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PUBLIC CONSUMPTION OVER THE BUSINESS CYCLE

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ABSTRACT

What fraction of the business cycle volatility of government purchases is accounted for as endogenous reactions to overall macroeconomic conditions? We answer this question in the framework of a neoclassical representative household model where the provision of a public consumption good is decided upon endogenously and in a time-consistent fashion. A simple frictionless version of such a model with aggregate productivity as the sole driving force can explain almost all the volatility of U.S. non-defense government consumption expenditures. However, such a model fails to match other important features of the business cycle dynamics of public consumption, which comes out as not persistent enough and too synchronized with the cycle. We add implementation lags and implementation costs in the budgeting process to the model, plus taste shocks for public consumption relative to private consumption, and achieve a substantially better match to the data. All these ingredients are essential to improve the fit. Depending on the precise specification of the flow utility function over private consumption, public consumption and leisure, 25-40 percent of the variance of public consumption is driven by aggregate productivity shocks.

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1 Introduction

Public consumption has interesting business cycle properties, which we carefully document in this paper. It is about as volatile as private consumption, roughly as persistent as aggregate output and, unlike any other component of aggregate demand, contemporaneously acyclical, but mildly procyclical with one- and two-year lags. Overall, public consumption is the least procyclical component of aggregate demand. We define public consumption as the counterpart of private consumption within government purchases: "government expenditures on consumption and investment goods", as stipulated in the NIPA accounts.

Standard business cycle analysis typically treats government purchases as an *exogenous* stochastic process. As such they appear in at least three different strands of the literature: as a wedge and potential driving force of aggregate fluctuations (see Baxter and King, 1992, Chari et al., 2007, or Leeper et al., 2010, for instance); in the vast empirical literature on the sign and magnitude of the government spending multiplier as a source of an exogenous shock to be identified (see Shapiro and Ramey, 1998, Blanchard and Perotti, 2002, Mountford and Uhlig, 2009, or Ramey, 2011, for example); and in the optimal fiscal policy literature (see Chari and Kehoe, 1999, and Kocherlakota, 2010, for an overview), where there is an exogenous stream of government purchases that needs to be financed by either taxes or debt.

In this paper we reverse the perspective and ask: once we allow for a mechanism of *endogenous* public good provision, what fraction of the business cycle fluctuations of government purchases is accounted for as endogenous reactions to overall macroeconomic conditions? And how much volatility is generated through shocks directly related to the provision of public goods, for example preference shocks between private and public consumption?

To answer this question we draw on previous work by Klein, Krusell and Rios-Rull (2008) (KKR henceforth). The KKR framework is a natural starting point for our quantitative analysis, because it features a standard neoclassical modeling of the macroeconomic business cycle and adds to that time-consistent public policy.[1] The model has a government that cannot commit ex ante to a path of future public consumption, but takes into account this path and how it depends on current decisions. The solution concept for the game between successive governments is the Markov-perfect equilibrium. Public consumption is financed by linear income taxes. We abstract from government debt and transfers. We discipline the exercise by requiring that the model match the business cycle features of public consumption described above.

As a first step, we add conventional aggregate productivity shocks as the sole aggregate driving force to the KKR framework, thus making it as close as possible to a standard real business

[1] In the words of Kocherlakota (2010): "These literatures [on time-consistency and dynamic political economy] examine the properties of equilibrium outcomes of particular dynamic games. Hence, they are trying to model *actual* behavior of governments." (emphasis in the original).

cycle model.[2] Our first result is that such a model can explain almost all the volatility of government consumption in the data, but it falls short in terms of persistence and it makes government consumption almost perfectly and contemporaneously correlated with the cycle.

Motivated by the dynamic correlation pattern of government consumption in the data, we add an implementation lag to the physical environment: today's government can only decide about public consumption tomorrow and tomorrow's government is bound by this decision. Implementation lags are a realistic feature of the budgeting process given the numerous bureaucracies involved with government expenditures. This helps us push the peak correlation of public consumption and output away from contemporaneous, but still leaves us with far too high a dynamic correlation and too low persistence.

We then include a taste shock for public consumption (relative to private consumption) in the flow utility of the representative household, which leads to a decoupling of economic aggregates and government consumption. On its own, such a shock does not lead to sizeable output fluctuations or realistic business cycles. Moreover, this second shock makes government consumption too volatile and reduces persistence further compared to the data.

We remedy this, finally, by introducing implementation costs (in addition to the implementation lags). We thus assume that it is costly for governments to deviate too much from previous budgets. One interpretation for these somewhat reduced-form adjustment costs are budget planning costs. Implementation lags and costs are modeled similarly to, respectively, time-to-build and convex adjustment costs for capital in standard macroeconomic models.

Our second result is that within the class of models we are studying only the model with two aggregate shocks and two implementation frictions (in addition to the "no commitment"-friction) can produce a reasonable fit to all three dimensions of the business cycle dynamics of government purchases: volatility, persistence and dynamic comovement.

Our final result is the answer to our original research question: depending on the specification of the felicity function over private consumption, public consumption and leisure, 25 to 40 percent of the fluctuations of public consumption are explained by endogenous reactions to macroeconomic conditions.

Our paper has also a methodological contribution: we show how global solution methods can be applied to models of time-consistent public good provision with multiple aggregate shocks and that parameterizations of the equilibrium law of motion that include higher order terms of the aggregate state variables can improve the accuracy of the solution.

[2]Most of the related literature on endogenous public policy in a dynamic environment so far has focussed on long-run steady state questions: in addition to KKR, see Krusell et al. (1997), Krusell and Rios-Rull (1999), Hassler et al. (2003), Hassler et al. (2005), Song et al. (2009), Corbae et al. (2009), Azzimonti (2011), Bai and Lagunoff (2011). Models with aggregate shocks are featured in Azzimonti et al. (2010), Battaglini and Coate (2008), Barseghyan et al. (2010), Debortoli and Nunes (2010), Bachmann and Bai (2011).

The reminder of the paper is organized as follows: the next section documents the business cycle facts for government consumption. Section 3 sets up the model and discusses its computation and calibration. Section 4 presents the results and explains in detail how each of the model features contributes to fitting the model to the observed dynamics of public consumption. A final section concludes. Details are relegated to various appendices.

2 Facts

Table 1: BUSINESS CYCLE FACTS – GOVERNMENT CONSUMPTION

Moment	GSLC	GNDC	GND	GC	G	CNDS
$std(\cdot)$	0.783	0.777	0.943	1.173	1.362	0.705
$rho(\cdot)$	0.296	0.218	0.412	0.461	0.534	0.362
$correl(\cdot, Y)$	0.003	-0.013	0.235	-0.034	0.049	0.862
$correl(\cdot, Y_{-1})$	0.306	0.209	0.428	0.206	0.340	0.223
$correl(\cdot, Y_{-2})$	0.375	0.364	0.397	0.390	0.433	-0.283
$correl(\cdot, CNDS)$	0.217	0.104	0.263	-0.056	0.016	-
$correl(\cdot, CNDS_{-1})$	0.302	0.199	0.430	0.150	0.292	-
$correl(\cdot, CNDS_{-2})$	0.320	0.381	0.438	0.446	0.480	-

Notes: data source is the BEA (NIPA data). All variables are annual, the sample goes from 1960-2006. They are deflated by their corresponding deflators, logged and filtered with a Hodrick-Prescott filter with smoothing parameter 6.25. 'GSLC' stands for state and local government consumption. 'GNDC' denotes total non-defense consumption, 'GND' total non-defense purchases and 'GC' total government consumption. 'G' is total government purchases. '$std(\cdot)$' denotes the time series volatility of an aggregate variable, $rho(\cdot)$ its first-order autocorrelation. '$correl(\cdot, Y)$' denotes the contemporaneous correlation with aggregate GDP, '$correl(\cdot, Y_{-1})$ ' and '$correl(\cdot, Y_{-2})$' the correlation with aggregate GDP one and two years lagged, respectively. '$CNDS$' stands for nondurable and services consumption.

