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COMMODITY AND ASSET PRICING MODELS:
AN INTEGRATION

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ABSTRACT

We present a simple methodology that integrates commodity and asset pricing models. Given current evidence on the financialization of commodity markets, valuable information about commodity risk premiums can be extracted from asset pricing models and used to substantially improve the estimates of expected spot prices provided by current commodity price models. The methodology can be used with any pair of commodity and asset pricing models. An implementation of the methodology is presented using the Schwartz and Smith (2000) two-factor commodity price model and the CAPM. Reasonable expected spot prices are obtained without negative consequences in the model's fit to futures prices.

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1. Introduction

Stochastic models of commodity prices have evolved considerably during recent years. Using multiple factors, different specifications and modern estimation techniques, these models have been able to accurately fit commodity futures' term structures and their dynamics. While this fit implies that the parameters that determine the risk-adjusted process seem adequate, risk premiums (which affect the dynamics under the physical measure) are far from robust. Thus the expected spot prices obtained from these models may be highly unreliable.

To illustrate this issue consider the model presented in Schwartz and Smith (2000). Calibrating this model with COMEX copper data from January 2009 to February 2012, the five-year futures and expected five-year spot prices for each date are presented in Figure 1-1. It can be seen that for this example results are unreasonable, as it is very unlikely for expected spot prices in five years to be around 5 times the corresponding five-year futures price today, as shown in the figure.

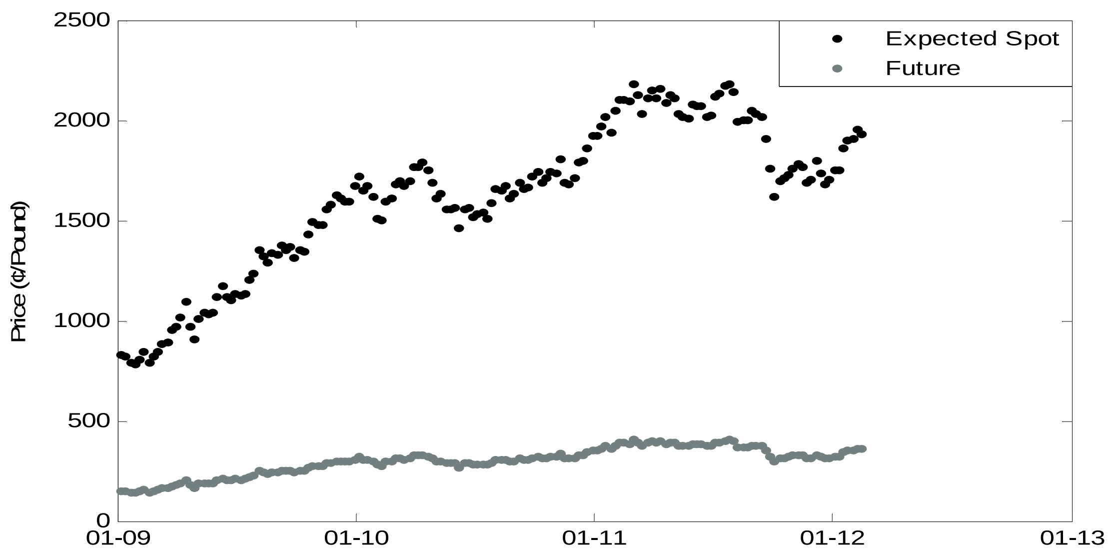

Figure 1-1: Five year Expected Spot and Futures prices for Copper using the Schwartz and Smith (2000) model.

It is well known that expected spot and futures prices should differ only on the risk premiums since futures prices are expected spot prices under the risk neutral measure. Thus, if these risk premiums are not well estimated, even though futures prices may not be affected, expected spot prices under the physical (true) measure will be[1].

Under a risk neutral framework, asset valuation can be done using futures prices to estimate cash flows and then discounting them at the risk free rate. This makes future expected spot prices unnecessary for valuation purposes. While this is true, the commodity price distribution under the physical measure is still important. The reason for this is twofold. First, the true distribution is useful for purposes other than valuation, for example, for risk management (i.e. calculations of the *Value at Risk*).

Second, many practitioners still do not use the risk neutral approach for valuing natural resource investments, but instead use commodity price forecasts and then discount the expected cash flows at the weighted average cost of capital[2]. Thus, not only the risk adjusted process is of interest for users of commodity models, but also the dynamics under the physical measure. Moreover, the fact that commodity models may provide such unreasonable estimations of expected spot prices limits the credibility and practical use of these commodity models altogether.

There is a separate strand on the finance literature that deals with asset pricing models which has been largely ignored in the commodity pricing literature. One explanation for this may be that commodity futures in the past were generally traded by non-financial institutions and therefore didn't behave as a classic financial asset. However, this has changed in recent years. Commodities have attracted a growing interest from the financial world and have started to be viewed as an asset class on their own. This issue has generated a large literature on the financialization of commodity markets. While the debate is still ongoing, there is considerable empirical evidence that commodity futures are behaving more like classic financial assets and are being included as an asset class in portfolio allocation algorithms.

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[1] In an independent work, Heath (2013) also finds that a futures panel is well suited for estimating the cost of carry, relevant for futures prices, but not the risk premiums, required for expected spot prices, as will be described later.

[2] The International Valuation Standards Council (IVSC) released the discussion paper *Valuation in the Extractive Industries* in July 2012. Different questions about valuation methodologies where stated in this paper which industry participants were invited to answer. These answers where published and can be accessed at http://www.ivsc.org/comments/extractive-industries-discussion-paper. Respondents include the Valuation Standards Committee of the SME, The VALMIN Committee, the CIMVal committee and the American Appraisal Associates among others. Most of the respondents stated that their main method of valuation was a discounted cash flow analysis (DCF) using various methods of price forecasting. For the discount factor the most widely used method was a weighted average cost of capital (WACC) based on the Capital Asset Pricing Model (CAPM).

Therefore, considering the rise of commodities as an asset class and their financialization, commodity pricing models should not ignore information about risk premiums that could be obtained from classic asset pricing models. This paper proposes to integrate these two types of models by extracting information from the latter and using it in stochastic commodity pricing models. We show that it is possible to obtain more reliable estimates not only of futures prices but also of expected spot prices, thus making commodity models more credible.

The remaining of the paper is as follows: Section 2 presents evidence on the financialization of commodity futures markets, Section 3 gives a short review on different commodity and asset pricing models, Section 4 describes the integration methodology that we propose and Section 5 presents the results of implementing our methodology for Copper and Oil futures. Finally, Section 6 concludes.

2. Evidence on the Financialization of Commodity Futures Markets

Extensive debate has emerged recently about the financialization of commodity markets. According to Henderson et al. (2012) financialization is the process by which the financial sector has gained influence relative to the real sector over the behavior of commodity prices. The authors point out two strands of the literature about financialization: one which describes changes in the trading and positions of investors in the commodity markets, while the other one analyzes changes in the price dynamics that might be explained by the new inflow of financial investors.

2.1. Changes in Positions and Volume

Commodity futures markets have experienced big changes since the beginning of the 21st century. Open interest in commodity futures markets were significantly larger in 2010 than those observed a decade earlier [Büyüksahin and Robe (2012b)]. Figures 2-1 through 2-3 show the open interest for three commodities: WTI Oil, Wheat and Copper which grew 212%, 270% and 99%, respectively, during the decade.

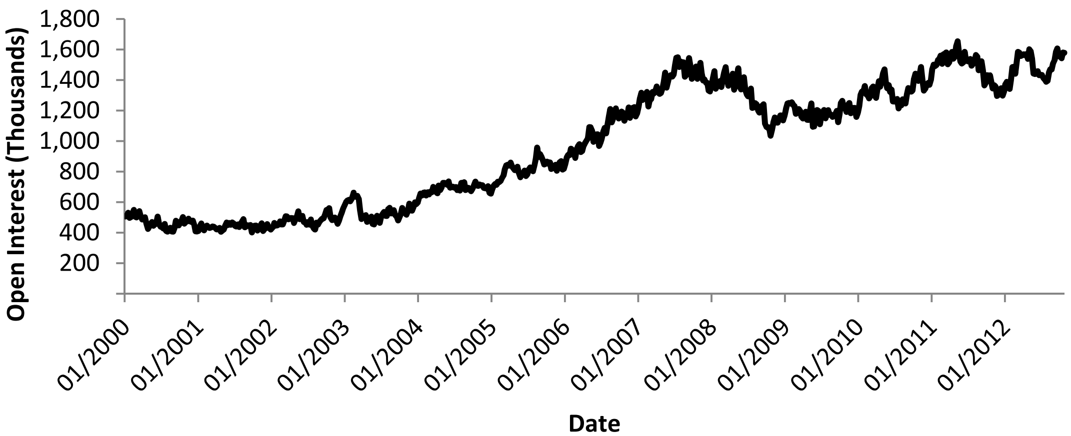

Figure 2-1: Crude Oil WTI Open Interest (NYMEX).
Source: CFTC

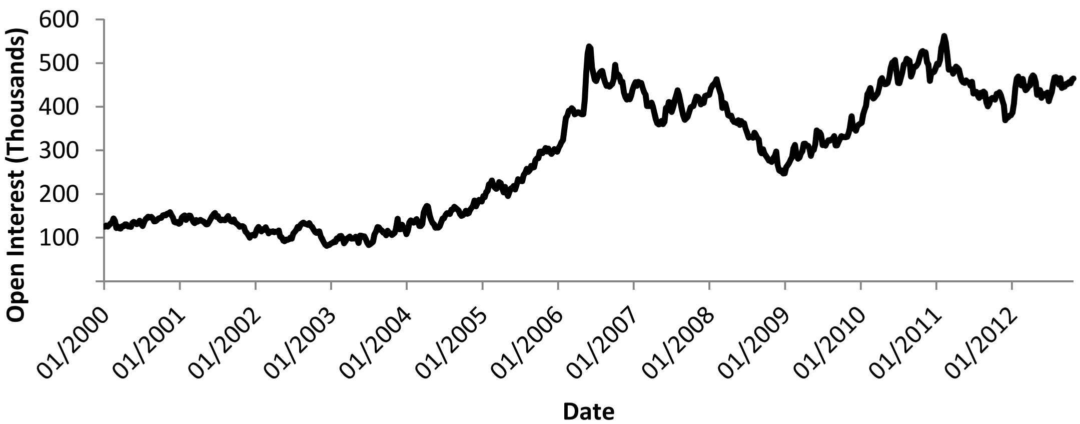

Figure 2-2: Wheat Open Interest (CBOT).
Source: CFTC

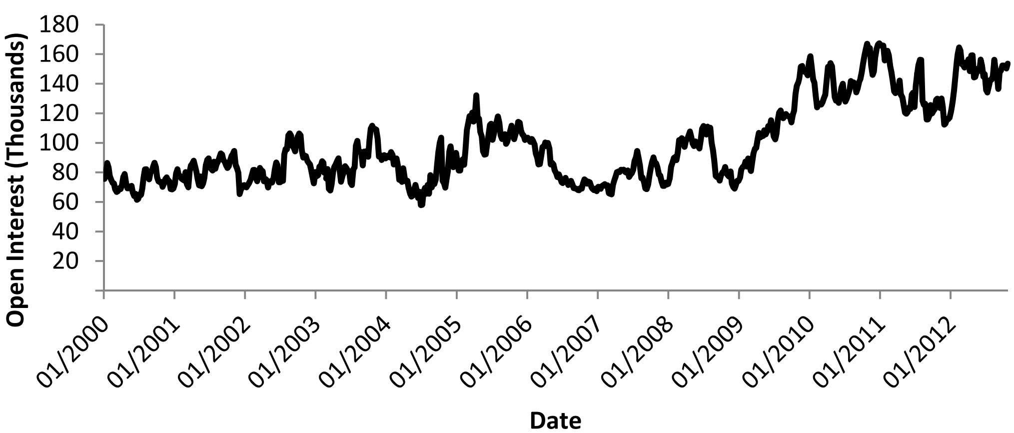

Figure 2-3: Copper Open Interest (COMEX).
Source: CFTC

Figure 2-4 and Figure 2-5 show the traded volume for the three shortest maturity contracts between 2000 and 2012 for oil and copper, respectively. Both figures show a relatively constant volume for the first years of the decade and a sharp increasing trend starting in 2007 for oil and in 2009 for copper. Similar behavior can be observed for Commodity Exchange Traded Funds (Commodity ETF). Figure 2-6 and Figure 2-7 show transaction volume for two different commodity ETF's. Again in line with the financialization process, both figures show very low volume for the first years and a significantly higher volume since mid- 2009.

