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**ABSTRACT**

What are the welfare effects of the information contained in macroeconomic statistics, central-bank communications, or news in the media? We address this question in a business-cycle framework that nests the neoclassical core of modern DSGE models. Earlier lessons that were based on "beauty contests" (Morris and Shin, 2002) are found to be inapplicable. Instead, the social value of information is shown to hinge on essentially the same conditions as the optimality of output stabilization policies. More precise information is unambiguously welfare-improving as long as the business cycle is driven primarily by technology and preference shocks—but can be detrimental when shocks to markups and wedges cause sufficient volatility in "output gaps." A numerical exploration suggests that the first scenario is more plausible.

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# 1 Introduction

Market participants pay close attention to public signals regarding the state of the economy, like those embodied in central-bank communications, macroeconomic statistics, surveys of consumer confidence, or news in the media. Given how noisy these signals can be, the market's heightened response to them often feels "excessive". In an influential *AER* article, Morris and Shin (2002) have argued that this is due to the sunspot-like role that public news play in environments with dispersed information: by helping agents coordinate their choices, noisy public signals can have a destabilizing effect on the economy, contributing to higher volatility and lower welfare.

Motivated by these observations, this paper seeks to understand the welfare effects of information within the context of business cycles. To this goal, we abandon the "beauty-contest" framework used by Morris and Shin (2002) and much of the subsequent literature.[1] Although the findings of this work are customarily interpreted in a macroeconomic context, the lack of micro-foundations in this prior work renders such interpretations largely premature. By contrast, we employ a micro-founded framework that nests the neoclassical backbone of modern DSGE models. In so doing, we develop a clean theoretical benchmark for the welfare effects of information within the context of business cycles—one that provides a novel, and sharp, answer to the question of interest.

**Framework.** We follow the New-Keynesian paradigm in allowing for product differentiation and monopolistic competition. We nevertheless abstract from nominal rigidities, thereby focusing, a fortiori, on flexible-price allocations. We do so because this is always an excellent benchmark for normative questions: the welfare properties of sticky-price allocations hinge on the welfare properties of the underlying flexible-price allocations. Finally, we allow for multiple type of shocks, including shocks to technologies and preferences, as well as shocks to monopoly markups and labor wedges.

This last modeling choice brings our analysis close to the pertinent DSGE paradigm (Christiano, Eichenbaum, and Evans, 2005, Smets and Wouters, 2007). Most importantly, it helps accommodate two distinct notions of fluctuations. Technology and preference shocks capture fluctuations that involve no variation in the "output gap" between flexible-price and first-best allocations. By contrast, shocks to markups and labor wedges capture fluctuations that are tied to market distortions and that manifest with variation in the aforementioned "output gap". This distinction is known to be a key determinant of both the desirability of output-stabilization policies and the welfare costs of the business cycle: see, inter alia, Goodfriend and King (1997), Rotemberg and Woodford (1997), Adao, Correia and Teles (2003), Khan, King and Wolman (2003), Benigno and Woodford (2005), and Gali, Gertler, and Lopez-Salido (2007). As we show in this paper, this distinction is also central to understanding the welfare effects of information.

Turning to the information structure, we let agents observe, not only noisy signals of the exogenous shocks, but also noisy indicators of the endogenous economic activity. This is key to both

[1] This includes three sequels in *AER*: a critique by Svensson (2005), a response by Morris et al. (2005), and a recent article by James and Lawler (2011).

the robustness and the applicability of our insights. Macroeconomic statistics and financial prices are noisy indicators of the choices and opinions of other agents. The information that these signals contain regarding the underlying structural shocks is thus endogenous to equilibrium behavior. This gives rise to informational externalities which, in general, can have important implications for the normative properties of the economy (Amador and Weill, 2010, 2011; Vives, 2008, 2011). Our analysis takes care of these complications and permits us to interpret public information as a signal of either the exogenous shocks or, more realistically, of the endogenous state of the economy.

**Results.** Our first result decomposes equilibrium welfare in terms of discrepancies, or "gaps", between equilibrium and first-best allocations. This decomposition, which extends related results from complete-information models (e.g., Woodford, 2003b, Gali, 2008), is instrumental in characterizing the welfare effects of either the underlying shocks or the available information. Under complete information, the aforementioned discrepancies obtain only because of product and labor-market distortions. With incomplete information, they obtain also because of noise in the available information. Either way, the consequent welfare losses manifest at the macro level as volatility in the aggregate output gap, and in the cross section as excess dispersion in relative prices.

Consider now the case where the business cycle is driven by technology and preference shocks. If information were complete, these shocks would cause efficient fluctuations: both the volatility of aggregate output gaps and the excess dispersion in relative prices would have been zero. It follows that, when information is incomplete, the welfare effects of information hinge entirely on the impact of noise: any correlated noise contributes to volatility in aggregate output gaps, while any idiosyncratic noise contributes to excess dispersion in relative prices.

As more precise public information motivates agents to pay less attention to their private information, the dispersion in relative prices falls, contributing to higher welfare. However, as agents pay more attention to public information, aggregate output gaps may become more volatile, contributing to lower welfare. The overall effect thus looks ambiguous at first glance. Nonetheless, we show that the beneficial effect on price dispersion necessarily outweighs any potentially adverse effect on aggregate volatility, guaranteeing that welfare necessarily improves with more information.

This result holds true despite the coordinating role of public information. The aggregate demand externalities that are embedded in our business-cycle framework induce a form of strategic complementarity that is akin to the one in the "beauty contests" studied by Morris and Shin (2002) and subsequent work. Nonetheless, the welfare lessons of this prior work are inapplicable: public information is found to be welfare improving no matter the degree of strategic complementarity.

Two observations are key to understanding these findings. First, the negative welfare effect of public information within "beauty contests" a la Morris and Shin (2002) hinges entirely on the presumption that its coordinating role is socially undesirable. By contrast, the coordinating motives that originate from aggregate demand externalities and that are thus embedded in conventional DSGE models like ours are fully warranted from a social perspective. Second, as long as the underlying market distortions are state-invariant, these distortions do not interfere with the response of the economy to any noisy information about the underlying preference and technology shocks. The first observation explains why the insights of Morris and Shin (2002) are inapplicable; the second explains why information about technology and preference shocks is necessarily welfare improving.

With these insights in mind, we next study the social value of information when the business cycle is driven by shocks to monopoly power and labor-market wedges—that is, by shocks that interfere with the level of market inefficiency. For this case, we show that more noise *reduces* the welfare losses associated with the volatility of aggregate output gaps and the excess dispersion in relative prices. In this sense, information is detrimental for welfare.

At the same time, information has a countervailing effect though the mean level of economic activity: by reducing the uncertainty faced in equilibrium, more precise information brings the mean value of aggregate output closer to the optimal one, contributing to higher welfare. The strength of this countervailing effect, however, depends on the severity of the market distortion: the smaller the level of the distortion, the smaller the benefits of any given reduction in it. We thus show that, for the case of inefficient fluctuations, the overall effect of public information on welfare is negative as long as the mean level of monopoly or other market distortions is not too large.

A simple punchline thus emerges: the welfare effects of information hinge essentially on the same conditions as the optimality of flexible-price allocations. Our results thus provide a direct mapping from the view one may hold regarding the need for output stabilization to the inference one should make regarding the welfare effects of information regarding the state of the economy, whether this information is provided by policy makers, statistical agencies, the media, or the markets.

A numerical exercise is used to shed further light. In line with our theoretical results, the *sign* of the welfare effects of information is pinned down by the relative contribution of technology and markup shocks. Nonetheless, aggregate demand externalities—and the strategic complementarity obtains from them—emerge as a key determinant of the *magnitude* of these effects. Finally, when the contribution of the various shocks in our model is matched to the one estimated by Smets and Wouters (2007) for the US economy, technology shocks are sufficiently prevalent that the social value of information is positive: more information improves welfare.

**Related literature.** Our paper adds to a large literature that followed the influential contribution of Morris and Shin (2002). Much of this work—including Svensson (2005), Angeletos and Pavan (2007, 2009), Morris and Shin (2007), Cornand and Heinemann (2008), Myatt and Wallace (2009), and James and Lawler (2011)—continues to employ the same abstract game as Morris and Shin, or certain variants of it. By contrast, our paper studies the welfare effects of information within the class of micro-founded DSGE economies that rest at the core of modern macroeconomic theory. The discipline imposed by the micro-foundations of this particular class of economies explains the sharp contrast between our results and those of the aforementioned work.

The framework we use is borrowed from Angeletos and La'O (2009); similar micro-foundations underlie, inter alia, Woodford (2003a) and Lorenzoni (2010). This prior work shows how dispersed information can have profound implications on the positive properties of the business cycle. Here, we shift focus to the normative question of how information impacts welfare. We thus complement Hellwig (2005) and Roca (2010), which are concerned with the same question but focus on monetary shocks—shocks that matter only when prices are sticky and monetary policy fails to offset them. Related are also Amador and Weill (2010, 2011) on the interaction between public information and social learning, as well as Angeletos and La'O (2011) and Wiederholt and Paciello (2011) on optimal monetary policy with informational frictions. We elaborate on these relations in Sections 7 and 9.

Finally, the methodological approach we take in this paper builds on Angeletos and Pavan (2007). That paper was the first to highlight that, in general, the welfare effects of information hinge on the relation between equilibrium and efficient allocations, and to indicate the potential importance of different types of shocks. Nevertheless, the analysis of that paper was confined within an abstract class of linear-quadratic games. By contrast, the contribution of the present paper is squarely on the applied front. To the best of our knowledge, our paper is the first to study the welfare effects of information along the flexible-price allocations of a canonical, micro-founded DSGE model.

**Layout.** The rest of the paper is organized as follows. Section 2 introduces our framework. Section 3 characterizes the equilibrium and Section 4 decomposes welfare. Sections 5 and 6 study the welfare effects of information for the cases of, respectively, technology/preference shocks and markup/labor-wedge shocks. Section 7 extends the analysis from signals of the exogenous shocks to signals of the endogenous state of the economy. Section 8 conducts a suggestive numerical exercise. Section 9 concludes with a translation of our results to sticky-price models, and with a discussion of directions for future research.

# 2 The model

Our framework builds on Angeletos and La'O (2009), adding incomplete information to the flexible-price allocations of an elementary DSGE model. As in Lucas (1972), there is a continuum of "islands", which define the "geography" of information: information is symmetric within islands, but asymmetric across islands. Each island is inhabited by a continuum of firms, which specialize in the production of differentiated commodities. Finally, there is a representative household, or "family", consisting of a consumer and a continuum of workers, one worker per island. Islands are indexed by $i \in I = [0, 1]$; firms and commodities by $(i, j) \in I \times J$; and periods by $t \in \{0, 1, 2, ...\}$.

Each period has two stages. In stage 1, workers and firms decide how much labor to, respectively, supply and demand in their local labor market, and local wages adjust so as to clear any excess demand. At this point, workers and firms know their local fundamentals, but imperfect information regarding the fundamentals and the level of economic activity in other islands. After employment and production choices are sunk, workers return home and the economy transitions to stage 2. At this point, all information that was previously dispersed becomes publicly known, and centralized markets operate for the consumption goods. This two-stage structure permits us to introduce dispersed information while maintaining the convenience of a representative consumer.

**Households.** The utility of the representative household is given by

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left[ U(C_t) - \int_I S_{i,t}^n V(n_{i,t}) di, \right]$$

where

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \qquad \text{and} \qquad V(n) = \frac{n^{1+\epsilon}}{1+\epsilon}.$$

with $\gamma, \epsilon \geq 0$. Here, $n_{i,t}$ is labor effort on island $i$ during stage 1 of period $t$, $S_{i,t}^n$ is an island-specific shock to the disutility of labor, and $C_t$ is aggregate consumption. The latter, which also defines the numeraire used for wages and commodity prices, is given by the following nested CES structure:

$$C_t = \left[ \int_I S_{i,t}^c (c_{i,t})^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}}$$

where $S_{i,t}^c$ is an island-specific shock to the utility of the goods produced by island $i$ and $c_{i,t}$ is a composite of the goods produced by island $i$, defined by

$$c_{i,t} = \left[ \int_J (c_{ij,t})^{\frac{\eta_{it}-1}{\eta_{it}}} dj \right]^{\frac{1}{\eta_{it}}},$$

with $c_{ij,t}$ denoting the quantity consumed in period $t$ of the commodity $j$ from island $i$. Here, $\rho$ identifies the elasticity of substitution across the consumption composites of different islands, while $\eta_{it}$ identifies the elasticity of substitution across the goods produced within a given island $i$.

As will become clear shortly, $\rho$ controls the strength of aggregate demand externalities (the sensitivity of optimal firm profits to aggregate output), while $\eta$ controls the degree of monopoly power. The pertinent literature often imposes (implicitly) that $\rho = \eta$, thereby confounding the notion of demand externalities with the notion of monopoly power. By contrast, we let $\rho \neq \eta$ so that we can separate these two distinct notions. Finally, by letting $\eta$ be random and abstracting from any heterogeneity within islands, we introduce a pure form of markup shocks that cause variation in equilibrium allocations without affecting first-best allocations.

Households are diversified in the sense that they own equal shares of all firms in the economy. The budget constraint of household $h$ is thus given by the following:

$$\int_I \int_J p_{ij,t} c_{ij,t} dj dj + B_{t+1} \leq \int_I \int_J \pi_{ij,t} didj + \int_I (1-\tau_{i,t}) w_{it} n_{i,t} di + R_t B_t + T_t,$$

Here, $p_{ij,t}$ is the period-$t$ price of the commodity produced by firm $j$ on island $i$, $\pi_{ij,t}$ is the period-$t$ profit of that firm, $w_{it}$ is the period-$t$ wage on island $i$, $R_t$ is the period-$t$ nominal gross rate of return on the riskless bond, and $B_{h,t}$ is the amount of bonds held in period $t$.

The variables $\tau_{i,t}$ and $T_t$ are exogenous to the representative household and the firms, but satisfy the following restriction at the aggregate:

$$T_t = \int_I \tau_{it} w_{it} n_{it} di.$$

One can thus readily interpret $\tau_{it}$ as an island-specific tax on labor income and $T_t$ as the resulting aggregate tax revenues that are distributed back to households in lump-sum transfers. Alternatively, we can consider a variant of our model with monopolistic labor markets as in Blanchard and Kiyotaki (1987), in which case $\tau_{it}$ could re-emerge as an island-specific markup between the wage and the marginal revenue product of labor. In line with much of the DSGE literature, we interpret $1 - \tau_{it}$ more generally as a "labor wedge" or a "labor-market distortion".

The objective of each household is simply to maximize expected utility subject to the budget and informational constraints faced by its members. Here, one should think of the worker-members of each family as solving a team problem: they share the same objective (family utility) but have different information sets when making their labor-supply choices. Formally, at the beginning of stage 1 the household sends off its workers to different islands with bidding instructions on how to supply labor as a function of (i) the information that will be available to them at that stage and (ii) the wage that will prevail in their local labor market. In stage 2, the consumer-member collects all the income that the worker-members have earned and decides how much to consume in each of the commodities and how much to save (or borrow) in the riskless bond.

**Firms.** The output of firm $j$ on island $i$ during period $t$ is given by

$$y_{ij,t} = A_{i,t} n_{ij,t}$$

where $A_{i,t}$ is the productivity in island $i$ and $n_{ij,t}$ is the firm's employment.[2] The firm's realized profit is given by $\pi_{ij,t} = p_{ij,t} y_{ij,t} - w_{i,t} n_{ij,t}$. Finally, the objective of the firm is to maximize its expectation of $U'(C_t)\pi_{ij,t}$, the representative consumer's valuation of its profit.

**Markets.** Labor markets operate in stage 1, while product markets operate in stage 2. The corresponding clearing conditions are as follows:

$$\int_J n_{ij,t} dj = n_{i,t} \ \forall i \qquad \text{and} \qquad c_{ij,t} = y_{ij,t} \ \forall (i,j)$$

Asset markets also operate in stage 2, when all information is commonly shared. This guarantees that asset prices do not convey any information. The sole role of the bond market in the model is then to price the risk-free rate. Moreover, because our economy admits a representative consumer, allowing households to trade risky assets in stage 2 would not affect any of the results.

