

NBER WORKING PAPER SERIES

COVAR

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Working Paper 17454  
<http://www.nber.org/papers/w17454>

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
October 2011

Special thanks go to Daniel Green and Hoai-Luu Nguyen for outstanding research assistance. The authors also thank Paolo Angelini, Gadi Barlevy, René Carmona, Stephen Brown, Robert Engle, Mark Flannery, Xavier Gabaix, Paul Glasserman, Beverly Hirtle, Jon Danielson, John Kambhu, Arvind Krishnamurthy, Burton Malkiel, Maureen O'Hara, Andrew Patton, Matt Pritsker, Matt Richardson, Jean-Charles Rochet, José Scheinkman, Jeremy Stein, Kevin Stiroh, René Stulz, and Skander Van den Heuvel for feedback, as well as seminar participants at numerous universities, central banks, and conferences. We are grateful for support from the Institute for Quantitative Investment Research Europe. Brunnermeier also acknowledges financial support from the Alfred P. Sloan Foundation. The views expressed in this paper are those of the authors and do not necessarily represent those of the Federal Reserve Bank of New York, the Federal Reserve System, or the National Bureau of Economic Research.

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CoVaR

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NBER Working Paper No. 17454

September 2011

JEL No. G17,G21,G22

**ABSTRACT**

We propose a measure for systemic risk: CoVaR, the value at risk (VaR) of the financial system conditional on institutions being under distress. We define an institution's contribution to systemic risk as the difference between CoVaR conditional on the institution being under distress and the CoVaR in the median state of the institution. From our estimates of CoVaR for the universe of publicly traded financial institutions, we quantify the extent to which characteristics such as leverage, size, and maturity mismatch predict systemic risk contribution. We also provide out of sample forecasts of a countercyclical, forward looking measure of systemic risk and show that the 2006Q4 value of this measure would have predicted more than half of realized covariances during the financial crisis.

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# 1 Introduction

During times of financial crises, losses tend to spread across financial institutions, threatening the financial system as a whole.<sup>1</sup> The spreading of distress gives rise to systemic risk—the risk that the intermediation capacity of the entire financial system is impaired, with potentially adverse consequences for the supply of credit to the real economy. In systemic financial events, spillovers across institutions can arise from direct contractual links and heightened counterparty credit risk, or can occur indirectly through price effects and liquidity spirals. As a result of both, measured comovement of institutions’ assets and liabilities tends to rise above and beyond levels purely justified by fundamentals. Systemic risk measures capture the potential for the spreading of financial distress across institutions by gauging this increase in tail comovement.

The most common measure of risk used by financial institutions—the value at risk (*VaR*)—focuses on the risk of an individual institution in isolation. The  $q\%$ -*VaR* is the maximum dollar loss within the  $q\%$ -confidence interval; see Kupiec (2002) and Jorion (2006) for overviews. However, a single institution’s risk measure does not necessarily reflect systemic risk—the risk that the stability of the financial system as a whole is threatened. First, according to the classification in Brunnermeier, Crocket, Goodhart, Persaud, and Shin (2009), a systemic risk measure should identify the risk to the system by “individually systemic” institutions, which are so interconnected and large that they can cause negative risk spillover effects on others, as well as by institutions that are “systemic as part of a herd.” A group of 100 institutions that act like clones can be as precarious and dangerous to the system as the large merged identity. Second, risk measures should recognize that risk typically builds up in the background in the form of imbalances and bubbles and materializes only during a crisis. Hence, high-frequency risk measures that rely primarily on contemporaneous price movements are potentially misleading. Regulation based on such contemporaneous measures tends to be procyclical.

The objective of this paper is twofold: First, we propose a measure for systemic risk. Second, we outline a method to construct a countercyclical, forward looking systemic risk measure by

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<sup>1</sup>Examples include the 1987 equity market crash, which was started by portfolio hedging of pension funds and led to substantial losses of investment banks; the 1998 crisis, which was started with losses of hedge funds and spilled over to the trading floors of commercial and investment banks; and the 2007-09 crisis, which spread from SIVs to commercial banks and on to investment banks and hedge funds. See e.g. Brady (1988), Rubin, Greenspan, Levitt, and Born (1999), Brunnermeier (2009), and Adrian and Shin (2010a).

predicting future systemic risk using current institutional characteristics such as size, leverage, and maturity mismatch. To emphasize the systemic nature of our risk measure, we add to existing risk measures the prefix “*Co*,” which stands for *conditional*, *contagion*, or *comovement*. We focus primarily on *CoVaR*, where institution  $i$ ’s *CoVaR* relative to the system is defined as the *VaR* of the whole financial sector conditional on institution  $i$  being in distress.<sup>2</sup> The difference between the *CoVaR* conditional on the distress of an institution and the *CoVaR* conditional on the “normal” state of the institution,  $\Delta CoVaR$ , captures the marginal contribution of a particular institution (in a non-causal sense) to the overall systemic risk.

There are several advantages to the  $\Delta CoVaR$  measure. First, while  $\Delta CoVaR$  focuses on the contribution of each institution to overall system risk, traditional risk measures focus on the risk of individual institutions. Regulation based on the risk of institutions in isolation can lead to excessive risk-taking along systemic risk dimensions. To see this more explicitly, consider two institutions,  $A$  and  $B$ , which report the same *VaR*, but for institution  $A$  the  $\Delta CoVaR = 0$ , while for institution  $B$  the  $\Delta CoVaR$  is large (in absolute value). Based on their *VaRs*, both institutions appear equally risky. However, the high  $\Delta CoVaR$  of institution  $B$  indicates that it contributes more to system risk. Since system risk might carry a higher risk premium, institution  $B$  might outshine institution  $A$  in terms of generating returns in the run up phase, so that competitive pressure might force institution  $A$  to follow suit. Regulatory requirements that are stricter for institution  $B$  than for institution  $A$  would break this tendency to generate systemic risk.

One could argue that regulating institutions’ *VaR* might be sufficient as long as each institution’s  $\Delta CoVaR$  goes hand in hand with its *VaR*. However, this is not the case, as (i) it is not welfare maximizing that institution  $A$  should increase its contribution to systemic risk by following a strategy similar to institution  $B$  and (ii) empirically, there is no one-to-one connection between an institution’s  $\Delta CoVaR$  (y-axis) and its *VaR* (x-axis), as Figure 1 shows.

Another advantage of our co-risk measure is that it is general enough to study the risk spillovers from institution to institution across the whole financial network. For example,  $\Delta CoVaR^{j|i}$  captures the increase in risk of individual institution  $j$  when institution  $i$  falls into

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<sup>2</sup>Just as *VaR* sounds like variance, *CoVaR* sounds like covariance. This analogy is no coincidence. In fact, under many distributional assumptions (such as the assumption that shocks are conditionally Gaussian), the *VaR* of an institution is indeed proportional to the variance of the institution, and the *CoVaR* of an institution is proportional to the covariance of the financial system and the individual institution.

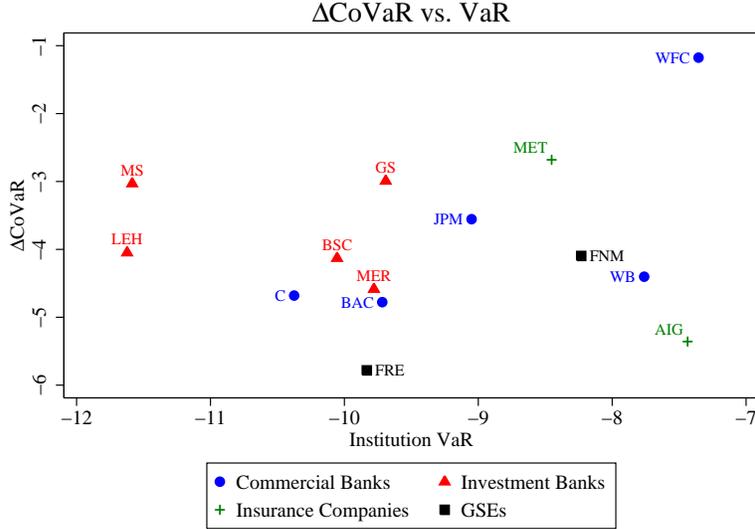


Figure 1: The scatter plot shows the weak link between institutions’ risk in isolation, measured by  $VaR^i$  (x-axis), and institutions’ contribution to system risk, measured by  $\Delta CoVaR^i$  (y-axis). The  $VaR^i$  and  $\Delta CoVaR^i$  are unconditional 1% measures estimated as of 2006Q4 and are reported in weekly percent returns for merger adjusted entities.  $VaR^i$  is the 1% quantile of firm returns, and  $\Delta CoVaR^i$  gives the percentage point change in the financial system’s 1%  $VaR$  when a particular institution realizes its own 1%  $VaR$ . The institutions used in the figure are listed in Appendix D.

distress. To the extent that it is causal, it captures the risk spillover effects that institution  $i$  causes on institution  $j$ . Of course, it can be that institution  $i$ ’s distress causes a large risk increase in institution  $j$ , while institution  $j$  causes almost no risk spillovers onto institution  $i$ . That is, there is no reason why  $\Delta CoVaR^{j|i}$  should equal  $\Delta CoVaR^{i|j}$ .

So far, we have deliberately not specified how to estimate the  $CoVaR$  measure, since there are many possible ways. In this paper, we primarily use quantile regressions, which are appealing for their simplicity and efficient use of data. Since we want to capture all forms of risk, including not only the risk of adverse asset price movements, but also funding liquidity risk (which is equally important), our estimates of  $\Delta CoVaR$  are based on (weekly) changes in (market-valued) total assets of all publicly traded financial institutions. However,  $\Delta CoVaR$  can also be estimated using methods such as *GARCH* models, as we show in the appendix.

Our paper also addresses the problem that (empirical) risk measures suffer from the rarity of “tail observations”. After a string of good news, risk seems tamed, but, when a new tail event

occurs, the estimated risk measure may sharply increase. This problem is most pronounced if the data samples are short. Hence, regulatory requirements should be based on forward looking risk measures. We propose the implementation of a *forward- $\Delta CoVaR$*  that is constructed to be forward looking and countercyclical.

We calculate unconditional and conditional measures of  $\Delta CoVaR$  using the full length of available data. We use weekly data from 1986Q1 to 2010Q4 for all publicly traded commercial banks, broker-dealers, insurance companies, and real estate companies. While the unconditional  $\Delta CoVaR$  estimates are constant over time, the conditional ones model variation of  $\Delta CoVaR$  as a function of state variables that capture the evolution of tail risk dependence over time. These state variables include the slope of the yield curve, the aggregate credit spread, and implied equity market volatility from *VIX*. We first estimate  $\Delta CoVaR$  conditional on the state variables. In a second step we use panel regressions, and relate these time-varying  $\Delta CoVaRs$ —in a predictive, Granger causal sense—to measures of each institution’s characteristics like maturity mismatch, leverage, market-to-book, size, and market beta.

We show that the predicted values from the panel regressions (which we call “forward  $\Delta CoVaRs$ ”) exhibit countercyclicality. In particular, consistent with the “volatility paradox” that low volatility environments breed the build up of systemic risk, the forward  $\Delta CoVaRs$  are strongly negatively correlated with the contemporaneous  $\Delta CoVaRs$ . We also demonstrate that the “forward- $\Delta CoVaRs$ ” have out of sample predictive power for realized correlation in tail events. In particular, the forward- $\Delta CoVaRs$  estimated using data through the end of 2006 predicted half of the cross sectional dispersion in realized covariance during the financial crisis of 2008.

The forward- $\Delta CoVaR$  can be used to monitor the buildup of systemic risk in a forward looking manner. It indicates which firms are expected to contribute most to systemic financial crisis, based on current firm characteristics. The forward- $\Delta CoVaR$  can thus be used to calibrate the systemic risk capital surcharges. A capital surcharge based on (forward) systemic risk contribution changes ex-ante incentives to conduct activities that generate systemic risk. In addition, it increases the capital buffer of systemically important financial institutions, thus protecting the financial system against the risk spillovers and externalities from systemically important financial institutions.

