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ABSTRACT

We develop a model of endogenous maturity structure for financial institutions that borrow from multiple creditors. We show that a maturity rat race can occur: an individual creditor can have an incentive to shorten the maturity of his own loan to the institution, allowing him to adjust his financing terms or pull out before other creditors can. This, in turn, causes all other lenders to shorten their maturity as well, leading to excessively short-term financing. This rat race occurs when interim information is mostly about the probability of default rather than the recovery in default, and is most pronounced during volatile periods and crises. Overall, firms are exposed to unnecessary rollover risk.

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One of the central lessons of the financial crisis of 2007-09 is the importance of maturity structure for financial stability. In particular, the crisis vividly exposed the vulnerability of institutions with strong maturity mismatch—those who finance themselves short-term and invest long-term—to disruptions in their funding liquidity. This raises the question of why financial institutions use so much short-term financing.

In this paper we develop a model of the equilibrium maturity structure of a financial institution. We show that for financial institutions that can borrow at differing maturities from multiple creditors to finance long-term investments, the equilibrium maturity structure will in general be inefficiently short-term—leading to excessive maturity mismatch, unnecessary rollover risk, and inefficient creditor runs.

Intuitively, a financial institution that cannot commit to an aggregate maturity structure can have an incentive to approach one of its creditors and suggest switching from a long-term to a short-term (rollover) debt contract. This dilutes the remaining long-term creditors: If negative information comes out at the rollover date, the short-term creditor increases his face value. This reduces the payoff to the long-term creditors in case of ultimate default, whose relative claim on the firm is diminished. On the other hand, if positive information is revealed at the rollover date, rolling over short-term debt is cheap. While this benefits the remaining long-term debtholders in case the financial institution defaults, typically bankruptcy is less likely after positive news than after negative news. Hence, in expectation the long-term creditors are worse off—they suffer a negative externality. This means that the financial institution has an incentive to shorten its maturity whenever interim information received at rollover dates is mostly about the *probability of default.* Whenever this is the case, rollover financing is the unique equilibrium, even though it leads to inefficient rollover risk. In contrast, when interim information is mostly about the *recovery given default*, long-term financing can be sustained.

The same logic extends to settings in which short-term credit can be rolled over multiple times before an investment pays off. In fact, when multiple rollover dates are possible the contractual externality between short-term and long-term debt can lead to a successive unraveling of the maturity structure: If everyone's debt matures at time T, the financial institution has an incentive to start shortening an individual creditor's maturity, until everyone's maturity is of length $T-1$. Yet, once everyone's maturity is of length $T-1$, there would be an incentive to give some creditors a maturity of $T-2$. Under certain conditions, the maturity structure thus successively unravels to the very short end—a *maturity rat race*.

In our model, the fundamental incentive to shorten the maturity structure can emerge whenever a financial institution borrows from multiple creditors. However, since the externality that causes maturity shortening stems from the short-term creditors' ability to update their financing terms in response to interim information, the incentive to shorten the maturity structure is particularly strong during times of crisis, when investors expect a lot of default-probability relevant information to be released before the financial institution's investments mature.

The maturity rat race is inefficient. It leads to excessive rollover risk and causes inefficient liquidation of the long-term investment project after negative interim information. Moreover, because creditors anticipate the costly liquidations that occur when rolling over short-term debt is not possible, some positive NPV projects do not get started in the first place. This inefficiency stands in contrast to some of the leading existing theories of maturity mismatch. For example, Diamond and Dybvig (1983) highlight the role of maturity mismatch in facilitating long-term investment projects while serving investors' liquidity needs that are individually random, but deterministic in aggregate. Calomiris and Kahn (1991) and Diamond and Rajan (2001) demonstrate the role of short-term financing and the resulting maturity mismatch as a disciplining device for bank managers. In all of these theories, maturity mismatch is an integral and desirable part of financial intermediation.

Our model has very different implications. In particular, to the extent that maturity mismatch results from our 'rat race' mechanism, a regulator may want to impose restrictions on short-term financing to preserve financial stability and reduce rollover risk. In this respect our paper thus complements Diamond (1991) and Stein (2005) in arguing that financing may be excessively short-term. However, while the driving force in these models is asymmetric information about the borrower's type, a mechanism that is also highlighted in Flannery (1986), our model emphasizes the importance of contractual externalities among creditors of different maturities.

The key friction in our model is the inability to commit to an aggregate maturity structure. This friction applies particularly to financial institutions, rather than to corporates in general: When offering debt contracts to its creditors, it is almost impossible for a financial institution to commit to an aggregate maturity structure. While corporates that tap capital markets only occasionally may be able to do this through covenants or seniority restrictions, committing to a maturity structure is much more difficult (and potentially undesirable) for financial institutions. Frequent funding needs, opaque balance sheets, and their more or less constant activity in the commercial paper market makes committing to a particular maturity structure virtually impossible.

Of course, the incentive to shorten the maturity structure may be affected by priority rules or covenants. While in our baseline model we assume unsecured debt with equal priority among creditors and no covenants, we discuss the impact of other priority rules and the introduction of covenants. Seniority restrictions or covenants can weaken the maturity rat race, but generally do not eliminate it. Moreover, even when they could, financial institutions may not be willing to counteract the maturity rat race through covenants or restrictive seniority clauses, because they attach a high value to financial flexibility.[1] Finally, even when by virtue of seniority restrictions or covenants an equilibrium with long-term financing exists, the financial institution may still get caught in a 'short-term financing trap,' an inefficient short-term financing equilibrium that continues to exist even when long-term financing is also an equilibrium. The reason is that, given that all other lenders are only providing short-term financing, it is not individually rational for the financial institution to move an individual creditor to a longer maturity. In fear of getting stuck while others withdraw their funding or adjust their financing terms at rollover, the lone long-term creditor would require a correspondingly large face value, such that the financial institution is better off under all short-term financing (even though that maximizes the institution's rollover risk).

Our paper relates to a number of recent papers on short-term debt and rollover risk. Brunnermeier and Yogo (2009) provide a model of liquidity risk management in the presence of rollover risk. Their analysis shows that liquidity risk management does not necessarily coincide managing duration risk. Acharya, Gale, and Yorulmazer (2010) show how rollover risk can reduce a security's collateral value. In contrast to our paper, they take short-term financing of assets as given, while we focus on why short-term financing emerges in the first place. In He and Xiong (2009) coordination problems among creditors with debt contracts of random maturity can lead to the liquidation of financially sound firms. Given a fixed expected rollover frequency, they show that each creditor has an incentive to raise his individual rollover threshold, inducing others to raise theirs as well. Unlike

[1] In practice, very little financing of financial institutions contains covenants. See Flannery (1994) for reasons why it is often hard or even undesirable for financial institutions to use covenants.

their dynamic global games setting, in which interest rates and maturity structure are exogenous, we focus on the choice of maturity with endogenous interest rates. Farhi and Tirole (2009) show how excessive maturity mismatch emerges through a collective moral hazard that anticipates untargeted ex-post monetary policy intervention during systemic crises. Unlike their paper, in our model shows that excessive maturity mismatch may arise even in the absence of an anticipated ex-post intervention by the central bank.

