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THE TERM STRUCTURE OF INTEREST RATES: EVIDENCE AND THEORY

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ABSTRACT

The term structure of interest rates is an old topic. Over the years, both the hypotheses debated and the research techniques used have changed considerably. Two fairly recent developments which distinguish current research are the widespread adoption of rational expectations and the integration of the term structure with the general theory of asset pricing. This survey reviews previous work from this perspective. The main objective is to catalog available evidence about term premia and to interpret this evidence in light of alternative models of term premia determination.

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THE TERM STRUCTURE OF INTEREST RATES: EVIDENCE AND THEORY

Financial markets are characterized by a wide array of fixed-income securities, each bearing its own particular rate of interest. The study of the relationship among these various yields, as opposed to their overall level, falls under the rubric of the term structure of interest rates. The distinction between models which determine relative yields and those which determine the general level of interest rates is often forced. The best formulated models should provide an integrated explanation of both. Nonetheless, a well established tradition of research has focused on the apparently simpler problem of determining only relative yields.

This paper has two main objectives (i) to survey the literature on the term structure of interest rates with particular attention to the empirical evidence and methodology; and (ii) to catalog available evidence regarding term premia and to interpret this evidence in light of alternative models of their determination.

The literature on the term structure of interest rates has grown very large and unwieldy. Over the years, both the hypotheses debated and the techniques employed by researchers have changed considerably. This shifting focus complicates the task of those seeking an introduction to this literature. An important goal of this paper is to provide an historical guide that reduces the barriers to entry for both students and non-specialists.

Two fairly recent developments which distinguish current research are the widespread adoption of rational expectations and the integration of the term structure with recent advances in the general theory of asset pricing. The important change in perspective brought about by these developments requires a re-evaluation of the earlier literature. This survey attempts to identify earlier studies that are still relevant to the current research agenda, as well as to summarize the main conclusions of recent investigations.

Economists have had a long interest in the term structure. According to the conventional wisdom, the central bank mainly affects short-term interest rates such as call money rates or the yields on Treasury bills. Real economic activity, on the other hand, is more closely linked to the yield on bonds with the same maturity as physical capital, say in the order of 10 to 20 years. According to this view, it is crucial that we understand the factors which influence the relative yields on these different types of securities, in order to understand the impact of central bank actions on the real side of the economy.[1]

The price of a bond should presumably depend upon its features. Important characteristics include (i) the maturity of the bond; (ii) the size and timing of its coupons; (iii) the provision of options to call, extend or convert the bond; and (iv) factors which affect the probability of timely payment, such as the credit worthiness of the issuer. The principal concern in the mainstream economics literature has been with the pricing of bonds identical in every respect except for maturity. In particular, economists have studied the pricing of pure discount bonds, that are not only free of default risk, but also free of call or other options.[2] Almost all of the empirical work has dealt with Treasury or high-grade corporate securities. This emphasis on a very simple and specialized aspect of bond pricing has been productive, but not without its costs. Until recently, pure discount bonds did not exist, except at short maturities. As a result, prior to the empirical testing of term structure models, actual data on the prices of heterogeneous, coupon-bearing bonds were processed into an estimate of the yield curve for pure discount bonds. This preliminary data analysis is laden with difficulty.[3]

Because of its historical importance, the expectations model of the term structure is the central focus of this survey. The literature on this subject is extremely large and often confusing. Despite the immense research activity, it may appear that we have learned little. Professional opinion has vacillated and the quality of much of the empirical research is questionable.[4] When I started my own research in this area, one of my colleagues warned that altogether too much has been written on the topic already and that we should agree to allow the entire literature to die a quiet death.

One of the conclusions of this survey is that frustration as to the implications of existing empirical research about the expectations model is largely unwarranted. Historically, most of the confusion has been due to the lack of a professional consensus about how to model expectations. If one adopts the current view that expectations are rational, in the sense of Muth (1961), the implications of existing research become much clearer. The papers which are consistent with rational expectations and exercise care in the examination of high quality data speak with an almost uniform voice.

The main developments and empirical conclusions, discussed in detail in the text, can be broadly summarized as follows.

The substantive prediction of the expectations hypothesis is that term premia are time invariant. Until the early 1970s, this prediction was not seriously challenged and the central question that dominated empirical research was the relationship between the average (unconditional) term premia and maturity. Indeed, much ink was spilled on whether or not these term premia were in fact zero. With the adoption of rational expectations, a consensus was established that term premia have been generally positive and increasing (but not monotonically) with maturity.[5]

Subsequently, the focus of research shifted to the question of whether or not movements in the yield curve are due entirely to revisions in expectations about the level of future short rates brought about by the arrival of new information.[6] In other words, if we maintain that expectations are rational can we conclude that term premia are time invariant?

The earliest empirical studies provide evidence to reject this hypothesis about term premia at the short end of the maturity spectrum and subsequent research confirms this conclusion. Using data for longer maturity bonds, however, many authors investigated and failed to reject the expectations model. Nonetheless, as described in Section 5, care in the selection of the alternative hypothesis and in the collection of data have recently resulted in the accumulation of convincing empirical evidence. The best documented result is that holding premia on long bonds have been positively correlated with the spread between long and short rates.

The paper is organized as follows. In Section 2, the expectations model is discussed and compared to competing models of the term structure. The objective is to survey different well-formulated approaches to modelling term premia. Recent research has focused on whether or not the stylized facts about term premia can be accounted for by models which treat them as rewards to bearing risk.[7] As much of this work is in its early stages, I provide only a brief discussion of the preliminary empirical findings. In general, the empirical evidence about term premia is presented on an historical basis. In Section 3, the main controversies which were debated up until the mid 1960s are reviewed. In Section 4, the rather confusing literature that followed Meiselman's (1962) suggestion of divorcing expectations from subsequent realizations is assessed. The discussion extends to the general adoption of rational expectations in the 1970s. Section 5 surveys recent evidence concerning the time variation of term premia. Following a well established tradition, the paper ends with a brief conclusion.

2. ALTERNATIVE MODELS OF PRICE DETERMINATION

Most of the research on the term structure of interest rates has focused on one of the many variants of the expectations model. However, many alternative frameworks have been proposed that also characterize equilibrium restrictions on expected asset yields. The purpose of this section is to review quickly these alternatives, since several of them are unfamiliar except to specialists in finance, and to provide a common framework for comparing them.

Consider the following class of models

$$\begin{aligned} E_t\, H_t(n) &= H(R_t(1),\, X_t) \\ &= R_t(1) + T_t(n) \qquad n = 2,3\ldots \end{aligned} \tag{2.1}$$

where $H_t(n)$ denotes the one-period holding yield (coupon plus any capital gains or losses) on an n period bond; $R_t(m)$ denotes the yield to maturity on an m-period bond; and X_t is a vector of relevant variables which will be described in more detail below. $T_t(n)$ denotes a term premium.

The left hand side of (2.1) denotes the market's expectation of $H_t(n)$. It is generally agreed that the market's expectation cannot be directly measured.[8] One of the central objectives of researchers has been to construct an empirical counterpart to the unobservable market expectation. Opinion on the merits of various suggestions has varied considerably, and debate continues.

It is important to stress that if it stands alone, the relationship described by (2.1) is a tautology. It simply expresses an accounting identity and is void of empirical content. The model becomes interesting only when we specify explicit and refutable models for expectations and for term premia. With only a model of expectation formation, (2.1) simply defines the term premium. Similarly, a model of term premium determination allows us to construct via (2.1) a model of the market's expectation.

Current opinion favours viewing E_t as a conditional expectation operator with respect to an information set $\Omega_t \subset \Omega_{t+1}$. It is usually assumed that Ω_t includes at least current and past yields on bonds of all maturities. The merits of rational as opposed to "reasonable" expectations remains an area of controversy. Nonetheless, in this paper it will be assumed that the true model (2.1) obtains with rational expectations and the evaluation of available empirical evidence will be from this perspective.

