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UNEMPLOYMENT CRISES

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ABSTRACT

A search and matching model, when calibrated to the mean and volatility of unemployment in the postwar sample, can potentially explain the large unemployment dynamics in the Great Depression. The limited response of wages to labor market conditions from credible bargaining and the congestion externality from matching frictions cause the unemployment rate to rise sharply in recessions but decline gradually in booms. The frequency, severity, and persistence of unemployment crises in the model are quantitatively consistent with U.S. historical time series.

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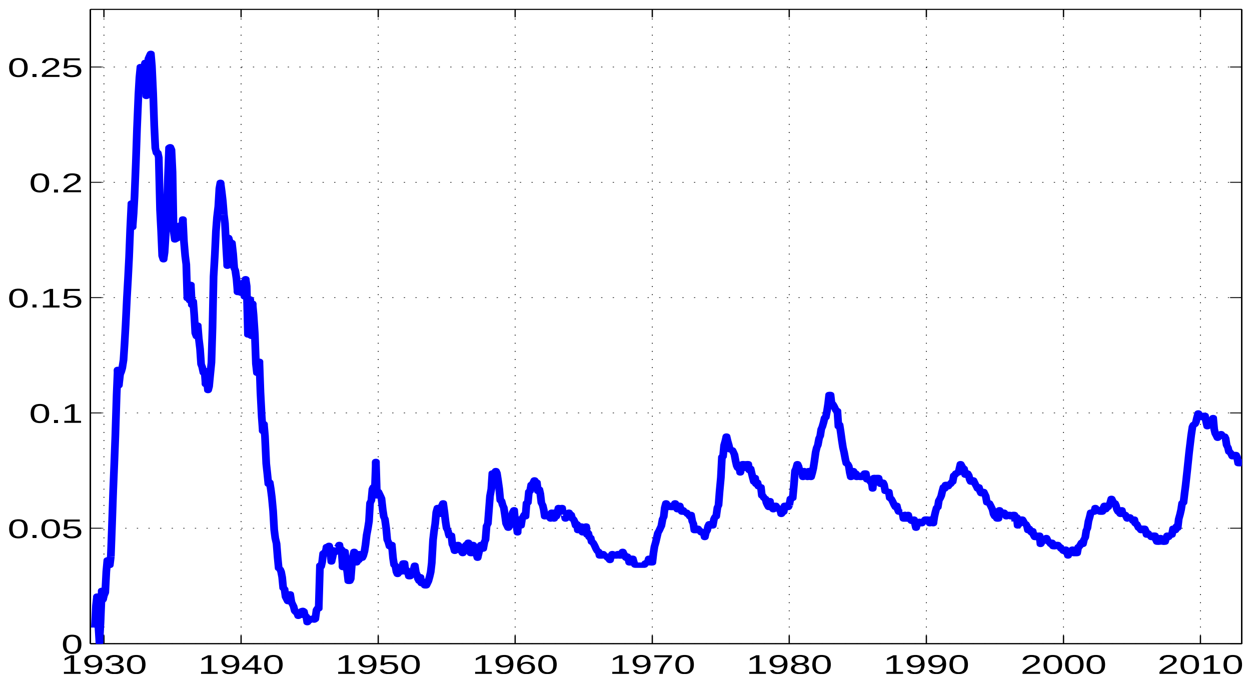
Figure 1 : Monthly U.S. Unemployment Rates, April 1929 to December 2012



1 Introduction

Figure 1 plots the monthly U.S. unemployment rates from April 1929 to December 2012 (see Section 2 for sample construction). The mean unemployment rate is 6.97%, and the median rate is 5.70%. The most striking feature of the figure is the extraordinarily high levels of unemployment in the 1930s, known as the Great Depression. From January 1931 to December 1939, the average unemployment rate is 17.57%, and the highest unemployment rate reaches 25.59% in May 1933. In contrast, such large unemployment dynamics are absent in the postwar sample. We fit a three-state Markov chain on the unemployment rate series via maximum likelihood. Identifying months in which the unemployment rate is above 20% as a crisis state, we estimate the unconditional probability of an unemployment crisis to be 2.76% and its persistence (the probability of a crisis next period conditional on a crisis in the current period) to be 88.89%.

We ask whether a Diamond-Mortensen-Pissarides model of equilibrium unemployment, when calibrated to the mean and volatility of unemployment in the postwar sample, can explain the large unemployment dynamics in the Great Depression. Perhaps surprisingly, the answer is affirmative. Unemployed workers search for vacancies posted by a representative firm. A matching function takes vacancies and unemployed workers as inputs to produce the number of new hires in the labor market. Because of matching frictions, the vacancy filling rate decreases with the tightness of the labor market measured as the ratio of the number of vacancies over the number of unemployed workers. Deviating from the standard wage determination via a generalized Nash bargaining game, we follow Hall and Milgrom (2008) to derive the equilibrium wage as the outcome of a credible bargaining game. In this setup, both parties make alternating offers that can be accepted, rejected to make a counteroffer, or rejected to take outside options. Relative to the Nash wage, the credible bargaining wage is more insulated from conditions in the labor market.

Our key finding is that the model, calibrated to match the mean of 5.84% and the volatility of 13.48% for the unemployment rate in the postwar sample, can match the persistence and the unconditional probability of the crisis state in the full sample. The persistence of the crisis state is 86.96% in the model, and its unconditional probability is 3.18% (with a cross-simulation standard deviation of 7.72%). The volatility of unemployment in the model's crises samples is 0.233, which is more than twice as large as 0.109 in normal periods. Remarkably, the model moment is close to the unemployment volatility of 0.219 in the long U.S. sample from April 1929 to December 2012.

The unemployment dynamics in the model are large and extremely nonlinear. In booms unemployment declines only gradually, whereas in recessions unemployment rises drastically. The model's empirical stationary distribution of unemployment is highly skewed with a long right tail. Also, impulse responses are stronger in recessions than in booms. In response to a negative one-standard-deviation shock to log labor productivity, the unemployment rate increases only by 0.05% if the economy starts at the 95 percentile of the model's stationary distribution of employment and productivity. In contrast, the unemployment rate jumps by 1.06% (more than 20 times larger) if the economy instead starts at the 5 percentile of the bivariate distribution.

The probability of breakdown in wage bargaining and the delaying cost incurred by the firm during each round of alternating offers (which are two key parameters in the credible bargaining game) are quantitatively important for the crisis dynamics in the model. A higher probability of breakdown, in which both parties take outside options, brings credible bargaining closer to Nash bargaining and makes the equilibrium wage more response to labor market conditions. In recessions, labor productivity falls, but the wage also drops, making profits less sensitive to downturns and providing the firm with incentives to create jobs. As such, crisis dynamics in unemployment are dampened. In contrast, a higher delaying cost makes the equilibrium wage more insulated from labor market conditions. As output falls in recessions, the wage does not fall as much, meaning that profits must drop disproportionately more. As such, crisis dynamics are strengthened.

The congestion externality also plays a role in driving our results. The congestion arises because in booms many vacancies compete for a small pool of unemployed workers. An extra vacancy can cause a large drop in the vacancy filling rate, raising the marginal cost of hiring. In contrast, in recessions many unemployed workers compete for a small pool of vacancies. An extra vacancy is quickly filled, and the vacancy filling rate can hardly increase further. As such, the marginal cost of hiring runs into a downward rigidity in recessions. This effect is also buttressed by the fixed matching cost that puts a constant component into the marginal cost of hiring.

Consider a large negative shock to labor productivity. Output falls, but profits plummet more because the credible bargaining wage is relatively insulated from aggregate conditions. To make matters worse, the marginal cost of hiring runs into the downward rigidity, failing to decline to counteract the impact of plummeting profits on the firm's incentives of hiring. As a result, unemployment rises drastically, giving rise to crises. Quantitatively, the log productivity levels that are two and three unconditional standard deviations below its unconditional mean would lead to unemployment rates of 13% and 28%, respectively. To reach the crisis threshold of 20% unemployment rate, the log productivity must drop 2.62 unconditional standard deviations below the unconditional mean. Most important, the search and matching model endogenizes unemployment crises similar to the Great Depression, even though the exogenous driving force is a first-order autoregressive process with homoscedastic lognormal shocks as in the standard business cycle literature.

To evaluate the quantitative importance of credible bargaining in the model, we experiment with the baseline search model with the equilibrium wage determined from the standard Nash bargaining. We find that the model can quantitatively explain the same crisis dynamics in unemployment, but only with either an excessively high flow value of unemployment activities or an extremely high fixed matching cost. Both calibrations do not appear realistic. As such, we conclude that the credible bargaining framework is essential in providing a realistic microfoundation for unemployment crises.

The theoretical foundations for the search model of unemployment are provided by Diamond (1982), Mortensen (1982), and Pissarides (1985). Shimer (2005) contributes a simple yet profound insight that the unemployment volatility in the baseline search model is too low relative to that in the data.[1] As noted, our model is built on Hall and Milgrom (2008), who show that replacing the Nash wage with the credible bargaining wage helps explain the volatility puzzle. Going beyond the volatility puzzle in the labor market, our central contribution is to push the search model of unemployment to explain the crisis dynamics of unemployment in the Great Depression.

Kuehn, Petrosky-Nadeau, and Zhang (2013) embed the search model of unemployment into an equilibrium asset pricing framework. With recursive preferences, their work focuses on asset prices and related consumption disasters. Our work differs by focusing on labor market dynamics (in particular, unemployment crises as well as their impact on the second moment of the labor market). Most important, we move beyond the Nash bargaining to the Hall and Milgrom (2008) credible bargaining to provide a rich microfoundation for unemployment crises.

The rest of the paper is organized as follows. Section 2 documents the nonlinear dynamics in the

[1] A large subsequent literature has developed to tackle the volatility puzzle. In particular, Hall (2005) shows how wage stickiness, which satisfies the condition that no worker-firm pair has any unexploited opportunity for mutual gain, increases labor market volatilities. Mortensen and Nagypál (2007) and Pissarides (2009) show that the fixed recruiting cost can explain the volatility puzzle. Hagedorn and Manovskii (2008) show that a calibration with small profits and a low bargaining power for the workers can produce realistic volatilities. Petrosky-Nadeau and Wasmer (2013) show how incorporating financial frictions can generate higher labor market volatilities.

historical U.S. unemployment rates. Section 3 presents the search and matching model. Section 4 calibrates the model and discusses its accurate solution algorithm. Section 5 quantifies the model's crisis dynamics in unemployment. Finally, Section 6 concludes.

2 Evidence on Unemployment Crises

We construct a monthly time series for the U.S. unemployment rate stretching back to April of 1929 by drawing from NBER macrohistory files. We access the data via Federal Reserve Economic Data (FRED) at Federal Reserve Bank of St. Louis. We concatenate four different series of the U.S. unemployment rate: (i) the seasonally adjusted unemployment rate from April 1929 to February 1940 (NBER data series m08292a, FRED series ID: M0892AUSM156SNBR); (ii) the seasonally adjusted unemployment rate from March 1940 to December 1946 (NBER data series m08292b, FRED series ID: M0892BUSM156SNBR); (iii) the unemployment rate (not seasonally adjusted) from January 1947 to December 1947 (NBER data series m08292c, FRED series ID: M0892CUSM156NNBR); and (iv) the seasonally adjusted civilian unemployment rate from January 1948 to December 2012 from Bureau of Labor Statistics at U.S. Department of Labor (FRED series ID: UNRATE).

To model the tail behavior in the U.S. unemployment rate series plotted in Figure 1, we follow Chatterjee and Corbae (2007) to fit a three-state Markov chain model via maximum likelihood. The aggregate state of the economy, $\eta \in \{g, b, c\}$, evolves through good (g), bad (b), and crisis (c) states with different employment prospects. Let the transition matrix of the Markov Chain be given by:

$$\Lambda = \begin{bmatrix} \lambda_{gg} & \lambda_{bg} & \lambda_{cg} \\ \lambda_{gb} & \lambda_{bb} & \lambda_{cb} \\ \lambda_{gc} & \lambda_{bc} & \lambda_{cc} \end{bmatrix}, \tag{1}$$

in which, for example, $\lambda_{gb} \equiv \text{Prob}\{\eta_{t+1} = g | \eta_t = b\}$ is the probability of the economy being in state g next period conditional on the economy being in state b in the current period.

