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A PROBLEM OF FINANCIAL MARKET EQUILIBRIUM WHEN THE TIMING OF TAX PAYMENTS IS INDETERMINATE

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ABSTRACT

If firms are indifferent about the timing of dividends, the government's cash flow from taxes on dividends is indeterminate. In an earlier paper, I showed in the context of a world without uncertainty that variations in tax receipts from this source would have no real effects. The extension of the analysis to a world of risk turns out to involve new elements that may be of some general interest. In particular, the conditions for neutrality seem less likely to be fulfilled in a practical context.

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A Problem of Financial Market Equilibrium When the Timing of Tax Payments is Indeterminate

by

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1. Introduction.

This paper concerns an aspect of the question: When do government deficits matter? It takes as a starting point previous work (1981) showing how endogenously generated deficits might have no real effect, a result obtained under an assumption of perfect substitutability between government and private debt. The extension to a world of risky debt where the latter no longer holds turns out to involve new elements that may be of some general interest. In particular, the conditions for neutrality seem less likely to be fulfilled in practice.

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The underlying idea is that it should not matter when taxes are paid, provided there is an appropriate compensating interest element in the postponed liability. This notion conflicts with the assumption often employed that it is the government's cash flow balance that counts, even though current deficits may be offset by correspondingly larger liability for future tax payments, and surpluses may reflect drawing down liabilities for future taxes. This issue arises especially strongly in the context of analysis of proposals for consumption-type taxes, where there is a choice between a literal consumption tax and a tax on wage and transfer receipts. Typically the two approaches generate the same liabilities in a present value sense, but very different cash flows. In some systems (for example, the Cash Flow Tax analysed in Bradford et al. (1984)), the taxpayer has wide latitude to choose between the approaches.

The specific case I analyze here presents the same issues in a particularly clear way. It involves a tax on distributions by corporations to equity holders, in essence, a dividend tax. If such a tax is assessed at a flat rate which is not expected to change, and if negative distributions (sales of new equity) are included (i.e., subsidized), the case is quite compelling in a partial equilibrium setting that the level of the tax should have no influence on real or financial transactions of a corporation acting in the interest of its stockholders. The reason is simply that the flat tax changes proportionately the consequences of all decisions as far as stockholder outcomes are concerned. In particular, the trade-off in the after-tax dollars for the shareholders between a larger distribution today and the consequently smaller distribution at some future time is unaffected by the rate of tax.

In the absence of all taxes (and transactions costs) the various versions of the Modigliani-Miller (1958) Theorem tell us the corporation will be indifferent between debt and equity finance. An implication is that the timing of dividend payments is a matter of indifference. Since, as I have just argued informally, a flat tax on dividends has no effect on the optimal financial policy, something like the Modigliani-Miller Theorem should continue to hold. However, the choice of pay-out affects the government's cash flow. Government receipts will be determined by the whims of corporate managers; private wealth-maximizing calculations are insufficient to fix the path of revenues. Will the consequent fluctuations in tax receipts have real effects?

In Bradford (1981) I spelled out an overlapping-generations model in which government debt does generally influence the rational expectations equilibrium path, but in which variations in government debt attributable to variations in distribution-tax receipts do not matter. The indifference about financial policy at the level of the firm, in spite of the tax on distributions, carries over to neutrality of the economy's path to the choice of financial policy, even though the flow of tax receipts is affected.

The key question is whether there are general equilibrium effects on the rate of interest. The basis for the neutrality conclusion may be sketched as follows: The capital investment level chosen by the firm is governed by the going interest rate. Therefore, a decision to issue an extra dollar of debt implies a decision to distribute an extra dollar to shareholders. This in turn implies extra tax receipts of t dollars (where t is the rate of tax). If real government spending is fixed, the extra t dollars are devoted to reducing the public debt. The

result thus far is a net addition of 1-t dollars to the supply of financial assets. There remains, however, the equity interest in the corporation, the value of which is predicted to fall, not by one dollar, as a result of the extra debt cum distribution, but by 1-t dollars, in reflection of the government's claim to a fraction t of all cash flows to shareholders. As a consequence, all markets continue to clear. The firm's decision has no real effect, even though tax receipts are increased.

