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LIFE INSURANCE OF THE ELDERLY: ADEQUACY AND DETERMINANTS

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ABSTRACT

Despite a general reduction in poverty among the aged, roughly one third of elderly nonmarried women are officially poor. Many of these women are widows. The fact that poverty rates are significantly larger for widows than for married women suggests that many households may fail to buy sufficient life insurance. This paper considers the adequacy and determinants of life insurance among the elderly. Its principal conclusions are:

(1) Combined private and public life insurance is inadequate for a significant minority of elderly households;

(2) Of those elderly households in which the husband's future income represents a significant fraction of total household resources, roughly half are inadequately insured;

(3) Households do not significantly offset Social Security's provision of survivor insurance by reducing their private purchase of life insurance; and

(4) The actual determinants of the purchase of life insurance appear to differ greatly from those predicted by economic theory.

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I. Introduction

In the past several decades there has been a significant improvement in the general economic position of the elderly. In contrast to 1960, when over a third of elderly households had incomes below the poverty line, the current figure is roughly 15 percent. Despite the general reduction in poverty among the aged, the poverty rate amoung elderly nonmarried women, including widows, remains high. Today roughly one third of elderly nonmarried women are officially poor.

The fact that 31 percent of elderly nonmarried women, but only 8 percent of elderly married women are in poverty suggests a significant economic risk from the dissolution of marriage through divorce or death. While divorce insurance is unavailable, life insurance is readily available. However, many households may fail to buy enough life insurance. Presumably, concern about insufficient life insurance underlies the provision of survivor and death benefits by the Social Security system. This paper examines the adequacy of the life insurance protection provided by the combination of private and public insurance. It also investigates the determinants of private life insurance purchases. There are three central questions addressed in the study:

(1) How large are private life insurance holdings relative to the amounts needed to maintain prior living standards of surviving spouses?

(2) Do Social Security survivor benefits significantly increase the amount of life insurance protection?

(3) Is the pattern of private insurance purchases in general accord with the predictions of economic theory, particularly the proposition that Social Security survivor insurance should substitute (under assumptions specified below) dollar for dollar for private life insurance?

The answers to these questions are clearly very important for understanding the cause of poverty among widows, the extent of the government's intervention in the life insurance market, and the effectiveness of that intervention in increasing the sum of private plus public life insurance.

The data set chosen for this study, the Retirement History Survey (RHS), is attractive because it focuses on the elderly and because it permits the observation of household economic status before and after the death of a spouse. These data are, however, deficient in several respects for the study of life insurance. First they include the face value, but not the cash value of life insurance policies. Second, while there are a variety of questions about retirement plans and expected future income, many of these questions were not answered by a considerable number of respondents. Understanding the size of expected future income, particularily labor earnings, is obviously of great importance for assessing the adequacy of life insurance holdings. Given these data problems our results should be viewed cautiously. However, in some important cases, correcting biases arising from missing data would likely strengthen our conclusions.

The principal inferences we draw from this study are:

(1) Combined private and public life insurance is inadequate for a significant minority of elderly households.

(2) Almost one half of households at risk (those for whom a significant portion of household resources take the form of earnings and benefits that cease with the death of the husband or wife) are inadequately insured.

(3) Empirical estimation of the demand for life insurance produces many results that are greatly at odds with theorectical predictions.

(4) Households do not appear to significantly offset Social Security's provision of survivor insurance by reducing their private purchase of life insurance.

There are four remaining sections in the paper. The next presents general descriptive information about the extent and adequacy of life insurance. Adequacy of life insurance is assessed in terms of the ability of surviving spouses to maintain their previous living standards. Comparisons of previous with current living standards are made for households in which a spouse dies between 1969 and 1971. In addition, for all married households in 1969 similar comparisons are made for hypothetical surviving spouses. After presenting these comparisons of pre- and post-death living standards of actual and potential surviving spouses we discuss six potential biases in these comparisons. In our view, these six biases, on net, lead us to understate the inadequacy of life insurance holdings.

The third section examines optimal choice of life insurance holdings within a simple two period model. The substitutability of private and public insurance is considered as well as the proper valuation of future income streams under the assumption of incomplete life insurance and annuity markets.

The model illustrates the interdependent choices of life insurance of husbands and wives. As stressed in this section, the insurance demand of one spouse depends on whether the other spouse has positive or zero life insurance. The model assumes that private annuities are unavailable. As pointed out by Yaari (1965), purchasing private annuities is effectively equivalent to having negative holdings of life insurance. Negative holdings of life insurance are ruled out because annuities appear available on the private market only at extremely unfair rates (Friedman and Warshawsky 1985). None of the households in our RHS sample reported holdings of private annuities.

The theoretical model of section III motivates the econometric specification of a two indicator switching regressions model, which is presented in section IV. Section V discusses the empirical findings and uses the results to evaluate Social Security's impact on the purchase of private insurance. The final section summarizes the paper's findings and suggests ideas for additional research.

II. The Adequacy of Life Insurance

A. Conceptual Framework

This section considers the adequacy of life insurance by comparing living standards before and after either the actual or hypothetical death of a spouse. The definition of living standard is obviously arbitrary. By living standard we mean the sustained level of consumption of goods and services that can be afforded, based on the household's current assets and current and future income. Calculating affordable consumption annuities both prior to and after the death of a spouse requires information on the net worth, future labor earnings, private pensions, and Social Security benefits available to the couple when both spouses are alive as well as to actual or hypothetical surviving spouses. Life insurance obviously raises the resources available to surviving spouses, and its purchase can protect surviving spouses from a reduction in their affordable standards of living.

The size of consumption streams that can be financed from a given amount of resources depends on actuarial factors, such as the interest rate, the extent to which annuities are implicitly, if not explicitly available, and household economies to scale in joint consumption. "Economies to scale" refers to the "two can live cheaper than one" proposition. Obviously many goods, such as heating, lighting, and other housing services, are jointly consumed by married couples. Other goods, such as food and clothing, do not have this public goods feature.

To see the importance of the economies of scale issue, consider at one extreme that all goods consumed by couples are local public goods like heating. In this case, for surviving spouses to maintain prior affordable living standards, they need to be able to purchase the same commodities when single that they and their spouse would have purchased when married. To do so obviously requires the same economic resources. By full insurance of the survival-contingent income stream of each spouse, the living standard of the surviving spouse will be fully insured.

While fully insuring the survival contingent income stream is required to maintain living standards of survivors when all household consumption is joint, this is not true in the absence of significant economies to scale. Consider the case of no joint consumption by married couples. In this setting the surviving spouse will suffer a drop in affordable living standard only if the uninsured decedent's survival-contingent income stream would have financed more than his or her own stream of consumption; i.e., fully insuring the surviving spouse requires buying insurance equal to the difference between the value of the decedent's future income stream and the value of his or her future consumption. If this difference is negative, i.e., the value of the future income stream the decedent would have earned is less than the potential future value of his or her consumption, the surviving spouse's living standard will be greater than it would have been had his or her spouse not died.

Since we do not know the precise extent of economies to scale, we present our adequacy of life insurance calculations assuming no economies to scale and then discuss the likely bias arising from ignoring scale economies. "Living standard" is measured in the calculations as the level annuity that could be financed with available resources. Prior to the actual or hypothetical death of a spouse, we calculate the combined resources of the couple and compute the level annuity, A_m, that could be purchased for each spouse under the assumption that each spouse receives an equal annuity. Next we determine the annuity that could be afforded by the surviving spouse, A_s. The ratio of the second annuity to the first annuity (A_s/A_m) is our measure of the adequacy of insurance. Ratios below .75 are described as "inadequate."

More formally, let PVR_m be the present value of resources of the couple when they are both alive, and let PVR_s be the present value of resources of the surviving spouse. We calculate A_s/A_m, where A_m, the annuity of the surviving spouse when married and A_s, the annuity of the surviving spouse after the partner's death, are determined by:

$$(1m) \qquad PVR_m = (D_h + D_w)A_m$$

$$(1s) \qquad PVR_s = D_s A_s, \; s = h,w$$

In (1) D_h and D_w are discount factors for the husband and wife; D_h equals the present value of $1 received annually by the husband until his death; D_w is correspondingly defined. D_s represents the discount factor for the surviving spouse, where s = h or w.

As discussed by Yaari (1965), Kotlikoff and Spivak (1981), and Bernheim (1986) and as indicated in the next section, in the presence of life span uncertainty, the proper valuation of future income streams depends critically on the nature of explicit and implicit insurance arrangements. At one extreme one could assume a perfect market in annuities and life insurance in which insurance premia are actuarially fair. In this case PVR_m and PVR_s would correspond to the present expected values of resources of married couples and surviving spouses, where the expectation is taken over survival probabilities. Similarily, D_h, D_w, and D_s would be the expected value discount factors.

Even if there were no public market in annuities, Kotlikoff and Spivak (1981) indicate that risk sharing amoung family members (e.g., parents and children) can closely approximate perfect annuity insurance even when the number of family members is as few as four. While families are not as effective in hedging the loss of future income streams (i.e., providing life insurance) as they are in hedging the duration of future consumption streams (i.e., providing annuity insurance), the combination of life insurance that is close to actuarially fair plus family annuity insurance arrangements may approximate the situation of perfect life and annuity insurance. In this case, using actuarially fair discounting in forming PVR_m, PVR_s, D_h, and D_w would be roughly appropriate.