The paper is also related to the literature on debt dilution, either by issuing senior debt to dilute existing debt (see, e.g., Fama and Miller (1972)), by borrowing from more lenders (White (1980), Bizer and DeMarzo (1992), Parlour and Rajan (2001)), or by preferentially pledging collateral to some creditors. While our paper shares the focus on dilution, the mechanism, maturity structure, is different. First, as our model shows, shortening the maturity of a subset of creditors is not equivalent to granting seniority, and only works in the favor of the financial institution under certain conditions. Second, in contrast to borrowers in the sequential banking papers by Bizer and DeMarzo (1992) and Parlour and Rajan (2001), the financial institution in our model can commit to the aggregate amount borrowed, it just cannot commit to whether the amount borrowed is financed through long-term or rollover debt.

The remainder of the paper is structured as follows. We describe the setup of our model in Section 1. In Section 2 we then characterize the equilibrium maturity structure and show that opaque balance sheets and the inability to commit to an aggregate maturity structure can lead to excessive short-term financing. In Section 3 we show that the rat race leads to excessive rollover risk and underinvestment, and discuss the impact of adding seniority restrictions and covenants to our model. Section 4 concludes.

1 Model Setup

Consider a risk-neutral financial institution that can invest in a long-term project. The investment opportunity arises at $t = 0$, is of fixed scale, and we normalize the required $t = 0$ investment cost to 1. At time T, the investment pays off a random non-negative amount θ, distributed according to a distribution function $F(\cdot)$ on the interval $[0, \infty)$. Seen from $t = 0$, the unconditional expected payoff from investing in the long-term project is $E[\theta] = \int_0^\infty \theta dF(\theta)$, and its net present value is positive when $E[\theta] > 1$. For simplicity we assume that there is no time discounting.

Once the project has been undertaken, over time more information is learned about its profitability. At any interim date $t = 1, ..., T-1$, a signal s_t realizes. We assume that for any history of signals $\{s_1, .., s_t\}$, there is a sufficient statistic S_t, conditional on which the distribution of the project's payoff is given by $F(\theta|S_t)$, and its expected value accordingly by $E(\theta|S_t)$. For the remainder of the text, we will loosely refer to S_t, the sufficient statistic for the signal history, as the signal at time t. We assume that $F(\cdot)$ satisfies the strict monotone likelihood ratio property with respect to the signal S_t. This implies that when $S_t^A > S_t^B$, the updated distribution function $F(\theta|S_t^A)$ dominates $F(\theta|S_t^B)$ in the first-order stochastic dominance sense (Milgrom (1981)). The signal S_t is distributed according to the distribution function $G_t(\cdot)$. We refer to the highest possible signal at time t as S_t^H, and the lowest possible signal as S_t^L.

The long-term project can be liquidated prematurely at time $t < T$ with a continuous liquidation technology that allows to liquidate all or only part of the project. However, early liquidation yields only a fraction of the conditional expectation of the project's payoff, $\lambda E(\theta|S_t)$, where $\lambda < 1$. This implies that early liquidation is always inefficient—no matter how bad the signal realization S_t turns out to be, in expectation it always pays more to continue the project rather than to liquidate it early. These liquidation costs reflect the deadweight costs generated by shutting down the project early, or the lower valuation of a second-best owner, who may purchase the project at an interim date.[2]

The financial institution has no initial capital and needs to raise the financing for the long-term project from a competitive capital market populated by a continuum of risk-neutral lenders. Each lender has limited capital, such that the financial institution has to borrow from multiple creditors to undertake the investment. Financing takes the form of debt contracts. We take debt contracts as given and do not derive their optimality from a security design perspective. Debt contracts with differing maturities are available, such that the financial institution has to make a choice about its maturity structure when financing the project. A debt contract specifies a face value and a

[2] Of course, in practice early liquidation must not always be inefficient. In this case, if the financial institution may want to continue inefficient projects because of private benefits or empire building motives, some amount of short-term financing may be desirable, because it may help force liquidation in states where this is efficient (see for example Eisenbach (2010)). We intentionally rule out this possibility for the remainder of the paper in order to restrict the analysis to situations in which short-term debt has no inherent advantage, and then show that under reasonable assumptions short-term financing will nevertheless emerge as the equilibrium outcome.

maturity date at which that face value is due. We refer to a debt contract with maturity T as a long-term debt contract. This long-term debt contract matches the maturity of the assets and liabilities of the financial institution and has a face value of $D_{0,T}$ to be paid back at time T. By definition, long-term debt contracts do not have to be rolled over before maturity, which means that long-term debtholders cannot adjust their financing terms in response to the signals observed at the interim dates $t < T$.

In addition to long-term debt, the financial institution can also issue debt with shorter maturity, which has to be rolled over at some time $t < T$. A short-term debt contract written at date 0 specifies a face value $D_{0,t}$ that comes due at date t, at which point this face value has to be repaid or rolled over. When short-term debt is rolled over at t, the outstanding face value is adjusted to reflect the new information contained in the signal S_t. In terms of notation, if debt is rolled over from time t to time $t + \tau$, we denote the rollover face value due at $t + \tau$ by $D_{t,t+\tau}(S_t)$.

Short-term debtholders are atomistic and make uncoordinated rollover decisions at the rollover date. If short-term debtholders refuse to roll over their obligations at date t, some or even all of the long-term investment project may have to be liquidated early to meet the repayment obligations to the short-term debtholders. If the financial institution cannot repay rollover creditors by offering new rollover debt contracts or repaying them by liquidating part or all of the long-term investment, the financial institution defaults. In the case of default at time $t \leq T$, long-term debt is accelerated and that there is equal priority among all debtholders. Consistent with U.S. bankruptcy procedures, we do not draw a distinction between principal and accrued interest in the case of default. Equal priority then implies that in the case of default the liquidation proceeds are split among all creditors, proportionally to the face values (principal plus matured interest). Holders of non-matured debt do not have a claim on interest that has not accrued yet.

We now describe the main novel assumption of our model: We assume that the financial institution's maturity structure is *opaque*. This opacity has two important effects. First, when dealing with the financial institution, individual creditors can only observe the financing terms offered to them. They cannot observe the financing terms and maturities offered to other creditors, nor can they observe the financial institution's aggregate maturity structure. Second, as a result of this opacity, it is impossible for the financial institution to commit to a particular maturity structure (for example by promising to issue only long-term debt contracts with maturity T) when raising financing for the long-term investment. Hence, when raising financing at date 0, the financial institution thus simultaneously offers debt contracts to a continuum of individual creditors without being able to commit to an aggregate maturity structure.

The assumption of an opaque maturity structure is motivated by a fundamental difference between corporates and financial institutions. While corporates raise financing only occasionally, financial institutions more or less constantly finance and refinance themselves in the commercial paper and repo markets. Relative to corporates, this makes it much harder to ascertain a financial institution's maturity structure and rollover needs at any point in time. This, in turn, makes it extremely difficult, if not impossible, for financial institutions to commit to a particular maturity structure. Moreover, even if such commitment were possible, financial institutions may often not find it optimal to bind themselves to a particular maturity structure in order to keep financial flexibility. We thus view the limited commitment assumption as a natural friction that arises when a financial institution deals with many dispersed creditors. In our model, this limited commitment is the key friction that generates equilibrium maturity structures that are excessively short-term.