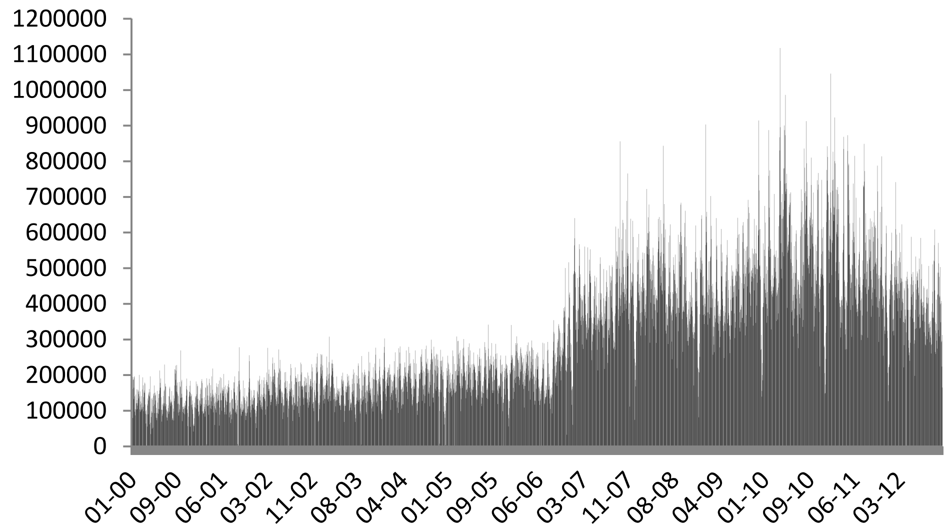

Figure 2-4: Transaction Volume for Oil.
Traded volume for the three shortest contracts available for each day. Source: Bloomberg.

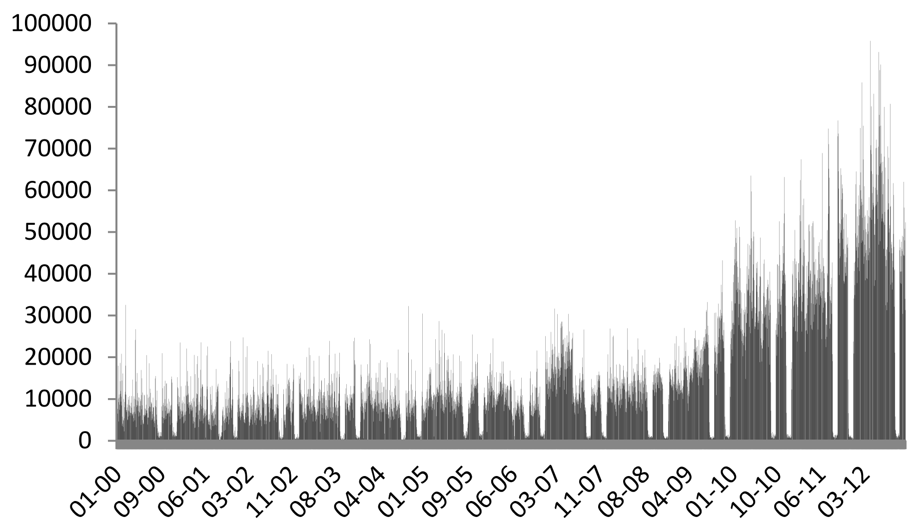

Figure 2-5: Transaction Volume for Copper.
Traded volume for the three shortest contracts available for each day. Source: Bloomberg.

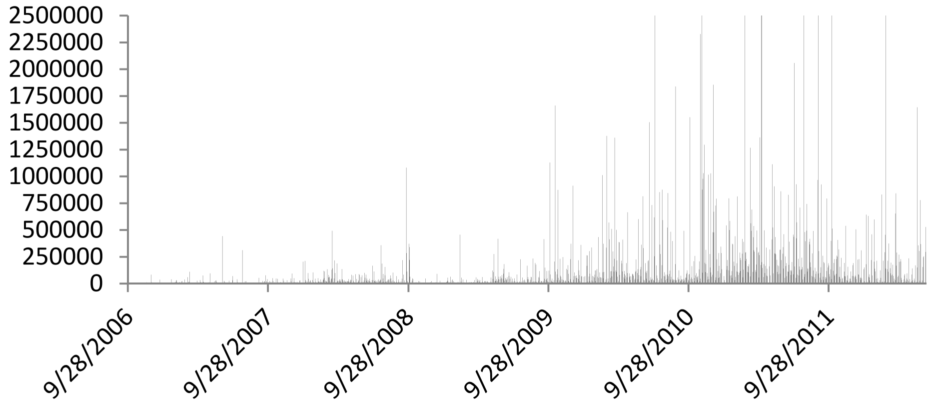

Figure 2-6: Daily Transaction Volume for ETFS All Commodities DJ-AIGCISM (AIGC).
This ETF tracks the DJ-AIG Commodity Index SM.

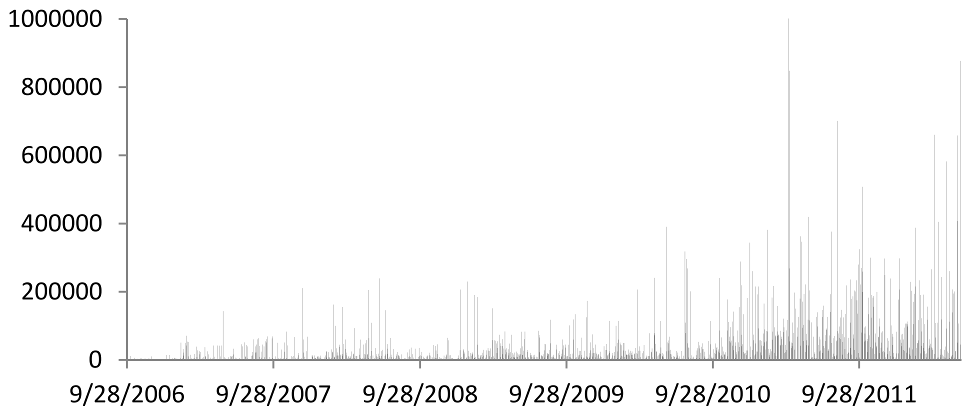

Figure 2-7: Daily Transaction Volume for ETFS Copper (COPA).
This ETF tracks the DJ-AIG Copper Sub-Index SM.

This increase in commodity futures market activity can be partly explained by the use by financial institutions and investors from the financial sector of commodities as a new asset class to be included in their investment portfolios. New interest for investing in commodities has been motivated in part by empirical research that found positive historical returns together with low or even slightly negative equity-commodity correlations and positive inflation-commodity correlations [Gorton and Rouwenhorst (2006), Erb and Harvey (2006)], increasing the appeal of these assets.

Not only did open interest in commodity futures market increase, but also the proportion of participants from the financial sector taking positions on commodity futures rose sharply. For example, the market share of financial traders in WTI oil market went from less than 20% in 2000 to more than 40% in 2010 [Büyüksahin and Robe (2012a)]. This change in investor-type distribution is largely explained by increased commodity index funds investments [Irwin and Sanders (2011)] and money managers (hedge funds) positions [Büyüksahin and Robe (2012b)].

Some additional information that has been made public by the Commodity Futures Trading Commission (CFTC) is summarized in Figures 2-8 to 2-10. These figures show how the positions in the corresponding commodity are distributed between different types of investors[3]. Even though this information only dates back to 2006[4] some interesting trends can still be observed. For WTI oil, positions by the "producer/merchant/processor/user" category dropped from more than 30% in 2006 to less than 18% in 2012. This drop had its counterpart in the other three categories that have seen their share grow during the same time period. A similar trend can be seen for wheat futures. Even though for copper the drop in the "producer/merchant/processor/user" category isn't as sharp as for the other commodities, the increase in the "money manager" positions is considerable going from around 16% in 2006 to more than 24% in 2012. All of this clearly point towards the idea of the financialization of commodities, as financial institutions are having a higher presence in commodity markets.

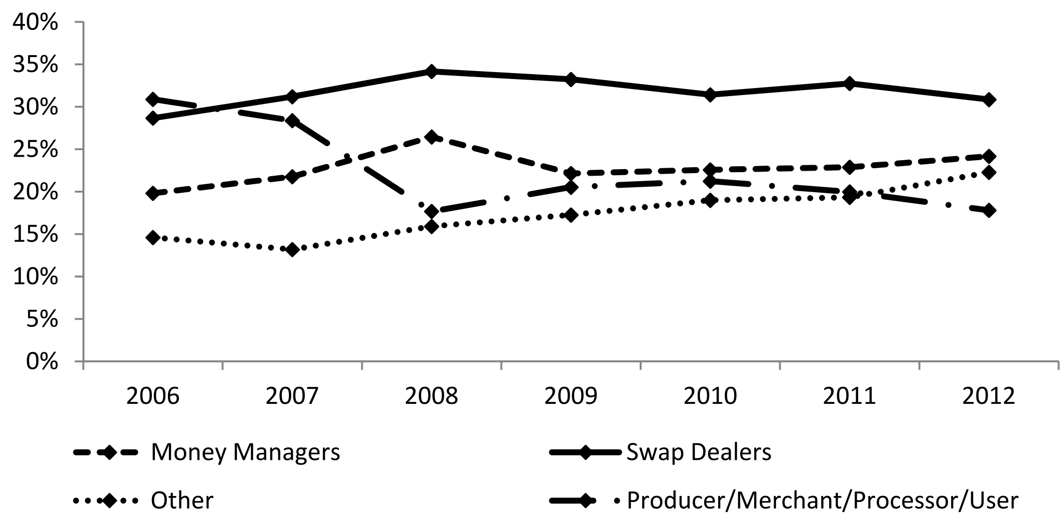

Figure 2-8: Crude Oil WTI Distribution of Positions by Trader Category (NYMEX).
Source: Calculated from CFTC Data

[3] Further explanation on how the raw CFTC data was processed to obtain these figures is available in appendix A.
[4] Other work such as the cited Büyüksahin and Robe (2012b) has non-public data available which dates further back than 2006.