**Related Literature.** Our *co-risk measure* is motivated by theoretical research on externalities across financial institutions that give rise to amplifying liquidity spirals and persistent distortions. *CoVaR* tries to capture externalities, together with fundamental comovement. *CoVaR* also relates to econometric work on contagion and spillover effects.

*Spillovers* and “externalities” can give rise to excessive risk taking and leverage in the run-up phase. The externalities arise because each individual institution takes potential fire-sale prices as given, while as a group they cause the fire-sale prices. In an incomplete market setting, this pecuniary externality leads to an outcome that is not even constrained Pareto efficient. This result was derived in a banking context in Bhattacharya and Gale (1987) and a general equilibrium incomplete market setting by Stiglitz (1982) and Geanakoplos and Polemarchakis (1986). Prices can also affect borrowing constraints. These externality effects are studied in within an international finance context by Caballero and Krishnamurthy (2004), and most recently shown in Lorenzoni (2008), Acharya (2009), Stein (2009), and Korinek (2010). Runs on financial institutions are dynamic co-opetition games and lead to externalities, as does banks’ liquidity hoarding. While hoarding might be microprudent from a single bank’s perspective it need not be macroprudent (fallacy of the commons). Finally, network effects can also lead to externalities, as emphasized by Allen, Babus, and Carletti (2010).

*Procyclicality* occurs because risk measures tend to be low in booms and high in crises. The margin/haircut spiral and precautionary hoarding behavior outlined in Brunnermeier and Pedersen (2009) and Brunnermeier and Sannikov (2009) led financial institutions to shed assets at fire-sale prices. Adrian and Shin (2010b) and Gorton and Metrick (2010) provide empirical evidence for the margin/haircut spiral. Borio (2004) is an early contribution that discusses a policy framework to address margin/haircut spirals and procyclicality.

Recently a number of systemic risk measures complementary to *CoVaR* have recently been proposed. Huang, Zhou, and Zhu (2010) develop a systemic risk indicator measured by the price of insurance against systemic financial distress, based on credit default swap (CDS) prices. Acharya, Pedersen, Philippon, and Richardson (2010) focus on high-frequency marginal expected shortfall as a systemic risk measure. Like our “exposure CoVaR”, they switch the conditioning and addresses the question which institutions are most exposed to a financial crisis as opposed to which institution contributes most to a crisis. Importantly, their analysis focuses on the cross

sectional comparison across financial institutions and do not address the problem of procyclicality that arises from contemporaneous risk measurement. In other words, they do not address the stylized fact that risk is building up in the background during boom phases characterized by low volatility and materializes only in crisis times. Billio, Getmansky, Lo, and Pelizzon (2010) propose a systemic risk measure that relies on Granger causality among firms. Giglio (2011) uses a nonparametric approach to derive bounds of systemic risk from CDS prices. A number of recent papers have extended the *CoVaR* method and applied it to additional financial sectors. For example, Adams, Füss, and Gropp (2010) study risk spillovers among financial institutions, using quantile regressions; Wong and Fong (2010) estimate *CoVaR* for the CDS of Asia-Pacific banks; Gauthier, Lehar, and Souissi (2009) estimate systemic risk exposures for the Canadian banking system.

The *CoVaR* measure is related to the literature on volatility models and tail risk. In a seminal contribution, Engle and Manganelli (2004) develop *CAViaR*, which uses quantile regressions in combination with a *GARCH* model to model the time varying tail behavior of asset returns. Manganelli, Kim, and White (2011) study a multivariate extension of *CAViaR*, which can be used to generate a dynamic version of the *CoVaR* systemic risk measure.

The *CoVaR* measure can also be related to an earlier literature on contagion and volatility spillovers (see Claessens and Forbes (2001) for an overview). The most common method to test for volatility spillover is to estimate multivariate *GARCH* processes. Another approach is to use multivariate extreme value theory. Hartmann, Straetmans, and de Vries (2004) develop a contagion measure that focuses on extreme events. Danielsson and de Vries (2000) argue that extreme value theory works well only for very low quantiles.

Another important strand of the literature, initiated by Lehar (2005) and Gray, Merton, and Bodie (2007), uses contingent claims analysis to measure systemic risk. Bodie, Gray, and Merton (2007) develop a policy framework based on the contingent claims. Segoviano and Goodhart (2009) use a related approach to measure risk in the banking system.

**Outline.** The remainder of the paper is organized in four sections. In Section 2, we outline the methodology and define  $\Delta CoVaR$  and its properties. In Section 3, we outline the estimation method via quantile regressions. We also introduce time-varying  $\Delta CoVaR$  conditional on

state variables and present estimates of these conditional  $\Delta CoVaR$ . Section 4 shows how to use  $\Delta CoVaR$  to implement preemptive macroprudential supervision and regulation by demonstrating that institutional characteristics such as size, leverage, and maturity mismatch can predict systemic risk contribution in the cross section of institutions. We conclude in Section 5.

## 2 $CoVaR$ Methodology

### 2.1 Definition of $CoVaR$

Recall that  $VaR_q^i$  is implicitly defined as the  $q$  quantile, i.e.,

$$\Pr(X^i \leq VaR_q^i) = q,$$

where  $X^i$  is the variable of institution  $i$  for which the  $VaR_q^i$  is defined. Note that  $VaR_q^i$  is typically a negative number. In practice, the sign is often switched, a sign convention we will not follow.

**Definition 1** We denote by  $CoVaR_q^{j|i}$  the VaR of institution  $j$  (or the financial system) conditional on some event  $\mathbb{C}(X^i)$  of institution  $i$ . That is,  $CoVaR_q^{j|i}$  is implicitly defined by the  $q$ -quantile of the conditional probability distribution:

$$\Pr\left(X^j \leq CoVaR_q^{j|\mathbb{C}(X^i)} \mid \mathbb{C}(X^i)\right) = q.$$

We denote institution  $i$ 's contribution to  $j$  by

$$\Delta CoVaR_q^{j|i} = CoVaR_q^{j|X^i=VaR_q^i} - CoVaR_q^{j|X^i=Median^i}.$$

For most of the paper we focus on the conditioning event of  $\{X^i = VaR_q^i\}$  and simplify the notation to  $CoVaR_q^{j|i}$ . Furthermore, we primarily study the case where  $j = system$ , i.e., when the return of the portfolio of all financial institutions is at its  $VaR$  level. In this case, we drop the superscript  $j$ . Hence,  $\Delta CoVaR^i$  denotes the difference between the  $VaR$  of the financial system conditional on the distress of a particular financial institution  $i$  and the  $VaR$  of the financial system conditional on the median state of the institution  $i$ .

The more general definition of  $CoVaR^{j|i}$ —i.e., the  $VaR$  of institution  $j$  conditional on institution  $i$  being at its  $VaR$  level—allows the study of spillover effects across a whole financial network. Moreover, we can derive  $CoVaR^{j|system}$ , which answers the question of which institutions are most at risk should a financial crisis occur.  $\Delta CoVaR^{j|system}$  reports institution  $j$ 's increase in value-at-risk in the case of a financial crisis. We call  $\Delta CoVaR^{j|system}$  the “exposure  $CoVaR$ ,” because it measures the extent to which an individual institution is affected by systemic financial events.<sup>3</sup>

## 2.2 The Economics of Systemic Risk

Systemic risk has two important components. First, it builds up in the background during credit booms when contemporaneously measured risk is low. This buildup of systemic risk during times of low measured risk gives rise to a “volatility paradox.” The second component of systemic risk relates to the spillover effects that amplify initial adverse shocks in times of crisis.

The contemporaneous  $\Delta CoVaR^i$  measure quantifies these spillover effects by measuring how much an institution adds to the overall risk of the financial system. The spillover effects can be direct, through contractual links among financial institutions. This is especially the case for institutions that are “too interconnected to fail.” Indirect spillover effects are quantitatively more important. Selling off assets can lead to mark-to-market losses for all market participants who hold a similar exposure—common exposure effect. Moreover, the increase in volatility might tighten margins and haircuts forcing other market participants to delever as well (margin spiral). This can lead to crowded trades which increases the price impact even further.

The notion of systemic risk that we are using in this paper captures direct and indirect spillover effects and is based on the tail covariation between financial institutions and the financial system. Definition 1 implies that financial institutions whose distress coincides with the distress of the financial system will have a high systemic risk measure. Systemic risk contribution gauges the extent to which financial system stress increases conditional on the distress of a particular firm, and thus captures spillover effects. It should be noted, however, that the approach taken in this paper is a statistical one, without explicit reference to structural economic

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<sup>3</sup>Huang, Zhou, and Zhu (2010) and Acharya, Pedersen, Philippon, and Richardson (2010) propose systemic risk measures that reverse the conditioning of  $CoVaR$ . These alternative measures thus use the same conditioning logic as that for the “exposure  $CoVaR$ ”.

models. Nevertheless, we conjecture that the  $\Delta CoVaR^i$  measure would give rise to meaningful time series and cross sectional measurement of systemic risk in such economic theories.

Many of these spillovers are externalities. That is, when taking on the initial position with low market liquidity funded with short-term liabilities—i.e. with high liquidity mismatch, each individual market participant does not internalize that his subsequent individually optimal response in times of crisis will cause a (pecuniary) externality on others. As a consequence the initial risk taking is often excessive in the run-up phase.

In section 4, we construct a “forward  $\Delta CoVaR$ ”. This forward measure captures the stylized fact that systemic risk is building up in the background, especially during in low volatility environments. As a result, contemporaneous systemic risk measures are not suited to fully capture the buildup component of systemic risk. Our “forward  $\Delta CoVaR$ ” measure avoids the “procyclicality pitfall” by estimating the relationship between current firm characteristics and future spillover effects, as proxied by  $\Delta CoVaR^i$ .

### 2.3 Properties of $CoVaR$

**Cloning Property.** Our  $CoVaR$  definition satisfies the desired property that, after splitting one large “individually systemic” institution into  $n$  smaller clones, the  $CoVaR$  of the large institution (in return space) is exactly the same as the  $CoVaRs$  of the  $n$  clones. Put differently, conditioning on the distress of a large systemic institution is the same as conditioning on one of the  $n$  clones.

**Causality.** Note that the  $\Delta CoVaR$  measure does not distinguish whether the contribution is causal or simply driven by a common factor. We view this as a virtue rather than as a disadvantage. To see this, suppose a large number of small hedge funds hold similar positions and are funded in a similar way. That is, they are exposed to the same factors. Now, if only one of the small hedge funds falls into distress, this will not necessarily *cause* any systemic crisis. However, if the distress is due to a common factor, then the other hedge funds—all of which are “systemic as part of a herd”—will likely be in distress. Hence, each individual hedge fund’s co-risk measure should capture the notion of being “systemic as part of a herd” even in the absence of a direct causal link. The  $\Delta CoVaR$  measure achieves exactly that. Moreover, when

we estimate  $\Delta CoVaR$ , we control for lagged state variables that capture variation in tail risk not directly related to the financial system risk exposure.

**Tail Distribution.** *CoVaR* focuses on the tail distribution and is more extreme than the unconditional *VaR*, as *CoVaR* is a *VaR* that conditions on a “bad event”—a conditioning that typically shifts the mean downwards, increases the variance, and potentially increases higher moments such as negative skewness and kurtosis. The *CoVaR*, unlike the covariance, reflects shifts in all of these moments. Estimates of *CoVaR* for different  $q$  allow an assessment of the degree of systemic risk contribution for different degrees of tailness.

**Conditioning.** Note that *CoVaR* conditions on the event  $\mathbb{C}$ , which we mostly assume to be the event that institution  $i$  is at its *VaR* level, occurs with probability  $q$ . That is, the likelihood of the conditioning event is independent of the riskiness of  $i$ 's strategy. If we were to condition on a return level of institution  $i$  (instead of a quantile), then more conservative (i.e., less risky) institutions could have a higher *CoVaR* simply because the conditioning event would be a more extreme event for less risky institutions.

**Endogeneity of Systemic Risk.** Note that each institution's *CoVaR* is endogenous and depends on other institutions' risk taking. Hence, imposing a regulatory framework that internalizes externalities alters the *CoVaR* measures. We view as a strength the fact that *CoVaR* is an equilibrium measure, since it adapts to changing environments and provides an incentive for each institution to reduce its exposure to risk if other institutions load excessively on it.

