

NBER WORKING PAPER SERIES

INVESTORS' AND CENTRAL BANK'S UNCERTAINTY EMBEDDED IN INDEX  
OPTIONS

Alexander David  
Pietro Veronesi

Working Paper 16764  
<http://www.nber.org/papers/w16764>

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
February 2011

Previously circulated as "Investor and Central Bank Uncertainty and Fear Measures Embedded in Index Options." For comments and suggestions, we thank Francisco Palomino, Burton Hollifield, Amir Yaron, as well as seminar participants at Duke, UNC Chapel Hill, HEC Montreal, Penn State, Rice, Swiss Finance Institute (Lugano), the American Economic Association Meetings, the Society of Financial Studies Finance Cavalcade, and the Western Finance Association Meetings. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2011 by Alexander David and Pietro Veronesi. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Investors' and Central Bank's Uncertainty Embedded in Index Options  
Alexander David and Pietro Veronesi  
NBER Working Paper No. 16764  
February 2011, Revised December 2013  
JEL No. G12,G13,G18

**ABSTRACT**

Shocks to equity options' ATM implied volatility (ATMIV) are followed by persistently lower short-term rates. Shocks to the ratio of OTM puts' over OTM calls' implied volatilities (P/C) are followed by persistently higher rates. The stock's and Treasury-bond's ATMIV indices, which measure market and policy uncertainty, are counter-cyclical while the P/C index, which measures downside risk, is pro-cyclical. An equilibrium model where investors and the central bank learn about composite regimes on economic and policy variables explains these options' dynamics, linking them to a learning-based, forward-looking Taylor rule. The model produces several predictions on the relation between options, monetary policy variables, and beliefs that find support in the data.

Alexander David  
University of Calgary  
alexander.david@haskayne.ucalgary.ca

Pietro Veronesi  
University of Chicago  
Booth School of Business  
5807 South Woodlawn Avenue  
Chicago, IL 60637  
and NBER  
pietro.veronesi@chicagobooth.edu

# 1. Introduction

When the 2008 crisis hit, the implied volatility of options on stocks and Treasury bonds skyrocketed as stocks suffered massive losses and the Federal Reserve dropped its target rate in its attempt to stabilize the economy. The Fed funds rate has been near zero ever since. This dramatic event underscores the active role of monetary policy in the economy, and its potential impact on asset prices. Indeed, while it is intuitive that corporate stock and option prices react to corporate news, empirical evidence also suggests that they react to monetary policy shocks.<sup>1</sup> This empirical evidence though spurs fundamental questions about the relation between option prices, corporate fundamentals, and the actions of the central bank. In this paper, we provide an equilibrium model that links option prices to fundamentals and monetary policy, and provide a dynamic and time consistent methodology to extract investors beliefs on the macroeconomic and central bank policy regime.

To motivate our approach, we begin by presenting some empirical relations between option prices, the state of the economy, and monetary policy. Our analysis focuses on two popularly quoted indices constructed from options prices. The first index is the implied volatility of at-the-money options, henceforth ATMIV. We compute such index both for equities, namely from options on the S&P 500 index, and for long-term Treasury bonds, namely, from options on the 10-year Treasury bond futures. The equity ATMIV index has long been considered an “investor fear gauge” (see e.g. CBOE Bulletin on VIX, 2003), as it typically increases during bad times. The T-bonds’ ATMIV index is especially useful in our context to discuss the drivers of uncertainty about monetary policy. The second index is the ratio of the implied volatilities of out-of-the-money puts over out-of-the-money calls, henceforth referred to as the P/C index. This index is designed to measure the market assessment of downside risk versus upside risk, and it has been studied extensively since the work of Bates (1991) to gauge investors’ worries about a market decline.

Quarterly time series plots of these indices, computed from three-months options, are shown in the left panels of Figure 1 for the 24-year period 1988 - 2011. Comparing the stock’ ATMIV index in panel A with the P/C index in Panel C we see that, surprisingly perhaps, ATMIV and the P/C are negatively related, with the ATMIV (P/C) being generally counter (pro) cyclical. While it is intuitive that implied volatility ATMIV is high during

---

<sup>1</sup>For example, Bernanke and Kuttner (2005) report that monetary policy surprises affect the stock market, while Rigobon and Sack (2003) show that the monetary policy responds to stock returns with a greater reaction during times of higher volatility, and more recently Bekaert, Hoerova, and Duca (2010) find a significant reaction of options prices to lead and lag measures of monetary policy.

downturns, it is less obvious why the downside-risk index  $P/C$  is high during booms and low during recessions. Our model explains why by tying this variation to investors' beliefs about business cycle and monetary policy dynamics. Comparing instead Panel A and E, we see that the ATMIV indices of stocks and bonds appear both counter-cyclical, with the bonds' ATMIV index especially volatile in the last decade.

It is revealing to see how shocks to these options' indices impact monetary policy. By way of motivation, we estimate pairwise Vector Auto Regressions (VAR) of each option index with the three-month T-bill rate, which we take as proxy of monetary policy.<sup>2</sup> The right panels of Figure 1 report resulting impulse responses for the historical series over the options subsample of 1988 – 2011. The results are striking and all in one direction: shocks to both equity ATMIV and to  $P/C$  lead to sustained impacts on future monetary policy (short-term rate). In contrast, we do not find that monetary policy has any sustained impacts on the two options indices (results not shown). Moreover, and more interestingly, Panel B shows that the 3-month Treasury rate decreases for up to eight quarters in response to a shock to the ATMIV index. Even more interestingly, Panel D shows a shock to the downside-risk index  $P/C$  induces the 3-month T-bill rate to increase for up to eight quarters in the future. Under the interpretation of Bates (1991), the latter result implies that when investors become more worried about a stock market decline, future short-term rates increase. What is the economic mechanism generating this empirical observation? What does this evidence tell us about the central bank's reaction to economic news and its interest rate policy?<sup>3</sup>

We provide a dynamic equilibrium model of learning that links equity and Treasury options to investors' and central banks' uncertainty about fundamentals. In order to have a model amenable to the empirical investigation, we follow the recent macro-finance term-structure literature (e.g. Ang and Piazzesi (2003) and Ang, Piazzesi, and Wei (2006)) and posit a structural econometric model for the equilibrium dynamics of fundamental variables along with the specification of a forward looking Taylor rule, which links the central bank's expected future inflation and expected future capacity utilization to its target interest rate.<sup>4</sup>

---

<sup>2</sup>In our empirical analysis, we use the 3-month T-bill rate as our short-term rate, rather than the Federal Funds rate, as the latter is affected by banks' default premium, which is absent in our model. The 3-month T-bill rate and the Fed Funds rate are very highly correlated.

<sup>3</sup>Interestingly, we find that the put-to-call ratio  $P/C$  strongly predicts future interest rates, while we do not find such relation with other measures of crash risk, such as the difference in implied volatility of out-of-the-money puts versus at-the-money puts. Our fitted model is consistent also with this evidence. Panel F of Figure 1 shows that shocks to bond ATMIV also do not predict future interest rates, which is not inconsistent with our model either, as we will see.

<sup>4</sup>The New Keynesian Economics approach shows the optimality of such rules in settings where price stickiness implies deviations from short run full employment and capacity utilization [see, e.g. Woodford (2003)]. Gallmeyer, Hollifield, and Zin (2005), Gallmeyer, Hollifield, Palomino, and Zin (2007), and Bekaert,

We impose no-arbitrage restrictions by also positing the equilibrium dynamics for the state price density, which we use to price all traded assets from the fundamental variables, endogenous investors' beliefs about the economy, and the Taylor-rule-based riskless rate.

We generalize the Taylor rule by introducing three key features, which we provide evidence for: First, we specify an unobserved regime switching model with composite regimes of macroeconomic and policy fundamental variables. The composite regime formulation introduces low frequency comovement of fundamentals and monetary policy variables, which affect the dynamics of asset prices. Second, we specify a learning-based Taylor rule, in which neither the central bank nor investors observe the true trend growths of nominal as well as real variables. Agents in the economy (investors and the central bank) are econometricians in the sense of Hansen (2007), that is, they attempt to learn about the drift regimes of fundamentals from the observation of past and current fundamentals. Their Bayesian learning dynamics about the regime of the economy are the key drivers of our results, as explained below. Finally, to extend the current understanding of the effects of monetary policy on the stock market we follow the suggestions in Lucas (2007) to allow money growth to affect transitions between fundamental drift regimes.<sup>5</sup> The observations of money growth affect investors' beliefs about future fundamental regimes, which the econometrician can extract from fundamentals and price data.

Our model sheds light on the compelling dynamic one-way relation between options' ATMIV and P/C and monetary policy, discussed earlier in Figure 1. The strong evidence that these option-based indices lead policy variables, but not the reverse, is not driven by differences in information between investors and the central bank, as our model assumes they observe the same data and have the same information. Instead, our model highlights

---

Cho, and Moreno (2010) build term-structure models using policy variables.

<sup>5</sup> Lucas (2007) complains about the lack of use of monetary aggregates in recent models of monetary policy and recommends their use in information extraction:

One source of this concern is the increasing reliance of central bank research on New-Keynesian modeling. New-Keynesian models define monetary policy in terms of a choice of money market rate and so make direct contact with central banking practice. Money supply measures play no role in their estimation, testing or policy simulation. A role for money in the long run is sometimes verbally acknowledged, but the models themselves are formulated in terms of deviation from trends that are themselves somewhere off stage. It seems likely that these models could be reformulated to give a unified account of trends, including trends in monetary aggregates, and deviations about trend but so far they have not been. This remains an unresolved issue on the frontier of monetary theory. Until it is resolved, monetary information should continue to be used as a kind of add-on or cross-check, just as it is in the ECB policy formulation today.

Coenen, Levin, and Wieland (2005) and Beck and Wieland (2008) show that money growth can help predict real activity when the real output and real money are economically linked but the central bank, which partially controls money growth, receives noisy information on the former.

the role of beliefs about the economy that drive both option prices as well as monetary policy through the forward looking Taylor rule. In particular, our model shows that a higher stock and bond ATMIV occurs when investors are uncertain about the current regime. The reason is that from Bayes' formula high uncertainty leads to faster revision of beliefs to news and thus higher return volatility. During such times, expected economic growth – which also depends on beliefs – is lower than during good times, and in particular such times are characterized by lower expected future capacity utilization. These beliefs thus induce the central bank to react, through its forward looking Taylor rule, and lower the target real rate of interest in order to maintain economic stability. This explains why shocks to ATMIV are related to lower future rates, through the movement in beliefs about economic growth.

Indeed, in the last decade, the economy has suffered not only from the possibility of lower economic growth, but also from fears of deflation, which bring about low inflation, low economic growth, and low capacity utilization. Because the forward looking Taylor rule depends on both expected future inflation and expected future capacity utilization, in the last decade deflation fears sparked a very aggressive reaction of central bank into dramatically lowering real rates. The same movement in beliefs increase the ATMIV index.

Similarly, our model shows why increases in the P/C index predict future increases in short rates. In good times investors' perceive greater downside risk in stocks than in bad times, as positive fundamentals news have little impact on revising investors' beliefs about a boom regime, but negative fundamental news may lead to a large downward revision of such beliefs. A large downward revision of beliefs to be in a booming regime would lead to substantially lower stock prices. Thus, in good times, stock returns tend to be negatively skewed. The negatively skewed return distribution rises the price of OTM put options relative to OTM call options, the P/C index. When P/C raises it is an indication (for the econometrician) that the economy is moving to a booming regime, which has regular capacity utilization and regular inflation. As a consequence, the forward looking Taylor rule predicts a tightening of monetary policy back to regular levels and hence higher future interest rates.

These effects also explain why ATMIV and the P/C are negatively correlated (see Figure 1): In periods of strong growth with stable policy variables, investors' overall belief volatility is relatively low, and so is the ATMIV. At these good times, however, the P/C index is high, due to the relative increase in downside risk, as explained.

Finally, turning to bonds, we find that the model produces a positive relation between stock ATMIV and bond ATMIV, as shows in Figure 1. This common variation stems from the commonality in the uncertainty about economic growth and inflation regime. Because

especially the last decade has been characterized by fear of deflation (low inflation and low economic growth), uncertainty about economic growth and about inflation would move together. Indeed, David and Veronesi (2013) show that this comovement generates not only high stock and bond return volatility around the recessions during this period, but also a negative covariance between stocks and bonds. David and Veronesi (2013) do not use options, nor link the variation in interest rates to monetary policy, but their evidence is consistent with our evidence in this paper.

Our model makes a number of additional predictions, which we test in the data and find support for. First, similar impulse response functions that relate shocks to ATMIV and P/C to future interest rates (see right panels of Figure 1) should be apparent also when performed on capacity utilization (CU), which is a key variable of monetary policy. Indeed, we find that the data support this prediction, as shocks to ATMIV predict lower future CU, while shocks to P/C predict higher future CU.

Second, our model predicts that option prices should be related to beliefs about economic growth and inflation. In particular, for stocks, the model predicts that ATMIV (P/C) should be positively (negatively) related to the probability of a recession, to economic uncertainty, and the probability of a deflationary period. Instead, expected inflation and inflation uncertainty should not be significant predictors of either ATMIV or P/C. To test such predictions in the data, we exploit the probability forecasts from the Survey of Professional Forecasters (SPF). In fact, the SPF asks its forecasters not only the point forecast of the future value of a given variable, but also to provide a probability assessment that such variable will lie in some given intervals. We exploit such survey-based SPF probabilities to test the model's predictions described above, and find support for all of them. In addition, the model predicts that P/C should be negatively related to ATMIV ( $R^2 = 62\%$ ) and we find the same in the data ( $R^2 = 36\%$ ), which is of course not surprising given Figure 1. For bonds, our model predicts that ATMIV should be positively related to the recession probability, economic uncertainty, inflation uncertainty, and the probability of deflation. We test such predictions using the same survey-based SPF probabilities and find support for all of these predictions, except for inflation uncertainty that has the right sign, but is insignificant. Finally, the model also predicts that bonds and stock ATMIV should be positively correlated ( $R^2 = 32\%$ ) and we find exactly the same result in the data ( $R^2 = 26\%$ ).

Third, our model has a number of additional predictions for the behavior of option prices themselves. For instance, it is known in the literature that high volatility periods corresponds also to high “volatility of volatility” periods, a fact that is inconsistent with

standard option pricing models, such as Heston (1993) (see e.g. Jones (2003)). Our model in contrast predicts exactly such relation, both for stocks and bonds. Indeed, we find that in the data, the correlation between implied volatility and absolute changes in volatility is 38% for stocks and 39% for bonds, and indeed the model implies correlations of 48% and 35% for stocks and bonds, respectively.

Why is this evidence consistent with our learning-based model? The reason is that it is a property of Bayesian learning that during periods of high uncertainty beliefs react more to news. It follows that during periods of high uncertainty not only the implied volatility should be high, but also its volatility of volatility should be high. Indeed, not only the behavior of volatility of volatility in the data is consistent with the one in the model, but it is also related to beliefs, as the model predicts. In fact, we find that for both stocks and bonds the absolute changes in implied volatility should be positively related to the recession probability, to economic uncertainty, and the probability of deflation. For bonds, we also find that expected inflation should be negatively related to the volatility of volatility. We find that most of these predictions are supported in the data when again we proxy beliefs using the SPF probabilities.

Finally, our model also provides the proper dynamics for the stock and bond implied volatility premium (IVP), that is, the difference between implied volatility and expected future volatility. The fluctuation over time of the IVP are driven by beliefs in our model, and we find similar implications from the beliefs from survey data, especially for stocks.

In terms of methodology, we fit the parameters of our structural model with an overidentified Simulated Method of Moments (SMM) procedure, which uses the likelihood of observing the fundamentals to extract investors' beliefs, and then use such beliefs to compute pricing errors for stock, Treasury bond, and option prices. It is important to note that our estimation methodology ensures that the extracted beliefs are time-consistent and respect Bayes formula over the whole sample period. This implies that the estimated dynamics of uncertainty is also time consistent and is the outcome of the realization of fundamentals. This distinguishes our work from related work on options with learning that resets the model uncertainty in each period to some proxy of uncertainty in the data and focuses on conditional reactions in options prices.<sup>6</sup> Indeed, we find that our model-estimated beliefs are highly

---

<sup>6</sup> For example Guidolin and Timmermann (2005) and Buraschi and Jiltsov (2006) study option prices and volume in models with learning about fundamentals. Dubinsky and Johannes (2006) study the reaction of options prices on individual stocks to news about earnings. Benzoni, Collin-Dufresne, and Goldstein (2005) show that the increase in investors' perception about the average jump size of stock prices led to a steepening of the implied volatility smirk after the stock market crash of 1987, but do not study its time variation in subsequent years. In a paper related to ours, Shaliastovich (2009) models investors' non-Bayesian



correlated with the probabilities extracted from the Survey of Professional Forecasters.

Besides the literature on option prices with learning (see footnote 6), this paper contributes to a small set of papers that provides economic explanations of the implied volatility curve for options.<sup>7</sup> Bollen and Whaley (2004) and Garleanu, Pederson, and Poteshman (2008) find that net buying pressure affects the prices of options for several days as market makers fail to provide options at no-arbitrage prices, but charge for the residual risk due to the limits to arbitrage. In addition to focus on lower frequency data and explaining the entire time series of options prices, we do not depart from the no-arbitrage framework. Among theoretical explanations for smirks, Liu, Pan, and Wang (2005) study the implications for ambiguity about rare event risk that raise the prices of puts relative to calls. Drechsler (2008) and Du (2010) provide calibrated models with time-varying ambiguity and with habit formation preferences, respectively, to generate the left skewed implied volatility smile, but neither paper studies the time series properties of the smile, nor their interaction with monetary policy. Bollerslev, Tauchen, and Zhou (2009), Drechsler and Yaron (2010) and Eraker (2008) construct equilibrium models with “long run risks” in the consumption process to understand the size of the Variance Risk Premium and some of its unconditional moments. None of these papers consider monetary policy and learning as we do.

The layout of the paper is as follows. Section 2. outlines the model and derives the key pricing results. In Section 3. we estimate the parameters of our model and discuss the results related to the dynamics of beliefs and asset prices. Section 5. focuses on monetary policy and options, and we test for additional model’s predictions. In Section 6., we show the model also sheds light on other properties of options, such as the dynamics of the volatility of volatility and the implied volatility premium. Section 7. concludes. Two technical appendices provide proofs of technical results and the estimation methodology, respectively.

---

(behavioral) learning about the long run drift of consumption to generate the smile, but does not study its time series fit to data series.

<sup>7</sup> There is also a large literature that explains the volatility smile by assuming exogenous processes for stock prices, volatilities, and jumps. Indeed, since the classic work of Black and Scholes (1977) the major innovations have been the addition of stochastic volatility [see, e.g., Hull and White (1987) and Heston (1993)], jumps in prices [see e.g. Bates (1996) and Bates (2000), and Pan (2002)], and jumps in volatility [see, e.g. Eraker, Johannes, and Polson (2003)]. A tremendous amount of empirical work has been done on these extensions of the BS formula that has enriched our understanding of stock price dynamics, and of options returns. Bakshi, Cao, and Chen (1997) provides a specification analysis of some of these models. Among more recent innovations, Christoffersen, Jacobs, Ornathanalai, and Wang (2008) build multi-factor stochastic volatility models, and somewhat related to our paper, Polson, Johannes, and Stroud (2008) price options when exogenously specified volatility follows an unobserved process that investors learn about. Constantinides, Jackwerth, and Perrakis (2008) find that several exogenously specified volatility models, such as GARCH, can be rejected as possible data generating processes for S&P 500 index options.

## 2. Structure of the Model

Our main assumption throughout the paper is that the drift rates of the fundamental processes are driven by an  $N$ -regime, continuous time, hidden Markov chain process. It is useful to describe this process first. We denote by  $s_t$  the regime at time  $t$ , where  $s_t \in \{s^1, \dots, s^N\}$ , and we let  $\Lambda$  denote the Markov chain infinitesimal generator matrix. That is, over the infinitesimal time interval of length  $dt$

$$\lambda_{ij}dt = \text{prob}(s_{t+dt} = s^j | s_t = s^i), \quad \text{for } i \neq j, \quad \lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}.$$

All agents in our economy, both investors and the central bank, do not observe the realizations of  $s_t$  but learn about it from the observation of numerous signals, including realized fundamental variables. Given an information filtration  $\{\mathcal{F}_t\}$  generated by such signals, we denote agents' common beliefs at time  $t$  about regime  $s^i$  as

$$\pi_{it} = \text{prob}(s_t = s^i | \mathcal{F}_t), \quad i = 1, \dots, N \quad (1)$$

Lemma 1 below characterizes the dynamics of the vector  $\pi_t = \{\pi_{1t}, \dots, \pi_{Nt}\}$ , but before we introduce the learning result, we need to introduce the rest of the model.

There is a single homogeneous good in the economy whose price,  $Q_t$ , follows:

$$\frac{dQ_t}{Q_t} = \beta(s_t)dt + \sigma_Q dW_t, \quad (2)$$

where  $W_t = (W_{1t}, W_{2t}, W_{3t}, W_{4t}, W_{5t})'$  is a 5-dimensional vector of independent Weiner processes, inflation volatilities are summarized in  $1 \times 5$  constant vector  $\sigma_Q = (\sigma_{Q,1}, 0, 0, 0, 0)$ , and the drift rate  $\beta(s_t)$  depends on the realization of the (hidden) regime  $s_t$ .

The main real corporate fundamental in the economy is the process of real earnings,  $E_t$ , which follows the jump-diffusion process

$$\frac{dE_t}{E_t} = (\theta(s_t) - \kappa \xi_1)dt + \sigma_E dW_t + (e^{Y_{1t}} - 1)dL_t \quad (3)$$

where volatilities,  $\sigma_E = (0, \sigma_{E,2}, 0, 0, 0)$ , are constant, the drift rate  $\theta(s_t)$  depends on the realization of the regime  $s_t$ ,  $L_t$  is the counter of a Poisson process with constant intensity  $\kappa$ , i.e.  $\text{Prob}(dL_t = 1) = \kappa dt$ , the jump size  $Y_{1t}$  is i.i.d. normal with mean  $\mu_1$  and volatility  $\sigma_1$ , and  $\xi_1 = e^{\mu_1 + 0.5\sigma_1^2} - 1$ . The regime process,  $s_t$ , the Brownian motions,  $W_t$ , and the

jump process  $L_t$  are all independent of each other. Under the assumption of continuous observation of fundamentals, and hence their quadratic variation processes, investors can perfectly observe jumps. In our model, jumps to earnings play two important roles: First, their inclusion permits a better estimation of the earnings process, which has some large outcomes in our sample. Second, negative mean jumps will be shown to increase the average put-to-call implied volatility ratio (P/C). We model i.i.d. jump sizes and constant jump intensity, however, and therefore the modeled jumps in themselves are unable to explain the time series variation in either the ATMIV or P/C, which is the subject of our paper.

The next important fundamental in the economy is de-meaned industrial capacity utilization (CU),  $K_t$ , which follows the process

$$dK_t = \rho(s_t) dt + \sigma_K dW_t, \quad (4)$$

where  $\sigma_K = (\sigma_{K,1}, 0, 0, \sigma_{K,4}, 0)$ , are constants, and the drift  $\rho(s_t)$  depends on the realization of regime  $s_t$ . Unlike the other state variables, CU is stated in levels, and hence can become negative. The use of CU improves the term structure fit of our model. We will comment on the nonzero instantaneous correlation between CU and inflation in Section 3..

The final state variable is aggregate real money in the economy,  $H_t$ , which follows

$$\frac{dH_t}{H_t} = \omega(s_t) dt + \sigma_H dW_t, \quad (5)$$

where  $\sigma_H = (0, 0, 0, 0, \sigma_{H,5})$  and the drift  $\omega(s_t)$  depends on the regime  $s_t$ . We emphasize that  $H_t$  is the equilibrium quantity of real money in the economy determined both by its demand and supply. It is also useful to note that while ours is not a full structural model in which the quantity of money is endogenously determined, the statistical properties of  $dH_t/H_t$  affect agents' beliefs' dynamics, and thus equilibrium prices.

## 2.1. The Central Bank Policy Rule

All agents, investors and central bank, observe the same data and thus have the same information about the regime of the economy. Thus, the regime probabilities  $\pi_{it}$  defined in (1) are common across all agents. The central bank sets the real rate of the economy  $\bar{\phi}_t$  by using a forward looking Taylor rule, namely

$$\bar{\phi}_t = \alpha_0 + \alpha_\beta \mathbb{E} \left[ \frac{dQ_t}{Q_t} | \mathcal{F}_t \right] + \alpha_\rho \mathbb{E} [dK_t | \mathcal{F}_t]. \quad (6)$$

where the expectations are taken with respect to the information available at time  $t$ ,  $\mathcal{F}_t$ .<sup>8</sup> The second and third terms of the real rate capture the essential elements of the Taylor rule, which posits that the central bank increases rates in response to increases in expected inflation and the expected real slack in the economy [see Taylor (1993)]. Our policy rule is hence ‘forward-looking’ in the sense of Clarida, Gali, and Gertler (2000), who suggested replacing current and/or lagged values of inflation and the output gap by their forward-looking conditional expectations. A significant contribution of our analysis is to jointly estimate the expectations from corporate earnings as well as regular macroeconomic variables, so that there is interaction between uncertainty in the corporate sector and central bank policy. In addition, following the assumption in Rudebusch and Wu (2008) we use the industrial capacity utilization series obtained from the Federal Reserve Board rather than the output gap, in the original Taylor rule.

