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THE PRODUCTION OF HUMAN CAPITAL: ENDOWMENTS, INVESTMENTS AND FERTILITY

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ABSTRACT

We study how endowments, investments and fertility interact to produce human capital in childhood. We begin by providing empirical support for two key features of existing models of human capital: that investments and existing human capital are complements in the production of later human capital (dynamic complementarity) and that parents invest more in children with higher endowments due to the complementarity between endowments and investments (static complementarity). For the former, we exploit an exogenous source of investment, the launch of Head Start in 1966, and estimate greater gains from preschool in the IQ of those with the highest stocks of early human capital, consistent with dynamic complementarity. For the latter, we are able to overcome the potential endogeneity and measurement error associated with traditional measures of endowment based on health at birth. When we do, we find that parents invest more in highly endowed children. Moreover, we find that the degree of reinforcement increases with family size. Thus, an increase in quantity leads not only to a decline in average quality (the quantity-quality tradeoff) but to an increase in the variation in quality, due to both greater variation in endowments (from more children) and greater reinforcing investments. These findings can be explained by extending the quantity-quality trade-off model to include heterogeneous child endowments and parental preferences that feature complementarity between quality and quantity and moderate aversion to inequality in child human capital within the household.

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1 Introduction

Growing evidence points to the important role which conditions in early childhood play in determining adult human capital and earnings. Experimental evidence shows that enriching the early environments of disadvantaged children has large social and economic benefits, both in the U.S. context (see, e.g., Olds et al., 2002; Campbell et al., 2008; Heckman et al., 2010, Conti et. al, 2012) as well as in the context of developing countries (Grantham-McGregor et al., 1994; Behrman et al., 2009). Measures of human capital at ages 6-8 can explain 12 (20) percent of the variation in adult educational attainment (wages) (Currie and Thomas, 1999; McLeod and Kaiser, 2004).

Why would early conditions matter so much? Existing models of the production of human capital provide some explanation. In a seminal paper, Becker and Tomes (1986) present a model of the intergenerational transmission of human and financial capital. In their model, childhood lasts one period and the production function of human capital exhibits complementarity between parental investments and the child's endowment. This is referred to as "static complementarity" and generates an incentive for parents to invest more in the highly endowed (referred to as "reinforcing investment"). More recently, Cunha and Heckman (2007) develop a multi-period model of human capital formation, central to which is the notion that investments during different periods of childhood are complements in the production of human capital. This is referred to as dynamic complementarity and implies that existing stocks of human capital of children complement parental investments (and also that the earlier the investment, the greater the potential returns.) While the notions of static and dynamic complementarity are widely accepted in the literature, it is difficult to establish direct empirical support for them due to challenging identification issues. In this paper we examine the production of human capital in early childhood. In so doing, we both provide empirical support for these two key features of existing models of human capital production and extend the analysis to incorporate fertility, which has largely been treated separately.

We first provide evidence of dynamic complementarity. Identification in this context is difficult due to the endogeneity of investments and the lack of data on human capital at multiple points over childhood. We are able to exploit exogenous variation in investment in the form of preschool enrollment across children in the same household for identification. Specifically, our data consist of a low income sample of siblings that includes multiple measures of human capital over several periods of childhood and spans the launch of Head Start in 1966. We use this exogenous increase in preschool availability (our measure of investment) to identify the impact of investment on child IQ and cognitive achievement as well as any complementarities with stocks of human capital in a family fixed effect/sibling comparison framework. We find that preschool enrollment has a positive and significant impact on four-year IQ for all, but that the impact is largest for those with higher early stocks of cognitive human capital as measured by cognitive development at eight months of age. By seven years, the effect of preschool on IQ and achievement has faded for all but highly developed infants for whom the impact persists.

We then test for static complementarity not directly, but by testing for the implication of static complementarity: reinforcing investment. When the production function exhibits static complementarity, an exogenous increase in the endowment of a child increases the marginal returns to investing in that child's human capital, generating an incentive for parents to invest more in highly endowed children. Thus, if parental investments are an increasing function of endowments, then it must be the case that the production function exhibits static complementarity.

To examine this empirically, we introduce two innovations to the analysis. The first addresses the endogeneity and measurement error that often plague empirical analyses of the relationship between endowments and parental investments that rely on a single measure of newborn health as the measure of endowment. The endogeneity of endowment as measured by newborn health arises from the fact that newborn health is comprised of both the true endowment and prenatal investments, the latter of which are typically unobserved and likely correlated with postnatal investments, generating a significant source of omitted variable bias. In our analysis, we can control for prenatal investments, thereby eliminating this source of bias. To address measurement error, we factor analyze the orthogonal components to prenatal investments of multiple measures of health at birth. The second innovation is our introduction of a new measure of parental investment that is an assessment of the quality of the interaction of the mother and child as rated by a psychologist. We present evidence that parents invest more in children with higher endowments, consistent with strong static complementarity in the production of human capital.

Having provided empirical support for two key features of existing models, we then extend the analysis to consider how fertility affects the relationship between endowments and investments. We find that investments are more reinforcing in large families. This finding, in combination with our previous findings with respect to complementarity in the production function, implies that the variability of child quality (within family) increases in large families. This is exactly what we find. More specifically, our data show that the highest quality child in a large family is comparable to the highest quality child in a small family. In contrast, the lowest quality child in a large family has much lower human capital than the lowest quality child in a small family. Importantly, these relationships cannot be explained by birth order, though we do find birth order effects, and are found in other datasets as well.

Finally, we show that our finding that higher fertility is realted to greater reinforcing investment and an increase in the variation in child quality within a family can be rationalized by a quantity-quality model as in Becker and Lewis (1973), which features heterogeneity in endowments as in Becker and Tomes (1976) and parental aversion to inequality as in Behrman, Pollak, and Taubman (1982). For the model to be able to qualitatively replicate our findings, it is necessary for parental preferences to exhibit (1) complementarity between the quantity and quality of children and (2) a moderate aversion to inequality. Complementarity explains the divergent profile of maximum and minimum human capital as a function of fertility: as fertility increases, it is cheaper to increase quality by investing more in children with higher endowments. The moderate aversion to inequality represents a compromise between two forces working in opposite directions. On the one hand, if parents have great aversion to inequality, then parental preferences would overcome the complementarities in the production function and parental investments would compensate, not reinforce, differences in human capital and endowments. On the other hand, if parents have no concerns about inequality (or are inequality loving), then the combination of complementarities in the technology of skill formation and in the utility function would lead maximum human capital stock to increase strongly as fertility increases.

Our theoretical and empirical results have important implications for our understanding of the production of human capital in childhood and how initial levels of human capital, investments and fertility interact to affect not only average levels of human capital but its distribution within a family. In addition, by providing new estimates of the impact of Head Start on multiple measures of cognitive ability and achievement that exploit the exogenous variation in Head Start availability within the family, our results also contribute to the growing literature on the impact of Head Start and other high-quality interventions in early childhood (e.g., Currie and Thomas, 1995; Garces, Currie and Thomas, 2002; Ludwig and Miller, 2007; Behrman et. al., 2009, Heckman et. al, 2010).

The rest of the paper is organized in five sections. In Section II, we empirically test the key features of the models of human capital production, establishing that preschool investments and early human capital are complements in the production of late human capital (dynamic complementarity). In Section III, we investigate the nature of the relationship between parental investments and endowments and find that they are reinforcing (static complementarity). We then estimate the extent to which fertility influences the allocation of investments and human capital within a household. In Section IV, we present a simple model of fertility and investments in children and derive sufficient conditions to replicate the qualitative features of the data. In the last section, we conclude.

2 Are Investments and Early Human Capital Complements in the Production of Later Human Capital?

2.1 Background

Let $h_{i,t}$ denote the human capital of child i at age t. Let $x_{i,t}$ represent parental investment in child i's human capital at age t. The variable ε_i is the child's endowment that, unlike $h_{i,t}$, is time invariant and unaffected by parental investments. For example, ε_i captures the child's genes while $h_{i,t}$ is the expression of genes (see, e.g., Caspi et al., 2002). Consider the following production of human capital:

$$h_{i,t} = f\left(h_{i,t-1}, x_{i,t}, \varepsilon_i\right)$$

where f is increasing, concave, and twice differentiable in each of its arguments. By static complementarity, we mean that $\frac{\partial^2 f}{\partial x_{i,t}\partial \varepsilon_i} > 0$ for all t. This is the nature of the complementarity in Becker and Tomes (1986). By dynamic complementarity, we mean that $\frac{\partial^2 f}{\partial x_{i,t}\partial h_{i,t-1}} > 0$. As noted by Cunha and Heckman (2007), dynamic complementarity, unlike static complementarity, implies that investments in one period increase the marginal productivity of investments in a later period:

$$\frac{\partial^2 h_{i,t}}{\partial x_{i,t}\partial x_{i,t-1}} = \frac{\partial^2 h_{i,t}}{\partial x_{i,t}\partial h_{i,t-1}}\frac{\partial h_{i,t-1}}{\partial x_{i,t-1}} > 0 \Leftrightarrow \frac{\partial^2 f}{\partial x_{i,t}\partial h_{i,t-1}} > 0.$$

There is very little work showing direct evidence in favor of dynamic complementarity. Cunha, Heckman, and Schennach (2010), who use the CNLSY/79 data, find estimates for the elasticity of substitution between cognitive skills and investments that range between 0.562 and 0.847, consistent with complementarity between early human capital and investments. Heckman et al. (2010) find that the Perry Preschool Program had the largest effects on cognitive achievement among those at the top of the distribution. They argue that the stronger effects at the top of the distribution are consistent with complementarity of early human capital and investments.

The present study differs from previous work in that we have measures of early and late human capital and we exploit a plausibly exogenous measure of investment – preschool enrollment as affected by the creation of Head Start, a fully subsidized preschool program established in 1965-66 for low-income children. Simply by virtue of being born after 1962, some of the children in our sample had access to a fully subsidized preschool program, while their siblings, by virtue of being born prior to 1962, did not. Moreover, we have multiple measures of human capital taken for each child at birth, eight months, four years and seven years.

2.2 Data

The National Collaborative Perinatal Project (NCPP) contains comprehensive information on maternal and paternal characteristics, prenatal conditions, birth outcomes and follow-up information through age seven for a cohort of roughly 59,000 births between 1959 and 1965 (of which 17,000 are siblings) in 12 sites (located in 11 central cities) throughout the US. Mothers were recruited for participation in the NCPP primarily through public clinics associated with academic medical centers. As such, they are characterized by greater poverty and less education than the general population at the time. Sample characteristics are presented in Appendix Table 1. Follow-up information on the children was collected at ages eight months, one year, four years and seven years and includes the results of extensive physical, pathological, psychological, and neurological examinations.

At birth, the measures of human capital available in the data include birth weight, head circumference, body length, weeks of gestation, and whether the doctor confirms or suspects a neurological abnormality in the neonate.

At eight months of age, three measures of human capital are taken: the mental, motor, and social Bayley scores of development. The eight-month mental development score is our preferred measure of early human capital as it is the most closely related to our later measures of human capital (IQ, math and reading test scores). To generate this score, the examiner presents a series of test materials to the child and observes the child's responses and behaviors and evaluates her along three scales (mental, motor and social). [1] In our sample, the scores in the mental scale vary from 0 to 99, with an average of 79 and a standard deviation of 6. Within families, the average difference is 4, or two-thirds of the cross-sectional standard deviation (Figure 1).

Later measures of child cognitive human capital are collected at ages four (IQ) and seven (IQ, reading and math achievement). There is considerable variation in these measures, both across and within families. For example, the average seven-year IQ is 96 with a standard deviation of 15. Within families, the average difference between siblings is 12 points (Figure 2).

To support our use of the eight-month Bayley as a measure of early human capital, we compare its ability to predict future cognitive ability/achievement with that of birth weight, which has been used extensively in the literature as a measure of early human capital (e.g., Behrman and Rosenzweig, 2004; Datar et al., 2010). We find that the eight-month mental scale is more predictive of nearly every measure of cognitive human capital at later ages than is birth weight. In both OLS and family-fixed effect settings, the eight-month mental scale is either similar to or more predictive of any cognitive delay at age one, speech delay at age three, IQ at ages four and seven and math achievement at age seven (Appendix Table 2). We attribute these findings to the fact that the mental scale is a more precise measure of cognitive human capital than birth weight, which is a more general measure of human capital.

2.3 Empirical Strategy

To test the hypothesis of dynamic complementarity, we estimate the following production function:

$$h_{2,i,j} = \gamma_1 h_{1,i,j} I_{2,i,j} + \gamma_2 I_{2,i,j} + \gamma_3 h_{1,i,j} + \gamma_4 X_{i,j} + u_j + \nu_{i,j}$$

where late human capital of child i in family j ($h_{2,i,j}$) is measured as IQ at age four, or IQ, reading and math achievement at age seven; investment ($I_{2,i,j}$) is preschool enrollment at age four, and child early human capital ($h_{1,i,j}$) is measured by the eight-month mental Bayley test score. The main effects of investment and early human capital are included, as is the interaction term $h_{1,i,j}I_{2,i,j}$, which captures the presence, if any, of dynamic complementarity in early human capital and investments in the production of late human capital. Also included are u_j, a family-specific fixed effect, and $X_{i,j}$, a vector of characteristics that varies across siblings within a family and includes child gender, birth

[1] The mental scale evaluates several types of abilities: sensory/perceptual acuities, discriminations, and response; acquisition of object constancy; memory learning and problem solving; vocalization and beginning of verbal communication; basis of abstract thinking; habituation; mental mapping; complex language; and mathematical concept formation (see Appendix A for the individual items). The motor development scale assesses muscle control (control of the body) and large and fine motor coordination.

order, maternal age at birth, income at birth and marital status at time of birth. The inclusion of the family fixed effect allows us to control for any unobserved differences across families that might be correlated with both children's early human capital and investment (e.g., in our data, more educated mothers are more likely to enroll their children in preschool and their children also have higher IQs).

In general, it is not straightforward to obtain consistent estimates of γ_1. Investment is likely endogenous and may, for example, be correlated with parental characteristics as well as child-specific characteristics that the parents observe about their children, but the psychologist and the researcher do not. We argue that variation in our measure of investment (preschool enrollment at age four) is likely exogenous as it appears to be driven by the launch of Head Start as an eight-week summer program in 1965, which was then expanded in 1966 to a part-day nine-month program.[2] In our sample, preschool enrollment increases significantly and discontinuously among four year olds in 1966 and continues to increase slightly each year through 1970, the end of our study period (Figure 3). The sudden increase in preschool enrollment observed (from 7 to 12.5 percentage points, an increase of 73 percent, between 1965 and 1966), combined with the fact that our sample is a low-income urban sample, suggests that the arguably exogenous launch of Head Start in 1965/1966 is largely responsible for this growth in preschool enrollment.[3]

Since our sample includes siblings born to the same family before and after 1962 (four years before the start of full year Head Start in 1966), within a given family, some children had no access to Head Start at age four, while others, by virtue of being born after 1962, did. This, we argue, provides the exogenous variation in investment within family that we need for identification. To control for the fact that access to Head Start increases with birth order, we control for birth order and its interaction with preschool enrollment in the regression as well.