If public insurance markets are far from perfect such that market insurance is effectively unavailable and if family arrangements do not arise, then simple discounting by only the interest rate is appropriate (assuming no constraints on borrowing and lending). Between the case of perfect insurance and zero insurance are a range of partial insurance environments in which future streams are priced (discounted) using survival probabilities to some extent depending on the availability and pricing of particular insurance policies. The next section examines such cases.

Since assessing the precise degree to which the insurance market is complete is difficult, if not impossible, we examine the A_s/A_m ratios assuming, at one extreme, perfect insurance, and, at the other extreme, no insurance. In our view the assumption of perfect insurance is a closer approximation to the reality experienced by the RHS sample than the assumption of zero insurance. This assessment is based on the fact that 65 percent of our 1969 RHS sample of elderly couples report positive life insurance for both spouses and for another 22 percent of couples the life insurance of at least one spouse is positive. At least some of this life insurance surely represents term insurance, and, as demonstrated by Yaari (1965), buying additional life insurance is equivalent to selling an annuity. Hence, for at least those couples in which both spouses have term life insurance, one can argue that annuities were available at the margin; had these couples purchased less term insurance they would have had more annuities. In addition, the ability of parents implicitly to insure longevity risk with their children leads us to view the perfect insurance benchmark as more appropriate than the zero insurance benchmark.

B. Variable Definitions, Data, and Sample Selection

The terms PVR_m and PVR_s are defined as:

(2m) $$PVR_m = NW_m + PVE_h + PVE_w + PVP_h + PVP_w + PVB_h + PVB_w + PVS_h + PVS_w$$

(2s) $$PVR_s = NW_s + PVE_s + PVP_s + PVB_s + PVS_s + F_d,$$

where NW stands for net worth, PVE stands for the present expected value of labor earnings, PVP stands for the present expected value of non-Social Security pension benefits, PVB stands for the present expected value of Social Security retirement benefits, PVS stands for the present expected value of Social Security survivor benefits, and F represents the decedents' insurance. The subscripts m,h,w, s and d stand for the married couple, the husband, the wife, the surviving spouse, and the decedent spouse. When s = h, d = w, and when s = w, d = h.

As indicated in (2s), in calculating hypothetical annuities for surviving spouses in 1969, PVR_s includes the life insurance of the hypothetical decedent spouse, F_d. In the comparison of the 1971 value of A_s with the 1969 value of A_m, reported NW_s presumably includes unspent proceeds from the decedent spouses' life insurance; hence F_d is not included in forming PVR_s. This formulation of (2) treats reported life insurance as if it were only term insurance. Below we indicate how the distribution of the A_s/A_m ratios would be affected by making the opposite assumption, namely that the unreported cash value of life insurance equals the reported face value.

Net worth equals the sum of the reported values of assets less the reported values of liabilities. Observations were deleted if the market value of real estate was not reported, if the value of mortgages were not reported, or if more than two types of financial assets had missing values.

1. Calculating the Present Value of Earnings and Non-Social Security Pensions

The present value of earnings was calculated by assuming current real earnings continue to the reported expected age of retirement. Respondents indicating they would never retire were assigned an expected retirement age, calculated in the following manner. Using the RHS Social Security earnings histories, we determined the actual retirement ages for those nongovernment workers who stated in 1969 that they would never retire, but did in fact stop working prior to 1975. With these data, we were able to calculate age-specific probabilities of retiring at each age between ages 58 and 67. We assumed that those who did not retire by age 67 retired, on average, at age 70. Using this information, we calculated the expected age of retirement for those stating they never intended to retire as a function of age in 1969.

Similar calculations were made for respondents with positive 1969 labor earnings who indicated that they were partially retired, but did not state an expected age of complete retirement. Also included in this group were 1969 respondents who reported positive labor earnings, but stated that they were retired. Since spouses were not asked in 1969 when they expected to retire the same technique was used to estimate the expected retirement ages for spouses with positive labor earnings in 1969. In this case we calculated age-specific retirement probabilities between the ages 51 and 67 and again assumed an average retirement age of 70 for spouses not retiring before age 67.

For employed respondents and spouses, current earnings equals 1968 Social

Security reported earnings (obtained from the RHS Social Security earnings records) valued in 1969 dollars provided these earnings were below the 1968 earnings ceiling, but above the 1968 earnings reported by the respondent and spouse. If 1968 Social Security reported earnings were above the earnings ceiling or below 1968 self-reported earnings, 1968 self-reported earnings valued in 1969 dollars are used.

The stream of non-social security pension benefits equals currently reported pension benefits prior to the expected retirement age and the larger of current pension benefits and reported expected retirement benefits after the expected retirement age. Since there is no information on whether these pensions will provide joint survivor benefits, we assume that these benefits accrue solely to the husbands. It is our understanding that prior to the passage of ERISA, joint survivor annuities were relatively rare. Since pension benefits of the wife are not reported in the 1969 survey, we include only those of the husband in the analysis. This omission biases downwards our calculation of A_w/A_m and biases upwards our calculation of A_h/A_m.

2. Calculating the Present Value of Social Security Benefits

The stream of future Social Security benefits was calculated by first determining the age 62 Primary Insurance Amount (PIA) for each respondent and spouse. Earnings prior to 1969 used in this PIA calculation were obtained from the social security earnings records. Earnings between 1962 and the year in which the respondent or spouse reaches age 62 are projected as just described. The wife's social security benefit is the larger of her own benefit and the dependent benefit, once the wife is eligible for the dependent benefit (ie., she is older than 62 and her husband is receiving benefits or is dead). Prior to eligibility for the dependent benefit the wife may be eligible to collect her own benefit. For respondents and spouses who either indicated they were already retired, expected to retire prior to age 62, or were earning less than $1680 (the social security exempt amount), the actuarially reduced stream of benefits commences at age 62. For respondents and spouses expecting to retire after age 62 or earning above $1680, actuarially reduced benefits commmence at the indicated expected retirement age assuming that age is less than or equal to 65. Between ages 65 and 72 respondents and spouses who have not yet retired (according to stated or imputed expectations) are given earnings-tested benefits in our calculations. After age 72 non earnings-tested benefits are available for all respondents and spouses.

In the contingency that the husband is dead survivor benefits are available. As described above, these benefits are provided on an actuarially reduced basis and are earnings-tested where appropriate. While our calculations of total benefits follow Social Security law, we define the stream of social security survivor benefits to be the excess of the surviving wife's benefit over what she would receive were her husband not dead.

In projecting future real Social Security benefits we treat households as assuming that current Social Security law will continue, except that benefits will be increased in the future to adjust for inflation.

3. Data Characteristics

After deletion of observations with missing data the 1969 sample of married households numbers 5131. A total of 4295 husbands and 3389 wives reported positive life insurance. Table 1 presents mean values of life insurance holdings as well as components of PVR cross-classified by level of PVR, the present value of family resources. In forming present values all future earnings, Social Security, and pension streams are actuariarily discounted using mortality probabilities and assuming a 4 percent real interest rate. The overall average holdings of life insurance of husbands is $9360, which may be compared with the mean value of PVR, $108,886. Life insurance is small relative to PVR, but is it small relative to the amount of the husband's future earnings and social security and other pension benefit streams that could be insured?

The answer is that the mean value of the husband's insurance is less than one fifth the sum of the mean values of such insurable future income streams. Insurance holdings of wives is even smaller relative to their insurable streams; the ratios of these means is less than 6 percent. These figures would be inconsequential were future income streams only a trivial fraction of PVR. This is not the case. Future income streams of husbands are 44 percent of our sample households' total PVR; and those of wives are 19 percent. The table also indicates that Social Security survivor benefits represent almost one fourth of combined private plus public insurance on the life of the husband. However, even ignoring any private insurance offset, they make only a very small contribution to reducing the gap between husbands' insurable streams and their private life insurance.

Table 1
Mean Values of Components of Family Wealth: 1969
(By 1969 PVR Class)

	Present Value of Resources						
	<10K	10-25K	25-50K	50-100K	100-250K	250K+	Total
No. Obs	55	179	714	2040	1922	221	5131
Husband's							
Earnings	307	1099	5316	16822	33312	42431	21772
Percent of PVR	5%	6%	13%	22%	24%	9%	20%
Benefits	2059	6959	13138	17304	20025	18971	17269
Percent of PVR	36%	38%	32%	23%	14%	4%	16%
Pension	0	759	2561	5670	13663	29357	9020
Percent of PVR	0%	4%	6%	8%	10%	6%	8%
Insurance	1247	2057	2582	5042	12057	55914	9360
Percent of PVR	22%	11%	6%	7%	9%	12%	9%
Wife's							
Earnings	67	973	2044	5981	12271	11864	7804
Percent of PVR	1%	5%	5%	8%	9%	3%	7%
Benefits	922	3348	6354	9506	12154	11389	9824
Percent of PVR	16%	18%	16%	13%	9%	2%	9%
Surv Bens	571	1531	2818	3551	3505	4133	3348
Percent of PVR	10%	8%	7%	5%	3%	1%	3%
Insurance	251	487	687	1018	1419	1260	1103
Percent of PVR	4%	3%	2%	1%	1%	0%	1%
Couple's							
Net Worth	1825	3803	8484	16329	44975	352834	39849
Percent of PVR	32%	21%	21%	22%	32%	75%	37%
PVR	5751	18472	40715	75163	139905	470978	108886

The table suggests insurance is much more adequate for those with less than $25,000 in PVR. For couples with PVR above $250,000 the husband's insurance is also larger relative to PVR; for this group the husband's future income streams are less than one fifth of PVR. The concern about inadequate insurance is, therefore, much more of an issue for middle class households with PVR between $25,000 and $250,000.

