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ABSTRACT

Many leading asset pricing models predict that the term structures of expected returns and volatilities on dividend strips are strongly upward sloping. Yet the empirical evidence suggests otherwise. This discrepancy can be reconciled if these models replace their exogenously specified dividend dynamics with processes that are derived endogenously from capital structure policies that generate stationary leverage ratios. Under this policy, shareholders are being forced to divest (invest) when leverage is low (high), which shifts risk from long-horizon to short-horizon dividend strips. This framework also generates stock volatility that is higher than long-horizon dividend volatility, even with constant market prices of risk.

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1 Introduction

Recent advances in consumption-based asset pricing models have provided frameworks for capturing salient features of security prices (e.g., high excess returns and volatilities for stocks, low levels and volatilities for interest rates). Primitive inputs to these endowment economies include agent's preferences, consumption dynamics, and dividend dynamics. These first two inputs identify the pricing kernel, which provides information on the risk free rate and market prices of risk. The pricing kernel and dividend dynamics combine to determine the prices, excess returns and volatilities of stocks.

While successful in capturing the asset pricing properties mentioned above, many leading asset pricing models, such as Campbell and Cochrane (CC, 1999) and Bansal and Yaron (BY, 2004), predict that the term structure of expected returns and volatilities on dividend strips (i.e., claims to dividends paid over some prespecified interval) are strongly upward sloping. Yet the empirical evidence reported in Binsbergen, Brandt and Koijen (BBK, 2011) suggests otherwise. While Boguth, Carlson, Fisher and Simutin (2011) have questioned some of the findings of BBK, both papers seem to agree that the strongly upward sloping term structures predicted by BY and CC are inconsistent with the historical evidence on returns of dividend strips.[1]

In this paper we demonstrate that the counterfactual implications of these leading asset pricing models can be eliminated without altering their proposed preferences or consumption processes (and thus, their pricing kernels). Rather, all that is necessary to make these models consistent with the empirical findings of BBK is to replace their proposed dividend dynamics with processes that are both more economically justifiable and more consistent with the empirical features of dividend dynamics that we identify below. Importantly, we show that our proposed changes to dividend dynamics do not impact these frameworks' abilities to capture the salient properties of stock returns and interest rates mentioned previously.

Instead of specifying an exogenous dividend process as in BY and CC, in this paper we investigate a framework in which dividend dynamics are derived *endogenously* from capital structure policies that generate stationary leverage ratios.[2] We show that these

[1]See also Binsbergen, Hueskes, Koijen, and Vrugt (2011) for updated and additional empirical evidence using international data.

[2]There are at least two reasons why leverage ratios for an aggregate index are stationary. First, the literature on optimal dynamic capital structure (e.g., Fischer, Heinkel and Zechner (1989), Goldstein, Ju and Leland (2001)) shows that firms which wish to maximize shareholder value will follow such a strategy. Second, firms which perform poorly will be eliminated from the index long before they default.

internally consistent dividend dynamics have a first order impact on both the empirical properties of dividends and on the excess returns and volatilities of dividend strips.

By specifying dividend dynamics that are internally consistent with stationary leverage ratios, our model is able to capture two important properties which, at first blush, seem contradictory. First, compared to unleveraged cash flows of a firm (which we will refer to as EBIT), dividends are a leveraged cash flow. It is thus not surprising that claims to dividends (i.e., equity) are more volatile and have higher average historical returns than claims to EBIT (i.e., debt and equity). Yet, over long horizons, EBIT and dividends should be cointegrated in that, path-by-path, dividends and EBIT should share the same long run growth rate. Hence, at long horizons, dividends are no riskier than EBIT.

This apparent contradiction can be explained by noting that when a firm rebalances its debt levels over time to maintain a stationary leverage process, shareholders are being forced to divest (invest) when leverage is low (high). Thus, even if investors follow a static strategy of holding a fixed supply of stock, their position is effectively being managed by the capital structure decisions of the firm. Below, we show that these imposed investments/divestments conceal the leveraged nature of dividends in that, even though dividends are correctly interpreted as leveraged cash flows, over the long run, EBIT and dividends are equally risky.

Interestingly, leading asset pricing models either ignore the leveraged nature of dividends, or its cointegration with unleveraged cash flows, or both. Moreover, even if they do account for leverage, they do so in a reduced-form way by introducing free parameters that are not directly tied down to observed leverage ratios. For example, CC specify consumption and dividends as iid with the same drift, and therefore disregard leverage. BY capture leverage by assuming that dividends have greater exposure to shocks in expected growth rates than does consumption. However, their model does not capture cointegration. Abel (1999, 2005) models cash flows to be of the form y^{λ}, where $\lambda = 0$ for fixed income securities, $\lambda = 1$ for EBIT, and $\lambda > 1$ for dividends. This framework also does not capture cointegration (in levels).

To demonstrate the impact of capital structure policies on the properties of dividends in leading asset pricing models, we investigate modified versions of the BY and CC economies. Instead of specifying dividends exogenously, we exogenously specify an unleveraged cash flow (i.e., EBIT) process (with the same functional forms as the dividend processes in BY and CC) and combine it with a dynamic capital structure strategy that produces stationary leverage ratios. These two ingredients generate an endogenously obtained dividend process that is internally consistent with the EBIT process. Claims to this dividend process (i.e., equity) have higher expected returns and higher volatilities than claims to EBIT (i.e., equity plus debt). Yet, this framework generates dividend and EBIT processes that are cointegrated.

Our main findings can be summarized as follows. First, our framework generates a term structure of expected returns and volatilities for dividend strips that are decreasing in horizon, consistent with the empirical findings of BBK, and in contrast to the baseline models of BY and CC. Intuitively, this is due to the implicit divestments (investments) that the firm's capital structure policy imposes on stockholders in good (bad) times. As such, long-maturity dividend strips are not as risky as typically imagined – rather, they are about as risky as long-maturity EBIT strips, since dividends and EBIT are cointegrated. However, claims to *all* future dividends (i.e., equity) are riskier than claims to EBIT (ie., equity plus debt). The implication is that dynamic capital structure decisions that generate stationary leverage ratios shift the risk in dividends from long-horizons to short horizons, and thus generate a downward shift in the slope of the term structure of dividend strip returns compared to the slope of EBIT strips. We demonstrate that this impact is very large for both the CC and BY models. Indeed, calibrating a simple parsimonious model assuming separation of investment and capital structure decisions, we obtain downward sloping term structures for dividend strip returns for both the modified BY and CC models in spite of the fact that their term structures for EBIT strip returns are upward sloping.[3]

Second, our framework generates stock return volatility that is higher than long-horizon dividend volatility, even if we specify a constant market price of risk. This result is in contrast to the standard Gordon growth model prediction that long horizon dividend volatility equals stock return volatility, and in stark contrast to the long-run risk model of BY, which predicts that stock returns are less volatile than long-horizon dividends. The intuition for this result is that, since dividends are cointegrated with EBIT, its long-horizon volatility is equal to the volatility of (unleveraged) EBIT. In contrast, stock return volatility is pushed up by a "leverage factor" $\left(\frac{1}{1-L}\right)$. Thus, for an average leverage ratio of approximately 40%, the stock price volatility is about 67% higher than the long-run dividend volatility in a Gordon growth model framework.[4]

[3]Moreover, in a robustness section we demonstrate that even if investment is tied to debt issuance, we still obtain similar results.

[4]We note that this effect alone does not explain the entire excess volatility puzzle identified by Shiller

Third, our framework predicts that dividend variance ratios should be a decreasing function of horizon. We find empirical support for this in the data. This prediction differs from both CC, where iid dividend dynamics generates constant variance ratios, and BY, where long run risk generates variance ratios that increase with horizon. In our model, downward sloping dividend variance ratios are due to the fact that dividends are a leveraged cash flow in the short run, but are cointegrated with EBIT in the long run.

Our work is closely related to the large literature on consumption-based asset pricing.[5] As discussed in BBK, even though most of the models in this literature do not attempt to study the pricing of dividend strips, they do provide theoretical predictions about their values. We focus on the predictions from BY and CC given the importance of these papers. Lettau and Wachter (2007), Croce, Lettau and Ludvigson (2009) and Kogan and Papanikolaou (2012) are able to generate a downward sloping term structure of returns on dividend strips, but their frameworks are substantially different from BY and CC. We propose a framework that is nearly identical to BY and CC, but we emphasize the importance of endogenous dividend dynamics that generate dividend and capital structure policies that are internally consistent.

Second, our approach is also related to the literature exploring the implications of cointegration restrictions for asset prices. As discussed in Engle and Granger (1987), cointegration implies predictability. This is important, since many researchers have reported that dividends mostly follow a random walk. However, our variance ratio tests show that this random walk assumption is not supported by the data. Models such as Menzly, Santos and Veronesi (2004) and Santos and Veronesi (2006) directly model cointegration between consumption and dividends, but their mechanism is through labor share, and not stationary leverage ratios, which generates our results here. Many other papers investigate the asset pricing implications of cointegration between dividends and consumption. A non-comprehensive list includes Bansal, Dittmar, and Lundblad (2001, 2005), Hansen, Heaton and Li (2008), Bansal, Dittmar and Kiku (2009), Bansal, Kiku, and Yaron (2010). Our paper differs because we focus on the cointegration between dividends and EBIT and investigate its implications for dividend strips. Further, we derive the cointegration relationship endogenously through capital structure policies.

Finally, our paper also relates to the extant literature on the time variation of cor-

(1981). Some amount of time variation in the market price of risk is still needed.

[5]See Campbell (2003) and Cochrane (2006) for comprehensive review of this literature. Ludvigson (2011) reviews the empirical literature on consumption-based asset pricing.

porate cash flows and discount rates.[6] While the early literature on "excess volatility" focused on the ratio of equity volatility to *short*-horizon dividend volatility (e.g., Shiller (1981), LeRoy and Porter (1981)), the fact that managers can (and do, see e.g., Chen (2009)) smooth dividends suggests that the more economically relevant property is the ratio of equity volatility to *long*-horizon dividend volatility. (Marsh and Merton (1986), Shiller (1986)). Our paper focuses on this property, and shows how modifying the dividend process of the BY framework eliminates some of its counterfactual predictions noted by Beeler and Campbell (2012).

Other related papers include Campbell and Shiller (1988), who find that variation in dividend yield is driven mostly by changes in discount rates. However, others have questioned the power of return predictability (Stambaugh (1999), Campbell and Yogo (2006)). Further, Larrain and Yogo (2008) find that discount rates do not need to be so volatile when focusing on the overall cash flows of the firm rather than just dividends, which is also consistent with the findings in Boudoukh, Michaely, Richardson, and Roberts (2007). The issue of dividend growth predictability and smoothing has been investigated in Chen (2009) and Chen, Da and Priestley (2011). Our paper adds to this literature by pointing out long-run variations in dividends are significantly impacted by the capital structure decisions of the firm. Aydemir et al (2007) investigate the effect of leverage in a habit formation model, in particular, how much of the variation in stock volatility can be explained by time variation in leverage. Their focus is thus very different from ours.

The rest of the paper is as follows. In Section 2 we propose a very simple two period binomial model in order to intuitively demonstrate that imposing stationary leverage ratios shifts risk from long horizon dividend strips to short horizon. In Section 3 we provide empirical evidence that dividend variance ratios decrease with horizon. We also show that, consistent with our model, leverage ratios are stationary. We investigate a model that captures long-run risk similar to BY in Section 4. We then demonstrate the robustness of our findings by applying it to a model of habit formation similar to CC in Section 5. In both cases, even though the term structures of EBIT strip returns are upward sloping, the term structures of dividend strip returns are downward sloping, consistent with the empirical evidence of BBK. We conclude in Section 6. Proofs are

[6]For recent reviews of the literature on return and cash flow predictability see the special issue in the Review of Financial Studies (Spiegel, 2008), Koijen and Van Nieuwerburgh (2011), Lettau and Ludvigson (2010) and Cochrane (2006, 2011).

found in the Appendix.

2 A Two Period Binomial Model

In this section we demonstrate within a simple framework the impact that stationary leverage ratios have on the term structure of dividend strips. In particular, we show that imposing stationary leverage ratios tends to increase short horizon (and decrease long horizon) dividend volatility relative to a setup in which managers do not maintain stationary leverage ratios.

We investigate a two period (i.e., three date) binomial framework with no bankruptcy costs or tax benefits, so we are in a Modigliani-Miller (1958) world where capital structure decisions do not affect firm value. The exogenously specified EBIT process (denoted by Y) is given in Figures 1 and 2. The firm liquidates at date-2. We also assume that we are in a complete markets framework, and that the Arrow-Debreu price of a security that pays \$1 iff an up (down) state occurs is $\frac{1}{3}$ $\left(\frac{2}{3}\right)$. Note that this implies that the risk free rate is zero. Finally, assume that the probability of an up state is one-half for all states.

We will compare two firms: one which maintains a constant level of outstanding debt, and one which maintains a constant leverage ratio (which is an an extreme case of maintaining a stationary leverage ratio).

Consider first a firm that has previously issued a bond that pays \$25 in period-1, and will cover these cash flows by issuing another one period bond with face value of \$25. Note that in all states of nature, bondholders are paid off in full, so the corporate bond is riskless. Moreover, since the risk free rate is zero, the bond price is $B_t = \$25$ in all states of nature. The dividend paid (denoted by $\mathcal{D}$) in any state is equal to the sum of EBIT plus the change in the value of the bond position (i.e. $\mathcal{D}_t = Y_t - (B_t - B_{(t-1)})$. The firm's equity value V_t can be calculated in all states by backward induction as $\mathrm{E}_t^Q[\mathcal{D}_{(t+1)} + V_{(t+1)}]$. We can verify that at time 0 the equity value is equal to the enterprise value ($P_0 = \mathrm{E}^Q[Y_1 + Y_2] = 100$) minus the debt value ($B_0 = 25$), i.e., $V_0 = P_0 - B_0 = 75$, consistent with Modigliani-Miller's theorem. We report these numbers in Figure 1.

Now, let us compare this to an otherwise identical firm which follows a dynamic capital structure policy that leads to stationary (in fact, constant) leverage ratios. In particular, assume that the firm maintains a 25% leverage ratio. In order to do so,

$$\mathcal{D}_{\uparrow\uparrow} = 246,\ Y_{\uparrow\uparrow} = 271$$

$$B_{\uparrow} = 25,\ \mathcal{D}_{\uparrow} = 35,\ Y_{\uparrow} = 35,\ V_{\uparrow} = 116 \rightarrow \mathcal{D}_{\uparrow\downarrow} = 51,\ Y_{\uparrow\downarrow} = 76$$

$$V_0 = 75,\ B_0 = 25$$

$$B_{\downarrow} = 25,\ \mathcal{D}_{\downarrow} = 20,\ Y_{\downarrow} = 20, V_{\downarrow} = 17 \rightarrow \mathcal{D}_{\downarrow\uparrow} = 51,\ Y_{\downarrow\uparrow} = 76$$

$$\mathcal{D}_{\downarrow\downarrow} = 0,\ Y_{\downarrow\downarrow} = 25$$

Figure 1: Cash flows and security prices of a firm which maintains a constant one-period debt level of \$25, by reissuing new one-period debt in period 1. Arrow-Debreu state prices are $q_{\uparrow} = 1/3$ and $q_{\downarrow} = 2/3$ which implies that the risk-free rate is $r = 0$. The equity value can be solved as $V_t = \mathrm{E}_t^Q[\mathcal{D}_{t+1} + V_{t+1}]$ with $V_2 = 0$ (the firm is liquidated in period 2). Debt value (B_t) is risk-free. Modigliani-Miller holds, so that the enterprise value $P_t = V_t + B_t$. Actual probabilities are $p_{\uparrow} = p_{\downarrow} = 1/2$.

it must change its level of outstanding debt at date-1. For example, if an up-state occurs, then after paying off the old debt \$25, it will issue \$35.25 of new debt (which is 25% of the claim to EBIT: $P_{\uparrow} = \$141$), and distribute net difference plus EBIT as a dividend ($\mathcal{D}_{\uparrow} = 35 + 35.25 - 25 = 45.25$). Analogously, if a down state occurs, then after paying off the old debt \$25, it will issue \$10.50 of new debt (which is 25% of the claim to EBIT: $P_{\downarrow} = \$42$), and distribute net difference plus EBIT as a dividend ($\mathcal{D}_{\uparrow} = 20 + 10.50 - 25 = 5.50$). Date-2 dividend payments are determined analogously after noting that there is no new debt issuance at date-2. The relevant security values and cash flows are shown in Figure 2.