Table 1 shows the business cycle moments for state and local government consumption (GSLC), our baseline government purchases aggregate, as well as other subaggregates of total government purchases. All variables are annual, logged and detrended with a Hodrick-Prescott filter with a smoothing parameter of 6.25 (see Ravn and Uhlig, 2002). We find:

1. GSLC is at least as volatile as private consumption expenditures, measured as spending on nondurables and services, and roughly half as volatile as GDP (1.34%).

2. GSLC is fairly persistent, at least as persistent as GDP (0.292).

3. GSLC is contemporaneously acyclical.

4. GSLC is dynamically procyclical.

State and local government consumption belongs by definition to the non-defense category, which is a plausible candidate for endogenous expenditures. The structural vector autoregressions literature often takes the same view and uses military purchases to identify exogenous government spending shocks. Focussing on the state and local level also allows us to abstract from government debt, which would complicate the model and the computation considerably. Furthermore, GSLC is roughly 10 percent of GDP and slightly under 50 percent of total government purchases, which makes it the largest individual category at this level of aggregation.

In any event, Table 1 also shows that total non-defense government consumption (GNDC), which includes federal consumption expenditures, has very similar business cycle properties to GSLC. Persistence is if anything low in GSLC, compared to other subaggregates. And the dynamic correlation pattern of state and local government consumption can also be found in other aggregates, such as non-defense purchases, total public consumption and total government purchases.[3] We view this as at least suggestive that the causes of the business cycle of government purchases should be sought in aggregate factors.

We use the Hodrick-Prescott filter with a smoothing parameter of 6.25 to capture very narrowly the business cycle dynamics of government purchases. Bachmann and Bai (2011) argues that government purchases also have important medium-frequency dynamics that we want to exclude here. Table 9 in Appendix A indeed shows that, using a bandpass filter which lets through frequencies from 2 to 8 years, we get similar results to the HP(6.25) filter. Finally, Figures 2 to 4 in the same appendix show, using data from the Annual Survey of State Government Finances, that the dynamic correlation pattern for aggregate state and local government consumption with GDP also holds for most U.S. states individually.

The evidence taken together leads us to treat the four properties of GSLC from the beginning of this section as *new stylized business cycle facts*. They are also suggestive of some of the model ingredients we use in the quantitative exercise that follows. The fact that the dynamic correlogram between public consumption and output/private consumption is tilted towards public consumption lagging the cycle suggests implementation lags. We will also show that without a second shock a representative agent model overshoots the level of the correlogram (see Bachmann and Bai, 2011, for an alternative story in a heterogeneous agent framework). Finally, persistence suggests the budget implementation costs we use.

[3] Table 8 in Appendix A shows that this is also true for a functional disaggregation of government purchases.

3 The Model

The environment is a neoclassical representative household one-sector growth model with valued public consumption. The government finances the provision of the public good with a flat rate income tax and adheres to a balanced budget rule, which for government consumption approximates well most U.S. states' constitution. The government cannot commit ex ante to future public policy. Government consumption is chosen to maximize the welfare of the representative household. The equilibrium is subject to a time-consistency requirement.

3.1 The Economic Environment

The economy is populated by a unit mass continuum of infinitely lived identical households. In each period, the household is endowed with $\tilde{l}$ units of time. She values private consumption, c, leisure, $\tilde{l} - l$, and government consumption, G, according to the following felicity function:

$$u(c, l, G) = \eta\Big(\theta \log(c) + (1-\theta)\log(G)\Big) + (1-\eta)\log(\tilde{l} - l). \tag{1}$$

Life time utility follows the standard expected utility form with a discount factor β. θ, the parameter that governs the relative preferences for private consumption versus public consumption, is assumed to be time-varying. We interpret this taste shock as a shock that directly affects the provision of consumption in form of private versus public goods, but otherwise does not generate realistic economic business cycle fluctuations. For example, this $\theta-$shock does not cause any sizeable output fluctuations. Specifically, we assume that $\theta = \bar{\theta}\hat{\theta}$, where $\hat{\theta}$ follows a two-state symmetric Markov chain with support $[1-\epsilon_\theta, 1+\epsilon_\theta]$ and transition matrix $\left(\begin{smallmatrix} \rho_\theta & 1-\rho_\theta \\ 1-\rho_\theta & \rho_\theta \end{smallmatrix}\right)$. ϵ_θ governs the volatility of this process, ρ_θ its persistence.

Notice that, with a time-varying θ, we implicitly assume here that the relative taste shock is primarily between private and public *consumption* with only an indirect leisure effect. We want to highlight the time-varying tastes in the population between private and public provision of physical commodities and use this formulation as our baseline case.[4]

The household owns capital, k, and rents it out in a perfectly competitive market. Capital depreciates at rate δ. The budget constraint of the household is given by:

$$c + k' = (1-\delta)\,k + (1-\tau)\,(wl + rk)\,, \tag{2}$$

[4]With three commodities in the felicity function there is another formulation where the taste shock is between public consumption and the private bundle *including* leisure. We explore this specification as well as one with inelastic labor supply in Section 4.3: $u(c, l, G) = \theta\Big(\eta \log(c) + (1-\eta)\log(\tilde{l} - l)\Big) + (1-\theta)\log(G)$.

where k' is the capital carried over to the next period, τ the flat income tax rate, w the real wage and r the rental rate for capital. k' is restricted to lie in $[0, +\infty)$.

Aggregate output, Y, is produced by a representative firm according to an aggregate Cobb-Douglas production function: $Y = zK^{\alpha}L^{1-\alpha}$, where K and L are the aggregate capital stock and the aggregate labor input, respectively. z denotes aggregate productivity and is the baseline source of aggregate uncertainty in this economy that generates realistic economic business cycles. Its natural logarithm evolves according to a Gaussian AR(1) process. The firm rents capital and hires labor from the household at the rental rate r and the wage rate w. Competitive factor markets guarantee the usual factor pricing conditions: $w(K, L, z) = (1-\alpha)(K/L)^{\alpha}$ and $r(K, L, z) = \alpha z (K/L)^{\alpha-1}$.

Government consumption is decided one period ahead. We assume that the current government is legally bound by this decision and in this sense there is a one-period-ahead commitment. This feature captures implementation lags in the budget process. In addition, the budget authority pays a quadratic adjustment cost for changing next period's government consumption. Both government consumption of the current period and the adjustment costs are financed by the flat tax on current income through a balanced budget requirement.

$$\tau Y = G + \frac{\Omega}{2}\left(G' - G\right)^2. \tag{3}$$

The flat income tax rate is thus implicitly defined as a function of $\left(K, L, z, G, G'\right)$:

$$\tau\left(K, L, z, G, G'\right) = \frac{G + \frac{\Omega}{2}\left(G' - G\right)^2}{zK^{\alpha}L^{1-\alpha}}. \tag{4}$$

Aggregate output is used for private and public consumption, plus budget adjustment costs, as well as private investment:

$$C + G + \frac{\Omega}{2}\left(G' - G\right)^2 + K' = (1-\delta)K + zK^{\alpha}L^{1-\alpha}. \tag{5}$$

3.2 Equilibrium with Endogenous Public Policy

Tomorrow's government consumption is chosen to maximize the welfare of the representative household today. When deciding tomorrow's G, the government does not have commitment power into the future beyond tomorrow. Without a commitment device, it is well known that the commitment equilibrium in our environment is not time-consistent. Time consistency thus requires imposing a subgame-perfect restriction with successive governments and the households as game players. Following KKR, we focus on a subclass of subgame-perfect equilibrium with Markov strategies, i.e., Markov-Perfect Equilibrium (MPE). The formal definition follows.

