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# MONETARY POLICIES IN INTERDEPENDENT ECONOMIES WITH STOCHASTIC DISTURBANCES: A STRATEGIC APPROACH

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#### ABSTRACT

This paper analyzes strategic monetary policies using a standard two country stochastic macro model. Three noncooperative equilibria, namely Cournot, Stackelberg, and Consistent Conjectural Variations, are considered. The Pareto Optimal equilibrium, where aggregate joint costs are minimized is also considered, and all strategic equilibria are compared to the perfectly fixed and flexible exchange rate regimes. The main conclusions obtained are: (i) Demand shocks are much less problematical than supply disturbances from the viewpoint of macro stabilization; (ii) the gains from cooperation are typically small; (iii) the strategic equilibria all show substantial margins of superiority over the fixed and flexible regimes.

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### 1. INTRODUCTION

Western economies have become increasingly interdependent during recent years. An important consequence of this is that the effects of macroeconomic policies within these economies have become more closely related. A policy implemented in one country will generate effects abroad, while the impacts of this policy on the domestic economy are modified by the behavior and policies of the foreign economies with which it is interacting. Thus policy making in a multicountry context necessarily involves strategic behavior.

Analysis of strategic behavior within the context of international macroeconomic policy began with the pioneering work of Hamada (1976), who investigated strategic behavior under the assumptions of Cournot and Stackelberg behavior. His analysis is based on a fixed exchange rate regime and the objective function involves the tradeoff between inflation and the balance of payments. More recently, Canzoneri and Gray (1985) consider alternative strategic monetary policies within the context of two economies subject to a mutual oil disturbance.<sup>1</sup>

This paper continues the analysis of strategic monetary policy. The framework it employs is a two country stochastic macro model in which both economies are subjected to stochastic demand and supply shocks and expectations are rational.<sup>2</sup> The policy makers in the two countries seek to optimize their respective objective functions, which are taken to be functions of unanticipated movements in output and in the consumer price index. The model begins by determining the usual Cournot and Stackelberg equilibria for this model. However, these represent just two possible equilibria, and a number of alternatives are also considered. In particular, in the derivation of the Cournot equilibrium, each agent takes the behavior of his opponent as given, and therefore assumes that his rival does not react to his actions. On the other hand, each agent is shown to respond in accordance with a reaction function, so that ex post, the assumption of no response is incorrect. By contrast, we also consider a Consistent Conjectural Variations equilibrium (CCV) in which each policy maker, in determining his own actions, correctly conjectures the response of his opponent. The requirement of consistency is an appealing one, and we therefore find this equilibrium concept to be a particularly interesting one.<sup>3</sup>

These three equilibrium concepts are all non-cooperative; agents behave in their own self interests under alternative strategic assumptions. We also consider a number of cooperative solutions. The first of these is where the agents choose to maximize their joint aggregate welfare. However, as Canzoneri and Gray and others have argued, cooperative equilibria may be hard to enforce in that the individual agents may have incentives to cheat and break the rules of the game.

Finally, two alternative forms of monetary regimes are considered, namely, perfectly flexible and perfectly fixed exchange rates. These represent the traditional regimes in both international macroeconomic theory and policy discussions. And although they are not usually viewed in this way, they can be regarded as representing cooperative behavior. In the first of these, the policy makers in the two economies agree to do nothing, allowing the exchange rate to respond freely to market pressures. In the latter, they agree to intervene mutually in the exchange market to maintain a fixed rate. In fact, this rate can be pegged by coordinating their policies in an infinite number of ways, and one natural alternative is considered in this paper.

An important objective of the analysis is to compare the relative merits of the various equilibria. In the process of doing this, we touch upon the old debate of fixed versus flexible rates, but our analysis can be viewed as embedding this

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discussion within a larger class of equilibrium concepts. Our analysis also addresses the more topical issue concerning the gains from cooperation over the alternative noncooperative equilibria.

The theoretical framework is outlined in Section 2, with the four strategic equilibria being discussed in Sections 3-6. Insofar as possible, our study is conducted analytically. Because of their complexity, the formal expressions characterizing the optimal policies and equilibria provide only limited insight and to obtain further understanding we combine the formal analysis of the model with numerical simulations and sensitivity analysis.<sup>4</sup> These numerical procedures are outlined in Section 7, while Sections 8 and 9 undertake the numerical solutions and the sensitivity analysis. The main conclusions and general comments are given in Section 10.

## 2. THE THEORETICAL FRAMEWORK

The analysis is based on the following two country macroeconomic model, which is a direct extension of the recent stochastic rational expectations open economy framework; see e.g., papers in Bhandari (1985). It describes two identical economies, each specializing in the production of a distinct good and trading a single common bond.<sup>5</sup> In deviation form, it is expressed by the following equations:

$$Y_{t} = d_{1}Y_{t}^{*} - d_{2}[I_{t} - [\pounds_{t}(P_{t+1}) - P_{t}]] + d_{3}(P_{t}^{*} + E_{t} - P_{t}) + u_{t}$$
(1)

$$Y_{t}^{*} = d_{1}Y_{t} - d_{2}[I_{t}^{*} - [\xi_{t}(P_{t+1}^{*}) - P_{t}^{*}]] - d_{3}(P_{t}^{*} + E_{t} - P_{t}) + u_{t}^{*}$$

$$0 < d_{1} < 1, d_{2} > 0, d_{3} > 0$$

$$(1')$$

$$M_{t} - P_{t} = \alpha_{1}Y_{t} - \alpha_{2}I_{t}$$
(2)  
$$\alpha_{1} > 0, \alpha_{2} > 0$$
  
$$M_{t}^{\star} - P_{t}^{\star} = \alpha_{1}Y_{t}^{\star} - \alpha_{2}I_{t}^{\star}$$
(2')

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$$I_{t} = I_{t}^{*} + \xi_{t}(E_{t+1}) - E_{t}$$
(3)

$$Y_{t} = \gamma [P_{t} - \xi_{t-1}(P_{t})] + v_{t}$$
(4)

$$Y_{t}^{*} = \gamma [P_{t}^{*} - \beta_{t-1}(P_{t}^{*})] + v_{t}^{*}$$
(4')

$$C_{t} = \delta P_{t} + (1 - \delta)(P_{t}^{*} + E_{t})$$

$$\frac{1}{2} < \delta < 1$$
(5)

$$C_{t}^{*} = \delta P_{t}^{*} + (1 - \delta)(P_{t} - E_{t})$$
(5')

where

Y = real output, measured as a deviation about its natural rate level,

- P = price of domestic output, expressed in logarithms,
- C = consumer price index, expressed in logarithms,
- E = exchange rate (measured in terms of units of domestic currency per unit of foreign currency), measured in logarithms,
- I = nominal interest rate, expressed in natural units,
- M = nominal money supply, expressed in logarithms,
- $\xi_{+}$  = expectation, conditioned on information at time t,
- u, = stochastic shift in demand,
- $v_{+}$  = stochastic shift in supply.

Domestic variables are unstarred; foreign variables are denoted with asterisks. We shall also refer to these as Country 1 and Country 2, respectively.

Equations (1) and (1') describe equilibrium in the two goods markets. Output depends upon the real interest rate, output in the other country, the relative price, and the stochastic shift in demand. The corresponding effects across the two economies are identical and the relative price influences demand in exactly offsetting ways. The money market equilibrium conditions in the two

countries are standard and described by (2) and (2') respectively.<sup>6</sup> It is trivial to modify these relationships to allow for shifts in the money demand analogous to  $u_t$  and  $v_t$ . Such shifts can simply be absorbed in the money supply M and accommodated for directly in any money supply adjustment rule.<sup>7</sup> The perfect substitutability between domestic and foreign bonds is described by the interest rate condition (3). Equations (4) and (4') describe the consumer price index (CPI) in the two economies. They embody the assumption that in each country a proportion  $\delta$  of income is spent on the respective home good. We assume that residents of each country have a preference for their own good, so that  $\delta > 1/2$ . Note that the real interest rate in (1) and (1'), and the real money supplies in (2) and (2') are deflated by the output price of the respective economies. Little would be changed, except for additional detail, if the deflators were in terms of their respective CPI's.<sup>8</sup> Finally, equations (5) and (5') describe outputs in the two economies in terms of standard Lucas supply functions; the deviation in output from its natural rate is postulated to be a positive function of the unanticipated movement in the price of output, together with the stochastic shift in supply.

The stochastic variables,  $u_t$ ,  $v_t$ ,  $u_t^*$ ,  $v_t^*$ , are assumed to be independently distributed with zero means. If in addition, as in fact turns out to be the case,  $M_t$  and  $M_t^*$  depend only on these current disturbances, then as is well known, the rational expectations solution to the system (1) - (5) implies that

$$\xi_{t}(P_{t+s}) = \xi_{t}(P_{t+s}^{*}) = \xi_{t}(E_{t+s}) = 0 \text{ for all } t, s$$
 (6)

The exchange rate and price level in all future periods are both expected to remain constant. The fact that this constant is zero, is simply a consequence of specifying the system in deviation form. In particular, setting s = 1 in (6), yields

$$\xi_{t}(P_{t+1}) = \xi_{t}(P_{t+1}^{*}) = \xi_{t}(E_{t+1}) = 0 \text{ for all } t$$
 (6')

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One further important feature is that the shifts  $u_t$ ,  $u_t^*$ ,  $v_t$ ,  $v_t^*$  are assumed to be observed at time t and therefore to determine the policy makers' decisions at that time. Indeed, these shifts are what generate the strategic problem.

