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DEBT FRAGILITY AND BAILOUTS

Russell Cooper

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ABSTRACT

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Russell Cooper
Department of Economics
The Pennsylvania State University
611 Kern
State College, PA 16802
and NBER
russellcoop@gmail.com

Debt Fragility and Bailouts*

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March 30, 2015

Abstract

This paper studies debt fragility and the sharing of the resulting strategic uncertainty through *ex post* bailouts. Default arises in equilibrium because of both fundamental shocks and beliefs. The probability of default depends on borrowing rates and, in equilibrium, on the beliefs of lenders about this probability. This interaction creates a strategic complementarity and thus the basis for strategic uncertainty. The paper analyzes the role of credible *ex post* bailouts as a means of sharing both fundamental and strategic uncertainty. While bailouts may occur in some states, debt fragility remains.

1 Motivation

This paper provides a framework for understanding **credible** guarantees in the presence of strategic uncertainty over the valuation of sovereign debt. The analysis is motivated by the recent experience of European countries who have witnessed large movements in the price (bond spreads) of their government debt and the consequent policy discussion of debt guarantees and bailout. What are the sources of the variations in these spreads? What are the effects of credible bailouts on the valuation of sovereign debt?

The framework addresses these questions.[1] The model highlights the interactions between fundamental and strategic uncertainty underlying the volatility of spreads. The first form of uncertainty arises from the stochastic nature of underlying economic variables, such as productivity. The second source of uncertainty reflects the interaction of investors, each responding to the beliefs of others. In this model, as in Calvo (1988), the power of expectations reflects a strategic complementarity linking the beliefs of investors about repayment, interest rates and default probabilities.[2] If investors believe that default is likely, a large

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†Department of Economics, The Pennsylvania State University; Research Associate, NBER, russellcoop@gmail.com

[1] This paper builds upon Cooper (2012). It has essentially the same model of fragility for a single country but it has new results regarding conditions for bailout. This paper also adds a third country, following the suggestion of Guillermo Ordonez.

[2] A similar logic is present in Cole and Kehoe (2000) though that paper has a dynamic element missing in this formulation. Eaton (1987) studies capital flight in a model with multiple equilibria generated by a complementarity through the government budget constraint. More recently, Corsetti and Dedola (2012) also use a variant of the Calvo model to look at the European debt situation. While that model shares foundations with Calvo (1988), one of the distinguishing features of this paper is the construction of a multi-country environment to study bailouts and guarantees and thus the sharing of strategic uncertainty.

premium will arise in equilibrium. Given this large premium, a country is indeed more likely to default. Taken together, these forms of uncertainty underlie the pricing of sovereign debt and its fragility.

As its main contribution, the framework is used to evaluate bailouts as a means to deal with volatile debt prices. Here a bailout entails the assumption of the debt obligations of one country by a federation to which it belongs. The analysis emphasizes a couple of points.

The provision of guarantees, i.e. an *ex ante* commitment to bailout, as in the standard models of deposit insurance, provide a means of sharing strategic uncertainty.[3] **But, these guarantees are valuable only if they are credible.**

The framework is used to provide sufficient conditions for the provision of an *ex post* bailout. There are two gains to a bailout. The first entails the avoidance of a default cost through the spreading of the debt obligation across federation members. The second gain, following the consumption smoothing motive highlighted in Cooper, Kempf, and Peled (2008), arises when a bailout yields a more equitable consumption allocation across households in countries within a federation compared to default. One leading case for bailouts is when the country on the verge of default has lower output and consumption, both with and without a bailout, than the country providing the bailout. In this case, the bailout leads to consumption smoothing across the countries in a federation.

Despite the gains to *ex post* bailouts, the debt of a country is still subject to strategic uncertainty. This is because bailouts are state contingent and thus do not occur for all realizations of fundamentals. As a consequence, default occurs in equilibrium and strategic uncertainty remains.

Overall, the results indicates that credible debt guarantees do matter: in some states a bailout occurs and this reduces default risk. In this sense, the strategic uncertainty is shared. But generally these guarantees, because they are required to be credible, do not eliminate default in all states. Consequently debt prices remain influenced by strategic uncertainty

2 Model

The model studies the issues of debt valuation and bailout in a multi-country setting with two periods.[4] The analysis focuses on the pricing of a predetermined stock of government debt as a function of the repayment decision of a second country issuing the debt and the bailout decision of second country (or a coalition of countries as in the European Union). The first and second country, of population size N_i for $i = 1, 2$ belong to the same coalition. The third country, of size N_3, consists of households outside of the coalition of countries involved in the bailout.[5]

[3] Bigelow, Cooper, and Ross (1993) and Cooper and Ross (1999) study guarantee funds in a variety of interactions in which complementarities are prominent.

[4] The interaction can be embedded in a dynamic equilibrium model allowing for the choice of B, for example, as in Cooper, Kempf, and Peled (2008). But the essence of determining the interest rate on the outstanding debt and the conditions for bailout are aptly captured by the interactions described here.

[5] The main results in Calvo (1988) only have insiders though international lending is mentioned in Section III. Guillermo Ordonez made the invaluable suggestion to amend the model to include households outside of the coaltion.

2.1 Households within the Coalition

Consider first the optimization problem of households in countries 1 and 2. These households are distinct from those in country 3 as they are part of a coalition in which a bailout is possible.

In the initial period, households make portfolio decisions. Two factors influence the return to debt holders. First, there is fundamental uncertainty over the future tax base of the borrower. This is represented through a productivity shock that determines the level of output and ultimately the tax base. Second, there is strategic uncertainty: the likelihood of repayment depends, in part, on the beliefs of the lenders. In the model, the fundamental and strategic uncertainty interact to determine asset prices.

At the start of the second period, the government in the country 1 decides to default or not. Repayment of the debt entails the reallocation of tax revenues from domestic to foreign debt holders. Default avoids the repayment but is costly. The potential of a bailout influences this choice.

More formally, household utility over period 2 consumption is represented by $u(c)$, which is continuously differentiable with $u'(c) > 0$ and $u''(c) \leq 0$. Agents have a fixed endowment of ω in period 1, allocated between a riskless asset with a return of x and risky government debt. Holdings of both assets must be non-negative. Agents in country i have a random period 2 endowment denoted A_i, with a cdf given by $F(\cdot)$. These endowment draws are independent across countries. Suppose that the domain of A_1 is given by $A \in [A^-, A^+]$ with $A^- > 0$.

Government debt has a market determined return of R, paid to debt holders in all period 2 repayment states. In the event of default, the return on government debt is 0. For simplicity there is no partial default.