Figure 2-9: Wheat Distribution of Positions by Trader Category (CBOT). Source: Calculated from CFTC Data

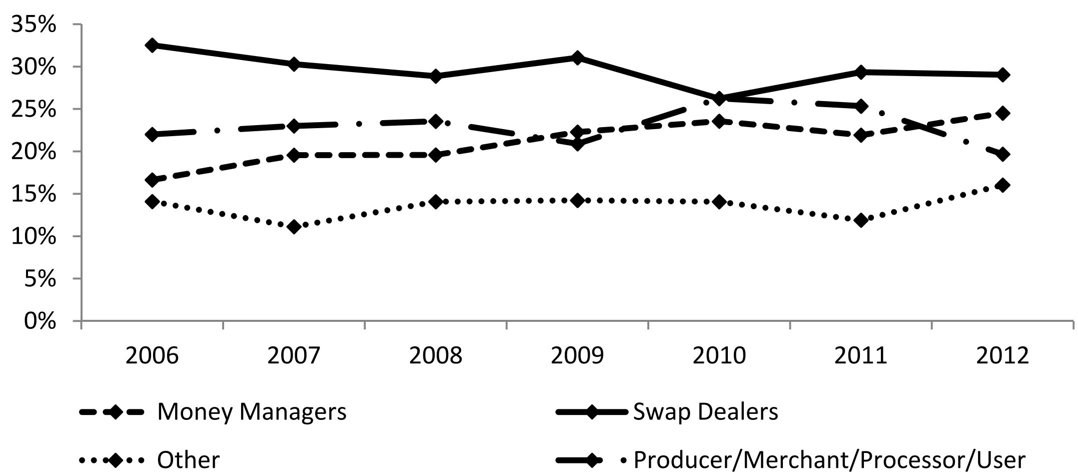

Figure 2-10: Copper Distribution of Positions by Trader Category (COMEX). Source: Calculated from CFTC Data

2.2. Changes in Price Dynamics

At the same time when changes in investor positions occurred, and perhaps because of it, there has been a change in commodity futures price behavior. In particular, four effects have been documented: increases in price volatility [Tang and Xiong (2012)]; increases in the correlation between the returns of different commodities [Tang and Xiong (2012)]; increases in the correlation of commodity returns with various market factors, such as stock market returns [Büyüksahin and Robe (2012b)]; and changes in the pricing of risk [Hamilton and Wu (2013a)]

Of particular interest is the increase in the correlation between commodity and equity markets since there is a substantial change in relation to what was observed in previous studies [Gorton and Rouwenhorst (2006), Erb and Harvey (2006)] which showed that equity-commodity correlation was rather low. Until May 2008, the correlation between commodity and equity indexes had not experienced any significant change, maintaining their fairly variable behavior over time [Büyüksahin et al. (2010)][5]. However, in more recent work Büyüksahin and Robe (2012b) show that from September 2008 the correlation between stocks and commodities has experienced a sharp increase remaining at a high level until the end of the time window considered (year 2010). Tang and Xiong (2012) and Silvennoinen and Thorpe (2013) find similar results.

This correlation increase should be reflected in the β coefficient of the Capital Asset Pricing Model (CAPM) applied to commodity futures. A time series of β coefficients for oil and copper are presented in Figure 2-11 and Figure 2-12, respectively. The values are obtained using a two-year rolling window of weekly returns[6]. The figures show the results of the estimation for contracts of three different maturities, in which a stable growing trend can be observed from 2009 to 2012[7].

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[5] On the other hand, Chong and Miffre (2010) using data that comprises the period 1980-2006, conclude that the correlation between commodities and stock indexes have declined over time.

[6] Further explanations of the calculation are presented in section 4.1.

[7] Considering the two-year rolling window the data is from 2007 to 2012

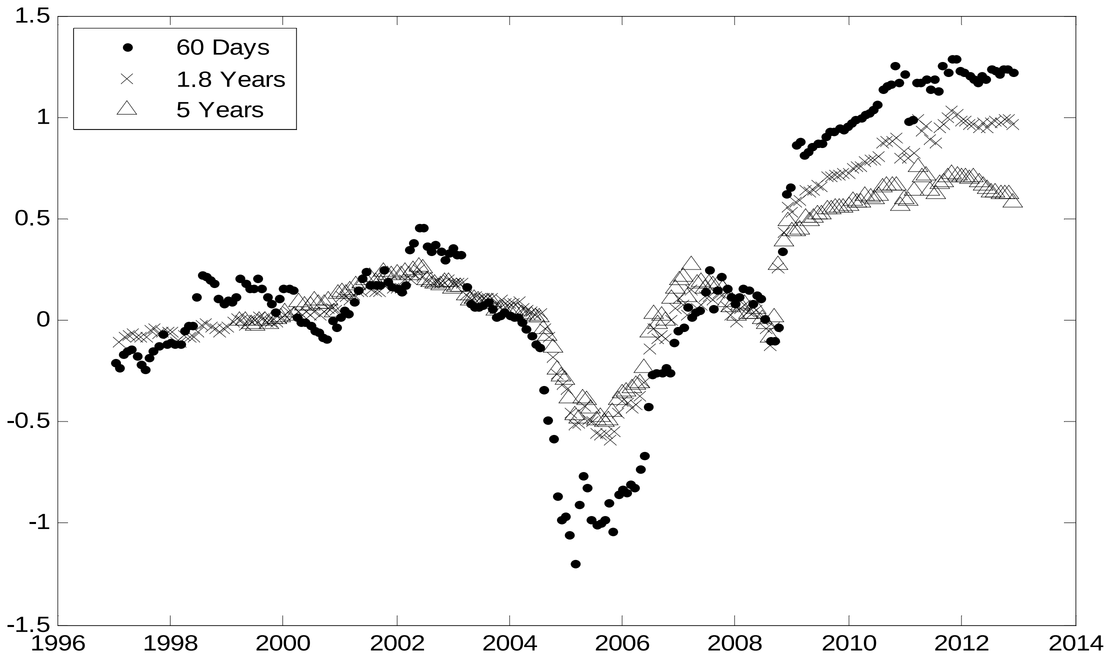

Figure 2-11: β Evolution for Oil.
2-year rolling window β coefficients for contracts of different maturities.

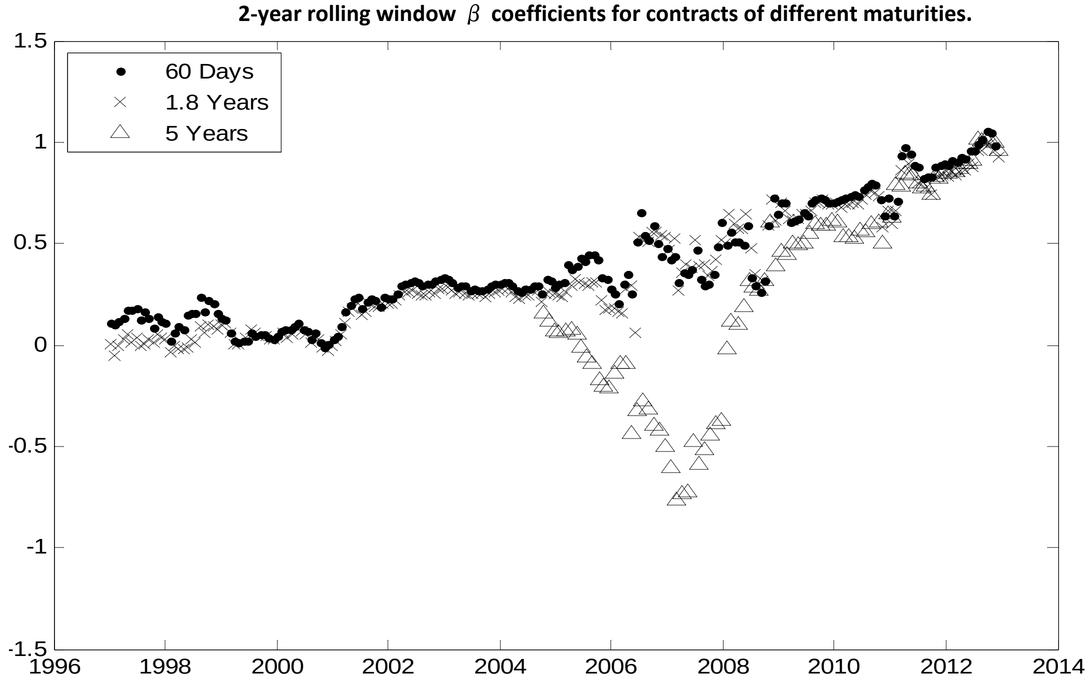

Figure 2-12: β Evolution for Copper.
2-year rolling window β coefficients for contracts of different maturities.

There is an extensive literature that studies the linkage of investor positions and price dynamics. Empirical research in this area has reached different conclusions. Büyüksahin and Harris (2011) find little evidence that traders' activity caused price changes in crude oil futures market from 2000-2008. Similarly, Brunetti et al. (2011), with data for 2005-2009, conclude that positions of different types of investors (including swap dealers and hedge funds) have no effect on prices but are effective in reducing volatility. Similarly, Sanders and Irwin (2011a and 2011b) find that swap dealers and index trader's positions did not help predict returns for most of the commodities under study and Hamilton and Wu (2013b) conclude that there is little evidence that index-fund investing has a considerable impact on commodity futures prices.

In contrast, Mou (2011), using data from 2000 to March 2010, finds out that index traders' activity, when rolling over between contracts of different maturities, has a significant impact on price levels. Additionally, Mayer (2012), using Granger causality tests, concludes that the positions of commodity index investments caused changes in prices for several commodities (soybeans, soybean oil, oil and copper) throughout their sample period (July 2006 - June 2009), while the positions of hedge funds only affected copper and oil during what they considered the crisis period (June 2007 - June 2008). In turn, Singleton (2012) shows that changes in spread positions of hedge funds and index traders in medium-term futures caused price changes in the period September 2006 - January 2010.

In terms of the correlation increase between equity and commodity futures' returns Büyüksahin and Robe (2012b) conclude that hedge funds' activity in commodity futures helps explain the rise in their correlation with equity during the 2000-2010 period. Furthermore, they find that hedge funds that actively trade in both markets are especially relevant while positions from investors outside of the hedge fund category have little explanatory power over the equity-commodity correlation. In turn, Tang and Xiong (2012) find that this correlation rises more sharply for futures belonging to indexes usually used as benchmark (GSCI y DJ-UBSCI) than for those that don't belong to these indices.

While these last studies seem to provide solid evidence on the effects of changes in investor behavior, these findings should be taken with caution because of some problems arising from the causality tests used [Irwin and Sanders (2011)] and how index traders' positions are computed or approximated [Irwin and Sanders (2011, 2012), Singleton (2012)].

In summary, while debate is still ongoing about the relation between investors' positions and price levels, evidence on the influence of the financial sector over commodity-equity correlation seems to be strong, supporting the financialization of commodity futures markets and the emergence of commodities as an asset class.

3. Some Alternative Approaches for Modeling Prices

There have been two main approaches for modeling prices and returns: Stochastic Pricing Models, which use no-arbitrage arguments to define price dynamics, and Asset Pricing Models, which estimate risk premiums that should be earned by investors in equilibrium.

The first type of models has been the main approach used for commodity futures. Several of these models have been proposed during the last decades. Their specification varies considerably depending on the number and interpretation of the state variables that model the underlying risk [Gibson and Schwartz (1990), Schwartz (1997), Schwartz and Smith (2000), Cortazar and Schwartz (2003), Cortazar and Naranjo (2006)].

These models are calibrated using futures panel data[8]. They assume that there are no-arbitrage opportunities in trading within these contracts and that the underlying process for commodity prices may be derived using only futures prices. These models have gained wide acceptance because of their success in accurately fitting the observed futures term structure and its dynamics. However, while the estimation of futures prices is adequate, the estimation of risk premiums may be unreasonable, such as those presented in Figure 1-1. In addition it is often the case that risk premium parameters estimated with these models are statistically insignificant.

Singleton (2012) points out that commodity excess return is given by the risk premium minus the convenience yield. Because of this an accurate model of commodity price dynamics must capture the effect of these two variables. As in Heath (2013), we argue that futures contracts data contains enough information to ensure a correct estimation of the cost of carry[9] (which is relevant for futures prices) but not necessarily of the risk premiums (which are required for obtaining expected future spot prices).

[8] Futures prices for contracts with different maturities and dates.
[9] The cost of carry (c_t) is given by $c_t = r_t - \delta_t$, where r_t is the risk free rate and δ_t is the convenience yield.