2.1 The Expectations Model of the Term Structure

Perhaps the simplest assumption we can make is that the term premia are time invariant,

$$E_t H_t(n) = R_t(1) + T(n) \qquad n = 2,3,\ldots \tag{2.2}$$

There have been many traditions in the term structure literature. Although comparing holding period yields on short and long bonds goes back at least to Keynes (1930), empirical work based on (2.2) is relatively new. Most of the original work compared forward rates to subsequent spot rates. Subsequently, authors tended to emphasize the relationship between the yield to maturity on long bonds and the sequence of future short rates. Cox, Ingersoll and Ross (1981) provide a review of these different approaches. In addition, they show that these three variants of the expectations model are logically incompatible, strictly speaking. This is moderately bothersome. Shiller (1979) and Shiller, Campbell and Schoenholtz (1983) have shown, however, that the three versions of the expectations model are not substantively dissimilar, as they are well approximated (within the range of historical variation) by a family of linear approximations which is internally consistent.[9] In particular, the holding period yield is highly correlated with the approximation $h_t(n)$ satisfying

$$h_t(n) = R_t(n) + (D(n)-1)\ [R_t(n) - R_{t+1}(n-1)] \qquad n = 2,3,\ldots \tag{2.3}$$

where $D(n) = (1-g^n)/(1-g)$ denotes the "duration" of the long bond, and g is a "typical" discount rate.

If we ignore the distinction between the holding period yield and its approximation then we are led to a constant coefficient stochastic difference equation which we can solve as

$$R_t(n) = \sum_{k=0}^{n-1} \frac{g^k}{D(n)} E_t\ R_{t+k}(1) + V(n) \qquad n = 2,3,\ldots \tag{2.4}$$

with $V(n) = \sum_{1}^{n} g^k\ T(k)/D(n)$.[10] This form of the expectations model relates the long rate to a weighted average of current and expected future short rates. The formula given in (2.4) is meant for coupon bearing bonds and as a consequence the weights decline into the future. For pure discount bonds the weights become n^{-1} yielding the familiar arithmetic approximation.

Early empirical work on the expectations model focused on forward rates $F_t(n)$. The forward rates for pure discount bonds are defined by the relationship

$$(1+F_t(n)) = (1+R_t(n+1)^{n+1}/(1+R_t(n))^n \tag{2.5}$$

Arbitrage arguments based on expectations held with certainty or risk neutrality led to the model

$$F_t(n) = E_t\, R_{t+n}(1) + L(n) \qquad n = 1,2,3,\ldots \tag{2.6}$$

With short selling, forward rates for pure discount bonds constitute the implicit one period rate at which agents can contract today to borrow or lend n periods into the future. The appropriate definition of forward rates for coupon bearing bonds is not clear. Shiller, Campbell and Schoenholtz (1983) provide a useful extension of the usual definition. Their definition of a forward rate is well approximated by

$$f_t(n) = \frac{D(n+1)\ R_t(n+1) - D(n)\ R_t(n)}{D(n+1) - D(n)} \tag{2.7}$$

If we ignore the approximation errors (i.e., treat (2.2) and (2.6) as obtaining with $h_t(n)$ and $f_t(n)$ replacing $H_t(n)$ and $F_t(n)$), then it can be shown that the three variants of the expectations model are equivalent in the sense that any one implies the other two. This is extremely useful since empirical investigation has proceeded under all three definitions. The accuracy of the linear approximations gives us some justification in treating all evidence symmetrically as pertaining to <u>the</u> expectations model.

2.2 Capital Asset Pricing Model

The workhorse of security pricing in the finance literature has been the capital asset pricing model (CAPM) developed by Sharpe (1964) and Lintner (1965). As pointed out by Roll (1971) and McCallum (1975), CAPM implies a term premium of the form[11]

$$T_t(n) = \beta_t(n)*E_t(RM_t - R_t(1)) \tag{2.8}$$

where RM_t denotes the rate of return on the market portfolio and $\beta_t(n)$ is the ratio of the conditional covariance of $H_t(n)$ and RM_t to the conditional variance of the market return.

There are many derivations of the CAPM model, but necessary and sufficient conditions for market equilibrium to obey the restrictions of CAPM are not known. The usual derivation begins from a set of sufficient conditions which includes mean variance preferences defined over end of period wealth, as originally posited by Markowitz (1952).

Although many of the predictions of the CAPM appear to be broadly correct (see Jensen (1972)), professional opinion on the model's merits is increasingly negative (see Ross(1978)).

Research has uncovered several empirical anomalies. For example, it is now well documented that idiosyncratic risk (Fama and Macbeth (1973)) and small-firm (or "January") effects (Reinganum (1981)) appear in the pricing relationship, in addition to the effect of systematic risk as measured by market proxies. It must be noted that there exist serious problems in deciding how to interpret this evidence. In an important paper, Roll (1977) has raised serious doubts as to the adequacy of empirical studies which use market proxies. Since it is generally argued that the return to the market portfolio of all assets, including stocks, bonds, real estate and human capital, can only be measured approximately, Roll's critique raises doubts as to whether the CAPM has any testable implications at all.

Theoretical unease with the CAPM is also widespread. An important criticism focuses on the assumption of mean-variance preferences used to generate asset demands. The debate involves more than just functional form. Stiglitz (1970) was one of the first to remind the profession that the fundamental reason for holding assets is to facilitate consumption plans. Rather than end of period wealth, a more traditional approach for economists is to posit that agents have preferences defined over distributions of uncertain consumption paths or sequences. At each point in time (whether continuous or discrete) agents have to choose their portfolio, amongst other things, keeping in mind its immediate return as well as its implications for future consumption/investment choices.

Fama (1970a) and Hakansson (1970, 1971) provide conditions under which this more general problem yields portfolio rules similar to the static (or atemporal) CAPM. These restrictions are quite severe and limit the usefulness of the model to account for the stylized facts about term premia. According to the CAPM, the holding premia on long term bonds are explained by the covariance of bond returns with those of the market portfolio. Variations in the premia can be accounted for by allowing this covariance to change over time (see Bollerslev, Engle and Wooldridge (1985)). However, a pattern of predictable time varying risk would seem to be exactly the sort of thing to make the assumption of myopic optimization, upon which the CAPM rests, untenable.

2.3 Structural Models of Demand and Supply

An important development in the last ten years has been the empirical implementation of structural models of demand and supply, as suggested by Brainard and Tobin (1968). The most notable contributions have been made by Ben Friedman (1977, 1980) and V. Vance Roley (1981, 1982).

These models begin by postulating risk averse investors with mean variance preferences defined over end of period wealth, and hence are intimately related to the CAPM. However, by not imposing all of the assumptions required by CAPM, they avoid the empirically embarassing prediction that agents all hold the same risky portfolio. They also differ by postulating that costs of adjustment introduce a wedge between the desired portfolio allocation, which the mean variance analysis predicts, and observed portfolios. The disaggregated models allow for different speeds of adjustment as well as quite different target portfolios.

In order to get predictions about asset yields, demand equations with the structure described above are combined with some specification for asset supplies (often that they are exogenous) and the the assumption that the expected holding period yields[12] on assets adjust to achieve market equilibrium. As a result, the implied "term premium" will depend upon (i) the level of expected yields on exogenous securities; (ii) the distribution of asset holdings across investor classes; (iii) the distribution of flows of new wealth across investor classes; (iv) the quantity of securities extant.[13] Factors (i) - (iv) were emphasized by Culbertson (1957) as important determinants of asset yields, and the Friedman-Roley work is often viewed as the state of the art implementation of the market segmentation hypothesis.

The structural models have achieved an enviable level of empirical success. They explain within sample variations of asset yields about as well as any competitor. Nonetheless, most of the profession seems to have decided to reserve judgement on both the results and the value of this research programme.

The various criticisms which have been raised against mean- variance preferences apply to the current generation of structural models as well. Myopic behaviour can be justified under certain conditions. However, the transactions costs which are at the heart of the adjustment models and therefore the dynamics of the structural demand equations require an intertemporal view. The model's dynamics should be derived from a more explicit approach to the agent's choice problem.

