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TAIL RISK AND ASSET PRICES

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ABSTRACT

We propose a new measure of time-varying tail risk that is directly estimable from the cross section of returns. We exploit firm-level price crashes every month to identify common fluctuations in tail risk across stocks. Our tail measure is significantly correlated with tail risk measures extracted from S&P 500 index options, but is available for a longer sample since it is calculated from equity data. We show that tail risk has strong predictive power for aggregate market returns: A one standard deviation increase in tail risk forecasts an increase in excess market returns of 4.5% over the following year. Cross-sectionally, stocks with high loadings on past tail risk earn an annual three-factor alpha 5.4% higher than stocks with low tail risk loadings. These findings are consistent with asset pricing theories that relate equity risk premia to rare disasters or other forms of tail risk.

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1 Introduction

Recent models of time-varying disasters in output or consumption offer a theoretical solution to a range of asset pricing puzzles. They show that the mere potential for infrequent events of extreme magnitude can have important effects on economic activity and asset prices.

Since at least Mandelbrot (1963) and Fama (1963) a separate literature has developed arguing that unconditional return distributions are heavy-tailed and aptly described by a power law. More recent empirical work suggests that the return tail distribution varies over time.[1] We show that empirical studies of fat-tailed stock return behavior and theoretical models of tail risk in the "real" economy are closely linked.

Our primary goal is to investigate the effects of time-varying extreme event risk in asset markets. The chief obstacle to this investigation is a viable measure of tail risk over time. Ideally, one would directly construct a measure of aggregate tail risk dynamics from the time series of, say, market returns or GDP growth rates, in analogy to dynamic volatility estimated from a GARCH model. But dynamic tail risk estimates are infeasible in a univariate time series model due to the infrequent nature of extreme events.

To overcome this problem, we devise a panel estimation approach that captures common variation in the tail risks of individual firms. If firm-level tail distributions possess similar dynamics, then the cross section of crash events for individual firms can be used to identify the common component of tail risk at each point in time.

Our empirical framework centers on a reduced form description for the tail distribution of returns. The time t lower tail distribution is defined as the set of return events falling below some extreme negative threshold u_t. We assume that the lower tail of asset return i

[1] A seminal paper documenting variation in the power law tail of returns is Quintos, Fan and Phillips (2001), with additional evidence presented in Galbraith and Zernov (2004), Werner and Upper (2004), and Wagner (2003).

behaves according to

$$P(R_{i,t+1} < r \mid R_{i,t+1} < u_t \text{ and } \mathcal{F}_t) = \left(\frac{r}{u_t}\right)^{-a_i/\lambda_t}, \tag{1}$$

where $r < u_t < 0$. Equation (1) states that extreme return events obey a power law. The key parameter of the model, a_i/λ_t, determines the shape of the tail and is referred to as the tail exponent. Because $r < u_t < 0$, $r/u_t > 1$. This implies that $a_i/\lambda_t > 0$ to ensure that the probability $(r/u_t)^{-a_i/\lambda_t}$ always lies between zero and one. High values of λ_t correspond to "fat" tails and high probabilities of extreme returns.[2]

In contrast to past power law research, Equation (1) is a model of the *conditional* return tail. The $1/\lambda_t$ term in the exponent may vary with the conditioning information set $\mathcal{F}_t$. Although different assets can have different levels of tail risk (determined by the constant a_i), dynamics are the same for all assets because they are driven by the common process λ_t. Thus we refer to λ_t as "tail risk," and we refer to the tail structure in (1) as a dynamic power law.

We build a tail risk measure from the dynamic power law structure (1). The identifying assumption is that tail risks of individual assets share similar dynamics. Therefore, in a sufficiently large cross section, enough stocks will experience individual tail events each period to provide accurate information about the prevailing level of tail risk. Applying Hill's (1975) power law estimator to the time t cross section recovers an estimate of λ_t.[3]

We find that the time-varying tail exponent is highly persistent. We estimate λ_t separately each month, so there is no mechanical persistence in this series, yet we find a monthly AR(1) coefficient of 0.927. Thus, λ_t has strong predictive power for future extreme returns of individual stocks, offering a first indication that λ_t is a potentially important determinant

[2] A convenient heuristic for the tail fatness of a power law is the following. The m^{th} moment of a power law variable diverges if $m \geq a_i/\lambda_t$.

[3] This allows us to isolate common fluctuations in individual firms' tails over time. This procedure avoids having to accumulate years of tail observations from the aggregate series in order to estimate tail risk, and therefore avoids using stale observations that carry little information about current tail risk.

of asset prices. We also find a high degree of comovement among the tail risks of disjoint sets of firms, supporting our assumption of common firm-level tail dynamics. For example, when we estimate separate tail risk series for each industry, we find time series correlations in their tail risks ranging from 57% to 87%.

We find strong predictive power of tail risk for market portfolio returns and individual stock returns. First, we test the hypothesis that tail risk forecasts aggregate stock market returns. Predictive regressions show that a one standard deviation increase in tail risk forecasts an increase in annualized excess market returns of 4.5%, 4.0%, 3.7% and 3.2% at the one month, one year, three year and five year horizons, respectively. These are all statistically significant with t-statistics of 2.1, 2.0, 2.4 and 2.7, based on Hodrick's (1992) standard error correction. These results are robust out-of-sample, achieving a 4.5% R^2 at the annual frequency, compared to 6.1% in-sample. The forecasting power of tail risk is also robust to controlling for a broad set of alternative predictors, outperforming the dividend-price ratio and other common predictors surveyed by Goyal and Welch (2008).

The tail exponent also has substantial predictive power for the cross section of average returns. We run predictive regressions for each stock, then sort stocks based on their predictive tail risk exposures. Stocks in the highest quintile earn annual value-weighted three-factor alphas 5.4% higher than stocks in the lowest quintile over the subsequent year. This tail risk premium is robust to controlling for other priced factors and characteristics, including momentum (Carhart (1997)), liquidity (Pastor and Stambaugh (2003)), individual stock volatility (Ang, Hodrick, Xing and Zhang (2006)) and downside beta (Ang, Chen and Xing (2006)). We also find a strong association between our tail risk measure and the crash insurance premium on deep out-of-the-money equity put options.

We then investigate the mechanism linking tail risk to equity premia. Model (1) is a description of tail distributions for individual firms. Since discount rates are determined by *aggregate* risk exposure, why might *individual* return tail distributions be tied to equity risk premia? We propose two reasons why aggregate risks (and therefore risk premia) are linked to the common component in firm-level tail risks.

First, power law distributions are stable under aggregation: A sum of idiosyncratic power law shocks inherits the tail behavior of the individual shocks.[4] This implies that firm-level tail distributions are informative about the likelihood of market-wide extremes. Aggregate tail risks, which we expect to have important pricing implications, are thus linked to common dynamics in idiosyncratic tails. Because direct estimation of tail dynamics for univariate series is infeasible, our approach jointly models individual tails to indirectly infer the aggregate tail.

A second link between individual firm risks and aggregate effects arises from the impact of uncertainty shocks on real outcomes. Bloom (2009) argues that, due to capital and labor adjustment costs, an increase in uncertainty raises the value of a firm's "real options," such as the option to postpone investment decisions. In his framework, idiosyncratic uncertainty fluctuates in concert across all firms. Thus a rise in uncertainty depresses aggregate economic activity by inducing all firms to simultaneously reduce investment and hiring. While Bloom focuses on uncertainty in the form of volatility, his rationale also implies that common changes in firm-level tail risk can have important aggregate real effects.[5] Because we find common fluctuations in tail risk across firms, firm-level *tail uncertainty* shocks may be transmitted to aggregate real outcomes, representing a second potential channel through which tail risk impacts equity premia.

We explore both of these mechanisms empirically. Because it is built from individual stock data, it is important to investigate whether our tail estimator also describes the tail risk of the market portfolio. Options data, though only available for the last twenty years,

[4] Gabaix (2009) provides a summary of aggregation properties for variables with power law tails. Power law tails are conserved under addition, multiplication, polynomial transformation, min, and max. Further details and derivations are found in Jessen and Mikosch (2006).

[5] Gourio (2012) presents a theoretical model showing that shocks to aggregate tail risk induce qualitatively similar business fluctuations as the volatility uncertainty studied in Bloom (2009). Our focus is instead on firm-level tail risks.

provide an opportunity to compare our measure to option-implied tail risk for the S&P 500 index. We find that our tail measure has significant correlation of 33% with option-implied kurtosis and −30% with option-implied skewness, suggesting that our measure is closely associated with lower tail risks perceived by option market participants. Furthermore, our tail risk series has significant predictive power for future risk-neutral skewness and kurtosis even after controlling for their own lags. Thus, options data corroborate the power law aggregation property that firm-level tail distributions contain information about the likelihood of aggregate extreme events.

Motivated by the uncertainty shocks argument, we investigate whether there is evidence of time-varying tail risk in firms' fundamentals. We apply our estimation approach to the panel of firm-level sales growth and show that dynamics in stock return tails share a significant correlation of 31% with fluctuations in the tail distribution of cash flows (p-value of 0.008). Furthermore, we find that economic activity is highly sensitive to tail risk shocks. Aggregate investment, output and employment drop significantly following an increase in tail risk. These facts provide a bridge between empirical studies of fat-tailed stock return behavior and theoretical models of tail risk in the "real" economy.

Our research question draws on several literatures. Recently, researchers have hypothesized that heavy-tailed shocks to economic fundamentals help explain certain asset pricing behavior that has proved otherwise difficult to reconcile with traditional macro-finance theory. Examples include the Rietz (1988) and Barro (2006) rare disaster hypothesis and its extensions to dynamic settings by Gabaix (2011), Gourio (2012) and Wachter (2013), as well as extensions of Bansal and Yaron's (2004) long run risks model that incorporate fat-tailed endowment shocks (Eraker and Shaliastovich (2008), Bansal and Shaliastovich (2010, 2011), and Drechsler and Yaron (2011)).[6] Model calibrations show that this class of models matches a number of focal asset pricing moments. Ours is the first paper to directly

[6]These long run risks extensions build on a large literature that models extreme events with jump processes, most notably the widely used affine class of Duffie, Pan and Singleton (2000).

document time-varying tail risk in fundamentals. We also provide direct estimates of the association between tail risk and risk premia (as opposed to model calibrations). There are two key equity premium implications from this class of models, and we find that tail risk significantly relates return data in the manner predicted. First, tail risk positively forecasts excess returns. Because investors are tail risk averse, increases in tail risk raise the return required by investors to hold the market, thereby inducing a positive predictive relationship between tail risk and future returns. The second implication applies to the cross section of expected returns. High tail risk is associated with bad states of the world and high marginal utility. Hence, assets that hedge tail risk are more valuable (have lower expected returns) than those that are adversely exposed to tail risk.

There are two extant approaches to measuring tail risk dynamics for stock returns: One based on option price data and another on high frequency return data. Examples of the option-based approach include Bakshi, Kapadia and Madan (2003) who study risk-neutral skewness and kurtosis, Bollerslev, Tauchen and Zhou (2009) who examine how the variance risk premium relates to the equity premium, and Backus, Chernov and Martin (2012) who infer disaster risk premia from index options. Tail estimation from high-frequency data is exemplified by Bollerslev and Todorov (2012). These approaches are powerful but subject to data limitations (a sample horizon of at most 20 years). Also, they are not generalizable to direct estimation of cash flow tails. In contrast, our tail risk series is estimated using returns and sales growth data since 1963, and may be used in any setting where a large cross section is available.[7]

[7] The cross section procedure that we propose has subsequently been adopted as a measure of systemic banking sector risk by Allen, Bali, and Tang (2011).

2 Empirical Methodology

2.1 The Tail Distribution of Returns

We posit that returns obey the dynamic power law structure in Equation (1). An extensive literature in finance, statistics and physics has thoroughly documented power law tail behavior of equity returns.[8] Evidence suggests that the key parameter of this power law may vary over time (Quintos, Fan and Phillips (2001)). We propose a novel specification for equity returns in which the tail distribution obeys a potentially time-varying power law. Modeling dynamic tail risk is challenging because observations that are informative about tails occur rarely by definition. To overcome this challenge, our approach relies on commonality in the tail risks of individual assets, which in turn exploits the comparatively rich information about tail risk in the cross section of returns. We allow for a different level of firm-specific tail risk across assets, but assume that tail risk fluctuations for all assets are governed by a single process. This structure implies that firms have different unconditional tail risks, but their tail risk dynamics are similar (we provide evidence below that supports this assumption). As described in Kelly (2011), this mechanism is convenient for modeling common tail risk variation even when the true tails possess some additional idiosyncratic dynamics.

Conditional upon exceeding some extreme lower "tail threshold," u_t, and given information $\mathcal{F}_t$, we assume that an asset's return obeys the tail probability distribution

$$P(R_{i,t+1} < r \mid R_{i,t+1} < u_t, \mathcal{F}_t) = \left(\frac{r}{u_t}\right)^{-a_i/\lambda_t},$$

where $r < u_t < 0$.[9]

[8] See, for example, Mandelbrot (1963), Fama (1963, 1965), Officer (1972), Blattberg and Gonedes (1974), Akgiray and Booth (1988), Hols and de Vries (1991), Jansen and de Vries (1991), Kearns and Pagan (1997), Gopikrishnan et al. (1999), and Gabaix et al. (2006).

[9] This specification is motivated by the Pickands-Balkema-de Haan limit theorem, which states that for a wide class of heavy-tailed distributions for $R_{i,t+1}$, $P(R_{i,t+1} < r \mid R_{i,t+1} < u_t)$ will converge to a generalized power law distribution as u_t approaches the support boundary of $R_{i,t+1}$. To operationalize this limit result, we follow the extreme value statistics literature and treat the power law specification as an exact relationship.