Previous work with commodity models that add new information, in addition to futures prices, include Schwartz (1997) and Casassus and Collin-Dufresne (2005), which include bond prices and Geman and Nguyen (2005) that incorporate inventory data. Also Cortazar et al. (2008) and Cortazar and Eterovic (2010) formulate multi-commodity models which use prices from one commodity to estimate the dynamics of another, and Trolle and Schwartz (2009) use commodity option prices to calibrate an unspanned stochastic volatility model.

A second and separate approach for modeling commodity prices and returns are asset pricing models which estimate investor risk premiums. A number of different asset pricing models have been applied to commodity returns. The starting point of this line of research can be found in Dusak (1973) who studied risk premiums under the Capital Asset Pricing Model (CAPM). Dusak's work focused on three agricultural commodities and found β coefficients close to zero for all of them

In other related research Bodie and Rosansky (1980) estimate β coefficients for different commodities and find that the CAPM doesn't hold. Carter et al. (1983) discuss the validity of Dusak's selection of the S&P 500 index as the market proxy and state that another index should be used. They also find systematic risk significantly different from zero (for the same contracts studied by Dusak) when β is allowed to be stochastic and it is specified as a function of net market position of large speculators. Chang et al. (1990) finds significant systematic risk for copper, platinum and silver, differing from previous work done on agricultural commodities.

Furthermore, Bessembinder and Chan (1992) and Bjorson and Carter (1997) find that treasury bill yields, equity dividend yields and the 'junk' bond premium have forecasting power in various commodity future markets. Bessembinder (1992) presents results for single and multiple β models[10] while Erb and Harvey (2006) apply a variation of Fama and French (1993) five factor model to various commodities and commodity portfolios. In both works no factor is consistently significant across commodities. Bessembinder (1992) also uses his single and multiple β models to test for market integration. He finds

[10] In the single β model the explanatory variable is the return on a market index while in the multiple β model six macroeconomic variables are also considered besides the market index.

no statistical evidence to reject the market integration hypothesis[11] while on a different test finds out that hedging pressure has an impact on commodity and currency futures but not on financial futures[12].

De Roon et al. (2000) show that hedging pressure on futures contracts and also hedging pressure on other markets (cross-hedging pressures) have significant influence on futures return.

In more recent research Khan et al. (2008) report results for a three factor model which considers a market proxy, an inventory variable and a hedging pressure variable. The model is applied to copper, crude oil, gold and natural gas presenting mixed results. While the hedging pressure variable holds explanatory power across the four commodities, the other two variables are not statistically significant in all of them.

Moreover Hong and Yogo (2010) study the predictability of commodity futures returns. They use a commodity futures portfolio composed of 30 products from the agriculture, energy, livestock and metal sectors. They find that the short rate, the yield spread, the aggregate basis[13] and the open interest growth rate helps to predict commodity futures returns.

Finally Dhume (2010) studies commodity futures returns using a consumption-based asset pricing model developed by Yogo (2006) which extends the classic consumption CAPM (CCAPM) to include durable goods. Dhume finds out that the high correlation between commodities and durable goods consumption growth can explain commodity returns. This finding contrast with Jagannathan (1985) who found that the CCAPM (not including durable goods) was rejected for agricultural commodities.

4. A Simple Methodology for Integrating Commodity and Asset Pricing Models

Given the inability of commodity pricing models to provide reliable estimations of expected spot prices, and the new relevance of asset pricing models due to the financialization of commodity markets, we propose integrating these two approaches.

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[11] This is done by studying the uniformity of risk premiums across assets and futures with an adaptation of the traditional Fama and MacBeth (1973) methodology. He recognizes that the test performed has relatively low power.

[12] The impact of hedging pressure is observed when residual risk, conditional on a hedging pressure variable, is used. This is consistent with Hirshleifer (1988)

[13] Interesting to note here is that the basis has been found to be related to inventory levels and to the risk premium [Gorton et al., 2013]

The methodology is divided into three steps: (i) Estimation of expected futures returns using an asset pricing model. (ii) Derivation of the parameter restrictions on the commodity pricing model required to obtain a given expected futures return (iii) Estimation of the commodity pricing model satisfying the parameter restrictions.

This methodology requires choosing an asset pricing model and a commodity pricing model. To illustrate its implementation we use the CAPM as the asset pricing model, and the Schwartz and Smith (2000) model as the commodity pricing model. We use copper and oil data to perform the estimations. The methodology naturally applies to alternative choices of asset pricing models and of commodities.

4.1. First Step: Estimation of expected futures returns using an asset pricing model

The basic output of an asset pricing model applied to commodity futures is the expected return of a futures contract on asset i with time to maturity T ($E(R_{i,T})$). How to implement this step obviously depends on the asset pricing model selected. As it was mentioned in the previous section, the implementation of the methodology that will be presented here is done considering the CAPM. In its classical form the CAPM is defined as:

$$E_t(R_i) = R_f + \beta_i[E_t(R_m) - R_f] \quad (1)$$

where R_i is the return on asset i, R_m is the return on the market portfolio, R_f is the risk free rate, β_i is the systematic risk of asset i and $E_t(\cdot)$ is the expectation operator conditional on the information available at time t.

Futures contracts are a special case of assets as they represent zero investment positions. Following Chang et al. (1990) and Bessembinder (1992) the CAPM for futures contracts is defined as:

$$E_t(R_{i,T}) = \beta_{i,T}[E_t(R_m) - R_f] \quad (2)$$

where $R_{i,T}$ is the return on the futures price for a contract on the underlying asset i that matures at time $t + T$. Two important details about this specification are worth mentioning. First, for a particular commodity multiple β coefficients can be estimated depending on the time to maturity (T) of the futures contract chosen. Second, this relation implies that the expected return earned by a holder of a long position in the futures contract is only given by the expected risk premium.

When estimating β coefficients from equation 1 the following regression is typically run[14]:

$$R_t - R_{f,t} = a + \mathrm{b}\left[R_{m,t} - R_{f,t}\right] + \epsilon_t \quad (3)$$

where R_t is the realized return of the asset for time period t, $R_{m,t}$ is the realized return on the market portfolio for time period t, $R_{f,t}$ is the risk free rate at time period t, ϵ_t is an error term and b is the estimate of β. Also, if the CAPM holds, a should not be statically different from zero. However, when applied to future contracts the coefficients to be estimated are those of Equation 2, therefore, the following regression is estimated:

$$R_{T,t} = a_T + \mathrm{b}_{\mathrm{T}}\left[R_{m,t} - R_{f,t}\right] + \epsilon_t \quad (4)$$

where the same terms of Equation 3 can be found, with the exception that in this case $R_{T,t}$ is the realized return for time period t of a future contract that matures at time $t + T$, ϵ_t is an error term and b_{T} is the estimated value of β_{T} present in Equation 2.

Note that to perform the regression specified by Equation 4 a futures contract with exact time to maturity T should be available for each time period ($t \rightarrow t + \Delta t$). This is not the case as one futures contract matures each month. Because of this a rolling strategy must be followed in order to hold a contract that has an approximate maturity of T. At the end of each month the futures contract that has the closest time to maturity to the defined value T is selected. This futures contract is held for the next month and by the end of the month the same process is repeated. Once the futures contract is selected, the price of this contract is used to calculate the futures return. Defining $F_{t+T,t}$ as the price at time t of a futures contract that matures at time $t + T$, the return is defined as the log difference of consecutive futures prices[15]:

$$R_{T,t} = \ln\left(F_{t+T,t+\Delta t}\right) - \ln\left(F_{t+T,t}\right) \quad (5)$$

In addition to an estimate of the β_{T} coefficient, an estimate of the expected market risk premium, $RP = \left[E(R_m) - R_f\right]$, is needed in order to use Equation 2. Damodaran (2009) suggests that there are

[14] For simplicity sub-index i will be dropped from the notation from this point on.
[15] Note that the return is computed for consecutive (separated by a time period of Δt) futures prices that mature at the same date ($t + T$).

three alternative approaches to estimate the equity risk premium: (i) survey investors, managers or academics, about their expectations, (ii) use the historical premium (over a certain period of time) as the market expectation and (iii) use implied methods that try to extract the expectations from market prices or rates. For simplicity the survey approach will be used in this work.

Two types of surveys are available in the literature: those that ask academics (Fernandez (2009), Welch (2001 and 2008)) and those that ask CFO's (Graham and Harvey (2005)). In an unpublished work, Graham and Harvey (2012) update their 2005 work providing quarterly results for the average expected market risk premium since 2000. This is the data set that will be used to compute the commodity futures expected return. According to Equation 6, the expected return on a futures contract of maturity T is:

$$E(R_T) = \beta_T \cdot RP \tag{6}$$

A final issue is the time period used to estimate β_T. Two alternative methods will be used: a static approach and a dynamic approach. In the static approach a single β_T coefficient is estimated using return data from the same time window considered for the model calibration. In contrast, for the dynamic approach different β_T^t coefficients are calculated for every time instant t using two-years back looking rolling windows.

The main differences between these approaches are: (i) The static approach uses the same coefficient for the whole time window while the dynamic method considers a time series of beta coefficients. (ii) The imputed value of the static approach uses information of the whole time window so the coefficient that corresponds to a certain time t includes information before t, but also information between t and the end of the time window, while the β_T^t of the dynamic method is only computed with information prior to time t.

4.2. Second Step: Derivation of the parameter restrictions on the commodity pricing model

This step uses the chosen commodity pricing model to compute expected futures returns as a function of the model parameters. This will allow in the third step to restrict parameter values to induce the expected futures return computed in the first step.

Similar to Equation 5, the expected futures return at time t for a contract with time to maturity T can be expressed as:

$$E_{\mathrm{t}}(R_T) = E\left(\ln(F_{t+T,\mathrm{t}+\Delta t}) - \ln(F_{t+T,\mathrm{t}})\right) \quad (7)$$

In general $F_{t+T,t}$ will be a function of the state variables (X_t) and the model´s parameters (ψ). Regardless of the number of factors considered, Equation 7 will only be a function of the model parameters, the time to maturity (T) and the time step considered for the return calculation (Δt)[16].

Thus by equating the expected futures return from step 1 with the expression resulting from Equation 7 a restriction on the commodity pricing model parameters is obtained. More precisely, let ER_T be the first step expected return (Equation 6) and $g(\psi, \mathrm{T}, \Delta t)$ the function obtained from deriving the expression presented in Equation 7 for a given stochastic model. We impose the following restriction:

$$g(\psi, \mathrm{T}, \Delta t) = ER_T \quad (8)$$

By adding this restriction one degree of freedom is lost and, as the right hand side of Equation 8 is the value resulting from the first step, one can easily express one of the parameters as a function of the first step value and the remaining parameters. Thus by adding the restriction one of the risk premiums $\lambda_j \epsilon \{\lambda_1, \lambda_2, \ldots \lambda_N\}$ is estimated.

Given that in an N-factor model there are N risk premiums to be estimated, we propose allowing each λ_j to be expressed as a function of the remaining parameters (ψ') and using N different futures contract maturities $T_j \epsilon \{T_1, T_2, \ldots T_N\}$ to compute N expected futures returns in step 1.

Thus:

$$\lambda_j = \lambda_j(\psi', \bar{T}, \overline{ER_T}, \Delta t) \quad (9)$$

where

$$\bar{T} = \{T_1, T_2, \ldots T_N\}$$

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[16] This is shown in appendix B where the expression of equation 7 is derived for the Cortazar and Naranjo (2006) N-factor Gaussian model which generalizes several models previously found in the literature.

$$\overline{ER_T} = \{ER_{T_1}, ER_{T_2}, \dots ER_{T_N}\}$$

$$\psi' = \psi - \{\lambda_1, \lambda_2, \dots \lambda_N\}$$

As it was mentioned earlier, the methodology presented here can be used with any of the available stochastic models of commodity prices. As an illustration the two-factor Schwartz and Smith (2000) model is used. The first state variable (ξ_t) of this model represents the long term equilibrium (log) price level while the second state variable (χ_t) represents the short term mean reverting variations.