2 The Equilibrium Maturity Structure

Given our setup, two conditions must be met for a maturity structure to constitute an equilibrium.[3] First, since capital markets are competitive, a zero profit condition applies, such that in any equilibrium maturity structure all creditors must just break even in expectation.[4] Given that all creditors just break even, the financial institution thus has to issue a combination of debt contracts of potentially different maturities that have an aggregate expected payoff equal to the initial cost of undertaking the investment.

However, while creditor breakeven is a necessary condition for equilibrium, it is not sufficient. A second condition arises because the financial institution deals bilaterally with multiple creditors and cannot commit to an aggregate maturity structure when entering individual debt contracts. Hence, for a maturity structure to be an equilibrium, at the breakeven face values the financial institution must have no incentive to deviate by forming a coalition with an individual creditor (or

[3] For a formal definition of our equilibrium concept see Definition 1 below.

[4] In Parlour and Rajan (2001) lenders make positive profits in competitive equilibrium. This is due to a moral hazard problem that is not present in our setting.

a subset of creditors) and changing this creditor's maturity, while holding fixed everybody else's financing terms and beliefs about the institution's aggregate maturity structure.

To illustrate this second requirement, consider for example a conjectured equilibrium in which all creditors expect financing to be in the form of long-term debt that matures at T. The 'no-deviation' requirement is violated when the financial institution has an incentive to move one of the long-term creditors to a shorter maturity contract, given that all remaining creditors anticipate financing to be purely long-term and set their financing terms such that they would just break even under all long-term financing. If this deviation is profitable, the all long-term financing outcome cannot be an equilibrium maturity structure.

We now examine the break-even and no-deviation conditions in turn. For simplicity, in what follows we will initially focus on the financial institution's maturity structure choice when there is only one potential rollover date t. Later on we will show that the analysis can be extended to accommodate multiple rollover dates.

2.1 Creditor Break-Even Conditions

Assume for now that there is only one rollover date, $t < T$. Consider first the rollover decision of creditors whose debt matures at the rollover date t, and denote by α the fraction of creditors that has entered such rollover contracts. In order to roll over the maturing short-term debt at time t, the financial institution has to issue new debt of face value $D_{t,T}(S_t)$, which, conditional on the signal S_t, must have the same value as the amount due to a matured rollover creditor, $D_{0,t}$. This means that the rollover face value must be set such that

$$\int_0^{\bar{D}_T(S_t)} \frac{D_{t,T}(S_t)}{\bar{D}_T(S_t)} \theta dF(\theta|S_t) + D_{t,T}(S_t) \int_{\bar{D}_T(S_t)}^{\infty} dF(\theta|S_t) = D_{0,t}, \tag{1}$$

where $\bar{D}_T(S_t) = \alpha D_{t,T}(S_t) + (1-\alpha) D_{0,T}$ denotes the aggregate face value due at time T.

The interpretation of equation (1) is as follows. If default occurs at time T, the creditors rolling over at t receive a proportional share of the projects cash flows, $\frac{D_{t,T}(S_t)}{\bar{D}_T(S_t)}\theta$. When the financial institution does not default, the entire face value $D_{t,T}(S_t)$ is repaid. Equation (1) thus says that for rollover to occur, $D_{t,T}(S_t)$ must be set such that in expectation the creditors receive their outstanding face value $D_{0,t}$.[5]

Short-term debt can be rolled over as long as the project's future cash flows are high enough such that the financial institution can find a face value $D_{t,T}(S_t)$ for which (1) holds. Given equal priority at time T, the maximum the financial institution can pledge to the short-term creditors at time t is the entire expected future cash flow from the project. This is done by setting $D_{t,T}(S_t)$ to infinity, in which case rollover creditors effectively become equity holders and long-term debtholders are wiped out. Hence, rolling over short-term debt becomes impossible when the expected future cash flows conditional on the signal S_t are smaller than the maturing face value $\alpha D_{0,t}$ owed to the short-term creditors at t. This is the case when

$$\alpha D_{0,t} > E\left[\theta | S_t\right]. \tag{2}$$

First-order stochastic dominance implies that the amount of pledgeable cash flow the financial institution has at its disposal to roll over debt at time t is increasing in the signal realization S_t. Hence, there is a critical signal $\tilde{S}_t(\alpha)$ for which (2) holds with equality:

$$\alpha D_{0,t} = E\left[\theta | \tilde{S}_t(\alpha)\right] \tag{3}$$

When the signal realization S_t is below $\tilde{S}_t(\alpha)$, the financial institution cannot roll over its short-term obligations. This is because the bank's dispersed creditors make their rollover decision in an uncoordinated fashion. They will thus find it individually rational to pull out their funding when $S_t < \tilde{S}_t(\alpha)$ in a 'fundamental bank run:' When the financial institution cannot offer short-term creditors full repayment via rollover, each individual creditor will prefer to take out his money in order to be fully repaid that way. In this case, the financial institution has to liquidate the entire project and defaults. Note that the critical signal realization below which the project is liquidated is a monotonically increasing in α, the fraction of overall debt that has been financed short-term.

The above argument implicitly assumes that short-term debt cannot be restructured at the rollover date, such that uncoordinated rollover decisions lead to inefficient liquidation at the rollover date. This assumption reflects the fact that in the presence of multiple creditors such debt restruc-

[5] Note that both $\bar{D}_T(S_t)$ and $D_{t,T}(S_t)$ are also functions of α, the fraction of creditors with debt contracts that need to be rolled over at time t. For ease of notation we will generally suppress this dependence in the text.

turings are often difficult or even impossible to achieve, mainly because of the well-known holdout problem that arises in debt restructuring. Essentially, since the Trust Indenture Act prohibits changing the timing or the payment amounts of public debt, debt restructuring must take the form of exchange offers, which usually require consent of a specified fraction of debtholders to go through. If each debtholder is small, he will not take into account the effect of his individual tender decision on the outcome of the exchange offer. This means that assuming that a sufficient number of other creditors accept the restructuring, an individual creditor prefers not to accept in order to be paid out in full.[6]

Anticipating potential early liquidation that arises when the financial institution cannot roll over its short-term obligations, the rollover face value from 0 to t must satisfy

$$\int_{S_t^L}^{\tilde{S}(\alpha)} \lambda E\left[\theta|S_t\right] dG\left(S_t\right) + \left[1 - G\left(\tilde{S}_t\left(\alpha\right)\right)\right] D_{0,t} = 1. \tag{4}$$

The interpretation of (4) is as follows. When $S_t < \tilde{S}\left(\alpha\right)$, the short-term creditors withdraw their funding at the rollover date and the financial institution defaults. Long-term debt is accelerated, and each rollover creditor receives $\lambda E\left[\theta|S_t\right] = \lambda \int_0^\infty \theta dF\left(\theta|S_t\right)$.[7] When $S_t \geq \tilde{S}\left(\alpha\right)$, short-term creditors roll over, in which case they are promised a new face value $D_{t,T}\left(S_t\right)$, which in expectation has to be worth $D_{0,t}$. Together, these two terms must be equal to the setup cost for rollover creditors to break even.

