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# THE EFFICIENCY OF THE SUPPLY OF PUBLIC EDUCATION

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# The Efficiency of the Supply of Public Education

## ABSTRACT

The question of whether governments spend too much or too little has been a frequent subject of debate, but has been infrequently analyzed. This paper proposes and then applies a methodology which checks to see whether the "Samuelson condition" for the efficient provision of local public education is satisfied, i.e. whether the sum over the school district of individual marginal rates of substitution between public education and a private numeraire equals the marginal rate of technical substitution between these two goods. The econometric methodology uses a micro-based approach to the estimation of marginal rate of substitution functions, which accounts for possible biases associated with the selection of school districts by individual households.

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## I. Introduction

The question of whether governments spend too much or too little is a fine subject for popular debate. We do not expect to displace this issue from the realm of rhetoric, but we think that there are some feasible empirical tests that may suggest whether it is likely that any particular local government is spending more or less than a Pareto optimal amount on a specified activity. We will explain the theory behind one such test and apply it to the case of local school expenditures in Michigan school districts.

The procedure is very simple in principle. We seek to check whether the standard "Samuelson first order condition" (Samuelson, 1954) for efficient provision of public goods is satisfied. The Samuelson condition requires that the sum over the community of individual marginal rates of substitution between a public good and a private numeraire equals the marginal rate of technical substitution between these two goods. This condition is necessary for an interior Pareto optimum and in a well-behaved convex economy is also sufficient.

Bowen (1943) proposed a model in which the amount spent on public goods is the median of the quantities desired by voter-taxpayers, each of whom realizes that in return for the benefits of additional public expenditures, the burden will be predetermined share of the extra cost. Bowen showed that if marginal rates of substitution are symmetrically distributed and all citizens have the same tax shares, then majority rule leads to a Pareto efficient supply of public goods. Barlow (1970) suggests that the Bowen condition is typically not satisfied and argues that, in the case of local school expenditures in Michigan, the median quantity demanded is less than Pareto optimal. Bergstrom (1979) extends the domain of the Bowen efficiency theorem to more "realistic" cases.

It has been argued that voters systematically underestimate the benefits of public goods. Galbraith (1958) attributes this to the effect of private advertising. Downs (1960) argues that because information is costly and a single voter has a negligible effect on public outcomes, it is rational for voters to be less than fully informed about the effects of public goods. This, Downs argues, leads to a systematic underestimation of benefits, which are poorly understood. Others have suggested that local governments supply too little public goods because there are "spillovers" in benefits from one city to another. These effects are analyzed by Brazer (1961), Weisbrod (1964) and Williams (1966).

Taking a somewhat different point of view, Romer and Rosenthal (1979), Brennan and Buchanan (1971), Denzau and Mackay (1982), and others have suggested that bureaucrats may manipulate the choices offered to voters in such a way as to lead to greater expenditure than the median of the most preferred amounts of voters. Shapiro and Sonstelie (1982) found some evidence to support the hypothesis of bureaucratic manipulation, while Courant, Gramlich and Rubinfeld (1979) suggested that public employee market power might also lead to inefficient levels of public provision.[1]

Finally, the intermediate efficiency position is supported by Brueckner (1982) who suggests that an efficient provision of public goods is likely to arise if communities maximize aggregate property values. His empirical analysis of a sample of Massachusetts communities provides mild support for that view.[2]

The tests that we consider are designed to test for undersupply of the kind suggested by Barlow or oversupply due to bureaucratic manipulation. Since we deal with consumers' reported preferences about expenditures in their own jurisdictions, we will not be able to detect undersupply or oversupply that occurs because individuals do not know what is good for them. Our tests will also be unable to discover whether there is undersupply because of unrewarded spillovers from one city to another.[3] Furthermore, our results can tell us nothing about whether efficiency would require a different assignment of people to cities. The tests are useful only for finding out whether the existing population of a city could make a Pareto improvement for its members by increasing or decreasing its public expenditures.

Section II outlines the methodological underpinnings of the theory. The description of the data and empirical results appear in Section III. Section IV contains some brief conclusions. Details concerning the construction of some of our variables appear in the appendix.

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[1] See Inman (1980) for an alternative theoretical approach and Gramlich and Rubinfeld (1982) for some empirical evidence.

[2] Brueckner regresses aggregate property values on the level of public goods provision, arguing that a zero coefficient on the public good variable is a necessary condition for efficiency. Brueckner assumes (restrictively) that individuals have identical tastes, and that the allocation of land between residential and business use is fixed. The Brueckner test is, in addition, a test for average efficiency among all jurisdictions, so that oversupply in one community can be balanced against undersupply in another.

[3] If the system of grants correctly accounts for these spillovers, then our test for intrajurisdictional efficiency can tell us something about interjurisdictional efficiency as well.