As discussed in Chatterjee and Corbae (2007), the maximum likelihood estimate of λ_{kj}, which is the (j, k)th element of the aggregate state transition matrix, is the ratio of the number of times the economy switches from state j to state k to the number of times the economy is in state j. Let, for example, $\mathbf{1}_{\{\eta_t=j\}}$ denote the indicator function that takes the value of one if the economy in period t is in state j and zero otherwise. The maximum likelihood estimate of λ_{kj} is given by:

$$\hat{\lambda}_{kj} = \frac{\sum_{t=1}^{T-1} \mathbf{1}_{\{\eta_{t+1}=k\}} \mathbf{1}_{\{\eta_t=j\}}}{\sum_{t=1}^{T-1} \mathbf{1}_{\{\eta_t=j\}}}. \tag{2}$$

In addition, the asymptotic standard error for $\hat{\lambda}_{kj}$ is given by:

$$\mathrm{Ste}(\hat{\lambda}_{kj}) = \sqrt{\frac{\hat{\lambda}_{kj}(1-\hat{\lambda}_{kj})}{\sum_{t=1}^{T} \mathbf{1}_{\{\eta_t=j\}}}}. \tag{3}$$

In practice, we identify the good state, g, as months in which the U.S. unemployment rate is below the median unemployment rate of 5.70%. We define the crisis state, c, as months in which the U.S. unemployment rate is above or equal to 20%. The bad state, b, is then identified as months in which the unemployment rate is below 20% but above or equal to the median of 5.70%. We choose the crisis cutoff rate of 20% judiciously such that (i) the cutoff rate is relatively high; but (ii) there are still a sufficient number of months in which the economy hits the crisis state so that the transition probability estimates can be (relatively) precise.

Table 1 reports the estimated aggregate state transition matrix. The crisis state is persistent in that the probability of the economy remaining in the crisis state conditional on it being in the crisis state is 88.9%. This estimate is also precise, with a small standard error of 0.061. With a probability of 11.1%, the economy switches from the crisis state to the bad state. Unconditionally, the tail probability of the economy being in the crisis state is estimated to be 2.76%.

3 The Model

We construct a search model of unemployment embedded with credible bargaining in determining the equilibrium wage as in Hall and Milgrom (2008).

Table 1 : Estimated Aggregate State Transition Matrix and Unconditional Probabilities of the Three Economic States, April 1929–December 2012

This table reports the estimated state transition matrix in equation (1). The transition probabilities are defined as in equation (2), and the standard errors (in parentheses) are in equation (3). The last row reports the unconditional probabilities of the states, calculated by raising the transition matrix to the power 1,000.

	Good	Bad	Crisis
Good	0.9590	0.0410	0
	(0.0090)	(0.0090)	(0)
Bad	0.0389	0.9550	0.0061
	(0.0087)	(0.0094)	(0.0035)
Crisis	0	0.1111	0.8889
	(0)	(0.0605)	(0.0605)
Unconditional probability	0.4733	0.4992	0.0276

3.1 The Environment

The model is populated by a representative household and a representative firm that uses labor as the single productive input. Following Merz (1995) and Andolfatto (1996), we use the representative family construct, which implies perfect consumption insurance. The household has a continuum with a unit mass of members who are, at any point in time, either employed or unemployed. The fractions of employed and unemployed workers are representative of the population at large. The household pools the income of all the members together before choosing per capita consumption and asset holdings. Finally, the household is risk neutral with a time discount factor of β.

The representative firm posts a number of job vacancies, V_t, to attract unemployed workers, U_t. Vacancies are filled via a constant returns to scale matching function, $G(U_t, V_t)$, specified as:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^\iota + V_t^\iota)^{1/\iota}}, \tag{4}$$

in which $\iota > 0$ is a constant parameter. This matching function,specified as in Den Haan, Ramey, and Watson (2000), has the desirable property that matching probabilities fall between zero and one.

Define $\theta_t \equiv V_t/U_t$ as the vacancy-unemployment (V/U) ratio. The probability for an unemployed worker to find a job per unit of time (the job finding rate), denoted $f(\theta_t)$, is:

$$f(\theta_t) \equiv \frac{G(U_t, V_t)}{U_t} = \frac{1}{\left(1 + \theta_t^{-\iota}\right)^{1/\iota}}. \tag{5}$$

The probability for a vacancy to be filled per unit of time (the vacancy filling rate), denoted $q(\theta_t)$, is:

$$q(\theta_t) \equiv \frac{G(U_t, V_t)}{V_t} = \frac{1}{(1 + \theta_t^{\iota})^{1/\iota}}. \tag{6}$$

It follows that $q'(\theta_t) < 0$: An increase in the scarcity of unemployed workers relative to vacancies makes it harder to fill a vacancy. As such, θ_t is labor market tightness from the firm's perspective.

The representative firm incurs costs in posting vacancies. Following Mortensen and Nagypàl (2007) and Pissarides (2009), we incorporate a fixed component in the unit cost per vacancy:

$$\kappa_t \equiv \kappa_0 + \kappa_1 q(\theta_t), \tag{7}$$

in which κ_0 is the proportional cost, κ_1 is the fixed cost, and both are nonnegative. The proportional cost is standard in the literature. The fixed cost captures training, interviewing, and administrative setup costs of adding a worker to the payroll, costs that are paid after a hired worker arrives but before wage bargaining takes place. The marginal cost of hiring arising from the proportional cost, $\kappa_0/q(\theta_t)$, is time-varying, but that arising from the fixed cost is constant, κ_1.

Jobs are destroyed at a constant rate of $s > 0$ per period. Employment, N_t, evolves as:

$$N_{t+1} = (1 - s)N_t + q(\theta_t)V_t, \tag{8}$$

in which $q(\theta_t)V_t$ is the number of new hires. The size of the population is normalized to be unity, $U_t = 1 - N_t$. As such, N_t and U_t are also the rates of employment and unemployment, respectively.

The firm takes aggregate labor productivity, X_t, as given. The law of motion for $x_t \equiv \log(X_t)$ is:

$$x_{t+1} = \rho x_t + \sigma \epsilon_{t+1}, \tag{9}$$

in which $\rho \in (0,1)$ is the persistence, $\sigma > 0$ is the conditional volatility, and ϵ_{t+1} is an independently and identically distributed standard normal shock. The firm uses labor to produce output, Y_t, with a constant returns to scale production technology,

$$Y_t = X_t N_t. \tag{10}$$

The dividends to the firm's shareholders are given by:

$$D_t = X_t N_t - W_t N_t - \kappa_t V_t, \tag{11}$$

in which W_t is the equilibrium wage rate. Taking $q(\theta_t)$ and W_t as given, the firm posts an optimal number of job vacancies to maximize the cum-dividend market value of equity, denoted S_t:

$$S_t \equiv \max_{\{V_{t+\triangle t}, N_{t+\triangle t+1}\}_{\triangle t=0}^{\infty}} E_t \left[\sum_{\triangle t=0}^{\infty} \beta^{\triangle t} \left[X_{t+\triangle t} N_{t+\triangle t} - W_{t+\triangle t} N_{t+\triangle t} - \kappa_{t+\triangle t} V_{t+\triangle t} \right] \right], \tag{12}$$

subject to the employment accumulation equation (8) and a nonnegativity constraint on vacancies:

$$V_t \geq 0. \tag{13}$$

Because $q(\theta_t) > 0$, this constraint is equivalent to $q(\theta_t)V_t \geq 0$. As such, the only source of job destruction in the model is the exogenous separation of employed workers from the firm.[2]

Let λ_t denote the multiplier on the nonnegativity constraint $q(\theta_t)V_t \geq 0$. From the first-order conditions with respect to V_t and N_{t+1}, we obtain the intertemporal job creation condition:

$$\frac{\kappa_t}{q(\theta_t)} - \lambda_t = E_t \left[\beta \left[X_{t+1} - W_{t+1} + (1-s) \left[\frac{\kappa_{t+1}}{q(\theta_{t+1})} - \lambda_{t+1} \right] \right] \right]. \tag{14}$$

Intuitively, the marginal cost of hiring at time t equals the marginal value of employment to the firm, which in turn equals the marginal benefit of hiring at period $t+1$, discounted to t. The marginal benefit at $t+1$ includes the marginal product of labor, X_{t+1}, net of the wage rate, W_{t+1},

[2] The constraint is occasionally binding in simulations from the baseline search model with Nash bargaining. Because a negative vacancy makes no economic sense, we impose the nonnegativity constraint throughout the paper.

plus the marginal value of employment, which equals the marginal cost of hiring at $t+1$, net of separation. Finally, the optimal vacancy policy also satisfies the Kuhn-Tucker conditions:

$$q(\theta_t)V_t \geq 0, \quad \lambda_t \geq 0, \quad \text{and} \quad \lambda_t q(\theta_t)V_t = 0. \tag{15}$$

Let C_t denote consumption. In equilibrium, the goods market clearing condition says:

$$C_t + \kappa_t V_t = X_t N_t. \tag{16}$$

3.2 Equilibrium Wages with Credible Bargaining

To close the model, we need to specify how the wage rate, W_t, is determined in equilibrium. In the standard Diamond-Mortensen-Pissarides model, the wage rate is derived from the sharing rule per the outcome of a generalized Nash bargaining process between the employed workers and the firm (e.g., Pissarides (2000, Section 1.4)). Let $0 < \eta < 1$ be the workers' relative bargaining weight and b the workers' value of unemployment activities. The Nash-bargained wage rate is:

$$W_t = \eta\left(X_t + \kappa_t \theta_t\right) + (1-\eta)b. \tag{17}$$

Although analytically simple, the baseline search model with the Nash wage requires an extremely high replacement ratio (i.e., the value of unemployment activities over the average marginal product of labor) to reproduce realistic labor market volatilities (e.g., Hagedorn and Manovskii (2008)).

We adopt the credible bargaining wage proposed by Hall and Milgrom (2008). Built on Binmore, Rubinstein, and Wolinsky (1986), Hall and Milgrom place a crucial distinction between a threat point and an outside option in the wage bargaining game. Bargaining takes time. Both parties make alternating offers which can be accepted, rejected to make a counteroffer, or rejected to abandon the bargaining altogether. In the standard Nash bargaining, disagreement leads immediately to the abandonment of the bargaining game, meaning that the relevant threat point is the outside options for both parties. In contrast, in the more realistic alternating bargaining, disagreement only leads to another round of alternating offers. The threat point is the payoff from another round of alternating offers, and outside options are taken only when abandoning the bargaining altogether.

The outside option for a worker is the value of unemployment, and that for the firm is zero. During a period in which both parties engage in another round of alternating offers, the worker receives the flow value of unemployment activities, b, and the firm incurs the cost of delaying, $\chi > 0$. During this period, the negotiation can also break down with a probability of δ.

With this setup of the alternating bargaining game, the indifference condition for a worker when considering a wage offer, W_t, from the firm is:

$$J_{Nt}^{W} = \delta J_{Ut} + (1-\delta)\left(b + E_t[\beta J_{Nt+1}^{W'}]\right), \tag{18}$$

in which $J_t \equiv J(N_t, X_t)$ is the indirect utility function of the representative household, J_{Nt}^{W} is the marginal value of an employed worker to the household when accepting the wage offer from the employer, J_{Ut} is the marginal value of an unemployed worker to the household, and $J_{Nt+1}^{W'}$ is the marginal value of an employed worker to the household when rejecting the firm's wage offer to make a counteroffer of W'_{t+1} in the next period. The indifference condition in equation (18) says that the payoff to the worker when accepting the wage offer from the firm, J_{Nt}^{W}, is just equal to the payoff from rejecting the offer. After rejecting the offer, with a probability of δ, the negotiation breaks down, and the worker returns to the labor market, leaving the household with the marginal value of an unemployed worker. With the probability of $1-\delta$, the worker receives the flow value of unemployment, b, for the current period, and makes a counteroffer of W'_{t+1} to the firm in the next period.

The indifference condition for the firm when considering the worker's counteroffer, W'_t, is:

$$S_{Nt}^{W'} = \delta \times 0 + (1-\delta)\left(-\chi + E_t[\beta S_{Nt+1}^{W}]\right), \tag{19}$$

in which $S_{Nt}^{W'}$ is the marginal value of an employed worker to the firm when accepting the worker's counteroffer, and S_{Nt+1}^{W} is the marginal value of an employed worker to the firm when rejecting the worker's offer to make a counteroffer of W_{t+1} in the next period. Intuitively, equation (19) says that the firm is just indifferent between the payoff from accepting the worker's offer W_t' and the payoff from rejecting the offer to have an opportunity to make a counteroffer of W_{t+1} in the next period. When rejecting the offer, the firm pays the delaying cost of χ if the bargaining does not break down. When the negotiation does break down, the firm's payoff is zero.

The two indifference conditions collapse to the indifference conditions for the standard Nash bargaining when the probability of breakdown, δ, equals one. During the alternating bargaining, it is optimal for each party to make a just acceptable offer. As in Hall and Milgrom (2008), we assume that the firm makes the first offer, which the worker accepts. As such, W_t is the equilibrium wage, and the delaying cost, χ, is never paid in equilibrium.

