Boissay et al. (2015) proposed, treating the model as a true data-generating process and testing whether parsimonious logit regressions could yield warnings as accurate as the model's in terms of the fractions of Type-1 and Type-2 errors in crises prediction.[28] In the results reported in Section G of the Appendix, we show that a regression using the ratio of total credit to GDP produces fractions of both errors similar to those produced by the model, which is consistent with the findings of Boissay et al. (2015).

[28]Type-1 errors occur when a warning is not issued at t but a crisis occurs at $t+1$, and Type-2 errors occur when a warning is issued at t but a crisis does not occur at $t+1$.

In summary, the results for the two simple rules we examined highlight both the benefits and dangers of simple macroprudential policies: If institutional limitations prevent regulators from using optimal, state-contingent macroprudential policy instruments, it is possible to end up with environments in which the policy is welfare-reducing. This contrasts sharply with the results in Bianchi (2011), because in his setup fixed taxes do not have negative effects on borrowing capacity across states, whereas here they do because of the forward-looking nature of asset prices.

4 Conclusions

This paper performed a normative analysis of a dynamic stochastic general equilibrium model of financial crises in which a collateral constraint limits access to intertemporal debt and working capital to a fraction of the market value of assets. This constraint introduces financial amplification via the classic Fisherian debt-deflation mechanism, affecting both aggregate demand and supply, and produces a pecuniary externality, because agents do not internalize the effects of individual borrowing decisions on asset prices that determine aggregate borrowing capacity.

We compared theoretically the unregulated competitive equilibrium with the one attained by a constrained-efficient financial regulator unable to commit to future policies. This regulator faces the collateral constraint but internalizes the pecuniary externality, taking into account how its current borrowing choices affect current and future asset prices and the borrowing decisions of future regulators, so that the optimal macroprudential policy is endogenously time-consistent. In contrast, we showed that the forward-looking nature of asset prices causes the optimal policy under commitment to be time-inconsistent. We also showed that the optimal, time-consistent allocations can be decentralized as a competitive equilibrium using a state-contingent schedule of debt taxes. We labeled the component of these taxes associated with the future value of collateral a macroprudential debt tax, and proved that it is strictly positive. Capital requirements or loan to value ratios can be used with similar results (see Bianchi, 2011).

We conducted a quantitative analysis in a version of the model calibrated to OECD data. The competitive equilibrium features strong financial amplification and large pecuniary externalities, resulting in financial crises that are markedly more severe and more frequent than with the optimal policy. As a result, the optimal policy yields a welfare gain of roughly 0.3 of a percent, with a debt tax that is about 3.6 percent on average, half as volatile as output and with a correlation with leverage of 0.7.

We recognize that despite these findings, macroprudential policy also faces serious hurdles. One is the state-contingent complexity of the optimal policy. We showed that simpler rules can be effective too, but they can be counterproductive if they are not designed carefully. Fixed debt taxes are at best ineffective, because when targeted to maximize their welfare gain they have negligible effects on welfare and on the magnitude and frequency of crises, and if set higher than that target (including at the average of the optimal tax) they produce outcomes *worse* than the unregulated equilibrium. A macroprudential Taylor rule that makes the tax a function of the ratio of debt to a target, with an elasticity optimized to maximize the welfare gain, is more effective, albeit still less than the optimal policy.

There are several other hurdles that we did not study in this paper. One has to do with the complex heterogeneity of actual financial markets, which include a large set of financial constraints affecting various types of borrowers (e.g. households, nonfinancial firms, financial intermediaries). Hence, effective macroprudential policy faces severe informational requirements in terms of both coverage and timeliness of leverage and debt positions. A second hurdle relates to incomplete information and innovation in capital markets. In Bianchi et al. (2012) we showed that the effectivenes of macroprudential regulation weakens in an environment in which financial innovation occurs but agents, including the regulator, are imperfectly informed about it. Finally, there are also hurdles related to possible conflicts with the overall stance of fiscal policy. In particular, cutting macroprudential taxes when a crisis hits can be hard, because often these are times when governments run large deficits, which make it hard to lower taxes, especially of a financial nature.

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Online Apppendix to
Optimal Time-Consistent Macroprudential Policy

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Contents

A Theoretical Analysis and Proofs

This section provides theoretical proofs of our analysis in Bianchi and Mendoza (2015).

A.1 Optimal Time Consistent Planner's Problem

We explain here the analytical formulation of the markov perfect equilibrium and show that the recursive social planner's problem under discretion is given by (12). To do this, we first provide a complete formulation of the planner's problem and then establish that solving a relaxed planner problem that include only a subset of the constraints of the time consistent planner problem as in (12) is equivalent to solving the full optimal time consistent planner problem.

Construction of the Planner's Problem—- As described in Section 2.3, we consider a social planner that chooses b_{t+1} on behalf of the representative firm-household while the households retain the choices of consumption, labor supply, intermediate inputs, land holdings taking as given asset prices, wages, and future government policies. Hence, the planner's problem can be seen as choosing all allocations and prices subject to implementability constraints, as we'll describe below. To derive the implementability constraints, we analyze the problem of the households, which consist of choosing $\{c_t, h_t, k_{t+1}\}$ taking as given prices $\{q_t, w_t\}$ and the planner's transfers T_t:

$$\max_{\{c_t,h_t,k_{t+1}\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t - G(h_t)$$

$$s.t. \quad c_t + q_t k_{t+1} = k_t q_t + z_t F(k_t, h_t, v_t) - p_v v_t + T_t$$

$$\frac{b_{t+1}}{R} - \theta p_v v_t \geq -\kappa_t q_t k_t$$

First-order conditions are

$$z_t F_h(k_t, h_t, v_t) = G'(h_t) \tag{A.1}$$

$$z_t F_v(k_t, h_t, v_t) = p_v(1 + \theta\mu_t/u'(t)) \tag{A.2}$$

$$q_t u'(t) = \beta \mathbb{E}_t \left\{ u'(t+1) \left(z_{t+1} F_k(k_{t+1}, h_{t+1}, v_{t+1}) + q_{t+1} \right) + \kappa q_{t+1} \mu_{t+1} \right\} \tag{A.3}$$

which correspond to conditions (4),(5),(7). These three conditions together with complementary slackness conditions $\mu \geq 0$ and $\mu \left(\frac{b'}{R} - \theta p^v v + \kappa q \right) = 0$ constitute the implementability constraints in the optimal planner's problem, as described below. Notice that combining the household budget constraint with the government budget constraint $T_t = b_t - \frac{b_{t+1}}{R_t}$ we arrive to the resource constraint

$$c + \frac{b'}{R} = b + zF(1, h, v) - p^v v. \tag{A.4}$$

The planner's problem consists of maximizing expected lifetime utility (1) subject to (4),(5),(7), (A.4), and complementary slackness conditions, taking as given future planner's policies policies $\{\mathcal{B}(b,s), \mathcal{C}(b,s), \mathcal{H}(b,s), \mathcal{V}(b,s), \boldsymbol{\mu}(b,s), \mathcal{Q}(b,s)\}$

Problem 1. *The recursive optimal time consistent planner's problem is given by the following problem:*

$$\mathcal{V}(b,s) = \max_{c,b',q,\mu,h,v} u(c - G(h)) + \beta \mathbb{E}_{s'|s} \mathcal{V}(b',s') \tag{SP}$$

$$c + \frac{b'}{R} = b + zF(1,h,v) - p^v v \tag{SP1}$$

$$\frac{b'}{R} - \theta p^v v \geq -\kappa q \tag{SP2}$$

$$qu'(c - G(h)) = \beta \mathbb{E}_{s'|s} u'(\mathcal{C}(b',s') - G(\mathcal{H}(b',s')))(\mathcal{Q}(b',s') + z' F_k(1, \mathcal{H}(b',s'), \boldsymbol{v}(b',s')))$$

$$+ \kappa' \boldsymbol{\mu}(b',s') \mathcal{Q}(b',s') \tag{SP3}$$

$$zF_h(1,h,v) = G'(h) \tag{SP4}$$

$$zF_v(1,h,v) = p^v \left(1 + \frac{\theta \mu}{u'(c - G(h))}\right) \tag{SP5}$$

$$\mu \geq 0 \tag{SP6}$$

$$\mu \left(\frac{b'}{R} - \theta p^v v + \kappa q\right) = 0 \tag{SP7}$$

Proposition 2 (Relaxed Planner Problem)**.** *Consider the planning problem A.1. Constraints (SP4)-(SP7) do not bind.*

Proof:

The proof proceeds by analyzing a relaxed planner problem where the planner is not subject to (SP4)-(SP7) and then showing that those conditions are satisfied.

Consider the following relaxed problem (i.e., Problem 12 in the text):

$$\begin{aligned}
\mathcal{V}(b,s) &= \max_{c,b',q,\mu,h,v} u(c - G(h)) + \beta \mathbb{E}_{s'|s} \mathcal{V}(b',s') \\
c + \frac{b'}{R} &= b + zF(1,h,v) - p^v v \\
\frac{b'}{R} - \theta p^v v &\geq -\kappa q \\
qu'(c - G(h)) &= \beta \mathbb{E}_{s'|s} u'(\mathcal{C}(b',s') - G(\mathcal{H}(b',s')))(\mathcal{Q}(b',s') + z' F_k(1, \mathcal{H}(b',s'), v(b',s'))) \quad \text{(A.5)} \\
&\quad + \kappa' \boldsymbol{\mu}(b',s') \mathcal{Q}(b',s')
\end{aligned}$$

Let $\lambda \geq 0$ be the multiplier on the resource constraint, $\mu^* \geq 0$ be the multiplier on the collateral constraint, $\xi \geq 0$ be the multiplier on the asset pricing implementability constraint.

First order conditions and envelope condition in Problem 12 yields:

$$c :: \qquad \lambda = u'(c - G(h)) - \xi u''(c - G(h))q \tag{A.6}$$

$$b' :: \qquad \lambda = \beta R \mathbb{E}_{s'|s}[\mathcal{V}_b(b', s') + \xi\hat{\Omega}] + \mu^* \tag{A.7}$$

$$q :: \qquad \mu^*\kappa = \xi u'(c - G(h)) \tag{A.8}$$

$$h :: \qquad \lambda z F_h(1, h, v) = u'(c - G(h))G'(h) - \xi q u''(c - G(h))G'(h) \tag{A.9}$$

$$v :: \qquad zF_v(1, h, v) = p^v \left(1 + \frac{\mu^*}{\lambda}\theta\right) \tag{A.10}$$

$$KT :: \qquad \mu^* \left(\frac{b'}{R} - \theta p^v v + \kappa q\right) = 0 \tag{A.11}$$

$$EC :: \qquad \mathcal{V}_b(b', s') = \lambda \tag{A.12}$$

where $\hat{\Omega} \equiv \frac{1}{R}\big[u''(\mathcal{C}(b', s') - G(\mathcal{H}(b', s'))) \{\mathcal{Q}(b', s') + z' F_k(1, \mathcal{H}(b', s'), v(b', s'))\}\cdot$ $\{\mathcal{C}_b(b', s') - G'(\mathcal{H}(b', s'))\mathcal{H}_b(b', s')\} + u'(\mathcal{C}(b', s') - G'(\mathcal{H}(b', s')))\{\mathcal{Q}_b(b', s') + z'[F_{kh}(1, \mathcal{H}(b', s'), v(b', s'))\mathcal{H}_b(b', s') +$ $F_{kv}(1, \mathcal{H}(b', s'), v(b', s'))v_b(b', s')]\} + \kappa'[\mu_b(b', s')\mathcal{Q}(b', s') + \mu(b', s')\mathcal{Q}_b(b', s')]\big]$

Set $\mu = \frac{u'(c-G(h))}{\theta}\left(\frac{zF_v(1,h,v)}{p^v} - 1\right)$.[29] Rearranging (A.10), we have $\mu^* = \frac{\lambda}{\theta}\left(\frac{zF_v(1,h,v)}{p^v} - 1\right)$ and combining this with the expression for μ, we have $\mu_t = \frac{\mu_t^*}{\lambda_t}u'(c_t - G(h_t))$. Since $u'(.) > 0$, the KT conditions of the relaxed problem $\mu^* \geq 0$ and (A.11) imply (SP6) and (SP7), the KT conditions of the original problem. By definition of μ, we have that (SP5) is satisfied. Finally, substituting (A.6) into (A.9) yields the original optimality condition with respect to employment (SP4).