The tail distribution's shape is governed by the power law exponent. As a_i/λ_t falls, the tail of the return distribution becomes fatter. The threshold parameter u_t is chosen by the econometrician and defines where the center of the distribution ends and the tail begins. It represents a suitably extreme quantile of the return distribution such that any observations below this cutoff are well described by the specified tail distribution. In practice, we fix the threshold at the 5^{th} percentile of the cross section distribution period-by-period, following standard practice in the extreme value literature. As a result, the threshold varies as the cross section distribution fans out and compresses over time, which mitigates undue effects of volatility on tail risk estimates. We discuss volatility considerations further in Appendix A.

The common time-varying component of return tails, λ_t, may be a general function of time t information. Kelly (2011) specifies λ_t as an autoregressive process updated by recent extreme return observations, and develops the properties of maximum likelihood estimation under this assumption. For purposes of the asset pricing tests presented in this paper, we use a simpler and more transparent estimation approach that produces the same qualitative (and nearly identical quantitative) results as the more sophisticated estimator. In particular, we estimate the tail exponent month-by-month by applying Hill's (1975) power law estimator to the set of daily return observations for all stocks in month t. Applied to the pooled cross section each month, it takes the form[10]

$$\lambda_t^{Hill} = \frac{1}{K_t} \sum_{k=1}^{K_t} \ln \frac{R_{k,t}}{u_t}$$

where $R_{k,t}$ is the k^{th} daily return that falls below u_t during month t and K_t is the total number of such exceedences within month t.[11] The extreme value approach constructs Hill's

[10] For simplicity, the Hill formula is written as though the cross-sectional u-exceedences are the first K_t elements of R_t. This is without loss of generality because the elements of R_t are exchangeable from the perspective of the estimator.

[11] We work with arithmetic returns, but the estimator may also be applied if R is a log return. At the daily frequency, this distinction is trivial because even extreme returns are typically small enough magnitude

measure using only those observations that exceed the tail threshold (observations such that $R_{i,t}/u_t > 1$, referred to as "u-exceedences") and discards non-exceedences. To understand why this is a sensible estimate of the exponent, first note that non-exceedences are part of the non-tail domain, thus they need *not* obey a power law and are appropriately omitted from tail estimates. Next, because u-exceedences obey a power law with exponent a_i/λ_t, *log* exceedences are exponentially distributed with scale parameter a_i/λ_t. By the properties of an exponential random variable, $E_{t-1}[\ln(R_{i,t}/u_t)] = \lambda_t/a_i$. When all stocks have the same ex ante probability of experiencing a threshold exceedence, the expected value of λ_t^{Hill} becomes the cross-sectional harmonic average tail exponent:[12]

$$E_{t-1}\left[\frac{1}{K_t}\sum_{k=1}^{K_t}\ln\frac{R_{k,t}}{u_t} \mid \lambda_t, R_{k,t} < u_t\right] = \lambda_t\frac{1}{\bar{a}}, \quad \text{where} \quad \frac{1}{\bar{a}} \equiv \frac{1}{n}\sum_{i=1}^{n}\frac{1}{a_i}. \tag{2}$$

Equation (2) states that, in expectation, the Hill estimator is equal to the true common tail risk component λ_t times a constant multiplicative bias term. Thus, expected value of period-by-period Hill estimates is perfectly correlated with λ_t.

2.2 Other Empirical Considerations

A potential empirical concern is contamination of tail estimates due to dependence arising, for example, from a common factor structure in returns. This can be mitigated by first removing common return factors, then estimating the tail process from return residuals. We implement this strategy by removing common return factors with Fama and French (1993)

that the approximation $\ln(1+x) \approx x$ is highly accurate. We find nearly identical quantitative results with log returns.

[12] In Appendix A we consider the case in which different stocks have different ex ante probabilities of experiencing threshold exceedence. The left hand side of Equation (2) is an average over the entire pooled cross section due to the fact that the identities of the K_t exceedences are unknown at time $t-1$. Although the identities of the exceedences are unknown, the number of exceedences *is* known because the tail is defined by a fixed fraction of the pool size (the most extreme 5% of observations that month). In different periods, different stocks will experience tail realizations, which will affect period-by-period tail measurement due to heterogeneity in the set of a_i coefficients entering the tail calculation over time. However, the conditional expectation of the Hill measure is unaffected by this heterogeneity because *ex ante* it is unknown which stocks will be in the tail.

three-factor model regressions and then estimating tail risk from the residuals.[13]

Next, because the tail threshold varies over time, common time-variation in volatility is largely taken into account in the construction of our tail estimates. This mitigates the potential contamination of the tail risk time series by volatility dynamics. The threshold u_t is selected as a fixed $q\%$ quantile of the cross section,

$$\hat{u}_t(q) = \inf_i \left\{ R_{(i),t} \in R_t : \frac{q}{100} \leq \frac{(i)}{n} \right\}$$

where (i) denotes the i^{th} order statistic of the $(n \times 1)$ vector R_t. Thus, the threshold expands and contracts with volatility so that a fixed fraction of the most extreme observations is used for estimation each period, helping to nullify the effect of volatility dynamics on tail estimates. Our estimates use $q = 5$.[14]

In Appendix A we discuss additional potentially confounding issues that can arise when estimating tail risk. We show via simulation that Hill estimates appear consistent amid common forms of dependence and heterogeneity known to exist in return data, including factor structures and cross-sectional differences in volatilities and tail exponents. The simulations corroborate theoretical results from the extreme value literature (see Hill (2010)).

2.3 Hypotheses

Our hypothesis is that investors' marginal utility (and hence the stochastic discount factor) is increasing in tail risk and that tail risk is persistent. These hypotheses have two testable

[13] These results are very similar to tail estimates based on raw returns.

[14] Threshold choice can have important effects on results. An inappropriately mild threshold will contaminate tail exponent estimates by using data from the center of the distribution, whose behavior can vary markedly from tail data. A very extreme threshold can result in noisy estimates resulting from too few data points. Although sophisticated methods for threshold selection have been developed (Dupuis (1999) and Matthys and Beirlant (2000), among others), these often require estimation of additional parameters. In light of this fact, Gabaix et al. (2006) advocate a simple rule that fixes the u-exceedence probability at 5% for unconditional power law estimation. We follow these authors by applying a similar simple rule in the dynamic setting. Unreported estimates suggest that ranging q between 1 and 5 produces similar empirical results.

asset pricing implications. The first applies to the equity premium time series. Because investors are averse to tail risk, a positive tail risk shock increases the return required by investors to hold any tail risky portfolio, including the market portfolio. Tail risk persistence is a necessary condition for time series effects because investors will only dynamically adjust their portfolio positions (or, equivalently, their discount rates) in response to shocks that are informative about future levels of risk. Empirically, we test whether tail risk positively forecasts market returns.

Second, assets that hedge tail risk will command a relatively high price and earn low expected returns, whereas assets that are particularly susceptible to tail risk shocks will be more heavily discounted and earn higher expected returns. This implication may be tested in the cross section by comparing average returns of stocks to their estimated tail risk sensitivities.

Marginal utility and discount rates are determined by *aggregate* risk exposure. The key question is therefore how our estimated tail risk series, which describes tail distributions for individual firms, is tied to aggregate risk. A variety of models can potentially generate the hypothesized association between tail risk and risk premia. Rather than specifying a detailed model of preferences and fundamentals, we discuss two general mechanisms that give rise to asset pricing effects of tail risk. We then provide a simple example that illustrates both of these mechanisms.

A first link comes from the fact that power law distributions are stable under aggregation. A sum of idiosyncratic power law shocks inherits the tail behavior of the individual shocks. If the summands have different power law exponents, the heaviest-tailed summand determines the tail of the sum. Jessen and Mikosch (2006) show that this so-called "inheritance mechanism," employed by Gabaix (2006, 2009) among others, is quite general and also applies to weighted sums, products, order statistics and in some cases even infinite sums of power law variables. These aggregation properties offer an approach to inferring aggregate tail risk by understanding the common tail behavior of the individuals that comprise the aggregate. It implies that the tail distribution of shocks to the market return share similar dynamics to tails of firm-level shocks.

The real business cycle literature suggests a second channel by which shifts in idiosyncratic risk impact investors' marginal utility and therefore asset prices. Bloom (2009) argues that an increase in uncertainty raises the value of a firm's "real options." Because firms face capital and labor adjustment costs, higher uncertainty makes the option to postpone investment more valuable. This can produce aggregate effects if uncertainty at the firm-level tends to rise and fall in unison across firms. Bloom (2009) focuses on uncertainty in the form of volatility, and Bloom et al. (2012) provide evidence that firm-level volatility tends to rise for many firms during economic downturns, depressing aggregate investment, hiring and output. If investors are unable to smooth consumption across waves of high idiosyncratic uncertainty and falling output, idiosyncratic risk can impact investors' marginal utility via the uncertainty shock channel.

An additional implication of the uncertainty shock channel is that tail risk should be associated not only with equity premia, but also with aggregate economic activity. We test this implication by estimating the response of macroeconomic activity such as output, investment, and employment to a shock to tail risk (while controlling for other potential sources of uncertainty shocks as in Bloom (2009)).

2.3.1 Example

To bolster the intuition behind these hypotheses, we consider a highly stylized example economy. It emphasizes the roles of power law aggregation and uncertainty shocks to illustrate how idiosyncratic tail risk can have effects on risk premia and aggregate economic activity.

There are N ex ante identical firms with capital endowment K that have access to two production technologies. The first is a risky constant returns to scale technology that yields output A_i per unit of investment. Investment in the risky technology, denoted I, incurs a standard quadratic adjustment cost, $0.5(I/K)^2 K$. The firm also has a risk-free storage technology with return $1 - \delta$. All output is consumed at the end of the period, and at the start of the period the firm maximizes its value.

The first key feature of this economy is that all production shocks A_i obey a power law and are completely idiosyncratic (*i.i.d.*). In particular,

$$P(A_i < a) = a^{1/\lambda}, \text{ with } \lambda \in (0,1) \text{ and } A_i \in [0,1]. \tag{3}$$

A_i is a multiplicative productivity shock and is therefore bounded below by zero. This distribution embeds precisely the same slow probability decay for extreme downside events as a standard power law with infinite support, except that in this case extreme events are those approaching zero (when invested capital is wiped out).

A representative agent's consumption growth depends on the aggregation of firm-level shocks, $N^{-1} \sum_i A_i$. With standard preferences, power law aggregation implies that the stochastic discount factor shock inherits A_i's power law for low output realizations. Instead of modeling consumer preferences, we directly specify the discount factor as $M = \bar{A}^{-1}$. We assume that $\bar{A}$ follows the same power law as A_i in order to mimic the economy's aggregation of firm-level shocks, while the functional form of M is motivated by log utility.[15] We assume (conditional on knowing the level of tail risk) that $\bar{A}$ is independent of each A_i, which emphasizes the pricing effects of tail risk even when firms' shocks are purely idiosyncratic.

The distribution in (3) implies that

$$E[A_i|\lambda] = \frac{1}{1+\lambda} \quad \text{and} \quad E[M|\lambda] = \frac{1}{1-\lambda}.$$

[15] We follow Berk et al. (1999) and Zhang (2005) in our use of an analytically tractable discount factor that is exogenously specified yet economically motivated. This allows us to obtain closed form pricing expressions since the precise distribution of $\sum_i A_i$ is not generally known when A_i is a power law. While we cannot exactly characterize the distribution of the sum, our specification is motivated by the fact that the lower tail of the sum is approximated by a power law with the same exponent as A_i.

The second key feature of this economy is uncertainty about the distribution of the tail parameter. This is the tail analogue of Bloom's (2009) volatility uncertainty model.[16] We assume the tail parameter takes one of two values λC_H or λC_L with equal probability, where C_H and C_L are constants that satisfy $1 > C_H > C_L > 0$. The baseline tail risk value, λ, is known. This structure for tail risk uncertainty is meant to resemble persistence in tail risk. As λ increases, the high and low possible tail risk values both increase.[17]

Consider the return on risky investment excluding adjustment costs, defined as $R_i = A_i I / E[M A_i I]$. The associated risk premium $E[R_i]/R_f$, which captures how steeply investors discount the future output shock under the risk-neutral measure relative to its objective expectation, may be written as

$$\frac{E[M]E[A_i]}{E[MA_i]} = \frac{1}{2}\left[1 + \frac{2 - 2\lambda^2 C_L C_H}{2 - \lambda^2(C_L^2 + C_H^2)}\right]. \tag{4}$$

This equation highlights the role of investors' uncertainty about future tail risk. If the tail distribution is perfectly known, then $C_L = C_H$ and the risk premium is simply one. If there is *any* uncertainty about the tail distribution, then the risk premium rises above one.[18]

We can also see how changes in the baseline level of tail risk λ impact the equity premium:

$$\frac{\partial}{\partial \lambda}\left(\frac{E[M]E[A_i]}{E[MA_i]}\right) = \frac{2\lambda(C_H - C_L)^2}{(2 - \lambda^2(C_H - C_L)^2)^2} > 0$$

This captures the intuition behind return predicability on the basis of tail risk: When tail risk is high, future expected returns are also high. Again, the key to this result is that investors have some ex ante uncertainty about tail risk. Panel A of Figure 1 plots the equity premium

[16]It also shares similar logic as the production-based rare disaster economy of Gourio (2012), who argues that shocks to the probability of a disaster produce business cycle effects. Our setting differs in that we are relying on purely idiosyncratic shocks to generate pricing and production effects, but similar in our focus on extreme event risk.