## II. Methodology

Suppose that we observe a number of communities each of which supplies a single local public good and that the public good can be purchased at constant cost in the "national" market. We choose units of measurement so that the price of unit of public goods is one unit of private good. Each citizen $i$ of community $j$ has a utility function of the form

$$u_i(Y_i, A_j, H_i, Z_j) \tag{1}$$

$Y_i$ is $i$'s disposable income (consumption of private good), $A_j$ is the amount of the public good supplied in $j$, $H_i$ is a vector of personal characteristics of $i$ (such as age, sex, family status, etc.) and $Z_j$ is a vector of characteristics of community $j$ (such as its population, climate, proximity to other cities). Consumer $i$'s marginal rate of substitution between the public and private good will be a function of the form

$$m_i = m(Y_i, A_j, H_i, Z_j) \tag{2}$$

The Samuelson condition for community $j$ is just

$$\sum_i m_i = 1 \tag{3}$$

If we knew the exact nature of the function $m(\,)$, the value of $Y_i$ and $H_i$ for each of $n$ citizens in the community, the community characteristics $Z_j$ and public goods supply $A_j$, then we could check to see if (3) was satisfied by direct substitution. Of course, we will have to settle for a statistical estimate of the function $m(\,)$.

In the followig section, $m(\,)$ is estimated using a 1979 survey of Michigan households (see Courant, Gramlich and Rubinfeld [1979]). With the estimated $m(\,)$ and characteristics of a school district's population, the sum of marginal rates of substitution between public and private goods can be computed for any district. It is important to realize that the estimates of the marginal rate of substitution functions come from a statewide sample rather than just from those members of the sample who live in the specific community for which we estimate the summed marginal rates. If all of the observations came from a single community, we would not be able to identify a marginal rate of substitution function, since there would be no variation in expenditure levels experienced by respondents.

Information about the number of individuals with various personal characteristics is available from the census. However, information about the joint distribution of these characteristics with income or with each other typically is not. Therefore, in order to estimate the sum or marginal rates of substitution, we will have to make some restrictive assumptions about the functional form of $m(\,)$. In particular, we assume that individual marginal rate of substitution functions are of the form:

$$m(Y_i, A_j, H_i, Z_j) = \beta_0 + \beta_1 \ln A_j + \beta_2 \ln Y_i + \sum_{l=1}^{L} \beta_{3l} \ln Z_{jl} + \sum_{k=i}^{K} \beta_{4k} H_{ik} + \epsilon_i \tag{4}$$

This form was chosen not only because of its econometric convenience, but also because, as Bergstrom and Cornes (1983) have shown, a marginal rate of substitution function which is linear in disposable income is sufficient for the Pareto efficient amount of public goods to be determined by total income independently of other parameters of the income distribution. The stochastic term $\epsilon_i$ is thought to be the unmeasured explanatory variables and it is distributed $N(0, \sigma)$ and is orthogonal to the right-hand side explanatory variables.

From the Courant, Gramlich, Rubinfeld survey, we can calculate the respondent's tax price $(t)$. It would seem straightforward to estimate the parameters of (4) since the observed tax price would be equated with the marginal rate of substitution. But, for various reasons, this optimization condition may not be met; for instance, public goods are complementary with private goods and community choices with desired combinations of public and private goods may be limited, or alternatively, households may simply misperceive or make mistakes.[4] Since public goods choices are made with error, the observed relationship is

$$t_i = m(\,) + v_i \tag{5}$$

The random variable $v_i$ represents the difference between the household's marginal rate of substitution and its tax price.

If $v_i$ is uncorrelated with variables included in the mrs function, the mismatch between $t_i$ and $m_i$ would cause no econometric problem. In such a case the least squares estimator of the parameters of equation (4), using the tax price as a dependent variable, would be unbiased. If people sorted

[4] In Bergstrom, Rubinfeld, and Shapiro (1982) we discuss some of the problems associated with the measurement of tax price.

themselves perfectly among communities and were able to equate their marginal rates of substitution with their tax price, $v_i$ would be zero for all $i$. In this case of Tiebout-Lindahl equilibrium, a least squares estimation procedure would be appropriate. However we do not know whether or not people are able to equate $t$ with $m$ or more importantly whether or not the error $v_i$ is uncorrelated with the right-hand variables in equation (4). It seems plausible that ability to match (the size of $v_i$) is related, for instance, to income and education, and perhaps the level of public good supplied - the variables that affect the marginal rate of substitution.

If matching errors $v_i$ depend on the same explanatory variables as does the marginal rate of substitution, the correct econometric specification is more complicated than a simple linear regression of tax price on the individual and community explanatory variables.

The central variable involves the survey questions which describe individual attitudes towards education. They reveal whether individuals want expenditures to be more (M), the same (S), or less (L). Unlike data on market behavior, from which purchases reveal the relationship between quantities consumed and marginal rates of substitution, the qualitative survey responses can only give an approximation of the true value of the marginal rate of substitution. Economic theory tells us if preferences are convex, a person wants greater spending if the mrs is greater than the tax price and less spending if it is smaller than the tax price. We will show that the survey responses along with substantial variations in tax price and expenditures, provide enough information to estimate the parameters of the mrs equation (4).

The details of the estimation technique are presented in two previous papers.[5] Here we present an outline of the estimation model for the case in which tax price is endogenous. Results for the more complete case in which the level of public good expenditure as well as tax price is endogenous are described briefly later.