The main qualm about the structural modelling approach appears to be the informational requirements and the size and difficulty of the research programme which it suggests. Structural modelling requires a great deal of work, patience, and resources. The larger models cannot be estimated in their full generality. A myriad of choices including parameter restrictions must be made in order to obtain a tractable model. Although each of these choices may appear reasonable when viewed individually, the resulting model is often far removed from the original derivation and it is extremely difficult to evaluate the effects of the modelling choices taken as a group.

2.4 Multifactor Models

A generalization of the CAPM is the "multifactor" or "multi-beta" model where the term premium satisfies

$$T_t(n) = \sum_{j=1}^{J} \beta_{jt}(n)\, \theta_{jt} \tag{2.9}$$

where θ_{jt} denotes the "factors" which summarize all systematic uncertainty in the financial markets, and $\beta_{jt}(n)$ are the weights which these factors receive in determining the excess return.

The term premium model given by (2.9) can be derived as either a consequence of the intertemporal capital asset pricing model (ICAPM) or the arbitrage pricing theory (APT).

The ICAPM was introduced in a pioneering paper by Merton (1973). It is a fairly explicit equilibrium model of asset return determination. In his paper, Merton postulated that the economy's uncertainty could be summarized by a finite dimensional state vector. Agents were presumed to make consumption and portfolio choices continuously so as to maximize the expectation of a time additive utility functional. Application of theorems of control for such environments allows for the determination of asset demands and returns in terms of the value of the state vector and the parameters characterizing its evolution. Equation (2.9) appears as an implication of equilibrium on instantaneous asset returns. Merton shows that the number of factors will be (at most) equal to one plus the number of fundamental economic sources of uncertainty.

In an interesting paper, Breeden (1979) shows that the ICAPM can be expressed in the form of a single beta model with the marginal rate of substitution of aggregate consumption replacing the market return.[14] The insight provided is that agents should be willing to pay more for securities which have high payoffs in those states where consumption must be reduced. Since it is rather easy to construct economies in which the market return and aggregate consumption behave quite differently, the ICAPM can have potentially quite different predictions than the static CAPM.

The ICAPM can be reduced to the traditional CAPM, however, in a variety of ways. The simplest way to achieve this is to postulate that the sources of uncertainty at different times are independent. Combined with the assumption of time additive preferences, this breaks the choice problem up into a sequence of independent problems so that myopic behaviour is optimal.

Cox, Ingersoll and Ross (1981) compare the characterization (2.9) implied by the ICAPM with continuous time versions of the expectations model. They show that the expectations model is inconsistent with (2.9), and therefore inconsistent with the equilibrium framework used to derive it, unless the term premia are zero. They also show that risk neutrality is not sufficient to generate zero term premia, at least in continuous time.

Empirical work based on the ICAPM is recent and still exploratory. It has been hampered by the difficulties involved in translating the restrictions on the continuous time processes into restrictions on observed data. Long (1974) and Lucas (1978) provide discrete time versions of the ICAPM. Hansen and Singleton (1983) overwhelmingly reject this model. Grossman, Melino and Shiller (1985) report some qualitative success but basically reach a similiar conclusion about the continuous time version. The covariance of returns with consumption growth is high for stocks and less so for bonds, which is consistent with the fact that stocks have earned on average a higher return. However, the difference between the average return on stocks and bonds appears to be too high to explain except by postulating an incredibly high aversion to risk.

A related approach which also implies the representation (2.9) is the arbitrage pricing theory (APT) developed originally by Ross (1976).[15] The APT is not an explicit general equilibrium model in the sense that it does not describe how preferences, opportunities and information sets interact to determine market equilibrium. It begins with the empirical proposition that the realized holding period return on any asset can be written as a linear combination of, say, J common factors plus an idiosyncratic component. The key intuition is that with a large number of assets (strictly speaking an infinite number), we can invoke central limit theorems to show that portfolios can be constructed which are independent of idiosyncratic risk. If wealth is valued, it follows that the reward for bearing idiosyncratic risk should be zero in equilibrium.[16]

The generality of APT is at once its main attraction and its main fault. Since it is not an explicit general equilibrium theory, it provides us with no guidance as to the identities of the common factors, their number, or how the return generating process will change with shifts in the economic environment.[17]

Most of the empirical testing of APT has examined stock return data (Chen (1983); Lehman and Modest (1985)). The results are encouraging in that many of the pricing anomalies that plague the CAPM are accounted for by APT with a relatively small number of factors. The main exception is the small firm effect. Brennan and Schwartz (1979, 1982) have investigated a continuous time version of the APT for bond returns under the assumption that the number of factors is quite small (1 or 2) and that there is no idiosyncratic risk. They report some success in modelling the yield curve and in pricing more complicated debt instruments. However, they also conclude that a larger number of factors is needed. Unfortunately, this leads to some technical difficulties which have not yet been resolved.

3. THE EXPECTATIONS MODEL - EARLY EVIDENCE AND DEBATE

The literature on the term structure is extensive. Debate was first organized around the pattern of term premia under the maintained assumption that they did not vary over time. From the current perspective, it is useful to think about the early literature as pertaining to the pattern of the average or unconditional term premia. In this section, we review the main theories and relevant evidence, up to about the early 1960s. To anticipate, the main empirical conclusions of the early literature are that at the very short end of the maturity spectrum forward rates are not accurate predictors of subsequent spot rates, forward premia are not zero on average, and other factors (perhaps taxes and transactions costs) have significant effects on the yield curve.

The substantive prediction of the expectations model is that the term premia are constant. Historically, discussion first centered around the forward rate expression of the expectations model, namely,

$$F_t(n) = E_t R_{t+n}(1) + L(n) \qquad n = 1,2,3,\ldots \tag{3.1}$$

but of course statements about $L(n)$ can be translated into statements about the term premia $T(n)$ or $V(n)$ and vice versa.

In Section 2.1, we distinguished various traditions of the expectations models on the basis of focusing on forward rates, yields to maturity or holding period yields. This is a rather recent view. Initially, participants in the literature emphasized different models of the relationship of the term premia with the horizon n.

The hypothesis that the term premium $L(n)$, is zero for all maturities is usually attributed to Fisher (1930) or Lutz (1940),[18] and is commonly referred to as the pure expectations theory, or PET.[19] Lutz motivated PET on the basis of frictionless markets, and investors possessed with single valued and accurate expectations.[20] Early critics of PET were often content with showing that the expectations embedded in the term structure did not coincide with subsequent realizations and in fact that the two looked quite different.

Competitors to PET were quickly formulated. Hicks (1939) argued that forward rates should exceed subsequent spot rates, on average, and that the difference should increase with maturity.[21] Hicks based his argument on the assumption that most borrowers looking to finance long lived investments would prefer to borrow long, but lenders preferred the liquidity and absence of capital risk provided by short term securities. This imbalance of desired maturities for borrowing and lending, Hicks argued, would require issuers of long-term bonds to increase their promised rate of return by a positive "liquidity premium" in order to induce borrowers to purchase their securities. The longer the instrument, the greater the liquidity premium would need to be. This hypothesis was formalized as $0 \leq L(1) \leq L(2) \leq \ldots \leq L(n)$, and is usually referred to as the liquidity preference theory.

Market participants viewed both the pure expectations and the liquidity preference theories as just so much academic nonsense. In an influential paper, Culbertson (1957) articulated the market segmentation hypothesis. The basic idea was that financial markets determined market yields by the familiar process of supply and demand. Arbitrage across the maturity spectrum was limited. Flows of wealth and the relative supplies of securities played the most important role in determining security returns. A straw man version of this hypothesis maintained that expectations played no role in determining relative yields. Although much discussed, empirical implementation of the market segmentation hypothesis was elusive.

Modigliani and Sutch (1966, 1967) integrated several of the ideas of the market segmentation hypothesis with the expectations model. They argued that liquidity premia could be positive or negative and that there was no need for them to follow any systematic pattern with maturity.[22] Their preferred habitat hypothesis recognized that heterogeneous groups of borrowers and lenders preferred securities of different maturities. Life insurance companies, for example, are observed to purchase mainly long lived securities. Matching demands and supplies for bonds in their world of heterogeneous preferred habitats could generate any conceivable pattern of liquidity premiums.[23] Although Modigliani and Sutch argued that the pattern of term premia would depend upon the changing wealth and preferences of investor categories, as well as upon the maturity distribution of securities offered, they were unable to find any empirical evidence that these additional factors generated noticeable variations in the pattern of term premia. In empirical research, the richness of the preferred habitat hypothesis was reduced to the proposition that term premia need not follow any systematic pattern.