[17]We require $\lambda \in (0, 1)$ for productivity shocks to have well-defined first moments. The assumption that $A_i < 1$ is for convenience and easily generalized.

[18]We have $C_L^2 + C_H^2 > 2C_L C_H$ since $(C_H - C_L)^2 > 0$.

for the firm's total return as a function of λ.[19] It is straightforward to extend this setting to incorporate heterogeneity in tail risk across firms, for example in the form of firm-level tails being described by λ/a_i. This has the intuitive implication that firms with higher tail risk have higher sensitivity to tail risk uncertainty, producing cross sectional differences in expected returns (and aligning with Equation (1)).

In this economy, a rise in tail risk also impacts investment. The standard solution to the firm's problem is $1-\delta+\frac{I}{K}=\frac{E[MA_i]}{E[M]}$. The expression for investment implies that investment is decreasing in tail risk.[20] Panel B of Figure 1 plots this association at various parameter values.

This highly stylized example is meant to capture the economic effects of heavy-tailed shocks. Tail risk can impact a firm's equity risk premium and investment even when firm shocks are purely idiosyncratic. For this to be the case, two conditions must be met. First, aggregate and idiosyncratic tail risks must have similar dynamics. We expect this to be the case by the properties of power law aggregation as long as firm-level tails risks commove (we document this commovement in Section 3). Second, investors must possess some uncertainty about future tails, which introduces higher-order dependence between the SDF and idiosyncratic shock and generates a risk premium. If λ is persistent so that information about today's tail distribution is informative about future tails, then λ will predict future returns with a positive sign. Value-maximizing behavior of managers leads to an impact of

[19] The equity premium corresponding to the total return incorporates not only the return on risky investment but also adjustment costs, depreciation of stored capital, and the ex ante value of stored capital. Its behavior is qualitatively the same as the risky investment risk premium though with more complicated expressions.

[20] The risk free storage technology is important for this result since investors have precautionary savings motives. Without the risk-free technology, investors are forced to invest more in the risky technology to meet their demand for precautionary savings. The solution for investment per unit of capital is

$$\frac{I}{K}=\frac{1/(2(C_H^2\lambda^2-1))+1/(2(C_L^2\lambda^2-1))}{1/(2(C_H\lambda-1))+1/(2(C_L\lambda-1))}+\delta-1$$

and its derivative with respect to tail risk is

$$\frac{\partial I/K}{\partial\lambda}=-\frac{(C_H+C_L)(C_H^3C_L\lambda^4+C_H^2\lambda^2+C_HC_L^3\lambda^4-6C_HC_L\lambda^2+C_L^2\lambda^2+2)}{(C_H\lambda+1)^2(C_L\lambda+1)^2(C_H\lambda+C_L\lambda-2)^2}<0.$$

tail risk uncertainty on investment decisions. Common tail fluctuations in the cross section imply that firms' investment will rise and fall in unison, leading to aggregate fluctuations in investment, hiring and output.

3 Empirical Results

3.1 Tail Risk Estimates

We estimate the dynamic power law exponent using daily CRSP data from January 1963 to December 2010 for NYSE/AMEX/NASDAQ stocks with share codes 10 and 11. Large data sets are crucial to the accuracy of extreme value estimates since only a small fraction of data are informative about the tail distribution. Because our approach to estimating the dynamic power law exponent relies on the cross section of returns, we require a large panel of stocks in order to gather sufficient information about the tail at each point in time. The number of stocks in CRSP varies dramatically over time.[21] We focus on the 1963–2010 sample due to the cross section expansion of CRSP beginning in August 1962.[22] To further increase the sample size and reduce sampling noise we estimate the tail exponent monthly, pooling all daily observations within the month.

Figure 2 plots the estimated tail risk series alongside the market return over the subsequent three-year period (both series scaled for comparison). The tail risk series appears countercyclical. Our sample begins just after a 28% drop in the aggregate US stock market during the first half of 1962. This major market decline was the first in the post-war era. Estimated tail risk is high at this starting point, but begins to decline steadily until December 1968, when it reaches its lowest level in the sample. This tail risk minimum corresponds to

[21] The period-wise Hill approach to the dynamic power law in Section 2 naturally accommodates changes in cross section size over time.

[22] The sample begins with just under 500 stocks in 1926 and has fewer than 1,000 stocks for the next 25 years. In July 1962, the sample size roughly doubles to nearly 2,000 stocks with the addition of AMEX, then in December 1972 NASDAQ stocks enter the sample raising the stock count above 5,000.

a late 1960's bull market peak, the level of which is not reached again until the mid-1970's. Tail risk rises throughout the 1970's, accelerating its ascent during the oil crisis. It fluctuates above its mean for several years. Tail risk recedes in the four bull market years leading up to 1987, rising quickly in the months following the October crash. During the technology boom, tail risk retreats sharply but briefly, rising to its highest post-2000 level amid the early 2003 market trough. At this time the value-weighted index was down 49% from its 2000 high and NASDAQ was 78% off its peak. During the last half of the decade, tail risk hovers close to its mean, and is roughly flat through the 2007-2009 financial crisis and recession. The absence of an increase in measured tail risk during the recent financial crisis may be surprising *prima facie*, but is potentially consistent with the account of the recent financial crisis in Brownlees, Engle and Kelly (2011). They argue that the financial crisis was characterized by soaring volatility, but that this volatility was predictable over short horizons using standard volatility forecasting models and that volatility-adjusted residuals do not appear extreme compared to their historical distribution. This argument is also consistent with Figure 3, which plots the cross section tail threshold series $\hat{u}_t$ (in absolute value) alongside monthly realized volatility of the CRSP value-weighted index. The lower tail threshold has a 60% correlation with market volatility. During the crisis period, the threshold, which measures the dispersion of the cross section distribution, spikes drastically along with market volatility. A fixed percentile is used to define the tail region for exactly this reason. If volatility rises dramatically but the shape of return tails is unchanged, then a widening of the threshold will absorb the effect of volatility changes and leave estimates of the tail exponent unaffected.

The tail series is highly persistent, possessing a monthly AR(1) coefficient of 0.927. Because the Hill measure is estimated month-by-month with non-overlapping data, this autocorrelation is strong evidence that the severity of extreme returns is highly predictable.[23]

[23] Tail risk estimates are inherently noisy. The AR(1) coefficient is thus likely to be downward biased due to the fact that estimation noise presumably mean reverts more quickly than the true tail process. This also helps explain significant return predictability at multi-year horizons despite mean reversion in the measured tail series.

That is, a high tail risk estimate in month t significantly forecasts relatively severe tail risk in stock returns in month $t+1$. The estimated persistence in tail risk is on par with that of equity volatility. Because tail shocks are persistent, they have the potential to weigh significantly on equilibrium prices.

Our hypotheses rely on a close association between tail risk dynamics estimated from individual stock returns and tail risk of the aggregate market portfolio. Validating this association is a challenge because the latter is difficult (if not infeasible) to estimate from the time series of market returns alone, which is the original motivation for our panel-based estimator. S&P 500 index options present one potential way to measure aggregate market tail risk directly, albeit for a comparatively short sample (only beginning in 1996) and under the risk neutral rather than physical measure.

In Table 1, we compare our tail risk estimates to various options-based measures of tail risk during the 15 year subsample in which options are available. First, we compare against risk-neutral skewness and kurtosis estimated from S&P 500 index options, following Bakshi, Kapadia and Madan (2003).[24] We find correlations of -30% and 33%, respectively, indicating that when tail risk rises the risk-neutral market return distribution also becomes more negatively skewed and more leptokurtic (p-value of 0.02 and 0.01, respectively, based on Newey-West (1987) standard errors with twelve lags).[25]

Next, we compare our tail risk time series to the slope of the implied volatility smirk for out-of-the-money S&P 500 put options. We estimate the slope in a regression of OTM put-implied volatility on option moneyness (strike over spot) using options with Black-Scholes delta greater than -0.5 and one month to maturity. A more negative slope of the smirk

[24]We only use options with positive open interest when calculating risk neutral skewness and kurtosis and the smirk slope. Each of these measures is estimated separately for two sets of options with maturities closest to 30 days (one set for the maturity just greater than 30 days, and one set for the maturity just less than 30 days), then the estimates are linearly interpolated to arrive at a measure with constant 30-day maturity.

[25]We also find that our tail risk series forecasts risk-neutral skewness and kurtosis one month ahead after controlling for lagged skewness and kurtosis. Forecast coefficients on tail risk are significant with p-values of 0.06 and 0.04, respectively.

means that OTM puts are especially expensive relative to ATM puts and indicates that investors are willing to pay more to insure against downside market risk. Tail risk has a significant correlation of -17% with the slope of the smirk indicating that OTM puts become especially expensive when tail risk is high (though this estimate is insignificant with Newey-West p-value of 0.15). Finally, we compare tail risk against the CBOE put/call ratio (Pan and Poteshman (2006)). This ratio measures the number of new put contracts purchased by non-market makers relative to new calls purchases, which depends in part on crash risk perceived by investors. We find a correlation of 42% (p-value of 0.01) between tail risk and the put/call ratio, indicating that high tail risk is associated with above average purchases of puts.[26]

Collectively, the strong correlation between our tail risk series and a range of S&P 500 option-based tail risk proxies suggest that our measure is closely associated with aggregate market crash risk perceived by option market participants.

The key feature of our tail specification in Equation (1) is that the tail risk of all assets share a common factor. This is motivated by the empirical fact that dynamic tail risk estimates are highly correlated across firms. We demonstrate this fact by splitting the sample of CRSP stocks into non-overlapping subsets and applying cross-sectional tail risk estimator to each subset.

Because our estimation approach requires a large cross section, we split stocks into reasonably large subsets. First, we group stocks into five industries according to the SIC code classification of Fama and French. Within each industry we calculate the cross section lower tail Hill estimate pooling daily observations within a month, as in our main tail series construction above. Table 2, Panel A shows that industry-level tail risks are highly correlated over time, ranging between 57% and 87%. Panel B conducts the same test but instead

[26]We use daily put/call ratios from 1996 to 2010 are for all option contracts traded on the Chicago Board of Options Exchange and compute monthly averages. Data are available at http://www.cboe.com/data/PutCallRatio.aspx. Put/call ratios for the S&P 500 index are also available, but the series only begin in 2010.

groups stocks into equally-spaced size (market equity) quintiles each month. Time series correlations of size quintile tails range between 38% and 86%. All correlation estimates in Table 2 are highly statistically significant ($p < 0.001$). In summary, dynamic tails estimated from entirely distinct subsets of CRSP data display a high degree of comovement, providing empirical support for the specification in Equation (1).

3.2 Predicting Stock Market Returns

We first test the hypothesis that tail risk forecasts returns of the aggregate market portfolio. Because our tail risk series is persistent, it has the potential to impact returns at both short and long horizons. A preliminary visual inspection of Figure 2 shows that the monthly tail risk series possesses very similar dynamics to the the compounded market return over the subsequent three-year period, highlighting a close correspondence between tail risk and realized future returns.

To investigate this hypothesis we estimate a series of predictive regressions for market returns based on the estimated tail risk series. All regressions are conducted at the monthly frequency, meaning that observations are overlapping for the one, three and five year analyses. We conduct inference using the Hodrick (1992) standard error correction for overlapping data.[27]

The dependent variable is the return on the CRSP value-weighted index at frequencies of one month, one year, three years and five years. To illustrate economic magnitudes, all reported predictive coefficients are scaled to be interpreted as the effect of a one standard deviation increase in the regressor on future annualized returns. Table 3 shows that tail risk

[27] Richardson and Smith (1991), Hodrick (1992) and Boudoukh and Richardson (1993) (among others) have noted the inferential problems concomitant with overlapping horizon predictive regressions. Overlapping return observations induce a moving average structure in prediction errors, distorting the size of tests based on OLS, and even Newey-West (1987), standard errors. Ang and Bekaert (2007) demonstrate in a Monte Carlo study that the standard error correction of Hodrick (1992) provides the most conservative test statistics relative to other commonly employed procedures, maintaining appropriate test size over horizons as long as five years. We also find that Hodrick's correction produces the most conservative results in our analysis.

has large, significant forecasting power over all horizons. A one standard deviation increase in lower tail risk predicts an increase in future excess returns of 4.5%, 4.0%, 3.7% and 3.2% per annum, based on data for one month, one year, three year and five year horizons, respectively. The corresponding Hodrick t-statistics are 2.1, 2.0, 2.4 and 2.7.[28]

Table 3 compares the forecasting power of tail risk with a large set of alternative forecasting variables studied in a survey by Goyal and Welch (2008).[29] Tail risk forecasts returns strongly and consistently over all horizons, with performance comparable to the aggregate dividend-price ratio. The long term bond return strongly predicts one month returns, but its effect dies out at longer horizons. The long term yield is successful at long horizons, but has weak short horizon predictability.

We next run bivariate regressions using lower tail risk alongside each Goyal and Welch variable to assess the robustness of tail risk's return forecasts after controlling for alternative predictors. Table 4 presents these results. Conclusions regarding the predictive ability of tail risk are unaffected by including alternative regressors. For one month forecasts, the tail risk predictive coefficient remains above 4% when combined with each of the Goyal and Welch variables, with a t-statistic above 1.8 in all cases. At longer horizons, the performance of tail risk relative to alternatives becomes stronger. At the five year horizon, the t-statistic is always above 2.2, except when included with the long term yield when it is 1.74. Tail risk, when combined with the dividend-price ratio, achieves impressive levels of predictability, reaching R^2 values of 38% at three years and 54% at five years.