**Directionality.** *CoVaR* is directional. That is, the *CoVaR* of the system conditional on institution  $i$  does not equal the *CoVaR* of institution  $i$  conditional on the system.

**Exposure CoVaR.** The direction of conditioning that we consider is  $\Delta CoVaR_q^{system|i}$ . However, for risk management questions, it is sometimes useful to compute the opposite conditioning,  $\Delta CoVaR_q^{i|system}$ , which we label exposure “*Exposure CoVaR*”. The *Exposure CoVaR* is a measure of an individual institution's exposure to system wide distress, and is similar to the stress tests performed by individual institutions.

**CoES.** Another attractive feature of *CoVaR* is that it can be easily adopted for other “corisk-measures.” One of them is the co-expected shortfall, *Co-ES*. Expected shortfall has a number of advantages relative to  $VaR^4$  and can be calculated as a sum of  $VaR$ s. We denote the  $CoES_q^i$ , the *expected shortfall* of the financial system conditional on  $X^i \leq VaR_q^i$  of institution  $i$ . That is,  $CoES_q^i$  is defined by the expectation over the  $q$ -tail of the conditional probability distribution:

$$E [X^{system} | X^{system} \leq CoVaR_q^i]$$

Institution  $i$ 's contribution to  $CoES_q^i$  is simply denoted by

$$\Delta CoES_q^i = E [X^{system} | X^{system} \leq CoVaR_q^i] - E [X^{system} | X^{system} \leq CoVaR_{50\%}^i].$$

Ideally, one would like to have a co-risk measure that satisfies a set of axioms as, for example, the Shapley value does (recall that the Shapley value measures the marginal contribution of a player to a grand coalition).<sup>5</sup>

## 2.4 Market-Valued Total Financial Assets

Our analysis focuses on the  $VaR_q^i$  and  $\Delta CoVaR_q^i$  of growth rates of market-valued total financial assets. More formally, denote by  $ME_t^i$  the market value of an intermediary  $i$ 's total equity, and by  $LEV_t^i$  the ratio of total assets to book equity. We define the growth rate of market valued total assets,  $X_t^i$ , by

$$X_t^i = \frac{ME_t^i \cdot LEV_t^i - ME_{t-1}^i \cdot LEV_{t-1}^i}{ME_{t-1}^i \cdot LEV_{t-1}^i} = \frac{A_t^i - A_{t-1}^i}{A_{t-1}^i}, \quad (1)$$

where  $A_t^i = ME_t^i \cdot LEV_t^i$ . Note that  $A_t^i = ME_t^i \cdot LEV_t^i = BA_t^i \cdot (ME_t^i / BE_t^i)$ , where  $BA_t^i$  are book-valued total assets of institution  $i$ . We thus apply the market-to-book equity ratio to transform book-valued total assets into market-valued total assets.<sup>6</sup>

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<sup>4</sup>Note that the VaR is not subadditive and does not take distributional aspects within the tail into account. These concerns are however more of theoretical nature since the exact distribution within the tails is difficult to estimate.

<sup>5</sup>Tarashev, Borio, and Tsatsaronis (2009) elaborate the Shapley value further, and Cao (2010) shows how to use Shapley values to calculate systemic risk contributions of *CoVaR*. See also Brunnermeier and Cheridito (2011).

<sup>6</sup>There are several alternatives to generating market valued total assets. One possibility is to use a structural model of firm value in order to calculate market valued assets. Another possibility is to add the market value of equity to the book value of debt. We did not find that any of these alternative ways to generate market valued total assets had a substantial impact on the qualitative outcomes of the subsequent analysis.

Our analysis uses publicly available data. In principle, a systemic risk supervisor could compute the  $VaR_q^i$  and  $\Delta CoVaR_q^i$  from a broader definition of total assets which would include off-balance-sheet items, exposures from derivative contracts, and other claims that are not properly captured by the accounting value of total assets. A more complete description of the assets and exposures of institutions would potentially improve the measurement of systemic risk and systemic risk contribution. Conceptually, it is straightforward to extend the analysis to such a broader definition of total assets.

We focus on the  $VaR_q^i$  and  $\Delta CoVaR_q^i$  of total assets as they are most closely related to the supply of credit to the real economy. Ultimately, systemic risk is of concern for economic welfare as systemic financial crisis have the potential to inefficiently lower the supply of credit to the nonfinancial sector.

Our analysis of the  $VaR_q^i$  and  $\Delta CoVaR_q^i$  for market valued assets could be extended to compute the risk measures for equities or liabilities. For example, the  $\Delta CoVaR_q^i$  for liabilities captures the extent to which financial institutions rely on debt funding—such as repos or commercial paper—that can collapse during systemic risk events. Equity is the residual between assets and liabilities, so the  $\Delta CoVaR_q^i$  measure applied to equity can give additional information about the systemic risk embedded in the asset-liability mismatch. The study of the properties of  $\Delta CoVaR_q^i$  for these other items of intermediary balance sheets is a potentially promising avenue for future research.

## 2.5 Financial Institution Data

We focus on publicly traded financial institutions, consisting of four financial sectors: commercial banks, security broker-dealers (including the investment banks), insurance companies, and real estate companies. We start our sample in 1986Q1 and end it in 2010Q4. The data thus cover three recessions (1991, 2001, and 2007-09) and several financial crisis (1987, 1998, 2000, and 2008). We obtain daily market equity data from CRSP and quarterly balance sheet data from COMPUSTAT. We have a total of 1226 institutions in our sample. For bank holding companies, we use additional asset and liability variables from the FR Y9-C reports. Appendix C provides a detailed description of the data.

### 3 CoVaR Estimation

In this section we outline *CoVaR* estimation. In Section 3.1, we describe the basic time-invariant regressions that are used to generate Figure 1. In Section 3.2, we describe estimation of the time-varying, conditional *CoVaR*. Details on the econometrics are given in Appendix A. Section 3.3 provides estimates of *CoVaR* and discusses properties of the estimates.

#### 3.1 Estimation Method: Quantile Regression

We use quantile regressions to estimate *CoVaR*.<sup>7</sup> To see the attractiveness of quantile regressions, consider the predicted value of a quantile regression of the financial sector  $\hat{X}_q^{system,i}$  on a particular institution or portfolio  $i$  for the  $q^{th}$ -quantile:

$$\hat{X}_q^{system,i} = \hat{\alpha}_q^i + \hat{\beta}_q^i X^i, \tag{2}$$

where  $\hat{X}_q^{system,i}$  denotes the predicted value for a particular quantile conditional on institution  $i$ .<sup>8</sup> In principle, this regression could be extended to allow for nonlinearities by introducing higher order dependence of the system return as a function of returns to institution  $i$ . From the definition of value at risk, it follows directly that

$$VaR_q^{system} | X^i = \hat{X}_q^{system,i}. \tag{3}$$

That is, the predicted value from the quantile regression of the system on institution  $i$  gives the value at risk of the financial system conditional on  $X^i$ , since the  $VaR_q$  given  $X^i$  is just the conditional quantile. Using a particular predicted value of  $X^i = VaR_q^i$  yields our  $CoVaR_q^i$  measure (for the conditioning event  $\{X^i = VaR_q^i\}$ ). More formally, within the quantile regression

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<sup>7</sup>The *CoVaR* measure can be computed in various ways. Quantile regressions are a particularly efficient way to estimate *CoVaR*. It should be emphasized, however, that quantile regressions are by no means the only way to estimate *CoVaR*. Alternatively, *CoVaR* can be computed from models with time-varying second moments, from measures of extreme events, or by bootstrapping past returns. In Appendix B we provide a comparison to estimation using a bivariate *GARCH* model.

<sup>8</sup>Note that a median regression is the special case of a quantile regression where  $q = 50\%$ . We provide a short synopsis of quantile regressions in the context of linear factor models in Appendix A. Koenker (2005) provides a more detailed overview of many econometric issues.

While quantile regressions are used regularly in many applied fields of economics, their applications to financial economics are limited.

framework, our specific *CoVaR* measure is simply given by

$$CoVaR_q^{system|X^i=VaR_q^i} := VaR_q^{system|VaR_q^i} = \hat{\alpha}_q^i + \hat{\beta}_q^i VaR_q^i. \quad (4)$$

The  $\Delta CoVaR_q^i$  is then given by

$$\Delta CoVaR_q^{system|i} = \hat{\beta}_q^i (VaR_q^i - VaR_{50\%}^i). \quad (5)$$

The unconditional  $VaR_q^i$  and  $\Delta CoVaR_q^i$  estimates for Figure 1 are based on equation (5), where an asset's estimated  $VaR_q^i$  is simply the  $q^{th}$ -quantile of its returns. In the remainder of the paper, we use conditional *VaR* and  $\Delta CoVaR$  estimates that explicitly model the time variation of the joint distribution of asset returns as a function of lagged systematic state variables.

### 3.2 Time Variation Associated With Systematic State Variables

The previous section presented a methodology for estimating *CoVaR* that is constant over time. To capture time variation in the joint distribution of  $X^i$  and  $X^{system}$ , we estimate the conditional distribution as a function of state variables. We indicate time-varying  $CoVaR_t$  and  $VaR_t$  with a subscript  $t$  and estimate the time variation conditional on a vector of lagged state variables  $M_{t-1}$ . We run the following quantile regressions in the weekly data (where  $i$  is an institution):

$$X_t^i = \alpha^i + \gamma^i M_{t-1} + \varepsilon_t^i, \quad (6a)$$

$$X_t^{system} = \alpha^{system|i} + \beta^{system|i} X_t^i + \gamma^{system|i} M_{t-1} + \varepsilon_t^{system|i}. \quad (6b)$$

We then generate the predicted values from these regressions to obtain

$$VaR_t^i(q) = \hat{\alpha}_q^i + \hat{\gamma}_q^i M_{t-1}, \quad (7a)$$

$$CoVaR_t^i(q) = \hat{\alpha}^{system|i} + \hat{\beta}^{system|i} VaR_t^i(q) + \hat{\gamma}^{system|i} M_{t-1}. \quad (7b)$$

Finally, we compute  $\Delta CoVaR_t^i$  for each institution:

$$\Delta CoVaR_t^i(q) = CoVaR_t^i(q) - CoVaR_t^i(50\%) \quad (8)$$

$$= \hat{\beta}^{system|i} (VaR_t^i(q) - VaR_t^i(50\%)). \quad (9)$$

From these regressions, we obtain a panel of weekly  $\Delta CoVaR_t^i$ . For the forecasting regressions in Section 4, we generate a quarterly time series by summing the risk measures within each quarter.

The systematic state variables  $M_{t-1}$  are lagged. They should not be interpreted as systematic risk factors, but rather as conditioning variables that are shifting the conditional mean and the conditional volatility of the risk measures. Note that different firms can load on these risk factors in different directions, so that particular correlations of the risk measures across firms—or correlations of the different risk measures for the same firm—are not imposed by construction.

**State variables:** To estimate the time-varying  $CoVaR_t$  and  $VaR_t$ , we include a set of state variables  $M_t$  that are (i) well known to capture time variation in conditional moments of asset returns, and (ii) liquid and easily tradable. We restrict ourselves to a small set of risk factors to avoid overfitting the data. Our factors are:

(i) *VIX*, which captures the implied volatility in the stock market reported by the Chicago Board Options Exchange.<sup>9</sup>

(ii) A short term “*liquidity spread*,” defined as the difference between the three-month repo rate and the three-month bill rate. This liquidity spread measures short-term liquidity risk. We use the three-month general collateral repo rate that is available on Bloomberg, and obtain the three-month Treasury rate from the Federal Reserve Bank of New York.<sup>10</sup>

(iii) The change in the three-month Treasury bill rate from the Federal Reserve Board’s H.15. We use the change in the three-month Treasury bill rate because we find that the change,

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<sup>9</sup>The *VIX* is available only since 1990. We use the *VXO* for the 1986-90 period by running a regression of the *VIX* on the *VXO* for the 1990-2010 period and then using the predicted value from that regression for the 1986-89 period.