An excellent survey of empirical studies of the term structure prior to 1965 is provided by Malkiel (1966).[24] Much of this work, although lacking in econometric sophistication, remains highly relevant. The focus in many of the early studies is on the relationship between the predictions of future yields embedded in the term structure and subsequent realizations. This makes these studies entirely consistent with the assumption of rational expectations. Although the data analysis of these early studies is fairly simple (sometimes amounting to little more than the presentation of descriptive statistics) this is often more than compensated by the quality and quantity of the data examined.

Early work on the expectations model focused on the accuracy of forward rates as predictors of subsequent spot rates. Macaulay (1938) observed that before the establishment of the Federal Reserve System in 1915 there existed a pronounced and well known seasonal in the call money rate. Macaulay found that time money rates (from one to six months) did indeed anticipate the seasonal, but there was little additional evidence of successful forecasting.[25] In fact, the forward rate constructed from the term structure of very short-term securities was found to be useless in predicting the qualitative change in the spot rate. On balance, the difference between the forward rate and the spot rate was negatively correlated with observed changes in the spot rate.

In a very carefully executed study, Kessel (1965) confirmed Macaulay's findings also using very short term data. Kessel found that a stable seasonal pattern in very short term yields had emerged once again in the late fifties.[26] Using data on 27 and 55 day bills over the period 1959-61, he also concluded that the seasonal component of call money rates was anticipated. Using data on 14, 28, 42, 56, 63 and 91-day bill rates, Kessel constructed a series of implied forward rates. He found that they systematically over predicted subsequent spot rates. Like Macaulay, Kessel also found that the forward rates provided poor and, on balance, misleading qualitative predictions about the change in rates. Kessel suggested that the forward rate should be viewed as the market's expectation of the subsequent spot rate plus a term premium which varied positively with the level of the current spot rate.[27] He found that adjusting the forward rate by subtracting an estimate of the term premium provided a qualitatively accurate predictor of rate changes.[28]

Most of the early research involved the relationship between forward rates and subsequent spot rates as a test of the expectations model, but there were exceptions. Culbertson (1957) computed and graphed holding period yields (coupons plus capital gains or losses) for short and various long term Treasury securities. He considered holding periods of one week and three weeks.[29] The realized holding period yields were very different from observed spot rates and Culbertson concluded that the Lutz hypothesis of accurate expectations was totally unjustified. Culbertson went further and remarked that it was difficult for him to believe that such large discrepancies would be possible if professional speculators were attempting to arbitrage the yield differences.

Several historical episodes discussed in the early literature provide useful evidence on the relative importance of expectations in determining yields. It is worthwhile to review them.

Kessel (1965) reports prolonged periods in 1959 and 1960 when the computed one week ahead forward rate was negative.[30] Since interest rates can never be negative (such a bond being dominated by cash), negative forward rates cannot be representing the market's expectation. Furthermore, since on average the term premium for the one week maturity was positive over the sample period, this episode provides clear cut evidence that term premia have varied. It would be interesting to know if negative forward rates are a common phenomena at very short maturities. This topic does not appear to have been systematically explored.[31]

A less clear cut but perhaps more important piece of evidence about the expectations model is provided by the behaviour of U.S. rates in the forties. During the period 1942-1947 and to some extent until the Accord of 1951, the Treasury and the Federal Reserve System pursued a policy of pegging the term structure.[32] The stated aims of the policy were to reduce speculation that rates would rise, so that Treasury offerings would be well received, and to help keep the costs of war finance low. To be precise, the rates were not pegged but ceilings on yields were imposed.[33] The ceilings corresponded roughly to the term structure extant in 1941, and were maintained successfully from 1942 to 1947. Over this period, short-term securities traded at their limits. Long rates bumped against the ceiling until 1944 and then fell by about 30-40 basis point.

Walker (1954) argues that the term structure in late 1941 indicated expectations of higher rates. If so, then a credible policy of effectively fixing the path of short-term rates should have had drastic consequences. We should have either observed a precipitous fall in long rates or a dramatic shift in the maturity composition of private portfolios toward the long end of the spectrum. Neither occurred, although the latter seems closer to describing actual events.[34] Private agents did lengthen their portfolios, and purchases of new Treasury Bills were almost exclusively by the Federal Reserve System. It seems difficult to describe the portfolio shifts as dramatic, however. For example, banks continued to keep a large fraction of their portfolios in still short-term but higher yielding certificates of indebtedness.

Modigliani and Sutch (1967) argue that the success in maintaining interest rate ceilings for such a long time constitutes prima facie evidence that expectations cannot be the only determinant of yield differentials. In particular, they argue that the maturity composition of the securities supplied by the Treasury must also be very important. Kessel (1967) in his comment on their paper argues that such a conclusion is unwarranted. All we can learn from this period, he argues, is that private agents are quite willing to change the maturity composition of their portfolios in response to perceived yield differentials.

The safest conclusion appears to be that the behaviour of the term structure before the Accord remains a remarkable but relatively unexplored source of evidence.

4. FROM MEISELMAN TO RATIONAL EXPECTATIONS

By 1960, opinion as to the merits of the expectations model was almost uniformly negative. In a striking contribution, Meiselman (1962) provided a most influential and eloquent defense of the pure expectations model that revitalized the debate. Meiselman pointed out that accurate forecasting was not a necessary condition for zero term premia. They could be generated from an equilibrium model in which well financed risk neutral speculators eliminated any differential in ex ante yields. It followed, therefore, that one could not conclude that forward rates did not represent the market's expectation simply by demonstrating that they were poor predictors. It is difficult to overestimate Meiselman's impact on the term structure literature and for almost a decade his hypothesis of zero term premia remained the focus of debate.

The number of papers contributing to the literature on the expectations model since Meiselman is staggering. Unfortunately, a review up to about the mid seventies reveals few substantive results. Almost all of the empirical work during this period contains at least one of several common flaws that make them irrelevant to current debate. The important contributions during this period are almost entirely methodological.

The main problem with the empirical work from this period is the treatment of expectations. Having noted that the best forecasts possible are often far off the mark, a researcher who thinks in terms of rational expectations would expect the debate to shift to whether or not there is evidence of systematic forecasting error. However, Meiselman seemed to take the position that we can infer nothing about whether or not forward rates represented the market's expectations from comparing the implied forward rates to the subsequent expectations. He suggested a "plausible" rule for how expectations should evolve over time and then showed that forward rates behaved in a manner that was more or less consistent with it. Although Meiselman's error learning model is a very interesting mechanical rule, most of the literature which followed his lead in divorcing expectations from realizations can only be described as confusing and confused.

Meiselman's error learning model postulated that forward rates evolved according to the rule.

$$F_t(n) - F_{t-1}(n+1) = \alpha_n + \beta_n(R_t(1) - F_{t-1}(1)) \tag{4.1}$$

Using the Durand annual data on high grade corporate bond yields from 1901-1954, he estimated the relationship (4.1) for $n = 1,\ldots,9$. He found that the estimated α_n were not individually different from zero. He also found that the estimated β_n were all less than one and declined with n. Finally, the R^2 of his regressions were high for low n but declined from about 0.8 to 0.3 for $n=9$.