We finally note that in standard Taylor rules, the central bank sets the nominal interest rate.<sup>9</sup> In our model, the inflation risk premium is constant, so the policy rule can equivalently be written as setting of the nominal rate by adding expected inflation and the inflation risk premium on both sides of equation (6).

## 2.2. No Arbitrage Pricing

To build the policy rule of the central bank into a no-arbitrage framework, we follow Ang and Piazzesi (2003) and Piazzesi (2005) in specifying a state price density to price all cash flows in our model. Let  $M_t$  be the state price density at date  $t$ . As in the modern classic asset pricing theory (see, e.g. Cochrane (2001)), a generic random real cash flow  $\{D_t\}$  is priced as

$$M_t P_t = \mathbb{E} \left[ \int_t^\infty M_s D_s ds | \mathcal{F}_t \right]. \quad (7)$$

It is convenient to first write the process of the state price density in terms of the original hidden Markov process  $s_t$  and Brownian motions  $W_t$ . We specify  $M_t$  taking the form

$$\frac{dM_t}{M_t} = (-\phi(s_t) - \kappa \xi_2) dt - \sigma_M dW_t + (e^{Y_{2t}} - 1) dL_t, \quad (8)$$

---

<sup>8</sup>We allowed for a generalization the Taylor rule to let interest rates directly be impacted by money growth but did not estimate a significant effect.

<sup>9</sup>The original Taylor rule [see Taylor (1993)] is  $i_t = \pi_t + r_t^* + a_\pi(\pi_t - \pi_t^*) + a_y(y_t - \bar{y}_t)$ , where  $i_t$  is the target nominal rate,  $\pi_t$  is the realized rate of inflation,  $r_t^*$  is the assumed equilibrium real rate of interest,  $\pi_t^*$  is the desired inflation rate,  $y_t$  is the log of GDP, and  $\bar{y}_t$  is the log of potential GDP.

where  $\phi(s_t)$  denotes the real rate *conditional* on observing the regime (see discussion below),  $\sigma_M = (\sigma_{M,1}, \sigma_{M,2}, \sigma_{M,3}, \sigma_{M,4}, \sigma_{M,5})$  is a  $1 \times 5$  constant vector of the market prices of risk,  $L_t$  is the same Poisson counter as in the earnings process in (3),  $Y_{2t}$  has an i.i.d. normal distribution with mean  $\mu_2$  and volatility  $\sigma_2$  and perfectly correlated with  $Y_{1t}$ , and  $\xi_2 = e^{\mu_2 + 0.5\sigma_2^2} - 1$ . Note that jumps in earnings in equation (3) are systematic since they are perfectly correlated with the pricing kernel. We note that constant prices of risk also arise in a simple Lucas (1978) economy with no government where the representative agent has constant relative risk aversion, and where the fundamental volatility of consumption (dividends) is constant. This assumption along with the homoskedasticity of fundamentals ensure that all fluctuations in volatilities and option indices arise endogenously only due to learning.

To ensure no-arbitrage, the expected drift rate of the state price density must equal the real rate  $\bar{\phi}_t$  in (6), so that we impose

$$\mathbb{E} \left[ \frac{dM_t}{M_t} | \mathcal{F}_t \right] = -\bar{\phi}_t dt$$

Since investors and the central bank have the same information, this no arbitrage restriction is naturally obtained by requiring that regime by regime:<sup>10</sup>

$$\phi(s_t) = \alpha_0 + \alpha_\beta \beta(s_t) + \alpha_\rho \rho(s_t). \quad (9)$$

### 2.3. Learning Dynamics

For notational convenience, we stack the fundamental processes (2), (3), (4), and (5) that are observed by the econometrician as signals in a vector  $dX_t = \left( \frac{dQ_t}{Q_t}, \frac{dE_t}{E_t}, dK_t, \frac{dH_t}{H_t} \right)'$ , so that

$$dX_t = \varrho(s_t) dt + \Sigma_4 dW_t + J_{4t} dL_t, \quad (10)$$

where the drift vector process is  $\varrho(s_t) = (\beta(s_t), \theta(s_t) - \kappa\xi_1, \rho(s_t), \omega(s_t))'$ , the volatility matrix is  $\Sigma_4 = (\sigma'_Q, \sigma'_E, \sigma'_K, \sigma'_H)'$ , and the vector of jump sizes is  $J_{4t} = (0, e^{Y_{1t}} - 1, 0, 0, )$ . In particular, we assume the econometrician does not observe investors' state price density  $M_t$ . Agents in the economy, instead, observe both signals  $dX_t$  and  $dM_t$  and we denote the full set of signals as  $dZ_t = \left( dX_t', \frac{dM_t}{M_t} \right)'$ , which has the drift vector  $\nu(s_t) = (\varrho(s_t)', -\phi(s_t) - \kappa\xi_2)'$ , volatility matrix  $\Sigma = (\Sigma'_4, \sigma'_M)'$ , and jump size of  $J_t = (J'_{4t}, e^{Y_{2t}} - 1)'$ .

The following Lemma characterizes the dynamics of beliefs  $\pi_{it} = \text{prob}(s_t = s^i | \mathcal{F}_t)$ . For notational convenience, we denote the drift of the signal vector  $dZ_t$  in regime  $i$  by  $\nu^i = \nu(s^i)$ .

---

<sup>10</sup>Indeed, from (8):  $-\mathbb{E} \left[ \frac{dM_t}{M_t} | \mathcal{F}_t \right] = \mathbb{E} [\phi(s_t) | \mathcal{F}_t] = \alpha_0 + \alpha_\beta \mathbb{E} [\beta(s_t) | \mathcal{F}_t] + \alpha_\rho \mathbb{E} [\rho(s_t) | \mathcal{F}_t]$ , which yields (6).

**Lemma 1.** *Given an initial condition  $\pi_0 = \hat{\pi}$  with  $\sum_{i=1}^N \hat{\pi}_i = 1$  and  $0 \leq \hat{\pi}_i \leq 1$  for all  $i$ , the vector of probabilities  $\pi_t = (\pi_{1t}, \dots, \pi_{Nt})'$  satisfies the  $N$ -dimensional system of stochastic differential equations:*

$$d\pi_t = \Lambda' \pi_t dt + \Sigma(\pi_t) d\tilde{W}_t, \quad (11)$$

in which the  $i$  row of  $\Sigma(\pi)$  is

$$[\Sigma(\pi)]_i = \sigma_i(\pi_t) = \pi_{it} [\nu^i - \bar{\nu}(\pi_t)]' \Sigma'^{-1}, \quad (12)$$

$$\bar{\nu}(\pi_t) = \sum_{i=1}^N \pi_{it} \nu^i = \mathbb{E}_t(dZ_t | \mathcal{F}_t),$$

$$\text{and} \quad d\tilde{W}_t = \Sigma^{-1}[dZ_t - J_t dL_t - \bar{\nu}(\pi_t)] = \Sigma^{-1}(\nu_t - \bar{\nu}(\pi_t))dt + dW_t. \quad (13)$$

Moreover, for every  $t > 0$ ,  $\pi_{it} \geq 0$  and  $\sum_{i=1}^N \pi_{it} = 1$ .

This filtering result is a straightforward extension of the Wonham filter (see Wonham (1964)), which characterizes the Bayesian learning about the hidden drift with Brownian noise.<sup>11</sup> In the setup here, the observed fundamental vector process has observable jumps in some elements, which do not affect investors' beliefs about the hidden drift. In particular, the high frequency variation in investors' beliefs is driven by investors' inferred shocks,  $d\tilde{W}$ , in equation (13) as opposed to the true shocks,  $dW$ , which affect fundamentals. It is also possible to write the fundamental process vector  $dZ_t = \nu_t dt + \Sigma dW + J_t dL_t = \bar{\nu}(\pi_t) dt + \Sigma d\tilde{W} + J_t dL_t$ . The right hand side of (13) also reveals that the inferred shocks process  $d\tilde{W}$ , does not depend on the jump parameters, since investors are able to observe jumps which thus do not affect their inference about  $s_t$ .

## 2.4. Stock Prices and the Term Structure of Interest Rates

We now obtain the price-earnings (henceforth P/E) ratio and the nominal bond price:

**Proposition 1.** (a) *The P/E ratio at time  $t$  is*

$$\frac{P}{E}(\pi_t) = \sum_{j=1}^N C_j \pi_{jt} \equiv C \cdot \pi_t, \quad (14)$$

---

<sup>11</sup>The first application of the Wonham filter in financial economics, as well as several properties of the filtering process, are derived in David (1997). We find it useful to recall that a main advantage of this modeling strategy as opposed to the more commonly used Kalman filter is that investors uncertainty (conditional variance of expectations about the drift terms) fluctuates forever, while in the Kalman filter, this uncertainty converges to a constant. The fluctuating confidence (inverse of the conditional variance) is the driver of the options' indices that we seek to explain in this paper.

where the vector  $C = (C_1, \dots, C_N)$  has  $C_i = \mathbb{E} \left[ \int_0^\infty \frac{M_\tau E_\tau}{M_t E_t} d\tau \mid \nu_t = \nu^i \right]$  and it satisfies  $C = c A^{-1} \cdot \mathbf{1}_N$ , with  $c$  being a constant dividend payout ratio, and

$$A = \text{Diag}(\phi^1 - \theta^1 + \sigma_M \sigma'_E - \kappa(\xi_3 - \xi_1 - \xi_2), \dots, \phi^N - \theta^N + \sigma_M \sigma'_E - \kappa(\xi_3 - \xi_1 - \xi_2)) - \Lambda. \quad (15)$$

and  $\xi_3 = e^{\mu_1 + \mu_2 + 0.5(\sigma_1 + \sigma_2)^2} - 1$ .

(b) The price of a nominal zero-coupon bond at time  $t$  with maturity  $\tau$  is

$$B(\pi_t, \tau) = \sum_{i=1}^N \pi_{it} B_i(\tau), \quad (16)$$

where the  $N \times 1$  vector valued function  $B(\tau)$  with element  $B_i(\tau) = E \left[ \frac{M_{t+\tau}}{M_t} \cdot \frac{Q_t}{Q_{t+\tau}} \mid \nu_t = \nu^i \right]$  is given by

$$B(\tau) = \Omega e^{\omega \tau} \Omega^{-1} \mathbf{1}_N. \quad (17)$$

In (17),  $\Omega$  and  $\omega$  denote the matrix of eigenvectors and the vector of eigenvalues, respectively, of the matrix  $\hat{\Lambda} = \Lambda - \text{Diag}(r^1, r^2, \dots, r^N)$ , where each  $r^i = k^i + \beta^i - \sigma_M \sigma'_Q - \sigma_Q \sigma'_Q$ , is the nominal rate that would obtain in the  $i^{\text{th}}$  regime, were the regimes observable. In addition,  $e^{\omega \tau}$  denotes the diagonal matrix with  $e^{\omega_i \tau}$  in its  $(i, i)$  position.

The proof for stocks is in the appendix. The proof for bonds follows from a simple extension of the proof in a similar setting in David and Veronesi (2013). The stock price formula has a similar form to that developed in the pure diffusion setup of David and Veronesi (2013). The major difference is the presence of jumps in earnings, which are priced and thus decrease the P/E ratio, and the fact that our regimes here involve the drift rates of policy variables, such as capacity utilization. Indeed, the constant  $C_i$  in (14) is the P/E as in the Gordon growth model conditional on regime  $i$  being known, and such regime depends on policy variables (inflation drift and CU drift) in that regime. Because the true economic regime is not known to investors and central bank alike, the P/E ratio is the beliefs-weighted average of such conditional P/E ratios. The interpretation for the bond pricing formula (16) is similar to the one for the P/E ratio. In contrast to stocks, bond prices do not jump since the belief processes are continuous and the main bond fundamental, inflation, is continuous.

Let  $P_t^n = P_t \cdot Q_t$  be the nominal value of stock, where  $P_t$  is the real value of stocks in Proposition 1. Using the dynamics of the inflation and earnings processes under the observed filtration, we now formulate the nominal return processes for stocks and bonds.

**Proposition 2.** *The nominal stock return process under the investor's filtration is given by*

$$\frac{dP_t^n}{P_t^n}(\pi_t) = (\mu^n(\pi_t) - \delta(\pi_t) - \kappa \xi_1) dt + \sigma^n(\pi_t) d\tilde{W}_t + (e^{Y_{1t}} - 1) dL_t,$$

where  $\delta(\pi_t) = c/(C \cdot \pi_t)$  is the dividend yield,  $\mu^n(\pi) = r_t^n + \sigma^n(\pi) (\sigma_M + \sigma_Q) - \kappa(\xi_3 - \xi_1 - \xi_2)$  is the nominal expected return. The nominal stock price volatility is

$$\sigma^n(\pi_t) = \sigma_E + \sigma_Q + \frac{\sum_{i=1}^N C_i \pi_{it} (\nu_i - \bar{\nu}(\pi_t))' (\Sigma')^{-1}}{\sum_{i=1}^N C_i \pi_{it}}. \quad (18)$$

The proof follows from an application of Ito's formula for jump-diffusions. Asset volatilities have exogenous as well as learning-based components, which depends on the volatility of each regime probability  $\pi_i$ . A similar expression holds for bonds, but we do not report it for brevity.

## 2.5. Return Volatility and its Dynamic Properties

A key variable for understanding a number of features of options prices is the volatility of stock variance. We develop its properties here. We start by introducing the following notation. Let

$$\pi_{it}^\circ = \frac{\pi_{it} C_i}{\sum_{j=1}^N \pi_{jt} C_j} \quad (19)$$

As in Veronesi (2000), we call  $\pi_t^\circ = (\pi_{1t}^\circ, \dots, \pi_{nt}^\circ)$  the *value-weighted probabilities* (notice that  $\pi_{it}^\circ \geq 0$  for each  $i$  and  $\sum_{i=1}^N \pi_{it}^\circ = 1$ ). From now on, a ‘‘o’’ denotes a quantity computed with respect to the distribution  $\pi_t^\circ$ . For example,  $\bar{\theta}_t^\circ$  denotes the mean of the drift vector  $\theta$  computed using the distribution  $\pi_t^\circ$  (whereas e.g.  $\bar{\theta}_t$  denotes the mean drift vector computed using the original distribution  $\pi_t$ ), and

$$\sigma_{\theta\beta t} = \sum_{i=1}^N \pi_{it} (\theta_i - \bar{\theta}_t) (\beta_i - \bar{\beta}_t); \text{ and } \sigma_{\theta\beta t}^\circ = \sum_{i=1}^N \pi_{it}^\circ (\theta_i - \bar{\theta}_t^\circ) (\beta_i - \bar{\beta}_t^\circ) \quad (20)$$

are the covariances of the drift vectors  $\theta$  and  $\beta$  computed using  $\pi$  and  $\pi^\circ$ , respectively. In addition we denote  $\sigma_{\theta\nu}$  and  $\sigma_{\theta\nu}^\circ$  to be the vectors of covariances of  $\theta$  with each element of the vector  $\nu$  using the two sets of probabilities respectively. We then have:

**Proposition 3** (a) *Stock return variance is given by*

$$V_t = \sigma^n(\pi_t) \sigma^{n'}(\pi_t) = (\sigma_E + \sigma_Q) (\sigma_E + \sigma_Q)' + (\bar{\nu}_t^\circ - \bar{\nu}_t)' (\Sigma \Sigma')^{-1} (\bar{\nu}_t^\circ - \bar{\nu}_t) + 2 [(\bar{\theta}_t^\circ - \bar{\theta}_t) + (\bar{\beta}_t^\circ - \bar{\beta}_t)] \quad (21)$$

(b) *Return variance  $V_t$  follows the process  $dV_t = \mu_{Vt} dt + \sigma_{Vt} d\tilde{W}_t$ , where*

$$\sigma_{Vt} = 2 \left[ \sum_i ([\pi_{it}^\circ (\nu_i - \bar{\nu}_t^\circ) - \pi_{it} \nu_i]' (\Sigma \Sigma')^{-1} (\bar{\nu}_t^\circ - \bar{\nu}_t) (\nu_i - \bar{\nu}_t)') + (\sigma_{\theta\nu t}^\circ - \sigma_{\theta\nu t})' + (\sigma_{\beta\nu t}^\circ - \sigma_{\beta\nu t})' \right] \Sigma'^{-1} \quad (22)$$



(c) *The volatility of stock volatility is*

$$\sigma_{\sigma t} = 0.5 \frac{\sigma_{Vt}}{\sqrt{V_t}} \quad (23)$$

Similar expressions hold for bonds, and we do not report them for brevity. We finally show that the stock price process in our model satisfies important regularity conditions, which guarantee the solutions to the option pricing partial differential equation as well as estimation of the likelihood function. These conditions will be useful to compare the properties of our model with standard option pricing models in Section 6.

**Proposition 4** *The stock price process,  $P_t^n$ , in Proposition 2 satisfies global Lipschitz and growth conditions.*

## 2.6. Option Prices

Unfortunately, closed form formulas for option prices are not available. However, we can easily compute stocks and bonds' option prices using Monte Carlo simulations. We need to find the process for stocks and bonds under the risk-neutral measure first.<sup>12</sup>

**Proposition 5** *The stock and zero-coupon bond prices under the risk-neutral measure follow:*

$$\begin{aligned} \frac{dP_t^{n*}}{P_t^{n*}} &= (r(\pi_t^*) - \delta(\pi_t^*) - \kappa^* \xi_1^*) dt + \sigma^n(\pi_t^*) d\tilde{W}_t^* + (e^{Y_{1t}^*} - 1) dL_t^*, \\ \frac{dB_t^*(\tau)}{B_t^*(\tau)} &= r(\pi_t^*) dt + \sigma_B(\pi_t^*) d\tilde{W}_t^*, \\ d\pi_t^* &= (\Lambda' \pi_t^* - \vartheta(\pi_t^*)) dt + \Sigma(\pi_t^*) d\tilde{W}_t^*, \end{aligned}$$

where  $d\tilde{W}_t^*$  is  $5 \times 1$  vector Brownian motion,  $L_t^*$  is the counter of a Poisson process with intensity  $\kappa^* = \kappa \cdot e^{\mu_2 + 0.5\sigma_2^2}$ ,  $Y_{1t}^*$  is distributed  $N(\mu_1 + \sigma_1\sigma_2, \sigma_1^2)$ , and  $\xi_1^* = e^{\mu_1 + \sigma_1\sigma_2 + 0.5\sigma_1^2} - 1$ . Finally the market price of risk of the belief of regime  $i$ , which is the covariance of  $\pi_i$  with the nominal pricing kernel is given by

$$\vartheta_i(\pi_t^*) = \pi_{it}^* ((\beta_i - \bar{\beta}(\pi_t^*)) + (\phi_i - \bar{\phi}(\pi_t^*))). \quad (24)$$

---

<sup>12</sup>In Proposition 5 we report the risk neutral dynamics of zero coupon bonds. The appendix discusses how we modify the model to price options on futures on coupon bonds, which correspond to our data.

The proof is in Appendix 1. We appeal to the Feynman-Kac formula to use Monte-Carlo simulations to evaluate the risk-neutral expectation

$$f(t, \pi_t, P_t^n) = \mathbb{E}^Q \left[ \exp \left( - \int_{s=t}^T r(\pi_s) ds \right) g(P_T^n, \pi_T) \right]. \quad (25)$$

## 2.7. A Three-Regime Example

Before we take the model to the data, it is useful to illustrate the impact of our learning model with monetary policy on stock and bond option prices by using a simple three-regime model. Table 1 provides the drift rate parameters for three composite regimes: Regime 1 is a regular boom, with relatively low inflation, high earnings growth, and close to average capacity utilization (0%, as the series is demeaned in our model). Regime 2 is a “deflationary” regime, in which inflation is slightly negative, earnings growth is very negative, and capacity utilization is far below average. Finally, Regime 3 is a regular boom like Regime 1, except that capacity utilization is again below average. These regimes characterize part of the option’s sample, 1988 - 2011, and the parameters in Table 1 are in fact those we estimate in Section 4.<sup>13</sup>

Table 1 also contains the conditional P/E ratios across the three regimes, the short-term rate, and the long-term bond prices. The short-term rate depends on monetary policy through the Taylor rule (9). Thus, the short-term rate is the highest in Regime 1, the smallest in the deflationary Regime 2, and intermediate in Regime 3, when the economy is in a boom, but capacity utilization is below average. The impact of monetary policy on stock prices is now apparent from comparing the conditional P/E ratio across Regimes 1 and 3. In fact, although both Regimes 1 and 3 have the same growth rate of real earnings and inflation rate, Regime 3 has lower capacity utilization, and thus lower real rate, which in turn increases the P/E ratio compared to Regime 1.

Figure 2 plots the stock and bonds ATMIV, and the stock P/C, in two cases. In the first case (Panels A and C), we consider the impact of uncertainty between the regular boom Regime 1 and the recession Regime 2. In particular, we set the probability of the low-capacity boom Regime 3 to zero,  $\pi_3 = 0$ . Panel A shows that as the probability of Regime 1 decreases, the ATMIV of both stocks (solid line) and bonds (dashed line) increase, to a maximum

---

<sup>13</sup>The remaining parameters of the model are as estimated in Table 2 with the exception of the infinitesimal generator, which here is assumed as follows  $\Lambda = \begin{pmatrix} -0.1 & 0.1 & 0.0 \\ 0.1 & -0.2 & 0.1 \\ 0 & 0.1 & -0.1 \end{pmatrix}$ . That is, booms are longer than recessions, and the recession can be accessed by either boom regime.

around the point of maximum uncertainty ( $\pi_1 = 0.5$ ) and then decline again. Intuitively, higher real economic uncertainty increases the volatility of stocks, while uncertainty about monetary policy, induced by uncertainty about inflation and capacity utilization, increase the volatility of bonds. Panel C plots the P/C ratio against the probability of the boom Regime 1,  $\pi_1$ . In good times ( $\pi_1 \approx 1$ ) the P/C ratio is very high, while in bad times the P/C ratio is very low. The intuition is that in good times, there is a larger probability of stock price drop than in bad times, as prices are affected not only by i.i.d. shocks (random Brownian motions or jumps), but also by the possibility of bad earnings news that would decrease the probability to be in a boom, thereby giving a double downward kick to the stock prices. Thus, OTM put options become relatively more expensive than OTM calls and the P/C is high. The opposite argument holds when  $\pi_1 \approx 0$ .

Panels B and D of Figure 2 highlight instead the impact of monetary policy on option prices. Indeed, in this case we keep the probability of a recession constant at zero,  $\pi_2 = 0$ , but only examine the uncertainty between the two “boom” regimes, namely, one with average capacity utilization (Regime 1) and one with low capacity utilization (Regime 3). Panel B shows that the patterns of ATMIVs of stock and bonds are qualitatively similar to the ones in Panel A, with higher monetary policy uncertainty increasing ATMIV of both stocks and bonds. While the pattern is similar, comparing the magnitudes in Panel A and B, we see the impact of just monetary policy uncertainty on stocks’ and bonds’ ATMIV is smaller than when we have both economic and monetary policy uncertainty (in Panel A).<sup>14</sup>

To understand the source of variation in ATMIV, it is useful to look at the expression of variance  $V(\pi_t)$  in (21), specialized to the case in which there are two regimes  $i$  and  $j$  for which  $\pi_i = 1 - \pi_j$ . In this case, we obtain

$$\begin{aligned}
V(\pi_t) &= (\sigma_Q + \sigma_E)(\sigma_Q + \sigma_E)' + \left\{ 2\pi_{it}(1 - \pi_{it}) \frac{(C_i - C_j) [(\beta_i - \beta_j) + (\theta^i - \theta^j)]}{P/E(\pi_t)} \right\} + \\
&+ \left\{ [\pi_{it}(1 - \pi_{it})]^2 \frac{(C_i - C_j)^2 h}{P/E(\pi_t)^2} \right\} \tag{26}
\end{aligned}$$

where  $h = (\nu_i - \nu_j)'(\Sigma\Sigma')^{-1}(\nu_i - \nu_j)$  is a positive constant. Given the log pricing function  $\log(P^n) = \log(E_t Q_t) + \log\left(\sum_j C_j \pi_j\right)$ , the variance of stock returns (26) is given by three terms. The first term is induced directly by fundamental shocks, and it is constant. The second term is induced by the comovement of inflation and earnings news with the log P/E ratio, that is, the volatility of returns due to  $Cov[d\log(E_t Q_t), d\log(P/E(\pi_t))]$ . This term is large when uncertainty  $\pi_{it}(1 - \pi_{it})$  is large, when the difference of conditional P/E ratios

---

<sup>14</sup>We emphasize that this is not however a general result. Parameters can be found that increase the impact of monetary policy uncertainty on stock option’s volatility.

across regimes ( $C_i - C_j$ ) is large, and when news have an impact on expected future inflation and earnings, which occur if the difference in regime values  $\beta_i - \beta_j$  and  $\theta_i - \theta_j$  are large. Finally, the last term in (26) is induced by conditional variance of  $\log(P/E(\pi_t))$  itself. This last term is also large when both  $\pi_{it}(1 - \pi_{it})$  and  $(C_i - C_j)$  are large. In addition, this term increases with the multiplier  $h$ , which itself depends on distances of drift differences across regimes, weighted by the informativeness of the signals  $\Sigma^{-1}$ . For bond returns, the expression for variance is just given by the third term in (26), with conditional bond prices  $B_i(\tau)$  replacing conditional price/earnings ratios  $C_i$ .