In order to simplify the presentation, let $x_i = (\ln A_j, \ln Y_i, \ln Z_j, H_i)$ be the vector of right-hand variables of the mrs equation. Equation (4) is rewritten

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[5] Bergstrom, Rubinfeld and Shapiro (1982) and Rubinfeld, Shapiro and Roberts (1986).

$$m_i = \beta' X_i + \epsilon_i \tag{6}$$

Individual survey respondents answer more (M) if their mrs is ***sufficiently*** larger than their tax price and less (L) if it is ***sufficiently*** smaller. If there is an insufficient difference between mrs and $t$, the answer is same (S). The concept of sufficient difference is formalized with a parameter $\delta$ such that the response model can be written

| **Response** | **Condition** |
|---|---|
| M: | $m > t + \delta$ |
| S: | $t + \delta \geq m \geq t - \delta$ |
| L: | $m < t - \delta$ |

This simple model of response allows us to express the probability of any response conditional on $t$ and $x$ as

$$\begin{aligned} P(M|t, x) &= P(\epsilon > t - \beta' x + \delta) \\ P(S|t, x) &= P(t - \beta' x + \delta \geq \epsilon \geq t - \beta' x - \delta) \\ P(L|t, x) &= P(\epsilon < t - \beta' x - \delta) \end{aligned} \tag{7}$$

Since $\epsilon \sim N(0,\sigma)$, the probabilities can be expressed in terms of the standard normal cumulative density function F( ) in the following way:

$$P(M|t, X) = 1 - F\left[\frac{t - \beta' x + \delta - E(\epsilon|t, X)}{\sigma(\epsilon|t, x)}\right]$$

$$P(S|t, X) = F\left[\frac{t - \beta' X + \delta - E(\epsilon|t, X)}{\sigma(\epsilon|t, x)}\right] - F\left[\frac{t - \beta' X - \delta - E(\epsilon|t, X)}{\sigma(\epsilon|t, x)}\right] \tag{8}$$

$$P(L|t, X) = F\left[\frac{t - \beta' X - \delta - E(\epsilon|t, X)}{\sigma(\epsilon|t, x)}\right]$$

As long as $\epsilon$ is distributed independently of $t$ and $X$, $E(\epsilon|t, x) = 0$. Maximum likelihood estimation using these probability functions is then a straightforward problem, which yields consistent and efficient estimators of the parameters $\beta$ and $\delta$. However, because there is reason to suspect that

$\epsilon$, $t$ and $X$ are not independently distributed, the estimation is more complex. From equation (5) and (6) we can write

$$t_i = \beta' X_i + \epsilon_i + v_i \tag{9}$$

One way to express the possibility that $v_i$ might depend on $X$ is to make it a linear function of $X$ in the following way:

$$v_i = \gamma_1' X_i + \gamma_2' \hat{X}_i + u_i \tag{10}$$

In this formulation, $\hat{X}_i$ is a vector of variables that explains the mismatch, but does not enter the marginal rate of substitution function; and $\epsilon$ is a random error uncorrelated with $X$ and $\hat{X}$. The random error $u_i$ is assumed to be distributed $N(0, \sigma_u)$.

The tax price equation is therefore

$$t_i = (\beta' + \gamma_1') X_i + \gamma_2' \hat{X}_i + \omega_i \tag{11}$$

where $\omega_i = \epsilon_i + u_i$. Because of the normality of $\epsilon_i$ and $u_i$, $\omega_i \sim N(0, \sigma_\omega)$.

Elsewhere (Rubinfeld, Shapiro and Roberts [1986]) we have proven that

$$E(\epsilon|t, X) = \lambda[t - (\beta' + \gamma_1') X - \gamma_2' \hat{X}]$$

where $\lambda$ is a parameter proportional to the covariance of $\epsilon$ and $\omega$. Values of $\lambda$ significantly different from zero indicate that sorting or matching is important, and that an alternative to least-squares estimation is necessary.

Equations (8) and (11) constitute a system of simultaneous equations. The first set explains the probability of response conditional on a particular tax price and the second explains the tax price in relation to the determinants of the marginal rate of substitution and the systematic mismatch between price and mrs. Under the distributional assumptions, the log-likelihood function is

$$\begin{aligned} L = & \sum_{i \in Less} \log F\left(\theta_0^L + \frac{1-\lambda}{\sigma_e} t_i - \frac{\beta - \lambda\theta_1}{\sigma_e} X_i - \frac{\lambda\gamma_2}{\sigma_e} X_i\right) \\ & + \sum_{i \in Same} \log\left\{F\left(\theta_0^M + \frac{1-\lambda}{\sigma_e} t_i - \frac{\beta - \lambda\theta_1}{\sigma_e} X_i - \frac{\lambda\gamma_2}{\sigma_e} X_i\right)\right. \\ & \left. - F\left(\theta_0^L + \frac{1-\lambda}{\sigma_e} t_i - \frac{\beta - \lambda\theta_1}{\sigma_e} X_i - \frac{\lambda\gamma_2}{\sigma_e} X_i\right)\right\} \end{aligned} \tag{12}$$

$$+ \sum_{i \in More} \left\{ 1 - F\left[ \theta_0^M + \frac{1-\lambda}{\sigma_e} t_i - \frac{\beta - \lambda\theta_1}{\sigma_e} X_i - \frac{\lambda\gamma_2}{\sigma_e} X_i \right] \right\}$$

$$- \Sigma \log 2\pi\sigma_w^2 - \frac{1}{2\sigma_w^2} \Sigma (t - \theta_1 X_i - \gamma_2 X_i)^2$$