Equations (1) - (5), together with (6) describe the structure of the two economies. The policy makers in these economies are assumed to minimize quadratic cost functions specified in terms of unanticipated deviations in output and the CPI from their respective expected levels. Under the assumptions on the underlying stochastic variables, these expectations are all zero. Thus the respective functions to be optimized are simply

$$\Omega \equiv aY_{t}^{2} + (1-a)C_{t}^{2}$$

$$\Omega^{*} \equiv aY_{t}^{*2} + (1-a)C_{t}^{*2}$$
(7)
(7)

where a and 1-a are the relative weights assigned to output stability on the one hand, and price stability on the other.

Using (6'), equations (1) - (5) may be solved for  $Y_t$ ,  $Y_t^*$ ,  $E_t$  as follows

$$Y_{t} = \phi_{1}M_{t} + \phi_{2}M_{t}^{*} + \phi_{3}u_{t} + \phi_{4}u_{t}^{*} + \phi_{5}v_{t} + \phi_{6}v_{t}^{*}$$
(8a)

$$Y_{t}^{*} = \phi_{2}M_{t} + \phi_{1}M_{t}^{*} + \phi_{4}u_{t} + \phi_{3}u_{t}^{*} + \phi_{6}v_{t} + \phi_{5}v_{t}^{*}$$
(8b)

$$E_{t} = \beta_{1}(M_{t} - M_{t}^{*}) + \beta_{2}(u_{t} - u_{t}^{*}) + \beta_{3}(v_{t} - v_{t}^{*})$$
(8c)

where

$$\phi_{1} \equiv \frac{\gamma}{2} \left( \frac{d_{2}}{D} + \frac{d_{2}^{+2d}}{D} \right) > 0 ; \qquad \phi_{2} \equiv \frac{\gamma}{2} \left( \frac{d_{2}}{D} - \frac{d_{2}^{+2d}}{D} \right)$$

$$\phi_{3} \equiv \frac{\alpha_{2}\gamma}{2} \left( \frac{1}{D} + \frac{1}{D^{\dagger}} \right) > 0 ; \qquad \phi_{4} \equiv \frac{\alpha_{2}\gamma}{2} \left( \frac{1}{D} - \frac{1}{D^{\dagger}} \right) > 0$$

$$\phi_{5} \equiv \frac{(1+\alpha_{2})}{\gamma} \phi_{1} > 0 ; \qquad \phi_{6} \equiv \frac{(1+\alpha_{2})}{\gamma} \phi_{2}$$

$$\beta_{1} = \frac{\gamma(1+d_{1}) + d_{2} + 2d_{3}}{D'} > 0 ; \qquad \beta_{2} = \frac{-(\gamma\alpha_{1}+1)}{D'} < 0$$

$$\beta_{3} = \frac{(1+d_{1}) - \alpha_{1}(d_{2}+2d_{3})}{D'}$$

$$D = \gamma[(1-d_{1})\alpha_{2} + d_{2}\alpha_{1}] + d_{2}(1+\alpha_{2}) > 0$$

$$D' = \gamma[(1+d_{1})\alpha_{2} + (d_{2}+2d_{3})\alpha_{1}] + (d_{2}+2d_{3})(1+\alpha_{2}) > 0$$

Equations (8a) - (8c) have several interesting features. First, the symmetry of the underlying economies is reflected in the symmetry of these reduced form solutions. As expected, an increase in  $M_r$ , as well as positive disturbances in domestic demand  $u_t$  or supply  $v_t$  lead to increases in domestic output. A positive foreign demand shock  $u_t^*$  also leads to an increase in domestic output, although by a lesser amount than when the demand shock is of domestic origin. The reason for the positive spillover is that an increase in  $u_t^*$  leads to an appreciation of the foreign currency (depreciation of the domestic currency), thereby stimulating demand for domestic output and domestic output itself. The effect of an increase in the foreign money supply on domestic output,  $\phi_2$ , and vice versa, is highly indeter-This is because, on the one hand, an increase in  $M_t^*$  raises foreign outminate. put and demand, giving rise to the usual positive spillover onto domestic demand and output. But at the same time, the foreign monetary expansion leads to a depreciation of the foreign currency (appreciation of the domestic currency). This leads to an increase in the relative price of domestic goods, thereby leading to a contraction in domestic demand and output. This "negative transmission" mechanism is a familiar one, dating back to early work by Mundell (1963).The direction of the net effect is given by

 $sgn \phi_2 = sgn\{\alpha_2[d_2d_1 - d_3(1 - d_1)]\}$ 

For plausible parameter values, we find that the negative effect dominates. Indeed, the small numerical magnitude of  $\phi_2$  turns out to be very important for the numerical comparison of the alternative strategic equilibria. When  $\phi_2$  is small or negative, this turns out to reduce the quantitative degree of interaction between the policy instruments in the two economies. In this case, the strategic elements in the determination of optimal policy are minimal and the various strategic equifibria all tend to be numerically close. When the relative price effect  $d_3$  is small, so that  $\phi_2 > 0$  and larger numerically, the interaction between the policy instruments increases in importance. Also, the role of prices in market clearing decreases in importance, thereby increasing the scope for discretionary monetary policy. Greater numerical variation between the various equilibria is obtained. For this reason, it is important in our numerical work below to make the distinction between "large" and "small" values of  $d_3$ .<sup>9</sup>

The other interesting point to note is that the exchange rate responds purely to differentials between the domestic and foreign variables. Any shock, or policy change, which is common to both economies, leaves the exchange rate unchanged.

Combining the solutions (8a)-(8c) with (4), (4'), (6), (6') and substituting into (5), (5'), the solutions for the CPI are

$$C_{t} = \eta_{1}M_{t} + \eta_{2}M_{t}^{*} + \eta_{3}u_{t} + \eta_{4}u_{t}^{*} + \eta_{5}v_{t} + \eta_{6}v_{t}^{*}$$
(9a)

$$C_{t}^{*} = \eta_{2}^{M} t + \eta_{1}^{M} t + \eta_{4}^{u} t + \eta_{3}^{u} t + \eta_{6}^{v} t + \eta_{5}^{v} t$$
(9b)

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where

$$\begin{split} n_1 &\equiv [\delta \phi_1 + (1-\delta)\phi_2]/\gamma + \beta_1(1-\delta) > 0 \ ; \ n_2 &\equiv [\delta \phi_2 + (1-\delta)\phi_1]/\gamma - \beta_1(1-\delta) \\ n_3 &\equiv [\delta \phi_3 + (1-\delta)\phi_4]/\gamma + \beta_2(1-\delta) \ ; \ n_4 &\equiv [\delta \phi_4 + (1-\delta)\phi_3]/\gamma - \beta_2(1-\delta) \\ n_5 &\equiv [\delta(\phi_5 - i) + (1-\delta)\phi_6]/\gamma + \beta_3(1-\delta) \ ; \ n_6 &\equiv [\delta \phi_6 + (1-\delta)(\phi_5 - 1)]/\gamma - \beta_3(1-\delta) \\ \end{split}$$
By raising domestic output demand and causing the domestic currency to depreciate, a domestic monetary expansion raises the domestic CPI. By contrast, a foreign monetary expansion causes the domestic currency to appreciate, which together with the likely negative output transmission effects, generally (but not always) causes the domestic CPI to fall.

### 3. OPTIMAL STRATEGIES

The optimal policy problem confronting cach of the policy makers is to choose their respective nominal money supplies to minimize their cost functions (7), (7') subject to the constraints (8a), (8b), (9a), (9b). A key feature in the determination of the equilibrium concerns the strategic behavior and the following equilibria will be derived.

### A. Cournot

Under the Cournot assumption, each policy maker chooses his money supply so as to minimize his respective cost function, taking the behavior of his opponent as remaining fixed. Taking partial derivatives of (7), (7'), respectively, this gives rise to the optimality conditions.<sup>10</sup>

$$\frac{\partial \Omega}{\partial M} \equiv aY \frac{\partial Y}{\partial M} + (1-a)C \frac{\partial C}{\partial M} = 0$$
(10a)

$$\frac{\partial \Omega^{\star}}{\partial M^{\star}} \equiv aY^{\star} \frac{\partial Y^{\star}}{\partial M^{\star}} + (1-a)C^{\star} \frac{\partial C^{\star}}{\partial M^{\star}} = 0$$
(10b)

Substituting (8a), (8b), (9a), (9b), as well as the appropriate partial derivatives, into (10a), (10b) yields the pair of linear equations in M, M\*,

$$\frac{\partial\Omega}{\partial M} = \Psi_{11}M + \Psi_{12}M^* + \Psi_{13}u + \Psi_{14}u^* + \Psi_{15}v + \Psi_{16}v^* = 0$$
(11a)

$$\frac{\partial \Omega^{\star}}{\partial M^{\star}} = \Psi_{12}^{M} + \Psi_{11}^{M^{\star}} + \Psi_{14}^{u} + \Psi_{13}^{u^{\star}} + \Psi_{16}^{v} + \Psi_{15}^{v^{\star}} = 0$$
(11b)

where

$$\Psi_{1j} \equiv a\phi_1\phi_j + (1-a)\eta_1\eta_j \qquad j = 1, \dots, 6$$

Equations (11a), (11b) define the reaction curves for Country 1 (the domestic economy) and Country 2 (the foreign economy), respectively. The slopes of these curves in M-M\* space are

$$\left(\frac{dM^{*}}{dM}\right)_{1} = -\frac{\Psi_{11}}{\Psi_{12}}; \left(\frac{dM^{*}}{dM}\right)_{2} = -\frac{\Psi_{12}}{\Psi_{11}}$$

and with  $\Psi_{11} > 0$ , these depend upon sgn  $\Psi_{12} = \text{sgn} \{a\phi_1\phi_2 + (1-a)\eta_1\eta_2\}$ . Given  $\phi_1$ ,  $\eta_1 > 0$ , and with  $\phi_2$ ,  $\eta_2$ , tending to be negative (the latter strongly so),  $\Psi_{12} < 0$ . This in fact turns out to be so for 49 of the 50 parameter sets we consider. Thus taking  $\Psi_{12} < 0$ , the reaction curves are positively sloped, implying that a monetary expansion in one country induces a monetary expansion in the other.