Definition 1 *A Markov-Perfect Equilibrium for the economy is a set of functions, including a government policy function $G' = \Psi(K,G,z,\theta)$, a transition function $K' = H\left(K,G,z,\theta,G'\right)$, an aggregate labor supply function $L(K,G,z,\theta,G';\Psi,H)$, an equilibrium continuation value function $v(k,K,G,z,\theta;\Psi,H)$, a best-response value function $J\left(k,K,G,z,\theta,G';\Psi,H\right)$ and a best-response decision rule $k' = h\left(k,K,G,z,\theta,G';\Psi,H\right)$ and $l = l(k,K,G,z,\theta,G';\Psi,H)$, such that*

(a) For any given G', the value functions and decision rules solve the household problem

$$\begin{aligned} J\left(k,K,G,z,\theta,G';\Psi,H\right) &= \max_{\{c,l,k'\}} \left\{u(c,l,G) + \beta E\left[v\left(k',K',G',z',\theta';\Psi,H\right)|z,\theta\right]\right\} \\ & s.t. \\ & c \geq 0, k' \geq 0, 0 \leq l \leq \tilde{l} \\ & c + k' = (1-\delta)k + \left(1-\tau\left(K,L,z,G,G'\right)\right)\left(w(K,L,z)\,l + r(K,L,z)\,k\right), \\ & K' = H\left(K,G,z,\theta,G'\right), \\ & L = L\left(K,G,z,\theta,G';\Psi,H\right). \end{aligned}$$

In addition, $v(k,K,G,z,\theta;\Psi,H) = J(k,K,G,z,\theta,\Psi(K,G,z,\theta);\Psi,H)$.

(b) $H\left(K,G,z,\theta,G'\right) = h\left(K,K,G,z,\theta,G';\Psi,H\right)$ and $L\left(K,G,z,\theta,G';\Psi,H\right) = l\left(K,K,G,z,\theta,G';\Psi,H\right)$.

(c) $\Psi(K,G,z,\theta)$ maximizes the welfare of the representative household on the equilibrium path, i.e.,

$$\Psi(K,G,z,\theta) = \arg\max_{G'}\left\{J\left(K,K,G,z,\theta,G';\Psi,H\right)\right\}. \tag{6}$$

The first part of the equilibrium definition says that the household decision rules should be the best response to an arbitrary decision on G', when the future follows the equilibrium path, a so called one-shot deviation best response. J denotes the value function corresponding to these optimal household decisions. In addition, the best-response value function should coincide with the equilibrium continuation value function when evaluated at the equilibrium policy $G' = \Psi(K,G,z,\theta)$.

The second part of the equilibrium definition requires that the evolution of the aggregate capital stock and labor supply are both generated by the household's best responses. This reflects rational expectations on the household side for both the on- and off-equilibrium path. On the equilibrium path, this requirement reduces to the familiar consistency restriction in a Recursive Competitive Equilibrium. The third part specifies the constitutional rule for the choice of public consumption tomorrow.

3.3 Computation and Calibration

We use numerical methods to characterize and analyze the Markov-Perfect equilibrium of the specified economy. As already intimated in the equilibrium definition, we use a global method to iterate on the capital transition function and policy rule (H, Ψ) until a fixed point is reached. The fixed point of H takes the following form:

$$\log K' = a_0(z,\theta) + a_1(z,\theta)\log K + a_2(z,\theta)\log G + a_3(z,\theta)\log G' + \\ a_4(z,\theta)(\log G')^2 + a_5(z,\theta)(\log G')^3 + a_6(z,\theta)\log G \log G'; \quad (7)$$

and that of Ψ takes the form

$$\log G' = b_0(z,\theta) + b_1(z,\theta)\log K + b_2(z,\theta)\log G. \quad (8)$$

Notice that these functions depend, through the coefficients $a_i(\cdot,\cdot)$ and $b_i(\cdot,\cdot)$, on the level of aggregate productivity and the taste for private versus public consumption. As for the functional form in (7), we started with a simple log-linear rule instead of (7), but found the $R2$ to be somewhat low, at least for some specifications of the model. After some experimentation, (7) turned out to be a good compromise between numerical stability and accuracy. Notice that H has to have good predictive power not only on-equilibrium, but also for a grid of off-equilibrium proposals for G'. The average $R2$ over the discrete number of aggregate states improves from 0.9748 to 0.9999 for the baseline model, when we add nonlinear terms, and for the model with the alternative felicity function (see Section 4.3) from 0.9368 to 1.0000.[5]

We set the output elasticity of capital, $\alpha = 0.36$ and the labor scale $\tilde{l} = 1$. For other parameters, the model is calibrated to match important features of the U.S. economy from 1960 to 2006. Annual data on government consumption correspond closely to the yearly nature of government budgeting and therefore we calibrate our model to this frequency. This choice implies three parameter selections: the depreciation rate, δ, is set to 0.1; the discount rate, β, is fixed at 0.96. Following Tauchen (1986), we model aggregate productivity, z, as a five-state Markov chain that approximates a Gaussian log-AR(1) process with an autocorrelation coefficient of 0.8145 (i.e. 0.95 to the power of four) and - in the baseline calibration - conditional standard deviation of 0.0123. This standard deviation is chosen to make our models approximately match the annual percentage standard deviation of GDP in the data, 1.34%. This paper is not concerned with explaining output volatility from a measured exogenous shock series, as the RBC

[5] See Appendix B for an outline of the algorithm, the coefficients of the equilibrium law of motion and the government policy function for the baseline case in Tables 10 and 11, and (in Table 12) the comparison in fit between the baseline version where we use (7) and one where we use only the terms until $a_3(z,\theta)\log G'$ for the parameterization of H.

tradition which uses fluctuations in the Solow residual to generate a large part of observed output fluctuations. Rather, this paper is about explaining government consumption dynamics (and other components of aggregate demand), given the correct output fluctuations.

The two parameters in the felicity function are calibrated as follows: $\bar{\theta} = 0.8512$, the average love-of-private-consumption parameter is picked to match the time-averaged $\frac{G}{Y}$-ratio based on aggregate state and local government consumption, i.e. roughly 10.2%.[6] η, the parameter specifying the relative weight between the private-public-consumption-composite and leisure, is chosen to make average labor hours 0.33.

Three non-standard parameters remain to be calibrated, ρ_θ, ϵ_θ and Ω. We fix ρ_θ at 0.75, which means that a given taste for government consumption remains operative for four years on average. ϵ_θ and Ω are chosen to minimize a weighted quadratic form in the following summary statistics for the dynamics of public and private consumption: the standard deviations and first-order autocorrelations of public and private consumption; the contemporaneous and one- and two-year lagged correlations of public consumption with GDP and private consumption; and the contemporaneous and one-year lagged correlations between private consumption and GDP. These statistics (numbers can be found in Table 1) summarize the joint business cycle dynamics of public and private consumption as well as GDP.

Specifically, let M be the collection of the aforementioned business cycle moments in the data; and let $\hat{M}_i$ be the same collection of moments from the $i-th$ simulation of the model. Then we minimize: $||\frac{M-\frac{1}{190}\sum_{i=1}^{190}\hat{M}_i}{W}||$, where W denotes the conforming collection of standard deviations of the twelve time series moments in the data (see Table 13 in Appendix B for details), and $||\cdot||$ is the Eucledian norm.[7] We use 190 simulations of length 40 to compute the model-based moments.

4 Results

4.1 Main Results

Table 2 summarizes two of our three results. A ‘Simple Model’ with no implementation lags, no implementation costs and only aggregate productivity shocks can generate the volatility of public consumption observed in the data. However, it would be wrong to conclude from this

[6]To take into account the higher distortion from higher government expenditures that in reality include federal spending, investment spending, transfers, etc., we also study a calibration where we posit a fixed amount of wasteful government spending that is not decided over, in order to also match the ratio of total government revenues to GDP in the data: 0.287. While the details of the calibration are somewhat different, our basic results do not change under this specification. They are available on request from the authors.