On the basis of his empirical findings, Meiselman concluded that (i) term premia were zero; and (ii) forward rates behaved as expectations should. We now know that both conclusions were unwarranted. Wood (1963) and Kessel (1965) were quick to point out that one could not rule out increasing liquidity premia, even if the intercepts in (4.1) were truly zero.[35] A more obvious complaint is that it is hard to see what we learn from the significant correlation between forward rate changes and innovations in the spot rate. From today's perspective, innovations in almost any variable are conceivably useful in revising expectations, and while innovations in the spot rate are a plausible source of information, they are certainly not a summary statistic. If we believe that expectations are rational, the question to ask is if observed correlations between forward rate changes and innovations in the spot rate are consistent with the stochastic properties of the latter, <u>not</u> whether these correlations are non-zero.[36]

The error learning model is in fact closely related to rational expectations. A few clarifying remarks on this score may be useful. If the short rate follows a univariate stationary process whose innovations are orthogonal to the history of publicly available information, then optimal forecasts will be updated exactly as the error learning model predicts. Of course, in this case the R^2 from the regression (4.1) should be unity. If the short rate is one member of a perhaps large information set of covariance stationary processes, then the error learning model can be shown to be consistent with optimal forecasting. However, in this more general case, the parameters and the R^2 of Meiselman's regressions can take on any pattern with the horizon n, and no testable implications are implied by his model of expectation formation.[37]

Meiselman initiated several other traditions. One of the most important was the switch to the investigation of much longer maturities. His empirical work looked at one- to nine-year-ahead one year forward rates. Previously, the implications of the expectations model for this part of the yield curve had been virtually ignored.

Unfortunately, the emphasis on longer forward rates and the more distant future carried with it a sharp deterioration in the quality of the data examined. At the short end of the spectrum, yield data are available from prices of existing and traded securities. Questions about other parts of the yield curve have to deal with missing and incomplete data. Empirical work is often based on estimated yield curves. The quality of these estimates have been highly variable. It is now generally accepted, for example, that the Durand data used by Meiselman (1962) and later by Nelson (1972) are completely unreliable for the study of forward rates.[38] Despite the problems encountered, the preliminary processing of bond price data into a yield curve estimate continues to be the norm in empirical investigation of the expectations model.

In their influential papers, Modigliani and Sutch (1966, 1967) followed Meiselman's initiative of breaking the link between expectations and realizations. In their original formulation, they began by postulating that expected holding period yields were equated to the short rate plus a term premium, as in (2.2). They hypothesized that the expected capital gains could be written in terms of a fixed coefficient distributed lag of current and past short rates. In their subsequent paper, they motivated their work on the hypothesis that expectations of the short rate were formed from its own past history, and then investigated the expression (2.4). Both approaches lead to an expression for the long rate of the form

$$R_t(n) = \sum_{j=0}^{L} \beta_j R_{t-j}(1) + Z_t\delta + \varepsilon_t \qquad (4.2)$$

where β_j and δ are parameters. Modigliani and Sutch treated the distributed lag as representing the effects of expectations. All other variables, denoted by Z_t, were treated as representing the term premium.

The Modigliani-Sutch framework turned out to be a very popular one, and literally dozens of papers were written on the specification of the lag coefficients alone.[39] In retrospect, however, the conclusion seems to be that most of this research effort was misspent.[40]

The criteria used to distinguish variables related to expectations from those related to term premia is arbitrary and implausible. The important paper by Modigliani and Shiller (1973) demonstrates the awkwardness of the working hypothesis that only the past history of a process is useful in predicting its future values. Modigliani-Shiller discovered that the rate of inflation helped improve the prediction of subsequent spot rates, at least since the late sixties. Therefore, they added a distributed lag of current and past inflation to the right hand side of the Modigliani-Sutch specification. However, their main argument to justify why inflation belonged on the right hand side of (4.2) seems to have been that viewing nominal rates as a real rate plus inflation made it natural to forecast the sum using the past history of both these variables. Once begun, this line of reasoning seems impossible to restrain. Why not view the nominal rate as the sum of the after tax rate plus a tax premium and include a distributed lag of the latter in the long rate regression?

Clearly, some better way of identifying the effect of variables on expectations and term premia was necessary. It is impossible to distinguish these two effects from consideration of a reduced form equation such as (4.2) alone. Modigliani and Shiller recognized this and they were careful, as were many authors, to buttress their arguments with direct evidence from the forecasting equation for the short rate. Their conclusion that the effects of inflation in the long rate equation reflected expectations alone also invoked a direct comparison of these coefficients with those in the short rate equation. This procedure is exactly what current practice dictates. The problem was to make these comparisons in a more formal and potentially testable manner and this was solved by explicitly incorporating the structure provided by the rational expectations hypothesis.[41]

Nelson (1972) introduced the idea of identifying expectations by first estimating a univariate ARMA process for the short rate and then solving for the coefficients on lagged short rates implied by the representation (2.4).[42] The difference between the current long rate and its predicted value given past short rates was computed and used as a proxy for the term premium. In the second stage, the estimated term premium was regressed on a vector of variables to uncover patterns of term premium behaviour. This two step procedure can be adapted fairly readily to larger information sets. Pesando (1978) uses a bivariate representation of short rates and inflation to generate his forecasts. He finds, for example, that at the short end of the maturity spectrum, the rolling premia are positively correlated with the level of rates, but that the correlation is reversed for longer horizons.[43] Pesando also finds some evidence that relative supplies of securities are correlated with his term premia estimates.

Although it is an improvement over the earlier methodology, the two step procedure suffers from some problems. For example, as usually implemented, there is a bias towards accepting the null hypothesis of time invariant term premia.[44]

The use of univariate or bivariate models to generate forecasts does not eliminate entirely the problems of identification which plague the reduced form approach of (4.2). For example, we do not know if the significant correlation between term premia and security supplies reported by Pesando reflects an expectational effect or a correlation with term premia. In the two step approach, we can provide some clarification by testing if security supplies provide a significant reduction in the errors of forecasting the short rate. This is a consistent but not very powerful test.

The major conclusion that seems to be drawn from the Modigliani-Sutch paper and subsequent related literature is that it is difficult to find variables which provide a significant improvement in explaining long rates once the correlation with other interest rates has been taken into account.

Although the coefficients of estimated reduced form equations such as (4.2) vary over time, within sample multiple correlations of a long rate on a distributed lag of short rates are uniformly high and the estimated standard errors of these equations are small. Moreover, the deterioration in the forecast performance of the estimated equations is often not serious for several years after the end of the sample used to estimate the regression parameters. It is sometimes argued that estimated reduced form equations such as (4.2) are therefore useful in forecasting and for short-run policy evaluation.

Lucas (1976) provides a convincing argument against using reduced form estimates of equations such as (4.2) for policy evaluation. In general, we should view the reduced form coefficients as a combination of deeper parameters characterizing agents' preferences and the stochastic environment which they face. Changes in policy rules amount to changes in the environment, so that reduced form estimates will not be a reliable guide for policy makers.

The problem of parameter variation over time is serious and pervasive in economics. In many cases, we have no alternative but to ignore it. Taking seriously the objective of uncovering agents' objectives and constraints will not eliminate parameter variation. The hope is that it will help us to correct for some systematic and predictable shifts.

These various remarks suggest that a linear time invariant stochastic representation for short and long rates may be a poor approximation. Fortunately, if we invoke rational expectations, we can study term premium behaviour even if the rules which agents use to form their forecasts vary over a sample. Since the assumption of time invariant expectational rules is easily relaxed when looking at forward rates or holding period yields, this may be an advantage to concentrating research within these two traditions rather than looking at the yield to maturity expression (2.4).

In general, adoption of rational expectations in the term structure literature was a significant advance and preceded its general acceptance in macroeconomics. Several authors made important contributions. Roll (1970) adopted the martingale model of Samuelson (1965) to the forward rate. Nelson (1972) appears to have been the first to introduce and exploit the techniques of time series analysis to identify optimal predictors. Sargent (1972) provided an exhaustive discussion of the implications of rational expectations for the expectations model in a world where short rates are sufficient statistics. Modigliani and Shiller (1973) confirmed the usefulness of rational expectations, helped shift attention towards multivariate information sets, and demonstrated the importance of the law of iterated projections for empirical work.

In contrast to the methodological advances, most of the empirical results of research conducted during the period 1962-1973 do not appear to merit serious review. Of course, there are some notable exceptions.