2.4 Results

2.4.1 Evidence of the Exogeneity of Preschool Enrollment

Before presenting the results of estimating equation (1), we present two pieces of evidence to support our contention that preschool enrollment is exogenous in this sample. First, we link preschool enrollment in our sample with local (county) levels of Head Start funding by regressing an indicator for preschool enrollment at age four in each year (1963-1970) on county-level funding of Head Start

[2] In 1960, there were 3.97 million four year olds (the primary age of those served by Head Start). By 1968 Head Start served 733,000 children in its summer program and 212,000 children in its full-year program.

[3] Moreover, evidence presented by Ludwig and Miller (2007) shows that although Head Start was launched in 1966, it continued to expand in the years after (owing largely to continual recruitment of providers in the early years), which would explain why the trend in preschool enrollment observed in our data jumps discontinuously in 1966 but then continues to increase in the years immediately after. For example, in 1966, only $8 million was spent on Head Start's nine-month program (serving 20,000 children), but by 1968, $239 million had been allocated to serve 212,000 children.

in 1968, the only year with credible data (Table 1).[4] We find that Head Start spending per poor person in the county of residence in 1968 does not predict preschool enrollment in our sample in 1963, 64 or 65, but that it does predict preschool enrollment in 1966 – 1970 (Table 1, Panel A), though many of the estimates are imprecise.[5] However, the results are larger and more precise for those most likely to be eligible for Head Start: when we restrict our sample to mothers with a high school diploma or less (90% of our sample), the estimated relationship between local Head Start spending and preschool enrollment increases and becomes significant (Table 1, Panel B). Finally, we include maternal fixed effects in a regression of preschool enrollment on a variable that is the interaction between local Head Start spending (in 1968) and an indicator equal to one in all years after Head Start was established.[6] We continue to find a strong relationship between local Head Start spending and the probability of preschool enrollment within families (Table 1, Panel C).

A second piece of evidence of the exogeneity of preschool enrollment is that preschool attendance is uncorrelated with early human capital. In the cross section (Table 2, Panel A) and within family (Table 2, Panel B), there is no significant relationship between preschool attendance and any of our measures of early human capital (birth weight, gestation, eight-month mental Bayley score and social development score, abnormal language reception or expression at age 3), consistent with exogenous preschool enrollment resulting from the creation of Head Start.[7]

2.4.2 Preschool Attendance and Human Capital at Four Years

Estimates of equation 1 including maternal fixed effects show that (1) preschool enrollment is highly productive of four-year IQ ($\gamma_2 > 0$) and that (2) preschool enrollment and early human capital are indeed complements in the production of four-year IQ ($\gamma_1 > 0$). Specifically, a child who attends preschool has an IQ at age four that is 16 percent of a standard deviation higher than a sibling within the same family who did not go to preschool (Table 3A). If that child also had a high level of early human capital, then the effect of preschool attendance on four-year IQ would be even larger.

[4]These data on Head Start spending at the county level in 1968 were generously provided by Jens Ludwig and Doug Miller. For the earliest years of the program, they found that only county funding levels for 1968 and 1972 were credible, which is why we use only the 1968 data (1972 is beyond our time frame). While Ludwig and Miller calculate per capita Head Start funding for their analysis, because our sample is a low-income sample, we calculate spending per poor person in the county.

[5]Head Start funding in 1968 for these 11 cities ranges from \$4 to \$29 per poor person (in 1968 dollars) and preschool enrollment in 1968 ranges from 6 to 15 percentage points among mothers with no more than a high school diploma in our sample. The results from our analysis suggest that a doubling of Head Start funding (across cities) increases the probability of enrollment in preschool by 50%. Ludwig and Miller (2007) find that doubling Head Start funding in low-income counties increases Head Start enrollment by 100%. However, our results are not directly comparable. Ludwig and Miller focus on differences across low-income counties, while our sample is drawn from moderate-income urban counties where we might expect the impact to differ.

[6]In other words, this is equal to zero in all years prior to 1966 and equal to local Head Start spending in all years after 1966. The main term of local Head Start spending in 1968 is subsumed by the maternal fixed effect. We do not include year dummies (which reduces precision), but rather a quadratic time trend.

[7]Maternal characteristics (education in particular) are, however, correlated with preschool enrollment in the cross section, necessitating the need to include a maternal fixed effect. Without a maternal fixed effect, there is a large and significant impact of preschool enrollment on child IQ at age four for all children.

For example, evaluated at the average within family difference in eight-month Bayley scores, a sibling with a higher Bayley score who attended preschool would have a four year IQ that was 33 percent of a standard deviation higher than his siblings with lower early human capital (about five IQ points). Since birth order is also correlated with preschool enrollment in these data, we also include an interaction between birth order and early human capital in these regressions, which has no effect on four-year IQ and which allows us to rule out the possibility that the interaction term preschool*early human capital simply reflects a preschool*birth order effect.

2.4.3 Preschool Enrollment and Human Capital at Seven Years (IQ and Achievement)

The estimated impact of preschool on human capital fades by age seven for all but those with the highest levels of initial human capital. For seven-year IQ and math achievement, the main effects of preschool enrollment and early human capital decline considerably, but not their interaction, which remains large. Thus, for those with higher early human capital, the impact of preschool lasts significantly longer than for others.[8]

To explore other potential sources of heterogeneity in the effect of preschool on seven-year IQ, we interact preschool with birth weight, birth order, and gender and find no significant effects (Table 3B). In results not presented here, we also find that the effect of preschool does not vary with maternal characteristics (education, age or income). We do, however, find a significant interaction effect for another measure of early human capital: advanced social/emotional development at eight months of age which is both positively related to seven-year IQ and interacts positively and significantly with preschool enrollment in the production of seven-year IQ. When we include both terms and their interaction with preschool (eight-month Bayley*preschool and advanced emotional development*preschool), the former is unchanged while the latter effect declines slightly and is no longer significant. Moreover, it should be noted that only 203 children are classified as socially/emotionally advanced in these data, and the eight-month mental Bayley score and emotional development are highly correlated, which is consistent with existing psychological research establishing that cognitive and emotional development in infancy and early childhood are closely related. [9]

We conclude that the estimated complementarity between early human capital and investments made during the preschool period is empirically meaningful and has important implications because it provides incentives for parents to invest in a reinforcing manner due to higher returns. Such an investment strategy would exacerbate differences in human capital among siblings. However, if parents have a preference for equality among offspring (as posited by some existing theoretical models), this would suggest that parents face a tradeoff in their investment decisions: compensating investments to achieve equality vs. reinforcing to increase their overall returns. In the next section, we explore how parental investment decisions react to a child's endowment.

[8]Unfortunately, we cannot with these data estimate whether the effect eventually fades for all, though existing evidence suggests that IQ is stable by age 10, just three years after our measure of IQ.

[9]literature, some argue that emotional development is a function of cognitive development and others that the relationship is more mutual (e.g., Lazarus, 1984).

3 Endowments and the Allocation of Parental Investments

3.1 Background

Existing evidence with respect to the question of whether parents invest in a compensatory or reinforcing manner is often limited by both lack of data on initial endowments and few measures of parental investments that vary within household and do not reflect decisions made by the child. Existing work based on data that do not include measures of initial endowments includes Hanushek (1992) who finds that having a sibling with higher measured achievement is positively correlated with own achievement (which he argues is inconsistent with reinforcing investment). Adhvaryu and Nyshadham (2012) find that investments – as measured by vaccination and breastfeeding – are higher for children who received iodine supplementation while in utero and interpret this finding as evidence of reinforcing investments. Ashenfelter and Rouse (1998) and Behrman, Rosenzweig and Taubman (1994) base their identification on differences in education and earnings of identical twins relative to fraternal twins, arguing that (unobserved) endowments of identical twins are more similar. They find that differences in earnings and schooling are greater for fraternal twins who have more dissimilar endowments and interpret this as evidence of reinforcing investments. Datar, Kilburn, and Loughran (2010) use birth weight as a measure of initial endowment and show that parental investment increases with endowment. Rosenzweig and Wolpin (1988) rely upon a residual in a human capital production function as a proxy for endowment – a procedure that we refine below – and find that children with better health endowments are more likely to be breastfed. Pitt, Rosenzweig, and Hassan (1990) follow a similar procedure and find that the more highly endowed receive more nutrition in a developing country setting.

There are three main innovations of our analysis of whether parental investments compensate or reinforce initial endowments. First, we develop an alternative measure of parental investment that captures the quality of the mother-child interaction as evaluated by a psychologist. This builds on existing work in economics that has focused less on parental time and more on the quality of time as measured by parenting skills (Paxson and Schady, 2007; Todd and Wolpin, 2007; Cunha, Heckman, and Schennach, 2010). Second, we address the possibility that traditional measures of human capital at birth (i.e., birth weight) are both measured with error and potentially endogenous because they already reflect prenatal investments. Third, we explore how the investment decision interacts with the fertility decision. We discuss each innovation below.

3.2 A New Measure of Investment: Parenting

In this subsection we describe our measure of parental investment. Because it differs from more traditional measures of investment (e.g., time), we follow with arguments to justify this measure.

3.2.1 Construction of the Measure of Parental Investment from the NCPP Data

Our specific measure of investment is derived from a psychologist's rating of the interaction between mother and child at eight months of age along the following six dimensions: maternal expression of affection (negative to extravagant), handling of the child (rough to very gentle), management of the child (no facilitation to over-directing), responsiveness to the needs of the child (unresponsive to absorbed), her focus during the child's examination (self to child), and her own evaluation of the child (critical to effusive). A final, 7th dimension is the child's appearance (unkempt to overdressed).

We assume that these variables are error-ridden measures of investment. More formally, let $Z_{x,i,j,k}$ denote the k^{th} error-ridden measure of investment on child i in family j. We assume that $Z_{x,i,j,k}$ is linked to actual investment on child i, $x_{i,j}$, by the following equation:

$$Z_{x,i,j,k} = \alpha_{x,k} x_{i,j} + \epsilon_{x,i,j,k},\ k = 1, ..., K_x. \tag{1}$$

In our data, $K_x = 7$. The parameters $\alpha_{x,k}$ are the factor loadings and the variables $\epsilon_{x,i,j,k}$ constitute the measurement error in $Z_{x,i,j,k}$. As usual in factor analysis, we assume that $\epsilon_{x,i,j,k}$ is independent from $\epsilon_{x,i,j,l}$, $l \neq k$, and $\epsilon_{x,i,j,k}$ is independent from $x_{i,j}$.

To produce an estimator of $x_{i,j}$, we factor analyze $Z_{x,i,j} = \{Z_{x,i,j,k}\}_{k=1}^{K_x}$ (Table 4). This produces estimates of the factor loadings $\alpha_{x,k}$ as well as the variance of $\epsilon_{x,i,j,k}$. We use this information to compute the relative importance of each of the $K_x = 7$ dimensions of parenting quality. We find that responsiveness and affection toward the child are the measures with the highest share of true to total variance, while appearance and handling are the ones with the lowest share.

To produce an estimate of $x_{i,j}$, we use the Bartlett Method (Bartlett, 1938). More precisely, let $\alpha_x = (\alpha_{x,1}, ..., \alpha_{x,K_x})'$ and define the vectors $\epsilon_{x,i,j}$ and $Z_{x,i,j}$ accordingly. Let $\Theta_x = Var(\epsilon_{x,i,j})$. The Bartlett factor score, $\hat{x}^B_{i,j}$, is defined as:

$$\hat{x}^B_{i,j} = (\alpha'_x \Theta_x \alpha_x)^{-1} \alpha'_x \Theta_x Z_{x,i,j}$$

For the single measure of investment generated in this fashion, which varies from -11.2 to 11.3 in the sibling subsample, (with a higher value indicating greater investment), 68% of the sample receive the same score (.089) corresponding to average or normal values for all seven measures, but there is still variance. Figure 4 displays the distribution of this measure of investment in the cross section in the first graph and within family differences in the second graph. Within family, for exactly half the sample, there is no difference in parenting across siblings. But for those families that exhibit different parenting across siblings, the differences can be quite large. This is consistent with existing work that has shown that in one-third to two-thirds of families, parents "differentiate in terms of closeness, support and comfort" beginning in early childhood (Suitor et al., 2008, page 334). Throughout the text and tables, we refer to this measure of investment as "quality of parenting," though it can also be thought of as a measure of favoritism.

3.2.2 Justification

We argue that the above measure of investment is preferable to more traditional measures such as parental time, nutrition or education. With respect to parental time, not only is time spent with a specific child in a household difficult to measure, but much of the variation in parental time between siblings is driven by birth order and/or maternal work, both of which likely exert independent effects on child outcomes (Price, 2009). In contrast, evidence suggests that between one-third and two-thirds of parents exhibit preferential treatment toward one sibling and that this does not vary systematically with birth order (Suitor et al, 2008). With respect to nutrition, variation within households exists and has been measured in developing countries, but in the US, there is less evidence of nutritional variation within households due in part to insufficient data. Finally, variation in educational attainment suffers from the fact that children are also involved in the decision (and even the financing, at higher levels), so that it does not just reflect parental investments.

A second argument for using "parenting" as a measure of investment comes from extensive research in developmental psychology and neurobiology showing that the quality of maternal-child attachments in the first years of life is an important determinant of a child's development, especially cognitive development. The theoretical foundation of this research derives from "attachment theory," which stipulates that a strong bond between the child and primary the care-giver serves to provide a secure base from which an infant can explore the world. More specifically, having a secure base enables the infant to "engage in a variety of adult-supervised learning experiences [including] exploratory interactions with objects and social partners that lead to eventual mastery of these domains" (Seifer and Schiller, 1995). Key to the establishment of this "secure base" is a high degree of maternal sensitivity and responsiveness to infant signals. In a review of the empirical research on the relationship between early maternal-infant attachment and later child outcomes, Ranson and Urichuk (2008) conclude that the evidence strongly supports a strong relationship between maternal-infant attachment in infancy with later cognitive outcomes (e.g., IQ, reading and GPA), though establishing a causal relationship is more difficult. [10]

More recently, neurobiologists have posited that strong attachment in infancy fosters brain growth and development, providing a biological basis for a widely accepted psychological theory of "attachment" (Schore, 2001). The experimental research in neurobiology generally supports a strong role for early attachment in the neurobiology of brain development. [11]

A third and final justification of our use of parenting as a measure of parental investment is our finding that it is correlated with two other more traditional measures of investment – parental time and the Home Observation for Measurement of the Environment (HOME) score, collected as part

[10] The research also supports a strong relationship between attachment and social-emotional and mental health outcomes.