We also investigate the out-of-sample predictive ability of tail risk. Using data only through month t (beginning at $t = 120$ to allow for a sufficiently large initial estimation period), we run univariate predictive regressions of market returns on tail risk. This coefficient is used to forecast the $t + 1$ return. The estimation window is then extended by one

[28] We find that Goyal and Welch (2008) bootstrap standard errors produce even stronger statistical results than those based on the Hodrick correction.

[29] We thank Amit Goyal for providing the data from Goyal and Welch (2008), updated through 2010.

month to obtain a new predictive coefficient, and an out-of-sample forecast of the following month's return is constructed. This procedure is repeated until the full sample has been exhausted. Because coefficients are based only on data through t, this procedure mimics the information set an investor would work with in real-time. Using the forecast errors from this approach, we calculate the out-of-sample R^2 as $1 - \sum_t (r_{m,t+1} - \hat{r}_{m,t+1|t})^2 / \sum_t (r_{m,t+1} - \bar{r}_{m,t})^2$, where $\hat{r}_{m,t+1|t}$ is the out-of-sample forecast of the $t+1$ return based only on data through t, and $\bar{r}_{m,t}$ is the historical average market return through t. A negative R^2 implies that the predictor performs worse than setting forecasts equal to the historical mean. This recursive out-of-sample forecast approach is also performed using each of the alternative predictors considered in the preceding tables.[30] The results from this analysis are reported in Table 5. Tail risk forecasts demonstrate similar predictive success out-of-sample. At the one month, one year, three year and five year horizons, the tail risk out-of-sample R^2 is 0.3%, 4.5%, 15.7% and 20.1%, versus 0.7%, 6.1%, 16.6% and 20.9% in-sample. We conduct tests of out-of-sample predictive power based on Clark and McCracken (2001), which is the benchmark out-of-sample predictive test in the forecasting literature. According to this test, only tail risk and the long term yield demonstrate statistically significant out-of-sample performance at multiple horizons (at the 5% significance level or better).

In summary, predictive regressions suggest that tail risk is positively and significantly related to market discount rates.

3.3 Tail Risk and the Cross Section of Expected Stock Returns

We next test the hypothesis that tail risk helps explain differences in expected returns across stocks, consistent with the priced tail risk hypothesis. If investors are averse to tail risk, stocks with high predictive loadings on tail risk will be discounted more steeply and thus have higher expected returns going forward. On the other hand, stocks with low or negative

[30] Due to the short time series for the variance risk premium, out-of-sample forecasts based on VRP are infeasible and thus omitted.

tail risk loadings serve as effective hedges and therefore will have comparatively higher prices and lower expected returns.

In line with the aggregate predictive analysis above, we estimate tail risk sensitivities of individual stocks with regressions of the form $E_t[r_{i,t+1}] = \mu_i + \beta_i \lambda_t$. Consistent with the intuition from aggregate tail risk predictive regressions, stocks with high values of β_i are those that are most sensitive to tail risk, and thus are deeply discounted when tail risk is high and have high expected returns going forward. On the other hand, stocks with low or negative β_i are good tail risk hedges because, when tail risk rises, their prices rise contemporaneously and their expected future returns fall. Each month, we estimate the tail loading for each stock in regressions that use the most recent 120 months of data.[31] Stocks are then sorted into quintile portfolios based on their estimated tail risk loadings. We track twelve month post-formation value-weighted and equal-weighted quintile portfolio returns, which are reported in Panel A of Table 6. Portfolio returns are truly out-of-sample; there is no overlap between data used for loading estimation and the post-formation performance period.

Stocks in the highest tail risk loading quintile earn value-weighted average annual returns 4.2% higher than stocks in the lowest quintile, with a t-statistic of 2.2 based on Newey-West standard errors using twelve lags. The equal-weighted high minus low tail risk portfolio average return is 4.0% per annum (t=2.5). Average portfolios' returns demonstrate a stable monotonic pattern that is increasing in tail risk.

We next test if the high average return for the long/short tail risk portfolio is robust to considering alternative priced factors. Panel A reports alphas from regressions of portfolio returns on the three Fama-French factors, alphas with respect to the Fama-French-Carhart four factor momentum model, and alphas with respect to the Fama-French-Carhart model plus the Pastor and Stambaugh (2003) traded liquidity factor as a fifth control. Alphas of

[31] This analysis uses all NYSE/AMEX/NASDAQ stocks with CRSP share codes 10 and 11 and at least 36 months out of 120 with non-missing returns. Portfolios are reconstituted each month.

the value-weighted high minus low tail risk portfolio are large and statistically significant for each of these models. For the three-factor model the alpha is 5.4% per annum (t=3.0). On an equal-weighted basis, the high minus low tail risk portfolio alpha is 4.0% for the three-factor model (t=2.9). Portfolio alphas retain the same stable monotonicity that was observed for average portfolio returns.

Panel B reports one-month post-formation returns. These results show that short horizon portfolio returns have the same qualitative behavior as annual returns. The value-weighted three-factor alpha for the high minus low tail risk portfolio is 5.5% annualized (t=2.6), whereas the equal-weighted three-factor alpha is 3.6% annualized (t=2.2).

We also examine the robustness of tail risk's cross section return explanatory power to controlling for other individual stock characteristics that are potentially associated with return tails. We test whether the return spread between high and low tail risk portfolios is robust to controlling for three alternative firm characteristics. The first characteristic we examine is firm size, measured as equity market value at the time of portfolio formation, which may be an important driver of tail risk if smaller firms are particularly susceptible to tail risk shocks. Next, because our tail measure is derived from tail events among individual firms, we explore its association with the idiosyncratic volatility effect of Ang, Hodrick, Xing and Zhang (2006). We measure firm volatility as the standard deviation of daily residuals from the Fama-French three factor model in the month prior to portfolio formation. The results are qualitatively unchanged if we use raw returns rather than factor model residuals or different window lengths to calculate firms' volatility. Lastly, because our tail risk measure captures an asymmetric downside risk, we investigate how tail risk interacts with the downside beta of Ang, Chen and Xing (2006). Downside beta is estimated as the regression coefficient of firm returns on market returns based only on months in which the market return was negative, using the most recent 120 months of data prior to portfolio formation.

Results from independent two-way portfolio sorts are reported in Table 7. We report annual four-factor post-formation alphas. Within each alternative characteristic quartile we calculate the average returns on the high minus low tail risk portfolio and the corresponding Newey-West t-statistic with twelve lags. Results are broadly consistent with findings reported thus far. Value-weighted spreads within size quartiles range are above 3.6% per annum for all but the smallest stocks, are between 2.2% and 5.4% within volatility quartiles, and are between 2.4% and 4.0% within downside beta quartiles.

3.4 Crash Insurance

The preceding analysis shows that stocks with low tail risk exposure have low average returns, consistent with the view that investors value the ability of such stocks to hedge against fluctuations in tail risk. We next examine the relative values of contracts explicitly designed to hedge against tail risk. We form portfolios of individual equity put options on the basis of option moneyness following the approach of Frazzini and Pedersen (2012).[32] Moneyness is defined as absolute value of the Black-Scholes delta of an option, and the five portfolios are deep out-of-the-money (DOTM, $|\Delta| < 0.20$), out-of-the-money (OTM, $0.20 \leq |\Delta| < 0.40$), at-the-money (ATM, $0.40 \leq |\Delta| < 0.60$), in-the-money (ITM, $0.60 \leq |\Delta| < 0.80$) and deep-in-the-money (DITM, $0.80 \leq |\Delta|$). Portfolios are rebalanced corresponding to the monthly expiration schedule for exchange-listed options (the Saturday immediately following the third Friday of the month). Our option sample covers 1996 to 2010.

We compute the return of selling a put with one month to maturity on the first trading day following each expiration date and holding it to the next month's expiration. Each put position is delta-hedged daily. We use the standard put return calculation, incorporating the change in option value, the profit or loss from the delta hedge, and interest on the margin

[32] We use data from OptionMetrics and apply data filters that include dropping all observations for which the bid-ask spread is smaller than the minimum tick size, the bid is zero, open interest is zero, embedded leverage is in the top or bottom 1% of the distribution, or time value is below 5%. The time value filter controls for the American exercise feature as discussed in Frazzini and Pedersen (2012).

account. We recalculate our monthly tail risk measure to correspond to the expiration schedule, so there is no timing overlap between tail risk in month t and option portfolio returns in $t+1$. We then estimate a predictive regression of each portfolio's return on lagged tail risk. Due to the relatively short sample for options data we estimate a single in-sample predictive coefficient for each portfolio.

Panel A of Table 8 reports predictive tail betas and average monthly returns on delta-hedged put option portfolios. An investor that is willing to sell crash protection in the form of DOTM puts earns a massive insurance premium. The average DOTM return is 19.5% per month and falls monotonically with moneyness. The difference between DOTM and DITM short put returns is 16.7% per month (t=3.6), and cannot be accounted for by standard risk factors. The exposure of option portfolios to tail risk is also monotonically decreasing in moneyness. The difference in tail risk coefficients for the DOTM portfolio versus DITM is 7.2 (t=2.4), meaning that a one standard deviation increase in tail risk predicts an increase in the expected return spread (DOTM−DITM) of over 7% in the next month.[33] The far right column reports the correlation between tail beta and portfolio alpha across the five portfolios. There is a 94% correlation between exposures and average portfolio returns.

Equity positions can be levered as much as twenty times using out-of-the-money options. Frazzini and Pedersen (2012) argue that much of the spread in Panel A is due to a premium that financially constrained investors are willing to pay to hold implicitly levered positions. To remove this "embedded leverage" effect from our analysis, we modify weights in our portfolio construction to equalize the embedded leverage of each portfolio. This procedure, described in Karakaya (2013), is a direct extension of the Frazzini-Pedersen portfolio approach that additionally scales positions by the elasticity of an option's price with respect to the underlying price. Since embedded leverage also magnifies risk exposure and expected returns, de-leveraged return magnitudes are more easily compared to our earlier equity port-

[33]In all regressions, tail risk is first standardized to have unit variance for ease of interpreting the estimated coefficients.

folio results.

Panel B reports leverage-adjusted average returns and tail exposures for short put portfolios. The DOTM portfolio returns 1.6% per month, exceeding returns on the DITM portfolio by 1.1% per month (t=2.4). Portfolio returns and tail risk exposures decrease monotonically with moneyness. The difference in DOTM and DITM coefficients corresponds to a predicted increase in the return spread of 0.8% (t=2.4) in the following month for a one standard deviation increase in tail risk. The correlation between tail risk exposures and average returns across portfolios is 81%.

These results suggest that a large portion of the premium for stock market crash insurance is associated with the ability of these contracts to hedge fluctuations in tail risk, and cannot be explained by exposures to standard risk factors or differences in embedded leverage alone.

4 Tail Risk Shocks and the Real Economy

In Section 2.3 we discuss two mechanisms that can give rise to asset pricing effects of tail risk. The first mechanism is the stability of power law distributions under aggregation, which is supported by a high degree of correlation between options-based measures of tail risk and our panel-based estimates.

The second channel derives from the real business cycle literature, which suggests that shifts in idiosyncratic risk can impact aggregate real activity. The discussion and example in Section 2.3 imply that tail risk should manifest itself not only in returns, but also in firms' fundamental growth rate shocks. We check this implication directly by testing for comovement between tail risk in firm-level sales growth and tail risk measured from stock returns. We estimate sales growth tail risk by applying our cross section Hill estimator approach to the panel of quarterly sales growth data from Compustat. To ensure a sufficiently large cross section, we pool all reported sales data that occur within the same calendar quarter and use data beginning in 1975.[34]

Figure 4 reports correlations between stock return tail risk in quarter t, and sales growth tail risk in quarters $t-4$ to $t+4$. Despite the coarseness of quarterly sales data, we still find that fundamental cash flow tail risk shares a significant contemporaneous correlation of 23% with the stock return tails (Newey-West p-value of 0.024). Return tails are most strongly correlated with sales growth tails one quarter ahead (31%, $p = 0.008$), and remains significantly correlated up to three quarters ahead. The notion that return tail risk leads tail risk measured from sales growth is perhaps unsurprising given the comparatively rapid response of market prices to news and the infrequent reporting of accounting data.

To have pricing effects via the uncertainty shocks channel outlined in Section 2.3, tail risk measured from the cross section must ultimately be associated with aggregate real economic outcomes. Bloom (2009) provides a useful framework to gauge the influence of uncertainty on economic activity and shows that the evolution of uncertainty (measured by stock market volatility) has a large influence on industrial production and employment.

We examine the impact of time-varying tail risk on macroeconomic aggregates in a monthly vector autoregression (VAR) that extends Bloom's (2009) econometric model to include tail risk. In our VAR ordering, stock market volatility is first, followed by tail risk, the Federal Funds Rate, log average hourly earnings, the log consumer price index, hours, log employment, and log industrial production. The resulting impulse responses, however, are robust to different orderings of the variables. Since our sample period coincides largely with Bloom (2009), we estimate the VAR using monthly data from July 1963 to June 2008 (as available from Bloom) so that we can quantify the incremental impact of tail risk relative to volatility.