[7]We have also experimented with a mean absolute deviation criterion with similar results.

finding that business cycle fluctuations of government consumption are exclusively the result of aggregate productivity shocks or, more generally, constrained optimal reactions to changing macroeconomic conditions. The 'Simple Model' delivers basically no persistence and the wrong correlogram for public consumption. The 'Baseline Model' with an implementation lag and calibrated implementation costs as well as a taste shock does substantially better in matching the data.[8]

Table 2: BASELINE RESULT

Business Cycle Moment	Baseline Model	Simple Model	Data
$std(G)$	0.699	0.737	0.783
$rho(G)$	0.230	0.076	0.296
$correl(G,Y)$	-0.136	0.983	0.003
$correl(G,Y_{-1})$	0.564	0.137	0.306
$correl(G,Y_{-2})$	0.307	-0.175	0.375
$correl(G,C)$	0.184	0.970	0.217
$correl(G,C_{-1})$	0.300	-0.018	0.302
$correl(G,C_{-2})$	0.121	-0.317	0.320
$std(C)$	0.569	0.475	0.705
$rho(C)$	0.124	0.174	0.362
$correl(C,Y)$	0.838	0.933	0.862
$correl(C,Y_{-1})$	0.258	0.289	0.223

Notes: the 'Baseline Model' features both a one-year implementation lag and implementation costs ($\Omega = 15$), as well as $\epsilon_\theta = 0.005$. The 'Simple Model' has no implementation lags or costs ($\Omega = 0$), and $\epsilon_\theta = 0$. All time series for both actual and model-simulated data are logged and HP(6.25)-filtered. The model-based moments have been computed as the average from 190 simulations of length 40. Both models have the standard deviation of aggregate productivity at 0.0123. Public consumption in the data refers to 'GSLC' (state and local government consumption). Private consumption in the data refers to '$CNDS$' (nondurable and services consumption). 'std' denotes the standard deviation and 'rho' the first-order autocorrelation of the corresponding time series.

Table 3 displays our third result, a variance decomposition for public consumption in the baseline model. When we run models with the same parametrization as the 'Baseline Model', but shut down, respectively, the taste shocks between private and public consumption and the aggregate productivity shocks, we generate, respectively, 41% and 49% of the variance of public consumption in the 'Baseline Model'. That these variances do not quite add up to unity is in-

[8]We do not literally match the fact that the peak correlation in the data is, albeit just barely, at two years of output lags. Including a two-year implementation lag into the model would mean a substantial computational burden without much additional insight.
Table 14 in Appendix C shows that the 'Baseline Model' also does well in matching the same statistics, when we replace public and private consumption with their respective ratios over aggregate output.
The business cycle moments of other macroeconomic aggregates are standard and basically the same across model specifications.

dicative of small endogenous interaction effects in the joint response of public consumption to these shocks. This means that a substantial fraction of government consumption fluctuations over the business cycle can be viewed as endogenous reactions to changing macroeconomic conditions. It also means that the bulk of the business cycle fluctuations of public consumption, however, is driven by shocks that directly affect the optimal mix of private versus public provision of consumption goods. We nevertheless note that aggregate productivity shocks are necessary to generate realistic GDP fluctuations, as the model without z-shocks basically generates no GDP volatility.

Table 3: VARIANCE DECOMPOSITION - BASELINE MODEL

Contribution of z-shocks	Contribution of θ-shocks	Both
40.82%	49.49%	90.31%

Notes: see notes to Table 2. The first column displays the fraction of the time series *variance* of public consumption in the 'Baseline Model', when the θ-shocks are shut down, but the model is parameterized the same otherwise. The second column shuts down the aggregate productivity shocks. The third column is simply the sum of these variances.

4.2 Explaining the Mechanism

How do the various elements of the baseline model – implementation lags and implementation costs as well as taste shocks between private and public consumption – contribute towards the model's fit to the data? We address this question in two steps: Table 4 stays within the class of models with implementation lags, but, one step at a time, removes implementation costs and the taste shocks for public consumption from the baseline calibration, keeping all other parameters the same. Table 5 then shows how a model without implementation lags fails to reproduce the initially increasing correlogram between public consumption and output/private consumption in the data. This is the case even when the model is recalibrated to minimize the same quadratic form as the baseline calibration.

Starting from the fifth column in Table 4 we see that a model with no implementation costs and only aggregate productivity shocks fails to match the data in two important dimensions. The model delivers no persistence of public consumption and overstates the level of the dynamic correlation between public consumption and lagged private consumption/output. It does do a good job in producing the right amount of volatility for public consumption. Introducing the taste shocks (column four) into the economy remedies the second failure, but worsens the persistence problem and leads, not surprisingly, to excess volatility in public con-

Table 4: THE ROLE OF IMPLEMENTATION COSTS AND TASTE SHOCKS

Business Cycle Moment	Baseline Model	No Taste Shock	No Implementation Costs	No Taste Shock No Implementation Costs	Data
$std(G)$	0.699	0.446	1.221	0.707	0.783
$rho(G)$	0.230	0.322	-0.013	0.085	0.296
$correl(G,Y)$	-0.136	-0.204	-0.059	-0.052	0.003
$correl(G,Y_{-1})$	0.564	0.841	0.568	0.974	0.306
$correl(G,Y_{-2})$	0.307	0.461	0.096	0.149	0.375
$correl(G,C)$	0.184	0.154	0.150	0.248	0.217
$correl(G,C_{-1})$	0.300	0.964	0.163	0.965	0.302
$correl(G,C_{-2})$	0.121	0.361	0.025	-0.003	0.320
$std(C)$	0.569	0.511	0.564	0.505	0.705
$rho(C)$	0.124	0.180	0.115	0.165	0.362
$correl(C,Y)$	0.838	0.931	0.839	0.935	0.862
$correl(C,Y_{-1})$	0.258	0.295	0.239	0.277	0.223

Notes: see notes to Table 2. The 'Baseline Model' features both a one-year implementation lag and implementation costs ($\Omega = 15$), $\epsilon_\theta = 0.005$. The 'No Taste Shock' model is identical to the 'Baseline Model', but sets $\epsilon_\theta = 0$. The 'No Implementation Costs' model is identical to the 'Baseline Model', but sets $\Omega = 0$. The 'No Taste Shock - No Implementation Costs' model is a combination of columns three and four.

sumption. Conversely, introducing budget implementation costs only (column three) fixes persistence and, somewhat, the oversynchronisation issue between public consumption and the other macroeconomic aggregates, but causes insufficient volatility. Combining both features leads to a model that matches the data reasonably well.

This is interesting for two reasons: the fact that the "simpler" model in column five gets the volatility of public consumption approximately right, but fails in two other important dimensions of business cycle fluctuations, shows that within the class of models studied *both* additional features – implementation costs and taste shocks – are required by the data. Starting from the "simpler" model in column five, any additional shock that affects public consumption will always lead to excess volatility, whereas implementation costs will lead to insufficient volatility of public consumption. A combination of the two ingredients is therefore necessary to match the data. This means, secondly, that the physical environment studied here features a standard amplification-propagation trade-off. There is, however, a priori no reason to believe that this trade-off can be reconciled with the data in a way such that the dynamic oversynchronisation between public consumption and the overall cycle is sufficiently, but not excessively dampened.

We next study the role of implementation lags. The government decides now about G, not G'. The government flow budget constraint changes as follows:[9]

$$\tau Y = G + \frac{\Omega}{2}\left(G - G_{-1}\right)^2. \tag{9}$$

Taking away implementation lags from the baseline model increases the volatility of public consumption and makes it less persistent (column three in Table 5). Implementation lags thus play a similar role as implementation costs (see Table 4). Their main effect, however, is to get the rough shape of the correlogram between public consumption and private consumption/output right.