This completes the proof that (SP4)-(SP7) do not bind. This proves Proposition 2. ☐

A.2 Proof of Proposition 1

Define the tax as:

$$1 + \tau_t = \frac{\beta R \mathbb{E}_t \left\{u'(\mathcal{C}(b_{t+1}, z_{t+1})) - \xi_{t+1}u''(\mathcal{C}(b_{t+1}, z_{t+1}))\mathcal{Q}(b_{t+1}, z_{t+1}) + \xi_t\Omega_{t+1}\right\} + \xi_t u''(c_t)q_t}{\beta R \mathbb{E}_t u'(\mathcal{C}(b_{t+1}, z_{t+1}))} \tag{A.13}$$

The constrained efficient equilibrium can be characterized by sequences $\{c_t, k_{t+1}, b_{t+1}, q_t, \lambda_t, \mu_t\}_{t=0}^{\infty}$ that satisfy (3), (13), (14), (15), $k_t = 1$ together with complementary slackness conditions.

The regulated decentralized equilibrium is characterized by a sequence $\{c_t, k_{t+1}, b_{t+1}, q_t, \lambda_t, \mu_t\}_{t=0}^{\infty}$

[29]Notice the distinction between μ, which is the shadow value of relaxing the collateral constraint for individual agents, and is choice variable for the planner, and μ^*, which is the shadow value of relaxing the collateral constraint from the planner's perspective (i.e. the shadow value in the planner's KT problem).

that satisfy (2), (3), (A.6), (16),

$$u'(c_t) = \beta R \left(1+\tau_t\right) \mathbb{E}_t u'(c_{t+1}) + \mu_t, \tag{A.14}$$

together with complementary slackness conditions. Using the expression for the tax (A.13) and (A.14), yields condition (14) and identical conditions characterizing the two equilibria.

$$u'(t) = \beta R \mathbb{E}_t \left[u'(t+1) - \xi_{t+1} u''(t+1) \mathcal{Q}_{t+1} + \xi_t \hat{\Omega}_{t+1} \right] + \xi_t u''(t) q_t + \mu_t^*$$

When the collateral constraint is not binding, we obtain the following macro-prudential debt tax:

$$\tau_t = \mathbb{E}_t \frac{-\xi_{t+1} u''(t+1) \mathcal{Q}_{t+1}}{\mathbb{E}_t u'(t+1)} \geq 0 \tag{A.15}$$

A.3 Optimal Tax on Debt Problem: Equivalence Result

Consider the following planner's problem that seeks an optimal tax on debt, given that future taxes $\mathcal{T}(B,s)$ are chosen by future planners, which are associated with policies $\{\mathcal{B}(b,s), \mathcal{C}(b,s), \mathcal{H}(b,s),$
$\mathcal{V}(b,s), \boldsymbol{\mu}(b,s), \mathcal{Q}(b,s)\}$. The planner chooses the tax on debt to maximize utility considering the optimal response of households. That is, the planner chooses sequentially a tax on debt subject to all competitive equilibrium conditions including $u'(c) = \beta R(1+\tau)\mathbb{E}\left[u'(\mathcal{C}(b',z'))\right] + \mu$

$$\mathcal{V}(b,s) = \max_{c,b',q,\mu,h,v,\tau} u(c - G(h)) + \beta \mathbb{E}_{s'|s} \mathcal{V}(b',s') \tag{A.16}$$

$$c + \frac{b'}{R} = b + zF(1,h,v) - p^v v$$

$$\frac{b'}{R} - \theta p^v v \geq -\kappa q$$

$$\begin{aligned} q u'(c - G(h)) = {} & \beta \mathbb{E}_{s'|s} u'(\mathcal{C}(b',s') - G(\mathcal{H}(b',s')))(\mathcal{Q}(b',s') + z' F_k(1, \mathcal{H}(b',s'), \boldsymbol{v}(b',s'))) \\ & + \kappa' \boldsymbol{\mu}(b',s') \mathcal{Q}(b',s') \end{aligned}$$

$$zF_h(1,h,v) = G'(h)$$

$$zF_v(1,h,v) = p^v \left(1 + \frac{\theta \mu}{u'(c - G(h))}\right)$$

$$\mu \geq 0$$

$$\mu \left(\frac{b'}{R} - \theta p^v v + \kappa q\right) = 0$$

$$u'(c - G(h)) = \beta R(1+\tau)\mathbb{E}\left[u'(\mathcal{C}(b',z') - G(\mathcal{H}(b',z')))\right] + \mu \tag{A.17}$$

Relative to problem (SP), problem (A.16) contains one additional restriction A.17 and one additional policy instrument τ. Let τ^* the optimal tax that solves (A.16), the MPE condition implies that $\tau(B,s)=\mathcal{T}(B,s)$

Proposition 3. *A sequence of allocations and prices constitute a constrained efficient equilibrium if and only if they are the outcome of a markov perfect equilibrium where the government chooses sequentially a tax on debt.*

Proof

Consider solving the relaxed problem of maximizing the objective function dropping constraint (A.17). Notice that the resulting problem is the same as (12), after applying Proposition 2. Hence, allocations and prices satisfy (12) if and only if they satisfy the optimal tax problem. In addition, notice that τ only appears in (A.17). Hence, setting $1+\tau=\frac{u'(c)}{\beta RE[u'(\mathcal{C}(b',z'))]+\mu}$ imply that condition (A.17) is satisfied. □

A.4 Non-Negative Tax on Debt

We solve for the optimal tax on debt, as in problem (A.16) but consider taxes on debt that cannot be negative. This is natural because subsidies on debt require lump sum taxes. So we rule out lump sum taxes but not lump sum transfers. As we showed in Section 2.4, the tax is non-negative when the constraint is not binding, but is possibly negative when the constraint binds. Hence, it is possible that the non-negativity constraint on τ would be binding.

The problem of this planner is the same as in (A.16) but with an additional implementability constraint given by:

$$1+\tau_t=\frac{u'(c_t-G(h_t))-\mu_t}{\beta R_t\mathbb{E}\left[u'(c_{t+1}-G(h_{t+1}))\right]}\geq 1$$

Following the same steps as in (A.1) and (A.3), the regulator's problem can be written as:

$$\mathcal{V}(b,s)=\max_{c,b',q,\mu,h,v} u(c-G(h))+\beta\mathbb{E}_{s'|s}\mathcal{V}(b',s') \tag{A.18}$$

$$c+\frac{b'}{R}=b+zF(1,h,v)-p^v v$$

$$\frac{b'}{R}-\theta p^v v\geq-\kappa q$$

$$qu'(c-G(h))=\beta\mathbb{E}_{s'|s}u'(\mathcal{C}(b',s')-G(\mathcal{H}(b',s')))(\mathcal{Q}(b',s')+z'F_k(1,\mathcal{H}(b',s'),\boldsymbol{v}(b',s')))$$
$$+\kappa'\boldsymbol{\mu}(b',s')\mathcal{Q}(b',s')$$

$$\frac{u'(c-G(h))}{\beta R_t\mathbb{E}\left[u'(\mathcal{C}(b',s')-G(\mathcal{H}(b',s')))\right]}\geq 1$$

Let $\gamma_t^p\geq 0$ denote the lagrange multipliers on the last constraint. Following the same steps as above yields that the optimal tax is given by:

$$
\begin{aligned}
1+\tau_t^{tax\geq 0} = & \frac{1}{\beta R \mathbb{E}_t u'(\mathcal{C}(b_{t+1},z_{t+1}) - G(\mathcal{H}(b',s')))} \beta R \mathbb{E}_t \Bigg[u'(\mathcal{C}(b_{t+1},z_{t+1}) - G(\mathcal{H}(b_{t+1},z_{t+1}))) \\
& \xi_{t+1}^p \mathcal{Q}(b_{t+1},z_{t+1}) u''(\mathcal{C}(b_{t+1},z_{t+1})) + \gamma_{t+1}^p \left(\frac{u''(\mathcal{C}(b_{t+1},z_{t+1}) - G(\mathcal{H}(b_{t+1},z_{t+1})))}{\beta R \mathbb{E}_t [u'(\mathcal{C}(b_{t+1},z_{t+1}) - G(\mathcal{H}(b_{t+1},z_{t+1}))]} \right) \\
& + \xi_t^p \Omega_{t+1}^p + \gamma_t^p \phi_{t+1}^p \Bigg] - \mu_t(\varphi_t^p + 1) + \mu_t^p \\
& + \xi_t^p q_t u''(c_t) - \gamma_t^p \left(\frac{u''(c_t - G(h_t)}{\beta R \mathbb{E}_t [u'(\mathcal{C}(b_{t+1},z_{t+1}) - G(\mathcal{H}(b_{t+1},z_{t+1}))]} \right)
\end{aligned} \tag{A.19}
$$

Now suppose that both the non-negative tax constraint and the collateral constraint are slack today but might bind tomorrow with positive probability, i.e. $\gamma_t^p = 0$ and $\mu_t^* = 0$. In this case, A.19 becomes:

$$
\begin{aligned}
1+\tau_t^{tax\geq 0} = & \frac{1}{\beta R \mathbb{E}_t u'(\mathcal{C}(b_{t+1},z_{t+1}))} \Bigg[\beta R \mathbb{E}_t \Bigg[u'(\mathcal{C}(b_{t+1},z_{t+1})) - \xi_{t+1}^p \mathcal{Q}(b_{t+1},z_{t+1}) u''(\mathcal{C}(b_{t+1},z_{t+1})) \\
& + \gamma_{t+1}^p \left(\frac{u''(\mathcal{C}(b_{t+1},z_{t+1}))}{\beta R \mathbb{E}_t [u'(\mathcal{C}(\mathcal{C}(b_{t+1},z_{t+1})))]} \right) \Bigg] \Bigg]
\end{aligned} \tag{A.20}
$$

where $\phi_{t+1}^p \equiv \frac{\mu_t - u'(c_t)}{\beta^2 R_t} \mathbb{E}_t \left[\frac{u''(\mathcal{C}(b_{t+1},z_{t+1}))\mathcal{C}_b(b_{t+1},z_{t+1})}{(u'(\mathcal{C}(b_{t+1},z_{t+1})))^2} \right]$

Notice that the tax has the same form as above but now it has an additional term given by the possible non-negativity constraint on future tax rates.

A.5 Derivation of Collateral Constraint

We provide a derivation of the collateral constraint (3) as an incentive compatibility constraint resulting from a limited enforcement problem. Debt contracts are signed with creditors in a competitive environment. Financial contracts are not exclusive, i.e., agents can always switch to another creditor at any point in time. Households borrow at the beginning of the period, before the asset market open. Within period, households can divert future revenues and avoid any costs from defaulting next period when debt becomes due. At the end of the period, there are no more opportunities for households to divert revenues and repayment of previous bonds is enforced. Financial intermediaries can costlessly monitor diversion activities at time t. If creditors detect the diversion scheme, they can seize a fraction κ_t of the household assets. After defaulting, a household regains access to credit markets instantaneously and a repurchases the assets that investors sell in open markets. Given this environment, a household that borrows $\tilde{d}_{t+1}$ and engage in diversion activities gains $\tilde{d}_{t+1}$ and loses a $\kappa_t q_t k_t$.

Formally, let V^R and V^d be the value of repayment and default respectively, and V be the continuation value. If a household raises $\tilde{d}$ resources by borrowing $\frac{b'}{R}$ at the beginning of the period, and defaults, it gets

$$V^d(\tilde{d}, b, k, X) = \max_{b',k',c,v} u(c - G(h)) + \beta \mathbb{E}_{s'|s} V(b', k', B', s') \tag{A.21}$$

$$s.t. \quad q(B,s)k' + c + \frac{b'}{R} = \tilde{d} + q(B,s)k(1-\kappa) + b + zF(k,h,\nu) - p_\nu \nu$$

$$-\frac{b'}{R} + \theta p_\nu \nu \leq \kappa q(B,s)k$$

where the budget constraint reflects the fact that the household regain access to credit markets and can borrow b' and the collateral constraint reflects that agents buy back the assets from investors. If the household does not default, it gets the utility from current consumption plus the continuation value of starting next period with debt b' as stated in 10.

$$V^r(b, k, X) = \max_{b',k',c,h} u(c - G(n)) + \beta \mathbb{E}_{s'|s} V(b', k', B', s') \tag{A.22}$$

$$s.t. \quad q(B,s)k' + c + \frac{b'}{R} = q(B,s)k + b + zF(k,h,\nu) - p_\nu \nu$$

$$-\frac{b'}{R} + \theta p_\nu \nu \leq \kappa q(B,s)k$$

A simple inspection at the budget constraints implies that households repay if and only if $\tilde{d}_{t+1} \leq \kappa_t q_t k_t$

Notice that for the constrained-efficient equilibrium, the derivation of the feasible credit positions is analogous to the case in the decentralized equilibrium. If the planner engages in diversion, creditors can seize a fraction κ_t of assets in the economy. Moreover, households can buy back the assets at the market price q_t. This implies that the same collateral constraint applies in the constrained-efficient equilibrium in this environment.