Roll (1970) continued the earlier tradition of studying forward rates and term premiums for very short maturities. He looked at weekly data from October 1946 to December 1964.[45] Roll's data were carefully collected from dealer quote sheets and were usually based on the Tuesday price for Thursday delivery. Invoking rational expectations allowed him to estimate historical term premia from sample averages of the weekly figures. Roll showed that forward premia are generally positive and, while they tend to increase with maturity, that this relationship is not monotonic.[46]

It is hard to understand, in retrospect, the fury with which Meiselman's hypothesis of zero term premia was debated for almost a decade. However, by the early seventies, a general consensus emerged that term premia existed and even that they were usually positive. Subsequently, the expectations model was identified with the proposition that term premia are time invariant. The average relationship between term premia and maturity or horizon is still investigated, but it is viewed as an empirical question and no longer excites debate.

5. TIME VARYING TERM PREMIA: RECENT EVIDENCE

Having agreed upon the generic existence of term premia, debate next turned to whether or not they varied over time. Most research activity from the early seventies to the early eighties centered around the null hypothesis of rational expectations and time invariant term premia. It is now common practice to refer to this null hypothesis simply as the expectations model, and this will be the convention followed in this section. Loosely speaking, the expectations model contends that movements in the term structure are due almost entirely to the arrival of new information and the associated revision in expectations about the future course of short-term interest rates.

For about a decade, the expectations model was often referred to in the literature as the efficient markets theory.[47] This latter choice of nomenclature was unfortunate. It sometimes left the impression that evidence of time varying term premia constituted evidence of improperly functioning capital markets. As we saw in Section 2, there is no such implication. Several asset pricing theories predict that variations in the structure or rewards to bearing risk can account for time varying term premia.

There is currently a great deal of research activity dealing with time varying term premia, so the empirical conclusions that can be offered in this survey are necessarily incomplete. However, several important results are available.

As we saw in Section 3, at the short end of the spectrum, there has been available for some time convincing evidence of time variation in forward premia. Moreover, these forward premia tend to covary positively with the level of short rates. More recent research confirms this evidence and documents that movements in forward premia at the short end of the maturity spectrum are large in a substantive sense, and account for a good deal of the variation in that portion of the maturity spectrum.

Evidence about premia for longer maturity bonds is less complete. It was much more difficult to reject the expectations hypothesis for yields on long term bonds. In part, this was due to the technical difficulty of testing the yield to maturity rather than holding period expression of the expectations model. More fundamentally, the sharp reduction in the signal to noise ratio that occurs as we move towards the longer end of the maturity spectrum makes it very difficult to distinguish between competing hypotheses. However, there is now convincing evidence that holding premia on long term bonds do vary over time. In particular, they covary positively with the long-short spread and the movements in holding premia are large relative to movements in the spread.

Initial investigations usually conveyed the impression that the expectations theory was an excellent approximation to the truth. There are several stylized facts which seem to support this position. Looking at long bonds, one finds that the first difference of their yields are well approximated as a martingale with respect to their own past history. More generally, most of the observed variance of the change in long rates cannot be predicted ex ante. Neither of these properties are implied, strictly speaking, by the expectations model but it is argued that they are close approximations to the model's predictions. In any event, the same statements can be used to describe observed excess holding period yields on long bonds.

It seems almost tautological to say that the inability to predict long rate changes must be interpreted as implying that movements in the long rate are due almost entirely to the arrival of new information. However, it could be news that generates a reassessment about the structure and rewards to risk rather than the path of short-term interest rates that is driving the change in the long rate. There are many reasons for believing that this alternative hypothesis is in fact the more plausible. Historically, movements in the long rate have been much larger than the ex post realizations of the weighted average of future short rates which appears in (2.4), and it is difficult to reconcile observed movements in the long rate with the historical properties of short rates. The errors made in using the current long rate to forecast the weighted average of future short rates have been systematically and positively related to the level of rates. Research also reveals that while excess holding period yields are extremely erratic, they are not totally unpredictable. In particular, the excess yield is positively correlated with both the general level of rates and with the long-short spread.

The evidence favourable to the expectations hypothesis at shorter maturities most often cited is that regression estimates of realized short rates on forward rates result in a coefficient that is pretty close to one. Moreover, the forecast errors implied by the forward rates are well approximated as a martingale with respect to their own past history. The stronger proposition that the forecast errors are orthogonal to all publicly available information has never been generally accepted, and studies which cast doubt on this prediction are numerous.

Pesando (1978) suggests that if the efficient markets model is correct, then the yield to maturity on a long bond should be well approximated as a martingale. In order to understand the intuition which supports this idea, it is useful to invoke Shiller's approximation to the holding period yield

$$E_t h_t(n) = R_t(1) + T_t(n) \tag{5.1}$$

or

$$E_t R_{t+1}(n-1) - R_t(n) = \frac{1}{D(n) - 1}((R_t(n) - R_t(1)) - T_t(n)) \tag{5.2}$$

For long bonds, it seems safe to ignore the distinction between E_t $R_{t+1}(n-1)$ and E_t $R_{t+1}(n)$ and treat (5.2) as a statement about predicted changes in the long rate. Even if the term premium is time invariant, it is clear that the expression on the right hand side of (5.2) will not be a constant. However, predicted changes in the long rate for spreads within the historical range of variation should be small. An example may be useful to provide some perspective. Using quarterly data and assuming that on average bonds carry a coupon of about 9 per cent, it turns out that the duration of a 15 year bond trading at par is about 28 1/2 quarters, or a little over seven years. A difference of about 100 basis points for the spread (expressed at annual rates) would amount to about a 4 basis point predicted change. Pesando estimated observed quarterly differences of Canadian long bonds to have a root mean square of about 60 basis points. If we use this figure to complete the example, we cannot escape the conclusion that predicted changes in the long rate will be very small compared to observed movements.

Pesando uses the long rate for an index of Government of Canada bonds of 10 years maturity or over to test the hypothesis that the change in the long rate is unpredictable. There are an infinity of choices for variables which could conceivably be useful in predicting the change in the long rate. Inspired by Modigliani-Sutch (1966, 1967) and Modigliani-Shiller (1973), he regresses the change in the long rate on a distributed lag of the changes in the short rate, and then on distributed lags of the changes in both the short rate and the rate of inflation. He finds that only the contemporaneous change in the short rate matters, and no evidence to contradict the martingale hypothesis.[48]

What are we to conclude from these results? The most obvious conclusion is that the Modigliani-Sutch and Modigliani-Shiller specifications provide no improvement over the martingale model without estimates of subsequent spot rates that exploit insider or non-publicly available information. It is not possible to conclude anything about the validity of the expectations model from these results.

The expectations model per se makes absolutely no prediction about the accuracy with which we can predict changes in the long rate. In fact, in the extreme case of no sources of uncertainty, we would be able to predict changes in the long rate with perfect accuracy. By the same token, we can construct models consistent with the expectations theory in which the percentage of the variation in the long rate which we can hope to predict is arbitrarily close to zero. Since the expectations model says absolutely nothing about the per cent of the variation of changes in the long rate which we can hope to predict, evidence that this percentage is small has no bearing whatsoever on the merits of the model. It is also clear that looking at the magnitude of predicted versus actual changes in the long rate is not a very useful way of investigating the issue of time varying term premia. From (5.2), it appears that substantial variation in term premia is also consistent with an inability to predict more than a small fraction of observed changes, particularly if the covariance between the term premium and the long-short spread is positive, as appears to be the case.

The testable implication of the expectations model is that the change in the long rate corrected for the effect due to the long-short spread should be unpredictable, given only publicly available information. Pesando does not take into account the correction, which turns out to be very important. Shiller (1979) demonstrates that the long-short spread is a statistically significant predictor of the change in the long rate. However, the effect is negative rather than positive as required by (5.2). This is only possible if the holding premium is positively correlated with the long-short spread, and if a 1 basis point change in the spread is usually accompanied by a more than 1 basis point change in the premium.

The inability to predict more than just a small proportion of the variance of long rate differences does seem to imply that movements in the long rate are dominated by the arrival of new information. However, this information may be about future term premia - the structure of non-diversifiable risk and the rewards for bearing it - as well as about the course of future short rates. The evidence favours the idea that the movements in long rates cannot be justified by observed historical patterns of short rate behaviour.