The relation of the spot price (S_t) with the state variables is given by Equation 10, while the stochastic processes (under the physical measure) followed by the state variables is given by Equations 11 and 12, where μ_ξ, κ, σ_ξ and σ_χ are parameters of the model.

$$\ln(S_t) = \chi_t + \xi_t \quad (10)$$

$$d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi \quad (11)$$

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi \quad (12)$$

Furthermore, dz_ξ and dz_χ are correlated Brownian motions with correlation $\rho_{\chi\xi}$ such that:

$$dz_\chi dz_\xi = \rho_{\chi\xi} dt \quad (13)$$

Under the risk neutral measure the processes followed by the state variables are given by Equations 14 to 16, where λ_χ and λ_ξ are the risk premiums of the respective state variables.

$$d\xi_t = (\mu_\xi - \lambda_\xi)dt + \sigma_\xi dz_\xi^Q \quad (14)$$

$$d\chi_t = (-\kappa\chi_t - \lambda_\chi)dt + \sigma_\chi dz_\chi^Q \quad (15)$$

$$dz_\chi^Q dz_\xi^Q = \rho_{\chi\xi} dt \quad (16)$$

Some relevant results of the Schwartz and Smith (2000) model are the expected value of the state variables, their covariance matrix and the (log) expected value of the spot price. These are presented in Equations 17 through 19, respectively. Furthermore the price of a futures contract at time t that matures at time $t+T$ ($F_{t+T,t}$) is given by the expected spot price under the risk neutral measure (${E_t}^Q[S_{t+T}]$), therefore the (log) futures price can be expressed as shown in Equation 20.

$$E_t\left(\begin{bmatrix}\chi_{t+T}\\ \xi_{t+T}\end{bmatrix}\right) = \begin{bmatrix}e^{-\kappa T}\chi_t\\ \mu_\xi T\end{bmatrix} \tag{17}$$

$$Cov_t\left(\begin{bmatrix}\chi_{t+T}\\ \xi_{t+T}\end{bmatrix}\right) = \begin{bmatrix}(1-e^{-2\kappa T})\dfrac{\sigma_\chi^2}{2\kappa} & (1-e^{-\kappa T})\dfrac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\\ (1-e^{-\kappa T})\dfrac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} & \sigma_\xi^2 T\end{bmatrix} \tag{18}$$

$$\begin{aligned}\ln\big(E_t(S_{t+T})\big) = {} & e^{-\kappa T}\chi_t + \xi_t + \mu_\xi T\\ & + \frac{1}{2}\left((1-e^{-2\kappa T})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 T + 2(1-e^{-\kappa T})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right)\end{aligned} \tag{19}$$

$$\ln(F_{t+T,t}) = e^{-\kappa(T)}\chi_t + \xi_t + A(T) \tag{20}$$

where

$$A(T) = (\mu_\xi - \lambda_\xi)T - (1-e^{-\kappa T})\frac{\lambda_\chi}{\kappa} + \frac{1}{2}\left((1-e^{-2\kappa T})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 T + 2(1-e^{-\kappa T})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right)$$

Using Equations 7 and 20 and following the general procedure presented in Appendix B, the expected futures return becomes

$$E_{\mathrm{t}}(R_T) = \mu_\xi * \Delta\mathrm{t} + A(T-\Delta t) - A(T) \tag{21}$$

$$\begin{aligned}E_{\mathrm{t}}(R_T) = {} & \left(\lambda_\xi - \frac{1}{2}\sigma_\xi^2\right)\Delta t - e^{-\kappa T}\left(1-e^{\kappa\Delta t}\right)\frac{\lambda_\chi}{\kappa} + \frac{1}{2}\left(e^{-2\kappa T}\left(1-e^{2\kappa\,\Delta\,t}\right)\frac{\sigma_\chi^2}{2\kappa}\right)\\ & + e^{-\kappa T}\left(1-e^{\kappa\Delta t}\right)\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\end{aligned} \tag{22}$$

Equation 22 corresponds to $g(\psi, \mathrm{T}, \Delta\, t)$ for the Schwartz and Smith (2000) model. Therefore, following Equation 8 and using contracts with two different maturities, two restrictions must be set so that λ_ξ and λ_χ can be expressed as in Equation 9. In this way, the problem arising from the risk premium estimation can be solved as they will now depend on the other parameters and the imputed information from the first step.

Finally, two important considerations must be noted about the methodology described above. (i) The time interval ($\Delta\, t$) used to calculate the CAPM β coefficient (Equation 5) must be the same as the one considered for Equation 7[17]. (ii) The expected market risk premium (RP from Equation 6) must also correspond to time interval Δt[18].

4.3. Third Step: Estimation of the commodity pricing model satisfying the parameter restrictions

The stochastic model of commodity prices will be estimated by maximum likelihood and the Kalman filter. This estimation must include the restrictions derived in the previous step to ensure expected futures returns are consistent with those obtained in step 1 from the asset pricing model.

The Kalman filter requires specifying two equations. The first one is the transition equation, which describes the evolution of the state variables for a determined time step Δt:

$$X_t = GX_{t-1} + c + \omega_t \tag{23}$$

From Equation 17, for the specific case of the Schwartz and Smith (2000) model, the terms presented above are:

$$X_t = \begin{bmatrix} \chi_t \\ \xi_t \end{bmatrix}$$

$$c = \begin{bmatrix} 0 \\ \mu_\xi \Delta t \end{bmatrix}$$

$$G = \begin{bmatrix} e^{-\kappa \Delta t} & 0 \\ 0 & 1 \end{bmatrix}$$

and ω_t is a 2×1 vector of serially uncorrelated, normally distributed errors with mean zero and covariance given by Equation 18.

The second equation is the measurement equation, which describes the relationship between the state variables and the observed futures prices:

$$Y_t = J_t X_t + d_t + v_t \tag{24}$$

where[19]

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[17] Hawawini (1983) points out that β coefficients shift when the return time interval changes. The reason for this is "the existence of intertemporal relationships between the daily returns of individual securities and those of the general market". Because of this, as the application that will be given to the β coefficient is for time interval Δt (Equations 7, 21 and 22), then Δt used for the CAPM must be the same.

[18] In the empirical application that follows Δt corresponds to one week and the two futures contracts maturities are 60 days and 1 year.

[19] T_1 ... T_n are the maturities of the future contracts.

$$Y_t = \begin{bmatrix} \ln\left(F_{t,t+T_1}\right) \\ \vdots \\ \ln\left(F_{t,t+T_n}\right) \end{bmatrix}$$

and, from Equation 20, for the specific case of the Schwartz and Smith (2000) model, the terms presented above are:

$$d_t = \begin{bmatrix} A(T_1) \\ \vdots \\ A(T_n) \end{bmatrix}$$

$$J_t = \begin{bmatrix} e^{-\kappa T_1} & 1 \\ \vdots & \vdots \\ e^{-\kappa T_n} & 1 \end{bmatrix}$$

Also, v_t is a $n \times 1$ vector of serially uncorrelated, normally distributed errors with mean zero and diagonal variance-covariance matrix (R_t).

As X_t and v_t are normally distributed random variables, Y_t is also normally distributed. Thus the probability distribution of Y_t can be determined and the likelihood of the observed futures prices can be computed. This allows estimating the set of parameters by maximum likelihood. Further explanation about the estimation method can be found in Schwartz and Smith (2000) and Cortazar and Naranjo (2006).

In addition to including the parameter restrictions derived in the previous steps we estimate the model following Schwartz and Smith (2000) with one important difference. Our data set is much larger and includes a variable number of futures contracts. Thus the dimension of our R_t matrix is time varying, as opposed to constant in Schwartz and Smith (2000). Given the much higher dimensionality of our problem, instead of associating a different volatility parameter for each maturity, contracts were classified in five groups according to their maturity[20] and the same volatility parameter was associated to each contract within a determined group. Therefore, considering that σ_j is the volatility parameter associated to the jth group, R_t has the following structure:

[20] The maturities considered in each group varied for each estimation time window depending on the distribution of future contracts maturities within the time period.

$$\overbrace{\qquad\qquad}^{1^{st}\ \text{Group}} \qquad \overbrace{\qquad\qquad}^{i^{th}\ \text{Group}} \qquad \overbrace{\qquad\qquad}^{5^{th}\ \text{Group}}$$

$$R_t = \begin{bmatrix} \sigma_1^2 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \ddots & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \sigma_1^2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \sigma_j^2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sigma_j^2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sigma_5^2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \ddots & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \sigma_5^2 \end{bmatrix}$$

5. Implementation and Results

5.1. Data

The model was estimated for two commodity data sets: copper and oil. The data used can be divided into three: (i) Commodity futures, (ii) Market information and (iii) Market Surveys.

Regarding commodity futures, copper data was obtained from the Commodity Exchange, Inc (COMEX) and oil information from the New York Mercantile Exchange (NYMEX). Copper data was complemented with London Metal Exchange (LME) long term contracts[21]. Weekly futures prices contracts from January 1995 until December 2012 were used. For oil, the number of contracts traded each date ranged from 12 to 78[22], while for copper between 12 and 40. Figures 5-1 and 5-2 show a time series of futures term structures for each commodity.

Market information consists of a time series of weekly closing prices for the Standard & Poor's 500 Index (S&P 500) and for the three-month Treasury bill rate. These were used as proxies for the equity market and for the risk free rate necessary for estimating the futures risk premiums on the first step.

[21] One or two contracts with maturities at least one year over the longest COMEX contract were added.

[22] Before February 2006 the number of contracts available at a single date was rarely more than 35. Since February 2006 contracts available in the data set went to more than 70. Given the high number of contracts for each date from February 2006, a sample of contracts was selected. The selection always considered the first five futures and then one in every two contracts were also selected, making sure that the longest maturity contract was always in the estimation set.