[11] See the book by the National Research Council and the Institute of Medicine (2000) for a review.

of the PSID Child Development Supplement (CDS).[12] The CDS does not include the exact same measure of parenting that we use, but it does include the parental warmth scale which is based on interviewer observations as to whether the parent shows verbal, physical, and emotional affection toward the child and whether the parent interacts by joking, playing, participating in activities with the child or showing interest in the child's activities. We argue that this measure of warmth is sufficiently similar to our measure of parenting and that by showing its positive correlation with the two other more traditional measures of parental investment, HOME score and time, both across and within families in the PSID (Appendix Table 3), we provide further justification of our use of parenting as a measure of parental investment.

3.3 Measures of Endowments That Address Measurement Error and Endogeneity

Our second innovation is to construct an alternative measure of endowment that addresses both measurement error and potential endogeneity associated with more commonly used measures of endowment such as birth weight. Endogeneity may arise from the fact that typical measures of human capital at birth (e.g., birth weight) might reflect not only endowment but also prenatal investments. If so, correlation between human capital at birth and post-natal investments might simply reflect serial correlation in investments. In fact, in our data, prenatal investments including nutrition (as measured by weight gain) and smoking abstinance during pregnancy are positively correlated with our measure of investment (parenting) during the postnatal period, even when maternal fixed effects are included.

To address this concern, we construct a measure of human capital at birth that we argue is plausibly net of maternal investments during the prenatal period. To do so, we follow Rosenzweig and Wolpin (1988) and consider a production function for human capital at birth that includes the following inputs: the initial endowment of the child, maternal prenatal investments (nutrition and smoking), whether the mother was trying to conceive (a measure of the "wantedness" of the child), a family-specific term (to capture, for example, genetics) and an idiosyncratic child-specific error term. Because we have measures of maternal prenatal investments that differ for children within the same family, we can estimate the above production function and calculate the residual, which we argue consists of the child's endowment and an idiosyncratic child specific error term.

More formally, let $y_{i,j,k}$ denote the birth outcome k (birth weight, head circumference, body length and gestation) of child i born in family j. Let $c_{i,j}$ denote a quadratic in the number of cigarettes that mother j smoked while pregnant with child i. Let $w_{i,j}$ denote the weight of mother j when she became pregnant with child i and $g_{i,j}$ denote the weight gain while pregnant with child i. Let $WANT_{i,j}$ reflect whether the mother reports she was trying to conceive child i. Let η_j denote

[12] Among the many items that constitute the HOME score are the number of books available to the child, how often the child goes to museums, how often the child goes to the theater, and how often the mother reads to or with the child.

the maternal fixed effect. The term $\varepsilon_{i,j}$ denotes the endowment of child i. Let $\epsilon_{\varepsilon,i,j,k}$ denote the idiosyncratic component of birth outcome k. Assume that:

$$y_{i,j,k} = \beta_{0,k} + \beta_{1,k}c_{i,j} + \beta_{2,k}w_{i,j} + \beta_{3,k}g_{i,j} + \beta_{4,j}WANT_{i,j} + \delta_k\eta_j + \alpha_{\varepsilon,k}\varepsilon_{i,j} + \epsilon_{\varepsilon,i,j,k}$$

Our goal is to obtain an estimate of $\varepsilon_{i,j}$, the child endowment. A simple fixed-effect procedure allows us to obtain $\beta_{0,k}$, $\beta_{1,k}$, $\beta_{2,k}$, $\beta_{3,k}$, and $\beta_{4,k}$. The estimated coefficients for each of the four measures of conditions at birth in the data are presented in Table 5. Once we know these components, we can predict the residual term $Z_{\varepsilon,i,j,k}$ for each birth outcome k:

$$Z_{\varepsilon,i,j,k} = \alpha_{\varepsilon,k}\varepsilon_{i,j} + \epsilon_{\varepsilon,i,j,k}$$

As argued by Rosenzweig and Wolpin (1988), each predicted residual term approximates the endowment of the child net of the presumably most important maternal prenatal investments (smoking and nutrition). On the other hand, the residual term suffers from measurement error $\epsilon_{\varepsilon,i,j,k}$. In our data, we can exploit the fact that we have multiple measures of health at birth (birth weight, gestation, head circumference and body length) to conduct a factor analysis of $Z_{\varepsilon,i,j,k}$ to extract the single common underlying endowment $\varepsilon_{i,j}$. Table 5 presents the estimates of the newborn health production function, and Table 6 shows the estimated factor loadings, $\alpha_{\varepsilon,k}$, and the variance of the uniquenesses $\epsilon_{\varepsilon,i,j,k}$. The factor explains 67% of the cross-sectional variance of birth weight, about 50% of the variance in head circumference and body length, and 26% of the variance in weeks of gestation. The inverse ordering is true for the variance of measurement error: weeks of gestation has the largest amount of measurement error (which is unsurprising given that it is approximated), followed by body length and head circumference. Birth weight has the least amount of measurement error.

Let $\alpha_\varepsilon = (\alpha_{\varepsilon,1}, ..., \epsilon_{\varepsilon,i,j,K_\varepsilon})'$ and define the vectors $\epsilon_{\varepsilon,i,j}$ and $Z_{\varepsilon,i,j}$ in similar fashion. Let $\Theta_\varepsilon = Var(\epsilon_{\varepsilon,i,j})$ and $\sigma^2_\varepsilon = Var(\varepsilon_{i,j})$. We estimate $\varepsilon_{i,j}$ from the factor scores using the Regression method (Thurnstone, 1934), $\hat{\varepsilon}^R_{i,j}$

$$\hat{\varepsilon}^R_{i,j} = \sigma^2_\varepsilon \boldsymbol{\beta}' \left(\boldsymbol{\beta}\sigma^2_\varepsilon\boldsymbol{\beta}' + \Theta_\varepsilon\right)^{-1}.$$

In the next subsection, we explain the reason why parental investment, $x_{i,j}$, is predicted using the Bartlett score $\hat{x}^B_{i,j}$ while the child endowment, $\varepsilon_{i,j}$, is predicted using the score produced by the Regression method, $\hat{\varepsilon}^R_{i,j}$.

3.4 Factor Score Fixed Effect Regression

To test whether children with a higher initial endowment, $\varepsilon_{i,j}$, receive greater investments, $x_{i,j}$, we estimate models of the following form:

$$x_{i,j} = \gamma\varepsilon_{i,j} + u_j + \nu_{i,j}, \tag{2}$$

where i indexes each child within family j. To focus on the important ideas, we abstract from observed control variables that are included in our empirical investigation of (2).

Assumption A1: Let $\varepsilon_j = \{\varepsilon_{i,j}\}_{i=1}^n$ denote the vector of endowments in family j. We assume that $\nu_j = \{\nu_{i,j}\}_{i=1}^n$ satisfies the following orthogonality condition: $E(\nu_j | \varepsilon_j, u_j) = 0$.

Our goal is to obtain consistent estimators of γ. If we observed the vectors x_j and ε_j directly, then under *Assumption A1* we could employ the usual fixed-effect estimator of γ:

$$\hat{\gamma}_{FE} = \left[\sum_{j=1}^{J} \varepsilon_j' (Q\varepsilon_j)\right]^{-1} \sum_{j=1}^{J} \varepsilon_j' (Qx_j)$$

where Q is the symmetric and idempotent matrix:

$$Q = \begin{bmatrix} 1-\frac{1}{n} & -\frac{1}{n} & \cdots & -\frac{1}{n} \\ -\frac{1}{n} & 1-\frac{1}{n} & \cdots & -\frac{1}{n} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{n} & -\frac{1}{n} & \cdots & 1-\frac{1}{n} \end{bmatrix}.$$

Unfortunately, we don't observe x_j or ε_j directly. Instead, we observe K_x and K_ε error-ridden measures of parental investment ($Z_{x,i,j,k}$) and endowment ($Z_{\varepsilon,i,j,k}$), respectively. As discussed above, we used these measures to produce estimates $\hat{x}_{i,j}^B$ and $\hat{\varepsilon}_{i,j}^R$. We now show that the estimator that replaces ε_j with $\hat{\varepsilon}_{i,j}^R$ and x_j with $\hat{x}_{i,j}^B$ produces a consistent estimator of γ. To do so, let $\epsilon_{x,j} = (\epsilon_{x,1,j}, \epsilon_{x,2,j}, ..., \epsilon_{x,n,j})'$ and define the vectors $\epsilon_{\varepsilon,j}$ and ν_j analogously.

Assumption A2a: $E(\epsilon_{x,j} | \epsilon_{\varepsilon,j}) = 0$.

Assumption A2b: $E(\nu_j | \epsilon_{\varepsilon,j}) = 0$.

Assumption A2a rules out correlation between the measurement error in $x_{i,j}$ and measurement error in $\varepsilon_{i,j}$. If *Assumption A2a* does not hold, then the dependence between between $Z_{x,j}$ and $Z_{\varepsilon,j}$ cannot separately identify the dependence between $x_{i,j}$ and $\varepsilon_{i,j}$ from the dependence between $\epsilon_{x,j}$ and $\epsilon_{\varepsilon,j}$.

Assumption A2b ensures that the measurement error in $\varepsilon_{i,j}$ is not correlated with the residuals in (2). *Assumption A2b* does not hold, for example, if there are multiple dimensions of investments and that some of these dimensions are correlated with unobserved inputs captured in ν_j.

Theorem 1 *Let $\hat{x}_{i,j}^B$ denote the estimate of $x_{i,j}$ produced by the Bartlett method. Let $\hat{\varepsilon}_{i,j}^R$ denote the estimate of $\varepsilon_{i,j}$ produced by the Regression method. If Assumptions A2a and A2b hold, then*

$$\lim_{J\to\infty} \left[\sum_{j=1}^{J} \left(\hat{\varepsilon}_{i,j}^R\right)' \left(Q\hat{\varepsilon}_{i,j}^R\right)\right]^{-1} \sum_{j=1}^{J} \left(\hat{\varepsilon}_{i,j}^R\right)' \left(Q\hat{x}_{i,j}^B\right) = \gamma.$$

Proof. See appendix B. ■

The main result of Theorem 1 is that when that when the regressand in (2) is replaced by

$\hat{x}_{i,j}^{B}$ and the regressor is replaced by $\hat{\varepsilon}_{i,j}^{R}$, then the fixed-effect estimator that uses these predicted values is consistent. In other words, the choice of which scores to use actually matter for γ to be consistently estimated. If we had replaced $x_{i,j}$ with $x_{i,j}^{R}$ or $\varepsilon_{i,j}$ with $\varepsilon_{i,j}^{B}$, then the resulting estimator for γ would not be consistent. A similar Theorem has been proved by Skrondal and Laake (2001) in the context of OLS regressions. We simply extend it to the fixed-effect framework and we rely on it to provide a formal explanation for our choices of factor scores for parenting and endowments.

3.5 Results: Endowments and the Allocation of Investments within Families

To estimate how parental investments respond to child endowments within family, we regress our measure of investment, (the quality of parenting at eight months of age), on different measures of endowment, as constructed above, and including maternal fixed effects. The first measure of endowment is birth weight. Within family, the child with higher birth weight receives more investment in the form of higher-quality parenting. The results suggest that a one-standard-deviation difference in birth weight between siblings can explain 10% percent of the average difference in investment between siblings within families.

When we consider that birth weight may be endogenous because it already reflects maternal prenatal investments and use instead the residual from a birth weight production function that includes maternal smoking, pre-pregnancy weight and weight gain as regressors (Table 5), we find that the relationship still holds, though it is somewhat attenuated (Table 7, column 2). In columns 3-8 we present estimates based on different measures of health at birth (gestation, body length, head circumference) and their corresponding residual measures and the same pattern emerges. In the last two columns we present estimates for a measure of endowment based on factor scores of multiple measures of health at birth (birth weight, head circumference, body length, gestation), and factor scores based on the four residuals from the newborn health production function. The results based on factor scores of the four measures of health at birth (column 9) are similar, though slightly smaller than the birth weight result in column 1, and the results based on factor scores of the four residuals is still positive and significant, but only 30% of the original estimate based on birth weight.

The results suggest that postnatal investments are greater for more highly endowed children – consistent with reinforcing investments. However, some but not all of this relationship captures serial correlation in prenatal and postnatal investments.

3.6 How Fertility Affects the Distribution of Investments Within a Family

We follow with an exploration of whether and how this reinforcing investment behavior varies with fertility. To do so, we estimate the ability of initial endowment to predict both parental investments at eight months and future human capital (seven-year IQ) within family, stratified by fertility. We find that the relationship between initial endowments and postnatal investments within family is stronger for larger families (Table 8, Panel A). This is true for each of the four measures of initial endowment that we use, though we only present the results for the residual birth weight used in Table 7. We interpret this finding as evidence that investments tend to be more reinforcing the larger the family.

One testable implication of this interpretation is that the relationship between a child's initial endowment and later human capital should be stronger in large families.[13] We test this empirically by examining how the relationship between endowment and later human capital (measured by seven-year IQ) varies with fertility. We find that the positive correlation between a child's endowment and his/her later human capital increases with family size (Table 8, Panel B).[14]

A second testable implication is that the spread in child human capital should likewise increase with family size. We show this in Figure 5, where we present the maximum and minimum IQ at age seven by family size.[15] One can clearly see that the difference between minimum IQ and maximum IQ within a family increases with family size. For a two-child family, this difference is 40% of a standard deviation; for a four-child family, the difference increases to 100% of a standard deviation. Interestingly, this increase in the difference is driven entirely by declines in the minimum. The maximum human capital of children in large families is the same as the maximum in small families, but the minimum is much lower. It has been well documented that average human capital is lower in large families and the same is true in these data. However, the results here show that this difference is driven by differences at the bottom of the distribution of child human capital, not the top. This pattern can be explained by two forces: 1) the fact that the variance in endowment increases with family size (Figure 5, second panel), and 2) greater reinforcing investments in large families (Figure 5, third panel, and Table 8).

We corroborate these findings with another data source - the Children of the National Longitudinal Survey of the Youth 1979 (CNLSY-79). The CNLSY-79 includes data on birth weight and child human capital (PPVT and math scores). We see the same pattern in the NLSY (Figure 6A

[13] Previous work examining whether investments are compensatory or reinforcing has relied on this strategy (e.g., Behrman, Rosenzweig, and Taubman, 1994). Previous work did not have measures of endowment, but rather assumed endowments were more similar between identical twins than fraternal twins and examined whether sibling differences in outcomes were greater among fraternal twins relative to identical twins.

[14] While this evidence is consistent with greater reinforcing investment in large families, it could also be consistent with different production functions based on family size, which we cannot rule out.

[15] We limit this analysis to families in which all births are observed in the data. We show, however, that our findings are replicated in the Children of the National Longitudinal Survey of Youth (1979) data.

and 6B): both the spread in endowments and child human capital increase with family size, with the increase driven by declines at the bottom of the distribution. We also use the NLSY data to explore whether birth order can explain the patterns observed (Figure 6C). While we observe significant birth order effects, they only explain a third of the difference between the maximum and minimum child human capital within family.