B Reinterpretation: Model with Households and Firms

We describe here a setup where households and firms are modeled independently and is isomorphic to the baseline model. This rationalizes the approach of consolidating households and firms within a single problem as well as the calibration approach.

B.1 Household Problem

Households choose consumption, holdings of stocks, bond holdings and labor supply to maximize

$$\max_{\{c_t,h_t,b^H_{t+1},s_{t+1}\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t - G(h_t) + \phi_t k^H)$$

subject to

$$\begin{aligned} s_{t+1}p_t + c_t + \frac{b^H_{t+1}}{R_t} &\leq s_t(d_t + p_t) + b^H_t + w_t h_t && \text{(B.1)} \\ b^H_{t+1} &\geq -\kappa_t q_t k^H && \text{(B.2)} \end{aligned}$$

where ϕ captures preference for housing, s_t represents the holdings of firm shares and p_t represents the price of firm shares. We assume that the stock of housing owned by households is constant.

First order conditions:

$$\begin{aligned} p_t u'(t) &= \beta\mathbb{E}_t u'(t+1)(d_{t+1} + p_{t+1}) && \text{(B.3)} \\ G'(h_t) &= w_t && \text{(B.4)} \\ u'(t) &= \beta R_t \mathbb{E}_t u'(t+1) + R\mu^H_t, && \text{(B.5)} \\ \mu^H_t(b^H_{t+1} + \kappa_t q_t k^H) &= 0 && \text{(B.6)} \\ \mu^H_t &\geq 0 && \text{(B.7)} \end{aligned}$$

where μ^H is the non-negative Lagrange multiplier on the household borrowing constraint.

B.2 Firms

Et problem of the firm is to choose capital, labor, intermediate inputs, dividends and bond holdings to maximize equity value, which can be expressed as:

$$\max_{\{d_t,b^F_{t+1},k^F_{t+1},n_t,v_t\}} \sum_{t=0}^{\infty} u'(c_t - G(h_t))d_t$$

$$\begin{aligned} d_t + q_t k^F_{t+1} + \frac{b^F_{t+1}}{R_t} &\leq k^F_t q_t + F(z_t, k^F_t, n_t, v_t) - w_t n_t - p^v_t v_t + b^F_t && \text{(B.8)} \\ b^F_{t+1} - \theta p^v_t v_t &\geq -\kappa_t q_t k^F_t && \text{(B.9)} \end{aligned}$$

First order conditions are:

$$\begin{aligned}
u'(t)q_t &= \mathbb{E}_t u'(t+1)(F_k(t+1)+q_{t+1}) + \beta \mathbb{E}_t \mu^F_{t+1} q_{t+1} && \text{(B.10)}\\
F_n(t) &= w_t && \text{(B.11)}\\
F_v(t) &= p^v \left(1+\theta \frac{\mu^F_t}{u'(t)}\right) && \text{(B.12)}\\
u'(t) &= \beta R_t \mathbb{E}_t u'(t+1)) + R\mu^F_t, && \text{(B.13)}\\
\mu^F_t (b^F_{t+1} + \theta p^v_t v_t + \kappa_t q_t k^F_t) &= 0 && \text{(B.14)}\\
\mu^F_t &\geq 0 && \text{(B.15)}
\end{aligned}$$

where μ^F is the non-negative Lagrange multiplier on the firm collateral constraint.

B.3 Market Clearing and Competitive Equilibrium

Market clearing requires:

$$h_t = n_t \qquad \text{(B.16)}$$

$$s_t = 1 \qquad \text{(B.17)}$$

$$k^F_{t+1} + k^H = \bar{K} \qquad \text{(B.18)}$$

Notice that (B.13) and (B.5) imply that $\mu^H_t = \mu^F_t$.

A competitive equilibrium in this economy is $\{c_t, h_t, b^H_{t+1}, s_{t+1}, k^H, d_t, b^F_{t+1}, k^F_{t+1}, n_t, v_t, \mu^H_t\}$ such that conditions (B.1)-(B.15) equations hold.

B.4 Household-Firm problem

$$\max_{\{c_t, b_{t+1}, k_{t+1}, n_t, v_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t - G(n_t) + \phi_t k^H)$$

$$\begin{aligned}
c_t + k_{t+1} q_t + \frac{b_{t+1}}{R_t} &\leq k_t q_t + F(z_t, k_t, n_t, v_t) + b_t && \text{(B.19)}\\
b_{t+1} - \theta p_v v &\geq -\kappa q_t (k^H + k_t) && \text{(B.20)}
\end{aligned}$$

First-order conditions:

$$\begin{aligned}
u'(t)q_t &= \beta \mathbb{E}_t u'(t+1)(F_k(t+1)+q_{t+1}) + \beta \mathbb{E}_t \mu_{t+1} q_{t+1} && \text{(B.21)}\\
F_n(t) &= G'(n_t) && \text{(B.22)}\\
u'(t) &= \beta R_t \mathbb{E}_t u'(t+1) + R_t \mu_t, && \text{(B.23)}\\
\mu_t(b_{t+1} - \theta p_v v + \kappa_t q_t (k^H + k_t)) &= 0 && \text{(B.24)}\\
\mu_t &\geq 0 && \text{(B.25)}
\end{aligned}$$

where μ_t denotes the Lagrange multiplier on the consolidated collateral constraint.

A competitive equilibrium in this economy is $\{c_t, n_t, b_{t+1}, k^F_{t+1}, \mu_t\}$ such that conditions (B.19)-(B.25) hold.

B.5 Equivalence

Proposition 4. *If* $c_t, h_t, b^H_{t+1}, s_t, k^H, d_t, b^F_{t+1}, k^F_{t+1}, n_t, v_t, w_t, p_t\}$ *is a competitive equilibrium allocation in the economy with separated households and firms, then* $\{c_t, n_t, b_{t+1}, k^F_{t+1}\}$ *is a competitive equilibrium allocation in an economy with a representative household-firm with* $b_{t+1} = b^H_{t+1} + b^F_{t+1}, k_{t+1} = k^F_{t+1}$.

Conversely, if $\{c_t, n_t, b_{t+1}, k^F_{t+1}\}$ *is a competitive equilibrium allocation in an economy with a representative household-firm, then there exists prices* w_t, q_t, p_t *such that* $c_t, h_t, b^H_{t+1}, s_t, k^H, d_t, b^F_{t+1}, k^F_{t+1}, n_t, v_t$ *is a competitive equilibrium allocation in the economy with separated households and firms, with* $b_{t+1} = b^H_{t+1} + b^F_{t+1}, k_{t+1} = k^F_{t+1}$.

Proof: " $\rightarrow$" Suppose $\{c_t, h_t, b^H_{t+1}, s_{t+1}, k^H, d_t, b^F_{t+1}, k^F_{t+1}, n_t, v_t\}$is a competitive equilibrium allocation in an economy with a separated households and firms. Then we have to show that conditions (B.1)-(B.15) imply that (B.19)-(B.25) also hold. This follows from simple inspection: Budget constraint of the household firm (B.19) follows by combining (B.1),(B.17) and (B.8). The household-firm collateral constraint (B.20) follows from combining (B.2),(B.9) and $b_{t+1} = b^H_{t+1} + b^F_{t+1}$. Labor market condition (B.22) follows from (B.4) and (B.11). Euler equations and complementary slackness conditions (B.23)-(B.25) follow from (B.5)-(B.7).

□

Proof: " $\leftarrow$" Suppose $\{c_t, n_t, b_{t+1}, k^F_{t+1}\}$ is a competitive equilibrium allocation in an economy with a representative household-firm. Then, we have to show that (B.19)-(B.25) imply that equations (B.1)-(B.15) also hold. This follows from simple inspection after constructing prices as follows $w = G'(h)$, q_t so that $p_t u'(t) = \beta \mathbb{E}_t u'(t+1)(d_{t+1} + p_{t+1}), u'(t) q_t = \beta \mathbb{E}_t u'(t+1)(F_k(t+1) + q_{t+1}) + \beta \mathbb{E}_t \mu_{t+1} q_{t+1}$ Construct b^H_{t+1}, b^F_{t+1} such that $b^H_{t+1} \leq \kappa q_t k^H, b^F_{t+1} - \theta p^v_t v_t \geq -\kappa_t q_t k^F_t$ and $b^F_{t+1} = b_{t+1} - b^H_{t+1}$. Construct $s_{t+1} = 1$ and $d_t = F(z_t, k^F, n_t, v_t) - w_t n_t - p^v_t v_t + b^F_t - \frac{b^F_{t+1}}{R_t}$. Market clearing in labor market (B.16) and optimality of labor demand (B.11) follows by taking $w = G'(h)$. and (B.22). Euler equation and complementary slackness condition (B.5)-(B.7) follow from (B.23)-(B.25).

C Investment

This section shows that the qualitative insights of the model that features capital in exogenous supply extend to a model with capital adjustment cost.

In order to have k_{t+1} units of capital ready for production in period $t+1$, an agent with k_t^o units of used capital needs to employ $\phi(k_{t+1}, k_t^o)$ units of the consumption good at date t. There is a competitive market for used capital, where agents can buy and sell capital at the price q_t^o, after production has taken place. To simplify the exposition, we assume no intermediate inputs and a borrowing constraint $\frac{b_{t+1}}{R_t} \geq -\kappa_t q_t \bar{K}$ where $\bar{K}$ is the aggregate capital stock at steady state and q is the price of newly installed capital readily available to produce in the following period.

The budget constraint is as follows

$$c_t + \frac{b_{t+1}}{R_t} + q_t^o k_t^o + \phi(k_{t+1}, k_t^o) + q\tilde{k}_{t+1} \leq b_t + z_t F(k_t, h_t) + q_t^o k_t + q k_{t+1}$$

According to this, households sell old capital k_t and newly installed capital k_{t+1}, and use bonds and production to buy newly installed capital $\tilde{k}_{t+1}$, old capital k_t^o, consume and issue new bonds. This allows for the possibility of $k_t^o \neq k_t$ and $k_{t+1} \neq \tilde{k}_{t+1}$. However, market clearing in the used capital market requires $k_t^o = k_t, k_{t+1} = \tilde{k}_{t+1}$.

Recursive representation— Let $X = (B, K, s)$ denote the aggregate state of the economy. The recursive optimization problem of agents is given by:

$$V(b, k, X) = \max_{c, k^o, \tilde{k}', b', h, k_{t+1}} u(c - G(h)) + \beta \mathbb{E}_{s'|s} V(b', k', X') \tag{C.1}$$

$$c + \frac{b'}{R_t} + \phi(k', k^o) + q^o(X)k^o + q(X)\tilde{k}' = b + zF(k, h) + q^o(X)k + q(X)k' \tag{C.2}$$

$$\frac{b'}{R_t} \geq -\kappa q(X)\bar{K} \tag{C.3}$$

Optimality conditions are:

$$u'(c_t - G(h_t)) = \beta R \mathbb{E}_t[u'(c_{t+1} - G(h_{t+1}))] + \mu \tag{C.4}$$

$$G'(h) = F_h(k, h) \tag{C.5}$$

$$q^o(X) = \phi_2(\tilde{k}'^o) \tag{C.6}$$

$$q_t = \phi_1(k_{t+1}, k_t^o) \tag{C.7}$$

$$u'(c_t - G(h_t))q_t = \beta \mathbb{E}_t[u'(c_{t+1} - G(h_{t+1}))\{q_{t+1}^o + z_{t+1}F_k(k_{t+1}, h_{t+1})\}] \tag{C.8}$$

Market clearing implies that the resource constraint is

$$c + \frac{b'}{R} + \phi(k', k^o) = b + zF(k, h) \tag{C.9}$$

C.1 Optimal Time Consistent Planner's Problem

As described in Section 3 and Section of the Appendix, the planner chooses directly borrowing on behalf of the households, and let all markets clear competitively. That is, the planner chooses allocations and prices subject to a set of implementability constraints given by (C.5)-(C.9).