The equilibrium wage, W_t, and the worker's counteroffer wage, W_{t+1}', can be characterized further. First, we note that the marginal value of an unemployed worker to the household is:

$$J_{Ut} = b + E_t \left[\beta \left(f_t J_{Nt+1}^W + (1 - f_t) J_{Ut+1} \right) \right], \tag{20}$$

in which $f_t \equiv f(\theta_t)$ is the job finding rate. The equation says that the value of unemployment equals the flow value of unemployment activities, b, plus the discounted expected value in the next period. With a probability of f_t, the unemployed worker lands a job, which delivers the value of J_{Nt+1}^W. Otherwise, the worker remains unemployed with a value of J_{Ut+1}.

In addition, the marginal value of an employed worker to the household is:

$$J_{Nt}^W = W_t + E_t \left[\beta \left((1 - s) J_{Nt+1}^W + s J_{Ut+1} \right) \right]. \tag{21}$$

The equation says that the value of employment equals the flow value of employment, W_t, plus the discounted expected value in the next period. With a probability of s, the employed worker separates from the firm, and returns to the labor market as an unemployed worker with a value of J_{Ut+1}. Otherwise, the worker remains on the job, which delivers the value of J_{Nt+1}^W.

The wage offer by the firm to the worker, W_t, can be expressed as (see Appendix A):

$$W_t = b + (1-\delta)\beta E_t \left[J^{W'}_{Nt+1} - J_{Ut+1} \right] - (1 - s - \delta f_t)\, \beta E_t \left[J^{W}_{Nt+1} - J_{Ut+1} \right] \tag{22}$$

Intuitively, the wage offer from the firm increases in the flow value of unemployment activities b. The second term in equation (22) says that if the bargaining does not breakdown, the wage offer also increases in the surplus that the worker would enjoy after making a counteroffer, W'_{t+1}, to the firm. From the last term in equation (22), the equilibrium wage, W_t, also increases in the separation rate, s. As s goes up, the expected duration of the job shortens. As such, the worker requires a higher wage to remain indifferent between accepting and rejecting the wage offer. Finally, W_t increases in the job finding rate, f_t. As f_t rises, the worker's outside job market prospects improve, and the firm must offer a higher wage to make the worker indifferent. However, this impact of labor market conditions on W_t becomes negligible as the probability of breakdown in the bargaining, δ, goes to zero.

The wage offer of a worker to the firm, W'_t, can be expressed as:

$$W'_t = X_t + (1-\delta)\chi + \beta E_t \left[(1-s) S^{W'}_{Nt+1} - (1-\delta) S^{W}_{Nt+1} \right] \tag{23}$$

Intuitively, W'_t increases in labor productivity, X_t, and the cost of delay to the firm, χ. A higher χ makes the firm more likely to accept a higher wage offer from the worker to avoid any delay. As W'_t contains a higher constant proportion because of a higher χ, W'_t becomes more insulated from labor market conditions. Furthermore, because W'_t is the flow value of $J^{W'}_{Nt}$ shown in equation (21), $J^{W'}_{Nt}$ also becomes more insulated. More important, as $J^{W'}_{Nt}$ enters the second term in equation (22), the equilibrium wage, W_t, becomes less sensitive to aggregate conditions as a result of a higher χ.

From the last term in equation (23), an increase in the separation rate reduces the wage offer from the worker to the firm, W'_t. As s rises, the present value of profits produced by the worker drops. To make the firm indifferent, the worker must reduce the wage offer. Also, the worker's offer, W'_t, increases (naturally) in the firm's surplus from accepting the offer, $S^{W'}_{Nt+1}$. In contrast, the worker's offer would be lower if the firm's surplus, S^W_{Nt+1}, from rejecting the offer to make a counteroffer, W_t, is higher. However, the quantitative importance of this channel would be negligible if the breakdown probability, δ, goes to one. As such, W'_t increases with δ.

Finally, the two parties of the credible bargaining game would agree to accept the equilibrium wage if the joint surplus of the match is greater than the joint value of the outside options, J_{Ut}, as well as the joint present value of continuous delaying:

$$S^W_{Nt} + J^W_{Nt} > \max\left(J_{Ut}, E_t\left[\sum_{\triangle t=0}^{\infty} \beta^{\triangle t}(b-\chi)\right]\right) = J_{Ut}. \tag{24}$$

The last equality holds because the flow value of unemployment, b, is higher than $b-\chi$ (the delaying cost is positive). We verify that this condition holds in simulations.

4 Calibration and Computation

We calibrate the model in Section 4.1 and discuss its solution algorithm in Section 4.2.

4.1 Calibration

Our calibration strategy is to match the mean and the volatility of the unemployment rate in the postwar sample, before quantifying the model's performance in explaining the nonlinear dynamics documented in Section 2. Calibrating to the unemployment dynamics in normal periods seems sensible as the common practice in the existing literature. Because of the strong nonlinearity in the model, steady state relations hold very poorly in simulations. As such, we do not use these relations.

Table 2 lists the parameter values for the monthly calibration of the benchmark model. The time discount factor, β, is set to be $e^{-5.524/1200} = 0.9954$. This value implies a discount rate of 5.524% per annum, which is the leverage-adjusted aggregate discount rate in the 1951–2012 sample.[3] To calibrate the log labor productivity, we set its persistence, ρ, to be $0.95^{1/3} = 0.983$ as in

[3] We obtain monthly series of the value-weighted stock market returns, one-month Treasury bill rates, and inflation rates from January 1951 to December 2012 from Center for Research in Security Prices. The average real interest rate (one-month Treasury bill rates minus inflation rates) is 0.895% per annum. The equity premium (the value-weighted market returns in excess of one-month Treasury bill rates) is on average 6.807%. Because we do not

Table 2 : Parameter Values in the Monthly Calibration for the Benchmark Model

Notation	Parameter	Value
β	Time discount factor	$e^{-5.524/1200}$
ρ	Aggregate productivity persistence	$0.95^{1/3}$
σ	Conditional volatility of productivity shocks	0.00635
s	Job separation rate	0.045
ι	Elasticity of the matching function	1.25
b	The value of unemployment activities	0.71
δ	Probability of breakdown in bargaining	0.1
χ	Cost to employer of delaying in bargaining	0.25
κ_0	The proportional cost of vacancy posting	0.15
κ_1	The fixed cost of vacancy posting	0.1

Gertler and Trigari (2009). We then calibrate its conditional volatility, σ, to match the standard deviation of labor productivity in the data. We measure the labor productivity as seasonally adjusted real average output per job in the nonfarm business sector (Series id: PRS85006163) from the Bureau of Labor Statistics. The sample is quarterly from 1951 to 2012. We detrend the series as the Hodrick-Prescott (1997, HP) filtered cyclical component of proportional deviations from the mean with a smoothing parameter of 1,600. The standard deviation of the detrended series is 0.012 in the data. With a value of 0.00635 for σ, the implied standard deviation from the model is 0.013.

We set the job separation rate, s, to be 4.5%, which is higher than 3.78% from the Job Openings and Labor Turnover Survey (JOLTS). However, Davis, Faberman, Haltiwanger, and Rucker (2010) show that the JOLTS sample overweights establishments with stable hiring and separation and underweights volatile establishments with rapid growth or contraction. Adjusting for this bias, Davis et al. (Table 5.4) estimate the separation rate to be 4.96%. For the elasticity parameter in the matching function, ι, we set it to be 1.25, which is close to that in Den Haan, Ramey, and Watson (2000).

Following Hall and Milgrom (2008), we calibrate the value of unemployment activities to be 0.71. The probability of breakdown in bargaining, δ, is 0.1 in our monthly frequency, and is close to Hall and Milgrom's value of 0.0055 in their daily calibration (with 20 working days per month).

model financial leverage, we calculate the leverage-adjusted equity premium as $(1 - 0.32) \times 6.807\% = 4.629\%$, in which 0.32 is the aggregate market leverage ratio of U.S. corporations reported in Frank and Goyal (2008). Taken together, the leverage-adjusted aggregate discount rate in the data is $4.629\% + 0.895\% = 5.524\%$.

The delaying cost parameter, χ, is set to be 0.25, which is close to 0.27 in Hall and Milgrom.

To calibrate the recruiting cost parameters, κ_0 and κ_1, we target the first and the second moments of the unemployment rate in U.S. postwar sample. As noted, the mean unemployment rate in the sample from January 1951 to December 2012 is 5.84%. To calculate the second moment, we follow a similar procedure in Shimer (2005). We first convert the monthly series to a quarterly series by taking the monthly average unemployment rate within a given quarter. We detrend the quarterly series as the HP-filtered cyclical component of proportional deviations from the mean with a smooth parameter of 1,600. The standard deviation of the detrended series is 13.48% in the data. We end up with the recruiting cost parameters, $\kappa_0 = 0.15$ and $\kappa_1 = 0.1$, which imply a mean of 5.75% and a standard deviation of 10.94% for the unemployment rate in the non-crisis states in the model.

4.2 Computation

Because we focus on the higher moments of unemployment, the standard loglinearization cannot be used. We instead adapt the projection algorithm first developed in Kuehn, Petrosky-Nadeau, and Zhang (2013) to our setting. The state space consists of employment and log productivity, (N_t, x_t). The goal is to solve for the optimal vacancy function, $V_t = V(N_t, x_t)$, the multiplier function, $\lambda_t = \lambda(N_t, x_t)$, and the equilibrium wage, $W_t = W(N_t, x_t)$, from the following five functional equations:

$$\begin{aligned}
\frac{\kappa_t}{q(\theta_t)} - \lambda(N_t, x_t) &= E_t\left[\beta\left[X_{t+1} - W_{t+1} + (1-s)\left[\frac{\kappa_{t+1}}{q(\theta_{t+1})} - \lambda(N_{t+1}, x_{t+1})\right]\right]\right] && (25)\\
W(N_t, x_t) &= b + (1-\delta)\beta E_t\left[J_N^{W\prime}(N_{t+1}, x_{t+1}) - J_U(N_{t+1}, x_{t+1})\right] \\
&\quad - (1 - s - \delta f_t)\,\beta E_t\left[J_N^W(N_{t+1}, x_{t+1}) - J_U(N_{t+1}, x_{t+1})\right] && (26)\\
J_U(N_t, x_t) &= b + E_t\left[\beta\left(f_t J_N^W(N_{t+1}, x_{t+1}) + (1-f_t) J_U(N_{t+1}, x_{t+1})\right)\right] && (27)\\
J_N^W(N_t, x_t) &= W_t + E_t\left[\beta\left((1-s) J_N^W(N_{t+1}, x_{t+1}) + s J_U(N_{t+1}, x_{t+1})\right)\right] && (28)\\
J_N^{W\prime}(N_t, x_t) &= W_t' + E_t\left[\beta\left((1-s) J_N^{W\prime}(N_{t+1}, x_{t+1}) + s J_U(N_{t+1}, x_{t+1})\right)\right]. && (29)
\end{aligned}$$

In addition, $V(N_t, x_t)$ and $\lambda(N_t, x_t)$ must also satisfy the Kuhn-Tucker condition (15).

We parameterize the conditional expectation in the right-hand side of equation (25) as $\mathcal{E}_t \equiv \mathcal{E}(N_t, x_t)$, as well as four other functions, $W(N_t, x_t)$, $J_U(N_t, x_t)$, $J_N^W(N_t, x_t)$, and $J_N^{W'}(N_t, x_t)$. As in Christiano and Fisher (2000), we then exploit a convenient mapping from $\mathcal{E}_t$ to policy and multiplier functions to eliminate the need to parameterize the multiplier function separately.[4] We approximate the log productivity process, x_t, in equation (9) based on the discrete state space method of Rouwenhorst (1995).[5] We use 17 grid points to cover the values of x_t, which are precisely within four unconditional standard deviations from the unconditional mean of zero. For the N_t grid, we set the minimum value to be 0.001 and the maximum 0.99. The range is sufficiently large so that N_t never hits one of the bounds in simulations. On each grid point of x_t, we use cubic splines with 20 basis functions on the N_t space to approximate the five functions. We use extensively the approximation toolkit in the Miranda and Fackler (2002) CompEcon Toolbox in MATLAB. To obtain an initial guess, we use the loglinear solution to a simplified model without the fixed matching cost.

Figure 2 reports the approximation errors for the five functional equations from (25) to (29). The error for each equation is the left-hand side minus the right-hand side of the equation. The errors, in the magnitude no greater than 10^{-13}, are extremely small. As such, our projection algorithm does an accurate job in characterizing the competitive search equilibrium.

5 Quantitative Results

We examine the model's stationary distribution in Section 5.1, and quantify the model's performance in matching higher moments of unemployment in Section 5.2. Section 5.3 studies the second moments of the labor market. Section 5.4 contains nonlinear impulse response functions. Section 5.5 presents an array of comparative statics to illustrate the intuition behind the results.