<sup>10</sup>The three-month repo rate is available on Bloomberg only since 1990. We use the three-month Libor rate as reported by the British Bankers Association for the 1986-90 period by running a regression of the repo rate on the libor rate for the 1990-2010 period and then using the predicted value from that regression for the 1986-89 period.

not the level, is most significant in explaining the tails of financial sector market-valued asset returns.

In addition, we consider the following two fixed-income factors that capture the time variation in the tails of asset returns:

(iv) The change in the *slope of the yield curve*, measured by the yield spread between the ten-year Treasury rate and the three-month bill rate obtained from the Federal Reserve Board's H.15 release.

(v) The change in the *credit spread* between BAA-rated bonds and the Treasury rate (with the same maturity of ten years) from the Federal Reserve Board's H.15 release.

We further control for the following equity market returns:

(vi) The weekly equity market return from CRSP.

(vii) The weekly real estate sector return in excess of the market return (from the real estate companies with SIC code 65-66).

The following table give summary statistics for the state variables. We also report the 1% stress level, which is the variable's mean conditional on the financial system being in its historical 1% tail.

[Table 1 here]

Table 1 provides summary statistics of the state variables. The 1% stress level is the level of each respective variable during the 1% worst weeks for financial system asset returns. For example, the average of the VIX during the stress periods is 51.66, as the worst times for the financial system include the times when the VIX was highest. Similarly, the stress level corresponds to a high level of the liquidity spread, a sharp decline in the Treasury bill rate, sharp increases of the term and credit spreads, and large negative equity return realizations. In general, by comparing the extreme values of the state variables to the numbers of standard deviations away from their mean, we can see that the distributions appear highly skewed.

### **3.3 *CoVaR* Summary Statistics**

Table 2 provides the estimates of our weekly conditional 1%-*CoVaR* measures that we obtain from using quantile regressions. Each of the summary statistics constitutes the universe of

financial institutions.

[Table 2 here]

Line (1) of Table 2 give the summary statistics for the market-valued total asset growth rates; line (2) gives the summary statistics for the  $VaR_t^i$  for each institution; line (3) gives the summary statistics for  $\Delta CoVaR_t^i$ ; lines (4) gives the summary statistics for the 1%-stress level of  $\Delta CoVaR_t^i$ ; and line (5) gives the summary statistics for the financial system value at risk,  $VaR_t^{system}$ . The stress  $\Delta CoVaR_t^i$  is estimated by substituting the worst 1% of state variable realizations into the  $\Delta CoVaR_t^i$  estimates.

Recall that  $\Delta CoVaR_t^i$  measures the marginal contribution of institution  $i$  to overall systemic risk and reflects the difference between the value at risk of the financial universe conditional on the stressed and the median state of institution  $i$ . We report the mean, standard deviation, and number of observations for each of the items in Table 2. All of the numbers are expressed in weekly percent returns. We have a total of 1226 institutions in the sample, with an average length of 645 weeks. The institution with the longest history spans all 1300 weeks of the 1986Q1-2010Q4 sample period. We require institutions to have at least 260 weeks of asset return data in order to be included in the panel. In the following analysis, we focus primarily on the 1% and the 5% quantiles, corresponding to the worst 13 weeks and the worst 65 weeks over the sample horizon, respectively. It is straightforward to estimate more extreme tails following the methodology laid forward by Chernozhukov and Du (2008), an analysis that we leave for future research. In the following analysis, we find results that are largely qualitatively similar for the 1% and the 5% quantiles.

[Table 3 here]

We obtain time variation of the risk measures by running quantile regressions of asset returns on the lagged state variables. We report average  $t$ -stats of these regressions in Table 3. A higher VIX, higher repo spread, and lower market return tend to be associated with more negative risk measures. In addition, increases in the three-month yield, declines in the term spread, and increases the credit spread tend to be associated with larger risk. Overall, the average significance of the conditioning variables reported in Table 3 show that the state variables do indeed proxy for the time variation in the quantiles and particularly in  $CoVaR$ .

### 3.4 *CoVaR* versus *VaR*

Figure 1 shows that, *across institutions*, there is only a very loose link between an institution’s  $VaR^i$  and its contribution to systemic risk as measured by  $\Delta CoVaR^i$ . Hence, imposing financial regulation solely based on the risk of an institution in isolation might not be sufficient to insulate the financial sector against systemic risk. Figure 2 repeats the scatter plot of  $\Delta CoVaR^i$  against  $VaR^i$  for 240 portfolios, grouped by 60 portfolios for each of the four financial industries.<sup>11</sup> We do so, since one might argue that firms change their risk taking behavior over the sample span of 1986 to 2010. Using portfolios,  $\Delta CoVaR^i$  and  $VaR^i$  have only a weak relationship in the cross section. However, they have a strong relationship in the time series. This can be seen in Figure 3, which plots the time series of the  $\Delta CoVaR^i$  and  $VaR^i$  for the portfolio of large broker dealers over time. We note that the cross sectional average of  $\Delta CoVaR^i$  including all institutions has a very close time series relationship with the value at risk of the financial system,  $VaR^i$ , per construction. One way to interpret the  $\Delta CoVaR^i$ s is by viewing them as cross sectional allocation of system wide risk to the various institutions.

## 4 *Forward- $\Delta CoVaR$*

In this section, we calculate forward looking systemic risk measures that can be used for financial stability monitoring, and as a basis for (countercyclical) macroprudential policy. We first present the construction of the “*forward- $\Delta CoVaR$ ”*, and then present out of sample tests. We finally discuss how the measures could be used as a basis for capital surcharge calibrations.

Instead of tying financial regulation directly to  $\Delta CoVaR$ , we propose to link it to financial institutions’ characteristics that predict their future  $\Delta CoVaR$ . This addresses two key issues of systemic risk regulation: measurement accuracy and procyclicality. Any tail risk measure, estimated at a high frequency, is by its very nature imprecise. Quantifying the relationship between  $\Delta CoVaR$  and more easily observable institution-specific variables, such as size, leverage, and maturity mismatch, allows for more robust inference than measuring  $\Delta CoVaR$  directly. Furthermore, using these variables to predict *future* contributions to systemic risk addresses the inherent procyclicality of market-based risk measures. This ensures that  $\Delta CoVaR$ -based

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<sup>11</sup>The portfolios are constructed from quintiles by market-valued assets, 2- year market-valued asset growth, maturity mismatch, equity volatility, leverage, and market-to-book ratio for each of the four industries.

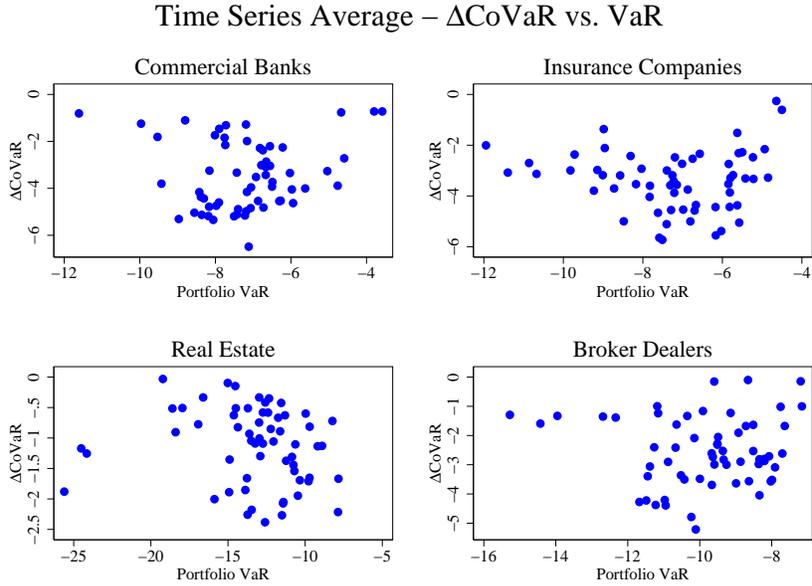


Figure 2: The scatter plot shows the weak cross-sectional link between the time-series average of a portfolio’s risk in isolation, measured by  $VaR^i$  (x-axis), and the time-series average of a portfolio’s contribution to system risk, measured by  $\Delta\text{CoVaR}^i$  (y-axis). The  $VaR^i$  and  $\Delta\text{CoVaR}^i$  are in units of weekly percent returns to total market-valued financial assets and measured at the 1% level.

financial regulation is implemented in a forward-looking way that counteracts the procyclicality of current regulation.

#### 4.1 Constructing the *Forward- $\Delta\text{CoVaR}$*

We relate estimates of time-varying  $\Delta\text{CoVaR}$  to characteristics of financial institutions. We collect the following set of characteristics:

1. leverage, defined as *total assets / total equity* (in book values);
2. maturity mismatch, defined as *(short term debt - cash) / total liabilities*;
3. market-to-book, defined as the ratio of the market to the book value of total equity;
4. size, defined by the log of total book equity;
5. equity return volatility, computed from daily equity return data within each quarter;
6. equity market beta calculated from daily equity return data within each quarter.

[Table 4 here]

[Table 5 here]

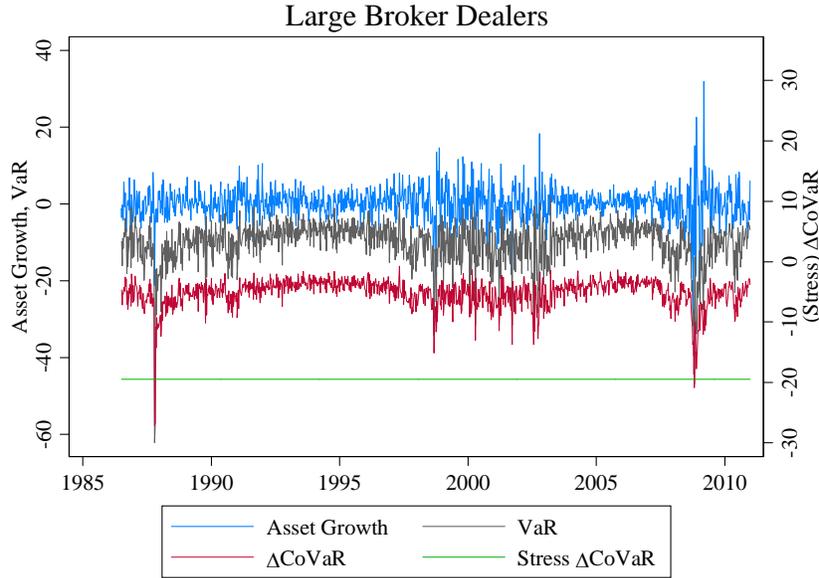


Figure 3: This figure shows the market-valued asset returns (blue), the 1%- $VaR$  (gray), and the 1%- $\Delta CoVaR$  (red) for a portfolio of the 20% of largest investment banks. The 1%-stress  $\Delta CoVaR$  is also plotted. All risk measures are in percent weekly returns to total market-valued assets.

Table 4 provides the summary statistics for  $\Delta CoVaR$  at the quarterly frequency, and the quarterly firm characteristics. In Table 5, we ask whether systemic risk contribution can be forecast cross sectionally by lagged characteristics at different time horizons. Table 5 shows that firms with higher leverage, more maturity mismatch, and larger size tend to be associated with larger systemic risk contributions one quarter, one year, and two years later, both at the 1% and the 5% levels. These  $\Delta CoVaR$  regressions are run with risk measures that are time-aggregated by summing the weekly measures within each quarter. The coefficients in Table 5 are sensitivities of  $\Delta CoVaR$  with respect to the characteristics expressed in units of basis points of systemic risk contribution. For example, the coefficient of  $-6.7$  on the leverage forecast at the two-year horizon implies that an increase in leverage (say, from 15 to 16) of an institution is associated with an increase in systemic risk contribution of 6.7 basis points of quarterly asset returns at the 5% systemic risk level. For an institution that has \$1 trillion of total market-valued assets, that translates into \$67 billion of systemic risk contribution. Columns (1)-(3) and (4)-(6) of Table 5 can be understood as a “term structure” of systemic risk contribution if

read from right to left. The comparison of Panels A and B provide a gauge of the “tailness” of systemic risk contribution.