The Cournot equilibrium is attained at the intersection of the two reaction curves (lla), (llb), namely,

$$M = -\frac{\alpha_2}{d_2}u - \frac{(1+\alpha_2)}{\gamma}v - \frac{(\Psi_{13} - \frac{\alpha_2}{d_2}\Psi_{11})}{\Psi_{11} - \Psi_{12}}(u-u^*)$$
(12a)

$$+ \frac{(1-a)\eta_{1}}{\gamma(\Psi_{11}^{2}-\Psi_{12}^{2})} [\Psi_{11}v - \Psi_{12}v^{*}]$$

$$M^{*} = -\frac{\alpha_{2}}{d_{2}}u^{*} - \frac{(1+\alpha_{2})}{\gamma}v^{*} + \frac{(\Psi_{13} - \frac{\alpha_{2}}{d_{2}}\Psi_{11})}{\Psi_{11} - \Psi_{12}}(u-u^{*})$$

$$+ \frac{(1-a)\eta_{1}}{\gamma(\Psi_{11}^{2}-\Psi_{12}^{2})} [\Psi_{11}v^{*} - \Psi_{12}v]$$
(12b)

The shifts in domestic demand and supply impact directly upon domestic output through the aggregate demand and the aggregate supply functions, respectively. Equations (12) indicate that one component of the optimal monetary policies in the two economies is to contract the money supply sufficiently to ensure that the domestic interest rate rises so as to neutralize exactly these effects on output.<sup>11</sup> These adjustments are described by the terms  $-\alpha_2 u/d_2 - (1+\alpha_2)v/\gamma$  in (12a) and  $-\alpha_2 u^*/d_2 - (1+\alpha_2)v^*/\gamma$  in (12b). We shall refer to these as being the "direct shift" component of the optimal policy rules. But, in addition, the shocks are transmitted across the two economies through movements in the exchange rate (relative price movements) and the policy responses themselves. These effects are incorporated in the terms involving (u-u\*), ( $\Psi_{11}v - \Psi_{12}v*$ ) for the domestic economy, and the analogous expressions for the foreign country. We shall refer to these as being the "interactive" component of the optimal policy rule.

#### A. Demand Shocks

It is interesting to note that these interactive components of the demand shifts require totally symmetric adjustments in the two economies. It can be verified by direct evaluation that  $(\Psi_{13} - \frac{\alpha_2}{d_2} \Psi_{11})/(\Psi_{11} - \Psi_{12}) < 0$ , so that in response to a positive domestic demand shock, say, the domestic monetary authority should expand its money supply in response to the interactive effect, thereby offsetting (but only partially) the initial contraction in response to the direct shift effect.<sup>12</sup> At the same time, the foreign monetary authority should contract its money supply, doing so by an amount which exactly matches the interactive component of the domestic authority's response.

The reason for this is as follows. From the reduced form solutions, (8), it is seen that the net effect of the initial expansion in domestic demand, together with the direct monetary contraction, on domestic output is  $dY = \phi_3 - \alpha_2 \phi_1/d_2 < 0$ . By neglecting the relative price effect and policy interaction, the domestic monetary authority overcontracts its money supply and domestic output falls. At the same time, both the demand expansion and the initial domestic monetary contraction cause the domestic exchange rate to appreciate; i.e., the foreign currency depreciates, thereby inducing an expansion abroad. In order to stabilize the foreign economy, the foreign monetary authority contracts its money thereby now causing an appreciation of the foreign currency, with positive spillovers to the domestic economy. Furthermore, by now expanding its money supply, the domestic monetary authority is able to correct for the initial overcontraction which occurred.

The second demand shock of interest is that of a worldwide expansion, shared equally by the two economies, so that u = u\*. In this case, the optimal response is simply for each policy maker to contract his money supply to neutralize the effects of the disturbance in his economy. There are in effect no relative price spillover effects to be taken into account.

The solutions for output and inflation in the two economies are reported in equations (A.1) and (A.6) of the Appendix. From these equations we see that after each policy maker has accommodated the direct disturbances in his own economy, output, inflation, and therefore welfare costs, in the two economies depend upon the difference between the domestic and foreign demand shocks, viz (u-u\*). This again is a consequence of the symmetry of the two economies. Consider a domestic demand shock u > 0. After allowing for the contractions in the money supply which occur both at home and abroad, output in the home economy rises, while output abroad falls. At the same time, the domestic CPI falls and the CPI abroad increases. Basically, this is because of the appreciation of the domestic currency, which more than offsets the positive effects of the domestic demand expansion on the price of domestic output.

The most interesting feature of these results is the perfect symmetry of the demand effects across the two economies. A unit shift in demand in Country 1, say, has precisely equal and opposite effects on output and the CPI in the two economies. The welfare effects, as measured by the quadratic objective functions, are therefore equally borne by both economies. A further consequence is that if the two economies are both subjected to identical shifts in demand, reflecting a worldwide demand shift, then the monetary authorities need simply neutralize the direct effects of these shocks in their respective economies. This will ensure that the output level and CPI in the two economies remain pegged at their respective equilibrium levels. Welfare costs are minimized at zero; both economies will attain their respective Bliss points.

### B. Supply Shocks

The adjustments to the interactive components of the supply shocks are not perfectly symmetric. A positive shock in domestic supply requires a domestic

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monetary expansion, in order to adjust for the effects of the initial (direct) monetary contraction, namely  $-(1+\alpha_2)v/\gamma$ . The reasoning for this is essentially analogous to that for demand shocks, described above. The combined effects of the positive supply shock and the direct monetary contraction on output are  $\phi_5 - \phi_1(1+\alpha_2)/\gamma = 0$ ; i.e., they are exactly offsetting. On the other hand, they lead to an appreciation of the domestic currency, leading to a reduction in the domestic CPI. Given the quadratic cost function, domestic welfare is improved by now increasing the money supply, thereby moderating the reduction in the domestic CPI and requiring output to increase somewhat.

The overall effects of the domestic supply shock on the domestic money supply depends critically upon a, the relative cost of output stability in the welfare function. If this is large, the expansionary component is dominated by the direct contractionary component and on balance, the money stock in the domestic economy falls. However, if the objective function is weighted towards price stability, the expansionary effect is the dominant one, leading to an overall increase in the domestic money supply.

The response abroad, or equivalently the domestic response to a foreign supply shock, depends upon whether  $\Psi_{12} \gtrless 0$ . Taking the more likely case where  $\Psi_{12} \lt 0$ , so that the reaction functions are positively sloped, we see that the increase in the domestic money supply resulting from the interactive component, leads to a monetary expansion abroad.

The net effect of a positive domestic supply shock on the two economies is seen in equations (A.4). Output at home and abroad both increase, with the domestic effect being larger. At the same time, the CPI's in the two economies will fall, with again the greater effect being in the domestic economy. Thus, in contrast to the perfectly symmetric effects of a single country demand shock, a domestic supply shock has significantly greater effect on the output and CPI

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of the domestic economy than it does on the foreign economy. The welfare costs are therefore borne more heavily by the economy in which the shock is taking place.

Worldwide supply shifts, experienced equally by both economies, generate equal output and CPI effects in the two countries. The nominal exchange rate remains unchanged. The effects of the shocks in the two economies compound one another, making the attainment of the Bliss point (zero welfare costs) impossible. In this respect, worldwide supply shocks impose a much more serious stabilization policy problem than do worldwide demand shocks.

### 4. STACKELBERG EQUILIBRIUM

In deriving the Stackelberg equilibrium we shall treat Country 1 as the leader, with Country 2 being the follower. The procedure is familiar. Country 1 optimizes its welfare function subject to the reaction curve of the follower. The solutions for the optimal policies and the outputs and inflation rates are of the same general form as for the Cournot case. However, since the expressions turn out to be rather involved, and not particularly enlightening, we merely summarize their main qualitative aspects.

First, part of the adjustment of the policy instruments should be to accommodate to the "direct shift" terms, just as was the case in the Cournot equilibrium. Secondly, the demand shifts u, u\* enter the "interactive" component of the optimal rules symmetrically, although in contrast to the Cournot case, the coefficients are different for the leader than for the follower. Thirdly, because of this symmetry, if both economies are subject to identical shifts in demand, then the monetary authorities in each country need simply neutralize the direct effects of the demand shifts in their own economy. This will ensure the attainment of the Bliss point.

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Supply shifts are more complicated. First, domestic and foreign disturbances enter asymmetrically. Moreover, they impact differently on the leader from the follower. A variety of patterns regarding the effects of the supply shifts on outputs and inflation in the two countries are found and are noted further in Section 8 below.

# 5. CONSISTENT CONJECTURAL VARIATIONS

Under Cournot behavior, each policy maker assumes that the other does not respond to his actions. In fact, however, each policy maker will respond in accordance with his reaction curve. The Cournot equilibrium is therefore consistently wrong in predicting the response of the rival. The CCV equilibrium assumes that each agent, in choosing his own strategy, takes the response of his rival into account. Furthermore, the response is correctly anticipated and hence the solution corresponds to a rational expectations equilibrium.

