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TESTS FOR LIQUIDITY CONSTRAINTS:  
A CRITICAL SURVEY

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ABSTRACT

This paper surveys recent empirical work on tests for liquidity constraints. The focus of the survey is on the tests based on the Euler equation. After examining the technical aspects of the recent tests on aggregate time-series data and on micro data, the survey tries to evaluate their economic significance. The paper concludes that for a significant fraction of the population the behavior of consumption over time is affected in a way predicted by credit rationing and differential borrowing and lending rates. However, the available evidence is shown to have failed in providing information necessary to calculate the response of consumption to changes in the time profile of income. The paper attributes the failure to the fact that not much attention in the literature has been paid to the cause of liquidity constraints.

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## 1. Introduction

The issue of liquidity constraints comes up in several areas of economics. The main ingredient in modern theories of business cycles is the consumers who execute intertemporal optimization through trading in perfectly competitive asset markets. Traditionally, the Life Cycle-Permanent Income Hypothesis has been the label of such consumer behavior. Some authors have argued that the observed comovements of consumption and income (or the lack thereof) can best be explained by examining the role of liquidity constraints as the additional constraint in the consumers' decision problem.<sup>1</sup> The notion that consumers are unable to borrow as they desire is also used to argue against the Ricardian doctrine of the equivalence of taxes and deficits. In the literature on implicit labor contracts, the assumption is often made that workers are unable to borrow against future earnings. Liquidity constraints have even been used in some instances as an excuse to focus on static single-period analyses.

Despite its popularity, the term liquidity constraints has not yet gained a precise and unique definition. To some the term might be associated with agents facing the cash-in-advance constraint. The most widely accepted definition, however, is that consumers are said to be liquidity constrained if they face quantity constraints on the amount of borrowing (credit rationing) or if the loan rates available to them are higher than the rate at which they could lend (differential rates). In this survey we will employ this definition of liquidity constraints, thereby abstracting from the interesting and important issue of why people hold money.

The survey is selective in other ways too. We will ignore the possible connection between consumption and income arising from the consumption-leisure choice. This is justified if consumption and leisure are separable in the utility function. We will also ignore the large literature on econometric studies of the consumption function. Our focus therefore is on what has come to be called the Euler equation approach which has been the rapidly growing segment of the literature on consumer behavior. Our choice of being selective is motivated by the recent exhaustive survey ably done by Mervyn King (1983).

The questions we would hope to be answered by the available empirical evidence may be divided into three groups. First of all, we would like to know if the Life Cycle-Permanent Income Hypothesis can be rejected in favor of the hypothesis of liquidity constraints. Although there exist many studies rejecting the hypothesis, another careful scrutiny may be warranted. Second, if liquidity constraints are shown to exist, how do we proceed to identify the preference parameters under liquidity constraints? The identification of the structural parameters is a necessary prelude to the construction of macro models that would allow us to study business cycles and analyze policy interventions. Third, which of the standard conclusions derived under no liquidity constraints will survive and which will not? More specifically, under liquidity constraints how does consumption respond to temporary income changes? Does the Ricardian Equivalence Theorem cease to hold? The available empirical work will be examined critically on these three scores.

The organization of the paper is as follows. Section 2 outlines the test for liquidity constraints based on the Euler equation and contrasts it with the approach based on the consumption function. Problems associated with the Euler equation approach are also discussed. Section 3 examines the

available empirical evidence from aggregate data and micro data. The discussion of technical issues is contained in this section. In section 4, which contains original material, we will consider three specific models of liquidity constraints and argue that the economic implication of the available evidence cannot be determined unless the cause of liquidity constraints is identified. Section 5 is a brief conclusion.

## 2. Intertemporal Optimization with and without Liquidity Constraints

Throughout this section and section 4, we will focus for expositional ease on the conventional two-period model of intertemporal optimization, although most of the discussion can be readily extended to the many-period case. The objective function of the consumer is the expectation of lifetime utility that is time separable:

$$(2.1) \quad u(c_1) + \beta E_1 u(c_2),$$

where  $c_t$  is consumption in period  $t$  ( $t=1,2$ ),  $u(\cdot)$  is the instantaneous utility function,  $\beta$  is the discount factor,  $E_1$  is the expectations operator conditional on information available to the consumer in period 1. Let  $A_t$  and  $w_t$  be nonhuman wealth and after-tax labor income in period  $t$ . Then  $A_t$  follows:

$$(2.2) \quad A_2 = (1+r)(A_1 + w_1 - c_1),$$

where  $r$  is the market risk-free real rate. The constraint to the consumer is that debt be eventually paid back, which means under no bequests that, for any realization of (possibly stochastic) future labor income  $w_2$ ,

$$(2.3) \quad c_2 = A_2 + w_2.$$

Combining (2.2) and (2.3) we obtain the lifetime budget constraint:

$$(2.4) \quad c_1 + c_2/(1+r) = A_1 + w_1 + w_2/(1+r).$$

The important observation to be made here is that the consumer is constrained only by the lifetime budget constraint, so that consumption can be shielded from period-to-period fluctuations in income through borrowing and lending. Any changes in the configuration of  $(w_1, w_2)$  lead to revisions in the optimal consumption plan  $(c_1^*, c_2^*)$  only in so far as they change the distribution of  $w_1 + w_2 / (1+r)$ . Thus the MPC (marginal propensity to consume) out of a temporary increase in  $w_1$  (which leaves unaltered the distribution of  $w_2$ ) will be much smaller than the MPC out of a permanent increase in  $w_1$  (which shifts the distribution of  $w_2$  by the amount of increase in  $w_1$ ).<sup>2</sup>

However, as recent research to be surveyed in the next section indicates, consumption appears to be more sensitive to current income than is implied by intertemporal optimization. One explanation that has often been mentioned is the existence of liquidity constraints or imperfect loan markets. It means either that consumers are credit rationed (so that there is a lower bound on nonhuman wealth) or that the loan rates available to consumers are higher than the lending rate (the market interest rate). The consequence of liquidity constraints can be seen most easily for the deterministic case in which the consumer has a point expectation  $w_2^e$  about future labor income  $w_2$ . Figure 1(a) is the familiar diagram showing that the optimal consumption plan  $(c_1^*, c_2^*)$  in the absence of liquidity constraints is the point where the marginal rate of substitution  $u'(c_1) / [Bu'(c_2)]$  is equated to the marginal rate of transformation  $1+r$ . As long as total wealth (the sum of nonhuman wealth  $A_1$  and human wealth  $w_1 + w_2^e / (1+r)$ ) is held constant, changes in the configuration of  $(w_1, w_2^e)$  have no influence whatsoever on current consumption. Panels (b) and (c) of Figure 1 illustrate the two versions of liquidity constraints: in Figure 1(b) the consumer is credit rationed with the amount of rationing being

$c_1^* - A_1 - w_1$ , while in Figure 1(c) the consumer faces a schedule of loan rates as an increasing function of the loan quantity. Under liquidity constraints consumption is excessively sensitive to income in the following sense. If the consumer is credit rationed and if the amount of rationing is constant, the optimal consumption plan moves from point A to B in Figure 1(b) as current income increases from  $w_1$  to  $w_1'$ . So the MPC out of a temporary current income increase is unity. It is less than but still close to unity when the consumer faces an upward sloping borrowing rate schedule. It is also clear that under liquidity constraints current consumption is not invariant to changes in the configuration of  $(w_1, w_2^e)$  that hold total wealth constant.