C. Annuity Ratios

Table 2 compares the annuity that actual surviving wives could purchase in 1971 with the corresponding annuity they could have purchased in 1969 when their husbands were alive (assuming, as stated, the purchase of an annuity of equal amount for their husbands). In this table as well as Tables 3, 4, 5, 6, 7 the assumption of perfect insurance is maintained; hence, the annuity ratios are based on discounting by both mortality and interest rates. Over 1/3 of surviving spouses are unable to afford an annuity as large as 75 percent of what was affordable while married. For 15 percent of the sample the affordable annuity is less than one half of what could have been purchased (assuming perfect insurance) while married. At the other extreme, a sizeable fraction of widows appear economically better off after the death of their husbands. For over one quarter of the sample of surviving widows the 1971 affordable annuity is over twenty-five percent larger than the 1969 affordable annuity.

Table 3 presents similar calculations for men who were widowed between 1969 and 1971. Although the sample is small, it appears that a smaller percentage of widowers than widows experienced a large drop in consumable resources from the death of their spouse. Only 13 percent of the widowers have annuity ratios less

Table 2

Widows' Annuity Ratios: 1971
(By 1969 PVR Class)

	Present Value of Resources						
	<10K	10-25K	25-50K	50-100K	100-250K	250K+	Total
No. Obs	5	5	19	42	16	2	89
	Fraction of Potential 1969 Annuity						
<.1							
Number	0	0	0	0	1	0	1
Percent	0%	0%	0%	0%	6%	0%	1%
.1-.25							
Number	0	0	0	1	0	1	2
Percent	0%	0%	0%	2%	0%	50%	2%
.25-.5							
Number	1	0	1	4	5	0	11
Percent	20%	0%	5%	10%	31%	0%	12%
.5-.75							
Number	0	1	5	10	2	1	19
Percent	0%	20%	26%	24%	13%	50%	21%
.75-1.0							
Number	1	1	5	12	3	0	22
Percent	20%	20%	26%	29%	19%	0%	25%
1.0-1.25							
Number	0	1	2	7	1	0	11
Percent	0%	20%	11%	17%	6%	0%	12%
1.25-1.5							
Number	0	0	3	3	3	0	9
Percent	0%	0%	16%	7%	19%	0%	10%
1.5-1.75							
Number	0	0	0	3	0	0	3
Percent	0%	0%	0%	7%	0%	0%	3%
1.75-2							
Number	1	0	0	2	1	0	4
Percent	20%	0%	0%	5%	6%	0%	4%
>2							
Number	2	2	3	0	0	0	7
Percent	40%	40%	16%	0%	0%	0%	8%

Table 3

Widowers' Annuity Ratios: 1971
(By 1969 PVR Class)

	Present Value of Resources						
	<10K	10-25K	25-50K	50-100K	100-250K	250K+	Total
No. Obs	0	6	7	21	13	0	47
	Fraction of Potential 1969 Annuity						
<.1							
Number	0	0	0	0	0	0	0
Percent	0%	0%	0%	0%	0%	0%	0%
.1-.25							
Number	0	0	0	0	1	0	1
Percent	0%	0%	0%	0%	8%	0%	2%
.25-.5							
Number	0	0	0	0	1	0	1
Percent	0%	0%	0%	0%	8%	0%	2%
.5-.75							
Number	0	0	0	3	1	0	4
Percent	0%	0%	0%	14%	8%	0%	9%
.75-1.0							
Number	0	1	2	8	2	0	13
Percent	0%	17%	29%	38%	15%	0%	28%
1.0-1.25							
Number	0	2	2	6	4	0	14
Percent	0%	33%	29%	29%	31%	0%	30%
1.25-1.5							
Number	0	1	0	3	4	0	8
Percent	0%	17%	0%	14%	31%	0%	17%
1.5-1.75							
Number	0	1	1	0	0	0	2
Percent	0%	17%	14%	0%	0%	0%	4%
1.75-2							
Number	0	0	0	0	0	0	0
Percent	0%	0%	0%	0%	0%	0%	0%
>2							
Number	0	1	2	1	0	0	4
Percent	0%	17%	29%	5%	0%	0%	9%

than .75, and 30 percent have ratios greater than 1.25.

Table 4 considers the entire 1969 sample of households and compares the annuity that could be purchased at the time of the RHS interview with the annuity that the surviving wife could have purchased had her husband expired immediately after the interview. This distribution of hypothetical surviving wives by their annuity ratios is quite similar to that of Table 2. About 25 percent of the sample has an annuity ratio below .75; almost 50 percent have a ratio less than 1. A sizeable fraction, 33 percent, has an annuity ratio above 1.25.

Turning to hypothetical surviving husbands (Table 5) we find a dramatically different situation. Only 2 percent of hypothetical widowers have annuity ratios below .75; 95 percent have ratios above 1, and 73 percent have ratios above 1.5. Clearly there is little reason for general concern about inadequate life insurance of wives.

Another way of examining the adequacy of insurance coverage is to limit investigation to those couples where significant insurance would be required to keep a surviving spouse from suffering a large drop in consumable resources. This would exclude couples with most of their wealth held in current net worth, since the death of a spouse in such cases would have little effect on total family resources (excluding insurance) available to the survivor. We therefore repeat, in Tables 6 and 7, the calculations of Tables 4 and 5 for the subsamples of husbands and wives who are "at risk", which we define to be those for whom the other spouse's survival-contingent resources (labor earnings, pension benefits, and Social Security benefits) constitute over half of the family's total resources.

Table 4
Wives' Annuity Ratios if Husbands Die: 1969
(By 1969 PVR Class)

Resources:	<10K	10-25K	25-50K	50-100K	100-250K	250K+	Total
	Present Value of Resources						
No. Obs	55	179	714	2040	1922	221	5131
	Fraction of Potential 1969 Annuity						
<.1							
Number #	1	0	5	1	0	0	7
Percent	2%	0%	1%	0%	0%	0%	0%
.1-.25							
Number	0	4	9	10	3	1	27
Percent	0%	2%	1%	0%	0%	0%	1%
.25-.5							
Number	3	19	75	155	83	1	336
Percent	5%	11%	11%	8%	4%	0%	7%
.5-.75							
Number	13	42	156	391	280	4	886
Percent	24%	23%	22%	19%	15%	2%	17%
.75-1.0							
Number	7	35	183	514	418	9	1166
Percent	13%	20%	26%	25%	22%	4%	23%
1.0-1.25							
Number	3	8	131	458	478	16	1094
Percent	5%	4%	18%	22%	25%	7%	21%
1.25-1.5							
Number	6	18	56	306	329	29	744
Percent	11%	10%	8%	15%	17%	13%	15%
1.5-1.75							
Number	4	15	37	114	213	56	439
Percent	7%	8%	5%	6%	11%	25%	9%
1.75-2							
Number	8	11	23	57	83	70	252
Percent	15%	6%	3%	3%	4%	32%	5%
>2							
Number	10	27	39	34	35	35	180
Percent	18%	15%	5%	2%	2%	16%	4%

Table 5
Husbands' Annuity Ratios if Wives Die: 1969
(By 1969 PVR Class)

	Present Value of Resources						
	<10K	10-25K	25-50K	50-100K	100-250K	250K+	Total
No. Obs	55	179	714	2040	1922	221	5131
Fraction of Potential 1969 Annuity							
<.1							
Number	1	2	3	0	0	0	6
Percent	2%	1%	0%	0%	0%	0%	0%
.1-.25							
Number	1	2	0	3	0	0	6
Percent	2%	1%	0%	0%	0%	0%	0%
.25-.5							
Number	2	3	12	8	2	1	28
Percent	4%	2%	2%	0%	0%	0%	1%
.5-.75							
Number	0	6	9	39	8	1	63
Percent	0%	3%	1%	2%	0%	0%	1%
.75-1.0							
Number	0	7	29	76	30	0	142
Percent	0%	4%	4%	4%	2%	0%	3%
1.0-1.25							
Number	4	22	58	133	116	1	334
Percent	7%	12%	8%	7%	6%	0%	7%
1.25-1.5							
Number	11	37	151	313	256	3	771
Percent	20%	21%	21%	15%	13%	1%	15%
1.5-1.75							
Number	16	46	217	596	457	23	1355
Percent	29%	26%	30%	29%	24%	10%	26%
1.75-2							
Number	5	30	158	634	621	63	1511
Percent	9%	17%	22%	31%	32%	29%	29%
>2							
Number	15	24	77	238	432	129	915
Percent	27%	13%	11%	12%	22%	58%	18%

Table 6
Wives' Annuity Ratios if Husbands Die: 1969
Wives at Risk
(By 1969 PVR Class)