**Shocks and information.** Each island is subject to multiple shocks: technology shocks are captured by $A_{it}$, preference shocks by $S^n_{it}$ and $S^c_{it}$, markup shocks by $\eta_{it}$, and labor-wedge shocks by $\tau_{it}$. These shocks have both aggregate and idiosyncratic components.

[2]The assumption of linear returns to labor is consistent with the idea that, at business-cycle frequencies, the variation in the stock of capital is negligible and that the rate of capital utilization is proportional to hours. In any event, this assumption is only for expositional simplicity: the results extend directly to decreasing returns. In fact, they also extend to increasing returns, provided that the curvature that is introduced in the typical firm's problem by the downward slopping demands is enough to preserve the convexity of that problem and, in so doing, to also preserve the uniqueness of the equilibrium.

Turning to the information structure, we note that, once all agents meet in the centralized commodity market that takes place in stage 2, they can aggregate their previously dispersed information and can therefore reach common knowledge about the aforementioned shocks. Nonetheless, such common knowledge may be missing in the beginning of the period, when firms and workers have to make their employment and production choices.

Our specification of the information structure is otherwise flexible enough to allow for multiple private and public signals, some of which could be the product of noisy indicators of aggregate economic activity. We nevertheless restrict the information structure to be Gaussian in order to obtain a closed-form characterization of equilibrium allocations and welfare. Without this restriction, our results can be re-interpreted as local approximations, pretty much as in Woodford (2003) and most other log-linearized DSGE models. We spell out the details of this Gaussian specification in due course, making clear the particular results that depend on it.

**Aggregates and equilibrium.** We henceforth normalize nominal prices so that the "ideal" price index is constant:

$$P_t \equiv \left[ \int p_{it}^{1-\rho} di \right]^{\frac{1}{1-\rho}} = 1.$$

We also define aggregate output $Y_t$ and employment $N_t$ as follows:

$$Y_t \equiv \left\{ \int S_{it}^c y_{it}^{\frac{\rho-1}{\rho}} di \right\}^{\frac{\rho}{\rho-1}} \quad \text{and} \quad N_t \equiv \int n_{it} di.$$

Our definition of aggregate output is thus consistent with the usual reinterpretation of the Dixit-Stiglitz framework that lets the differentiated commodities be intermediate inputs in the production of single final consumption good. Finally, an equilibrium is defined as a collection of wages, commodity prices, and employment, production, and consumption plans such that (i) wages clear the local labor markets that operate in each island during stage 1; (ii) commodity prices clear the centralized product markets that operate during stage 2; (iii) employment and production levels are optimally chosen conditional on the information that is available in stage 1; and (iv) consumption levels are optimally chosen conditional on the information that is available on stage 2.

## 3 Equilibrium and first-best allocations

To characterize the equilibrium, consider the behavior of firms and workers in a given island $i$ and a given period $t$.[3] When firms decide how much labor to employ and how much to produce during stage 1, they understand that they are going to face a downward-slopping demand in stage 2. They therefore seek to to equate the local wage with the expected marginal revenue product of their labor, targeting an optimal markup over marginal cost. Workers, on their part, equate their disutility of effort with the expected marginal value of the extra income they can provide their family. It follows

[3] The characterization of the equilibrium follows from Angeletos and La'O (2009). The contribution of the present paper rests in the welfare analysis of the subsequent sections.

that the period-$t$ equilibrium production levels of any given island $i$ are pinned down by the following condition:

$$S_{it}^{n}V'\left(\frac{y_{it}}{A_{it}}\right)=(1-\tau_{i,t})\left(\frac{\eta_{it}-1}{\eta_{it}}\right)\mathbb{E}_{it}\left[S_{it}^{c}U'\left(Y_t\right)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\right]A_{it}, \tag{1}$$

To interpret this condition, note that the LHS is the marginal disutility of effort, while the RHS is the product of the labor wedge, times the reciprocal of the monopoly markup, times the expected marginal value of the marginal product of labor. This condition therefore equates private costs and benefits. Finally, note that expectations are taken over $Y_t$: firms and households alike are uncertain about the ongoing aggregate economic activity because, and only because, information is dispersed.

For comparison, the first-best allocation satisfies the following condition:

$$S_{it}^{n}V'\left(\frac{y_{it}}{A_{it}}\right)=S_{it}^{c}U'\left(Y_t\right)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}A_{it}. \tag{2}$$

It follows that equilibrium allocations can deviate from first-best allocations, not only because of monopoly power and labor-market distortions, but also because of errors in expectations of aggregate output—and hence because of the noise in the available information. Our subsequent analysis will therefore explore how the resulting welfare losses vary with the precision of the available information.

Before proceeding to this, however, it is worth highlighting a certain isomorphism between our business-cycle framework and the class of coordination games studied by Morris and Shin (2002) and others. Assuming a Gaussian information structure, we can restate condition (1) as follows:

$$\log y_{it}=const+(1-\alpha)\,f_{it}+\alpha\,\mathbb{E}_{it}\left[\log Y_t\right] \tag{3}$$

where $f_{it}\equiv\frac{1}{\epsilon+\gamma}\log\left[A_{it}^{1+\epsilon}\frac{S_{it}^{c}}{S_{it}^{n}}\left(1-\tau_{i,t}\right)\left(\frac{\eta_{it}-1}{\eta_{it}}\right)\right]$ is a composite of the underlying shocks and

$$\alpha\equiv\frac{\frac{1}{\rho}-\gamma}{\frac{1}{\rho}+\epsilon}<1. \tag{4}$$

The general equilibrium of our economy can therefore be interpreted as the PBE of a "beauty contest" among the different islands of the economy, with an island's best response given by condition (3) and the corresponding degree of strategic complementarity given by $\alpha$.

At first glance, this isomorphism might suggest that the welfare lessons of Morris and Shin (2002) apply to our framework and more generally to conventional business-cycle models; we explain why this is not the case in the following sections. At the same time, this isomorphism captures an important truth: the incentives of the typical economic agent—whether a firm, a worker, or a consumer—crucially depend on expectations of the aggregate choices of all other agents.

This kind of interdependence—a form of strategic complementarity—is embedded in any modern DSGE model and reflects the combination of at least two kinds of general-equilibrium effects. On the one hand, an increase in macroeconomic activity raises the demand faced by each firm, which other things equal stimulates firm profits, production, and employment. On the other hand, an increase in income discourages labor supply and raises real wages which, other things equal, has the opposite effect on firm profits, production, and employment. The former effect formalizes the familiar Keynesian notion of aggregate demand externalities; the latter captures the reaction of wages on the labor-supply side. The overall feedback is thus positive ($\alpha > 0$) if and only if the demand-side effect dominates—a property that seems likely to hold in practice and rests at the heart of Keynesian thinking. Although not required for our key results,[4] we henceforth impose this restriction in order to simplify the exposition.

**Assumption.** *The equilibrium features strategic complementarity in the sense that local output increases with beliefs of aggregate output (that is, $\alpha > 0$).*

# 4 Welfare

We now move to the core of our contribution, starting with a decomposition of the channels through which the various structural shocks, and the available information about them, affect welfare.

Take any feasible allocation and let $y_{it}$ and $Y_t$ denote the associated local and aggregate levels of output. As standard, welfare is defined by the ex-ante utility of the representative household. Since $C_t = Y_t$ and $n_{it} = y_{it}/A_{it}$, this implies that welfare is given by the following:

$$\mathcal{W} \equiv \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left\{ U(Y_t) - \int S_{it} V\left(\frac{y_{it}}{A_{it}}\right) di \right\}\right]$$

Next, let $y^*_{it}$ and $Y^*_t$ denote the (full-information) first-best levels of, respectively, local and aggregate output. Define then the local and aggregate output gaps by the following:

$$\log \tilde{y}_{it} \equiv \log y_{it} - \log y^*_{it} \qquad \text{and} \qquad \log \tilde{Y}_t \equiv \log Y_t - \log Y^*_t$$

Finally, let $\theta_{it} \equiv \log(A_{it}, S^c_{it}, S^n_{it})$, and let $\Theta_t$ denote the cross-sectional average of $\theta_{it}$. We impose the following joint restriction on these shocks and the allocation under consideration.

**Assumption.** *The aggregate variables $(\Theta_t, \log Y_t)$ and the local variables $(\theta_{it}, \log y_{it})$ are jointly Normal, and the latter are i.i.d. across $i$ conditional on the former.*

As will become clear shortly, this joint log-normality restriction is automatically satisfied by the equilibrium allocation as long as the underlying information structure is Gaussian. The next result, however, provides us with a convenient characterization of welfare that holds true irrespectively of whether the allocation under consideration is an equilibrium one or not and that obtains in any given allocation. This characterization is exact as long as the aforementioned assumption holds, but can also be interpreted more generally as a second-order approximation.

[4] As shown in the Appendix, our decomposition of welfare in the next section (Proposition 1) and our key welfare lessons (Theorems 1 and 2) hold true even if $\alpha < 0$.

**Proposition 1.** *Welfare is given by*

$$\mathcal{W} = \mathbb{E}\left[\sum_t \beta^t W_t^* \Delta_t \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\}\right]$$

*where $W_t^*$ is the period-t utility that obtains at the first best, $\Delta_t$ captures the first-order welfare losses that obtain from suboptimality in the mean level of aggregate output, and $\Lambda_t$ captures the second-order welfare losses that obtain from uncertainty.*[5]

*For the first-order losses, we have that*

$$\Delta_t = \Delta(\delta_t) \equiv \frac{U(\delta_t) - V(\delta_t)}{U(1) - V(1)}$$

*where $\delta_t$ is the ratio between $\mathbb{E}[Y_t|\Theta_{t-1}]$, the mean (predictable) level of aggregate output that obtains in the allocation under consideration, and the one that would have maximized welfare.*

*For the second-order losses, we have that*

$$\Lambda_t = \Lambda(\Sigma_t, \sigma_t) \equiv \Sigma_t + \frac{1}{1-\alpha}\sigma_t$$

*where $\alpha < 1$ is defined as in (4) and where*

$$\Sigma_t \equiv Var[\tilde{Y}_t|\Theta_{t-1}] \qquad and \qquad \sigma_t \equiv Var[\tilde{y}_{it}|\Theta_t]$$

*measure, respectively, the volatility of aggregate output gaps and the cross-sectional dispersion of local output gaps (or, equivalently, the excessive dispersion in relative prices).*

This proposition generalizes related results that have been established in canonical, complete-information, macroeconomic models (e.g., Woodford, 2003b, Gali, 2008). The only key difference is that welfare losses relative to the first best may now originate, not only in markup and labor-wedge shocks, but also in noisy information. This difference, however, does not interfere with the insight that *any* type of welfare loss can conveniently be represented in terms of the associated gaps between equilibrium and first-best allocations. Establishing the robustness of this simple, but important, insight to imperfect information is the first key result of our paper.

To elaborate on the origins of these welfare losses, let us henceforth focus on equilibrium allocations and, to start, suppose that there are no shocks at either the macro or the micro level. In this case, there is no variation in output gaps at either the aggregate or the cross section ($\Sigma_t = \sigma_t = 0$), guaranteeing that all second-order welfare losses vanish ($\Lambda_t = 0$). Yet, first-order losses obtain as long as there is a positive markup or a positive labor wedge (in which case $\delta_t < 1$ and $W_t^* \Delta_t < W_t^*$).[6]

Next, consider the welfare implications of uncertainty. To start, suppose that information about the underlying shocks is perfect—the conventional scenario in the pertinent literature. In this case,

[5] By uncertainty here we mean both the one associated with the underlying structural shocks and the one associated with the noise in the available information.

[6] When $\gamma < 1$, $W_t^*$ is positive and $\Delta_t$ is strictly concave in $\delta_t$. When instead $\gamma > 1$, $W_t^*$ is negative and $\Delta_t$ is strictly convex. Either way, however, the product $W_t^* \Delta_t$ is strictly concave and reaches its unique maximum at $\delta = 1$.

the response of equilibrium allocations to productivity and taste shocks is efficient. This is the familiar result that flexible-price allocations are efficient in standard New-Keynesian models when the business cycle is driven only by productivity or taste shocks. It follows that the gap between equilibrium and first-best allocations can vary, whether in the aggregate or in the cross-section, only when there are shocks to monopoly markups and/or to labor wedges. Any *aggregate* variation in monopoly markups and labor wedges is then manifested in $\Sigma_t$, the volatility of aggregate output gaps, while any *idiosyncratic* variation in such markups and wedges is manifested in $\sigma_t$, the inefficient component of the dispersion in relative prices.

Finally, consider the case where information about the underlying shocks is imperfect. In this case, $\Sigma_t$ and $\sigma_t$ continue to capture the volatility and dispersion effects of any markup and labor-wedge shocks, to the extent that these shocks are present and the economy reacts to them. However, this effect now interacts with the incompleteness of information about these shocks. For example, less information has the potential of *reducing* both $\Sigma_t$ and $\sigma_t$ to the extent that it dampens the response of equilibrium allocations to the underlying markup and labor-wedge shocks. Intuitively, if agents have little information about this kind of shocks, they will "fail" to react to them, which is undesirable from a private perspective but desirable from a social perspective.

At the same time, less information has the potential of *raising* $\Sigma_t$ to the extent that it dampens the response of equilibrium allocations to the underlying productivity and taste shocks. Intuitively, if agents have little information about this kind of shocks, they will again "fail" to react to them–but now this failure increases the gap between equilibrium and first best and hence is socially undesirable. Finally, any noise in information by itself can contribute to variation in either aggregate or local output gaps, whether this information regards markup and labor wedge shocks, or productivity and taste shocks. Noise by itself can thus play a role akin to markup or labor-wedge shocks.

Furthermore, how the aforementioned effects will get manifested in $\Sigma_t$ and $\sigma_t$ is bound to depend on whether information is public or private—or, equivalently, on whether the associated noise is correlated or idiosyncratic. For example, an increase in the precision of the public information is likely to reduce the noise-driven dispersion in relative prices by motivating people to shift their attention away from private sources of information. At the same time, the volatility of aggregate output gaps may actually increase as people pay more attention to noisy public news—and perhaps the more so if, because of the underlying aggregate demand externalities, such public signals end up playing a coordinating role akin to sunspots.

This discussion suggests that, in general, the effects of information on volatility, dispersion, and welfare are likely to hinge on (i) the nature of the underlying shocks, (ii) the extent to which information is public or private, and (iii) the strength of aggregate demand externalities, as summarized in $\alpha$. We explore, and qualify, these intuitions in the subsequent sections.

## 5 Shocks to technologies and preferences

In this section we study the welfare effects of information for the special case in which the business cycle is driven only by preference and technology shocks. In particular, we impose that $\eta_{it} = \bar{\eta}$ and $\tau_{it} = \bar{\tau}$ for all islands, dates, and states, thereby ruling variation in either the monopolistic markup or the labor wedge. This helps us capture more generally the scenario in which the business cycle would have been efficient—in the sense that the output gap would be constant—had information been complete. It is this scenario that rests at the heart of the "divine coincidence" in modern New-Keynesian models; and it is this scenario that we focus on in this section.

To obtain closed-form solutions, we henceforth impose a Gaussian information structure. We also start with the case where agents observe only signals of the exogenous shocks. Finally, to simplify the exposition, we focus on technology shocks; the analysis for preferences shocks is identical modulo a change in notation/interpretation.

The log of local productivity, $a_{it} \equiv \log A_{it}$, is given by

$$a_{it} = \bar{a}_t + \xi_{it},$$

where $\bar{a}_t \equiv \log \bar{A}_t$ is the aggregate productivity shock and $\xi_{it}$ is an idiosyncratic productivity shock. The latter is Normally distributed with unconditional mean 0 and variance $\sigma_\xi^2 \equiv 1/\kappa_\xi$, orthogonal to $\bar{a}_t$, and i.i.d. across islands and time. Aggregate productivity, on the other hand, is given by

$$\bar{a}_t = \chi_t + \nu_t,$$

where $\chi_t = \chi(\bar{a}_{t-1}, \bar{a}_{t-2}, ...) \equiv \mathbb{E}[\bar{a}_t | \bar{a}_{t-1}, \bar{a}_{t-2}, ...]$ is the component of the current productivity shocks that is predictable on the basis of past productivity shocks, and $\nu_t$ is a Normal innovation, i.i.d. over time, independent of any other shock, and with mean 0 and variance $\sigma_a^2 \equiv 1/\kappa_a$.