Coming back to understanding the impact of uncertainty on volatility, when uncertainty is about inflation and earnings regimes, as in the first case (Panel A), both the second and third term in (26) are non-zero. The second term is especially important. For instance, a negative earnings news impacts the price twice: both because earnings are now smaller and also because that news implies a downward revision of the boom probability  $\pi_{1t}$ , which in turn lowers the P/E ratio. Thus, uncertainty leads to a relatively high volatility of stock returns. When the uncertainty is purely about monetary policy, i.e. between Regime 1 and 3, then the second term in (26) is instead zero. In fact, in this case a negative earnings news, for instance, has no direct impact on the probability of future earnings, as both Regimes 1 and 3 have the same earnings drift. In this case, only news about capacity utilization matter as they move the probability  $\pi_{1t}$  and thus the P/E ratio. For instance, a negative news on capacity utilization has no direct impact the stock price, but only an indirect effect as it increases the probability of Regime 1, which *increases* the price/earnings ratio (see Table 1). Economically, the bad news about capacity utilization implies a reaction from the central bank to lower interest rates, which in turn increases prices. This type of variation is implied by the third term in (26), and uncertainty about the CU regime generates higher implied volatility for both stocks and bond options.

Finally, Panel D of Figure 2 shows that CU uncertainty has very small impact on the P/C ratio, compared to the case in Panel C, which features both economic and monetary policy uncertainty. The intuition is the same as above: In either case we are in a boom, and thus in both cases there is a relatively high probability of negative economic news in case of an increased perceived probability of transitioning in the deflationary Regime 2.

How does all this translate on the relation between ATMIVs and P/C and interest rates, which are set by the forward looking Taylor rule and thus depend on beliefs? In each Panel A through C of Figure 3, we plot each options' index against the short-term rate, when both are computed on a grid of beliefs  $(\pi_{1t}, \pi_{2t}, \pi_{3t})$  on the unit simplex (with  $\pi_{it} \in [0, 1]$  and

$\sum_{i=1}^3 \pi_{1t} = 1$ .) Panel A shows that the three composite regime model implies that stock's ATMIV is broadly decreasing with short term-rates. That is, in this example, periods with very high interest rates tend to be mainly related to low implied volatility periods. However, the relation between ATMIV and interest rates is not one-to-one and much noise should be observed in the data when we relate monetary policy to implied volatilities. Panel B shows that the P/C ratio is mainly increasing in interest rates. That is, good times (high interest rates) are correlated with a high P/C ratio, consistently with the intuition provided above, namely, that in good times it becomes relatively more likely that stocks may drop due to learning about a potential regime switch into a recessionary period. Finally, bond return volatility is typically lower for higher interest rates, although there is much dispersion of bond ATMIV across various levels of interest rates, and in fact even more so than for stocks in Panel A.

This example shows that even with only three composite regimes, the relation between option indices, ATMIV and P/C, and monetary policy is quite complex and it depends on current beliefs about composite regimes themselves.<sup>15</sup>

### 3. Estimation

#### 3.1. Data

Our data sample for fundamentals runs from 1967 to 2011. The definitions of fundamental series are as follows. Aggregate quarterly earnings for the economy are approximated as the operating earnings of S&P 500 firms, and these data are obtained from Standard and Poor's. The other three fundamentals, the Consumer Price Index (CPI), Industrial capacity utilization (CU) and money (M1) are obtained from the Federal Reserve Board.

Stock prices are obtained from Standard & Poor and P/E ratio is estimated as the equity value of these firms divided by their operating earnings. The time series of zero-coupon yields and returns on Treasury bonds of different maturities are obtained from Gurkaynak, Sack, and Wright (2007). Options data are obtained from two sources. We obtain transactions data on S&P 500 index options from 1986:Q2 to 1996:Q1 from the CBOE. These data are no longer available from 1996:Q2, and therefore, we use data on these same options from

---

<sup>15</sup>While the relations between options and monetary policy described in this example are intuitive, we should remember they depend on the three regimes used. As shown in Section 3., in reality there are many additional composite macroeconomic - policy regimes, such as stagflation regimes, whose implications for the relation between monetary policy and options may differ from what discussed in the simple example.

Option Metrics from 1996:Q2 to 2011:Q4. It is important to note that Option Metrics provide the average of bid and ask prices at the end of each trading day, and not prices based on actual transactions. Prices at the beginning of each quarter are fitted with fundamental data available at the end of the previous quarter. At-the-money implied volatilities on options on 10-year Treasury bond futures are from Mueller, Vedolin, and Yen (2013) and are available from 1985Q3 to 2011Q4.<sup>16</sup> We restrict the option sample to the post October 1987 crash period, when the option smile became relevant and P/C increased substantially to above one (e.g. Bates (1991)).

Survey data are from the Survey of Professional Forecasters (SPF) at the Federal Reserve Bank of Saint Louis. We obtain survey data for consensus forecasts of inflation (GDP deflator), GDP growth, the probability of GDP decline (“anxiety index”), and the consensus probabilities that inflation (GDP deflator) will lie in pre-specified intervals in following quarter. Data are available at the quarterly frequency for the sample 1968 to 2011. We follow the same procedure as David and Veronesi (2013) to obtain SPF probabilities for inflation regimes. Finally, survey data on capacity utilization for the sample 1998 to 2011 are from Bloomberg.

## 3.2. Methodology

Except for the data used, the empirical methodology is close to the one used in David (2008) and especially David and Veronesi (2013). While we leave the details to the Appendix, we provide a brief summary of the methodology in this section to highlight its benefit and especially to better interpret the empirical results in Section 4.

We use a Simulated Method of Moments that combines both a maximum likelihood estimation of the fundamentals and pricing errors from asset prices. More specifically, denote the parameters to estimate by  $\Phi$  and let  $\mathcal{L}(\Phi)$  be the likelihood function of fundamental variables (CPI, real earnings, CU, and money growth). We construct the quarterly likelihood function by simulations at daily frequency, as in Brandt and Santa-Clara (2002), and then compute the scores the likelihood function  $\frac{\partial \mathcal{L}}{\partial \Phi}$ . For given path of beliefs  $\pi_t$  we can compute the model-implied prices, namely, the P/E ratio, the 3-month rate, the slope of the term structure of interest rates, as well as the ATMIV of stocks and bonds, and finally the OTM Put/Call implied volatility ratio. We can thus compute the model’s pricing errors, stack them together with the scores of the likelihood function, and minimize the usual GMM

---

<sup>16</sup>We thank Mueller, Vedolin, and Yen for providing the data.

criterion function.

We highlight three important features of our procedure. First, it is important for the goal of this paper to extract investors' beliefs in a dynamic, forward-looking, and time-consistent manner. The SMM procedure ensures we achieve this goal, as the structural parameters are held constant through time, but the arrival of fundamental news lead to updates of posterior probabilities about the underlying composite regimes. Importantly, our simulation method ensures that the estimated beliefs  $\pi_t$  are mainly driven by fundamentals (CPI, real earnings, CU, and Money Growth) and that Bayes formula in equation (11) exactly holds. That is, the beliefs that we estimate in our model are consistent with shocks to fundamental data throughout the sample, as the evidence in Section 4.1. will show.

Second, the use of stocks, bonds and options also ensures the identification of composite regimes that matter for asset prices. That is, as discussed in David and Veronesi (2013) asset prices depend on beliefs about regimes that may or may not occur in sample (i.e. classic "Peso Problem" situations). Our SMM procedure that combines fundamental shocks with asset prices allow us to estimate composite regimes that would not be precisely estimated if only fundamentals were used. For instance, as we are going to see, the existence of a potential deflationary period is important for asset prices, and explains extremely low interest rates and high stocks and bonds' implied volatilities. However, it may not necessarily be the case that such regime has been realized on an ex-post basis. Finally, our methodology allows us deal with potential time-aggregation issues that arise from our availability of only annual earnings at the quarterly frequency. In the SMM procedure, we simulate the same annual averages at quarterly frequencies and thus we can compute the proper likelihood function.

We conclude this section with a couple of remarks about the selection of the number of composite regimes, and the constraints we impose to avoid parameter proliferation. We select  $N = 8$  composite regimes, which are described below. While this number of composite regimes may appear large at first, we note that even allowing just two regimes for each of the four fundamental series would lead to a cross-product of  $2^4 = 16$  composite regimes. In addition, for each individual series we can actually detect three or four regimes, whose cross-product would lead to between 81 ( $= 3^4$ ) and 256 ( $= 4^4$ ) composite regimes. Clearly, not only such a large number of regimes would be intractable, but such an analysis would be mostly useless, as the vast majority of such composite regimes have zero probability of occurring in sample. Our choice of  $N = 8$  composite regimes allow us to still detect the low-frequency common variation across series, but in a more parsimonious model. In addition, the use of asset prices to detect regimes allow us to compute such regimes with reasonable precision.

Finally, we also impose a number of parameter restrictions to the estimation driven by the results of a preliminary un-restricted estimation.

A second point to emphasize is that our model with eight regimes implies seven state variables (beliefs  $\pi_t$ ), which follow a vector autoregressive process with stochastic volatility (equation (11)). Beliefs are driven only by five Brownian motions (four fundamental news plus pricing kernel shocks), which implies ours is a five-factor model.<sup>17</sup> Moreover, four of the five shocks that drive beliefs are in fact observable by the econometrician, as they are just fundamental shocks. This is important as it ensures our beliefs have an obvious economic meaning, and they are not just “hidden factors” (e.g. level, slope, and curvature) that are extracted from prices. Indeed, we present evidence in Section 4.1. that our beliefs extracted from fundamentals and asset prices as described in the previous paragraphs are in fact consistent with survey beliefs.

## 4. Estimation Results

Table 2 contains the parameter estimates. The top panel contains the estimates of the eight composite regimes, with evocative names for easier reference.<sup>18</sup> Overall, we have four regimes characterized by high (or very high) earnings growth (referred to as booms hereafter), and four regimes with negative earnings growth (recessions). Recessions are accompanied either by medium or high inflation, or by deflation. Finally, capacity utilization and monetary policy vary across regimes: For instance, Regime 1 and Regime 7 are identical, except that capacity utilization is far lower in the latter than in the former, while money growth is higher. Regime 1 is a “Regular Boom,” which, as we shall see, characterizes most of the sample, while Regime 7 really only characterizes the last decade. We return on discussing these regimes in the next Section, as we discuss the probabilities.

The middle panel of Table 2 contains the estimates of the diffusion matrix, and the jump parameters. The latter show that jumps in earnings and the pricing kernel are important, and large. Jumps occur with intensity 7.52%, implying a jump every 13 years or so. The size of the jump is substantial, as the average jump size is negative 6.30% with and standard

---

<sup>17</sup>The number of “factors” in a factor model depends on the number of uncorrelated Brownian motions. The number of state variables may be larger than the number of factors driving them.

<sup>18</sup>The first six regimes are similar to those in David and Veronesi (2013), who only fit inflation and real earnings, but no monetary policy variables. Indeed, note that the last two regimes (7 and 8) are identical to regime 1 and 2 for inflation and earnings, but differ in their monetary policy regime. Compared to David and Veronesi (2013), the two additional fundamental series to fit (CU and MG) required an expansion of the number of composite regimes from six to eight. See discussion in previous section.



deviation of 34%. While this number is very large, it should be compared with the massive jump in earnings growth during the crisis, as visible in Figure 4.

The Taylor rule coefficients are reasonable, as the interest rate rule parameters suggest that the real rate in the economy depends positively on both expected inflation and expected change in CU:  $\alpha_\beta = 0.2278$  and  $\alpha_\rho = 0.2188$ . These estimates are similar to estimates of the Taylor rule in many other papers. In interpreting the results, however, we should remember that the left-hand-side of the Taylor rule is the real rate, that we use industrial capacity utilization rather than the output gap used by Taylor, and that the rates depend on the expected drifts of the variables rather than the variable realizations themselves.

Panels A to D of Figure 4 plots the fundamental data, together with the model’s expected fundamental drift for each variable. Panels A to B also reports consensus forecasts for inflation (GDP deflator) and for economic growth (real GDP growth) from the Survey of Professional Forecasters. As it can be seen, in both cases the model’s expectation tracks well the consensus forecasts. Panel C also plots Bloomberg consensus forecasts of capacity utilization, whose series only starts in 1998. The figure shows that the model expected drift rate of CU tracks well the realized level, as well as the Bloomberg forecasts for the common sample. Panel D reports only money growth and the model expected drift rate, which again track well each other. Unfortunately, no survey data on money growth are available.

Table 3 reports the regression of realized fundamental variables and their expected value according to the fitted model. We find that the regression coefficients are all strongly significant, and that the expected fundamental explain realized fundamentals with a relatively high  $R^2$ , ranging from 17.4% for real earnings growth, to 85.3% for capacity utilization. Real earnings growth is clearly the harder series to predict, as it is has large volatility and it is also affected by large i.i.d. jumps, consistently with the model.

## 4.1. The Dynamics of Beliefs

Figure 5 plots the dynamics of the fitted beliefs  $\pi_{it}$  over the sample. The left panels (A, C, E, and G) are the “boom” regimes, while the right panels (B, D, F, and H) are the recessionary regimes. The description of the regimes is in Table 2. Panel A shows the beliefs of a “regular boom,” with low inflation, high growth, average capacity utilization, and slightly decreasing money growth. The posterior probability of this regime hovers around 70% in the sample, with significant dips around the NBER-dated recessions (the shaded areas), or during booms that are characterized by a over-heating economy (with higher CU, as in Panel

C), or exceptionally strong earnings growth (Panel E), or very low capacity utilization (Panel G). The latter regime characterizes in fact the last decade. The right panels plot beliefs of recessionary regimes, whose probabilities indeed increase around the shaded areas. Notable in these panels are the spikes in the last decade of the probability of a deflationary regime, visible in Panel F.

While the fitted posterior probabilities increase and decrease around historical periods in a reasonable way, we check in Figure 6 whether their variation correspond to probabilities that can be extracted from surveys. The first five panels (A - E) of this figure uses probabilities of inflation and economic growth from the Survey of Professional Forecasters (SPF). Indeed, in this survey, professional forecasters are not only asked a point forecast about a given economic variables (e.g. inflation), but also their probability assessment that next quarter such economic variable will be in given intervals. SPF also provides the average probabilities from the professional forecasters.

Panels A to D compare the model's beliefs of high, medium, low, and zero inflation regime with the probabilities from SPF, where for SPF we define high, medium, low, and zero inflation intervals by using the middle points of our estimated regimes. The correlations between SPF probabilities and the model-fitted probabilities are 71%, 51%, 80% and 36% for high, medium, low, and zero inflation, respectively. Similarly, Panel E plots the SPF probability of a decline in GDP growth next quarter with the fitted model probability of a recession. The correlation between the two series is 57%. These results provide comfort that the model's "state variable" resemble real probabilities.

Panels F to G still of Figure 6 provide evidence that the model-fitted beliefs on capacity utilization are also similar to survey-based forecasts. Unfortunately, Bloomberg forecasts only provide point forecasts of next month CU, but they do not ask their forecasters about probability assessments. Thus, in this case we use the dispersion of forecasters' forecasts to elicit a distribution of forecasts, and obtain an implied forecast probability of high, medium, and low CU, using the middle points in our regimes' estimates to determine the ranges. One additional fact about Bloomberg forecasts is that such forecasts are projection for the current month' CU reading, which thus dramatically limit the dispersion. One implication is that such forecasts dispersion-based beliefs are concentrated at zero or one. Still, Panels F and G show that such beliefs are very close to the model-fitted beliefs, which again lend support to our fitted model.

To conclude this section, this evidence is meant to underscore that the state variables in our model – the posterior beliefs – are mostly driven by four *observable* fundamental shocks

(inflation, earnings, capacity utilization, and money growth) and updated according to Bayes rule (equation (11)), and they are not merely inverted from asset prices. Asset prices are used in the estimation to determine the model's parameters, which indirectly affect the beliefs. But the full dynamics of beliefs is largely determined by observable shocks.

## 4.2. The Dynamics of Asset Prices

We now move to discuss the model's fit to asset prices. Figure 7 shows the fit of the six asset pricing series used in the estimation. In particular, the left panels A, C, and E plot the realized P/E ratio, the 3-month T-bill rate, and the slope of the term structure, respectively, along with their model-fitted counterparts. In each panel, the solid gray line represents the data, while the dashed, black line the fitted model. As it can be seen, the model performs rather well, especially if we remember that model prices only depend on beliefs  $\pi_t$ , which are driven mostly by observable fundamental shocks, as discussed in the previous section. Admittedly, the model's fit has some shortcomings, such as its inability to match the P/E ratio during the dot-com boom, or the sizable slope of the term structure in early 1990s and early 2000. Still, Table 3 reports the results of a regression of realized prices on their fitted counterparts, and we find very significant slope coefficients with  $R^2$  of 47.5%, 45.4%, and 50.8% for P/E ratio, 3-month T-bill, and slope, respectively. That is, the dynamics of beliefs in Figure 5 capture the variation of asset prices at the proper frequency.

The right panels B, D, and F plot the option-related series, namely, stock ATMIV index, P/C index, and the 10-year bond ATMIV index, for the shorter option sample 1988 - 2011. As it can be seen, the model's quantities, which we emphasize are only driven by beliefs  $\pi_t$  and thus by fundamental shocks, track the realized prices well. Indeed, Table 3 shows that the  $R^2$ 's of regressions of data on model-fitted prices are 37%, 32.5% and 30.5% for stock's ATMIV, the P/C index, and the bond's ATMIV, respectively. The slope coefficients are strongly significant.

## 5. Monetary Policy and Option Prices

The previous section shows that the model fits well the data. We now turn to investigate the relation between option prices and monetary policy in light of our model.

## 5.1. Understanding Impulse Responses

Panel A of Figure 8 plots the impulse response of the 3-month T-bill rate to a unit shock to stock ATMIV, in the data (solid line), and in the fitted model (dashed line). The dotted lines correspond to the 95% confidence interval. As it can be seen, the impulse response obtained from the fitted model is very similar to the one in the data. This finding is important, as recall that the model-fitted ATMIV and 3-month rate are solely functions of beliefs  $\pi_t$ , which themselves are mostly driven by observable shocks to fundamentals through Bayes formula. That is, conditional on the parameter estimates, shocks to fundamentals and Bayes formula capture the correct dynamics of ATMIV and short-term interest rates to generate the observation that a shock to ATMIV tends to decrease future interest rates, whereas both are driven by a belief process. Within our model, the interpretation is that adverse shocks to fundamentals change the beliefs of *both* the investors, whose uncertainty increases the implied volatility of options, and of the central bank, whose beliefs affect the interest rates through the Taylor rule in expression (6).

A similar comment pertains to the impulse response of the 3-month T-bill rate to a unit shock to the P/C index, plotted in Panel C of Figure 8. As in Panel A, the impulse response computed from the fitted model (dashed line) is very close to the one computed from the data (solid line), once again suggesting that the slow movement of beliefs driven by fundamental shocks captures the proper dynamics to explain monetary policy action. Finally, Panel E of Figure 8 reports the impulse response of the 3-month T-bill rate to unit shock to the bond ATMIV. Differently from the previous two cases, while in the data it appears that bond ATMIV has not much of an impact on the 3-month T-bill rate, whose impulse response function converges quickly to zero, the pattern in the fitted data is a bit different, although it still lies within the 95% confidence interval. The main reason of this different behavior appears to be that the model generates a slightly too high covariance between bond ATMIV and stock ATMIV, as documented in Table 4 and further discussed in the next section, compared to the data.

To provide further evidence of the relation between option prices and monetary policy, the right panels of Figure 8 plot the impulse response of the capacity utilization – a key policy variable – to a unit shock to stock ATMIV (Panel B), P/C (panel D), and bond ATMIV (panel F). In particular, in Panel B we see that a unit shock to stock ATMIV decreases capacity utilization for the following eight quarters. Moreover, the fitted model produces a very similar result. This additional evidence supports the channel that increases in economic uncertainty through variation in beliefs (the drivers of ATMIV) affect monetary policy, as

the forward looking Taylor rule implies that the expectation of lower capacity utilization generates lower future interest rates.

Panels D and F report similar evidence on P/C and bond ATMIV. A shock to P/C tends to raise future capacity utilization, both in the data and in the fitted model. The similar pattern indicates that the same variation in beliefs that increases P/C also affects expected future capacity utilization, which in turn affects the short-term rate through the Taylor rule. Economically, during good times, bad shocks to fundamentals decrease the P/C, as the reaction of beliefs to bad news pushes down stock prices and decreases the probability of further sharp declines, and it also decreases expected capacity utilization, which in turn decreases the short-term rate (as in Panel C). Interestingly, in Panel F we also find that a shock to bond ATMIV tends to decrease future capacity utilization, for the same reason as in Panel A. The fitted model and the data are in fact very similar. Insofar as expected lower CU affect interest rates, this finding is consistent with the overall message of the model.

## 5.2. Option Prices and Beliefs

The previous section shows that indeed shocks to option prices have an impact on both the short-term rate and capacity utilization in a way that is similar to the model, whose quantities are mostly driven by beliefs. In this section we document that indeed option prices and beliefs are in fact quite correlated, both in the model and in the data.

Table 4 provides the results of the following regression

$$(\text{Option Index})_{t+1} = \alpha + \beta X_t + \epsilon_{t+1} \quad (27)$$

where “Option Index” is the stock ATMIV index (Panel A), the P/C index (Panel B), and the bond ATMIV index (Panel C). The independent variable is a belief-related variable described by the name on each row, and whose definition is discussed below. We lag the independent variables to deal with potential concerns of reverse causality in the data.

The first five columns of Table 4, under the heading “Model”, only use model-based quantities, both for the right-hand-side and for the left-hand-side of (27). For instance, in Panel A the “Option Index” ATMIV is the stock ATM implied volatility  $ATMIV(\pi_t)$  which is fitted through the 1988 - 2011 sample, and only depend on the model’s filtered beliefs  $\pi_t$ , depicted in Figure 5. Similarly, the same model’s filtered beliefs are used to construct the independent variables  $X_t = f(\pi_t)$  as described below. This exercise is important because both the left-hand-side and the right-hand-side of (27) depend on beliefs that are observable

to the econometrician, and thus provide the exact model’s implications about the source variation in option prices.

The second set of five columns, under the heading “Data”, perform regression (27) only on data, where the left-hand-side is one of the option indices constructed from traded options, and the right-hand-side variable  $X_t$  depends on beliefs that are obtained from the Survey of Professional Forecasters. We already used such survey-based beliefs to compare model’s beliefs to survey beliefs in Panels A - E of Figure 6.

### 5.2.1. Stock’s ATMIV and Beliefs

Turning to the result, the first row in Panel A shows the result of a regression of stock ATMIV on the probability of a recession  $ProbRecess$ . In the model, such probability is simply given by the probability to be in a recession regime, that is, the sum of the beliefs of Regime 2, 4, 6, and 8. In the data, such probability is given by the probability of a GDP decline next quarter from the Survey of Professional Forecasters. Both probabilities are plotted in Panel E of Figure 6. The model predicts that stock ATMIV should be strongly positively relative with the recession probability, with an  $R^2 = 38\%$ . This result is of course related to usual finding in the literature that volatility should be higher in recessions, and our fitted model predicts exactly that. Consistently with our model, the data-based regression produces very similar results, with a strongly significant slope coefficient (t-stat = 3.59), and  $R^2 = 32\%$ . Moreover, we also see that the slope coefficient itself is very similar to the model’s prediction.