For both of these firms, which differ only in their capital structure decisions in period 1, we compute i) the standard deviations of period-1 and period-2 dividends, ii) the date-0 prices of dividend strips $V^t(0) = \mathrm{E}_0^Q[\mathcal{D}_t]$, iii) their expected returns and variances, and iv) the expected returns and return variances for the stocks. We report these statistics in Table 1 below.

Comparing the results across rows in Table 1 confirms our main insight: imposing stationary leverage ratios tends to increase the variance of short-horizon dividends and decrease the variance of long-horizon dividends. In particular, the one-period variance increases from 56 to 395, and the two-period variance decreases from 8860 to 7491, as we move from the constant debt model to the constant leverage ratio model. As we show

$$\mathcal{D}_{\uparrow\uparrow} = 235.75,\ Y_{\uparrow\uparrow} = 271$$

$$B_{\uparrow} = 35.25,\ \mathcal{D}_{\uparrow} = 45.25,\ Y_{\uparrow} = 35,\ V_{\uparrow} = 105.75 \quad \mathcal{D}_{\uparrow\downarrow} = 40.75,\ Y_{\uparrow\downarrow} = 76$$

$$V_0 = 75,\ B_0 = 25$$

$$B_{\downarrow} = 10.50,\ \mathcal{D}_{\downarrow} = 5.50,\ Y_{\downarrow} = 20,\ V_{\downarrow} = 31.5 \quad \mathcal{D}_{\downarrow\uparrow} = 65.50,\ Y_{\downarrow\uparrow} = 76$$

$$\mathcal{D}_{\downarrow\downarrow} = 14.50,\ Y_{\downarrow\downarrow} = 25$$

Figure 2: Cash flows and security prices of a firm which maintains a constant leverage ratio of one-period debt $B_t/(V_t + B_t) = 25\%$ when re-issuing new debt in period 1. All parameters are as in Figure 1.

in the theoretical models discussed below, this result is quite general and not specific to the particular example discussed here.

Table 1 also shows that this shift in risk from long horizons to short horizons due to the stationary leverage ratio policy impacts the the properties of dividend strip returns. Specifically, compared to the firm that holds a constant amount of debt, the firm that maintains a constant leverage ratio experiences an increase in the expected return and variance on the one period dividend strip and a decrease in the expected return and variance on the two period dividend strip.

Interestingly, note that the one-period expected return and return variance of the stock at date-0 are not impacted by future capital structure policies, as reported in the last two rows of Table 1. This result is consistent with Modigliani-Miller's "dividend irrelevance" theorem. Looking ahead, this dividend irrelevance will imply that the change in dividend dynamics due to imposing stationary leverage ratios will not negatively impact the ability of the CC and BY models to capture salient features of stock returns.

3 Empirical Support

In this section, we provide empirical support for the two most fundamental properties of the model that drive our results. First, we show that dividend variance ratios are decreasing with horizon. That is, long horizon dividends are not as risky as iid models would predict, and much less risky than what 'long-run risk' models (which, as we show

Statistic	Definition	Constant Debt level	Stationary Leverage Ratio
Variance Dividend 1	$\text{Var}_0\left[\mathcal{D}(1)\right]$	56	395
Variance Dividend 2	$\text{Var}_0\left[\mathcal{D}(2)\right]$	8860	7491
Dividend 1 Strip Price	$V^1(0) = E^Q[\mathcal{D}_1]$	25	18.75
Dividend 2 Strip Price	$V^2(0) = E^Q[\mathcal{D}_2]$	50	56.25
Expected return strip 1	$\text{E}\left[\frac{\mathcal{D}(1)-V^1(0)}{V^1(0)}\right]$	0.1	0.35
Variance strip 1	$\text{Var}\left[\frac{\mathcal{D}(1)-V^1(0)}{V^1(0)}\right]$	0.09	1.12
Expected return strip 2	$\text{E}\left[\frac{V^2(1)-V^2(0)}{V^2(0)}\right]$	0.33	0.22
Variance strip 2	$\text{Var}\left[\frac{V^2(1)-V^2(0)}{V^2(0)}\right]$	0.98	0.44
Expected stock return	$\text{E}\left[\frac{V(1)+\mathcal{D}(1)-V(0)}{V(0)}\right]$	0.25	0.25
Variance of stock	$\text{Var}\left[\frac{V(1)+\mathcal{D}(1)-V(0)}{V(0)}\right]$	0.58	0.58

Table 1: Calculations of various key statistics corresponding to two different leverage policies represented in figures 1 and 2

below, generate dividend variance ratios that increase with horizon) predict. Second, we provide support for the assumption that the aggregate leverage ratio is stationary. In addition, we also characterize certain business cycle properties of leverage that will be used to calibrate the modified versions of BY and CC below. In particular, we focus on the correlation between leverage and the main state variables of these models (namely, expected cash flows in BY, market price of risk in CC).

3.1 Data

The two main variables required for our empirical work are: i) the dividends on the aggregate stock market, and ii) the aggregate leverage ratio. In this section we explain how these variables are constructed.

We consider three alternative measures of aggregate dividends to help establish the robustness of the findings. We perform the analysis using annual data to avoid the seasonality in dividend payments.[7] The use of an annual dividend series implies that we need to take a stance on how dividends received within a particular year are reinvested. We consider two alternative reinvestment strategies. In the first strategy, we assume the monthly dividends are reinvested in the aggregate stock market. As in Binsbergen and Koijen (2009), we refer to this dividend series as market-invested dividends. This measure of dividends is by far the most common in the dividend-growth and return-forecasting literature, and thus we focus on this definition for the main part of our analysis.[8] In the second strategy, we invest the monthly dividends in cash, and obtain a time series of annual dividends which we call cash-invested dividends. As shown by Binsbergen and Koijen (2010) and Chen (2009), the two dividend series have different time series properties in the post-war sample period.

We obtain the data for the two dividend series from Long Chen's webpage (the data is used in Chen (2009)). We use this dataset because it covers a long sample period from 1873 to 2008, thus covering the pre Center for Research in Security Prices (CRSP) period. Focusing on this long sample allows us to obtain more robust results (and also to address Merton's (1987) concern about the lack of research in the pre-CRSP period). To construct the two dividend series, Chen (2009) combines the pre-CRSP data compiled by Schwert (1990) with the data from the CRSP (NYSE/Amex/Nasdaq) value-weighted market portfolio at monthly frequency. We refer the reader to Chen (2009) for additional details on the construction of the two dividend series.

In addition to the previous two dividend series, we investigate a third alternative measure of dividends that includes share repurchases. The data for this alternative dividend series is available from Motohiro Yogo's webpage (the data is used in Gomes, Kogan and Yogo (2009)), and covers a relatively shorter sample period from 1927 to 2007. Examining this alternative definition of dividends is motivated by the observation that firms have increased the fraction of payouts to shareholders via repurchase programs compared to dividends in recent years (Fama and French (2001), Grullon and Michaely (2002)). As discussed in Lettau and Ludvigson (2005), still, large firms with high earnings have continued to increase traditional dividend payouts over time (DeAngelo, DeAngelo and Skinner, 2004). The impact on aggregate dividends is therefore

[7] For a similar approach, see also Cochrane (1994), Lettau and Ludvigson (2005), and Binsbergen and Koijen (2010).

[8] A non comprehensive list of studies that use this measure of dividends includes Lettau and Ludvigson (2005), Cochrane (2008), and Lettau and Van Nieuwerburgh (2008).

unclear. To show that our main findings are not altered by adjusting dividends to account for share repurchase activity, since 1971, we consider a dividend series augmented with equity repurchases using Compustat's statement of cash flows.

We transform all nominal dividends into real dividends by deflating the annual dividends by the consumer price index (CPI), which is available from Robert Shiller's webpage.

Finally, to construct the time series of the aggregate leverage ratio, we use data from the Flow of Funds Accounts of the United States (Board of Governors of the Federal Reserve System, 2005). The aggregate leverage ratio is defined as the ratio of total value of liabilities to the sum of the total value of liabilities and the total market value of equity. Liabilities are the sum of accounts payable; bonds, notes, and mortgages payable; and other liabilities. The data is for the nonfarm, nonfinancial corporate sector and is available annually since 1946. Larrain and Yogo (2008) extend the data back to 1927. We use this dataset which is available on Motohiro Yogo's webpage, but this data ends in 2004. As such, we update the data to 2008 by collecting the updated aggregate total liabilities data from the Flow of Funds Accounts, and by constructing the total market value of equity in the nonfarm, nonfinancial corporate sector by replicating the approach in Larrain and Yogo (2008). We refer the reader to Larrain and Yogo (2008) for further details on the data construction.

3.2 Dividend Variance Ratios

If dividends follow a random-walk, then the variance of dividend growth increases linearly with the observation interval. That is, for example, the variance of two year dividend growth will equal twice the variance of one year dividend growth, implying that the ratio of the two variances per unit of time equals unity. Following the approach of Lo and MacKinlay (1988), we construct the dividend variance ratio statistic (VR) across horizons from one to twenty years for each of the three alternative dividend series. We then show that dividend variance ratios are decreasing with horizon.

To compute the VR statistic, we directly apply the test formulas from Lo and MacKinlay (1988) (see their Section 1). For completeness, and to help in the calibration of the theoretical model proposed below, we also report the dividend volatilities at each horizon. We define dividend volatility over a given horizon using two different approaches.[9] First, the more standard approach is to specify dividend volatility over a

[9]These formulas are not the ones used in the variance ratio test of Lo and MacKinlay (1988), who

horizon T as

$$\sigma^T_{D,1} \;=\; \sqrt{\left(\frac{1}{T}\right) \mathrm{Var}_0 \left[\log\left(\frac{D(T)}{D(0)}\right)\right]}. \tag{1}$$

We also consider a second definition:

$$\sigma^T_{D,2} \;=\; \sqrt{\left(\frac{1}{T}\right) \log\left[\frac{\mathrm{E}_0\left[D^2(T)/D^2(0)\right]}{\left(\mathrm{E}_0\left[D(T)/D(0)\right]\right)^2}\right]}. \tag{2}$$

Note that for the case of log-normal (i.e., iid random walk) dynamics

$$\frac{dD}{D} \;=\; g\,dt + \sigma\,dz, \tag{3}$$

which in integral form can be expressed as

$$D(T) \;=\; D(0)e^{(g-\sigma^2/2)T+\sigma\,z(T)}, \tag{4}$$

both definitions produce the result $\sigma^T_{D,1} = \sigma^T_{D,2} = \sigma$ for all horizons T. The reason we consider the second definition is that it is defined even if dividends are negative (that is, if equity issuances are larger than dividend payments.)

Table 2 reports the VR test results for the three alternative measures of dividends. It reports the per-year variance of dividend growth across each horizon T for the two alternative definitions of dividend variance ($\sigma_{D,1}$ and $\sigma_{D,2}$). In addition, it reports the VR test statistic at each horizon, the corresponding standard errors (s.e.(VR)), and its p-value. The p-value is for the test of the null hypothesis that dividends follow a random walk, in which case the VR test statistic is 1. In specifying the null hypothesis, we consider the most general case in which the shocks to dividends can be heteroskedastic, not necessarily iid.

Table 2 shows that dividends do not follow a random walk. The VR test statistic decreases strongly with horizon for the three alternative dividend measures, implying predictability in dividends. Both definitions of dividend variance show that the variance of dividend growth is much smaller at long horizons than at short horizons. Regardless of the measure of dividends used and of how dividend variance is computed, the difference between short (1-year)- and long (10 or 20 years)-run dividend volatility is always greater

instead use unbiased estimators of the variance by appropriately adjusting for the degrees of freedom. As a result, the variances reported here do not exactly match the variances used in the reported VR statistics, but the difference between the two is minimal.

than 5.2%. The conclusion that dividend variance decreases with horizon thus seems to be robust to how the monthly dividends are reinvested during the year, and to the inclusion of share repurchases in the measurement of dividends.[10]

For the first measure of dividends (market-invested dividends), Table 2 shows that the VR test statistic rejects the hypothesis that dividends follow a random walk at the 10% significance level for the 4-and 15-year horizon, and at the 5% significance level for the 6-, 8- and 10-year horizon. Using the first definition of dividend variance, the volatility is 15.0% for the one year horizon, but only 7.5% for the 20-year horizon, a large difference of 7.5%. Using the second definition ($\sigma_{D,2}$), the difference between short- and long-run dividend volatility is even larger. The volatility is 14.7% for the one year horizon, and only 6% for the 20-year horizon, a difference of 8.7%. For the other two alternative measures of dividends, the statistical rejection of the random walk hypothesis is weaker, but it is clear that the volatility of dividends decreases significantly with the horizon as well. For the second measure of dividends (cash-invested dividends), the difference between short- and long-run dividend volatility is 6.2% using the first definition of dividend variance, and is 7.3% using the second definition of dividend variance. For the last measure of dividends (with equity repurchases), the corresponding differences using the two dividend variance definitions is 8.3% and 9.1%, respectively.

The previous results have implications for the evaluation of leading asset pricing models. To illustrate the implications in a clear manner, Figure 3 graphically demonstrates the results in Table 2. The figure focuses on the main definition of dividends (market-invested dividends). The large difference between short- and long-run dividend volatility implies that we can strongly reject the random walk assumption of CC in specifying dividend dynamics, which naturally implies a dividend variance that is constant across the different horizons and hence a VR test statistic that is always equal to one. Moreover, we can reject even more strongly the long run risk dividend dynamics posited

[10]Table 2 shows that the variance of dividend growth for the first two measures of dividends (market-invested and cash-invested) is very similar. This result seems in contrast with the descriptive statistics reported in Binsbergen and Koijen (2010) who show that the volatility of the cash-invested dividend growth is almost half the volatility of the market-invested dividend growth. The difference is the sample period. In Binsbergen and Koijen (2010) the sample period is from 1946 to 2007, whereas we examine a larger sample from 1873 to 2008. When we restrict the analysis to the shorter sample from 1946 to 2007, we confirm the Binsbergen and Koijen (2010) results using Chen's (2009) measure of cash-invested dividends. The larger volatility of cash-invested dividends in the pre-1946 period makes the properties of the two dividend series more similar in the full sample. Chen (2009) reports a similar sub-sample analysis and confirms that the different properties of the two series varies across sub-samples.

in BY (using their calibration), which we also plot in Figure 3. Here, due to long run risk, dividend growth volatility *increases* with horizon, in sharp contrast with the data. These results confirm (using more data) the analysis of Beeler and Campbell (2012), who show that dividend variance ratios in the U.S. aggregate stock market increase with horizon in the long run risk model, but not in the real data, using a sample of annual dividend data for the 1930 to 2008 period.

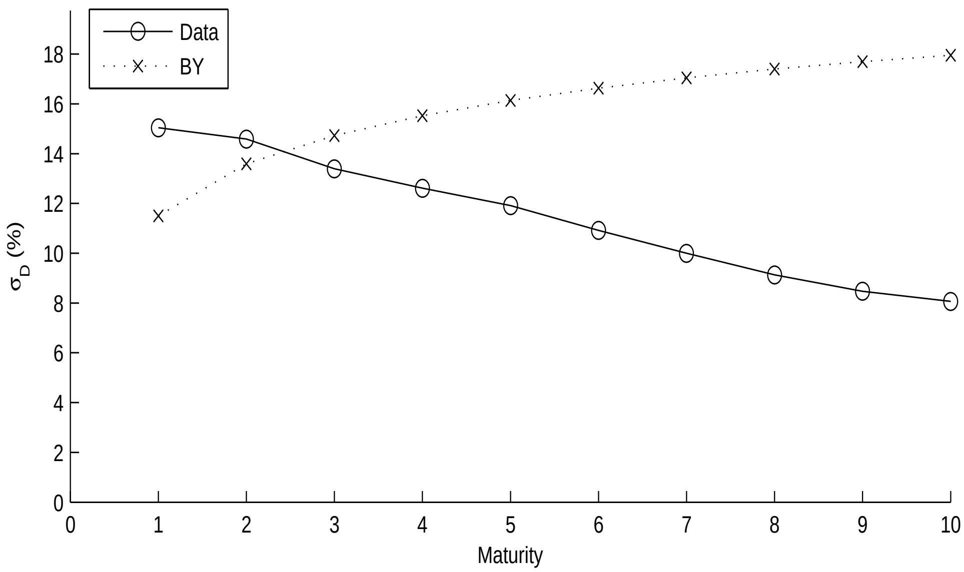

Figure 3: Expected dividend growth volatility as a function of horizon in the data and in Bansal and Yaron (2004). The data are annual from 1873 to 2008.