The regression coefficients of Table 5 can be used to weigh the relative importance of various firm characteristics. To make this more explicit, consider the following example: Suppose a small bank is subject to a tier-one capital requirement of 7%. That is, the “leverage ratio” cannot exceed 1 : 14.<sup>12</sup> Our analysis answers the question of how much stricter the capital requirement should be for a larger bank with the same leverage, assuming that the small bank and the large bank are allowed a fixed level of systemic risk contribution  $\Delta CoVaR$ . If the larger bank is 10 percent larger than the smaller bank, then the size coefficient predicts that its  $\Delta CoVaR$  per unit of capital is 27 basis points larger than the small bank’s  $\Delta CoVaR$ . To ensure that both banks have the same  $\Delta CoVaR$  per unit of capital, the large bank would have to reduce its maximum leverage from 1 : 14 to 1 : 10. In other words, the large bank should face a capital requirement of 10% instead of 7%. The exact trade-off between size and leverage is given by the ratio of the two respective coefficients of our forecasting regressions. Of course, in order to achieve a given level of systemic risk contribution per units of total assets, instead of lowering the size, the bank could also reduce its maturity mismatch or improve its systemic risk profile along other dimensions. Similarly, for a Pigouvian taxation scheme, the regression coefficients should determine the weight of leverage, maturity mismatch, size, and other characteristics in forming the tax base.

This methods allows the connection of macroprudential policy with frequently and robustly measured characteristics.  $\Delta CoVaR$ —like any tail risk measure—relies on relatively few extreme-crisis data points. Hence, adverse movements, especially followed by periods of stability, can lead to sizable increases in tail risk measures. In contrast, measurement of characteristics such as size are very robust, and they can be measured more reliably at higher frequencies. The debate on “too big to fail” suggests that size is the all-dominating variable, indicating that large institutions should face a more stringent regulation compared to smaller institutions. As mentioned above, unlike a co-risk measure, the “size only” approach fails to acknowledge that many small institutions can be “systemic as part of a herd.” Our solution to this problem is to

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<sup>12</sup>We are loose here, since the Basel capital requirement refers to ratio between equity capital and risk weighted assets, while our study simply takes total assets.

combine the virtues of both types of measures by projecting the spillover risk measure  $\Delta CoVaR$  on multiple, more frequently observable variables.

This method can also address the procyclicality of contemporaneous risk measures. Systemic risk builds up before an actual financial crisis occurs and any regulation that relies on contemporaneous risk measure estimates would be unnecessarily tight after adverse events and unnecessarily loose in periods of stability. In other words, it would amplify the adverse impacts after bad shocks, while also amplifying balance sheet expansions in expansions.<sup>13</sup> Hence, we propose to focus on variables that can be reliably measured at a quarterly frequency and predict future, rather than contemporaneous,  $\Delta CoVaR$ .

## 4.2 *Forward- $\Delta CoVaR$ for Bank Holding Companies*

Ideally, one would like to link macroprudential policies to more institutional characteristics than simply size, leverage, maturity mismatch etc. If one restricts the sample to bank holding companies, we have more characteristic data to extend our method. On the asset side of banks' balance sheets, we use loans, loan-loss allowances, intangible loss allowances, intangible assets, and trading assets. Each of these asset composition variables is expressed as a percent of total book assets. The cross-sectional regressions with these asset composition variables are reported in Panel A of Table 6. In order to capture the liability side of banks' balance sheets, we use interest-bearing core deposits (IBC), non-interest-bearing deposits (NIB), large time deposits (LT), and demand deposits. Each of these variables is expressed as a percent of total book assets. The variables can be interpreted as refinements of the maturity mismatch variable used earlier. The cross-sectional regressions with the liability aggregates are reported in Panel B of Table 6.

[Table 6 here]

Panel A of Table 6 shows which types of liability variables are significantly increasing or decreasing systemic risk contribution. Bank holding companies with a higher fraction of non-interest-bearing deposits have a significantly higher systemic risk contribution, while interest bearing core deposits and large time deposits are decreasing the forward estimate of  $\Delta CoVaR$ .

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<sup>13</sup>See Estrella (2004), Kashyap and Stein (2004), and Gordy and Howells (2006) for studies of the procyclical nature of capital regulation.

Non-interest-bearing deposits are typically held by nonfinancial corporations and households, and can be quickly reallocated across banks conditional on stress in a particular institution. Interest-bearing core deposits and large time deposits, on the other hand, are more stable sources of funding and are thus decreasing the systemic tail risk contribution (i.e., they have a positive sign). The share of deposits is not significant. The maturity-mismatch variable that we constructed for the universe of financial institutions is no longer significant once we include the more refined liability measures for the bank holding companies. In fact, in some specifications, the maturity mismatch variable is significant with the wrong sign.

Panel B of Table 6 shows that loan-loss allowances and trading assets are particularly good predictors for the cross-sectional dispersion of future systemic risk contribution. The fraction of intangible assets is marginally significant. Conditional on these variables, the size of total loans as a fraction of book equity tends to decrease systemic risk contribution, which might be due to the accounting treatment of loans: loans are held at historical book value, and deteriorating loan quality is captured by the loan-loss reserves. By including loan-loss reserves, trading assets, and intangible assets in the regression gives rise to lower estimates of systemic risk contribution.

In summary, the results of Table 6, in comparison to Table 5, show that more information about the balance sheet characteristics of financial institutions can potentially improve the estimated forward  $\Delta CoVaR$ . We expect additional data that capture particular activities of financial institutions, as well as supervisory data, to lead to further improvements in the estimation precision of forward systemic risk contribution.

### 4.3 Out of Sample *Forward- $\Delta CoVaR$*

We compute “forward  $\Delta CoVaR$ ” as the predicted value from the panel regression reported in Table 5. We generate this forward  $\Delta CoVaR$  “in sample” through 2000, and then out of sample by re-estimating the panel regression each quarter, and computing the predicted value. Since one cannot use time effects in an out of sample exercise, we use the macro state variables to capture common variation across time.

We plot the  $\Delta CoVaR$  together with the two-year forward  $\Delta CoVaR$  for the average of the largest 50 financial institutions in Figure 4. The figure clearly shows the strong negative correlation of the contemporaneous  $\Delta CoVaR$  and the forward  $\Delta CoVaR$ . In particular, during the

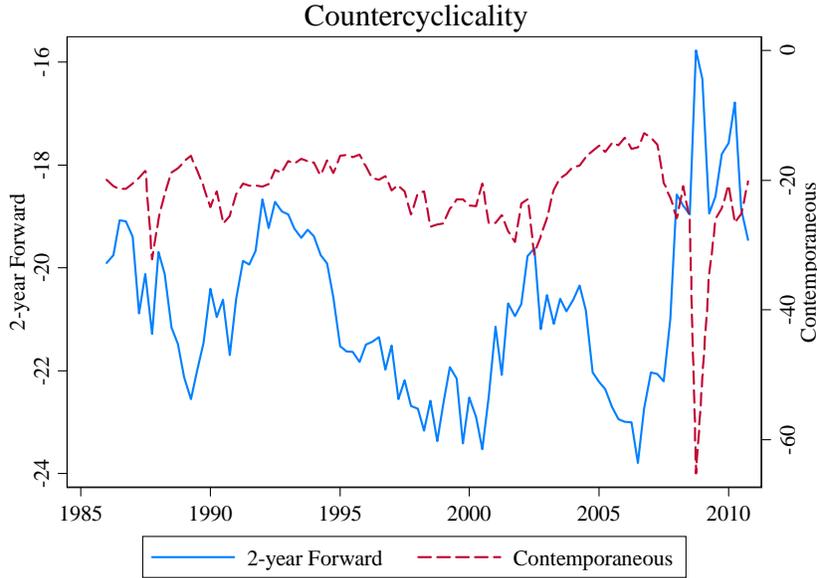


Figure 4: **Out-of-Sample Forward  $\Delta CoVaR$** : This figure shows average forward and contemporaneous 5%  $\Delta CoVaR$  estimated out-of-sample for the top 50 financial institutions estimated out-of-sample. First, weekly contemporaneous  $\Delta CoVaR$  is estimated out-of-sample starting in 2000Q1 at one quarter increments with an expanding window. Forward  $\Delta CoVaR$  is generated as described in the paper but in an out-of-sample fashion, again beginning in 2000Q1. The forward  $\Delta CoVaR$  at a given date uses the data available at that time to predict  $\Delta CoVaR$  two years in the future.

credit boom of 2003-06, the contemporaneous  $\Delta CoVaR$  is estimated to be small (in absolute value), while the forward  $\Delta CoVaR$  is large (in absolute value). Macroprudential regulation based on the forward  $\Delta CoVaR$  are thus countercyclical.

Next, we extend the in sample panel estimates reported in Tables 5 and 6 to out of sample estimates. In particular, we show that the forward  $\Delta CoVaR$  predicts the cross section of systemic risk realizations out of sample. In order to show the out of sample performance, we need a measure of realized systemic risk contribution. As a proxy, we compute covariances of financial institutions with the financial system during the financial crisis. In particular, we estimate this crisis covariance as the realized covariance from weekly data for 2007Q2 - 2009Q1. We use the forward  $\Delta CoVaR$  estimated with data as of 2006Q4 in order to forecast the cross section of realized crisis covariances. We use the 5% level, though we found that the 1% gives very similar results.

[Table 7 here]

Table 7 shows that the forward  $\Delta CoVaR$  as of the end of 2006 was able to explain a little bit over 50% of the cross sectional covariance during the crisis. We view this result as a very strong one. Comparison of columns (1)-(3) shows that the forward horizon did not matter much in terms of cross sectional explanatory power. Furthermore, column (4) shows that the information contained in the estimated forward measures captures most of the ability of firm characteristics to predict crisis covariance, as the individual characteristics only generate a slightly higher  $R$ -squared statistic. The forward  $\Delta CoVaR$  thus summarizes in a single variable for each firm the extent to which it is expected to contribute to future systemic risk.

## 5 Conclusion

During financial crises or periods of financial intermediary distress, tail events tend to spill across financial institutions. Such spillovers are preceded by a risk-buildup phase. Both elements are important contributors to financial system risk.  $\Delta CoVaR$  is a parsimonious measure of systemic risk that complements measures designed for individual financial institutions.  $\Delta CoVaR$  broadens risk measurement to allow a macroprudential perspective. The *forward- $\Delta CoVaR$*  is a forward looking measure of systemic risk contribution. It is constructed by projecting  $\Delta CoVaR$  on lagged firm characteristics such as size, leverage, maturity mismatch, and industry dummies. This forward looking measure can potentially be used in macroprudential policy applications.

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# Appendices

## A CoVaR Estimation via Quantile Regressions

This appendix explains how to use quantile regressions to estimate *VaR* and *CoVaR*. Suppose that returns  $X_t^i$  have the following linear factor structure

$$X_t^j = \phi_0 + M_{t-1}\phi_1 + X_t^i\phi_2 + (\phi_3 + M_{t-1}\phi_4 + X_t^i\phi_5)\varepsilon_t^j, \quad (10)$$

where  $M_{t-1}$  is a vector of state variables. The error term  $\varepsilon_t$  is assumed to be i.i.d. with zero mean and unit variance and is independent of  $M_{t-1}$  so that  $E[\varepsilon_t^j | M_{t-1}, X_t^i] = 0$ . Returns are generated by a process of the “location scale” family, so that both the conditional expected return  $E[X_t^j | M_{t-1}, X_t^i] = \phi_0 + M_{t-1}\phi_1 + X_t^i\phi_2$  and the conditional volatility  $Vol_{t-1}[X_t^j | M_{t-1}, X_t^i] = (\phi_3 + M_{t-1}\phi_4 + X_t^i\phi_5)$  depend on the set of state variables  $M_{t-1}$  and on  $X_t^i$ . The coefficients  $\phi_0$ ,  $\phi_1$ , and  $\phi_3$  could be estimated consistently via OLS of  $X_t^j$  on  $M_{t-1}$  and  $X_t^i$ . The predicted value of such an OLS regression would be the mean of  $X_t^j$  conditional on  $M_{t-1}$  and  $X_t^i$ . In order to compute the *VaR* and *CoVaR* from OLS regressions, one would have to also estimate  $\phi_3$ ,  $\phi_4$ , and  $\phi_5$ , and then make distributional assumptions about  $\varepsilon_t^j$ .<sup>14</sup> The quantile regressions incorporate estimates of the conditional mean and the conditional volatility to produce conditional quantiles, without the distributional assumptions that would be needed for estimation via OLS.