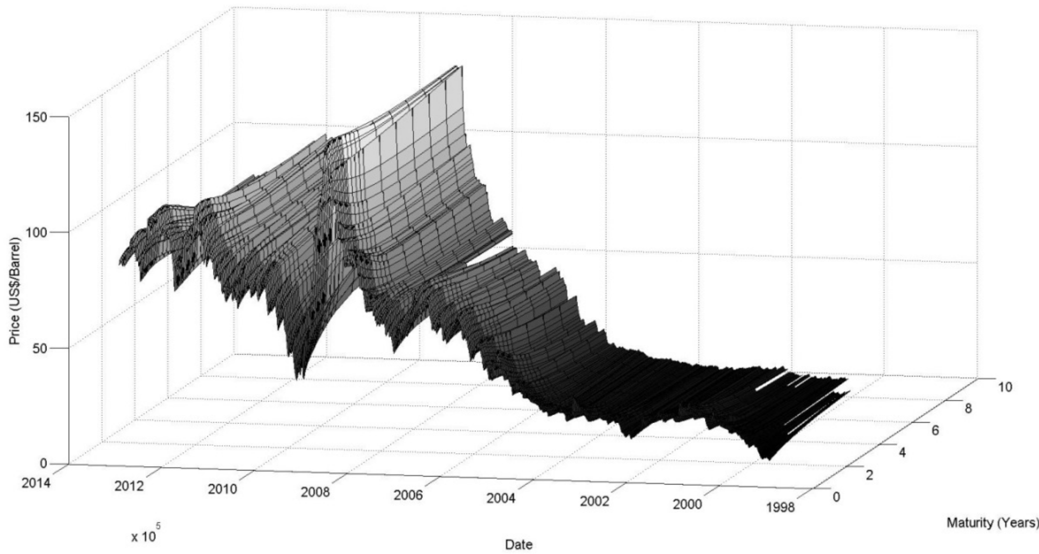

Figure 5-1: WTI Oil (NYMEX) Futures Term Structure

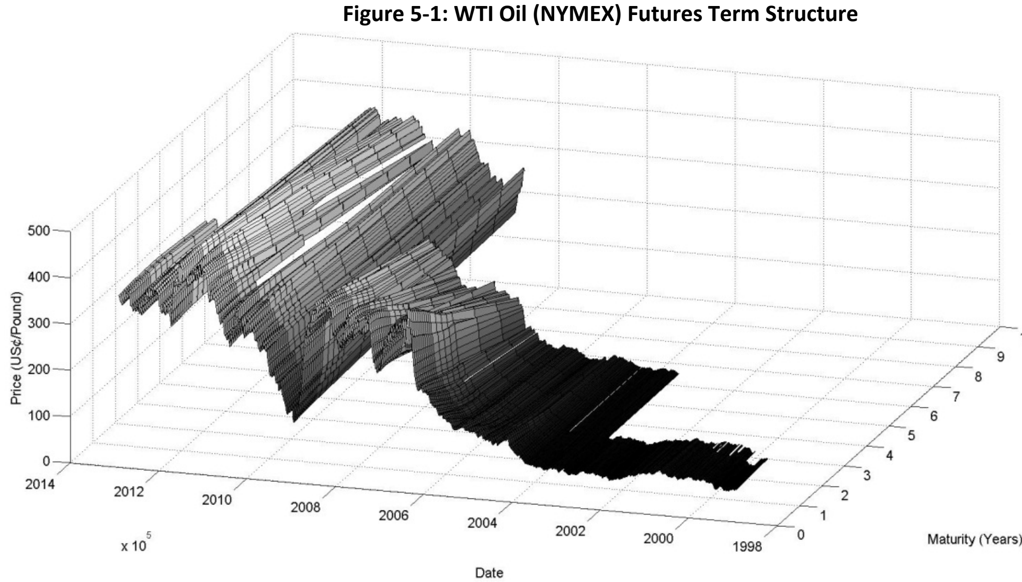

Figure 5-2: Copper (COMEX) Futures Term Structure

Finally the survey information on expected market risk premiums was obtained from Graham and Harvey (2012). Figure 5-3 presents the quarterly surveys results on the expected market risk premium from Chief Financial Officers (CFOs) for the period June 2000 to March 2012[23]. Weekly expected equity risk premiums are obtained by linear interpolation.

23 The exact question asked to CFOs was about the average expected market return over the next 10 years.

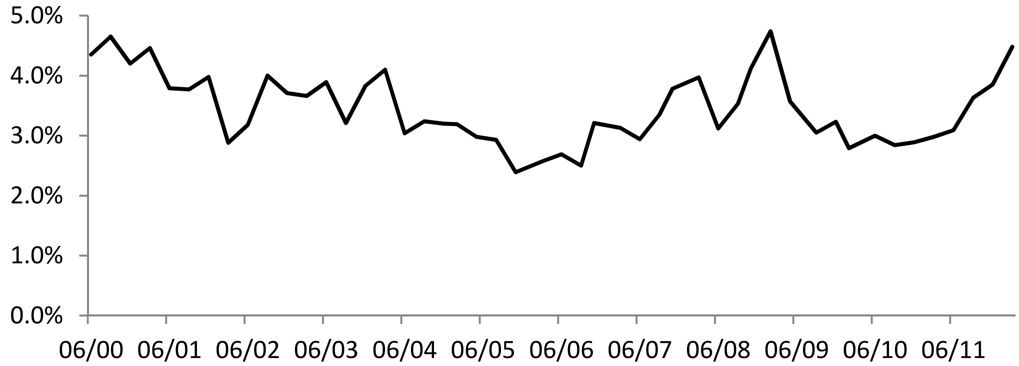

Figure 5-3: Expected market risk premium from Graham and Harvey (2012). Data is obtained from surveys to CFO's.

5.2. Parameter Results

The model was estimated for two five-year[24] windows (2001-2006 and 2006-2011) and one additional three-year window (January 2009 to February 2012) that does not include the financial crisis. Data between February and December 2012 was used for out-of-sample tests.

Tables 5-1 to 5-6 show copper and oil models' parameters for each time window. In every table, results for the dynamic, static and non-restricted parameter estimations are shown[25]. The first two parameter estimations correspond to restricting the model to generate expected futures returns consistent with the asset pricing model using the dynamic or static approach for estimating β_T. The non-restricted parameter estimation shows the result of ignoring asset pricing models and using only information from future contracts to estimate the model, as it has traditionally been done in the commodity pricing literature.

Note that instead of estimating μ_ξ and λ_ξ, we follow Schwartz and Smith (2000) and estimate μ_ξ and μ_ξ^Q with $\mu_\xi = \mu_\xi^Q + \lambda_\xi$, which is equivalent. Thus, the expected return restrictions imposed on λ_ξ are actually reflected in the values of μ_ξ.

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24 The actual length is 5 years and one month as it was the case in Schwartz and Smith (2000)

25 The results for μ_ξ and λ_χ in the restricted cases are time varying because they depend on the other parameters (which are constant) but also on the asset pricing model expected returns which are time varying as a consequence of the time variation in the expected market risk premium information and, for the dynamic case, time variation of the estimated β coefficient. The results presented in the tables correspond to the value for the last time instant of each window.

It can be observed from the tables that estimates for μ_ξ and λ_χ have significant differences between the non-restricted and the restricted cases, indicating that the CAPM restriction has a considerable impact in their estimation. In contrast, the impact of the integration methodology on the other parameters (ψ') is much lower.

Finally, regarding the statistical significance of the parameters, it can be seen that in the non-restricted case either μ_ξ or λ_χ, the parameters that define the risk premiums, are not statistically significant, which is typical of commodity pricing models. On the other hand, the results for the restricted estimations show that for most[26] cases the application of either the static or dynamic approach achieves statistically significant estimates for both[27] μ_ξ and λ_χ.

Parameter	Dynamic			Static			Non-Restricted		
	Estimate	S.D	T-Test	Estimate	S.D	T-Test	Estimate	S.D	T-Test
κ	0.475	0.006	80.859	0.475	0.006	80.860	0.475	0.006	80.867
σ_X	0.218	0.010	22.789	0.218	0.010	22.785	0.218	0.010	22.719
λ_X	0.025	0.023	1.064	0.022	0.004	6.157	-0.006	0.028	-0.213
μ_ε	-0.018	0.018	-0.98	-0.014	0.002	-6.878	0.194	0.083	2.325
σ_ε	0.206	0.009	22.089	0.205	0.009	22.085	0.204	0.009	22.057
μ_ε^Q	-0.026	0.002	-12.820	-0.026	0.002	-12.816	-0.026	0.002	-12.825
$\rho_{X,\varepsilon}$	-0.395	0.054	-7.299	-0.395	0.054	-7.286	-0.405	0.054	-7.553

Table 5-1: Copper estimated parameters, standard deviation (S.D) and t-Test. 2001-2006 time window.

26 The only exception is for the 2009-2012 copper time window, where the integration methodology only achieves a statistically significant estimate for μ_ξ. Anyway, this is still an improvement compared to the non-restricted case where both μ_ξ and λ_χ are not significant. Furthermore, for the 2001-2006 copper time window the results for the dynamic approach are counterintuitive because the t-statistic worsens. This may be due to the high volatility of the estimates for the corresponding β_T parameters, a problem that isn't present in the static approach.

27 These results must be taken with caution as the procedure used to estimate the standard deviation for these two parameters has some shortcomings because the imputed expected futures returns were estimated separately from the model parameters. The standard deviations for μ_ξ and λ_χ in the restricted cases were estimated using the delta method which is used to estimate standard deviations for functions of estimators, as is the case here, where μ_ξ and λ_χ are functions of the other parameters (ψ') and the imputed expected futures return. The method linearizes the function with first order partial derivatives. For these calculations the variance and covariance of the two β_T parameters were also considered, but a possible covariance between these and the set of parameters ψ' are ignored as they come from two separate estimations.

Parameter	Dynamic			Static			Non-Restricted		
	Estimate	S.D	T-Test	Estimate	S.D	T-Test	Estimate	S.D	T-Test
κ	0.101	0.003	29.775	0.098	0.004	27.908	0.103	0.004	26.410
σ_X	1.162	0.024	48.294	1.181	0.026	45.203	1.152	0.027	43.405
λ_X	0.177	0.040	4.460	0.182	0.038	4.778	0.233	0.049	4.749
μ_ε	-0.300	0.016	-19.004	-0.305	0.010	-29.021	0.178	0.143	1.246
σ_ε	1.003	0.043	23.114	1.019	0.045	22.803	0.795	0.044	18.107
μ_ε^Q	-0.215	0.033	-6.599	-0.215	0.033	-6.506	-0.037	0.025	-1.505
$\rho_{X,\varepsilon}$	-0.948	0.006	-158.841	-0.950	0.006	-162.763	-0.915	0.014	-67.362

Table 5-2: Copper estimated parameters, standard deviation (S.D) and t-Test. 2006-2011 time window.

Parameter	Dynamic			Static			Non-Restricted		
	Estimate	S.D	T-Test	Estimate	S.D	T-Test	Estimate	S.D	T-Test
κ	0.160	0.007	23.834	0.160	0.007	22.827	0.111	0.012	9.513
σ_X	0.707	0.020	35.955	0.708	0.021	34.405	0.910	0.069	13.180
λ_X	0.121	0.080	1.506	0.111	0.082	1.352	0.036	0.096	0.369
μ_ε	-0.132	0.026	-5.019	-0.139	0.009	-16.256	0.266	0.145	1.833
σ_ε	0.556	0.112	4.942	0.568	0.121	4.694	0.605	0.143	4.240
μ_ε^Q	-0.108	0.058	-1.869	-0.114	0.064	-1.798	-0.043	0.056	-0.764
$\rho_{X,\varepsilon}$	-0.877	0.047	-18.561	-0.881	0.048	-18.408	-0.903	0.048	-18.905

Table 5-3: Copper estimated parameters, standard deviation (S.D) and t-Test. 2009-2012 time window.

Parameter	Dynamic			Static			Non-Restricted		
	Estimate	S.D	T-Test	Estimate	S.D	T-Test	Estimate	S.D	T-Test
κ	1.224	0.010	121.157	1.216	0.010	119.452	1.216	0.010	119.410
σ_X	0.732	0.010	74.887	0.726	0.010	73.401	0.726	0.010	73.384
λ_X	0.256	0.028	9.207	0.240	0.037	6.413	0.233	0.156	1.489
μ_ε	-0.082	0.010	-8.057	0.013	0.026	0.514	0.208	0.083	2.503
σ_ε	0.228	0.011	20.578	0.198	0.011	17.409	0.197	0.011	17.224
μ_ε^Q	-0.036	0.003	-14.059	-0.029	0.002	-12.656	-0.029	0.002	-12.591
$\rho_{X,\varepsilon}$	-0.188	0.120	-1.559	0.333	0.116	2.859	0.341	0.116	2.943

Table 5 4: Oil estimated parameters, standard deviation (S.D) and t-Test. 2001-2006 time window.

Parameter	Dynamic			Static			Non-Restricted		
	Estimate	S.D	T-Test	Estimate	S.D	T-Test	Estimate	S.D	T-Test
κ	0.278	0.004	78.443	0.277	0.004	77.841	0.277	0.004	77.790
σ_X	0.551	0.005	112.337	0.550	0.005	111.760	0.551	0.005	111.762
λ_X	0.195	0.018	10.825	0.215	0.016	13.136	0.163	0.045	3.629
μ_ε	-0.064	0.008	-8.328	-0.075	0.005	-15.511	0.081	0.113	0.719
σ_ε	0.292	0.016	17.948	0.278	0.015	18.389	0.276	0.015	18.436
μ_ε^Q	-0.017	0.005	-3.614	-0.013	0.004	-3.070	-0.012	0.004	-2.992
$\rho_{X,\varepsilon}$	-0.453	0.074	-6.088	-0.395	0.083	-4.779	-0.384	0.085	-4.546

Table 5-4: Oil estimated parameters, standard deviation (S.D) and t-Test. 2006-2011 time window.