Under discretion, planner takes future policies for capital $\mathcal{K}$, consumpion $\mathcal{C}$, bonds $\mathcal{B}$ as given and solves the following problem:

$$\begin{aligned}
\mathcal{V}(b,k,s) &= \max_{c,k',b',h} u(c-G(h)) + \beta \mathbb{E}_{s'|s}\mathcal{V}(b',k',s') \\
c + \frac{b'}{R} + \phi(k',k^o) &= b + zF(k,h) \\
\frac{b'}{R} &\geq -\kappa\phi_1(k',k)k_t \\
G'(h) &= F_h(k,h) \\
u'(c-G(h))\phi_1(k',k) &= \beta\mathbb{E}_{s'|s}[u'(\mathcal{C}(b',k',X') - G(\mathcal{H}(b',k',X'))) \{\phi_2(\mathcal{K}(b',k',X'),k')) \\
&\quad + z'F_k(k',\mathcal{H}(b',k',X'))\}]
\end{aligned}$$

Using first-order conditions and envelope condition, the Euler equation for bonds is:

$$\begin{aligned}
u'(c_t - G(h_t)) &= \beta R\mathbb{E}_t\big[u'(\mathcal{C}(b_{t+1},k_{t+1}) - G(\mathcal{H}(b_{t+1},k_{t+1})) \\
&\quad -\xi_{t+1}\phi_1(\mathcal{K}(b_{t+1},k_{t+1},X),k_{t+1})u''(\mathcal{C}(b_{t+1},k_{t+1},X) - G(\mathcal{H}(b_{t+1},k_{t+1},X))) + \xi_t\Omega_t\big] \\
&\quad +\xi_t\phi_1(k_{t+1},k_t)u''(c_t - G(h_t)) + \mu_t^*
\end{aligned}$$

where $\Omega \equiv \mathbb{E}_{s'|s}[u''(\mathcal{C}(b',k',X') - G(\mathcal{H}(b',k',X')))\{\phi_2(\mathcal{K}(b',k',X'),k')) + z'F_k(\bar{K},\mathcal{H}(b',k',s'))\}(\mathcal{C}_b(b',k',s')$ $-G'(\mathcal{H}(b',k',s'))\mathcal{H}_b(b',k',s')) + u'(\mathcal{C}(b',k',s') - G(\mathcal{H}(b',k',s')))(\phi_{21}(\mathcal{K}(b',k',s'),k')\mathcal{K}_b(b',k',s')$ $+z'\{F_{kk}(\bar{K},\mathcal{H}(b',k',s'))\mathcal{K}_b(b',k',s')) + F_{kh}(\bar{K},\mathcal{H}(b',k',s'))\mathcal{H}_b(b',k',s'))\}]$.

Suppose that the implementability constraint is slack today but binds tomorrow, i.e $\xi_t = 0$ but $\xi_{t+1} \geq 0$. This yields

$$u'(c_t - G(h_t)) = \beta R\mathbb{E}_t[u'(\mathcal{C}(b_{t+1},k_{t+1},X) - G(\mathcal{H}(b_{t+1},k_{t+1},X))) \tag{C.10}$$

$$-\xi_{t+1}\phi_1(\mathcal{K}(b_{t+1},k_{t+1},X),k_{t+1})u''(\mathcal{C}(b_{t+1},k_{t+1},X)] \tag{C.11}$$

Just like in the condition (14) of the baseline model, there is a positive wedge between the marginal cost of borrowing from the planner and households when the constraint is not binding at t but is expected to bind at $t+1$.

D Commitment

This section provides more details about the analysis under commitment. There are two sections. Section D.1 derives some theoretical results and D.2 provides a numerical analysis.

D.1 Theoretical Results

Under commitment, the planner chooses at time 0 once and for all $\{c_t, b_{t+1}, q_t, h_t, v_t, \mu_{t+1}\}_{t\geq 0}$ to maximize expected lifetime utility:

$$\max_{\{c_t,b_{t+1},q_t,h_t,v_t,\mu_t\}_{t\geq 0}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(c_t - G(h_t))$$

$$\begin{aligned}
c_t + \frac{b_{t+1}}{R} &\leq b_t + z_t F(1, h_t, v_t) - p_t^v v_t && \text{(D.1)}\\
z_t F_h(1, h_t, v_t) &= G'(h_t) && \text{(D.2)}\\
z_t F_v(1, h_t, v_t) &= p_t^v \left(1 + \frac{\theta \mu_t}{u'(c_t - G(h_t))}\right) && \text{(D.3)}\\
\frac{b_{t+1}}{R} - \theta p_t^v v_t &\geq -\kappa_t q_t && \text{(D.4)}\\
u'(c_t - G(h_t))q_t &= \beta \mathbb{E}_t \Big[u'(c_{t+1} - G(h_{t+1}))(q_{t+1} + z_{t+1} F_k(1, h_{t+1}, v_{t+1})) \\
&\quad + \kappa_{t+1}\mu_{t+1}q_{t+1} \Big] && \text{(D.5)}\\
\mu_t \left(\frac{b_{t+1}}{R} - \theta p_t^v v_t + \kappa_t q_t\right) &= 0 && \text{(D.6)}\\
\mu_t &\geq 0 && \text{(D.7)}
\end{aligned}$$

Let λ_t, ζ_t^h, ζ_t^v, μ_t^*, ξ_t, ν_t and χ_t denote the lagrange multipliers on constraints (D.1)-(D.7) respectively. First-order conditions with respect to c_t, b_{t+1}, q_t, h_t, v_t, and μ_t are:

$$c_t :: \qquad \lambda_t = u'(t) - \xi_t u''(t) q_t + \xi_{t-1} u''(t)(q_t + z_t F_k(t) + \kappa_t \mu_t q_t) \tag{D.8}$$

$$b_{t+1} :: \qquad \lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t^* + \mu_t \nu_t \tag{D.9}$$

$$q_t :: \qquad \xi_t = \xi_{t-1}(1 + \kappa_t \mu_t) + \frac{\kappa_t(\mu_t \nu_t + \mu_t^*)}{u'(t)} \tag{D.10}$$

$$h_t :: \qquad z_tF_h(t) = G'(h_t) + \frac{1}{\lambda_t}\left[\zeta_t^h[z_tF_{hh}(t) - G''(h)] + \zeta_t^v z_tF_{vh}(t) - \xi_{t-1}u'(t)z_tF_{kh}(t)\right] \tag{D.11}$$

$$v_t :: \qquad z_tF_v(t) = \frac{1}{\lambda_t}\left[p_t^v[\lambda_t + \theta\mu_t^* + \theta\mu_t\nu_t] + \zeta_t^h z_tF_{hv}(t) + \zeta_t^v z_tF_{vv}(t) - \xi_{t-1}u'(t)z_tF_{kv}(t)\right] \tag{D.12}$$

$$\mu_t \ :: \qquad \xi_{t-1}\kappa_t q_t u'(t) + \chi_t + \zeta_t^v p^v\theta = -\nu_t\left(\frac{b_{t+1}}{R} - \theta p_t^v\nu_t + \kappa_t q_t\right) \tag{D.13}$$

Notice that conditions (D.8)-(D.10) correspond to conditions (19)-(21 in Section 2.5.

Complementary slackness conditions is:

$$\mu_t^*\left(\frac{b_{t+1}}{R} - \theta p_t^v v_t + \kappa_t q_t\right) = 0 \tag{D.14}$$

$$\chi_t\mu_t = 0 \tag{D.15}$$

The Euler equation for bonds is:

$$\begin{aligned} u'(t) \;=\; & \beta R\mathbb{E}_t[u'(t+1) - \xi_{t+1}u''(t+1)q_{t+1} + \xi_t u''(t+1)(q_{t+1}(1+\kappa_{t+1}\mu_{t+1}) + z_{t+1}F_k(t+1))\ ] \\ & + \xi_t u''(t)q_t - \xi_{t-1}u''(t)(q_t(1+\kappa_t\mu_t) + z_tF_k(t)) - \frac{\zeta_t^\nu p_t^v\theta\mu_t u''(t)}{u'(t)^2} + \mu_t^* + \nu_t\mu_t \end{aligned} \tag{D.16}$$

Following the same steps as in A.2 and assuming that the implementability constraints (D.11), (D.12), and (D.13) are not binding, the macro-prudential tax on debt under commitment $\tau_t^{MP,C}$ is given by:

$$\tau_t^{MP,C} = \frac{-\mathbb{E}_t\frac{\kappa\mu_{t+1}}{u'(c_{t+1})}u''(c_{t+1})q_{t+1} + \xi_{t-1}(\mathbb{E}_t u''(c_{t+1})z_{t+1} - z_t u''(c_t))}{\mathbb{E}_t u'(c_{t+1})}$$

Compared to the tax in the markov perfect equilibrium (17), the tax under commitment features another term that relates to previous commitments as given by the second term on the right hand side of (D.1).

D.2 Numerical Results

A numerical solution requires of the model under commitment requires to find appropriate state values to keep track of previous promises. Notice that ξ follows an increasing sequence over time and hence is not an appropriate state variable for a numerical solution.

Building on Kydland and Prescott (1980), one can transform the planner's problem to express it recursively expanding the states to include consumption and asset prices. For simplicity, we consider here a production function of the form $F(k) = zk$, a collateral constraint that depends on the market value of aggregate assets and only productivity shocks. The recursive problem for

$t > 0$ can be expressed as follows

$$\begin{aligned} V(b,q,c,s) &= \max_{b',q(s'),m(s')} u(c) + \beta \mathbb{E}_{s'} V(b', q(s'), c(s'), z') \\ c \ + b'/R &\leq b + z \\ b' &\geq -\kappa q \\ q u'(c) &\geq \beta \mathbb{E} c(s')(z' + q(s')) \end{aligned} \tag{D.17}$$

with states $(b,q,c,s) \in A$ where A is the largest set of points (b,q,c,s) such that we can find a sequence b_{t+1}, c_t, q_t satisfying all the constraints.

At$t = 0$, the planner is not bound by past promises of consumption and asset prices. The time 0 problem consists of choosing $q(b,s), c(b,s)$ that maximize $V(b,q,c,s)$.

$$\max_{c,q} V(b,q,c,s) \tag{D.18}$$

To solve the model, we construct a grid for the three endogenous state variables b, q, c of dimension NB, NQ and NC respectively and a grid of dimension NS for the exogenous state variables. Notice that since the planner is choosing asset prices and consumption for each possible value of the shock tomorrow, the dimension of the control space for each combination of state variables is $NS \times NB \times NQ \times NC + 1$. To keep the numerical solution manageable, we use relatively coarse grids of $NS = 2, NB = 20, NQ = 10, NC = 20$. Finally, the set A is determined through an iterative procedure.

Figure 9 shows the value function, and policy functions for asset prices, consumption, and bonds at $t = 0$ and comparing them to the decentralized equilibrium. That is, we first solve (D.17) and obtain the policy function $b'(b,q,c,s)$ and value function $V'(b,q,c,s)$, and then find c^*, q^* that solve (D.18). Figure 9 plots the solution for c^*, q^* together with the associate value function, and the bond policy that solves (D.17) for c^*, q^* that solve (D.18) .