Following the lead of Hall (1978), recent tests for liquidity constraints have utilized the "Euler equations" (first-order conditions characterizing the optimal consumption plan) rather than the consumption function (optimal contingency rule that relates optimal current consumption to the set of information currently available to the consumer). As seen above, the implication of the lifetime budget constraint is that consumption is invariant to changes in income if total wealth is controlled for. The test for liquidity constraints based on the consumption function exploits this by regressing consumption on total wealth and current income and by examining the significance of the income coefficient. There are several reasons against this consumption function approach. We mention two of them.<sup>3</sup> First, when future income is uncertain, a closed-form optimal contingency rule cannot in general be derived, which renders the notion of "total wealth" unoperational. Even if a closed-form solution is available, the definition of total wealth is not preference-free. For instance, if the instantaneous utility function is quadratic, the consumption function is



$$(2.5) \quad c_1^* = a_0 + a_1 \times \{A_1 + w_1 + E(w_2)/(1+r)\},$$

where  $a_0$  and  $a_1$  depend on  $r$  and the parameters characterizing the instantaneous utility function. If the instantaneous utility function exhibits a constant absolute risk aversion, the consumption function is

$$(2.6) \quad c_1^* = b_0 + b_1 \times \{A_1 + w_1 + [\ln(E_1(\exp(-\mu w_2)))]^{-\mu/(1+r)}\},$$

where  $\mu$  is the constant degree of absolute risk aversion. The definition of total wealth, which is the expression in the braces, depends on the utility function. This example also highlights the second difficulty with the consumption function approach: total wealth (if it is well-defined) cannot be calculated without data on the distribution of future income. Such data are not typically available.

The Euler equation approach exploits another implication of intertemporal optimization subject to the lifetime budget constraint, namely that at optimum the marginal rate of substitution between current and future consumption is set equal to the marginal rate of transformation (see Figure 1(a)). The beauty of this approach is that it can easily accommodate stochastic real rates as well as stochastic labor income:

$$(2.7a) \quad u'(c_1) = \beta E_1[(1+r)u'(c_2)], \text{ or}$$

$$(2.7b) \quad E_1[(1+r)\beta u'(c_2)/u'(c_1)] = 1, \text{ or}$$

$$(2.7c) \quad E_1(e_2) = 0 \text{ where } e_2 = 1 - (1+r)\beta u'(c_2; \theta)/u'(c_1; \theta),$$

where  $\theta$  is a parameter vector characterizing the utility function. The interpretation of the Euler equation is familiar: the left hand side of (2.7a) is the marginal utility benefit of increasing  $c_1$  by one unit, while the right hand side is the marginal utility cost of a reduction in  $c_2$  arising from the reduced current saving. Equation (2.7b) indicates that, ex ante, the marginal rate of substitution and the marginal rate of transformation are equated. Ex post, the two rates can differ because the realization of future income and real rates are not perfectly foreseen. The discrepancy is represented by the consumption innovation  $e_2$ . The most attractive feature of the Euler equation approach is that it allows a direct estimation of preference parameters  $(\theta, \beta)$  as done by Hansen and Singleton (1982). If  $x_1$  is a vector of variables in the period 1 information set, (2.7c) implies that the conditional expectation  $E(e_2|x_1)$  is zero, which in turn means  $E(e_2x_1) = 0$ . Under rational expectations, the consumer's subjective distribution about future stochastic variables agrees with the objective distribution, so that the orthogonality condition  $E(e_2x_1) = 0$  (and hence  $\text{Cov}(e_2, x_1) = 0$ ) must hold on data. This is precisely the situation for which Hansen's (1982) GMM (generalized methods of moments) estimation is designed to estimate the unknown preference parameters under the orthogonality condition.

The Euler equation does not hold in the presence of liquidity constraints, because consumers who would like to borrow at the market rate but who are prevented from doing so consume relatively less in period 1 and more in period 2 than in the absence of liquidity constraints. Thus under liquidity constraints there should be a negative correlation between the marginal rate of substitution and  $A_1 + w_1$  or any variable that reduces the severity of liquidity constraints (see Figure 1(b) and (c)). This is the

basic strategy of testing for liquidity constraints by the Euler equation. In its most sophisticated form, the procedure is Hansen's (1982) test of overidentifying restrictions: estimate the preference parameters  $(\theta, \beta)$  from the nonlinear Euler equation (2.7c) by the GMM where the set of instruments  $x_1$  in the period 1 information set excludes variables (like  $A_1$  and  $w_1$ ) pertinent to the consumer's liquidity, estimate by the GMM where  $x_1$  is expanded to include liquidity variables and compare the two estimates. If they significantly differ, liquidity constraints must be binding. This test takes a familiar form for some commonly used utility functions, because the Euler equation can be made linear. In the case of quadratic utility [ $u(c) = -(\alpha - c)^2$ ] with a deterministic interest rate  $r$ , the Euler equation is

$$(2.8) \quad c_2 = [1 - \beta^{-1}(1+r)^{-1}]\alpha + \beta^{-1}(1+r)^{-1}c_1 + \varepsilon_2, \quad \varepsilon_2 = c_2 - E_1(c_2).$$

In the case of a constant absolute risk aversion [ $u(c) = -\exp(-\mu c)$ ,  $\mu > 0$ ], it is

$$(2.9) \quad c_2 - c_1 = \mu^{-1}\ln(\beta) + \mu^{-1}\ln(1+r) + \mu^{-1}e_2,$$

where  $e_2$  is defined in (2.7c).<sup>4</sup> In the case of a constant relative risk aversion [ $u(c) = c^{1-1/\sigma}$ ,  $\sigma > 0$ ], it is

$$(2.10) \quad \ln(c_2) - \ln(c_1) = \sigma\ln(\beta) + \sigma\ln(1+r) + \sigma e_2.$$

This  $\sigma$  is called the elasticity of intertemporal substitution. We can test for liquidity constraints by adding a set of variables in the period 1 information set that represent the consumer's liquidity to the Euler

equation. Since the consumption innovation  $e_2$  and  $\varepsilon_2$  is uncorrelated with any variable in the period 1 information set, the regression estimate of the liquidity variable coefficients should be insignificant if the consumer is not liquidity constrained.<sup>5</sup>

Before turning to a survey of recent empirical work, we point out three nontechnical problems with the Euler equation approach; technical problems will be discussed in the next section. The last two are also shared by the consumption function approach. The first problem, which is completely obscured by our focus on the two-period model, is that the Euler equation does not exhaust all the implication of intertemporal optimization subject only to the lifetime budget constraint. Although it captures the important implication that under rational expectations the change in the marginal utility of consumption (the consumption innovation) is uncorrelated with any variable (like anticipated income changes, permanent or temporary) in the period 1 information set, the Euler equation does not by itself place any restrictions on the relation between the consumption innovation and unanticipated income changes. The Euler equation will be satisfied even if the consumer is myopic in that he or she cares only about the first two periods of the multi-period life. Even though the consumer's planning horizon is the entire lifetime, the likelihood of future liquidity constraints effectively shortens the horizon.<sup>6</sup> For example, if the consumer expects that he or she will face a binding constraint of a ban on borrowing  $n$  periods from now, the optimal consumption plan will be such that nonhuman wealth in that period is zero. So the consumer will act as if the horizon is only  $n$  periods. From the Euler equation alone we cannot tell how the consumer would react to a unanticipated temporary income change. This problem can be alleviated by the use of the Euler equation between  $c_1$  and

$c_{t+n}$ :  $u'(c_1) = \beta^n E_1[(1+r_n)^n u'(c_{1+n})]$  where  $r_n$  is the  $n$ -period real rate. If the Euler equation is satisfied for all  $n$ ,  $1 \leq n \leq T$ , then the effective horizon is longer than  $T$  periods. However, the horizon length to be tested is limited if the data are a short panel. Another solution is to make an auxiliary assumption about the stochastic process generating labor income and derive a theoretical relationship involving the horizon length between the consumption innovation  $e_2$  and innovations in labor income.<sup>7</sup> This forms the basis of what we will call the excess sensitivity test.

Second, the derivation of the Euler equation and the consumption function has ignored the nonnegativity constraint  $c_t \geq 0$ . This is justified if disutility of zero consumption is prohibitive (i.e.,  $u'(0) = +\infty$ ) and if the consumer has to go through zero consumption in the event of default. No plan allowing defaults can be chosen. Otherwise, the consumer may plan to default when the second period labor income turns out to be insufficient to repay the loan, which will either put a premium in the loan rate or limit the quantity of the loan available to the consumer. For example, if the loan market can provide only risk-free loans, the constraint that  $c_2 = A_2 + w_2 \geq 0$  (see (2.3)) for any realization of the stochastic variable  $w_2$  implies that the loan repayment ( $-A_2$ ) must be less than or equal to the sure part of  $w_2$ . This certainly blurs the distinction between intertemporal optimization with and without liquidity constraints. This important issue will be discussed later at some length in a separate section (section 4).