At a fundamental level, the finding that the dividend variance decreases with horizon must reflect negative serial correlation in the dividend growth series. To show this formally, we consider a simple econometric approach based on linear regression. Specifically, we investigate if past values of dividend growth help predict future dividend growth by running a regression of the form:

$$d_{t+1} - d_t \quad = \quad a + \sum_{k=1}^{K} b_k \left(d_{t+1-k} - d_{t-k} \right) + \varepsilon_t, \tag{5}$$

where d_t is log dividend at time t, and K is the number of lagged observations of dividend growth included in the regression. We consider K=1 and 2 (the main conclusion is robust to including other lags). By construction, this test is designed to capture the existence of serial correlation in dividend growth, which is ruled out by the random walk assumption.

The results reported in Table 3 show that past values of dividend growth help predict future dividends. In particular, in specification 2, the twice-lagged value of dividend growth helps forecasting dividends growth (the slope coefficient of b_2 is significantly different from zero with a p-value of 1%). When the one- and two-year lagged values of dividend growth are included (specification 2), the chi-squared test rejects the hypothesis that all the slope coefficient are zero with a p-value of 4%. Finally, the slope coefficient on the lagged values of dividend growth are negative. Thus, an unusually high value of dividends growth today predicts lower dividend growth. It is this negative autocorrelation that drives the decreasing pattern of dividend volatility across maturities.

3.3 Properties of Aggregate Leverage

In this section we provide empirical support for the assumption that the aggregate leverage ratio is stationary. In addition, we characterize the relationship between aggregate leverage and the two state variables in BY and CC models.

Previous studies show that leverage ratios are stationary. As discussed in Collin-Dufresne and Goldstein (2001), at an aggregate (industry) level, leverage ratios have remained within a fairly narrow band even as equity indices have increased ten-fold over the past thirty years. At the firm level, Opler and Titman (1997) provide empirical support for the existence of target leverage ratios within an industry.[11] Further, dynamic models of optimal capital structure by Fischer, Heinkel, and Zechner (1989), and Goldstein, Ju and Leland (2001) find that firm value is maximized when a firm acts to keep its leverage ratio within a certain band.

Our empirical measure of aggregate leverage ratio is stationary as well. To demonstrate this, we run a regression of changes in log aggregate leverage ratio on lagged values of the log aggregate leverage ratio. We obtain the following results (Newey-West corrected t-statistics in parenthesis):

$$\Delta \text{Lev}_{t+1} = \underset{(-2.92)}{-0.129} - \underset{(-2.99)}{0.137} \times \text{Lev}_t + e_{t+1}, \qquad \text{R}^2 = 5.90\%, \qquad \sigma(e_{t+1}) = 0.12.$$

The negative slope coefficient on the lagged value of leverage implies mean reversion in the aggregate leverage ratio.

[11] Additional studies providing empirical support for the claim that leverage ratios are stationary at the firm level include Flannery and Rangan (2006) and Fama and French (2002).

The time-series properties of the aggregate leverage ratio are an important input for the calibration of the theoretical models that we present below. In addition, it is important to characterize the relationships between aggregate leverage ratio and the state variables in the BY and CC economies that we study in this paper. We discuss these two models in detail in the theoretical section, but report here the relevant empirical links to help in the calibration of these models.

The relevant summary statistics of aggregate leverage are the following. The mean leverage ratio in our sample is 39.5% (in logs, the mean is -0.957 ± 0.05). The standard deviation of the aggregate leverage ratio is 9.69% (0.242 in logs) and the first order autocorrelation is 87.8% (in logs the autocorrelation is 86.6%).

In the theoretical sections below we will be modeling log-leverage dynamics in continuous time as an AR1 process similar to:

$$d\ell \;=\; \kappa(\bar{\ell} - \ell)\, dt + \sigma_{\ell}\, dz. \tag{6}$$

The data above allows us to calibrate this model with the parameter estimates: ($\bar{\ell} = -0.957 \pm 0.05$, $\kappa = 0.147 \pm 0.046$, $\sigma_{\ell} = 0.12 \pm 0.01$.)

In BY, the main state variable is the time-varying expected growth rate of cash-flows (which, in BY, is measured by consumption). Following BY, we denote expected growth rate of cash-flows by the variable x. In the one-channel version of the long-run risk BY model that we study here, the x variable is the main business cycle variable and the main source of systematic risk in the model. We use the time-series estimate of x reported in Bansal, Kiku, and Yaron (2007).[12] The correlation between changes in (log) leverage and changes in x_t is -38%. Since positive shocks to x are good news, this correlation suggests that the leverage ratio is countercyclical, that is, leverage is higher in bad economic times. This is consistent with intuition and with the fact that equity is riskier than debt.

Turning to the CC model, the main state variable is the consumption surplus ratio, which is the source of variation in the price of risk. Following Wachter (2006), the consumption surplus is constructed as a smoothed average of the past 40 quarters of the consumption of nondurables and services real growth rate:

$$\text{CSPLS}_t = \Sigma_{j=1}^{40} \phi^j \Delta c_{t-j}, \quad \text{with } \phi = 0.97.$$

We annualize the quarterly data by taking the observation in the last quarter of each year as the annual value. High values of consumption surplus are associated with low

[12]We thank Bansal, Kiku, and Yaron for sharing their data with us.

degrees of risk aversion, and hence with good economic times. The correlation between shocks to CSPLS_t (measured as the growth rate in CSPLS) and changes in (log) leverage is -37%. As in the BY model, this negative correlation also suggests that the leverage ratio is countercyclical, that is, leverage is higher in bad economic times.

4 Endogenous Dividend Dynamics in a 'Long Run Risk' Model

Here we investigate a model which captures the essential features of the "one-channel long-run risk model" of Bansal and Yaron (BY, 2004). In particular, we specify a state price density and aggregate cash-flow processes that correspond to a continuous time version of the exponential affine (approximate) solution presented in BY (2004).[13] BY demonstrate that their model can capture high expected returns, volatility and Sharpe ratios of stocks even with moderate levels of risk aversion. However, rather than exogenously specifying dividend dynamics as BY did, here we specify EBIT dynamics that are similar to BY's dividend dynamics, and then combine that with a dynamic capital structure policy in order to determine dividend dynamics endogenously. Specifying EBIT dynamics and leverage dynamics separately is consistent with the standard approach in the capital structure literature of assuming a separation of investment and capital structure policies. In the Appendix we show that allowing for debt-financed investment does not significantly impact our main results.

4.1 EBIT Dynamics

We specify the dynamics for log-EBIT y_t to have a small but persistent shock to its expected growth x_t:

$$dy = \left(g + x - \frac{\sigma_y^2}{2}\right) dt + \sigma_y \, dz_1 \tag{7}$$

$$dx = -\kappa_x x \, dt + \sigma_{x1} \, dz_1 + \sigma_{x2} \, dz_2. \tag{8}$$

[13]BY show that the affine approximation is very accurate relative to the numerical approximation of the exact model. See also Appendix C1 in Chen, Collin-Dufresne and Goldstein (2009), who provide a continuous time approximation of the original Epstein-Zin representative agent economy similar to the one used here.

Since state vector dynamics are affine, it follows that date-t expectations of the first two moments of date-T EBIT take the exponential affine forms:

$$\mathrm{E}_t\left[e^{y_T}\right] = e^{y_t + A_0(T-t) + x_t A_1(T-t)} \tag{9}$$

$$\mathrm{E}_t\left[e^{2y_T}\right] = e^{2y_t + A_2(T-t) + x_t A_3(T-t)}, \tag{10}$$

where the deterministic coefficients $\{A_0(\tau),\, A_1(\tau),\, A_2(\tau),\, A_3(\tau)\}$ are given in the Appendix.

The term structure of EBIT expected growth rates over horizon τ is defined as

$$\begin{aligned} g_{y,\tau} &\equiv \left(\frac{1}{\tau}\right)\log\left(\mathrm{E}_0\left[e^{y_\tau - y_0}\right]\right) \\ &= \left(\frac{1}{\tau}\right)\left[A_0(\tau) + x_0 A_1(\tau)\right]. \end{aligned} \tag{11}$$

Similarly, define the term structure of EBIT volatilities to be

$$\begin{aligned} \sigma_{y,\tau} &\equiv \sqrt{\left(\frac{1}{\tau}\right)\log\left[\frac{\mathrm{E}_0\left[e^{2(y_\tau - y_0)}\right]}{\left(\mathrm{E}_0\left[e^{y_\tau - y_0}\right]\right)^2}\right]} \\ &= \sqrt{\left(\frac{1}{\tau}\right)\left[A_2(\tau) - 2A_0(\tau)\right]}. \end{aligned} \tag{12}$$

Interestingly, we find $\sigma_{y,\tau}$ is independent of x. This is due to $A_3(\tau) = 2A_1(\tau)$.

We plot these term structures at their long run mean ($x_t = 0$) in Figure (4) using the parameter values in Table 1 below. Note that the term structure of volatilities is upward sloping. Intuitively, this is because over short horizons, the random variable x_T does not differ too much from its current value x_0, and therefore log-EBIT approximately follows a random walk (with volatility approximately equal to σ_y). Over longer horizons, however, the future value of x_T becomes more uncertain (hence the name, "long run risk"), in turn generating an increasing term structure of volatilities.

4.2 EBIT Strips

BY demonstrate that their specified endowment dynamics combined with recursive preferences (Epstein Zin (1989)) generate pricing kernel dynamics that are well-approximated by constant market prices of risk:[14]

$$\frac{d\Lambda}{\Lambda} = -r\,dt - \theta_1\,dz_1 - \theta_2\,dz_2. \tag{13}$$

[14]For simplicity, we have set the risk free rate to a constant since it has no bearing on the issues at hand.

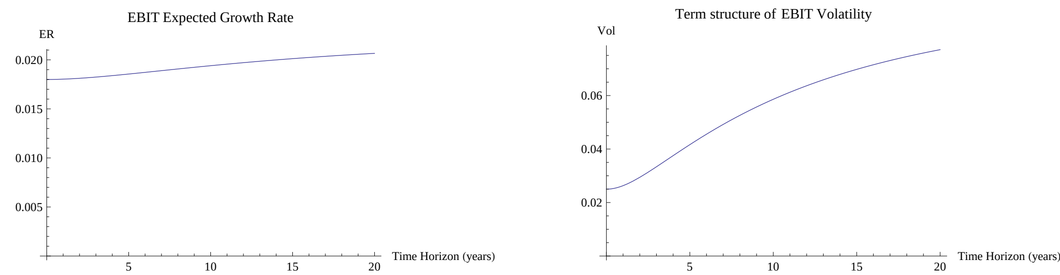

Figure 4: Term structure of EBIT expected growth rate (equation (11)) and volatilities (equation (12)) for the BY economy. Parameters are set as in Table 4.

This implies that risk-neutral dynamics are:

$$dy = \left(g^Q + x - \frac{\sigma_y^2}{2}\right) dt + \sigma_y \, dz_1^Q \tag{14}$$

$$dx = \kappa_x (\overline{x}^Q - x) \, dt + \sigma_{x1} \, dz_1^Q + \sigma_{x2} \, dz_2^Q, \tag{15}$$

where we have defined $g^Q \equiv (g - \sigma_y \theta_1)$, $\overline{x}^Q \equiv -\left(\frac{\theta_1 \sigma_{x1} + \theta_2 \sigma_{x2}}{\kappa_x}\right)$.

The date-t price $P^T(t, x_t, y_t)$ of the security whose payoff is the date-T EBIT flow e^{y_T} is:

$$P^T(t, x_t, y_t) = e^{-r(T-t)} \mathrm{E}_t^Q \left[e^{y_T}\right]. \tag{16}$$

The solution takes the exponential affine form:

$$P^T(t, x_t, y_t) = e^{y_t + F(T-t) + G(T-t) \, x_t}, \tag{17}$$

where the deterministic functions $(F(\tau), G(\tau))$ are derived in the Appendix.

Expected excess returns on the EBIT strips satisfy

$$\begin{aligned} \frac{1}{dt} \mathrm{E}\left[\frac{dP^T(t, x_t, y_t)}{P^T(t, x_t, y_t)} - r \, dt\right] &= -\frac{1}{dt} \mathrm{E}\left[\frac{d\Lambda}{\Lambda} \frac{dP^T(t, x_t, y_t)}{P^T(t, x_t, y_t)}\right] \\ &= \theta_1 \left[\sigma_y + G(T-t)\sigma_{x1}\right] + \theta_2 \left[G(T-t)\sigma_{x2}\right]. \end{aligned} \tag{18}$$

EBIT strip volatility is

$$\begin{aligned} \sigma^{P,\tau} &\equiv \sqrt{\frac{1}{dt}\left(\frac{dP^{t+\tau}(t, x_t, y_t)}{P^{t+\tau}(t, x_t, y_t)}\right)^2} \\ &= \sqrt{\left(\sigma_y + G(\tau)\sigma_{x1}\right)^2 + \left(G(\tau)\sigma_{x2}\right)^2}. \end{aligned} \tag{19}$$

We calibrate this model using the parameter values in Table 4, and plot the resulting term structures in Figure (5). We choose the parameter $\theta_1 = 0.0$ to be small and $\theta_2 = 0.4$ to be large in order to capture the intuition of BY that compensation for consumption risk is low – it is uncertainty related to future expected consumption growth that agents with Epstein-Zin (1989) preferences are extremely averse to.[15]

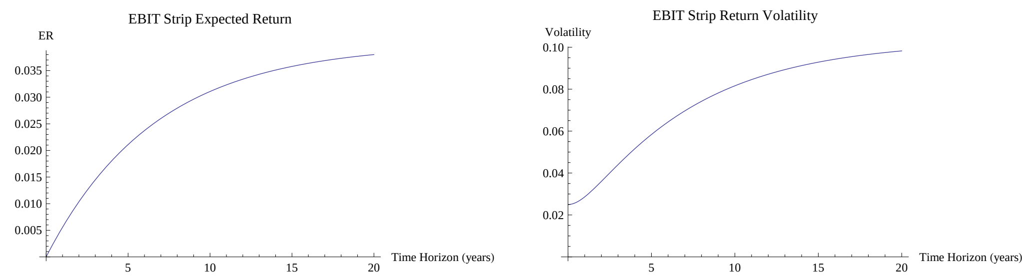

Figure 5: Term structure of EBIT strip excess returns (equation (18)) and volatilities (equation (19)) for the BY economy. Parameters are given in Table 4.

As noted in Binsbergen, Brandt and Koijen (2011), this long-run risk model generates an upward sloping term structure of expected returns and volatilities. That is, the return variances on EBIT strips have inherited the upward sloping term structure associated with the variance ratios of the EBIT cash flows.

The enterprise value of the firm is equal to the present value of the claim to all EBIT strips:

$$P(x_t,\, y_t) \;=\; e^{y_t} \int_t^{\infty} dT\, e^{F(T-t)+G(T-t)\, x_t}. \tag{20}$$

As noted by Bansal and Yaron (2004) and others, this can be well-approximated by a log-linear approximation:

$$P(x_t,\, y_t) \;\approx\; e^{y_t + F + G\, x_t}, \tag{21}$$

where the coefficients (F, G) are given in Table 5 below.[16] Figure 6 plots the exact and

[15]These parameters are similar to the parameters obtained in the exponential affine approximation of an Epstein-Zin-utility representative agent economy with the same aggregate output dynamics (see, e.g., appendix C1 in Chen, Collin-Dufresne and Goldstein (2009)).

[16]Since we know the closed-form solution in equation (20), we choose the parameters of the exponential affine approximation in equation (21) to minimize the expected squared difference between the two. See the Appendix for more details. We use the approximate solution instead of the exact solution because it simplifies our calculations of the levered equity value below. An alternative would be to change the definition of the EBIT process so that the solution in equation 21 is exact. This can be accomplished by choosing EBIT to be of the form $(a_0 + a_1 x)e^{y+bx}$ for a suitable choice of $(a_0,\, a_1,\, b)$.

approximate solution, and shows the accuracy of the log-linear approximation.

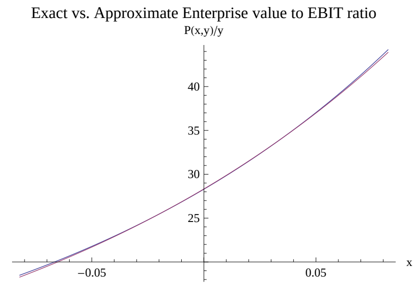

Figure 6: Exact and approximate solutions to the enterprise value as given in respectively equations (20) and (21) for the parameters given in Table 4. The x-axis covers −4 to +4 standard deviations of the unconditional distribution of x.