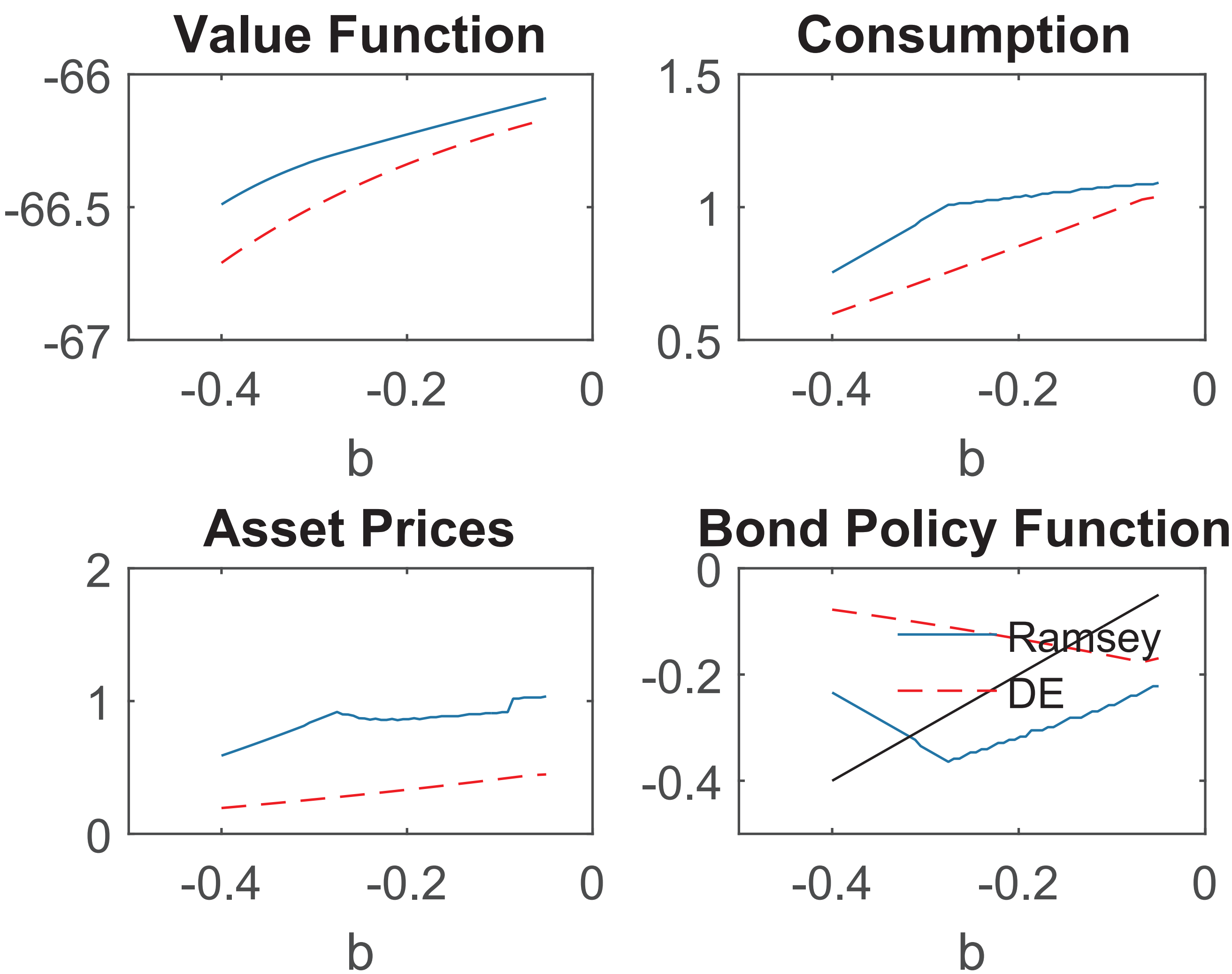


Figure 9: Policy Functions under Commitment vs Decentralized Equilibrium

Note: Dashed lines represent optimal macroprudential policy under commitment.

E Data Appendix

E.1 Data Sources

- Net Foreign Asset Position (NFA): *Flow of Funds*
- Total Credit: *Survey of Terms of Business Lending* and *Flow of Funds*
- Intermediate Inputs: United Nations *UNdata*
- GDP: OECD National Accounts Statistics

E.2 Frequency and Duration of Financial Crises

To construct estimates of the duration and frequency of financial crises, we applied the methodology proposed by Forbes and Warnock (2012) to identify the timing and duration of sharp changes in financial conditions. A financial crisis is defined as an event in which the cyclical component of the linearly-detrended current account is above two-standard deviations from its mean. Since the current account is the overall measure of financing of the economy vis-a-vis the rest of the world, this unusually large current accounts represent unusually large drops in foreign financing. The starting (ending) dates of the events are set in the year within the previous (following) two years in which the current account first rose (fell) above (below) one standard deviation. Using the data for all OECD countries over the 1984-2012 period, we obtained financial crises with a frequency of 4 percent and a mean duration of 1 year.

Table 4 indicates the list of all crises events identified with this methodology and Figures E.2-E.3 show the data for current account for all countries.

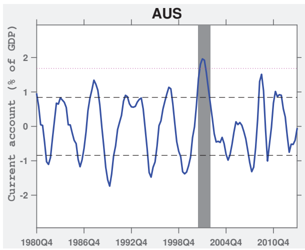
Table 4: Financial Crises Episodes.

Australia	[2001Q2 - 2002Q3]
Austria	[2002Q3 - 2003Q2], [2008Q1 - 2009Q1]
Belgium	[2010Q2 - 2011Q2]
Canada	[1996Q1 - 1997Q1]
Chile	
Czech Republic	[2005Q3 - 2006Q3]
Denmark	[2005Q4 - 2006Q3], [2010Q3 - 2011Q2]
Estonia	[2009Q3 - 2010Q4]
Finland	[1995Q1 - 1996Q2]
France	[1999Q1 - 2000Q1]
Germany	[1989Q1 - 1990Q4]
Greece	
Hungary	[2001Q4 - 2002Q3], [2009Q4 - 2011Q1]
Iceland	
Ireland	[2003Q2 - 2004Q4]
Israel	[2006Q2 - 2007Q3], [2010Q1 - 2010Q4]
Italy	
Japan	[2007Q2 - 2008Q2]
Korea, Republic of	[2004Q1 - 2005Q1]
Luxembourg	[2005Q1 - 2005Q3]
Mexico	[1983Q2 - 1984Q3], [1995Q4 - 1997Q2]
Netherlands	[2006Q1 - 2007Q1]
New Zealand	[2009Q3 - 2010Q4]
Norway	[2000Q3 - 2002Q1]
Poland	
Portugal	[2003Q1 - 2004Q3]
Slovak Republic	[1995Q4 - 2000Q4]
Slovenia	
Spain	
Sweden	[2006Q4 - 2008Q4]
Switzerland	[2010Q1 - 2011Q1]
Turkey	[1994Q4 - 1995Q2], [2001Q4 - 2002Q3], [2009Q2 - 2010Q2]
United Kingdom	
United States	[1991Q1 - 1992Q2], [2009Q2 - 2010Q2]

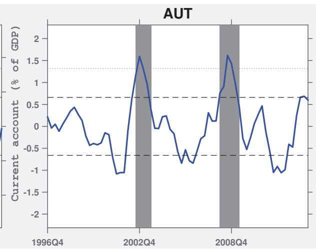
Note: Financial Crises episodes.

Figure E.1: Financial Crises

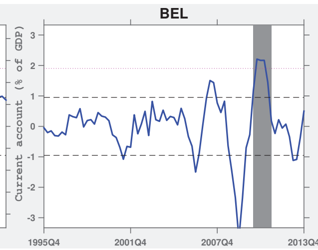
(a) Australia



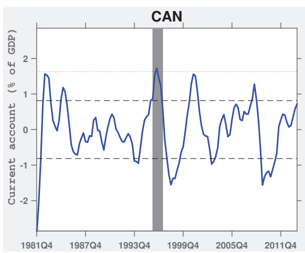
(b) Austria



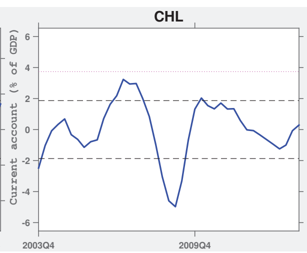
(c) Belgium



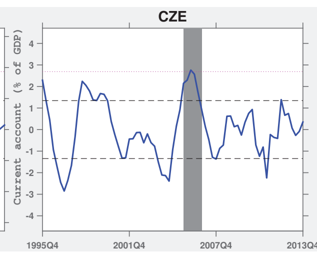
(d) Canada



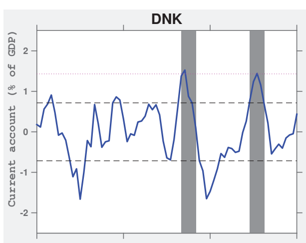
(e) Chile



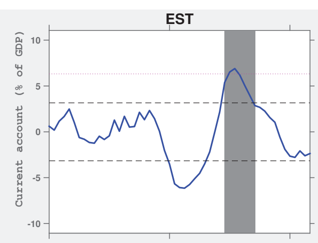
(f) Czech



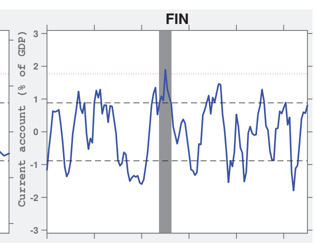
(g) Denmark



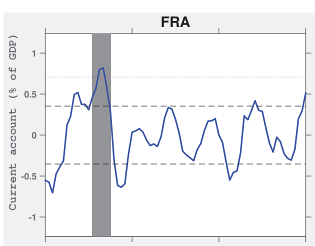
(h) Estonia



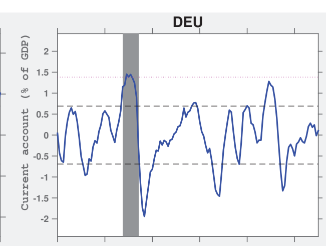
(i) Finland



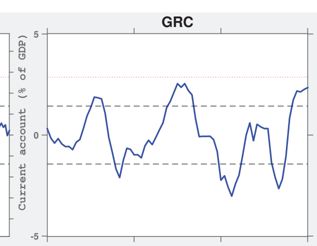
(j) France



(k) Germany



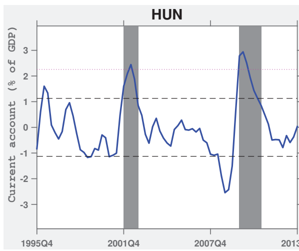
(l) Greece



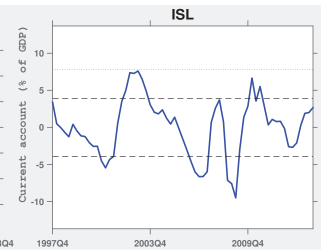
Ratio of cumulated net capital inflows over the last four quarters to cumulated GDP over the last four quarters. Shaded areas describe periods of Financial Crises