Now turn to the break-even condition for the long-term creditors. Since long-term creditors enter their debt contracts at $t = 0$ and cannot change their financing terms after that, they must break even taking an expectation across all signal realizations at the rollover date. When $S_t < \tilde{S}_t\left(\alpha\right)$, the project is liquidated at time t, long-term debt is accelerated, and the long-term creditors receive their share of the liquidation proceeds, $\lambda E\left[\theta|S_t\right] = \lambda \int_0^\infty \theta dF\left(\theta|S_t\right)$. When $S_t \geq \tilde{S}_t\left(\alpha\right)$ the project is not liquidated at time t, and the long-term creditors receive either their proportional share of the cash flow $\frac{D_{0,T}}{\bar{D}_T(S_t)}\theta$ if the financial institution defaults at time T, or they are paid back their entire face value $D_{0,T}$. Taking an expectation across all signal realizations at the rollover date t, this leads

[6] The holdout problem in debt restructuring is analyzed in more detail in Gertner and Scharfstein (1991). See also the parallel discussion on takeovers in Tirole (2006).

[7] Since long-term debtholders do not have a claim on non-matured interest, when default occurs at date 1, all creditors are treated symmetrically in bankruptcy and the cash flow from liquidation is split equally among all creditors.

to the long-term break-even condition

$$\int_{S_t^L}^{\tilde{S}_t(\alpha)} \lambda E\left[\theta|S_t\right] dG\left(S_t\right) + \int_{\tilde{S}_t(\alpha)}^{S_t^H} \left[\int_0^{\bar{D}(S_t)} \frac{D_{0,T}}{\bar{D}_T\left(S_t\right)} \theta dF\left(\theta|S_t\right) + D_{0,T} \int_{\bar{D}(S_t)}^{\infty} dF\left(\theta|S_t\right)\right] dG\left(S_t\right) = 1. \tag{5}$$

2.2 Profit to the Financial Institution and No-Deviation Condition

Consider the expected profit for the financial institution. As the residual claimant, the financial institution receives a positive cash flow at time T if two conditions are met. First, the project must not be liquidated at t, which means that the financial institution only receives a positive cash flow when $S_t \geq \tilde{S}_t\left(\alpha\right)$. Second, conditional on survival until T, the realized cash flow θ must exceed the aggregate face value owed to the creditors of different maturities, $\bar{D}_T\left(S_t\right)$. This means that we can write the expected profit the to the financial institution as

$$\Pi = \int_{\tilde{S}_t(\alpha)}^{S_t^H} \int_{\bar{D}_T(S_t)}^{\infty} \left[\theta - \bar{D}_T\left(S_t\right)\right] dF\left(\theta|S_t\right) dG\left(S_t\right). \tag{6}$$

The inner integral of this expression is the expected cash flow to the institution given a particular signal realization S_t. The outer integral takes the expectation of this expression over signal realizations for which the project is not liquidated at time t.

To find the no-deviation condition, we now calculate the payoff to the financial institution of moving one additional creditor from a long-term debt contract to a short-term debt contract. Following McAfee and Schwartz (1994), when observing this out-of-equilibrium contract offer, the creditor keeps his beliefs about all other contract offers by the financial institution unchanged.[8] The deviation condition payoff can then be calculated by differentiating the financial institution's profit (6) with respect to the fraction of rollover debt α, keeping in mind that $\bar{D}_T\left(S_t\right) = \alpha D_{t,T}\left(S_t\right) + \left(1-\alpha\right) D_{0,t}$. This yields

$$\frac{\partial \Pi}{\partial \alpha} = \int_{\tilde{S}_t(\alpha)}^{S_t^H} \int_{\bar{D}_T(S_t)}^{\infty} \left[D_{0,T} - D_{t,T}\left(S_t\right)\right] dF\left(\theta|S_t\right) dG\left(S_t\right). \tag{7}$$

[8] This *passive beliefs* restriction proposed by McAfee and Schwartz (1994) is the standard refinement used in games with unobservable bilateral contracts (see, for example, Chapter 13 in Bolton and Dewatripont (2005)). In essence, it means that when observing an out-of-equilibrium contract, a creditor believes that all other contracts have remained unchanged.

The intuitive interpretation for this expression is as follows. On the margin, the gain from moving one long-term creditor to a rollover contract is given by the differences of the marginal cost of long-term and short-term debt. Because there is one less long-term creditor, the financial institution saves $D_{0,T}$ in states in which it is the residual claimant, i.e. when $S_t \geq \tilde{S}_t(\alpha)$ and $\theta > \bar{D}_T(S_t)$. This gain has to be weighed against the marginal cost of short-term credit in those states, which is given by $D_{t,T}(S_t)$. Note that in deriving this expression we made use of the fact that the derivatives with respect to the lower integral boundaries drop out, since in both cases the term inside the integral equals zero when evaluated at the boundary.

If starting from any conjectured equilibrium maturity structure, in which all creditors just break even, we have

$$\frac{\partial \Pi}{\partial \alpha} > 0, \tag{8}$$

the financial institution has an incentive to move an additional creditor from long-term financing to a shorter maturity, keeping everybody else's financing terms fixed. The no-deviation condition thus implies that an equilibrium maturity structure will either be characterized by $\frac{\partial \Pi}{\partial \alpha} = 0$ (with the appropriate second order condition holding), or it will be a extreme maturity structure, either with all long-term debt ($\alpha = 0$ and $\frac{\partial \Pi}{\partial \alpha}\big|_{\alpha=0} \leq 0$), or all short-term rollover debt ($\alpha = 1$ and $\frac{\partial \Pi}{\partial \alpha}\big|_{\alpha=1} \geq 0$).[9]

Definition 1 *An equilibrium maturity structure is characterized by a fraction of rollover debt* α^* *and face values* $\{D_{0,T}(\alpha^*), D_{0,t}(\alpha^*), D_{t,T}(S_t, \alpha^*)\}$ *such that the following conditions are satisfied:*

1. *Creditors correctly conjecture the fraction of rollover debt* α^*.

2. *Face values* $D_{0,T}(\alpha^*)$, $D_{0,t}(\alpha^*)$ *and* $D_{t,T}(S_t, \alpha^*)$ *are set such that all creditor's break even.*

3. *The financial institution has no incentive to deviate from* α^* *by changing the maturity of one (or a subset of) individual creditors.*

[9] Note also that the discussion in the text focuses on local deviations. We do so, since in our setup local deviations are profitable if and only if global deviations are profitable. We discuss this in more detail in the proof of Proposition 1 in the appendix.

2.3 Interim Information and Maturity Shortening

Before stating our result in the general setup outlined above, we first present two simple examples to build intuition. The first example illustrates the mechanism that leads to the unraveling of short-term financing: short-term debt imposes an contractual externality on the remaining long-term creditors and long-term financing cannot be an equilibrium. The second example highlights that not any type of interim information leads to an incentive to shorten the maturity structure. In particular, when information at the rollover date is exclusively about the recovery in default, but does not affect the default probability, there is no incentive for the financial institution to deviate from long-term financing. Hence, for maturity shortening to be privately optimal for the financial institution, the signal at the rollover date must thus contain sufficient information about the financial institution's default probability, a restriction that we will make more precise when we discuss the general case in Section 2.4.