The third problem is closely related to the second. In the test for liquidity constraints described above, no intertemporal optimization problem under liquidity constraints is explicitly spelled out. One possible specification is to assume that the loan rate available to the consumer is an increasing function of the loan quantity  $c_1 - A_1 - w_1$ , which delivers an

Euler equation under liquidity constraints that the ex ante marginal rate of substitution equals one plus the loan rate on marginal loans. Since the marginal loan rate is a function of the loan quantity under liquidity constraints, we can test for liquidity constraints by examining the relationship between the marginal rate of substitution and the loan quantity. This is essentially the test for liquidity constraints discussed above. One could further proceed to estimate under this specification of liquidity constraints both preference parameters and the loan rate schedule using this Euler equation. But it leaves unanswered the question of why there exists a gap between the loan rate and the risk-free rate. If the gap is a premium that compensates for possible defaults, the rate of return on a loan is no longer exogenous to the consumer in that its realization depends on the loan quantity: it equals the contracted loan rate if the consumer repays the loan in full and minus one if the consumer defaults on the loan. Then the Euler equation under liquidity constraints will take a different form because the level of the second period consumption in the event of default is unaffected by marginal changes in the loan quantity. Thus, the estimate of preference parameters under liquidity constraints is sensitive to the nature of the loan market underlying the loan rate schedule. An example in which this is the case will be provided in section 4.

### 3. Recent Empirical Work

#### A. Tests for Liquidity Constraints using Aggregate Time-Series Data

Two types of tests can be distinguished in the literature. The first test, which may be called the orthogonality test, checks whether the consumption innovation  $e_t$  (defined in (2.7) for  $t = 2$ ) is orthogonal to any variables in the information set  $\Omega_{t-1}$  available to the consumer in period  $t-1$ . Recent studies (see e.g., Dunn-Singleton (1984)) have extended the Hansen-Singleton (1982) paper by including durables or by examining several asset returns simultaneously. Typically, the overidentifying restrictions are strongly rejected.<sup>8</sup> This, however, cannot be taken as evidence in favor of liquidity constraints, because, the estimation of preference parameters being their primary concern, these studies did not specifically include liquidity variables in the set of additional variables used for the test of overidentifying restrictions. Most time-series tests for liquidity constraints assume constant real rates.<sup>9</sup>

The second test may be called the excess sensitivity test. Since under constant real rates labor income is the only source of uncertainty, the consumption innovation must be proportional to the labor income innovation. Now make the auxiliary assumption that labor income follows a univariate autoregressive process:

$$(3.1) \quad w_t = \mu + \rho_1 w_{t-1} + \rho_2 w_{t-2} + \dots + \rho_{t-n} w_{t-n} + u_t, \quad E_{t-1}(u_t) = 0.$$

Then, as shown by Flavin (1981) for the case of quadratic utility with  $\beta(1+r) = 1$ , we obtain the following relation between the consumption and labor income innovations when the horizon length is infinite:  $\Delta c_t = k u_t$  where  $k = (r/(1+r)) [1 - \rho_1(1+r)^{-1} - \rho_2(1+r)^{-2} - \dots - \rho_n(1+r)^{-n}]$ . If an estimate of

$k$  is greater than this expression, consumption is more sensitive to current labor income than is justified by intertemporal optimization without liquidity constraints. The failure of the orthogonality test is sufficient, but not necessary, for the excess sensitivity test to fail, because myopic consumers whose horizon is short but longer than two periods will also satisfy the Euler equation.

In Flavin's (1981) testing procedure, the lagged income coefficients in the regression of the consumption innovation on  $\Omega_{t-1}$  have a certain structural interpretation, as the following example shows. Suppose, as Hall (1978) suggested, that there are two types of consumers. The first group (the "rule of thumb" consumers) simply consume all of its disposable income, either because they face a binding constraint of a ban on debt or because they are myopic. If this group earns a fraction  $\lambda$  of aggregate disposable income  $y$ , the change in their consumption is  $\lambda(y_t - y_{t-1})$ . Consumers in the second group follow the Euler equation (2.8) with  $\beta(1+r) = 1$ . Namely, consumption by the second group is a random walk. Then aggregate consumption is described by

$$(3.2) \quad \Delta c_t \equiv c_t - c_{t-1} = \lambda \Delta y_t + \varepsilon_t.$$

It is incorrect to estimate  $\lambda$  in (3.2) by regressing  $\Delta c_t$  on  $\Delta y_t$  because  $\Delta y_t$ , not necessarily in  $\Omega_{t-1}$ , can be correlated with  $\varepsilon_t$ . To extract (part of) the disposable income change that is forecastable on the basis of  $\Omega_{t-1}$ , write the least squares projection of  $y_t$  on lagged disposable income as

$$(3.3) \quad y_t = \mu + \rho y_{t-1} + v_t.$$



By construction,  $v_t$  is uncorrelated with lagged disposable income. Since there may be other variables in  $\Omega_{t-1}$  that help predict  $y_t$ , this  $v_t$  is not necessarily the true innovation to disposable income (i.e.,  $E_{t-1}(e_t)$  may not be equal to 0). The consumption equation (3.2) can be rewritten as

$$(3.4) \quad \Delta c_t = \lambda\mu + \lambda(\rho-1)y_{t-1} + (\varepsilon_t + \lambda v_t).$$

Now the error term  $\varepsilon_t + \lambda v_t$  is uncorrelated with lagged disposable income. The parameters  $(\lambda, \mu, \rho)$  can be estimated from (3.3) and (3.4) by the multivariate regression with the cross-equations restriction that the same autoregression coefficient  $\rho$  appears in both equations. This estimate of  $\lambda$  is numerically identical to the estimate obtained from (3.2) by the instrumental variables technique with  $y_{t-1}$  as the instrument for  $\Delta y_t$ . The test statistic for the hypothesis  $\lambda = 0$  is numerically identical to the  $t$  statistic in the regression of  $\Delta c_t$  on  $y_{t-1}$ . Flavin's estimate of  $\lambda$  based on detrended quarterly U.S. data on nondurables and disposable income was so large that almost all of aggregate consumption was attributable to the "rule of thumb" consumers.

One technical and potentially serious problem can be pointed out at this junction: the use of detrended data biases the test toward rejection of the hypothesis that  $\lambda = 0$ , if disposable income is a random walk.<sup>10</sup> As noted by Hall (1978), the model consisting of (3.3) and (3.4) becomes unidentifiable if disposable income is a random walk (so that  $\rho = 1$ ), because the lagged income coefficient  $\lambda(\rho-1)$  is zero no matter what the value of  $\lambda$  is. Now consider what happens when  $\lambda$  is zero and detrended data are used. Since the consumption innovation  $\varepsilon_t$  is proportional to the labor income innovation, the consumption and disposable income series will be

highly correlated random walks. Furthermore, detrended series from random walks exhibit spurious cycles. Thus detrended consumption and disposable income will move up and down together in a cyclical fashion. Mankiw-Shapiro (1984) have shown that if such series are used to estimate (3.4), the lagged income coefficient is likely to be significant.

Other empirical studies that assume constant real rates include Bilson (1980), Hayashi (1982), and Flavin (1985). Bilson use data from the U.S., the U.K. and West Germany. Because of data limitations his consumption concept is total consumption expenditure (which includes durables expenditure), while Hayashi, using U.S. annual data, excludes durables expenditure and includes service flows from durables. He estimates  $\lambda$ , the fraction of the "rule of thumb" consumers, by the instrumental variables technique. Flavin (1985) finds that the change in the unemployment rate is highly significant if it is included in the consumption equation (3.2). Her interpretation is that the "rule of thumb" consumers are liquidity constrained rather than myopic. Overall, the studies surveyed so far point to rejection of the hypothesis of intertemporal optimization without liquidity constraints.