The foregoing analysis support two conclusions. First, a tax on corporate distributions may not have the often-assumed incentive effects with respect to real and financial allocations. And second, variations in government receipts attributable to changes in corporate distribution policy may have no effect on the real path of the economy. The present investigation concerns how the neutrality results are affected if the perfect substitutability among financial assets, used in the argument above and attributable to the assumption of certainty, is replaced in the context of an explicit treatment of uncertainty.

As it turns out, the earlier results carry over without significant complication when only equity is risky at the margin, while corporate debt and marginal public debt are risk free and therefore perfect substitutes. The restriction on government financial choices is a special case of the requirement that applies when the risk characteristics of corporate debt are unrestricted. In the more general case, the neutrality conclusions require that the government policy be describable as one of issuing a certain pre-specified risky debt together with the purchase of a fraction of the private debt supply equal to the tax rate.

These conditions on government behavior are not as arbitrary as might appear. As I stressed in my earlier paper, the proportional tax on corporate distributions in effect gives the government a fractional ownership in the firm. Neutrality will follow if the government uses the tax revenue incident upon an incremental distribution to purchase bonds of the firm. This is just what is needed to preserve the pre-distribution portfolio of the government, taking into account its implicit ownership of corporate equity. The government's behavior, in other words, is exactly what we would--in Modigliani-Miller fashion--expect of a shareholder that receives a dividend.

In Section 2 below I review the uncertainty results as a way of introducing the basic model. The extension to a model with uncertainty is presented in Section 3. Section 4 contains concluding remarks.

2. Review of the Certainty Model.

The model underlying the analysis is in the Samuelson (1959) consumption loan tradition. Individuals live for two periods in an infinite time horizon world. In the first life period each individual works (offering one unit of labor inelastically), consumes, and saves for retirement. "Retirement" describes the second life period, when each individual dissaves and consumes, leaving nothing to his heirs.

All production takes place in the consolidated corporate sector which is modeled as a single price-taking firm. Production conditions are described by a neoclassical production function of capital and labor employed, with constant returns to scale. The capital available to the corporation in any period is inherited from the previous period and is thus fixed in amount before the time of actual production. The output of

a period may either be consumed or frozen into infinitely durable capital. Investment is regarded as reversible.

Savings may be held in three forms: bonds issued by the corporation, shares of its common stock, or bonds issued by the government. A given generation of individuals acquires these financial assets at the end of its first life period, after production for that period has been completed, and after the investment and financial plans of the corporation have been realized.

The holders of its common stock "own" the corporation. The owners at the beginning of a period control that period's production and the real investment which determines the amount of capital which will be available for use in production at the beginning of the next period. The owners of the firm at the beginning of a period specify as well the financial policy for that period, which means they set the amount of funds to be distributed to themselves as dividends and the amount of corporate borrowing. At the end of the period the current owners sell the equity to the young generation of savers.

The government's real spending program is assumed fixed, for simplicity at a zero level. The government is thus modeled as a mechanical cash flow manager: In each period the inherited debt obligation must be paid off, with any shortfall of tax receipts covered by the issue of new debt. Since the distribution tax is the only tax, there is nothing else for the government to do. Note that by allowing the government to make lump-sum transfers the model could be used to permit the government to engage in (pre-announced) intertemporal redistributions. This would involve issue of government debt in amounts larger or smaller than the difference between old debt obligations and distribution tax receipts.

Notation.

To describe the results of the analysis formally, I use the following notation (involving minor changes from that of the 1981 paper to facilitate extension to incorporate uncertainty):

L: total number of labor units available for application during the period (equals the number of individuals born in the period, exogenously given).