2.3.1 Example 1: Information about Default Probability

Consider a setting in which the final cash flow θ can only take two values, θ^H and θ^L. Assume that the high cash flow is sufficiently large to repay all debt at time T, whereas the low cash flow realization leads to default (i.e., $\theta^L < 1$). The probability of default at date T is thus equal to the probability of the low cash flow. At the rollover date, additional information is revealed about this probability of default: Seen from date 0, the probability of the high cash flow realization is given by p_0, and the probability of default by $1 - p_0$. At the rollover date t the probability of the high cash flow realization is updated to p_t.

For this example, we focus on the initial deviation from all long-term financing (i.e., from a conjectured equilibrium in which the fraction of short-term financing is given by $\alpha = 0$). If all financing is long-term, the break-even condition for the long-term creditors (5) can be rewritten as

$$(1 - p_0)\,\theta^L + p_0 D_{0,T} = 1, \tag{9}$$

which implies a face value for long-term debt of $D_{0,T} = \frac{1-(1-p_0)\theta^L}{p_0}$.

But is financing with all long-term debt an equilibrium maturity structure? To determine this, we need to check the no-deviation condition derived above. Since $\theta^L > 0$, the first short-term creditor can always be rolled over at time t, which implies that $D_{0,t} = 1$. For the first rollover creditor, the time-t rollover condition (1) then reduces to

$$(1 - p_t)\frac{D_{t,T}}{D_{0,T}}\theta^L + p_t D_{t,T} = 1, \tag{10}$$

which implies a rollover face value of $D_{t,T} = \frac{1-(1-p_0)\theta^L}{\theta^L p_0 + \left(1-\theta^L\right)p_t}$.

The financial institution has an incentive to deviate from all long-term financing when

$$\frac{\partial \Pi}{\partial \alpha} = p_0 D_{0,T} - E\left[p_t D_{t,T}\right] > 0. \tag{11}$$

Using the face values calculated above we can rewrite (11) as

$$1 > E\left[\frac{p_t}{\theta^L p_0 + \left(1-\theta^L\right)p_t}\right]. \tag{12}$$

A simple application of Jensen's inequality shows that this condition is always satisfied when $0 < \theta^L < 1$.[10] All long-term financing can thus *not* be an equilibrium outcome—starting from a conjectured equilibrium in which financing is all long-term, the financial institution has an incentive to shorten the maturity structure.

The intuition for why the financial institution has an incentive to deviate from long-term financing is illustrated in Figure 1. Panel A shows the face value of long-term debt, and the rollover face value, as a function of the interim signal p_t. As the figure shows, the rollover face value is a convex function of the realization of p_t, which means that unconditionally, the expected rollover face value is higher than the long-term face value. The financial institution, however, does not care about face values per se, but about the marginal cost of financing from the equityholder's perspective, i.e., the face values multiplied by the probability of being the residual claimant. This is depicted in Panel B. Note that once we multiply the face values by the survival probability p_t, the marginal cost of rollover financing becomes a concave function. Taking an expectation over all possible realizations

[10] The expression inside the expectation is a strictly concave function when $\theta^L < 1$. From Jensen's inequality we thus know that

$$E\left[\frac{p_t}{\theta^L p_0 + \left(1-\theta^L\right)p_t}\right] < \frac{E\left[p_t\right]}{\theta^L p_0 + \left(1-\theta^L\right)E\left[p_t\right]} = 1,$$

where the final equality uses the fact that $E\left[p_t\right] = p_0$.

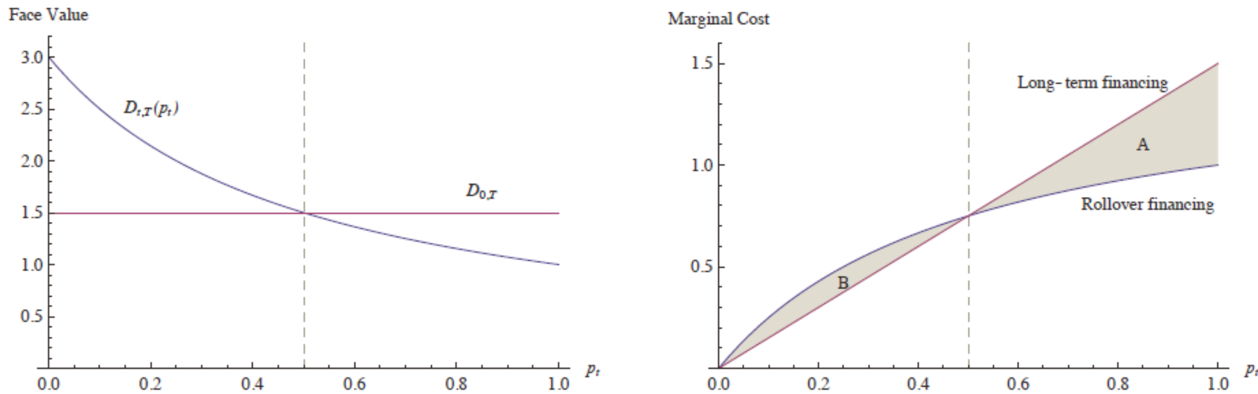

Figure 1: **Illustration: News about Default Probability.** The left panel shows the long-term face value $D_{0,T}$ and the rollover face value $D_{t,T}(p_t)$. While the long-term face value is fixed, the face value charged by the first rollover creditor is a convex function of p_t. The right panel shows that even though the expected rollover face value exceeds the long-term face value, the marginal cost of rollover finance is, in expectation, less than the marginal cost of long-term financing. Hence, an incentive to shorten the maturity structure arises. For this illustration we set $p_0 = 0.5$.

of p_t (and using that $E[p_t] = p_0$) we see that from the equityholder's perspective rollover financing is cheaper than long-term financing, which makes the deviation profitable.

The incentive to deviate from all long-term financing is driven by the concavity of the marginal cost of rollover debt. This implies that the incentive to shorten the maturity structure is stronger, the higher the variance of the signal p_t. To see this, assume that assume that p_t can only take two values, $p_0 + \sigma$ or $p_0 - \sigma$. The concavity of the marginal cost of rollover financing depicted in the right panel of Figure 1 then implies that the deviation from long-term financing becomes more profitable when σ increases. This means that in this example the incentive to shorten the maturity structure depends on the amount of interim updating of the default probability. This is consistent with the maturity shortening during financial crisis. For example, Krishnamurthy (2010) shows that maturities in the commercial paper market shortened substantially in September 2008, when, in the aftermath of Lehman's default, investors were expecting to learn which other institutions might also default.