Note that local productivity $a_{it}$ is itself a private signal of aggregate productivity $\bar{a}_t$. Each agent (or island) may have other sources of private information about the underlying productivity shock. We summarize all the private (local) information of island $i$ regarding the current aggregate productivity shock $\bar{a}_t$ in a sufficient statistic $x_{it}$ such that

$$x_{it} = \bar{a}_t + u_{it},$$

where $u_{it}$ is idiosyncratic noise, Normally distributed, orthogonal to $\bar{a}_t$ and i.i.d. across $i$ and $t$, with mean 0 and variance $\sigma_x^2 \equiv 1/\kappa_x$. In addition to this private information, every agent has also access to public information about the aggregate shock $\bar{a}_t$. This public information is summarized in a statistic $z_t$ such that

$$z_t = \bar{a}_t + \varepsilon_t, \tag{5}$$

where $\varepsilon_t$ is noise, Normally distributed with mean 0 and variance $\sigma_z^2 \equiv 1/\kappa_z$, and orthogonal to all other variables. The scalars $\kappa_x$ and $\kappa_z$ parameterize the precisions of available private and public information regarding the underlying productivity shock; the question of interest therefore reduces to the comparative statics of equilibrium welfare with respect to $\kappa_z$.

The above log-normal structure permit us to obtain a closed-form solution to both first-best and equilibrium allocations. In particular, it is easy to check that the first best satisfies[7]

$$\log y^*_{it} = \Psi \bar{a}_t + \psi (a_{it} - \bar{a}_t) \qquad \text{and} \qquad \log Y^*_t = \Psi \bar{a}_t \tag{6}$$

where $\Psi \equiv \frac{1+\epsilon}{\epsilon+\gamma}$ measures the response of equilibrium output to aggregate productivity shocks, while $\psi \equiv (1-\alpha)\Psi \equiv \frac{1+\epsilon}{\epsilon+1/\rho}$ measures the response to idiosyncratic shocks. The equilibrium, on the other hand, satisfies

$$\log y_{it} = \varphi_a a_{it} + \varphi_x x_{it} + \varphi_z z_t + \varphi_{-1} \bar{a}_{t-1}, \tag{7}$$

where $\varphi_a, \varphi_x, \varphi_z$, and $\varphi_{-1}$ are positive scalars that are determined by $\alpha, \Psi$, and the information structure (see the Appendix for a detailed proof and the characterization of these coefficients). The corresponding aggregate level of output is given by

$$\log Y_t = \Phi \bar{a}_t + \varphi_z \varepsilon_t + \varphi_{-1} \bar{a}_{t-1}, \tag{8}$$

where $\Phi \equiv \varphi_a + \varphi_x + \varphi_z$ is positive but lower than $\Psi$.

The fact that $\Phi < \Psi$ means that incomplete information dampens the response of the economy to the underlying aggregate productivity (or taste) shocks. This property is akin to how incomplete information dampens the response of nominal prices to monetary shocks in Woodford (2003), Nimark (2008), and Lorenzoni (2010). But whereas the results of those papers rest entirely on the failure of monetary policy to replicate flexible-price allocations, the fact we document here underscores how incomplete information may distort the response of the economy to technology and preference shocks even when prices are flexible—or, equivalently, when prices are sticky but monetary policy succeeds in replicating flexible-price allocations, which is actually the optimal thing to do as long as the business cycle is driven only by these kind of shocks.[8]

This subdued response of equilibrium allocations to the underlying productivity shocks means that the aggregate output gap is now variable and negatively correlated with these shocks, or equivalently positively correlated with the business cycle: the booms and recessions caused by these shocks are too shallow relative to the first best. At the same time, the noise in the public signal $z_t$ (or more generally any correlated errors in people's beliefs about aggregate economic activity) add independent variation in output gaps: a fraction of the booms and recessions is now driven by pure noise. Both of these effects contribute towards volatility in aggregate output gaps. Finally, the idiosyncratic noise in the private signals $x_{it}$ (or more generally any idiosyncratic errors in the aforementioned beliefs) causes variation in local output gaps in the cross-section of the economy,

[7]The formulas in conditions (6) through (8) are correct up to constants that are spelled out in the Appendix but are omitted in the main text for expositional simplicity.

[8]The positive implications of these insights are explored in Angeletos and La'O (2009), while the optimality of flexible-price allocations in the presence of informational frictions is studied in Angeletos and La'O (2011).

contributing to ineffcent dispersion in relative prices. The welfare effects of public information therefore hinge on the comparative statics of $\Sigma_t$ and $\sigma_t$, which we study next.

**Proposition 2.** *(i) An increase in the precision of public information reduces relative-price dispersion and has a non-monotone effect on the volatility of aggregate output gaps:*

$$\frac{\partial \sigma_t}{\partial \kappa_z} < 0 \text{ necessarily} \qquad \text{and} \qquad \frac{\partial \Sigma_t}{\partial \kappa_z} > 0 \text{ iff } \kappa_z < \hat{\kappa}$$

*where* $\hat{\kappa} \equiv (1-\alpha)\kappa_x - \kappa_0$.

*(ii) A stronger demand externality (stronger complementarity) raises the volatility of aggregate output gaps and has a non-monotone effect on relative-price dispersion:*

$$\frac{\partial \sigma_t}{\partial \alpha} < 0 \text{ iff } \alpha > \hat{\alpha} \qquad \text{and} \qquad \frac{\partial \Sigma_t}{\partial \alpha} > 0 \text{ necessarily}$$

*where* $\hat{\alpha} \in (1/2, 1)$.

To understand part (i), note that an increase in the precision of public information induces firms and workers to reduce their reliance on their private signals, which in turn reduces the contribution of idiosyncratic noise to cross-sectional dispersion in local output choices and relative prices. At the same time, because these agents increase their reliance on noisy public news, the contribution of the noise in these news to aggregate output gaps is ambiguous. On the one hand, the increase in the precision of public information means that the level of this noise is smaller, which tends to reduce $\Sigma_t$ for any given reaction to this noise. On the other hand, people are now reacting more to this information, which tends to raise $\Sigma_t$ for any given level of noise. Which effect dominates depends on how large the noise is: when the precision of public information is small enough, a (marginal) increase in it ends up contributing to more volatile output gaps.

The perverse effect of public information on aggregate output gaps depends, in part, on the coordinating role that public information plays in our framework: firms and households use the available public signals, not only predict the underlying fundamentals, but also to coordinate their choices. Furthermore, as people do so, aggregate output ends up moving away from the first best.

Formalizing this last insight, part (ii) documents that a higher $\alpha$ raises $\Sigma_t$: the stronger the underlying aggregate demand externalities and the associated coordinating role of public information, the greater the volatility of the resulting output gaps. At the same time, one can show that the overall volatility of equilibrium output actually *falls* with $\alpha$. This qualifies the precise sense in which the coordinating role of public information contributes to macroeconomic volatility: it is only the output gap, not output per se, that becomes more volatile as $\alpha$ increases.

This finding opens the door to the possibility that more precise public information ends up reducing welfare by raising the volatility of output gaps. Nonetheless, part (ii) of the above proposition also establishes that more precise public information helps dampen the excess dispersion in relative prices, which contribute to higher welfare. As it turns, this second, beneficial effect on relative price dispersion always dominates any potentially perverse effect on volatility. What is more, the overall positive effect on welfare is higher the higher $\alpha$.

**Proposition 3.** *An increase in the precision of public information necessarily reduces the joint welfare losses of the volatility and dispersion in output gaps:*

$$\frac{\partial \Lambda_t}{\partial \kappa_z} < 0 \tag{9}$$

*Furthermore, the corresponding welfare benefit of public information increases with the degree of strategic complementarity:*

$$\frac{\partial^2 \Lambda_t}{\partial \kappa_z \partial \alpha} < 0 \tag{10}$$

This result contrasts sharply Morris and Shin (2002), where the social value of information turns negative when the degree of complementarity is sufficiently large. In the class of economies we are concerned with, a higher $\alpha$ means, not only a stronger coordinating motive and thereby more volatile output gaps, but also a lower contribution of this volatility to welfare losses relative to that of cross-sectional dispersion. As a result, the reduction in dispersion $\sigma_t$ that obtains with more precise public information is more valuable when $\alpha$ is higher, which in turn helps this effect dominate the potentially perverse effect that public information may have on volatility $\Sigma_t$. In fact, not only is the overall effect necessarily positive, but it is also *increasing* in the degree of strategic complementarity. By contrast, the relative welfare contribution of volatility and dispersion are invariant to the degree of strategic complementarity in Morris and Shin (2002).[9] Their result therefore applies only to economies in which coordination motives are misalligned with social preferences—a scenario that does not apply to the flexible-price allocations of a canonical DSGE model.

An alternative, and perhaps more powerful, intuition to our results is also the following. Suppose there are no market distortions such as externalities, monopoly power, and labor wedges—or that the right policy instruments are in place to correct them. In this case, the equilibrium is first-best efficient when information is complete (commonly shared), but ceases to be so once information is incomplete (dispersed); the noise in the available information causes equilibrium allocations to diverge from their first-best counterparts. Nevertheless, equilibrium allocations remain *constrained efficient* in the sense that they coincide with the solution to a planning problem where the planner can dictate any allocation he wishes subject to the same resource and informational constraints as the market.[10] Note then that, by Blackwell's theorem, this planner cannot possibly be worse off with more information—if that were the case, he could simply have ignored the additional information. It is then immediate that, as long as the equilibrium attains the same allocations as this planner, equilibrium welfare *has* to increase with more precise public information.

This principle leaves outside economies where monopoly power or other market distortions create a discrepancy between equilibrium and constrained efficient allocations. Nonetheless, as long as this

[9] Translating this to our context, this is the same as imposing an ad hoc welfare objective in which $\Sigma_t$ and $\sigma_t$ enter $\Lambda_t$ symmetrically (as if $\alpha$ were zero), even though the equilibrium features a coordination motive (a positive $\alpha$). With such an ad hoc objective, we would also have obtained that the perverse effect of public information on $\Sigma_t$ dominates in some cases.

[10] See Angeletos and La'O (2009, 2011) for a definition and a proof of this kind of constrained efficiency.

discrepancy takes the form of a fixed (state-invariant) wedge between the relevant marginal rates of substitution and transformation, the social value of public information is bound to remain positive. This is because a fixed wedge affects the mean level of economic activity, but does not interfere with the response of the economy to either the underlying business-cycle disturbances or the available information about them.

Mapping this insight to our earlier welfare decomposition, this means that the aforementioned wedge implies positive first-order welfare losses ($\Delta_t < 1$), but these losses are invariant to the precision of the available public information ($\partial\Delta_t/\partial\kappa_z = 0$). The effects we documented for second-order welfare losses therefore directly translate to overall welfare: an increase in the precision of public information necessarily improves welfare, and the more so the higher the $\alpha$.

Symmetric results hold if we consider the effects of private rather than public information. In particular, a higher $\kappa_x$ can raise $\sigma_t$, the inefficient dispersion in relative prices, as agents pay more attention to private signals. At the same time, a higher $\kappa_x$ necessarily reduces $\Sigma_t$, the volatility of aggregate output gaps, as agents shift their attention away from public news. Finally, the latter effect always dominates, guaranteeing that welfare increases with the precision of private information. Our findings can thus be summarized as follows.

**Theorem 1.** *Suppose the economy is hit only by shocks to technologies and preferences. Depending on whether it is public or private, more precise information can have an adverse effect on either the volatility of the output gap or the dispersion in relative prices—but not on both at the same time. All in all, welfare necessarily increases with the precision of either public or private information.*

## 6 Shocks to markups and wedges

We now shift focus from technology and preference shocks to shocks in monopoly markups and labor wedges. The distinctive characteristic of this type of shocks is that they are a source of inefficient fluctuations under complete information.

Indeed, suppose for a moment that information were complete. If a benevolent planner had access to the necessary policy instruments, he or she would completely eliminate the fluctuations generated by shocks to monopoly markups and labor wedges. With flexible prices, this could be achieved only with a state-contingent subsidy on aggregate output (or some other regulatory or tax instrument that induces the same incentives). With sticky prices, monetary policy may also play a similar role. Either way, to the extent that the available policy instruments cannot perfectly offset these shocks, the residual fluctuations generated by these shocks contribute to welfare losses relative to the first best. It is this kind of fluctuations that are the focus of this section.

To explore how incomplete information interacts with the magnitude and the welfare consequences of this kind of fluctuations, we now shut down technology and preference shocks. For expositional simplicity, we further focus on the case of markup shocks alone. The case of labor-wedge shocks is identical, modulo a change of notation/interpretation.

Let $\mu_{it} \equiv \log\left(\frac{\eta_{it}}{1-\eta_{it}}\right)$ denote the log of the local markup of island $i$ in period $t$. This is given by

$$\mu_{it} = \bar{\mu}_t + \xi_{it},$$

where $\bar{\mu}_t$ captures an aggregate markup shock and $\xi_{it}$ is a purely idiosyncratic shock. The latter is Normally distributed with mean 0 and variance $\sigma_\xi^2$, orthogonal to $\bar{\mu}_t$, and i.i.d. across islands.[11] The aggregate markup shock, on the other hand, is given by

$$\bar{\mu}_t = \chi_t + \nu_t,$$

where $\chi_t = \chi(\bar{\mu}_{t-1}, \bar{\mu}_{t-2}, ...) \equiv \mathbb{E}[\bar{\mu}_t | \bar{\mu}_{t-1}, \bar{\mu}_{t-2}, ...]$ is the component of the current markup shock that is predictable on the basis of past public information and $\nu_t$ is a Normal innovation, with mean 0 and variance $\sigma_\mu^2 \equiv 1/\kappa_\mu$, i.i.d. over time. Finally, the information structure is the same as the one we assumed for the case with productivity shocks, except that the shocks themselves have a different meaning: the private and public information are summarized in, respectively,

$$x_{it} = \bar{\mu}_t + u_{it} \qquad \text{and} \qquad z_t = \bar{\mu}_t + \varepsilon_t,$$

where $u_{it}$ is idiosyncratic noise, with variance $\sigma_x^2 \equiv 1/\kappa_x$, while $\varepsilon_t$ is aggregate noise, with variance $\sigma_\varepsilon^2 \equiv 1/\kappa_z$. The precision of public information is thus parameterized, once again, by $\kappa_z$.

Clearly, the first best is invariant to markup shocks (and to any information thereof). The equilibrium, however, responds to these shocks (and to any information thereof). The above log-normal structure then permits us, once again, to obtain a closed-form solution to the equilibrium allocations and the associated gaps. In particular, the complete-information equilibrium satisfies

$$\log y_{it} = \Psi' \bar{\mu}_t + \psi(\mu_{it} - \bar{\mu}_t) \qquad \text{and} \qquad \log Y_t^* = \Psi' \bar{\mu}_t$$

where $\Psi' \equiv -\frac{1}{\epsilon+\gamma} < 0$ measures the response of equilibrium output to aggregate markup shocks, while $\psi \equiv (1-\alpha)\Psi \equiv -\frac{1}{\epsilon+1/\rho} < 0$ measures the response to idiosyncratic markup shocks. The incomplete-information equilibrium, on the other hand, satisfies

$$\log y_{it} = \varphi_\mu \mu_{it} + \varphi_x x_{it} + \varphi_z z_t + \varphi_{-1} \bar{a}_{t-1},$$

where $\varphi_\mu, \varphi_x, \varphi_z$, and $\varphi_{-1}$ are *negative* scalars that are determined by $\alpha, \Psi'$, and the information structure (see the Appendix for a detailed proof and the characterization of these coefficients). The corresponding aggregate level of output is given by

$$\log Y_t = \Phi' \bar{\mu}_t + \varphi_z \varepsilon_t + \varphi_{-1} \bar{\mu}_{t-1},$$

where $\Phi' \equiv \varphi_a + \varphi_x + \varphi_z$ is negative but higher (i.e., smaller in absolute value) than $\Psi'$.

Not surprisingly, local output decreases with either the local markup or any information about the aggregate markup. As a result, aggregate output decreases with either the true aggregate

[11] For expositional simplicity, we henceforth ignore the restriction that $\mu_{it}$ cannot be negative.

markup or any correlated error in people's beliefs about it. This captures a simple but important fact. Market distortions such as monopoly power and labor wedges have a powerful effect in the economy, not only due to their direct impact on individual payoffs and incentives, but also because of a powerful general-equilibrium feedback: a firm that expects the rest of the economy to experience an increase in monopoly power or other market distortions will find it optimal to reduce its own employment and production even if its own monopoly power or other local fundamentals remain unchanged. In this sense, the mere expectation of an inefficient recession may suffice for triggering an actual inefficient recession.