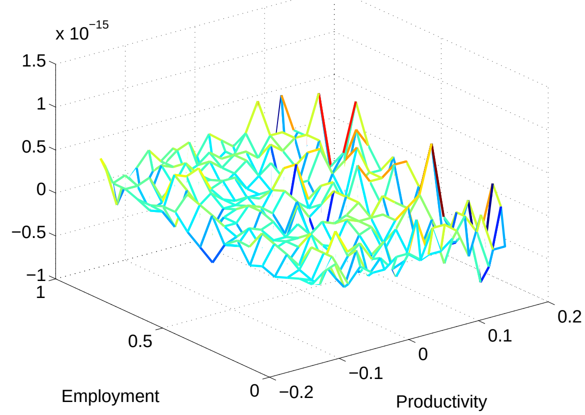
[4] Specifically, after obtaining the parameterized $\mathcal{E}_t$, we first calculate $\tilde{q}(\theta_t) \equiv \kappa_t/\mathcal{E}_t$. If $\tilde{q}(\theta_t) < 1$, the nonnegativity constraint is not binding, we set $\lambda_t = 0$ and $q(\theta_t) = \tilde{q}(\theta_t)$. We then solve $\theta_t = q^{-1}(\tilde{q}(\theta_t))$, in which $q^{-1}(\cdot)$ is the inverse function of $q(\cdot)$ from equation (6), and $V_t = \theta_t(1 - N_t)$. If $\tilde{q}(\theta_t) \geq 1$, the nonnegativity constraint is binding, we set $V_t = 0$, $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_t - \mathcal{E}_t$.

[5] Kopecky and Suen (2010) show that the Rouwenhorst method is more accurate than other methods in approximating highly persistent autoregressive processes.

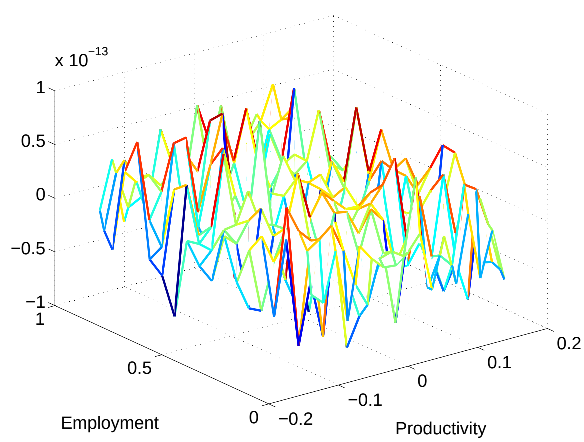
Figure 2 : Approximation Errors

The approximation errors are for the five functional equations from (25) to (29). The error for each equation is the left-hand side minus the right-hand side of the equation.

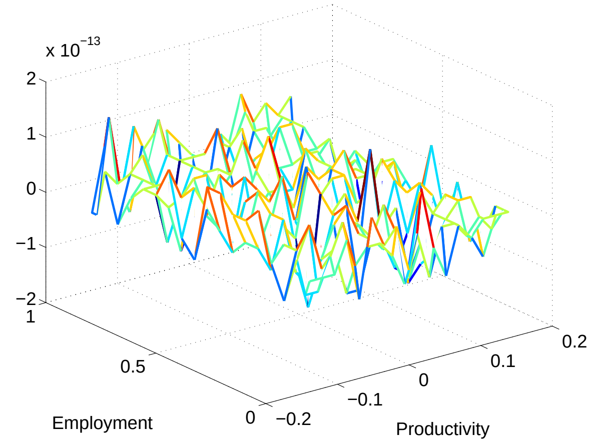
Panel A: The $\mathcal{E}(N_t, x_t)$ error



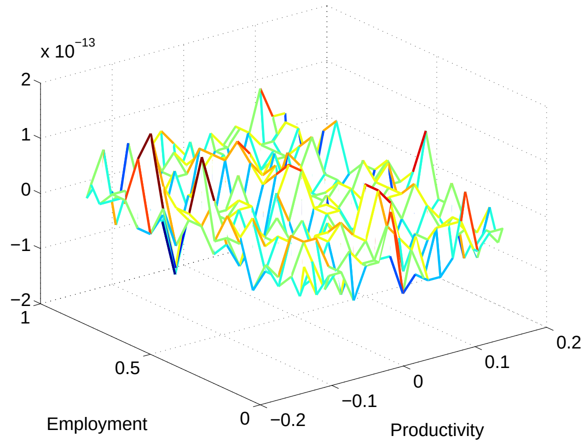
Panel B: The $W(N_t, x_t)$ error



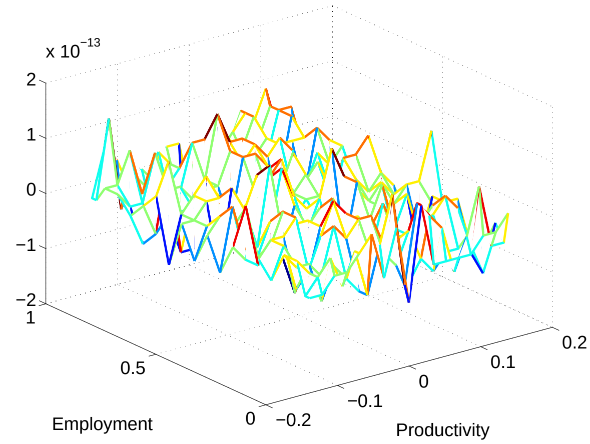
Panel C: The $J_U(N_t, x_t)$ error



Panel D: The $J_N^W(N_t, x_t)$ error



Panel E: The $J_N^{W'}(N_t, x_t)$ error



5.1 Properties of the Stationary Distribution

We simulate the economy for one million monthly periods from the model's stationary distribution. To reach the stationary distribution, we start at the initial condition of zero for log labor productivity and 0.90 for employment, and simulate the economy for 6,000 months. We perform a battery of diagnostics on the one-million-month simulated sample to study the model's stationary distribution.

Panel A of Figure 3 plots the unemployment rate against labor productivity in simulations. The relation is strongly nonlinear. When labor productivity is above its mean of unity, unemployment goes down only slightly. However, when labor productivity is below its mean, unemployment goes up drastically. The correlation between unemployment and productivity is -0.654.

Panel B plots labor market tightness, θ_t, against productivity. Although the relation is nonlinear, the nonlinearity is not nearly as dramatic as that of unemployment in Panel A. The main source of nonlinearity in Panel B seems the nonnegativity constraint on vacancy. When the constraint is binding with lower values of labor productivity, θ_t is fixed at zero. In contrast, when the constraint is not binding with higher values of productivity, the θ_t-productivity relation is (virtually) linear.

Figure 4 further illustrates the nonlinearity. Using the one-million-month simulated data, we report the empirical cumulative distribution functions of unemployment and labor market tightness. From Panel A, unemployment is highly skewed with a long right tail. The skewness of the unemployment rate is 6.12. The 2.5 percentile, 4.73%, is close to the median of 5.32%, but the 97.5 percentile is far away, 11.34%. Also, the 1 percentile is 4.69%, but the 99 percentile is 15.65%. In contrast, Panel B shows that the empirical distribution of the labor market tightness is largely symmetric. Finally, the minimum unemployment rate is 4.55%, but the maximum rate is 47.64%.

5.2 Explaining Higher Moments of Unemployment Quantitatively

Can the search and matching model explain quantitatively the nonlinear unemployment dynamics in the data, including the aggregate state transition matrix and the tail probability of the crisis state in Table 1? To this end, from the model's stationary distribution, we repeatedly simulate 100,000

Figure 3 : The Unemployment-productivity Relation and the Labor Market Tightness-productivity Relation

From the model's stationary distribution, we simulate one million monthly periods, and present the scatter plots of the unemployment rate versus productivity and labor market tightness versus productivity.

Panel A: Unemployment | Panel B: Labor market tightness

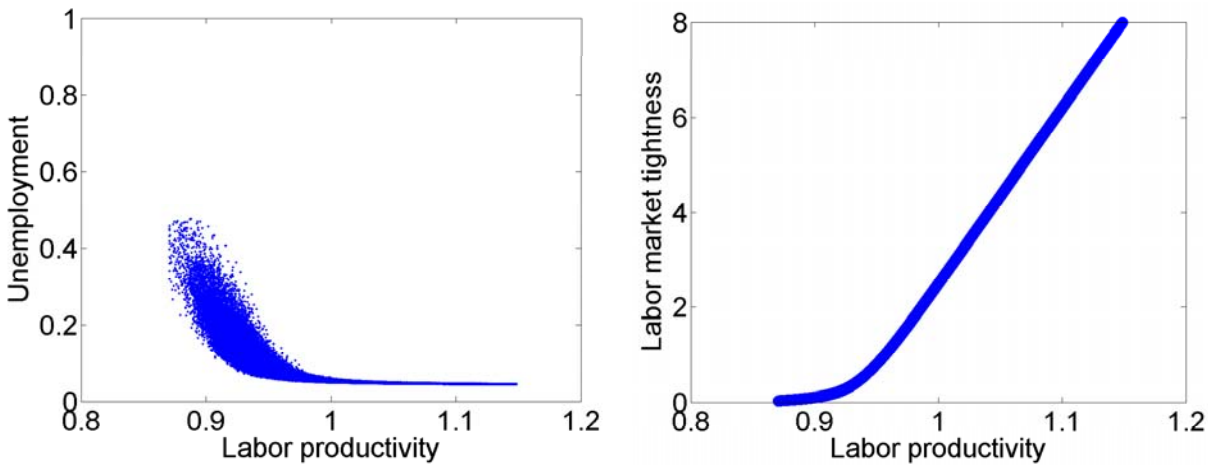
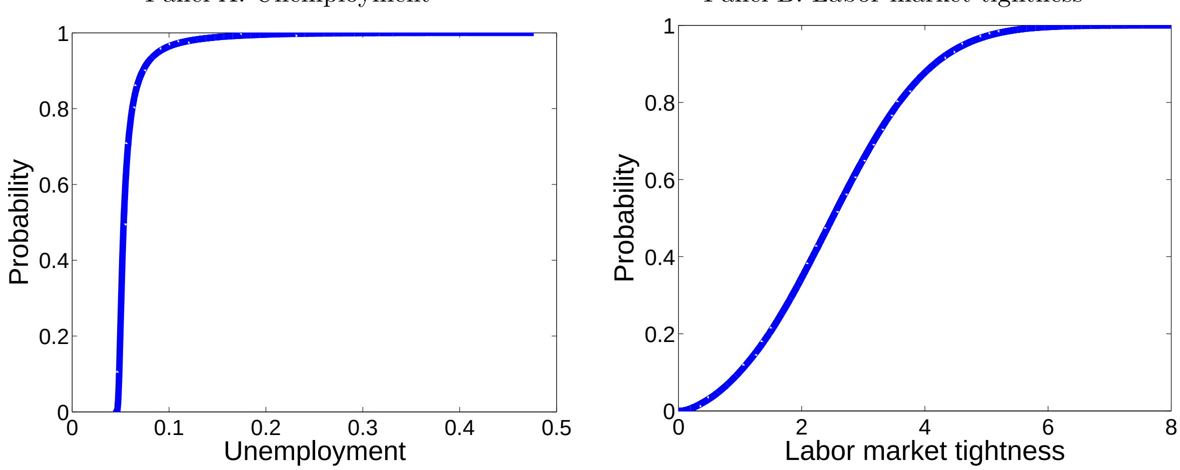


Figure 4 : Empirical Cumulative Distribution Functions of Unemployment and Labor Market Tightness

From the model's stationary distribution, we simulate one million monthly periods, and plot the empirical cumulative distribution functions for unemployment and labor market tightness.

Panel A: Unemployment | Panel B: Labor market tightness



artificial samples, each of which contains 1,005 monthly periods. The sample length matches the number of months in the data from April 1929 to December 2012.

Because crises, which are rare by definition, do not occur in every simulated sample, we split the 100,000 samples into two groups, non-crisis samples and crisis samples. If the maximum unemployment rate in an artificial sample is greater than or equal to 20%, we categorize it as a crisis sample (otherwise a non-crisis sample). The cutoff threshold of 20% is consistent with our empirical procedure in Section 2. Out of the 100,000 simulations, we have in total 17,412 crisis samples (17.41%). On each crisis sample (i.e., conditional on at least one crisis), we calculate the state transition matrix and unconditional probabilities of the states using the exactly the same procedure as in Table 1. We then report the cross-simulation averages and standard deviations across the crisis samples.[6]

Table 3 reports the model output. A comparison with Table 1 shows that the model does a good job in explaining the large unemployment dynamics in the data. In particular, the crisis state is almost as persistent in the model as in the data. The probability of the economy remaining in the crisis state next period conditional on the crisis state in the current period is 86.96%, which is close to 88.89% in the data. The switching probability from the crisis state to the bad state in the model is 12.92%, which is close to 11.11% in the data. In addition, the unconditional probability of the crisis state in the model is 3.18%. Although somewhat higher than 2.76% in the data, the cross-simulation standard deviation of this estimate is 7.72%. This high standard deviation is perhaps not surprising for a tail probability estimate. As such, the model's estimate seems empirically plausible.