In period 1, country $i = 1, 2$ agents choose debt holdings to solve

$$max_{b \geq 0} E_{A_i, \tilde{R}} u((\omega - b)x + b\tilde{R} + A_i(1 - \tau_i)). \tag{1}$$

The optimal choice of bonds by country i agents, denoted b_i, satisfies

$$E_{A_i, \tilde{R}} (\tilde{R} - x) u'((\omega - b_i)x + b_i\tilde{R} + A_i(1 - \tau_i)) = 0. \tag{2}$$

Here the expectation is taken over the endowment A_i as well as the random return on sovereign debt, $\tilde{R}$ where $\tilde{R} = R$ in repayment states and $\tilde{R} = 0$ under default. Variations in the return on government debt, as explained below, reflect the uncertainty in A_i, through the effect of this endowment on the tax base, and strategic uncertainty over default.

In this problem, τ_i is a tax rate levied on household income in country i. If country 1 repays its debt in period 2,

$$\tau_1 = \frac{BR}{A_1} \tag{3}$$

where B is the predetermined stock of country 1 debt outstanding per country 1 agent. In the event of repayment, there is no tax on country 2 households, $\tau_2 = 0$. If instead there is a bailout of country 1, then τ_i for $i = 1, 2$. The revenues collected from country 2 households are used to finance transfers to country 1 under a bailout as detailed below.

In solving their optimization problems, households take the distribution of $\tilde{R}$ as given. It is determined in equilibrium of the period 1 market for government debt and reflects expectations about the default and bailout decisions of period 2. The holding of the country 1 debt is also determined in equilibrium.

2.2 External Households

Households in country 3 are risk neutral. They consume in both periods of life. These agents have a large endowment in period 1 which they can store for period 2 consumption. As with households in countries 1 and 2, they can either store through a storage technology at a return of x or hold country 1 government debt. These households will play a primary role in the pricing of country 1 debt.

3 Equilibria without Bailout

This section studies equilibria without bailout. An equilibrium has three components. First, the default decision of the government in country 1, taken in period 2 is optimal. Second, households in all countries optimally choose their holdings of debt and storage in period 1 taking as given the state contingent default decision of the country 1 government in period 2. Third, the market for country 1 debt clears in period 1.

Let b_i be the demand for government debt by a representative country i household and θ_i the fraction of the debt held by country i households, i.e. $\theta_i = \frac{b_i N_i}{B N_1}$. Further define $\theta = (\theta_1, \theta_2, \theta_3)$. It is useful to determine an equilibrium through these shares. The condition for market clearing is that $\sum_i \theta_i = 1$ with $\theta_i \geq 0$.

3.1 Period 2 Repayment Decision

In period 2, country 1 decides to repay its debt or default. It does so in the interest of households in that country. Debt is repaid through a tax on the period 2 endowment of the households in country 1.

If the debt is repaid, the representative household in country 1 has consumption of

$$V^r \equiv (\omega - b_1)x + b_1 R + A_1(1 - \tau_1) = (\omega - b_1)x + A_1 - BR(1 - \theta_1). \tag{4}$$

The value if repayment, V^r, comes from consuming the after tax level of output, $(1 - \tau_1)A_1$ plus the return to the portfolio of assets. The representative household owns a fraction θ_1 of the per capita debt, B, and hence recoups $BR\theta_1$ when the debt is repaid. Using the government budget constraint, (3), the consumption if repayment can be rewritten as in the second term of (4).

If there is default, then consumption of the representative country 1 household is given by

$$V^d \equiv (\omega - b_1)x + A_1(1-\gamma). \tag{5}$$

In this case, the country defaults on both internal and external debt. A fraction of output, γ, is lost. This is a conventional assumption in the debt default literature meant to capture a wide range of costs stemming from default including a reduction in trade and risk sharing opportunities, exclusion from markets in the future, etc. The details of those costs do not concern us at this stage, it is only important they are present in the analysis.

The values of repayment and default and the default decision are calculated *ex post*, after the realization of the fundamental uncertainty through the productivity shock.[6] *Ex ante* A_1 is not known. It is common knowledge that the shock is drawn from $F(\cdot)$.

Using (4) and (5), the debt will be repaid iff $A_1 \geq \hat{A}$, where

$$\hat{A} \equiv \frac{BR(1-\theta_1)}{\gamma}. \tag{6}$$

From (6), $\hat{A}$ depends on (B, R, θ_1) as well as the default costs, γ. If enough debt is held internally, so that θ_1 is near 1, $\hat{A}$ can be small enough that default will not occur: i.e. $A^- > \hat{A} > 0$.

3.2 Period 1 Pricing of Government Debt: Risk Neutrality

Assume that households in countries 1 and 2, like households in country 3, are risk neutral. Let p denote the probability of repayment of country 1 debt. With risk neutral households, (2) becomes a standard arbitrage condition

$$pR = x. \tag{7}$$

As long as (7) holds, households in all three countries are indifferent with respect to the composition of their portfolios. So the share of government debt held by residents of the three countries, θ_i for $i = 1, 2, 3$, is not determined in equilibrium. As we shall see though, θ will influence bailout and default decision.

The probability of repayment in (7) is determined by a comparison of V^r and V^d conditional on the realized value of A. Since the loan is made *ex ante*, p is the probability that A falls outside the default range determined by (6), i.e. $p = 1 - F(\hat{A})$.

The strategic uncertainty comes from the dependence of the probability of repayment on the interest rate together with the dependence of the interest rate on that same probability. There is a natural strategic complementarity here: if investors think that default is likely, they demand a high interest rate and thus

[6] Calvo (1988) excludes fundamental uncertainty and focuses solely on the endogenous determination of a repudiation rate by a government that lacks commitment.

there is default unless a very high A_1 is realized.[7] This is consistent with the high default probability believed by investors. This interaction between beliefs, interest rates and default decisions is brought out by the equilibrium analysis.

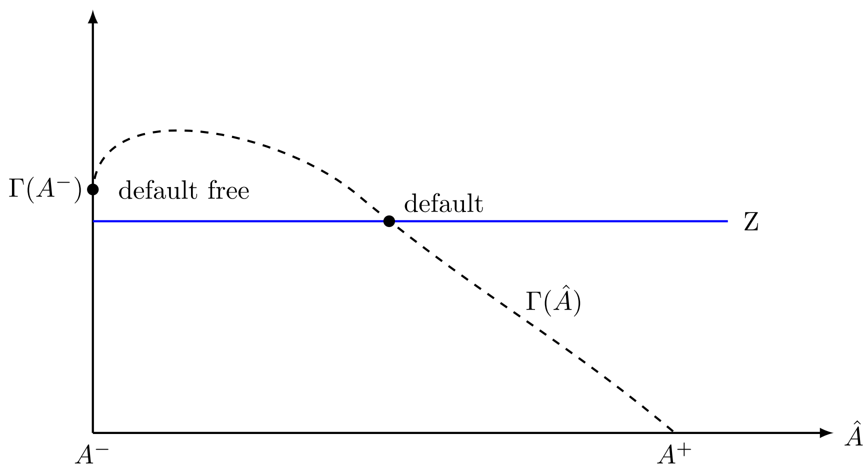
An equilibrium is determined by the values of $(\hat{A}, R)$ that jointly satisfy (6) and (7), with $p = (1-F(\hat{A}))$. If (7) holds, the distribution of government debt holdings, θ is not determined. Yet θ influences $\hat{A}$ from (6). The following equilibrium characterization is for a given value of θ.