K: stock of corporate capital at the beginning of the period (used for production during the period).

B: total stock of corporate indebtedness at the beginning of the period, which must be repaid during the period.

$F(K,L)$: the production function, characterized by constant returns to scale.

D: total distribution made by the corporation during the period.

B^g: total stock of government indebtedness at the beginning of period, which must be repaid during the period.

t: rate of tax on corporate distributions to stockholders (which may be negative).

There are two ordinary prices in the model (current output is numeraire), and one price-like "valuation function:"

w: wage.

d: discount factor, the current-period price of a bond paying one dollar next period.

$V(K_+, B_+)$: a function relating the "ex-dividend" value of equity, that is, the value at the end of a period, after

production is complete, to the financial and investment decisions of the current owners.

For any variable, X, let X_+, X_{++}, etc., represent its value in succeeding periods: X_- its value in the preceding period. In order to determine how to value the firm's equity, individuals must form expectations about the prices and valuation function one period hence. Let w_+^e denote the value of w expected to obtain in the next period, and similarly for d and V. I assume that everyone agrees about w_+^e, d_+^e and V_+^e; forecasts more than one period into the future are not needed.

There are three classes of agents in the model, two displaying maximizing behavior. The "young" try to maximize $U(c^1, c_+^2)$ (labor is supplied inelastically), where c^i is consumption durin the i^{th} period of the life cycle. The "old" manage the corporation to maximize c^2, which means maximize the sum of after-tax distributions and the proceeds from sale of equity, $(1 - t)D - V(K_+, B_+)$. The third agent is the government which simply manages the cash flow according to $dB_+^g = B^g - tD$. (Reminder: d is a price, not the differential operator.)

Evolution of the Economy.

The situation inherited from the past is described by (K, B, B^g, L), with L evolving exogenously along a known path. The requirements placed on the model world are that the evolution to (K_+, B_+, B_t^g, L_+) be determined by clearing of competitive spot markets for labor, corporate bonds, government bonds, corporate equity and goods, and that the price expectations on which the value of the firm depends be "rational." The general notion of rational expectations, attributed to Muth (1961), here encompasses two properties: Expectations are correct, and they are

determinate, in the sense that they are governed by knowledge of the economic structure and the current state of the economy. For rational expectations to make sense, there must be an appropriate degree of determinacy of the model as a whole, including its expectations-formation. Solving the model involves showing that the endogenous variables, including prices and expectations, can be expressed as stationary functions of the state variables, K, B, and B^g, given the known path of L. Properly, there should also be a demonstration that the proposed equilibrium path of prices is at least locally unique--otherwise, why should the economic agents pick the required expectations-forming rule?

My previous paper described an equilibrium path of the economy sketched out above. (I did not succeed in demonstrating local uniqueness.) The neutrality result concerning the rate of tax on corporate distributions followed from the conclusion that the valuation function on the equilibrium path is given by

$$(1) \qquad V(K_+, B_+) = (1 - t)d(F_+(K_+, L_+) - w_+^e L_+ - B_+ + K_+) \quad .$$

The objective of the owners of the firm is to set employment together with K_+, B_+ and D to maximize $(1-t)D + V(K_+, B_+)$. If we substitute for D in the objective function, using accounting relationship (2) among the outlays by the firm, B_+ drops out.

$$(2) \qquad D = F(K, L) - wL - B + K + dB_+ - K_+ \quad .$$

Maximizing values of the other variables are independent of t, while the financial structure of the firm is indeterminate.