Turning now attention to how the available information impacts the magnitude of such inefficient fluctuations, we observe that the incompleteness of information dampens the response of aggregate output to the true aggregate markup shocks. This is similar to the property we observed earlier for the case of productivity shocks, except for one important difference. Whereas in that case this dampening effect contributed towards more volatile output gaps, now it contributes to the opposite: because the first best is now constant, dampening the response of equilibrium output to the underlying markup shocks helps stabilize the output gap.

This indicates that raising the precision of public information may now have an adverse effect on the volatility of the aggregate output gap, despite the reduction in the level of noise. At the same time, raising the precision of public information is likely to induce agents to reduce their reliance on any private information about the underlying aggregate distortions, which in turn may help reduce any inefficient dispersion in relative prices. We verify these intuitions in the next proposition, where, for comparison to the case of productivity shocks, we study the comparative statics of $\Sigma_t$ and $\sigma_t$ with respect to both the precision of public information and the strength of the coordinating motives.

**Proposition 4.** *(i) An increase in the precision of public information reduces the dispersion in relative prices and raises the volatility of the aggregate output gap:*

$$\frac{\partial \sigma_t}{\partial \kappa_z} < 0 \qquad \textit{and} \qquad \frac{\partial \Sigma_t}{\partial \kappa_z} > 0$$

*(ii) A stronger aggregate demand externality (stronger complementarity) reduces both the dispersion in relative prices and the volatility in aggregate output gaps:*

$$\frac{\partial \sigma_t}{\partial \alpha} < 0 \qquad \textit{and} \qquad \frac{\partial \Sigma_t}{\partial \alpha} < 0$$

The intuition behind part (i) was already discussed. To understand part (ii), note first that a stronger coordination motive (higher $\alpha$) induces people to react less to private information, which dampens the impact of idiosyncratic noise on equilibrium allocations and thereby reduces the excess dispersion in relative prices. This effect is thus the same as in the case of productivity shocks. By contrast, the effect of $\alpha$ on the volatility of output gaps is different for essentially the same reason that the effect of public information is also different. As in the case of productivity shocks, a higher $\alpha$ amplifies the contribution of noise to output gaps. Yet, unlike the case of productivity shocks, a higher $\alpha$ helps stabilize output gaps by raising the anchoring effect of the common prior and thereby dampening the overall response of the output gap to the underlying markup shocks (formally, $\Psi'$ falls with $\alpha$). This effect is strong enough to guarantee that $\Sigma_t$ falls with $\alpha$. Thus, in the case of markup shocks, stronger coordination motives help stabilize output gaps.

Returning to the social value of public information, the conflicting effects on volatility and dispersion raises the possibility that the joint effect on welfare is ambiguous. The next proposition establishes that this is not the case: the adverse effect on volatility necessarily dominates.

At the same time, public information now impacts welfare, not only through volatility and dispersion, but also through the suboptimality of the mean level of output. The overall welfare effect of public information thus hinges on the comparative statics of both $\Lambda_t$ and $\Delta_t$.

**Proposition 5.** *An increase in the precision of public information necessarily increases the second-order welfare losses that obtain from volatility and dispersion:*

$$\frac{\partial \Lambda_t}{\partial \kappa_z} > 0$$

The joint effect of public information on volatility and dispersion is therefore the exact opposite than in the case of productivity shocks. At the same time, public information now impacts welfare, not only through volatility and dispersion, but also through the mean level of output.

**Proposition 6.** *An increase in the precision of public information necessarily reduces the inefficiency in the mean level of output and therefore reduces the associated first-order welfare losses:*

$$\frac{\partial (W_t^* \Delta_t)}{\partial \kappa_z} > 0$$

This finding can be explained as follows. When firms and workers face more uncertainty about the underlying aggregate shocks, the mean level of equilibrium output tends is lower. This effect is present irrespectively of whether the shocks are in preferences and technologies or in markups and labor wedges; it follows from convexity in preferences and technologies. However, the normative consequences of this effect hinge on the nature of the underlying shocks. In the case of productivity or taste shocks, the impact of uncertainty on equilibrium reflects social incentives: a benevolent planner would have reacted to the increase in uncertainty in exactly the same way as the equilibrium. This explains why $\delta_t$ is invariant to the precision of public information in the case of productivity or taste shocks. By contrast, in the case of markup or labor-wedge shocks, the impact of this uncertainty on the equilibrium is stronger than the socially optimal one. By reducing this uncertainty, more precise public information then helps reduce the associated inefficiency in the mean level of output, and thereby also to reduce first-order welfare losses.

By the envelope theorem, one may expect the gains from raising the mean level of output to be small when $\delta_t$ is close enough to 1: when the distortion is small, a marginal change in this distortion has a trivial welfare effect. This intuition suggests that the welfare losses due to volatility and dispersion are likely to dominate as long as the mean distortion is not too large. Finally, as in the case with productivity shocks, we expect the effects of private information to be symmetric to those of public. We verify these intuitions in the following theorem.

**Theorem 2.** *Suppose that the economy is hit only by shocks to monopoly markups and labor wedges. There exists a threshold $\hat{\delta} \in (0,1)$ such that welfare decreases with the precision of either public or private information if and only if $\delta_t > \hat{\delta}$.*

To interpret this result, recall that $\delta_t$ is the ratio of $\mathbb{E}_{t-1}[Y_t]$ to the level of output that would have maximized welfare. Hence, as long as $\delta_t < 1$, the condition $\delta_t > \hat{\delta}$ means, in effect, that the average value of the aggregate output gap is not too large. Furthermore, note that $\delta_t$ is a decreasing function of the mean value of the monopoly markup and the labor wedge. By contrast, the threshold $\hat{\delta} \in (0,1)$ is invariant to the monopoly markup and the labor wedge.[12] Thus, whether one measures the distortion by the underlying wedges or the resulting output gaps, the message remains the same: in the case of inefficient fluctuations, more precise information reduces welfare as long as the mean distortion is small enough.[13]

# 7 Macroeconomic statistics

The preceding analysis has studied the welfare effects of information under a particular specification of the information structure: we summarized the available information in exogenous signals of the shocks hitting the economy. While standard practice in the related literature, this specification is not best suited for practical questions. Consider, in particular, the following question: how does welfare depend on the quality of the macroeconomic statistics publicized by the government, or of the financial news disseminated by the public media? Unfortunately, such sources of information are *not* direct signals of the exogenous shocks hitting the economy. Rather, they are only noisy indicators of the *behavior* of other agents—they are signals of the *endogenous* state of the economy.

To capture this fact, we modify the preceding analysis as follows. In addition to (or in place of) any exogenous signals of the underlying shocks, firms and workers can now observe a noisy public indicator of the ongoing level of aggregate output. This indicator is given by

$$\omega_t = \log Y_t + \varepsilon_t \tag{11}$$

where $\varepsilon_t \sim N\left(0, \sigma_\omega^2\right)$ represents classical measurement error and $\sigma_\omega > 0$ parameterizes the level of this measurement error. The results are identical if we consider other indicators of aggregate economic activity, such as noisy public signals of aggregate employment and consumption, or a survey of opinions regarding any of these macroeconomic variables. The scalar $\kappa_\omega \equiv \sigma_\omega^{-2}$ can thus

[12] See the Appendix for the exact characterization of both $\hat{\delta}$ and $\delta_t$.

[13] Clearly, this condition is automatically satisfied if the government has access to a subsidy or some other policy instrument that permits to eliminate any predictable distortion and thereby to induce $\delta_t = 1$ $(> \hat{\delta})$.

be interpreted more generally as the precision, or quality, of the available macroeconomic statistics and of any other public information regarding the endogenous state of the economy.

No matter whether the underlying shocks are in preference and technologies or in markups and labor wedges, the aforementioned macroeconomic indicator will serve, in equilibrium, as a signal of these shocks: variation in equilibrium output signals variation in the underlying fundamentals. However, the informational content of this indicator—formally, the signal to noise ratio—depends crucially on agents' behavior. It follows that, pretty much as in other rational-expectations settings (e.g., Lucas, 1972, Grossman and Stiglitz, 1980), the equilibrium must now be understood as a fixed point between the information structure and the equilibrium allocations.

In general, this fixed-point problem can be intractable. However, the combination of a power-form specification for preferences and technologies and a log-normal specification for the exogenous shocks guarantees the existence of a log-linear Rational Expectations equilibrium.

To understand this, suppose that the economy is hit only by productivity shocks. Take any allocation for which local output can be expressed as a log-linear function of the local shocks and the private information, and let $\varphi_a$ and $\varphi_x$ denote the corresponding sensitivities of local log output. That is, suppose

$$\log y_{it} = \varphi_a a_{it} + \varphi_x x_{it} + h_t,$$

where $h_t$ is an arbitrary function of time, of the past aggregate productivity shocks, and of any other variable that is common knowledge as of the beginning of period $t$. Then, aggregate output is given by $\log Y_t = (\varphi_a + \varphi_x)\bar{a}_t + h_t$ and, since $\varphi_a, \varphi_x$ and $h_t$ are commonly known, observing the statistic $\omega_t$ is equivalent to observing a signal of the form

$$\omega_t' \equiv \frac{\omega_t - h_t}{\varphi_a + \varphi_x} = \bar{a}_t + \varepsilon_t', \quad \text{where} \quad \varepsilon_t' \equiv \tfrac{1}{\varphi_a + \varphi_x}\varepsilon_t.$$

This is akin to the exogenous public signal we had assumed in the preceding analysis, except for one important difference: the errors in this signal are inversely proportional to the sum $\varphi_a + \varphi_x$ and, in this sense, the precision of this signal is endogenous to the allocation under consideration. This captures a more general principle: the signal-to-noise ratio in macroeconomic statistics is pinned down by the sensitivity of equilibrium allocations to the underlying shocks to fundamentals.

Notwithstanding this important qualification, note that that this endogenous signal is Gaussian, just as the exogenous ones. It follows that the available public information can be summarized in a Gaussian sufficient statistic whose precision is given by the sum of the precisions of the exogenous and endogenous signals. In effect, this means that our preceding analysis goes through once we let

$$\kappa_z = \tilde{\kappa}_z + (\varphi_a + \varphi_x)^2 \kappa_\omega, \tag{12}$$

where $\tilde{\kappa}_z$ measures the precision of the exogenous public signal and $\kappa_\omega$ parameterizes the quality of macroeconomic statistics. We conclude that the equilibrium is now the fixed point between the allocation described in condition (7) and the precision of information described in (12).

With these observations at hand, we can now determine the social value of macroeconomic statistics simply by studying how $\kappa_z$ varies with $\kappa_\omega$. This is because the derivative of welfare with respect to $\kappa_\omega$ is equal to the derivative of welfare with respect to $\kappa_z$, which can be read off directly from Theorems 1 and 2, times the derivative of $\kappa_z$ with respect to $\kappa_\omega$.

Clearly, if the sum $\varphi_a + \varphi_x$ were fixed, $\kappa_z$ would increase one-to-one with $\kappa_\omega$. That is, if the response of equilibrium output to the underlying productivity shocks were invariant to the information structure, an increase in the quality of macroeconomic statistics would translate one-to-one to an increase in the precision of the public information. Any increase in $\kappa_z$, however, implies a reduction in the equilibrium value of $\varphi_x$: as the available public information becomes more precise, firms and households alike pay less attention to private information. By itself, this effect contributes to a lower signal-to-noise ratio in the observed macroeconomic statistic and therefore to a lower equilibrium value for $\kappa_z$: as people pay less attention to their private signals, the efficacy of social learning falls. Nonetheless, this effect is never strong enough to offset the aforementioned direct effect of $\kappa_\omega$ on $\kappa_z$. To see this, suppose, towards a contradiction, that the overall effect were negative. If that were the case, agents would have found it optimal to put *more* weight on their private signals, which would have implied more social learning. Both the direct and the indirect effects would then have contributed to an increase in $\kappa_z$, contradicting the original claim.

We conclude that an increase in the quality of macroeconomic statistics necessarily increases $\kappa_z$, which in turn permits us to translate all the results of the preceding analysis to the more realistic scenario in which public information regards indicators of aggregate economic activity rather than direct signals of the underlying structural shocks. The same is true for the case of markup shocks, modulo a re-interpretation of the shocks and the signals—the welfare effects of $\kappa_z$ are now different, but the positive relation between $\kappa_z$ and $\kappa_\omega$ holds true no matter whether the underlying shocks are in technologies, markups or other fundamentals.

**Proposition 7.** *Theorems 1 and 2 continue to hold if the precision of public information is re-interpreted as the quality of macroeconomic statistics.*

This result need not hinge on the details of the available macroeconomic statistics: we could replace the aforementioned signal of aggregate output with a signal of aggregate employment or consumption, a survey of people's forecast of these macroeconomic outcomes, or any combination of the above. This result may nevertheless depend on the absence of private forms of social learning. By this we mean the following. Suppose that, in addition to the aforementioned publicly observable macroeconomic statistics, each island also observes a noisy private signal of the output or employment level of a neighboring island, or some other private signal of the actions of other agents in the economy. This renders the *private* information that is available to each agent endogenous to the choices of other agents, pretty much as in the case of public information above. But now note that an improvement in the quality of the macroeconomic statistics may reduce the efficacy of these private forms of social learning: as each agent pays more attention to these public signals and less to his private information, the signal-to-noise ratio in the aforementioned private signals deteriorates. This deterioration, in turn, tends to have the exact opposite effect on welfare than the improvement in macroeconomic statistics: it contributes towards lower welfare in the case of productivity shocks, and towards higher welfare in the case of markup shocks.

This possibility, which is at the core of Amador and Weill (2010, 2011), qualifies the robustness of Proposition 7. Nonetheless, note that Theorems 1 and 2 characterize the welfare effects of the available information *no matter* where this information originates from. Therefore, even if one seeks to study environments that endogenize either the collection or the aggregation of information, our results remain indispensable: the welfare implications of any change in the environment that involves, directly or indirectly, a change in the available information hinges on the effects we have documented in Theorems 1 and 2.[14]

# 8 A numerical exploration

Our framework is too stylized to permit a serious quantitative analysis.[15] This qualification notwithstanding, we now consider a numerical exercise that permits us to further explore the determinants of the social value of information within the context of business cycles.

To this goal, we let the economy be hit by both productivity and markup shocks. Both shocks are log-normally distributed: $\bar{a}_t \sim \mathcal{N}(0, \sigma_a^2)$ and $\bar{\mu}_t \sim \mathcal{N}(0, \sigma_\mu^2)$, where $\sigma_a$ and $\sigma_\mu$ parameterize the volatilities of the two shocks.[16] The local productivity and markup shocks are log-normally distributed around the corresponding aggregates and, for simplicity, comprise the entire private information of an island. Finally, the public information consists of a noisy statistic of aggregate output, defined again as in (11): $\omega_t = \log Y_t + \varepsilon_t$, where $\varepsilon_t \sim N\left(0, \sigma_\omega^2\right)$ is a measurement error and $\sigma_\omega > 0$ is its standard deviation.

This statistic now reveals information about *both* types of shocks: there exist scalars $\lambda_a, \lambda_\mu, \lambda_\varepsilon \in \mathbb{R}_+$ and $\lambda_0 \in \mathbb{R}$ such that, in equilibrium,

$$\omega_t = \lambda_0 + \lambda_a \bar{a}_t - \lambda_\mu \bar{\mu}_t + \lambda_\varepsilon \varepsilon_t \tag{13}$$

[14]For example, this observation is key to understanding how the insights of Amador and Weill (2010, 2011) may apply to the class of business-cycle economies we are interested in: those papers point out why more precise public information might slow down private learning, but it is only the analysis of our paper that informs one how this will affect output gaps, relative price dispersion, and overall welfare in a canonical business-cycle model. A similar point applies to the design of optimal policy when information is endogenous (see the discussion in the end of Section 9).

[15]Among other simplifications, we have ruled out from all forms of dynamic interdependence, such as investment and adjustment costs, thereby also abstracting from how current economic activity depends on expectations of future economic activity. Furthermore, we have assumed that the state of the economy become common knowledge by the end of each period, thereby abstracting from the rich informational dynamics that can emerge when learning takes place slowly over time. While none of these simplifications is likely to impact the essence of our theoretical insights, they are certainly relevant for quantitative questions.