With some algebra the critical value of A, denoted $\hat{A}$, determining the bound of the default region solves

$$\Gamma(A) \equiv [1 - F(A)]A = \frac{Bx(1-\theta_1)}{\gamma} \equiv Z. \tag{8}$$

From (6), default occurs for $A_1 < \hat{A}$ where $\hat{A}$ is a solution to (8). The value of R is then determined from (7). Solutions to (8) will be called interior equilibria.

Figure 1: Locally Safe



This figure show the existence of a locally isolated equilibrium without default as well as other equilibria with default.

Note that $\Gamma(A^-) = A^-$ and $\Gamma(A^+) = 0$. The product of $(1 - F(A))$ and A may not be monotone in A. This is the source of the multiplicity of interior equilibria.

In addition to the equilibria solving (8), there can be other more extreme equilibria. If $\Gamma(A^-) \geq Z$, then there is an equilibrium without default. Further $\Gamma(A^+) = 0$ so there is always an equilibrium with certain default and hence a zero value of debt.

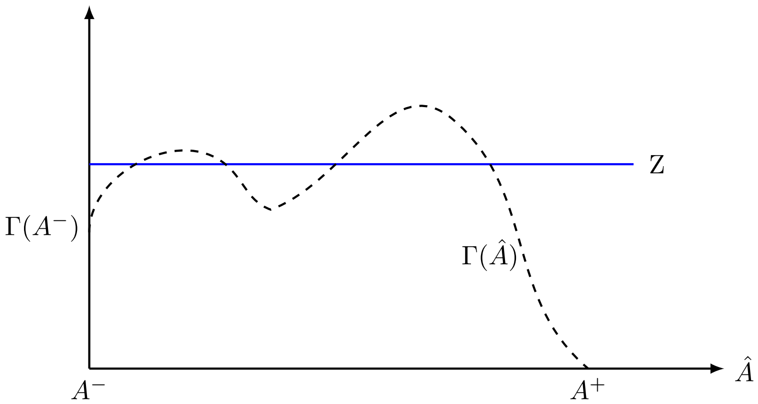
Figure 1 shows the determination of an equilibrium for the model when $\Gamma(A^-) = A^- > Z$. In this case, there is an equilibrium without default, implying $R = x$. There is another equilibrium at a much higher value of $\hat{A}$ with $R > x$. The second equilibrium has default for $A \in [A^-, \hat{A})$. Clearly, strategic uncertainty is present even in the case where there is an equilibrium without default. In this case, there is a single

[7]The complementarity takes a dynamic form in Cole and Kehoe (2000). The prospect of a future default increases the cost of rolling over the current stock of debt and thus makes current default more likely.

interior equilibrium. The set of equilibria is robust to small variations in Z.

For there to be an equilibrium without default requires, all else the same, a relatively low value of Z. From (8), a low value of Z comes from a small debt burden, i.e. BR is small relative to output. Also, if θ_1 is large so that most of the debt is held internally, the incentive to default is reduced and Z is low as well.

Figure 2: Multiple Interior Equilibria



This figure show the existence of multiple equilibria with default.

There are other cases to consider. Figure 2 shows a case with four interior equilibria. Here Z is sufficiently high so that there is no default-free equilibrium. There are multiple interior equilibria with different critical values of $\hat{A}$ satisfying (8) and therefore different default probabilities and associated values of R. The multiplicity of interior equilibria reflects the non-monotonicity of $\Gamma(A)$.[8] And, as in Figure 1, there is the equilibrium in which default always occurs.

This case is one where the debt burden is high enough relative to the lower support of A that repayment with certainty is not possible. In this case, the fundamental uncertainty is large enough that no promises of certain repayment are credible. Interestingly this creates at least two interior equilibria, in addition to the equilibrium with certain default.

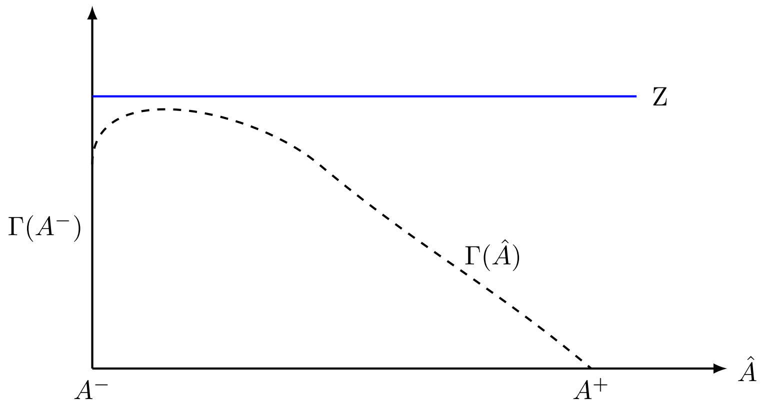
If Z is sufficiently high and $\Gamma(A)$ does not increase rapidly enough, then there may be no equilibria with repayment. This is shown in Figure 3.

As noted earlier, this analysis takes θ as given. Since θ is not pinned down as long as (7) holds, this implies that there is another dimension to the multiplicity. Clearly, the more debt held externally, the lower is θ, and the higher is Z. This means that an economy can transition from a situation of certain repayment to strategic uncertainty as external debt is substituted for internally held debt.

For the analysis of bailout equilibria that follows, it is useful to summarize results on multiplicity given the holdings of debt. The analysis assumes there exists an equilibrium without default and builds other

[8]Examples like this can be generated from a uniform distribution of A so that $\Gamma(A) = (1 - A)A$.

Figure 3: Equilibrium with Certain Default



This figure shows an economy where there is no equilibrium with repayment, just certain default.

equilibria from it.[9]

Lemma 1. *If $\Gamma(A^-) = A^- > Z$, then there exists an equilibrium without default and another with a positive probability of default. For some distributions, $F(\cdot)$, there exist multiple equilibria with positive probabilities of both repayment and default.*

Proof. $\Gamma(A^-) > Z$ implies the existence of an equilibrium without default. There is always a solution to (8) in which debt has no value with $R = \infty$. By continuity, there always exists at least one other equilibrium with R finite. For some distributions $F(\cdot)$, $\Gamma(A) = [1 - F(A)]A$ is not monotonically decreasing in A. In that case, multiple interior equilibria, i.e. solutions to (8), can be constructed by the appropriate choice of θ and hence Z. □

To be clear, Lemma 1 provides sufficient but not necessary conditions for multiplicity. Figure 2 shows a case in which there is no equilibrium with certain repayment, though there are multiple equilibria with positive probabilities of default.