The second row regresses ATMIV on economic uncertainty, which is defined as the posterior variance of earnings growth, that is,  $EconUnc = V_t[\theta] = \sum_{j=1}^8 \pi_{jt} (\theta_j - E_t[\theta])^2$ . Again, ATMIV is strongly positively related to economic uncertainty, as its slope coefficient is strongly significant (t-stat = 5.11) and  $R^2 = 54\%$ . In the data, we define economic uncertainty from the SPF probability of a decline in GDP. That is, we proxy  $EconUnc = ProbRecess \times (1 - ProbRecess)$ . Indeed, in a regime switching model with two regimes, this quantity is in fact proportional to posterior variance.<sup>19</sup> The empirical results strongly support a positive relation between stock ATMIV and economic uncertainty, with a slope t-stat of 3.02 and  $R^2 = 23\%$ .

The next three regressions consider inflation probabilities, that is, those for which we

---

<sup>19</sup>In such case, we have  $EconUnc = \sum_{j=1}^2 \pi_{jt} (\theta^j - E_t[\theta])^2 = \pi_{1t}(1 - \pi_{1t})(\theta^1 - \theta^2)^2$ . We note that because in the data we regress only on the quantity  $\pi_{1t}(1 - \pi_{1t})$ , the slope coefficient also includes  $(\theta^1 - \theta^2)^2$ , and thus regression coefficients in data and model are not directly comparable.

actually have empirical counterparts in the SPF data. The model suggests that expected inflation should be negatively related to stock ATMIV, although only mildly. Indeed, the SPF data show that there is only a marginally significant relation between these two variables. Interestingly, in the model inflation uncertainty, which is defined analogously as economic uncertainty, is not a significant explanatory variable for stock ATMIV. We find the same result using option data and SPF probabilities.<sup>20</sup>

Given that since the turn of the millennium, deflation fears have resurfaced and indeed led the Federal Reserve to slash rates in the aftermath of the 2001 and 2008 recessions, it is interesting to check how beliefs of deflation correlate with stock ATMIV, both in the model and in the data. In the last row of Panel A we perform such test. We find that the model ATMIV is positively related to the probability of a deflationary regime (regime 6), with a strong slope coefficient and  $R^2 = 23\%$ . A similar result occurs in the data. In this case, we proxy for the probability of deflation as the SPF beliefs of very low inflation (see Panel D in Figure 6), and we find a positive and significant slope coefficient (t-stat = 2.89) with  $R^2 = 13\%$ . That is, fears of deflation seem to have contributed to the increase in implied volatility over the option sample, as the model predicts.

### 5.2.2. The OTM Put-to-Call Implied Volatility Index and Beliefs

Panel B of Table 4 investigates the relation between the put-to-call OTM implied volatility ratio (P/C) and beliefs. The fitted model shows that P/C should be negatively related to the probability of recession ( $R^2 = 27\%$ ), to economic uncertainty ( $R^2 = 23\%$ ), to the probability of deflation ( $R^2 = 26\%$ ). Exactly the same results are observable in the data, as *ProbRecess*, *EconUnc*, and *ProbDef* are all significant (t-stats from -2.14 to -3.42), with the expected sign, and with  $R^2$ 's ranging from 14% (*ProbRecess*) to 16% (*EconUnc*). Instead, *ExpInf* and *UncInf* are not significant.

Moreover, the model also predicts that P/C should be negative related to stock ATMIV ( $R^2 = 62\%$ ), and indeed, the same relation holds in the data ( $R^2 = 36\%$ ), as we would expect from Figure 1. This matching of the inverse variation of ATMIV and P/C in the model is important, as recall that ATMIV and P/C in the model are only driven by beliefs  $\pi_t$ , which themselves are mostly driven by fundamental shocks. This empirical finding is consistent with the intuition provided in the simple three-regime model in Section 2.7. and the related

---

<sup>20</sup>Specifically, we compute  $InfUnc = \sum_{j=1}^J p_{jt}^{SPF} (I_j - E_t[I])^2$  where  $p_{jt}^{SPF}$  is the SPF probability that inflation will be in the unit interval centered at  $I_j$ . We compute the probabilities on unit intervals by interpolating the SPF inflation probabilities, as discussed in the David and Veronesi (2013).

Figure 3.

### 5.2.3. Bonds' ATMIV and Beliefs

Finally, Panel C considers the relation between the bonds ATMIV index and beliefs. The model predicts that ATMIV should be positively related to the recession probability, economic uncertainty, inflation uncertainty, and the probability of deflation. The data confirm that all of these four belief-related quantities are in fact significant regressors in the data, with t-stats ranging between 1.97 (*ProbDef*) and 3.41 (*ProbRecess*), and  $R^2$  between 4% (*UncInf*) and 21% (*ProbRecess*). The model also predicts a positive relation between bond implied volatility and stock implied volatility ( $R^2 = 32\%$ ), which is in fact verified in the data (t-stat = 3.94 and  $R^2 = 26\%$ ).

Overall, this section provides strong evidence that beliefs' dynamics about the economy and the inflation affect option prices. Because under the forward looking Taylor rule, beliefs enter into the determination of the short-term interest rate, the evidence provided in these last two sections support the belief-based explanation of the time variation of options and interest rates.

## 6. Additional Properties of Options' Dynamics

In this section we discuss features of observed option prices that are not directly fitted by our empirical methodology. The ability of our model to replicate these additional features provides further support for the economic mechanism that determines option prices in our model. Moreover, we see that beliefs dynamics enter as an important explanatory variable, as predicted by the model.

### 6.1. The Volatility of ATM Implied Volatility

In the previous section we saw that our model ATMIV was able to explain about 37 percent of the variation in the data ATMIV. In the model, the implied volatility is to a large part determined by the endogenous volatility of stock prices, which increases during periods of greater investor and central bank uncertainty. Looking again at Panel A of Figure 1, we see that during episodes of high volatility around the three NBER dated recessions in



the options subsample, ATMIV also fluctuated by large amounts. The positive relation between volatility and the volatility of volatility is noted in Jones (2003) who further notes that it cannot arise in the Heston (1993) stochastic volatility model, which has been the workhorse of the option pricing literature. To obtain the level dependence of volatility, Jones (2003) proposes a generalization of the Constant Elasticity of Variance (CEV) model of Chan, Karolyi, Longstaff, and Sanders (1992). One drawback of the volatility processes in such models is that they do not satisfy global growth and Lipschitz conditions, which are commonly used sufficient conditions for a number of important results. In contrast, our model, which satisfies these two regularity conditions (see Proposition 4), is able to provide an economic explanation of the positive relation between volatility and the volatility of volatility.

Indeed, the top panels of Figure 9 show the scatter plots of implied volatility and absolute changes in implied volatility for the data and model series. Both show a positive association of similar magnitude between these variables with correlations of 38 percent and 48 percent, respectively, and these correlations are statistically significant. The economic explanation offered from the model can be readily seen in expression (11). In particular, the Bayesian learning mechanism that drives volatility in our model implies that investors revise their beliefs faster during periods of high uncertainty as they have low confidence in their estimates of the current regime of the fundamentals.

If the mechanism implied by the model is correct, we should see a similar positive association between the volatility of return volatility and absolute changes in the implied volatility. We construct a time series of the model's volatility-of-volatility (VV) using expression (23) and evaluate it at each date using the filtered beliefs in Figure 5. The scatter plot of absolute changes in implied volatility (data and model) with this VV series are shown in the bottom panels. As seen, the model volatility of volatility is highly correlated with both the data and model absolute changes in implied volatility with correlations of 33 and 36 percent respectively. Note that the model series measures the ex-ante volatility of volatility at each date and is compared to the ex-post realized absolute changes in ATMIV and our model predicts a positive but not one-to-one association between these variables. This is highlighted by the fact that the correlation between these variables is only about 36 percent correlation even when both variables are generated by our model.

Additional supporting evidence in favor of our learning mechanism comes from performing the same exercise but for bond volatility. Figure 10 shows the scatter plots of bonds' implied volatility and absolute changes in implied volatility for the data and model series. Once

again, we see that the model and the data behave similarly, with correlations of 39 and 35 percent, respectively. In addition, the bottom panels show that, as for stocks, the model-based volatility of bond return volatility and the absolute changes in volatility are positively correlated, with correlations of 26 and 39 percent in the data and the model, respectively.

Finally, to provide further evidence on the beliefs-based explanation of the time varying volatility of volatility, Table 5 reports results from the regression

$$|\Delta ATMIV(t + 1)| = \alpha + \beta X_t + \epsilon_{t+1} \quad (28)$$

where  $|\Delta ATMIV(t + 1)|$  is the absolute changes in ATMIV for both stocks (Panel A) and bonds (Panel B), and  $X_t$  are the beliefs-based explanatory variables already used in Table 4. Again, as in this latter table, the first five columns are results from the fitted model for which beliefs variation is the *only* driver of implied volatility and absolute changes in implied volatility, while the second five columns are the results in the data, where beliefs use the Survey of Professional Forecasters probabilities.

Panel A of Table 5 shows that both in the model and in the data, the recession probability and economic uncertainty are both important drivers of the absolute changes in volatility. In all cases, the slope coefficient is strongly significant, and the  $R^2$  are high, in the 30 - 35% range for the model, and 12-16% range for the data. Interestingly, expected inflation and inflation uncertainty are not significant drivers of the absolute changes in volatility in either the model or the data. Finally, the probability of deflation, ProbDef, is significant for both model and data.

Panel B shows similar results for the absolute changes in ATMIV for Treasury bonds. Consistently with the Bayesian learning story, we find that absolute changes in ATMIV is positively related to the recession probability and economic uncertainty, both in the model and in the data, with strong t-statistics and high  $R^2$ . In the model, higher expected inflation implies a lower volatility of volatility, although such evidence is not present in the data. Like before, inflation uncertainty is not significant for either the model or the data. However, we find that the probability of a deflationary state is strongly significant explanatory variable of the absolute changes in volatility, while it is not as strong in the data. While the sign of the regression is correct, the coefficient is insignificant.

In sum, the evidence from this section provides strong support for the learning-based mechanism proposed in this paper as to the dynamics of option prices.

## 6.2. The Implied Volatility Premium

In this last section we investigate the model's implication for the dynamics of the implied volatility premium, for both stocks and bonds. The implied volatility premium is an ex-ante measure of the market volatility forecast of investors' priced into options relative to a volatility forecast under the objective ( $P$ ) measure see, and it has attracted much attention in the literature lately (see e.g. Bollerslev, Tauchen, and Zhou (2009), Drechsler (2008), Drechsler and Yaron (2010), and Mueller, Vedolin, and Yen (2013)). Our fitted model also provides predictions about the relation between the implied volatility premium (IVP) and beliefs, which we can test in the data.

More formally, we define the implied volatility premium as

$$\text{IVP}(t) = \text{ATMIV}(t, t + \tau) - E_t[\text{VOL}(t, t + \tau)]$$

where now we emphasize the maturity of the options underlying the ATMIV, and  $\text{VOL}(t, t + \tau)$  is the realized return volatility during the life of the option. As in Drechsler and Yaron (2010) we use  $\text{ATMIV}(t, t + \tau)$  as an information variable to compute the expectation, controlling for lagged realized volatility, that is, we use the projection:

$$E_t[\text{VOL}(t, t + \tau)] = \beta_0 + \beta_1 \text{ATMIV}(t, t + \tau) + \beta_2 \text{VOL}(t - \tau, t)$$

Given our use of three month options, we thus regress realized volatility during the quarter  $(t, t + \tau)$  on the implied volatility  $\text{ATMIV}(t, t + \tau)$  and lagged realized volatility. While for bonds, lagged volatility is significant, for stocks lagged volatility is not significant (t-stat close to zero) and so we omit it in the forecasting regression. We thus obtain the forecasting regressions:

$$\begin{aligned} \text{VOL}_S(t, t + \tau) &= -1.66 & + & 0.92 \text{ATMIV}_S(t, t + \tau) & \bar{R}^2 = 66.0\% \\ &[-9.70] & & [6.73] \end{aligned}$$

$$\begin{aligned} \text{VOL}_B(t, t + \tau) &= 1.46 & + & 0.35 \text{ATMIV}_B(t, t + \tau) & + & 0.43 \text{Vol}_B(t - \tau, t); & \bar{R}^2 = 38.50\% \\ &[2.11] & & [2.51] & & [4.13] \end{aligned}$$

The result for stocks are consistent with Drechsler and Yaron (2010), who also find high  $R^2$  using similar regressions. The  $R^2$  for bond return volatility is not as high, but we verified that adding lags of predicting variables does not improve the forecasting power.

We similarly construct a model based IVP series by taking the difference between the model implied volatility and the model forecast of volatility under the P-measure using simulation methods as described in equation (52) in Appendix 2.

To check whether the model captures the variation of the IVP, we first regress the IVP from the data on the IVP from the model, through a contemporaneous regression.

$$\begin{aligned} \text{Stocks : } IVP_S^{Data}(t) &= 3.45 + 0.29 IVP_S^{Model}(t); & \bar{R}^2 = 13.77\% \\ & [20.91] \quad [2.14] \\ \text{Bonds : } IVP_B^{Data}(t) &= 0.11 + 0.23 IVP_B^{Model}(t); & \bar{R}^2 = 4.58\% \\ & [1.05] \quad [1.88] \end{aligned}$$

The regression slope for stock IVP is significant, with a reasonable  $R^2$ . Given that the model IVP is only driven by beliefs (and hence fundamental shocks), it is reassuring that the model is capturing some of the time variation in the true IVP. However, the regression coefficients suggest that the model's stock IVP is too small compared to the data. Indeed, the average model IVP for stocks is negative 1.04%, against a positive 3.16% in the data.<sup>21</sup> The slope coefficient for the bond IVP is only marginally significant, and the  $R^2$  is small. In terms of magnitudes, though, the model bond IVP averages to 0.17% which is very close to its counterpart in the data of 0.15%.

Recalling that our estimation methodology does not target the level of the IVP as one of the moments, it may not be too surprising that we fail to exactly match the level of stock IVP. However, our model suggests some specific predictions on the relation between IVP and beliefs, which we can test in the data. Table 6 reports the results of the regression:

$$IVP(t+1) = \alpha + \beta X_t + \epsilon_t \tag{29}$$

for both stocks (Panel A) and bonds (Panel B), where the explanatory variables  $X_t$  are the beliefs-based variables already used in Tables 4 and 5. Again, as in previous tables, the first five columns are the results from the fitted model for which beliefs variation is the *only* driver of IVP, while the second five columns are the results in the data, where we proxy for beliefs by using the Survey of Professional Forecasters probabilities.

Turning to Panel A, the model shows that stock's IVP should be positively related to the probability of a recession *ProbRecess*, inflation uncertainty *UncInf*, and the probability of deflation *ProbDef*. The data show that both probability of a recession and the probability of deflation are significant predictors of stock IVP, with  $R^2$  of *ProbRec* of 32%. Instead, inflation uncertainty is not significant in the data, but economic uncertainty *EconUnc* is

---

<sup>21</sup>The variance risk premium, defined as the difference between expected variance under Q and P, is positive in our model. It appears that ATMIV does not capture well the tails of the risk neutral distribution due to the large risk neutral jump sizes.

significant (t-stat = 3.02 and  $R^2 = 23\%$ ). Looking at bond's IVP in Panel B, we see that the model again predicts that bond IVP should be positively related to the probability of a recession, economic and inflation uncertainty, and the deflation probability. Of these predictions, the data only support inflation uncertainty as a driver of bond's IVP. In addition, in the data, expected inflation is also significant (higher inflation leads to a higher IVP), as is the probability of deflation, the latter though with the opposite sign compared to the model's prediction.

Overall, while we find mixed evidence on the model's predictions of the dynamics of bond IVP, we do find quite strong evidence on the model's predictions of the dynamics of stock IVP and, especially, its relation to beliefs.

## 7. Conclusions

Option prices provide key forward looking information on investors' expectations, and market attention is often focused on two uncertainty measures from options, the at-the-money implied volatility (ATMIV) and the ratio of implied volatilities of out-of-the-money puts and calls (P/C). The former is measure of market turbulence, while the latter is a measure of downside risk. We show that both measures are empirically relevant for monetary policy, but in opposite direction: a positive shock to equity ATMIV leads to a decline in future rates, while the opposite is true for a positive shock to P/C.

We provide a model in which stock, bond, and option prices, are functions of investors' beliefs of the composite regimes of macroeconomic and policy fundamentals through a forward-looking Taylor rule. The model is able to shed light on the counter-cyclicality of the stock and bond ATMIV and pro-cyclicality of the P/C index. In addition, the model shows why shocks to stock ATMIV tend to be followed by an expansionary monetary policy with lower future rates, while positive shocks to the downside risk (P/C) index tend to be followed by an increase in interest rates.

The fitted model produces numerous predictions about the relation between option prices and beliefs, that we test using the Survey of Professional Forecasters. Differently from previous literature, we do not consider dispersion of beliefs, but rather actual forecasters average beliefs about future economic growth and inflation. The model's predictions that stocks and bonds ATMIV indices should be positively related to beliefs about recession, economic uncertainty, and the probability of a deflationary regime, are strongly supported

in the data. In addition, our model predicts that the down-side risk P/C measure should be negatively related to such variables, and we find so in the data.

Moreover, our model produces also predictions for quantities that we did not use in the estimation of the model, such as the variation in volatility of implied volatility and the implied volatility premium. In particular, we show our model is able to explain the positive correlation between ATMIV and absolute changes in ATMIV (a feature that is not consistent with standard option pricing models), both for stocks and bonds, as well as the time variation in the implied volatility premium. More importantly, our model also provides additional predictions about the relation between such quantities and beliefs. Using again beliefs extracted from the Survey of Professional Forecasters we show that such predictions are supported in the data. This evidence provides additional support for the learning mechanism proposed in the paper.

Our reduced form model for equilibrium fundamental processes suggests some further support for the Taylor type rules, but also some additional factors to be worked on in future macro research such as the direct impact of uncertainty on interest rates and the role of money in monetary policy, which has been conspicuously absent in recent modeling. Indeed, the further understanding of the potential feedback relation between asset prices and the behavior of the central bank seem also a fruitful and important area of future research.

## Appendix 1

For proving Proposition 1 we will need the following lemma.

**Lemma 2** *Given the process of earnings in (3) and the SPD in (8), over a small interval of time  $\Delta$  we have*

$$\mathbb{E} \left[ \frac{M_{t+\Delta} E_{t+\Delta}}{M_t X_t} \middle| \nu_t = \nu_i \right] = e^{[\theta_i - \phi_i - \sigma_M \sigma'_E + \kappa(\xi_3 - \xi_1 - \xi_2)]\Delta} + o(\Delta),$$

where  $\xi_3 = e^{\mu_1 + \mu_2 + 0.5(\sigma_1 + \sigma_2)^2} - 1$ .

**Proof.** From (3) and (8) we have

$$\begin{aligned} \frac{E_s}{E_t} &= \exp \left( \int_t^s \theta_s - \kappa \xi_1 - 0.5 \sigma_E \sigma'_E du + \sigma_E (W_s - W_t) + \sum_{j=L_{t+1}}^{L_s} Y_{1j} \right) \\ \frac{M_s}{M_t} &= \exp \left( \int_t^s -\phi_s - \kappa \xi_2 - 0.5 \sigma_M \sigma'_M du - \sigma_M (W_s - W_t) + \sum_{j=L_{t+1}}^{L_s} Y_{2j} \right). \end{aligned}$$

Multiplying the two equations we have

$$\begin{aligned} \frac{E_s M_s}{E_t M_t} &= \exp \left( \int_t^s [\theta_s - \phi_s - \sigma_E \sigma'_M - \kappa(\xi_1 + \xi_2) - 0.5(\sigma_E \sigma'_E + \sigma_M \sigma'_M)] du + (\sigma_E - \sigma_M)(W_s - W_t) \right) \\ &\times \exp \left( \sum_{j=L_{t+1}}^{L_s} Y_{1j} + Y_{2j} \right). \end{aligned}$$

Now for a small interval of time  $\Delta$  and the fact that jumps in the drift processes and  $L_t$  are independent of each other and each occurs with probability of order  $O(\Delta)$ , we have

$$\begin{aligned} \mathbb{E} \left[ \frac{E_{t+\Delta} M_{t+\Delta}}{E_t M_t} \middle| \nu_t = \nu_i \right] &= e^{[\theta_i - \phi_i - \sigma_E \sigma'_M - \kappa(\xi_1 + \xi_2)]\Delta} \cdot \mathbb{E} \left[ e^{\sum_{j=L_{t+1}}^{L_{t+\Delta}} Y_{1j} + Y_{2j}} \right] \\ &= [1 + (\theta_i - \phi_i - \sigma_E \sigma'_M - \kappa(\xi_1 + \xi_2))\Delta][1 - \kappa\Delta + \kappa\Delta(1 + \xi_3)] + o(\Delta) \\ &= 1 + [\theta_i - \phi_i - \sigma_E \sigma'_M + \kappa(\xi_3 - (\xi_1 + \xi_2))]\Delta + o(\Delta) \\ &= e^{[\theta_i - \phi_i - \sigma_M \sigma'_E + \kappa(\xi_3 - \xi_1 - \xi_2)]\Delta} + o(\Delta), \end{aligned}$$

as claimed. Note in the first equality above we have used the independence property of the drift process and the jump process, while in the second we have used the definition of  $e^x = 1 + x + x^2/2! + \dots$ .

■

**Proof of Proposition 1:** The price-dividend ratio at time  $t$  satisfies

$$\begin{aligned} \frac{P_t}{D_t} &= \mathbb{E} \left[ \int_t^\infty \frac{M_s D_s}{M_t D_t} ds \middle| \mathcal{F}_t \right] = \mathbb{E} \left[ \int_t^\infty \frac{M_s E_s}{M_t E_t} ds \middle| \mathcal{F}_t \right] \\ &= \sum_{i=1}^N \pi_{it} \mathbb{E} \left[ \int_t^\infty \frac{M_s E_s}{M_t E_t} ds \middle| \nu_t = \nu_i \right] \equiv \sum_{i=1}^N \pi_{it} V_{it}. \end{aligned}$$

Let  $\hat{\theta}_i = \theta_i - \phi_i - \sigma_M \sigma'_E + \kappa(\xi_3 - \xi_1 - \xi_2)$ . Using Lemma 2 to evaluate the expectations over a time interval  $\Delta$ , we have

$$\begin{aligned} V_{i,t} &= \mathbb{E} \left[ \int_t^{t+\Delta} \frac{M_s E_s}{M_t E_t} ds \middle| \nu_t = \nu_i \right] + \mathbb{E} \left[ \frac{M_{t+\Delta} E_{t+\Delta}}{M_t E_t} \int_{t+\Delta}^{\infty} \frac{M_s E_s}{M_{t+\Delta} E_{t+\Delta}} ds \middle| \nu_t = \nu_i \right] \\ &= \int_t^{t+\Delta} e^{\hat{\theta}_i ds} + e^{\hat{\theta}_i \Delta} \mathbb{E} \left[ \int_{t+\Delta}^{\infty} \frac{M_s E_s}{M_{t+\Delta} E_{t+\Delta}} ds \middle| \nu_t = \nu_i \right] \\ &= \frac{e^{\hat{\theta}_i \Delta} - 1}{\hat{\theta}_i} + e^{\hat{\theta}_i \Delta} \left[ (1 + \lambda_{ii} \Delta) V_{i,t+\Delta} + \sum_{j \neq i} \lambda_{ij} \Delta V_{j,t+\Delta} \right]. \end{aligned}$$

Since  $V_{i,t}$  is time homogeneous, we have  $V_{i,t} = V_{i,t+\Delta} = V_i$ . Now collecting terms and taking the limit as  $\Delta \rightarrow 0$ , we get

$$\begin{aligned} V_i \frac{1 - e^{\hat{\theta}_i \Delta}}{\Delta} &= \frac{e^{\hat{\theta}_i \Delta} - 1}{\hat{\theta}_i \Delta} + e^{\hat{\theta}_i \Delta} \left[ \lambda_{ii} V_i + \sum_{j \neq i} \lambda_{ij} V_j \right] \\ -\hat{\theta}_i V_i &= 1 + \left[ \lambda_{ii} V_i + \sum_{j \neq i} \lambda_{ij} V_j \right]. \end{aligned}$$

In vector form we can write this equality as

$$\left( \text{Diag}(-\hat{\theta}) - \Lambda \right) V = \mathbf{1}_N,$$

whose solution is  $V = A^{-1} \cdot \mathbf{1}_N$ . Finally, defining by  $c = D/E$  a constant dividend payout ratio, the P/E ratio is mechanically given by  $P/E = c P/D \sum_{i=1}^n V_i \pi_{it}$ . Thus,  $C = c V = c A^{-1} \cdot \mathbf{1}_N$  as in the statement of the proposition. ■

For proving Proposition 3 we will use the algebraic result stated in the following lemma.