Figure E.2: Financial Crises

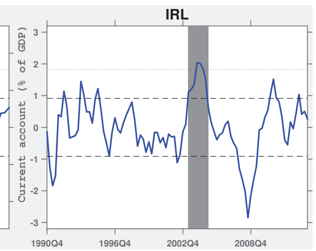
(a) Hungary



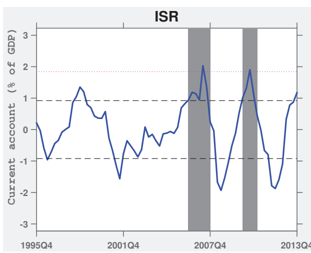
(b) Iceland



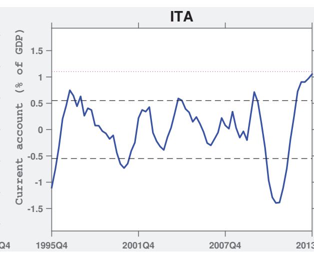
(c) Ireland



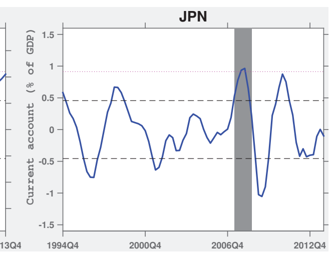
(d) Israel



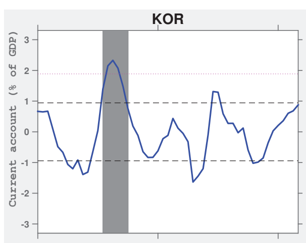
(e) Italy



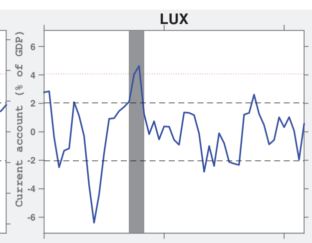
(f) Japan



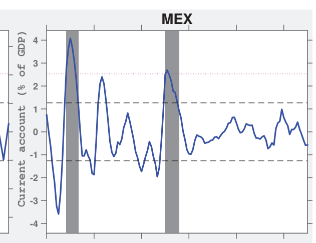
(g) Korea



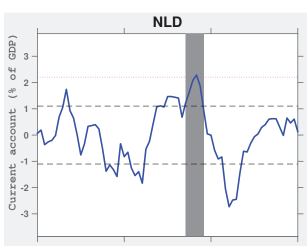
(h) Luxembourg



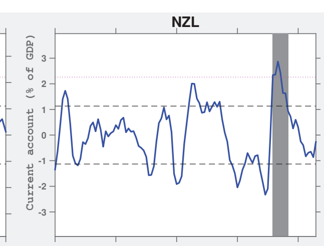
(i) Mexico



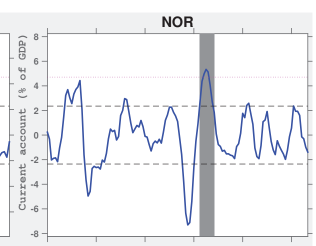
(j) Netherlands



(k) New Zealand

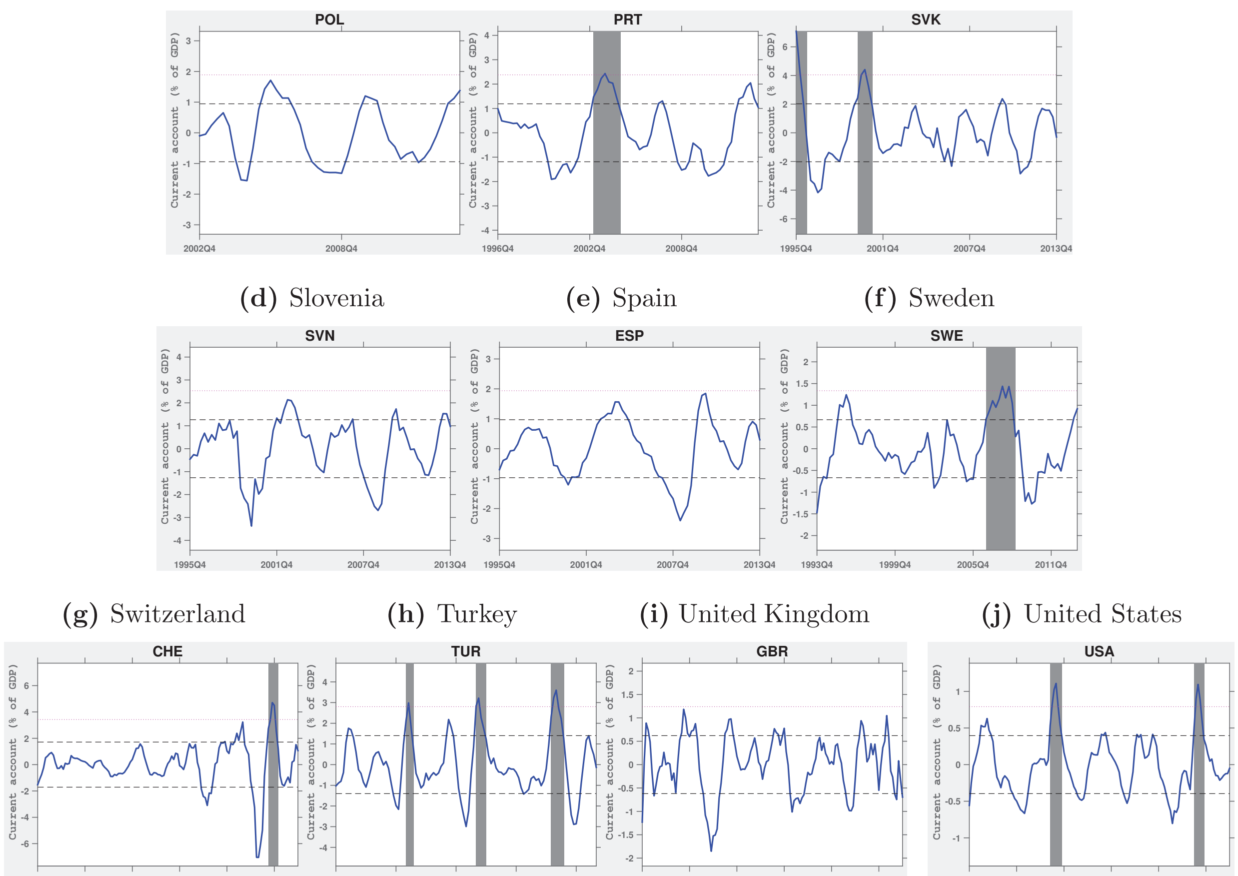


(l) Norway



Ratio of cumulated net capital inflows over the last four quarters to cumulated GDP over the last four quarters. Shaded areas describe periods of Financial Crises

Figure E.3: Financial Crises



(a) Poland **(b)** Portugal **(c)** Slovak Republic

(d) Slovenia **(e)** Spain **(f)** Sweden

(g) Switzerland **(h)** Turkey **(i)** United Kingdom **(j)** United States

Ratio of cumulated net capital inflows over the last four quarters to cumulated GDP over the last four quarters. Shaded areas describe periods of Financial Crises

F Great Recession Experiment

In section 3.2, we conduct ed an event analysis designed to show how average financial crises look in DE and conduct counterfactual experiments to show the effectiveness of macroprudential policy. In this section, we examine instead the DE model's predicted time-series dynamics leading up to and including the global financial crisis event using a window spanning the 2000-2009 period. We compare these dynamics with the observed dynamics in the United States and Europe, and with a counterfactual of what the event would have looked like under the SP's optimal policy.[30] The results for three of the model's key variables (asset prices, output and the current account) are shown in Figure 1.

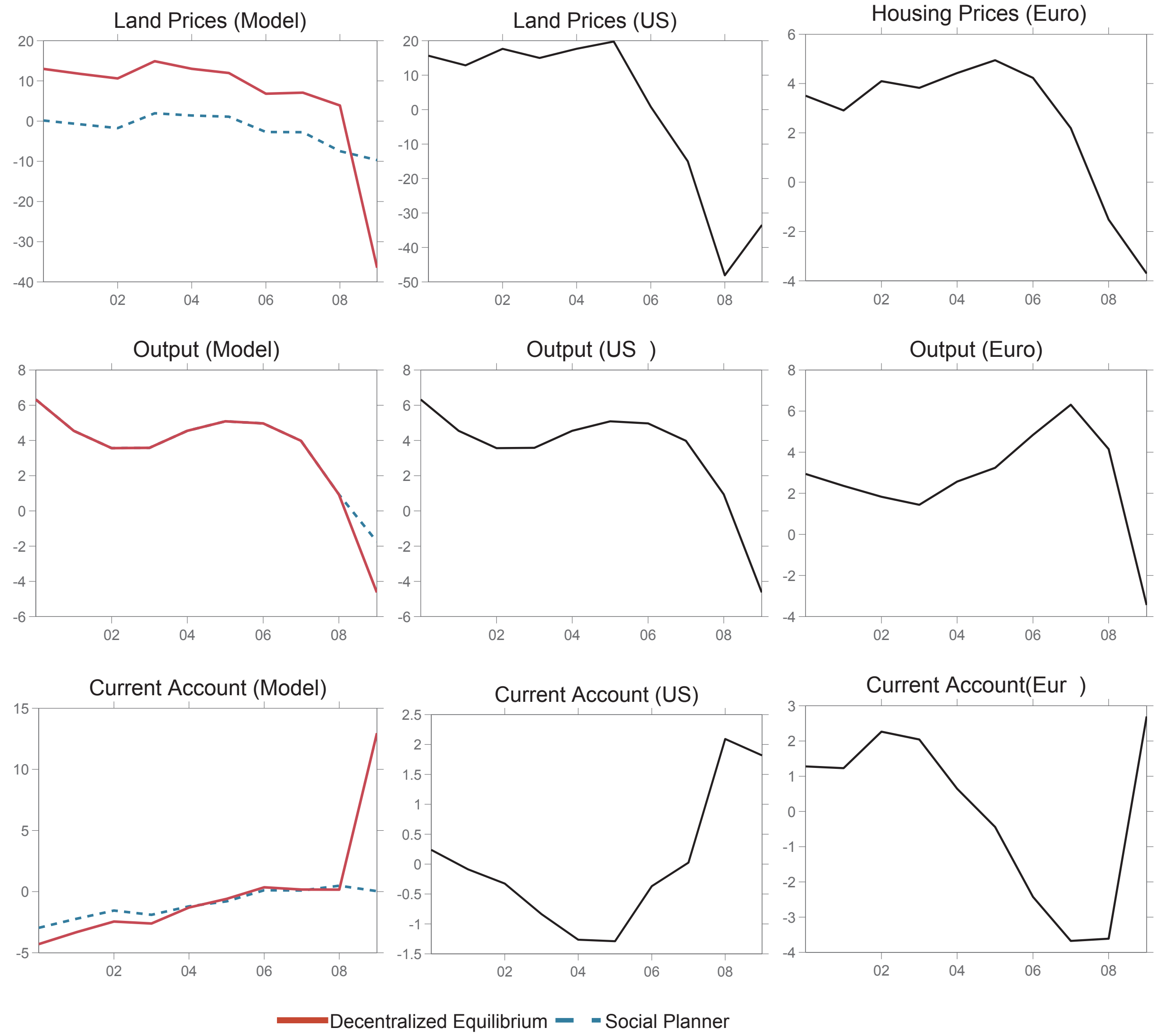


Figure F.1: Comparison of Crises Dynamics

Note: In the model, Land Prices and output expressed as a percentage deviation of mean values for decentralized equilibrium. Data values are expressed as deviation from a linear trend over the period 1984-2010.

[30]Data for Europe represents simple average of European Union—the source is Eurostat.

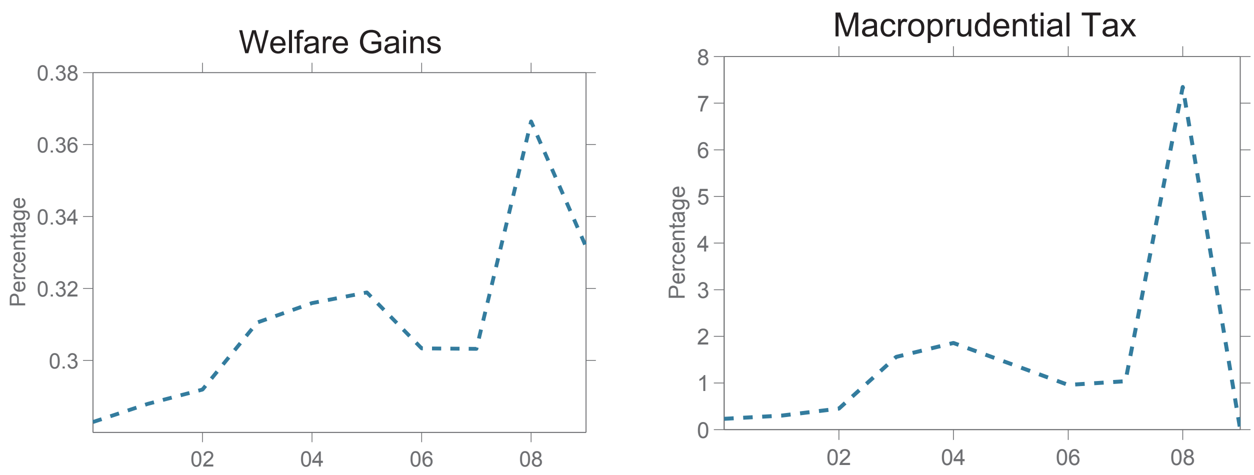


Figure F.2: Macroprudential Tax and Welfare Gains

To generate the simulated data for the DE and SP we need to set an initial condition for b, and values for the realizations of TFP, R and κ for the ten years in the event window. The initial condition for b is set equal to the private NFA-GDP ratio of the United States observed in 2000, which was -11.6 percent. The values of the interest-rate shocks are set to their observed realizations during 2000-2009 and the values of the TFP shocks are set so as to match the observed deviation from linear trend of the U.S. real GDP in the same period—we use interpolation over the realizations included in the Markov approximation of the AR(1) process of TFP). The values of κ are set to κ^H for 2000-2008 and κ^L for 2009. The DE and SP decision rules for bonds, together with the recursive functions that map the values of bonds and the shocks into equilibrium prices and allocations, are then used to generate the plots shown in the first column of Figure 1.

Comparing the DE model's crisis dynamics with the U.S. and European data, Panels (a)-(c) of the Figure 1 show that the model does quite well at tracking the dynamics of asset prices, particularly for the United States, which is the country used to set the initial conditions and shock realizations for the model simulations. The timing of the crash in the asset price is off by one year, because we set the change to κ^L and the lowest realization of TFP in 2009, to be consistent with the fact that the lowest deviation from trend in GDP was observed in 2009. We could calibrate to 2008 instead and then the crash in the asset price would be in the same year as in the data. The current account in the DE and the U.S. data share the feature that the financial crisis is associated with a sharp current account reversal. The reversal, however, is much large in the model than in the data, which is partly due to the fact that debt in the model is only one-period debt, while actual U.S. net foreign assets include significant positions in long-term instruments.[31] For the same reason, the large current account reversal implies a decline in private consumption larger than what was observed in U.S. data.