These studies use different consumption concepts. Although inclusion of durables expenditure in the consumption concept is unwarranted because it is service flows from durables that yield utility, the focus on a particular consumption category can be justified if the instantaneous utility function is separable across commodities. That is, if  $u(c_t) = u_1(c_{1t}) + u_2(c_{2t}) + \dots + u_n(c_{nt})$  (where  $n$  here is the dimension of the consumption vector  $c_t$ ), the Euler equation holds for each consumption component. The rejections reported in the empirical studies may be attributable to nonseparability across commodities. Bernanke (1985) studied a simultaneous determination of

nondurables and durables purchases. The quadratic instantaneous utility function he estimated is:

$$(3.5) \quad u = -(1/2)(\bar{c}-c_t)^2 - (a/2)(\bar{K}-K_t)^2 - m(\bar{c}-c_t)(\bar{K}-K_t) - (d/2)(K_{t+1}-K_t)^2,$$

where  $c_t$  is nondurables (plus services) and  $K_t$  is the stock of consumer durables. The third term captures the interaction between nondurables and durables. Adjustment costs in changing the stock of durables are also introduced by the fourth term. If  $m = 0$ , the Euler equation for nondurables is (2.8) and does not involve  $K_t$ . If  $m = a$  and  $d = 0$ , then nondurables and durables are perfect substitutes, so the correct consumption concept must include service flows from the stock of durables. Bernanke rejected the hypothesis of intertemporal optimization without liquidity constraints because consumption is too sensitive to labor income innovations. His estimate of  $a$ ,  $m$  and  $d$  is too imprecise to determine what the relevant consumption concept should be. His rejection of the hypothesis, however, may be due to his use of detrended data.

As mentioned above, under constant real rates the consumption innovation should be proportional to the labor income innovation. Results in Kotlikoff-Pakes (1984) for the U.S. and Weissenberger (1985) for the U.K. and West Germany show that the labor income innovations estimated from univariate time-series models explain only a very small fraction of the consumption innovation. This suggests that changes in real rates and "transitory" consumption (i.e., shocks to the utility function and measurement errors in consumption) are important determinant of consumption changes. Even if real rates are constant, simultaneity bias caused by transitory consumption is sufficient to invalidate the orthogonality tests.

Suppose, for example, there is a white noise taste shock  $\eta_t$  to the quadratic utility function:  $u(c_t) = -(\alpha + \eta_t - c_t)^2$ . As shown by Flavin (1981), the Euler equation with  $\beta(1+r) = 1$  is:

$$(3.6) \quad \Delta c_t = \varepsilon_t + \eta_t / (1+r) - \eta_{t-1}.$$

So consumption is no longer a random walk. If  $\eta_{t-1}$  is correlated with  $y_{t-1}$  through general equilibrium interactions, lagged income will be significant in the regression of  $\Delta c_t$  on  $y_{t-1}$ . Even if there is no transitory consumption, the neglect of changes in real rates may lead econometricians to erroneously conclude that the excess sensitivity test fails. Consider, for example, Lucas's (1978) model of asset prices where agents intertemporally optimize without liquidity constraints. Assume that endowments are white noise, so that all endowment changes are temporary. Since there is no saving in equilibrium, observed consumption perfectly tracks income!

Another reason for the random walk hypothesis to appear to fail is time aggregation. There is no reason that the decision interval coincides with the data sampling interval. Christiano (1984) shows using quarterly U.S. data on nondurables plus services and disposable income that the random walk hypothesis (in levels and in logs), while it can be rejected if the decision interval is taken to be the sampling interval, is consistent with the quarterly data if the decision interval is semi-quarterly.

And then there is the question of aggregation across consumers. Unless the utility function takes a specific form like a quadratic form, the Euler equation does not aggregate across consumers. What then is it that we have been estimating on aggregate data? As Constantinides (1984) has shown, at

least in Arrow-Debreu economies there exists a fictitious representative consumer who maximizes a utility function defined over the aggregate of individual consumptions generated by consumers with heterogeneous preferences. But since in general that representative consumer's utility function depends on income distribution, the preference parameters estimated on aggregate data is not invariant to changes in policy rules that induce redistribution of income.<sup>11</sup>

This list of caveats suggests that the time-series evidence is far from conclusive. Furthermore, key parameters (preference parameters and the  $\lambda$  parameter) have not been sharply estimated. Our interest, therefore, naturally turns to the wealth of information contained in micro data. By using micro data, we may be able to avoid problems associated with simultaneity, aggregation, and nonstationarity that are inherent in aggregate time-series data. However, as we will see, micro data have their own problems.

## B. Tests using Micro Data

To implement the Euler equation approach at micro level, we need panel data because the Euler equation involves consumption at two points in time. A typical panel data set has information for a large number ( $N$ ) of households but the number of periods covered ( $T$ ) is small. If  $x_{it}$  is the value of  $x$  for household  $i$  at time  $t$  and if the population of households from which the sample is drawn is represented by a uniform distribution over the unit interval, the (population) mean of  $x_{it}$  is  $\int_0^1 x_t(\omega) d\omega$ , which can be consistently estimated by the cross-section average  $N^{-1} \sum_{i=1}^N x_{it}$ . The variance and the covariance are defined accordingly. A very useful discussion of the econometrics of panel data can be found in Chamberlain (1984).

Hall-Mishkin (1982) were the first implementation of the Euler equation approach on panel data. They examine the relation between consumption innovations and income innovations. The data come from the University of Michigan's Panel Study of Income Dynamics (PSID) which contains information on food consumption (including expenditure in restaurants) in an average week of the year and income over several years. The following is a simplified version of the model. Make the auxiliary assumption that labor income  $w_t$  is described by

$$(3.7) \quad \Delta w_t \equiv w_t - w_{t-1} = u_t + \Delta v_t,$$

where  $u$  and  $v$  are serially and mutually uncorrelated shocks to labor income. Thus  $u$  and  $v$  are permanent and temporary shocks. Under the assumption of quadratic utility and  $\beta(1+r) = 1$  the change in consumption under no liquidity constraints is directly tied to these shocks as

$$(3.8) \quad \Delta c_t = \alpha u_t + \alpha k v_t + \Delta \xi_t,$$

where  $\xi$  is an additive measurement error in consumption and  $\alpha$  is the marginal expenditure share of food.<sup>12</sup> The temporary income coefficient  $k$  should be close to 0. Under an infinite horizon, it equals  $r/(1+r)$  (see the expression for  $k$  right below (3.1)). This model, however, turned out to be inconsistent with the data because it failed the orthogonality test: the lagged income change was negative and significant in the regression of  $\Delta c_t$  on  $\Delta y_{t-1}$ . So the model is augmented to encompass the "rule of thumb" consumers (whose consumption simply tracks income) as

$$(3.9) \quad \Delta c_t = (1-\lambda)(\alpha u_t + \alpha k v_t) + \lambda(\alpha u_t + \alpha \Delta v_t) + \Delta \xi_t.$$

(3.7) and (3.9) imply that each element of the covariance matrix of the vector  $(\Delta c_1, \dots, \Delta c_T, \Delta w_1, \dots, \Delta w_T)$  is a function of the parameters of the model (which include  $\alpha$ ,  $k$ ,  $\lambda$ , and the variance of  $u, v, \xi$ ). Hall-Mishkin (1982) use the maximum likelihood procedure assuming normality. The normality assumption is unwarranted if a constant fraction  $\lambda$  of the population (rather than of consumption) is assumed to follow the rule of thumb, because then  $\Delta c_t$  will have a binomial element. If the distribution is not normal, their point estimate is consistent but standard errors are biased probably downwards (see Camberlain (1984)). They use disposable income for  $w$ , presumably because under constant real rates there should be no shocks to property income. Their estimates indicate that more than 90 percent of the variance in  $\Delta c$  is accounted for by the consumption measurement error. Their estimate of  $k$  of 0.17 is somewhat larger than the theoretical prediction.  $\lambda$  is estimated to be 0.20 with a  $t$  value of about 3. Bernanke (1984) applied this methodology to data on automobile expenditure (University of Michigan's Survey of Consumer Finances). His estimate of  $\lambda$  does not indicate the presence of rule of thumb expenditure. This may be explained by the fact that automobile expenditure can easily be financed.