**Lemma 3**

$$\frac{\partial \bar{\theta}^\circ}{\partial \pi_i} = \frac{C_i (\theta_i - \bar{\theta}^\circ)}{\left( \sum_j \pi_j C_j \right)}.$$

**Proof of Lemma 3:**

$$\begin{aligned} \frac{\partial \bar{\theta}^\circ}{\partial \pi_i} &= \frac{\partial \left( \frac{\sum_j \pi_j C_j \theta_j}{\sum_j \pi_j C_j} \right)}{\partial \pi_i} = \frac{C_i \theta_i \left( \sum_j \pi_j C_j \right) - C_i \left( \sum_j \pi_j C_j \theta_j \right)}{\left( \sum_j \pi_j C_j \right)^2} \\ &= \frac{C_i \theta_i}{\left( \sum_j \pi_j C_j \right)} - \frac{C_i \left( \sum_j \pi_j C_j \theta_j \right)}{\left( \sum_j \pi_j C_j \right)^2} = \frac{C_i \theta_i}{\left( \sum_j \pi_j C_j \right)} - \frac{C_i \bar{\theta}^\circ}{\left( \sum_j \pi_j C_j \right)} \\ &= \frac{C_i (\theta_i - \bar{\theta}^\circ)}{\left( \sum_j \pi_j C_j \right)}, \end{aligned}$$



which completes the proof. ■

**Proof of Proposition 3:** Let the second term in the variance equation be  $V_2 = (\bar{\nu}^\circ - \bar{\nu})'(\Sigma\Sigma')^{-1}(\bar{\nu}^\circ - \bar{\nu})$ . Then, using Lemma 3 on each element of the drift vector  $\nu$  we have

$$\frac{\partial V_2}{\partial \pi_i} = 2 \left[ \frac{C_i(\nu_i - \bar{\nu}^\circ)}{\sum_j \pi_j C_j} - \nu_i \right]' (\Sigma\Sigma')^{-1}(\bar{\nu}^\circ - \bar{\nu}).$$

Then, using the volatilities of the beliefs process in equation (12), we have  $dV_2 = \mu_{V,2}dt + \sigma_{V,2}$ , where

$$\begin{aligned} \sigma_{V,2} &= \sum_i \frac{\partial V_2}{\partial \pi_i} \sigma_i \\ &= 2 \sum_i \pi_i \left[ \frac{C_i(\nu_i - \bar{\nu}^\circ)}{\sum_j \pi_j C_j} - \nu_i \right]' (\Sigma\Sigma')^{-1}(\bar{\nu}^\circ - \bar{\nu})(\nu_i - \bar{\nu})' \Sigma'^{-1} \\ &= 2 \sum_i [\pi_i^\circ (\nu_i - \bar{\nu}^\circ) - \pi_i \nu_i]' (\Sigma\Sigma')^{-1}(\bar{\nu}^\circ - \bar{\nu})(\nu_i - \bar{\nu})' \Sigma'^{-1}. \end{aligned}$$

Similarly, let the third term in the variance equation be  $V_3 = 2[(\bar{\theta}^\circ - \bar{\theta}) + (\bar{\beta}^\circ - \bar{\beta})]$ . Then we have  $dV_3 = \mu_{V,3}dt + \sigma_{V,3}$ , where

$$\begin{aligned} \sigma_{V,3} &= \sum_i \frac{\partial V_3}{\partial \pi_i} \sigma_i = 2 \sum_i \frac{\partial[(\bar{\theta}^\circ - \bar{\theta}) + (\bar{\beta}^\circ - \bar{\beta})]}{\partial \pi_i} \sigma_i \\ &= 2 \sum_i \pi_i \left( \left[ \frac{C_i(\theta_i - \bar{\theta}^\circ)}{\sum_j \pi_j C_j} - \theta_i \right] + \left[ \frac{C_i(\beta_i - \bar{\beta}^\circ)}{\sum_j \pi_j C_j} - \beta_i \right] \right) (\nu_i - \bar{\nu})' \Sigma'^{-1} \\ &= 2 [(\sigma_{\theta_\nu}^\circ - \sigma_{\theta_\nu})' + (\sigma_{\beta_\nu}^\circ - \sigma_{\beta_\nu})'] \Sigma'^{-1}, \end{aligned}$$

where the second equality follows from Lemma 3, the third the definition of  $\pi_i^\circ$ , and the fourth from the fact that

$$\sum_j \pi_j^\circ (\theta_j - \bar{\theta}^\circ)(\beta_j - \bar{\beta}) = \sum_j \pi_j^\circ \theta_j \beta_j - \bar{\theta}^\circ \bar{\beta} = \sigma_{\theta\beta}^\circ,$$

and analogous terms for the other elements of  $\nu$ . Now summing  $\sigma_{V,2}$  and  $\sigma_{V,3}$  provides the statement of (b). ■

**Proof of Proposition 4** Since  $\|\sigma^n(\pi)\|$  in (18) is a continuous function of  $\pi$  on the  $N$  dimensional simplex, which is a compact set, it has a maximum and minimum, which we denote by  $\|\bar{\sigma}^n\|$  and  $\|\underline{\sigma}^n\|$ . Therefore,  $\|S_1\sigma^n(\pi_1) - S_2\sigma^n(\pi_2)\| \leq (\|\bar{\sigma}^n\| - \|\underline{\sigma}^n\|) \cdot |S_1 - S_2|$  so that the Lipschitz condition is satisfied for the stock price. Similarly,  $\|S\sigma^n(\pi)\|^2 \leq \|\bar{\sigma}^n\|^2 S^2 < (1 + \|\bar{\sigma}^n\|^2 S^2)$ , so that the growth condition holds as well. Similarly the norm of the volatility of beliefs in (12) is bounded by  $\|\bar{\sigma}_i\|$  and  $\|\underline{\sigma}_i\|$  and both conditions hold for the beliefs processes, which completes the proof. ■

**Proof of Proposition 5:** The change of measure with respect to the Brownian motions in the context of the filtering setup has been derived in David (2008). For brevity, we only provide the proof of the change of measure for the jump component.

$$\begin{aligned}
\kappa \mathbb{E}\left[\frac{M^+ - M}{M} \frac{S^+ - S}{S} \middle| \mathcal{F}_t\right] &= \kappa \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{Y_1} e^{Y_2} f(Y_1, Y_2) dY_2 dY_1 \\
&= \kappa \int_{-\infty}^{\infty} e^{Y_1} f(Y_1) \int_{-\infty}^{\infty} e^{Y_2} f(Y_2|Y_1) dY_2 dY_1 \\
&= \kappa \int_{-\infty}^{\infty} e^{Y_1} e^{\mu_2 + \frac{\sigma_2}{\sigma_1}(Y_1 - \mu_1)} f(Y_1) dY_1 \\
&= \kappa e^{\mu_2 - \frac{\sigma_2}{\sigma_1}\mu_1} e^{\mu_1(1 + \frac{\sigma_2}{\sigma_1}) + 0.5(1 + \frac{\sigma_2}{\sigma_1})^2 \sigma_1^2} \\
&= \kappa e^{\mu_2 + .5\sigma_2^2} e^{\mu_1 + \sigma_1\sigma_2 + 0.5\sigma_1^2} \\
&= \kappa^* \mathbb{E}^*[e^{Y_1}].
\end{aligned}$$

In the above, the second equality arises from the definition of a conditional expectation, the third because the two jump processes are perfectly correlated, and the fourth from the moment generating function of a normal distribution. ■

## Appendix 2

### 1. SMM Estimation of the Regime Switching Jump-Diffusion Model

We start by providing here the details of the SMM estimation procedure, which is used to estimate the model. The procedure uses the SML methodology of Brandt and Santa-Clara (2002), which has already been extended to learning framework in the pure diffusion setting in David (2008). We provide here the extension to the case of observable jumps in the fundamental processes. Piazzesi (2005) has extended the procedure to a setting with jump-diffusions.

Using the definition of the inferred shocks (13) we can write the variables observed by the econometrician in (10) as perceived by the investors as  $\frac{dY_t}{Y_t} = \varrho(\pi_t)dt + \Sigma_4 dW_t + J_{4t}dL_t$ . Similarly the pricing kernel in (8) under investors' filtration can be written as  $dM_t/M_t = (-\bar{\phi}(\pi_t) - \kappa\xi_2)dt - \sigma_M d\tilde{W}_t - (e^{Y_{2t}} - 1)dL_t$ , where the real rate in the economy,  $\bar{\phi}(\pi_t)$ , is the expected value of  $\phi_t$  in (6) conditional on investors' filtration. Since fundamentals are stationary in growth rates, we start by defining logs of variables:  $y_t = \log(Y_t)$ , and  $m_t = \log(M_t)$ . Using these characterizations we can write

$$dy_t = (\bar{\varrho}(\pi_t) - \frac{1}{2} \text{diag}(\Sigma_4 \Sigma_4'))dt + \Sigma_4 d\tilde{W}_t + J_{4t}dL_t, \quad (30)$$

$$dm_t = (-\bar{\phi}(\pi_t) - \kappa\xi_2 - \frac{1}{2} \sigma_M \sigma_M')dt - \sigma_M d\tilde{W}_t + Y_{2t}dL_t, \quad (31)$$

where  $\text{diag}(x)$  is a column vector composed of the diagonal elements of a square matrix  $x$ . It is immediate that investors' beliefs  $\pi_t$  completely capture the state of the system  $(y_t, m_t)$  for forecasting future growth rates. The specification of the system is completed with the belief dynamics in (11).

The econometrician has data series  $\{y_{t_1}, y_{t_2}, \dots, y_{t_K}\}$ . Let  $\Psi$  be the set of parameters of the model. Let

$$\mathcal{L}(\Psi) \equiv p(y_{t_1}, \dots, y_{t_K}; \Psi) = p(\pi_{t_0}; \Psi) \prod_{k=1}^K p(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi),$$

where  $p(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi)$  is the marginal density of fundamentals at time  $t_{k+1}$  conditional on investors' beliefs at time  $t_k$ . Since  $\{\pi_{t_k}\}$  for  $k = 1, \dots, K$  is not observed by the econometrician, we maximize

$$E[\mathcal{L}(\Psi)] = \int \cdots \int \mathcal{L}(\Psi) f(\pi_{t_1}, \pi_{t_2}, \dots, \pi_{t_K}) d\pi_{t_1}, d\pi_{t_2}, \dots, d\pi_{t_K}, \quad (32)$$

where the expectation is over all sample paths for the fundamentals,  $\tilde{y}_t$ , such that  $\tilde{y}_{t_k} = y_{t_k}$ ,  $k = 1, \dots, K$ . In general, along each path, the sequence of beliefs  $\{\pi_{t_k}\}$  will be different.

As a first step, we need to calculate  $p(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi)$ . Following Brandt and Santa-Clara (2002), we simulate paths of the state variables over smaller discrete units of time using the Euler discretization scheme (see also Kloeden and Platen (1992)):

$$\tilde{y}_{t+h} - \tilde{y}_t = (\bar{\varrho}(\pi_t) - \frac{1}{2}(\sigma_Q \sigma'_Q, \sigma_E \sigma'_E)') h + \Sigma_2 \sqrt{h} \tilde{\epsilon}_{2t} + \mathbf{1}_{\tilde{u}_t < \kappa h} \tilde{\epsilon}_{2t}, \quad (33)$$

$$m_{t+h} - m_t = (-\bar{\phi}(\pi_t) - \kappa \zeta_2 - \frac{1}{2} \sigma_M \sigma'_M) h - \sigma_M \sqrt{h} \tilde{\epsilon}_{1t} + \mathbf{1}_{\tilde{u}_t < \kappa h} \tilde{\epsilon}_{2t}, \quad (34)$$

$$\pi_{t+h} - \pi_t = \mu(\pi_t) h + \sigma(\pi_t) \sqrt{h} \tilde{\epsilon}_{1t}, \quad (35)$$

where  $\tilde{\epsilon}_{1t}$  and  $\tilde{\epsilon}_{2t}$  are 5- and 1- dimensional standard normal variables, respectively,  $\tilde{u}_t$  is uniformly distributed, and  $h = 1/M$  is the discretization interval. The Euler scheme implies that the marginal conditional density of the  $4 \times 1$  fundamental growth vector  $y_t$  over  $h$  is 4-dimensional normal.

We approximate  $p(\cdot | \cdot)$  with the density  $p_M(\cdot | \cdot)$ , which obtains when the state variables are discretized over  $M$  subintervals. Since the drift and volatility coefficients of the state variables in (11), and (30) to (31) are infinitely differentiable, and  $\Sigma \Sigma'$  is positive definite, Lemma 1 in Brandt and Santa-Clara (2002) implies that  $p_M(\cdot | \cdot) \rightarrow p(\cdot | \cdot)$  as  $M \rightarrow \infty$ . First consider the case where earnings are observed quarterly. The Chapman-Kolmogorov equation implies that the density over the interval  $(t_k, t_{k+1})$  with  $M$  subintervals satisfies

$$p_M(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi) = \int \int \phi(y_{t_{k+1}} - y; \Psi) \times p_M(y - y_{t_k}, \pi, m, t_k + (M-1)h | \pi_{t_k}, t_k) d\pi dy, \quad (36)$$

where  $\phi(y; \psi)$ , denotes the mixture-of-normals density given as:

$$\phi(y; \psi) = N(\varrho(\pi_t)h, \Sigma_4 \Sigma'_4 h) \quad \text{with probability } \kappa h, \quad (37)$$

$$= N(\varrho(\pi_t)h + (0, 1, 0, 0)' \mu_1, \Sigma_4 \Sigma'_4 h + i_2 \sigma_1^2) \quad \text{with probability } 1 - \kappa h, \quad (38)$$

where  $i_2$  is the  $4 \times 4$  square matrix with zero in all elements except the (2, 2) element, which is 1. Now  $p_M(\cdot | \cdot)$  can be approximated by simulating  $L$  paths of the state variables in the interval  $(t_k, t_k + (M-1)h)$  and computing the average

$$\hat{p}_M(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi) = \frac{1}{L} \sum_{l=1}^L \phi(y_{t_{k+1}} - y^{(l)}; \Psi). \quad (39)$$

The Strong Law of Large Numbers (SLLN) implies that  $\hat{p}_M \rightarrow p_M$  as  $L \rightarrow \infty$ .

To compute the expectation in (32), we simulate  $S$  paths of the system (33) to (35) “through” the full time series of fundamentals. Each path is started with an initial belief,  $\pi_{t_0} = \pi^*$ , the stationary beliefs implied by the generator matrix  $\Lambda$ . In each time interval  $(t_k, t_{k+1})$  we simulate  $(M-1)$  successive values of  $\tilde{y}_t^{(s)}$  using the discrete scheme in (33), and set  $\tilde{y}_{t_k}^{(s)} = y_{t_k}$ . The results

in the paper use  $M = 90$  for quarterly data, so that shocks are approximated at roughly a daily frequency. The pricing kernel and beliefs along the entire path of the  $s^{th}$  simulation are obtained by iterating on (34) and (35). We approximate the expected likelihood as

$$\hat{\mathcal{L}}^{(S)}(\Psi) = \frac{1}{S} \sum_{s=1}^S \prod_{k=0}^{K-1} \hat{p}_M(y_{t_{k+1}}^{(s)} - y_{t_k}^{(s)}, t_{k+1} | \pi_{t_k}^{(s)}, t_k; \Psi), \quad (40)$$

where  $\hat{p}_M(\cdot|\cdot)$  is the density approximated in (39). The SLLN implies that  $\hat{\mathcal{L}}^{(S)}(\Psi) \rightarrow E[\mathcal{L}(\Psi)]$  as  $S \rightarrow \infty$ . We often report  $\bar{\pi}_{t_k} = 1/S \sum_{s=1}^S \pi_{t_k}^{(s)}$ , which is the econometrician's expectation of investors' belief at  $t_k$ .

So far above we assumed that quarterly earnings growth is observed. In fact, S&P provides the four quarter moving average of earnings, and hence the observed vector contains the growth rate of four-quarter moving average of earnings. Our simulated system of observables in (33) instead computes the quarterly growth rate of all fundamentals when aggregated over all the subintervals. Let  $Y_t^c$  denote the vector of all observed variables other than earnings. To deal with the aggregation of earnings, we instead compute

$$\phi \left( \hat{y}_{t_{k+1}}^c - y^c, \hat{e}_{t_{k+1}} - 1/4 (e + e_{t_k}^{(s)} + e_{t_{k-1}}^{(s)} + e_{t_{k-2}}^{(s)}) \right), \quad (41)$$

where  $e_{t_k}^{(s)}$  is the model's simulated quarterly earnings growth rate in the interval ending at time  $t_k$  along the  $s$ -th sample path in the previous paragraph, and  $y^c$  denotes the simulated growth rate for the period ending at  $t_{k+1}$  after  $(M - 1)$  subintervals.<sup>22</sup>

To extract investors' beliefs from data on price levels and volatilities in addition to fundamentals we add overidentifying moments to the SML method above. From Proposition 1, we can compute the time series of model-implied price-earning ratios and bond yields at the discrete data points  $t_k$ ,  $k = 1, \dots, K$  as

$$\widehat{P/E}_{t_k} = C \cdot \bar{\pi}_{t_k}, \quad \hat{i}_{t_k}(\tau) = -\frac{1}{\tau} \log(B(\tau) \cdot \bar{\pi}_{t_k}).$$

We note that the constants  $C$ s and the functions  $B(\tau)$  both depend on the parameters of the fundamental processes,  $\Psi$ . Hence, we let the pricing errors be denoted

$$e_{t_k}^P = \left( \widehat{P/E}_{t_k} - P/E_{t_k}, \hat{i}_{t_k}(0.25) - i_{t_k}(0.25), (\hat{i}_{t_k}(5) - \hat{i}_{t_k}(1)) - (i_{t_k}(5) - i_{t_k}(1)) \right).$$

We similarly formulate the errors from options prices as

$$e_{t_k}^O = \left( \widehat{V}_{t_k} - V_{t_k}, \widehat{(P/C)}_{t_k} - (P/C)_{t_k} \right),$$

where  $V$  is the ATMIV, and  $P/C$  is the put-call ratio as discussed. The model-implied options prices are calculated using Monte-Carlo simulations as described below.

---

<sup>22</sup>We log linearize the model's growth rate of the moving average. In particular the first order approximation of the growth rate is

$$\log \left[ \frac{\exp(w + z + y + x) + \exp(y + z + w) + \exp(z + w) + \exp(w)}{\exp(y + z + w) + \exp(z + w) + \exp(w) + 1} \right] \simeq \frac{1}{4}(w + z + y + x).$$

For the subset of the earnings data, where we have the quarterly growth rates available, the approximation leads to growth rate very close to the growth rate of the moving average.

To estimate  $\Psi$  from data on fundamentals as well as financial variables, we form the overidentified SMM objective function

$$c = \left( \frac{1}{T} \sum_{t=1}^T \epsilon_t \right)' \cdot \Omega^{-1} \cdot \left( \frac{1}{T} \sum_{t=1}^T \epsilon_t \right). \quad (42)$$

The moments used are the scores of the log likelihood function from fundamentals, and the pricing errors from stock, Treasury bond, and options prices. The number of scores in  $\frac{\partial \log(\hat{\Sigma})}{\partial \Phi}(t_k)$  equals the number of parameters driving the fundamental processes in  $\Psi$ . There are seven pricing errors (stocks' P/E, short rate, term structure slope, stocks' ATMIV and P/C, bond's ATMIV, and the unconditional Sharpe ratio on stocks, which we set equal to 0.3). The pricing kernel has seven parameters (two prices or risk, three parameters for the interest rate rule, and two for the jumps in the kernel), which we collectively call  $\Phi$ . Finally, we have 14 equality constraints in the generator matrix, which we discuss below. Overall, the statistic  $c$  in (42) has a chi-squared distribution with 14 degrees of freedom. We correct the variance covariance matrix for autocorrelation and heteroskedasticity using the Newey-West method [see, for example, Hamilton (1994) equation 14.1.19] using a lag length of  $q = 8$ .

The asymptotic distribution of the constrained GMM estimator satisfies

$$\sqrt{T}(\hat{\theta} - \theta_0) \sim N[\mathbf{0}, B^{-1/2} M B^{-1/2}],$$

where  $M = I - B^{-1/2} A' (A B^{-1} A')^{-1} A B^{-1/2}$ ,  $A = \nabla_{\theta} a(\theta_0)$ ,  $B = G' \Omega^{-1} G$ , and  $G = E[\nabla_{\theta} g(z_t, \theta_0)]$ .  $a(\theta_0)$  is the  $2 \times k$  vector of constraints on the parameters, and  $g(z_t, \theta_0)$  is the vector of moment conditions using data point  $z_t$ . The estimate of  $G$  as

$$G_T = \begin{bmatrix} \frac{1}{T} \sum_{t=1}^T \frac{\partial e'}{\partial \Phi} & \frac{1}{T} \sum_{t=1}^T \frac{\partial e'}{\partial \Psi} \\ \mathbf{0} & \frac{1}{T} \sum_{t=1}^T \left[ \frac{\partial \Sigma_1}{\partial \Psi}(t)' \frac{\partial \Sigma_1}{\partial \Psi}(t) \right] \end{bmatrix} \quad (43)$$

We end the description of our estimation methodology with two important details. First, for determining the number of specified regimes we do not use likelihood ratio tests, which are computationally extremely demanding and beyond the scope of this paper (see Garcia (1998)). Instead, we follow the simpler and more practical methodology of using the overidentified SMM objective to determine a stopping rule on the number of regimes used in a number of papers modeling regimes shifts (e.g. Gray (1996), Bansal and Zhou (2002)). Second, to reduce the number of parameters, we follow a two-step procedure. First we estimate an unrestricted generator matrix and rank all its elements into 7 bins. All elements in the smallest bin (whose values we estimated to be below 0.001) were set to zero. Elements in each of the remaining bins were constrained to be equal in our second step estimation.