In this “one-channel” model, the expected return and volatility of the claim to EBIT are constant under the log-linear approximation:

$$(\mu^P - r)_{BY} \approx \theta_1 \left(\sigma_y + G\sigma_{x1}\right) + \theta_2 \left(G\sigma_{x2}\right). \tag{22}$$

$$\sigma^P_{BY} \approx \sqrt{\left(\sigma_y + G\sigma_{x1}\right)^2 + \left(G\sigma_{x2}\right)^2}. \tag{23}$$

Their values, given the calibrated parameters, are given in the Table 5.

4.3 Dividend Dynamics

At this point we could choose to close the model by exogenously specifying dividend dynamics so that, in the short run, dividends are riskier than EBIT, but over longer horizons, dividends and EBIT are cointegrated. In unreported results, we investigate such a model and calibrate it to match the downward sloping dividend variance ratios from the previous section. We find that such a model generates term structures of dividend strip excess returns and volatilities that are downward sloping, consistent with the empirical finding of BBK.

Instead, however, here we attempt to be a bit more ambitious and provide an economic mechanism that explains *why* dividends are riskier than EBIT in the short run, and why variance ratios are downward sloping. Specifically, we argue that a payout policy consistent with a firm maintaining a stationary leverage ratio will generate these predictions endogenously. As such, we first specify a capital structure policy that leads to stationary leverage ratios. Then, we combine this policy with the EBIT dynamics specified above to endogenously determine dividend dynamics.

Assume that at all dates-t, the firm issues riskless debt that matures at date-$(t+dt)$ with present value equal to

$$\begin{aligned} B(\ell_t, x_t, y_t) &= e^{\ell_t + y_t + F + Gx_t} \\ &\approx e^{\ell_t} P(x_t, y_t). \end{aligned} \tag{24}$$

We interpret $e^{\ell_t} \approx \frac{B(\ell_t, x_t, y_t)}{P(x_t, y_t)}$ as the leverage of the firm. Since it is riskless, the firm must pay $e^{r\,dt} B(\ell_t, x_t, y_t)$ at date $(t+dt)$. It does so by issuing at this time debt with face value $B(\ell_{t+dt}, x_{t+dt}, y_{t+dt})$, with all residual cash flows paid out as dividends. As such, dividends $dD(t) = D(t+dt) - D(t)$ paid out at date-$(t+dt)$ are

$$\begin{aligned} dD(t) &= B(\ell_{t+dt}, x_{t+dt}, y_{t+dt}) - e^{r\,dt} B(\ell_t, x_t, y_t) + e^{y_{t+dt}}\, dt. \\ &\overset{\mathcal{O}(dt)}{=} dB(\ell_t, x_t, y_t) - rB(\ell_t, x_t, y_t)\, dt + e^{y_t}\, dt. \end{aligned} \tag{25}$$

We choose the dynamics of log-leverage so that i) it is mean-reverting, and ii) dividend payments are locally deterministic, that is, cumulative dividend dynamics are of the form: $dD(t) = \mathcal{D}_t\, dt$, with no diffusion term. We emphasize that these conditions are chosen for parsimony and impose a lot of structure on the model, as they significantly reduce the number of free parameters in leverage dynamics to three drift parameters $(\kappa_\ell, \overline{\ell}^P, \alpha)$ and zero diffusion parameters.[17] In particular, we choose[18]

$$\begin{aligned} d\ell_t &= \kappa_\ell \left(\overline{\ell}^P + \alpha x - \ell\right) dt - \left(G\sigma_{x1} + \sigma_y\right) dz_1 - G\sigma_{x2}\, dz_2 \\ &= \kappa_\ell \left(\overline{\ell}^Q + \alpha x - \ell\right) dt - \left(G\sigma_{x1} + \sigma_y\right) dz_1^Q - G\sigma_{x2}\, dz_2^Q, \end{aligned} \tag{26}$$

where

$$\overline{\ell}^Q = \overline{\ell}^P + \left(\frac{1}{\kappa_\ell}\right) \left[\theta_1 \left(G\sigma_{x1} + \sigma_y\right) + \theta_2 G\sigma_{x2}\right]. \tag{27}$$

Since the combination $(d\ell + dy + G\,dx)$ is locally deterministic, dividends paid out over the interval $(t, t+dt)$ are equal to $\mathcal{D}(t)\, dt$, where

$$\begin{aligned} \mathcal{D}(t) &= e^{y_t} \left[1 + e^{\ell_t + F + Gx_t} \left(\kappa_\ell \left(\overline{\ell} + \alpha x - \ell\right) + g + x - \frac{\sigma_y^2}{2} - G\kappa_x x - r\right)\right] \\ &= e^{y_t} \left[1 + e^{\ell_t + F + Gx_t} \left(\kappa_\ell \left(\overline{\ell}^Q + \alpha x - \ell\right) + g^Q + x - \frac{\sigma_y^2}{2} + G\kappa_x \left(\overline{x}^Q - x\right) - r\right)\right] \end{aligned} \tag{28}$$

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[17] We emphasize that by imposing dividend dynamics to be locally deterministic, not only are we forcing our model to look like the rest of the literature, but we are also making it more difficult to generate a downward sloping term structure of dividend variance ratios. Indeed, dividend dynamics that are not locally deterministic have very high short-horizon volatilities.

[18] Note that for D to be locally deterministic it is sufficient that B (and therefore $\log B$) be locally deterministic. Since $d\log B_t = d\ell_t + dy_t + Gdx_t$, it is clear that our choice below achieves this objective.

Note that the terms inside the square bracket follow a stationary process. Hence, dividends are cointegrated with EBIT e^{y_t}.[19]

We calibrate the leverage ratio parameters as in Table 6. Note that our parameters ($e^{\bar{\ell}} = 0.35$, $\kappa_\ell = 0.11$) are well within a one standard deviation estimate of the empirical results ($e^{\bar{\ell}} = 0.39$, $\kappa_\ell = 0.14$), although admittedly our implied leverage volatility $\sigma_\ell = 0.06$ is significantly lower than the empirical observation $\sigma_\ell = 0.12$. Moreover, in this model the conditional correlation between log-leverage and the x process is -0.95. This is in contrast to the empirical estimate of -0.38 reported in the previous section. In the Appendix, we present a model with debt-financed investment which allows us to improve our fit for both leverage volatility and this correlation.

Consistent with the analysis in the empirical section, we define the term structures of i) expected growth rates and ii) standard deviations of dividends over horizon T as:

$$g_{D,T} \equiv \left(\frac{1}{T}\right) \log\left(\mathrm{E}_0\left[\frac{\mathcal{D}_T}{\mathcal{D}_0}\right]\right) \tag{29}$$

$$\sigma_{D,T} \equiv \sqrt{\left(\frac{1}{T}\right) \log\left[\frac{\mathrm{E}_0\left[\mathcal{D}_T^2\right]}{\left(\mathrm{E}_0\left[\mathcal{D}_T\right]\right)^2}\right]}. \tag{30}$$

We plot these term structures for $x_t = 0$ in Figure (7). Note that, in contrast to the upward sloping variance ratios of EBIT, the dividend variance ratios are downward sloping, consistent with our reported empirical results.

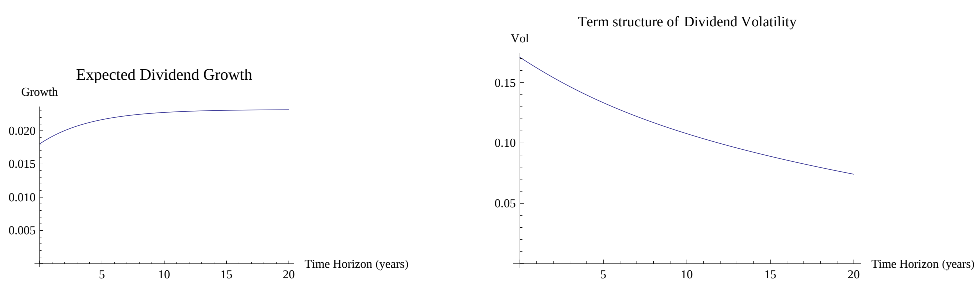

Figure 7: Term structure of expected dividend growth rate (equation (29)) and dividend volatility ($\sigma_{D,T}$ in equation (30)) for the BY economy. Parameters are set as in Table 4.

In this section we have reported the term structure of *instantaneous* dividends estimated at the long-run mean of the state vector. This allows us to make use of the

[19] Our model does not restrict leverage to be less than unity, or dividends to be positive. In the Appendix we discuss an extension of the model which imposes these more realistic restrictions. We find that the main quantities of interest (dividend variance ratios, strip expected returns and volatilities), are very similar to the results presented here.

closed-form solutions for all the prices and moments of returns. In contrast, in the empirical section we investigated dividends that were time-aggregated over a period of one year. In the Appendix, we show that aggregating dividends in this model does not significantly impact our results.

4.4 Dividend Strips

Here we provide a closed-form expression for the price of dividend strips, defined as:

$$V^T(t) \;=\; \mathrm{E}_t^Q\left[e^{-r(T-t)}\mathcal{D}(T)\right]. \tag{31}$$

As noted previously, at date-T, the firm will issue risk-free debt of value $e^{\ell_T+y_T+F+Gx_T}$, and will retire debt of value $e^{r\,dt}e^{\ell_{T-dt}+y_{T-dt}+F+Gx_{T-dt}}$. The date-$t$ value of these claims are:

$$\begin{aligned}
W^T(t,\ell_t,x_t,y_t) &= \mathrm{E}^Q\left[e^{-r(T-t)}e^{\ell_T+y_T+F+Gx_T}\right] \\
U^T(t,\ell_t,x_t,y_t) &= \mathrm{E}^Q\left[e^{-r(T-t)}e^{r\,dt}e^{\ell_{T-dt}+y_{T-dt}+F+Gx_{T-dt}}\right] \\
&= W^{T-dt}(t,\ell_t,x_t,y_t).
\end{aligned} \tag{32}$$

It therefore follows that the date-t present value of a claim to date-T dividends is:[20]

$$\begin{aligned}
V^T(t,\ell_t,x_t,y_t)\,dt &= W^T(t,\ell_t,x_t,y_t) - W^{T-dt}(t,\ell_t,x_t,y_t) + dt\,\mathrm{E}^Q\left[e^{-r(T-t)}e^{y_T}\right] \\
&= \left[\frac{\partial}{\partial T}W^T(t,\ell_t,x_t,y_t) + P^T(t,x_t,y_t)\right]dt.
\end{aligned} \tag{33}$$

From its definition, $e^{-rt}W^T(t,\ell_t,x_t,y_t)$ is a Q-martingale, implying that

$$\begin{aligned}
0 \;=\; & -rW + W_t + W_\ell\kappa_\ell\left(\overline{\ell}^Q + \alpha x - \ell\right) + W_x\kappa_x(\overline{x}^Q - x) + W_y\left(g^Q + x - \frac{\sigma_y^2}{2}\right) + \frac{\sigma_\ell^2}{2}W_{\ell\ell} \\
& + \frac{\sigma_x^2}{2}W_{xx} + \frac{\sigma_y^2}{2}W_{yy} - W_{\ell x}\left[\sigma_{x1}\left(G\sigma_{x1}+\sigma_y\right) + G\sigma_{x2}^2\right] - W_{\ell y}\sigma_y\left(G\sigma_{x1}+\sigma_y\right) + W_{xy}\sigma_{x1}\sigma_y,
\end{aligned} \tag{34}$$

[20] There are at least three alternative approaches to derive the solution to the dividend strip in our context. First, one can directly estimate $V^T(t,x_t,y_t) = \mathrm{E}_t^Q\left[e^{-r(T-t)}\mathcal{D}(T)\right]$, where $\mathcal{D}(t)$ is defined in equation (28). Second, the solution can also be computed using equation (33). Given the log-linear approximation used above, the two closed-form solutions will not agree exactly. However, we have verified that the difference between the two is very small (at our parameter values, the difference is less than 10^{-14}). A third approach is to compute the present value of future dividends as the difference between the spot stock price $V(t) = P(x_t,\, y_t)(1-e^{\ell_t})$ and the futures price $F^T(t) = \mathrm{E}_t^Q[V(T)]$ using the cash-and-carry formula: $V(t) = e^{-r(T-t)}F^T(t) + \int_t^T V^s(t)\,ds$. In turn, differentiating with respect to T gives yet another expression for Dividend strip: $V^T(t) = re^{-r(T-t)}F^T(t) - e^{-r(T-t)}\frac{\partial F^T(t)}{\partial T}$. All of these approaches would give exactly the same values if the log-linear approximation to the enterprise value is not used. In practice, for our parameter choices, the differences between these three approaches are negligible.

where we have defined

$$\begin{aligned} \sigma_{\ell}^2 &\equiv \left(G\sigma_{x1} + \sigma_{y}\right)^2 + \left(G\sigma_{x2}\right)^2 \\ \sigma_{x}^2 &\equiv \sigma_{x1}^2 + \sigma_{x2}^2. \end{aligned} \tag{35}$$

Since the dynamics of the state vector are affine, it is known (e.g., Duffie and Kan (1996)) that the solution takes an exponential-affine form:

$$W^T(t, \ell_t, x_t, y_t) = e^{y_t + H(T-t) + I(T-t)\ell_t + J(T-t)x_t}, \tag{36}$$

where the deterministic coefficients are determined in the Appendix.

Expected excess returns on the dividend strips satisfy

$$\begin{aligned} \frac{1}{dt}\mathrm{E}\left[\frac{dV^T(t, \ell_t, x_t, y_t)}{V^T(t, \ell_t, x_t, y_t)} - r\,dt\right] &= -\frac{1}{dt}\mathrm{E}\left[\left(\frac{d\Lambda}{\Lambda}\right)\left(\frac{dV^T(t, \ell_t, x_t, y_t)}{V^T(t, \ell_t, x_t, y_t)}\right)\right] \\ &= \theta_1\Omega_1(\tau, \ell_t, x_t, y_t) + \theta_2\Omega_2(\tau, \ell_t, x_t, y_t), \end{aligned} \tag{37}$$

where the terms $\Big(\Omega_1(\tau, \ell_t, x_t, y_t),\, \Omega_2(\tau, \ell_t, x_t, y_t)\Big)$ are given in the Appendix. Dividend strip volatility is

$$\begin{aligned} \sigma^{P,\tau} &\equiv \sqrt{\frac{1}{dt}\left(\frac{dV^T(t, x_t, y_t)}{V^T(t, x_t, y_t)}\right)^2} \\ &= \sqrt{\Omega_1^2(\tau, \ell_t, x_t, y_t) + \Omega_2^2(\tau, \ell_t, x_t, y_t)}. \end{aligned} \tag{38}$$

We report the dividend strip expected return and volatility term structures in Figure (8) below. This figure shows one of our main results: whereas the term structures of EBIT strips (and dividend strips in the original BY model) as shown in Figure (5) are upward sloping, in our modified BY framework, we obtain downward sloping term structures for dividend strips, consistent with the empirical findings of BBK.

4.5 Equity Returns

The value of equity equals the claim to all dividends:

$$\begin{aligned} V(\ell_t, x_t, y_t) &= \int_t^\infty dT\, V^T(t, \ell_t, x_t, y_t) \\ &= \int_t^\infty dT\, \left[\frac{\partial}{\partial T}W^T(t, \ell_t, x_t, y_t) + P^T(t, x_t, y_t)\right] \\ &= P(x_t, y_t) - B(\ell_t, x_t, y_t) \\ &\approx P(x_t, y_t)\left(1 - e^{\ell_t}\right). \end{aligned} \tag{39}$$

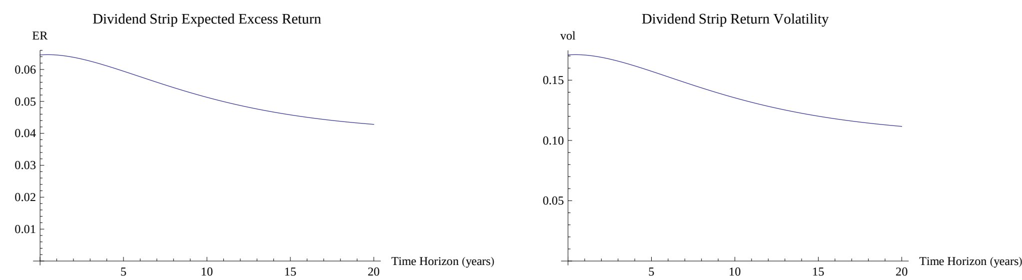

Figure 8: Term structures of dividend strip excess returns (equation (37)) and volatilities (equation (38)) for the BY economy. Parameters are given in Tables 4 and 6.