It is also instructive to look at the two polar cases when either $\theta^L = 0$ or when $\theta^L = 1$. It turns out that in either of these cases, the deviation ceases to be profitable. When $\theta^L = 0$, there is nothing to be distributed among the creditors in the default state. Thus, the rollover creditor cannot gain at the expense of the long-term creditors by adjusting his face value at the rollover date when default is more likely. When $\theta^L = 1$, on the other hand, all debt becomes safe. In this case, default will never occur, again preventing the rollover creditor from diluting the existing long-term creditors by increasing his face value. These polar cases illustrate that it is the rollover creditor's ability to increase his face value in states when default is more likely in order to appropriate more of the bankruptcy mass θ^L that makes the deviation profitable.

2.3.2 Example 2: Information about Recovery Value

We now present a second example, in which long-term financing can be sustained as an equilibrium. In contrast to the example in Section 2.3.1, in which information released at the rollover date was exclusively about the probability of default, in this second example, interim information only affects the recovery in default, but not the default probability.

Again, assume that the final cash flow can take two values, θ^H or θ^L. However, this time we keep the probability of the high cash flow fixed at p, whereas the value of the low cash flow θ^L is random seen from date 0, and its realization is revealed at the rollover date t. Assume that θ^L is always smaller than one, such that the financial institution defaults when the low cash flow realizes, regardless of what value θ^L takes. Information revealed at date t is thus exclusively about the recovery in default.

The face value of long-term debt, assuming all long-term financing, is determined by rewriting (5) as

$$(1-p) E\left[\theta^L\right] + pD_{0,T} = 1, \tag{13}$$

which implies that $D_{0,T} = \frac{1-(1-p)E\left[\theta^L\right]}{p}$. The face value the first rollover creditor would charge can be determined by rewriting the breakeven condition for rollover creditors (1) as

$$(1-p) \frac{D_{t,T}}{D_{0,T}} \theta^L + pD_{t,T} = 1, \tag{14}$$

which, after substituting in for $D_{0,T}$, yields $D_{t,T}\left(\theta^L\right) = \frac{1-(1-p)E\left[\theta^L\right]}{p+p(1-p)\left(\theta^L - E\left[\theta^L\right]\right)}$.

Given these face values, we can now check whether the financial institution has an incentive to deviate from all long-term financing by checking the no-deviation condition. It turns out that in

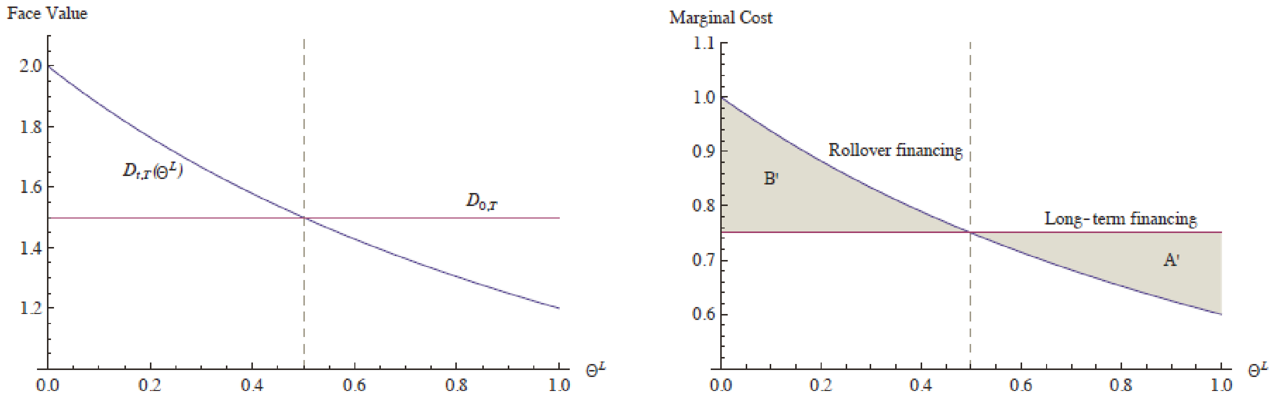

Figure 2: **Illustration: News about Recovery in Default.** The left panel shows the long-term face value and the rollover face value. Again, the long-term face value is fixed, while the rollover face value is a convex function of the realized recovery in default θ^L. In this case, the probability of default is unchanged, such that the expected marginal cost of rollover finance is higher than the expeted marginal cost of long-term financing (right panel). For this illustration we set $p = 0.5$.

this example, the financial institution has no incentive to shorten the maturity structure because

$$\frac{\partial \Pi}{\partial \alpha} = pD_{0,T} - pE\left[D_{t,T}\left(\theta^L\right)\right] < 0 \tag{15}$$

Again, this follows from a simple application of Jensen's inequality. In contrast to the earlier example, the marginal cost of rollover financing $pD_{t,T}\left(\theta^L\right)$ is now a convex, decreasing function of the recovery value θ^L, which implies that $E\left[pD_{t,T}\left(\theta^L\right)\right] > pD_{t,T}\left(E\left[\theta^L\right]\right) = pD_{0,T}$. In words, from the financial institution's perspective the marginal cost of rollover financing now exceeds the marginal cost of long-term financing, such that the deviation from all long-term financing is unprofitable. Hence, when interim information is purely about the recovery value in default, all long-term financing is an equilibrium.

This is illustrated in Figure 2. As before, the rollover face value is a convex function of the signal at the rollover date, such that the expected rollover face value is higher than the face value of long-term debt. In contrast to the earlier example, however, when multiplying the face values by the repayment probability p, the marginal cost of rollover financing remains a convex function, such that the expected marginal cost of rollover debt is higher than the expected marginal cost of long-term debt.

This second example shows that the introduction of rollover debt does not always dilute remaining long-term debt. In fact, in this example the remaining long-term creditors are *better off* after the introduction of a rollover creditor. This raises the question how this counter-example differs from the simple example in Section 2.3.1, in which shortening the maturity structure is profitable? Clearly, the difference lies in the type of information that is revealed at the rollover date. In the example in 2.3.1 the information revealed at the rollover date is purely about the probability of default, while the recovery value in default is held fixed. In 2.3.2, on the other hand, information learned at the rollover date is purely about the recovery in default, while the probability of default is held fixed. This shows that the incentive to shorten the maturity structure depends on the type of information that is revealed by the rollover date. More precisely, the signal at the rollover date must contain sufficient information about the probability of default, as opposed to the recovery in default, a notion we will make more precise in Section 2.4.

2.4 Maturity Structure Shortening: The General Case

Of course, the above examples are both special cases. First, in both examples the final cash flow was restricted to only take two values, and interim information was either about the probability of default or the recovery in default. Below, we allow the final cash flow to follow a general distribution function, and the interim signal to affect both, probability of default and recovery. Second, in both examples we only considered the initial deviation starting from a conjectured equilibrium with all long-term financing. Below we generalize the analysis to conjectured equilibrium maturity structures with both long-term debt and rollover debt. This will allow us to characterize under which condition the equilibrium maturity structure unravels to all short-term rollover debt.

To show this, we need to demonstrate that under certain conditions there is a profitable deviation for the financial institution starting from any maturity structure that involves some amount of long-term debt ($\alpha < 1$). The unique equilibrium maturity structure then exhibits all rollover financing ($\alpha = 1$): All creditors receive short-term contracts and roll over at time t. When this is the case, the equilibrium maturity structure leads to strictly positive rollover risk, such that the long-term project has to be liquidated at the rollover date with positive probability. The financial institution's incentive to shorten the maturity structure thus leads to a real inefficiency.