## 2. Options Prices

As for the likelihood function we formulate options prices as expected discounted values of their terminal payoffs under the risk-neutral measure. Because for Treasury bonds, our options on futures on the 10-year coupon bond, we need to first obtain the value of a coupon bond, as

opposed to a zero coupon bond, and then simulate futures on coupon bonds. We use the following proposition

**Proposition A.1** *The price of a coupon bond at  $t$  with maturity  $\tau$  paying semi-annual coupons of  $0.5c$  is given by*

$$B(\pi_t, c, \tau) = \sum_{i=1}^N \pi_{it} B_i(c, \tau), \quad (44)$$

where the  $N \times 1$  vector valued function  $B(c, \tau)$  is

$$B(c, \tau) = 0.5c \sum_{i=1}^{2\tau} B(0.5i) + B(\tau). \quad (45)$$

Under the risk neutral probabilities, futures are martingales, and therefore, the futures price with delivery time  $T$  of a  $\tau$ -maturity bond is given by

$$F(\pi_t^*) = \mathbb{E}_t^* [B(\pi_T^*, c, \tau)] = \sum_{i=1}^n \mathbb{E}_t^* [\pi_{iT}^*] B_i(c, \tau)$$

where  $\pi_{it}^*$  follows the risk neutral process described in (24). Unfortunately, the evaluation of  $\mathbb{E}_t^* [\pi_{iT}^*]$  requires either additional MC simulations, or the solution to a non-linear system of equations. Given the short-maturity of the options (three months) compared to the maturity of the bonds (10 years), and the fact that we only need the volatility of futures for simulation, as futures are martingales, we approximate such futures volatility with the risk neutral volatility of the coupon bonds themselves, i.e. with

$$\sigma_B(c, \pi_t^*) = \frac{\sum_{i=1}^N B_i(c, \tau) \pi_{it}^* (\nu_i - \bar{\nu}(\pi_t^*))' (\Sigma')^{-1}}{\sum_{i=1}^N B_i(c, \tau) \pi_{it}^*}. \quad (46)$$

Expectations are approximated using Monte Carlo simulation while discretizing the dynamics of the state variables of our system along the  $sth$  sample path under the risk-neutral measure as:

$$\pi_{t+h}^{*(s)} - \pi_t^{*(s)} = \left( \mu(\pi_t^{*(s)}) - \vartheta(\pi_t^{*(s)}) \right) h + \sigma(\pi_t^{*(s)}) \sqrt{h} \tilde{\epsilon}_{1t}^{*(s)}, \quad (47)$$

$$\begin{aligned} P_{t+h}^{n*(s)} &= P_t^{n*(s)} \exp \left[ \left( r(\pi_t^{*(s)}) - 0.5 \|\sigma^n(\pi_t^{*(s)})\|^2 - \delta(\pi_t^{*(s)}) - \kappa^* \xi_1^* \right) h + \right. \\ &\quad \left. + \sigma^n(\pi_t^{*(s)}) \sqrt{h} \tilde{\epsilon}_{1t}^{(s)*} + \mathbf{1}_{\tilde{u}_t < \kappa^* h} \tilde{\epsilon}_{2t}^{(s)*} \right], \end{aligned} \quad (48)$$

$$F_{t+h}^{n*(s)} = F_t^{n*(s)} \exp \left[ -0.5 \|\sigma_B(c, \pi_t^{*(s)})\|^2 dt + \sigma_B(c, \pi_t^{*(s)}) \sqrt{h} \tilde{\epsilon}_{1t}^{(s)*} \right], \quad (49)$$

$$B_{t+h}^{*(s)} = B_t^{*(s)} \exp[ -r(\pi_t^{*(s)}) h ], \quad (50)$$

where  $\tilde{\epsilon}_{1t}^*$  and  $\tilde{\epsilon}_{2t}^*$  are 5- and 1- dimensional standard normal variables, respectively,  $\tilde{u}_t$  is uniformly distributed, and  $h = 1/M$  is the discretization interval. On each sample the process for the state variables is simulated starting with  $\pi_t^{*(s)} = \pi_t$ , the assumed beliefs of investors at time  $t$ . Then the value of a European call option at time  $t$  when investors have beliefs  $\pi_t$  that matures at  $t + T$  is given by

$$C^{M*}(t, T, \pi_t) = \frac{1}{S} \sum_{s=1}^S B_{t+T}^{*(s)} \max \left[ P_{t+T}^{n*(s)} - K, 0 \right].$$

We report option prices for  $M = 90$ . To reduce the time of computations we use three variance reduction techniques: the first two, antithetic and control variate (with Black-Scholes prices), are well known. In addition, we use the expected martingale simulation technique of Duan et. al. The volatility forecast under the Q-measure is approximated from the path of forecasted beliefs under this measure as

$$\sigma^{M^*}(t, T, \pi_t) = \sqrt{\frac{1}{S} \sum_{j=1}^{(T-t)M} \sigma^n(\pi_{t+jh}^{*(s)}) \sigma^n(\pi_{t+jh}^{*(s)})' h}. \quad (51)$$

Similarly, using the discretized beliefs processes as in (35), volatility forecasts under the objective measure are analogously constructed as

$$\sigma^n(t, T, \pi_t) = \sqrt{\frac{1}{S} \sum_{j=1}^{(T-t)M} \sigma^n(\pi_{t+jh}^{(s)}) \sigma^n(\pi_{t+jh}^{(s)})' h}. \quad (52)$$

## References

- Ang, A., and M. Piazzesi, 2003, “A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables,” *Journal of Monetary Economics*, 50, 745–787.
- Ang, A., M. Piazzesi, and M. Wei, 2006, “What Does the Yield Curve Tell Us About GDP Growth?,” *Journal of Econometrics*, 131, 359–403.
- Bakshi, G., C. Cao, and Z. Chen, 1997, “Empirical Performance of Alternative Option Pricing Models,” *Journal of Finance*, 52(5), 2003–2051.
- Bansal, R., and H. Zhou, 2002, “Term Structure of Interest Rates with Regime Shifts,” *Journal of Finance*, 57(5), 1997–2044.
- Bates, D.S., 1991, “The Crash of ’87: Was it Expected? The Evidence from Options Markets,” *Journal of Finance*, 46(3), 1009–1044.
- , 1996, “Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in PHLX Deutsche Mark Options,” *Review of Financial Studies*, 9(1), 69–107.
- , 2000, “Post-’87 Crash Fears in the S&P 500 Futures Option Market,” *Journal of Econometrics*, 94, 181–238.
- Beck, G., and V. Wieland, 2008, “Central Bank Misperceptions and the Role of Money in Interest Rate Rules,” *Journal of Monetary Economics*, 55, 35–68.
- Bekaert, G., S. Cho, and A. Moreno, 2010, “New-Keynesian Macroeconomics and the Term Structure,” *Journal of Money Credit and Banking*, 42, 33–62.
- Bekaert, G., M. Hoerova, and M.L. Duca, 2010, “Risk, Uncertainty and Monetary Policy,” NBER Working Paper # 16397.
- Benzoni, L., P. Collin-Dufresne, and R.S. Goldstein, 2005, “Can Standard Preferences Explain the Prices of Out-of-the-Money S&P 500 Put Options,” Working Paper, University of Minnesota.
- Bernanke, B.S., and K.N. Kuttner, 2005, “What Explains the Stock Market’s Reaction to Federal Reserve Policy?,” *Journal of Finance*, 60, 1221–1257.
- Bollen, N.P.B., and R.E. Whaley, 2004, “Does Net Buying Pressure Affect the Shape of Implied Volatility Functions,” *Journal of Finance*, 59, 711–753.



- Bollerslev, T., G. Tauchen, and H. Zhou, 2009, “Expected Stock Returns and Variance Risk Premia,” *Review of Financial Studies*, pp. 4462–4492.
- Brandt, MW., and P. Santa-Clara, 2002, “Simulated Likelihood Estimation of Diffusions with an Application to Exchange Rate Dynamics in Incomplete Markets,” *Journal of Financial Economics*, 63, 161–210.
- Buraschi, A., and A. Jiltsov, 2006, “Model Uncertainty and Options Markets with Heterogeneous Beliefs,” *Journal of Finance*, 61, 2841–2898.
- Chan, K., GA. Karolyi, FA. Longstaff, and AB. Sanders, 1992, “An Empirical Comparison of Alternative Models of the Short-Term Interest Rate,” *Journal of Finance*, 47, 1209–1227.
- Christoffersen, PF., K. Jacobs, C. Ornathanalai, and Y. Wang, 2008, “Option Valuation with Long-run and Short-run Volatility Components,” *Journal of Financial Economics*, 90, 272–297.
- Clarida, R., J. Gali, and M. Gertler, 2000, “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory,” *Quarterly Journal of Economics*, 140, 147–180.
- Cochrane, JH., 2001, *Asset Pricing*. Princeton University Press, Princeton, NJ.
- Coenen, G., AT. Levin, and V. Wieland, 2005, “Data Uncertainty and the Role of Money as an Information Variable in Monetary Policy,” *European Economic Review*, pp. 975–1006.
- Constantinides, G., JC. Jackwerth, and S. Perrakis, 2008, “Mispricing of S&P 500 Index Options,” *Review of Financial Studies (Forthcoming)*.
- David, A., 1997, “Fluctuating Confidence in Stock Markets: Implications for Returns and Volatility,” *Journal of Financial and Quantitative Analysis*, 32(4), 427–462.
- , 2008, “Inflation Uncertainty, Asset Valuations, and the Credit Spreads Puzzle,” *Review of Financial Studies*, 21, 2487–2534.
- David, A., and P. Veronesi, 2013, “What Ties Return Volatilities to Price Valuations and Fundamentals?,” *Journal of Political Economy*, 121(4), 682 – 746.
- Drechsler, I., 2008, “Uncertainty, Time-Varying Fear, and Asset Prices,” Job market paper, Wharton School of the University of Pennsylvania.
- Drechsler, I., and A. Yaron, 2010, “What’s Vol Got to Do With It?,” *Review of Financial Studies (Forthcoming)*.

- Du, D., 2010, “General Equilibrium Pricing of options with habit formation and event risks,” *Journal of Financial Economics (Forthcoming)*.
- Dubinsky, A., and M. Johannes, 2006, “Earnings Announcements and Option Prices,” Working Paper, Columbia University.
- Eraker, B., 2008, “The Volatility Premium,” Working Paper, University of Wisconsin Madison.
- Eraker, B., M. Johannes, and N. Polson, 2003, “The Impact of Jumps in Equity Index Volatility and Returns,” *Journal of Finance*, 58, 1269–1300.
- Gallmeyer, M., B. Hollifield, F. Palomino, and S. Zin, 2007, “Arbitrage-Free Bond Pricing with Dynamic Macroeconomic Models,” *Federal Reserve of St. Louis Review*.
- Gallmeyer, MF., B. Hollifield, and SE. Zin, 2005, “Taylor Rules, McCallum Rules and the Term Structure of Interest Rates,” *Journal of Monetary Economics*, 52, 921–950.
- Garcia, R., 1998, “Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models,” *International Economic Review*, 39(3), 763–788.
- Garleanu, N., LH. Pederson, and AM. Poteshman, 2008, “Demand-Based Option Pricing,” *Review of Financial Studies (Forthcoming)*.
- Gray, S., 1996, “Modeling the Conditional Distribution of Interest Rates as a Regime Switching Process,” *Journal of Financial Economics*, 42, 27–62.
- Guidolin, M., and ATimmermann, 2005, “Option Prices Under Bayesian Learning: Implied Volatility Dynamics and Predictive Densities,” *Journal of Economic Dynamics and Control*, 27, 717–769.
- Gurkaynak, RS., B. Sack, and JH. Wright, 2007, “The U.S. Treasury Yield Curve: 1961 to the Present,” *Journal of Monetary Economics*, 54, 2291 – 2304.
- Hamilton, JD., 1994, *Time Series Analysis*. Princeton University Press, Princeton, New Jersey.
- Hansen, LP., 2007, “Beliefs, Doubts and Learning: Valuing Macroeconomic Risk,” *American Economic Review*, 97, 1–30.
- Heston, SL., 1993, “A Closed-Form Solution for Options with Stochastic Volatility and Applications to Bond and Currency Options,” *Review of Financial Studies*, 6(2), 326–343.

- Hull, J.C., and A. White, 1987, "The Pricing of Options on Assets with Stochastic Volatility," *Journal of Finance*, 42(2), 281–300.
- Jones, C.S., 2003, "The Dynamics of Stochastic Volatility: Evidence From Underlying and Options Markets," *Journal of Econometrics*, 116, 181–224.
- Kloeden, P.E., and E. Platen, 1992, *Numeric Solutions of Stochastic Differential Equations*. Springer-Verlag, New York.
- Liu, J., J. Pan, and T. Wang, 2005, "An Equilibrium Model of Rare Event Premia and its Implications for Option Smirks," *Review of Financial Studies*, 19, 131–164.
- Lucas, R.E., 1978, "Asset Prices in an Exchange Economy," *Econometrica*, 46, 1429–1446.
- , 2007, "Central Banking: Is Science Replacing Art," in *Monetary Policy: A Journey from Theory to Practice*. European Central Bank.
- Mueller, P., A. Vedolin, and Y.-M. Yen, 2013, "Bond Variance Risk Premiums," Working paper, London School of Economics.
- Pan, J., 2002, "The Jump-Risk Premia Implicit in Options: Evidence from an Integrated Time-Series Study," *Journal of Financial Economics*, 63, 3–50.
- Piazzesi, M., 2005, "Bond Yields and the Federal Reserve," *Journal of Political Economy*, 113, 311–344.
- Polson, N., M. Johannes, and J. Stroud, 2008, "Learning about Jumps and Stochastic Volatility: Filtering Stochastic Differential Equations with Jumps," *Review of Financial Studies* (Forthcoming).
- Rigobon, R., and B. Sack, 2003, "Measuring the Reaction of Monetary Policy to the Stock Market," *Quarterly Journal of Economics*, 118, 639–669.
- Rudebusch, G.D., and T. Wu, 2008, "A Macro-Finance Model of the Term-Structure, Monetary Policy, and the Economy," *Economic Journal*, 118, 906–926.
- Shaliastovich, I., 2009, "Learning, Confidence and Option Prices," Working Paper Wharton School of the University of Pennsylvania.
- Taylor, J.B., 1993, "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.

Veronesi, P., 2000, “How Does Information Quality Affect Stock Returns?,” *Journal of Finance*, 55(2), 807–839.

Wonham, WJ., 1964, “Some Applications of Stochastic Differential Equations to Optimal Filtering,” *SIAM Journal of Control*, 2, 347–368.

Woodford, M., 2003, *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton, New Jersey.

Table 1: Example with Three Composite Regimes

Regime	$\beta^i$ (%)	$\theta^i$ (%)	$\rho^i$ (%)	$(P/E)_i$	$\phi_i$ (%)	$B_i(10)$ ( $\times 100$ )	$y_i(10)$ (%)
$i = 1$	2.33	5.65	-0.80	15.45	4.33	72.54	3.21
$i = 2$	-0.04	-5.27	-6.78	12.47	0.12	84.60	1.67
$i = 3$	2.33	5.65	-6.78	17.46	3.03	78.48	2.42

Notes: The first three columns contain the composite regimes for inflation rate  $\beta^i$ , real earning growth  $\theta^i$ , and de-meaned capacity utilization  $\rho^i$  for a simple 3-regime switching model. Conditional P/E ratios  $(P/E)_i$ , real rate  $\phi_i$ , 10-year bond price  $B_i(10)$  and yield  $y_i(10)$  are computed by using the same parameters as in Table 2, except for the infinitesimal generator, assume here to be

$$\Lambda = \begin{pmatrix} -0.1 & 0.1 & 0.0 \\ 0.1 & -0.2 & 0.1 \\ 0 & 0.1 & -0.1 \end{pmatrix}$$

Table 2: Parameter Estimates

Regime Name	Composite Regimes					Conditional P/E and Rates			
	#	Infl $\beta$ (%)	Earn $\theta$ (%)	CapUt $\rho$ (%)	Money $w$ (%)	P/E $C_i$	Conditional Rates (%)		
							$r_i^n$	$y_i(1)$	$y_i(10)$
Regular Boom	1	2.33 (0.03)	5.65 (3.07)	-0.80 (0.25)	-0.57 (0.14)	14.41	4.33	5.04	6.36
Regular Recession	2	6.71 (0.14)	-5.27 (1.45)	-0.80 (0.25)	3.98 (0.28)	9.68	9.70	9.61	8.25
Overheating Boom	3	6.71 (0.14)	5.65 (3.07)	4.66 (0.17)	-0.57 (0.14)	13.69	10.90	10.09	7.49
Stagflation	4	9.87 (0.34)	-5.27 (1.45)	4.66 (0.17)	-5.82 (2.11)	9.41	14.78	13.85	9.54
New Economy Growth	5	2.33 (0.03)	6.73 (3.23)	-0.80 (0.25)	-1.87 (1.44)	33.14	4.33	4.32	4.42
Deflation	6	-0.04 (0.00)	-5.27 (1.45)	-6.75 (0.123)	3.98 (0.28)	9.77	0.12	0.67	3.37
Low Capacity Boom	7	2.33 (0.03)	5.65 (3.07)	-6.75 (0.12)	3.98 (0.28)	14.45	3.03	3.17	4.34
Deep Recession	8	6.71 (0.14)	-5.27 (1.45)	-6.75 (0.12)	-1.87 (1.44)	10.37	8.40	7.90	6.38

		Diffusion Matrix(%)				Jump (%)		Taylor Rule ( $\times 100$ )	
Inflation	1.76 (0.01)	0.00	0.00	0.00	0.00	$\kappa$ 7.53 (0.030)	$\alpha_0$ 1.67 (0.06)		
Earnings	0.00	7.93 (0.724)	0.00	0.00	0.00	$\mu_1$ -6.30 (0.044)	$\alpha_\beta$ 22.78 (0.79)		
Kernel	0.00	26.73 (1.541)	46.41 (7.901)	0.00	0.00	$\sigma_1$ 34.77 (0.813)	$\alpha_\rho$ 21.88 (0.63)		
Capacity Utilization	1.86 (0.06)	0.00	0.00	3.31 (0.03)	0.00	$\mu_2$ -81.41 (8.593)			
Money	0.00	0.00	0.00	0.00	3.67 (0.14)	$\sigma_2$ -129.92 (3.502)			

Regime	Infinitesimal Generator								Estimates		
	1	2	3	4	5	6	7	8			
1	$-\sum_j \lambda_{1j}$	$\lambda_4$	$\lambda_6$	0	$\lambda_2$	$\lambda_1$	$\lambda_3$	0	$\lambda_1$	0.499	(0.094)
2	$\lambda_4$	$-\sum_j \lambda_{2j}$	$\lambda_3$	$\lambda_5$	$\lambda_1$	$\lambda_2$	0	$\lambda_2$	$\lambda_2$	1.636	(0.0531)
3	$\lambda_6$	$\lambda_2$	$-\sum_j \lambda_{3j}$	$\lambda_2$	0	$\lambda_1$	$\lambda_3$	$\lambda_2$	$\lambda_3$	5.656	(0.851)
4	$\lambda_4$	$\lambda_5$	$\lambda_4$	$-\sum_j \lambda_{4j}$	0	0	0	$\lambda_2$	$\lambda_4$	9.495	(1.956)
5	$\lambda_2$	0	0	0	$-\sum_j \lambda_{5j}$	$\lambda_1$	0	0	$\lambda_5$	10.167	(1.274)
6	$\lambda_3$	$\lambda_3$	0	$\lambda_2$	0	$-\sum_j \lambda_{6j}$	$\lambda_3$	0	$\lambda_6$	20.439	(23.601)
7	$\lambda_4$	0	0	0	0	$\lambda_3$	$-\sum_j \lambda_{7j}$	$\lambda_3$			
8	$\lambda_4$	$\lambda_3$	$\lambda_2$	$\lambda_2$	0	$\lambda_3$	$\lambda_4$	$-\sum_j \lambda_{8j}$			

Notes: Simulated Methods of Moments (SMM) estimates of the regime-switching model's parameters. The methodology combines the scores of the (simulated) likelihood function from fundamentals (inflation, real earnings, capacity utilization, and money growth) with pricing errors from financial variables (S&P500 index P/E ratio, 3-months Treasury Rate, Treasury Slope, Stock ATM Implied Volatility, 5% OTM Put-to-Call Implied Volatility Ratio, and 10-year Treasury Bond Futures ATM Implied Volatility) The last four columns of top panel also report the conditional P/E ratios and conditional yields across the eight composite regimes. The data sample is 1962 - 2011, except for options, whose sample is 1988 - 2011. Newey-West adjusted standard errors are in parenthesis.

Table 3: Model Fit and Data

	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$
Inflation	-2.16	1.68	-3.67	9.68	60.3
Real Earnings	-9.20	2.88	-2.89	4.88	17.4
Capacity Utilization	-32.34	1.41	-4.19	14.77	85.3
Money Growth	0.71	2.61	1.17	5.87	31.5
P/E Ratio	-4.64	1.40	-1.66	7.86	47.5
3-month Treasury Rate	-0.19	0.89	-0.17	5.42	45.4
Term Structure Slope	1.15	0.73	8.91	8.86	50.8
Stock Implied Volatility	5.45	0.72	1.59	3.84	37.0
Put-to-Call Ratio	0.37	0.75	2.07	5.13	32.5
Bond Implied Volatility	4.53	0.39	8.05	4.49	30.5

Notes: Results of the regressions

$$\begin{aligned}
 (\text{Fundamentals})_t &= b_0 + b_1 E_t[\text{Fundamentals}] + \epsilon_t \\
 (\text{Financial Variable})_t^{Data} &= b_0 + b_1 (\text{Financial Variable})_t^{Model} + \epsilon_t
 \end{aligned}$$

where “Fundamentals” is either inflation, real earnings growth, capacity utilization, or money growth, and financial variables are identified in each row. In these regressions, both expected fundamentals and model-implied financial variables are conditional on the fitted beliefs. The sample is 1967 - 2011, except for option-based quantities, whose sample is 1988-2011. All t-statistics are Newey-West adjusted for heteroskedasticity and autocorrelation using four lags.

Table 4: Beliefs and Options

Panel A: Stock ATM Implied Volatility										
	Model					Data				
	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$
ProbRecess	16.62	30.56	17.64	3.47	38.12	14.21	27.93	9.30	3.59	32.43
EconUnc	14.76	59.72	14.82	5.11	53.84	11.76	0.60	4.98	3.02	22.69
ExpInf	24.53	-181.60	5.71	-1.51	5.77	24.78	-209.83	7.19	-1.85	6.33
UncInf	17.72	52.37	8.79	0.83	1.34	19.07	1.82	8.21	0.02	0.00
ProbDef	17.93	26.57	17.87	3.58	23.49	16.69	33.77	12.14	2.89	12.51

Panel B: OTM Put-to-Call Implied Volatility Ratio										
	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$
	ProbRecess	1.31	-0.45	56.78	-5.03	26.75	1.39	-0.36	39.52	-3.42
EconUnc	1.32	-0.68	47.76	-3.20	22.56	1.45	-0.01	27.54	-3.09	15.90
ExpInf	1.17	3.28	14.64	1.50	6.13	1.24	3.23	19.66	1.38	3.86
UncInf	1.29	-0.93	30.65	-0.78	1.37	1.36	-2.66	24.96	-0.98	1.14
ProbDef	1.29	-0.49	62.53	-5.41	26.30	1.37	-0.56	40.49	-2.14	8.85
Stock ATMIV	1.53	-0.01	38.39	-7.00	61.69	1.55	-0.01	27.27	-4.59	35.69

Panel C: Bond ATM Implied Volatility										
	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$
	ProbRecess	5.35	9.59	15.06	4.49	33.40	6.80	4.66	17.86	3.41
EconUnc	4.83	17.81	12.40	5.25	42.63	5.66	0.10	10.23	2.57	16.54
ExpInf	4.47	55.21	2.53	1.07	4.75	7.04	-5.38	10.73	-0.30	0.11
UncInf	3.75	90.66	5.86	5.01	35.83	6.54	27.95	13.22	1.21	1.25
ProbDef	5.79	7.60	15.30	5.53	17.11	6.62	3.94	22.47	1.97	4.36
Stock ATMIV	2.51	0.19	2.53	3.83	32.25	4.96	0.10	8.78	3.94	26.44

Notes: The table reports t-statistics and  $R^2$  of the regression

$$(\text{Option Index})_{t+1} = \alpha + \beta X_t + \epsilon_{t+1}$$

where ‘‘Option Index’’ is the ATM implied volatility of stocks (Panel A), the put-to-call 5% OTM implied volatility ration (Panel B), and the ATM implied volatility of options on the 10-year Treasury bond futures. The independent variable  $X_t$  is identified by each row’s name. The first five columns report results for the fitted model, while the next five columns report results for the data. For the fitted model, ProbRecess is the probability to be in a recession (probability of Regime 2, 4, 6, 8), EconUnc is economic uncertainty, computed as the posterior variance of real earnings growth, ExpInf is expected inflation, UncInf is inflation uncertainty, computed as the posterior variance of inflation drifts, and ProbDef is the probability to be in the deflationary regime (Regime 6). For the data, ProbRecess is the probability of GDP decline the next quarter from the Survey of Professional Forecasters (SPF), EconUnc is the economic uncertainty computed from ProbRec, ExpInf is the consensus forecast from SPF, UncInf is inflation expectation computed as the conditional variance of next-year inflation obtained from the SPF forecasters’ beliefs, and ProbDef is the probability of a deflation according to SPF beliefs. The sample is 1988 - 2011. All t-statistics are Newey-West adjusted for heteroskedasticity and autocorrelation using four lags.



Table 5: Absolute Changes in ATMIV and Economic Uncertainty

Panel A: Stock Absolute Change in ATM Implied Volatility										
	Model					Data				
	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$
ProbRecess	0.95	13.50	3.98	3.47	0.33	2.05	10.83	3.66	4.50	15.73
EconUnc	0.33	23.64	1.29	3.89	0.38	0.97	0.24	0.82	2.46	12.08
ExpInf	3.95	-63.77	2.25	-1.21	0.03	5.36	-52.28	3.84	-1.12	1.27
UncInf	1.55	18.81	2.49	0.77	0.01	3.69	19.79	4.03	0.38	0.08
ProbDef	1.53	11.66	4.53	4.36	0.20	3.16	10.94	6.17	2.86	4.24

Panel B: Bond Absolute Change in ATM Implied Volatility										
	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$
	ProbRecess	0.55	6.08	7.53	9.05	0.47	0.46	2.33	4.25	4.45
EconUnc	0.37	9.21	3.50	3.88	0.40	0.24	0.05	1.24	3.09	15.75
ExpInf	2.51	-49.13	3.29	-2.20	0.13	1.04	-6.51	3.45	-0.73	0.58
UncInf	1.01	1.23	5.44	0.15	0.00	0.76	8.27	4.63	0.78	0.41
ProbDef	0.78	5.98	7.01	13.37	0.37	0.71	2.23	5.99	1.41	5.21

Notes: The table reports the results of the regression

$$(\text{Absolute Change in ATMIV})_{t+1} = \alpha + \beta X_t + \epsilon_{t+1}$$

where “Absolute Change in ATMIV” is for stocks (Panel A), and for the 10-year Treasury bonds futures options (Panel B). The independent variable  $X_t$  is identified by each row’s name. The first five columns report results for the fitted model, while the next five columns report results for the data. For the fitted model, ProbRecess is the probability to be in a recession (probability of Regime 2, 4, 6, 8), EconUnc is economic uncertainty, computed as the posterior variance of real earnings growth, ExpInf is expected inflation, UncInf is inflation uncertainty, computed as the posterior variance of inflation drifts, and ProbDef is the probability to be in the deflationary regime (Regime 6). For the data, ProbRecess is the probability of GDP decline the next quarter from the Survey of Professional Forecasters (SPF), EconUnc is the economic uncertainty computed from ProbRec, ExpInf is the consensus forecast from SPF, UncInf is inflation expectation computed as the conditional variance of next-year inflation obtained from the SPF forecasters’ beliefs, and ProbDef is the probability of a deflation according to SPF beliefs. The sample is 1988 - 2011. All t-statistics are Newey-West adjusted for heteroskedasticity and autocorrelation using four lags.