The optimality conditions for the two countries under the assumption of CCV are given by

$$\frac{\partial\Omega}{\partial M} + \frac{\partial\Omega}{\partial M^{\star}} \left(\frac{dM^{\star}}{dM}\right)_{2} = 0$$
(13a)

$$\frac{\partial \Omega^{*}}{\partial M^{*}} + \frac{\partial \Omega^{*}}{\partial M} \left(\frac{dM}{dM^{*}}\right)_{1} = 0$$
(13b)

where  $(dM*/dM)_2$ ,  $(dM/dM*)_1$ , denote the correcly conjectured response on the part of the opponent to each policy maker's decisions. For notational convenience we let

$$\left(\frac{dM^{\star}}{dM}\right)_2 \equiv x_2; \left(\frac{dM}{dM^{\star}}\right)_1 \equiv x_1$$

Performing the partial differentiation in (13a) and (13b), the optimality conditions in the two contries are, respectively

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$$(\Psi_{11} + x_2 \Psi_{12})M + (\Psi_{12} + x_2 \Psi_{22})M^* = -(\Psi_{13} + x_2 \Psi_{23})u - (\Psi_{14} + x_2 \Psi_{24})u^*$$
$$- (\Psi_{15} + x_2 \Psi_{25})v - (\Psi_{16} + x_2 \Psi_{26})v^*$$
(14a)

$$(\Psi_{12} + x_1\Psi_{22})M + (\Psi_{11} + x_1\Psi_{12})M^* = -(\Psi_{14} + x_1\Psi_{24})u - (\Psi_{13} + x_1\Psi_{23})u^*$$
$$- (\Psi_{16} + x_1\Psi_{26})v - (\Psi_{15} + x_1\Psi_{25})v^*$$
(14b)

where

$$\Psi_{2j} \equiv a\phi_{2}\phi_{j} + (1-a)\eta_{2}\eta_{j} \qquad j = 1, \dots, 6$$

From (14a) and (14b) we obtain

$$\left(\frac{dM}{dM\star}\right)_{1} \equiv x_{1} = -\left[\frac{\Psi_{12} + x_{2}\Psi_{22}}{\Psi_{11} + x_{2}\Psi_{12}}\right]$$
(15a)  
$$\left(\frac{dM\star}{dM}\right)_{2} \equiv x_{2} = -\left[\frac{\Psi_{12} + x_{1}\Psi_{22}}{\Psi_{11} + x_{1}\Psi_{12}}\right]$$
(15b)

The slope of the reaction curve of each policy maker depends upon the slope of the reaction function of his rival. These two equations provide a pair of equations in  $x_1$  and  $x_2$ . It is immediately seen that the solutions are

$$\mathbf{x}_1 = \mathbf{x}_2 = \mathbf{x}$$

where x satisfies the quadratic equation

$$\Psi_{12}x^{2} + (\Psi_{11} + \Psi_{22})x + \Psi_{12} = 0$$
(16)

Thus the optimal monetary policies under CCV, obtained by solving (14a), (14b), are given by

$$M = -\frac{\alpha_2}{d_2} u - (\frac{1+\alpha_2}{\gamma})v - \frac{\left[(\Psi_{13} - \frac{\alpha_2}{d_2}\Psi_{11}) + x(\Psi_{23} - \frac{\alpha_2}{d_2}\Psi_{12})\right]}{(\Psi_{11} + x\Psi_{12}) - (\Psi_{12} + x\Psi_{22})} (u-u^*) + \frac{(1-a)(\eta_1 + x\eta_2)}{\gamma\left[(\Psi_{11} + x\Psi_{12})^2 - (\Psi_{12} + x\Psi_{22})^2\right]} \left[(\Psi_{11} + \Psi_{12}x)v - (\Psi_{12} + \Psi_{22}x)v^*\right] (17a)$$

$$M^* = -\frac{\alpha_2}{d_2} u^* - (\frac{1+\alpha_2}{\gamma})v^* + \frac{\left[(\Psi_{13} - \frac{\alpha_2}{d_2}\Psi_{11}) + x(\Psi_{23} - \frac{\alpha_2}{d_2}\Psi_{12})\right]}{(\Psi_{11} + x\Psi_{12}) - (\Psi_{12} + x\Psi_{22})} (u-u^*) + \frac{(1-a)(\eta_1 + x\eta_2)}{2} \left[(\Psi_{11} + \chi_{12}) - (\Psi_{12} + \chi_{22})\right] (u-u^*) (17b)$$

$$+ \frac{1}{\gamma [(\Psi_{11} + x\Psi_{12})^{2} - (\Psi_{12} + x\Psi_{22})^{2}]} [(\Psi_{11} + \Psi_{12}x)v^{*} - (\Psi_{12} + \Psi_{22}x)v]$$
(17b)

where x is the solution to (16). The optimal policies are of the same general form as (12a), (12b). Observe that since x is the solution to a quadratic equation, there are two roots. Denoting these two roots by say  $x^1$ ,  $x^2$ , we see that these roots are (i) real with

(ii) 
$$x^{1}x^{2} = 1$$

$$x^{1} + x^{2} = -(\Psi_{11} + \Psi_{22})/\Psi_{12}$$

If  $\Psi_{12} < 0$ , then  $x^1 > 1$ ,  $0 < x^2 < 1$ ; if  $\Psi_{12} > 0$ , the two roots lie in the range  $x^1 < -1$ ,  $-1 < x^2 < 0$ . Thus there are two equilibria which correspond to consistent conjectural variations. In the case of country-specific supply shocks these equilibria may give rise to conflicts in welfare for the two economies. One solution is better for one country, while the other equilibrium is preferable for the other. In this case we choose solution having lower aggregate welfare costs as the equilibrium.

The solutions for output and CPI in the two countries are obtained by . substituting (17a), (17b) into (8a,b), (9a,b), with the resulting expressions

being reported in the equations (A.2) of the Appendix. Since the qualitative properties of both the optimal rules and the behavior of the economy are similar to those under Cournot, we do not discuss these expressions further at this time.

### 6. COOPERATIVE EQUILIBRIA

The solutions discussed so far are all noncooperative. We now consider alternative forms of cooperative equilibria.

#### A. Pareto Optimal

Assume that the two policy makers collude to minimize their aggregate joint cost function.

$$\Omega + \Omega^* \equiv a(Y^2 + Y^{*2}) + (1 - a)(C^2 + C^{*2})$$
(18)

Differentiating (18) with respect to M, M\* yields the optimality conditions

$$a\left[Y\frac{\partial Y}{\partial M} + Y^*\frac{\partial Y^*}{\partial M}\right] + (1-a)\left[C\frac{\partial C}{\partial M} + C^*\frac{\partial C^*}{\partial M}\right] = 0$$
(19a)

$$a\left[\gamma \frac{\partial Y}{\partial M^*} + \gamma^* \frac{\partial Y^*}{\partial M^*}\right] + (1-a) \left[C \frac{\partial C}{\partial M^*} + C^* \frac{\partial C^*}{\partial M^*}\right] = 0$$
(19b)

with the optimal policies being

$$M = -\frac{\alpha_2}{d_2} u - (\frac{1+\alpha_2}{\gamma})v - \frac{\left[(\Psi_{13} - \frac{\alpha_2}{d_2}\Psi_{11}) - (\Psi_{23} - \frac{\alpha_2}{d_2}\Psi_{12})\right]}{\Psi_{11} + \Psi_{22} - 2\Psi_{12}} (u-u^*)$$

$$-\frac{(1-a)\left[2\eta_2\Psi_{12} - \eta_1(\Psi_{11} + \Psi_{22})\right]}{\left[(\Psi_{11} + \Psi_{22})^2 - 4\Psi_{12}^2\right]\gamma} v - \frac{(1-a)\left[2\eta_1\Psi_{12} - \eta_2(\Psi_{11} + \Psi_{22})\right]}{\left[(\Psi_{11} + \Psi_{22})^2 - 4\Psi_{12}^2\right]\gamma} v^*$$
(20a)

-

$$M^{*} = -\frac{\alpha_{2}}{d_{2}}u^{*} - (\frac{1+\alpha_{2}}{\gamma})v^{*} - \frac{\left[(\Psi_{13} - \frac{\alpha_{2}}{d_{2}}\Psi_{11}) - (\Psi_{23} - \frac{\alpha_{2}}{d_{2}}\Psi_{12})\right]}{\Psi_{11} + \Psi_{22} - 2\Psi_{12}}(u^{*}-u)$$

$$-\frac{(1-a)\left[2\eta_{2}\Psi_{12} - \eta_{1}(\Psi_{11} + \Psi_{22})\right]}{\left[(\Psi_{11} + \Psi_{22})^{2} - 4\Psi_{12}^{2}\right]\gamma}v^{*} - \frac{(1-a)\left[2\eta_{1}\Psi_{12} - \eta_{2}(\Psi_{11} + \Psi_{22})\right]}{\left[(\Psi_{11} + \Psi_{22})^{2} - 4\Psi_{12}^{2}\right]\gamma}v$$
(20b)

As in the previous forms of strategic behavior, the optimal rules require the neutralization of what we have termed the direct shift terms  $-\alpha_2^{u/d_2}$  $-(1+\alpha_2)v/\gamma$ ,  $-\alpha_2^{u*/d_2} - (1+d_2)v*/\gamma$ . In addition, the interaction components exhibit the same general characteristics as in the other equilibria, although the magnitudes of the adjustments are modified.<sup>13</sup>

(i) <u>Demand Shocks</u>: The interactive components of the demand shocks are perfectly symmetrical, with the coefficient of (u-u\*) in the domestic (foreign) monetary policy function being positive (negative), but smaller in magnitude than the direct effect. Thus both the domestic and foreign economies should contract their respective money supplies in response to a domestic demand shock, with the same reasoning as for the Cournot equilibrium continuing to apply. However, in contrast to the Cournot equilibrium, relatively more of the adjustment is shifted to the foreign economy.