where

$$\theta_0^L = \frac{\delta - (1-\lambda)\beta_0}{\sigma_e}, \quad \theta_0^M = -\frac{\delta + (1-\lambda)\beta_0}{\sigma_e}, \quad \theta_1 = \beta + \delta_1$$

and

$$\sigma_e = \sigma(\epsilon | t, X)$$

The first three sums of equation (12) are the standard probit likelihood function, while the final two terms are the equivalent of a standard linear regression of $t$ on $X_1$ and $X_2$. If one were to estimate the parameters of the mrs function with a simple regression of $t$ on $X_1$ and $X_2$, the parameters would not be the expected mrs parameters, $\beta$, but the sum of these and the matching parameters $\gamma$ (see Ladd and Christopherson [1983]). On the other hand, if one were to use a probit estimation as was done in Bergstrom, Rubinfeld and Shapiro (1982), the resulting estimators would generally be inconsistent unless we have estimated the parameters by maximizing the full likelihood function given in (12).

## III. Testing for Efficiency in the Provision of Education

The first step in the efficiency test is to estimate the parameters of the likelihood function (12). The data used to estimate these parameters was a subsample of 1093 homeowners from a survey of Michigan voters residing in many different school districts. The amount of education provided in each school district was obtained from the Michigan Department of Education; other community characteristics were taken from the 1970 U.S. Census First Count and Fourth Count School District data tapes. The definitions of all variables utilized in the estimation procedure are given in Table 1.

The mrs function is specified as linear in the logarithm of the continuous variables and a set of relevant dummy variables. The explanatory variables were chosen first with respect to economic theory which suggests that with convex preferences, the marginal rate of substitution will vary

TABLE 1
Definition of Variables

| | |
|---|---|
| PRICE: | Individual's tax price |
| LNA: | Log of actual per pupil educational expenditures |
| LNY: | Log of household disposable income |
| BLACK: | Dummy variable 1 if black 0 otherwise |
| AGE 1-5: | Number of children younger than six years old |
| AGE 6-11: | Number of children between 6 and 11 |
| PRIVSCH: | Dummy variable 1 if child in private school 0 otherwise |
| COLGRAD: | Dummy variable 1 if a college graduate 0 otherwise |
| LTHS: | Dummy variable 1 if did not graduate from high school 0 otherwise |
| FEMALE: | Dummy variable 1 if female 0 otherwise |
| RETDA: | Dummy variable 1 if retired or disabled 0 otherwise |
| AGE 65+: | Dummy variable 1 if age over 65 0 otherwise |
| UNEMPL: | Dummy variable 1 if unemployed 0 otherwise |
| TRANSFER: | Dummy variable 1 if transfer payments received 0 otherwise |
| LNENRL: | Log of total school district enrollment |
| LNPUP-B: | Log of number of students per school |
| LNCOTEACH: | Log of county average teachers salary |
| LNCY: | Log of number of students per school |
| LNCW: | Log of county average wage rate |
| YDIST: | Fraction of households with income within 30 percent of median (YDIST is thought of as a measure of population homogeneity) |
| CCITY: | Dummy variable 1 if in central city 0 otherwise |
| SMSA: | Dummy variable 1 if in SMSA 0 otherwise |

inversely with the level of expenditures (A) and directly with the disposable income (Y). The remaining variables were included either because they might reasonably affect underlying preferences or because they might affect the real price of education.

Race (BLACK) is one of the variables intended to account for differences in preference. The only division we used here was between Black and Non-Black. A respondent with children (Ages 1-5 and Ages 6-11) could reasonably be expected to have a higher desired expenditure level than a similar respondent without children. Conversely, a respondent with a child in a private school (PRIVSCH) could be expected to have a smaller desired expenditure. The dummy variables for education of the respondent were included as indicators of taste. Educated (COLGRAD) people can reasonably be expected to have stronger preferences for education than uneducated ones (LTHS). We had no expectation about how gender (FEMALE), retirement (RETDA), employment status (UNEMPL), welfare status (TRANSFER) or age (AGE65+) affect taste for public education.

The continuous variables were included as descriptors of the school district in order to account for differences in quality per unit of expenditure. School district size (LNENRL) and average school size (LNPUP-B) were used to account for potential economies or diseconomies of scale at the level of the school district and the individual schools. The average teacher's salary (LNCOTEACH) gives a measure of the amount of teachers' services that can be purchased per dollar of expenditure. The higher the salary, the lower the number of hours of teacher's time that can be purchased. However, salaries might also be an index of teacher quality and in equilibrium interjurisdictional wage difference makes no difference to the unit of educational quality per dollar of expenditure. The variables average income (LNCY) and wage rates (LNCW) are included to account for cost of living difference between jurisdictions. We might reasonably expect that income affects demand for products and the wage rate the supply. The higher these are, the higher prices of all goods are likely to be.