Table 5: THE ROLE OF IMPLEMENTATION LAGS

Business Cycle Moment	No Implementation Lag Recalibrated	No Implementation Lag Param. from Baseline	Baseline Model	Data
$std(G)$	0.718	1.033	0.699	0.783
$rho(G)$	0.257	0.193	0.230	0.296
$correl(G,Y)$	0.387	0.322	-0.136	0.003
$correl(G,Y_{-1})$	0.280	0.202	0.564	0.306
$correl(G,Y_{-2})$	0.070	0.030	0.307	0.375
$correl(G,C)$	0.178	-0.031	0.184	0.217
$correl(G,C_{-1})$	0.122	0.034	0.300	0.302
$correl(G,C_{-2})$	0.019	0.036	0.121	0.320
$std(C)$	0.537	0.562	0.569	0.705
$rho(C)$	0.142	0.116	0.124	0.362
$correl(C,Y)$	0.851	0.823	0.838	0.862
$correl(C,Y_{-1})$	0.277	0.253	0.258	0.223

Notes: see notes to Table 2. The second column shows the results of a model where public consumption is decided on contemporaneously, but implementation costs and the volatility of the relative taste shock between private and public consumption have been calibrated to minimize the same quadratic form as the 'Baseline Model': $\Omega = 25$, $\epsilon_\theta = 0.004$. The third column shows the results of a model where public consumption is decided on contemporaneously, but the implementation costs parameter and the volatility of the relative taste shock are set equal to those in the 'Baseline Model': $\Omega = 15$, $\epsilon_\theta = 0.005$.

Column three displays the results of a model simulation where current G is decided on in the current period, but the parameters for implementation costs and the standard deviation of the taste shocks are fixed at their values from the baseline model with implementation lags. Without implementation lags the volatility of public consumption shoots up, its persistence

[9] G_{-1} denotes last period's public consumption. Notice that for the computation the public consumption that was decided on last period remains a state variable as long as $\Omega > 0$. Therefore, in the definition of the equilibrium functions G replaces G' and G_{-1} replaces G as long as $\Omega > 0$. If $\Omega = 0$, then we have one state variable, G_{-1}, less.

goes down and any correlation with private consumption at all horizons is eliminated. From this result it can be expected that recalibration of Ω and ϵ_θ to minimize the same weighted quadratic form as the baseline model, but under the assumption of no implementation lags, will lead to a combination of higher implementation costs and/or lower variance of the taste shock. This is indeed the case: the recalibrated model (column two) has $\Omega = 25$ (up from $\Omega = 15$) and $\epsilon_\theta = 0.004$ (down from $\epsilon_\theta = 0.005$). This model gets the volatility and persistence of public consumption right, even a little better than the baseline model, but fails to deliver the dynamic correlogram between public consumption and other macroeconomic aggregates qualitatively; i.e., in this model the dynamic correlation is lower than the contemporaneous one. Moreover, the average deviation of the model-generated business cycle moments from their data counterparts as a fraction of their standard deviations is 1.44 in the recalibrated model with no implementation lags, whereas it is 1.16 in the baseline model.

4.3 Alternative Model Specifications

In this section we discuss the sensitivity of our results to the specification of the felicity function over private consumption, public consumption and leisure. In our baseline specification the taste shock was directly between private and public consumption, see (1). There is another possible grouping of commodities in which the $\theta-$shock becomes a taste shock between public consumption and the private consumption *bundle* consisting of physical goods *as well as* leisure:

$$u(c,l,G) = \theta\Big(\eta \log(c) + (1-\eta)\log(\tilde{l} - l)\Big) + (1-\theta)\log(G). \tag{10}$$

In this specification, an increase in θ not only leads to a (persistent) expansion in private consumption, but also to a (persistent) reduction in labor supply and therefore output. This potentially means that a $\theta-$shock is a much more potent driver of aggregate fluctuations in this felicity specification than in the baseline one.

This is indeed confirmed by comparing the third and the fifth column of Table 6, which show that the model with the alternative felicity function under the same parameterization as the baseline model exhibits excess volatility both compared to the baseline model and the data. A lower standard deviation for the $\theta-$shock in the recalibrated version of the alternative model is required (see column two of Table 6); ϵ_θ declines from 0.005 to 0.003. It also means that the relative contribution of the aggregate productivity shock to the business fluctuations of public consumption is considerably reduced, from 41 percent in the baseline specification to 24 percent (see Table 7). We also study a specification with perfectly inelastic labor supply, which behaves very similarly to the baseline model, both in terms of fit to the data, but also in terms

Table 6: THE ROLE OF THE FELICITY FUNCTION AND LABOR SUPPLY

Business Cycle Moment	Alternative Felicity Recalibrated	Alternative Felicity Param. from Baseline Model	Perfectly Inelastic Labor Supply	Baseline Model	Data
$std(G)$	0.765	1.464	0.740	0.699	0.783
$rho(G)$	0.252	0.180	0.223	0.230	0.296
$correl(G,Y)$	-0.161	-0.125	-0.102	-0.136	0.003
$correl(G,Y_{-1})$	0.486	0.394	0.569	0.564	0.306
$correl(G,Y_{-2})$	0.298	0.194	0.291	0.307	0.375
$correl(G,C)$	0.150	0.163	0.169	0.184	0.217
$correl(G,C_{-1})$	0.331	-0.019	0.313	0.300	0.302
$correl(G,C_{-2})$	0.147	-0.000	0.125	0.121	0.320
$std(C)$	0.525	0.551	0.609	0.569	0.705
$rho(C)$	0.165	0.131	0.120	0.124	0.362
$correl(C,Y)$	0.872	0.783	0.868	0.838	0.862
$correl(C,Y_{-1})$	0.292	0.272	0.245	0.258	0.223

Notes: see notes to Table 2. The first column shows the results of a model where the felicity function over private consumption, public consumption and leisure is given by (10) instead of (1). Implementation costs and the volatility of the relative taste shock between private and public consumption have been calibrated to minimize the same quadratic form as the 'Baseline Model': $\Omega = 25$, $\epsilon_\theta = 0.003$. $\bar{\theta} = 0.94$. The second column shows the results of a model where the felicity function over private consumption, public consumption and leisure is given by (10) instead of (1), but the implementation costs parameter and the volatility of the relative taste shock are set equal to those from the 'Baseline Model': $\Omega = 15$, $\epsilon_\theta = 0.005$. $\bar{\theta} = 0.94$. The third column shows the results of a model with inelastic labor supply, i.e. $\eta = 1$. In this case the minimizing Ω and ϵ_θ happen to be the same as in the baseline case, but in order to match the volatility of output the standard deviation of the aggregate productivity shock has to be increased to 0.0180. $\bar{\theta} = 0.86$ in this case.

of the variance decomposition exercise. Inelastic labor supply only requires us to increase the volatility of the exogenous aggregate productivity shock necessary to match the volatility of output in the economy. As usual, elastic labor supply amplifies aggregate fluctuations.

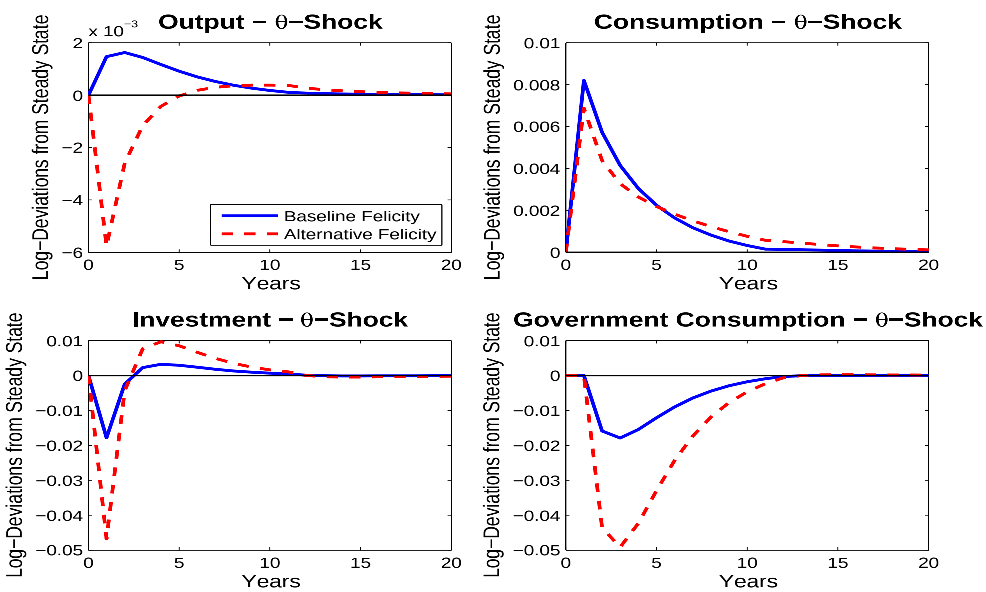
Table 7: VARIANCE DECOMPOSITION - VARIOUS MODELS

	Contribution of z-shocks	Contribution of θ-shocks	Both
Baseline	40.82%	49.49%	90.31%
Alternative Felicity	24.28%	70.37%	94.65%
Inelastic Labor Supply	41.65%	57.09%	98.74%

Notes: see notes to Tables 2, 3 and 6.