3.3 Period 1 Pricing of Government Debt: Risk Aversion

If households in either country 1 or country 2 are risk averse, then their demand for risky government debt will be zero. The valuation of government debt will be set by the arbitrage of the risk neutral agents in country 3 and the risk averse households will set $\theta_i = 0$ for $i = 1, 2$. The bond market will then clear based upon the demands of country 3 agents as long as their endowment is large enough to purchase BN_1.

The default decision of country 1 is again determined by (6), evaluated at $\theta_1 = 0$. Repayment occurs iff $A_1 \geq \hat{A} = \frac{BR}{\gamma}$. Using the Lemma 1, it is straightforward to construct multiple equilibria in this case as well.

[9]This follows the construction in Camous and Cooper (2014).

4 Bailout

The prospect of default opens the possibility of a bailout of country 1 debt by the coalition of countries 1 and 2.[10] If the federation was able to simply commit to a bailout of country 1 debt, then the strategic uncertainty would simply disappear. But, of course, such an ironclad guarantee of debt repayment is not feasible.

Accordingly, this section addresses two questions. First, under what conditions will a bailout be provided *ex post*? Second, will the bailout eliminate strategic uncertainty?

To study bailouts, the model must be enhanced to be specific about the interaction between the countries. A key issue is whether a bailout will be provided. The analysis focuses on two motives for bailout. The first is avoiding default costs and the second is consumption smoothing across countries within the federation. For both motives, sufficient conditions for bailout are provided. Importantly bailout is generally state contingent, i.e. the federation prefers bailouts for some but not all realizations of (A_1, A_2).

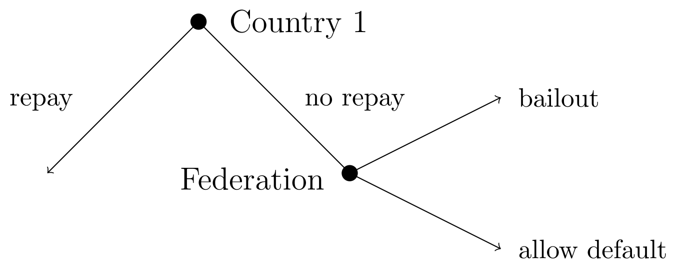
The bailout decision occurs in the last period and thus takes θ as given. To construct equilibria, we then return to the period 1 choices of households in the three countries. In this way we address the second question about the impact of strategic uncertainty when (state-contingent) bailouts are provided.

4.1 Bailout Incentives

The federation has two members. One country 1, whose choice of default or repay was analyzed above. The other member is a composite of the other countries in the federation, denoted country 2 in the preceding analysis. Country 3 households, while they hold government debt, do not have any direct influence on the choices in of the country or the federation.

The interaction between country 1 and the federation is shown in Figure 4. Country 1 chooses first and can repay its debt or default. If it chooses not to repay, then the federation has an option of a bailout or allowing default. The default cost is incurred by country 1 only if the federation chooses not to bailout that country.

Figure 4: Repayment Game



The underlying assumption in this game is that the Federation is always able to come to the rescue

[10]As noted earlier, country 2 represents all other countries integrated into the single federation.

of the country. Regardless of the many stages of "negotiation" over repayment or not, ultimately if the country is not going to repay its debt, then the Federation has an option to bailout. This is highlighted in this extensive form game.[11]

We first make explicit the consumption allocations for households in countries 1 and 2 under the alternative outcomes of debt repayment, debt default and bailout. We use these to construct equilibria. The consumption for households in country 3 does not factor into these choices.

To recall notation, N_i is the population in country $i = 1, 2, 3$. Let $N = N_1 + N_2$ be the population of the coalition. Recall that B is the total debt outstanding of country 1 per capita of that country. Further θ_i is defined through the debt holdings of a representation country i household: $b_i = \frac{\theta_i B N_1}{N_i}$. Let $\theta \equiv (\theta_1, \theta_2, \theta_3)$ be the vector of debt holdings. As A_i is the realized productivity in country i, $Y_i = N_i A_i$ is total country i output and $\bar{Y} = \sum_{i=1,2} Y_i$ is total output of the federation.

Repayment Looking first at the representative household in country 1, using (4), the consumption allocation under repayment is $c_1^r = A_1(1-\tau_1) + b_1 R + (\omega - b_1)x = A_1 - BR(1-\theta_1) + (\omega - b_1)x$. When $\theta_1 = 1$, the repayment of debt is just a reshuffling within a household's budget constraint without any real implications. Using (5), the consumption allocation for country 1 agents under default is $c_1^d = (1-\gamma)A_1 + (\omega - b_1)x$.

Similar expressions hold for the consumption of agents in country 2 under repayment and default on country 1 debt. If there is repayment, $c_2^r = A_2 + b_2 R + (\omega - b_2)x$, while if country 1 defaults, $c_2^d = A_2 + (\omega - b_2)x$.

Bailout In the event of a complete bailout, a common tax rate is set and levied on agents throughout the federation. The tax rate, denoted $\bar{\tau}$, is sufficient to cover the total debt obligations of country 1: $\bar{\tau}\bar{Y} = N_1 BR$. The consumption of country $i = 1, 2$ agents under a bailout is $c_i^b = A_i(1-\bar{\tau}) + b_i R + (\omega - b_i)x$.

Objective The last ingredient of the model is the objective of the federation. Assume that the federation has an objective of the weighted average of utility from the consumption levels of the households in the two countries: $W(c_1, c_2) = \sum_i \Delta_i v(c_i)$ where Δ_i is a welfare weight and $v(\cdot)$ is strictly increasing and concave. Normalize so that $\sum_i \Delta_i = 1$. The weights might coincide with population shares or could reflect broader objectives of the federation.

The bailout incentive arises from the curvature in the objective function of the federation. This curvature could come directly from the risk aversion of the households, so that $v(\cdot) = u(\cdot)$. More generally, it could reflect the views of the policymakers that take into account the joint welfare of all households in the federation or a political process that puts more value on equality of consumption across members of the federation.

[11] Cooper, Kempf, and Peled (2008) studies the opposite timing and hence provide somewhat different conditions for bailout. The timing adopted here highlights the effects of federation bailout on the choices of country 1.

This maximization of a weighted sum of the strictly concave utilities obtained from the member countries, creates, as in Cooper, Kempf, and Peled (2008), a consumption smoothing motive for bailout. In the leading case where Δ_i equals the population share of country i, the federation prefers to equalize consumption across agents, given a fixed amount of resources.

4.2 Risk Neutral Households

Suppose all households are risk neutral and are treated so in the federation objective: $v(c) = c$. We first look at bailout incentives and then study the presence of strategic uncertainty.