Instead of using OLS regressions, we use quantile regressions to estimate model (10) for different percentiles. We denote the cumulative distribution function (cdf) of  $\varepsilon^j$  by  $F_{\varepsilon^j}(\varepsilon^j)$ , and its inverse cdf by  $F_{\varepsilon^j}^{-1}(q)$  for percentile  $q$ . It follows immediately that the inverse cdf of  $X_t^j$  is

$$F_{X_t^j}^{-1}(q | M_{t-1}, X_t^i) = \alpha_q + M_{t-1}\gamma_q + X_t^i\beta_q, \quad (11)$$

where  $\alpha_q = \phi_0 + \phi_3 F_{\varepsilon^j}^{-1}(q)$ ,  $\gamma_q = \phi_1 + \phi_4 F_{\varepsilon^j}^{-1}(q)$ , and  $\beta_q = \phi_2 + \phi_5 F_{\varepsilon^j}^{-1}(q)$  for quantiles  $q \in (0, 1)$ . We call  $F_{X_t^j}^{-1}(q | M_{t-1}, X_t^i)$  the conditional quantile function. From the definition of

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<sup>14</sup>The model (10) could otherwise be estimated via maximum likelihood using a stochastic volatility or GARCH model if distributional assumptions about  $\varepsilon$  are made. The quantile regression approach does not require specific distributional assumptions for  $\varepsilon$ .

$VaR$ , we obtain

$$VaR_q^j = \inf_{VaR_q} \{ \Pr (X_t \leq VaR_q | M_{t-1}, X_t^i) \geq q \} = F_{X_t^j}^{-1} (q | M_{t-1}, X_t^i).$$

The conditional quantile function  $F_{X_t^j}^{-1} (q | M_{t-1}, X_t^i)$  is the  $VaR_q^j$  conditional on  $M_{t-1}$  and  $X_t^i$ . By conditioning on  $X_t^i = VaR_q^i$ , we obtain the  $CoVaR_q^{j|i}$  from the quantile function:

$$CoVaR_q^{j|i} = \inf_{VaR_q} \{ \Pr (X_t \leq VaR_q | M_{t-1}, X_t^i = VaR_q^i) \geq q \} = F_{X_t^j}^{-1} (q | M_{t-1}, VaR_q^i). \quad (12)$$

We estimate the quantile function as the predicted value of the  $q$ -quantile regression of  $X_t^i$  on  $M_{t-1}$  and  $X_t^j$  by solving

$$\min_{\alpha_q, \beta_q, \gamma_q} \sum_t \begin{cases} q \left| X_t^j - \alpha_q - M_{t-1} \beta_q - X_t^i \gamma_q \right| & \text{if } \left( X_t^j - \alpha_q - M_{t-1} \beta_q - X_t^i \gamma_q \right) \geq 0 \\ (1-q) \left| X_t^j - \alpha_q - M_{t-1} \beta_q - X_t^i \gamma_q \right| & \text{if } \left( X_t^j - \alpha_q - M_{t-1} \beta_q - X_t^i \gamma_q \right) < 0 \end{cases}.$$

See Bassett and Koenker (1978) and Koenker and Bassett (1978) for finite sample and asymptotic properties of quantile regressions. Chernozhukov and Umantsev (2001) and Chernozhukov and Du (2008) discuss  $VaR$  applications of quantile regressions.

## B Robustness Checks

### B.1 GARCH $\Delta CoVaR$

One potential shortcoming of the quantile estimation procedure described in Section 3 is that it does not allow for estimation of the time-varying nature of systemic exposure to firm risk. An alternative approach is to estimate bivariate GARCH models to obtain the time-varying covariance between institutions and the financial system. However, such an approach requires strong distributional assumptions and takes the form of a complex optimization problem which can be difficult to estimate. As a robustness check we estimate  $\Delta CoVaR$  using a bivariate diagonal GARCH model (DVECH) and find that this method produces estimates quite similar to the quantile regression method, leading us to the conclusion that the quantile regression framework is sufficiently flexible to estimate  $\Delta CoVaR$ . We begin by outlining a simple Gaussian framework under which  $\Delta CoVaR$  has a closed-form expression, and then discuss the estimation

results.

**Gaussian Model** Assume firm and system returns follow a bivariate normal distribution:

$$\begin{pmatrix} X_t^i, X_t^{system} \end{pmatrix} \sim N \left( 0, \begin{pmatrix} (\sigma_t^i)^2 & \rho_t \sigma_t^i \sigma_t^{system} \\ \rho_t \sigma_t^i \sigma_t^{system} & (\sigma_t^{system})^2 \end{pmatrix} \right) \quad (13)$$

By properties of the multivariate normal distribution, the distribution of system returns conditional on firm returns is also normally distributed:

$$X_t^{system} | X_t^i \sim N \left( \frac{X_t^i \sigma_t^{system} \rho_t}{\sigma_t^i}, (1 - \rho_t^2) (\sigma_t^{system})^2 \right) \quad (14)$$

We can define  $CoVaR_t^i(q, p)$  as the  $q\%$ -VaR of the financial system given firm  $i$  is at its  $p\%$ -VaR level. It is defined implicitly by:

$$\Pr \left( X_t^{system} < CoVaR_t^i(q, p) | X_t^i = VaR_t^i(p) \right) = q \quad (15)$$

which, rearranged, gives:

$$\Pr \left( \left[ \frac{X_t^{system} - X_t^i \rho_t \sigma_t^{system} / \sigma_t^i}{\sigma_t^{system} \sqrt{1 - \rho_t^2}} \right] < \frac{CoVaR_t^i(q, p) - X_t^i \rho_t \sigma_t^{system} / \sigma_t^i}{\sigma_t^{system} \sqrt{1 - \rho_t^2}} | X_t^i = VaR_t^i(p) \right) = q \quad (16)$$

where  $\left[ \frac{X_t^{system} - X_t^i \rho_t \sigma_t^{system} / \sigma_t^i}{\sigma_t^{system} \sqrt{1 - \rho_t^2}} \right] \sim N(0, 1)$ . The firm value-at-risk is given by  $VaR_t^i(p) = \Phi^{-1}(p) \sigma_t^i$ . Combining, we can write:

$$CoVaR_t^i(q, p) = \Phi^{-1}(q) \sigma_t^{system} \sqrt{1 - \rho_t^2} + \Phi^{-1}(p) \rho_t \sigma_t^{system} \quad (17)$$

Because  $\Phi^{-1}(50\%) = 0$ , solving for  $\Delta CoVaR$  gives:

$$\Delta CoVaR(q, q) = \Phi^{-1}(q) \rho_t \sigma_t^{system} \quad (18)$$

The primary downfall of such a specification is that it uses an estimate of contemporaneous correlation with the market to gauge the size of potential tail spillover effects. In this sense, this specification is more contemporaneous than the quantile regression method, which uses

estimates of the tail correlation taken from the entire data history. Taking into account the heteroskedasticity of correlation in this way provides cross-sectional forecasting power where the simple Gaussian model cannot.

**Estimation** We estimate a bivariate diagonal vech GARCH(1,1) for each institution in our sample.<sup>15</sup> As a robustness check, we estimated the panel regressions of Section 4 on a matched sample of 145 institutions with \$10billion or more in total assets for which our GARCH estimates converged.

[Table 8 here]

The results in Table 8 show the coefficients on size, maturity mismatch, and leverage are quite similar between the GARCH and quantile estimation methods. The most notable difference, however, is that the R-squared values of the GARCH-based regressions are nearly double those of the quantile regressions. We found that this difference is driven entirely by the time fixed effects in the regressions, which implies that there is much less cross-sectional variation in the GARCH  $\Delta CoVaR$  than in the quantile regression based estimation. In fact, a large portion of the time variation in GARCH  $\Delta CoVaR$  comes from the estimated time-varying system volatility, while the time variation in the quantile method comes from firm-specific loadings on time-varying risk factors. As expected, controlling for firm instead of time fixed effects significantly increases the R-squared for the quantile  $\Delta CoVaR$  forecast regressions but reduces the R-squared for the regressions using GARCH  $\Delta CoVaR$ .

The differences between the quantile and the GARCH  $\Delta CoVaR$  are least pronounced for the largest institutions, which have persistently high correlation with the financial system. Figure 5 shows the close comovement of the quantile and the GARCH based systemic risk measures for the four largest financial institutions. Despite the fact that the estimation methods differ sharply, we can see that the two approaches generate very similar time patterns of systemic risk contribution. The GARCH based estimators do seem to pick up tails a bit more strongly.

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<sup>15</sup>We were able to get convergence of the Garch model for 75% of firms. We found that convergence of the models in our data is very sensitive to both missing values and extreme returns. Truncation of returns generally, but not consistently, resulted in an increase in the fraction of the models that converged.

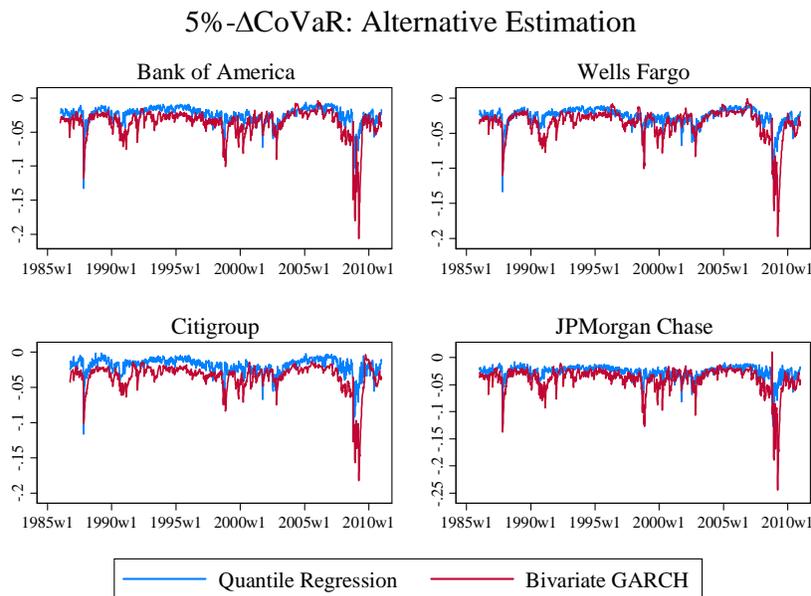


Figure 5: Alternative Estimation of  $\Delta CoVaR$ : This figure shows estimates of 5%  $\Delta CoVaR$  for the four largest commercial banks: Bank of America, Wells Fargo, Citigroup, and JP Morgan Chase. The time series and magnitudes of these estimates are remarkably similar, indicating that the time-varying correlation of the GARCH model adds little information for these institutions.

## B.2 Financial System Return Variable

The financial system return variable  $X_t^{system}$  used in the paper is the weekly return on the market-valued total assets of the financial system, as proxied by the universe of financial institutions. This measure is generated by taking average market valued asset returns, weighted by lagged market valued total assets. One concern with this methodology is that it might introduce a mechanical correlation between each institution and the financial system proportional to the relative size of the financial institution. We check to see if such a mechanical correlation is driving our results by reestimating institutions'  $\Delta CoVaR$  using system return variables formed from the value weighted returns of all other institutions in the sample, leaving out the institution for which  $\Delta CoVaR$  is being estimated.

[Table 9 here]

We find a very strong correlation across institutions, and across time, for the two different systemic risk measures. In fact, even for the largest institutions do we find a very strong

correlation between the baseline system return variable and the modified system return, with correlation coefficients over 99%. Table 9 reports the forward CoVaR regressions for the 5% level using both specifications. The coefficients under the two specifications are statistically indistinguishable, indicating that this mechanical correlation is not driving our results.