Resources:	<10K	10-25K	25-50K	50-100K	100-250K	250K+	Total
	Present Value of Resources						
No. Obs	28	99	446	1201	972	30	2776
Fraction of Potential 1969 Annuity							
<.1							
Number	1	0	5	1	0	0	7
Percent	4%	0%	1%	0%	0%	0%	0%
.1-.25							
Number	0	4	9	10	3	1	27
Percent	0%	4%	2%	1%	0%	3%	1%
.25-.5							
Number	3	19	75	155	83	1	336
Percent	11%	19%	17%	13%	9%	3%	12%
.5-.75							
Number	13	42	156	391	280	4	886
Percent	46%	42%	35%	33%	29%	13%	32%
.75-1.0							
Number	7	25	152	454	376	7	1021
Percent	25%	25%	34%	38%	39%	23%	37%
1.0-1.25							
Number	2	3	35	146	171	8	365
Percent	7%	3%	8%	12%	18%	27%	13%
1.25-1.5							
Number	0	4	7	36	41	5	93
Percent	0%	4%	2%	3%	4%	17%	3%
1.5-1.75							
Number	1	2	5	3	9	1	21
Percent	4%	2%	1%	0%	1%	3%	1%
1.75-2							
Number	0	0	0	2	6	2	10
Percent	0%	0%	0%	0%	1%	7%	0%
>2							
Number	1	0	2	3	3	1	10
Percent	4%	0%	0%	0%	0%	3%	0%

Table 7
Husbands' Annuity Ratios if Wives Die: 1969
Husbands at Risk
(By 1969 PVR Class)

	Present Value of Resources						
	<10K	10-25K	25-50K	50-100K	100-250K	250K+	Total
No. Obs	5	24	62	180	101	2	374
Fraction of Potential 1969 Annuity							
<.1							
Number	1	2	3	0	0	0	6
Percent	20%	8%	5%	0%	0%	0%	2%
.1-.25							
Number	1	2	0	3	0	0	6
Percent	20%	8%	0%	2%	0%	0%	2%
.25-.5							
Number	2	3	12	8	2	1	28
Percent	40%	13%	19%	4%	2%	50%	7%
.5-.75							
Number	0	6	9	39	8	1	63
Percent	0%	25%	15%	22%	4%	50%	17%
.75-1.0							
Number	0	7	27	74	29	0	137
Percent	0%	29%	44%	41%	29%	0%	37%
1.0-1.25							
Number	0	4	10	52	54	0	120
Percent	0%	17%	16%	29%	53%	0%	32%
1.25-1.5							
Number	0	0	1	3	8	0	12
Percent	0%	0%	2%	2%	8%	0%	3%
1.5-1.75							
Number	1	0	0	1	0	0	2
Percent	20%	0%	0%	1%	0%	0%	1%
1.75-2							
Number	0	0	0	0	0	0	0
Percent	0%	0%	0%	0%	0%	0%	0%
>2							
Number	0	0	0	0	0	0	0
Percent	0%	0%	0%	0%	0%	0%	0%

Over half of the wives in our full sample are, by this measure, at risk. Of this group, over 45 percent have an annuity ratio of less than .75. For wives and husbands at risk who are in poorer households the extent of underinsurance is more significant. Consider, for example, wives at risk with household PVR between 25 and 50 thousand dollars in Table 6. Fifty-five percent of this group have an annuity ratio below .75, and 20 percent have a ratio below .5. Table 7 indicates that 28 percent of husbands at risk have hypothetical annuity ratios below .75. However, the number of "at risk" husbands is small. These results reinforce our previous ones that underinsurance, particularly of husbands, is a potentially serious problem indeed.

D. Potential Biases in the Annuity Ratio Calculations

1. Ignoring Cash Value of Life Insurance

To see how excluding the unobserved cash value of life insurance should affect Tables 3 and 4, consider again equations (1) and (2). In the case of hypothetical surviving wives PVR_m is too small by an amount equal to the cash value of the husband's and wife's insurance, while PVR_s, which includes the face value and thus the cash value of the decedent spouse's insurance (F_d), is too small by an amount equal to just the cash value of the surviving spouse's insurance. Since the average value of the husbands' insurance is over 8 times larger than that of wives, one would expect the cash value of the husbands' insurance greatly to exceed that of the wives. As a consequence the omission of cash value implies that the ratio PVR_w/PVR_m is biased upwards and the ratio PVR_h/PVR_m is biased downwards. This implies an upwards bias in the calculation of the hypothetical surviving wives' annuity ratios (A_w/A_m) and a downward bias in the calculated annuity ratios of surviving husbands (A_h/A_m).

To consider the possible extent of this bias we recalculated Table 3 under the assumption that the cash value of husbands' and wives' insurance equals the face value; i.e., there is no term insurance. This assumption increases from 25 percent to 27 percent the fraction of hypothetical surviving wives with annuity ratios below .75. The bias with respect to the values of A_h/A_m in Table 5 is in the opposite direction; here making the extreme assumption of no term insurance increases the annuity ratio for hypothetical widowers. Again the potential bias is small; the fraction of hypothetical widowers with annuity ratios above 1.5 rises from 73 percent to 74 percent.

2. Economies of Scale

Ignoring economies of scale in household consumption biases upwards both widows' and widowers' annuity ratios, if the annuity is viewed as the level stream of consumption that can be financed. Suppose, for example, that household consumption was a pure public good. In this case the consumption stream that could be financed with A_m would equal two times A_m provided both spouses remain alive. For a widow with an annuity ratio of .75, the death of her spouse, according to this reasoning, means a 62.5 percent ((2-.75)/2) decline in consumption and an adjusted annuity ratio of .375 (.75/2). Hence, Tables 2 through 7 may significantly understate the potential welfare decline experienced by surviving spouses.

3. Bequests to Children and End of Life Expenses

The hypothetical annuity ratios also ignore possible bequests to children, end of life uninsured medical and funeral expenses, and, for those few observations with considerable wealth, estate taxes. Inclusion of these factors would reduce the hypothetical annuity ratios below the reported values. This point is supported by the finding that actual annuity ratios of surviving spouses in Tables 2 and 3 are smaller than those of hypothetical surviving spouses in Tables 4 and 5; 36 percent of actual widows, but only 25 percent of hypothetical widows have ratios below .75, and 14 percent of actual widowers, but only 2 percent of hypothetical widowers have ratios below .75.

4. Valuing Future Streams if Insurance is Imperfect

The annuity distributions of Tables 2 through 7 are quite sensitive to the use of perfect insurance valuation. For example, if one discounts future streams only by the interest rate, which would be appropriate absent any explicit or implicit insurance arrangement, the fraction of those at risk hypothetical surviving wives with inadequate insurance protection drops from over 45 percent to only 20 percent. For at risk hypothetical surviving husbands, on the other hand, the fraction with inadequate insurance protection rises from 28 percent to 40 percent. The direction of these changes reflects the fact that Dh/Dw is larger when discounting by only the interest rate than when discounting by both interest and mortality rates; husbands in the sample are older and have larger age specific death rates than their wives. While we present these alternative calculations to permit the reader to draw his or her own conclusion, in our view the calculations based on the assumption of close to perfect insurance arrangements better approximate the insurance environment of the RHS sample.

5. Income Taxes and Choice of Real Interest Rate

In calculating the annuity ratios we did not attempt to estimate taxes that would be paid on earnings and pension streams. Nor did we estimate the marginal effective income tax rate to form an after tax rate of return for discounting future income streams. We believe that these adjustments would, on net, lower the annuity ratios. Present value calculations of this kind are highly sensitive to the choice of discount rate. Realistic inclusion of tax factors would lead to discounting by an after tax real return substantially below 4 percent, which would raise considerably the present values of those income streams that would be lost in the event of a spouse's death. Since the ratio D_H/D_W is a decreasing function of the discount rate, if the husband is older than the wife adjusting for taxes would lower the annuity ratios of surviving wives by more than that of surviving husbands.

6. Earnings Uncertainty

The annuity ratio calculations assume future real earnings are certain with the exception of earnings uncertainty due to death. While we have not closely examined the bias from ignoring other types of earnings uncertainty, we believe that, roughly speaking, uncertain future earnings should be discounted by a risk adjusted discount rate -- a rate that could well be higher than the 4 percent real rate used here. Hence, by ignoring earnings uncertainty we are probably biasing downward the calculated annuity ratios and, on this score, exaggerating somewhat the need for additional life insurance.

III. A Model of Life Insurance Demand

This section develops an estimable model of the demand for life insurance by married couples, based on the assumption of expected utility maximization. The model focuses on life cycle consumption of husbands and wives and ignores possible parental bequest motives and longevity risk sharing between parents and children. It also ignores earning uncertainty. The purposes of estimating a model of life insurance demand are twofold: first, to determine whether the actual purchase of life insurance is in accord with predictions of economic theory, and second, to determine the extent to which households reduce their private purchase of life insurance in response to Social Security's provision of survivor insurance. Assuming the husband's life insurance is positive, the theory predicts that properly valued income streams of the wife, including her labor earnings, public and private pension benefits, and survivor benefits, should substitute at the margin dollar for dollar for the husband's life insurance. Similar arbitrage relationships should hold at the margin between the wife's insurance and properly valued income streams of the husband.