Figure 1 sheds additional light on the role of endogenous labor supply in the felicity function for the propagation of the θ-shocks into the economy. It shows for aggregate output, investment, private and public consumption the theoretical impulse response functions, in log deviations from a steady state, to a standardized taste shock towards private consumption (increase in θ) for the model with the baseline felicity and the model with the alternative felicity. In the baseline case, an increase in θ leads to an increase in labor supply and therefore contemporaneously to a (small) increase in aggregate output. This means that public consumption does not need to fall as much, in order to satisfy the increased taste in private consumption goods. Conversely, in the alternative specification, a positive taste shock towards the private consumption bundle leads to less labor supply and therefore contemporaneously to a (large) fall in aggregate output which is propagated through a reduction in capital accumulation. The effects of a taste switch on public consumption are much more severe in this case and, therefore, the θ-shock is much more potent in generating fluctuations of government consumption.

Figure 1: Theoretical Impulse Response Functions to a θ-Shock



Notes: this Figure shows the theoretical impulse responses – expressed in percentage deviations from a steady state – to the same θ-shock for the baseline model (blue solid lines) and the model with the alternative felicity function (10) (dashed red lines), both with $\Omega = 15$, $\epsilon_\theta = 0.005$ (see Table 6 for details). Specifically, we set $z = 1$ and keep the economy at the lower value for θ until it reaches a steady state. We then increase θ to its upper value and let θ drop back probabilistically, according to its transition matrix. The reported IRF is the average over those time paths.

5 Conclusion

We document the business cycle behavior of various subaggregates of government purchases, in particular state and local government consumption. We provide a tractable workhorse model that is as close as possible to standard quantitative macroeconomic models in order to match important business cycle features of public consumption. We argue that both implementation lags and implementation costs in the budgeting process plus taste shocks for public consumption relative to private consumption are essential to generate this match. We then use this model to decompose the variance of public consumption into fluctuations that are endogenous responses of the policy maker to changing macroeconomic conditions versus fluctuations that are the direct result of taste shocks in the populace between private and public consumption. Depending on the precise specification of the felicity function over private consumption, public consumption and leisure, 25 to 40 percent of the variance of public consumption is explained by aggregate productivity shocks. Some model features used here are rather stylized and need a better microfoundation. We view this paper as a first step into using quantitative macroeconomic reasoning to explain observed fluctuations of public policy variables.

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A Data Appendix

Table 8: BUSINESS CYCLE FACTS - GOVERNMENT PURCHASES - FUNCTIONAL DISAGGREGATION

Moment	$rho(\cdot)$ 1st-order	$correl(\cdot, Y)$	$correl(\cdot, Y_{-1})$	$correl(\cdot, Y_{-2})$	Frac. of GSL
General public service	0.316	0.050	0.190	0.2138	10.72 %
Public order and safety	-0.021	-0.091	0.308	0.501	13.90%
Economic affairs	0.481	0.450	0.455	0.059	19.40%
Transportation	0.466	0.489	0.371	-0.002	15.17%
Other economic affairs	0.107	0.163	0.486	0.208	4.20%
Housing & comm. serv.	0.037	0.003	0.266	0.600	3.77%
Health	0.004	-0.348	0.014	0.179	3.51%
Recreation and culture	0.219	-0.290	0.348	0.490	1.98%
Education	0.477	0.264	0.459	0.334	42.98%
Elementary and secondary	0.460	0.235	0.349	0.350	34.90%
Higher	0.083	0.238	0.496	0.076	6.57%
Libraries and other	0.190	0.090	0.365	0.336	1.76%
Income security	0.198	0.205	0.117	-0.049	3.88%

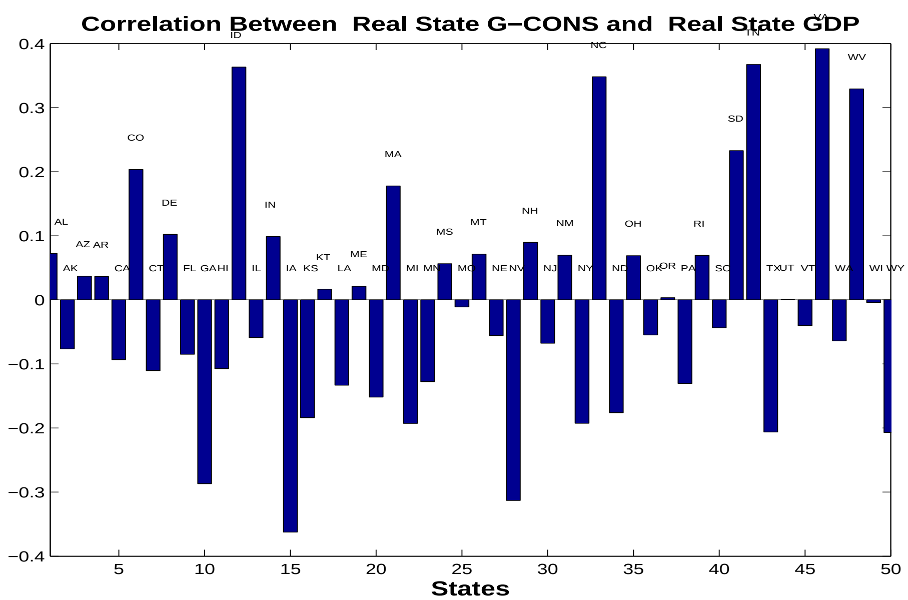
Notes: data source is the BEA (NIPA data). All variables are annual, the sample goes from 1960-2006. They are deflated by their corresponding deflators, logged and filtered with a Hodrick-Prescott filter with smoothing parameter 6.25. '$rho(\cdot)$' denotes the first-order autocorrelation of an aggregate variable. '$correl(\cdot, Y)$' denotes the contemporaneous correlation with aggregate GDP, '$correl(\cdot, Y_{-1})$' and '$correl(\cdot, Y_{-2})$' the correlation with aggregate GDP one and two years lagged, respectively. 'Frac. of GSL' denotes the fraction of the corresponding aggregate with respect to total state and local government purchases (there is not consumption/investment distinction in the functional disaggregation). 'Housing & comm. serv.' stands for 'Housing and community services'.