This equation is intuitive – it states that equity equals enterprise value minus debt outstanding. Indeed, it follows from Modigliani-Miller's capital structure irrelevance theorem ($V(t) = P(t) - B(t)$).

Excess return on equity is equal to excess return on EBIT (equation (22) scaled by the leverage factor:

$$\mu^V - r = \left(\frac{1}{1-e^{\ell_t}}\right)\left[\theta_1\left(\sigma_y + G\sigma_{x1}\right) + \theta_2\left(G\sigma_{x2}\right)\right]. \tag{40}$$

This equation captures "dividend irrelevance" in that future capital structure decisions do not impact equity returns (or equity value) today. Similarly, equity return volatility is scaled up by the same factor

$$\sigma^V = \left(\frac{1}{1-e^{\ell_t}}\right)\sqrt{\left(\sigma_y + G\sigma_{x1}\right)^2 + \left(G\sigma_{x2}\right)^2}. \tag{41}$$

With our current calibration, we find excess returns and volatilities on stocks to be well below excess returns and volatilities of short-maturity dividend strips, consistent with the findings of BBK.

4.6 Discussion

We have shown that if we start from an economy similar to that of BY, but endogenously derive dividend dynamics from an assumption about stationary (mean-reverting) leverage ratios, then, consistent with the empirical findings of BBK, short-maturity dividend strip returns have higher expected excess returns and higher volatilities than stock returns. Indeed, we find that the term structure of dividend strip return variances is downward sloping. This is in contrast to the term structure of EBIT (i.e., total firm cash flows) strip return volatilities, which are upward sloping. The downward shift in the slope of the term structure of dividend strip returns compared to the slope of EBIT strips is due to the implicit divestments (investments) that the firm imposes in good (bad) times on stockholders via capital structure decisions, which generates stationary leverage ratios. As such, long-maturity dividend strips are not as risky as typically imagined – rather, they are about as risky as long-maturity EBIT strips, since dividends and EBIT are cointegrated. However, claims to *all* future dividends (i.e., equity) are riskier than claims to EBIT (ie., equity plus debt). The implication is that dynamic capital structure decisions that generate stationary leverage ratios shift the risk in dividends from long-horizons to short horizons.

Interestingly, this model also generates long horizon 'excess volatility' in that stock return volatility $\sigma^V_{BY} = 0.126$ is higher than long-horizon dividend volatility $\sigma^D_{BY} = 0.10$. This prediction is more in line with observation compared to the BY model, which predicts that long-horizon dividend volatility is larger than stock volatility. This occurs despite the fact that we have a model with a constant market price of risk! As is well understood, (Campbell and Shiller (1987) and Cochrane (1991, 2007)) 'excess volatility' can be traced back to time variation in discount rates or predictability in dividends. In our framework, we have both, in that leverage predicts both future dividends (see equation (28)) and expected excess returns on equity (equation (40)). Expected excess returns on stocks are time-varying despite the fact that risk-premia are constant, simply due to the time variation in leverage.

5 Endogenous Dividend Dynamics in an 'External Habit Formation' Model

Here we investigate a modified version of the habit formation model of Campbell and Cochrane (CC, 1999). In contrast to the BY model, which has a constant market price of risk and cash flows that have a predictable component, the framework we consider here has no predictability in aggregate cash flows, but generates predictability in excess returns via time variation in risk-premia. Specifically, we assume that cash flows follow an iid process, and that shocks to the market price of risk are negatively correlated with shocks to these cash flows. But instead of modeling dividend dynamics exogenously as in CC, we specify EBIT dynamics and combine them with a dynamic capital structure policy that generates stationary leverage ratios in order to endogenously determine dividend dynamics. Because much of the theory is very similar to the BY framework, we present here only the main results, and relegate the derivations to the Appendix.

5.1 EBIT Dynamics

Instead of exogenously specifying dividend dynamics as in CC, we specify the dynamics for log-EBIT y_t to be iid

$$dy = \left(g - \frac{\sigma_y^2}{2}\right) dt + \sigma_y \, dz. \tag{42}$$

The term structure of EBIT expected growth rate over horizon τ is defined as

$$\begin{aligned} g_{y,\tau} &\equiv \left(\frac{1}{\tau}\right) \log\left(\mathrm{E}_0\left[e^{y_\tau - y_0}\right]\right) \\ &= g \qquad \forall \tau. \end{aligned} \tag{43}$$

Similarly, the term structure of EBIT volatility is defined as:

$$\begin{aligned} \sigma^2_{y,\tau} &\equiv \left(\frac{1}{\tau}\right) \log\left[\frac{\mathrm{E}_0\left[e^{2(y_\tau - y_0)}\right]}{\left(\mathrm{E}_0\left[e^{y_\tau - y_0}\right]\right)^2}\right] \\ &= \sigma_y^2. \end{aligned} \tag{44}$$

Note that the term structure of volatilities is flat.

5.2 EBIT Strips

CC provide a framework that generates a pricing kernel with a constant risk free rate and a countercyclical market price of risk. We approximate their model with the following dynamics:

$$\frac{d\Lambda}{\Lambda} = -r \, dt - \theta_t \, dz, \tag{45}$$

where innovations in the market price of risk are driven by the same Brownian motion that drives EBIT innovations:

$$d\theta = \kappa\left(\overline{\theta} - \theta_t\right) dt - \nu \, dz. \tag{46}$$

Thus, risk-neutral dynamics for the state variables follow

$$dy = \left(g - \frac{\sigma_y^2}{2} - \sigma_y \theta_t\right) dt + \sigma_y \, dz^Q \tag{47}$$

$$d\theta = \kappa^Q(\overline{\theta}^Q - \theta_t) \, dt - \nu \, dz^Q, \tag{48}$$

where we have defined $\kappa^Q \equiv (\kappa - \nu)$, and $\kappa^Q \overline{\theta}^Q \equiv \kappa\theta$.

The date-t price $P^T(t,\, \theta_t,\, y_t)$ of the security whose payoff is the date-T EBIT flow e^{y_T} is:

$$P^T(t, \theta_t, y_t) \;=\; e^{-r(T-t)} \mathrm{E}_t^Q \left[e^{y_T} \right]. \tag{49}$$

The solution takes the exponential affine form:

$$P^T(t, \theta_t, y_t) \;=\; e^{y_t + F(T-t) - G(T-t)\, \theta_t}, \tag{50}$$

where the deterministic functions $(F(\tau),\, G(\tau))$ are derived in the Appendix.

Expected excess returns on the EBIT strips satisfy $(\tau = (T-t))$:

$$\begin{aligned} \frac{1}{dt} \mathrm{E} \left[\frac{dP^T(t, \theta_t, y_t)}{P^T(t, \theta_t, y_t)} - r\, dt \right] &= -\frac{1}{dt} \mathrm{E} \left[\frac{d\Lambda}{\Lambda} \frac{dP^T(t, \theta_t, y_t)}{P^T(t, \theta_t, y_t)} \right] \\ &= \theta_t \left[\sigma_y + \nu G(\tau) \right]. \end{aligned} \tag{51}$$

EBIT strip volatility is

$$\sqrt{\frac{1}{dt} \left(\frac{dP^T(t, \theta_t, y_t)}{P^T(t, \theta_t, y_t)} \right)^2} \;=\; \left[\sigma_y + \nu G(\tau) \right]. \tag{52}$$

We calibrate this model using the parameter values in the following table. We report the results in Figure (9). As noted in Binsbergen, Brandt and Koijen (2011), the CC model generates an upward sloping term structure of expected returns and volatilities.

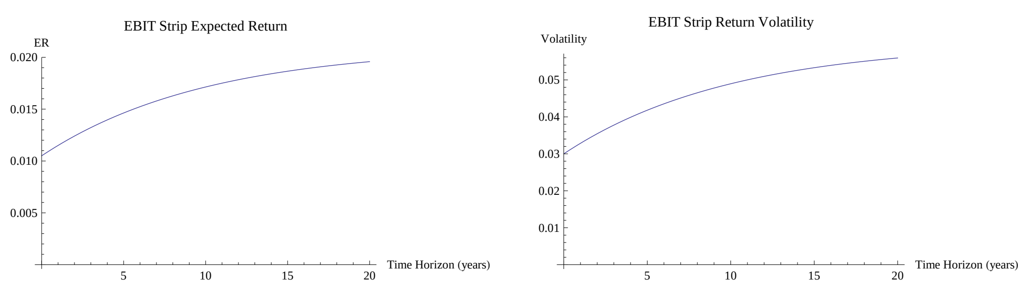

Figure 9: Term structure of EBIT Strip Expected Return (equation (51)) and volatility (equation (52)) for the CC economy. Parameters are set as in Table 8.

The enterprise value of the firm is equal to the present value of the claim to all EBIT strips:

$$P(\theta_t,\, y_t) \;=\; e^{y_t} \int_t^{\infty} dT\, e^{F(T-t) - G(T-t)\, \theta_t}. \tag{53}$$

This can be well-approximated by a log-linear approximation:

$$P(\theta_t,\, y_t) \;\approx\; e^{y_t + F - G\,\theta_t}, \tag{54}$$

where the coefficients (F, G) are given in Table 9 below. Figure 10 plots the exact and approximate solution, and shows the accuracy of the log-linear approximation.

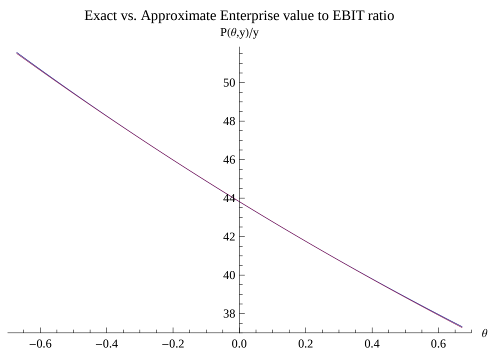

Figure 10: Exact and approximate solutions to the enterprise value as given in respectively equations (20) and (21) for the parameters given in Table 8. The x-axis covers -4 to $+4$ standard deviations of the unconditional distribution of θ.

Under this log-linear approximation, expected return and volatility of the claim to EBIT are:

$$(\mu^P - r)_{CC} \;\approx\; \theta_t\left(\sigma_y + \nu G\right) \tag{55}$$

$$\sigma^P_{CC} \;\approx\; \left(\sigma_y + \nu G\right). \tag{56}$$

Their values given the calibrated parameters are given in Table 9:

5.3 Dividend Dynamics

Assume that at all dates-t, the firm issues riskless debt that matures at date-$(t + dt)$ with present value equal to

$$\begin{aligned} B(\ell_t, \theta_t, y_t) &= e^{\ell_t + y_t + F - G\theta_t} \\ &\approx e^{\ell_t}\, P(\theta_t,\, y_t). \end{aligned} \tag{57}$$

As in the previous model, we interpret $e^{\ell_t} \approx \frac{B(\ell_t,\theta_t,y_t)}{P(\theta_t,y_t)}$ as the leverage of the firm. Using an argument analogous to the previous section, we specify leverage dynamics as:

$$\begin{aligned} d\ell_t &= \kappa_\ell\left(\bar{\ell} + \alpha\theta - \ell\right)\,dt - \left(\nu G + \sigma_y\right)\,dz \\ &= \kappa_\ell\left(\bar{\ell} + \alpha^Q\theta - \ell\right)\,dt - \left(\nu G + \sigma_y\right)\,dz^Q, \end{aligned} \tag{58}$$

where

$$\alpha^Q = \alpha + \frac{\nu G + \sigma_y}{\kappa_\ell}. \tag{59}$$

Since the combination $(d\ell + dy - G\,d\theta)$ is locally deterministic, dividends paid out over the interval $(t, t+dt)$ are equal to $\mathcal{D}(t)\,dt$, where

$$\begin{aligned}\mathcal{D}(t) &= e^{y_t}\left\{1 + e^{\ell_t + F - G\theta_t}\left[\kappa_\ell\left(\overline{\ell} + \alpha\theta - \ell\right) + g - r - \frac{\sigma_y^2}{2} - G\kappa\left(\overline{\theta} - \theta\right)\right]\right\} \qquad (60)\\ &= e^{y_t}\left[1 + e^{\ell_t + F - G\theta_t}\left[\kappa_\ell\left(\overline{\ell} + \alpha^Q\theta - \ell\right) + g - r - \frac{\sigma_y^2}{2} - \sigma_y\theta - G\kappa^Q\left(\overline{\theta}^Q - \theta\right)\right]\right\}.\end{aligned}$$

Note that the terms inside the square bracket follow a stationary process. Hence, dividends are cointegrated with EBIT e^{y_t}.

We define the term structures of expected growth rates and volatilities for dividends over horizon T as in equations (29)-(30). We plot these term structures for $\theta_0 = \overline{\theta}$ in Figure (11).

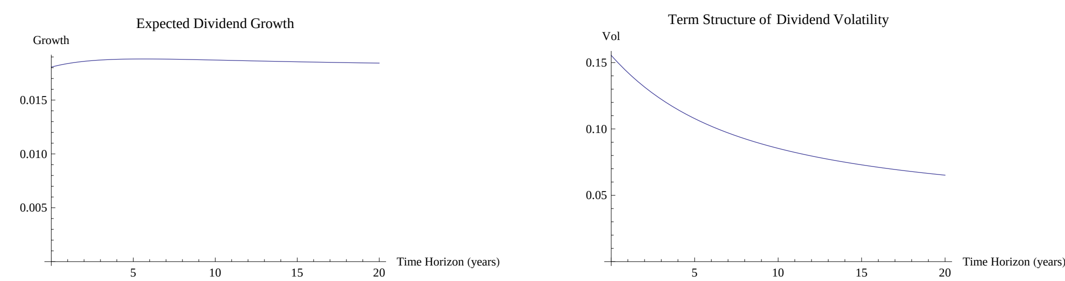

Figure 11: Term structure of expected dividend growth rate (equation (29)) and dividend volatility ($\sigma_{D,T}$ in equation (30)) for the CC economy. Parameters are set as in Table 8.

Note that the term structure of volatilities is downward sloping, consistent with our reported empirical evidence.

5.4 Dividend Strips

As in the previous section, we provide a closed-form expression for the price of dividend strips defined as:

$$V^T(t) = \mathrm{E}_t^Q[e^{-r(T-t)}\mathcal{D}(T)]$$

in terms of the claim:

$$W^T(t, \ell_t, \theta_t, y_t) = \mathrm{E}_t^Q\left[e^{-r(T-t)}e^{\ell_T + y_T + F - G\theta_T}\right],$$

which admits a closed-form exponential affine solution, which we derive in the Appendix:

$$W^T(t, \ell_t, \theta_t, y_t) \;=\; e^{y_t + H(T-t) + I(T-t)\ell_t - J(T-t)\theta_t}. \tag{61}$$

The date-t present value of a claim to date-T dividends is, as before,

$$V^T(t, \ell_t, \theta_t, y_t)\, dt \;=\; \left[\frac{\partial}{\partial T} W^T(t, \ell_t, \theta_t, y_t) + P^T(t, \theta_t, y_t)\right] dt. \tag{62}$$

Expected excess returns on the dividend strips satisfy

$$\begin{aligned} \frac{1}{dt}\mathrm{E}\left[\frac{dV^T(t, \ell_t, \theta_t, y_t)}{V^T(t, \ell_t, \theta_t, y_t)} - r\, dt\right] &= -\frac{1}{dt}\mathrm{E}\left[\left(\frac{d\Lambda}{\Lambda}\right)\left(\frac{dV^T(t, \ell_t, \theta_t, y_t)}{V^T(t, \ell_t, \theta_t, y_t)}\right)\right] \\ &= \theta\Omega(\tau, \ell_t, \theta_t, y_t) \end{aligned} \tag{63}$$

where $\Omega(\tau, \ell_t, \theta_t, y_t)$ is given in the Appendix. Dividend strip volatility is

$$\begin{aligned} \sigma^{P,\tau} &\equiv \sqrt{\frac{1}{dt}\left(\frac{dV^T(t, \theta_t, y_t)}{V^T(t, \theta_t, y_t)}\right)^2} \\ &= \Omega(\tau, \ell_t, \theta_t, y_t). \end{aligned} \tag{64}$$

We calibrate the leverage ratio parameters as in Table 10.

We report the dividend strip return and volatility term structures in Figure (12) below. This figure captures one of the main results of the paper, namely, that our modified version of the CC-model generates downward sloping term structures for both expected returns and volatilities of dividend strips, consistent with the empirical findings of BBK, even though Figure (9) shows that the term structure of EBIT strips expected returns and variances are upward sloping.