As figure F.1 shows, the policy not only prevents the asset price crash, but in addition it produces lower and more stable asset prices for the entire ten-year period. The output dynamics are identical across the two economies before the crisis, because they experience the same TFP shocks calibrated to replicate the path of output, but when the crisis hits output falls less in the SP because the collateral constraint is less binding, and hence implies a smaller cutback in working

[31]The model also abstracts from government policies that were put in place to offset the credit crunch (see e.g. Gertler and Kiyotaki (2010) and Bianchi (2012)).

capital financing. The DE shows slightly larger current account deficits than SP from 2002 to 2004, from then until the crisis hits the two are about the same, and then when the crisis hits the SP avoids the current account reversal completely. The larger initial deficits of the DE reflect the incentive to overborrow that private agents have because of the effect of the pecuniary externality. The differences in current accounts are small, which means that debt positions pre-crisis are not all that different, but as we show later, small differences in debt positions between the DE and SP result in large differences in macro outcomes, because the calibrated model features a strong financial amplification mechanism.

Figure F.2 shows the time-series dynamics of the optimal tax and the welfare gains (i.e. these are values conditional on each year instead of the averages shown in Table 2). The tax increases first gradually and then sharply to about 7 percent just before the crisis. The welfare gain of the optimal policy follows a very similar pattern, and reaches a maximum of 36 basis points the year before the crisis.

Table 5: Type-1 and Type-2 Errors in Crises Warnings

	Model Probability	Logit All Regressors	Logit Credit-Output
(i) 5 percent of Type-1 errors in the model			
Type 1	5.0	4.4	3.4
Type 2	63.7	69.1	73.2
(ii) 5 percent of Type-2 errors in the model			
Type 1	82.8	84.0	81.7
Type 2	5.0	7.9	9.2

Note: All values are expressed in percent. Warning probability cutoffs are 1.8 and 8.6 percent in scenarios (i) and (ii) respectively. "All Regressors" includes credit, asset prices and all exogenous shocks as explanatory variables

G Early Warnings

We examine whether it is feasible to construct a parsimonious statistical framework that yields accurate "early warnings" of financial crises by conducting an experiment similar to the one Boissay et al. (2015) proposed. We produce a 500,000-observations time-series simulation of the DE, which includes roughly 20,000 crisis events (since the probability of crises is 4 percent). This yields a time-series of the one-step-ahead probability of observing a crisis at t+1 conditional on date t. Then we select a cutoff value such that the model issues a crisis warning when the probability exceeds the cutoff. The criterion for setting the cutoff is that the warnings be statistically accurate, in the sense that Type-1 or Type-2 errors are in the 95 percentile.[32] Type-1 errors occur when the model does not issue a warning at t but a crisis occurs at $t+1$ in the simulated data (i.e. the model failed to predict a crisis). Type-2 errors occur when the model issues a warning at t but a crisis does not occur at $t+1$ (i.e. it wrongly predicted a crisis). At the 95 percentile for Type-1 (Type-2) errors, there should not be more than 5 percent of errors of that type. Table 4 shows that the model produces warnings with this accuracy if the probability cutoffs are set to 1.8 percent for Type-1 errors and 8.6 percent for Type-2 errors.

The above represents the "best" early warning system attainable, in the sense that the warnings are based on the true model's crisis probabilities. Since in practice these probabilities are unobservable, however, they cannot be used directly to build early-warning indicators, as Boissay et al. (2015) noted. Hence, we examine whether logit regressions using the model's fundamentals as explanatory variables can do as well as the model in terms of the fractions of Type-1 and Type-2 errors generated when issuing crisis warnings. In these regressions, the dependent variable is a

[32]Boissay et al. applied the cutoff only to Type-2 errors, but in principle it can be applied to both.

binary variable set to 0 when a crisis does not occur and 1 when it occurs (which is observable), and the independent variables enter in logs and with a one-period lag.

Table 4 shows Type-1 and Types-2 errors in crisis warnings obtained from two logit regressions using the 1.8 and 8.6 percent cutoffs produced by the model. One uses as regressors all of the model's state variables (TFP, interest rates, κ, and the bond position) together with GDP and asset prices, and the other uses only the ratio of total credit (bonds plus working capital) to GDP. Both of these regressions are good early-warning systems, because the fractions of Type-1 and Type-2 errors they produce are similar to the ones produced by the model, although the logit with all the regressors does slightly better. We also estimated alternative regressions with subsets of the regressors used in the first logit model, but they all produced larger fractions of Type-1 and Type-2 errors. This is in line with the finding of Boissay et al. (2015), showing that a logit regression using only the debt-GDP ratio approximates well model-based errors.

Table 6: Sensitivity Analysis

	Macroprudential Debt Tax	Welfare Gains	Crisis Prob. DE	Crisis Prob. SP	Asset Price Drop DE	Asset Price Drop SP	Equity Premium DE	Equity Premium SP
Baseline	3.6	0.30	4.0	0.02	−43.9	−5.4	4.8	0.8
Low WK ($\theta = 0.12$)	3.6	0.21	5.4	0.02	−37.1	−5.6	4.1	1.0
Endogenous R	3.7	0.28	3.88	0.00	−43.1	−6.6	4.8	0.7
Higher Patience	3.1	0.20	2.65	0.02	−51.0	−10.4	4.62	0.54

H Sensitivity

The supply side channel driven by working capital plays an important role for determining the large gains from macroprudential policy. Reducing θ by 25% reduces the welfare gains by about 33 %, i.e. welfare gains fall from 0.3 to 0.2 percentage points of permanent consumption. Hence, we conclude that large gains from macroprudential policy arise due to adverse effects of crises on economic activity, besides the gains from higher consumption smoothing. This links our results to Schmitt-Grohe and Uribe (2013) who show that capital controls achieve substantial gains by reducing average unemployment via reductions in volatility, and hence departing from standard Lucas welfare calculations.

In the second sensitivity experiment we modify the model to relax the assumption of a perfectly elastic supply of funds at an exogenous interest rate R_t. We argued that this is a natural assumption for many of the economies to which we calibrate our model, and is also a convenient assumption theoretically to abstract from redistribution effects. To see the robustness of our results to this assumption, we introduce a real interest rate which varies with aggregate bond holdings. In particular, we assume that the net interest rate is now given by $r(B') = r_t - \varrho(e^{-(B'-\bar{B})} - 1)$, where $\bar{B}$ denotes the average value of bond holdings and set $\varrho = 0.05$.[33] With $\varrho > 0$, the interest rate increases with the debt of the economy. In principle, this could work to attenuate the Fisherian deflation and the pecuniary externality, because of the endogenous self-correcting mechanism increasing the cost of borrowing as debt increases, but we found that quantitatively this did not result in large changes relative to the baseline.[34] This is shown in the last row of Table 6 for a value of $\varrho = 0.05$. With this value of ϱ, the real interest rate reaches a minimum of -1.5 percent in the simulations which is around the minimum value observed in the data between 1980 and 2012.

Finally, we consider an increase in the discount factor from $\beta = 0.95$ to $\beta = 0.96$. This makes agents less willing to borrow and the economy becomes relatively less exposed to crises. As a result, there are lower gains from macroprudential policy. Notice that higher patience also makes

[33] This specification is proposed by Schmitt-Grohe and Uribe (2003) to avoid the problem with the unit root in net foreign assets that arises when using perturbation methods to solve small open economy models. In our model, its purpose is only to approximate what would happen if the interest rate could respond to debt choices in a richer general equilibrium model.

[34] The average macroprudential debt tax is slightly higher because now the planner also internalizes how borrowing affects the interest rate, which is taken as exogenous by individual agents. The welfare gains of moving from the constrained efficient equilibrium with an exogenous interest rate to the decentralized equilibrium with the endogenous interest rate are about the same as in the baseline, which again suggests that the exogeneity of the interest rate does not have significant effects on the quantitative results.

agents relatively more precautionary about future fluctuations, which in principle could make macroprudential policy more desirable. Quantitatively, however, the first effect dominates, as can be seen in Table 6.

I Asset Pricing

We report here more details on the asset pricing implications of the models and the implications of macroprudential policy. Figure I.1 shows plots of six key asset pricing variables as functions of B in the DE and SP economies when TFP and the interest rate take their average values and $\kappa = \kappa^H$ (this is in contrast with Figure 2, which plotted policy functions for a "bad" state with low TFP and κ^L). The variables plotted are the expected return on assets, the price of assets, the Sharpe ratio, the volatility of returns, the risk premium, and the price of risk.

In this Figure DE experiences higher risk premia, return volatilities, risk prices and Sharpe ratios due to the fact that the DE is significantly more risky than the SP economy. Moreover, differences with SP become larger for lower values of B since this implies that it is more likely that the collateral constraint will bind at $t+1$. On the other hand, expected return are higher for SP. The higher risk premia in DE should in principle push asset prices down by reducing excess returns. At equilibrium, however, this effect is more than offset by the first-order effect of the SP's debt tax, which by arbitrage of returns between assets and bonds this tax increases the expected return on assets. In turn, higher excess returns contribute to explain the uniformly lower asset prices of the SP relative to the DE *for all* the domain of B in panel (b), in line with eq. (8).

It is also important to note in Figure I.1 the significant nonlinearities in the asset pricing variables within the DE itself (and keeping in mind we are looking at these variables for realizations of shocks in a "good" state as of date t). In particular, in the region with a positive probability of a crisis at $t + 1$, the Sharpe ratio, return volatility, risk premium and price of risk are steep decreasing functions of B, while they are virtually flat in the stable credit region. Similarly, asset prices are a steeper function of B in the positive crisis probability region than in the stable credit region.

Table 7 reports statistics that characterize the main properties of asset pricing behavior in the DE and SP. The Table shows expected excess returns ($\mathbb{E}_t[R^q_{t+1}]$) in column (1) and its two components, namely the after-tax risk free rate and the equity premium (R^{ep}_t) in columns (2) and (3) respectively. Using eq. (9), R^{ep}_t is decomposed into the two components that result from the effect of collateral constraints that bind at t (column (4)) or are expected to bind at $t+1$ (column (5)), and the standard risk premium component given by the covariance between the stochastic discount factor and asset returns (column (6)). The equity premium in column (3) is equal to the sum of these three components. In addition, the Table reports the market price of risk (column (7)), the log standard deviation of returns (column (8)) and the Sharpe ratio (column (9)). All of these statistics are reported for the unconditional long-run distributions of each economy as well as for distributions conditional on the collateral constraint being binding and not binding.

The unconditional equity premium is significantly higher in the DE than in the SP by about 400 basis points (4.8 v. 0.8 percent).[35] This difference is due to both higher risk premium (1.5 vs. 0.2 percent) and higher liquidity premium (4.7 vs. 1.8 percent). Moreover, the difference in equity premium can be attributed to both higher volatility (14.6 vs 5.2) and higher price of risk (14.6 vs 5.2). Higher price of risk in DE reflects the fact that consumption, and therefore the pricing kernel, fluctuate significantly more in the former than in the latter. Moreover, financial crises episodes introduce higher volatility in asset prices and asset returns. Finally, the unconditional Sharpe ratio of the DE is 0.5 v. 0.2 in the SP, implying that risk-taking is "overcompensated" in the competitive equilibrium relative to the compensation it receives under the optimal policy. The

[35] the sizable equity premium in the DE contrasts sharply with existing findings (e.g. Heaton and Lucas (1996)) showing that credit frictions without the Fisherian deflation mechanism do not produce large premia.