Three additional variables are chosen as explanatory values for $v$, the mismatch between tax price and marginal rate of substitution; as an identifying restriction the additional variables are assumed not to affect the mrs itself. We try to capture the homogeneity of the community by the

**TABLE 2**
**Maximum Likelihood Coefficients**

| Coefficients/ Variables | 1 | Coefficients/ Variables | 2 |
|---|---|---|---|
| $\Theta_0^L$/Constant | 8.152 | | |
| | (4.219) | | |
| $\Theta_0^M$/Constant | 9.891 | Constant | 1.613 |
| | (4.222) | | (2.892) |
| $\frac{1}{\sigma e}$/PRICE | 2.014 | | |
| | (1.105) | | |
| $\frac{\beta}{\sigma e}$ | | $\beta+\gamma_1$ | |
| /LNA | -1.118 | /LNA | -0.419 |
| | (0.597) | | (0.136) |
| /LNY | 0.413 | /LNY | 0.152 |
| | (0.180) | | (0.026) |
| /BLACK | 1.206 | /BLACK | 0.050 |
| | (0.190) | | (0.080) |
| /AGE 1-5 | 0.126 | /AGE 1-5 | -0.037 |
| | (0.073) | | (0.026) |
| /AGE 6-11 | 0.172 | /AGE 6-11 | 0.016 |
| | (0.057) | | (0.024) |
| /PRIVSCH | -0.190 | /PRIVSCH | 0.C38 |
| | (0.150) | | (0.064) |
| /COLGRAD | 0.337 | /COLGRAD | 0.110 |
| | (0.155) | | (0.046) |
| /LTHS | -2.116 | /LTHS | -0.023 |
| | (0.100) | | (0.044) |
| /FEMALE | 0.135 | /FEMALE | 0.005 |
| | (0.725) | | (0.033) |
| /RETDA | -0.353 | /RETDA | 0.063 |
| | (0.148) | | (0.059) |
| /AGE 65+ | 0.138 | /AGE 65+ | -0.002 |
| | (0.141) | | (0.064) |
| /UNEMPL | -0.258 | /UNEMPL | 0.046 |
| | (0.232) | | (0.103) |
| /TRANSFER | -0.422 | /TRANSFER | -0.130 |
| | (0.283) | | (0.108) |
| /LNENRL | -0.158 | /LNENRL | -0.040 |
| | (0.065) | | (0.026) |
| /LNPUP-B | 0.633 | /LNPUP- | 0.233 |
| | (0.290) | | (0.062) |
| /DETROIT | -0.334 | /DETROIT | -0.150 |
| | (0.313) | | (0.108) |
| /LNCOTEACH | 0.988 | /LNCOTEACH | 0.103 |
| | (0.749) | | (0.363) |

**TABLE 2 (continued)**

| | | | |
|---|---|---|---|
| /LNCY | 0.30 | /LNCY | 0.459 |
| | (1.035) | | (0.367) |
| /LNCW | 0.226 | /LNCW | -0.651 |
| | (1.240) | | (0.444) |
| $\frac{\lambda}{\sigma e}$ | 1.860 | $\gamma_2$/YDIST | 0.688 |
| | (1.107) | | (0.392) |
| | | /CCITY | 0.016 |
| | | | (0.062) |
| | | /SMSA | 0.059 |
| | | | (0.071) |

**TABLE 3**
**Marginal Rate of Substitution and Mismatch Parameters**

| Variable | (1) MRS Parameters ($\beta$) | (2) Mismatch Parameters ($\gamma$) |
|---|---|---|
| $\delta$ | 0.432 | |
| Constant | -4.480 | 6.093 |
| LNA | -0.555 | 0.136 |
| LNY | 0.205 | -0.553 |
| BLACK | 0.599 | -0.549 |
| AGE 1-5 | 0.063 | -0.100 |
| AGE 6-11 | 0.085 | -0.069 |
| PRIVSCH | -0.095 | 0.133 |
| COLGRAD | 0.167 | -0.057 |
| LTHS | -0.105 | 0.082 |
| FEMALE | 0.067 | -0.062 |
| RETDA | -0.175 | 0.238 |
| AGE 65+ | 0.068 | -0.070 |
| UNEMPL | -0.128 | 0.174 |
| TRANSFER | -0.210 | 0.070 |
| LNENRL | -0.078 | 0.038 |
| LNPUP-B | 0.314 | -0.354 |
| DETROIT | -0.166 | 0.004 |
| LNCOTEACH | 0.491 | -0.388 |
| LNCY | 0.015 | 0.444 |
| LNCW | 0.112 | -0.763 |
| YDIST | | 0.688 |
| CCITY | | 0.016 |
| SMSA | | 0.059 |

disperson of income (YDIST). We expect that the more homogeneous the community, the closer will be the match between desired and actual expenditures. Individual choice sets should also affect the ability to achieve the optimality condition. In urban areas (CCITY and SMSA), where there are many jurisdictions, individuals have more choices in the mix of tax prices and expenditures than in rural areas with fewer jurisdictions.

The maximum likelihood coefficients and the standard errors associated with the specification just described are given in Table 2. Each of the coefficients in column 1 represents the ratio of the marginal rate of substitution parameters ($\beta$'s) to the conditional standard deviation ($\sigma_e$). The consistent estimators of $\beta$ are found as the ratio of the coefficient of the explanatory variables to the coefficient of PRICE. These values are given in column 1 of Table 3. The coefficient of particular interest is the estimate of $\lambda/\sigma_e$. With a $t$ value of 1.68, the coefficient is significantly different from zero at a 10 percent level of significance. It appears that sorting of people by their preferences for public education can cause the coefficients of a simple probit maximum likelihood estimation to be biased.[6] Of the three variables hypothesized to affect the matching of $t$ and mrs, YDIST, CCITY and SMSA, only the first is significantly different from zero.