Life insurance transfers income across states of nature and thereby alters the amounts that can be consumed in different states. The optimal choice of life insurance is thus determined simultaneously with the optimal choice of state-contingent consumption. If insurance markets are complete and actuarially fair, life insurance will be purchased (or sold) up to the point that the marginal utility of consumption is equalized across each state of nature. Deviations of insurance pricing from actuarially fair values changes the effective prices of consuming in different states of nature and implies differences in the marginal utility of consumption across different states.

These points are illustrated in the equations below.

Our model has two periods. During the first period, both the husband and wife are alive. During the second period, there are four states of nature, corresponding to only the husband surviving, only the wife surviving, both surviving, and neither surviving. We denote consumption in the first period as C and, in the three states where at least one member of the family survives, as C_h, C_w, and C_{hw}, respectively. In the first period and in state hs in which both spouses are alive, each spouse separately consumes the amount C and C_{hw}, respectively. If there are no economies to scale, the couple spends 2C and $2C_{hw}$ in the first period and in the hw state, respectively. If consumption when married is a pure public good, the couple spends only C and C_{hw} in the two states, although each spouse still consumes C and, if they both survive, C_{hs}. Given the assumption of expected utility maximization, the family's problem is:

$$\text{(3)} \qquad \max\ p_h(1-p_w)U(C,C_h)+(1-p_h)p_wU(C,C_w)+p_hp_wU(C,C_{hw})+(1-p_h)(1-p_w)U(C,0)$$

where p_s is the probability that spouse s survives, and s equals h or w.

If both the husband and wife purchase positive amounts of insurance, then the following equations constrain the choice of consumption in the three states of nature in which at least one family member survives:

$$\text{(4h)} \qquad C_h = E_h + B_h + F_w + (A-\lambda C-\pi_hF_h-\pi_wF_w)(1+r)$$

$$\text{(4w)} \qquad C_w = E_w + B_w + B_w^* + F_h + (A-\lambda C-\pi_hF_h-\pi_wF_w)(1+r)$$

$$\text{(4hw)} \qquad \lambda C_{hw} = E_w + B_w + E_h + B_h + (A-\lambda C-\pi_hF_h-\pi_wF_w)(1+r),$$

where A is the family's tangible wealth in period 1, F_s is the face value of insurance purchased for spouse s, π_s is the corresponding premium paid per dollar of face value in the first period, E_s is spouse s's wages in the second period, B_s is the spouse's Social Security and pension benefits, and B_w^* is the survivor benefits to which the wife is entitled in the event of the husband's death. The terms r and λ are prices; r is the one period interest rate, and λ is the price of second period joint consumption. If $\lambda = 2$, there are no economies of scale in consumption; if $\lambda = 1$, household consumption is a pure public good.

These budget constraints are written under the assumptions that husbands will not be entitled to survivor benefits and that all private insurance is term insurance. The former assumption is consistent with the observations in our sample. The second is more problematic. To the extent that policies are "whole life" and not term policies, they will have a cash or asset value corresponding to the insurance policy's previous savings component, or "inside buildup." A whole life policy may be viewed as a combination of (1) a savings account with liquid assets equal to the policy's cash value, and (2) term insurance with a death benefit equal to the difference between the policy's face value and cash value. If we knew how much cash value each policy had, we would subtract this from the face value F in equations (4) and add it to A. Unfortunately, no such information is available. We defer further discussion of this data problem until the empirical implementation of the model is considered.

In the case of positive insurance purchases for both spouses, we may use expressions (4h), (4w) and (4hw) to eliminate F_h and F_w, obtaining a single expression in consumption levels that may be interpreted as "the" household budget constraint:

$$(5)\qquad \lambda C + \pi_h C_h + \pi_w C_w + \lambda[1/(1+r) - \pi_h - \pi_w]C_{hw}$$
$$= A + (E_h+B_h)[1/(1+r) - \pi_h] + (E_w+B_w)[1/(1+r) - \pi_w] + \pi_h B^*_w,$$

where λ, π_h, π_w and $\lambda[1/(1+r) - \pi_h - \pi_w]$ are the "prices" of the four consumption levels, and the right-hand-side is a weighted sum of the different resource components. We assume that insurance is actuarially fair. This implies that:

$$(6)\qquad \pi_s = (1-p_s)/(1+r), \qquad s = h,w$$

Combination of expressions (5) and (6) yields a simpler and more intuitive version of the budget constraint:

$$(7)\quad \lambda C + [(1-p_w)/(1+r)]C_h + [(1-p_h)/(1+r)]C_w + \lambda[p_h p_w - (1-p_h)(1-p_w)]/(1+r)C_{hw}$$

$$= A + [p_h/(1+r)](E_h+B_h) + [p_w/(1+r)](E_w+B_w) + [(1-p_h)/(1+r)]B^*_w$$

Note how this result differs from what would obtain in the presence of complete and actuarially fair markets for annuities and life insurance, which would permit state-contingent purchases of consumption. In that case, the present value of resources would equal the sum of the expected values, based on the associated survivor probabilities, of each of the components of wealth. The righthand side of (7) differs from the present expected value of resources in that the survivor benefit B^*_w is multiplied only by the husband's death probability rather than the product of his death probability and the wife's survival probability. This is because, without the availability of private annuities, resources that are available when the husband and wife both die are of no value. Put another way, the survivor benefit is of the same value as if it also paid off when both the husband and the wife die.

The implicit prices for second period state-contingent consumption also differ from the case of complete, actuarially fair insurance markets. The prices for C_h and C_w are higher, representing the fact that, in states in which one spouse dies, the family must commit resources regardless of whether the remaining spouse actually lives. This also makes the price of consumption lower in the state where both survive. The intuition is that by providing resources for the state in which both live, the family reduces the amount it must waste in the state when both die; i.e., increased expenditures for C_{hw} also increase consumption in the states with one surviving member, so that less direct expenditures are necessary.

Expected utility maximization by the household of (3) subject to (7) leads to an optimal consumption vector that is a function of the implicit prices and the present value of resources given by the right-hand side of expression (7). We label these q and PVR. We next derive expressions for the demands for life insurance. Subtracting expression (4hw) from (4w) yields:

(8h) $$F_h = (C_w - \lambda C_{hw}) + (E_h + B_h - B_w^*)$$

while from (4h) and (4hw) we obtain:

(8w) $$F_w = (C_h - \lambda C_{hw}) + (E_w + B_w)$$

Each expression has a clear interpretation, calling for the purchase of insurance for the husband or wife equal to the net loss in resources if that individual dies plus the additional consumption that must be financed. The latter term may well be negative, depending on the value of λ and the tastes of the household.

Substituting the optimal consumption demands into expressions (8h) and (8w) yields demand functions for insurance:

(9h) $F_h = H(q,PVR) + (E_h+B_h-B_w^*)$

(9w) $F_w = W(q,PVR) + (E_w+B_w)$

where H( ) and W( ) are the consumption demands for C_h and C_w in excess of joint consumption expenditures when both survive (i.e., $C_w - \lambda C_{hw}$ and $C_h - \lambda C_{hw}$, respectively).

Equations (9h) and (9w) are appropriate when both F_h and F_w are positive, since by assumption neither F_h nor F_w can be negative. We next consider regimes in which one or both spouses are constrained at zero in the purchase of life insurance. In such regimes, the couple faces an optimization problem of reduced dimension, with different implicit prices for consumption and different weights used to calculate the present value of resources. For example, suppose that the value of F_w satisfying (9w) is negative, requiring that it be constrained to zero. Then, in place of expression (8w) we have:

(8w') $0 = (C_h-\lambda Ch_w) + (E_w+B_w)$,

which implies that the family can no longer independently determine C_h and C_{hw}. Substituting this restriction into expression (7), we obtain a new budget constraint that omits C_{hw}:

(7w') $\lambda C + [p_h/(1+r)]C_h + [(1-p_h)/(1+r)]C_w$

$= A + [p_h/(1+r)](E_h+B_h) + [(1-p_h)/(1+r)](E_w+B_w+B_w^*)$

Note that the implicit price of the husband's consumption is now the probability of his own survival, rather than the probability of his wife's death. Likewise, the wife's wages and benefits are no longer weighted by her survival probability, but by her husband's death probability. Since only the husband may buy insurance, his insurance decision determines the allocation of resources between C_h and C_w, and the cost of this insurance determines their relative prices. Since the spouse cannot transfer resources to her husband through insurance, her survival probability does not enter into the budget constraint.

Letting q_h and PVR_h be the price vector and present value of resources given by the righthand side of (7w'), we obtain from (8h) the husband's demand for life insurance when his wife is constrained at zero life insurance:

$$(9h') \qquad F_h = H(q_h, PVR_h) + (E_h + B_h - B_w^*)$$

In an analagous fashion, we may derive prices q_w and the present value of resources PVR_w for the case where the husband's insurance is constrained to equal zero, and obtain the wife's demand function for insurance:

$$(9w') \qquad F_w = W(q_w, PVR_w) + (E_w + B_w)$$

We thus have four possible regimes:

A. Husband and wife unconstrained

$$F_h = H(q, PVR) + (E_h + B_h - B_w^*)$$
$$F_w = W(q, PVR) + (E_w + B_w)$$

B. Husband unconstrained, wife constrained

$$F_h = H(q_h, PVR_h) + (E_h + B_h - B_w^*)$$

$$F_w = 0$$

C. Wife unconstrained, husband constrained

$$F_h = 0$$

$$F_w = W(q_w, PVR_w) + (E_w + B_w)$$

D. Both constrained

$$F_h = 0$$

$$F_w = 0$$

Estimation of the demand for insurance across these regimes involves a switching regressions model with censored dependent variables. We discuss different estimation strategies below.