Table 9: Business Cycle Facts - Government Consumption - Bandpass Filter(2,8)

Moment	GSLC	GNDC	GND	GC	G	CNDS
$std(\cdot)$	0.719	0.742	0.871	1.041	1.195	0.627
$rho(\cdot)$	0.243	0.170	0.335	0.450	0.512	0.289
$correl(\cdot, Y)$	-0.122	-0.138	0.131	-0.066	0.024	0.856
$correl(\cdot, Y_{-1})$	0.254	0.252	0.427	0.245	0.381	0.121
$correl(\cdot, Y_{-2})$	0.413	0.453	0.495	0.425	0.476	-0.398
$correl(\cdot, CNDS)$	0.051	-0.015	0.158	-0.074	-0.008	-
$correl(\cdot, CNDS_{-1})$	0.202	0.195	0.399	0.105	0.263	-
$correl(\cdot, CNDS_{-2})$	0.377	0.483	0.547	0.400	0.457	-

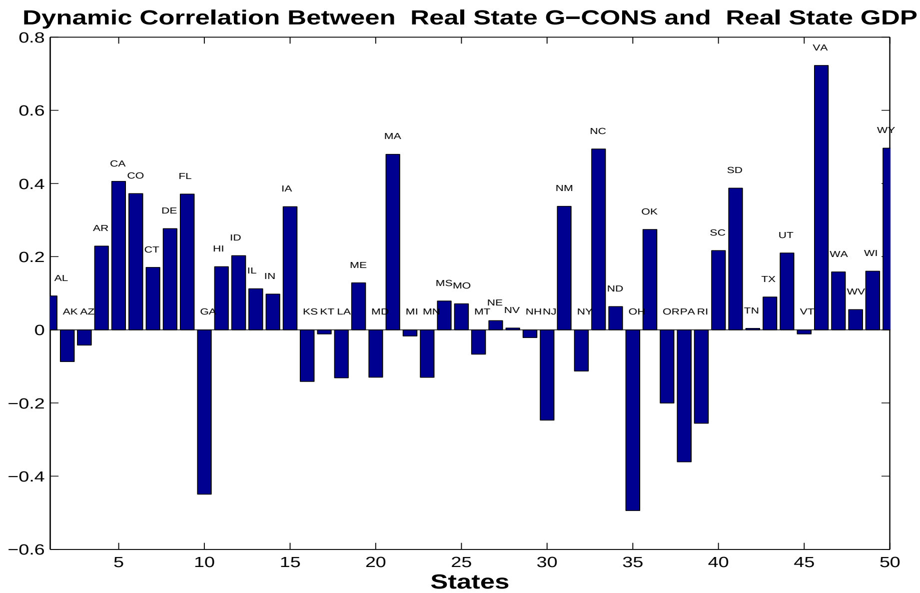
Notes: data source is the BEA (NIPA data). All variables are annual, they range from 1960-2006. They are deflated by their corresponding deflators, logged and filtered with a Bandpass Filter that defines business cycle frequencies as frequencies ranging from 2 to 8 years. 'GSLC' stands for state and local government consumption. 'GNDC' denotes total non-defense consumption, 'GND' total non-defense purchases and 'GC' total government consumption. 'G' is total government purchases.'$std(\cdot)$' denotes the time series volatility of an aggregate variable, $rho(\cdot)$ its first-order autocorrelation. '$correl(\cdot, Y)$' denotes the contemporaneous correlation with aggregate GDP, '$correl(\cdot, Y_{-1})$ ' and '$correl(\cdot, Y_{-2})$' the correlation with aggregate GDP one and two years lagged, respectively. '$CNDS$' stands for nondurable and services consumption.

Figure 2: Contemporaneous Correlation of GDP and Public Consumption by State



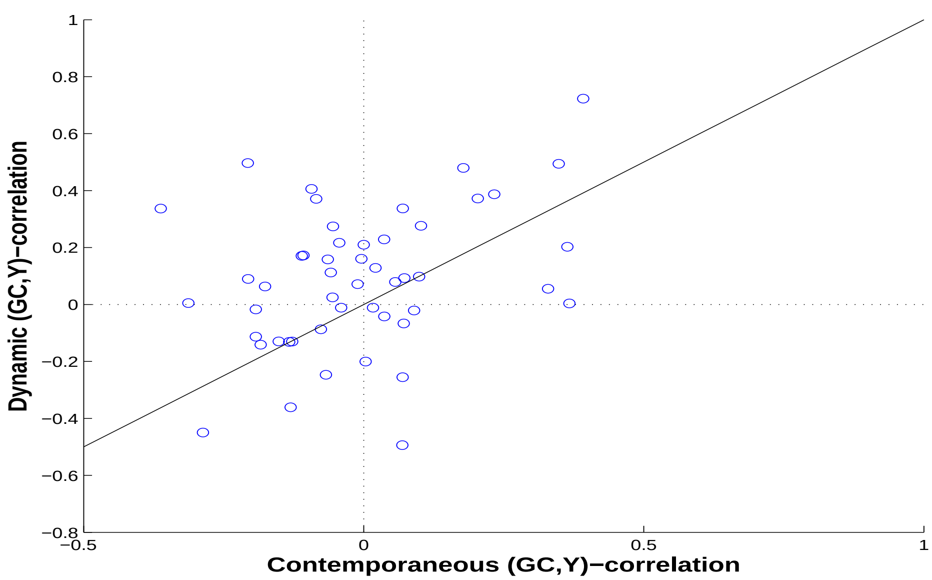
Notes: real GDP by state is taken from the BEA. Public consumption by state is measured as the 'Total Current Operations' category from the Annual Survey of State Government Finances from the Census, which we deflate by a state-specific deflator for government purchases, computed from BEA data on total nominal and real government purchases. All variables are annual, the sample goes from 1977-2006. They are logged and filtered with a Hodrick-Prescott filter with smoothing parameter 6.25.

Figure 3: Dynamic Correlation of GDP (one year lagged) and Public Consumption by State



Notes: see notes to Figure 2.

Figure 4: Dynamic Correlation of GDP (one year lagged) and Public Consumption Versus the Contemporaneous Correlation



Notes: see notes to Figure 2. Each point represents a U.S. state and the line is a 45-degree line.

B Numerical Appendix

Computational Algorithm

We solve the MPE using a fixed point iteration procedure from (H, Ψ) onto itself. The algorithm can be summarized as follows (for the baseline case):

Algorithm 1 *Fixed Point Iteration on* (H, Ψ)
Step 0: Select a grid for the capital stock K and government consumption G, and fix the functional forms for H and Ψ. Start from an initial guess of coefficients $\{a_0^0, \ldots, a_6^0\}, \{b_0^0, \ldots, b_2^0\}$ to get the initially conjectured functions $\left(H^0, \Psi^0\right)$. Set up a convergence criterion ε.
Step 1: In step n, imposing (H^n, Ψ^n) in the best-response optimization problem, use value function iteration to solve the household's parametric dynamic programming problem. Get the continuation value function $v^n(k, K, G, z; \Psi^n, H^n)$.
Step 2: Without imposing Ψ^n and instead varying G' freely on a finite grid, use H^n and $v^n(k, K, G, z; \Psi^n, H^n)$ to solve for the best-response value function $J^n\left(k, K, G, z, G'; \Psi^n, H^n\right)$ and decision rule $h^n\left(k, K, G, z, G'; \Psi^n, H^n\right)$.
Step 3: Simulate the economy using $N_H = 1$ households and T periods. In each period t of the simulation, calculate the equilibrium policy $G_t^{'eq.}$ by maximizing $J^n\left(K, K, G, z, G'; \Psi^n, H^n\right)$. Calculate the best response decision based on $h^n\left(K, K, G, z, G'; \Psi^n, H^n\right)$ for both equilibrium $G_t^{'eq.}$ and pre-specified N_G grid points of G', $\left(G_{t,i}^{'}\right)_{i=1}^{N_G}$. Gather a time series of $\left(K_{t+1}^{eq.}, \left(K_{t+1,i}\right)_{i=1}^{N_G}, G_t^{'eq.}, \left(G_{t,i}^{'}\right)_{i=1}^{N_G}\right)_{t=1}^{T}$, i.e. capital statistics both on ($K_{t+1}^{eq.}$) and off the equilibrium path $\left(\left(K_{t+1,i}\right)_{i=1}^{N_G}\right)$, with a total sample size of $T(1 + N_G)$.
Step 4: Use the gathered time series to get – separately for each value of the z-grid – OLS estimates of $\{\widehat{a}_0^n, \ldots, \widehat{a}_6^n\}, \{\widehat{b}_0^n, \ldots, \widehat{b}_2^n\}$, which with a slight abuse of notation we summarize as $\left(\widehat{H}^n, \widehat{\Psi}^n\right)$. Notice that $\widehat{H}^n$ is updated on both the on- and off-equilibrium paths, $\widehat{\Psi}^n$ only on the equilibrium path.
Step 5: If $|H^n - \widehat{H}^n| < \varepsilon$ and $|\Psi^n - \widehat{\Psi}^n| < \varepsilon$, stop. Otherwise, set

$$
\begin{aligned}
H^{n+1} &= \alpha_H \times \widehat{H}^n + (1 - \alpha_H) \times H^n, \\
\Psi^{n+1} &= \alpha_\Psi \times \widehat{\Psi}^n + (1 - \alpha_\Psi) \times \Psi^n,
\end{aligned}
$$

with $\alpha_H, \alpha_\Psi \in (0, 1]$, and go to step 1.[10]

[10] We choose $\varepsilon = 10^{-4}$ and $T = 10{,}000$, of which we discard the first 500 observations, when we update the transition and policy rules or compute summary statistics. To eliminate sampling error, we use the same series of aggregate shocks for all iterations in the algorithm and across all model simulations.