Probably the most serious criticism of the methodology just described is its neglect of income measurement error. Since the autocorrelation of income changes gets garbled by (possibly serially correlated) income measurement errors, it is difficult to model the true income process with confidence. Also, even under a given specification of the income process, the model becomes very difficult to identify. A small correlation between

consumption and income changes is consistent with intertemporally optimizing consumers partially responding to mostly transitory income changes. But it is also consistent with rule of thumb consumers weakly responding to noisy measure of income changes. The excess sensitivity test in micro data with income measurement error is practically impossible. The issue of income measurement error in the PSID data is taken up by Altonji-Siow (1985) who use the log-linear version (2.10) of the Euler equation. By allowing for a taste shock  $\eta$  in the constant relative risk aversion utility function  $u(c) = \exp(\eta/\sigma)c^{1-1/\sigma}$  and a multiplicative measurement error  $\xi$  in consumption, the error term in (2.10) becomes:  $\sigma e_2 + \Delta\eta_2 + \Delta\xi_2$ . Treating the real rate  $r$  as constant across consumers, the relation of the forecast error  $e_2$  with the current income change  $\Delta\ln y_2 \equiv \ln(y_2) - \ln(y_1)$  can be estimated by regressing  $\Delta\ln c_2 \equiv \ln(c_2) - \ln(c_1)$  on  $\Delta\ln y_2$  (provided, of course, that  $\eta$  and  $\xi$  are uncorrelated with  $\Delta\ln y_t$ ). But if  $y_t$  contains measurement error, we have the classical errors-in-variables problem that the regression estimate of the  $\Delta\ln y_2$  coefficient is biased toward zero. This can be avoided by the use of instrumental variables that are uncorrelated with the income measurement error. Altonji-Siow's regression estimate of the coefficient is 0.08 (see column 6 of their table 2). If such variables as the change in wage rates, hours of unemployed, past quits and layoffs are used as instruments, the estimate jumps to somewhere between 0.3 and 0.4 with a  $t$  value of above 4. Another indication of the importance of income measurement error is the low explanatory power of  $\Delta\ln y_2$  evidenced by a meagre  $R^2$  of below 0.5 percent.

Altonji-Siow also conducted the orthogonality test by regressing  $\Delta\ln c_2$  on variables dated 1. Contrary to Hall-Mishkin (1982), they found that no variables (not even lagged income changes) were significant as a group or individually. They note that the difference is attributable to their sample



selection rule of eliminating both high income and low income families due to the requirement that valid data be available on the variables used as instruments. This is an example in which treatment of extreme cases in micro data could influence results in an important way.

Exactly the type of orthogonality test for liquidity constraints described in the previous section is carried out by Runkle (1983) using the Denver Income Maintenance Experiment and by Zeldes (1984) using the PSID data. Both use the log-linear version (2.10). Zeldes finds that, in accordance with the hypothesis of liquidity constraints, the coefficient of lagged income  $y_1$  (to use the notation in the previous section) is negative and significant (with a  $t$  value of over 3) for low wealth families but not for the rest. Because of the cross-section difference in the marginal income tax rate, the after-tax real rate  $r$  in (2.10) differs across households in the sample. This permits the estimation of the intertemporal substitution elasticity  $\sigma$ . Zeldes's estimate from high wealth families is about 0.3 and insignificant. Runkle's estimate of  $\sigma$  is less precise. Runkle also finds the coefficient of lagged wealth  $A_1$  to be significant (with a  $t$  value of about 3) for nonhomeowners and low wealth families, but it has a wrong sign (positive). This may be explained by a measurement error in wealth. Unlike other studies, Runkle's consumption concept is annual expenditure on nondurables plus services. This measure is computed by subtracting durables expenditure from total spending which in turn is computed as the difference between annual disposable income and the change in wealth. Thus consumption inherits the wealth measurement error with a minus sign, causing  $A_1$  to be negatively correlated with  $c_1$  and positively with  $\Delta \ln c_2$ . This correlation will get amplified by the sample selection rule of including only low wealth families.

The last three studies cited above do not fully exploit the panel nature of the data. Instead of estimating  $T$  equations as a system where the  $t$ -th equation has  $\Delta \ln c_t$  as the dependent variable, they pooled the equations into one. Because the error term -- which consists of the consumption innovation (forecast error)  $e_t$ , the change in consumption measurement error  $\Delta \xi_t$  and the change in taste shock  $\Delta \eta_t$  -- is likely to be negatively serially correlated, the standard errors computed by those studies are likely to be biased upwards. Another technical problem, which applies to all the models that have the forecast error term as part of the error term, has been pointed out by Chamberlain (1984). The empirical studies have used the orthogonality condition that  $e_t$  is uncorrelated with  $x_{t-1}$  in the lagged information set  $\Omega_{t-1}$ , which justifies the use of the regression technique in the time-series context. Although it guarantees that a time average of  $e_t x_{t-1}$  converges to 0 as  $T \rightarrow \infty$ , the rational expectations orthogonality condition does not necessarily mean that a cross-section average converge to 0 as  $N \rightarrow \infty$ . Namely, it does not guarantee that  $\int_0^1 e_t(\omega) x_{t-1}(\omega) d\omega = 0$  (to use the notation introduced at the start of this subsection B). Therefore, all the significant coefficients of lagged variables discovered in the literature using panel data can in principle be explained away by the (cross-section) correlation between  $e_t$  and  $x_{t-1}$ . The practical importance of this problem, however, is hard to evaluate.<sup>13</sup> It is somewhat reassuring to note that this problem does not arise if the structure of the economy is such that the forecast error  $e_t$  is the sum of an economy-wide common shock (which can be separated from  $e_t$  as a constant across agents) and an idiosyncratic shock. The economy-wide shock, however, renders the original intercept term (e.g.,  $\sigma \ln(\beta)$  in (2.10)) unidentifiable.

The failure of the orthogonality test can also be explained by the often neglected distinction between consumption and expenditure which is important when the commodity is durable. The unanticipated part of an increase in income calls for an upward revision in the level of consumption and hence an increase in expenditure. But if the commodity is durable, the increased expenditure means a higher level of the stock of consumption to be carried over to the next period, which will depress expenditure in the next period. This explains the negative correlation of the change in expenditure with the lagged income level and change. This also shows that expenditure on durables cannot be a random walk (Mankiw (1982)). The issue of durability of a wide range of commodities was investigated by Hayashi (1985c) who used a Japanese panel data set on expenditure on several commodities (the 1982 Survey of Family Consumption, conducted by the Economic Planning Agency). He finds that nondurables and services excluding food are highly durable. His estimate of  $\lambda$ , the fraction of the "rule of thumb" consumers in the population consisting of wage earners, is about 0.15 with a  $t$  value of about 8. He was able to avoid the problem mentioned in the previous section because in his data set expectations are directly measured. The low estimate of  $\lambda$  is also evidenced in his regression where food expenditure responds to unanticipated income changes much more strongly than to anticipated income changes. The  $R^2$  of the regression, however, is less than 0.04.

Besides income measurement errors, there are a couple of explanations for the low explanatory power of current income changes in the equation for consumption changes reported in the literature. Changes in income, if either perfectly foreseen or fully insured, do not lead to revisions in consumption. But this is at variance with the result in Altonji-Siow (1985)

and Hayashi (1985c) that the income change coefficient is statistically significant. The other explanation is that consumption changes are dominated by changes in transitory consumption (consumption measurement errors and taste shocks). The standard deviation of the growth rate (measured as the change in logs) of consumption is 0.36 in Zeldes's (1984) data where the consumption concept is food expenditure and 0.33 in Runkle's (1983) data where the consumption concept is nondurables plus services. In the data set used by Hayashi (1985c) (where data are collected by interviewers actually visiting households in the survey), the ratio of the standard deviation of the change in food expenditure to the mean of the level is about 0.2. Using a Japanese monthly diary data set on hundreds of expenditure items (the Family Income and Expenditure Survey compiled by the Prime Minister's Office) where diaries are collected twice a month, Hayashi (1985b) calculated the standard deviation of the growth rate of quarterly food expenditure (including expenditure in restaurants) to be about 0.2. Since the measurement error in this monthly diary survey is likely to be small, we may conclude that close to a half of the food expenditure growth in the PSID data set (where at least some data in later waves are collected over the phone) is attributable to measurement error. As for the division of the remaining part of food expenditure changes into the forecast error and the taste shock, Hayashi (1985b) reports that the first order serial correlation coefficient of monthly food expenditure changes is roughly -0.5 on average. Because there is no durability in food, the change in food expenditure net of measurement errors is the sum of the forecast error (the random walk component) and a moving average of a taste shock (see (3.6)). It seems that the change in food expenditure is dominated by a taste shock that is close to a white noise. Even with an ideal micro data set we would

never be able to explain more than, say, 10 percent of changes in food expenditure by income changes.