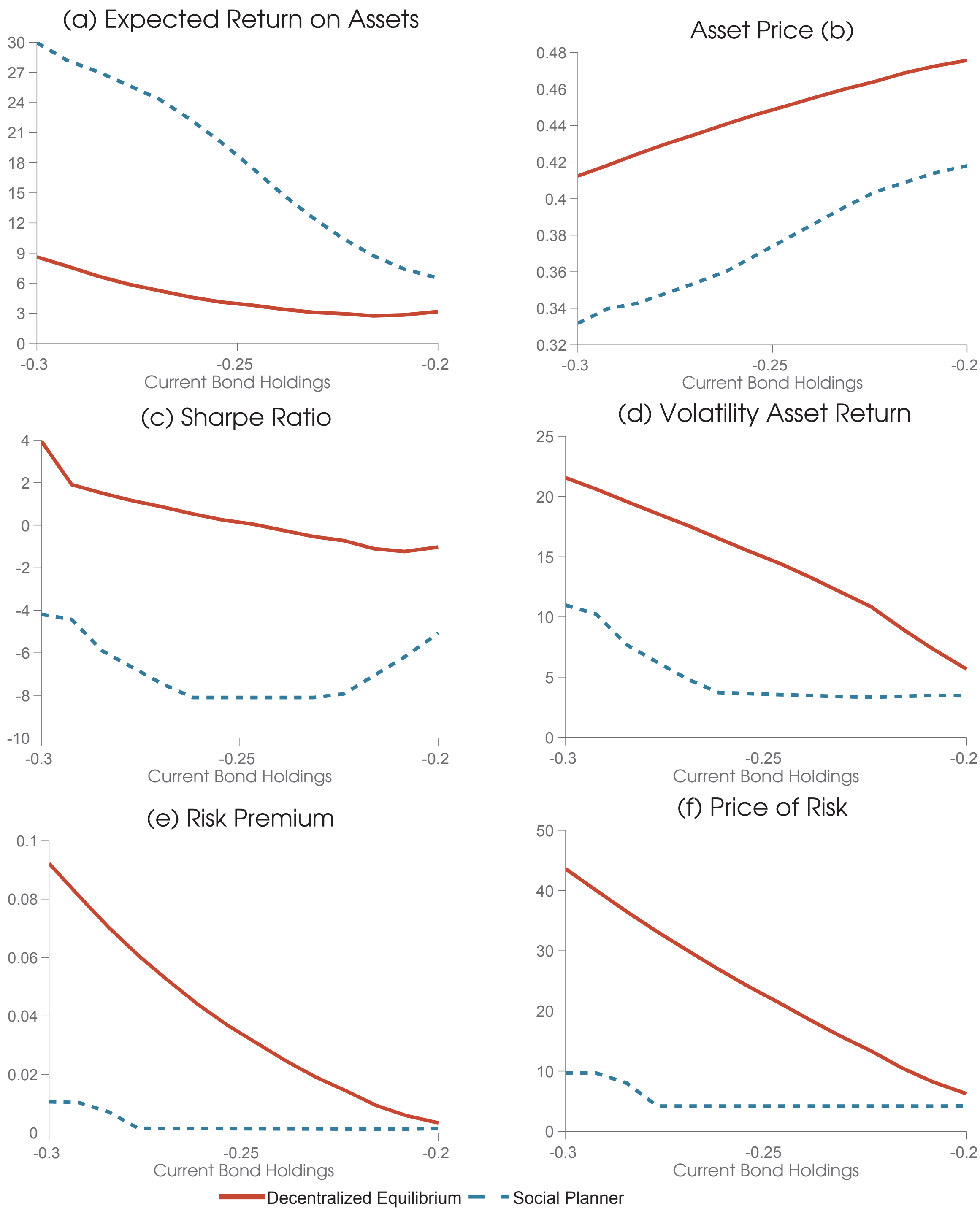


Figure I.1: Asset Pricing Variables in "Good" States of Nature

differences in asset pricing statistics between DE and SP apply to constrained and unconstrained region, as Table 7 shows.

Table 7: Asset Pricing Moments

	(1) Expected Return	(2) Risk-free Plus Tax	(3) Equity Premium	(4) Liquidity Premium	(5) Collateral Effect	(6) Risk Premium	(7) Price of Risk	(8) $\sigma_t(R^q_{t+1})$	(9) SR_t
			Decentralized Equilibrium						
Unconditional	6.0	1.2	4.8	4.7	1.4	1.5	14.6	9.1	0.5
Constrained	85.6	1.2	84.4	84.1	0.0	0.2	4.1	6.2	13.7
Unconstrained	1.3	1.2	0.1	0.0	1.5	1.6	15.3	9.3	0.0
			Social Planner						
Unconditional	4.1	3.3	0.8	1.8	1.2	0.2	5.2	3.8	0.2
Constrained	6.9	-21.8	28.7	28.5	0.0	0.2	5.1	3.8	7.6
Unconstrained	3.9	5.0	-1.2	0.0	1.3	0.2	5.2	3.8	-0.3

Note: The Sharpe ratio for each row is computed as the average of all corresponding equilibrium realizations of excess returns divided by the standard deviation of excess returns. All figures except the Sharpe ratios are in percent.

J Computational Algorithm

J.1 Numerical Solution Method for Decentralized Equilibrium

Following Bianchi (2011), we use Coleman (1990)'s time iteration algorithm, modified to address the occasionally binding endogenous constraint, that operates directly on the first-order conditions. Formally, the computation of the competitive equilibrium requires solving for functions $\{\mathcal{B}(b,s), \mathcal{Q}(b,s), \mathcal{C}(b,s), \nu(b,s), \mathcal{H}(b,s), \mu(b,s)\}$ such that:

$$\mathcal{C}(b,s) + \frac{\mathcal{B}(b,s)}{R} = zF(1, \mathcal{H}(b,s), \nu(b,s)) + b - p_v \nu(b,s) \tag{J.1}$$

$$-\frac{\mathcal{B}(b,s)}{R} + \theta p_\nu \nu(b,s) \leq \kappa \mathcal{Q}(b,s) \tag{J.2}$$

$$u'(\mathcal{C}(b,s) - G'(\mathcal{H}(b,s))) = \beta R \mathbb{E}_{s'|s}\left[u'(\mathcal{C}(\mathcal{B}(b,s), s') - G'(\mathcal{H}(\mathcal{B}(b,s), s))\right] + \mu(b,s) \tag{J.3}$$

$$\begin{aligned} zF_n(1, \mathcal{H}(b,s), \nu(b,s)) &= G'(\mathcal{H}(b,s)) && \text{(J.4)} \\ zF_\nu(1, \mathcal{H}(b,s), \nu(b,s)) &= p_\nu(1 + \theta\mu(b,s)/u'(\mathcal{C}(b,s))) && \text{(J.5)} \end{aligned}$$

$$\begin{aligned} qu'(c - G(h)) = {} & \beta \mathbb{E}_{s'|s}\{u'(\mathcal{C}(b', s') - G'(\mathcal{H}(b,s)))(\mathcal{Q}(b',s') + z'F_k(1, \mathcal{H}(b',s'), \nu(b',s'))) && \text{(J.6)} \\ & + \kappa'\mu(b',s')\mathcal{Q}(b',s')\} \end{aligned}$$

The algorithm follow these steps:

1. Generate a discrete grid for the economy's bond position $G_b = \{b_1, b_2, ...b_M\}$ and the shock state space $G_s = \{s_1, s_2, ...s_N\}$ and choose an interpolation scheme for evaluating the functions outside the grid of bonds. We use 300 points in the grid for bonds and interpolate the functions using a piecewise linear approximation.

2. Conjecture $\mathcal{B}_k(b,s), \mathcal{Q}_k(b,s), \mathcal{C}_k(b,s), \mathcal{H}_k(b,s), \nu_k(b,s), \mu_k(b,s)$ at time $K, \forall b \in G_b$ and $\forall s \in G_s$

3. Set $j = 1$

4. Solve for the values of $\mathcal{B}_{k-j}(b,s), \mathcal{Q}_{k-j}(b,s), \mathcal{C}_{k-j}(b,s), (b,s), \mu_{k-j}(b,s)$ at time $k-j$ using (J.1)-(J.7) and $\mathcal{B}_{k-j+1}(b,s), \mathcal{Q}_{k-j+1}(b,s), \mathcal{C}_{k-j+1}(b,s)$ $\mathcal{H}_{k-j+1}(b,s), \mu_{k-j+1}(b,s) \forall\ b \in G_b$ and $\forall\ s \in G_s$:

   (a) Assume collateral constraint (J.2) is not binding. Set $\mu_{k-j}(b,s) = 0$ and solve for $\mathcal{H}_{k-j}(b,s)$ and ν using (J.4) and (J.5). Solve for $\mathcal{B}_{k-j}(b,s)$ and $\mathcal{C}_{k-j}(b,s)$ using (J.1) and(J.3) and a root finding algorithm.

   (b) Check whether $-\frac{\mathcal{B}_{k-j}(b,s)}{R} + \theta p_\nu \nu_{k-j}(b,s) \leq \kappa \mathcal{Q}_{k-j+1}(b,s)$ holds. Notice that this step uses the asset price from the previous iteration to determine whether the collateral constraint is binding. Of course, once policies and prices converge, this becomes innocuous. The advantage from this formulation is that it avoids solving through iterations for an additional market clearing price, which may or may not be unique. We conducted several robustness checks in this dimension like starting from a different initial guess for the equilibrium. Moreover, it would be straightforward to alter our method by solving for possibly multiple Q that satisfy $-\frac{\mathcal{B}_{k-j}(b,s)}{R} + \theta p_{\nu}\nu_{k-j}(b,s) = \kappa \mathcal{Q}_{k-j+1}$ with a certain equilibrium selection, e.g. the one that maximizes the utility of the representative agent.

(c) If constraint is satisfied, move to next grid point.

(d) Otherwise, solve for $\mu(b,s), \nu_{k-j}(b,s), \mathcal{H}_{k-j}(b,s), \mathcal{B}_{k-j}(b,s)$ using (J.2, (J.3) and(J.4) with equality.

(e) Solve for $\mathcal{Q}_{k-j}(b,s)$ using (J.7)

5. Evaluate convergence. If $sup_{B,s}\|x_{k-j}(b,s) - x_{k-j+1}(b,s)\| < \epsilon$ for $x = \mathcal{B}, \mathcal{C}, \mathcal{Q}, \mu, \mathcal{H}$ we have found the competitive equilibrium. Otherwise, set $x_{k-j}(b,s) = x_{k-j+1}(b,s)$ and $j \rightsquigarrow j+1$ and go to step 4.

J.2 Numerical Solution Method for Constrained-Efficient Equilibrium

From a methodological standpoint, the solution method we developed is related to the literature using Markov perfect equilibria to solve for optimal time-consistent policy. In particular, we extended the methods proposed in Klein et al. (2008) and Klein et al. (2007) to models with an occasionally binding collateral constraint. The algorithm we propose uses a nested fixed point algorithm. Given future policies, we solve for policy functions and value functions using value function iteration as an inner loop. In the outer loop, we update future policies given the solution to the Bellman equation. The algorithm follows these steps:

1. Generate a discrete grid for the economy's bond position $G_b = \{b_{1,} b_2, ...b_M\}$ and the shock state space $G_s = \{s_{1,} s_2, ...s_N\}$ and choose an interpolation scheme for evaluating the functions outside the grid of bonds. We use 300 points in the grid for bonds and interpolate the functions using a piecewise linear approximation.

2. Guess policy functions $\mathcal{B}, \mathcal{Q}, \mathcal{C}, \nu, \mu$ at time K $\forall$ $b \in G_b$ and $\forall$ $z \in G_z$. We use as initial policies the policies of the decentralized equilibrium, and we check that we obtain the same equilibrium when starting from alternative policies.

3. For given $\mathcal{C}, \mathcal{Q}, \mathcal{H}, \nu, \mu$ solve for the value function and policy functions :

$$
\begin{aligned}
V(b', s') &= \max_{c, b', \mu, h, \nu} u(c - G(h)) + \beta \mathbb{E}_{s'|s} V(b', s') && \text{(J.7)} \\
c + \frac{b'}{R} &= b + zF(k, h, \nu) - p_\nu \nu && \text{(J.8)} \\
zF_h(k, h, \nu) &= G'(h) && \text{(J.9)} \\
zF_\nu(k, h, \nu) &= p_\nu \left(1 + \frac{\theta \mu}{u'(c - G(h))}\right) && \text{(J.10)} \\
\mu \left(\frac{b'}{R} - \theta p_\nu \nu + \kappa q\right) &= 0 && \text{(J.11)} \\
\frac{b'}{R} - \theta p_\nu \nu &\geq -\kappa q && \text{(J.12)}
\end{aligned}
$$

$$
\begin{aligned}
qu'(c - G(h)) &= \beta \mathbb{E}_{s'|s} \{u'(\mathcal{C}(b', s') - G'(\mathcal{H}(b, s)))(\mathcal{Q}(b', s') + z' F_k(1, \mathcal{H}(b', s'), \nu(b', s')) \\
&\quad + \kappa' \mu(b', s') \mathcal{Q}(b', s')\} && \text{(J.13)}
\end{aligned}
$$

   This recursive problem is solved using value function iteration. The value functions and policy functions are approximated using linear interpolation whenever the bond position is not in the grid. To solve the optimal choices in each state, we first assume the collateral constraint is not binding and use a Newton type algorithm to solve the optimization problem. If the collateral constraint is binding, we solve for every b', the combinations of c, h, ν, q, μ that satisfy these 6 conditions (J.8)-(J.13), with (J.13) holding with equality.

4. Denote by $\sigma^i, i = c, q, h, \nu, \mu$ the policy functions that solve the recursive problem in step (3) Compute the sup distance between $\mathcal{B}, \mathcal{Q}, \mathcal{C}, \nu, \mu$ and $\sigma^i, i = c, q, h, \nu, \mu$. If the sup distance is higher than 1.0e-6, update $\mathcal{B}, \mathcal{Q}, \mathcal{C}, \nu, \mu$ and solve again the recursive problem.

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