Indeterminacy of financial structure corresponds to indeterminacy of government tax receipts. The reason the equilibrium path of the economy's real variables and prices is nonetheless determinate is suggested by equilibrium condition (3), which describes equality between the value of demanded and supplied claims to future consumption:

$$(3)\quad dc_+^2(d, w)L = dB_+ + dB_+^g + (1 - t)d(F_+(K_+, L_+) - w_+L_+ - B_+ + K_+).$$

Here $c_+^2(d, w)$ is the retirement-period consumption (which will take place next period) demanded by a representative young person, the values of K_+ and B_+ are set by the current owners of the firm, and I have taken for granted $w_+^e = w_+$. The expression on the left is the value of claims demanded on the basis of life-cycle optimization by members of the young generation. The first two terms on the right are the values of bonds supplied by the firm and the government, and the third term is the value of corporate equity. Exploiting the government's budget constraint $dB_+^g = B^g - tD$, and the already mentioned accounting relationship (2) between D and the other variables, allows us to write condition (3) as

$$(4)\quad dc_+^2L = B^g - t(F - wL - B + K - K_+) + (1-t)d(F_+(K_+, L_+) - w_+L_+ + K_+).$$

Once again, B_+ has been eliminated, so the condition is independent of corporate financial policy.

Equation (5) expresses a further reformulation of the same condition, taking advantage of Euler's theorem:

$$(5)\quad dc_+^2 L = K_+ + B^g - t(F(K, L) - wL - B + K).$$

The left-hand side of (5) is, as before, the demand by the young generation for assets. The right-hand side, the economy's net supply of assets, is the sum of the capital stock and the difference between government bonds carried over from the past and the tax receipts that would be generated if the corporation were to be liquidated in the current period. The last term affects the real equilibrium path like an anticipated tax receipt "asset" of the government, offsetting explicit government debt. Through this relationship one obtains a clear sense of why deficits or surpluses due to variations in corporate distributions have no effect on the real path of the economy, even though government debt does matter. An increase in distributions simultaneously reduces government debt and the anticipated tax receipt asset by equal amounts.

3. Introducing Uncertainty.

There are various ways one might introduce uncertainty to this model. Taking advantage of the framework pioneered by Kenneth Arrow (see Arrow and Hahn (1971)), suppose that future production conditions depend upon the particular state (for example, weather conditions) occurring. Specifically, assume that the investment decision is fixed in the current period, but the actual production function is determined in the next period.

Let S stand for the set of possible states that might obtain in the current period, S_+ the set of possible states in the next period, and so on. The larger dimensionality of the problem requires some new notation. The following describes my compromise between

comprehensiveness and mnemonics. In general, variables are now understood to have a subscript to designate the state and time with which they are associated. Thus, w_i, $i \varepsilon S_+$, refers to the wage rate realized in state i next period. Since the story starts with a known current state, we can let variables with no subscript refer to the values currently realized.

I shall also assume that S is finite in each period, and that an ordering has been agreed upon for the states in each period, so we can use the subscript "+" to refer to the vector of values of a variable potentially obtaining in the next period. (I make an exception for K_+ and L_+, which keep their previous scalar interpretation.) Thus, w_+^e refers to the vector of wage rates presently anticipated for next period, with components w_i, $i \varepsilon S_+$, and has dimensionality equal to the number of states in S_+.

The firm and the government must now specify, instead of the single-dimensional bonds, vectors of state-specific claims. To simplify matters, I assume that markets exist for each of the possible one-period contingent claims. A unit of type i, $i \varepsilon S_+$, pays one dollar next period if state i is realized, and zero if another state is realized. To issue a riskless bond is equivalent to selling one unit each of claims of all types i, $i \varepsilon S_+$. The firm's bond financing is described by the vector, B_+, the i^{th} component of which, B_i $(i \varepsilon S_+)$, is the amount the firm will owe its bondholders if state i is realized next period.