Bailout Incentives From Figure 4, the federation chooses between bailout and default, given that country 1 has decided not to repay its debt. Under bailout, the welfare of the coalition is given by:

$$W^b = \Delta_1[A_1(1-\bar{\tau}) + x(\omega - b_1) + Rb_1] + \Delta_2[A_2(1-\bar{\tau}) + x(\omega - b_2) + Rb_2] \tag{9}$$

where $\bar{\tau} = \frac{RBN_1}{Y}$ is sufficient to repay the outstanding debt. If instead the federation allows default, then welfare of the coalition is given by

$$W^d = \Delta_1[A_1(1-\gamma) + x(\omega - b_1)] + \Delta_2[A_2 + x(\omega - b_2)]. \tag{10}$$

Using the budget constraint to determine $\bar{\tau}$, write the difference in values between bailout and default as

$$W^b - W^d = RBN_1\left[\frac{\Delta_1}{N_1}(\theta_1 - \frac{Y_1}{Y}) + \frac{\Delta_2}{N_2}(\theta_2 - \frac{Y_2}{Y})\right] + \gamma\Delta_1 A_1. \tag{11}$$

To highlight different influences on the bailout decision, assume $\Delta_i = N_i$ and normalize $N = 1$ so that population and welfare weights are the same. In this case, the desired social outcome is equal consumption across households in the federation. The following proposition provides a sufficient condition for a bailout to occur. The sufficient condition depends, in part, on who holds the debt of country 1 and the realization of country 1 output.

Lemma 2. *Given θ, if $\Delta_i = N_i$, then there exists a critical level of country 1 output, $\tilde{A}$, such that a bailout occurs iff $A_1 \geq \tilde{A}$.*

Proof. At $\Delta_i = N_i$, from (12), $W^b - W^d$ becomes

$$W^b - W^d = RBN_1(\theta_1 + \theta_2 - 1) + \gamma N_1 A_1 = \gamma N_1 A_1 - RBN_1\theta_3. \tag{12}$$

Let $\tilde{A} \equiv \frac{RB\theta_3}{\gamma}$. Clearly $W^b - W^d \geq 0$ iff $A_1 \geq \tilde{A}$.

□

Note that bailout is state contingent and occurs only for high values of output in country 1. With risk neutral households, bailout is only a way to reduce the total costs of default. The level of output in country 2 is irrelevant for determining the costs of default. Though the tax rate adjusts to variations in both A_1 and A_2, the total tax payment in the event of bailout is RBN_1 and thus independent of country i output.[12]

It is interesting to contrast the incentives of the coalition with those of country 1 acting alone. From (6), without the prospect of a bailout, country 1 chooses repayment iff $A_1 \geq \hat{A} \equiv \frac{RB(1-\theta_1)}{\gamma} = \frac{RB(\theta_2+\theta_3)}{\gamma}$.

Clearly $\tilde{A} \leq \hat{A}$, with equality if $\theta_2 = 0$ so that $1 - \theta_1 = \theta_3$. If $\theta_2 = 0$, Proposition 2 implies that the bailout option does not reduce the default probability. Instead, it is exactly those states in which country 1 wishes to default that the coalition is content to allow that default.

If, *ceteris paribus*, $\theta_2 > 0$, then debt holders in country 2 benefit from the repayment of debt. Hence $\tilde{A}(\theta_3) < \hat{A}$ when $\theta_2 > 0$. In this case, the possibility of bailout reduces the set of default states.

Taking as given the bailout decision of the federation, country 1 retains the option to repay rather than grant the option of a bailout to the federation. If $\theta_2 > 0$ so that $\tilde{A} < \hat{A}$, then there are states in which the federation prefers bailout while country 1 prefers default to repayment. Even if country 1 prefers default to bailout, it can only prevent a bailout by repaying its own debt. But clearly country 1 is always better off under bailout then repayment as bailout allows it to share the debt burden. Thus in these states, a bailout will occur. But, from Lemma 2, a bailout will not occur for low realizations of A_1.

With the prospect of bailout in some but not all states, will there nonetheless exist strategic uncertainty in the valuation of country 1 debt? The key, as in the analysis absent bailout, is in the potential for multiple solutions to the risk neutral pricing equation,(7), rewritten here as:

$$pr(r \cup b)R = x \tag{13}$$

where $pr(r \cup b)$ denotes the probability that the realization of (A_1, A_2) leads to either repayment by country 1 or bailout.

The following proposition uses the logic of Lemma 1 to argue there can be multiple solutions to (13). As highlighted earlier, there are two sources of multiplicity. First, if there exists an equilibrium without default, then by continuity there is an equilibrium with default in some states. Second, if the default probability is increasing in R, then multiple interior equilibria can be constructed. Both of these forces are present in Lemma 1 as well as the propositions that follow concerning the presence of strategic uncertainty along with bailouts.

Proposition 1. *If Lemmas 1 and 2 hold, $\theta_3 > 0$ and A^- is sufficiently low, multiple equilibria in the valuation of country 1 debt remain despite the prospect of a bailout.*

Proof. Under the hypothesis of Lemma 1, there is a solution to (13) with no default and $R = x$. There is

[12] Country 2 productivity would matter if there was a distortion in labor supply.

always an equilibrium with certain default: as R gets sufficiently large, $\tilde{A}$ does as well, exceeding A^+ so that bailout does not occur.

As shown in Lemma 2, bailout occurs iff $A_1 \geq \tilde{A}$. As long as the lower support of the distribution of the shocks is low enough, $\tilde{A} > A^-$. For $A_1 \in [A^-, \tilde{A}]$, country 1 chooses between default and repayment as a bailout is not forthcoming in these states. Since $\tilde{A} < \hat{A}$, country 1 will default in these states.

For this case, the arbitrage condition, (13), becomes

$$\left(1 - F(\tilde{A})\right) R = x. \tag{14}$$

As $\tilde{A} = \frac{RB\theta_3}{\gamma}$ is increasing in R, the probability of default is increasing in R. Using $R = \frac{\tilde{A}\gamma}{B\theta_3}$, the non-monotonicity in the left side is sufficient to generate multiple solutions to (14), following the construction in Lemma 1.

□

Here the existence of a default region requires $\theta_3 > 0$. If all debt is held within the federation, then $\tilde{A} = 0$ and repayment occurs for every realization of A_1. There is no reason to incur a default cost since the bailout entails a welfare neutral redistribution within the households of the federation.

The multiplicity arises despite the prospect of bailout. This has the same basis as the multiple solutions to (8). The key is the underlying complementarity: if investors are pessimistic about repayment, R will increase to compensate and this will increase the set of default states. This interaction arises even if bailout is possible because a credible bailout occurs iff $A_1 \geq \tilde{A}$ and $\tilde{A}$ is increasing in R.

Now consider the case in which the Δ_i differ from the population weights. In this case, the redistribution effects of bailout will matter. To highlight this, suppose $\theta_3 = 1$ so that all debt is held outside of the federation. In this case, (9) and (12) imply that a bailout is desired only if $\gamma \geq \bar{\tau}$. Otherwise, households in both countries are better off under default. Second, if a bailout is desired, it redistributes from country 2 to country 1. This also follows directly from $\gamma \geq \bar{\tau}$ so that the burden of the debt for country 1 is less than the default cost. Third, the welfare weights and the realized values of (A_1, A_2) will determine if the redistribution under a bailout increases federation welfare.