[16]Adding persistence to these shocks has small effects on our results. One should only interpret $\sigma_a$ and $\sigma_\mu$ as the volatilities of the innovations in these shocks.

where $\bar{a}_t$ and $\bar{\mu}_t$ are the aggregate productivity and markup shocks. A higher $\omega_t$ is thus interpreted partly as a signal of higher productivity and partly as a signal of lower markup. Either way, the impact on aggregate employment and output is positive: positive news about macroeconomic activity are bound to stimulate the economy irrespectively of whether these news reflect a higher aggregate productivity, a lower market distortion, or even a positive measurement error. The welfare consequences, however, crucially depend on the nature of the underlying shocks.

**Parameterization.** We interpret a period as a year and target a standard deviation of aggregate output growth equal to 0.02, which is consistent with US data. Together with the rest of the preference and technology parameters, this target pins down the overall volatility of the underlying shocks, but leaves undetermined the relative contribution of the two different types of shocks. We will later discuss how an estimate of the relative contribution of productivity and markup shocks can be obtained on the basis of the pertinent literature. To start with, however, we prefer to stay agnostic about this relative contribution. We thus proceed as follows. First, we measure the relative importance of the two shocks with the following statistic:

$$R \equiv \frac{(\Psi\sigma_a)^2}{(\Psi\sigma_a)^2 + (\Psi'\sigma_\mu)^2},$$

To interpret $R$, suppose for a moment that information were complete. Aggregate output would then be given (up to a constant) by $\log Y_t = \Psi\bar{a}_t + \Psi'\bar{\mu}_t$ and $R$ would thus coincide with the fraction of the volatility of aggregate output that is driven by productivity shocks. We thus think of $R \in [0, 1]$ as a measure of the relative importance of the two different types of shocks: a higher $R$ means that a larger fraction of the business cycle is efficient. We then let $R$ take any value in $[0, 1]$ and, for any given value of $R$, we choose $\sigma_a^2$ and $\sigma_\mu^2$, the volatilities of the shock innovations, so as to match both the particular value of $R$ and the target value of the standard deviation of output.

Next, to calibrate the standard deviation of the measurement error in the signal of aggregate output, we look at the BEA releases of quarterly GDP. For a given quarter, the BEA publishes different estimates of GDP as more information becomes available. We assume that the last estimate is the true value of GDP. We set $\sigma_\omega = 0.02$, which is close to the standard deviation of the error between the first and the last release found in the literature (e.g., Fixler and Grimm, 2005).

Next, we need to parameterized the idiosyncratic noise in private information. Clearly, this is challenging, because there is no obvious way to measure private information. We partly bypass this problem by imposing that private information consists only of the local productivity and markup shocks; this restriction is motivated by the idea that, for most firms and households, private information is likely to be limited to their idiosyncratic circumstances. The precision of private information is then pinned down by the variance of the innovations in the idiosyncratic shocks.[17]

[17] Another possibility would be to calibrate the private information to match the heterogeneity observed in surveys of forecasts. This approach, however, has its own problems because it is unclear how reliable these surveys are. Given the limited scope of the numerical exercise we conduct here, we leave further exploration of this issue to future work.

Even that last option, however, is not obvious how to implement. One of the main difficulties is that different assumptions about the markup charged by a firm imply different estimates of its Total Factor Productivity (TFP). To get a rough estimate of the idiosyncratic risk in TFP, we can refer to the NBER-CES Manufacturing Database, which computes a measure of TFP for all 6-digit NAICS manufacturing industries in the US is 2005. This gives an estimate of the standard deviation of idiosyncratic TFP growth of 0.06. One may argue using a higher value to take into account the fact that the firms in the NBER-CES dataset are a more homogeneous and less volatile subset of all the firms in the US economy. Looking at sales would also suggest a bigger number for the idiosyncratic risk faced by a firm. For the markups, on the other hand, we follow the international economics literature (e.g., Tybout, 2003, Epifani and Gancia, 2010) and use the standard deviation of price-cost margins (that is, sales net of expenditures on labor and materials) as a proxy for the idiosyncratic variation in market power. Doing so yields an estimate of the standard deviation of idiosyncratic markups of about 0.1.

Based on these observations, and lacking a better alternative, we choose 0.08 as our baseline value for the standard deviation of both types of idiosyncratic shocks. Together with our restriction that private information coincides with the local shocks, this means that the noise in private information regarding the aggregate state of the economy is roughly four times as large as the noise in public information. This sounds plausible, given that the main sources of information about aggregate economic activity are likely to be public. Svensson (2005) uses a similar argument to motivate his assumption that public information is likely to be far more precise than private information. In any event, our numerical findings do not appear particularly sensitive to the value of $\sigma_\xi$.

Turning to the remaining parameters of the model, consider $\epsilon$ and $\gamma$. The former gives the inverse of the Frisch elasticity of labor supply. The latter is typically interpreted as either the inverse of the elasticity of intertemporal substitution or the coefficient of relative risk aversion. Nevertheless, as emphasized by Woodford (2003), the main role of $\gamma$ in a model without capital is to control the income elasticity of labor supply.[18] Accordingly, we follow Woodford (2003) and set $\epsilon = .3$ and $\gamma = .2$. These values are such that the complete-information version of our model (which is, in effect, the basic RBC model without capital) replicates the empirical regularity that output and employment move closely together over the business cycle, while real wages are almost acyclical. We also set the average markup to 0.15, which is roughly consistent with estimates for the US economy.

The last parameter to set is the degree of strategic complementarity, $\alpha$. In our model, $\alpha$ identifies the elasticity of individual output to aggregate output for given local fundamentals; more generally, it captures the sensitivity of actual economic activity to expectations of economic activity. Causal observation suggests that this elasticity is high. Unfortunately, however, we are not aware of any estimate of this kind of elasticity—nor is it of course clear if the particular micro-foundations we

[18] To see this, take the representative-agent version of our model. Labor supply is then given by $n_t = w_t^{1/\epsilon} C_t^{-\gamma}$, which reveals the key role played by the parameters $\epsilon$ and $\gamma$. That been said, $\gamma$ also determines risk aversion, which plays evidently a role in the welfare effects of any kind of uncertainty. We return to this point below.

have considered here capture the complementarities that may be present in reality because of the far richer pattern of specialization, trade, and aggregate demand externalities.

Lacking a clear benchmark, we opt to set $\rho = 1$, which together with our choice of $\epsilon$ and $\gamma$ implies a degree of strategic complementarity $\alpha = 0.6$. The motivation behind this choice is the following. Depending on how one interprets the islands of our model—as industries, classes of intermediate inputs, or even individual firms—one could argue that these goods are either complements ($\rho < 1$) or substitutes ($\rho > 1$). By setting $\rho = 1$ (Cobb-Douglas), we take the middle ground. Nonetheless, given the uncertainty we face about the appropriate calibration of the strength of aggregate demand externalities and the degree of strategic complementarity, we will later consider the sensitivity of our numerical findings to a wide range of values for $\alpha$. As anticipated by our earlier theoretical results, the precise value of $\alpha$ is irrelevant for qualitative effects—but it is important for magnitudes.

**Results.** With the aforementioned parameterization at hand, the exercise we conduct is to compute the welfare consequences of eliminating the measurement error in the available macroeconomic statistic. More specifically, keeping all other parameters constant, we compute the welfare gain or loss of moving the economy from $\sigma_\omega = .02$ (our baseline value for the level of noise in macroeconomic statistics) to $\sigma_\omega = 0$ (which means, in effect, perfect public information). This welfare gain or loss is computed in consumption-equivalent units (i.e., as a fraction of the mean level of consumption) and is normalized by a measure of the welfare cost of the business cycle as in Lucas (1987).[19]

Figure 1 illustrates how the welfare effects of perfecting the macroeconomic statistics vary as one varies $R$, the relative contribution of productivity and markup shocks. The solid line in this figure represents the total welfare effect. The other two lines decompose the total welfare effect between the effect that obtains via the impact of information volatility and dispersion (i.e., via $\Lambda_t$) and the one that obtains via the impact of information on mean level of output (i.e., via $\Delta_t$)

In line with Theorem 1, we see that reducing the measurement error in the macroeconomic statistics—equivalently, increasing the precision of the available public information—improves welfare when productivity shocks drive a large enough fraction of the business cycle (i.e., for high values of $R$). Furthermore, the welfare gains can be non trivial, at least relative to a Lucas-type measure of welfare cost of the business cycle: when productivity shocks drive the business cycle, the welfare gains of reducing the noise in macroeconomic statistics are roughly equal to the welfare gains of removing aggregate consumption risk.

As for the case where the business cycle reflects mostly variation in monopoly markups or other

[19] More specifically, the norm we use is the consumption-equivalent welfare gain of eliminating the risk in aggregate consumption when information is complete and fluctuations are driven by productivity shocks. We choose such a normalization for two reasons. First, by abstracting from capital and choosing a low value for $\gamma$ in order to match the cyclical behavior of the economy, we have underestimated risk aversion and, in so doing, we have underestimated the welfare effects of any type of uncertainty. And second, the welfare effects of aggregate uncertainty are known to be notoriously small within the class of elementary DSGE models we are working with. Our normalization seeks to bypass these two issues by focusing on relative rather than absolute welfare effects. Enrichments of the model that increase the welfare costs of the business cycle are likely to increase as well the welfare effects we document here.

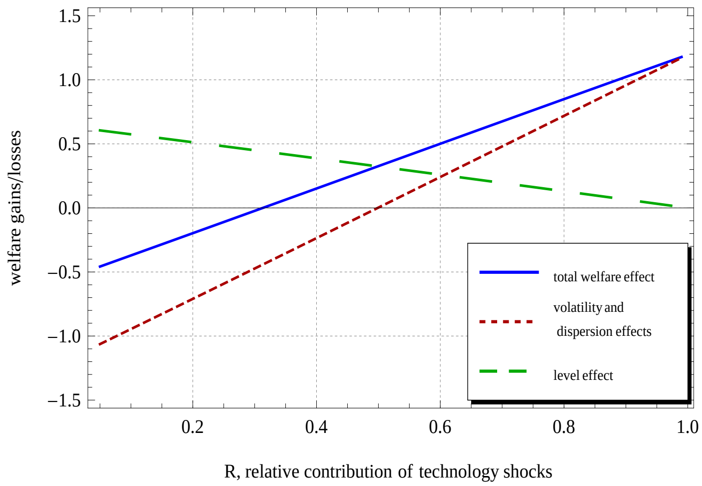


Figure 1: Welfare effects from eliminating the measurement error in the macroeconomic statistic as a function of $R$, the fraction of the business cycle that is driven by productivity shocks.

market distortions (i.e., for small values of $R$), recall from Theorem 2 that information reduces welfare if and only if its joint detrimental effect on volatility and dispersion outweigh the beneficial one on the predictable (mean) level of economic activity. Figure 1 then reveals that this is indeed the case for the parameterization under consideration.

To clarify this issue, the dashed and dotted lines in Figure 1 separate the welfare effect that obtains via the impact of information on the mean level of output from the one that obtains via its impact on volatility and dispersion. The latter effect is positive if and only if $R > \frac{1}{2}$, that is, if and only if productivity shocks explains at least a half of the business-cycle volatility in output. The effect via the mean level of output, on the other hand, is positive if and only if $R < 1$, that is, as long as there are markup shocks. Furthermore, this is decreasing in $R$, the relative contribution of productivity shocks, and vanishes at $R = 1$. It follows that there exists a threshold $\hat{R} < \frac{1}{2}$ such that the total welfare effect is positive if and only if $R > \hat{R}$.

Clearly, the threshold $\hat{R}$ is determined by the conflict between the beneficial effect that more precise public information has on the mean level of output and the adverse effect it has on output volatility and price dispersion when markup shocks are sufficiently prevalent. As anticipated in our earlier discussion of Theorem 2, the strength of the former effect crucially depends on how big the mean distortion is: the further away output is from its optimal level, the bigger the welfare gains from a given marginal increase in output.

The role of the mean level of market distortions is illustrated Figure 2. The left panel in that figure repeats the exercise of Figure 1 after resetting the mean markup to zero—or, equivalently, after introducing a subsidy that undoes the mean monopolistic distortion. The impact of information on the mean level of output now has a trivial welfare effect, implying that the total welfare effect is pinned down almost entirely by the one via volatility and dispersion.

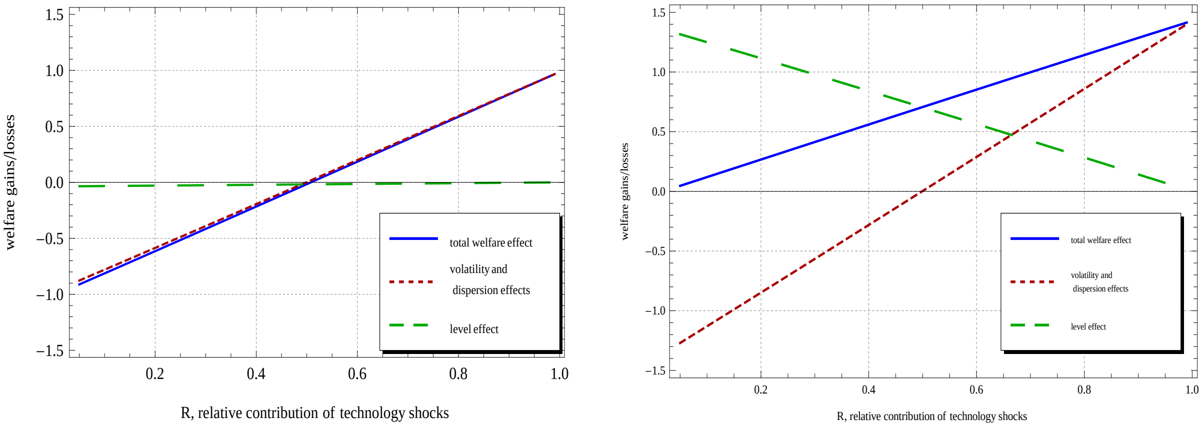


Figure 2: Welfare effects when the mean monopolistic distortion is reset to zero (left panel) or to a sufficiently large level (right panel).

Conversely, if we let the mean distortion to be sufficient severe—which, in our numerical example, translates to a mean markup above 36%—then the impact on the mean level of output is so strong that the welfare effect becomes positive for all $R$. This is the case illustrated in the right panel of Figure 2. We conclude that, other things equal, a sufficiently large mean distortion suffices for information to be welfare-improving even when the business cycle is driven entirely by variation in monopoly markups and other market distortions. If one takes into account tax distortions, this scenario might actually be empirically relevant.

Turning attention to the sensitivity of our findings to other parameters, Figure 3 studies the comparative statics of the threshold value $\hat{R}$ at which the welfare effects of information change sign, from negative for $R < \hat{R}$ (where the markup shocks are sufficiently prevalent) to positive for $R > \hat{R}$. As evident in this figure, the threshold $\hat{R}$ tends to increase with either a higher $\epsilon$ or a higher $\gamma$. This is because an increase in either $\epsilon$ or $\gamma$ tends to increases the welfare costs of volatility and dispersion relative to those of the mean distortion in output, which in turn means that the former end up playing a more important role in determining the total welfare effects of information. At the same time, we see that, for given $\epsilon$ and $\gamma$, this threshold is almost entirely invariant to $\alpha$, the degree of strategic complementarity (or, equivalently, the strength of aggregate demand externalities).