The effects of the demand shocks on output and CPI in the two economies are given by (A.3a), (A.3b). As before, the effects depend upon the difference the domestic and foreign demand shifts. The net effect of a shift in domestic demand is to increase domestic output and to reduce output abroad. The domestic CPI falls and the foreign CPI rises. The less balanced monetary adjustments in the two economies (relative to the Cournot equilibrium), and the fact that output is more sensitive to the domestic, rather than the foreign, money stock, means the rise in domestic output and fall in foreign output are both larger than under Cournot. A further consequence of the less balanced adjustment is that appreciation of the domestic currency is reduced so that the adjustments in the domestic and foreign CPI are reduced. In short, cooperation leads to more variation in output accompanied by less variation in the CPI than in the Cournot equilibrium. The same comparison holds with respect to the CCV equilibrium. The symmetry with respect to demand shifts across the two economies, obtained in the previous equilibria, holds in the cooperative solution as well.

(ii) <u>Supply Shocks</u>: As before, the symmetry associated with demand shifts does not apply. Moreover, the appropriate response of the money supply to both domestic and foreign shocks depends critically upon the relative weight, a, assigned to output stability in the objective function. The reasoning is basically as for the Cournot equilibrium, discussed above.

The net effect of the domestic supply shock, together with the policy responses, is to raise domestic output, with the effect on the foreign output being indeterminate. On the one hand, the expansion in the domestic economy generates positive spillovers abroad. On the other hand, the monetary contractions, typically conducted in the two economies, tend to generate contractionary effects. The supply shock always reduces the CPI in both economies, with the effect in the domestic economy being the numerically larger. Overall, the foreign adjustment to the domestic supply shock is less than that of the domestic economy, so that under cooperation, the greater instability, and therefore the greater welfare losses, are incurred by the domestic economy. Moreover, relative to the Cournot equilibrium, we find that domestic welfare is increased, while foreign welfare declines. The economy experiencing the supply shock therefore gains from cooperation, while its partner loses. This is a consequence of the fact that the shift is confined to one country; if it occurs in both, then both economies are better off under cooperation.

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### B. Flexible Exchange Rate

An important form of monetary cooperation arises when the policy makers in the two countries agree to do nothing. This of course is the case of a perfectly flexible exchange rate and is specified by

$$\mathbf{M} = \mathbf{M}^{\star} = \mathbf{0} \tag{21}$$

It serves as a useful benchmark case. 14

### C. Fixed Exchange Rate

The other extreme form of cooperation is where the monetary authorities in the two countries agree to maintain a fixed exchange rate, at E = 0 say. From (8c) this is attained when

$$\beta_1(M-M^*) + \beta_2(u-u^*) + \beta_3(v-v^*) = 0$$

In general there are an infinite number of combinations of intervention which will satisfy this condition, and these in general all produce different welfare costs. Given the symmetry of the two economies, we shall consider the most natural candidate,<sup>15</sup>

$$M = \frac{1}{2\beta_{1}} [\beta_{2}(u^{*}-u) + \beta_{3}(v^{*}-v)]$$

$$M^{*} = \frac{1}{2\beta_{1}} \left[\beta_{2}(u-u^{*}) + \beta_{3}(v-v^{*})\right]$$

(22)

## 7. NUMERICAL PROCEDURES

The parameters appearing in the optimal policies and solutions are themselves complex functions of the underlying parameters of the model. While we have been able to characterize the various equilibria in some detail, to gain further insight into the general welfare implications of the different regimes, we resort to numerical analysis.

Part A indicates two sets of base parameter values. These are chosen on the basis of reasonable empirical evidence. The elasticity of the demand for domestic output with respect to foreign output is  $d_1 = .3$ ; the semi-elasticity of the demand for output with respect to the real interest rate is  $d_2 = .5$ ; the income elasticity of the demand for money is  $\alpha_1 = 1$ ; the semi-elasticity of the demand for money with respect to the nominal interest rate is .5; the share of domestic output in domestic consumption  $\delta = .6$  for both economies; the slopes of the supply functions are 4/3; the relative weights given to output stabilization in the objective function is a = .75.

The choice of the relative price elasticity  $d_3$  is more problematical. Our initial chosen value was  $d_3 = 1$ , which is close to that assumed by others.<sup>16</sup> However, for  $d_3 = 1$ , we find that  $\phi_2$ ,  $n_2$ , both turn out to be very small numerically, relative to  $\phi_1$ ,  $n_1$ , a consequence of which is that both  $\Psi_{12}$ ,  $\Psi_{22}$ , are small relative to  $\Psi_{11}$ . The significance of all this is that the effects of policies within the economy overwhelm the effects of these policies abroad. The linkages between the economies is weak, so that the interaction between the policy makers is small. In this situation, the differences between the strategic equilibria are minimal. In all cases, the welfare differences between say the Cournot and the Pareto optimal cooperative equilibria are less than 1 percent for demand disturbances and 2 percent for supply disturbances; see Tables 2B, 3B, 5B. To a first approximation, each policy maker can act in isolation and it does not matter very much how his rival responds.

For values of  $d_3$  larger than 1, the strategic equilibria continue to be close and this is true for all variations of the other parameter sets. One conclusion of this is that for  $d_3 > 1$ , the gains from cooperation are extremely

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modest, a conclusion which is also consistent with some of the simulations of Oudiz and Sachs (1984). At the same time, the gains from any form of strategic behavior over simple rules such as fixed or flexible exchange rates, are significant. As  $d_3$ declines, we find that  $\Psi_{12}$ ,  $\Psi_{22}$ , increase in size relative to  $\Psi_{11}$  and greater divergence between the equilibria results. Thus as our preferred values we take  $d_3 = .1$ , although we recognize that this may be somewhat low.

While these values seem reasonable, they are arbitrary. In Part B of Table 1 we therefore consider variants of these values allowing the parameters to range between low and high values. To consider all combinations of these parameter values would be impractical. Our approach is therefore to begin with the base parameter set and to introduce one parameter change at a time. Combining these with the two values  $d_3 = .1$ ,  $d_3 = 1$ , gives a total of 50 parameter sets, which can be identified from Table 1. Introducing the parameter changes singly in this way enables a numerical form of comparative statics to be performed.

### 8. ALTERNATIVE EQUILIBRIA: BASE PARAMETER SET

Tables 2-5 summarize the equilibria resulting from various types of stochastic disturbances, for the base parameter sets. In addition to the four strategic equilibria, we also present the extremes of the perfectly flexible and perfectly fixed exchange rates. Results for the other parameter sets are available from the authors. These tables summarize the optimal monetary policies, the responses of the key macro variables Y, Y\*, E, C, C\*, as well as the implied welfare costs. In the case of the Stackelberg equilibrium Country 1 is the leader and Country 2 is the follower. We have already commented on how the CCV requirement gives rise to two equilibria. In the case of a single country supply shock, these equilibria may give rise to a conflict from a welfare point of view. We have chosen the Pareto superior solution.

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#### A. Demand Disturbances

Table 2 reports the numerical solution in the case where Country 1 is subject to a 10 unit positive random shock in demand.

Consider as a benchmark, the case of a perfectly flexible exchange rate. In the absence of intervention, the increase in demand in Country 1 leads to an increase in the output of Country 1, together with an appreciation of its currency. The latter is of sufficient magnitude to lead to a reduction in the domestic CPI. At the same time, the increase in domestic output and the appreciation of the domestic currency stimulates demand and output abroad, and puts upward pressure on the foreign CPI. Overall, the accommodation in the world economy to the monetary shock in Country 1 is accomplished by a relatively large quantity adjustment in Country 1, together with a relatively large price adjustment in Country 2.

Now suppose that each policy maker follows Cournot behavior. In particular, both countries respond to the stimulus in demand by decreasing their respective money stocks. This will tend to moderate the increase in output in Country 1 and in fact cause a decline in output in Country 2. At the same time, the relatively larger monetary contraction abroad moderates the depreciation of the foreign currency, thereby moderating the increase in its CPI. By shifting the relative adjustment away from output and towards the CPI in Country 1, and the reverse in Country 2, the welfare costs are reduced in both cases. This is an immediate consequence of the quadratic cost function.

In reaching the Cournot equilibrium, each policy maker assumes that his opponent will not react. In the CCV solution, each policy maker correctly takes account of his opponent's reaction. The slope of Country 2's reaction function is (for  $d_3 = .1$ )  $-\Psi_{12}/\Psi_{11} = .172$ , while the slope of the consistency conjectured reaction is x = .162, which is flatter. Thus with consistent conjectures,

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Country 1 correctly expects less monetary contraction on the part of Country 2 in response to its own contraction, and therefore contracts more itself. The reverse applies in Country 2. The slope of the consistently conjectured reaction of Country 1 is steeper than Country 1's reaction function. Country 2 expects a greater contraction by Country 1 than indicated by the reaction function, and therefore contracts less itself. The consequences of this are that the increase in output in Country 1, and the decrease in Country 2, are both moderated, relative to the Cournot equilibrium. The appreciation of Country 1's currency is increased and this leads to greater variations in the CPI. Given the quadratic cost functions the move towards less output variation and more price variation leads to lower welfare in both economies.

The Stackelberg solution involves a degree of cooperation in that each player assumes a specific role. The leader is aware of the follower's reaction function and the fact that the latter will contract his money supply, less than proportionately, in response to his own monetary contraction. This increases the appreciation of the domestic currency (relative to the Cournot equilibrium). Welfare in Country 1 is increased. Fluctuations in both output and CPI in Country 2 are increased, resulting in a welfare loss abroad.

All of the Cournot, Stackelberg, and CCV equilibria involve relatively low variations in output, accompanied by relatively large fluctuations in the CPI. Given the quadratic cost function, both countries can be made better off by cooperating, with Country 1 contracting less, and Country 2 contracting more. This arrangement leads to larger fluctuations in output, but smaller fluctuations in the exchange rate and CPI, leading to a higher overall level of welfare.