The left-hand plot in Panel A of Figure 5 shows the response of industrial production to

[34] Due to the quarterly nature of sales data, we are forced to use a substantially smaller number of observations to estimate the tail risk series. To increase the number of observation we define the sales growth tail threshold as the 7.5^{th} percentile of the cross section distribution each year. Quarterly stock return tails are calculated as an average of the monthly tail risk series within each calendar quarter.

a one-standard deviation shock to tail risk.[35] It indicates that industrial production displays an immediate decline of 0.6% within one year of the shock, with a subsequent recovery that peaks at two years. For comparison, the right plot in Panel A shows that a volatility shock produces a decline in industrial production of 1.4% with a similar pattern to that of tail risk.[36] These are distinct effects, however, as tail risk and volatility are weakly negatively correlated and included side-by-side in the VAR. Panel B estimates the impulse response for employment. These plots indicate that a shock to tail risk produces the same effect that it does for production, declining in the first year by just over 0.6% then rebounding at around two years.

Investment decisions play a central role in the production-based asset pricing example in Section 2.3. Unlike production and employment, investment is only available quarterly (and thus was omitted from Bloom's (2009) analysis). We estimate a quarterly trivariate VAR that includes stock market volatility, tail risk, and aggregate investment. We measure investment as either quarterly gross private domestic or private nonresidential fixed investment (as in Cochrane (1991, 1996)). Panels C and D indicate that, following a shock to tail risk, investment displays an immediate drop of 2.5% to 4% in the subsequent year, followed with a recovery by year three. The investment impact arising due to a volatility shock is smaller in magnitude (1.5% to 2.5%) than that arising from a tail risk shock. The response of investment to tail risk is larger in magnitude than the response of production or employment but is less precisely estimated (indicated by the relatively wide standard error bands), perhaps due to having one third as many observations as the monthly series.

[35] Because industrial production and employment are only calculated for the manufacturing sector, the VARs in Panels A and B use tail risk estimated from the cross section of manufacturing firms. The results are nearly identical when tails are estimated including non-manufacturing firms.

[36] Volatility produces a comparatively large effect due to our use of the volatility indicator constructed by Bloom (2009). It equals one when the peak of HP detrended volatility is more than 1.65 standard deviations above the mean. A "shock" is defined as a movement of this variable from zero to one, and thus represents an extreme shift in volatility. If instead we use raw stock market volatility in the VAR (to be more closely comparable to the tail risk measure that we use), the effect of a one standard deviation volatility shock is qualitatively similar, but quantitatively much smaller, producing a decline in IP growth of 0.4% after one year, while the effect of a tail risk shock is effectively identical to that reported in Figure 5.

In summary, after controlling for the impact of the volatility shocks as emphasized in the previous literature, we find that a positive shock to tail risk precedes an immediate and prolonged contraction in economic activity in the subsequent year. These effects on the real economy, coupled with the effects of tail risk on expected stock returns, suggest that tail risk plays an important role in the marginal utility of investors and in determining equilibrium asset prices.

5 Conclusion

A measure of extreme event risk is crucial for evaluating modern theoretical asset pricing paradigms. Estimates based on the univariate time series of aggregate market returns are incapable of accurately tracking conditional tail risk. We present a new dynamic tail risk measure that overcomes this difficulty. It uses the cross section of individual stock returns to estimate conditional tail risk at each point in time.

We provide evidence that tail risk has large predictive power for aggregate stock market returns over horizons of one month to five years, performing as well as the most successful alternative predictors considered in the literature. Furthermore, tail risk has substantial explanatory power for the cross section of stock and put option returns. Stocks that are effective tail risk hedges earn annual three-factor alphas that are 5.4% lower than their high tail risk counterparts.

These results can be understood from the perspective of structural models with heavy-tailed firm-level shock distributions that are preserved under aggregation. In this case, common fluctuations in tail risk across firms can lead them to simultaneously disinvest, which impairs aggregate economic activity even when firms' productivity shocks are idiosyncratic. Both power law aggregation and the real effects of uncertainty shocks represent channels through which firm-level tail risk can influence asset prices.

Internet Appendix

A Tail Estimation Amid Heterogeneous and Dependent Data

This subsection briefly addresses certain issues that can arise when estimating tail risk. Recent research in extreme value statistics shows that the Hill (1975) estimator is consistent in the presence of dependent and heterogeneously distributed observations. Implicit in our cross section application of the Hill estimator is an assumption that daily equity returns satisfy the conditions of consistency theorems in Hill (2010), Resnick and Stărică (1995) or Rootzen et al. (1998). Simulations provided in Appendix A support this assumption by showing that Hill estimates appear consistent amid forms of dependence and heterogeneity known to exist in return data, including factor structures and cross-sectional differences in volatilities and tail exponents.

Heterogeneity in individual stock volatilities affects the likelihood that a particular stock will exceed the tail threshold u_t and thus be included in the Hill estimate. To see this, let X be a power law variable such that $P(X < u) = bu^{-\lambda}$. Now consider a volatility rescaled version of this variable, $Y = \sigma X$. The exceedence probability of Y equals $b\left(\frac{u}{\sigma}\right)^{-\lambda}$, different than that of X. When $\sigma > 1$, Y has a higher exceedence probability than X. However, the *shape* of Y's u-exceedence distribution, and hence its power law exponent, is identical to that of X.

A reinterpretation of the estimator that allows for heterogeneous volatilities is easily established. Let each stock have unique u-exceedence probability p_i, and consider the effect of this heterogeneity on the expectation of the tail estimate. In this case, the expectation is no longer the harmonic average tail exponent, but is instead the *exceedence probability-weighted* average exponent,

$$E_{t-1}\left[\frac{1}{K_t}\sum_{k=1}^{K_t}\ln\frac{R_{k,t}}{u_t}\ |\lambda_t, R_{k,t} < u_t\right] = \lambda_t\sum_{i=1}^{n}\frac{\omega_i}{a_i}, \tag{5}$$

where stock i's weight in the average is $\omega_i = p_i/\sum_j p_j$. The entire estimation approach and consistency argument outlined above proceeds identically after establishing this point. The ultimate result is that the fitted λ_t series is no longer an estimate of the equal-weighted average exponent, but becomes a volatility-weighted average due to the effect that volatility has on the probability of exceeding threshold u_t.

In our setup, stocks are also allowed to have different levels of unconditional tail risk arising from heterogeneity in a_i coefficients. Because different subsets of stocks land in the cross section tail each period, differences in the cross-sectional tail shape from one period to the next may arise that are unrelated to fluctuations in λ_t. This sampling randomness introduces measurement noise, and may potentially bias our empirics against finding an effect of tail risk on prices. When the cross section is large, this noise becomes less severe. To ensure that a large number of threshold exceedences are entering the Hill estimate each period, we calculate the tail measure monthly by pooling all stocks' daily returns within that month.

An alternative way to address heterogeneity is to transform the data with a preliminary estimation step. Substantial measurement-related problems arise with this approach. First, at the individual stock level, variance standardization involves dividing returns by an estimate of volatility. Volatility-standardized returns can be extremely sensitive to estimation error in the divisor. For example, if sampling variation produces too low an estimate of volatility, volatility scaling can excessively inflate returns, making them appear as tail observations when they are not. The reverse is also true – volatility estimates can be substantially influenced by tail risk. If a stock experiences a tail event, this mechanically inflates its measured volatility, in which case scaling can over-shrink precisely those observations that are most informative about the tail distribution. Thus both over- and underestimating volatility is a genuine concern that can exacerbate measurement error in tail estimates.

This discussion and the simulations below suggest that avoiding preliminary data transformations may prove a less costly choice than attempting to volatility standardize returns.

A.1 Monte Carlo Analysis: Hill Estimates with Dependent and Heterogeneous Data

Econometric theory (e.g. Hill (2010), Resnick and Stărică (1995) and Rootzen et al. (1998)) proves that Hill's (1975) estimator may be applied to samples of dependent and heterogeneous data, but the conditions under which consistency obtains may be difficult to verify. The goal of this section is to demonstrate that our estimation procedure is accurate amid the forms of dependence and heterogeneity typically present in equity return data.

To show that dependence in returns has a small effect on estimates of the tail exponent, we perform a Monte Carlo experiment as follows. We generate data according to $Y = bX + e$, where Y, b and e are $n \times 1$ vectors and X is a scalar factor. We assume that X and e are Student t distributed with equal degrees of freedom, m. The volatility of e is set equal to 0.03, or nearly 50% per year. Note that a Student t variable with m degrees of freedom has a tail distribution that is approximately power law with exponent equal to $-m$. Simulations are performed under three factor structures. The first, independence, fixes $b = 0$. The second, equi-dependence, gives all elements of Y the same non-zero loading, $b = 1$. Third, we allow for heterogeneous dependence by assuming that for each element $b_i \in b$, $b_i \sim N(1, 1)$. Therefore, some elements will have near-zero or negative correlations with the factor (and with other members of the cross section), while other elements will be highly correlated. Note also that heterogeneity in b introduces heterogeneity into the variance of returns. We vary n and m to inspect how different sample sizes and tail weights affect estimates in the presence of dependence. In all cases we estimate the tail using data in the lowest 5 percent of the simulated return distribution.[37]

Results are shown in Panel A of Table A1. The way to read this table is to compare estimates under the "Equi-Dependence" and "Hetero-Dependence" cases to the "None" case in the same column.[38] Mean estimates under dependence are very close to the independent case. Estimates are hardly affected under equi-dependence, with the largest deviation from independence occurring when degrees of freedom equal two and the sample size is 500. In this case, the independent mean estimate is 2.64, while the equi-dependent mean estimate is 2.68. Deviations are marginally larger under hetero-dependence, with the largest deviation being 4.78 versus the mean independent estimate of 4.66 (when degrees of freedom are four and the sample size is 500).

The dependence concepts used in the two types of dependent simulations are substantial, and result in high average correlations among realizations.[39] Nonetheless, the Hill estimator is little affected, and for intuitive reasons. In the equi-dependent case, it is as though a constant value bX is added to each element of the error vector e. As a result, the distribution of Y is just a horizontally shifted version of e, without further distortion. When the 5% threshold is found, the distribution is re-centered and estimation essentially reverts to the independent setting. On the

[37] Note, the reported values of n are the pre-truncation sample sizes. The actual number of observations used to calculate the tail is approximately $0.05n$.

[38] Exponents are reported in absolute value in order drop minus signs from all estimates in the table.

[39] Correlation between variables Y_1 and Y_2, generated as described above, is equal to $\frac{b_1 b_2}{\sqrt{(b_1^2+1)(b_2^2+1)}}$, or 0.5 when $b_1 = b_2 = 1$.

other hand, when there is variety among loadings, data come closer to having a mixing character, and the consistency theorems mentioned above begin to take effect.

Next we consider the effects of heterogeneous volatility. Heterogeneity in volatility can affect selection of an appropriate threshold or the composition of firms that end up in the tail sample. The next Monte Carlo experiment is designed to examine how λ estimates can be affected by moderate to large differences in volatility across stocks and proceeds as follows. The vector e is again $n \times 1$ with m degrees of freedom, and this is transformed to $Y = b \odot e$, where b is $n \times 1$ and $\odot$ denotes element-wise multiplication. Three levels of volatility heterogeneity are considered: 1) no heterogeneity (fixing $b = 1$), 2) moderate heterogeneity ($b_i \sim U[0.5, 1.5]$), which allows stocks to have up to three times the volatility (nine times the variance) of other stocks in the cross section, and 3) severe heterogeneity ($b_i \sim U[0.1, 1.9]$), in which case the volatility ratio between stocks reaches as high as 19 (variance ratio of 361). Panel B of Table A1 shows that under moderate cross sectional variance dispersion, the Hill estimator is little affected. Even when heterogeneity is extreme, the estimator performs reasonably well. Some deterioration is detected as degrees of freedom increase: as tails become thinner, the estimator becomes more susceptible to confounding effects of cross sectional volatility dispersion. When tails are very fat, the extremity of events dominates differences in volatility among observations. The worst performance occurs when tails are thinnest (degrees of freedom are four) and the sample is smallest ($n = 500$), in which case the mean estimate under severe volatility heterogeneity is 4.33, versus 4.63 for the homogeneous case.

In light of the model in Equation 1, perhaps the most relevant dimension of heterogeneity is that of the tail exponent. The third Monte Carlo explores how tail estimates are affected when data contains observations with heterogeneous tail exponents. An $n \times 1$ vector of Student t variates are generated under three scenarios for cross sectional variation in degrees of freedom, m. These are 1) no heterogeneity (m the same for all observations), 2) moderate heterogeneity ($m \sim N(\bar{m}, 0.25^2)$), and 3) severe heterogeneity ($m \sim N(\bar{m}, 0.50^2)$). Simulation results in Panel C of Table A1 show that the Hill method accurately estimates the average tail exponent. Estimates under moderate heterogeneity are essentially unperturbed relative to the homogeneous case. Amid severe exponent heterogeneity, Hill estimates suffer most when tails are fattest (mean estimate of 2.57 under severe heterogeneity versus the 2.91 under homogeneity when degrees of freedom are three and $n = 500$). Nonetheless, mean estimates under all heterogeneity scenarios remain well within one standard error of the homogeneous estimate.

The conclusion from Table A1 is that the Hill estimator provides robust estimation of the (average) tail exponent under a variety of specifications, including rather extreme heterogeneity and dependence, and corroborates the theoretical results of Hill (2010). The scenarios that produce bias are those that use far fewer observations (only 500) than what we obtain from the CRSP cross section.