Next, we describe a simple model that allows us to interpret these findings.

4 Model

In what follows, we show that a very simple quality-quantity trade-off model as in Becker and Lewis (1973), with heterogeneity in endowments and static complementarity as in Becker and Tomes (1986), together with parental preferences as in Behrman, Pollak, and Taubman (1982) can reproduce the qualitative features of our findings. This is true as long as parental utility features (i) complementarity between quantity and average quality (human capital) of children and (ii) some, but not extreme, aversion to inequality among children.[16] As we show below, complementarity explains why the expected maximum human capital and expected minimum human capital move in opposite directions as quantity increases. The moderate inequality aversion is a compromise between two forces that go in opposite directions. First, investments reinforce differences in endowments, which is not possible if parents have an elevated aversion to inequality. Second, expected maximum human capital does not increase by much with fertility, which – given complementarity in preferences and production function – could not be rationalized if parents had no aversion to inequality.

The Production Function of Human Capital. Let ε_i denote the endowment of child i and x_i the parental investment on child i's human capital, h_i. The technology of skill formation is:

$$h_i = \varepsilon_i x_i. \tag{3}$$

In particular, note that technology (3) exhibits static complementarity.

Preferences. We assume that parental preferences are described by the following utility function:

$$u = c + \alpha_1 n - \frac{\alpha_2}{2} n^2 + \alpha_3 \left(\frac{1}{n} \sum_{i=1}^{n} h_i \right) - \frac{\alpha_4}{2} \left(\frac{1}{n} \sum_{i=1i}^{n} h_i^2 \right) + \alpha_5 n \left(\frac{1}{n} \sum_{i=1}^{n} h_i \right) - \frac{\alpha_6}{2} \left[\frac{1}{n} \sum_{i=1}^{n} \left(h_i - \bar{h} \right)^2 \right].$$

The preferences are linear in consumption (c) and quadratic in fertility (n) and human capital of child i. The parameter α_4 describes how fast the marginal utility decreases as h_i increases. The parameter α_5 represents the complementarity or substitutability between quantity and average

[16] It is possible to extend the model to multiple periods and introduce dynamic complementarity in the production function of human capital as in Cunha and Heckman (2007). We choose not to do so to keep the analysis as simple as possible.

quality in the utility function. The term $\frac{1}{n}\sum_{i=1}^{n}\left(h_i-\bar{h}\right)^2$ is the variance of human capital within the household. In this sense, the parameter $\alpha_6 \geq 0$ captures parental aversion to inequality in the spirit of Behrman, Pollak, and Taubman (1982). The higher α_6, the higher the psychic cost of inequality.

If there is no heterogeneity among children so that $h_i = h$ for all i (as in Becker and Lewis, 1973), then the utility function collapses to:

$$u = c + \alpha_1 n - \frac{\alpha_2}{2}n^2 + \alpha_3 h - \frac{\alpha_4}{2}h^2 + \alpha_5 nh.$$

Budget Constraint. Let y denote household income. Let p denote the cost of fertility and π the price of the investment good. The budget constraint is:

$$c + pn + \pi\sum_{i=1}^{n_j} x_i = y. \tag{4}$$

Our budget constraint is slighlty different from Becker and Lewis (1973). If $\varepsilon_i \neq \varepsilon_j$, then $x_i \neq x_j$ because parents will act on this heterogeneity by choosing investments that either reinforce or compensate differences in endowments.

Solving the Problem of the Parent. From (3), it is clear that $x_i = \frac{h_i}{\varepsilon_i}$. We can use this relationship to replace x_i in the budget constraint:

$$c + pn + \sum_{i=1}^{n_j}\pi\left(\varepsilon_i\right)h_i = y,$$

where $\pi\left(\varepsilon_i\right) = \frac{\pi}{\varepsilon_i}$ which is clearly decreasing in ε_i. The first-order condition for h_i is:

$$-\pi\left(\varepsilon_i\right) + \frac{\alpha_3}{n} - \frac{\alpha_4}{n}h_i + \alpha_5 - \frac{\alpha_6}{n}\left(h_i - \bar{h}\right) = 0.$$

We now discuss three different cases. First, we focus on the case in which children are homogeneous (Becker and Lewis, 1973). We then show that such a model is not consistent with our findings. Second, we allow for children to be heterogeneous, but parents have no aversion to inequality. The heterogeneity helps us understand why dispersion of human capital and investments increases with fertility, but does not reproduce the distinct profile of expected maximum and minimum quality with respect to quantity. Finally, we discuss the case in which children are heterogeneous and parents are averse to inequality which can explain the relationship between quantity and mamimum and minimum quality among children in a household that we observe in our data.

Case 2 *Assume that $\varepsilon_i = \varepsilon$ for all i so that children are homogeneous. Then, the relationship* *between human capital h and fertility n is given by:*

$$h_i = h = \frac{\alpha_3}{\alpha_4} + \frac{\alpha_5 - \pi}{\alpha_4} n \text{ for } i = 1, ..., n.$$

If household children are homogeneous, then an exogenous increase in n either increases the quality of all children (if $\alpha_5 - \pi > 0$), decreases the quality of all children (if $\alpha_5 - \pi < 0$), or quality does not change (if $\alpha_5 - \pi = 0$).

When children are homogeneous, an increase in the quantity of children affects the quality of all children in exactly the same way. Furthermore, the model is clearly unable to answer how the distribution of quality (and not only its first moment) is affected by quantity. Interestingly, it is not possible to separately identify α_5 from π. This means that one cannot test the Becker and Lewis (1973) model because one cannot test the hypothesis that $\pi = 0$. This non-identifiability result is not a product of our simple model. A general proposition was established by Rosenzweig and Wolpin (1980), but the intution is clear: If there is no variation in π, then it is not possible to separate the interaction between quantity and quality in the utility function (α_5) from the interaction in the budget constraint (π). One can, however, test whether quantity reduces the average human capital of children in the household. This amounts to testing whether $\alpha_5 - \pi < 0$ or $\alpha_5 - \pi \geq 0$.

Case 3 *Assume that children are heterogeneous within the household, but $\alpha_6 = 0$ so that parents don't have aversion to inequality. Then, the relationship between the human capital of child i and fertility is:*

$$h_i = \frac{\alpha_3}{\alpha_4} + \frac{\alpha_5 - \pi(\varepsilon_i)}{\alpha_4} n \text{ for } i = 1, ..., n. \tag{5}$$

If household children are heterogeneous and parents have no aversion to inequality, an exogenous increase in n increases the quality of all children with endowments such that $\pi(\varepsilon_i) < \alpha_5$ and decreases the quality of all children with endowments such that $\pi(\varepsilon_i) > \alpha_5$.

We now discuss the conditions under which Case 2 can generate the fact that the dispersion of human capital in the household increases as fertility increases. Define $h_{1:n} = \min\{h_1, ..., h_n\}$ and $h_{n:n} = \max\{h_1, ..., h_n\}$, that is, $h_{1:n}$ and $h_{n:n}$ are, respectively, the minimum and maximum human capital in a family with n children. Clearly, it follows that $h_{j:n} = \frac{\alpha_3}{\alpha_4} + \frac{\alpha_5 - \pi(\varepsilon_{j:n})}{\alpha_4} n$ for $j = 1, ..., n$; where $\varepsilon_{1:n}$ and $\varepsilon_{n:n}$ are the extreme order statistics for endowment in a family with n children. Assume that $\ln \varepsilon_i \sim N(0,1)$ so that $\frac{\pi}{\varepsilon_i}$ is also log-normally distributed with mean π and variance one. This is helpful because $E\left(\frac{\pi}{\varepsilon_{j:n}}\right) = \pi \frac{1}{E(\varepsilon_{j:n})} = \pi\left[E(\varepsilon_{j:n})\right]$. Then, it follows that:

$$E(h_{1:n}) = \frac{\alpha_3}{\alpha_4} + \frac{\alpha_5 - \pi\left[E(\varepsilon_{1:n})\right]}{\alpha_4} n, \tag{6}$$

$$E(h_{n:n}) = \frac{\alpha_3}{\alpha_4} + \frac{\alpha_5 - \pi\left[E(\varepsilon_{n:n})\right]}{\alpha_4} n. \tag{7}$$

An increase in n changes two components in equations (6) and (7). First, it changes n itself. Second, it also changes the expected values $E(\varepsilon_{1:n})$ and $E(\varepsilon_{n:n})$. Importantly, note that $E(\varepsilon_{1:n+1}) \leq E(\varepsilon_{1:n})$ for any $n \in \mathbb{N}$. At the same time, $E(\varepsilon_{n+1:n+1}) \geq E(\varepsilon_{n:n})$. This property of extreme order statistics generates the increase in dispersion that we document in our data, but not necessarily in the same way. As we show in Figures 5 and 6, $E(h_{n:n})$ moves very little (or hardly at all) and $E(h_{1:n})$ decreases sharply as n increases. Because the maximum human capital is a non-decreasing function of fertility, to match the pattern that we see in the data, we need to rule out $\alpha_5 < \pi[E(\varepsilon_{n:n})]$. If endowments are strictly positive random variables, then we conclude that $\alpha_5 \geq \pi[E(\varepsilon_{n:n})] > 0$. This implies that quantity and quality are complements in the parental utility function. If so, then our findings are consistent with recent evidence of small effects of quantity on average quality of children (Black, Deveraux, and Salvanes, 2005, 2007; Angrist, Lavy, and Schlosser, 2010).[17] At the same time, because the minimum human capital is a decreasing function of fertility, it must be the case that $\alpha_5 < \pi[E(\varepsilon_{1:n})]$ for any $n \geq 2$. Given that we observe family sizes up to $n = 6$, we conclude that $\pi[E(\varepsilon_{5:5})] \leq \alpha_5 < \pi[E(\varepsilon_{1:n})]$.

Unless specific distributional assumptions are imposed (i.e., a distribution for endowments such that the expected extreme order statistics satisfy both $E(\varepsilon_{n+1:n+1}) \approx E(\varepsilon_{n:n})$ and $E(\varepsilon_{1:n+1}) < E(\varepsilon_{1:n})$), the model above predicts that maximum human capital increases with fertility. One way to weaken this conclusion is to allow for parents to have aversion to inequality, which we discuss next.

Case 4 *Assume that children are heterogeneous within the household and parents are averse to inequality among children, so that $\alpha_6 > 0$. Then, the relationship between h_i and n is:*

$$h_i = \frac{\alpha_3}{\alpha_4} + \frac{\alpha_6}{\alpha_4}\sum_{j=1}^{n}\left(\frac{\alpha_5 - \pi(\varepsilon_j)}{\alpha_6 + \alpha_4}\right) + \left(\frac{\alpha_5 - \pi(\varepsilon_i)}{\alpha_6 + \alpha_4}\right) n. \tag{8}$$

Again, assume that $\ln \varepsilon_i \sim N(0,1)$. The expression above shows that when parents are averse to inequality, the endowments of all children in the household affect the parental choice of investment in child i and, ultimately, the human capital of child i, for $i \neq j$. The impact is larger the larger the size of α_6. In particular:

$$\lim_{\alpha_6 \to \infty} h_i = \frac{\alpha_3}{\alpha_4} + \sum_{j=1}^{n}\left[\frac{\alpha_5 - \pi(\varepsilon_j)}{\alpha_4}\right] \text{ for all } i,$$

and investments perfectly offset differences in endowments, contrary to what we find in our empirical

[17] Rosenzweig and Zhang (2009) argue that the procedure used by Black, Devereux, and Salvanes (2005) and Angrist, Lavy, and Schlosser (2010) – twinning at higher parities on the outcomes of older children – do not identify the effect of quantity on the quality of children because twins tend to have worse endowments and as close spacing as possible. They show that it is possible to bound the effects of quantity and quality by investigating twinning at different parities while holding constant the twin's endowments. Rosenzweig and Zhang (2009) find that an extra child decreases the schooling progress, the expected college enrolment, grades in school and the assessed health of all children in the family.

analysis. On the other hand, when $\alpha_6 = 0$, (8) becomes identical to (5).

We now investigate the behavior of $E(h_{1:n})$ and $E(h_{n:n})$ as fertility increases from n to $n+1$ when $\alpha_6 \in (0, \infty)$. Note that a change in n involves changes in several terms of equation (8). It is easy to derive the following equality:

$$E(h_{1:n+1}) - E(h_{1:n}) = \frac{\alpha_6}{\alpha_4} \sum_{j=1}^{n} \underbrace{\frac{\pi[E(\varepsilon_{j:n})] - \pi[E(\varepsilon_{j:n+1})]}{\alpha_6 + \alpha_4}}_{\leq 0} + \underbrace{\frac{\pi[E(\varepsilon_{1:n})] - \pi[E(\varepsilon_{1:n+1})]}{\alpha_6 + \alpha_4}}_{\leq 0} n \quad (9)$$

$$\frac{\alpha_6}{\alpha_4} \underbrace{\left[\frac{\alpha_5 - \pi[E(\varepsilon_{n+1:n+1})]}{\alpha_6 + \alpha_4}\right]}_{\geq 0} + \underbrace{\left[\frac{\alpha_5 - \pi[E(\varepsilon_{1:n+1})]}{\alpha_6 + \alpha_4}\right]}_{\leq 0}.$$

The first term on the right-hand side of (9) is non-positive because, for log-normal random variables, $E(\varepsilon_{j:n}) \geq E(\varepsilon_{j:n+1})$, which implies that $\pi[E(\varepsilon_{j:n})] \leq \pi[E(\varepsilon_{j:n+1})]$. The same reasoning explains why the second term is also non-positive. Following the previous discussion, $\alpha_5 \geq \pi[E(\varepsilon_{n+1:n+1})]$ and $\alpha_5 < \pi[E(\varepsilon_{1:n+1})]$. These restrictions imply that the third term is positive while the fourth term is negative. In any event, for the model to replicate the empirical patterns, it is necessary for the third term not to be "too large" so that $E(h_{1:n+1}) - E(h_{1:n}) < 0$.