Lemma 3. *If all debt is external to the federation, i.e. $\theta_3 = 1$, then there exists $\tilde{A}$ such that bailout occurs iff $A_1 \geq \tilde{A}$ with $\tilde{A}$ is increasing in the relative welfare weight of country 2, $\frac{\Delta_2}{\Delta_1}$, and decreasing in the relative size of country 2, $\frac{N_2}{N_1}$.*

Proof. If $\theta_3 = 1$, then debt market clearing implies $\theta_1 = \theta_2 = 0$. Consequently, the gain to bailout is proportional to

$$W^b - W^d = A_1\gamma - \bar{\tau}(A_1 + \frac{A_2\Delta_2}{\Delta_1}). \tag{15}$$

The first step shows that if there is a gain to bailout, then it is increasing in A_1. A necessary condition for $W^b \geq W^d$ is $\gamma > \bar{\tau}$. Under this condition, an increase in A_1 increases $W^b - W^d$ for a given $\bar{\tau}$. Further, using $\bar{\tau} = \frac{RBN_1}{\sum_i N_i Y_i}$, the tax rate falls as A_1 increases, thus further increasing the gain to bailout.

The second step characterizes the unique critical level of A_1 determining the bailout region. Define $\tilde{A}$ as the value of A_1 such that $W^b = W^d$. Using $\bar{\tau} = \frac{RBN_1}{\sum_i N_i Y_i}$, and setting $W^b = W^d$ yields:

$$\tilde{A} = \frac{RB}{\gamma} \left[\frac{1 + \hat{A}_2 \hat{\Delta}_2}{1 + \hat{A}_2 \hat{N}_2} \right] \tag{16}$$

where $\hat{\Delta}_2 = \frac{\Delta_2}{\Delta_1}$ is the relative welfare weight for country 2, $\hat{A}_2 = \frac{A_2}{\tilde{A}}$ is the productivity of country 2 relative to $\tilde{A}$ and $N_2 = \frac{N_2}{N_1}$ is the relative welfare weight for country 2.

The right side of (16) is monotone in $\hat{A}_2$ and thus in $\tilde{A}_1$. Whether it is increasing or decreasing in $\tilde{A}$ depends on the sign of $\hat{\Delta}_2 - \hat{N}_2$. If $\hat{\Delta}_2 = \hat{N}_2$, then $\tilde{A} = \frac{RB}{\gamma}$ as in Lemma 2 as here $\theta_3 = 1$. Thus there exists a unique $\tilde{A}$ solving (16) given A_2.

Since $W^b - W^d$ is increasing in A_1, bailout occurs for $A_1 \geq \tilde{A}$. From (16), $\tilde{A}$ is increasing $\frac{\Delta_2}{\Delta_1}$ and decreasing in $\frac{N_2}{N_1}$ given A_2. □

There is an interesting contrast between the conditions for bailout in Lemmas 2 and 3. In the first result of Lemma 2, the critical value of A_1 that determined the bailout region was independent of A_2. The relative productivities did not matter since the weights in the objective function mirrored the population weights used to determine the tax shares. But, in Lemma 3, $\tilde{A}$ depends on A_2, both directly and through the common tax rate in the event of bailout, $\bar{\tau}$. From (16), if $\hat{\Delta}_2 > \hat{N}_2$, then $\tilde{A}$ is increasing in A_2.

Overall, a bailout is more likely if country 1 has a larger relative welfare weight and is relatively small. Further when the relative welfare weight of country 1 is larger than its relative population, then bailout is less likely as A_2 increases. In this case, the welfare cost of the shared tax dominates the reduction in the tax rate for high country 2 output.

Proposition 2. *If Lemmas 1 and 3 hold and A^- is sufficiently low, multiple equilibria in the valuation of country* 1 *debt remain despite the prospect of a bailout.*

Proof. The proof follows the proof of Proposition 1. Under the hypothesis of Lemma 1, there is an equilibrium without default. If A^- is sufficiently low, then there is a non-empty default region, i.e. $A^- < \tilde{A}$. Further, the default region becomes larger as R increases as $\tilde{A}$ is increasing in R. As R increases, $\bar{\tau}$ increases and so does $\tilde{A} = \frac{\bar{\tau} A_2 \Delta_2}{\Delta_1(\gamma - \bar{\tau})}$. With the default range increasing in R, the resulting complementarity is sufficient to create multiple interior valuations of the debt following the argument of Lemma 1. □

4.3 Risk Averse Households

This section considers the case of strictly concave valuations of country i consumption, $v(c_i)$, in the federation's social welfare function. As noted earlier, this curvature may reflect the tastes of households in these countries or some elements in the determination of social welfare that implies the distribution of consumption matters. The following sections study these two cases.

For each, there are two types of results. First, the conditions for bailout are provided. Second, the dependence of the set of default states on R is shown to lead to multiple solutions of the debt valuation equation, (13). Throughout we focus on the importance of this curvature and ignore differences across countries due to size. Accordingly, assume $N_1 = N_2$ for this analysis.

4.3.1 All External Debt

First consider the case in which all households in countries 1 and 2 are risk averse and curvature matters in the federation objective, i.e. both $u(\cdot)$ and $v(\cdot)$ are strictly concave. Country 3 households remain risk neutral.

Bailout Incentives In equilibrium, the debt is priced by these risk neutral households and they hold all of the country 1 debt. In this case, $\theta_1 = \theta_2 = 0$ and $\theta_3 = 1$ and social welfare under bailout and default become:

$$W^b = \Delta_1 v(A_1(1-\bar{\tau}) + x\omega) + \Delta_2 v(A_2(1-\bar{\tau}) + x\omega) \tag{17}$$

$$W^d = \Delta_1 v(A_1(1-\gamma) + x\omega) + \Delta_2 v(A_2 + x\omega) \tag{18}$$

respectively.

As noted in the discussion leading to Lemma 3, these expressions immediately imply that a necessary condition for bailout is $\gamma > \bar{\tau}$. The bailout redistributes from country 2 to country 1. With risk averse households, the welfare effects of this redistribution will depend on the realizations of (A_1, A_2).

Lemma 4. *If the weight on country* 1*,* Δ_1*, is large enough, country* 1 *output,* A_1*, small enough and federation output,* Y*, large enough, then a bailout will be preferred to default.*

Proof. Assume $A_1 = A_2$ and $\Delta_1 = \Delta_2$ so that the two countries are symmetric in terms of size, output and the weight in federation welfare. From Lemma 2, if $\theta_3 = 1$, then $\tilde{A} = \frac{BR}{\gamma}$. From (17) and (18), at $A_1 = A_2 = \tilde{A}$, the total resources available to country 1 and 2 agents is the same under default and repayment. That is, $(A_1 + A_2)(1-\bar{\tau})$=$A_1(1-\gamma) + A_2$ at $A_1 = A_2 = \tilde{A}$ implying the level of federation output is such that $\gamma = 2\bar{\tau}$.