This last property underscores, once more, the contrast between our results and those of Morris and Shin (2002). In the "beauty contests" studied by them and much of the subsequent literature, the welfare effect of public information turns negative once the degree of strategic complementarity is sufficiently high and public information is sufficiently noisy. By contrast, in the class of business-cycle economies we are interested in, the degree of strategic complementarity appears to play no noticeable role in determining the *sign* of the welfare effects of information: whether information improves welfare or not depends on the relative contribution of the different shocks, not on the

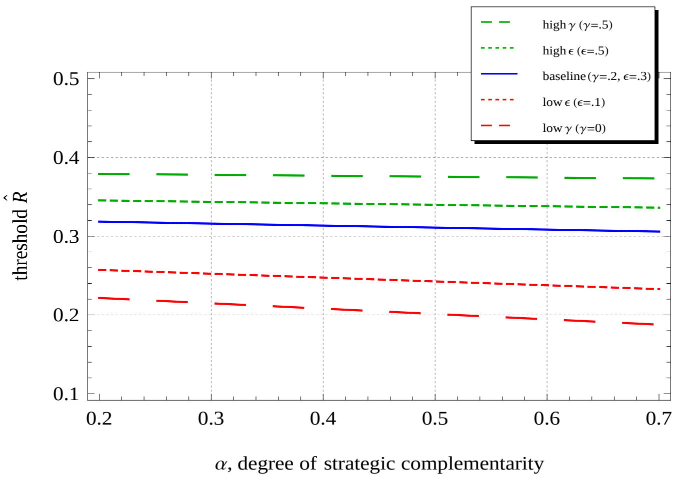


Figure 3: Sensitivity of the threshold $\hat{R}$ for the relative contribution of productivity shocks at which the welfare effects of information change sign, from negative for $R < \hat{R}$ to positive for $R > \hat{R}$.

strength of the aggregate demand externalities and the consequent coordination motives.[20]

That been said, the degree of strategic complementarity emerges as a key determinant of the *magnitude* of the welfare effects. This is evident in Figure 4, which studies the sensitivity of the welfare effects under the two extreme scenarios where the business cycle is driven either only by productivity shocks ($R = 1$, left panel in the figure) or only by markup shocks ($R = 0$, right panel). In either case, the absolute value of the welfare effects is increasing in $\alpha$: stronger coordination motives are associated with bigger welfare gains when the business cycle is driven by productivity shocks, and with bigger welfare losses when the business cycle is driven by markup shocks. Intuitively, this is because a higher $\alpha$ implies that firms and workers' decisions are more sensitive to the uncertainty they face about aggregate economic activity, which in turn amplifies the impact of noise on equilibrium allocations and thereby on welfare. Conversely, the welfare effects vanish as $\alpha$ approaches zero, for in that case expectations of aggregate economic activity—and hence any information either about the latter or about the underlying aggregate shocks—become entirely irrelevant for either equilibrium or welfare.[21]

Figure 4 also illustrates the sensitivity of the welfare effects to $\epsilon$ and $\gamma$. Whether the business cycle is driven by productivity shocks (left panel) or markup shocks (right panel), the welfare effects increase in absolute value when we reduce either $\epsilon$ or $\gamma$, that is, when the economy becomes more elastic to the underlying shocks. Yet, notwithstanding the uncertainty we face regarding the relevant

[20] As it turns out, the same is true for the level of noise: the threshold $\hat{R}$ is largely invariant to $\sigma_\omega$.

[21] Note, however, that $\alpha = 0$ is a knife-edge case. As long as $\alpha$ is *either* positive or negative, these expectations become crucial. As a result, the welfare effects of information turn out to be qualitatively the same irrespectively of whether $\alpha$ is positive or negative. For instance, suppose we take the extreme case in which the good of different islands are perfect substitutes, in which case $\rho = \infty$ and $\alpha = -2/3 < 0$. The welfare effects are then qualitatively identical to those of Figure 1, although they are smaller in magnitude.

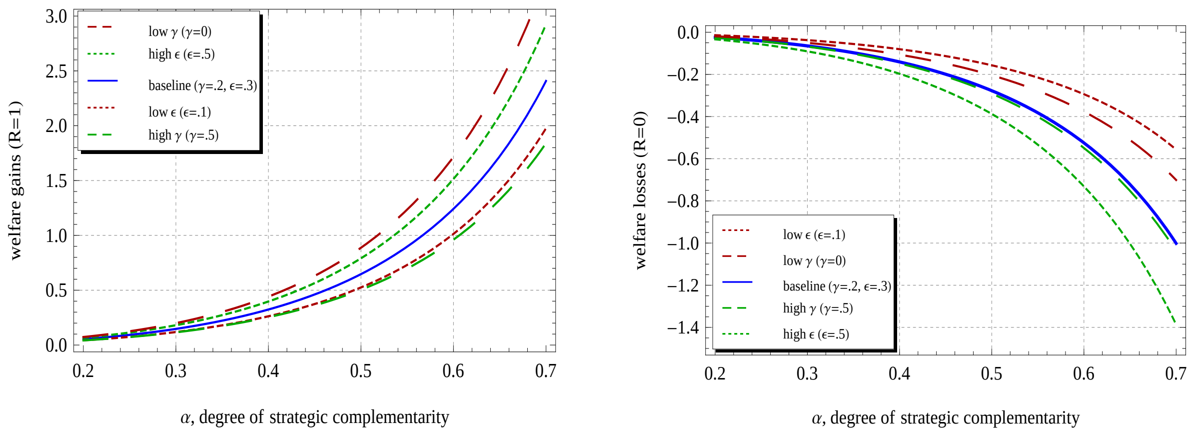


Figure 4: Sensitivity of welfare effects to $\alpha, \gamma$, and $\epsilon$. Left panel corresponds to $R = 1$ (productivity shocks), right panel corresponds to $R = 0$ (markup shocks).

range of values for $\alpha$, the impact of $\epsilon$ and $\gamma$ seems to be small relative to that of $\alpha$.

Figure 5 concludes our numerical exploration by examining the sensitivity of the welfare effects to $\sigma_\omega$ and $\sigma_\xi$. Not surprisingly, raising $\sigma_\omega$ amplifies the welfare effect of eliminating the measurement error in macroeconomic statistics: the larger the noise in macroeconomic statistics, the larger the welfare gains of eliminating that noise in the case of productivity shocks, and the larger the welfare losses in the case of mark up shocks. The impact of $\sigma_\xi$, on the other hand, appears to be non-monotonic in the case of productivity shocks. Furthermore, the magnitude of the welfare effects is not particularly sensitive to $\sigma_\xi$ as long as the latter exceeds 0.2, our baseline value for $\sigma_\omega$. Since it seems unlikely that the noise in private information can be smaller than the noise in public information, this suggests that the effects we document are not particularly sensitive to this parameter for which we have very little knowledge. That been said, we would like to emphasize, once more, the limitations of our numerical exercise and the challenge of obtaining reliable estimates of either the information structure or the degree of strategic complementarity. This seems a fruitful direction for future research.

Like our theoretical exercise, our numerical exploration has so far remained agnostic about the relative contribution of the different types of shocks. In so doing, we have sought to provide a direct mapping from one's view regarding the efficiency of the business cycle to one's inference about the social value of information. Putting such priors aside, one may seek an estimate of the relative contribution of different business-cycle disturbances in the pertinent empirical literature.

Thus consider Smets and Wouters (2007). The latter fit a DSGE model to the US economy and, among other things, estimate the contribution of different shocks to output volatility. At the one year horizon, a combination of their markup and labor-wedge shocks explains only half as much of output volatility as productivity shocks. Although their model is far richer than ours, and hence not directly comparable, their estimates suggest that we could have used $R = 2/3$ as a plausible

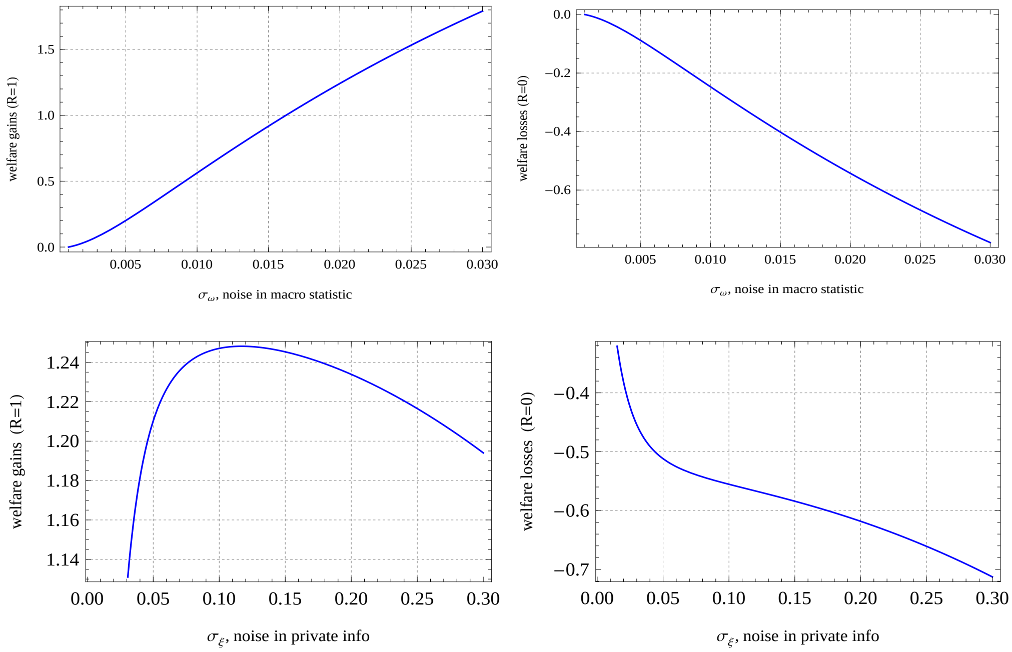


Figure 5: Sensitivity of welfare effects to $\sigma_\omega$ (top two panels) and $\sigma_\xi$ (bottom two panels). In each case, the left panel corresponds to $R = 1$ (only productivity shocks), right panel corresponds to $R = 0$ (only markup shocks).

benchmark in the numerical exercise of the preceding section.[22] Given that the threshold $\hat{R}$ that determines the sign of the welfare effects appears to be comfortably below 1/2, we conclude that $R > \hat{R}$ seems the most likely scenario. Therefore, notwithstanding the objections one may have either about our exercise or the precise meaning of the shocks identified in estimated DSGE models, the benchmark that emerges is one where productivity shocks are sufficiently prevalent that welfare improves with more precise information.

# 9 Discussion

The key lesson of our paper is that the social value of information hinges on the nature of the underlying shocks and the efficiency of the resulting fluctuations. Withholding the release of macroeconomic statistics, practicing "constructive ambiguity", or otherwise constraining the information that is available to the public makes sense when, and only when, a sufficiently high fraction of the business cycle is driven primarily by shocks that move the "output gap". For a plausible parameterization of our framework, this was not the case: more information was found to be welfare-improving.

[22] Clearly, a preferred alternative would be to (i) augment their model with the information structure we have considered here, (i) estimate the augmented model on US data, and (iii) use this to quantify the welfare effects of information. Such an exercise, however, is well beyond the scope of this paper and is left for future research.

Our analysis was based on the flexible-price allocations of an elementary DSGE model. As mentioned already, our focus on flexible prices was not accidental: studying the normative properties of flexible-price allocations is always the key step towards understanding the normative properties of sticky-price allocations. This principle, which is a cornerstone of the modern theory of optimal monetary policy, is also the key to the sharpness of the results we have delivered in this paper.

Needless to say, our results continue to apply if we introduce sticky prices but focus on monetary policies that replicate flexible-price allocations. Furthermore, for the case of technology or preference shocks, one can show that these policies are actually optimal (Angeletos and La'O, 2011). It follows that, for this particular case, our results extend directly to sticky prices as long as monetary policy is optimal. In the case of markup or labor-wedge shocks, on the other hand, flexible-price allocations are no more efficient and the optimal policy typically involves partial stabilization of the associated output gaps. Our results can then be interpreted as applying to the residual fluctuations that obtain once monetary policy, or other policy instruments, offset part of the underlying distortionary shocks.

These observations indicate how our results can be translated to richer settings with sticky prices. But they also underscore that this translation ought to hinge on the optimality of monetary policy—or lack thereof. More concretely, suppose that the economy is hit only by technology shocks, but, contrary to what would have been optimal, monetary policy fails to replicate flexible-price allocations. In this case, the suboptimal response of monetary policy to the underlying shocks introduces random variation in realized markups and output gaps. This opens the possibility that more precise information about the underlying productivity shocks may now be detrimental. In short, a suboptimal monetary policy might induce an otherwise innocuous productivity shock to have the same welfare implications as a distortionary markup shock.

By focusing on flexible-price allocations, we have deliberately abstracted from such confounding effects. By contrast, the suboptimality of monetary policy is playing a central role in the otherwise complementary work of Hellwig (2005) and Roca (2010). These papers assume sticky prices and study the welfare effects of information regarding exogenous shocks to the quantity of money. Clearly, monetary shocks that are unknown at the time firms set their prices cause equilibrium output to fluctuate away from the first best. To the extent that monetary policy fails to insulate the economy from such shocks, more precise public information can simply help the market do what monetary policy should have done in the first place: once these shocks become known at the time firms set their prices, prices adjust one-to-one to these shocks, guaranteeing that the shocks have no effect on real allocations and welfare. These observations help explain the results of the aforementioned two papers and the difference between their contribution and ours.

Putting aside the possibility of suboptimal monetary policy, preference and technology shocks can trigger inefficient fluctuations to the extent that market frictions impact the response of flexible-price allocations to the aforementioned shocks. For example, Blanchard and Gali (2007) show how this is a natural implication of the combination of search frictions and real-wage rigidities in the labor market. Intuitively, such rigidities make the labor wedge correlated with the technology shock.

A similar argument applies to credit-market frictions: see Buera and Moll (2011) for an analysis of how different forms of credit frictions manifest in different types of cyclical wedges. In the light of our results, one would then expect the welfare effects of information to hinge on how strongly the wedges covary with the technology shocks and thereby on the severity of the market frictions. Further exploring these ideas is left for future research.

Another interesting issue emerges if the available information interferes with the ability of a policy maker to stabilize the economy. James and Lawler (2011) make a related point within the context of the Morris-Shin “beauty contest”. Translating this insight in the context of business cycles hinges, once again, on the nature of the underlying shocks and the resulting “output gaps”.

To recap, although our analysis abstracts from a variety of issues that may interact in intriguing ways with the question of interest, it helps resolve the apparent confusion regarding the applicability of earlier results in the literature, and lays down a clean micro-founded benchmark for understanding the welfare effects of information within the context of business cycles—a benchmark that may help guide future work on the welfare consequences of informational frictions.

Our results may indeed prove instrumental towards different exercises than the particular one we conducted in this paper. Consider, for example, how informational frictions may impact the design of optimal policy. Ongoing work in this direction includes Angeletos and La’O (2011) and Wiederholt and Paciello (2011). Although the contributions of these papers are distinct, our findings help understand the key forces operating behind some of their results. In particular, Angeletos and La’O’s result that the optimal policy seeks to improve the aggregation of information through prices and macroeconomic statistics hinges on the fact that information is socially valuable when the business cycle is efficient. Similarly, Wiederholt and Paciello’s result that monetary policy seeks to induce agents to pay *less* attention to markup shocks hinges on the fact that information becomes detrimental in the case of such shocks. More generally, characterizing the social value of information—which was the contribution of our paper—is a key step towards answering any normative question that involves either an exogenous or an endogenous change in the available information.

# Appendix

**Proof of Proposition 1.** Welfare is given by

$$\mathcal{W} = \mathbb{E}\left[\sum \beta^t W_t\right]$$

where

$$W_t \equiv \mathbb{E}_{t-1}\left[\frac{Y_t^{1-\gamma}}{1-\gamma} - \tfrac{1}{1+\epsilon}\int S_{it}^n\left(\frac{y_{it}}{A_{it}}\right)^{1+\epsilon} di\right]$$

measures the period-$t$ utility flow. Clearly, the comparative statics of welfare with respect to the information structure are pinned down by those of $W_t$. We henceforth focus on the latter.

To simplify the exposition, we ignore the consumption taste shocks $S_{it}^c$; the proof extends to this case only at the cost of more tedious derivations. Furthermore, to simplify the notation, we drop the time index $t$. Finally, we let $E(X)$ and $V(X)$ short-cuts for the conditional expectation and variance of a random variable $X$ given the public information that is available at the end of period $t-1$ (which include all the period $t-1$ shocks). Similarly $Cov(X,Z)$ denotes the corresponding conditional covariance of $X$ and $Z$, and $Cov(X,Z|\Theta)$ their covariance once we condition, not only the end-of-period public information of last period, but also on the current aggregate shocks $\Theta$.