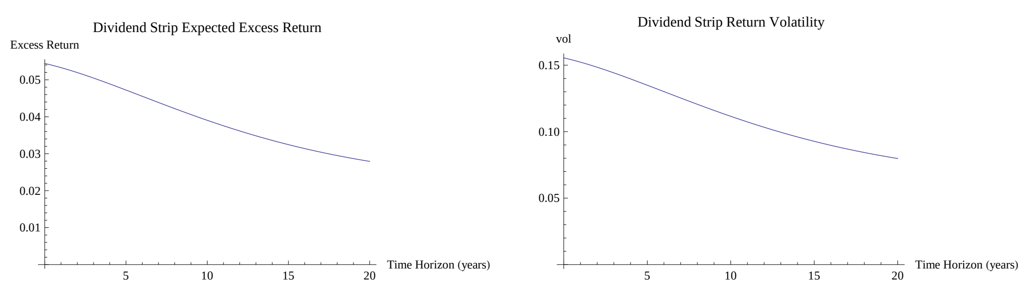

Figure 12: Term structure of Dividend Strip expected returns (equation (63)) and volatilities (equation (64)) for the CC economy. Parameters are set as in Table 8 and 10.

5.5 Equity Returns

The value of equity equals the claim to all dividends:

$$V(\ell_t, \theta_t, y_t) \approx P(\theta_t, y_t)\left(1 - e^{\ell_t}\right). \tag{65}$$

Excess return on equity is equal to excess return on EBIT (equation (55)) scaled by the leverage factor:

$$\mu^V - r = \left(\frac{1}{1 - e^{\ell_t}}\right)\left(\sigma_y + G\nu\right)\theta_t. \tag{66}$$

This equation captures "dividend irrelevance" in that future capital structure decisions do not impact equity returns (or equity value) today. Similarly, equity return volatility is scaled up by the same factor

$$\sigma^V = \left(\frac{1}{1 - e^{\ell_t}}\right)\left(\sigma_y + G\nu\right). \tag{67}$$

Results of our calibration are in Table 11.

5.6 Discussion

Even though in this section we focus on a different asset pricing framework than in the previous section, with time-varying expected return and no cash-flow predictability, we find similar patterns when looking at the term structure of dividend strip return volatilities. With endogenous dividend dynamics derived from a similar mean-reverting process for aggregate leverage, we find that the short term dividend claims are riskier than long-term claims. As a result they display higher volatility and expected returns than long-term claims.

6 Conclusion

Many leading asset pricing models such as Campbell and Cochrane (1999) and Bansal and Yaron (2004) predict that the term structure of excess returns and volatilities of dividend strips are strongly upward sloping. However, the empirical findings of Binsbergen, Brandt and Koijen (2011) suggest otherwise. We first show that, in contrast to the predictions of these leading models, empirical estimates for the variance ratios of dividends are decreasing with horizon. We then modify these leading models by retaining their pricing kernels, but replacing their dividend dynamics with processes that are consistent with this empirical fact. We show that this modification allows these models to retain their ability to match salient features of stock and bond returns while simultaneously generating and dividend strips with decreasing term structures of expected returns and volatilities.

More ambitiously, we provide an economic mechanism that explains why dividend variance ratios should be decreasing with horizon. Specifically, we determine 'endogenous' dividend dynamics consistent with exogenously specified unlevered cash flow (i.e., EBIT) dynamics, and with a dynamic capital structure policy that generates stationary leverage ratios. This approach generates dividends that are riskier than EBIT in the short run, but cointegrated with EBIT in the long run. Intuitively, this is because when a firm rebalances its debt levels over time to maintain a stationary leverage process, shareholders are being forced to divest (invest) when the firm does well (poorly). This interaction transfers risk from long horizon dividends to short horizon dividends, pushing downward the term structure of dividend strip volatilities.

Our modified dividend process also helps explain long-horizon "excess volatility" in that it generates models where stock returns are more volatile than long-horizon dividend volatility, even if the market prices of risk are constant. This prediction is more in line with observation, and eliminates a counterfactual prediction of the original BY model that stock return volatility should be significantly smaller than long horizon dividend volatility.

7 Appendix

7.1 Proof of Equation(9)

Since

$$M_{_1}^T(t, x_{_t}, y_{_t}) \equiv \mathrm{E}_{_t}\left[e^{y_T}\right] \tag{68}$$

is a P-martingale, it follows that

$$0 = M_{_t} - \kappa_{_x} x M_{_x} + M_{_y}\left(g + x - \frac{\sigma_{_y}^2}{2}\right) + \frac{\sigma_{_y}^2}{2} M_{yy} + \frac{\sigma_{_x}^2}{2} M_{xx} + \sigma_{_y}\sigma_{_{x1}} M_{_{xy}}. \tag{69}$$

Since state vector dynamics are exponentially affine, it is well known that the solution is of the form:

$$M^T(t, x_{_t}, y_{_t}) = e^{y_t + A_0(T-t) + x_t A_1(T-t)}. \tag{70}$$

Defining $\tau = (T - t)$, we identify initial conditions via

$$\mathrm{E}_{_T}\left[e^{y_T}\right] = e^{y_T + A_0(\tau=0) + x_t A_1(\tau=0)}, \tag{71}$$

and hence $A_{_0}(\tau = 0) = 0,\ A_{_1}(\tau = 0) = 0$. Plugging equation (70) into equation (69), collecting terms linear and independent of x, we obtain the ODE's

$$\begin{aligned} A_{_1}' &= 1 - \kappa_{_x} A_{_1} \\ A_{_0}' &= g + \frac{\sigma_{_x}^2}{2} A_{_1}^2 + \sigma_{_y}\sigma_{_{x1}} A_{_1}. \end{aligned} \tag{72}$$

The solutions are

$$\begin{aligned} A_{_1}(\tau) &= \frac{1}{\kappa_{_x}}\left(1 - e^{-\kappa_x \tau}\right) \\ A_{_0}(\tau) &= g\tau + \left(\frac{\sigma_{_y}\sigma_{_{x1}}}{\kappa_{_x}}\right)(\tau - A_{_1}(\tau)) + \left(\frac{\sigma_{_x}^2}{2\kappa_{_x}^2}\right)\left(\tau - A_{_1}(\tau) - \left(\frac{\kappa_{_x}}{2}\right) A_{_1}^2(\tau)\right). \end{aligned} \tag{73}$$

7.2 Proof of Equation(10)

Since

$$M^T(t, x_{_t}, y_{_t}) \equiv \mathrm{E}_{_t}\left[e^{2y_T}\right] \tag{74}$$

is a P-martingale, it follows that

$$0 = M_t - \kappa_x x M_x + M_y \left(g + x - \frac{\sigma_y^2}{2} \right) + \frac{\sigma_y^2}{2} M_{yy} + \frac{\sigma_x^2}{2} M_{xx} + \sigma_y \sigma_{x1} M_{xy}. \quad (75)$$

Since state vector dynamics are exponentially affine, it is well known that the solution is of the form:

$$M^T(t, x_t, y_t) = e^{2y_t + A_2(T-t) + x_t A_3(T-t)}. \quad (76)$$

Defining $\tau = (T-t)$, we identify initial conditions via

$$\mathrm{E}_T \left[e^{2y_T} \right] = e^{2y_T + A_2(\tau=0) + x_t A_3(\tau=0)}, \quad (77)$$

and hence $A_2(\tau = 0) = 0$, $A_3(\tau = 0) = 0$. Plugging equation (76) into equation (75), collecting terms linear and independent of x, we obtain the ODE's

$$\begin{aligned} A_3' &= 2 - \kappa_x A_3 \\ A_2' &= 2g + \sigma_y^2 + \frac{\sigma_x^2}{2} A_3^2 + 2\sigma_y \sigma_{x1} A_3. \end{aligned} \quad (78)$$

The solutions are

$$A_3(\tau) = \frac{2}{\kappa_x} \left(1 - e^{-\kappa_x \tau} \right) \quad (79)$$

$$A_2(\tau) = \left(2g + \sigma_y^2 \right) \tau + \left(\frac{2\sigma_y \sigma_{x1}}{\kappa_x} \right) (2\tau - A_3(\tau)) + \left(\frac{\sigma_x^2}{2\kappa_x^2} \right) \left(4\tau - 2A_3(\tau) - \left(\frac{\kappa_x}{2} \right) A_3^2(\tau) \right).$$

7.3 EBIT strip in the BY economy

Here we derive the solution to Equation (17):

$$P^T(t, x_t, y_t) = e^{-r(T-t)} \mathrm{E}_t^Q \left[e^{y_T} \right]. \quad (80)$$

Note that $e^{-rt} P^T(t, x_t, y_t)$ is a Q-martingale, implying that

$$\begin{aligned} 0 &= \mathrm{E}^Q \left[d \left(e^{-rt} P^T(t, x_t, y_t) \right) \right] \\ &= -rP + P_t + (g^Q + x) y P_y + \kappa_x (\overline{x}^Q - x) P_x + \frac{1}{2} y^2 \sigma_y^2 P_{yy} + \frac{\sigma_x^2}{2} P_{xx} + \sigma_y \sigma_{x1} y P_{xy} \end{aligned} \quad (81)$$

where we have defined $\sigma_x^2 \equiv \sigma_{x1}^2 + \sigma_{x2}^2$. Since the state vector dynamics are affine, it is well known (see, for example, Duffie and Kan (1996)) that the solution takes the exponential affine form:

$$P^T(t, x_t, y_t) = e^{y_t + F(T-t) + G(T-t)\, x_t}. \quad (82)$$

Plugging this functional form into equation (81) and then collecting terms linear and independent of x, we find that the deterministic functions $F(\tau)$ and $G(\tau)$ (where $\tau \equiv (T-t)$) satisfy the Ricatti equations:

$$G_{\tau} = 1 - \kappa_x G \tag{83}$$

$$F_{\tau} = (g^Q - r) + G(\sigma_y \sigma_{x1} + \kappa_x \overline{x}^Q) + \frac{\sigma_x^2}{2} G^2, \tag{84}$$

with boundary conditions $G(0) = 0$, $F(0) = 0$. The solutions are

$$\begin{aligned} G(\tau) &= \frac{1}{\kappa_x}\left(1 - e^{-\kappa_x \tau}\right) \\ F(\tau) &= (g^Q - r)\,\tau + \left(\frac{\sigma_y \sigma_{x1}}{\kappa_x} + \overline{x}^Q\right)\Big(\tau - G(\tau)\Big) + \left(\frac{\sigma_x^2}{2\kappa_x^2}\right)\Big(\tau - G(\tau) - \frac{\kappa_x}{2} G^2(\tau)\Big) \end{aligned} \tag{85}$$

7.4 Enterprise Value in BY Economy

Here we derive the constants F and G used in equation (21). Enterprise value can be determined via

$$P(x_t, y_t) = \mathrm{E}_t^Q \left[\int_t^{\infty} ds\, e^{-r(s-t)} e^{y_s}\right]. \tag{86}$$

The exact solution is

$$P(x_t, y_t) = \int_t^{\infty} P^T(t, x_t, y_t)\, dT. \tag{87}$$

This explicit solution can be approximated very accurately by an expression of the form:

$$P(x_t, y_t) \approx e^{y_t + F + G x_t}, \tag{88}$$

where the constants (F, G) can be derived by some local Taylor expansion argument as in Campbell-Shiller, or by minimizing some global error metric as in the appendix of Chen, Collin-Dufresne and Goldstein (2009). Since in our case, the closed-form solution is known, we simply minimize the mean-square error between the approximation and the closed-form solution. Results are shown in the main text. The approximation error (difference between exact and approximate solution) in absolute terms is less than 0.002, and in relative terms less than 0.005.

7.5 Solution for dividend strips V_t^T in the BY economy

Recall that

$$V^T(t,\ell_t,x_t,y_t) = \left[\frac{\partial}{\partial T} W^T(t,\ell_t,x_t,y_t) + P^T(t,x_t,y_t)\right].$$

where

$$W^T(t,\ell_t,x_t,y_t) = \mathrm{E}_t^Q\left[e^{-r(T-t)} e^{\ell_T + y_T + F + Gx_T}\right]. \tag{89}$$

Therefore all we need is an analytic expression for $W^T(t)$. Given that the state vector dynamics are affine, it is well known that the solution is of the form:

$$W^T(t,\ell_t,x_t,y_t) = e^{y_t + H(T-t) + I(T-t)\ell_t + J(T-t)x_t}, \tag{90}$$

where the functions H, I, J satisfy the PDE in equation (34). After defining time to maturity $\tau \equiv (T-t)$, we find that they solve the following system of equations:

$$\begin{aligned}
I'(\tau) &= -\kappa_\ell I \\
J'(\tau) &= \alpha\kappa_\ell I - \kappa_x J + 1 \\
H'(\tau) &= -r + \kappa_\ell \overline{\ell}^Q I + \kappa_x \overline{x}^Q J + g^Q + \frac{\sigma_\ell^2}{2} I^2 + \frac{\sigma_x^2}{2} J^2 - \left[\sigma_{x1}\left(G\sigma_{x1} + \sigma_y\right) + G\sigma_{x2}^2\right] IJ \\
&\quad -\sigma_y\left(G\sigma_{x1} + \sigma_y\right) I + \sigma_{x1}\sigma_y J.
\end{aligned} \tag{91}$$

The initial conditions are $H(0) = F$, $I(0) = 1$, $J(0) = G$. The solutions are

$$\begin{aligned}
I(\tau) &= e^{-\kappa_\ell \tau} \\
J(\tau) &= \left[G - \frac{1}{\kappa_x} - \left(\frac{\alpha\kappa_\ell}{\kappa_x - \kappa_\ell}\right)\right] e^{-\kappa_x \tau} + \frac{1}{\kappa_x} + \left(\frac{\alpha\kappa_\ell}{\kappa_x - \kappa_\ell}\right) e^{-\kappa_\ell \tau} \\
H(\tau) &= F + \left(g^Q - r\right)\tau + \left[\kappa_\ell \overline{\ell}^Q - \sigma_y\left(G\sigma_{x1} + \sigma_y\right)\right] \int_0^\tau ds\, I(s) + \left[\kappa_x \overline{x}^Q + \sigma_{x1}\sigma_y\right] \int_0^\tau ds\, J(s) \\
&\quad + \frac{\sigma_\ell^2}{2} \int_0^\tau ds\, I(s)^2 + \frac{\sigma_x^2}{2} \int_0^\tau ds\, J(s)^2 - \left[\sigma_{x1}\left(G\sigma_{x1} + \sigma_y\right) + G\sigma_{x2}^2\right] \int_0^\tau ds\, I(s)J(s),
\end{aligned} \tag{92}$$

where

$$
\begin{aligned}
\int_0^\tau ds\, I(s) &= \frac{1}{\kappa_\ell}\left(1-e^{-\kappa_\ell \tau}\right) \\
\int_0^\tau ds\, J(s) &= \frac{1}{\kappa_x}\left[G-\frac{1}{\kappa_x}-\left(\frac{\alpha\kappa_\ell}{\kappa_x-\kappa_\ell}\right)\right]\left(1-e^{-\kappa_x \tau}\right)+\frac{1}{\kappa_x}\tau+\left(\frac{\alpha\kappa_\ell^2}{\kappa_x-\kappa_\ell}\right)\left(1-e^{-\kappa_\ell \tau}\right) \\
\int_0^\tau ds\, I^2(s) &= \frac{1}{2\kappa_\ell}\left(1-e^{-2\kappa_\ell \tau}\right) \\
\int_0^\tau ds\, J^2(s) &= \frac{1}{2\kappa_x}\left[G-\frac{1}{\kappa_x}-\left(\frac{\alpha\kappa_\ell}{\kappa_x-\kappa_\ell}\right)\right]^2\left(1-e^{-2\kappa_x \tau}\right)+\left(\frac{\tau}{\kappa_x^2}\right)+\left(\frac{1}{2\kappa_\ell}\right)\left(\frac{\alpha\kappa_\ell}{\kappa_x-\kappa_\ell}\right)^2\left(1-e^{-2\kappa_\ell \tau}\right) \\
&\quad +\frac{2}{\kappa_x^2}\left[G-\frac{1}{\kappa_x}-\left(\frac{\alpha\kappa_\ell}{\kappa_x-\kappa_\ell}\right)\right]\left(1-e^{-\kappa_x \tau}\right)+\left(\frac{2}{\kappa_x\kappa_\ell}\right)\left(\frac{\alpha\kappa_\ell}{\kappa_x-\kappa_\ell}\right)\left(1-e^{-\kappa_\ell \tau}\right) \\
&\quad +2\left[G-\frac{1}{\kappa_x}-\left(\frac{\alpha\kappa_\ell}{\kappa_x-\kappa_\ell}\right)\right]\left(\frac{\alpha\kappa_\ell}{\kappa_x-\kappa_\ell}\right)\left(\frac{1}{\kappa_x+\kappa_\ell}\right)\left(1-e^{-(\kappa_x+\kappa_\ell)\tau}\right) \\
\int_0^\tau ds\, I(s)J(s) &= \left(\frac{1}{\kappa_x+\kappa_\ell}\right)\left[G-\frac{1}{\kappa_x}-\left(\frac{\alpha\kappa_\ell}{\kappa_x-\kappa_\ell}\right)\right]\left(1-e^{-(\kappa_x+\kappa_\ell)\tau}\right)+\left(\frac{1}{\kappa_x\kappa_\ell}\right)\left(1-e^{-\kappa_\ell \tau}\right) \\
&\quad +\left(\frac{\alpha}{2(\kappa_x-\kappa_\ell)}\right)\left(1-e^{-2\kappa_\ell \tau}\right). \qquad (9
\end{aligned}
$$