So, at $A_1 = A_2 = \tilde{A}$, $\bar{\tau} < \gamma$, else default would dominate for all households. Further, the curvature in $v(\cdot)$ implies $W^b > W^d$ since bailout implies a more equitable division of consumption across households in the federation compared to default.[13]

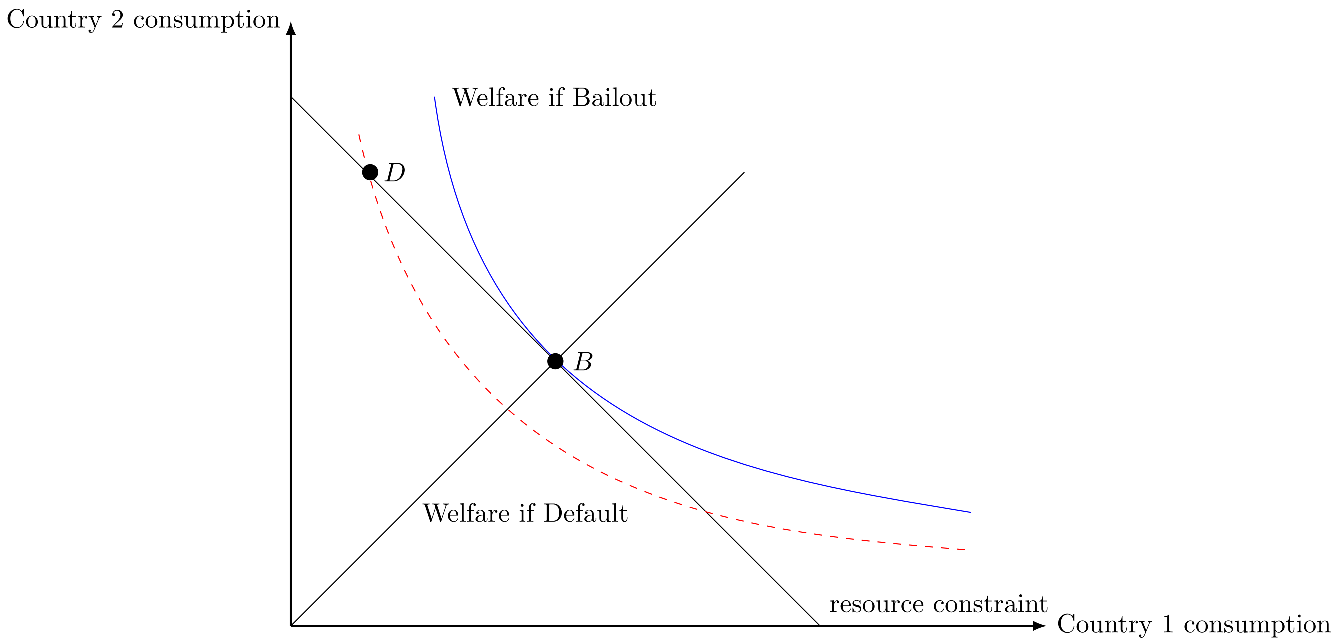
[13] In fact, as the two countries are symmetric, bailout generates the social welfare maximizing distribution of goods.

With $\bar{\tau} < \gamma$, bailout redistributes federation output from country 2 to country 1. Therefore, an increase in Δ_1 (relative to Δ_2) would increase the value of a bailout.

Further, suppose A_1 falls relative to A_2, leaving Y unchanged. Then the redistribution to country 1 under a bailout would be even more desirable. □

This result highlights the role of consumption smoothing, as in Cooper, Kempf, and Peled (2008), as a rational for bailout beyond the desire, highlighted in Lemma 2, of avoiding costly default. These gains are illustrated in Figure 5 when $A_1 = A_2 = \tilde{A}$. The bailout redistributes from country 2 to country 1 relative to default. As the social welfare function is strictly concave, this redistribution is welfare improving as the consumption of country 2 exceeds that of country 1 both under default and bailout.

Figure 5: Gains from Bailout



This figure shows two indifference curves. The solid (blue) curves is social welfare under a bailout. The dashed (red) curve is social welfare under default. The point D is the outcome under default and the point B is the outcome under a bailout. There is a common resource constraint as $\gamma = 2\bar{\tau}$.

To be clear, the result in Lemma 4 depends on the *ex post* realization of (A_1, A_2). This result does not imply there is a bailout in all states.

Lemma 5. *If the weight on country* 1, Δ_1, *is small enough, country* 1 *output,* $A_1 \leq \hat{A}$, *defined in* (6), *and federation output,* Y, *small enough, then country* 1 *will default.*

Proof. For $A_1 \leq \hat{A}$, with $\hat{A}$ defined in (6), country 1 prefers default over repayment. With Δ_1 and Y small, the common tax rate is very high and the redistribution from country 2 to country 1 is not desirable. So the federation will prefer default to bailout. In fact, for Y sufficiently low, $\bar{\tau} > \gamma$ and bailout is worse than default for both countries. □

Taken together, Lemmas 4 and 5 provide conditions such that bailout occurs in some states and default in others. Even if there is an equilibrium without default on country 1 debt, so that $R = x$, Lemma 1 implies the existence of other interior equilibria.

Proposition 3. *If $v(c_i)$ is strictly concave for households in country $i = 1, 2$ and linear for households in country 3, multiple interior equilibria in the valuation of country 1 debt remain despite the prospect of a bailout.*

Proof. The first step is to establish conditions for country 1 to prefer default to repayment. With $\theta_3 = 1$, a default is preferred to repayment by country 1 occurs if $A_1 \leq \frac{BR}{\gamma}$. The default states are increasing in R.

The second step is to establish conditions for the federation to prefer default over bailout. For realizations of (A_1, A_2) such that $Y < \frac{BR}{\gamma}$, $\gamma \leq \bar{\tau}$ and the federation prefers default. The set of states such that default is preferred to bailout by the federation is increasing in R.

From Lemma 1 multiple solutions to the debt valuation equation can be constructed. An increase in R implies that the set of default states is larger. Thus, using Lemma 1, multiple interior solutions can be constructed. □

4.3.2 Only Internal Debt

Within some federations, such as the EU, a large fraction of debt is held internally to both the country issuing the debt and the federation as a whole. This subsection studies this case by looking at a situation in which households in country 1 and 2 are risk neutral but country 3 households are risk averse.[14] In this case, risk neutral pricing remains and $\theta_3 = 0$.

We study the incentive for bailout under the assumption of risk aversion in the federation's welfare function, i.e. $v''(\cdot) < 0$. As above, the curvature of $v(\cdot)$ promotes a consumption smoothing motivation for bailout by the federation.