Finally, there are two studies based on cross-section data that specifically address the issue of liquidity constraints. Both use the 1963/64 Survey of Financial Characteristics of Consumers compiled by the Board of Governors of the Federal Reserve System. Mariger (1983) uses the implication of deterministic intertemporal optimization that the growth rate of consumption after adjustment for family size is constant over an interval between two successive occurrence of binding credit rationing. Given the age profile of income, this is sufficient to estimate from the level of current consumption and wealth the length of the horizon for each household in the sample. He estimates that 7 percent of the sample (which oversampled wealthy families) has a one-year horizon. His estimation procedure seems to depend critically on the assumption that the instantaneous utility is independent of age. Hayashi (1985a) splits the sample into high and low saving families, and finds that the correlation structure between consumption and other variables including income significantly differ between the two subsets of the sample even after a removal of the possible bias arising from the sample splitting. He interprets the difference as evidence for the presence of liquidity constraints on the ground that high saving families are not likely to be liquidity constrained.

The conclusions we may draw from micro studies are the following. First, at least a small fraction of the population appears to be liquidity constrained in that the Euler equation fails in a way predicted by the hypothesis of liquidity constraints. Second, because most tests on micro data are the orthogonality tests, we still do not know with confidence the average horizon of those who satisfy the Euler equation. That information

is necessary to calculate the response of consumption to a temporary income change and, more generally, to a change in the stochastic process for income. Third, the change in consumption is dominated by the transitory consumption component. Only a small fraction of the change is explainable by income changes. This suggests the fourth (and somewhat pessimistic) conclusion: the observed correlation of the change in consumption with lagged income is also attributable to a correlation between consumption measurement error and income measurement error or between consumption taste shock and leisure taste shock. The latter correlation can occur despite our basic maintained hypothesis that consumption and leisure are separable in the utility function. To identify the model we need variables that are uncorrelated with transitory consumption. Such variables are hard to come by.

#### 4. In What Sense is the Loan Market "Imperfect"?

It is not entirely clear what we do with the hard-won evidence that some consumers are liquidity constrained. Does a consensus estimate of  $\lambda$  (the share of "rule of thumb" consumers in the population) of (say) 0.15 imply that a debt-financed tax cut of \$100 for everyone increase per capita consumption by \$15? How is the size of the lagged income coefficient related to aggregate fluctuations? The problem stems from the vagueness of the term "liquidity constraints" or "imperfect loan markets" that we noted in section 2. We will argue by three examples that, unless the exact nature of the imperfection of the loan market is identified, the economic implication of the available evidence cannot be determined. In all three examples the Euler equation fails and so consumption shows excess sensitivity. The MPC (marginal propensity to consume) out of a temporary income increase varies across the examples. Only in the last example the Ricardian Equivalence Theorem fails to hold.

Consider a consumer in the two-period model whose second period labor income  $w_2$  is a random variable that takes with probability  $p$  a low value of  $w_2^L$  and with probability  $1-p$  a high value of  $w_2^H$ . We assume that  $u'(0) < +\infty$ , so that the consumer may choose a consumption plan that allows default with zero consumption. If an actuarially fair insurance is available, the risk averse consumer will engage in an insurance scheme that eliminates the income risk entirely. So the intertemporal optimization problem is exactly as in Figure 1(a) with the  $w_2^e$  in the figure replaced by  $pw_2^L + (1-p)w_2^H$ . The relevant marginal loan rate is the risk-free rate  $r$ . If we had data on the consumption and income changes for consumers whose utility function may differ in a way unrelated to the difference across consumers in the distribution of  $w_2$ , there should be no significant correlation between the

two variables. This is a test proposed by Scheinkman (1984) of the Arrow-Debreu complete markets paradigm. Note that this restriction is stronger than the Euler equation which by itself places no restrictions on the contemporaneous correlation of actual consumption changes with actual income changes.

In the following three examples, we assume that, for reasons to be discussed later, such income risk sharing is not available to the consumer. In the first example, lenders have the same opinion about the distribution of  $w_2$ . Let  $Z$  and  $R$  be the loan principal and the contracted repayment. Since  $w_2$  is at least  $w_2^L$  with probability one, the loan rate (the borrowing rate available to the consumer) must equal the market rate  $r$  under perfect competition among lenders when  $Z \leq w_2^L/(1+r)$ . However, when  $Z > w_2^L/(1+r)$ , the consumer will default with probability  $p$  on a marginal loan above  $w_2^L/(1+r)$ . The loan rate  $r^*$  on such a marginal loan satisfies

$$(4.1) \quad 1 + r^* = (1 + r)/(1 - p),$$

if lenders are risk neutral or if there are many consumers of the same characteristic. Thus the marginal loan rate schedule jumps up from  $r$  to  $r^*$  at  $Z = w_2^L/(1+r)$ . If the consumer defaults, the second period consumption is zero. So the expected lifetime utility under a loan contract  $(Z,R)$  is:

$$(4.2) \quad u(w_1+Z) + p\beta u[\max(0, w_2^L - R)] + (1-p)\beta u[\max(0, w_2^H - R)].$$

Since the focus of the paper is on liquidity constraints, we suppose that the value of  $(w_1, w_2^L, w_2^H)$  is such that the consumer facing this marginal loan rate schedule plans to default in the low-income state. Thus



the consumer behaves as if the middle term in (4.2) is absent. It is easy to show that the Euler equation is:

$$(4.3) \quad u'(c_1) = \beta(1+r^*)(1-p)u'(c_2^H),$$

where  $c_1 = w_1 + Z$  and  $c_2^H = w_2^H - R$ . This is a violation of the Euler equation without liquidity constraints because the latter requires:

$$(4.4) \quad u'(c_1) = \beta(1+r)[pu'(0) + (1-p)u'(c_2^H)].$$

It also is different from the Euler equation that would result if (as we will assume for the third example below) the gap between the loan rate and the risk-free rates were exogenously given and unrelated to defaults:

$$(4.5) \quad u'(c_1) = \beta(1+r')[pu'(c_2^L) + (1-p)u'(c_2^H)],$$

where  $1+r' = dA_2/d(A_1+w_1-c_1)$  and  $c_2^L$  is the second period consumption in the low-income state. Despite the existence in this first example of the loan rate schedule as an increasing function of the loan quantity, the preference parameters cannot be estimated by (4.5).

Because the Euler equation without liquidity constraints (4.4) fails, consumption shows excess sensitivity.<sup>14</sup> However, the Ricardian Equivalence Theorem still holds. To see this, suppose the government cuts taxes in real terms by  $x$  in exchange for a second period tax increase of  $(1+r)x$ . This increases  $w_1$  by  $x$  but reduces both  $w_2^L$  and  $w_2^H$  by  $(1+r)x$ . Thus the marginal loan rate schedule shifts to the left by exactly  $x$ . But the demand for private loans is also reduced by  $x$  because of the newly acquired government

loan. The net result of a debt-financed tax cut, therefore, is that the government loan thus provided crowds out the private loan market on a dollar-for-dollar basis and leaves the optimal consumption plan unaltered.

It is not at all clear why income insurance markets are not present in this example where both borrowers and lenders have common knowledge about the distribution of future income. The equilibrium loan contract  $(Z, R)$  is really a combination of two things: (i) an actuarially fair insurance whose payoff inclusive of the premium is  $(1+r)Z - w_2^L$  when  $w_2 = w_2^L$  and  $-(p/(1-p))[(1+r)Z - w_2^L]$  when  $w_2 = w_2^H$ , and (ii) a risk-free loan of principal  $Z$ . The insurance implicit in the loan is constrained so that  $c_2 = 0$  when  $w_2 = w_2^L$ . So the loan market cannot be a perfect substitute for insurance markets, although one would not call this loan market "imperfect".