Parameter	Dynamic			Static			Non-Restricted		
	Estimate	S.D	T-Test	Estimate	S.D	T-Test	Estimate	S.D	T-Test
κ	0.415	0.004	112.799	0.415	0.004	112.842	0.414	0.004	112.403
σ_X	0.577	0.005	124.526	0.577	0.005	124.299	0.579	0.005	124.744
λ_X	0.303	0.024	12.453	0.309	0.021	14.711	0.220	0.029	7.490
μ_ε	-0.049	0.011	-4.308	-0.056	0.006	-10.161	0.036	0.119	0.305
σ_ε	0.214	0.013	17.101	0.215	0.013	16.478	0.216	0.014	15.335
μ_ε^Q	-0.001	0.003	-0.255	-0.001	0.003	-0.311	-0.001	0.003	-0.333
$\rho_{X,\varepsilon}$	0.138	0.150	0.923	0.166	0.152	1.090	0.240	0.145	1.652

Table 5-5: **Oil estimated parameters, standard deviation (S.D) and t-Test. 2009-2012 time window.**

5.3. Model Fit

We now analyze the impact of the proposed approach on model fit. Tables 5-6 and 5-7 show the in-sample and out-of-sample mean absolute error for copper and oil, for each of the parameter estimation approaches and time windows. The errors are presented as percentage of the observed futures price.

Regarding futures prices in-sample fit, the three methodologies give the same good performance. In fact, considering both commodities, the mean absolute error is less than 1.5% for every time window. Moving to the out-of-sample fit, the mean absolute error for each time window is in general larger than for the in-sample test, but still errors for copper are always less than 2.5% and for oil less than 1.5% and basically the same regardless of the estimation methodology. Therefore, these results show that restricting parameter values has no significant effect in pricing futures.

Window	In Sample			Out of Sample		
	Dynamic	Static	Non-Restricted	Dynamic	Static	Non-Restricted
2001-2006	0.4%	0.4%	0.4%	2.2%	2.2%	2.2%
2006-2011	0.8%	0.7%	0.7%	0.9%	0.9%	0.9%
2009-2012	0.6%	0.6%	0.6%	0.5%	0.5%	0.5%

Table 5-6: Copper In-Sample and Out-of-Sample Mean Absolute Error for the three methods. Errors are calculated as percentage of the observed futures price.

Window	In Sample			Out of Sample		
	Dynamic	Static	Non-Restricted	Dynamic	Static	Non-Restricted
2001-2006	1.1%	0.9%	0.9%	1.3%	1.3%	1.3%
2006-2011	1.5%	1.3%	1.3%	0.8%	0.8%	0.8%
2009-2012	0.8%	0.8%	0.8%	1.4%	1.4%	1.4%

Table 5-7: Oil In-Sample and Out-of-Sample Mean Absolute Error for the three methods. Errors are calculated as percentage of the observed futures price.

We now study the effect of the above restrictions on expected spot prices ($E_t[S_T]$). Figures 5-4 and 5-5 show an example of a futures term structure and the corresponding expected spot prices for copper and oil, respectively. It can be seen that expected spot prices for the two restricted methods are similar while the expected spot price for the non-restricted method is considerably different showing a much higher (and fairly unreasonable) risk premium[28].

Figures 5-6 and 5-7 show the five year futures price and the expected 5-year spot price for both copper and oil over the last three years of the sample period. They are equivalent to Figure 1-1, only that now the restricted expected spot prices are also included. Results for the restricted estimations seem clearly more reasonable than those from the non-restricted case. Our results are consistent with those of Heath (2013) who reports that different risk premium specifications have an equivalent performance in fitting futures contracts, but provide considerably different price forecasts.

Therefore, to set restrictions on some parameter values such that expected futures returns are consistent with those of asset pricing models has the positive consequence of providing an expected spot price that incorporates new information in the estimation of the risk premiums, gives an expected spot price that is more reasonable and achieves this without losing the ability of adequately price futures contracts.

[28] The risk premium is the difference between the expected spot curve and the futures curve. As the futures curve is in fact the risk adjusted expected spot price (${E_t}^Q[S_T]$), the risk premium is $E_t[S_T] - {E_t}^Q[S_T]$.

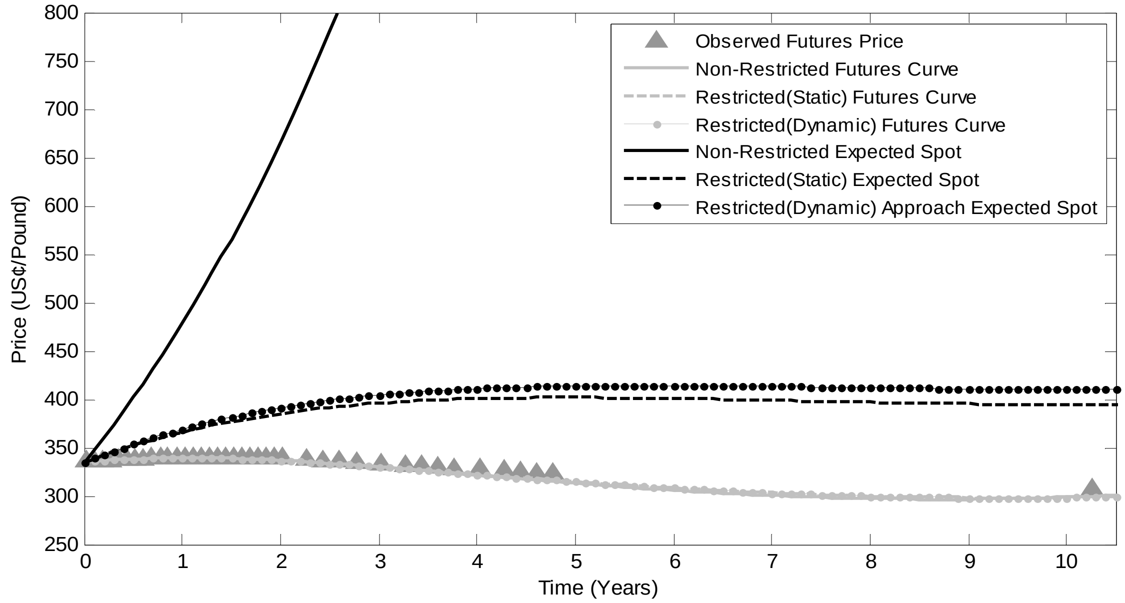

Figure 5-4: Copper Futures (grey) and Expected Spot Prices (black) for 12-20-2011. (2009-2012 parameters).

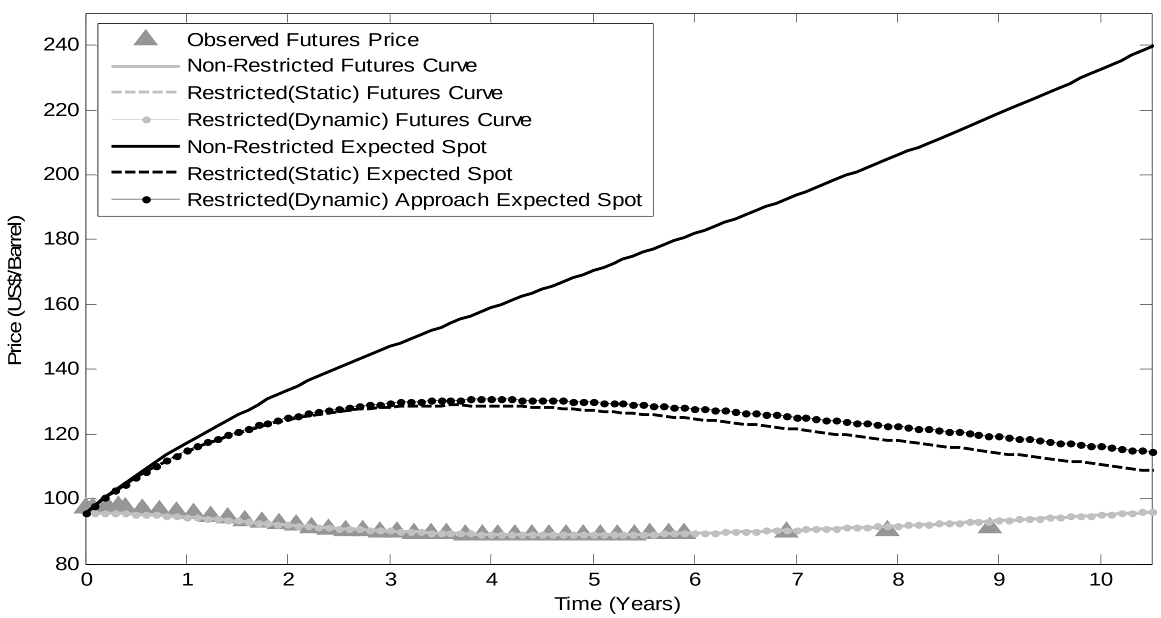

Figure 5-5: Oil Futures (grey) and Expected Spot Prices (black) for 12-20-2011. (2009-2012 parameters)

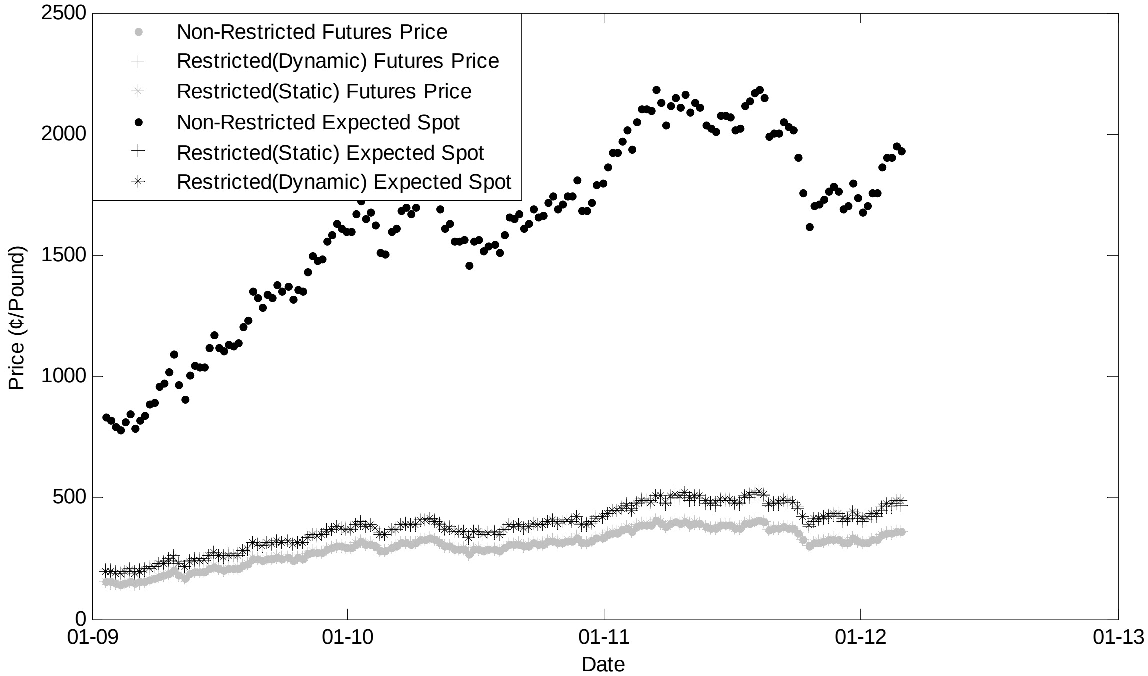

Figure 5-6: Five-year Copper Futures (grey) and Expected Spot Prices (black) for 2009-2012. (2009-2012 parameters)