There are a couple of ways to motivate this combination of household and federation preferences. First, households may indeed be risk neutral. Nonetheless, the political process within the federation leads to outcomes which balance the relative interests of the parties.[15] Or the federation may internalize preferences over consumption equality that competitive households ignore. In this case, the curvature in the welfare of the federation comes from internal political considerations rather than the risk aversion of households. Second households may be risk averse but funds are managed by risk neutral investment firms within countries 1 and 2. The households though ultimately own these firms and thus the proceeds from their

[14] If all households are risk averse, the of course (8) does not price the debt. It is possible to show, by continuity, that if there are multiple solutions to (8) under risk neutrality, then there are multiple equilibria for risk aversion close to zero using a continuity argument.

[15] For example, the Nash cooperative bargaining solution selects an allocation from a feasible set using a nonlinear objective. This same allocation maximizes the sum of strictly concave utilities of the two agents given welfare weights. Thanks to Alessandro Dovis for a discussion on this point.

activities.[16]

The conditions for bailout are given in the following lemmas. The first considers a symmetric situation and finds that a bailout is provided in all states. The second highlights conditions for default in some states. Throughout, assume $\Delta_1 = \Delta_2$.

Lemma 6. *If $\theta_1 = \theta_2$, the federation prefers bailout to default for all realizations of (A_1, A_2).*

Proof. Suppose default costs are zero. The consumption levels under default for $i = 1, 2$ are $c_i^d = A_i + (\omega - b_i)x$. The consumption levels for $i = 1, 2$ under a bailout are $c_i^b = A_i(1 - \bar{\tau}) + b_i R + (\omega - b_i)x$. Therefore the consumption gain from bailout is

$$c_i^b - c_i^d = b_i R - \bar{\tau} A_i = R(\theta_i - \frac{A_i}{A_1 + A_2}) \tag{19}$$

where the third term uses $\bar{\tau} = \frac{N_1 BR}{\sum_i (N_i A_i)}$ from the government budget constraint. With $N_1 = N_2$, $\bar{\tau} = \frac{BR}{\sum_i (A_i)}$.

Equation (19) implies that $\sum_i (c_i^b - c_i^d) = 0$. Bailout has only redistribution effects when $\gamma = 0$.

If $\theta_1 = \theta_2$, (19) implies that the redistribution is always in favor of the relatively poor country. This is welfare increasing as it leads to greater consumption equality. Thus bailout occurs in all states. If there are positive default costs, bailout is even more desirable. □

Interestingly, in the case of the closed federation, when $\theta_1 = \theta_2$ conditions for bailout are independent of R. Of course, bailout may also be undesirable if the redistribution is from poor to rich. This can arise when debt holdings are not equal, i.e. $\theta_1 \neq \theta_2$. The following lemma studies conditions for a bailout when country 1 debt is largely held by households in that country.

Lemma 7. *Suppose θ_1 large and γ small. If $A_1 > A_2$, then the federation prefers default to bailout. If $A_1 < A_2$ and A_1 is small enough, the federation prefers bailout to default.*

Proof. Assume $\gamma = 0$. From (19), for realizations of (A_1, A_2) such that $\theta_1 \geq \frac{A_i}{A_1 + A_2}$, country 1 gains from bailout even if $A_1 > A_2$. With $A_1 > A_2$, this redistribution implies consumption is less equitable under a bailout compared to a default and this reduces social welfare. For these realizations of (A_1, A_2), a bailout will not be provided.

With $A_1 < A_2$, consumption in country 1 is lower than that in country 2 under default. If A_1 is sufficiently low, so that $\theta_1 \leq \frac{A_i}{A_1 + A_2}$ even with θ_1 large, bailout redistributes from country 2 to country 1. This increases social welfare.

These arguments hold by continuity for small enough default costs. □

Again note the conditions for bailout do not depend on R. What matters is the direction of the redistribution, not its magnitude. Of course, the bailout must itself be feasible, i.e. $\bar{\tau} \in [0, 1]$. Thus for sufficiently large R, the bailout is not feasible and default will occur. Hence, Lemma 1 applies.

[16] Implicitly, the managers act in a risk neutral fashion despite the risk aversion of the owners.

Proposition 4. *If all debt is held by countries* 1 *and* 2 *and* $v(c_i)$ *is strictly concave for households in country* $i = 1, 2$*, and* θ_1 *is sufficiently large, then multiple interior equilibria in the valuation of country* 1 *debt remain despite the prospect of a bailout.*

Proof. The first step demonstrates that default occurs. Fix $A_2 < \hat{A}$, where $\hat{A}$ was defined in (6) as the value of A_1 such that country 1 defaults iff $A_1 < \hat{A}$. Using Lemma 7, either a bailout occurs or country 1 repays its own debt for realizations of $A_1 < \hat{A}$. For realizations of $A_1 \in [A_2, \hat{A}]$ there is no bailout and country 1 prefers to default. For $A_1 > \hat{A}$ country 1 repays.

Hence, for θ_1 large, there is a range of default: $A_1, A_2)$ such that $A_1 \in [A_2, \hat{A}]$. Further, since $\hat{A}$ is increasing in R, the probability of default is increasing in R.

Lemma 1 holds in this economy since for R large enough, bailout is not feasible. This, along with the fact that the default probability is increasing in R is sufficient for multiple equilibria.

□

5 Conclusion

This paper studies the conditions for debt runs. It does so in a model where default is possible due to fundamental shocks. Prices reflect this risk. In addition, the beliefs of investors have an independent influence on debt prices. Thus debt prices reflect both fundamental and strategic uncertainty.

The model is a vehicle to explore policy measures intended to stabilize debt prices. A key issue is the incentive for a federation to bailout the debt of a member country, particularly in the presence of strategic uncertainty. The analysis highlights the sharing of fundamental and strategic uncertainty as well as the conditions for a bailout.

There are a number of interesting extensions to consider. First, the model ignores an active monetary authority. If a monetary authority controls the real interest rate, then a channel exists between monetary policy and the volatility of debt prices. Integrating the model into a monetary framework would allow a more thorough investigation of this channel as well as the consideration of bailouts through monetary policy. Camous and Cooper (2014) studies monetary and fiscal actions within a country and provides a starting point for the introduction of monetary interventions.

Second, there is no banking sector in this economy. Informally, there seems to be another important complementarity at work between country debt and the banking system. As in Cooper and Nikolov (2013), if investors are pessimistic about debt repayment, the value of country debt falls and the balance sheets of banks worsen. This implies a higher liability for countries under deposit insurance schemes and/or guarantees (either implicit or explicit) of the banking system. But the assumption of these additional liabilities supports the initial pessimistic beliefs about debt repayment. Formalizing this interaction and using it to explore additional policy measures through active monetary interventions could prove insightful.

Third, moral hazard effects are ignored. These can arise as a consequence of a bailout in a couple of forms. As studied by Cooper, Kempf, and Peled (2010), for example, the prospect of bailout can induce countries to issue more debt. Moreover, for countries under pressure to reform, a bailout can relax that pressure and be counterproductive.

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