The next example we consider is similar to the model considered by Jaffee-Russel (1976). For each class of consumers indexed by the first period income  $w_1$ , there are two types (type L and type H) of consumers. Labor income in period 1 and 2 is  $(w_1, w_2^L)$  for type L and  $(w_1, w_2^H)$  for type H consumers. That is,  $p$  is unity for type L and zero for type H consumers. The type is private information: no one knows the type of other consumers but oneself. This eliminates private income insurance markets. But the loan market will still exist. In Figure 2 the horizontal and vertical axes represent consumption and income in the two periods. The origin for type L consumers is the point  $O^L$  on the vertical axis, reflecting the difference in the second period income between the two types. The same point E represents the initial endowments both for type L with  $(w_1, w_2^L)$  and for type H with  $(w_1, w_2^H)$ . The slope of the line ED is  $1+r$ , because if the loan principal is less than  $FD (= w_2^L/(1+r))$  no defaults will occur. Let  $\pi$  be the share of type L consumers in the population consisting of consumers with the same

first period income  $w_1$ . If the loan principal is greater than  $FD$  and if both types apply for the loan, only a fraction  $(1-\pi)$  of a marginal loan above  $w_2^L/(1+r)$  will be repaid, so that the marginal loan rate is  $r^*$  which satisfies the condition analogous to (4.1):  $1+r^* = (1+r)/(1-\pi)$ . Thus, the line  $DC$  with a slope of  $1+r^*$  along with the line  $ED$  with a slope of  $1+r$  represents the set of  $(c_1, c_2)$  available to type H consumers when both types apply for the same loan.

As we know from Rothschild-Stiglitz (1976) and Wilson (1977), there are two types of equilibrium in this informationally imperfect loan market. In a separating equilibrium, type L consumers choose the point A while the consumption plan for type H is the point B. Since type L consumers are indifferent between A and B (which translates into the point G as type L consumers will not repay the loan in full), they have no incentive to switch from A to B by claiming that they are of type H. Type H consumers are credit rationed in the sense that they would like to borrow more at the stated loan rate of  $r$ . No defaults occur. In a pooling equilibrium, both types choose the loan contract represented by the point C where an indifference curve for type H is tangent to the line  $DC$ . If type H prefers B to C, then a pair of loan contracts (A,B) is the separating equilibrium (Wilson's (1977) E1 equilibrium). Otherwise the point C is the equilibrium loan contract (Wilson's E2 equilibrium).<sup>15</sup> Since type L consumers prefer C to A, they also apply for the loan contract represented by the point C knowing that they will default. Clearly, the Euler equation fails to hold for both types in the pooling equilibrium and for type H in the separating equilibrium. In particular, for type H the MPC out of a temporary increase in current income is about unity in the separating equilibrium. We can think of type H consumers as ones experiencing a temporary drop in income.

Because they are mixed up with low income people in the loan market, their consumption is forced to be temporarily low. Thus their consumption appears to be tracking income.

It is now easy to see that, whichever equilibrium obtains, a debt-financed tax cut of quantity  $x$  in exchange for a tax increase in the second period of  $(1+r)x$  will have no real effects. The tax cut will move the initial endowment from  $E$  to  $E'$ . Now lenders realize that the fraction  $\pi$  of a marginal loan above  $w_2^L/(1+r) - x$  will be defaulted if both types apply for the loan. In the separating equilibrium, the amount of credit available is cut back by exactly  $x$ . In the pooling equilibrium, the loan principal at which the loan rate jumps up from  $r$  to  $r^*$  is also reduced by  $x$ . The equilibrium consumption plan is left unaltered. This irrelevance result remains valid even if type L consumers fail to pay the second period tax in full in the event of default, as long as the unpaid tax is borne by type H consumers of the same first period income. Although income redistribution between the two types occurs, the following argument for irrelevance does not assume homothetic preferences. Suppose the size of a debt-financed tax cut is  $FI$  in Figure 2. The second period tax to be paid by type L exceeds  $w_2^L$ . If the unpaid tax bill when type L defaults is to be picked up by type H consumers, the second period tax on type H is precisely  $EF$  plus  $IJ$  where the point  $J$  is the vertical projection of  $I$  on  $DB$ .<sup>16</sup> Thus the feasible set of  $(c_1, c_2)$  for type H when both types apply for the same loan is unchanged. If the initial equilibrium is the separating equilibrium  $(A, B)$ , then clearly this equilibrium is undisturbed by the tax cut.

In the preceding two examples, the excess sensitivity of consumption is not exploitable for stabilization purposes through substitution of taxes for the public debt. We now turn to the last example where the excess

sensitivity is exploitable. Here the basic premise is that the government is more efficient than the private loan market in arranging loans. This may arise if transactions costs for collecting private loans are higher than for collecting taxes, or if the court does not honor at least some private loan contracts. As shown by Barro (1984), a debt-financed tax cut will increase aggregate consumption because the government's increased share of lending activity raises the overall efficiency. In the extreme case where the private loan market is nonexistent because the legal system does not honor any private loan contracts, the MPC out of an increase in income induced by a debt-financed tax cut is exactly one, not zero. A model in which the Ricardian Equivalence Theorem does not hold but in which the gap between the loan rate and the risk-free rate is based on imperfect information, is also constructed by King (1984). Unlike our second example above, King assumes that lenders cannot observe the total loan quantity which a consumer borrows from various lenders.

## 5. Concluding Remarks

By way of concluding, let us see what answers have been provided by recent empirical work to the three questions raised at the beginning of this paper. The answer to the first question is positive. Some consumers are liquidity constrained in the sense of credit rationing or differential interest rates. But the same conclusion can be obtained from the following simple observation: according to the Federal Reserve Bulletin, the average rate on 24-month personal loans in 1982 was 18.65 percent, while the yield on 2-year U.S. Treasury notes in 1982 was 12.80 percent. This is a piece of evidence for liquidity constraints with a standard error of zero.

The estimation of the preference parameters under liquidity constraints is probably meaningless if done on aggregate data, because it would be impossible for economies with imperfect loan markets to induce the utility function of the representative consumer from heterogeneous consumers. If micro data are used, consumers who are likely to be liquidity constrained should be excluded from the sample because their first-order optimality condition depends on the specification of the loan market. The estimation of the preference parameters using a short panel is possible only when we can get cross-section variations in prices (e.g., after-tax interest rates).

The available evidence gives only a partial answer to the third question. The finding that the Euler equation fails for a fraction of the population does imply that consumption is excessively sensitive to temporary income changes. But that does not allow us to calculate quantitatively (even abstracting from the general equilibrium interaction running from consumption to income) the response of consumption to a hypothetical temporary increase in labor income. This is partly because the horizon of those who satisfy the Euler equation is unknown and partly because the

concomitant changes in the loan rate schedule depend on the specification of the loan market. As for the Ricardian Equivalence Theorem, the available evidence has no implication.

## Footnotes

1. Dornbusch-Fischer (1984, pp.186-87) cite liquidity constraints as the candidate to explain the excess sensitivity of consumption to income. DeLong-Summers (1984) credits the increased availability of consumer loans with the reduced variability of aggregate demand in the postwar U.S. Scheinkman-Weiss (1984) shows in an equilibrium model of business cycles inhabited by optimizing agents that borrowing constraints fundamentally alter the time-series properties of the model. In Walsh's (1984) general equilibrium model, anticipated money has real effects as it changes the probability of people being short of cash and lines of credit.
2. The statements in this paragraph about the MPC's remain valid if risky assets are introduced.
3. See Hayashi (1985a) for other reasons against the consumption function approach.
4. The derivation of (2.9) and (2.10) uses the approximation that  $\sum_{n=0}^{\infty} (1+x)^{-n} \approx x^{-1}$ .
5. If the realization of  $r$  is not known in period  $t$ , the consumption innovation is correlated with  $r$ . Thus  $r$  must be instrumented by some variable (e.g., lagged value of  $r$ ) in the period  $t$  information set.
6. See Muellbauer (1983), Mariger (1983) and Zeldes (1984). Rotemberg (1984) shows that expected future liquidity constraints can explain why people hold financial assets and liabilities simultaneously.
7. If the utility function is quadratic and if  $\beta(1+r) = 1$ , the consumption innovation is simply the change in consumption. Kotlikoff-Pakes (1984) show how to calculate the consumption innovation for general nonlinear utility functions.
8. However, Miron (1984) reports that when the seasonal fluctuations in consumption are explicitly included in the utility function the overidentifying restrictions cannot be rejected.
9. One exception is Summers (1982) who puts  $c - \lambda w$  (consumption minus a fraction of labor income) in place of  $c$  in the Euler equation and estimates  $\lambda$ .  $\lambda w$  is the part of aggregate consumption by liquidity constrained agents. His estimate of  $\lambda$  is too imprecise to draw any conclusions. See text below for the definition of  $\lambda$ .
10. DeLong-Summers (1984) and Mankiw-Shapiro (1984) show that disposable income in postwar U.S. is a random walk.
11. See, however, Eichenbaum-Hansen (1984) who show that a restriction on individual heterogeneity makes the representative agent's utility function free from income distribution. They also incorporate durability of commodities into the model.
12. The term  $\xi$  can also be interpreted as a preference shock. See (3.6).