Table 6: Implied Volatility Premium and Economic Uncertainty

Panel A: Stock Implied Volatility Premium										
	Model					Data				
	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$
ProbRecess	-0.01	0.03	-9.39	3.62	23.43	0.03	0.02	22.22	3.59	32.43
EconUnc	-0.01	0.03	-7.75	1.52	10.55	0.03	0.001	13.32	3.02	22.69
ExpInf	-0.01	0.15	-1.94	0.65	2.61	0.04	-0.17	12.91	-1.85	6.33
UncInf	-0.02	0.27	-9.19	4.96	23.32	0.03	0.00	16.70	0.02	0.00
ProbDef	-0.01	0.03	-8.97	3.73	19.81	0.03	0.03	26.50	2.89	12.51

Panel B: Bond Implied Volatility Premium										
	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$
	ProbRecess	0.00	0.04	-1.68	5.48	48.61	0.00	0.01	0.31	0.71
EconUnc	-0.003	0.07	-5.78	8.07	64.70	0.00	0.0002	-0.29	1.12	2.07
ExpInf	0.00	-0.08	0.89	-0.62	1.05	-0.01	0.38	-4.00	5.93	22.57
UncInf	0.00	0.15	-1.34	2.37	9.96	0.00	0.37	-1.80	4.06	9.29
ProbDef	0.00	0.03	0.43	7.51	23.74	0.00	-0.04	3.12	-3.90	16.44

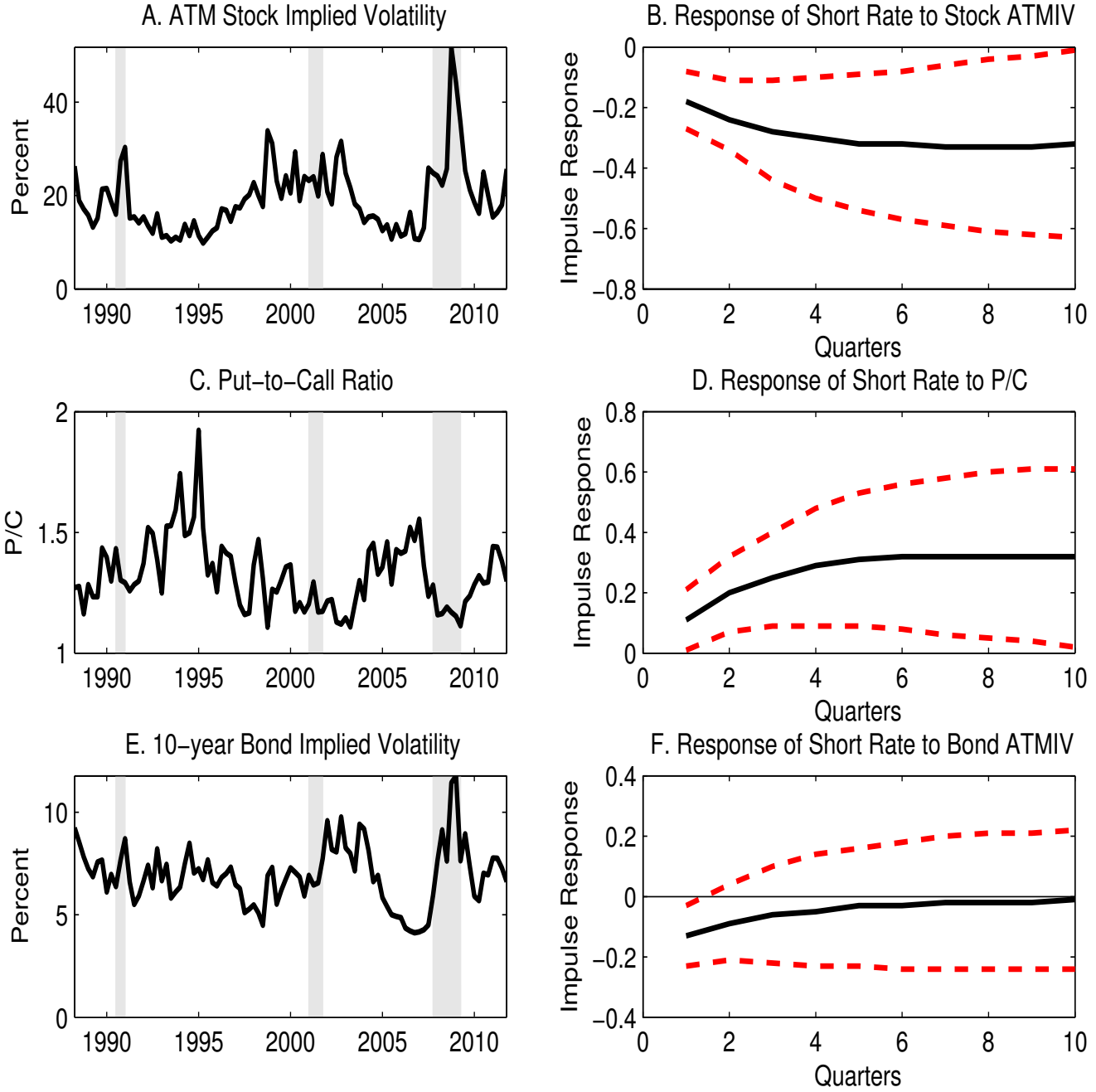
Notes: The table reports the results of the regression

$$(\text{Implied Volatility Premium})_{t+1} = \alpha + \beta X_t + \epsilon_{t+1}$$

where the “Implied Volatility Premium” (IVP) is for stocks (Panel A), and for the 10-year Treasury bonds futures options (Panel B). The independent variable  $X_t$  is identified by each row’s name. The first five columns report results for the fitted model, while the next five columns report results for the data. IVP is the difference between the ATM implied volatility and the forecast of future volatility obtained from regressing realized volatility on lagged volatility and lagged implied volatility (equations (??) and (??) in the text). For the fitted model, ProbRecess is the probability to be in a recession (probability of Regime 2, 4, 6, 8), EconUnc is economic uncertainty, computed as the posterior variance of real earnings growth, ExpInf is expected inflation, UncInf is inflation uncertainty, computed as the posterior variance of inflation drifts, and ProbDef is the probability to be in the deflationary regime (Regime 6). For the data, ProbRecess is the probability of GDP decline the next quarter from the Survey of Professional Forecasters (SPF), EconUnc is the economic uncertainty computed from ProbRec, ExpInf is the consensus forecast from SPF, UncInf is inflation expectation computed as the conditional variance of next-year inflation obtained from the SPF forecasters’ beliefs, and ProbDef is the probability of a deflation according to SPF beliefs. The sample is 1988 - 2011. All t-statistics are Newey-West adjusted for heteroskedasticity and autocorrelation using four lags.

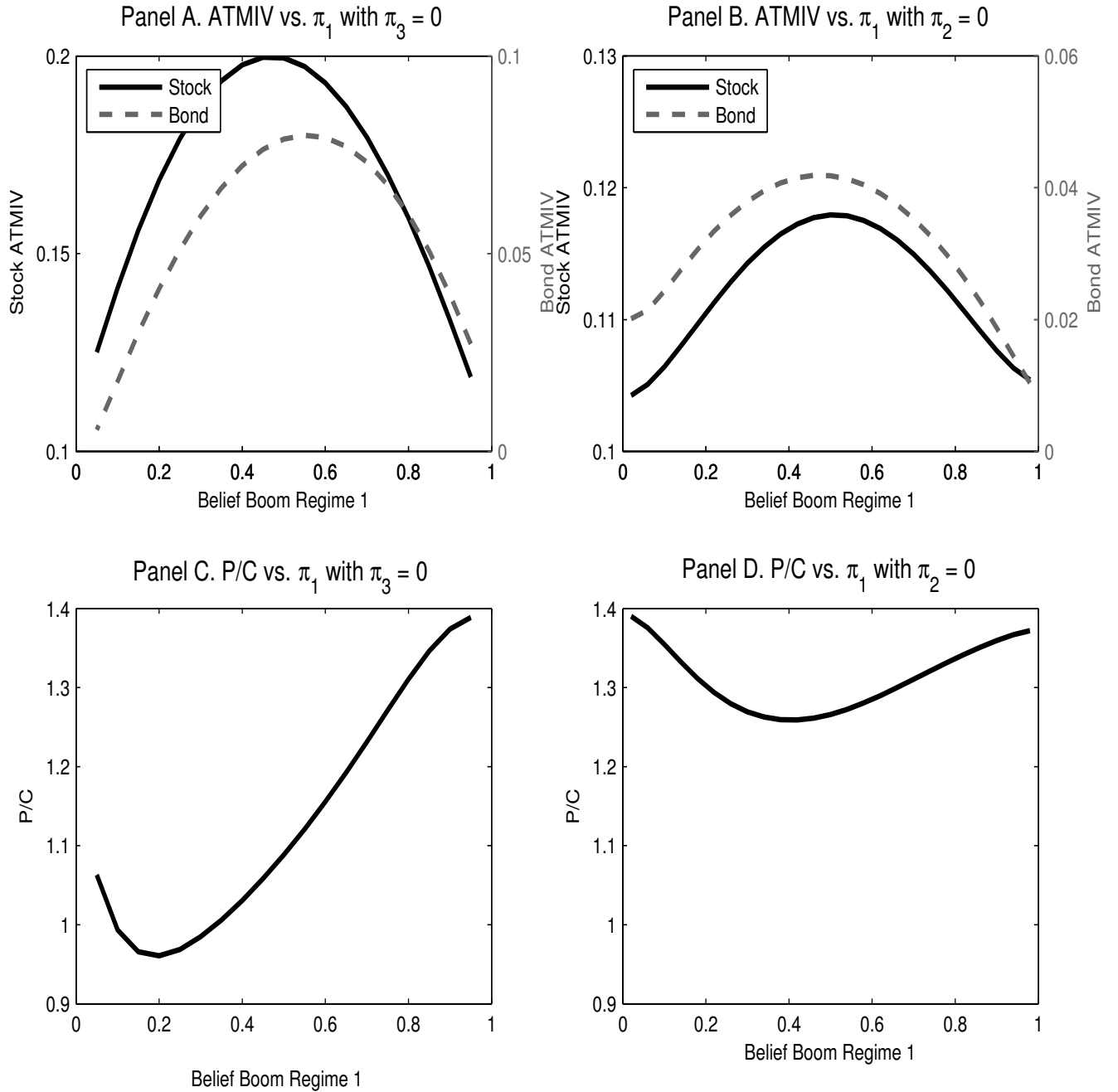


Figure 1: Option Prices and Impulse Responses



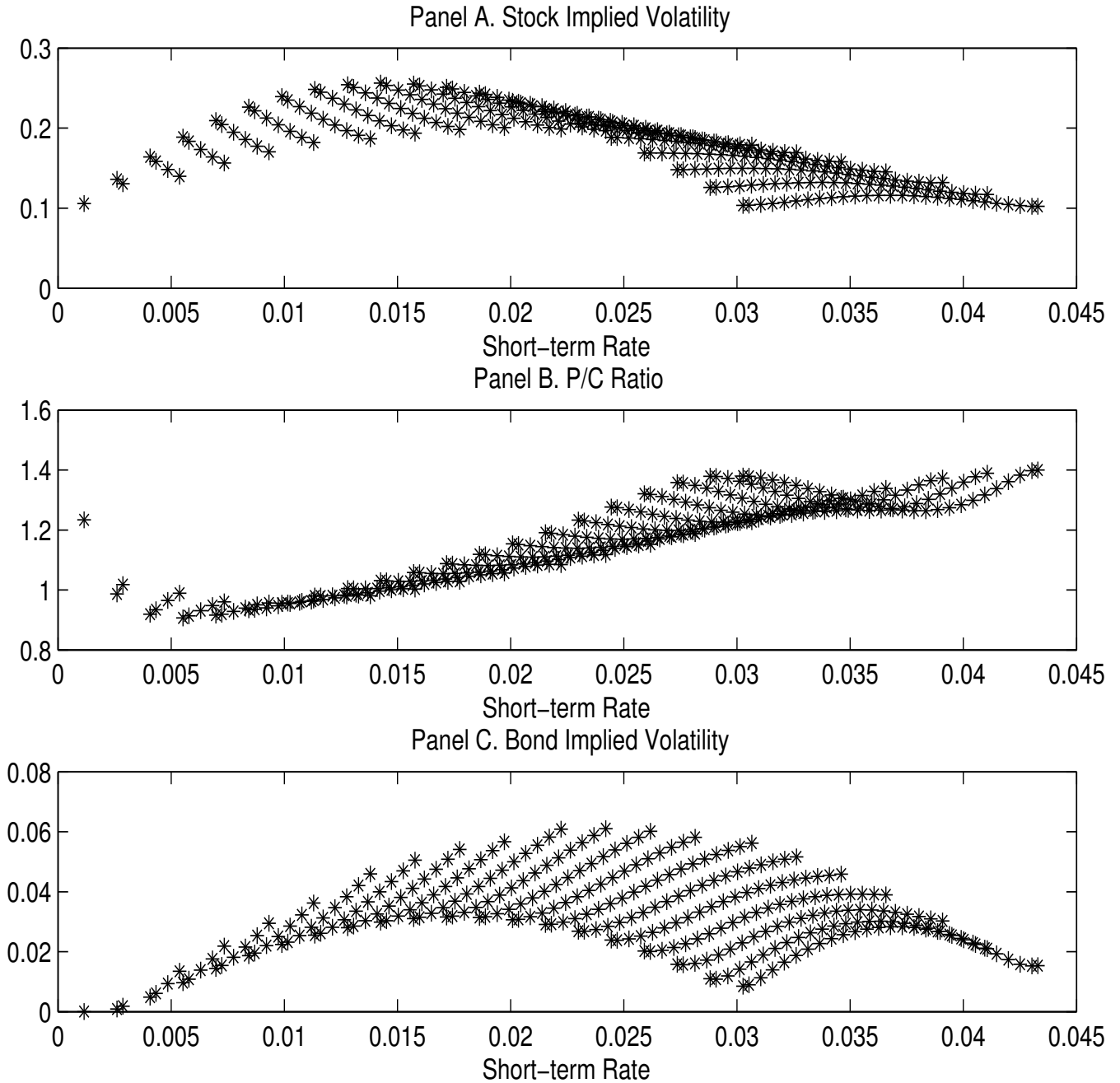
The left panels A, C, and E plot the time series of the stock at-the-money option implied volatility (ATMIV), the ratio of implied volatility of out-of-the-money puts over out-of-the-money calls (P/C), and the 10-year bond future option ATMIV, respectively. In each panel, the shaded areas correspond to NBER-dated recessions. The right panels B, D, and F plot the impulse response functions of a shock to, respectively, stock ATMIV, P/C, and bond ATMIV on the 3-month t-bill rate. The solid line is the impulse response function, and the dotted line are the 95% confidence interval.

Figure 2: Option Indices in a Three Regime Example



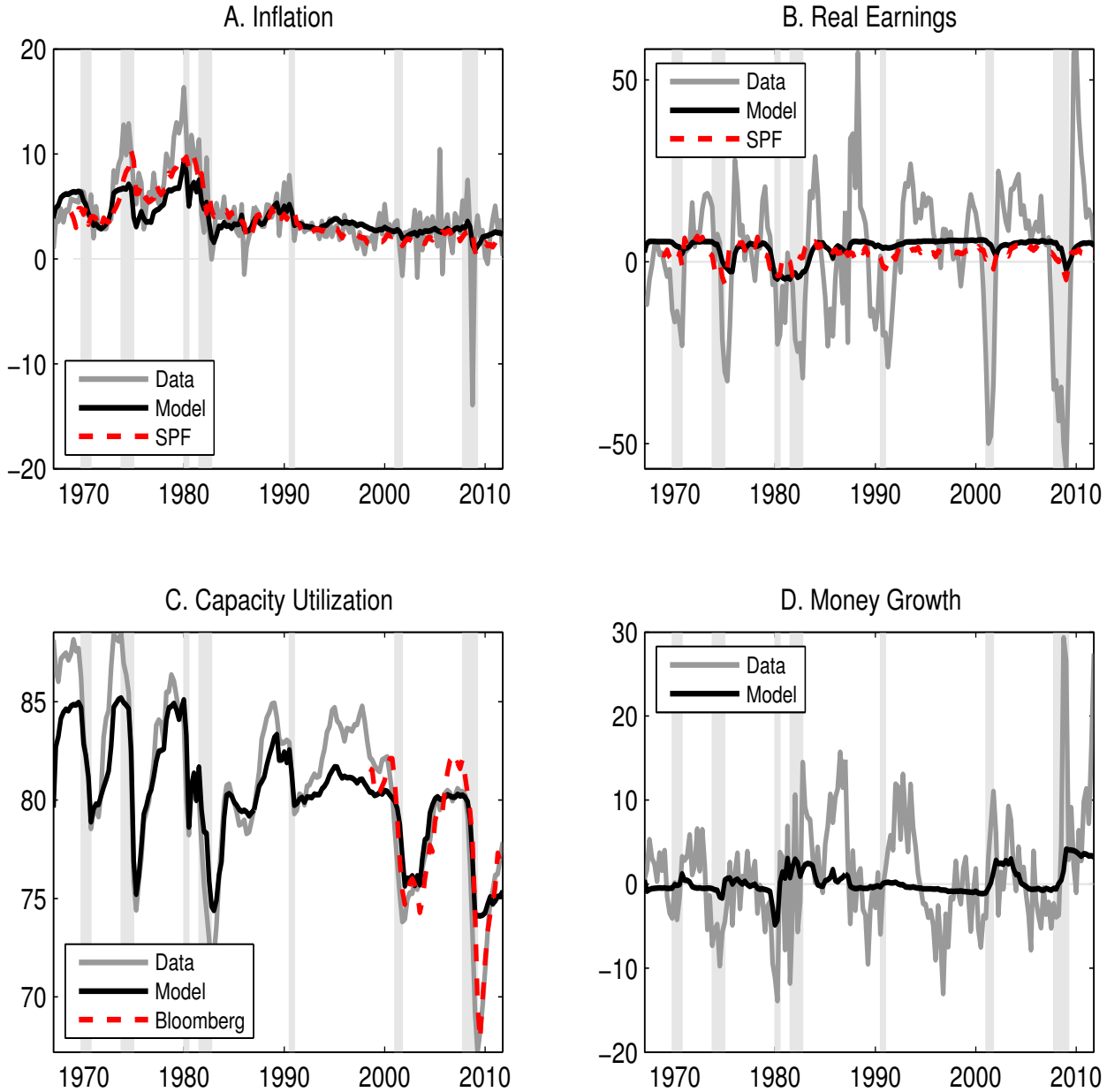
This figure shows the ATMIV of bonds and stocks and the stock put-to-call implied volatility ratio (P/C) for a three regime case. Regime 1 is a regular boom, with regular inflation, high earnings growth, and average capacity utilization. Regime 2 is a deflationary regime, with low inflation, negative earnings growth, and low capacity utilization. Regime 3 is similar to Regime 1, but with low capacity utilization, like Regime 2. In Panels A and C,  $\pi_3 = 0$ , and thus the uncertainty is between a regular boom and deflationary regime. In Panels B and D,  $\pi_2 = 0$ , and thus the uncertainty is between two boom regimes, one with average capacity utilization and one with low capacity utilization.

Figure 3: Option versus Interest Rates in a Three Regime Example



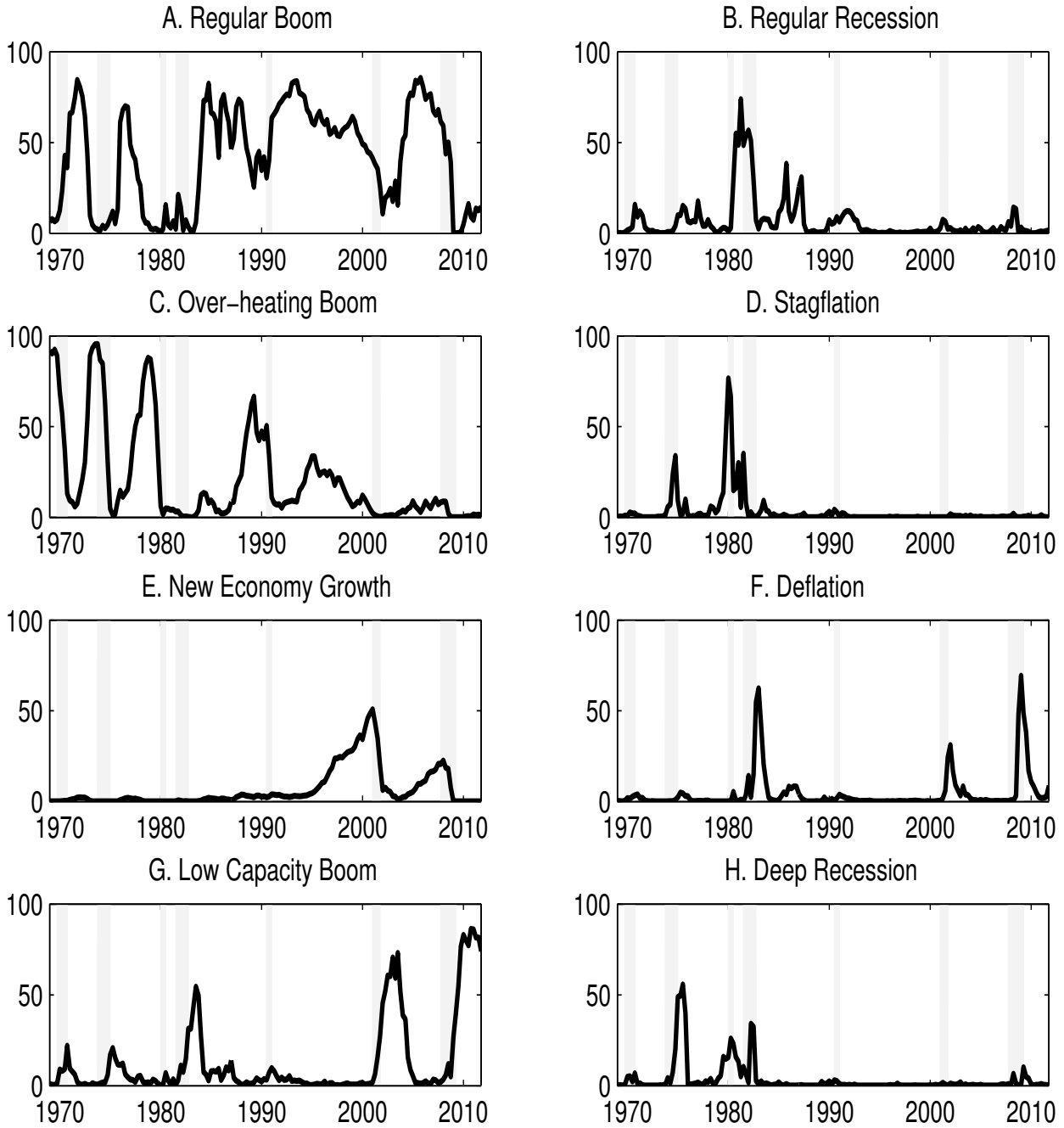
This figure shows scatterplots of stock ATMIV (Panel A), P/C (Panel B) and bond ATMIV (Panel C), plotted against the short term rate, for the three regime case in Table 1. Each panel is obtained by computing the respective option index and the short term rate for a large set of beliefs  $(\pi_{1t}, \pi_{2t}, \pi_{3,t})$  on a grid on the unit simplex, i.e. with  $\pi_{it} > 0$  and  $\sum_{j=1}^3 \pi_{jt} = 1$ , and then plot the corresponding scatterplot.

Figure 4: Model, Surveys and Fundamentals



Panel A plots the inflation data, the expected inflation rate from the fitted model, and the Survey of Professional Forecasters (SPF) consensus forecasts for GDP deflator-based inflation. Panel B plots real earnings data, the expected earnings growth from the fitted model, and the SPF consensus forecasts of real GDP growth. Panel C plots capacity utilization, the expected change from the model, and Bloomberg consensus forecasts of capacity utilization one quarter ahead. Finally, Panel D plots money growth and the model expected money growth rate. In all panels the solid grey line are the data, the solid black line is the model expectation, and the dashed line is the survey-based forecasts.