The mrs parameters, $\beta$'s are found by dividing the coefficients in column (1) by the coefficient on PRICE which itself is an estimate of $\frac{1}{\sigma_e}$. The coefficients as reported in column (1) reflect the effect of the listed variable on the probability of a more (M) response to the survey. For instance, the negative coefficient on LNA indicates that the larger the actual level of spending, the lower the probability of an M response. Similarly, the positive coefficient on LNY indicates that the higher the respondent's income, the more likely was an M response.

The mrs parameters $\beta$ as well as $\delta$ are given in column (1) of Table 2. $\delta$ and the constant are computed

$$\hat{\delta} = \frac{\hat{\theta}_0^M - \hat{\theta}_0^L}{2\left(\frac{\hat{1}}{\sigma_e}\right)} \tag{13}$$

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[6] Whether this source of bias will affect our efficiency test is a separate question.

$$Constant = \hat{\beta}_0 = \frac{\hat{\theta}_0^M + \hat{\theta}_0^L}{2\left(\frac{\hat{1}}{\sigma_e}\right)}$$

where the ' $\hat{}$ ' indicates maximum likelihood estimator. All the remaining $\beta$'s are calculated as the ratio of $(\beta/\sigma_e)$ to $(1/\hat{\sigma}_e)$ from column (1) in Table 2. The estimated value of the mismatch parameters ($\gamma$'s) are calculated as the difference between the estimated values of $(\beta + \gamma)$ given in column (2) of Table 2 and the estimated values of $\beta$ given in column (1) of Table 3. The resulting estimates are given in Table 3, column (2).

Computing the estimated values of the community marginal rates of substitution follows in a straight-forward way from equation (4). Replacing the $\beta$'s in (4) with their estimated values and summing (4) over all individuals in community $j$ yields

$$\sum_{i=1}^{N} m(Y_i, A_j, H_i, Z_j) = \hat{\beta}_0 + N(\hat{\beta}_1 \ln A_j + \sum_{l=1}^{L} \hat{\beta}_{3l} \ln Z_{jl}) + \hat{\beta}_2 \sum_{i=1}^{N} \ln Y_i + \sum_{i=1}^{N} \sum_{k=1}^{K} \hat{\beta}_{4k} H_{ik} \quad (14)$$

The variables $A_j$ and $Z_j$, which are the level of educational expenditure per pupil and other characteristics of community $j$, can be found in published sources. The components of the vector $H_k$ are zero-one variables specifying whether an individual is a homeowner, older than 65, non-white, etc. Therefore, the sum of these vectors is simply the vector whose components are the number of persons in a community with the corresponding characteristics.

For the actual calculations, we computed the community mean mrs and multiplied it by the population size. By the linearity of the mrs specification, equation (6), this computation is

$$\hat{m}_j = \sum_{i=1}^{M} \hat{m}_i = n\bar{m} = n(\hat{\beta}'\bar{x}) \quad (15)$$

where $\bar{m}$ is the estimate of the mean value of mrs and $\bar{x}$ is a vector of the average values of the explanatory variables $x$.

Our estimating equation is based on household income and tax price, and our estimates of marginal rates of substitution use Census data on family income and income of unrelated individuals in a school district. In order to construct a community mrs from family and individual data, we found the mean family mrs and the mean individual mrs and multiplied them by the number of families and the number of unrelated individuals respectively. The sum of these products is the estimated community mrs.

In the process of matching the census data to the needs of the model we confronted an additional difficulty with the income variable -- the distinction between families and individuals. The distribution of families in a school district was given for fifteen income brackets, allowing us to estimate median family income (we assumed a uniform distribution within each bracket). Assuming income is distributed log-normally,

$$mean(Ln\ Y^F) = median(Ln\ Y^F) = \log(median\ Y^F)$$

Since the only data available for unrelated individuals were aggregate income and the number of individuals, we took $Ln(mean(Y^I))$ as an approximation for mean $(Ln(Y^I))$. Other census data are not broken down into families and individuals but are given for the community as a whole. Assuming all children live in families, we let