The log-normality assumption allows us to rewrite the first component of $W$ (the one that corresponds to consumption) as follows:

$$E\left(Y^{1-\gamma}\right) = \left[E\left(Y\right)\right]^{1-\gamma}\exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right)V\left(\log Y\right)\right\}$$

Similarly, letting $q_i \equiv A_i^{1+\epsilon}/S_i^n$, denoting with $Q$ the cross-sectional mean of $q_i$, and noting that $Y \equiv \left(\int y_i^{\frac{\rho-1}{\rho}} di\right)^{\frac{\rho}{\rho-1}}$, we can express the second component of $W$ (the one that corresponds to leisure) as follows:

$$E\left(\int S_{it}^n\left(\frac{y_{it}}{A_{it}}\right)^{1+\epsilon} di\right) = E\left(\int \frac{y_i^{1+\epsilon}}{q_i} di\right) = \frac{\left[E\left(Y\right)\right]^{1+\epsilon}}{Q}\exp(G)$$

where

$$\begin{aligned} G \equiv\; & \tfrac{1}{2}\epsilon(1+\epsilon)V\left(\log Y\right) + \tfrac{1}{2}V\left(\log Q\right) - (1+\epsilon)Cov\left(\log Y, \log Q\right) \\ & + \tfrac{1}{2}\left(\epsilon + \tfrac{1}{\rho}\right)(1+\epsilon)V\left(\log y_i|\Theta\right) + \tfrac{1}{2}V\left(\log q_i|\Theta\right) - (1+\epsilon)Cov\left(\log y_i, \log q_i|\Theta\right) \end{aligned}$$

We infer that

$$W = \tfrac{1}{1-\gamma}\left[E\left(Y\right)\right]^{1-\gamma}\exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right)V\left(\log Y\right)\right\} - \tfrac{1}{1+\epsilon}\frac{\left[E\left(Y\right)\right]^{1+\epsilon}}{Q}\exp(G)$$

with $G$ defined as above.

Next, let us define $\hat{Y}$ as the value of $E\left(Y\right)$ that maximizes the aforementioned expression for $W$, taking as given $Q, G$, and $V\left(\log Y\right)$. Clearly, this is given by the solution to the following condition:

$$\hat{Y}^{1-\gamma}\exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right)V\left(\log Y\right)\right\} = \frac{\hat{Y}^{1+\epsilon}}{Q}\exp(G) \tag{14}$$

We can then restate $W$ as follows:

$$W = \left\{ \tfrac{1}{1-\gamma} \left[ \frac{E(Y)}{\hat{Y}} \right]^{1-\gamma} - \tfrac{1}{1+\epsilon} \left[ \frac{E(Y)}{\hat{Y}} \right]^{1+\epsilon} \right\} \frac{\hat{Y}^{1+\epsilon}}{Q} \exp(G)$$

If $E(Y)$ happens to equal $\hat{Y}$, then $W = \hat{W}$, where

$$\hat{W} \equiv \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)} \frac{\hat{Y}^{1+\epsilon}}{Q} \exp(G). \tag{15}$$

Letting

$$\delta \equiv \frac{E(Y)}{\hat{Y}} \qquad \text{and} \qquad \Delta \equiv \frac{U(\delta) - V(\delta)}{U(1) - V(1)} = \frac{\frac{1}{1-\gamma}\delta^{1-\gamma} - \frac{1}{1+\epsilon}\delta^{1+\epsilon}}{\frac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}},$$

we conclude that

$$W = \Delta \hat{W}. \tag{16}$$

$\Delta$ therefore identifies the wedge between actual welfare and the welfare that would have obtained if a planner had an instrument that permitted him to scale up and down the allocation under consideration by a factor $\delta$ and could choose this factor so at to maximize welfare.[23]

Next, from (14) we have that

$$\hat{Y} = Q^{\frac{1}{\epsilon+\gamma}} \exp\left\{ -\tfrac{1}{\epsilon+\gamma} \left[ G + \tfrac{1}{2}\gamma(1-\gamma) V(\log Y) \right] \right\},$$

which together with (15) gives

$$\hat{W} = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)} Q^{\frac{1-\gamma}{\epsilon+\gamma}} \exp\left\{ G - \tfrac{1+\epsilon}{\epsilon+\gamma} \left[ G + \tfrac{1}{2}\gamma(1-\gamma) V(\log Y) \right] \right\}$$

Equivalently,

$$\hat{W} = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)} Q^{\frac{1-\gamma}{\epsilon+\gamma}} \exp\left\{ -\tfrac{1}{2} \tfrac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma} \hat{\Omega}) \right\} \tag{17}$$

where

$$\begin{aligned} \hat{\Omega} &\equiv \tfrac{2}{1+\epsilon} G + \gamma V(\log Y) \\ &= (\epsilon+\gamma) V(\log Y) + \tfrac{1}{1+\epsilon} V(\log Q) - 2Cov(\log Y, \log Q) \\ &\quad + \left( \epsilon + \tfrac{1}{\rho} \right) V(\log y_i | \Theta) + \tfrac{1}{1+\epsilon} V(\log q_i | \Theta) - 2Cov(\log y_i, \log q_i | \Theta) \end{aligned}$$

Now, note that the first-best levels of output are given by the fixed-point to the following:

$$\log y_i^* = (1-\alpha) \tfrac{1}{\epsilon+\gamma} \log q_i + \alpha \log Y^*.$$

[23] To see this more clearly, note that $\Delta$ is strictly concave in $\delta_t$ and reaches its maximum at $\delta = 1$ when $\gamma < 1$, whereas it is strictly convex and reaches its minimum at $\delta = 1$ when $\gamma > 1$. Along with the fact that $\hat{W} > 0$ when $\gamma < 1$ but $\hat{W} < 0$ when $\gamma > 1$, this means that $\hat{W}\Delta$ is always strictly concave in $\delta$ and $\delta = 1$ is always the maximal point.

It follows that, up to some constants that we omit for notational simplicity,

$$\log Y^* = \tfrac{1}{\epsilon+\gamma} \log Q \qquad \text{and} \qquad \log y_i^* - \log Y^* = (1-\alpha)\tfrac{1}{\epsilon+\gamma}(\log q_i - \log Q)$$

Using this result towards replacing the terms in $\hat{\Omega}$ that involve $q_i$ and $Q$, we get

$$\begin{aligned}\hat{\Omega} \;=\; & (\epsilon+\gamma)V(\log Y) + \tfrac{(\epsilon+\gamma)^2}{(1+\epsilon)} V(\log Y^*) - 2(\epsilon+\gamma)Cov(\log Y, \log Y^*) \\ & + \left(\epsilon + \tfrac{1}{\rho}\right) V(\log y_i|\Theta) + \tfrac{(\epsilon+\gamma)^2}{(1+\epsilon)(1-\alpha)^2} V(\log y_i^*|\Theta) - 2\tfrac{\epsilon+\gamma}{1-\alpha} Cov(\log y_i, \log y_i^*|\Theta)\end{aligned} \tag{18}$$

Furthermore, the first-best level of welfare is given by

$$W^* = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)} Q^{\frac{1-\gamma}{\epsilon+\gamma}} \exp\left\{-\tfrac{1}{2}\tfrac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\Omega^*)\right\}$$

where $\Omega^*$ obtains from $\hat{\Omega}$ once we replace $y_i$ and $Y$ with, respectively, $y_i^*$ and $Y^*$. We conclude that

$$\hat{W} = W^* \exp\left\{-\tfrac{1}{2}\tfrac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\left(\hat{\Omega} - \Omega^*\right)\right\} \tag{19}$$

Finally, using the definitions of $\hat{\Omega}$ and $\Omega^*$ together with the fact that $1-\alpha = \frac{\epsilon+\gamma}{\epsilon+1/\rho}$, we have

$$\begin{aligned}\frac{\hat{\Omega} - \Omega^*}{\epsilon+\gamma} \;=\; & \{V(\log Y) + V(\log Y^*) - 2Cov(\log Y, \log Y^*)\} \\ & + \frac{1}{1-\alpha}\{V(\log y_i|\Theta) + V(\log y_i^*|\Theta) - 2Cov(\log y_i, \log y_i^*|\Theta)\}\end{aligned}$$

which together with the definitions of $\Sigma$ and $\sigma$ gives us

$$\frac{\hat{\Omega} - \Omega^*}{\epsilon+\gamma} = \Sigma + \frac{1}{1-\alpha}\sigma.$$

Combining this result with (16) and (19), and letting

$$\Lambda \equiv \Sigma + \frac{1}{1-\alpha}\sigma,$$

we conclude that

$$W = W^* \Delta \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\},$$

which completes the proof.

**Equilibrium with productivity shocks.** Suppose the equilibrium production strategy takes a log-linear form:

$$\log y_{it} = \varphi_0 + \varphi_a a_{it} + \varphi_x x_{it} + \varphi_z z_t + \varphi_{-1}\bar{a}_{t-1}, \tag{20}$$

for some coefficients $(\varphi_a, \varphi_x, \varphi_z, \varphi_{-1})$. Aggregate output is then given by

$$\log Y_t = \varphi_0 + X + \varphi_{-1}\bar{a}_{t-1} + (\varphi_a + \varphi_x)\bar{a}_t + \varphi_z z_t$$

where

$$X \equiv \frac{1}{2}\left(\frac{\rho-1}{\rho}\right) Var(\log y_{it}|\log Y_t) = \frac{1}{2}\left(\frac{\rho-1}{\rho}\right)\left[\frac{\varphi_a^2}{\kappa_\xi}+\frac{\varphi_x^2}{\kappa_x}+2\frac{\varphi_a\varphi_x}{\kappa_x}\right]$$

adjusts for the curvature in the CES aggregator. It follows that $Y_t$ is log-normal, with

$$\begin{aligned}\mathbb{E}_{it}\left[\log Y_t\right] &= \varphi_0 + X + \varphi_{-1}\bar{a}_{t-1} + (\varphi_a+\varphi_x)\,\mathbb{E}_{it}\left[\bar{a}_t\right] + \varphi_z z_t && (21)\\ Var_{it}\left[\log Y_t\right] &= (\varphi_a+\varphi_x)^2\, Var_{it}[\bar{a}_t] && (22)\end{aligned}$$

where, by standard Gaussian updating,

$$\begin{aligned}\mathbb{E}_{it}\left[\bar{a}_t\right] &= \frac{\kappa_a}{\kappa_a+\kappa_x+\kappa_z}\chi_t + \frac{\kappa_x}{\kappa_a+\kappa_x+\kappa_z}x_{it} + \frac{\kappa_z}{\kappa_a+\kappa_x+\kappa_z}z_t && (23)\\ Var_{it}[\bar{a}_t] &= \frac{1}{\kappa_a+\kappa_x+\kappa_z} && (24)\end{aligned}$$

Because of the log-normality of $Y_t$, the fixed-point condition (1) reduces to following:

$$\log y_{it} = (1-\alpha)(\Psi a_{it} - \Psi'\lambda) + \alpha\mathbb{E}_{it}[\log Y_t] + \Gamma \qquad (25)$$

where $\Psi \equiv \frac{1+\epsilon}{\epsilon+\gamma} > 0$, $\Psi' \equiv \frac{1}{\epsilon+\gamma} > 0$, $\lambda \equiv -\log\left[\left(\frac{\bar{\eta}-1}{\bar{\eta}}\right)(1-\bar{\tau})\right] \approx \bar{\mu}+\bar{\tau} > 0$ is the overall distortion caused by the monopoly markup and the labor wedge (which are both constant because we are herein focusing on the case with only productivity shocks), and

$$\Gamma = \tfrac{1}{2}\alpha\left(\tfrac{1}{\rho}-\gamma\right) Var_{it}\left[\log Y_t\right] = \tfrac{1}{2}\alpha^2\left(\tfrac{1}{\rho}+\epsilon\right) Var_{it}\left[\log Y_t\right] > 0$$

Next, combining (25) with (21) and (23), we obtain

$$\begin{aligned}\log y_{it} &= \Gamma - (1-\alpha)\Psi'\lambda + (1-\alpha)\,\Psi a_{it} + \alpha\left(\varphi_0 + X + \varphi_{-1}\bar{a}_{t-1} + \varphi_z z_t\right)\\ &\quad +\alpha\left(\varphi_a+\varphi_x\right)\left(\frac{\kappa_a}{\kappa_a+\kappa_x+\kappa_z}\chi_t + \frac{\kappa_x}{\kappa_0+\kappa_x+\kappa_z}x_{it} + \frac{\kappa_z}{\kappa_a+\kappa_x+\kappa_z}z_t\right)\end{aligned}$$

For this to coincide with our initial guess in (20) for every event, it is necessary and sufficient that the coefficients $(\varphi_0, \varphi_a, \varphi_x, \varphi_z, \varphi_{-1})$ solve the following system:

$$\begin{aligned}\varphi_0 &= \Gamma - (1-\alpha)\Psi'\lambda + \alpha(\varphi_0 + X)\\ \varphi_{-1} &= \alpha\varphi_{-1} + \alpha\left(\varphi_a+\varphi_x\right)\frac{\kappa_a}{\kappa_a+\kappa_x+\kappa_z}\\ \varphi_a &= (1-\alpha)\,\Psi\\ \varphi_x &= \alpha\left(\varphi_a+\varphi_x\right)\frac{\kappa_x}{\kappa_a+\kappa_x+\kappa_z}\\ \varphi_z &= \alpha\varphi_z + \alpha\left(\varphi_a+\varphi_x\right)\frac{\kappa_z}{\kappa_a+\kappa_x+\kappa_z}\end{aligned}$$

The unique solution to this system is given by the following:

$$\varphi_a = (1-\alpha)\,\Psi > 0, \quad \varphi_x = \frac{(1-\alpha)\,\kappa_x}{\kappa_a+(1-\alpha)\,\kappa_x+\kappa_z}\alpha\Psi > 0, \quad \varphi_z = \frac{\kappa_z}{\kappa_a+(1-\alpha)\,\kappa_x+\kappa_z}\alpha\Psi > 0,$$

$$\varphi_{-1} = \frac{\kappa_a}{\kappa_a+(1-\alpha)\,\kappa_x+\kappa_z}\alpha\Psi > 0, \qquad \text{and} \qquad \varphi_0 = -\Psi'\lambda + \tfrac{1}{1-\alpha}\left(\alpha X + \Gamma\right)$$

**Proof of Proposition 2.** Using the characterization of the equilibrium allocation in the preceding proof along with that of the first best in (6), we can calculate the equilibrium values of the volatility of the aggregate output gaps and of the cross-section dispersion of local output gaps as follows:

$$\Sigma = \frac{\varphi_z^2}{\kappa_z} + \frac{(\Phi - \Psi)^2}{\kappa_a} = \frac{\alpha^2 (\kappa_a + \kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^2} \Psi^2$$

$$\sigma = \frac{\varphi_x^2}{\kappa_x} = \frac{\alpha^2 (1-\alpha)^2 \kappa_x}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^2} \Psi^2$$

Taking the derivative of $\Sigma$ with respect to the precision of public information gives

$$\frac{\partial \Sigma}{\partial \kappa_z} = \frac{(1-\alpha)\kappa_x - (\kappa_a + \kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3} \alpha^2 \Psi^2,$$

which is positive if and only if $\kappa_z < (1-\alpha)\kappa_x - \kappa_0$, while taking the derivative of $\sigma$ gives

$$\frac{\partial \sigma}{\partial \kappa_z} = -2 \frac{\alpha^2 (1-\alpha)^2 \kappa_x}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3} \Psi^2,$$

which is necessarily negative.

Finally, taking the derivatives with respect to the degree of strategic complementarity, we obtain

$$\frac{\partial \Sigma}{\partial \alpha} = \frac{2 (\kappa_a + \kappa_z) (\kappa_x + \kappa_z + \kappa_a)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3} \alpha \Psi^2$$

which is necessarily positive, and

$$\frac{\partial \sigma}{\partial \alpha} = \left(\alpha^2 \kappa_x - (2\alpha - 1)(\kappa_x + \kappa_z + \kappa_a)\right) \frac{2\alpha (1-\alpha) \kappa_x \Psi^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3},$$

which is positive if and only if

$$\frac{2\alpha - 1}{\alpha^2} < \frac{\kappa_x}{\kappa_x + \kappa_z + \kappa_a}$$

Note that the RHS of the above condition is a number between 0 and 1. Next, let $g(\alpha)$ denote the LHS and note that $g : (0, 1] \to \mathbb{R}$ is strictly increasing in $\alpha$, with $g(1/2) = 0$ and $g(1) = 1$. It follows that there exists a unique $\hat{\alpha} \in (1/2, 1)$ such that the above condition is satisfied—and therefore $\sigma$ is locally increasing in $\alpha$—if and only if $\alpha < \hat{\alpha}$.