Table A1: Simulated Hill Estimates for Dependent and Heterogeneous Data

Panel A: Cross Sectional Dependence					Panel B: Volatility Heterogeneity					Panel C: Tail Exponent Heterogeneity				
d.o.f.	Type	n = 500	1000	5000	d.o.f.	Type	n = 500	1000	5000	d.o.f.	Type	n = 500	1000	5000
2	None	2.64	2.22	2.01	2	None	2.60	2.21	2.00	3	None	3.80	3.19	2.91
		(1.9)	(1.0)	(0.4)			(1.9)	(1.0)	(0.4)			(2.8)	(1.3)	(0.5)
2	Equi-Dep.	2.68	2.24	1.99	2	Moderate	2.66	2.22	2.00	3	Moderate	3.75	3.19	2.89
		(2.4)	(1.1)	(0.4)			(2.0)	(1.0)	(0.4)			(2.7)	(1.4)	(0.5)
2	Hetero-Dep.	2.69	2.23	2.02	2	Severe	2.59	2.21	1.99	3	Severe	3.47	2.89	2.57
		(2.4)	(1.1)	(0.4)			(1.9)	(1.0)	(0.4)			(2.7)	(1.3)	(0.5)
3	None	3.74	3.21	2.91	3	None	3.78	3.19	2.90	4	None	4.68	3.98	3.66
		(2.7)	(1.4)	(0.5)			(2.9)	(1.4)	(0.5)			(3.3)	(1.6)	(0.7)
3	Equi-Dep.	3.73	3.17	2.90	3	Moderate	3.64	3.14	2.88	4	Moderate	4.62	3.97	3.66
		(2.8)	(1.5)	(0.6)			(2.7)	(1.3)	(0.5)			(3.2)	(1.7)	(0.7)
3	Hetero-Dep.	3.81	3.24	2.93	3	Severe	3.59	3.08	2.84	4	Severe	4.46	3.81	3.49
		(3.0)	(1.5)	(0.7)			(2.5)	(1.3)	(0.5)			(3.2)	(1.6)	(0.6)
4	None	4.66	4.00	3.67	4	None	4.63	3.97	3.66	5	None	5.30	4.62	4.27
		(3.4)	(1.7)	(0.7)			(3.2)	(1.6)	(0.7)			(3.8)	(1.9)	(0.7)
4	Equi-Dep.	4.66	3.98	3.64	4	Moderate	4.50	3.85	3.56	5	Moderate	5.34	4.61	4.28
		(3.5)	(1.8)	(0.8)			(3.9)	(1.6)	(0.6)			(3.8)	(1.9)	(0.8)
4	Hetero-Dep.	4.78	4.04	3.72	4	Severe	4.33	3.75	3.48	5	Severe	5.26	4.49	4.17
		(4.4)	(1.8)	(0.9)			(3.1)	(1.5)	(0.6)			(3.9)	(1.8)	(0.7)

Notes: The table reports the mean and standard deviation (in parentheses) of Hill estimates for the tail exponent under various forms dependence and heterogeneity. Panel A shows results under factor structure dependence. Data are generated according to Y=bX+e, where Y, b and e are n×1vectors and X is a scalar factor. We assume that X and e are independent and Student t distributed with equal degrees of freedom. Simulations are performed under three factor structures. The first, independence, fixes b=0. The second, equi-dependence, gives all elements of Y the same loading, b=1. The third allows for heterogeneous dependence by assuming that elements of b are normally distributed with a mean and variance of one. Panel B presents results based on heterogeneous volatility among observations. In this case, data is generated according to Y=b·e, where Y, b and e are n×1 vectors and · denotes element-wise multiplication. We assume that e is Student t distributed, and simulations are performed under three different levels of volatility heterogeneity: 1) no heterogeneity (fixing b=1), 2) moderate heterogeneity (b∼U[0.5,1.5]), and 3) severe heterogeneity (b∼U[0.1,1.9]). Panel C presents results based on heterogeneity among tail exponents. An n×1 vector of Student t variates is generated under three scenarios for cross sectional variation in degrees of freedom. These are 1) no heterogeneity (d.o.f the same for all observations), 2) moderate heterogeneity (d.o.f.i∼N(d.o.f.,0.25)), and 3) severe heterogeneity (d.o.f.i∼N(d.o.f.,0.50)). Simulations are performed considering varying degrees of tail thickness (d.o.f) and cross section size (n). In all cases we estimate the tail index choosing the 2.5% of data in the upper tail of Y (therefore, the number of observations used in calculating this is approximately 0.025n).

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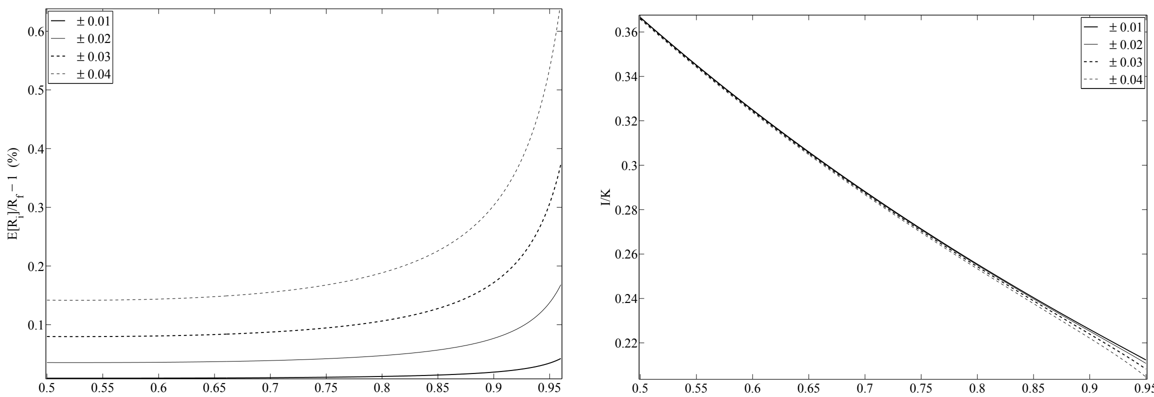
Figure 1: Risk Premium, Investment and Tail Risk

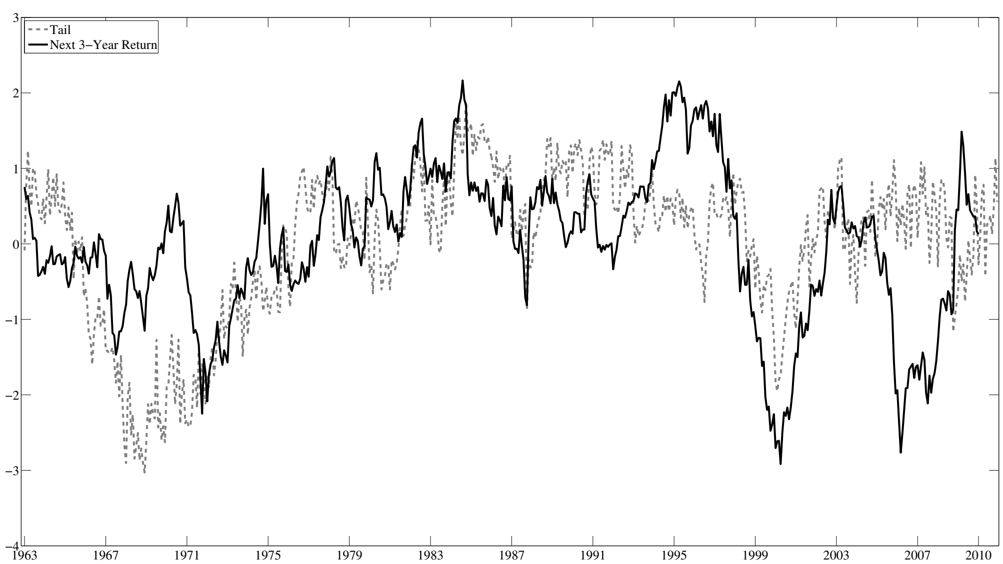
Panel A: Equity Risk Premium

Panel B: Investment



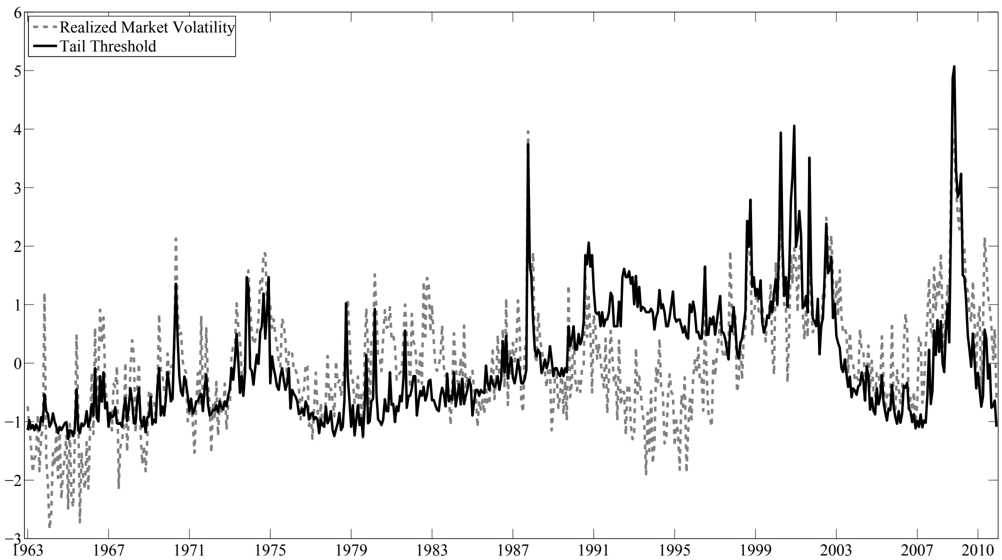
Notes: The figure plots the equilibrium equity risk premium (Panel A) and investment per unit of capital (Panel B) as a function of initial tail risk for the example economy presented in Section 2.3.

Figure 2: Tail Exponent Estimates and Subsequent Market Returns



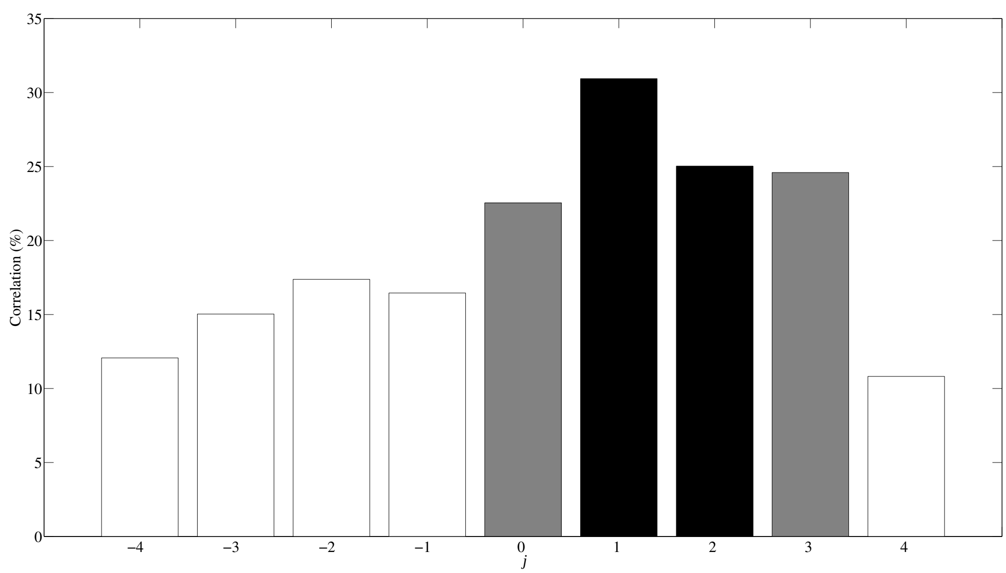
Notes: Plotted is the monthly estimated tail risk time series. Tail estimates are calculated each month by pooling daily returns of NYSE/AMEX/NASDAQ stocks. Also plotted in each month t is the realized market return over the three years following month t. To emphasize comparison, both series have been scaled to have mean zero and variance one.

Figure 3: Tail Threshold and Aggregate Market Volatility



Notes: Plotted is the monthly tail threshold series. The threshold is the absolute value of the fifth percentile of monthly pooled daily returns of NYSE/AMEX/NASDAQ stocks. Also plotted is the (annualized) monthly realized volatility of the CRSP value-weighted index. To emphasize comparison, both series have been scaled to have the mean zero and variance one.

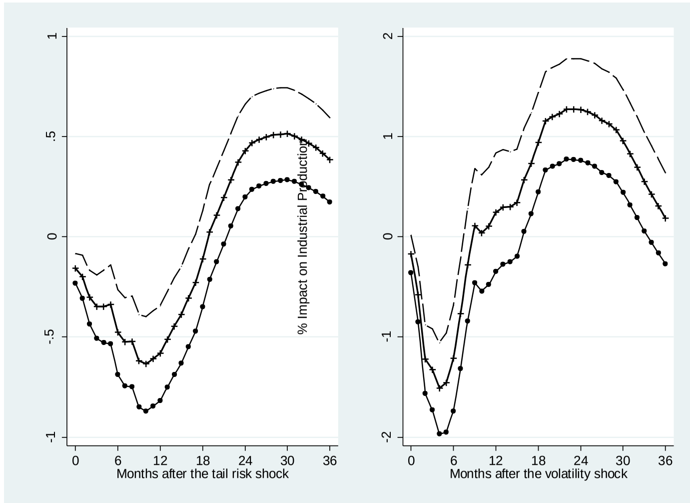
Figure 4: Correlogram: Sales Growth Tails and Stock Return Tails



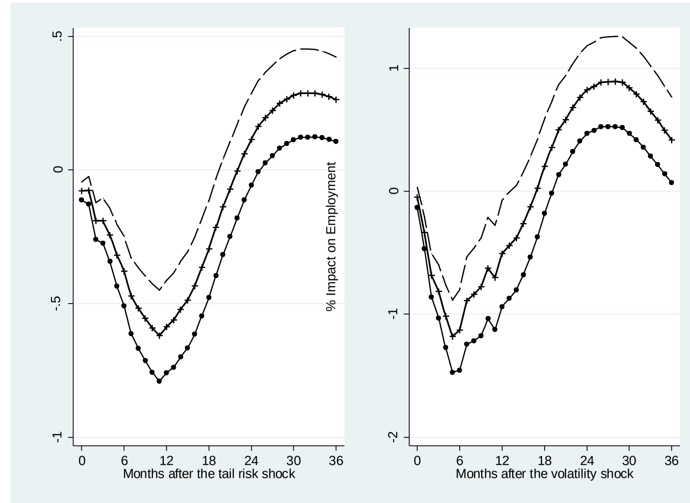
Notes: The figure shows the correlogram of the estimated return tail series in quarter t with the sales growth tail series in quarter $t + j$ for $j = -4, ..., 4$. Correlation estimates are reported in percentages and tests are conducted using Newey-West standard errors with four quarters of lags. Black and gray bars denote significant correlation at the 1% and 2.5% level, respectively.