Instead of the single discount factor d, we now have a vector of prices of unit claims contingent on the next period's state. Like the discount factor d in the certainty case, the vector d is understood as representing prices actually ruling in financial markets. (The prices

of unit contingent claims might be observable only by appropriately packaging available financial instruments.) If e_+ is the vector of ones of appropriate dimension, the inner product $e_+ \cdot d$ is the price of a riskless bond. Since we shall want to continue to use the ordinary subscript to refer to the state and time in which a particular variable is realized (for example, d_i, $i\varepsilon S_+$ is the vector of discount rates, applicable to claims on output two periods hence, realized if state i occurs next period), I shall use parentheses when I wish to identify a particular element of d. Thus, $(d)_i$, $i\varepsilon S_+$, is a scalar, namely, the present price of a claim to one unit if state i occurs next period. The symbol d_+ refers to the matrix of discount factor vectors, one for each possible state that might be realized next period.

We are now in a position to use the analogues in the world of risk to the temporary equilibrium relationships discussed earlier to explore the question whether changes in tax receipts due to changes in corporate distributions alter the equilibrium vector of discount factors. Of particular importance is the valuation function for corporate equity corresponding to (1), given by (6):

$$(6) \quad V(K_+, B_+) = (1 - t)d\cdot(F_+(K_+, L_+) - w^e_+L_+ - B_+ + e_+K_+) \quad .$$

This is simply the value of the vector of payoffs, contingent on the state realized, that the purchasers of the equity expect to obtain next period in the form of distribution plus proceeds from sale of the equity interest.

To analyze asset-market clearing in this case, it is not sufficient to look at the aggregate value of present claims, as we did in (3) above.

We must now look for the state-by-state equality of supplies of and demands for contingent claims. Recall that $c_+^2(w, d)$ is now to be interpreted as the vector of consumption plans by the representative young person for the retirement period, contingent on the state realized. Asset market clearing now requires the vector equation,

$$(7)\quad c_+^2(w, d)L = B_+ + B_+^g + (1 - t)(F_+(K_+, L_+) - w_+L_+ - B_+ + e_+K_+)$$

$$= tB_+ + B_+^g + (1 - t)(F_+(K_+, L_+) - w_+L_+ + e_+K_+),$$

where, as before, we are taking for granted the determination of K_+ and w_+ via other equilibrium relationships, given w and d.

In the certainty case we were able to use the government budget constraint, $d \cdot B_+^g = B^g - tD$, together with the accounting relationship between D and B, to eliminate both B_+^g and B_+ from the asset market clearing condition. Under uncertainty, the combination of these two relationships is no longer sufficient. Whereas before, constraining the value of the government's bond sales or purchases determined the quantity (given d), the government now may choose among various combinations of state-contingent claims (i.e., deal in bonds of different risk characteristics). Moreover, the same can be said of the firm. Thus, if we take as a starting point that the government only issues riskless bonds (buys riskless bonds in the case of negative government debt), we have still not pinned down temporary equilibrium because the risk characteristics of the firm's debt have not been determined.

If the corporation is restricted to riskless debt and the government is restricted to riskless debt at the margin, the argument goes through

much as it did in the risk-free analysis. In that case, extra bonds issued by the corporation generate not only the exact withdrawal in value of government bonds (as a consequence of extra taxes paid) required to maintain financial market equilibrium, but also the matching change in the state-by-state contingent claims. We can see this by inspection of equilibrium condition (7). If the firm issues another unit of bonds, it adds directly one dollar to each component of the vector, B_+, of claims supplied on the right-hand side, but subtracts an amount (1-t) from the anticipated recovery from owning equity in each state that might occur. If the government issues bonds delivering t in each state, the set of financial claims supplied will continue to balance the claims demanded.

Note that the condition calls for riskless bonds to be issued by the corporation, but for the government to issue riskless bonds "at the margin." The requirement specifies the way the government must react to variations in distribution-tax receipts. The government retains, however, freedom to set the risk characteristics of what we may call its basic debt issue, which we might specify as the debt issued if there were no corporate distributions in the period. The total value of the debt issued is determined by the budget constraint (which could itself be lifted by addition of, say, lump-sum taxes to the fiscal repertoire).