Hence, defining $\tau \equiv (T-t)$, we have

$$
\begin{aligned}
V^T(t,\ell_t,x_t,y_t) &= \frac{\partial}{\partial T}\left[e^{y_t+H(T-t)+I(T-t)\ell_t+J(T-t)x_t}\right]+P^T(t,x_t,y_t) \\
&= W^T(t,\ell_t,x_t,y_t)\left[-\kappa_\ell I(\tau)\ell_t+x_t\left(\alpha\kappa_\ell I(\tau)-\kappa_x J(\tau)+1\right)-r\right. \qquad (94) \\
&\quad +\kappa_\ell\overline{\ell}^Q I(\tau)+\kappa_x\overline{x}^Q J(\tau)+g^Q+\frac{\sigma_\ell^2}{2}I^2(\tau)+\frac{\sigma_x^2}{2}J^2(\tau)+P^T(t,x_t,y_t) \\
&\quad \left.-I(\tau)J(\tau)\left[\sigma_{x1}(G\sigma_{x1}+\sigma_y)+G\sigma_{x2}^2\right]-I(\tau)\sigma_y(G\sigma_{x1}+\sigma_y)-J(\tau)\sigma_{x1}\sigma_y\right].
\end{aligned}
$$

7.6 Identification of Ω_1 and Ω_2

Applying Itô's lemma to $V^T(t) \equiv V^T(t,\ell_t,x_t,y_t)$ we find

$$
dV^T(t) = rV^T(t)\,dt+\left(V_\ell^T(t)\,\sigma_{\ell1}+V_x^T(t)\,\sigma_{x1}+V_y^T(t)\,\sigma_y\right)\,dz_1^Q(t)+\left(V_\ell^T(t)\,\sigma_{\ell2}+V_x^T(t)\,\sigma_{x2}\right)\,dz_2^Q(t),
$$

where we have defined

$$
\begin{aligned}
\sigma_{\ell1} &= -(G\sigma_{x1}+\sigma_y) \qquad (95) \\
\sigma_{\ell2} &= -G\sigma_{x2}. \qquad (96)
\end{aligned}
$$

Therefore

$$\Omega_1 = \frac{(V^T_\ell(t)\,\sigma_{\ell 1} + V^T_x(t)\,\sigma_{x1} + V^T_y\,\sigma_y)}{V^T(t)} \tag{97}$$

$$\Omega_2 = \frac{(V^T_\ell(t)\,\sigma_{\ell 2} + V^T_x(t)\,\sigma_{x2})}{V^T(t)}. \tag{98}$$

All derivatives can be computed in closed form based on the closed-form expression for V above.

7.7 Solution to the EBIT strip price in the CC economy

Here we derive the solution to the EBIT strip price in the CC economy:

$$P^T(t, \theta_t, y_t) = e^{-r(T-t)} \mathrm{E}^Q_t \left[e^{y_T} \right]. \tag{99}$$

From equation (49), we see that $e^{-rt} P^T(t, x_t, y_t)$ is a Q-martingale, implying that

$$\begin{aligned} 0 &= \mathrm{E}^Q \left[d\left(e^{-rt} P^T(t, x_t, y_t) \right) \right] \\ &= -rP + P_t + (g - \frac{\sigma_y^2}{2} - \sigma_y \theta_t) P_y + \kappa^Q (\overline{\theta}^Q - \theta) P_\theta + \frac{\sigma_y^2}{2} P_{yy} + \frac{\nu^2}{2} P_{\theta\theta} - \sigma_y \nu P_{\theta y}, \end{aligned} \tag{100}$$

Since the state vector dynamics are affine, it is well known (see, for example, Duffie and Kan (1996)) that the solution takes the exponential affine form:

$$P^T(t, \theta_t, y_t) = e^{y_t + F(T-t) - G(T-t)\,\theta_t}. \tag{101}$$

Plugging this functional form into equation (100) and then collecting terms linear and independent of θ, we find that the deterministic functions $F(\tau)$ and $G(\tau)$ (where $\tau \equiv (T-t)$) satisfy the Ricatti equations:

$$G_\tau = \sigma_y - \kappa^Q G \tag{102}$$

$$F_\tau = (g - r) + G(\sigma_y \nu - \kappa^Q \overline{\theta}^Q) + \frac{\nu^2}{2} G^2, \tag{103}$$

with boundary conditions $G(0) = 0$, $F(0) = 0$. The solutions are

$$G(\tau) = \frac{\sigma_y}{\kappa^Q} \left(1 - e^{-\kappa^Q \tau} \right) \tag{104}$$

$$F(\tau) = (g - r)\,\tau + \left(\frac{\sigma_y \nu - \kappa^Q \overline{\theta}^Q}{\kappa^Q} \right) \left(\sigma_y \tau - G(\tau) \right) + \left(\frac{\nu^2}{2\kappa^Q} \right) \left[\left(\frac{\sigma_y}{\kappa^Q} \right) \left(\sigma_y \tau - G(\tau) \right) - \frac{1}{2} G^2(\tau) \right].$$

7.8 Enterprise Value in the CC Economy

The enterprise value can be determined via

$$P(\theta_t,\, y_t) \;=\; \mathrm{E}_t^Q\left[\int_t^\infty ds\, e^{-r(s-t)} e^{y_s}\right]. \tag{105}$$

The exact solution is:

$$P(\theta_t,\, y_t) = \int_t^\infty P^T(t, \theta_t, y_t)\, dT.$$

We approximate this exact solution using an expression of the form:

$$P(\theta_t,\, y_t) \approx e^{y_t + F - G\theta_t}. \tag{106}$$

The coefficients $(F,\, G)$ are chosen to minimize the mean-square error between the approximate and the closed-form solution. Results are shown in the main text. The approximation error (difference between exact and approximate solution) in absolute terms is less than 0.06, and in relative terms less than 0.0012.

7.9 Solution for dividend strips V_t^T in the CC economy

Recall that

$$V^T(t, \ell_t, \theta_t, y_t) = \left[\frac{\partial}{\partial T} W^T(t, \ell_t, \theta_t, y_t) + P^T(t, \theta_t, y_t)\right]. \tag{107}$$

where

$$W^T(t, \ell_t, \theta_t, y_t) = \mathrm{E}_t^Q\left[e^{-r(T-t)} e^{\ell_T + y_T + F - G\theta_T}\right]. \tag{108}$$

It can be shown that $W^T(t, \ell_t, \theta_t, y_t)$ satisfies the PDE

$$\begin{aligned} 0 \;=\; & -rW + W_t + W_\ell \kappa_\ell \left(\bar{\ell} + \alpha^Q \theta - \ell\right) + W_y \left(g - \frac{\sigma_y^2}{2} - \sigma_y \theta_t\right) + W_\theta \kappa^Q (\bar{\theta}^Q - \theta_t) \\ & + \frac{\sigma_\ell^2}{2} W_{\ell\ell} + \frac{\sigma_y^2}{2} W_{yy} + \frac{\nu^2}{2} W_{\theta\theta} - \sigma_\ell \sigma_y W_{\ell y} + \nu \sigma_\ell W_{\ell\theta} - \nu \sigma_y W_{y\theta}, \end{aligned} \tag{109}$$

where we have defined

$$\sigma_\ell \;\equiv\; \left(G\nu + \sigma_y\right). \tag{110}$$

Since the dynamics of the state vector are affine, the solution takes an exponential-affine form:

$$W^T(t, \ell_t, \theta_t, y_t) \;=\; e^{y_t + H(T-t) + I(T-t)\ell_t - J(T-t)\theta_t}, \tag{111}$$

To find the functions $(H(\tau),\ I(\tau),\ J(\tau))$ we plug equation (111) into equation (109) and then collect terms linear in θ, linear in ℓ, and independent of $(\theta,\ \ell)$ to obtain three coupled Riccati equations:

$$
\begin{aligned}
I'(\tau) &= -\kappa_\ell I \\
J'(\tau) &= \sigma_y - \kappa^Q J - \kappa_\ell \alpha^Q I \\
H'(\tau) &= g - r + (\nu\sigma_y - \kappa^Q \overline{\theta}^Q)J + (\kappa_\ell \overline{\ell} - \sigma_y \sigma_\ell)I + \frac{\nu^2}{2}J^2 + \frac{\sigma_\ell^2}{2}I^2 - \nu\sigma_\ell IJ. \quad (112)
\end{aligned}
$$

Initial conditions are $I(0) = 1,\ J(0) = -G,\ H(0) = F$. Solutions are

$$
\begin{aligned}
I(\tau) &= e^{\kappa_\ell \tau} \\
J(\tau) &= \left(\frac{\kappa_\ell \alpha_Q}{\kappa_\ell - \kappa^Q}\right) e^{-\kappa_\ell \tau} - \left(G + \left(\frac{\kappa_\ell \alpha_Q}{\kappa_\ell - \kappa^Q}\right) + \frac{\sigma_y}{\kappa^Q}\right) e^{-\kappa^Q \tau} + \frac{\sigma_y}{\kappa^Q} \quad (113) \\
H(\tau) &= (g - r)\tau + (\nu\sigma_y - \kappa^Q \overline{\theta}^Q) \int_0^\tau J(s)\, ds + (\kappa_\ell \overline{\ell} - \sigma_y \sigma_\ell) \int_0^\tau I(s) ds \\
&\quad + \frac{\nu^2}{2} \int_0^\tau J^2(s) ds + \frac{\sigma_\ell^2}{2} \int_0^\tau I^2(s) ds - \nu\sigma_\ell \int_0^\tau I(s)J(s) ds. \quad (114)
\end{aligned}
$$

All integrals can be obtained in closed-form but not reported for the sake of brevity. Finally $V^T(t, \ell_t, \theta_t, y_t)$ can be obtained in closed-form from the expression for $W^T(t)$ and $P(t)$ using equation (107) above.

Applying Itô's lemma to $V^T(t)$ we find:

$$
dV^T(t) = rV^T(t)\, dt + \left(V_y^T \sigma_y - V_\theta^T \nu - V_\ell^T(\nu G + \sigma_y)\right) dz_t. \quad (115)
$$

Thus

$$
\Omega(\tau, \ell, \theta, y) = \frac{(V_y^T \sigma_y - V_\theta^T \nu - V_\ell^T(\nu G + \sigma_y))}{V^T(t)}. \quad (116)
$$

7.10 Guaranteeing positive dividends and risk-free debt: Asset sales

Our specified dynamics for log-leverage, and in turn the endogenously implied dividend dynamics, have two shortcomings. First, log-leverage can become positive, which is inconsistent with the assumption that debt is risk-free if there is limited liability for equityholders. Second, our process does not guarantee that dividends remain positive at all times. While negative dividends (i.e., total payouts are cash-dividends plus share repurchases minus share issuances) are possible, empirically this is rarely observed at the aggregate level. Thus, to make the model more realistic, we extend it by introducing asset sales that (a) maintain a strictly negative log-leverage ratio, and (b) maintain positive dividends. We find that for the parameter vector used in the benchmark case, the differences between the closed-form presented in the text and solution with asset sales are small. In our simulation, we find that guaranteeing positive dividends seems to be sufficient to rule out positive log-leverage ratios, therefore we only discuss in detail how to guarantee positive dividends.[21] The approach to guarantee negative log-leverage is similar.

In order to guarantee positive dividends, we introduce a reflecting boundary near zero dividends. In particular, we assume that when the instantaneous dividend approaches zero, the firm sells assets in order to repurchase some outstanding debt, in turn lowering the leverage ratio. Thus, for example, if the current dividend is equal to zero, and the current state vector is (y_t, θ_t, ℓ_t), it follows that the current enterprise value and current value of debt is

$$\begin{aligned} P(y_t, \theta_t) &= e^{y_t + F - G\theta_t} \\ B(y_t, \theta_t, \ell_t) &= e^{\ell_t + y_t + F - G\theta_t}. \end{aligned} \tag{117}$$

Let us assume that the firm sells assets so that the new enterprise value is

$$\begin{aligned} P(y_t - \Delta y_t, \theta_t) &= e^{y_t - \Delta y_t + F - G\theta_t} \\ B(y_t - \Delta y_t, \theta_t, \ell_t - \Delta \ell_t) &= e^{\ell_t - \Delta \ell_t + y_t - \Delta y_t + F - G\theta_t}. \end{aligned} \tag{118}$$

All funds raised by the asset sale are used to reduce the outstanding debt, implying that

$$P(y_t - \Delta y_t, \theta_t) - P(y_t, \theta_t) = B(y_t - \Delta y_t, \theta_t, \ell_t - \Delta \ell_t) - B(y_t, \theta_t, \ell_t). \tag{119}$$

This implies that for a given $\Delta \ell$, the value of Δy is determined via

$$\Delta y = -\log\left(\frac{1 - e^{\ell_t}}{1 - e^{\ell_t - \Delta \ell}}\right). \tag{120}$$

[21] While intuitively, guaranteeing positive dividends would seem to be sufficient to insure that equity always has positive value, and therefore log-leverage would be negative, we were not able to prove this rigorously, due to the endogenous nature of our dividends, which are a function of the leverage dynamics, and depend on the assumption that debt is risk-free. It is thus simpler to impose that log-leverage remains negative to guarantee that this assumption is verified.

Hence, to maintain positive dividends, we modify the state vector dynamics for (y_t, x_t, ℓ_t) from equations (7), (8), (26) to:

$$dy = \left(g + x - \frac{\sigma_y^2}{2}\right) dt + \sigma_y \, dz_1 + \Delta y. \quad (121)$$

$$dx = -\kappa_x x \, dt + \sigma_{x1} \, dz_1 + \sigma_{x2} \, dz_2.$$

$$d\ell_t = \kappa_\ell \left(\bar{\ell}^P + \alpha x - \ell\right) dt - \left(G\sigma_{x1} + \sigma_y\right) dz_1 - G\sigma_{x2} \, dz_2 + \Delta\ell. \quad (122)$$

In our numerical implementation, we exogenously choose the the amount $\Delta\ell$ by which to 'reflect' the process. For the numerical results below, we choose $\Delta\ell = 10.0\sigma_\ell\sqrt{dt}$, where σ_ℓ is the diffusion volatility of the leverage process and dt is the numerical discretization time step.

In Figure 13 below we compare the term structures of dividend strip expected returns and volatilities as reported in the main text (for the CC economy), and compare them to the same quantities obtained via simulations for the model that implements asset sales to keep dividends positive and maintain leverage ratios below one. We see from the figure that the numerical solution are very similar to the closed-form solution, indicating that, for our parameter choices, the probability of dividends going negative does not significantly affect our results. In fact, the simulation also differs from the closed-form solutions because, to be closer to our empirical approach, we also time-aggregate dividends over one year, as we explain next. However, both effects, time aggregation and asset sales, appear to have minimal effects on our results relative to the closed-form solution reported in the text.

7.11 Time Aggregation of Dividends

In the theory part we present the term structure of dividend strips defined as claims to the instantaneous dividends, and estimated at the long-run mean of the state vector. This allows us to make use of the closed form solutions for all the prices and moments of returns. Instead, in empirical work typically, we consider dividends aggregated over one full year.