To what a low level must the log productivity process fall to trigger a crisis with the threshold level of 20% unemployment rate? To obtain a quantitative sense, we first use an illustrative crisis sample. Panel A of Figure 5 plots the unemployment rate, and Panel C plots the log productiv-

[6]We have experimented with simulating only 5,000 artificial samples, and the quantitative results are hardly changed under the benchmark calibration. However, we find that in comparative static experiments (Section 5.5) the results with 5,000 simulations can be sensitive to the increase of the number of simulations. Intuitively, the percentage of crisis samples can be small. As a result, the cross-simulation averages can be sensitive due to the small number of crisis samples being averaged over. To ensure precision in our quantitative results, we opt to work with 100,000 artificial samples for each of the parameterizations studied in the paper.

Table 3 : Aggregate State Transition Matrix and Unconditional Probabilities of the Three Economic States, the Credible Bargaining Model

From the model's stationary distribution, we simulate 100,000 artificial samples, each with 1,005 months. We split the samples into two groups: non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum rate is greater than or equal to 20%). On each crisis sample, we calculate the state transition matrix and unconditional probabilities of the states as in Table 1. We report the cross-simulation averages and standard deviations (in parentheses) across the crisis samples.

	Good	Bad	Crisis
Good	0.9798	0.0202	0
	(0.0067)	(0.0067)	(0)
Bad	0.0210	0.9765	0.0025
	(0.0070)	(0.0071)	(0.0012)
Crisis	0	0.1292	0.8696
	(0)	(0.1874)	(0.1896)
Unconditional probability	0.4931	0.4744	0.0318
	(0.0451)	(0.0474)	(0.0772)

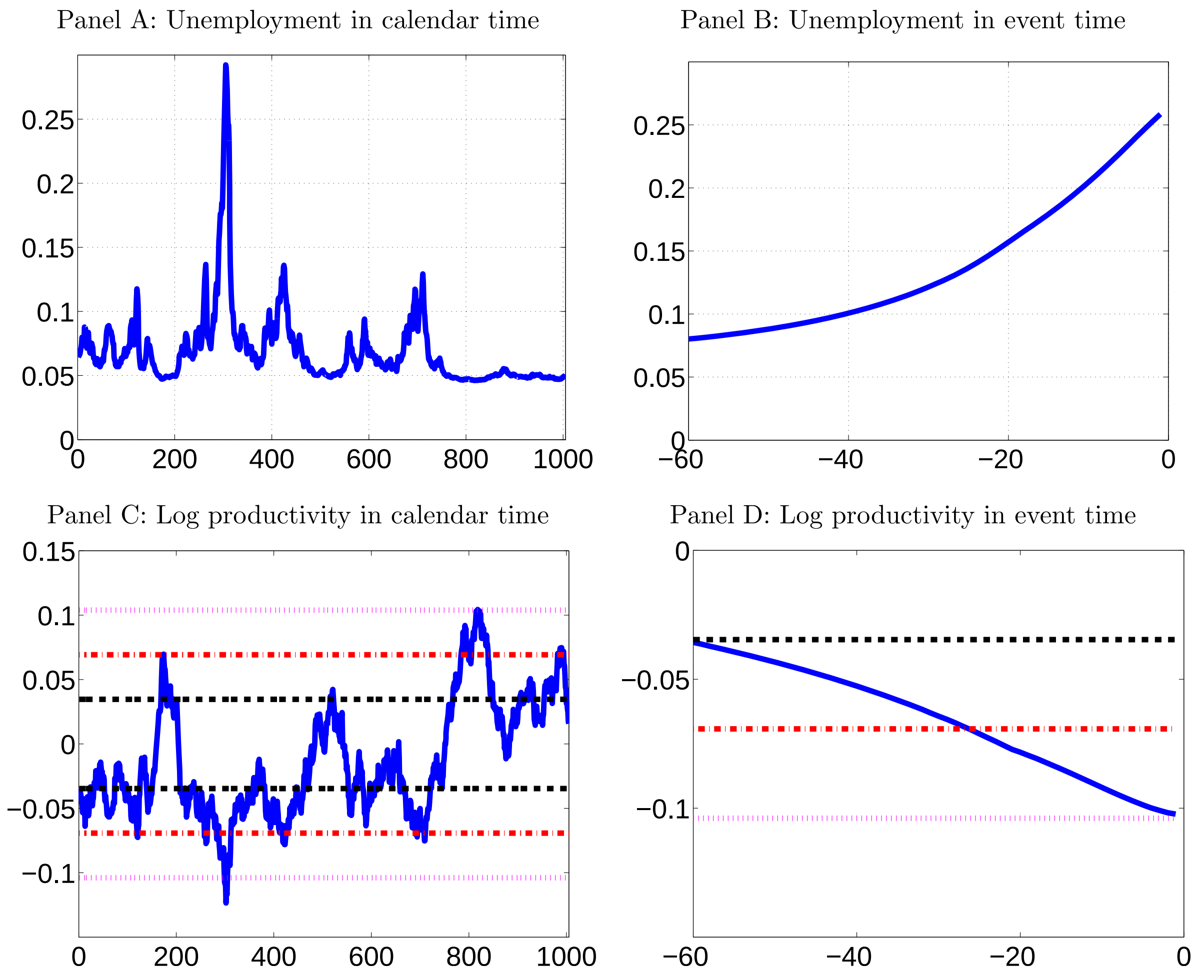
ity. A deep crisis occurs around the 300th month (the crisis sample has 1,005 months) with the log productivity drops more than three unconditional standard deviations below the unconditional mean of zero. In response, the unemployment rate hits almost 30%. Two other episodes with high unemployment rates around 13% occur shortly after the 400th month and the 700th month, when the log productivity dips just below the two-unconditional-standard-deviation bound.

The example is only illustrative. To provide a more precise answer, we conduct an event study using all the crisis samples out of the 100,000 simulations. In each crisis sample, we define the month in which the maximum unemployment rate occurs as event time 0. We record both the unemployment rates and the log productivity during 60 months prior to the event. We then average the pre-event unemployment rates and the pre-event log productivity across all the crisis samples. Panel B of Figure 5 plots the average unemployment, and Panel D plots the average log productivity, both in even time. By averaging across all the crisis samples, the event study makes the unemployment-log productivity relation (conditional on a crisis occurring) more precise.

Panels B and D are informative. When the log productivity falls one unconditional standard deviation below the unconditional mean of zero, the unemployment rate is about 8%. When the log

Figure 5 : An Illustrative Crisis Sample and An Event Study on Unemployment Crises

Using an illustrative crisis sample, we plot the unemployment rate in Panel A and log productivity in Panel C. For the event study, in each crisis sample, we define the period with the maximum unemployment rate as event time 0. We record the levels of both unemployment and log productivity during 60 months prior to the event (from month −60 to 0). We then average the pre-event unemployment rates and the pre-event log productivity across all the crisis samples. Panel B plots the average unemployment in event time, and Panel D plots the average log productivity in event time. In Panels C and D, the black dashed lines indicate one unconditional standard deviation above or below the unconditional mean of zero for log productivity. The red dashdot lines indicate two unconditional standard deviations above or below zero for log productivity, and the pink dotted lines indicate three unconditional standard deviations above or low zero.



productivity crosses the two-unconditional-standard-deviation bound, unemployment rises to 13%. The unemployment rate hits 28% when the log productivity falls three unconditional standard deviations below the mean. Using interpolation, we find that to reach the threshold of 20% unemployment, the log productivity needs to drop 2.62 unconditional standard deviations below zero.

5.3 The Impact on the Second Moments of the Labor Market

The nonlinear dynamics in the search model has important implications for the second moments such as volatilities and correlations, which are the traditional focus of the literature. In particular, it can be misleading to focus only on the second moments in normal periods (non-crisis samples). The second moments in the crisis samples can deviate greatly from those in normal periods.

We first report a standard set of second moments in the data as in Shimer (2005). Unlike the unemployment rate, the data for vacancies and labor productivity are available only for the postwar sample. We obtain seasonally adjusted monthly unemployment (thousands of persons 16 years of age and older) from Bureau of Labor Statistics (BLS), and seasonally adjusted help wanted advertising index from the Conference Board. The sample is from January 1951 to June 2006. (The Conference Board switched from help wanted advertising index to help wanted online index in June 2006. The two indexes are not directly comparable. We follow the common practice in the macro labor literature in using the longer time series before the switch.) We take quarterly averages of the monthly series to obtain quarterly observations. The average labor productivity is seasonally adjusted real average output per job in the nonfarm business sector from BLS.

It is customary to report all variables in log deviations from the HP-trend with a smoothing parameter of 1,600. In contrast, we detrend all variables as the HP-filtered cyclical component of proportional deviations from the mean with the same smoothing parameter. We do not take logs because vacancies can be zero in simulations when the nonnegativity constraint of vacancy is binding. (In the data, the two detrending methods yield quantitatively similar results.) Panel A of Table 4 reports the data moments in the postwar sample. The standard deviations of unemployment, vacancy, and the labor market tightness are 0.119, 0.134, and 0.255, respectively. Unemployment and vacancy have a correlation of -0.913, indicating a downward-sloping Beveridge curve.

To quantify the model's implications for the second moments of the labor market, we simulate 100,000 artificial samples, each with 1,005 months, from the model's stationary distribution. We again split the samples into two non-crisis samples and crisis samples, with the maximum unemployment rate of 20% as the threshold. We then apply the same empirical procedures as in Panel A of Table 4, and report the cross-simulation averages and standard deviations.

Panel B reports the results conditional on the non-crisis samples (normal periods). As a calibration target, the unemployment volatility is 0.109, which is close to 0.119 in the data. Although not a direct target, the standard deviation of vacancy is 0.146 in the model, which is also close to 0.134 in the data. However, the model predicts a standard deviation of labor market tightness of 0.185, which is lower than 0.255 in the data.[7] In addition, although negative, the unemployment-vacancy correlation is -0.572 in the model, which is lower in magnitude than -0.913 in the data. Finally, in contrast to the correlation of 0.299 between the labor market tightness and productivity in the data, the correlation is nearly perfect in the model, likely reflecting its single-shock structure.

However, focusing only on normal periods suffers from a sample selection bias that ignores the crisis samples. Panel C reports the quantitative results conditional on the crisis samples. The standard deviations of vacancy and labor market tightness are not much affected, 0.158 and 0.204 in the crisis samples versus 0.146 and 0.185 in the non-crisis samples, respectively. However, the unemployment volatility shoots up to 0.233, which is more than twice as large as 0.109 in normal periods. We apply the same procedure in Panel A of Table 4 on the unemployment rate series from April 1929

[7] The standard deviation of labor market tightness has traditionally been the focus of the macro labor literature, stimulated by Shimer (2005). Many studies argue that some modifications of the baseline search model can explain the volatility puzzle. However, reinforcing Shimer's key message, Petrosky-Nadeau and Zhang (2013) argue that once the model is solved accurately with a globally nonlinear algorithm, it is difficult to explain the volatility of labor market tightness without blowing up the volatility of unemployment. In particular, the authors show that loglinearization understates the standard deviation of unemployment, overstates the magnitude of the unemployment-vacancy correlation, and misses completely the nonlinear dynamics in unemployment. Petrosky-Nadeau and Zhang make their argument in the context of Hagedorn and Manovskii (2008). However, the argument is likely to apply more generally wherever a search model admits nontrivial nonlinear dynamics.

Table 4 : Labor Market Volatilities, Data and the Credible Bargaining Model

In Panel A, seasonally adjusted monthly unemployment (U, thousands of persons 16 years of age and older) is from the Bureau of Labor Statistics. The seasonally adjusted help wanted advertising index, V, is from the Conference Board. The sample is monthly from January 1951 to June 2006 (666 months). Both U and V are converted to quarterly averages of monthly series. We define the labor market tightness as $\theta = V/U$. The average labor productivity, X, is (quarterly) seasonally adjusted real average output per job in the nonfarm business sector from the Bureau of Labor Statistics. All variables are in HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600. In Panels B to D, we simulate 100,000 artificial samples from the model's stationary distribution, with 1,005 months in each sample. We split the samples into two groups: non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum rate is greater than or equal to 20%). We take the quarterly averages of monthly U, V, and X to convert to quarterly series. We implement the same procedure as in Panel A, and report cross-simulation averages and standard deviations (in parentheses). We do so conditionally on the non-crisis samples (Panel B) and on the crisis samples (Panel C) as well as unconditionally (Panel D).