To extend the intuition gained through the two examples above, we can use the relation

$E[XY] = E[X]E[Y] + cov[X,Y]$ to to rewrite the deviation payoff (7). This shows the financial institution has an incentive to shorten the maturity structure whenever

$$E_s\left[D_{0,T} - D_{t,T}(S_t)\,|S_t \geq \widetilde{S}_t(\alpha)\right] E_s\left[\int_{\bar{D}_T(S_t)}^{\infty} dF(\theta|S_t)\,|S_t \geq \widetilde{S}_t(\alpha)\right]$$
$$-cov\left(D_{t,T}(S_t), \int_{\bar{D}_T(S_t)}^{\infty} dF(\theta|S_t)\,|S_t \geq \widetilde{S}_t(\alpha)\right) \tag{16}$$

is positive.

From the breakeven conditions it can be shown that $E_s\left[D_{t,T}(s)\,|S_t \geq \widetilde{S}_t(\alpha)\right] > D_{0,T}$.[11] This implies that conditional on rollover at t, the expected promised yield for rollover debt is *higher* than the promised yield for long-term debt. The first term in (16) is thus strictly negative. This is the case because the rollover face value is convex in the signal S_t—i.e. it increases more after bad signals than it decreases after good signals (as illustrated in Figures 1 and 2). However, as the residual claimant the financial institution cares not about the face value conditional on rollover, but the face value conditional on rollover in states where the financial institution does not default. This is captured by the covariance term in (16): the financial institution has an incentive to shorten its maturity provided that the covariance between the rollover face value $D_{t,T}(S_t)$ and the survival probability $\int_{\bar{D}_T(S_t)}^{\infty} dF(\theta|S_t)$ is sufficiently negative. In other words, the deviation is profitable if after bad signals and a correspondingly high rollover face value, it is unlikely that the financial institution will be the residual claimant. Hence, equation (16) shows that in the general setup the deviation to shorten the maturity structure is profitable if the signal received at the rollover date contains sufficient information regarding the probability of default, rather than just the recovery given default, confirming the intuition gained from the examples above.

We now provide a simple and economically motivated condition on the signal structure that guarantees that interim information contains sufficient information about the probability of default. (Recall that up to this point we have only assumed that the signal S_t orders the updated distribution according to first-order stochastic dominance.)

Condition 1 $D_{t,T}(S_t)\int_{\bar{D}_T(S_t)}^{\infty} dF(\theta|S_t)$ *is weakly increasing in* S_t *on the interval* $S_t \geq \tilde{S}_t(\alpha)$.

Condition 1 restricts the distribution function $F(\cdot)$ to be such that, whenever rollover is possible,

[11] See Lemma 1 in the appendix for a proof.

the fraction of expected compensation that rollover creditors receive through full repayment rather than through default is weakly increasing in the signal realization. In other words, under Condition 1 a positive signal is defined as one that increases the amount that creditors expect to receive through full repayment at maturity, as opposed to repayment through recovery in default. This condition is satisfied whenever positive information is mostly about the *probability of default*, rather than about the expected *recovery in default*.

The condition thus directly relates to the intuition gained through the two examples above: Condition 1 is satisfied in the first example, in which all interim information is about the probability of default ($p_t D_{t,T}(p_t)$ is increasing in the realization of p_t), but violated in the second example, in which all interim information was about the recovery in default ($pD_{t,T}(\theta^L)$ is decreasing in θ^L). Condition 1 thus makes the intuition gained from the two examples precise: When Condition 1 holds, the signal received at the rollover date contains sufficient information about the default probability, as opposed to the recovery in default, such that the financial institution has an incentive to shorten the maturity structure starting from any conjectured equilibrium that involves some long-term debt. This allows us to state the following general proposition.

Proposition 1 ***Equilibrium Maturity Structure (A).*** *Suppose that Condition 1 holds. Then in any conjectured equilibrium maturity structure with some amount of long-term financing,* $\alpha \in [0, 1)$*, the financial institution has an incentive to increase the amount of short-term financing by switching one additional creditor from maturity* T *to the shorter maturity* $t < T$*. The unique equilibrium maturity structure involves all short-term financing.*

Why is the financial institution unable to sustain a maturity structure in which it enters into long-term debt contracts with all (or even just some) creditors? To see this, consider what happens when the institution moves one creditor from a long-term contract to a shorter maturity while keeping the remaining long-term creditors' financing terms fixed. The difference between long-term and short-term debt is that the face value of the short-term contract reacts to the signal observed at time t. When the signal is positive, rolling over the maturing short-term debt contract at time t is cheap. When, on the other hand, the signal is negative, rolling over the maturing short-term debt at t is costly or even impossible.

The reason why the deviation to short-term financing is profitable for the financial institution is that rolling over short-term financing is cheap exactly in those states in which the financial institution is likely to be the residual claimant. This means that benefits from an additional unit of short-term financing accrue disproportionately to the financial institution. On the other hand, the signal realizations for which rolling over short-term debt is costly or even impossible are the states in which the financial institution is less likely to be the residual claimant. The costs that arise from an additional unit of short-term financing are thus disproportionately borne by the existing long-term debtholders.

Note that when the financial institution moves an additional creditor to a short-term contract, the remaining long-term creditors do not lose on a state by state basis. However, under Condition 1 existing long-term creditors are worse off when the financial institution moves an additional creditor to short-term contract. This is because rollover creditors raise their face value whenever default is likely, while they lower their face value only in states when default is less likely.

Proposition 1 also shows that this rationale is not limited just to the initial deviation from a conjectured equilibrium in which all financing is through long-term debt, as in the examples above. Rather, under Condition 1 *any* maturity structure that involves some amount of long-term debt cannot be an equilibrium. Starting from any conjectured equilibrium that involves some amount of long-term debt, an additional rollover creditor imposes a negative contractual externality on the value of long-term debt, such that the financial institution gains from moving an additional creditor from a long-term to a short-term debt contract. The financial institution's maturity structure thus unravels to all short-term financing.[12]

Reversing Condition 1, on the other hand, provides a sufficient condition under which long-term financing is the unique equilibrium. This generalizes the intuition gained through our second example, in which all interim information was about the recovery rate. More specifically, when the amount that rollover creditors expect to receive through full repayment at time T is decreasing in

[12] The derivation of the deviation payoff uses the fact that the deviation is not observed by other creditors, both long-term and short-term. An alternative assumption would be that rollover creditors notice the deviation when they roll over their debt at t. In this case, the deviation payoff would have an additional term that captures the 'infra-marginal' effect of an extra unit of rollover financing on the cost of rolling over existing short-term debt. While it is possible to incorporate this into the model, the analysis becomes significantly less tractable, without much additional economic insight. The general effect of letting existing short-term creditors react to a deviation is a slight reduction in the incentive to shorten the maturity structure further when there is already some existing short-term debt. To quantify this effect, one has to revert to numerical analysis. Rollover financing remains the unique equilibrium when negative interim information is sufficiently correlated with increases in the financial institution's probability of default, but the required amount of correlation increases relative to the setup in the paper.

the signal S_t, then the financial institution has no incentive to deviate from long-term financing. Moreover, starting from any conjectured equilibrium with some amount of short-term debt, the financial institution would have an incentive to increase the fraction of long-term debt, such that long-term financing is the unique equilibrium. This shows that when most interim information is about recovery in default, as opposed to the probability of default, long-term financing is the unique equilibrium.