Figure 5: Tail Risk Impulse Response Functions

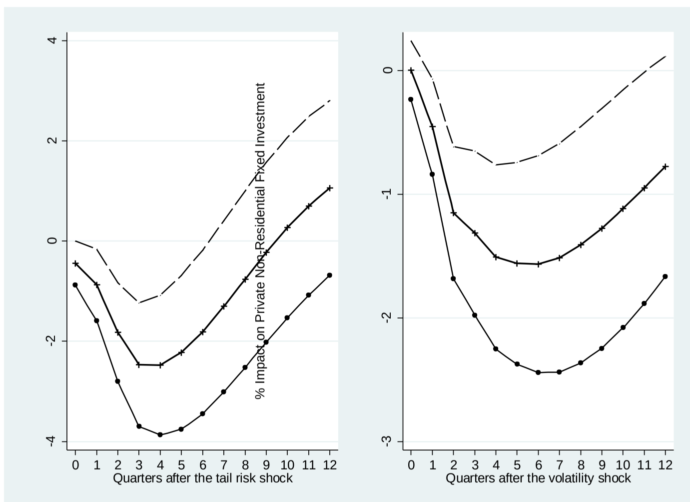
Panel A: Industrial Production



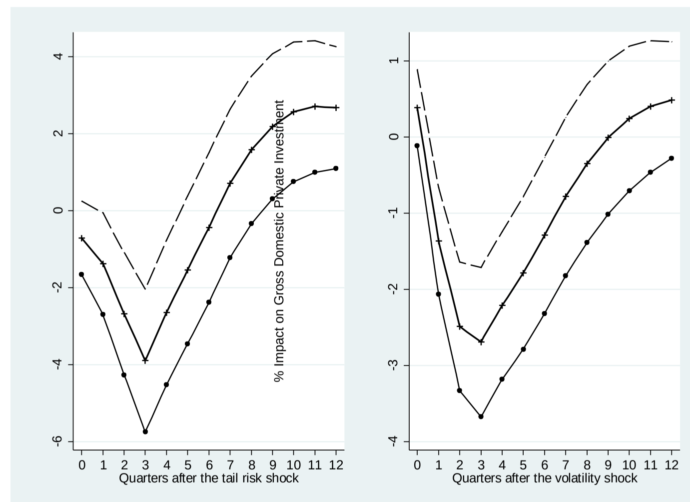
Panel B: Employment



Panel C: Gross Private Domestic Investment



Panel D: Private Nonresidential Fixed Investment



Notes: The figure plots the estimated impact of uncertainty shocks on industrial production (Panel A), employment (Panel B), gross private domestic investments (Panel C) and private nonresidential fixed investments (Panel D). Within each panel, the impulse response for a one standard deviation shock to tail risk on the left and for a one standard deviation shock to volatility on the right. For industrial production and employment we estimate a monthly VAR that includes stock market volatility, tail risk, Federal Funds Rate, log average hourly earnings, the log consumer price index, hours, log employment, and log industrial production over the period July 1963 to June 2008. Because investment is only available quarterly, Panels C and D are for quarterly trivariate VARs that includes stock market volatility, tail risk, and aggregate investment over the period 1963 Q3 to 2008 Q2. Because industrial production and employment are only calculated for the manufacturing sector, the VARs in Panels A and B use tail risk estimated from the cross section of manufacturing firms. Dashed lines are 1 standard-error bands.

Table 1: Correlations With Risk Measures Based on S&P 500 Index Options

		(1)	(2)	(3)	(4)	(5)	(6)
Tail	(1)	1.00					
		-					
R.N. Skewness	(2)	-0.30	1.00				
		0.02	-				
R.N. Kurtosis	(3)	0.33	-0.92	1.00			
		0.01	*<0.01*	-			
Put/Call Ratio	(4)	0.42	-0.49	0.38	1.00		
		0.01	*<0.01*	*0.01*	-		
OTM Put IV Slope	(5)	-0.17	0.22	-0.25	0.00	1.00	
		0.15	*0.06*	*0.05*	*0.97*	-	
Variance Risk Premium	(6)	0.04	0.10	-0.17	-0.22	-0.10	1.00
		0.67	*0.13*	*0.02*	*0.03*	*0.41*	-

Notes: The table reports monthly correlations between tail risk estimated from the cross section of returns on NYSE/AMEX/NASDAQ stocks, and various options-based risk measures derived from prices of S&P 500 index options from 1996 to 2010. Below each correlation estimate we report its p-value in italics (based on Newey-West (1987) standard errors with 12 lags). Risk-neutral skewness and kurtosis are estimated following Bakshi, Kapadia and Madan (2003). The slope of the implied volatility smirk for out-of-the-money S&P 500 put options is estimated from a regression of put-implied volatility on option moneyness (strike over spot) using options with Black-Scholes delta greater than -0.5 and one month to maturity. The put/call ratio measures the number of new put contracts purchased by non-market makers relative to new calls purchases and comes from the CBOE. The variance risk premium is the difference between the squared VIX and realized variance of the S&P 500 index and comes from Hao Zhou's website. Risk neutral moments and the IV slope only use options with positive open interest. These three measures are estimated separately for two sets of options with maturities closest to 30 days (one set for the maturity just greater than 30 days, and one set for the maturity just less than 30 days), then the estimates are linearly interpolated to arrive at a daily measure with constant 30-day maturity. Finally, all daily measures are averaged within the month to arrive at a monthly time series.

Table 2: Correlation of Dynamic Tail Risk Estimates Among Subgroups

Panel A: By Industry		(1)	(2)	(3)	(4)	(5)
Consumer	(1)	1.00				
Manufacturing	(2)	0.85	1.00			
Technology	(3)	0.81	0.75	1.00		
Healthcare	(4)	0.67	0.58	0.69	1.00	
Other	(5)	0.86	0.87	0.77	0.57	1.00

Panel B: By Size		(1)	(2)	(3)	(4)	(5)
Small	(1)	1.00				
	(2)	0.71	1.00			
	(3)	0.59	0.77	1.00		
	(4)	0.47	0.66	0.76	1.00	
Big	(5)	0.38	0.56	0.67	0.86	1.00

Notes: The table reports time series correlation between monthly tail risk series estimated from the cross section of stocks in each of the five Fama-French industry SIC classifications (Panel A) and in each size quintile (Panel B) for 1963 to 2010.

Table 3: MARKET RETURN PREDICTABILITY: UNIVARIATE PREDICTOR PERFORMANCE

	One month horizon			One year horizon			Three year horizon			Five year horizon		
	Coeff.	*t-stat.*	R^2	Coeff.	*t-stat.*	R^2	Coeff.	*t-stat.*	R^2	Coeff.	*t-stat.*	R^2
Tail	4.54	*2.08*	0.7	4.02	*2.04*	6.1	3.65	*2.40*	16.6	3.16	*2.65*	20.9
Book-to-market	2.49	*1.14*	0.2	3.12	*1.34*	3.7	2.26	*1.12*	6.3	2.76	*1.82*	15.5
Default return spread	2.96	*1.36*	0.3	0.43	*0.57*	0.1	0.28	*1.22*	0.1	0.02	*0.18*	0.0
Default yield spread	2.82	*1.29*	0.3	2.93	*1.63*	3.2	1.90	*1.19*	4.5	3.04	*2.80*	14.8
Dividend payout ratio	0.79	*0.36*	0.0	1.55	*0.90*	0.9	1.90	*1.38*	4.4	3.55	*3.68*	10.4
Dividend price ratio	4.24	*1.94*	0.7	4.75	*2.07*	8.5	4.34	*2.56*	23.1	4.19	*3.60*	36.5
Earnings price ratio	3.23	*1.48*	0.4	3.16	*1.48*	3.8	2.54	*1.65*	8.0	3.75	*2.90*	21.5
Inflation	-5.07	*-2.33*	0.9	-1.67	*-1.09*	1.1	0.40	*0.40*	0.2	0.81	*0.71*	1.2
Long term return	5.40	*2.48*	1.1	1.83	*3.04*	1.3	0.56	*2.16*	0.4	0.68	*2.69*	0.9
Long term yield	1.95	*0.89*	0.1	3.72	*1.70*	5.2	4.26	*3.48*	21.9	4.59	*5.17*	40.5
Net equity expansion	-0.71	*-0.33*	0.0	-0.03	*-0.01*	0.0	0.44	*0.27*	0.2	-0.14	*-0.13*	0.0
Stock volatility	-6.24	*-2.87*	1.4	0.61	*0.50*	0.1	0.07	*0.13*	0.0	0.01	*0.03*	0.0
Term Spread	2.28	*1.04*	0.2	2.57	*1.35*	2.5	2.53	*1.69*	7.7	2.18	*1.72*	9.1
Treasury bill rate	0.44	*0.20*	0.0	1.78	*0.80*	1.2	2.33	*1.48*	6.3	3.00	*2.56*	15.6
Var. Risk Prem.*	11.22	*3.45*	4.5	3.32	*2.18*	3.5	0.63	*0.33*	0.3	-1.24	*-1.90*	1.1
R.N. Skewness*	-0.74	*-0.17*	0.0	-1.13	*-0.34*	0.3	-0.35	*-0.18*	0.1	-0.32	*-0.37*	0.4
R.N. Kurtosis*	-1.82	*-0.43*	0.1	1.56	*0.53*	0.6	1.16	*0.48*	1.0	0.56	*0.59*	1.1

Notes: The table reports results from monthly predictive regressions of CRSP value-weighted market index returns over one month, one year, three year and five year horizons. The first row reports forecasting results based on our estimated tail risk time series. Next are results from predictors studied in the survey of Goyal and Welch (2008) (data from Amit Goyal's website), as well as the variance risk premium (Bollerslev, Tauchen and Zhou (2009), data from Hao Zhou's website) and risk neutral skewness and kurtosis based on S&P 500 index options. (*) denotes that a variable is available for a truncated sample: The variance risk premium is only available beginning in 1990 and risk neutral moments are only available beginning in 1996. Because overlapping monthly observations are used, test statistics are calculated using Hodrick's (1992) standard error correction for overlapping data with lag length equal to the number of months in each horizon. For comparison, reported predictive coefficients are scaled to be interpreted as the percentage change in expected annual market returns resulting from a one standard deviation increase in each predictor variable.

Table 4: Market Return Predictability: Bivariate Predictor Performance

	One month horizon					One year horizon				
	Tail Coeff.	*Tail t-stat.*	Coeff.	*t-stat.*	R^2	Tail Coeff.	*Tail t-stat.*	Coeff.	*t-stat.*	R^2
Book-to-market	4.68	*2.15*	2.73	*1.25*	1.0	4.19	*2.27*	3.34	*1.49*	10.3
Default return spread	4.57	*2.10*	3.01	*1.38*	1.1	4.02	*2.05*	0.46	*0.61*	6.2
Default yield spread	4.32	*1.97*	2.42	*1.10*	1.0	3.78	*2.01*	2.58	*1.51*	8.6
Dividend payout ratio	4.67	*2.13*	1.26	*0.58*	0.8	4.22	*2.19*	1.98	*1.14*	7.6
Dividend price ratio	4.32	*1.98*	4.00	*1.84*	1.3	3.77	*2.07*	4.54	*2.03*	13.8
Earnings price ratio	4.22	*1.92*	2.73	*1.25*	1.0	3.70	*1.92*	2.72	*1.31*	8.9
Inflation	4.12	*1.89*	-4.70	*-2.16*	1.5	3.90	*2.01*	-1.32	*-0.88*	6.7
Long term return	4.04	*1.85*	5.00	*2.29*	1.6	3.88	*1.99*	1.44	*2.82*	6.9
Long term yield	4.34	*1.91*	0.71	*0.31*	0.8	3.22	*1.54*	2.80	*1.23*	8.8
Net equity expansion	4.86	*2.10*	0.93	*0.40*	0.8	4.53	*2.31*	1.51	*0.53*	6.9
Stock volatility	4.04	*1.86*	-5.90	*-2.71*	2.0	4.10	*2.07*	0.95	*0.77*	6.4
Term Spread	4.27	*1.83*	0.80	*0.34*	0.8	3.56	*1.74*	1.33	*0.67*	6.7
Treasury bill rate	4.53	*2.07*	0.18	*0.08*	0.8	3.93	*1.99*	1.55	*0.74*	7.0