	U	V	θ	X		U	V	θ	X
	Panel A: Data					Panel B: Non-crisis samples			
Standard deviation	0.119	0.134	0.255	0.012		0.109	0.146	0.185	0.013
						(0.053)	(0.022)	(0.034)	(0.001)
Autocorrelation	0.902	0.922	0.889	0.761		0.774	0.727	0.773	0.773
						(0.063)	(0.049)	(0.039)	(0.038)
Correlation matrix		−0.913	−0.801	−0.224	U		−0.572	−0.618	−0.664
							(0.135)	(0.131)	(0.106)
			0.865	0.388	V			0.989	0.988
								(0.008)	(0.008)
				0.299	θ				0.997
									(0.004)
	Panel C: Crisis samples					Panel D: All samples			
Standard deviation	0.233	0.158	0.204	0.014		0.136	0.148	0.188	0.013
	(0.049)	(0.020)	(0.032)	(0.001)		(0.065)	(0.019)	(0.029)	(0.001)
Autocorrelation	0.860	0.717	0.780	0.783		0.797	0.729	0.778	0.778
	(0.043)	(0.043)	(0.031)	(0.030)		(0.060)	(0.041)	(0.031)	(0.031)
Correlation matrix		−0.329	−0.363	−0.497	U		−0.501	−0.544	−0.608
		(0.078)	(0.090)	(0.081)			(0.138)	(0.141)	(0.106)
			0.977	0.973	V			0.987	0.985
			(0.010)	(0.011)				(0.008)	(0.009)
				0.982	θ				0.994
				(0.010)					(0.008)

to December 2012, and calculate the unemployment volatility in this long sample to be 0.219. As such, the unemployment volatility in the crisis samples in the model is close to that in the long sample in the data. The unemployment-vacancy correlation also varies a lot between non-crisis samples and crisis samples, -0.572 in the non-crisis samples versus -0.329 in the crisis samples (Panel C).

These results are intrinsically linked to the nonlinear dynamics shown in Figures 3 and 4. Precisely because unemployment exhibits a long right tail, ignoring unemployment crises by focusing only on the second moments in normal periods understates drastically the unemployment volatility. Also, because the nonlinearity of vacancy (and labor market tightness) is substantially weaker than that of unemployment, ignoring the crisis periods overstates the unemployment-vacancy correlation, but the standard deviations of vacancy and labor market tightness are not materially affected.

5.4 Nonlinear Impulse Response Functions

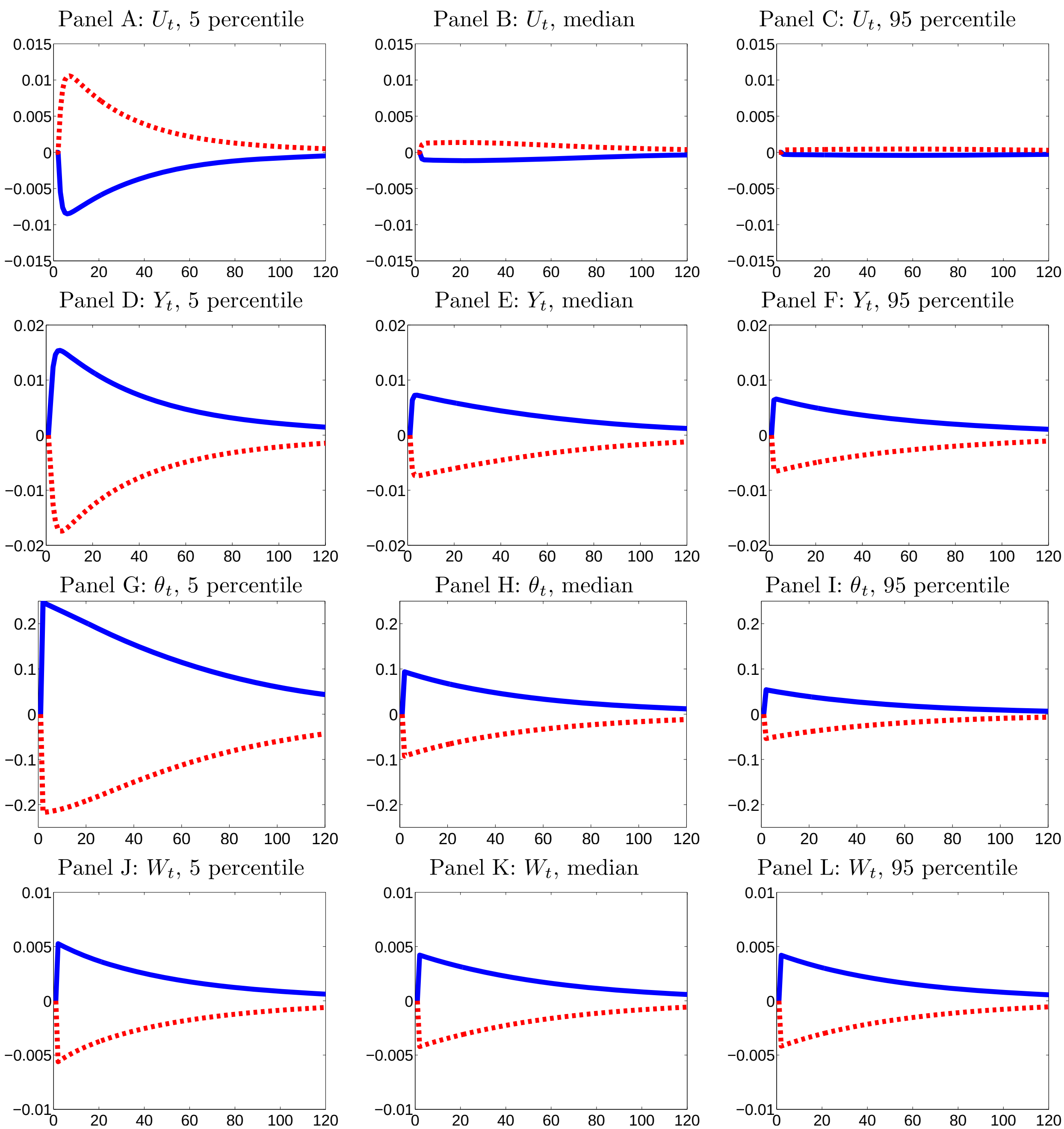
The nonlinear dynamics also impact on impulse response functions. The magnitude of the impulse responses depends on the initial point of simulations as well as the sign of the initial shock. We calculate the impulse responses from three different starting points: bad, median, and good economies. The bad economy is the 5 percentile of the model's bivariate stationary distribution of employment and log productivity ($U_t = 8.95\%, x_t = -0.0567$), the median economy is the median ($U_t = 5.32\%, x_t = 0$), and the good economy is the 95 percentile ($U_t = 4.78\%, x_t = 0.0564$). We compute the responses to a one-standard-deviation shock to log productivity, both positive and negative.

Figure 6 reports the nonlinear impulse response functions for the benchmark model. Two nonlinear patterns are clearly visible for unemployment. First, the impulse responses are substantially larger for the bad economy than for the good economy. After a negative one-standard-deviation shock, the unemployment rate shoots up by 1.06% in the bad economy (Panel A). This response is 20 times as large as the response of 0.05% in the good economy (Panel C). The response in the median economy is only 0.13% (Panel B), which is closer to that in the good economy.

Second, the responses to a negative shock are larger in magnitude than those to a positive shock

Figure 6 : Nonlinear Impulse Response Functions

We compute the impulse response functions from three different initial points: the five percentile, the median, and the 95 percentile of the model's bivariate distribution of employment and log productivity, respectively. The responses in unemployment are in levels. All the other responses are in percentage deviations from the respective values at a given initial point. The impulse responses are averaged across 100,000 simulations, each of which has 120 months. The blue solid (red broken) lines are the responses to a positive (negative) one-standard-deviation shock to log productivity.



of the same magnitude. As noted, in the bad economy, the unemployment rate shoots up by 1.06% in response to a negative one-standard-deviation shock. In contrast, with the same initial condition, the unemployment rate drops by only 0.85% in response to a positive one-standard-deviation shock, which is about 80% in magnitude of the response to a negative shock.

Similar asymmetric responses are present in output. In the bad economy, a negative one-standard-deviation shock to log productivity causes output to drop by 1.74% (Panel D). In contrast, in the good economy, output drops only by 0.65% (Panel F), which is about 37% of the response in the bad economy. In addition, the positive one-standard-deviation shock causes output to rise by 1.54%, which is somewhat smaller than the percentage drop in response to a negative shock.

The response in labor market tightness is also stronger in the bad economy than that in the good economy. However, in contrast to unemployment and output, the response to a positive shock is similar in magnitude to that to a negative shock, controlling for the same initial condition. In the bad economy, a negative impulse reduces the labor market tightness by up to 21.74%, and a positive impulse increases the tightness by 24.74% (Panel G). In contrast, in the good economy, a negative impulse reduces the tightness by 5.36%, and a positive impulse increases it by 5.39% (Panel I).

Panels J to L illustrate and quantify the limited response of wages to conditions in the labor market in the credible bargaining model. Even starting in the bad economy, a one-standard-deviation negative shock to log labor productivity only reduces the wage by a 0.56%. This wage response in the bad economy is quantitatively similar to that in the good economy, 0.42%.

5.5 Comparative Statics: What Drives Large Unemployment Dynamics?

To illustrate the intuition behind the large unemployment dynamics in the model, we conduct an array of comparative statics. We perform in total six computational experiments: (i) increasing the probability of breakdown in bargaining to $\delta = 0.15$; (ii) reducing the delaying cost to $\chi = 0.20$; (iii) reducing the proportional cost of vacancy posting to $\kappa_0 = 0.10$; (iv) removing the fixed cost of vacancy posting with $\kappa_1 = 0$; (v) reducing the job separation rate to $s = 0.035$; and (vi) reducing the matching function function parameter to $\iota = 0.625$. In each experiment, all the other parameters remain fixed as in the benchmark calibration. We quantify how the results reported in Tables 3 and 4 change as we vary each of the parameter values.

Table 5 reports the results for the crisis moments, and Table 6 for the second moments of the labor market. From Panel A of Table 5, increasing the probability of breakdown in negotiation, δ, weakens the crisis dynamics in the model. The percentage of the crisis samples out of 100,000 simulated samples drops from 17.41% under the benchmark calibration to only 0.15%. Conditional on the crisis samples, the persistence of crisis weakens somewhat from 0.87 to 0.84, and the unconditional crisis probability falls from 3.18% to 1.24%. Table 6 shows further that raising δ decreases the unemployment volatility in both non-crisis and crisis samples. A higher δ also leads to a higher magnitude of the unemployment-vacancy correlation, especially for normal times.

These results are intuitive. A higher probability of breakdown in negotiation brings credible bargaining closer to Nash bargaining. In the extreme case of $\delta = 1$, credible bargaining collapses to Nash bargaining, which implies a more flexible wage process. As such, a higher δ makes the equilibrium wage more responsive to labor market conditions. In bad times, productivity drops, but the wage also falls with the deteriorating state of the labor market for workers, providing the firm with incentives to create jobs. As such, the volatility of employment falls, consistent with Hall and Milgrom (2008). We push their argument further by showing that a more flexible wage process also substantially reduces the probability of unemployment crises.

The delaying cost is also quite important for matching the crisis moments. From Panel B of Table 5, reducing χ from 0.25 under the benchmark calibration to 0.20 lowers the percentage of the crisis samples out of 100,000 simulations from 17.41% to only 0.12%. Conditional on the crisis samples, the persistence of crisis weakens somewhat from 0.87 to 0.79, and the unconditional crisis probability falls from 3.18% to 1.44%. Panel B of Table 6 shows further that reducing χ lowers both the mean unemployment rate to 4.90% and the unemployment volatility to 0.027 in normal times. In the crisis

Table 5 : Comparative Statics, Aggregate State Transition Matrix and Unconditional Probabilities of the Three Economic States

We consider six comparative static experiments: (i) an increase in the probability of breakdown in bargaining to $\delta = 0.15$; (ii) a reduction in the delaying cost to $\chi = 0.20$; (iii) a reduction in the proportional cost of vacancy t0 $\kappa_0 = 0.10$; (iv) an elimination of the fixed cost of vacancy, $\kappa_1 = 0$; (v) a reduction in the job separation rate to $s = 0.035$; and (vi) a reduction in the curvature parameter of the matching function to $\iota = 0.625$. In each experiment, all the other parameters remain identical to those in the benchmark calibration. Under each alternative calibration, we simulate 100,000 artificial samples (each with 1,005 months) from the model's stationary distribution. We split the samples into two groups: non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum rate is greater than or equal to 20%). On each crisis sample, we calculate the state transition matrix and unconditional probabilities of the states per the procedure in Table 1, and report cross-simulation averages.