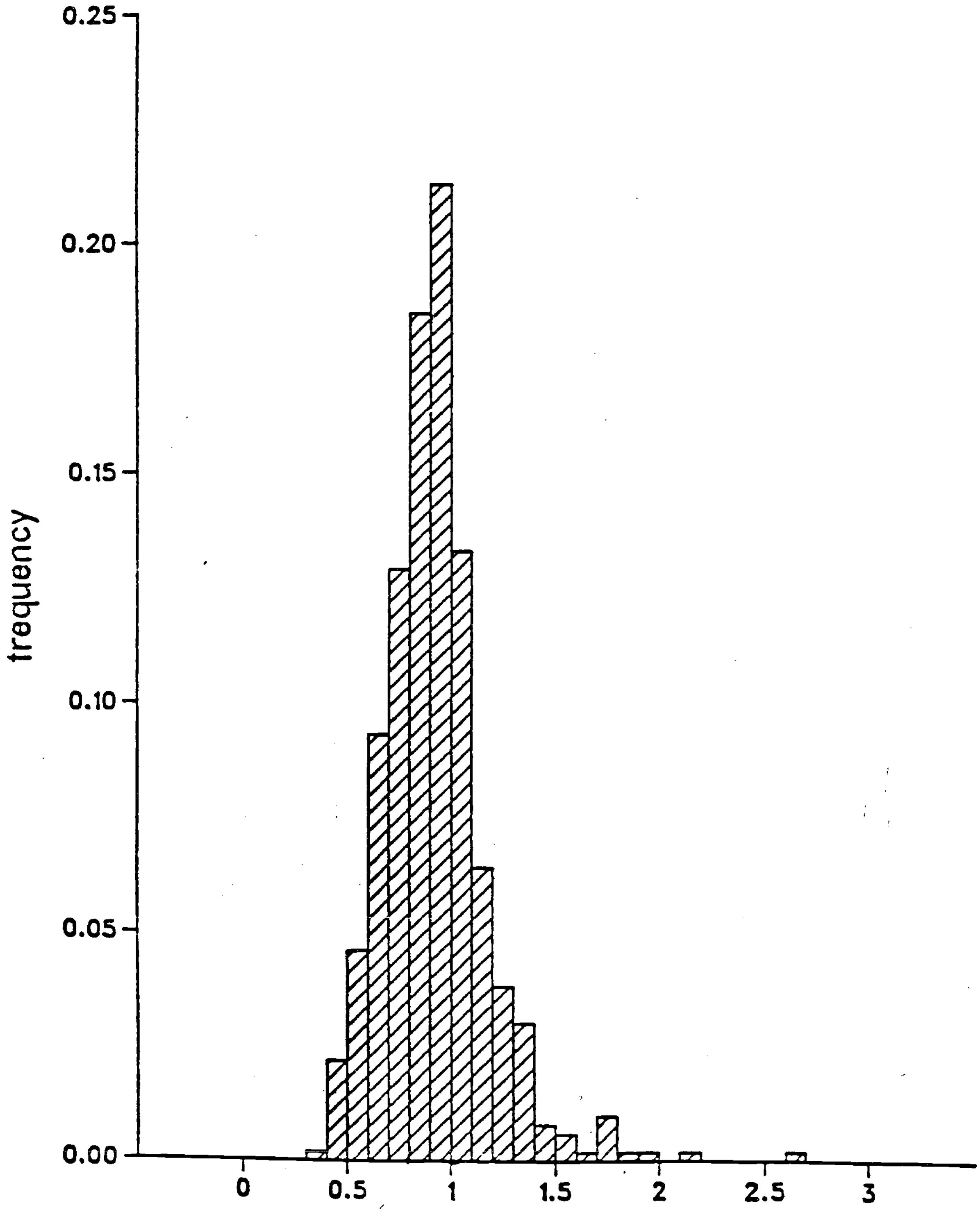
$$AGE\,1\text{—}5^I = 0$$
$$AGE\,1\text{—}5^F = (\#\ children\ age\ 0\text{—}5)/(\#\ families)$$

and similarly for AGE 6-11 and PRIVSCH. All the other demographic characteristics were assumed to be evenly distributed across families and individuals, e.g., COLGRAD = (# college grads)/(population 18+).

The distribution of the computed values of community mrs, $\hat{m}_j$, is shown in Figure 1. The average values for 576 Michigan school districts is 0.971 with a standard deviation of 0.276 and a range among individual school districts of 0.276 and 2.881. The apparent conclusion is that, on average, these communities supply very close to the efficient level of educational expenditures. However, this conclusion must be viewed as tentative for a number of important reasons. First, the standard deviation is sufficiently large so that we cannot rule out the possibility of either under- or oversupply of public education with substantial certainty. Second, the results are somewhat sensitive to the method of estimation and the specification of the relevant equations.

It is interesting to note that when the single equation maximization method is applied, the mean value of the community mrs's is .95 with a standard deviation of .28. While correction for the

# SUMMED MARGINAL RATES OF SUBSTITUTION:
## Distribution using Full Information Maximum Likelihood



endogeneity of tax-price does alter the estimates of the structural parameters substantially, it does little to our efficiency calculations.

We tried a number of alternative specifications for the simultaneous maximization approach, concentrating on the choice of instruments for the matching equation. We were concerned with the fact that the theory of sorting suggests the possibility of a nonlinear relationship between mismatch and the instruments, since the absolute value of $v_i$ is more important than its numerical value. To allow for the possibility of this nonlinearity we interacted the more, same, and less responses with each instrument (and kept the original instruments). Our calculated sum of the community mrs's fell to .92.[7]

Since the true marginal rate of transformation between public and private goods is equal to one, a test for social efficiency involves a test of the null hypothesis that $\hat{m}_j$ is equal to 1. However, the cost of education from the viewpoint of the school district may be less than 1 due to grants, commercial and industrial tax base, and more generally the prospect of tax exporting.