This freedom cannot, however, be unpredictably exercised. In order for the the agents of the model to be able to formulate rational expectations, the risk characteristics of the basic government debt issue must be specified in advance (for example, by a formula relating to the characteristics defining the different states of the world). This is a distinct addition to the model introduced by the extension to uncertainty. In the certainty case the government was wholly predictable because

it had no degree of freedom within the budget constraint. Uncertainty brings with it a range of options in each situation. Government policy will influence the course of the model economy (because the overlapping generations are, by assumption, not linked through bequests). In this case, the main effect of policy is to influence the choice among risky alternatives. It might be interesting to explore the question of optimal fiscal policy in this model; however, for present purposes, the point to stress is the requirement for predictability, involving pre-specification of the characteristics of the basic debt and the rule for responding at the margin to variations in distribution-tax receipts.

The latter requirement has thus far been spelled out for the case in which the corporation issues only riskless debt. However, we can readily formulate a more general rule relating private and marginal public debt to preserve the property of the model whereby the firm's financial policy has no real effect.

The investment and employment decisions in the model are essentially determined by the path of w and d, wages and contingent dollar-claim prices. Suppose we had a path of w and d such that (7) is continually satisfied when the firm is restricted to riskless debt and the government to riskless debt at the margin. Then equilibrium condition (7) provides us with a general condition on government finance that assures that the path is unaffected by other choices of private financial behavior, namely,

$$(8) \qquad B^g_+ = c^2_+(w,d)L - (1-t)(F_+(K_+,L_+) - w_+L_+ - e_+K_+) - tB_+ \quad .$$

In words, in order for corporate financial policy to have no real effect, government policy must be effectively to issue basic debt, consisting of a pre-determined package of contingent claims, $c_+^2(w,d)L - (1-t)(F_+(K_+,L_+) - w_+L_+ + e_+K_+)$, and to purchase tB_+, i.e., a fraction t, of the bonds issued by the firm. The requirement (7) of equilibrium implies that if any other government policy is followed, private financial policy will have real effects, and hence further restrictions on behavior are required to close the model.

4. Concluding Remarks.

With respect to the narrow question whether the neutrality results obtained earlier in an all certainty model carry over to a world of uncertainty, the analysis yields a clear conclusion. Uncertainty introduces degrees of freedom to government choices that must be balanced by restrictions. In particular, if corporate distributions are not to have real effects, the government must use the distribution-tax proceeds to purchase the debt of the distributing firm (or, equivalently, buy back a set of government-issued contingent claims to generate the same effect on the supply of each type in the market). As I have suggested in the introductory section of this paper, there is an economic rationale for such a policy on the part of the government. It really is the same policy that we would expect shareholders to follow in rebalancing their portfolios following a distribution. The distribution by the corporation changes the characteristics of the equity claim. The portfolio can be restored to its pre-distribution characteristics by using the dividend to buy the firm's bonds. When we recognize that the government is an

implicit equity owner, by virtue of the distribution tax, the behavior required for neutrality seems reasonable.

However, it is hard to imagine any actual government carrying out this program. The systems used to account for government are typically weak in the dimension of measuring either government liabilities to make future payments or government claims to payment of taxes in the future already established by existing policies. To take an example, existing accounting conventions in the United States record the loss in tax revenue due to contributions to tax-favored retirement plans, and make no allowance for the resulting increase in the present value of future liabilities implied by the same transaction (Kotlikoff (1984) develops this general theme more fully).

Unfortunately for the model described here, if the government does not follow the specified behavior, incremental distribution-tax revenues will have real effects. The problem this presents is not simply to describe these effects, but rather to understand how it is that equilibrium is determined at all. For the argument that corporate distributions are not determined in the model holds generally--it is simply a consequence of price-taking by private agents in financial markets. As a result, corporate distributions cannot be predicted. If an unpredictable aspect of the model has real consequences, how can agents have "rational expectations?" Thus I conclude this paper on a note of puzzlement.

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