Sargent (1979b) assumes that the first differences of short and long rates follow a fourth order bivariate autoregressive process. He then characterizes the restrictions on this process, assuming that the long rate is equal to the arithmetic average of expected future short rates. Using quarterly sampled data on three-month bill rates and the yield on five-year notes, Sargent originally concluded that the restrictions were in fact satisfied. However, there are several problems with this study. It turns out that Sargent imposed a weaker version of the full set of restrictions implied by the expectations model. Also, that first difference representation and the restrictions implied by the expectations model are an awkward combination. The two of them can hold simultaneously only if the data display a certain singularity.[49] In a subsequent paper with Lars Hansen (1981), these problems were corrected. The full set of restrictions were imposed. The singularity issue was sidestepped by postulating a covariance stationary representation for the first difference of the short rate and the long-short spread. Also, monthly values for the same two interest rates were collected, and a procedure which corrected the standard errors for the induced serial correlation was used. The restrictions implied by the expectations model were overwhelmingly rejected. The implication is that the difference between the long rate and the subsequently realized arithmetic average of future short rates was systematically related to current and lagged values of short and long rates.

The Hansen-Sargent results are a test of the joint hypothesis of the expectations model and the covariance stationary representation which they assume. As was discussed in the previous section, however, it is more convincing if we can use techniques that are robust to the assumption that such a representation exists. Shiller (1979) explicitly compares the long rate to the subsequently realized weighted average of future short rates from (2.4), which he calls the "ex post rational" rate. He uses quarterly data on four- to six-month prime commercial paper as his short rate, and recently offered Aaa utility bond yields for his long rate. The contrasts are startling. The ex post rational rate looks very much like a constant, moving over the period 1966:1 to 1977:1 in a range of about 50 basis points. By contrast, observed movements in the long rate gyrated within a range of about 800 basis points. Because of the moving average structure of the forecast errors, Shiller does not perform any formal test of the predictability of the difference between the ex post rational rate and the observed long rate. He does note, however, that the difference is almost perfectly correlated with the current level of the long rate.

Using six different data sets, Shiller provides convincing evidence that excess holding period yields while highly erratic are not totally unpredictable. He shows that if the realized excess yield is uncorrelated with the level of the long rate - as the expectations model requires - then it is possible to derive an upper bound on the ratio of the variance of the holding period yield on the long bond to the variance of the short rate. Shiller proposed to test the expectations model by assessing the sample violation of his variance bounds. Although the observed sample violations of the bounds seem large, the statistical properties of the volatility tests which he proposed are somewhat controversial.[50] In the same paper, however, Shiller used familiar regression tests to show that the correlation between the excess yield and the level of the long rate is positive, implies a substantial term premium and is statistically significant.[51] Shiller also demonstrates that the holding premium is significantly correlated with the long-short spread.[52] This particular finding appears to be the most robust and is confirmed by many other researchers (including Mankiw and Summers (1984); Campbell (1985b); Shiller, Campbell and Schoenholtz (1983)).

Convincing evidence that the expectations model does not adequately describe the behaviour of long-term interest rates is fairly recent. However, evidence of the failure of the expectations model to explain the evolution of short term yields has never been lacking. As we noted earlier, both Macaulay and Kessel found that forward rates provided qualitatively misleading predictions of the change in spot rates, at least in the samples which they examined. Kessel's finding of negative forward rates at very short maturities is clear cut evidence that term premia have varied and that expectations alone cannot explain the movements of all yields. Kessel also provided evidence that at short maturities, the term premia are positively correlated with the level of rates. More recently, Fama (1976) and Shiller, Campbell and Schoenholtz (1983) have looked at broader data sets and confirmed these earlier conclusions. Fama (1984a) and Campbell (1985b) also report that evidence of the relationship of excess holding period yields and the spread is even stronger at the short end of the spectrum.

Startz (1982) provides estimates of the size of the variation in forward premia. His rather involved procedure amounts to taking the explained sum of squares from a regression of, say, $F_t(1) - R_{t+1}(1)$ on various variables including $R_t(1)$ as an estimate of a lower bound for the variance of $L_t(1)$. He estimates that forward premia account for at least 44 per cent of the variation in the difference between the one-month forward rate and the one-month rate realized subsequently. He also estimates that over two-thirds of the difference between the forward rate prediction for the one-month rate eleven months in the future and the subsequent realization $(F_t(11) - R_{t+11}(1))$ can be attributed to variations in the forward premium. Forward premia do not seem to contribute much to the variance of the multiperiod forecast error. Campbell (1985b) reaches similar conclusions about holding premia. He estimates that holding premia account for at least 50% of the ex post variance of the excess return on two month over one month bills, and at least 20% of the ex post variance of the excess return on 20 year bonds over one month bills. He also provides evidence that holding premia on long bonds and for stocks are very collinear, although the latter are much larger.

6. CONCLUSION

Progress in improving our understanding of the term structure relationships has been uneven. Although much ink has been spilled on the subject, many of the main ideas and positions have changed remarkably little from the original discussion and debate. We have seen, however, important clarifications of hypotheses and a marked improvement in our ability to formalize these ideas into tractable models that are potentially refutable.

Recent research suggests as stylized facts that term premia do vary, that holding premia on long bonds tend to be positively correlated with the long-short spread, and that they account for a substantial part of the variation in yield curves at the short end of the spectrum.

Much important work remains to be done. Are bond holders efficiently rewarded for the risk they bear? How do the various actions of the monetary authority affect the structure of risk and returns? These are not new questions, but recent advances have left us in a much better position to attempt to answer them.

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Notes

[1]For an early expression of this view, see Keynes (1930) chapter 37.

[2]Of course, if we can price such bonds properly then it is a trivial matter to deal with coupon bearing bonds.

[3]In the earliest studies that provided estimates of the yield curve for long maturites, such as Durand (1942), the problems were mainly due to imposing too much structure on the shape of the curves (see Buse (1967)). Since McCulloch (1971), there has been a widespread adoption of more formal approximation theory and techniques that allow for very flexible yield curve shapes. Although the level of the yield curve is now estimated fairly accurately (given enough data), there still appear to be some difficulties for very long maturities, and the derivatives of the estimated yield curves (which are used to estimate forward rates) can often display erratic behavior (see Shea (1984)).

[4]Ed Kane (1970) writes, "It is generally agreed that, ceteris paribus, the fertility of a field is roughly proportional to the quantity of manure that has been dumped upon it in the recent past. By this standard, the term structure of interest rates has become in the last dozen years an extraordinarily fertile field indeed."

[5]These stylized facts refer to the term premia $L(n)$ defined in equation (2.6) below. Analogous conclusions can be stated for the other two forms of term premia, namely $T(n)$ and $V(n)$, described in Section 2. Specific evidence is described in Section 4.

[6]In the literature, this is often described as the "efficient market hypothesis". This is an unfortunate nomenclature since it suggests that evidence of time varying term premia constitutes evidence of improperly functioning markets. Careful authors always drew the distinction between a pricing model (such as time invariant term premia) and the hypothesis that markets are efficient if they quickly and fully reflect all available relevant information. According to this alternative usage, markets are efficient if expectations behave like rational expectations with respect to some postulated information set.

[7]Taxes and transactions costs are also agreed to be important considerations, but, it seems fair to say, they have not succumbed to a general treatment with empirical consequences.

[8]Some authors, eg. Friedman (1979) and Kane (1983), advocate the use of survey data on expectations.

[9]See Campbell (1985a) for further elaboration of this point.

[10]V(n) is sometimes referred to as the average or rolling term premium, while T(n) is referred to as the marginal or holding term premium, and L(n) is the forward premium.

[11]Roll (1971) dealt with the implications of CAPM for the forward premia $L_t(n)$ of (2.6). For (2.8) to obtain, it is assumed that $R_t(1)$ represents a riskless rate. Otherwise, $R_t(1)$ should be treated as the uncertain one period yield on any portfolio uncorrelated with the market. For details, see Black (1972).

Michaelsen (1965) appears to have been the first to attempt to use the CAPM to explain the pattern of term premia. Although he was somewhat informal in his application of the theory, it is surprising that this suggestion went largely unnoticed.