	Three year horizon					Five year horizon				
	Tail Coeff.	*Tail t-stat.*	Coeff.	*t-stat.*	R^2	Tail Coeff.	*Tail t-stat.*	Coeff.	*t-stat.*	R^2
Book-to-market	3.77	*2.56*	2.44	*1.40*	24.0	3.31	*2.55*	2.92	*2.31*	38.4
Default return spread	3.66	*2.40*	0.34	*1.61*	16.7	3.16	*2.65*	0.04	*0.39*	20.9
Default yield spread	3.51	*2.25*	1.57	*1.14*	19.6	2.83	*2.28*	2.57	*2.79*	31.2
Dividend payout ratio	3.86	*2.57*	2.27	*1.72*	22.8	3.36	*2.80*	3.97	*4.55*	33.9
Dividend price ratio	3.40	*2.47*	4.13	*2.65*	37.5	2.88	*2.47*	3.99	*3.66*	53.8
Earnings price ratio	3.40	*2.28*	2.14	*1.54*	22.2	2.84	*2.21*	3.38	*2.77*	38.2
Inflation	3.72	*2.47*	0.73	*0.83*	17.2	3.30	*2.61*	1.25	*1.26*	23.6
Long term return	3.63	*2.38*	0.18	*0.85*	16.6	3.13	*2.63*	0.28	*1.74*	21.1
Long term yield	2.63	*1.72*	3.46	*2.53*	29.7	1.99	*1.74*	3.94	*4.68*	48.0
Net equity expansion	4.31	*2.68*	1.91	*1.14*	20.6	3.73	*2.71*	1.62	*1.49*	24.4
Stock volatility	3.69	*2.41*	0.38	*0.67*	16.8	3.17	*2.66*	0.24	*0.66*	21.0
Term Spread	3.17	*2.03*	1.41	*1.06*	18.7	2.77	*2.46*	1.11	*0.99*	23.0
Treasury bill rate	3.51	*2.43*	2.07	*1.45*	21.6	2.98	*2.28*	2.77	*2.75*	34.2

Notes: The table reports results from monthly predictive regressions of CRSP value-weighted market index returns over one month, one year, three year and five year horizons. The table repeats the analysis of Table 3, but instead reports bivariate regressions that include each alternative predictor alongside the estimated tail risk process. For each horizon, the first two columns are the coefficient estimate and t-statistic for the tail risk process, while the third and fourth columns are the coefficient and t-statistic for the alternative predictor. Because overlapping monthly observations are used, test statistics are calculated using Hodrick's (1992) standard error correction for overlapping data with lag length equal to the number of months in each horizon. For comparison, reported predictive coefficients are scaled to be interpreted as the percentage change in expected annual market returns resulting from a one standard deviation increase in each predictor variable.

Table 5: MARKET RETURN PREDICTABILITY: OUT-OF-SAMPLE R^2 (%)

	One Month	One Year	Three Years	Five Years
Tail	0.3	4.5*	15.7*	20.1*
Book-to-market	-1.6	-9.9	-14.5	-34.0
Default return spread	-0.9	-0.6	-0.4	-0.3
Default yield spread	-0.6	-0.5	-9.2	3.0
Dividend payout ratio	-1.7	-23.6	-13.7	-46.4
Dividend price ratio	-1.3	-6.5	-4.4	15.8*
Earnings price ratio	-1.8	-15.6	-9.3	1.3
Inflation	-0.4	-4.0	-3.1	-11.4
Long term return	-0.1	0.9	-0.2	0.8
Long term yield	-0.9	-4.6	20.6*	10.2*
Net equity expansion	-0.8	-10.6	-6.6	-9.0
Stock volatility	-2.5	-25.0	-28.1	-22.1
Term Spread	-0.6	-2.9	-7.1	7.0
Treasury bill rate	-1.3	-9.2	5.8	-14.4

Notes: The table reports the out-of-sample forecasting R^2 in percent from predictive regressions of CRSP value-weighted market index returns over one month, one year, three year and five year horizons. In each month t (beginning at $t = 120$ to allow for a sufficiently large initial estimation period), we estimate rolling univariate forecasting regressions of monthly market returns on the estimated tail risk series and alternative predictors. Predictive coefficient estimates only use data through date t, which are then used to forecast returns at $t + 1$. The out-of-sample R^2 is calculated as $1 - \sum_t (r_{m,t+1} - \hat{r}_{m,t+1|t})^2 / \sum_t (r_{m,t+1} - \bar{r}_{m,t})^2$, where $\hat{r}_{m,t+1|t}$ is the out-of-sample forecast of the $t+1$ return based only on data through t, and $\bar{r}_{m,t}$ is the historical average market return through t. A negative R^2 implies that the predictor performs worse than setting forecasts equal to the sample mean. Due to the short time series for the variance risk premium out-of-sample forecasts are infeasible. A star (*) beside an estimate denotes that it is statistically significant at the 5% level or better based on the Clark and McCracken (2001) ENC-NEW test of out-of-sample predictability.

Table 6: Tail Beta-Sorted Portfolio Returns

	Low	2	3	4	High	High-Low	*t-stat*
Panel A: Twelve Month Returns							
			Equal-Weighted				
Average Return	1.14	1.24	1.31	1.39	1.48	0.33	*2.48*
CAPM Alpha	0.10	0.31	0.4	0.47	0.49	0.38	*2.63*
FF Alpha	-0.13	0.01	0.09	0.17	0.20	0.33	*2.85*
FF + Mom Alpha	0.04	0.10	0.15	0.22	0.26	0.21	*2.15*
FF + Mom + Liq Alpha	0.02	0.10	0.17	0.24	0.27	0.25	*2.54*
			Value-Weighted				
Average Return	0.86	0.99	1.03	1.14	1.21	0.35	*2.15*
CAPM Alpha	-0.17	0.05	0.12	0.20	0.18	0.35	*2.10*
FF Alpha	-0.20	-0.02	0.08	0.19	0.25	0.45	*3.00*
FF + Mom Alpha	-0.02	0.07	0.11	0.18	0.32	0.34	*2.24*
FF + Mom + Liq Alpha	-0.08	0.08	0.14	0.21	0.35	0.43	*2.93*
Panel B: One Month Returns							
			Equal-Weighted				
Average Return	1.14	1.24	1.28	1.4	1.45	0.31	*2.12*
CAPM Alpha	0.10	0.31	0.37	0.48	0.47	0.37	*2.52*
FF Alpha	-0.11	0.02	0.09	0.19	0.19	0.30	*2.22*
FF + Mom Alpha	-0.06	0.06	0.11	0.22	0.24	0.29	*2.14*
FF + Mom + Liq Alpha	-0.08	0.06	0.12	0.24	0.26	0.34	*2.50*
			Value-Weighted				
Average Return	0.84	0.96	0.98	1.18	1.20	0.36	*2.00*
CAPM Alpha	-0.19	0.03	0.08	0.25	0.18	0.37	*2.08*
FF Alpha	-0.19	-0.04	0.05	0.22	0.27	0.46	*2.58*
FF + Mom Alpha	-0.16	-0.03	0.03	0.18	0.30	0.45	*2.22*
FF + Mom + Liq Alpha	-0.21	-0.03	0.05	0.21	0.35	0.55	*2.78*

Notes: The table reports monthly return statistics for portfolios formed on the basis of tail risk beta. Each month stocks are sorted into quintile portfolios based on predictive tail loadings that are estimated from monthly data over the previous ten years. Portfolios are based on NYSE/AMEX/NASDAQ stocks with CRSP share codes 10 and 11. Panel A reports equal- and value-weighted average out-of-sample twelve month holding period portfolio returns and Panel B reports out-of-sample one month holding period portfolio returns. The table also reports portfolio alphas from regressions of portfolio returns using the Fama-French three-factor model as well as extended four and five-factor models controlling for momentum and liquidity (Pastor and Stambaugh (2003)) factors. The right-most columns report results for the high minus low zero net investment portfolio that is long quintile portfolio five and short quintile one and associated t-statistics. For twelve month returns, t-statistics use Newey-West (1987) standard errors based on twelve lags. Stocks with prices below \$5 at the portfolio formation date are excluded.

Table 7: Double-Sorted Portfolio Returns

	Low	2	3	High	High-Low	*t-stat*
Panel A: Firm Size and Tail Risk Beta						
			Equal-Weighted			
Small	0.10	0.16	0.26	0.15	0.05	*0.37*
2	-0.07	0.09	0.24	0.23	0.30	*2.54*
3	-0.07	0.11	0.15	0.24	0.31	*2.11*
Large	-0.08	0.03	0.08	0.27	0.35	*1.94*
			Value-Weighted			
Small	0.08	0.10	0.19	0.10	0.01	*0.10*
2	-0.07	0.06	0.23	0.24	0.31	*2.57*
3	-0.07	0.10	0.16	0.23	0.30	*2.03*
Large	-0.14	0.01	0.09	0.20	0.34	*1.69*
Panel B: Idiosyncratic Volatility and Tail Risk Beta						
			Equal-Weighted			
Low IV	0.15	0.16	0.22	0.30	0.15	*1.52*
2	0.10	0.17	0.25	0.33	0.22	*2.33*
3	0.11	0.13	0.24	0.30	0.19	*2.02*
High IV	-0.10	-0.01	0.08	0.11	0.21	*2.02*
			Value-Weighted			
Low IV	0.12	0.11	0.14	0.30	0.18	*1.53*
2	-0.01	0.09	0.16	0.30	0.30	*2.25*
3	-0.05	0.07	0.18	0.30	0.36	*2.32*
High IV	-0.17	-0.09	0.11	0.28	0.45	*2.61*
Panel C: Downside Beta and Tail Risk Beta						
			Equal-Weighted			
Low Down Beta	0.20	0.18	0.25	0.28	0.08	*0.76*
2	0.08	0.12	0.20	0.29	0.21	*1.92*
3	0.06	0.13	0.21	0.29	0.24	*2.43*
High Down Beta	-0.04	0.06	0.08	0.17	0.21	*1.97*
			Value-Weighted			
Low Down Beta	0.07	0.11	0.14	0.40	0.33	*2.15*
2	0.11	0.14	0.19	0.40	0.29	*1.72*
3	-0.15	0.01	0.03	0.16	0.31	*2.26*
High Down Beta	-0.04	-0.04	0.25	0.16	0.20	*0.94*

Notes: The table reports monthly 4-factor (Fama-French and momentum) alphas for double-sorted portfolios that are formed on the basis of tail risk loadings and size (Panel A), idiosyncratic volatility (Panel B), or downside beta (Panel C). Each month stocks are independently sorted into four quartile portfolios based each of these characteristics, and four quartiles along another dimension based on predictive tail loadings. Loading are estimated from monthly data over the previous ten years. Portfolios are based on NYSE/AMEX/NASDAQ stocks with CRSP share codes 10 and 11. The right-most columns report results for the high minus low zero net investment portfolio that is long quartile portfolio four and short quartile one. The portfolios are held for one year t-statistics use the Newey-West (1987) standard errors based on twelve lags. Stocks with prices below $5 at the portfolio formation date are excluded.

Table 8: Moneyness-Sorted Short Put Portfolios

	DOTM	OTM	ATM	ITM	DITM	DOTM−DITM	*t-stat*	R^2
Panel A: Delta-hedged								
Tail Risk Beta	7.17	4.20	2.89	1.07	0.01	7.17	*2.39*	-
Average Return	19.46	16.64	10.21	5.31	2.80	16.66	*3.55*	94.10
CAPM Alpha	18.50	15.88	9.74	5.01	2.53	15.96	*3.43*	93.91
FF Alpha	18.05	15.28	9.52	4.90	2.51	15.54	*3.31*	94.54
FF + Mom Alpha	17.87	15.21	9.51	4.96	2.62	15.25	*3.18*	94.36
FF + Mom + Liq Alpha	15.83	14.17	9.17	4.94	2.68	13.15	*2.82*	92.27
Panel B: Delta-hedged and Leverage-adjusted								
Tail Risk Beta	0.79	0.46	0.25	0.08	0.00	0.79	*2.43*	-
Average Return	1.63	1.57	1.28	0.99	0.56	1.07	*2.43*	79.15
CAPM Alpha	1.55	1.49	1.22	0.94	0.50	1.05	*2.29*	77.79
FF Alpha	1.55	1.44	1.20	0.91	0.50	1.04	*2.28*	80.79
FF + Mom Alpha	1.54	1.43	1.22	0.93	0.54	1.00	*2.07*	80.51
FF + Mom + Liq Alpha	1.29	1.32	1.16	0.94	0.56	0.73	*1.54*	65.97

Notes: We form portfolios of individual equity put options on the basis of option moneyness following the approach of Frazzini and Pedersen (2012) over 1996 to 2010. Moneyness is defined as absolute value of the Black-Scholes delta of an option, and the five portfolios are deep out-of-the-money (DOTM, $|\Delta| < 0.20$), out-of-the-money (OTM, $0.20 \leq |\Delta| < 0.40$), at-the-money (ATM, $0.40 \leq |\Delta| < 0.60$), in-the-money (ITM, $0.60 \leq |\Delta| < 0.80$) and deep-in-the-money (DITM, $0.80 \leq |\Delta|$). We also report results from the difference between the DOTM and DITM portfolios. Portfolio rebalancing dates and one month holding periods correspond to the monthly expiration schedule. We compute the return of selling an put with one month to maturity on the first trading day following the an expiration date and holding it to the next month's expiration. Each put position is delta-hedged daily. Tail betas are estimated from a predictive regression of each portfolio's return on lagged tail risk (which is recalculated to correspond to the option portfolio formation schedule and variance standardized). Due to the relatively short sample for options data we estimate a single in-sample predictive coefficient for each portfolio. The far right column reports the correlation between tail beta and portfolio alpha across the five portfolios. Panel A reports results for monthly returns on delta-hedged short put option portfolios. Panel B reports results for leverage-adjusted versions of these portfolios following the procedure in Karakaya (2013).