## C Data Description

### C.1 CRSP and COMPUSTAT Data

As discussed in the paper, we estimate  $\Delta CoVaR$  for the market-valued asset returns of financial institutions. We start with daily equity data from CRSP for all financial institutions with two-digit COMPUSTAT SIC codes between 60 and 67 inclusive, indexed by PERMNO. Banks correspond to SIC codes 60, 61, and 6712; insurance companies correspond to SIC codes 63-64, real estate companies correspond to SIC codes 65-6, and broker-dealers are SIC code 67 (except for the bank holding companies, 6712). All other financial firms in our initial sample are placed in an “other” category. We manually adjust the COMPUSTAT SIC codes to account for the conversions of several large institutions into bank holding companies in late 2008, but otherwise do not find time varying industry classifications. Following the asset pricing literature, we keep only ordinary common shares (which exclude certificates, ADRs, SBIS, REITs, etc.) and drop daily equity observations with missing or negative prices or missing returns. Our keeping only ordinary common shares excludes several large international institutions, such as Credit Suisse and Barclays, which are listed in the United States as American Depository Receipts.

The daily data are collapsed to a weekly frequency and merged with quarterly balance sheet data from the CRSP/COMPUSTAT quarterly dataset. The quarterly data are filtered to remove leverage and book-to-market ratios less than zero and greater than 100. We also apply 1% and 99% truncation to maturity mismatch.

Market equity and balance sheet data are adjusted for mergers and acquisitions using the CRSP daily dataset. We use a recursive algorithm to traverse the CRSP DELIST file to find the full acquisition history of all institutions in our sample. The history of acquired firms is collapsed into the history of their acquirers. For example, we account for the possibility that firm A was acquired by firm B, which was then acquired by firm C, etc. Our final panel therefore

does not include any firms that we are able to identify as having been ultimately acquired by another firm in our universe. Using the merger-adjusted data, we generate weekly leverage data as a linear interpolation of the quarterly data. Interpolated values of leverage are computed only between no more than two consecutive quarters of missing leverage data. Weekly leverage and market equity are used to generate market-valued asset returns. The final estimation sample is restricted to include firms with at least 260 weeks of non-missing market-valued asset returns. To construct the overall financial system portfolio (for  $j = system$ ), we simply compute the average market-valued returns of all financial institutions, weighted by the (lagged) market value of their assets.

## C.2 Bank Holding Company Y9-C Data

Balance sheet data from the FR Y-9C reports are incorporated into our panel data set using a mapping maintained by the Federal Reserve Bank of New York.<sup>16</sup> We are able to match data for 357 U.S. bank holding companies for a total of 17,382 bank-quarter observations. The link is constructed by matching PERMCOs in the linking table to RSSD9001 in the Y9-C data. We then match to the last available PERMCO of each institution in our CRSP/COMPUSTAT sample. It is important to note that our main panel of CRSP and COMPUSTAT data are historically merger-adjusted, but the Y9-C data is not. Performing such an adjustment for a large set of BHCs is infeasible because they mainly acquire smaller non-BHC entities with different data-reporting requirements and availability.

In the forecasting regressions of Table 6, these variables are expressed as a percentage of total book assets. All ratios are truncated at the 1% and 99% level across the panel. Detailed descriptions of the Y9-C variables listed above can be found in the Federal Reserve Board of Governors Micro Data Reference Manual.<sup>17</sup>

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<sup>16</sup>The mapping is available at [http://www.ny.frb.org/research/banking\\_research/datasets.html](http://www.ny.frb.org/research/banking_research/datasets.html).

<sup>17</sup><http://www.federalreserve.gov/reportforms/mdrm>

	Date Range	FR Y-9C Series Name
Trading Assets	1986:Q1–1994:Q4	bhck2146
	1995:Q1–2010:Q4	bhck3545
Loans Net Loan-Loss Reserves	1986:Q1–2010:Q4	bhck2122-bhck3123
Loan-Loss Reserve	1986:Q1–2010:Q4	bhck3123
Intangible Assets	1986:Q1–1991:Q4	bhck3163+bhck3165
	1992:Q1–2000:Q4	bhck3163+bhck3164
		+bhck5506+bhck5507
	2001:Q1–2010:Q4	bhck3163+bhck0426
Interest-Bearing Core Deposits	1986:Q1–2010:Q4	bhcb2210+bhcb3187+bhch6648
		-bhdma164+bhcb2389
Non-Interest-Bearing Deposits	1986:Q1–2010:Q4	bhdm6631+bhfn6631
Large Time Deposits	1986:Q1–2010:Q4	bhcb2604
Demand Deposits	1986:Q1–2010:Q4	bhcb2210

## D List of Financial Institutions for Figure 1<sup>18</sup>

### **Banks and Thrifts:**

Bank of America, Citigroup, JPMorgan Chase, Wachovia, Wells Fargo

### **Investment Banks:**

Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch, Morgan Stanley

### **GSEs:**

Fannie Mae, Freddie Mac

### **Insurance Companies:**

American International Group, Metlife

<sup>18</sup>Industry classifications are as of 2006Q4.

Table 1: **State Variable Summary Statistics.** The spreads and spread changes are expressed in basis points, returns in percent.

Variable	Mean	Std. Dev.	Min	Max	1% Stress Level
VIX	20.72	8.3	10.1	90.5	51.66
Liquidity Spread	21.56	28.7	-15.6	210.9	56.31
3-month Treasury Change	-0.55	10.9	-118.8	47	-23.1
Term Spread Change	0.12	12.71	-63.6	79.6	5.09
Credit Spread Change	0.03	7.4	-38.1	56.1	16.8
Equity Return	0.16	2.4	-15.3	13.827	-7.9
Real Estate Excess Return	-0.081	2.8	-14.5	21.330	-4.0

Table 2: **Summary Statistics for Estimated Risk Measures.** The table reports summary statistics for the asset returns and 1% risk measures of the 1226 financial firms for weekly data from 1986Q1-2010Q4.  $X^i$  denotes the weekly market-valued asset returns for the firms. The individual firm risk measures  $VaR^i$  and the system risk measure  $VaR^{system}$  are obtained by running 1-% quantile regressions of returns on the one-week lag of the state variables and by computing the predicted value of the regression.  $\Delta CoVaR^i$  is the difference between 1% -  $CoVaR^i$  and the 50% -  $CoVaR^i$ , where  $q - CoVaR^i$  is the predicted value from a  $q - \%$  quantile regression of the financial system asset returns on the institution asset returns and on the lagged state variables. The stress  $\Delta CoVaR^i$  is the  $\Delta CoVaR^i$  computed with the worst 1% of state variable realizations and the worst 1% financial system returns replaced in the quantile regression. All quantities are expressed in units of weekly percent returns.

Variable	Mean	Std. Dev.	Obs.
(1) $X_t^i$	0.34	7.3	791231
(2) $VaR_t^i$	-12.17	8.00	790868
(3) $\Delta CoVaR_t^i$	-1.16	1.30	790868
(4) Stress- $\Delta CoVaR_t^i$	-3.22	3.86	790868
(5) $VaR_t^{system}$	-6.24	3.53	1226

Table 3: **Average  $t$ -Statistics of State Variable Exposures.** The table reports average  $t$ -statistics from 1%-quantile regressions. For the risk measures  $VaR^i$  and the system risk measure  $VaR^{system}$ , 1-% quantile regressions are run on the state variables. For  $CoVaR^i$ , 1-% quantile regressions of the financial system returns are run on the state variables and the firms' asset returns.

Variable	$VaR^{system}$	$VaR^i$	$CoVaR^i$
VIX (lag)	(-16.18)	(-13.14)	(-20.76)
Repo spread (lag)	(-6.77)	(-0.84)	(-7.18)
Three month yield change (lag)	(-1.85)	(-0.69)	(-2.38)
Term spread change (lag)	(2.64)	(-0.59)	(3.10)
Credit spread change (lag)	(-6.70)	(-0.66)	(-6.96)
Market return (lag)	(16.50)	(4.29)	(18.64)
Housing (lag)	(5.02)	(2.72)	(2.39)
Portfolio asset return $X^i$			(8.10)
Pseudo- $R^2$	46.62%	24.02%	48.89%

Table 4: **Quarterly Summary Statistics.** The table reports summary statistics for the quarterly variables in the forward  $\Delta CoVaR$  regressions. The data are from 1986Q1-2010Q4, covering 1226 financial institutions.  $VaR$  is expressed in unites of quarterly basis points and  $\Delta CoVaR$  is expressed in quarterly basis points of systemic risk contribution. The rest of the variables are described in 4.1.

Variable	Mean	Std. Dev.	Obs.
(1) 1% $\Delta CoVaR_t^i$	-1494.9	1619.5	62689
(2) 5% $\Delta CoVaR_t^i$	-1026.7	970.9	62689
(3) 1% $VaR_t^i$	-15873.3	9414.9	62689
(4) 5% $VaR_t^i$	-9027.1	5071.1	62689
(5) Leverage	8.84	6.68	62624
(6) Maturity Mismatch	0.52	0.97	62689
(7) Market to Book	2.03	5.86	62633
(8) Log Equity	18.83	1.94	62637
(9) Volatility	3.09	2.79	59465
(10) Market $\beta$	0.74	0.98	62689

Table 5:  $\Delta CoVaR^i$  Forecasts for All Publicly Traded Financial Insititutions. This table reports the coefficients from forecasting regressions of the 5%  $\Delta CoVaR^i$  on the quarterly, one-year, and two-year lag of firm characteristics in Panel A and for the 1%  $\Delta CoVaR^i$  in Panel B. Each regression has a cross section of firms. The methodology for computing the risk measures  $VaR^i$  and  $\Delta CoVaR^i$  is given in the captions of Tables 2 and 3. FE denotes fixed effect dummies. All regressions include time effects. Newey-West standard errors allowing for up to five periods of autocorrelation are displayed in parentheses. One, two, and three stars denote significance at the 10, 5, and 1 percent levels, respectively.

Lagged Variables	Panel A: 5% $\Delta CoVaR^i$			Panel B: 1% $\Delta CoVaR^i$		
	(1) 2 Years	(2) 1 Year	(3) 1 Quarter	(4) 2 Years	(5) 1 Year	(6) 1 Quarter
VaR	-0.019*** (0.001)	-0.012*** (0.001)	-0.007*** (0.002)	-0.019*** (0.001)	-0.014*** (0.001)	-0.010*** (0.001)
Log Book Equity	-285.129*** (5.553)	-279.689*** (5.195)	-277.861*** (5.105)	-339.067*** (10.067)	-334.523*** (9.472)	-333.238*** (9.372)
Market $\beta$	-72.821*** (6.981)	-92.945*** (6.372)	-99.459*** (6.220)	-115.287*** (13.104)	-142.982*** (12.117)	-152.082*** (11.837)
Maturity Mismatch	-13.819* (7.398)	-12.059* (7.166)	-11.650* (6.866)	-46.644*** (14.057)	-43.797*** (13.901)	-43.048*** (13.386)
Market To Book	-18.480*** (3.490)	-17.571*** (3.400)	-16.672*** (2.988)	-22.827*** (5.543)	-21.457*** (5.415)	-20.084*** (4.856)
Volatility	-11.787*** (2.701)	-7.516*** (2.686)	-6.760* (3.550)	-5.779 (5.523)	-1.747 (5.512)	-4.016 (7.386)
Leverage	-6.765*** (1.965)	-7.220*** (1.866)	-7.229*** (1.811)	-6.924** (3.472)	-6.836** (3.256)	-7.187** (3.105)
Foreign FE	347.227*** (57.440)	328.697*** (54.077)	324.499*** (52.155)	334.195*** (87.040)	316.836*** (81.886)	307.916*** (78.677)
Insurance FE	91.849*** (25.307)	91.790*** (24.119)	95.932*** (23.583)	25.479 (45.142)	36.303 (42.953)	48.558 (41.763)
Real Estate FE	-68.005** (32.997)	-59.171* (31.808)	-56.493* (31.031)	-318.607*** (64.465)	-293.028*** (62.064)	-285.247*** (60.481)
Broker Dealer FE	-343.445*** (36.363)	-322.611*** (35.215)	-304.797*** (34.418)	-438.256*** (64.008)	-416.258*** (61.757)	-398.936*** (59.679)
Others FE	-52.677 (35.654)	-35.127 (33.646)	-20.558 (32.380)	10.638 (66.097)	32.362 (63.482)	48.961 (61.396)
Constant	4,419.804*** (126.317)	4,577.975*** (112.639)	4,621.168*** (112.157)	4,922.344*** (230.424)	5,217.287*** (205.215)	5,305.055*** (204.945)
Observations	49,351	54,127	57,750	49,351	54,127	57,750
Adjusted $R^2$	43.63%	43.05%	42.59%	25.78%	25.48%	25.01%

Table 6:  $\Delta CoVaR^i$  Forecasts For Bank Holding Companies. This table reports the coefficients from cross sectional predictive regressions of the 5%  $\Delta CoVaR^i$  on the quarterly, one year, and two year lag of liability characteristics in Panel A, and for asset characteristics in Panel B. Each regression has a cross section of bank holding companies. The methodology for computing the risk measures  $VaR^i$  and  $\Delta CoVaR^i$  is given in the captions of Tables 2 and 3. All regressions include time effects. IBC deposits denotes interest-bearing core deposits. NIB deposits denotes Non-Interest-Bearing deposits. LT deposits denote large time-deposits (greater than \$100,000). Newey-West standard errors allowing for up to 5 periods of autocorrelation are displayed in parentheses. One, two, and three stars denote significance at the 10, 5, and 1 percent levels, respectively.