[12]It is usually assumed by empirical researchers in this literature that expectations are unitary, i.e. that the current value is the best predictor of the future, so that expected capital gains are zero.

[13]Usually, in the Friedman-Roley work, government securities are treated as exogenously determined, but corporate bonds are endogenous variables.

[14]Hansen, Richard and Singleton (1982) provide a useful discussion of when and how a multifactor model can be reduced to a single beta model.

[15]See Chamberlain and Rothschild (1983) for some extensions. Rothschild (1985) as well as Dybvig (1983) provide some useful clarification.

[16]Ross (1976) discusses conditions under which this intuition is in fact correct.

[17]Cox, Ingersoll and Ross (1985) have recently provided an example of a completely specified general equilibrium model where asset prices exhibit the APT structure. While useful, the development of further example economies, especially those that incorporate monetary factors, is clearly needed.

[18]Actually, Lutz argued that the term structure would in general be upward sloping because of transactions costs. He believed that the premia would be zero after adjusting for this (small) bias.

[19]Following Malkiel (1966), some prefer the label "classical expectations hypothesis".

[20]Lutz made it clear that he envisaged agents acting as if they held single valued expectations. He also explicitly postulated that these forecasts were accurate, although he seemed uncomfortable with the idea.

[21]A pattern of increasing term premia is referred to as normal backwardation. The opposite pattern is called contango. The terms were borrowed from the commodity traders of the twenties and have nothing to do with sex.

[22]Lutz' (1940) criticism of the Hicksian liquidity preference theory amounts to saying that the preferred habitat theory is the more plausible alternative to PET.

[23]In order to avoid the association of the expression "liquidity premium" with the Hicksian theory, Nelson (1972) suggested the more agnostic "term premium". Current usage is about evenly split.

[24]A less detailed but informative survey of much the same literature is provided by Telser (1967).

[25]Macaulay speculated that the seasonal component of time money rates should have been larger given the observed magnitude of the seasonal in the call rate. Sargent (1971) repeated Macaulay's (and Kessel's (1965)) analysis using spectral techniques and confirmed these qualitative findings. Mankiw and Miron (1985), however, report that if we account for the seasonal component using dummy variables, the expectations imbedded in the term structure prior to the establishment of the Federal Reserve were accurate predictors. I can offer no explanation for this conflict.

[26]Diller (1971) estimates the seasonal movement between July and December constituted 20% of the average level of short rates from 1959-1961. By contrast, he estimates the seasonal movement in Macaulay's data to be about 35%.

[27]Friedman (1979) and Shiller (1979) also find that term premia are positively correlated with the level of rates. Nelson (1972) finds the opposite. Although Nelson's result is often cited, it is based on the Durand data and for that reason is probably best ignored.

[28]Using 28-day rates and monthly data from October 1949 - February 1961, Kessel estimates this premium to be about $0.22*R_t(1)$. Using weekly data on 91-day rates over just the last few years of this sample period, he estimates a term premium of about $.43*R_t(1)$.

[29]Shiller (1979) brought attention to this graph once again.

[30]Computed forward rates were based on ask prices. Kessel used quote sheets from three different brokers to confirm the evidence.

[31]I have been unable to find any other study which documents the existence of negative forward rates. Roll (1970) reports verifying computed negative forward rates as a data check. He does not tell us how often or when the negative rates occurred.

[32]Walker (1954) provides a useful historical summary.

[33]The ceilings were the following:

Security	Yield
3 month treasury bills	3/8 of 1 per cent
9-12 month certificates	7/8 of 1 per cent
7-9 year bonds	2 per cent
15 year or over bonds	2 1/2 per cent

[34]This opinion is based on my own casual inspection of the statistics from the Treasury Bulletin over this period.

[35]In particular $\alpha_n = 0$ is consistent with both $L(n) = 0$ and $L(n+1) = L(n) + \beta_n L(1)$.

[36]Malkiel (1966) reached pretty much the same conclusion in his review of Meiselman's contribution.

[37]See Sargent (1979a, chapter 10) for a discussion of the error learning model and of the various researchers who contributed to clarifying its relationship with optimal forecasts.

[38]The Durand data is an annual estimate of the yield curve for high grade corporate bonds. In an attempt to get at the riskless rate, it was drawn as an envelope curve, i.e., it was drawn below the observed scatter of points. Durand restricted his curves to be either level or monotonic. He also imposed several conditions to smooth his estimated curves. See Buse (1967) for a discussion of the pitfalls involved in making inferences from such data.

[39]Dobson et al. (1976) provide a survey of this literature.

[40]One of the biggest problems in interpreting the relevance for current debate of these empirical studies is the quality of their data. Modigliani and Sutch used quarterly averages of monthly figures, and the maturity of their long rate varied from 10-15 years over the sample. We know that both of these problems can sharply alter the dynamic properties of a series, and hence of optimal forecasts. Unfortunately, reversing the filter is analytically intractable.

[41]In his thesis, Sutch did compare the implied forecast equation for the short rate from his estimates of (4.2). He ignored several issues, such as non-uniqueness and the complications of time averaging his dependent variable. Nonetheless, he found the slope of the implied distributed lag to be qualitatively similar to the distributed lag obtained by estimating a short rate equation directly. Although Sutch did not report formal tests, he concluded that the two were broadly similar. Nelson (1972) opines that the difference is too large. It is interesting that the debate as to whether or not the coefficients on the distributed lag represented expectational effects continued as long as it did simply because of the technical difficulty involved in testing the issue. The debate would have been quickly resolved if Modigliani and Sutch had kept their original derivation in which the distributed lag was supposed to represent expected capital gains.

[42]Although Nelson's work was carefully executed, his decision to employ the Durand data renders his empirical results unreliable.

[43]Campbell and Shiller (1984) also report a negative relationship between the short rate and both the rolling and holding premia on long maturity bonds. Their conclusions are not subject to the qualifications which the two step approach necessitates.

[44]The test statistic for a constant term premium is asymptotically just nR^2, where n is the sample size and R^2 is the proportion of the variance explained in the regression of the forecast error from the first stage on the variables which are purported to explain term premia. The correct test statistic uses the R^2 from the regression which includes these variables <u>and</u> any variables used in the first stage to forecast short rates. [see Engle (1984). Testing for a zero term premium requires us to use the uncentered R^2.] Since including additional variables can never make the R^2 fall, and in general will cause it to rise, ignoring variables from the first stage in the second stage test biases the test towards accepting the null hypothesis.

[45]The first half of Roll's sample contains prices on bills from 1 to 13 weeks. Six month Treasury bills were first auctioned in February 1959, and after that date, prices for bills up to 26 weeks were collected.

[46]Michaelsen (1965) reached similiar conclusions about the holding premia T(n). Fama (1984b) provides a recent confirmation of Roll's findings and extends his analysis to include securities of longer maturity.

[47]Original discussion of the efficient markets theory associated it with the idea that expectational errors should be uncorrelated with publicly available information. It was therefore just a call to model expectations as rational and careful authors distinguished the model of expectations from the model of market equilibrium - in this case a time invariant term premium. With the general adoption of rational expectations, the efficient market hypothesis is now often understood to refer to the joint hypothesis described in the text. Purists may prefer to talk about the rational expectations model of the term structure, but this quickly becomes tiring. When there is no risk to confusion, we can simply speak of the expectations model.

[48]Including the contemporaneous change in the short rate instead of its innovation may introduce spurious results. Pesando repeats his test with only lagged values. Again, he finds no evidence against the martingale hypothesis. Although this last test is consistent, it would be more powerful if the innovation in the short rate were included as a regressor.

[49]These matters are elaborated in Melino (1983).

[50]See Flavin (1983) or Mankiw, Romer and Shapiro (1985) for a discussion.

[51]Mishkin (1978) points out that heteroscedasticity is an important problem for Shiller's regression. He obtains pretty much the same point estimate, but a larger standard error. The particular correction which he suggested, however, seems questionable. Another difficulty with this test is that the long rate appears to have a unit root so that the standard t-test is inappropriate.

[52]Bob Shiller informed me that this result was first pointed out to him many years earlier by Franco Modigliani.