**Proof of Proposition 3.** Using the results from the proof of Proposition 2, we have that

$$\Lambda_t = \Sigma_t + \frac{1}{1-\alpha} \sigma_t = \frac{\alpha^2 \Psi^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)}$$

from which it is immediate that second-order losses are decreasing in the precision of either public or private information. (Note that this would be true even if $\alpha$ were negative.) Furthermore,

$$\frac{\partial^2 \Lambda_t}{\partial \kappa_z \partial \alpha} = -\frac{2\alpha (\kappa_x + \kappa_z + \kappa_a) \Psi^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3}$$

which is negative (as long as $\alpha > 0$).

**Proof of Theorem 1.** The distortion in the mean level of output is given by

$$\delta_t = \left[\left(\frac{\bar{\eta}-1}{\eta}\right)(1-\bar{\tau})\right]^{\frac{1}{\epsilon+\gamma}} < 1$$

where $\frac{\bar{\eta}-1}{\eta}$ is the monopoly wedge (the reciprocal of the markup) and $1-\bar{\tau}$ is the labor wedge. Since $\delta_t$, and hence also $\Delta_t$, is invariant to the information structure, the welfare effects of public information are pinned down by comparative statics of $\Lambda_t$ alone, which were established before. It follows that welfare necessarily increases with the precision of either public or private information.

**Equilibrium with markup shocks.** This follows very similar steps as the characterization of equilibrium in the case with productivity shocks. Suppose equilibrium output takes a log-linear form:

$$\log y_{it} = \varphi_\mu + \varphi_\mu \mu_{it} + \varphi_x x_{it} + \varphi_z z_t + \varphi_{-1}\bar{\mu}_{t-1},$$

for some coefficients $(\varphi_\mu, \varphi_x, \varphi_z, \varphi_{-1})$. This guarantees that aggregate output is log-normal, which in turn implies that the fixed-point condition (1) now reduces to

$$\log y_{it} = (1-\alpha)(\Psi\bar{a} - \Psi'\mu_{it}) + \alpha\mathbb{E}_{it}[\log Y_t] + \Gamma$$

where $\Psi$, $\Psi'$, and $\Gamma$ are defined as in the case with productivity shocks. Following similar steps as in that case, we can then show that the unique equilibrium coefficients are given by the following:

$$\varphi_\mu = -(1-\alpha)\Psi' < 0, \quad \varphi_x = -\frac{(1-\alpha)\kappa_x}{\kappa_\mu+(1-\alpha)\kappa_x+\kappa_z}\alpha\Psi' < 0, \quad \varphi_z = -\frac{\kappa_z}{\kappa_\mu+(1-\alpha)\kappa_x+\kappa_z}\alpha\Psi' < 0,$$

$$\varphi_{-1} = -\frac{\kappa_\mu}{\kappa_\mu+(1-\alpha)\kappa_x+\kappa_z}\alpha\Psi' > 0 \qquad \text{and} \qquad \varphi_0 = \Psi\bar{a} + \frac{1}{1-\alpha}(\alpha X + \Gamma)$$

**Proof of Proposition 4.** With only markup shocks, first-best allocations are constant. The volatility of aggregate output gaps and the dispersion of local output gaps are thus given by the following:

$$\Sigma = \frac{\varphi_z^2}{\kappa_z} + \frac{\Phi^2}{\kappa_\mu} = \frac{\alpha^2\kappa_\mu\kappa_z + ((1-\alpha)\kappa_\mu + (1-\alpha)\kappa_x + \kappa_z)^2}{\kappa_\mu(\kappa_\mu + (1-\alpha)\kappa_x + \kappa_z)^2}(\Psi')^2$$

$$\sigma = \frac{\varphi_\mu^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2\frac{\varphi_\mu\varphi_x}{\kappa_x} = \frac{(1-\alpha)^2}{\kappa_\xi}(\Psi')^2 + \frac{\alpha(1-\alpha)^2(2\kappa_\mu + (2-\alpha)\kappa_x + 2\kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2}(\Psi')^2$$

Next, taking the derivatives with respect to the precision of public information, we obtain

$$\frac{\partial\Sigma}{\partial\kappa_z} = \frac{(2+\alpha)(1-\alpha)\kappa_x + (\kappa_z+\kappa_\mu)(2-\alpha)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^3}\alpha(\Psi')^2$$

which is necessarily positive, and

$$\frac{\partial\sigma}{\partial\kappa_z} = -\frac{2(1-\alpha)^2(\kappa_\mu + \kappa_x + \kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^3}\alpha(\Psi')^2,$$

which is necessarily negative.

Finally, taking the derivatives with respect to the degree of strategic complementarity, we obtain

$$\frac{\partial \Sigma}{\partial \alpha} = -\frac{2(1-\alpha)(\kappa_\mu + \kappa_x + \kappa_z)^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^3} (\Psi')^2$$

which is necessarily negative, and

$$\frac{\partial \sigma}{\partial \alpha} = 2(1-\alpha)(\Psi')^2 H,$$

where

$$H \equiv \frac{\alpha^2 (1-\alpha) \kappa_x^2 - \alpha(-2+3\alpha)\kappa_x ((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu) + (1-3\alpha)((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^3} - \frac{1}{\kappa_\xi}$$

Note that $\kappa_x \geq \kappa_\xi$, because $x$ is the sufficient statistic of the information contained both in the own shock and in other source of private information. Letting $k \equiv \frac{\kappa_x}{\kappa_z + \kappa_\mu}$, we thus have that

$$\begin{aligned} H &\leq \frac{\alpha^2 (1-\alpha) \kappa_x^2 - \alpha(-2+3\alpha)\kappa_x ((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu) + (1-3\alpha)((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^3} - \frac{1}{\kappa_x} \\ &= \frac{1}{\kappa_z + \kappa_\mu}\left(\frac{-(1+k)^2}{k((1-\alpha)k+1)^3}\right) = -\frac{(1+k)^2}{\kappa_x((1-\alpha)k+1)^3} \end{aligned}$$

Since the latter is clearly negative, we conclude that $\frac{\partial \sigma}{\partial \alpha}$ is also negative.

**Proof of Proposition 5.** From the preceding characterization of $\Sigma$ and $\sigma$, we have that

$$\Lambda = \frac{1-\alpha}{\kappa_\xi}(\Psi')^2 + \frac{(1-\alpha)\kappa_x + \kappa_z + (1-\alpha^2)\kappa_\mu}{\kappa_\mu((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)}(\Psi')^2$$

It follows that

$$\frac{\partial \Lambda}{\partial \kappa_z} = \frac{\alpha^2 (\Psi')^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2} > 0 \tag{26}$$

$$\frac{\partial \Lambda}{\partial \kappa_x} = \frac{(1-\alpha)\alpha^2 (\Psi')^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2} > 0, \tag{27}$$

which proves that second-order welfare losses necessarily increase with the precision of either public or private information. (Note that this would be true even if $\alpha$ were negative.)

**Proof of Proposition 6.** To study the impact of information on first-order losses $\Delta_t$, we first need to compute $\delta_t$. Using the equilibrium characterization and after some tedious algebra, we can show that the predictable component of aggregate output, $\mathbb{E}_{t-1}[Y_t]$, satisfies the following condition:

$$(\mathbb{E}_{t-1}[Y_t])^{1-\gamma} \exp\left(-\tfrac{1}{2}\gamma(1-\gamma)V(\log Y)\right) = \exp\left(\chi_t - \log(1-\bar{\tau}) + \tfrac{1}{2}D\right) \frac{(\mathbb{E}_{t-1}[Y_t])^{1+\epsilon}}{Q} \exp(\tilde{G})$$

where

$$\begin{aligned}
\tilde{G} &\equiv \tfrac{1}{2}\epsilon(1+\epsilon)V\left(\log Y\right) + \tfrac{1}{2}\left(\epsilon + \tfrac{1}{\rho}\right)(1+\epsilon)V\left(\log y_i|\Theta\right) \\
D &\equiv Var\left(\mu_{it}|\bar{\mu}_t\right) + 2\left(1+\epsilon\right)Cov\left(y_{it},\mu_{it}|\bar{\mu}_t\right) \\
&= \frac{1}{\kappa_\xi} + \frac{1}{\kappa_\mu} + 2\left(1+\epsilon\right)\left(\frac{\varphi_\mu}{\kappa_\xi} + \frac{\varphi_x}{\kappa_x} + \frac{\varphi_\mu + \varphi_x + \varphi_z}{\kappa_\mu}\right) \\
&= \frac{1}{\kappa_\xi} + \frac{1}{\kappa_\mu} - 2\left(1+\epsilon\right)\left(\frac{1-\alpha}{\kappa_\xi} + \frac{1}{\kappa_\mu} - \frac{\alpha^2}{\kappa_\mu + (1-\alpha)\kappa_x + \kappa_z}\right)\Psi'
\end{aligned}$$

From (14), on the other hand, we infer that the optimal $\hat{Y}_t$ solves the following condition:

$$\hat{Y}_t^{1-\gamma}\exp\left(-\tfrac{1}{2}\gamma\left(1-\gamma\right)V\left(\log Y\right)\right) = \frac{\hat{Y}_t^{1+\epsilon}}{Q}\exp(\tilde{G})$$

Combining these results, we infer that

$$\delta_t \equiv \frac{\mathbb{E}_{t-1}[Y_t]}{\hat{Y}_t} = \exp\left[-\Psi'\left(\chi_t - \log(1-\bar{\tau}) + \tfrac{1}{2}D\right)\right],$$

and therefore

$$\frac{\partial \delta_t}{\partial \kappa_z} = -\tfrac{1}{2}\delta_t\Psi'\frac{\partial D}{\partial \kappa_z} = \frac{\alpha^2\left(\Psi'\right)^2}{\left(\left(1-\alpha\right)\kappa_x + \kappa_z + \kappa_\mu\right)^2}\left(1+\epsilon\right)\delta_t > 0 \tag{28}$$

$$\frac{\partial \delta_t}{\partial \kappa_x} = -\tfrac{1}{2}\delta_t\Psi'\frac{\partial D}{\partial \kappa_x} = \frac{(1-\alpha)\alpha^2\left(\Psi'\right)^2}{\left(\left(1-\alpha\right)\kappa_x + \kappa_z + \kappa_\mu\right)^2}\left(1+\epsilon\right)\delta_t > 0. \tag{29}$$

It follows that, as long as $\delta_t < 1$, an increase in the precision of either public or private information reduces the mean distortion—it brings $\delta_t$ closer to 1—and thereby also reduces first-order welfare losses. (Once again, this result holds true irrespectively of whether $\alpha$ is positive or negative.)

**Proof of Theorem 2.** To obtain the overall welfare effect, recall that welfare is given by

$$W_t = W_t^*\Delta_t\exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\}$$

Consider first the case of public information. From the above, we have that

$$\frac{\partial W_t}{\partial \kappa_z} = W_t^*\exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\}\left(\frac{\partial \Delta_t}{\partial \delta_t}\frac{\partial \delta_t}{\partial \kappa_z} - \tfrac{1}{2}\Delta_t(1+\epsilon)(1-\gamma)\frac{\partial \Lambda_t}{\partial \kappa_z}\right)$$

From (26) and (28), we have that

$$\frac{\partial \delta_t}{\partial \kappa_z} = \frac{\partial \Lambda_t}{\partial \kappa_z}(1+\epsilon)\delta_t$$

It follows that

$$\frac{\partial W_t}{\partial \kappa_z} = W_t^*\exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\}\frac{\partial \Lambda_t}{\partial \kappa_z}H_t$$

where

$$\begin{aligned}
H_t &\equiv \frac{\partial \Delta_t}{\partial \delta_t}(1+\epsilon)\delta_t - \tfrac{1}{2}(1-\gamma)(1+\epsilon)\Delta_t \\
&= \tfrac{(1-\gamma)(1+\epsilon)}{2(\epsilon+\gamma)}\left[2(1+\epsilon)\left(\delta_t^{1-\gamma} - \delta_t^{1+\epsilon}\right) - \left((1+\epsilon)\delta_t^{1-\gamma} - (1-\gamma)\delta_t^{1+\epsilon}\right)\right] \\
&= \tfrac{(1-\gamma)(1+\epsilon)}{2(\epsilon+\gamma)}\left[(1+\epsilon)\delta_t^{1-\gamma} - (1+2\epsilon+\gamma)\delta_t^{1+\epsilon}\right],
\end{aligned}$$

and therefore

$$\frac{\partial W_t}{\partial \kappa_z} = \tfrac{(1-\gamma)(1+\epsilon)}{2(\epsilon+\gamma)} W_t^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\} \frac{\partial \Lambda_t}{\partial \kappa_z}\delta_t \left[(1+\epsilon) - (1+2\epsilon+\gamma)\delta_t^{\epsilon+\gamma}\right]$$

Note then that the sign of $W_t^*$ is the same as that of $(1-\gamma)$, which together with the facts that $\frac{\partial \Lambda_t}{\partial \kappa_z} > 0$ and $\delta_t > 0$ implies that the sign of $\frac{\partial W_t}{\partial \kappa_z}$ is the same as the sign of $(1+\epsilon)-(1+2\epsilon+\gamma)\delta_t^{\epsilon+\gamma}$. We conclude that

$$\frac{\partial W_t}{\partial \kappa_z} < 0 \quad \text{iff} \quad \delta_t > \hat{\delta},$$

where

$$\hat{\delta} \equiv \left(\frac{1+\epsilon}{1+2\epsilon+\gamma}\right)^{\frac{1}{\epsilon+\gamma}} \in (0,1).$$

Consider next the case of private information. From (27) and (29), we have that

$$\frac{\partial \delta_t}{\partial \kappa_x} = \frac{\partial \Lambda_t}{\partial \kappa_x}(1+\epsilon)\delta_t,$$

pretty much as in the case of public information. It follows that

$$\begin{aligned}
\frac{\partial W_t}{\partial \kappa_x} &= W_t^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\}\left(\frac{\partial \Delta_t}{\partial \delta_t}\frac{\partial \delta_t}{\partial \kappa_x} - \tfrac{1}{2}\Delta_t(1+\epsilon)(1-\gamma)\frac{\partial \Lambda_t}{\partial \kappa_x}\right) \\
&= W_t^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda_t\right\}\frac{\partial \Lambda_t}{\partial \kappa_x} H_t
\end{aligned}$$

where $H_t$ is defined as before. By direct implication,

$$\frac{\partial W_t}{\partial \kappa_x} < 0 \quad \text{iff} \quad \delta_t > \hat{\delta},$$

where $\hat{\delta}$ is the same threshold as the one in the case of public information.

**Proof of Proposition 7.** Consider the case where the economy is hit only by productivity shocks. From the analysis in the main text, we have that the equilibrium must satisfy

$$\kappa_z = \tilde{\kappa}_z + (\varphi_a + \varphi_x)^2 \kappa_\omega,$$

From the analysis of the equilibrium with productivity shocks in the beginning of this appendix, we have that

$$\varphi_a + \varphi_x = \frac{\kappa_a + \kappa_x + \kappa_z}{\kappa_a + (1-\alpha)\kappa_x + \kappa_z}(1-\alpha)\Psi,$$

Combining, we conclude that the equilibrium value of $\kappa_z$ is pinned down by the following fixed point:

$$\kappa_z = F(\kappa_z)$$

where

$$F(\kappa_z) \equiv \tilde{\kappa}_z + \left(\frac{\kappa_a + \kappa_x + \kappa_z}{\kappa_a + (1-\alpha)\kappa_x + \kappa_z}\right)^2 (1-\alpha)^2 \Psi^2 \kappa_\omega$$

Note that $F$ is continuous and decreasing in $\kappa_z$, with $F(0) > 0$. It follows that there exists a unique solution to $\kappa_z = F(\kappa_z)$, which means that the equilibrium is unique. Furthermore, since $F$ is increasing in $\kappa_\omega$, the equilibrium value of $\kappa_z$ is also increasing in $\kappa_\omega$. Along with the facts that $\kappa_\omega$ impacts welfare only through $\kappa_z$ and that welfare is increasing in $\kappa_z$, this proves that welfare is increasing in $\kappa_\omega$.

The same arguments apply to the case of markup shocks, modulo a change in notation/interpretation.

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