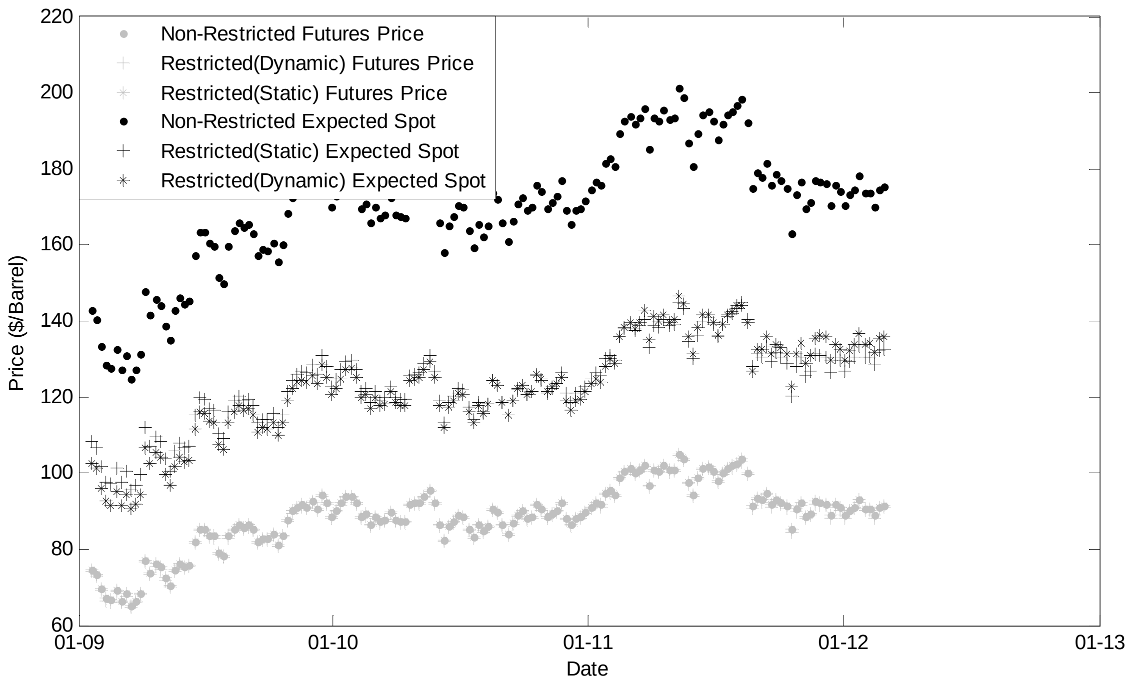

Figure 5-7: Five-year Oil Futures (grey) and Expected Spot Prices (black) for 2009-2012. (2009-2012 parameters)

6. Conclusion

We present a simple methodology that integrates commodity and asset pricing models. Given current evidence on the financialization of commodity markets, valuable information about the behavior of commodity prices can be extracted from asset pricing models.

Futures contracts data does not provide enough information for an accurate estimation of risk premiums, often resulting in highly unreasonable expected spot prices. Using information from asset pricing models provides a more robust estimation of risk premium parameters and therefore more credible expected spot prices, without compromising the model´s fit to futures price observations.

The methodology is general in the sense that it can be used with any pair of commodity and asset pricing models available in the literature. The procedure first defines the expected futures return implicit in the commodity pricing model as a function of the model's parameters. Then, it imposes a set of restrictions on the parameter estimation process so that these expected futures returns are consistent with those from the chosen asset pricing model. In this way new information, not available in traditional estimation of commodity models, is included which gives not only an excellent fit to observed futures prices, but also provides reasonable expectations for future spot prices.

To illustrate the methodology we selected the classic CAPM and the Schwartz and Smith (2000) model as the asset and commodity pricing model, respectively. We estimated the model for copper and oil futures contracts, considering different time windows between 2001 and 2012. We proposed two variations of the integration methodology: the static and the dynamic approach. For the static approach a single β_T coefficient is estimated using return data from the same time window considered for the model calibration. In contrast, for the dynamic approach different β_T^t coefficients are calculated for every time t using two-years back looking rolling windows. Both approaches provide similar results.

For comparison, we also estimated the commodity model ignoring the CAPM information, as is traditional in the literature. Our results show that the integration methodology has two important benefits relative to the traditional estimation. First, the statistical significance of the estimated risk premium parameters is improved. Second and more important, the expected spot prices implied by the restricted model are much more reasonable than those from the unrestricted case as the former are consistent with the asset pricing model.

Finally, the implementation of the proposed methodology presents no significant difference in the model's ability to fit future contracts, offering equivalent measures of in-sample and out-of-sample mean absolute error. Therefore, the above benefits are obtained without any negative consequence on model fit.

The proposed methodology makes commodity models more credible and useful and may expand the use of commodity pricing models among practitioners.

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Appendix A: CFTC Data Processing for Investor Type Distribution

The Disaggregated Commitments of Traders Report informs about open interest in each futures market separating positions in four categories: “Producer/Merchant/Processor/User”, “Swap Dealer”, “Money Manager” and “Other Reportables”. For the first category information is given for “short” and “long” positions, while for the other three categories information is provided for “long”, “short” and “spreading”. As the reports explanatory notes points out “spreading” is a computed amount equal to offsetting long and short position held by a trader. The computed amount of spreading is calculated as the amount of offsetting futures in different calendar months.

The goal of Figures 2-8 through 2-10 is to compare the presence that each of the four types of trader classification has on a certain market. Because the information is separated by “long”, “short” and “spreading” a grouping method must be used in order to have only one figure that represents market presence. In order to do this the number of “contract sides” that each type of trader has is computed. This means that for each “long” or “short” contract that is reported to a determined trader classification one “contract side” is counted, while for each “spreading” position reported two “contract sides” are counted as the spreading figure corresponds to the offsetting of long and short contracts. With this, the percentage of “contract sides” that each trader classification has is computed for each date and finally this is averaged for each year.

Appendix B: Expected Futures Return for an N-Factor Model

The expected return that will be derived here is based on the N-Factor model presented in Cortazar and Naranjo (2006). In this model the (log) spot price of the commodity can be expressed as:

$$\ln(S_t) = 1' X_t + \mu t \qquad \text{(B.1)}$$

where $\boldsymbol{X_t}$ is a $\boldsymbol{N} \times \mathbf{1}$ vector of state variables and $\boldsymbol{\mu}$ the constant long term growth rate. The vector of state variables follows the process:

$$dX_t = -KX_t dt + \Sigma dw_t \qquad \text{(B.2)}$$

where $\boldsymbol{K}$ and $\boldsymbol{\Sigma}$ are $\boldsymbol{N} \times \boldsymbol{N}$ matrices with the following form:

$$K = \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & \kappa_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \kappa_N \end{pmatrix} \qquad \Sigma = \begin{pmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_N \end{pmatrix}$$

Furthermore $(\boldsymbol{dw_t})'(\boldsymbol{dw_t}) = \boldsymbol{\Omega dt}$, where:

$$\Omega = \begin{pmatrix} 1 & \rho_{1,2} & \dots & \rho_{1,N} \\ \rho_{1,2} & 1 & \dots & \rho_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1,N} & \rho_{2,N} & \dots & 1 \end{pmatrix}$$

Given this and defining $\boldsymbol{\lambda}$ as a vector of constant risk premiums the risk adjusted process followed by the state variables is

$$dX_t = -(\lambda + KX_t)dt + \Sigma dw_t^* \qquad \text{(B.3)}$$

With this, it can be showed that the price of a future contract at time t and maturing at time $t + T$ is given by:

$$\ln\left(F_{t+T,t}\right) = X_1(t) + \sum_{i=2}^{N} e^{-\kappa_i T} X_i(t) + \mu t + \left(\mu - \lambda_1 + \frac{1}{2}\sigma_1^2\right)T - \sum_{i=2}^{N} \frac{1 - e^{-\kappa_i T}}{\kappa_i}\lambda_i + \frac{1}{2}\sum_{i \cdot j \neq 1}^{N} \sigma_i \sigma_j \rho_{i,j} \frac{1 - e^{-(\kappa_i + \kappa_j)T}}{\kappa_i + \kappa_j} \qquad \text{(B.4)}$$

Finally, the expected futures return needed is given by:

$$E_t(R_T) = E\left(\ln(F_{t+T,t+\Delta t}) - \ln(F_{t+T,t})\right) \tag{B.5}$$

Using expression B.4 and noting that $\ln(F_{t+T,t})$ is known at time t, the return from expression B.5 can be computed:

$$E_t(R_T) = E\left(\ln(F_{t+T,t+\Delta t})\right) - \ln(F_{t+T,t})$$

$$= E_t\left(X_1(t+\Delta t)\right) + \sum_{i=2}^{N} e^{-\kappa_i(T-\Delta t)} E_t\left(X_i(t+\Delta t)\right) + \mu(t+\Delta t)$$
$$+\left(\mu - \lambda_1 + \frac{1}{2}\sigma_1^2\right)(T-\Delta t) - \sum_{i=2}^{N} \frac{1-e^{-\kappa_i(T-\Delta t)}}{\kappa_i}\lambda_i + \frac{1}{2}\sum_{i\cdot j\neq 1}^{N} \sigma_i\sigma_j\rho_{i,j}\frac{1-e^{-(\kappa_i+\kappa_j)(T-\Delta t)}}{\kappa_i+\kappa_j}$$
$$-X_1(t) - \sum_{i=2}^{N} e^{-\kappa_i T} X_i(t) - \mu t - \left(\mu - \lambda_1 + \frac{1}{2}\sigma_1^2\right)T + \sum_{i=2}^{N}\frac{1-e^{-\kappa_i T}}{\kappa_i}\lambda_i$$
$$-\frac{1}{2}\sum_{i\cdot j\neq 1}^{N} \sigma_i\sigma_j\rho_{i,j}\frac{1-e^{-(\kappa_i+\kappa_j)T}}{\kappa_i+\kappa_j}$$

$$= X_1(t) + \sum_{i=2}^{N} e^{-\kappa_i(T-\Delta t)} e^{-\kappa_i \Delta t} X_i(t) + \mu(\Delta t) - \left(\mu - \lambda_1 + \frac{1}{2}\sigma_1^2\right)\Delta t$$
$$-\sum_{i=2}^{N} e^{-\kappa_i T}\frac{\left(1-e^{\kappa_i \Delta t}\right)}{\kappa_i}\lambda_i + \frac{1}{2}\sum_{i\cdot j\neq 1}^{N}\sigma_i\sigma_j\rho_{i,j}e^{-(\kappa_i+\kappa_j)T}\frac{1-e^{(\kappa_i+\kappa_j)\Delta t}}{\kappa_i+\kappa_j} - X_1(t) - \sum_{i=2}^{N} e^{-\kappa_i T} X_i(t)$$

which can be simplified to:

$$= \left(\lambda_1 - \frac{1}{2}\sigma_1^2\right)\Delta t - \sum_{i=2}^{N} e^{-\kappa_i T}\frac{\left(1-e^{\kappa_i \Delta t}\right)}{\kappa_i}\lambda_i + \frac{1}{2}\sum_{i\cdot j\neq 1}^{N}\sigma_i\sigma_j\rho_{i,j}e^{-(\kappa_i+\kappa_j)T}\frac{1-e^{(\kappa_i+\kappa_j)\Delta t}}{\kappa_i+\kappa_j} \tag{B.6}$$