VARIABLES	Panel A: Liability Variables			Panel B: Asset Variables		
	2 Year	1 Year	1 Quarter	2 Year	1 Year	1 Quarter
VaR	-0.021*** (0.002)	-0.015*** (0.002)	-0.011*** (0.002)	-0.021*** (0.002)	-0.015*** (0.002)	-0.011*** (0.002)
Log Book Equity	-289.647*** (9.848)	-285.051*** (8.955)	-287.521*** (8.715)	-290.695*** (9.586)	-285.526*** (8.693)	-288.208*** (8.469)
Market $\beta$	-93.437*** (14.906)	-124.607*** (12.526)	-129.171*** (11.686)	-94.249*** (14.989)	-126.479*** (12.585)	-133.240*** (11.736)
Maturity Mismatch	223.164*** (68.266)	199.567*** (65.458)	172.168*** (64.372)	173.155** (68.363)	133.971** (65.852)	95.607 (64.581)
Market to Book	-191.263*** (18.321)	-183.703*** (17.627)	-169.412*** (16.371)	-201.290*** (18.722)	-194.260*** (18.017)	-180.220*** (16.918)
Volatility	0.055 (8.170)	-7.431 (6.206)	-15.647** (6.888)	1.134 (8.123)	-5.084 (6.270)	-12.241* (6.941)
Leverage	-20.776*** (3.196)	-19.837*** (3.030)	-15.195*** (2.763)	-18.159*** (3.204)	-16.578*** (3.051)	-10.675*** (2.826)
IBC Deposits	2.581** (1.250)	2.530** (1.196)	2.791** (1.169)			
NIB Deposits	-16.706*** (3.422)	-17.455*** (3.151)	-18.405*** (2.988)			
LT Deposits	7.184*** (1.325)	6.388*** (1.250)	6.149*** (1.193)			
Demand Deposits	3.327 (3.600)	4.934 (3.296)	5.130* (3.118)			
Total Loans				4.375*** (1.160)	5.478*** (1.101)	5.992*** (1.045)
Loan Loss Reserves				-30.040 (30.066)	-72.695*** (26.489)	-92.941*** (25.021)
Intangible Assets				4.404 (8.369)	8.288 (7.792)	17.574** (7.330)
Trading Assets				-6.636* (3.945)	-4.842 (3.645)	-8.109** (3.463)
Constant	4,572.101*** (229.043)	4,814.428*** (200.422)	4,939.731*** (197.411)	4,367.466*** (224.295)	4,605.555*** (194.209)	4,718.841*** (191.312)
Observations	19,558	21,289	22,590	19,558	21,289	22,590
Adjusted $R^2$	51.68%	51.96%	51.38%	51.36%	51.77%	51.29%

Table 7:  $\Delta CoVaR^i$  Forecasts For Bank Holding Companies. This table reports a regression of the realized crisis covariance on the forward  $\Delta CoVaR$  for the universe of Bank Holding Companies. The crisis covariance is the realized covariance for the time span 2007Q1 - 2009Q1, estimated from weekly data. The forward  $\Delta CoVaR$  are the predictive values from a panel regression of  $\Delta CoVaR$  onto lagged balance sheet variables and lagged macro variables, as of 2006Q4. One, two, and three stars denote significance at the 10, 5, and 1 percent levels, respectively.

VARIABLES	(1)	(2)	(3)	(4)
		Crisis Covariance		
2Y Forward $\Delta CoVaR$	-0.709***			
1Y Forward $\Delta CoVaR$		-0.709***		
1Q Forward $\Delta CoVaR$			-0.710***	
Maturity Mismatch				-0.044*
Leverage				0.047*
Market Beta				0.312***
Log Book Equity				0.588***
Observations	832	832	832	827
$R^2$	50.3%	50.3%	50.4%	54.2%

Table 8:  $\Delta CoVaR^i$  Forecasts using GARCH estimation. This table reports the coefficients from forecasting regressions of the two estimation methods of 5%  $\Delta CoVaR^i$  on the quarterly, one-year, and two-year lag of firm characteristics. The methodology for computing the quantile regression  $\Delta CoVaR^i$  is given in the captions of Tables 2 and 3. FE denotes fixed effect dummies. The GARCH  $\Delta CoVaR^i$  is computed by estimating the covariance structure of a bivariate diagonal VECM GARCH model. All regressions include time effects. In this table  $VaR^i$  is estimated using a GARCH(1,1) model. Newey–West standard errors allowing for up to five periods of autocorrelation are displayed in parentheses. One, two, and three stars denote significance at the 10, 5, and 1 percent levels, respectively.

VARIABLES	2 Year		1 Year		1 Quarter	
	Quantile	GARCH	Quantile	GARCH	Quantile	GARCH
VaR (from GARCH)	-0.041*** (0.004)	-0.024*** (0.004)	-0.032*** (0.004)	-0.019*** (0.004)	-0.032*** (0.004)	-0.033*** (0.004)
Log Book Equity	-315.817*** (16.038)	-398.050*** (14.302)	-312.856*** (15.368)	-399.817*** (13.031)	-315.404*** (15.295)	-382.759*** (12.744)
Market $\beta$	-84.291*** (32.259)	-248.159*** (29.281)	-139.601*** (30.276)	-333.821*** (27.072)	-148.788*** (30.312)	-488.841*** (28.259)
Maturity Mismatch	-27.129 (38.386)	-37.249 (43.417)	-45.519 (38.461)	-45.328 (44.664)	-53.534 (38.432)	-19.636 (42.863)
Market to Book	-15.585*** (3.381)	-9.109*** (2.835)	-13.118*** (3.622)	-10.717*** (2.792)	-12.525*** (3.463)	-11.651*** (2.057)
Volatility	-29.611* (17.538)	35.273** (16.681)	-42.131*** (15.675)	59.179*** (18.629)	-73.635*** (19.603)	67.441*** (24.545)
Leverage	-4.783 (4.678)	-5.641* (2.905)	-4.669 (4.798)	-7.172** (2.823)	-3.520 (4.853)	-8.131*** (2.875)
Foreign FE	868.656*** (127.493)	492.486*** (105.187)	831.951*** (122.831)	484.021*** (96.542)	812.464*** (121.606)	485.913*** (92.098)
Insurance FE	195.261*** (59.633)	380.863*** (46.646)	178.982*** (58.922)	342.528*** (44.175)	182.955*** (59.656)	312.164*** (42.442)
Real Estate FE	-280.118*** (95.111)	271.617** (108.668)	-308.240*** (93.444)	230.222** (102.622)	-333.649*** (89.820)	184.723* (97.630)
Broker-Dealer FE	-581.106*** (86.882)	-447.003*** (65.064)	-525.245*** (88.249)	-347.995*** (63.062)	-507.003*** (88.640)	-290.299*** (61.052)
Others FE	240.091 (164.333)	409.705 (327.338)	211.535 (154.073)	408.266 (260.577)	237.068* (141.656)	366.190* (187.654)
Constant	3,283.473*** (358.088)	3,570.076*** (319.286)	4,311.172*** (328.797)	5,720.837*** (282.154)	4,402.390*** (327.029)	5,467.105*** (282.839)
Observations	8,873	8,873	9,423	9,423	9,823	9,823
Adjusted $R^2$	49.70%	81.22%	49.10%	81.54%	48.68%	82.16%

Table 9:  $\Delta CoVaR^i$  Forecasts using alternative system returns variable. This table reports the coefficients from forecasting regressions of the two estimation methods of 5%  $\Delta CoVaR^i$  on the quarterly, one-year, and two-year lag of firm characteristics. In the columns labeled  $X^{system}$ ,  $\Delta CoVaR$  is estimated using the regular system returns variable described in Section 3, while in columns labeled  $X^{system-i}$ ,  $\Delta CoVaR$  is estimated using a system return variable that does not include the firm for which  $\Delta CoVaR$  is being estimated. The methodology for computing the quantile regression  $\Delta CoVaR^i$  is given in the captions of Tables 2 and 3. FE denotes fixed effect dummies. All regressions include time effects. Newey–West standard errors allowing for five periods of autocorrelation are displayed in parentheses. One, two, and three stars denote significance at the 10, 5, and 1 percent levels, respectively.

VARIABLES	2 Year		1 Year		1 Quarter	
	$X^{system}$	$X^{system-i}$	$X^{system}$	$X^{system-i}$	$X^{system}$	$X^{system-i}$
5% $VaR$	-0.019*** (0.001)	-0.020*** (0.001)	-0.012*** (0.001)	-0.013*** (0.001)	-0.007*** (0.002)	-0.007*** (0.002)
Log Book Equity	-285.129*** (5.553)	-274.866*** (5.381)	-279.689*** (5.195)	-269.861*** (5.030)	-277.861*** (5.105)	-268.137*** (4.934)
Market $\beta$	-72.821*** (6.981)	-74.588*** (6.914)	-92.945*** (6.372)	-94.357*** (6.323)	-99.459*** (6.220)	-100.788*** (6.167)
Maturity Mismatch	-13.819* (7.398)	-16.075** (7.328)	-12.059* (7.166)	-14.292** (7.106)	-11.650* (6.866)	-13.744** (6.813)
Market to Book	-18.480*** (3.490)	-18.113*** (3.416)	-17.571*** (3.400)	-17.254*** (3.335)	-16.672*** (2.988)	-16.363*** (2.932)
Volatility	-11.787*** (2.701)	-9.714*** (2.648)	-7.516*** (2.686)	-6.024** (2.445)	-6.760* (3.550)	-5.303 (3.289)
Leverage	-6.765*** (1.965)	-5.475*** (1.907)	-7.220*** (1.866)	-5.980*** (1.819)	-7.229*** (1.811)	-6.107*** (1.773)
Foreign FE	347.227*** (57.440)	344.731*** (56.363)	328.697*** (54.077)	326.694*** (53.145)	324.499*** (52.155)	323.158*** (51.176)
Insurance FE	91.849*** (25.307)	84.374*** (25.016)	91.790*** (24.119)	84.463*** (23.871)	95.932*** (23.583)	88.143*** (23.358)
Real Estate FE	-68.005** (32.997)	-62.722* (32.801)	-59.171* (31.808)	-54.136* (31.653)	-56.493* (31.031)	-52.235* (30.893)
Broker-Dealer FE	-343.445*** (36.363)	-335.785*** (35.759)	-322.611*** (35.215)	-315.165*** (34.669)	-304.797*** (34.418)	-297.740*** (33.858)
Others FE	-52.677 (35.654)	-47.266 (35.251)	-35.127 (33.646)	-29.750 (33.275)	-20.558 (32.380)	-15.980 (32.038)
Constant	4,419.804*** (126.317)	4,196.287*** (122.764)	4,577.975*** (112.639)	4,374.726*** (109.305)	4,621.168*** (112.157)	4,421.007*** (108.720)
Observations	49,351	49,351	54,127	54,127	57,750	57,750
Adjusted $R^2$	43.63%	42.75%	43.05%	43.05%	42.59%	41.72%