A similar expression can be derived for the difference between $E(h_{n+1:n+1})$ and $E(h_{n:n})$:

$$E(h_{n+1:n+1}) - E(h_{n:n}) = \frac{\alpha_6}{\alpha_4} \sum_{j=1}^{n} \underbrace{\frac{\pi[E(\varepsilon_{j:n})] - \pi[E(\varepsilon_{j:n+1})]}{\alpha_6 + \alpha_4}}_{\leq 0} + \underbrace{\frac{\pi[E(\varepsilon_{n:n})] - \pi[E(\varepsilon_{n+1:n+1})]}{\alpha_6 + \alpha_4}}_{\geq 0} n$$

$$+ \underbrace{\frac{\alpha_5 - \pi[E(\varepsilon_{n+1:n+1})]}{\alpha_6 + \alpha_4}}_{\geq 0}.$$

As discussed above, the first term is non-positive because of the assumption of log-normality of endowments. The second term is non-negative for any distribution. Finally, the third term is also positive. As a result, whether the right-hand side is positive or negative depends on how large α_6 is. Note that:

$$\lim_{\alpha_6 \to \infty} E(h_{n+1:n+1}) - E(h_{n:n}) = \sum_{j=1}^{n} \frac{\pi[E(\varepsilon_{j:n})] - \pi[E(\varepsilon_{j:n+1})]}{\alpha_4} < 0.$$

On the other hand,

$$\lim_{\alpha_6 \to 0} E(h_{n+1:n+1}) - E(h_{n:n}) = \frac{\pi[E(\varepsilon_{n:n})] - \pi[E(\varepsilon_{n+1:n+1})]}{\alpha_4} n + \frac{\alpha_5 - \pi[E(\varepsilon_{n+1:n+1})]}{\alpha_4} > 0.$$

Our findings that $E(h_{n:n})$ increases little or not at all with n suggests that $\alpha_6 > 0$, which indicates that parents are sensitive to inequality among their children. However, we can rule out extreme inequality aversion because we also know that parents devote higher investments to children with higher endowments.

Thus, we are able to explain our empirical finding that reinforcing investment increases with fertility by simply modifying the quantity-quality trade-off model to allow 1) heterogeneity in endowments and static complementarity and 2) parental preferences that feature complementarity between quantity and average quality and some aversion to inequality among children.

5 Conclusions

This paper studies how endowments, investments, fertility, and parental preferences interact to produce human capital in childhood. We begin by providing empirical support for two key features of existing models of human capital. The first is that of dynamic complementarity: investments and existing human capital are complements in the production of later human capital. For this, we exploit an exogenous source of investment, the launch of Head Start in 1966, and estimate greater gains from preschool in the IQ of those with the highest stocks of early human capital, consistent with dynamic complementarity.

The second feature we examine empirically is that of static complementarity: parental investments and endowments are complements. We do not test this directly, but rather empirically examine a direct implication of this feature, that parents allocate greater investments to children with higher initial endowments. Unlike previous studies, our data enable us to overcome the potential endogeneity and measurement error associated with traditional measures of endowment based on health at birth. We find that parents invest more in highly endowed children.

We also find that the degree of reinforcement increases with family size. Thus, an increase in quantity leads not only to a decline in average quality (the quantity-quality tradeoff) but to an increase in the variation in quality, due to both greater variation in endowments (from more children) and greater reinforcing investments.

Finally, we show that our findings can be explained by extending the quantity-quality trade-off model to include heterogeneous child endowments and parental preferences that feature complementarity between quality and quantity and moderate aversion to inequality in child human capital within the household. The complementarity in preferences explains why the expected maximum human capital and expected minimum human capital move in opposite directions as quantity increases: parents can increase "average" quality by investing more in the children with the highest endowments. Complementarity also explains the findings in the literature that document small, if any, relationship between quality and quantity of children. The moderate inequality aversion arises because if parents had high aversion to inequality, then investments would not be reinforcing. On the other hand, if parents had no aversion to inequality, then expected maximum human capital would increase much more with fertility than what we find in our datasets.

Our theoretical and empirical results have important implications for our understanding of the production of human capital in childhood and how initial levels of human capital, investments, fertility, and parental preferences interact to affect not only average levels of human capital but its distribution within a family. In addition, by providing new estimates of the impact of Head Start on multiple measures of cognitive ability and achievement that exploit the exogenous variation in Head Start availability within the family, our results also contribute to the growing literature on the impact of Head Start and other high-quality interventions in early childhood.

A Individual Items of Bayley Scale of Mental Development at Age 8 Months

1. Social smiles.
2. Visually recognizes mother.
3. Eyes follow pencil.
4. Reacts to paper on face.
5. Searches with eyes for sound.
6. Vocalizes to social stimulus.
7. Manipulates ring.
8. Vocalizes two syllables.
9. Regards cube.
10. Glances from one object to another.
11. Makes anticipatory adjustment to lifting.
12. Reacts to dissapearance of face.
13. Reaches for ring.
14. Plays with rattle.
15. Fingers hand in play.

16/18. Follows vanishing ring/spoon

17. Is aware of strange situation.
19. Eyes follow ball across table.
20. Carries ring to mouth.
21. Manipulates table edge slightly.
22. Inspects own hands.
23. Closes on dangling ring.

24/25. Turns head to sound of bell/rattle.

26/30. Reaches for/Picks up cube.

27. Actively manipulates table.
28. Regards pellet.
29. Approaches mirror image.
31. Engages in exploitive paper play.
32. Retains two cubes.
33. Discriminates between strangers.
34. Vocalizes attitudes.
35. Recovers rattle in crib or playpen.
36. Reaches persistently.
37. Turns head after dropped objective.
38. Lifts cup.
39. Reaches for second cube.
40. Enjoys frolic play.
41. Transfers objects hand to hand.
42. Sustains inspection of ring.
43. Plays with string.
44. Picks up cube directly and easily.
45. Pulls string, secures ring.
46. Enjoys sound production.
47. Lifts cup by handle.
48. Retains two cubes.
49. Attends to scribbling.
50. Looks for dropped object.
51. Manipulates bell, shows interest in details.
52. Responds playfully to mirror.
53. Vocalizes four different syllables.
54. Pulls string purposefully to secure ring.

55/58. Responds to social play/name.

56. Attempts to secure three cubes.
57. Rings bell imitatively.
59. Says Da-Da or equivalent.
60. Uncovers toy.
61. Adjusts to words.
62. Fingers holes in peg board.
63. Puts cube in cup.
64. Looks for content of box.

B Proof of Theorem 1

Let $\Sigma_x = Var\left(x_j\right)$, $\Sigma_\varepsilon = Var\left(\varepsilon_j\right)$ and $\Sigma_\nu = Var\left(\nu_j\right)$. We observe K_x and K_ε error-ridden measures of parental investment $(Z_{x,i,j,k})$ and endowment $(Z_{\varepsilon,i,j,k})$, respectively. Assume that:

$$\begin{aligned} Z_{x,j,k} &= \boldsymbol{\alpha}_k x_j + \epsilon_{x,j,k},\ k = 1, ..., K_x. \\ Z_{\varepsilon,j,k} &= \boldsymbol{\beta}_k \varepsilon_j + \epsilon_{\varepsilon,j,k},\ k = 1, ..., K_\varepsilon. \end{aligned} \tag{10}$$

Note that $x_j = \left(x_{1,j}, ..., x_{n,j}\right)'$, that is, the measurement system is defined at the level of the family. The vectors $Z_{x,j,k}$, $Z_{\varepsilon,j,k}$, $\epsilon_{x,j,k}$, and $\epsilon_{\varepsilon,j,k}$ are defined analogously. The matrices of factor loadings, $\boldsymbol{\alpha}_k$ and $\boldsymbol{\beta}_k$, are diagonal matrices of dimension $(n \times n)$:

$$\boldsymbol{\alpha}_k = \begin{bmatrix} \alpha_k & 0 & \cdots & 0 \\ 0 & \alpha_k & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_k \end{bmatrix} \text{ and } \boldsymbol{\beta}_k = \begin{bmatrix} \beta_k & 0 & \cdots & 0 \\ 0 & \beta_k & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \beta_k \end{bmatrix}.$$

Now, let $\mathbf{Z}_{x,j} = \left(Z_{x,j,1}, ..., Z_{x,j,K_x}\right)'$ and define $\mathbf{Z}_{\varepsilon,j}$, $\boldsymbol{\epsilon}_{x,j}$, $\boldsymbol{\epsilon}_{\varepsilon,j}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\beta}$ in the same way. We can write the system as:

$$\begin{aligned} \mathbf{Z}_{x,j} &= \boldsymbol{\alpha} x_j + \boldsymbol{\epsilon}_{x,j} \\ \mathbf{Z}_{\varepsilon,j} &= \boldsymbol{\beta} \varepsilon_j + \boldsymbol{\epsilon}_{\varepsilon,j} \end{aligned}$$

Let $\Theta_x = Var\left(\epsilon_{i,j,x}\right)$ and $\Theta_\varepsilon = Var\left(\epsilon_{\varepsilon,i,j}\right)$. Our approach proceeds in three steps. In the first step, we factor analyze system (10) to obtain estimates of the factor loadings $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and variance matrices Θ_x and Θ_ε. In the second step, we compute the factor scores $\hat{x}_j^B$ by the Bartlett Method (Bartlett, 1938) and the factor scores $\hat{\varepsilon}_j^R$ by the Regression Method (Thurnstone, 1935). These factor scores are computed in the following way:

$$\begin{aligned} \hat{x}_j^B &= A^B \mathbf{Z}_{x,j},\ \text{where } A^B = \left(\boldsymbol{\alpha}' \Theta_x \boldsymbol{\alpha}\right)^{-1} \boldsymbol{\alpha}' \Theta_x, \\ \hat{\varepsilon}_j^R &= B^R \mathbf{Z}_{\varepsilon,j},\ \text{where } B^R = \Sigma_\varepsilon \boldsymbol{\beta}' \left(\boldsymbol{\beta} \Sigma_\varepsilon \boldsymbol{\beta}' + \Theta_\varepsilon\right)^{-1}. \end{aligned}$$

In the third step, we replace $x_{i,j}$ with $\hat{x}_{i,j}^B$ as well as $\varepsilon_{i,j}$ with $\hat{\varepsilon}_{i,j}^R$ and estimate γ by a fixed-effect regression. This is the factor score fixed-effect regression estimator, γ_{FSFE}:

$$\gamma_{FSFE} = \left[\sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' \left(Q\hat{\varepsilon}_j^R\right)\right]^{-1} \left[\sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' \left(Q\hat{x}_j^B\right)\right],$$

Now, let's focus on:

$$\begin{aligned}
\sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' \left(Q\hat{x}_j^B\right) &= \sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' \left(QA^B\mathbf{Z}_{x,j}\right) \\
&= \sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' \left[QA^B\left(\boldsymbol{\alpha}x_j + \boldsymbol{\epsilon}_{x,j}\right)\right] \\
&= \sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' \left[Q\left(\boldsymbol{\alpha}'\Theta_x\boldsymbol{\alpha}\right)^{-1}\boldsymbol{\alpha}'\Theta_x\left(\boldsymbol{\alpha}x_j + \boldsymbol{\epsilon}_{x,j}\right)\right] \\
&= \sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' Qx_j + \sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' \left[Q\left(\boldsymbol{\alpha}'\Theta_x\boldsymbol{\alpha}\right)^{-1}\boldsymbol{\alpha}'\Theta_x\boldsymbol{\epsilon}_{x,j}\right] \\
&= \sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' Q\left(\gamma\varepsilon_j + u_j + \nu_j\right) + \sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' \left[Q\left(\boldsymbol{\alpha}'\Theta_x\boldsymbol{\alpha}\right)^{-1}\boldsymbol{\alpha}'\Theta_x\boldsymbol{\epsilon}_{x,j}\right] \\
&= \gamma\sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' \left(Q\varepsilon_j\right) + \sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' Q\nu_j + \sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' \left[Q\left(\boldsymbol{\alpha}'\Theta_x\boldsymbol{\alpha}\right)^{-1}\boldsymbol{\alpha}'\Theta_x\boldsymbol{\epsilon}_{x,j}\right]
\end{aligned}$$

Now, consider the term $\sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' \left(Q\hat{\varepsilon}_j^R\right)$

$$\begin{aligned}
\sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' \left(Q\hat{\varepsilon}_j^R\right) &= \sum_{j=1}^{J} \left(B^R\mathbf{Z}_{\varepsilon,j}\right)' \left(QB^R\mathbf{Z}_{\varepsilon,j}\right) \\
&= \sum_{j=1}^{J} \left[B^R\left(\boldsymbol{\beta}\varepsilon_j + \boldsymbol{\epsilon}_{\varepsilon,j}\right)\right]' \left[QB^R\left(\boldsymbol{\beta}\varepsilon_j + \boldsymbol{\epsilon}_{\varepsilon,j}\right)\right]
\end{aligned}$$

Under *Assumptions A2a and A2b*, the following equalities hold:

$$\begin{aligned}
\lim_{J\to\infty} \frac{1}{J} \sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' Q\nu_j &= 0, \\
\lim_{J\to\infty} \frac{1}{J} \sum_{j=1}^{J} \left(\hat{\varepsilon}_j^R\right)' \left[Q\left(\boldsymbol{\alpha}'\Theta_x\boldsymbol{\alpha}\right)^{-1}\boldsymbol{\alpha}'\Theta_x\boldsymbol{\epsilon}_{x,j}\right] &= 0.
\end{aligned}$$

Furthermore,

$$\lim_{J\to\infty}\frac{1}{J}\sum_{j=1}^{J}\left(\hat{\varepsilon}_j^R\right)'(Q\varepsilon_j) = Cov\left(\hat{\varepsilon}_j^R,\varepsilon_j\right),$$

$$\lim_{J\to\infty}\frac{1}{J}\sum_{j=1}^{J}\left[B^R(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j})\right]'\left[QB^R(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j})\right] = Cov\left(B^R(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j}),QB^R(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j})\right)$$

As a result,

$$\lim_{J\to\infty}\gamma_{FSFE}=\gamma\left[Cov\left(B^R(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j}),QB^R(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j})\right)\right]^{-1}Cov\left(\hat{\varepsilon}_j^R,\varepsilon_j\right),$$

and we now show that

$$\left[Cov\left(B^R(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j}),QB^R(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j})\right)\right]^{-1}Cov\left(\hat{\varepsilon}_j^R,\varepsilon_j\right)=1.$$

Consider:

$$\begin{aligned}
Cov\left(\hat{\varepsilon}_j^R,\varepsilon_j\right) &= Cov\left(B^R\mathbf{Z}_{\varepsilon,j},\varepsilon_j\right)=Cov\left(B^R(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j}),Q\varepsilon_j\right)= \\
&= B^R\boldsymbol{\beta}Var(\varepsilon_j)Q=B^R\boldsymbol{\beta}\Sigma_\varepsilon Q \\
&= \Sigma_\varepsilon\boldsymbol{\beta}'(\boldsymbol{\beta}\Sigma_\varepsilon\boldsymbol{\beta}'+\Theta_\varepsilon)^{-1}\boldsymbol{\beta}\Sigma_\varepsilon Q
\end{aligned}$$

Next, note that:

$$\begin{aligned}
Cov\left(B^R(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j}),QB^R(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j})\right) &= B^RVar(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j})\left(B^R\right)'Q \\
&= \Sigma_\varepsilon\boldsymbol{\beta}'(\boldsymbol{\beta}\Sigma_\varepsilon\boldsymbol{\beta}'+\Theta_\varepsilon)^{-1}\boldsymbol{\beta}\Sigma_\varepsilon Q
\end{aligned}$$