For example, aggregating over one year, the computation of expected dividend growth

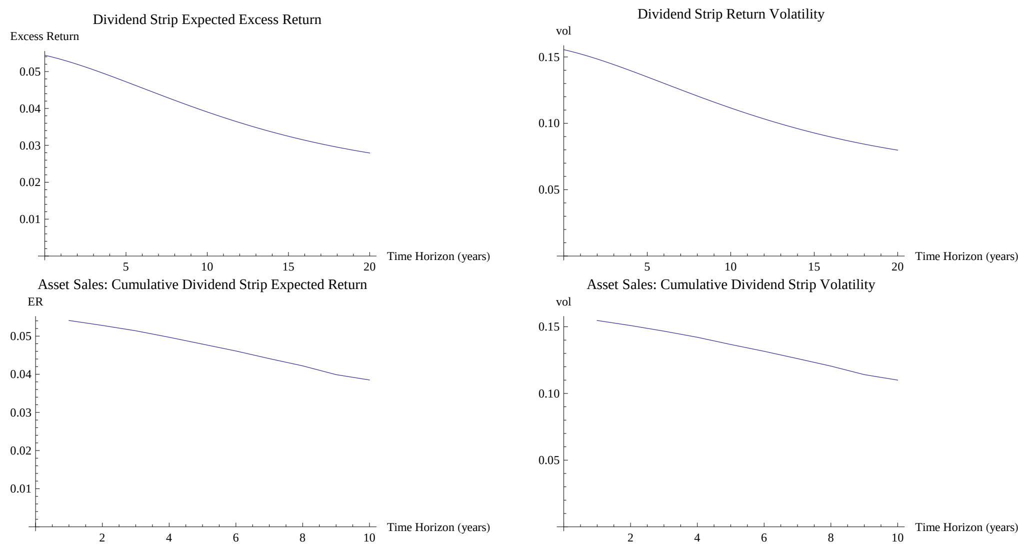

Figure 13: Term structure of Dividend Strip expected returns (equation (63)) and volatilities (equation (64)) for the CC economy versus similar term structure for claims in a model where asset sales are implemented whenever leverage reaches one to maintain positive dividends and guarantee that leverage cannot go above unity. Parameters are set as in table 8 and 10.

and variance ratio becomes:

$$\begin{aligned} g_{D,T} &\equiv \left(\frac{1}{T}\right)\log\left(\frac{\mathrm{E}_0\left[D_{T+1}-D_T\right]}{\mathrm{E}_0\left[D_1-D_0\right]}\right) \\ \sigma^2_{D,T} &\equiv \left(\frac{1}{T}\right)\log\left[\frac{\mathrm{E}_0\left[\left(D_{T+1}-D_T\right)^2\right]}{\left(\mathrm{E}_0\left[\left(D_{T+1}-D_T\right)\right]\right)^2}\right]. \end{aligned} \tag{123}$$

Also, the dividend strip claims considered by BBK are typically claims to the sum of all dividends paid out by the index over the year. In this section we compare the effects of aggregating dividends over one year for various statistics presented in the paper, and show that the difference is not economically significant. In Figure 13 above we compare the term structures of dividend strip expected returns and volatilities as reported in the main text, which refer to claims to future instantaneous dividend flows, to the expected returns and volatilities on claims to the **cumulative** dividends aggregated over one year before maturity (so the payout at the maturity T of the dividend strip is $\int_{T-1}^{T}\mathcal{D}(s)\,ds$). We see from the figure that the slopes on the cumulative dividend claims are almost indistinguishable from the claims to the instantaneous dividend strips (note that the cumulative dividend picture also differs from the main text model, because it imposes that dividends remain positive at all times, as explained in the previous section, but both effects have almost no impact on the predictions of the model).

7.12 Accounting for Investment

Our benchmark framework specifies an exogenous EBIT process and combines that with a capital structure policy that generates stationary leverage ratios. In this section, we demonstrate that modifying this framework to account for debt-financed investment still generates downward sloping variance ratios and term structures of dividend strips (in fact, even more downward sloping). This robustness check also has the added advantage that it allows us to better match both the historical one-year volatility of changes in (log) leverage $\sigma(e) = 0.12$ and the conditional correlation between changes in leverage and changes in x in the BY model that we estimated in the empirical section.

In order to capture debt-financed investment as parsimoniously as possible, we will assume all investments are zero net present value projects. In particular, we model investments (disinvestments) as a simultaneous increase (decrease) in both EBIT y and leverage ℓ. Intuitively, we are modeling an investment that is paid for by debt (hence, an increase in debt, and therefore leverage), and immediately generates a permanent increase in EBIT. Specifically, recall that enterprise value and the current value of debt are

$$\begin{aligned} P(y_t, x_t) &\approx e^{y_t + F + Gx_t} \\ B(y_t, x_t, \ell_t) &= e^{\ell_t + y_t + F + Gx_t}. \end{aligned} \tag{124}$$

Now, consider an investment opportunity that will permanently impact EBIT by Δy. In order to pay for this increase, debt is issued, which in turn increases the level of leverage by $\Delta\ell$. It follows that the new enterprise value and outstanding debt levels become:

$$\begin{aligned} P(y_t + \Delta y_t, x_t) &= e^{y_t + \Delta y_t + F + Gx_t} \\ B(y_t + \Delta y_t, x_t, \ell_t + \Delta\ell_t) &= e^{\ell_t + \Delta\ell_t + y_t + \Delta y_t + F + Gx_t}. \end{aligned} \tag{125}$$

For reasons of parsimony, we consider only investments (or disinvestments) that are zero NPV. As such, enterprise value changes by the same amount that outstanding debt value changes (leaving equity unchanged). Hence,

$$P(y_t + \Delta y_t, x_t) - P(y_t, x_t) = B(y_t + \Delta y_t, x_t, \ell_t + \Delta\ell_t) - B(y_t, x_t, \ell_t) \tag{126}$$

This implies that for a given $\Delta\ell$, the value of Δy is determined via

$$\begin{aligned}\Delta y &= \log\left(\frac{1-e^{\ell_t}}{1-e^{\ell_t+\Delta\ell}}\right) \\ &\overset{\mathcal{O}(\Delta\ell)^2}{=} \left(\frac{e^{\ell}}{1-e^{\ell}}\right)\Delta\ell + \frac{1}{2}\left(\Delta\ell\right)^2\left[\left(\frac{e^{\ell}}{1-e^{\ell}}\right)^2 + \left(\frac{e^{\ell}}{1-e^{\ell}}\right)\right]. \qquad (127)\end{aligned}$$

Hence, to account for investments, we modify state vector dynamics for (y_t, x_t, ℓ_t) from equations (7), (8), (26) to:

$$dy = \left(g + x - \frac{\sigma_y^2}{2}\right) dt + \sigma_y\, dz_1 + \Delta y. \qquad (128)$$

$$dx = -\kappa_x x\, dt + \sigma_{x1}\, dz_1 + \sigma_{x2}\, dz_2.$$

$$d\ell_t = \kappa_\ell\left(\bar{\ell}^P + \alpha x - \ell\right) dt - \left(G\sigma_{x1} + \sigma_y\right) dz_1 - G\sigma_{x2}\, dz_2 + \Delta\ell. \qquad (129)$$

We plot the results in Figure (14) for the case $\Delta\ell = 0.0\, dz_3$ and $\Delta\ell = 0.10\, dz_3$. All other parameters are the same as in the benchmark (although we impose positive dividends as discussed in the previous section.) We choose this parameter value to match the empirical one year volatility of log leverage changes. Interestingly, we find that the slope of the term structure of dividend strip volatilities becomes even more steep. Intuitively this is because long run dividend volatility is cointegrated with EBIT volatility, and the investment process we consider barely impacts long horizon EBIT volatility. However, the addition of another source of risk increases overall volatility. Stationary leverage ratios tend to shift this risk to the shorter horizons.

7.13 The basis between dividend strips extracted from options and dividend swaps

Given the discussion in the literature about dividend swaps (e.g., BBK), here we point out that dividend strip prices extracted from index options and dividend strip prices obtained directly from dividend swap strikes, should not, even in theory, be equal. Indeed, the former are a claim to the dividends as they get paid over the maturity of the option contract, whereas the latter are a claim to the *undiscounted* sum of dividends paid out over the maturity of the swap. Especially for long maturities this can lead to a substantial difference. And, of course, the sum of all dividend swap strikes should not be equal, in equilibrium, to the stock value. More specifically, the dividend strip

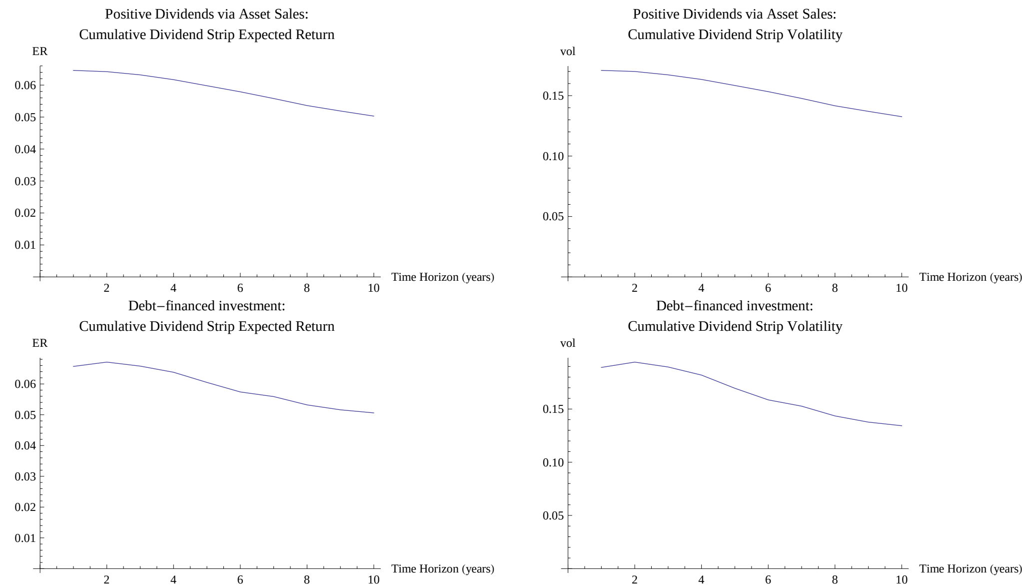

Figure 14: Term structure of Dividend Strip expected returns (equation (63)) and volatilities (equation (64)) for the BY economy. The upper panel presents the results for the case, where dividends are time-aggregated and we impose that dividends remain positive and log-leverage negative at all times, as explained in section 7.10. The bottom panel, presents the case where in addition debt may be issued to finance continuously arriving investment projects. Parameters are set as in table 4 and 6.

extracted from options or futures, as the difference between the spot price $V(t)$ and the discounted Futures prices $F^T(t) = \mathrm{E}_t^Q[V(T)]$ is, by absence of arbitrage, equal to:

$$V(t) - e^{-r(T-t)}F^T(t) = \int_t^T V^s(t)ds \equiv \mathrm{E}^Q[\int_t^T e^{-r(s-t)}\mathcal{D}(s)ds]. \tag{130}$$

Instead, dividend swaps pay the **undiscounted** sum of all dividends paid out over the life of the swap, minus the swap strike (say K_t^T) at maturity. Thus the arbitrage free swap strike satisfies:

$$K_t^T = \mathrm{E}^Q[\int_t^T \mathcal{D}(s)ds]. \tag{131}$$

In turn we see that the option implied (cumulative) dividend strips should always be lower than the dividend swap strike prices. Figure 15 shows that the difference becomes significant at longer horizons.

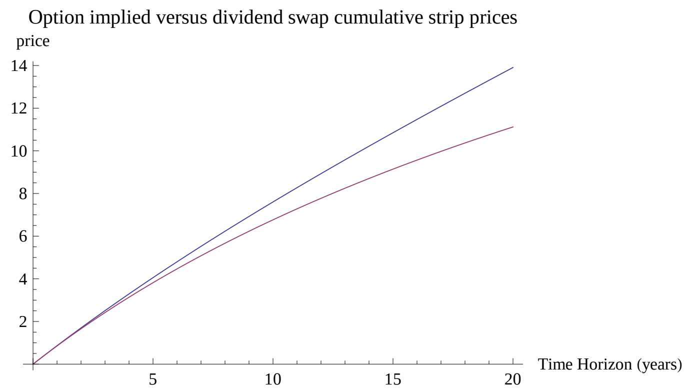

Figure 15: Term structure of dividend Strip prices extracted from options (or futures) compared to the fair dividend swap strike. Because the latter omits discounting it is always higher than the former. Parameters are set as in Table 8 and 10.

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Maturity (T)	1	2	4	6	8	10	15	20	Diff 1-10	Diff 1-20
Dividend Definition 1: Market-invested dividends										
$\sigma_{D,1}^T$	15.04	14.59	12.62	10.92	9.13	8.07	7.95	7.45	6.97	7.59
$\sigma_{D,2}^T$	14.74	14.06	11.82	9.96	8.21	7.14	6.70	5.99	7.60	8.87
VR	1.00	0.97	0.72	0.54	0.42	0.32	0.31	0.32	–	–
s.e.(VR)	–	0.09	0.17	0.23	0.28	0.31	0.37	0.48	–	–
p-value	–	0.74	0.10	0.04	0.04	0.03	0.06	0.15	–	–
Dividend Definition 2: Cash-invested dividends										
$\sigma_{D,1}^T$	13.27	13.88	12.44	10.63	9.21	8.00	7.91	7.10	5.27	6.17
$\sigma_{D,2}^T$	12.99	13.36	11.66	9.71	8.27	7.07	6.67	5.73	5.92	7.26
VR	1.00	1.10	0.87	0.64	0.51	0.38	0.40	0.35	–	–
s.e.(VR)	–	0.10	0.21	0.28	0.34	0.38	0.45	0.53	–	–
p-value	–	0.30	0.54	0.19	0.15	0.11	0.18	0.22	–	–
Dividend Definition 3: With equity repurchases										
$\sigma_{D,1}^T$	13.71	14.14	13.60	10.43	7.43	7.24	6.96	5.44	6.47	8.27
$\sigma_{D,2}^T$	13.40	13.62	12.73	9.58	6.70	6.36	5.80	4.28	7.04	9.12
VR	1.00	1.08	1.02	0.54	0.32	0.31	0.36	0.21	–	–
s.e.(VR)	–	0.13	0.23	0.30	0.35	0.39	0.49	0.54	–	–
p-value	–	0.53	0.92	0.12	0.05	0.08	0.19	0.14	–	–

Table 2: Dividend variance ratio test demonstrates that dividend volatility drops significantly with horizon in the data. We can reject the hypothesis that dividends follow a random walk. The data for dividend definitions 1 and 2 are annual from 1873 to 2008, and the data for dividend definition 3 are annual from 1927 to 2007.

		Parameter Estimates			Tests		
Spec.		a	b_1	b_2	R^2	$\chi^2(b=0)$	p-val(χ^2)
1	Slope	1.18	−0.04		−0.58	0.19	0.66
	p-val	0.38	0.67				
2	Slope	1.32	−0.05	−0.21	3.15	6.29	0.04
	p-val	0.32	0.61	0.01			

Table 3: Predictability regressions of real dividend growth on lagged values of dividend growth demonstrates that dividend growth is not a random walk. Data are annual from 1872 to 2008.

g	σ_y	κ_x	σ_{x1}	σ_{x2}	r	θ_1	θ_2
0.018	0.025	0.15	0.0	0.015	0.025	0.0	0.4

Table 4: Calibrated Parameters for the BY model.

F	G	$(\mu^P - r)_{BY}$	σ^P_{BY}
3.343	5.332	0.031	0.082

Table 5: Enterprise value expected return and volatility for the BY model with parameters set in equation 4.

$e^{\bar{\ell}}$	α	κ_ℓ
0.35	2.0	0.11

Table 6: Parameters for the log-leverage process in the BY model.

$(\mu^V - r)_{BY}$	σ^V_{BY}
0.048	0.126

Table 7: Stock return expected return and volatility for the modified BY model. Parameters are given in Table 4.

g	σ_y	$\bar{\theta}$	κ_θ	ν	r
0.018	0.03	0.35	0.2	0.1	0.025

Table 8: Calibrated Parameters for the CC model.

F	G	$(\mu^P - r)_{CC}$	σ^P_{CC}
3.780	0.241	0.019	0.054

Table 9: Enterprise value expected return and volatility for the CC model with parameters set in equation 4.

$e^{\bar{\ell}}$	α	κ_ℓ
.47	-0.5	0.11

Table 10: Parameters for the log-leverage process in the CC Model. (Note that the long run mean of the log-leverage process ℓ_t is $\bar{\ell} + \alpha\bar{\theta} = \log(0.4)$.)

$(\mu^V - r)_{CC}$	σ^V_{CC}	Sh^V_{CC}
0.036	0.103	.35

Table 11: Stock return expected return and volatility for the CC model with parameters set in Table 8.