Finally, the authorities can achive a perfectly fixed exchange rate with an appropriate monetary expansion in Country 1, matched by an equivalent contraction abroad. This shifts more the adjustment in Country 1 to output and less to

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the CPI, which given the relative weight in the objective function, reduces domestic welfare. On the other hand, the stable exchange rate eliminates the key mechanism whereby the domestic disturbance is transmitted abroad. Thus from the viewpoint of the foreign economy, the fixed exchange rate is the preferred regime.

The results where the demand disturbances occur in the foreign country are symmetrical and need not be discussed. Also, the case of a worldwide shift in demand, giving rise to identical shifts in the two countries, leads to the attainment of the Bliss point (zero cost equilibrium) as demonstrated previously.

### B. Supply Disturbances

The equilibria for positive supply disturbances are reported in Tables 3-5. Like the demand shocks these are assumed to be 10 units in magnitude. Turning first to the case of a domestic supply disturbance, it is clear that in all equilibria the adjustment is borne overwhelmingly by the domestic economy, with only modest effects being transmitted abroad.

To see why this is so, it is useful to begin with the benchmark case of a flexible exchange rate. The positive domestic supply shock leads to an increase in domestic output. The effect abroad can either be positive or negative, depending upon whether the positive direct spillover effect dominates the negative relative price effect. If the former dominates, foreign output rises, forcing up the foreign interest rate and causing the foreign currency to appreciate. If the latter dominates, foreign output falls and the foreign currency depreciates. In either case, the effect on foreign output is small quantitatively, relative to the domestic. At the same time, the expansion in domestic output puts downward pressure on the domestic CPI, this being larger when the dominance of the relative price effect causes the domestic currency to appreciate.

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Likewise, the foreign CPI falls, this being larger when the dominance of the direct spillover effect brings about an appreciation of the foreign currency.

Consider now the Cournot equilibrium. The domestic economy contracts its money supply, while the foreign economy expands. This tends to moderate the increase in output in Country 1, although exacerbating the fall in CPI. Given the weight in the objective function, this is a desirable tradeoff. The combination of a domestic monetary contraction coupled with a foreign expansion, generates a depreciation of the foreign currency, stimulating output abroad somewhat, but stemming the fall in foreign CPI substantially. This too is desirable from a welfare viewpoint.

For reasons discussed above, in the CCV equilibrium, the domestic monetary contraction is increased while the expansion abroad is decreased, relative to Cournot. This increases the appreciation of the domestic currency. Domestic and foreign outputs are less unstable; the fall in domestic CPI is increased, while the foreign CPI is stabilized. Welfare is improved in both economies. The Stackelberg equilibrium is close to the CCV, with both the leader and follower being better off than under Cournot.

The Cooperative equilibrium calls for less variation in output accompanied by greater variation in CPI. This can be achieved by the domestic economy increasing its monetary contraction and the foreign economy, reducing its rate of expansion, or even contracting its money supply modestly. Such an equilibrium is certainly welfare improving for the domestic economy, although the foreign economy is made worse off. Without compensation, the latter has an incentive to cheat. However, the gains to Country 1 are sufficient to enable it to compensate the foreign country and still make both better off.

Finally, the fixed rate, achieved by an equivalent contraction in 1 and expansion in 2 significantly destabilizes output in both economies. Welfare

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costs are increased, relative to all four strategic equilibria. Whether the fixed rate is worse than the flexible depends in part upon  $d_3$ .

In the situation where the supply shock occurs in the foreign economy, for all but the Stackelberg equilibrium, the responses are symmetric to those arising from supply disturbances in the domestic economy, and are not reported. The Stackelberg equilibrium in which the supply shock occurs in the follower economy is given in Table 4. The appropriate policies are approximately the same as if the shock occurs in the domestic economy; the money supply in the domestic economy should be contracted, while the money supply abroad (the country experiencing the shock) should be expanded. Most of the welfare costs are then forced onto the foreign economy.

Finally, Table 5 illustrates the case of a worldwide supply disturbance which impinges equally on the two economies. Except in the Stackelberg equilibrium, the symmetry of the shock leaves the exchange rate unchanged, so that the fixed and flexible regimes are identical. All strategic equilibria call for monetary contraction. The three noncooperative equilibria lead to insufficient monetary contraction, with too much variation in output and too little in the CPI. In the cooperative equilibrium, the increase in monetary contraction shifts the adjustment from output to inflation, resulting in welfare improvements to both economies.

### 9. SENSITIVITY ANALYSIS

Table 6 summarizes the welfare rankings of the alternative equilibria for the base parameter set. With one exception, the rankings of the four strategic equilibria, Cournot (N), Stackelberg (S), CCV and Cooperative (C) hold across all parameter sets, although in some cases the differences are quantiatively negligible. The exception is the case  $d_2 = .01$ , when for domestic demand disturbances, the Stackelberg leadership dominates the Cooperative equilibrium.

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The relative rankings of the fixed and flexible regimes are more parameter sensitive. For example, for extremely large values of d<sub>2</sub>, the fixed regime becomes the worst equilibrium for a country facing foreign demand disturbances, rather than the preferred equilibrium, as in the base case. Also, while the flexible rate generally does not perform particularly well, it is the preferred equilibrium for an economy confronting its own supply shocks, provided the objective is weighted primarily towards price stability.

In Table 7 we have summarized the qualitative effects of changes in the parameters across the sample sets, on the welfare costs in the two economies. These effects are straightforward and space limitations preclude any detailed discussion. However, the following general observations can be made.

(i) In almost all cases, the four strategic equilibria, N, S, CCV, and C, all respond similarly to a given parameter change. Exceptions arise with respect to changes in the money demand parameters  $\alpha_1$ ,  $\alpha_2$ . Welfare in the CCV and S equilibria are independent of these parameters. On the other hand, in the case of domestic demand or supply shocks, increases in these parameters have qualitatively opposite effects on the Cournot and Stackelberg equilibria in the foreign economy.

(ii) The qualitative effects of parameter changes are typically dependent upon the sources of the disturbances. Consider, for example, an increase in the degree of interdependence, as measured by an increase in  $d_1$ . In the case of a domestic demand shock, an increase in  $d_1$  reduces the welfare costs in both countries. In the case of a domestic supply shock, an increase in  $d_1$  improves domestic welfare, but lowers welfare abroad. In the case of a worldwide supply disturbance, both economies are worse off with increased interdependence.

(iii) With just two exceptions, increases in the parameter values have qualitatively the same welfare effects in the two economies, in the face of domestic demand shocks.<sup>17</sup> In the case of domestic supply shocks, on the other

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hand, the welfare effects on the two economies are generally opposite. However, this is not so in the case of changes in  $\gamma$ ; an increase in the slope of the supply curve is always welfare improving for both economies.

(iv) There are no entries for the qualitative effects of changes in the relative weight a in the objective function on the strategic equilibria N, S, CCV, and C. This is because the corresponding welfare cost functions are all nonlinear functions of a, being zero at the end points a = 0, a = 1, when each policy maker has only one objective, in which case, the strategic policy problem degenerates.

#### 10. CONCLUSIONS

Several general conclusions can be drawn from the analysis.

(i) Demand shocks are much less problematical than supply disturbances, from the viewpoint of macro stabilization. In all cases, a country-specific demand disturbance of a given magnitude gives rise to less aggregate welfare costs (as measured by the sum  $\Omega + \Omega^*$ ) than does a supply disturbance of equal magnitude. Moreover, worldwide demand shocks pose no problem whatsoever. Their effects can be eliminated entirely, provided each country simply adjusts its respective money supply so as to ensure that the interest rate in its economy rises sufficiently so as to exactly neutralize the effects of the shocks on aggregate demand. Worldwide supply shocks, on the other hand, are mutually compounding and their effects can never be eliminated.<sup>18</sup>

(ii) The superiority of the (Pareto optimal) Cooperative Equilibrium over the various non-cooperative equilibria (as measured by the aggregate welfare costs  $\Omega + \Omega^*$ ) is small. Indeed, if the relative price elasticity of demand exceeds unity say, it is almost negligible. But even for smaller values of this parameter it is never large. The reason for this is the old Mundell negative

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transmission mechanism which operates under flexible exchange rates and perfect capital mobility. Under these conditions, the effects of monetary policies on output and CPI abroad, as measured by  $\phi_2$  and  $n_2$ , are dominated by their effect domestically,  $\phi_1$ ,  $n_1$ , so that the interactions between the policy makers are small.

(iii) The strategic equilibria all show substantial margins of superiority over the traditional equilibria of fixed or flexible exchange rates. While fixed rates may be superior for one country in specific situations (such as when it faces foreign demand shocks), this is at the substantial cost of the other country, so that aggregate welfare is low.

(iv) One result of interest is the fact that, despite its use of superior knowledge, the CCV equilibrium may be dominated by the Cournot equilibrium from a welfare viewpoint. In the present analysis, this occurs with domestic demand shocks. The tendency for more contraction by the domestic economy and less abroad leads to too much variation in the CPI and too little adjustment in output, relative to the Cournot equilibrium.

In conclusion, we should note some of the limitations of the analysis and some directions for future work. First, the analysis is based on two identical economies and it would clearly be of interest to relax this assumption. More importantly, the model is purely static, with the disturbances being transitory white noise. Finally, it is clearly desirable to extend this type of analysis to a dynamic framework. To analyze intertemporal strategic behavior involves dynamic game theory. Some initial work along these lines has been undertaken using somewhat different models by Miller and Salmon (1985), Oudiz and Sachs (1985) and Currie and Levine (1985). In particular, an interesting issue is whether or not the small gains from cooperation, obtained in this static analysis, become larger over time.