For instance, the cost of public services is likely to be less than one in many school districts, since some of the tax base is commercial and industrial property. The argument that some taxes can be exported has been tested empirically by Ladd (1975), who estimated that roughly 50 percent of the industrial base and 79 percent of the commercial base was available to lower the effective cost of public services. Other explanations of the deviation of mrs from one are possible as well, including the prospect of matching grants. However, during 1970, when the Census data were collected, all state grants to education were non-matching, and thus inframarginal. State grants have not been incorporated into this part of the analysis.[8]

Rather than posit an arbitrary estimate for the reduction of price due to the commercial/industrial base, we attempted to estimate the effect. We assumed that communities can export a portion, $\alpha$, of their non-residential property tax base. Letting $pnr_j$ be the percent of non-

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[7] We also contemplated the possibility that both tax-price and school expenditure could be endogenous variables. Endogeneity of the expenditure variable can alter the calculations substantially because the expenditure coefficient appears in the denominator of the various ratios that are used to calculate the estimates of the structural parameters. When we reestimated and reevaluated our results with endogenous expenditures (as well as tax-price) we obtain a community sum of mrs's of 1.21.

[8] At the time of the survey matching grants were available but not perceived to affect tax price. See Bergstrom, Rubinfeld, and Shapiro (1982).

residential property value in community $j$, the community efficiency condition is

$$m_j = 1 - \alpha pnr_j \tag{16}$$

We could use this equation to calculate $\alpha$ but our calculated values $\hat{m}_j$ are measured with error. Furthermore, the public choice process is likely to lead to additional errors within each community. Therefore we postulate that the observed relationship to be

$$\hat{m}_j = 1 - \alpha pnr_j + u_j \tag{17}$$

where the random error $u_j$ is $N(\mu_0, \sigma_u)$. We allow for the possibility that community choices may be biased by the possibility of a non-zero mean. We also assume that $u_i$ is independent of the explanatory variables -- a crucial assumption here. The regression results are

$$\hat{m}_j = \underset{(0.021)}{1.121} - \underset{(0.054)}{0.430}\, pnr_j \qquad R^2 = 0.127 \tag{18}$$

This regression suggests that for every one percent non-residential tax base in a community, the sum of the marginal rates of substitution should fall by 0.43 percentage points. If public spending is chosen optimally, this result indicates that communities can export 43 percent of their non-residential tax base.

A more complete description of the community variables that explain the differences in marginal rates of substitution is given in Table 5, using the data described in Table 4. The results are from a regression using $\hat{m}_j$ as the dependent variable. They indicate that the more homogeneous the community (the larger the value of YDIST) the smaller the mrs. This is a curious result since an average value of 0.97 indicates some overspending. The significant TSAL coefficient implies that higher teachers' salaries lead to higher expenditures per pupil. Similarly the larger the fraction of blacks and unrelated individuals, the higher educational expenditures. Apparently poor communities, those with a large fraction of welfare recipients and unemployed, have a tendency to spend more than the efficient amount.

The final coefficient on PCRES is particularly interesting, for it indicates how community marginal rates of substitution change when residential value as a proportion of the tax base increases. If communities' educational expenditures are efficient, the coefficient of 0.31 indicates the amount of non-residential tax base that is exported. If none is exported, then changing PCRES would not affect

**TABLE 4**
**Definition of Variables**

| | |
|---|---|
| YDIST: | Fraction of households with income within 30 percent of median |
| PCOOC: | Fraction of owner occupied residences |
| SMSA: | Dummy variable for SMSA |
| TSAL: | Average teacher's salary |
| PCBLACK: | Fraction of population that is black |
| PC65UP: | Fraction of population at least 65 |
| PCINDS: | Fraction of population that are unrelated individuals |
| PUPILS: | Number of pupils in the school district |
| PCCOL: | Fraction of population with a college degree |
| PCHNDICP: | Fraction of population with handicap |
| PCWELFAR: | Fraction of population receiving welfare |
| PCSOCSEC: | Fraction of population receiving social security |
| RU: | Unemployment rate |
| PCRES: | Fraction of tax base that is residential (1-pnr) |

**TABLE 5**
**Explaining the Community**
**Marginal Rate of Substitution**

| | |
|---|---|
| Constant | -0.014 |
| | (0.202) |
| YDIST | -0.647 |
| | (0.211) |
| PCOOCC | 0.070 |
| | (0.104) |
| SMSA | -0.007 |
| | (0.027) |
| TSAL | -0.00011 |
| | (0.00002) |
| PCBLACK | -0.486 |
| | (0.149) |
| PC65UP | 0.740 |
| | (0.734) |
| PCINDS | -3.290 |
| | (0.246) |
| PUPILS | -0.00000078 |
| | (0.00000065) |
| PCCOL | -0.247 |
| | (0.228) |
| PCHNDICP | -0.316 |
| | (0.284) |
| PCWELFAR | -2.683 |
| | (0.824) |
| PCSOCSEC | -0.934 |
| | (0.857) |
| RU | -1.175 |
| | (0.192) |
| PCRES | 0.310 |
| | (0.045) |

$R^2 = 0.550$

mrs and the coefficient would be zero. If the entire tax on non-residential property were exported, then the coefficient would be one. In the 576 Michigan communities the average value of PCRES is approximately 65 percent. Therefore, the estimated value of the coefficient suggests that school districts act as if they can export approximately 11 percent (0.31 x 0.65) of their property tax.

## IV. Summary

We have provided a micro-based test for efficiency in the provision of local public schooling within school districts. Our results are generally consistent with the view that public education is efficiently supplied in Michigan. In addition, using the assumption that school districts supply education efficiently, we calculate that local residents bear approximately 90 percent of the local property tax burden, while 10 percent is exported. Further research on the model specification and on the relationship between educational expenditure and educational output would extend our work in useful directions.

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