13. Hayashi (1985c) gives a somewhat contrived example in which an unanticipated income tax reform causes a cross-section correlation between  $e_2$  and  $y_1$ .
14. A simple calculation under the assumption that  $\beta(1+r) = 1$  shows that the MPC out of a temporary income increase is  $(1+r^*)/(2+r^*)$ . Under a complete income risk pooling the MPC is  $(1+r)/(2+r)$ .
15. This E2 equilibrium seems to correspond to the situation referred to by Jaffee-Russel (1976) as credit rationing.
16. If type L consumers default, the additional tax to be levied on a type H consumer is  $DI \times (1+r)p/(1-p)$ . This equals KJ in Figure 2 because IJ equals  $DI \times (1+r^*) = DI \times (1+r)/(1-p)$  and IK equals  $DI \times (1+r)$ .

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Figure 1(a)

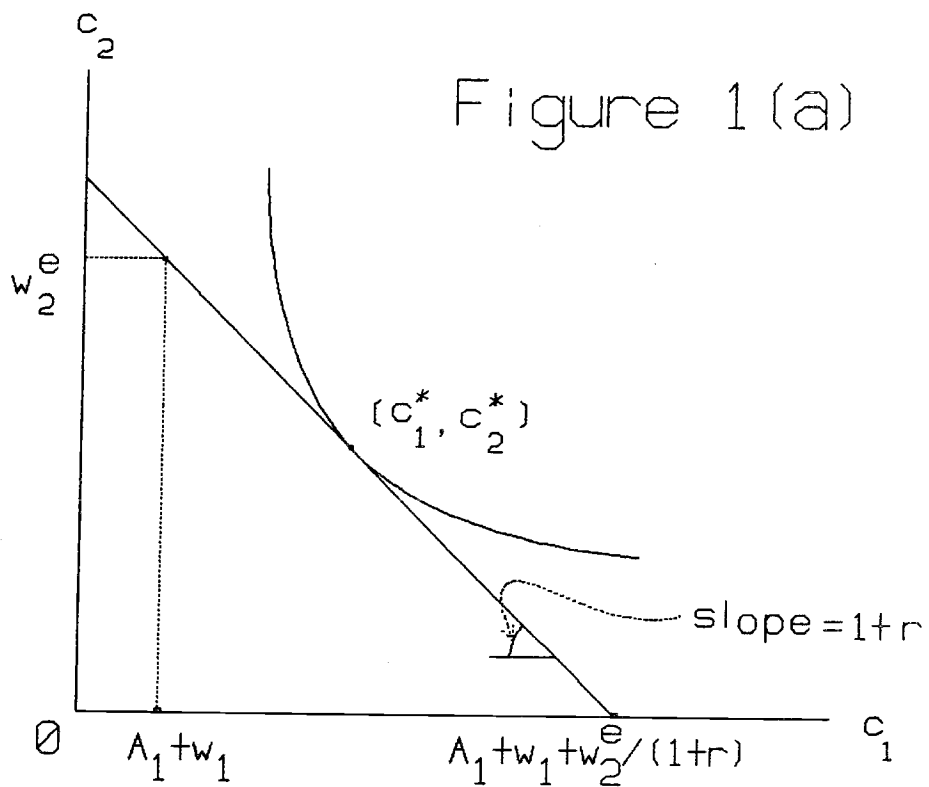


Figure 1(b)

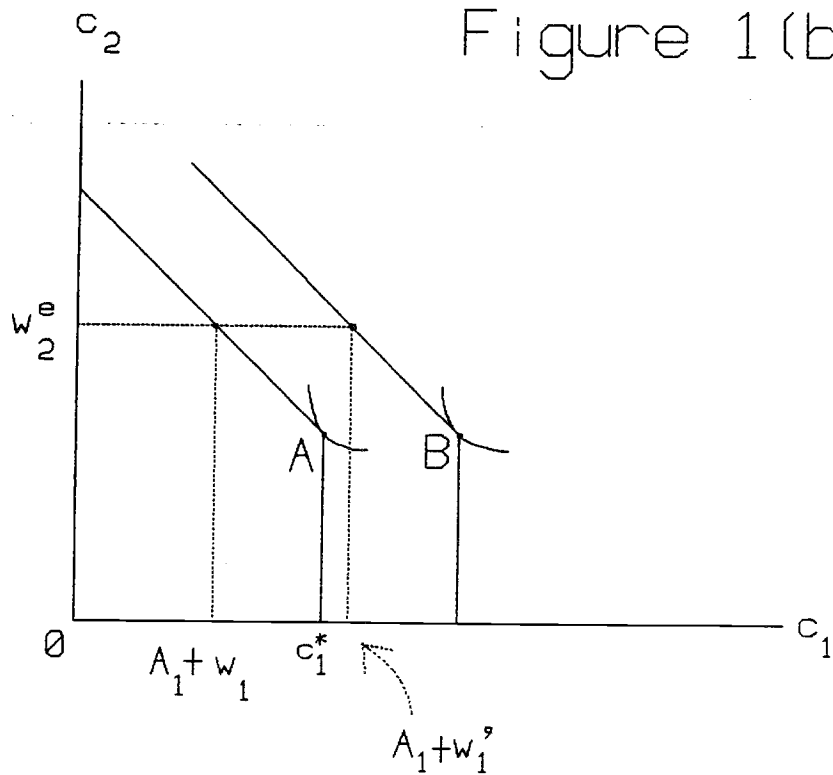


Figure 1(c)

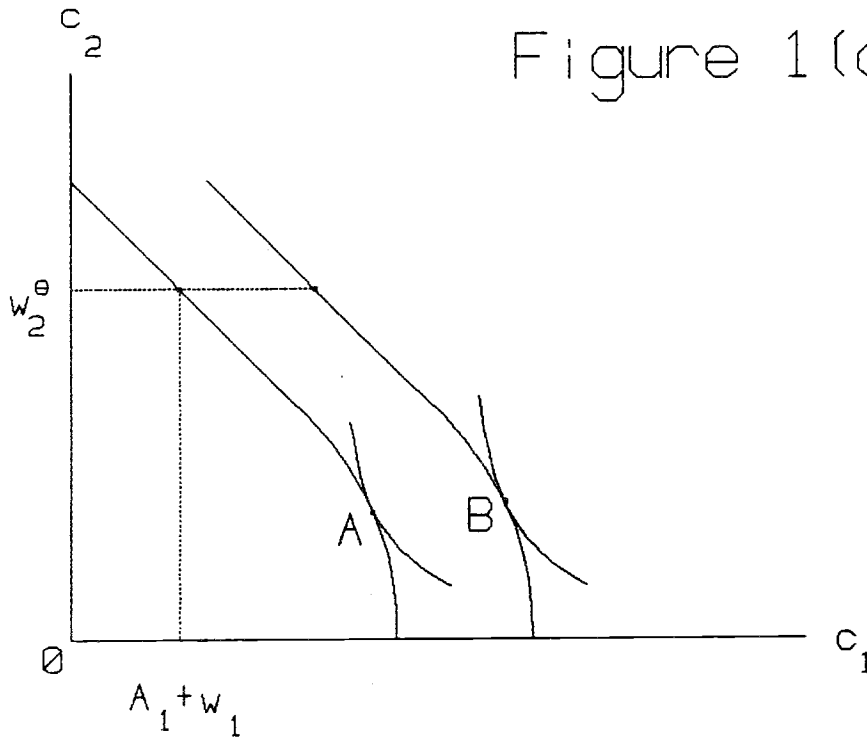


Figure 2.