A problem involved in estimating the insurance demand funtions (9h), (9h'), (9w), and (9w') is that our data does not report term insurance, which corresponds to F_h and F_w in the equations, but only the face value of insurance. An alternative approach that is robust to this particular problem is to estimate expressions (4h) and (4w) directly. Rearranging terms in (4w), and substituting in the demands for consumption, we obtain:

$$\text{(10h)} \qquad F_h + (A - \pi_h F_h - \pi_w F_w)(1+r) = \hat{H}(q, PVR) - (E_w + B_w + B_w^*)$$

where $\hat{H}(\ )$ is the expenditure on first period consumption times (1+r) plus the wife's second period consumption when widowed (i.e., $\lambda C(1+r) + C_w$). If we ignore the insurance premia or assume that assets are measured net of insured premia, and assume interest rates to be small, then we have in (10h) an expression for the sum of the husband's insurance plus family assets, which does not require the separation of cash value from term value of insurance. Equation (4h) provides a corresponding expression (10w) for F_w.

(10w) $$F_w + (A-\pi_h F_h-\pi_w F_w)(1+r) = \hat{W}(q,PVR) - (E_h+B_h)$$

As with the previous insurance demand equations, to estimate this model consistently one must allow for different regimes in which the husband's or wife's insurance demand may be constrained to zero. When the wife's insurance is zero, but the husband's is positive, $\hat{H}(q,PVR)$ is replaced by $\hat{H}(q_h,PVR_h)$, and when the husband's insurance is zero, but the wife's is positive, the function $\hat{W}(q,PVR)$ is replaced by $\hat{W}(q_w,PVB_w)$.

IV. Econometric Specification

The model presented in the previous section can be specified as a two-indicator switching regression model. Let I_h and I_w be zero-one indicators for zero versus positive values of the husband's and wife's insurance, respectively, and express

(11) $$I_h = -c_h + \mu_h$$

$$I_w = -c_w + \mu_w,$$

where μ_h and μ_w are mean zero errors, and the critical values c_h and c_w are linear combinations of observable economic and demographic characteristic vectors X_h and X_w:

(12) $$c_h = -B_h X_h$$

$$c_w = -B_w X_w$$

In (12) B_h and B_w are coefficient vectors. Referring to the discussions of equations (9) and (10), we express the choices of F_h and F_w in the four possible regimes as follows:

If $I_h > 0$ and $I_w > 0$:

(13) $$F_h = \gamma_h Z_h + \epsilon_h$$

$$F_w = \gamma_w Z_w + \epsilon_w$$

If $I_h > 0$ and $I_w \leqslant 0$:

(14) $$F_h = \theta_h Z'_h + \psi_h$$

$$F_w = 0$$

If $I_h \leqslant 0$ and $I_w > 0$:

(15) $$F_h = 0$$

$$F_w = \theta_w Z'_w + \psi_w$$

If $I_h \leqslant 0$ and $I_w \leqslant 0$:

(16) $$F_h = 0$$

$$F_w = 0$$

In (13), (14), and (15), Z_h, Z_w, Z'_h and Z'_w are vectors of explanatory variables, and γ_h, γ_w, θ_h and θ_w are coefficient vectors. We assume that the six error terms, μ_h, μ_w, ϵ_h, ϵ_w, ψ_h, and ψ_w, have zero means and are distributed joint normally. Denote by σ_{ij} the elements of the covariance matrix of this distribution where i,j references μ_h, μ_w, ϵ_h, ϵ_w, ψ_h, ψ_w.

Estimation Strategy

We can consistently estimate the econometric model represented by equations (11) through (16) by first estimating the choice of regimes (equations (11) and (12)) with a bivariate probit, and then using the results of this probit to correct for sample selection in the regressions for the levels of F_h and F_w in equations (13), (14), and (15). The appropriate Mills ratio selection correction factors differ in this case from that suggested by Heckman (1976) since they are based on a bivariate error process. To illustrate the appropriate Mills ratio formula, consider estimating the regression for F_h in (13). As derived in the appendix, the expected value of ϵ_h given $I_h > 0$ and $I_w > 0$ is:

$$E(\epsilon\ /I_h>0,\ I\ >0) = E(\epsilon_h/\mu_h>c_h,\ \mu_w>c_w) \tag{17}$$

$$= \frac{[1 - F(c_h)][1 - F(c_w)]}{1 - \Phi(c_h,c_w)}\ [E(\epsilon_h/\mu_h>c_h) + E(\epsilon_h/\mu_w>c_w)]$$

In (17) F( ) is the cumulative normal function, and Φ( ) is the bivariate cumulative normal. The last bracketed term on the righthand side of (17) contains the two univariate Mills ratios. If μ_h and μ_w were independent, the term $1 - \Phi(c_h,c_w)$ would equal $[1 - F(c_h)][1 - F(c_w)]$, and (17) would reduce to the sum of two separate Mills ratios. In this case one could run two separate probits for $I_h > 0$ and $I_w > 0$ to form $E(\epsilon_h/\mu_h > c_h)$ and $E(\epsilon_h/\mu_w > c_w)$. However, when μ_h and μ_w are not independent the term $\Phi(c_h,c_w)$ must be estimated from a bivariate probit.

V. Empirical Results

In this section, we report estimates for the demands for husbands' and wives' life insurance for the two models described in the previous section. We used William Green's LIMDEP routine to estimate the model. LIMDEP correctly calculates standard errors in the selection equations. The results are given in Table 8 for husbands and Table 9 for wives.

Of the 5110 total observations in the sample, there are 3351 households in which both spouses have insurance, 1024 households in which only the husband has insurance, 128 households in which only the wife has insurance, and 707 households in which neither family member has positive insurance.

We examine first the results for husbands' life insurance demands. We examine two samples of households in which the husband's insurance is positive: those in which wives also purchase positive amounts of insurance, and those in which wives purchase no insurance. The first two columns in Table 8 present estimates for Model 1 for the two samples, based on equation (9h) above. The last two columns present estimates for Model 2, based on equation (10h) above. Recall that the second model may be preferred because it does not require distinguishing the cash and face value of insurance. In all cases, we include three moments of the present value of family resources, husband's and wife's age, and interaction terms between these ages and the present value of family resources to account for the consumption demand functions H( ) and $\hat{H}$( ). The ages are meant to proxy for the survival probabilities that determine the state contingent prices of consumption. PVR_w, defined by the righthand side of equation (7), and PVR_h, defined by the

Table 8. Husband's Insurance Demand (Positive Levels Only)

	Model 1		Model 2	
	Wife's Ins.>0	Wife's Ins.=0	Wife's Ins.>0	Wife's Ins.=0
No. Obs.	3251	1024	3251	1024
Constant	-16425 (10130)	-53467 (32660)	-16030 (13360)	-427320 (163300)
Husband's:				
Earnings	-.0232 (.0428)	.656 (.1231)	--	--
SS Benefits	-.0254 (.05018)	.327 (.157)	--	--
Pension	-.02363 (.02266)	.288 (.075)		
Wife's:				
Earnings	--	--	-.05 (.038)	-.35 (.1611)
SS Benefits	--	--	-.0405 (.3329)	-4.39 (1.624)
Survivor Bens.	-.04199 (.07210)	.349 (.205)	.0697 (.0661)	-.434 (.196)
Net Worth	--	--	.0356 (.0207)	.029 (.065)
PVR(w)	-.18159 (.04930)	.299 (.1214)	-.1901 (.0339)	-.212 (.097)
PVR(h)**2	-.00387 (.00073)	.0043 (.0015)	-.0039 (.00073)	.0063 (.0015)
PVR(h)**3	.0000288 (.0000062)	-.00005 (.00001)	.000028 (.0000062)	-.000065 (.000011)
Husband's Age	-725.16 (194.7)	-99.69 (557.3)	-608.54 (321.7)	3352.3 (1614)
Wife's Age	397.97 (103.50)	627.7 (231.3)	395.28 (156.3)	1738.7 (538.3)
Husband's Age x PVR(w)	.66259 (.1040)	-.070 (.209)	.692 (.093)	.299 (.179)
Wife's Age x PVR(h)	.04926 (.07846	-.26 (.153)	-.014 (.0815)	.022 (.1513)
Mills Ratio Husband	6.2033 (.65)	4.07 (.86)	5.647 (.97)	-8.477 (3.8)
Mills Ratio Wife	2.9189 (.36)	2.05 (.68)	1.9127 (1.97)	-15.206 (5.8)
R squared	.28	.32	.29	.31

righthand side of equation (9w') is used for the regime B sample.

Consider first the results for Model 1 in Table 8. To evaluate the performance of the model, note that the components of husbands' receipts should each have a coefficient of 1, and the survivor's benefit should have a coefficient of -1. Each of these independent variables is calculated as the present expected value of the relevant income stream. For the sample in which the wife's insurance is positive, the coefficients of the husband's earnings, Social Security and pension benefits have the wrong sign, while survivor benefits has the right sign but is over 7 standard deviations from -1. The results are somewhat better in the sample in which the wife's insurance is zero; these coefficients, while positive and significant, are, however, significantly below 1. For this sample the wife's survivor benefits has the wrong sign. The two bivariate mills ratios are highly significant for both samples. Note that the standard errors of the coefficients tend, in general, to be quite small implying a fairly precise rejection of the theoretical model.