Table 10: THE EQUILIBRIUM LAW OF MOTION FOR CAPITAL - BASELINE MODEL - EQUATION (7)

		$\theta = 0.8469$			
Parameter	$z = 0.9384$	$z = 0.9687$	$z = 1$	$z = 1.0323$	$z = 1.0657$
$a_0(\cdot, \theta_1)$	-0.7499	-0.6696	-0.6028	-0.5397	-0.4897
$a_1(\cdot, \theta_1)$	0.8539	0.8543	0.8532	0.8511	0.8471
$a_2(\cdot, \theta_1)$	0.0778	0.0751	0.0728	0.0695	0.0658
$a_3(\cdot, \theta_1)$	-0.7691	-0.7041	-0.6519	-0.6022	-0.5660
$a_4(\cdot, \theta_1)$	-0.2742	-0.2539	-0.2378	-0.2225	-0.2110
$a_5(\cdot, \theta_1)$	-0.0269	-0.0249	-0.0233	-0.0219	-0.0208
$a_6(\cdot, \theta_1)$	0.0466	0.0458	0.0453	0.0445	0.0433
		$\theta = 0.8554$			
Parameter	$z = 0.9384$	$z = 0.9687$	$z = 1$	$z = 1.0323$	$z = 1.0657$
$a_0(\cdot, \theta_2)$	-0.7464	-0.6806	-0.6107	-0.5447	-0.4880
$a_1(\cdot, \theta_2)$	0.8552	0.8549	0.8531	0.8512	0.8474
$a_2(\cdot, \theta_2)$	0.0776	0.0743	0.0728	0.0694	0.0654
$a_3(\cdot, \theta_2)$	-0.7644	-0.7112	-0.6580	-0.6054	-0.5625
$a_4(\cdot, \theta_2)$	-0.2726	-0.2558	-0.2394	-0.2232	-0.2095
$a_5(\cdot, \theta_2)$	-0.0267	-0.0251	-0.0235	-0.0219	-0.0206
$a_6(\cdot, \theta_2)$	0.0464	0.0455	0.0450	0.0443	0.0429

Notes: this table displays the coefficients for the equilibrium law of motion for the (natural logarithm of the) aggregate capital stock, equation (7), for the baseline case. Recall equation (7):

$$\log K' = a_0(z,\theta) + a_1(z,\theta)\log K + a_2(z,\theta)\log G + a_3(z,\theta)\log G' + \\ a_4(z,\theta)(\log G')^2 + a_5(z,\theta)(\log G')^3 + a_6(z,\theta)\log G \log G';$$

Table 11: THE EQUILIBRIUM GOVERNMENT POLICY FUNCTION FOR G' - BASELINE MODEL - EQUATION (8)

			$\theta = 0.8469$		
Parameter	$z = 0.9384$	$z = 0.9687$	$z = 1$	$z = 1.0323$	$z = 1.0657$
$b_0(\cdot, \theta_1)$	-1.9466	-1.9037	-1.8808	-1.8882	-1.8926
$b_1(\cdot, \theta_1)$	0.2069	0.2178	0.2428	0.2714	0.2849
$b_2(\cdot, \theta_1)$	0.3526	0.3633	0.3674	0.3609	0.3540
			$\theta = 0.8554$		
Parameter	$z = 0.9384$	$z = 0.9687$	$z = 1$	$z = 1.0323$	$z = 1.0657$
$b_0(\cdot, \theta_2)$	-1.9444	-1.9133	-1.8824	-1.8827	-1.8853
$b_1(\cdot, \theta_2)$	0.2060	0.2159	0.2388	0.2711	0.2874
$b_2(\cdot, \theta_2)$	0.3587	0.3654	0.3720	0.3682	0.3622

Notes: this table displays the coefficients for the equilibrium government policy function for the (natural logarithm of) tomorrow's government consumption, equation (8), for the baseline case. Recall equation (8): $\log G' = b_0(z,\theta) + b_1(z,\theta)\log K + b_2(z,\theta)\log G$.

Table 12: DIFFERENT LAWS OF MOTION

Business Cycle Moment	Baseline Model	Linear Law of Motion	Data
$std(G)$	0.699	0.776	0.783
$rho(G)$	0.230	0.212	0.296
$correl(G,Y)$	-0.136	-0.121	0.003
$correl(G,Y_{-1})$	0.564	0.610	0.306
$correl(G,Y_{-2})$	0.307	0.294	0.375
$correl(G,C)$	0.184	0.181	0.217
$correl(G,C_{-1})$	0.300	0.339	0.302
$correl(G,C_{-2})$	0.121	0.114	0.320
$std(C)$	0.569	0.574	0.705
$rho(C)$	0.124	0.116	0.362
$correl(C,Y)$	0.838	0.847	0.862
$correl(C,Y_{-1})$	0.258	0.245	0.223

Notes: see notes to Table 2. The second column displays the results for the same parameters as the 'Baseline Model', except that the equilibrium law of motion for the (natural logarithm of the) aggregate capital stock, equation (7), only contains the first four, i.e. linear, terms with coefficients a_0 to a_3.

Table 13: WEIGHTING

Business Cycle Moment	Weighting	Data
$std(G)$	0.118	0.783
$rho(G)$	0.096	0.296
$correl(G,Y)$	0.201	0.003
$correl(G,Y_{-1})$	0.173	0.306
$correl(G,Y_{-2})$	0.168	0.375
$correl(G,C)$	0.194	0.217
$correl(G,C_{-1})$	0.190	0.302
$correl(G,C_{-2})$	0.162	0.320
$std(C)$	0.092	0.705
$rho(C)$	0.102	0.362
$correl(C,Y)$	0.024	0.862
$correl(C,Y_{-1})$	0.024	0.223

Notes: see notes to Table 2. The second column displays the standard deviations of the twelve business cycle moments used for the matching exercise from 2,000 nonparametric bootstrap simulations for GDP, private consumption ('CNDS') and public consumption ('GSLC') with non-overlapping blocks of eight years. They are the weighting coefficients in the quadratic form to be minimized (see Section 3.3).

C Results Appendix

Table 14: BASELINE RESULT - STATISTICS FOR $\frac{G}{Y}$- AND $\frac{C}{Y}$-RATIOS

Business Cycle Moment	Baseline Model	Data
$std(\frac{G}{Y})$	0.271	0.608
$rho(\frac{G}{Y})$	0.494	0.900
$correl(\frac{G}{Y}, Y)$	-0.697	-0.283
$correl(\frac{G}{Y}, Y_{-1})$	0.090	-0.050
$correl(\frac{G}{Y}, Y_{-2})$	0.263	0.152
$correl(\frac{G}{Y}, C)$	-0.494	-0.131
$correl(\frac{G}{Y}, C_{-1})$	0.151	-0.057
$correl(\frac{G}{Y}, C_{-2})$	0.313	0.149
$std(\frac{C}{Y})$	1.078	0.967
$rho(\frac{C}{Y})$	0.562	0.701
$correl(\frac{C}{Y}, Y)$	-0.703	-0.606
$correl(\frac{C}{Y}, Y_{-1})$	-0.010	-0.251

Notes: see notes to Table 2.