Therefore:

$$\begin{aligned}
\lim_{J\to\infty}\gamma_{FSFE} &= \gamma\left[Cov\left(B^R(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j}),QB^R(\boldsymbol{\beta}\varepsilon_j+\boldsymbol{\epsilon}_{\varepsilon,j})\right)\right]^{-1}Cov\left(\hat{\varepsilon}_j^R,\varepsilon_j\right) \\
&= \gamma\left[\Sigma_\varepsilon\boldsymbol{\beta}'(\boldsymbol{\beta}\Sigma_\varepsilon\boldsymbol{\beta}'+\Theta_\varepsilon)^{-1}\boldsymbol{\beta}\Sigma_\varepsilon Q\right]^{-1}\left[\Sigma_\varepsilon\boldsymbol{\beta}'(\boldsymbol{\beta}\Sigma_\varepsilon\boldsymbol{\beta}'+\Theta_\varepsilon)^{-1}\boldsymbol{\beta}\Sigma_\varepsilon Q\right] \\
&= \gamma.
\end{aligned}$$

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Table 1

Correlation between Local Head Start Spending Per Poor Person 1968 and Preschool Enrollment

Panel A: Full Sample

	1963-1964	1965	1966	1967	1968	1969	1970
Local HS spending per poor person in 1968	0.000309	0.0006	0.00452	0.00308	0.00361	0.00157	0.00132
	[0.000982]	[0.00113]	[0.00180]	[0.00177]	[0.00224]	[0.00185]	[0.00175]
Observations	6122	5099	5442	5571	6178	5616	1627
R-squared	0.012	0.021	0.021	0.019	0.017	0.034	0.048
Robust standard errors in brackets							

Panel B: Limiting to Mothers with no More than a High School Degree

	1963-1964	1965	1966	1967	1968	1969	1970
Local HS spending per poor person in 1968	0.000158	0.000692	0.00525	0.00392	0.0044	0.00342	0.00463
	[0.000754]	[0.00102]	[0.00176]	[0.00162]	[0.00187]	[0.00162]	[0.00169]
Observations	5637	4642	4875	4990	5538	4872	1308
R-squared	0.008	0.013	0.023	0.022	0.016	0.022	0.047

Panel C: Include Mother FE

	ALL	<=HS
Local HS Spending (=0 in years prior to 1966)	0.00183	0.00251
	[0.000978]	[0.00101]
Observations	0.894	0.882
R-squared	12938	11670

Robust standard errors in brackets.

OLS results in panels A and B of a regression of an indicator for preschool enrollment on a measure of local Head Start Spending includes controls for offspring gender, birth order dummies, maternal race, maternal education, maternal age, marital status and family income at birth. FE Regressions in panel C include controls for gender, birth order, maternal age, marital status, family income at birth and a quadratic in year of birth. Local (county) Head Start spending per poor person in 1968 ranges from $3 to $29.

Table 2
Determinants of Investment Across Families: Dependent Variable= Preschool

Panel A: OLS Regressions	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Standardized Birth Weight	-0.00247							-0.00297
	[0.00402]							[0.00419]
Low Birth Weight		0.00111						
		[0.0107]						
Weeks of Gestation at Birth			-0.000938					
			[0.00112]					
Premature birth				0.0151				
				[0.0113]				
Any abnormal language expression or reception - 3 years					-0.0175			
					[0.00981]			
Standardized 8-Month Mental Bayley						0.000834	0.00384	0.00158
						[0.00354]	[0.00448]	[0.00369]
Standardized 8-Month Motor Bayley							-0.00517	
							[0.00423]	
Maternal Education at Birth	0.00392	0.00391	0.00386	0.00387	0.00546	0.00391	0.00391	0.00392
	[0.000820]	[0.000820]	[0.000819]	[0.000819]	[0.00140]	[0.000819]	[0.000819]	[0.000820]
Maternal Age at Birth	0.000277	0.00027	0.000236	0.00026	0.00124	0.00024	0.000245	0.00027
	[0.000762]	[0.000762]	[0.000762]	[0.000762]	[0.00108]	[0.000761]	[0.000762]	[0.000762]
Family income (real) at pregnancy in $1000	0.000216	0.000212	0.000203	0.000202	0.00129	0.000211	0.000204	0.000216
	[0.000286]	[0.000286]	[0.000286]	[0.000286]	[0.000382]	[0.000286]	[0.000286]	[0.000286]
Married	-0.00713	-0.00722	-0.00859	-0.00848	-0.01	-0.00754	-0.0073	-0.0071
	[0.00987]	[0.00987]	[0.00987]	[0.00987]	[0.0130]	[0.00985]	[0.00986]	[0.00987]
Black	0.0208	0.0208	0.0197	0.0196	0.0461	0.0202	0.0211	0.0207
	[0.0479]	[0.0479]	[0.0478]	[0.0478]	[0.0586]	[0.0478]	[0.0478]	[0.0479]
White	-0.102	-0.102	-0.103	-0.103	-0.0775	-0.103	-0.103	-0.102
	[0.0472]	[0.0472]	[0.0472]	[0.0472]	[0.0578]	[0.0472]	[0.0472]	[0.0472]
Hispanic	-0.0611	-0.0614	-0.0617	-0.0618	-0.0178	-0.0616	-0.0613	-0.0611
	[0.0543]	[0.0543]	[0.0542]	[0.0542]	[0.0679]	[0.0542]	[0.0542]	[0.0543]
Male	-0.013	-0.0134	-0.0135	-0.0134	-0.00798	-0.0134	-0.0136	-0.0129
	[0.00662]	[0.00659]	[0.00658]	[0.00658]	[0.00902]	[0.00657]	[0.00658]	[0.00663]
First Birth	0.0373	0.0377	0.0369	0.037	0.0555	0.0373	0.0391	0.0368
	[0.0132]	[0.0132]	[0.0132]	[0.0132]	[0.0184]	[0.0132]	[0.0132]	[0.0132]
Second Birth	0.0349	0.0352	0.0348	0.035	0.0522	0.0349	0.0355	0.0347
	[0.0112]	[0.0112]	[0.0112]	[0.0112]	[0.0158]	[0.0112]	[0.0112]	[0.0112]
Third or Fourth Birth	0.0112	0.0114	0.0105	0.0107	0.0202	0.0112	0.0114	0.0111
	[0.00958]	[0.00957]	[0.00957]	[0.00958]	[0.0135]	[0.00957]	[0.00957]	[0.00958]
Observations	10156	10156	10132	10132	4885	10167	10160	10156
R-squared	0.047	0.047	0.048	0.048	0.055	0.047	0.047	0.047

Panel B: Maternal FE Regressions	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Standardized Birth Weight	0.00039							0.00151
	[0.00857]							[0.00880]
Low Birth Weight		-0.00928						
		[0.0191]						
Weeks of Gestation at Birth			-0.00209					
			[0.00190]					
Premature birth				0.00789				
				[0.0187]				
Any abnormal language expression or reception - 3 years					-0.00583			
					[0.0173]			
Standardized 8-Month Motor Bayley							-0.00298	
							[0.00680]	
Standardized 8-Month Mental Bayley						-0.00309	-0.00147	-0.00327
						[0.00569]	[0.00679]	[0.00584]
Observations	10157	10157	10133	10133	4886	10168	10161	10157
R-squared	0.739	0.739	0.739	0.739	0.758	0.738	0.738	0.739

Standard errors in brackets. AMC FE and year of birth indicators included in top panel. Maternal FE and year of birth indicators included in bottom panel. Note that abnormal language at age 3 is missing for half the sample for reasons unknown

Table 3A

Are Investments and Endowments Complements in the Production of Child Human Capital?

	IQ 4	**IQ 7**	**Read**	**Math**
Investment(preschool)*8-Month Bayley	0.165	0.104	0.0298	0.16
	[0.0420]	[0.0415]	[0.0473]	[0.0507]
Standardized 8-Month Mental Bayley	0.152	0.164	0.0199	0.0381
	[0.0298]	[0.0293]	[0.0315]	[0.0337]
Investment(Preschool)	0.163	0.0196	-0.0287	0.00878
	[0.0382]	[0.0395]	[0.0411]	[0.0440]
BO*8-Month Bayley	-0.00707	-0.0103	0.00393	0.00798
	[0.00709]	[0.00711]	[0.00763]	[0.00817]
Maternal Age at Birth	-0.0074	-0.0224	-0.00563	0.0177
	[0.0224]	[0.0226]	[0.0235]	[0.0252]
Family income (real) at pregnancy in $1000	0.000584	0.00101	-0.00134	8.57E-05
	[0.00117]	[0.00120]	[0.00125]	[0.00134]
Married	-0.0391	-0.0175	-0.0477	-0.101
	[0.0494]	[0.0503]	[0.0522]	[0.0558]
Male	-0.111	0.0282	-0.17	-0.0746
	[0.0214]	[0.0219]	[0.0227]	[0.0243]
First Birth	-0.202	-0.0144	-0.0598	-0.133
	[0.101]	[0.0852]	[0.0886]	[0.0948]
Second Birth	-0.0788	-0.00775	-0.0385	-0.00745
	[0.0789]	[0.0691]	[0.0717]	[0.0767]
Third or Fourth Birth	-0.0199	0.00099	0.013	0.0223
	[0.0539]	[0.0501]	[0.0519]	[0.0556]
Observations	9956	9229	9204	9205
R-squared	0.844	0.845	0.815	0.787

Standard errors in brackets. Maternal FE and all controls from previous table included in all regressions.

Table 3B
Heterogeneity in Impact of Preschool on 7 Year IQ

	OLS	FE	OLS	FE	OLS	FE	OLS	FE	OLS	FE	OLS	FE	OLS	FE
Preschool	0.193	0.01	0.197	0.0102	0.221	-0.0476	0.182	-0.00642	0.19	0.0223	0.00722	-0.112	0.0596	-0.0265
	[0.0257]	[0.0393]	[0.0256]	[0.0391]	[0.0492]	[0.0770]	[0.0353]	[0.0498]	[0.0251]	[0.0388]	[0.0925]	[0.121]	[0.109]	[0.152]
Preschool*Standardized Birth Weight			-0.0331	-0.00195									-0.0406	-0.0361
			[0.0303]	[0.0455]									[0.0315]	[0.0476]
Preschool*Birth Order					-0.0107	0.0158							-0.0161	0.0174
					[0.0132]	[0.0192]							[0.0128]	[0.0189]
Preschool*Male							0.0217	0.0366					0.0209	0.0465
							[0.0502]	[0.0681]					[0.0496]	[0.0683]
Preschool*8-Month Bayley									0.0679	**0.108**			0.0447	**0.114**
									[0.0301]	[0.0408]			[0.0347]	[0.0484]
Preschool*advanced social/emotional development											0.65	**0.614**	0.648	**0.403**
											[0.261]	[0.329]	[0.265]	[0.337]
Preschool*normal social/emotional development											0.195	0.127	0.18	-0.0392
											[0.0960]	[0.125]	[0.104]	[0.142]
Standardized Birth Weight			0.127	0.146									0.0758	0.113
			[0.0109]	[0.0211]									[0.0113]	[0.0218]
Standardized 8-Month Mental Bayley									0.186	0.127			0.168	0.121
									[0.00959]	[0.0148]			[0.0111]	[0.0167]
Advanced Social/Emotional Development											0.491	0.233	0.0926	0.017
											[0.0758]	[0.0997]	[0.0781]	[0.102]
Normal Social/Emotional Development											0.265	0.0989	0.00523	-0.045
											[0.0324]	[0.0446]	[0.0353]	[0.0476]
Observations	9339	9340	9330	9331	9228	9229	9339	9340	9339	9340	9325	9326	9205	9206
R-squared	0.289	0.839	0.3	0.842	0.289	0.84	0.289	0.839	0.322	0.844	0.298	0.84	0.327	0.848

Standard errors in brackets. Regressions include controls listed previously. All OLS regressions include controls for maternal characteristics (education, age, race, family income), offspring characteristics (birth order, gender, year of birth and AMC indicators). All FE regressions include controls for maternal age, family income, birth order, gender and year of birth indicators. Note: only 208 observations with advanced social/emotional development at 8 months of age.

Table 4
A Measure of Parental Investment: Factor Analysis

Variable	Factor Loadings	Uniqueness	Fraction of Variance Explained by Factor	Scoring Coefficients (Bartlett Method)
Appearance of Child	0.1751	0.9693	0.0378	0.0622
Responsivenes of Mother	0.5889	0.6532	0.3973	0.2860
Affection	0.6213	0.6140	0.4384	0.3146
Focus on Child	0.4744	0.7750	0.2650	0.1997
Management of Child	0.3324	0.8895	0.1336	0.1253
Attention to Child	0.3056	0.9066	0.1134	0.1143
Handling of Child	0.4941	0.7558	0.2863	0.2076

Number of observations = 31538

Table 5
Estimating Endowments from a Newborn Health Production Function

	Birth Weight		Head Circumference		Body Length		Gestational Length	
	OLS	**FE**	**OLS**	**FE**	**OLS**	**FE**	**OLS**	**FE**
Pre pregnancy weight	0.0165	0.0457	0.0178	0.0358	0.0185	0.0366	0.000169	0.0288
	[0.00170]	[0.00438]	[0.00179]	[0.00482]	[0.00173]	[0.00499]	[0.00121]	[0.00353]
Pre-pregnancy weight squared	-2.50E-05	-4.68E-05	-3.35E-05	-4.63E-05	-3.67E-05	-5.27E-05	8.14E-06	-2.48E-05
	[5.62e-06]	[1.34e-05]	[5.93e-06]	[1.48e-05]	[5.73e-06]	[1.53e-05]	[4.03e-06]	[1.09e-05]
Weight gain during pregnancy	0.0532	0.0689	0.0335	0.0413	0.0318	0.0382	0.0422	0.0595
	[0.00185]	[0.00249]	[0.00197]	[0.00286]	[0.00191]	[0.00296]	[0.00128]	[0.00192]
Weight gain squared	-0.000454	-0.000445	-0.000242	-0.000204	-0.000252	-0.000195	-0.000508	-0.000509
	[3.23e-05]	[4.06e-05]	[3.40e-05]	[4.51e-05]	[3.29e-05]	[4.66e-05]	[2.25e-05]	[3.22e-05]
Cigarettes per day	-0.0264	-0.000747	-0.0197	0.00294	-0.0235	0.000388	-0.00203	0.00704
	[0.00176]	[0.00371]	[0.00185]	[0.00406]	[0.00179]	[0.00420]	[0.00125]	[0.00300]
Cigarettes per day squared	0.000407	7.43E-05	0.000266	-8.84E-05	0.000388	7.14E-05	3.62E-05	-6.80E-05
	[5.59e-05]	[9.42e-05]	[5.86e-05]	[0.000103]	[5.67e-05]	[0.000107]	[3.97e-05]	[7.64e-05]
Trying to get pregnant	-0.0576	0.0644	-0.0335	0.0532	-0.0382	0.0427	-0.0242	0.0577
	[0.0200]	[0.0243]	[0.0210]	[0.0269]	[0.0203]	[0.0279]	[0.0141]	[0.0194]
Maternal age at birth	-0.0024	-0.0668	0.00238	-0.0451	0.000861	-0.0554	-0.00286	-0.08
	[0.00163]	[0.0148]	[0.00172]	[0.0163]	[0.00166]	[0.0168]	[0.00116]	[0.0120]
Family income (real) at pregnancy in $1000	0.00117	-0.000324	0.00231	0.00062	0.000627	-0.000567	-0.000139	0.000565
	[0.000594]	[0.000792]	[0.000624]	[0.000873]	[0.000603]	[0.000906]	[0.000420]	[0.000638]
Married	-0.00464	-0.0926	-0.0327	-0.0467	-0.0207	-0.0895	-0.000872	-0.0329
	[0.0204]	[0.0321]	[0.0214]	[0.0354]	[0.0207]	[0.0367]	[0.0145]	[0.0260]
Observations	15803	15804	15185	15186	15118	15119	15913	15914
R-squared	0.221	0.777	0.183	0.757	0.171	0.718	0.115	0.669

Standard errors in brackets. All Regressions include child gender, birth order indicators and year of birth indicators. OLS regressions also include city of birth FE and controls for maternal education.