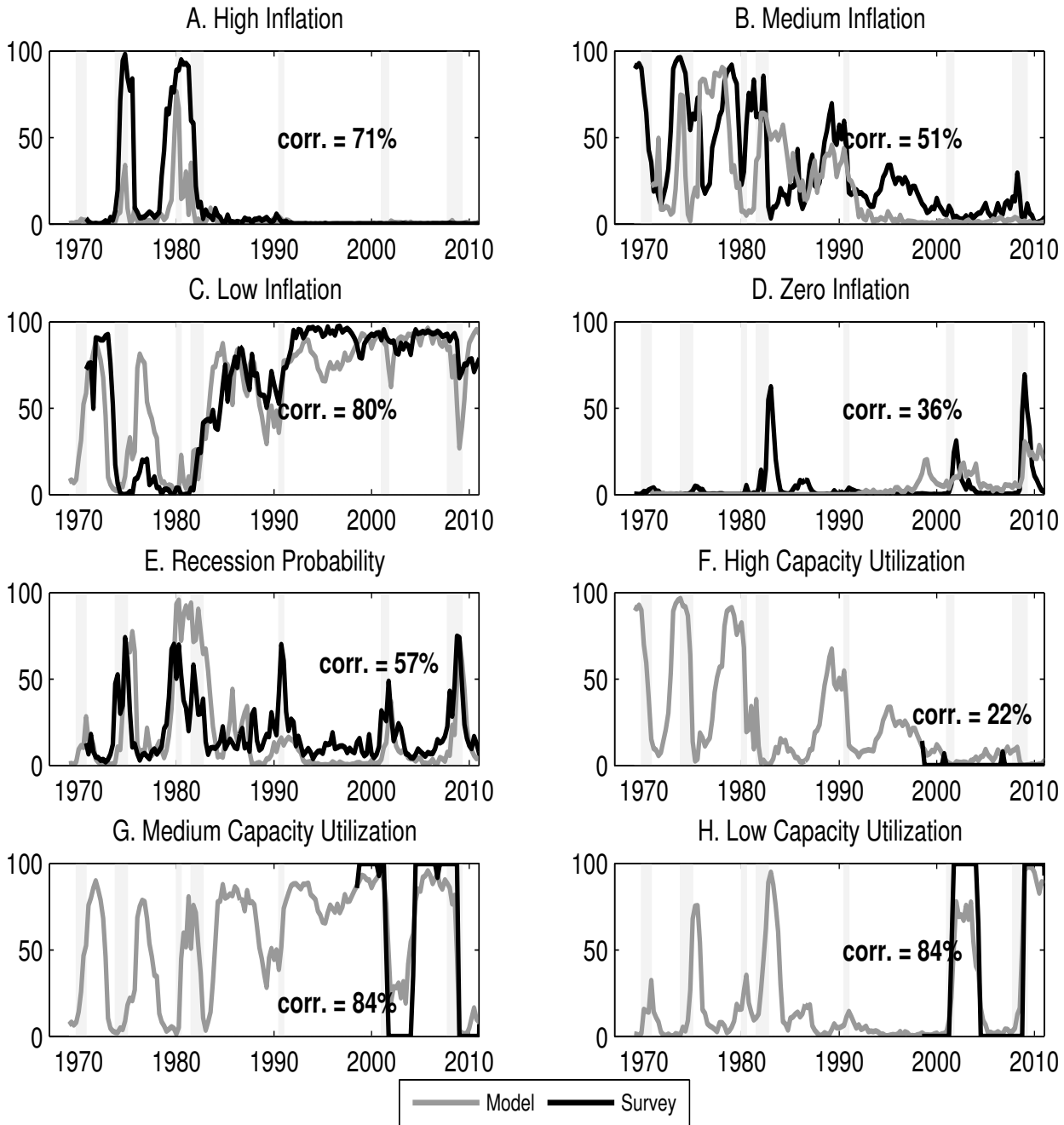
Figure 5: Composite Regime Probabilities



Model's fitted beliefs about each of eight composite regimes from 1967 to 2011. Shaded areas correspond to NBER-dated recessions. The estimates of the eight composite regimes are in Table 2.

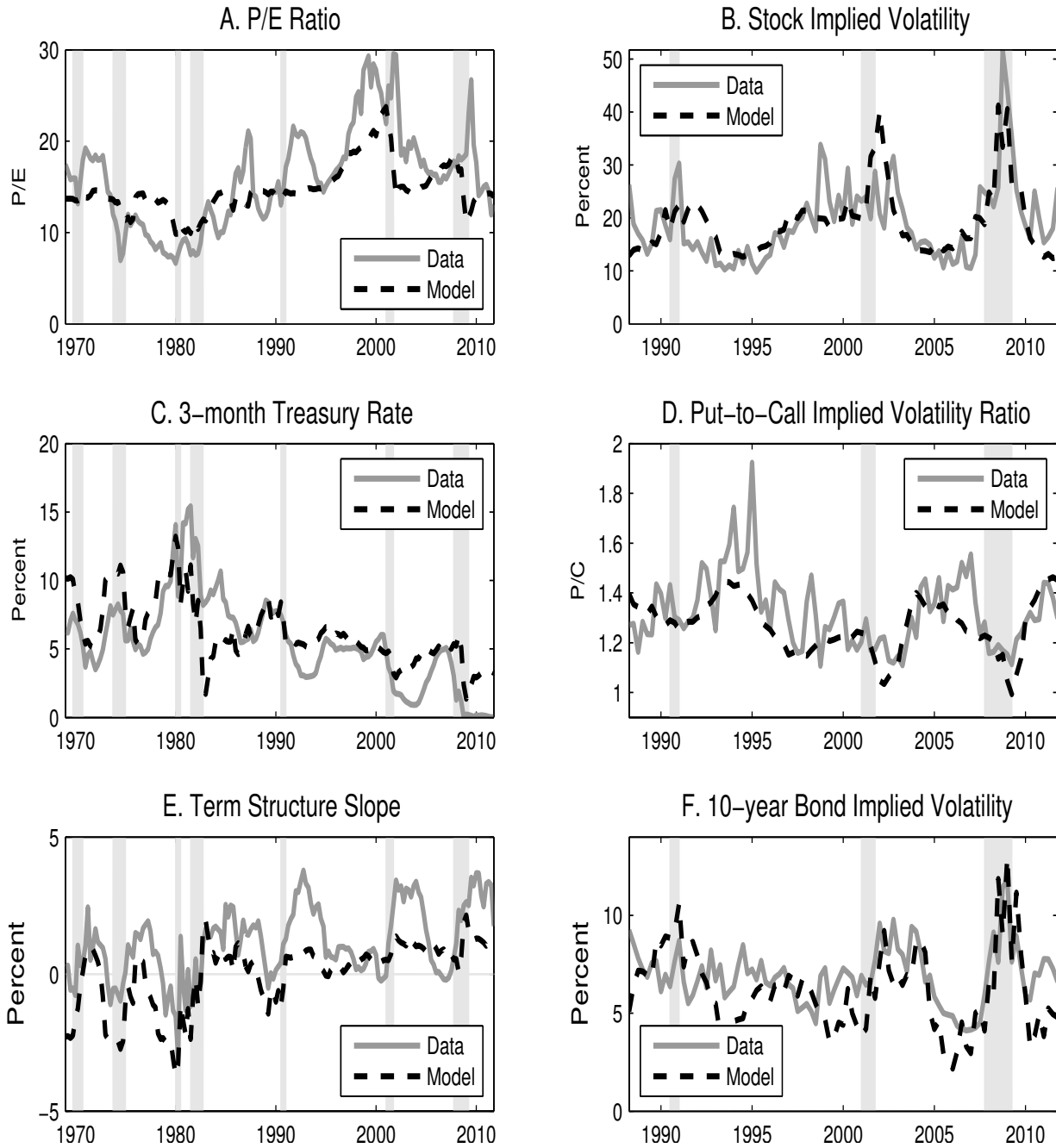


Figure 6: Marginal Probabilities: Model vs. Surveys



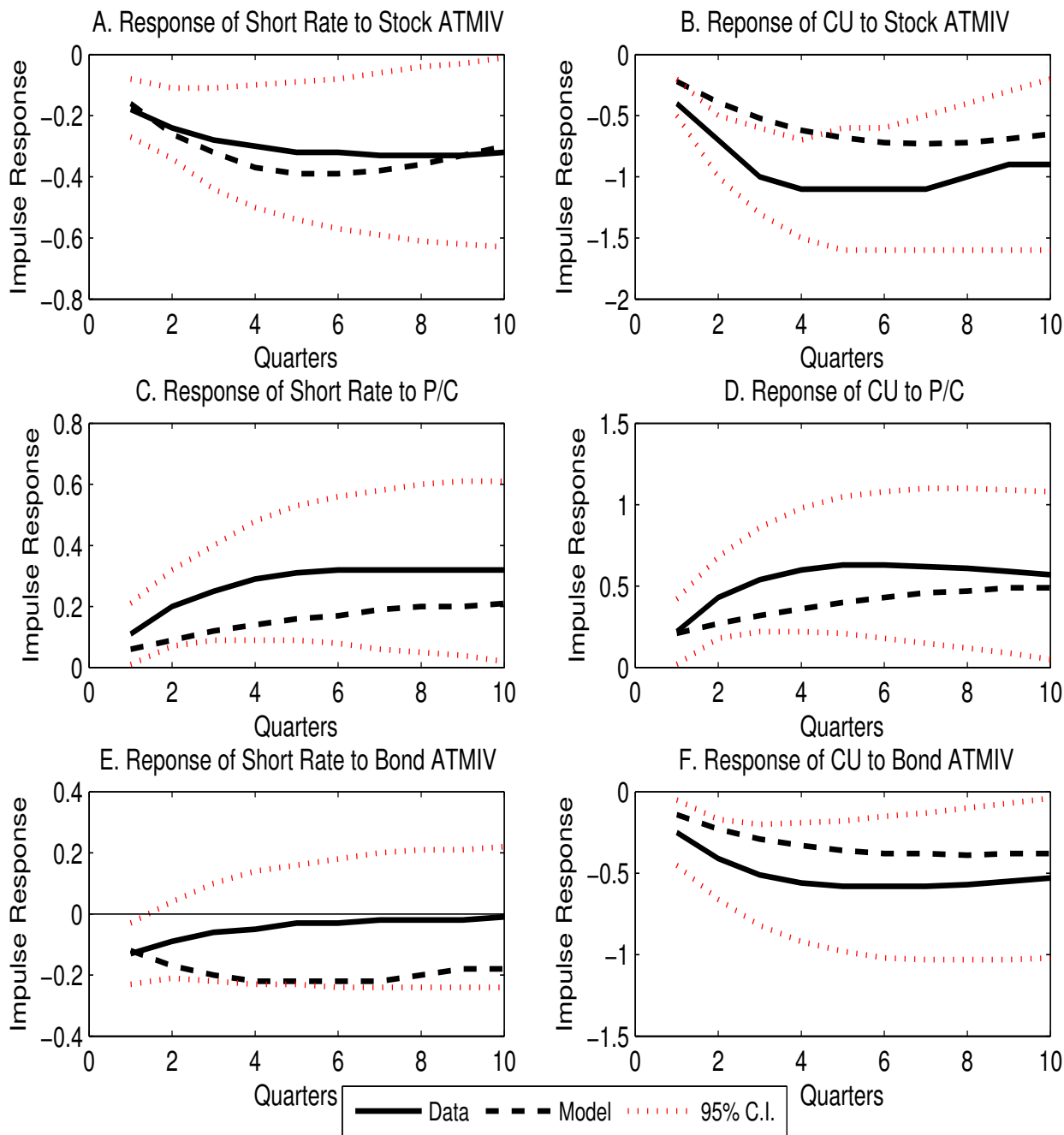
Panels A to D: Model's fitted marginal posterior probabilities about the four possible inflation regimes (black lines) and professional forecasters' probability assessments of similar levels of next-year inflation (grey lines). Panel E: Model's fitted marginal probability of a recession (black line) and professional forecasters probability assessment of a GDP decline the following quarter. Panels F to H: Model's fitted marginal posterior probabilities about high, medium, and low capacity utilization (black lines) and Bloomberg-based probability of the same three high, medium, low level of capacity utilization obtained from the distribution of Bloomberg forecasts. Shaded vertical bars are the NBER-dated recessions.

Figure 7: Fitted and Data Series



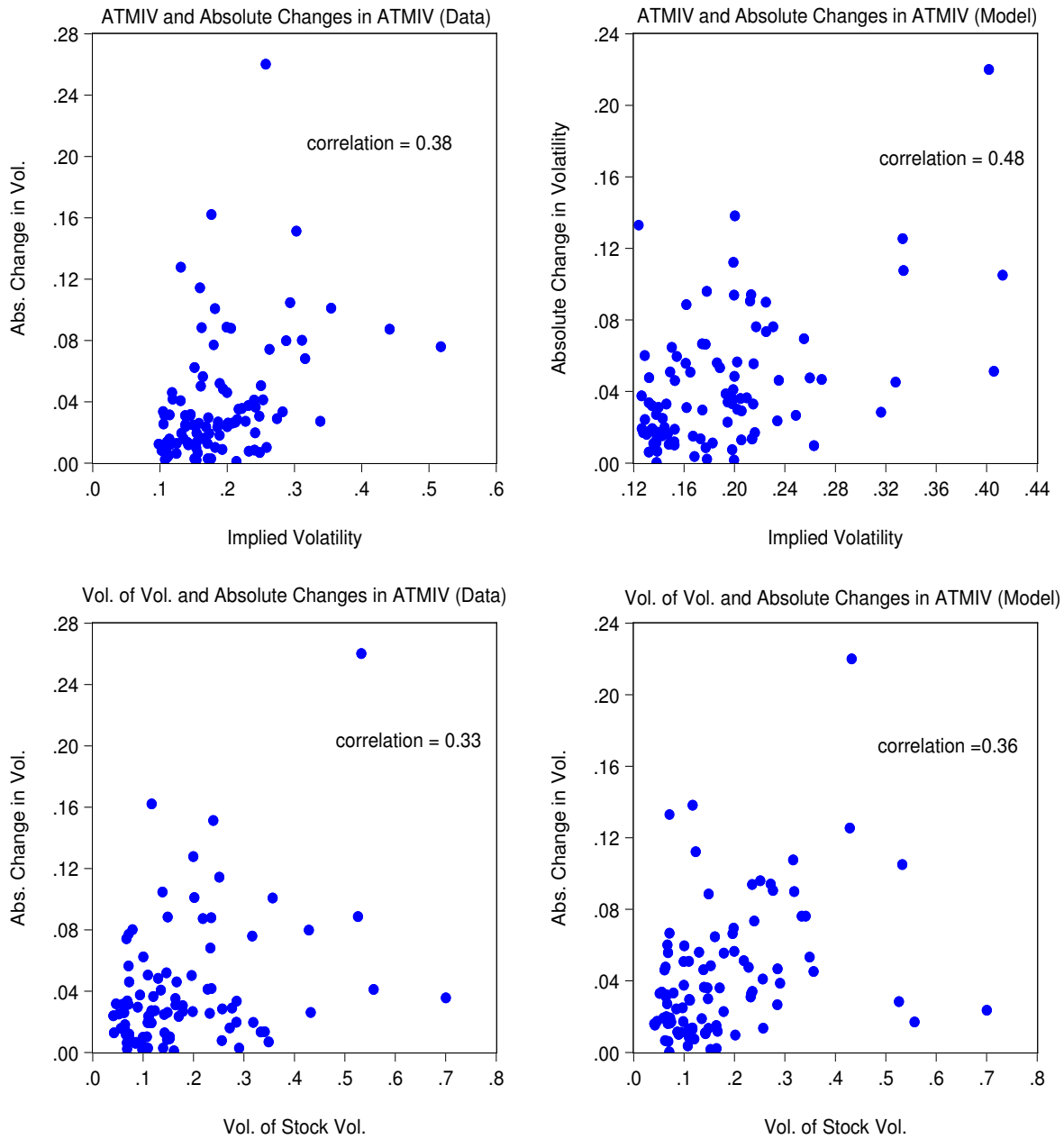
Panels A, C, and E plot the realized price/earnings ratio, 3-month T-Bill rate, and the slope of the term structure (10 year minus 1 year), respectively, and their model-fitted counterparts, over the sample 1967 - 2011. Panels B, D, and F plot the realized stock ATM IV, Put-to-Call implied volatility ratio, and 10 Bond futures option ATM IV, respectively, and their model-fitted counterparts over the option's sample 1988 - 2011. In all panels, the solid grey line is the data and the dashed black line is the model's fitted. Shaded areas correspond to NBER-dated recessions.

Figure 8: Impulse Responses: Data vs. Fitted Model



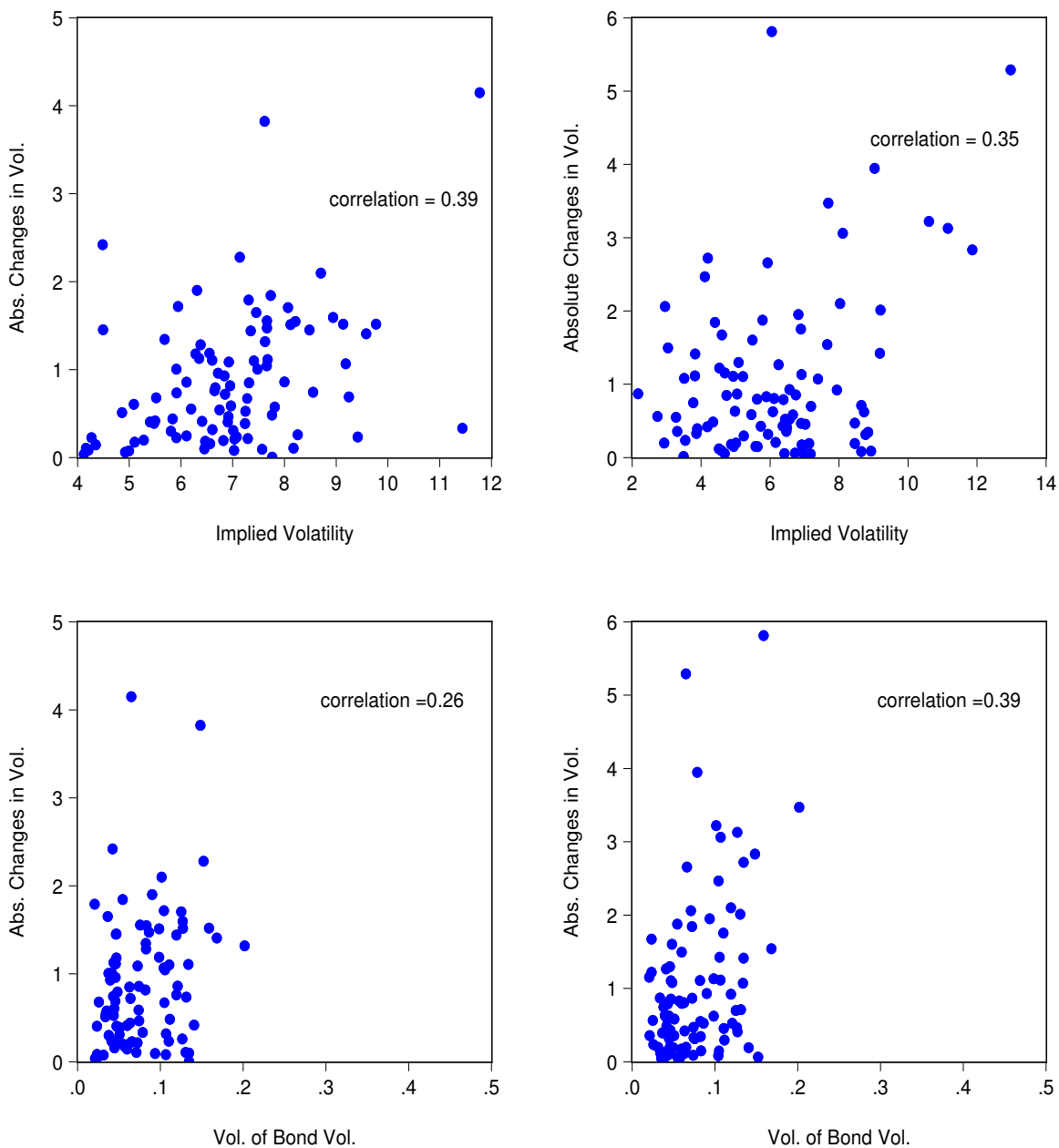
The left panels A, C, and E report the impulse response function from a shock to stock ATM IV, P/C ratio, and bond ATM IV, respectively, on the 3-month t-bill rate. The right panels B, D, and F report the impulse response function from a shock to stock ATM IV, P/C ratio, and bond ATM IV, respectively, on capacity utilization (CU). In each panel, the solid line is the impulse response, the dotted lines are the two-sided confidence bands, and the dashed line is the impulse response function computed on the fitted data.

Figure 9: Relationship Between Stocks ATM Implied Volatility and Absolute Changes in ATM Implied Volatility, (1988-2011)



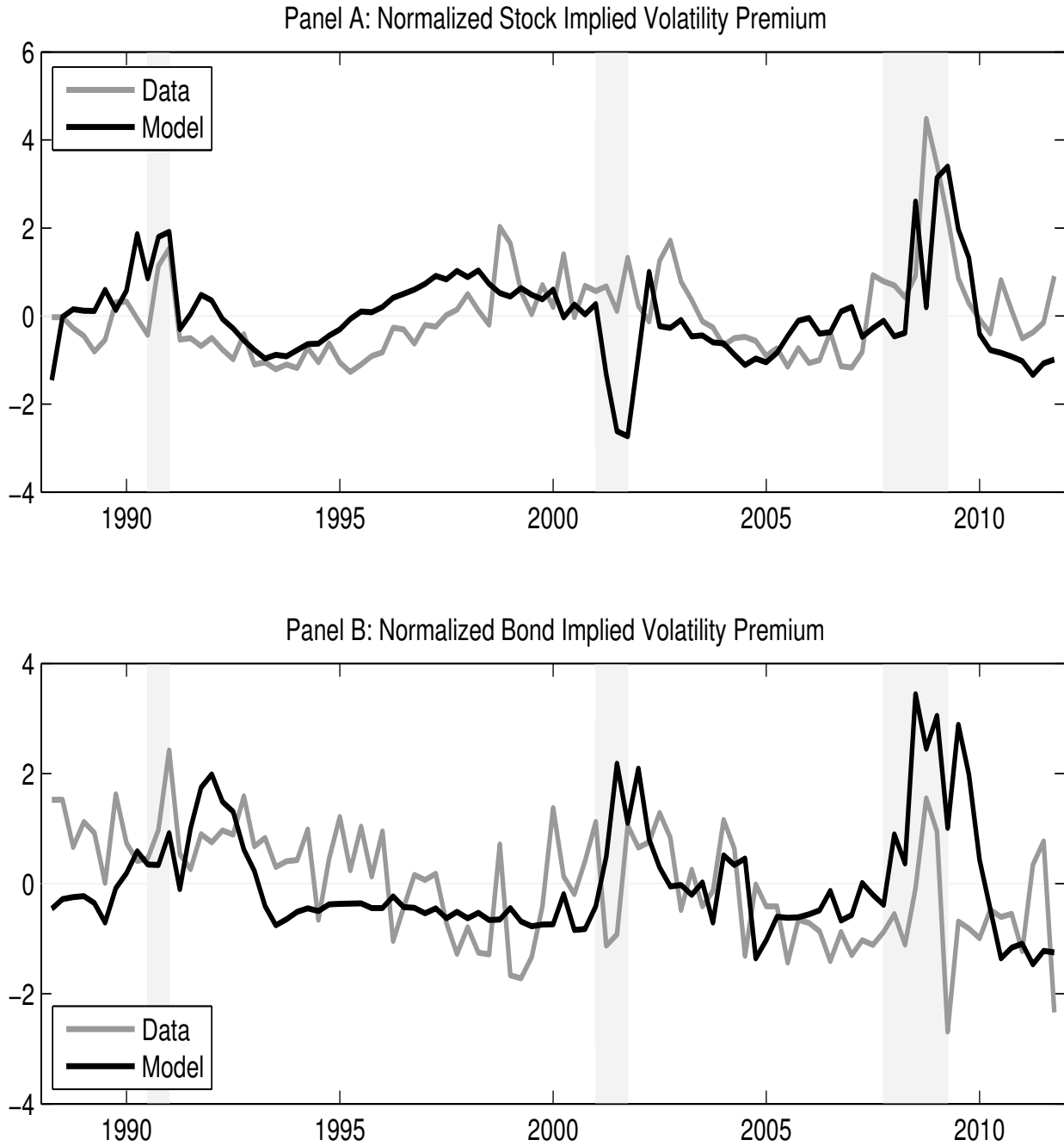
Data and model ATM implied volatility are shown in panel B of Figure 7. The model volatility of volatility is computed using (23) and the filtered belief series in Figure 5.

Figure 10: Relationship Between Bonds ATM Implied Volatility and Absolute Changes in ATM Implied Volatility, (1988-2011)



Data and model bond ATM implied volatility are shown in Panel F of Figure 7. The model volatility of volatility is computed using the analogous formula as in (23) but for bonds, and the filtered belief series in Figure 5.

Figure 11: Implied Volatility Premium



This figure plots the normalized implied volatility premium for stocks (Panel A) and bonds (Panel B), both in the data and implied from the model. The IVP in the data equals the ATM implied volatility minus the expected one-quarter ahead volatility obtained from a regression of realized volatility on lagged volatility and lagged implied volatility. The model implied volatility premium is computed using the model's implied volatility and the expected future volatility conditional on the filtered belief series in Figure 5.