The estimates for Model 2 are not much closer to those predicted by the theoretical model. The model predicts that the wife's social security benefits, survivor benefits, and earnings should all enter with a coefficient of -1; in the two subsamples only 4 of the 8 coefficients have the correct negative sign, and only 2 are significant. The large -4.39 coefficient in the model 2 $Fw < 0$ regression for the wife's social security benefits is hard to take seriously in light of these other results. The net worth variable (A) is positive in both samples, although very small in absolute value; recall its predicted value is -1. The overall goodness of fit for Model 2 is quite similar to that for Model 1.

Table 9 presents estimates for the analagous two models of wives' demands for life insurance. The two samples considered (all for wives who have positive life insurance) are for husbands who have insurance, and husbands who have no insurance. The second sample is quite small, as one would expect, containing only 128 households.

In the two models, the components of husbands' and wives' earnings and benefits enter with the correct signs in 8 of 10 cases, and 7 of these 8 coefficients are significant. However, none of these coefficients is close to 1 in absolute value. As in Table 8 standard errors are typically quite small. The results for Model 2 in which the husbands have zero insurance come closest to the theoretical prediction. For this sample each of the husand's streams as well as net worth have negative coefficients as predicted and are significant.

VI. Conclusions

A significant minority of elderly households appear to have inadequate amounts of life insurance. In addition, estimates of life insurance demand functions are, to a very large extent, at odds with theoretical predictions. There appears to be little systematic response of private life insurance holdings to social security's provision of survivor benefits, hence, social security is apparently effective in raising the welfare of widows and widowers through its provision of survivor insurance.

The surprisingly poor econometric results make us even more skeptical concerning the quality of the RHS data than when we started working on the paper. We intend to estimate the model using the SRI survey of the finances of higher income middle age and older households. The quality of these data may

Table 9. Wife's Insurance Demand (Positive Levels Only)

	Model 1		Model 2	
	Husb's Ins.>0	Husb's Ins.=0	Husb's Ins.>0	Husb's Ins.=0
No. Obs.	3251	128	3251	128
Constant	871.9 (3557)	-30244 (25900)	8140 (2420)	12266 (10810)
Husband's:				
Earnings	--	--	-.47 (.019)	-.41 (.17)
SS Benefits	--	--	-.0079 (.025)	-.44 (.19)
Pension	--	--	.0062 (.0177)	-.32 (.107)
Wife's:				
Earnings	.122 (.019)	.41 (.093)	--	--
SS Benefits	.207 (.077)	-.27 (.45)	--	--
Survivor Bens.	--	--	--	--
Net Worth	--	--	.0075 (.0179)	-.179 (.061)
PVR(w)	.081 (.021)	.43 (.1)	-.038 (.018)	.125 (.070)
PVR(w)**2	-.0002 (.00017)	-.00058 (.0012)	-.2E-03 (.17E-03)	-.0026 (.0028)
PVR(w)**3	.24E-05 (.15E-05)	.22E-05 (.2E-05)	-.22E-05 (.15E-05)	.71E-04 (.62E-04)
Husband's Age	-87.3 (81.9)	430 (447)	-77.802 (48.0)	-277 (194)
Wife's Age	-93.7 (34.7)	139 (147)	-15.8 (24.5)	-176 (199)
Husband's Age x PVR(w)	-.116 (.04)	-.6 (.18)	.093 (.020)	.166 (.082)
Wife's Age x PVR(w)	.012 (.018)	-.07 (.08)	-.010 (.019)	-.12 (.086)
Mills Ratio Husband	.901 (.228)	-.19 (1.04)	-.065 (.159)	-.127 (1.168)
Mills Ratio Wife	1.3 (.46)	-.38 (1.56)	.333 (.120)	-.581 (.812)
R squared	.16	.58	.15	.54

be superior to those of the RHS since they arose from a paid rather than a voluntary survey.

Given the significant poverty rates among elderly widows, this paper suggests the need for a reevaluation of the appropriate size of survivor benefits relative to retirement benefits under the social security system. It also suggests that poverty among elderly widows could be reduced by government programs aimed at increasing private purchase of life insurance.

Appendix

Derivation of the Bivariate Mills Ratio Sample Selection Correction

Let $x = (\mu_1, \mu_2, \epsilon) \sim \nu(0,\sum)$, where the elements of $\sum$ are σ_{ij}.

The correction for sample selection is:

(A1) $$E(\epsilon/\mu_1 > c_1 \text{ and } \mu_2 > c_2) = \frac{\int_{c_1}^{\infty}\int_{c_2}^{\infty}\int_{-\infty}^{\infty} \epsilon f(\mu_1,\mu_2,\epsilon)d\epsilon d\mu_1, d\mu_2}{\int_{c_1}^{\infty}\int_{c_2}^{\infty}\int_{-\infty}^{\infty} f(\mu_1,\mu_2,\epsilon)d\epsilon d\mu_1 d\mu_2}$$

$$= \frac{\int_{c_1}^{\infty}\int_{c_2}^{\infty}\int_{-\infty}^{\infty} \epsilon \exp(-1/2x'\sum^{-1}x)d\epsilon d\mu_1 d\mu_2}{1 - \Phi(c_1,c_2)},$$

where $\Phi(c_1,c_2)$ is the bivariate cumulative density function for (μ_1,μ_2).

Consider the numerator of (1); call it Q and write:

(A2) $$Q = \int_{c_1}^{\infty}\int_{c_2}^{\infty}\int_{-\infty}^{\infty} \epsilon$$

$$\times \exp -1/2\{\mu_1^2\sigma^{11} + 2\mu_1\mu_2\sigma^{12} + \mu_2^2\sigma^{22} + \epsilon^2\sigma^{33} + 2\epsilon\mu_1\sigma^{13} + 2\epsilon\mu_2\sigma^{23}\}d\epsilon d\mu_2 d\mu_1$$

$$= \int_{c_1}^{\infty}\int_{c_2}^{\infty} \exp -1/2[\mu_1^2\sigma_z^{11} + 2\mu_1\mu_2\sigma^{12} + \mu_2^2\sigma^{22}$$

$$- \frac{(\mu_1\sigma^{13}+\mu_2\sigma^{23})^2}{\sigma^{33}}] \int_{-\infty}^{\infty} \epsilon \exp -1/2\ \sigma^{33}[\epsilon + (\frac{\mu_1\sigma^{13}+\mu_2\sigma^{23}}{\sigma^{33}})]^2 d\epsilon d\mu_2 d\mu_1$$

where σ^{ij} is the $ij^{\underline{th}}$ element of $\sum^{-1}$.

Let $Z = \epsilon + (\frac{\mu_1\sigma^{13} + \mu_2\sigma^{23}}{\sigma^{33}})$, and let N be the last integral on the righthand side of (A2).

Then $N = \int_{-\infty}^{\infty} Z \exp\,[-1/2\sigma^{33}Z^2]dZ - \frac{\mu_1\sigma^{13} + \mu_2\sigma^{23}}{\sigma^{33}} \int_{-\infty}^{\infty} \exp[-1/2\sigma^{33}Z^2]dZ$

$$= 0 - \frac{\mu_1\sigma^{13} + \mu_2\sigma^{23}}{\sigma^{33}}, \text{ and}$$

$$\text{(A3)}\quad Q = -\int_{c_1}^{\infty}\int_{c_2}^{\infty}\left(\frac{\mu_1\sigma^{13}+\mu_2\sigma^{23}}{\sigma^{33}}\right) \exp -1/2[\mu_1^2(\sigma^{11} - \frac{\sigma^{13^2}}{\sigma^{33}}) + \mu_2^2(\sigma^{22} - \frac{\sigma^{23^2}}{\sigma^{33}}) + 2\mu_1\mu_2(\sigma^{12} - \frac{\sigma^{13}\sigma^{23}}{\sigma^{33}})]d\mu_1 d\mu_2$$

Let $\Delta = |\Sigma^{-1}|$; then

$$\frac{\sigma^{13}}{\sigma^{33}} = \frac{-1/\Delta(\sigma_{13}\sigma_{11}-\sigma_{23}\sigma_{12})}{1/\Delta(\sigma_{11}\sigma_{22}-\sigma_{12}^2)} = -\frac{\sigma_{13} - \sigma_{23}(\sigma_{12}/\sigma_{22})}{\sigma_{11} - \sigma_{12}(\sigma_{12}/\sigma_{22})} \equiv -\frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}}$$

Likewise, $\frac{\sigma^{23}}{\sigma^{33}} = -\frac{\sigma_{32\cdot 1}}{\sigma_{22\cdot 1}}$, and

$$\text{(A4)}\quad Q = \int_{c_1}^{\infty}\int_{c_2}^{\infty}\left[\frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}}\mu_1 + \frac{\sigma_{32\cdot 1}}{\sigma_{22\cdot 1}}\mu_2\right] \exp -1/2[\mu_1^2(\sigma^{11} - \frac{\sigma^{13^2}}{\sigma^{33}}) + \mu_2^2(\sigma^{22} - \frac{\sigma^{23^2}}{\sigma^{33}}) + 2\mu_1\mu_2(\sigma^{12} - \frac{\sigma^{13}\sigma^{23}}{\sigma^{33}})]d\mu_2, d\mu_1$$