Table 6
Factor Loadings and Variances of Uniqueness

Variable	Factor Loadings	Variance of Uniqueness	Fraction of Total Variance Explained by the Factor	Scoring Coefficients (Regression Method)
Residual of Birth Weight	0.805	0.351	0.665	0.46611
Residual of Head Circumference at Birth	0.701	0.509	0.544	0.28024
Residual of Body Length at Birth	0.645	0.585	0.489	0.22828
Residual of Weeks of Gestation	0.353	0.876	0.259	0.09346

Number of Observations = 13147

Table 7

How Endowments Affect Postnatal Investments (High-Quality Parenting) Within Family

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(11)	(12)
Standardized Birth Weight	0.0954 [0.0295]									
Standardized Birth Weight Residual Endowment		0.0275 [0.0106]								
Standardized Gestational Length			0.066 [0.0198]							
Standardized Gestational Length Residual Endowment				0.0259 [0.0106]						
Standardized Body Length					0.0358 [0.0213]					
Standardized Body Length Residual Endowment						0.0171 [0.0106]				
Standardized Head Circumference							0.0427 [0.0239]			
Standardized Head Circumference Residual Endowment								0.0171 [0.0107]		
Standardized Factor Scores of Birth Weight, Gestational Length, Body Length, and Head Circumference									0.072 [0.0250]	
Standardized Factor scores from Endowment Residuals of Birth Weight, Gestational Length, Body Length, and Head Circumference residual										0.0265 [0.0107]
Observations	12611	11526	12582	11496	12469	11402	12515	11444	12420	11354
R-squared	0.664	0.677	0.664	0.677	0.668	0.682	0.665	0.679	0.67	0.683

Standard errors in brackets. Also included are maternal fixed effect, indicators for child birth order, indicators for year of birth, controls for offspring gender, income and marital status at time of birth.

Table 8

How Do Reinforcing Investments Vary with Fertility?

Panel A: Outcome = Postnatal Investments	**2 kids**	**>2 kids**	**>3 kids**
Birth Weight Residual Endowment	0.05	0.0823	0.138
	[0.0255]	[0.0459]	[0.158]
Observations	9589	1937	262
R-squared	0.73	0.448	0.475

Panel B: Outcome = 7 Year IQ	**2 kids**	**>2 kids**	**>3 kids**
Birth Weight Residual Endowment	0.118	0.215	0.444
	[0.0246]	[0.0424]	[0.123]
Observations	8250	1776	251
R-squared	0.866	0.696	0.633

Standard errors in brackets. All regressions include full controls and maternal FE.

Appendix Table 1
Sample Means

	Overall		Std Dev. Within Family	Mean Difference Within Family			
				Raw		Standardized	
Maternal Characteristics	mean	Std Dev		mean	Std Dev	mean	Std Dev
Maternal Age	25.68	5.09					
Maternal Education	11.11	4.59					
Socioeconomic Index (Duncan)	50.44	21.25					
Married	0.86						
Black	0.42						
White	0.54						
Hispanic	0.03						
First Birth	0.13						
Second Birth	0.23						
Third or Fourth Birth	0.35						
Male	0.50						
Investments							
Maternal Investment	-0.07	0.83	0.48	0.54	0.9		
Preschool Attendance	0.13			0.15			
Cognitive Measures							
8 Month Bayley Score	79.32	6.04	3.30	4.6	5.8	0.75	0.93
4 year IQ	98.77	16.70	7.00	11.9	9.7	0.72	0.58
7 Year IQ	96.31	14.80	6.10	10.7	8.8	0.7	0.58
Newborn Health							
Birth Weight (kg)	3.18	0.57	0.23	0.44	0.49	0.66	0.73
Gestation at Birth (weeks)	39.23	3.00	1.70	3.2	5.2		
Lbw	0.11			0.14			
Premature	0.10			0.17			
Head Circumference	33.68	1.59	0.75				
Body Length	50.02	2.75	1.40				
Endowment Factor (standardized measures of newborn health)	0.00	0.93	0.41				
Endowment Factor (residuals from newborn health production function)	0.00	0.87	0.87				

Standardized: distribution relocated to mean zero and rescaled to variance one.

Appendix Table 2

Predictive Abilities of 8 Month Bayley and Birth Weight- OLS and FE Results

OLS	Any Cognitive delay - 1 year			Abnormal Language Reception - 3 year			Abnormal Language Expression - 3 Year			IQ - 4 year			IQ - 7 year			Reading - 7 year			Math - 7 year		
8 Month Mental Bayley - Standardized	-0.157		-0.154	-0.0461		-0.0407	-0.0508		-0.0474	0.205		0.184	0.191		0.174	0.0851		0.0711	0.141		0.13
	[0.00231]		[0.00237]	[0.00695]		[0.00714]	[0.00691]		[0.00709]	[0.00940]		[0.00977]	[0.00857]		[0.00888]	[0.00939]		[0.00976]	[0.00960]		[0.00998]
Birth Weight - Standardized		-0.0603	-0.0189		-0.034	-0.0239		-0.0263	-0.0148		0.135	0.0782		0.12	0.0667		0.0767	0.0555		0.081	0.0421
		[0.00309]	[0.00273]		[0.00751]	[0.00770]		[0.00753]	[0.00769]		[0.0105]	[0.0107]		[0.00979]	[0.0100]		[0.0102]	[0.0106]		[0.0105]	[0.0109]
Observations	11863	11851	11851	5375	5366	5366	5336	5327	5327	10303	10292	10292	10936	10925	10925	10897	10886	10886	10896	10885	10885
R-squared	0.29	0.042	0.293	0.079	0.074	0.08	0.047	0.04	0.048	0.264	0.242	0.267	0.293	0.271	0.296	0.163	0.161	0.165	0.111	0.099	0.113
test of equality of coefficients			F (1, 22841) = 1124			F (1,5351) = 2.1			F (1,5312) = 2.1			F (1,10280) = 42.31			F (1,10915) = 51.57			F (1, 10876) = 0.99			F (1, 10875) = 28.07
			(p=0.000)			(p=0.1476)			(p=0.0047)			(p=0.0000)			(p=0.0000)			(p=0.3206)			(p=0.0000)

Maternal race, education, income, marital status, age, child gender, birth order, AMC and year of birth also included

Maternal FE	Any Cognitive delay - 1 year			Abnormal Language Reception - 3 year			Abnormal Language Expression - 3 Year			IQ - 4 year			IQ - 7 year			Reading - 7 year			Math - 7 year		
8 Month Mental Bayley - Standardized	-0.148		-0.143	-0.0284		-0.0286	-0.0233		-0.0183	0.128		0.116	0.14		0.124	0.0422		0.0295	0.0923		0.0779
	[0.00389]		[0.00398]	[0.0121]		[0.0124]	[0.0121]		[0.0124]	[0.0138]		[0.0142]	[0.0124]		[0.0127]	[0.0138]		[0.0142]	[0.0152]		[0.0156]
Birth Weight - Standardized		-0.0801	-0.03		-0.00947	-0.000858		-0.0413	-0.0356		0.11	0.0709		0.143	0.1		0.0888	0.0789		0.116	0.0899
		[0.00689]	[0.00624]		[0.0176]	[0.0180]		[0.0181]	[0.0185]		[0.0197]	[0.0201]		[0.0183]	[0.0186]		[0.0195]	[0.0201]		[0.0215]	[0.0221]
Observations	11864	11852	11852	5376	5367	5367	5337	5328	5328	10304	10293	10293	10937	10926	10926	10898	10887	10887	10897	10886	10886
R-squared	0.735	0.662	0.736	0.761	0.76	0.761	0.75	0.75	0.75	0.839	0.837	0.84	0.837	0.835	0.838	0.801	0.802	0.802	0.757	0.757	0.759
test of equality of coefficients			F(1,4659) = 196			F (1,1719) = 1.36			F (1,1694) = 0.51			F (1,3875) = 2.80			F (1,4223) = 0.98			F (1,4200) = 3.29			F (1,4197) = 0.12
			(p=0.000)			(p=0.2437)			(p=0.4752)			(p=0.0944)			(p=0.3219)			(p=0.0700)			(p=0.7266)

Maternal age, marital status, income, child gender, birth order and year of birth also included

Standard errors in brackets

Appendix Table 3

Parental Time, Parental Warmth and the Home Score - Evidence from the PSID

Panel A: Correlation between Parental Time and Parental Warmth Scale

Dependent Variable= Standardized Parental Warmth Scale															
	OLS												Fixed Effect		
Age Range	One to Eight Months			One to Twelve Months			One to Sixteen Months			One to Twenty Months			All Ages		
Standardized Parental Quality	0.161			0.13			0.119			0.0894			0.092		
Time	(0.087)			(0.060)			(0.045)			(0.041)			(0.036)		
Standardized Parental Quality		0.117			0.111			0.103			0.0654			0.069	
Time (Weekday)		(0.080)			(0.056)			(0.041)			(0.039)			(0.035)	
Standardized Parental Quality			0.0748			0.107			0.0776			0.0766			0.062
Time (Weekend)			(0.088)			(0.066)			(0.053)			(0.043)			(0.031)
Observations	66	69	67	130	134	131	187	190	188	256	259	258	2663	2690	2703
R-squared	0.292	0.248	0.262	0.127	0.120	0.110	0.143	0.136	0.123	0.117	0.110	0.110	0.136	0.128	0.130

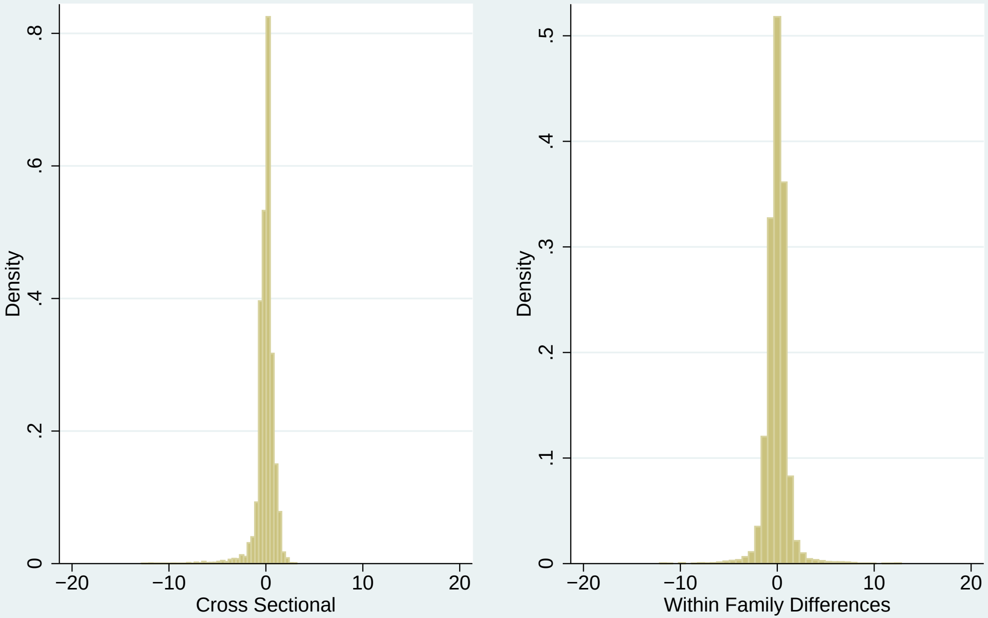
Panel B: Correlation between Home Score and Parental Warmth Scale

Dependent Variable= Standardized Parental Warmth Scale															
	OLS												Fixed Effect		
Age Range	One to Eight Months			One to Twelve Months			One to Sixteen Months			One to Twenty Months			All Ages		
Full HOME Score	0.394			0.478			0.402			0.350			0.006		
	(0.257)			(0.186)			(0.152)			(0.121)			(0.0391)		
HOME Cognitive Stimulation		0.521			0.469			0.465			0.453			0.0128	
Subscore		(0.221)			(0.169)			(0.145)			(0.116)			(0.026)	
HOME Emotional Support			-0.016			0.218			0.114			0.032			-0.004
Subscore			(0.204)			(0.155)			(0.117)			(0.089)			(0.035)
Observations	72	72	72	140	140	140	200	200	200	274	274	274	3334	3334	3334
R-squared	0.248	0.294	0.201	0.148	0.149	0.099	0.146	0.168	0.102	0.122	0.162	0.079	0.100	0.104	0.097

Robust standard errors in parentheses

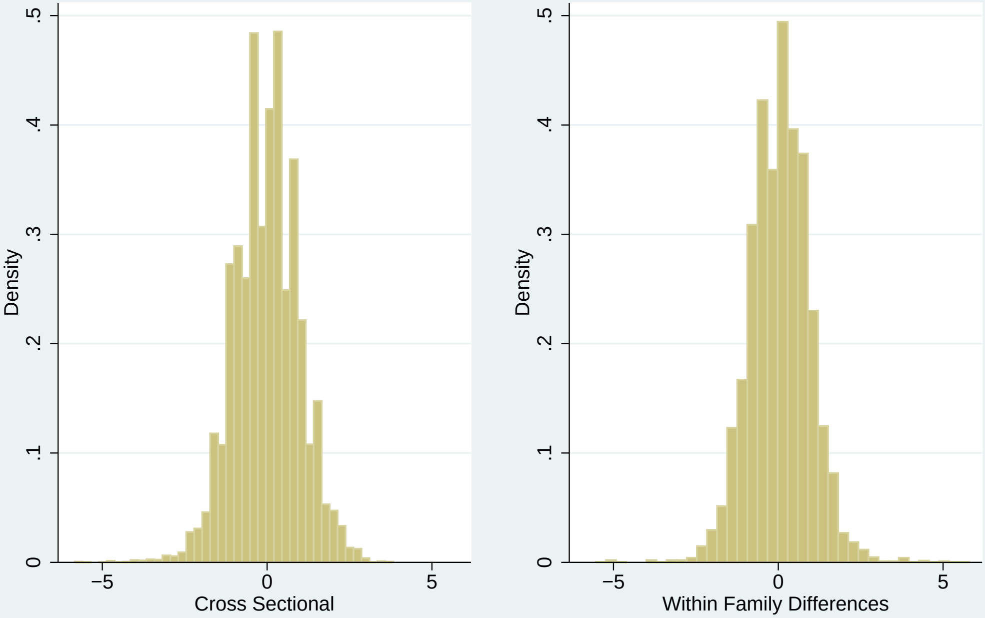
All regressions include controls for maternal characteristics (age, education, income, cognitive skills as measured by a reading test, and noncognitive skills as measured by the Rosenberg Self-Esteem Score and the Pearlin Self-Efficacy Scale) and offspring characteristics (race, gender, birth order, and age in months). In the fixed-effect specification, we add a polynomial of fourth-order in age as well as the interaction of age with marital status at birth, race of the child, and gender of the child.