	Good	Bad	Crisis	Good	Bad	Crisis
	Panel A: $\delta = 0.15$ (% crisis samples = 0.15)			Panel B: $\chi = 0.20$ (% crisis samples = 0.12)		
Good	0.9813	0.0187	0	0.9808	0.0192	0
Bad	0.0192	0.9788	0.0021	0.0195	0.9785	0.0020
Crisis	0	0.1552	0.8448	0	0.2103	0.7897
Unconditional probability	0.5002	0.4874	0.0124	0.4962	0.4894	0.0144
	Panel C: $\kappa_0 = 0.10$ (% crisis samples = 11.63)			Panel D: $\kappa_1 = 0$ (% crisis samples = 8.04)		
Good	0.9797	0.0203	0	0.9800	0.0200	0
Bad	0.0206	0.9768	0.0026	0.0203	0.9773	0.0024
Crisis	0	0.2025	0.7967	0	0.2257	0.7732
Unconditional probability	0.4907	0.4863	0.0224	0.4941	0.4866	0.0186
	Panel E: $s = 0.035$ (% crisis samples = 1.96)			Panel F: $\iota = 0.625$ (% crisis samples = 42.41)		
Good	0.9807	0.0193	0	0.9797	0.0203	0
Bad	0.0197	0.9781	0.0022	0.0215	0.9749	0.0036
Crisis	0	0.1461	0.8534	0	0.1256	0.8737
Unconditional probability	0.4932	0.4839	0.0225	0.4959	0.4686	0.0351

Table 6 : Comparative Statics, Labor Market Volatilities

We consider six experiments: (i) the probability of breakdown in bargaining, $\delta = 0.15$; (ii) the delaying cost, $\chi = 0.20$; (iii) the proportional cost of vacancy, $\kappa_0 = 0.10$; (iv) the fixed cost of vacancy, $\kappa_1 = 0$; (v) the job separation rate, $s = 0.035$; and (vi) the elasticity of the matching function, $\iota = 0.625$. For each experiment, we simulate 100,000 artificial samples from the model's stationary distribution, with 1,005 months in each sample. We split the 100,000 samples into two groups: non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum rate is greater than or equal to 20%). We implement the same procedure as in Panel A of Table 4, and report the cross-simulation averages. We do so conditionally on the non-crisis samples and on the crisis samples.

	U	V	θ	X		U	V	θ	X
	Non-crisis samples					Crisis samples			
	Panel A: $\delta = 0.15$ (mean U in non-crisis samples: 5.51%)								
Standard deviation	0.062	0.118	0.148	0.013		0.178	0.127	0.165	0.014
Autocorrelation	0.778	0.737	0.774	0.774		0.880	0.734	0.784	0.789
Correlation matrix		−0.674	−0.713	−0.747	U		−0.311	−0.334	−0.498
			0.994	0.992	V			0.985	0.973
				0.998	θ				0.976
	Panel B: $\chi = 0.20$ (mean U in non-crisis samples: 4.90%)								
Standard deviation	0.027	0.100	0.115	0.013		0.244	0.114	0.131	0.014
Autocorrelation	0.761	0.759	0.774	0.774		0.843	0.755	0.786	0.789
Correlation matrix		−0.765	−0.785	−0.785	U		−0.231	−0.258	−0.327
			0.998	0.998	V			0.987	0.988
				1.000	θ				0.996
	Panel C: $\kappa_0 = 0.10$ (mean U in non-crisis samples: 5.17%)								
Standard deviation	0.086	0.153	0.179	0.013		0.250	0.167	0.198	0.014
Autocorrelation	0.750	0.751	0.774	0.773		0.850	0.746	0.780	0.783
Correlation matrix		−0.567	−0.597	−0.633	U		−0.266	−0.289	−0.425
			0.996	0.994	V			0.989	0.980
				0.998	θ				0.983
	Panel D: $\kappa_1 = 0$ (mean U in non-crisis samples: 5.31%)								
Standard deviation	0.069	0.128	0.155	0.013		0.214	0.140	0.172	0.014
Autocorrelation	0.762	0.743	0.774	0.774		0.855	0.736	0.781	0.784
Correlation matrix		−0.640	−0.675	−0.695	U		−0.309	−0.338	−0.438
			0.995	0.994	V			0.985	0.983
				0.999	θ				0.991
	Panel E: $s = 0.035$ (mean U in non-crisis samples: 4.36%)								
Standard deviation	0.096	0.140	0.174	0.013		0.259	0.152	0.192	0.014
Autocorrelation	0.774	0.736	0.774	0.774		0.882	0.726	0.779	0.784
Correlation matrix		−0.581	−0.622	−0.665	U		−0.256	−0.282	−0.450
			0.992	0.990	V			0.980	0.970
				0.997	θ				0.974
	Panel F: $\iota = 0.625$ (mean U in non-crisis samples: 10.25%)								
Standard deviation	0.081	0.125	0.176	0.013		0.111	0.136	0.191	0.014
Autocorrelation	0.841	0.658	0.772	0.772		0.871	0.660	0.779	0.781
Correlation matrix		−0.664	−0.748	−0.799	U		−0.557	−0.615	−0.714
			0.966	0.971	V			0.954	0.965
				0.992	θ				0.981

samples, however, the unemployment volatility, 0.244, is barely changed from 0.233 under the benchmark calibration. As such, the difference across normal times and crisis samples becomes starker with a lower χ as the crises become infrequent. Another indication of the starker difference is the unemployment-vacancy correlation, which is -0.765 in normal times but -0.231 in crisis samples.

The proportional and the fixed costs of vacancy posting impact the results in the same direction as the cost of delaying, but to a lesser extent quantitatively. From Table 5, reducing κ_0 from 0.15 to 0.10 lowers the persistence of the crisis state from 0.87 to 0.80 and its unconditional probability from 3.18% to 2.24%. The percentage of the crisis samples out of 100,000 simulations also drops from 17.41% to 11.63% (Panel C). Similarly, reducing κ_1 from 0.10 to zero lowers the persistence of the crisis state to 0.77 and its unconditional probability to 1.86%. Also, the percentage of crisis samples drops to 8.04%. From Table 6, lowering κ_0 and κ_1 also reduces the volatility of unemployment, especially in normal times. These results are again intuitive. Lower vacancy costs stimulate the firm to create jobs to starve off unemployment crises. In particular, the fixed matching cost buttresses the downward rigidity in the marginal cost of hiring. Removing the fixed cost weakens the rigidity, allowing the marginal cost of hiring to decline and more jobs to be created in recessions.

From Panel E of Table 5, reducing the job separation rate, s, makes crises less frequent and less persistent. Reducing s from 4.5% to 3.5% reduces the persistence of the crisis state somewhat from 0.87 to 0.85 and the unconditional crisis probability from 3.18% to 2.25%. A lower s also reduces the mean unemployment rate from 5.75% to 4.36%, but leaves the second moments of the labor market largely unaffected (Table 6). Intuitively, because jobs are destroyed at a slower rate, all else equal, the economy is more capable of offsetting job destruction flows through job creation. As such, the mean unemployment rate is reduced, and the crisis dynamics dampened.

Finally, from Panel F of Table 5, halving the curvature of the matching function makes the crisis state more frequent. The persistence remains around 0.87, but the unconditional crisis probability goes up somewhat from 3.18% from 3.51%. Intuitively, a decrease in ι increases the elasticity of matching with respect to vacancies.[8] As such, the congestion effect becomes greater in recessions to reinforce the crisis dynamics of unemployment. Panel F of Table 6 shows that more severe matching frictions also increase the mean unemployment rate to 10.25% in normal times, but reduce the unemployment volatility in both normal times and crisis samples. Intuitively, matching frictions work as labor adjustment costs to dampen the unemployment volatility (e.g., Shimer (2010)).

5.6 The Nash Wage

To examine the role of the Hall and Milgrom (2008) credible bargaining, we report the quantitative results from the baseline search model with the Nash wage. Instead of equation (22), we assume that the wage rate is derived from the generalized Nash bargaining process between the employed workers and the firm. As noted, the Nash wage is given by equation (17). Apart from the equilibrium wage, the rest of the model remains identical to the credible bargaining model.

The computation of the Nash bargaining model is substantially simpler than that of the credible bargaining model. Instead of five functional equations from (25) to (29), we only need to solve equation (25). We continue to approximate the conditional expectation in the right-hand side of equation (25) as $\mathcal{E}_t \equiv \mathcal{E}(N_t, x_t)$. We use the discrete state space method of Rouwenhorst (1995) with 17 grid points to approximate x_t. The N_t grid is again from the minimum value of 0.01 to the maximum of 0.99. Because the Nash bargaining model sometimes exhibits strong nonlinearity, we choose to use cubic splines with 40 basis functions on the N_t space to approximate $\mathcal{E}(N_t, x_t)$ on each grid point of x_t. To obtain an initial guess, we use the loglinear solution without the fixed matching cost.

Table 7 reports the monthly parameter values for the Nash bargaining model. We consider two calibrations, one with a high value of unemployment activities (small surplus) and the other one with a high fixed matching cost. Panel A reports the parameters common to both calibrations. The time discount factor, β, the job separation rate, s, the elasticity parameter in the matching function, ι, as well as the persistence, ρ, and the conditional volatility, σ, of the log productivity

[8] To be precise, the elasticity of matches with respect to vacancies function is given by $1/(1+\theta_t^{\iota})$. A decrease in ι increases this elasticity for $\theta > 1$, which holds in the model's simulations.

Table 7 : Monthly Parameter Values in the Nash Bargaining Model, The Small Surplus Calibration and the Fixed Matching Cost Calibration

Notation	Parameter	Value
	Panel A: Common parameters	
β	Time discount factor	$e^{-5.524/1200}$
ρ	Aggregate productivity persistence	$0.95^{1/3}$
σ	Conditional volatility of productivity shocks	0.00635
η	Workers' bargaining weight	0.045
s	Job separation rate	0.045
ι	Elasticity of the matching function	1.25
	Panel B: The small surplus calibration	
b	The value of unemployment activities	0.9
κ_0	The proportional cost of vacancy posting	0.3
κ_1	The fixed cost of vacancy posting	0.3
	Panel C: The fixed matching cost calibration	
b	The value of unemployment activities	0.71
κ_0	The proportional cost of vacancy posting	0.05
κ_1	The fixed cost of vacancy posting	3.1

remain the same as in the credible bargaining model.

The value of b involves two components. Mulligan (2012) estimates a median replacement rate in the U.S. of 63%, which covers a variety of income support programs available to workers. We also assume a leisure component in b. In Panel B (the small surplus calibration), we follow Hagedorn and Manovskii (2008), who argue that in a perfectly competitive labor market, b should be close to the value of employment. We choose a high value of $b = 0.90$, but also conduct a comparative static experiment with $b = 0.85$. We set the workers' bargaining weight, η, to be 0.045 so that the wage elasticity to labor productivity in the model is 0.50, which is close to the 0.45 estimate in Hagedorn and Manovskii. To calibrate κ_0 and κ_1, we then target the mean and volatility of the unemployment rate in U.S. postwar sample. This procedure yields $\kappa_0 = \kappa_1 = 0.3$, which imply a mean of 5.81% and a volatility of 10.7% for unemployment in the non-crisis state in the model.

Several authors such as Mortensen and Nagypál (2007) and Hall and Milgrom (2008) argue that the Hagedorn and Manovskii (2008) calibration of b is too high. As such, we also examine an alternative calibration that emphasizes the fixed matching cost, following Pissarides (2009). In this

Table 8 : Aggregate State Transition Matrix and Unconditional Probabilities of the Three Economic States, The Nash Bargaining Model with the Small Surplus Calibration

Results are based on 100,000 artificial samples, each with 1,005 months. We split the samples into two groups: non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum unemployment rate is greater than or equal to 20%). On each crisis sample, we calculate the state transition matrix and unconditional probabilities of the states per the procedure in Table 1. We report the cross-simulation averages and standard deviations (in parentheses) across the crisis samples.

	Good	Bad	Crisis	Good	Bad	Crisis
	Panel A: $b = 0.90$ (% crisis samples = 43.53)			Panel B: $b = 0.85$ (% crisis samples = 0.11)		
Good	0.9792	0.0208	0	0.9808	0.0192	0
Bad	0.0220	0.9744	0.0036	0.0195	0.9785	0.0020
Crisis	0	0.1175	0.8818	0	0.2708	0.7292
Unconditional probability	0.4941	0.4673	0.0381	0.4936	0.4871	0.0194

calibration we set $b = 0.71$ as in Hall and Milgrom. We then choose κ_0 and κ_1 to bring the mean and volatility of unemployment in the non-crisis state in the model as close as possible to those in the postwar sample. We end up with $\kappa_0 = 0.05$ and $\kappa_1 = 3.1$, which imply a mean of 5.79% and a volatility of 9.8% in normal times in the model. We also conduct comparative statics on κ_1.