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# Table 1

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## Parameter Values

A. Base Sets

 $d_1 = .3; d_2 = .5; d_3 = .1$  or  $d_3 = 1; \alpha_1 = 1.0; \alpha_2 = .5;$ 

 $\delta = .6; \gamma = 4/3; a = .75$ 

B. Variants Additional 48 parameter sets

d<sub>1</sub>: 0, .2, .4, .6, .8

-

- d<sub>2</sub>: .01, .25, 1.0, 5
- $\alpha_1 = 0, 0.5$
- $\alpha_2 = .1, 1.0, 5$
- δ = .5, .75, .99
- $\gamma = 1, 2$

a = 0, .2, .4, .6, 1

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Domestic Demand Disturbance [u = 10]

A. $d_3 = .1$									
	M	M*	Y	Y*	E	С	C*	Ω	Ω*
Cournot	-4.499	-5.501	1.334	-1.334	-7.332	-2.733	2.733	3.201	3.201
Stackelberg	-4.710	-5.537	1.261	-1.349	-7.481	-2.830	2.764	3.195	3.274
CCV	-4.665	-5.335	1.279	-1.279	-7.615	-2.854	2.854	3.264	3.264
Соор	-3.694	-6.306	1.597	-1.597	-5.957	-2.143	2.143	3.062	3.062
Flexible	0	0	2 <b>.9</b> 40	•600	-8.187	-1.772	4.427	7.265	5.170
Fixed	4.795	-4.795	4.510	<b>9</b> 70	0	1.738	•917	16.01	.915

B.  $d_3 = 1$ 

	M	M*	Y	Y*	Е	C	C*	Ω	Ω*
Cournot	-5.100	<b>-4.9</b> 00	•377	377	-3.042	-1.160	1.160	.443	.443
Stackelberg	-5.122	-4.903	.369	378	-3.052	-1.168	1.168	•443	•443
CCV	-5.119	-4.881	.369	369	-3.062	-1.169	1.169	.444	•444
Соор	-4.979	-5.021	•428	428	-2.913	-1.101	1.101	•440	•440
Flexible	0	0	2.189	1.351	-2.935	.216	2.439	3.606	2.855
Fixed	2.756	-2.756	3.345	.195	0	1.564	1.091	9.002	•326

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Domestic Supply [v = 10]	Disturbance
$A \cdot d_{-} = d_{-}$	.1

			Α.	$d_3 = .$	1				
	<u>M</u>	M*	Y	Y*	E	С	C*	Ω	Ω*
Cournot	-4.793	1.112	2.215	•465	-2.937	-4.539	952	8.829	•388
Stackelberg	-5.144	1.052	2.095	•439	-3.185	-4.700	<b>9</b> 00	8.812	•347
CCV	-5.133	•990	2.097	•418	-3.122	-4.680	934	8.774	•349
Соор	-5.819	971	1.838	259	-2.034	-4.564	-1.751	7.741	•817
Flexible	0	0	3.833	•149	2.105	-1.888	-2.625	11.91	1.739
Fixed	-1.233	1.233	3.430	•553	0	-2.791	-1.722	10.77	•971

B.  $d_3 = 1$ 

				J					
	M	M*	Y	Y*	Е	C	C*	Ω	Ω*
Cournot	-6.372	•661	1.865	•096	-5.254	-5.734	296	10.83	•0 288
Stackelberg	-6.481	•646	1.823	.094	-5.304	-5.773	289	10.82	•0275
CCV	-6.480	•641	1.823	.091	-5.301	-5.773	291	10.82	.0275
Соор	-6.628	•162	1.792	214	-4.952	-5.738	577	10.64	.118
Flexible	0	0	4.350	367	-1.509	-3.257	-1.257	16.84	•496
Fixed	1.417	-1.417	4.944	962	0	-2.564	-1.950	19.97	1.644

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Foreign Supply Disturbance [v\* = 10]

			Α.	d <sub>3</sub> = .1					
	M	M*	Y	Y*	E	С	C*	Ω	Ω*
Stackelberg	-4.806	1.039	.439	2.210	2.885	986	-4.528	•388	8.787
			в.	$d_{2} = 1$					

			D.	°3 †					
	M	M*	Y	Y*	E	C	C*	Ω	Ω*
Stackelberg	•655 -	6.372	.094	1.864	5.252	298	-5.733	.0288	10.82

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World Supply Disturbance [v = v\* = 10]

			Α.	d <sub>3</sub> = .1	L				
	M	<u>M*</u>	Y	Y*	Е	C	C*	Ω	Ω*
Cournot	-3.681	-3.681	2.679	2.679	0	-5.491	-5.491	12.92	12.92
Stackelberg	-4.105	-3.754	2.534	2.649	300	-5.685	-5.428	12.90	12.63
CCV	-4.143	-4.143	2.516	2.516	0	-5.613	-5.613	12.62	12.62
Соор	-6.789	-6.789	1.579	1.579	0	-6.316	-6.316	11.84	11.84
Flexible	0	0	3.982	3.982	0	-4.513	-4.513	16.99	16.99
Fixed	0	0	3.982	3.982	0	-4.513	-4.513	16.99	16.99

B.  $d_3 = 1$ 

				3					
	M	M*	Y	Y*	E	С	C*	Ω	Ω*
Cournot	-5.711	-5.711	1.961	1.961	0	-6.029	-6.029	11.97	11.97
Stackelberg	-5.826	-5.826	1.917	1.958	053	-6.071	-6.023	11.97	11.94
CCV	-5.840	-5.840	1.915	1.915	0	-6.064	-6.064	11.94	11.94
Соор	-6.789	-6.789	1.579	1.579	0	-6.316	-6.316	11.84	11.84
Flexible	0	0	3.982	3.982	0	-4.513	-4.513	16.99	16.99
Fixed	0	0	3.982	3.982	0	-4.513	-4.513	16.99	16.99

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### Welfare Rankings of Alternative Equilibria Base Parameter Set

### Domestic Demand Disturbances

Domestic Economy: C > S > N > CCV > F > PForeign Economy: P > C > N > CCV > S > F

### Domestic Supply Disturbances

Domestic Economy: C > CCV > S > N > P > FForeign Economy: S > CCV > N > C > P > F

### Worldwide Supply Disturbance

Both Economies: C > CCV > S > N > P = F

Note: > denotes "is superior to"

- C = Cooperative
- S = Stackelberg
- N = Cournot

CCV = Consistent Conjectural Variations

- F = flexible rate
- P = fixed rate

# Qualitative Effects on Welfare Costs of Parameter Changes

Increase in	Disturbance	Ω <sup>N</sup>	ΩS	ΩCC	VΩC	Ω	Ω <sup>Ρ</sup>	Ω*	N s*S	Ω <sup>#CCV</sup>	Ω*C	_Ω*F	Ω*P
d <sub>1</sub>	<b>u &gt;</b> 0	-				-	-	-	-	_	-	-	-
	<b>v</b> > 0	-		-	-	-	-	+	+	+	+	+	+
	$\mathbf{v} = \mathbf{v}^* > 0$	+	+	+	+	+	+	+	+	+	+	+	+
d <sub>2</sub>	u > 0	-	-	-	-	-		-	-			-	
	<b>v</b> > 0	+	+	+	+	+	+	-	-	-	_	-	-
	$\mathbf{v} = \mathbf{v}^* > 0$	-	-	-	-	+	+	_		-	_	+	+
<sup>d</sup> 3	u > 0	-	-			[=		-		-			
	v > 0	-	-	-	-	-	-	+	+	+	+	+	+
	$v = v^* > 0$	-		_	0	+	+	_	_	-	0	+	+
°1	u > 0	-	-	0	0	-	-	_	+	0	0	+	-
	v > 0	÷	+	0	0	-	-	+	-	0	0	-	-
	$\mathbf{v} = \mathbf{v}^* > 0$	+	+	0	0	-	_	+	0	0	0		_
α <sub>2</sub>	u > 0	+	+	0	0	+	+	+		0	0	-	+
	<b>v</b> > 0	<b>1960</b>	-	0	0	-		-	÷	0	0	-	
	$\mathbf{v} = \mathbf{v} * > 0$	_		0	0	_	_	-		0	0	-	
δ	<b>u</b> > 0		-		1	-	+	-		-	-		
	$\mathbf{v} > 0$	+	+	+	+	+	+	-	-	-		-	-
	$\mathbf{v} = \mathbf{v} * > 0$		-	-	0	0	0	-		-	0	0	0
Ŷ	u > 0	+	+	+	+	+	+	+	+	+	+	-	
	v > 0	-	-	-	-	-	-	-	-	-	_	-	-migre
	$\mathbf{v} = \mathbf{v}^* > 0$	-		-	-	_	_	-		-	_		-
a	u > 0					+	+					-	+
	v > 0					+	+						-
	$\mathbf{v} = \mathbf{v}^* > 0$					_	_						

 $\Omega^*$ ,  $\Omega^{**}$  denotes welfare costs at home and abroad under regime X; X = N,S, CCV, C, F, P.

- + denotes welfare costs increase.
- denotes welfare costs decrease.
- 0 denotes welfare costs remain unchanged.

### APPENDIX

### Solutions for Output and CPI

Given the symmetry of the underlying economies, the solutions for output and the CPI for the domestic and foreign economies in the Cournot, CCV, and Cooperative equilibria are symmetric; the domestic and foreign shocks are simply reversed. Thus the solutions for only the domestic economy need be reported.