Figure 1: Histograms of 8 Month Bayley



Scores standardized mean 0, standard deviation 1

Figure 2: Histograms of 7 Year IQ



Scores standardized mean 0, standard deviation 1

Figure 3: Share in Preschool at Age 4

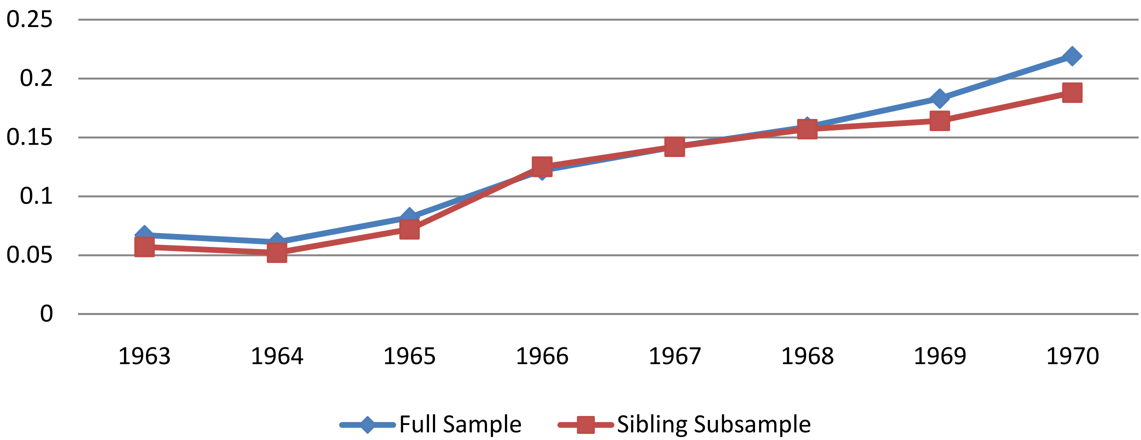
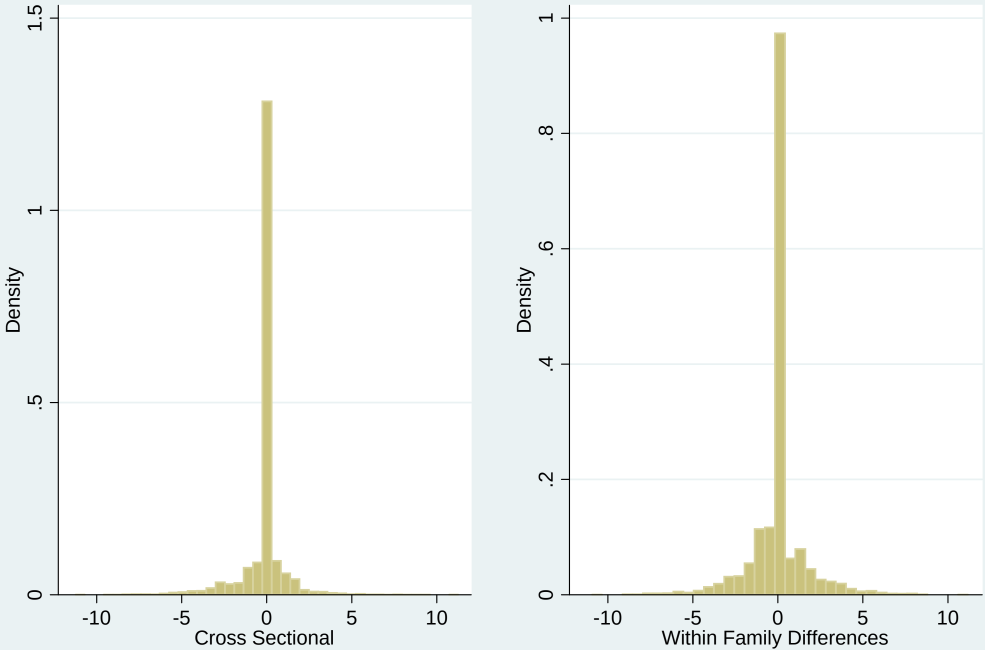


Figure 4: Histograms of Quality of Parenting



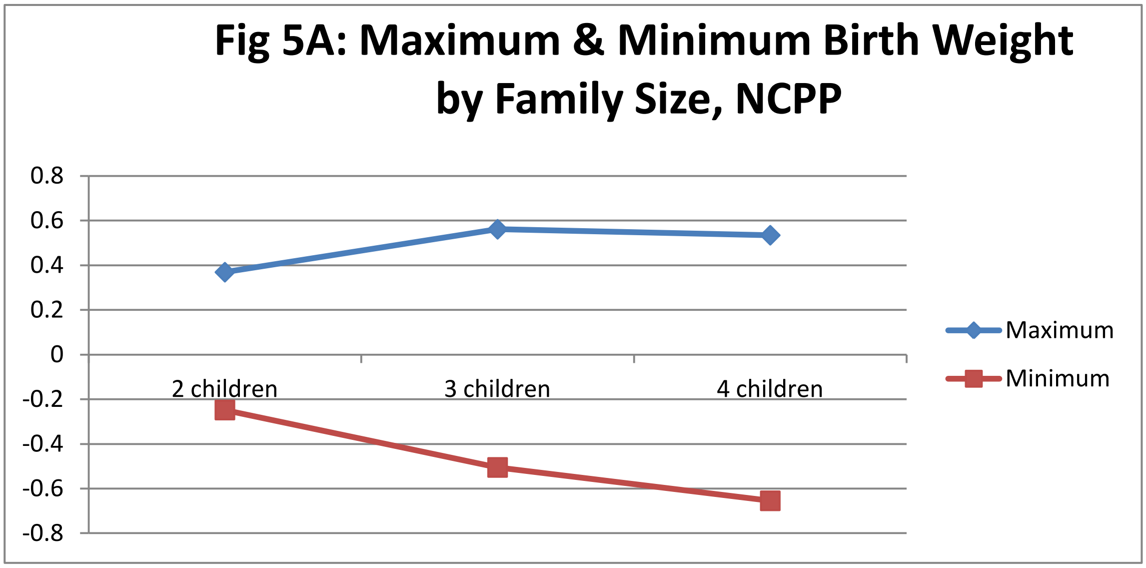


Fig 5A: Maximum & Minimum Birth Weight by Family Size, NCPP
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
2 children
3 children
4 children
Maximum
Minimum

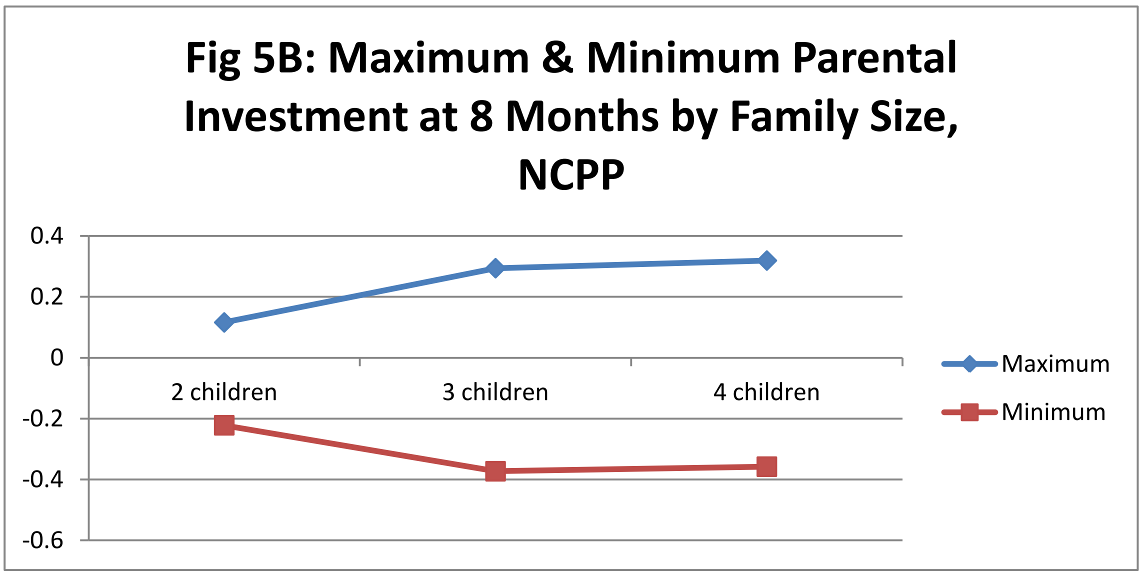


Fig 5B: Maximum & Minimum Parental Investment at 8 Months by Family Size, NCPP
0.4
0.2
0
-0.2
-0.4
-0.6
2 children
3 children
4 children
Maximum
Minimum

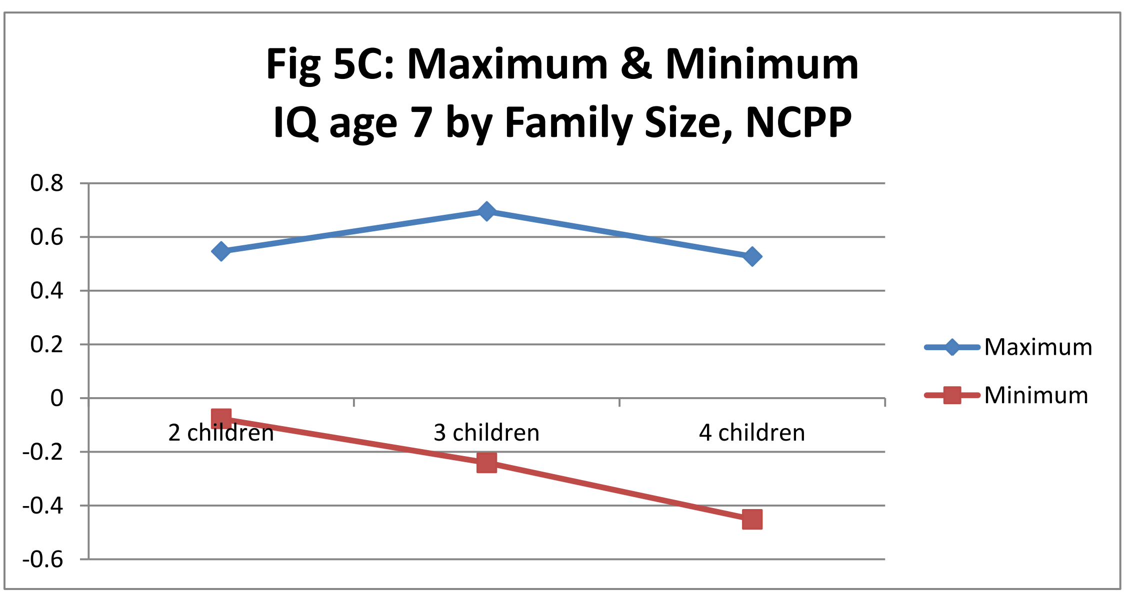


Fig 5C: Maximum & Minimum IQ age 7 by Family Size, NCPP
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
2 children
3 children
4 children
Maximum
Minimum

Figure 6A: Max and Min Birth Weight by Fertility, CNLSY-79

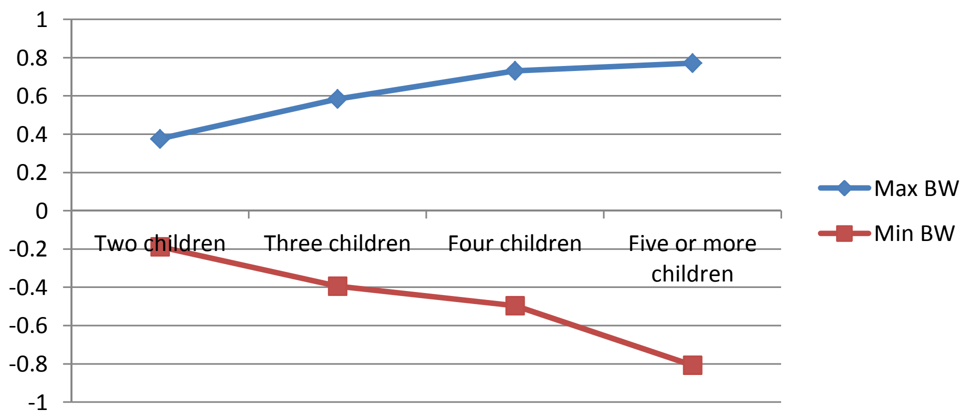


Figure 6B: Max and Min PPVT scores by Fertility, CNLSY-79

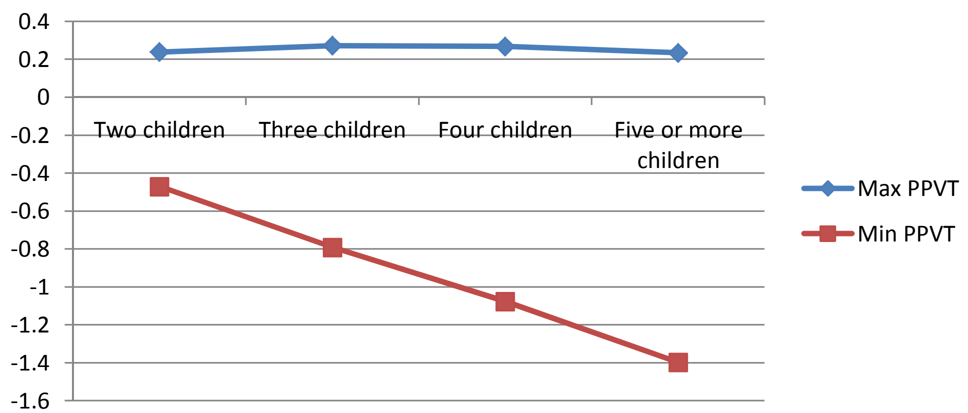


Figure 6C: Birth Order, PPVT and Math Scores within Family, CNLSY-79