Quantitative Results from the Small Surplus Calibration

Can the Nash bargaining model, when calibrated to the mean and volatility of unemployment in the postwar U.S. sample, explain quantitatively the higher moments of unemployment in the data (Table 1)? Panel A of Table 8 shows that the model with the small surplus calibration seems to do a good job. The persistence of the crisis state in the model is 0.88, which is very close to 0.89 in the data. The unconditional probability of the crisis state in the model is 3.81%. Although somewhat higher than 2.76% in the data, the cross-simulation standard deviation of this estimate is 7.40%. Finally, the percentage of crisis samples out of 100,000 simulations is 43.53%, which is higher than 17.41% in the credible bargaining model. As such, the Nash bargaining model seems to exhibit even stronger nonlinear unemployment dynamics than the credible bargaining model.

Panel A of Table 9 reports the second moments of the labor market both in normal times and

Table 9 : Labor Market Volatilities, the Nash Bargaining Model with the Small Surplus Calibration

All variables are in HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600. We simulate 100,000 artificial samples from the Nash bargaining model's stationary distribution, with 1,005 months in each sample. We split the samples into two groups: non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum rate is greater than or equal to 20%). We take the quarterly averages of monthly U, V, and X to convert to quarterly series. We implement the same procedure as in Panel A of Table 4, and report cross-simulation averages. We do so conditionally on the non-crisis samples and on the crisis samples.

	U	V	θ	X		U	V	θ	X
	Non-crisis samples					Crisis samples			
	Panel A: $b = 0.90$ (mean U in non-crisis samples: 5.81%)								
Standard deviation	0.107	0.128	0.165	0.013		0.451	0.163	0.189	0.014
Autocorrelation	0.757	0.709	0.772	0.772		0.824	0.636	0.781	0.781
Correlation matrix		−0.595	−0.657	−0.658	U		−0.262	−0.321	−0.354
			0.984	0.984	V			0.887	0.901
				1.000	θ				0.996
	Panel B: $b = 0.85$ (mean U in non-crisis samples: 4.97%)								
Standard deviation	0.022	0.085	0.100	0.013		0.244	0.099	0.115	0.014
Autocorrelation	0.770	0.757	0.774	0.774		0.778	0.735	0.792	0.792
Correlation matrix		−0.813	−0.834	−0.833	U		−0.242	−0.295	−0.292
			0.998	0.998	V			0.972	0.972
				1.000	θ				1.000

in crisis samples with $b = 0.90$. The unemployment volatility is 0.107 in normal times, which is not far from 0.135 in the data. However, similar to the credible bargaining model, the volatility of labor market tightness is 0.165, which falls short of 0.255 in the data. In the crisis samples, the unemployment volatility jumps up to 0.451, which is more than four times as large as that in normal times.

Unfortunately, the strong nonlinear dynamics in the Nash bargaining model hinge on the high value of $b = 0.90$. From Panel B of Table 8, reducing b from 0.90 to only 0.85 lowers drastically the percentage of crisis samples out of 100,000 simulations from 43.53% to 0.11%. Conditional on the crisis samples, the persistence of the crisis state drops from 0.88 to 0.73, and the unconditional crisis probability falls from 3.81% to 1.94%. Panel B of Table 9 shows further that with $b = 0.85$, the unemployment volatility is only 0.022 in normal times, which is substantially lower than 0.135 in the data. In untabulated results, we have also experimented with $b = 0.71$ as in Hall and Milgrom (2008) and Pissarides (2009). With $b = 0.71$, the model does not exhibit any crisis dynamics (the

Table 10 : Aggregate State Transition Matrix and Unconditional Probabilities of the Three Economic States, The Nash Bargaining Model with the Fixed Matching Cost Calibration

Results are based on 100,000 artificial samples, each with 1,005 months. We split the samples into two groups: non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum rate is greater than or equal to 20%). On each crisis sample, we calculate the state transition matrix and unconditional probabilities of the states per the procedure in Table 1. We report the cross-simulation averages and standard deviations (in parentheses) across the crisis samples.

	Good	Bad	Crisis	Good	Bad	Crisis
	Panel A: $\kappa_1 = 3.1$ (% crisis samples = 2.36)			Panel B: $\kappa_1 = 2.75$ (% crisis samples = 0.12)		
Good	0.9796	0.0204	0	0.9794	0.0206	0
Bad	0.0210	0.9768	0.0022	0.0203	0.9777	0.0021
Crisis	0	0.1186	0.8805	0	0.2476	0.7524
Unconditional probability	0.4938	0.4796	0.0261	0.4889	0.5017	0.0094

percentage of crisis samples out of 100,000 simulations is zero). Intuitively, a lower b increases the firm's profits and makes wages more elastic. When employment falls in recessions, wages drop as well, providing strong incentives of hiring. As such, crisis dynamics are dampened. Because a value of $b = 0.85$ is high relative to 0.71, we conclude that the Nash wage model with the small surplus calibration fails to provide a realistic microfoundation for unemployment crises.

Quantitative Results from the Fixed Matching Cost Calibration

We next ask whether the Nash bargaining model with a high fixed matching cost can explain the crisis dynamics of unemployment in the data. Panel A of Table 10 shows that this objective can be achieved when we push the fixed cost parameter, κ_1, to a high value of 3.1, while keeping the proportional cost parameter, κ_0, to be a low value of 0.05. The persistence of the crisis state in the model is 0.88, and its unconditional probability is 2.61%, both of which are close to those in the data. The percentage of crisis samples out of 100,000 simulations is 2.36%, which is not (very) close to zero. Panel A of Table 11 shows that the high fixed cost calibration also manages to fit the first two moments of unemployment in normal times. The mean of unemployment is 5.79%, and its standard deviation is 9.8% in the non-crisis state of the model.

Table 11 : Labor Market Volatilities, the Nash Bargaining Model with the Fixed Matching Cost Calibration

All variables are in HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600. We simulate 100,000 artificial samples from the Nash bargaining model's stationary distribution, with 1,005 months in each sample. We split the samples into two groups: non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum rate is greater than or equal to 20%). We take the quarterly averages of monthly U, V, and X to convert to quarterly series. We implement the same procedure as in Panel A of Table 4, and report cross-simulation averages. We do so conditionally on the non-crisis samples and on the crisis samples.

	U	V	θ	X		U	V	θ	X
	Non-crisis samples					Crisis samples			
	Panel A: $\kappa_1 = 3.1$ (mean U in non-crisis samples: 5.79%)								
Standard deviation	0.098	0.257	0.314	0.013		0.218	0.267	0.337	0.014
Autocorrelation	0.793	0.744	0.765	0.774		0.860	0.745	0.770	0.784
Correlation matrix		−0.473	−0.498	−0.771	U		−0.218	−0.231	−0.539
			0.997	0.897	V			0.994	0.853
				0.900	θ				0.851
	Panel B: $\kappa_1 = 2.75$ (mean U in non-crisis samples: 4.93%)								
Standard deviation	0.059	0.228	0.252	0.013		0.209	0.251	0.283	0.014
Autocorrelation	0.767	0.762	0.771	0.774		0.866	0.773	0.782	0.790
Correlation matrix		−0.509	−0.523	−0.709	U		−0.154	−0.162	−0.455
			0.999	0.958	V			0.998	0.903
				0.960	θ				0.903

A natural question is whether the high fixed cost calibration is empirically plausible. With $\kappa_0 = 0.05$ and $\kappa_1 = 3.1$, the marginal cost of hiring is on average 3.29, which is the average of $\kappa_0/q(\theta_t) + \kappa_1$ in simulations. Merz and Yashiv (2007) estimate the marginal cost of hiring in the data to be 1.48 (in terms of the average output per worker) with a standard error of 0.57. As such, 3.29 is too high because it is more than three standard errors from the empirical estimate of 1.48.

Panel B of Table 10 conducts a comparative static experiment when we vary κ_1 to 2.75. The marginal cost of hiring reduces to 3.10 in the model, which is still more than two standard errors from the empirical estimate of 1.48. Even with this extremely high fixed cost, the percentage of crisis samples in 100,000 simulations drops to only 0.12%. Conditional on a crisis sample, the persistence of the crisis state reduces to 0.75, and its unconditional probability to 0.94%. Panel B of Table 11 also shows that the unemployment volatility falls to 5.9% in the non-crisis sample, although the volatility of labor market tightness remains high, 25.2%. In untabulated results, we have also experimented with $\kappa_1 = 2.5$. The marginal cost of hiring is 3.02 in the model, which is still outside the two-standard-error bound of the empirical estimate. The model does not exhibit any crisis dynamics (the number of crisis samples out of 100,000 simulations is zero). As such, we conclude that the Nash bargaining model with the fixed cost calibration also fails to provide a realistic explanation of the crisis dynamics of unemployment in the U.S. data.

6 Conclusion

A search and matching model, when calibrated to the mean and volatility of unemployment in the postwar sample, can potentially explain the crisis dynamics of unemployment in the Great Depression. The key ingredient of the model is the Hall and Milgrom (2008) credible bargaining game for determinating the equilibrium wage. The limited impact of labor market conditions on the wage from credible bargaining and the congestion externality from matching frictions cause the unemployment rate to rise sharply in recessions but decline gradually in booms. The frequency, severity, and persistence of unemployment crises in the model are quantitatively consistent with U.S. historical time series from April 1929 to December 2012.

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A Derivations

To prove equation (22), we plug equations (20) and (21) into equation (18) to obtain:

$$W_t + E_t\left[\beta\left((1-s)J^W_{Nt+1} + sJ_{Ut+1}\right)\right] = b + \delta E_t\left[\beta\left(f_t J^W_{Nt+1} + (1-f_t)J_{Ut+1}\right)\right] + (1-\delta)E_t\left[\beta J^{W'}_{Nt+1}\right]. \tag{A.1}$$

Solving for W_t yields:

$$W_t = b + [\delta f_t - (1-s)]E_t\left[\beta J^W_{Nt+1}\right] + [\delta(1-f_t) - s]E_t\left[\beta J_{Ut+1}\right] + (1-\delta)E_t\left[\beta J^{W'}_{Nt+1}\right]. \tag{A.2}$$

Rearranging the right-hand side yields equation (22).

To characterize the worker's counteroffer, W'_t, as in equation (23), we first rewrite equation (12) recursively (while making explicitly the dependence of S_t on W_t with the notation S^W_t):

$$S^W_t = X_t N_t - W_t N_t - \kappa_t V_t + \lambda_t q(\theta_t)V_t + E_t\left[\beta S^W_{t+1}\right], \tag{A.3}$$

The first-order condition with respect to V_t yields:

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = E_t\left[\beta S^W_{Nt+1}\right]. \tag{A.4}$$

Also, replacing W_t with W'_t in equation (A.3) and differentiating with respect to N_t yield:

$$S^{W'}_{Nt} = X_t - W'_t + (1-s)E_t\left[\beta S^{W'}_{Nt+1}\right]. \tag{A.5}$$

Plugging equation (A.4) into the firm's indifference condition (19) yields:

$$S^{W'}_{Nt} = (1-\delta)\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t - \chi\right]. \tag{A.6}$$

Combining with equation (A.5) yields:

$$X_t - W'_t + (1-s)E_t\left[\beta S^{W'}_{Nt+1}\right] = (1-\delta)\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t - \chi\right]. \tag{A.7}$$

Isolating W_t' to one side of the equality:

$$W_t' = X_t + (1-\delta)\chi + (1-s)E_t\left[\beta S_{Nt+1}^{W'}\right] - (1-\delta)\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t\right] \tag{A.8}$$

$$= X_t + (1-\delta)\chi + (1-s)E_t\left[\beta S_{Nt+1}^{W'}\right] - (1-\delta)E_t\left[\beta S_{Nt+1}^{W}\right] \tag{A.9}$$

$$= X_t + (1-\delta)\chi + \beta E_t\left[(1-s)S_{Nt+1}^{W'} - (1-\delta)S_{Nt+1}^{W}\right], \tag{A.10}$$

which is identical to equation (23). Leading equation (A.6) by one period, plugging it along with equation (A.4) into equation (A.7), and solving for W_t' yield:

$$W_t' = X_t - (1-\delta)\left[\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t - \chi\right] - (1-s)E_t\left[\beta\left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda_{t+1} - \chi\right]\right]\right]. \tag{A.11}$$

We further characterize the agreement condition (24) as follows. Rewriting equation (A.5) with W_t and combining with equation (A.4) yield $S_{Nt}^W = X_t - W_t + (1-s)\left[\kappa_0/q(\theta_t) + \kappa_1 - \lambda_t\right]$. As such, the agreement condition becomes:

$$X_t - W_t + (1-s)\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t\right] + J_{Nt}^W > J_{Ut}. \tag{A.12}$$

Although equations (23) and (24) are easier to interpret, we implement equations (A.11) and (A.12) in our numerical algorithm.