## Cournot Equilibrium

$$Y = \frac{\gamma(1-a)\eta_{1}(1-\delta)}{(\Psi_{11}-\Psi_{12})D^{*}} (u-u^{*}) + \frac{(1-a)\eta_{1}(\phi_{1}-\phi_{2})}{(\Psi_{11}-\Psi_{12})\gamma} v$$
(A.1a)

$$C = \frac{-\gamma a_{\phi_1}(1-\delta)}{(\Psi_{11}^{-}\Psi_{12})D'} (u-u^*)$$

$$- \frac{a(1-a)\phi_1\eta_1\Theta}{(\Psi_{11}^{2}-\Psi_{12}^{2})\gamma} (v+v_*) - \frac{a\phi_1(\phi_1-\phi_2)}{(\Psi_{11}^{-}\Psi_{12})\gamma} v$$
(A.1b)

where

$$\Theta = \phi_2 \eta_1 - \phi_1 \eta_2 = \gamma^2 (1 - \delta) (1 + d_1) d_2 / DD' > 0$$

Consistent Conjectural Variations Equilibrium

$$Y = \frac{\gamma(1-a)(\eta_{1}+x\eta_{2})(1-\delta)}{[(\Psi_{11}+x\Psi_{12})^{-(\Psi_{12}+x\Psi_{22})}]D'} (u-u^{*})$$

$$+ \frac{(1-a)^{2}(\eta_{1}+x\eta_{2})^{2}\Theta}{[(\Psi_{11}+x\Psi_{12})^{2}-(\Psi_{12}+x\Psi_{22})^{2}]\gamma} (v+v^{*}) + \frac{(1-a)(\eta_{1}+x\eta_{2})(\phi_{1}-\phi_{2})}{[(\Psi_{11}+x\Psi_{12})-(\Psi_{12}+x\Psi_{22})]\gamma} v$$
(A.2a)

$$C = -\frac{\gamma a(\phi_{1} + x\phi_{2})(1-\delta)}{[(\Psi_{11} + x\Psi_{12})^{-(\Psi_{12} + x\Psi_{22})}]D'} (u-u^{*})$$

$$-\frac{a(1-a)(\phi_{1} + x\phi_{2})(\eta_{1} + x\eta_{2})\Theta}{[(\Psi_{11} + x\Psi_{12})^{2} - (\Psi_{12} + x\Psi_{22})^{2}]\gamma} (v+v^{*}) - \frac{a(\phi_{1} + x\phi_{2})(\phi_{1} - \phi_{2})}{[(\Psi_{11} + x\Psi_{12})^{-(\Psi_{12} + x\Psi_{22})}]\gamma} v$$
(A.2b)

where x is a solution to

$$\Psi_{12}x^2 + (\Psi_{11} + \Psi_{22})x + \Psi_{12} = 0$$

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Cooperative Equilibrium

$$Y = \frac{\gamma(1-a)(\eta_1-\eta_2)(1-\delta)}{(\Psi_{11}+\Psi_{22}-2\Psi_{12})D'} (u-u^*) + \frac{(\Psi_{11}-\Psi_{22})\Delta(1-a)}{[(\Psi_{11}+\Psi_{22})^2-4\Psi_{12}^2]\gamma} v$$

$$+ \frac{(1-a)\Omega[(1-a)(\eta_{1}^{2}-\eta_{2}^{2})-a(\phi_{1}^{2}-\phi_{2}^{2})]}{[(\Psi_{11}^{+}\Psi_{22}^{2})^{2}-4\Psi_{12}^{2}]\gamma} v^{*}$$
(A.3a)

$$C = \frac{-\gamma a(\phi_1 - \phi_2)(1 - \delta)}{(\Psi_{11} + \Psi_{22} - 2\Psi_{12})D'} (u - u^*) - \frac{2a\Delta\Theta}{[(\Psi_{11} + \Psi_{22})^2 - 4\Psi_{12}^2]\gamma} v^*$$

$$-\frac{a[(\Psi_{11}^{+\Psi}\Psi_{22}^{-})(\phi_{1}^{-\phi}\phi_{2}^{-})^{2}+2\phi_{1}\phi_{2}[\Psi_{11}^{+\Psi}\Psi_{22}^{-}\Psi_{12}^{-}]}{[(\Psi_{11}^{+\Psi}\Psi_{22}^{-})^{2}-4\Psi_{12}^{2}]\gamma} v$$
(A.3b)

where

$$\Delta \equiv \phi_{1^{\eta}1} - \phi_{2^{\eta}2} > 0$$

Note that qualitatively, all these solutions are of the form

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$$Y = f_{1}(u-u^{*}) + f_{2}v + f_{3}v^{*}$$
(A.4a)  

$$C = e_{1}(u-u^{*}) - e_{2}v - e_{3}v^{*}$$
(A.4b)

where for

Cournot and CCV

 $f_1 > 0, \quad f_2 > f_3 > 0$  $e_1 > 0, \quad e_2 > e_3 > 0$ 

Cooperative

$$f_1 > 0, \quad f_2 > 0, \quad f_2 > f_3, \quad f_3 < 0$$
  
 $e_1 > 0, \quad e_2 > e_3 > 0$ 

Writing the above solutions as

$$Y = Y(u, u^*, v, v^*)$$
 (A.5a)

 $C = C(u, u^*, v, v^*)$  (A.5b)

the corresponding solutions in the foreign economy may be summarized by

$$Y^* = Y(u^*, u, v^*, v)$$
 (A.6a)

$$C^* = C(u^*, u, v^*, v)$$
 (A.6b)

#### FOOTNOTES

 $^1$ See also the analysis included in Bryant (1980) and Jones (1983).

<sup>2</sup>A similar framework is employed in a recent paper by Canzoneri and Henderson (1985). However, since the purpose of their paper is primarily expositional, neither their model nor their analysis is as comprehensive as that undertaken here.

<sup>3</sup>See Bresnahan (1981) and Perry (1982) for applications of the consistent conjectural variations equilibrium to oligopoly theory. A recent paper by Brandsma and Hughes Hallett (1984) considers conjectural variations (which are not necessarily consistent) in a dynamic policy game framework. Although some game theorists view the consistent conjectural variations equilibrium with some skepticism (see Friedman (1982, Chapter 5)), it appears to be gaining their acceptance and interest (see Basar (1985)).

<sup>4</sup>The use of numerical simulations as a method of analyzing small macro models has been employed by a number of authors recently. See, e.g., Carlozzi and Taylor (1985), Taylor (1985), Oudiz and Sachs (1984) for policy simulations in two country macro models.

<sup>5</sup>The assumption of perfectly symmetric economies, being made in this paper, is made virtually uniformly throughout the two country policy coordination literature. One empirical investigation of coordination which allows for asymmetric economies has been undertaken by Hughes Hallett (1984).

<sup>6</sup>We maintain the usual assumption that residents of each country do not hold the currency of the other country.

<sup>7</sup>For example, if a shift term, w say, is added to the demand for money, our analysis remains unchanged by redefining M' = M - w.

<sup>8</sup>In fact we have carried out such simulations and the results are changed little.

<sup>9</sup>This characteristic of relatively weak impact of domestic monetary policy on foreign activity (and vice versa) arises, and for precisely the same reason, in the Carlozzi and Taylor (1985) paper.

 $^{10}$ Since the model is static, henceforth we shall delte the time subscript t.

<sup>11</sup>To show this, first set  $\xi_t(P_{t+1} = 0 \text{ in } (1) \text{ and } \xi_{t-1}(P_t) = 0 \text{ in } (4)$ , in accordance with (6'). Next substitute (4) into (1), to yield

 $(1+d_2/\gamma)Y = d_1^{Y*} - d_2I + d_3(P*+E-P) + u + d_2v/\gamma$ 

Clearly, the effects of u and v on aggregate demand are neutralized if the interest rate I is adjusted by

$$I = \frac{u}{d_2} + \frac{v}{\gamma}$$

Now substituting (4) into (2) gives

$$M = (\alpha_1 + 1/\gamma)Y - \alpha_2 I - v/\gamma$$

The implied adjustment in the money stock, which holds demand and output constant, is therefore

$$M = -\frac{\alpha_2}{d_2} u - \frac{(1+\alpha_2)}{\gamma} v$$

which is the first component of (12a).

<sup>12</sup>First, 
$$\Psi_{11} - \Psi_{12} = a\phi_1(\phi_1 - \phi_2) + (1 - a)\eta_1 - \eta_2 > 0$$
. Secondly,  
 $\Psi_{13} - \frac{\alpha_2}{d_2}\Psi_{11} = a\phi_1[\phi_3 - \frac{\alpha_2}{d_2}\phi_1] + (1 - a)\eta_1[\eta_3 - \frac{\alpha_2}{d_2}\eta_1]$ 

Substituting for  $\phi_i$  and  $\eta_i$  into this expression one can show  $\Psi_{13} - \frac{\alpha_2}{d_2} \Psi_{11} < 0$ .

<sup>13</sup>The symmetry of the optimal policies is a consequence of not only the symmetry of the underlying model, but also the fact that each country is weighted equally in the joint cost function. The case of equal weights is just a special case of the more general Pareto criterion, the minimization of  $\beta\Omega$  +  $(1-\beta)\Omega^*$ . 0 <  $\beta$  < 1.

<sup>14</sup>Our flexible exchange rate regime may also be described as being one of setting monetary targets.

<sup>15</sup>We have also considered another natural means of pegging the exchange rate,

$$M = -\frac{1}{\beta_1} [\beta_2 u + \beta_3 v]; \quad M^* = -\frac{1}{\beta_1} [\beta_2 u^* + \beta_3 v^*].$$

This requires each country to accommodate only to its own disturbances. This rule turns out to be inferior to (22).

<sup>16</sup>Miller and Salmon (1985) assume  $d_3$  to be 1, while Oudiz and Sachs take it to be somewhat larger, around 1.5. As noted, Carlozzi and Taylor run into similar problems regarding small linkage effects and take  $d_3 = .1$ . Currie and Levine (1985) take a slightly larger value of .3.

<sup>17</sup>The exceptions are the effects of changes in the monetary parameters  $\alpha_1$ ,  $\alpha_2$  in the Stackelberg equilibrium.

<sup>18</sup>The difficulty for stabilizing for supply shocks is also emphasized by Taylor (1984).

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