Note that:

$$\sigma^{11} - \frac{\sigma^{13^2}}{\sigma^{33}} = \frac{1 + \frac{\sigma_{12}^2}{\sigma_{11}\sigma_{22}-\sigma_{12}^2}}{\sigma_{11}} = \frac{1}{\sigma_{11}-\sigma_{12}^2/\sigma_{22}} = \frac{1}{\sigma_{11\cdot 2}},$$

$$\sigma^{22} - \frac{\sigma^{23^2}}{\sigma^{33}} = \frac{1}{\sigma_{22\cdot 1}}, \text{ and}$$

$$\sigma^{12} - \frac{\sigma^{13}\sigma^{23}}{\sigma^{33}} = \frac{1}{\sigma_{12}-(\sigma_{11}\sigma_{22}/\sigma_{12})}.$$

These expressions and the last formula for Q imply:

$$\text{(A5)}\quad Q = \int_{c_1}^{\infty}\int_{c_2}^{\infty}\left[\frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}}\mu_1 + \frac{\sigma_{32\cdot 1}}{\sigma_{22\cdot 1}}\mu_2\right]\exp -1/2\left[\frac{\mu_1^2}{\sigma_{11\cdot 2}} + \frac{\mu_2^2}{\sigma_{22\cdot 1}} + \frac{2\mu_1\mu_2}{\sigma_{12}-(\sigma_{11}\sigma_{22}/\sigma_{12})}\right]d\mu_2 d\mu_1$$

$$= \frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}}\int_{c_2}^{\infty}\int_{c_1}^{\infty}\mu_1 b(\mu_1,\mu_2)d\mu_1 d\mu_2 + \frac{\sigma_{32\cdot 1}}{\sigma_{22\cdot 1}}\int_{c_1}^{\infty}\int_{c_2}^{\infty}\mu_2 b(\mu_1,\mu_2)d\mu_2 d\mu_1,$$

where b( ) is the bivariate density function. Denote by A the first term on the righthand side of (A5).

$$A = \frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}}\int_{c_2}^{\infty}\exp\left[-1/2\,\frac{\mu_2^2}{\sigma_{22\cdot 1}}\right]\int_{c_1}^{\infty}\mu_1 \exp -1/2\left[\frac{\mu_1^2}{\sigma_{11\cdot 2}} + \frac{2\mu_1\mu_2}{\sigma_{12}-(\sigma_{11}\sigma_{22}/\sigma_{12})}\right]d\mu_1 d\mu_2$$

and define:

$$B = \int_{c_1}^{\infty}\mu_1 \exp -1/2\left[\frac{1}{\sigma_{11\cdot 2}}(\mu_1^2 + \frac{2\mu_1\mu_2\sigma_{11\cdot 2}}{\sigma_{12}-(\sigma_{11}\sigma_{22}/\sigma_{12})}\right]d\mu_1$$

It can be shown that: $\dfrac{\sigma_{11\cdot 2}}{\sigma_{12} - \dfrac{\sigma_{11}\sigma_{22}}{\sigma_{12}}} = \dfrac{-\sigma_{12}}{\sigma_{22}}$, and

$$B = \int_{c_1}^{\infty}\mu_1 \exp -1/2\left[\frac{1}{\sigma_{11\cdot 2}}(\mu_1^2 - 2\mu_1\mu_2\frac{\sigma_{12}}{\sigma_{22}})\right]d\mu_1$$

$$= \exp -1/2\left[-\frac{\sigma_{12}^2\mu_2^2}{\sigma_{22}^2\sigma_{11\cdot 2}}\right]\int_{c_1}^{\infty}\mu_1 \exp -1/2\left[\frac{1}{\sigma_{11\cdot 2}}[\mu_1 - \frac{\sigma_{12}}{\sigma_{22}}\mu_2]^2\right]d\mu_1$$

Thus,

$$A = \frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}}\int_{c_2}^{\infty}\exp -1/2\left[\mu_2^2\left(\frac{1}{\sigma_{22\cdot 1}} - \frac{\sigma_{12}^2}{\sigma_{22}^2\sigma_{11\cdot 2}}\right)\right]\int_{c_1}^{\infty}\mu_1\exp -1/2\left[\frac{1}{\sigma_{11\cdot 2}}(\mu_1-\frac{\sigma_{12}}{\sigma_{22}}\mu_2)^2\right]d\mu_1 d\mu_2$$

But,

$\dfrac{1}{\sigma_{22\cdot 1}} - \dfrac{\sigma_{12}^2}{\sigma_2^2\sigma_{11\cdot 2}} = \dfrac{1}{\sigma_{22}}$; hence:

$$A = \frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}} \int_{c_2}^{\infty} \exp -1/2 \frac{\mu_2}{\sigma_{22}} [\int_{c_1}^{\infty} \mu_1 \exp [-1/2 \frac{(\mu_1-(\sigma_{12}/\sigma_{22})\mu_2)}{\sigma_{11\cdot 2}}] d\mu_1] d\mu_2$$

Let $Z = \mu_1 - \frac{\sigma_{12}}{\sigma_{22}} \mu_2$, and P be last integral on the righthand side of the above equation. Then

$$V(Z) = \sigma_{11} - 2 \frac{\sigma_{12}^2}{\sigma_{22}} + \frac{\sigma_{12}^2}{\sigma_{22}} = \sigma_{11} - \frac{\sigma_{12}^2}{\sigma_{22}} = \sigma_{11\cdot 2}$$

Thus, $P = \int_{c_1}^{\infty} [Z + \frac{\sigma_{12}}{\sigma_{22}} \mu_2] \exp [-1/2 \frac{Z^2}{V(Z)}] dZ = \int_{c_1}^{\infty} Zf(Z)dZ + \frac{\sigma_{12}}{\sigma_{22}} \mu_2 \int_{c_1}^{\infty} f(Z)dZ$

$$= E(Z/Z > c_1)(1 - F(c_1)) + \frac{\sigma_{12}}{\sigma_{22}}\mu_2(1 - F(c_1)) = (1-F(c_1))[E(Z/Z>c_1) + \frac{\sigma_{12}}{\sigma_{22}}\mu_2]$$

Thus,

$$A = \frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}} \int_{c_2}^{\infty} [\exp - 1/2 \frac{\mu_2^2}{\sigma_{22}}][(1-F(c_1))[E(Z/Z_1>c_1) + \frac{\sigma_{12}}{\sigma_{22}} \mu_2]]d\mu_2$$

$$= \frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}} \cdot (1-F(c_1))[E(Z/Z>c_1)(1-F(c_2)) + \frac{\sigma_{12}}{\sigma_{22}} E(Z/Z>c_2)(1-F(c_2))]$$

$$= \frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}}(1-F(c_1)(1-F(c_2))[E(Z/Z>c_1) + \frac{\sigma_{12}}{\sigma_{22}} E(Z/Z>c_2)]$$

By symmetry, the second piece of Q is:

$$\frac{\sigma_{32\cdot 1}}{\sigma_{22\cdot 1}}(1-F(c_1))(1-F(c_2))[E(Z/Z>c_2) + \frac{\sigma_{12}}{\sigma_{11}} E(Z/Z>c_1)]$$

Thus,

$$Q = (1-F(c_1))(1-F(c_2))[E(Z/Z>c1)[\frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}} + \frac{\sigma_{32\cdot 1}}{\sigma_{22\cdot 1}} \cdot \frac{\sigma_{21}}{\sigma_{11}}] + E(Z/Z>c_2)[\frac{\sigma_{32\cdot 1}}{\sigma_{22\cdot 1}} + \frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}} \cdot \frac{\sigma_{12}}{\sigma_{22}}]$$

But

$$\frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}} + \frac{\sigma_{32\cdot 1}}{\sigma_{22\cdot 1}} \cdot \frac{\sigma_{21}}{\sigma_{11}} = \sigma_{22} \times \frac{\sigma_{31}-\sigma_{32}(\sigma_{12}/\sigma_{22})}{\sigma_{22}\,\sigma_{11} - \sigma_{12}^2} + \frac{\sigma_{32}-\sigma_{31}(\sigma_{12}/\sigma_{11})}{\sigma_{11}\,\sigma_{22} - \sigma_{12}^2} \cdot \sigma_{21} = \frac{\sigma_{31}}{\sigma_{11}},$$

and by symmetry,

$$\frac{\sigma_{32\cdot 1}}{\sigma_{22\cdot 1}} + \frac{\sigma_{31\cdot 2}}{\sigma_{11\cdot 2}} \cdot \frac{\sigma_{12}}{\sigma_{22}} = \frac{\sigma_{32}}{\sigma_{22}}.$$

Hence:

$$E(\epsilon/\mu_1 > c_1 \text{ and } \mu_2 > c_2) = \frac{[1-F(c_1)][1-F(c_2)]}{1 - \Phi(c_1,c_2)} [\frac{\sigma_{31}}{\sigma_{11}} E(Z/Z>c_1) + \frac{\sigma_{32}}{\sigma_{22}} E(Z/Z>c_2)]$$

Note that the terms in brackets are the two univariate Mills' ratios.

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