Proposition 2 ***Equilibrium Maturity Structure (B).*** *Suppose that Condition 1 is reversed, i.e.,* $D_{t,T}(S_t)\int_{\tilde{D}_T(S_t)}^{\infty} dF(\theta|S_t)$ *is weakly decreasing in* S_t *on the interval* $S_t \geq \tilde{S}_t(\alpha)$. *Then the unique equilibrium maturity structure involves all long-term debt.*

2.5 Successive Unraveling of the Maturity Structure

Up to now we have focused our analysis on a situation with just one possible rollover date t. In this section we show how in a setup with multiple rollover dates the deviation illustrated above can be applied repeatedly, successively unraveling the maturity structure to the very short end.

Successive unraveling of the maturity structure is illustrated in Figure 3. Consider starting in a conjectured equilibrium in which all debt is long-term, i.e., all debt matures at time T. From our analysis with just one rollover date, we know that if everyone's debt matures at time T, under Condition 1 the financial institution has an incentive to start shortening some creditor's maturity until everyone's maturity is only of length $T-1$. But now consider the same deviation again, but from a conjectured equilibrium in which everyone's maturity is $T-1$. Then, under a condition analogous to Condition 1 the financial institution has an incentive to shorten the maturity of some creditors to $T-2$. The financial institution would do this until all creditors have an initial maturity of $T-2$, after which the whole process would repeat again, in an analogous manner. This implies that in a model with multiple rollover dates, the maturity structure can unravel all the way to the extremely short end—the financial institution writes debt contracts of the shortest possible maturity with all creditors and rolls over its entire debt every period.

To state this more formally, we now generalize Condition 1. Condition 2 is the natural extension of Condition 1 to multiple rollover dates.

Condition 2 $D_{t-1,t}(S_{t-1})\int_{\widetilde{S}_t}^{\infty} dG(S_t|S_{t-1})$ *is increasing in* S_{t-1} *on the interval* $S_{t-1} \geq \widetilde{S}_{t-1}$.

Recall that $\widetilde{S}_{t-1}$ is the signal below which rollover fails at date $t-1$, while $\tilde{S}_t$ is the signal below which rollover fails at date t given successful rollover at date $t-1$. Hence, in the spirit of Condition 1, Condition 2 states that the amount that a creditor who is rolling over at $t-1$ expects to receive through successful rollover at the next rollover date t is increasing in the signal at $t-1$. Condition 2 thus directly extends Condition 1's notion of what constitutes a positive signal to a framework with multiple rollover dates.

Proposition 3 ***Successive Unraveling of the Maturity Structure.*** *Assume that Condition 2 holds. When many rollover dates are possible, successive application of the one-step deviation principle results in a complete unraveling of the maturity structure to the minimum rollover interval.*

Intuitively, this successive unraveling of the maturity structure is a direct extension of the one-step deviation principle stated in Proposition 1. Starting from any time τ at which all creditors roll over for the first time, if Condition 2 holds, it is a profitable deviation for the financial institution to move a creditor to a shorter maturity contract, keeping all other creditors' financing terms fixed. While in the original one-step deviation this increases the financial institution's expected payoff at time T, in this case the deviation increases the financial institution's expected continuation value at the rollover date τ. Save for this adjustment, the proof of sequential unraveling of the maturity structure is similar to the proof of the one-step deviation in Proposition 1.

Conceptually, Proposition 3 demonstrates the power of the simple contractual externality that arises when a financial institution cannot commit to an aggregate maturity structure. Not only does it result in a shortening of the maturity structure, it can result in a successive shortening to the very short end of the maturity structure. This successive unraveling maximizes rollover risk and the possibility of inefficient liquidation of the long-term project.

3 Discussion

In this section we discuss some of the economic implications that result from the maturity rat race. In Section 3.1 we show that the maturity rat race leads to rollover risk that is excessive from a social perspective and highlight that, if anticipated by the market, this can lead to underinvestment relative to first-best. Section 3.2 discusses the effects of introducing covenants and seniority

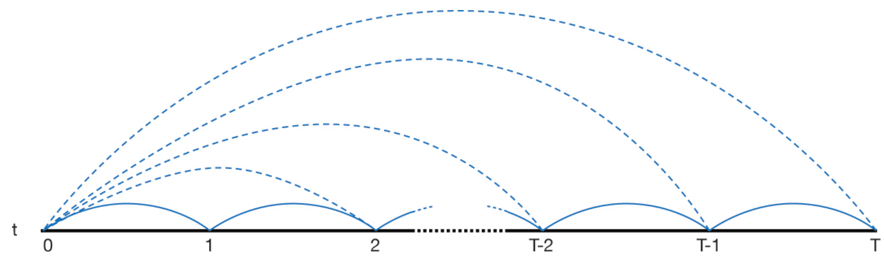

Figure 3: **Illustration of the Maturity Rat Race.** Start in a conjectured equilibrium in which all financing has maturity T (dashed line). In that case it is a profitable deviation for the financial institution to move some creditors to an initial maturity of $T-1$ and then roll over from $T-1$ to T. However, once all creditors' initial maturity is $T-1$, there is an incentive to move some creditors to an initial maturity of $T-2$. The process repeats until all financing has the shortest possible maturity and is rolled over from period to period.

restrictions into our model.

3.1 Excessive Rollover Risk and Underinvestment

The maturity mismatch that arises in our model is inefficient. Maturity mismatch does not help serve investors' interim liquidity needs, as in Diamond and Dybvig (1983). Nor does maturity mismatch serve a beneficial role by disciplining bank managers, as in Calomiris and Kahn (1991) or Diamond and Rajan (2001). In fact, we intentionally 'switched off' these channels, such that there is no reason to use rollover debt: our model is set up such that matching maturities by financing the long-term project via long-term debt is *always* efficient, whereas the short-term debt is inefficient because it leads to rollover risk and inefficient early liquidation.

This implies that the equilibrium maturity structure that emerges in our model when Condition 1 is satisfied is excessively short-term and inefficient. Specifically, the excessive reliance on short-term financing leads to excessive rollover risk and underinvestment, which is stated more formally in the following two corollaries. For simplicity, we state the two corollaries for the case with only one rollover date.

Corollary 1 ***Excessive rollover risk.*** *When Condition 1 holds, the equilibrium maturity structure ($\alpha = 1$) exhibits excessive rollover risk when, conditional on the worst interim signal, the expected cash flow of the project is less than the initial investment 1, i.e.* $\int_0^\infty \theta dF\left(\theta | S_t^L\right) < 1$.

Corollary 2 ***Some positive NPV projects will not get financed.*** *When Condition 1 holds, as a result of the maturity rat race, some positive NPV projects will not get financed. Only projects for which the NPV exceeds* $(1-\lambda)\int_{S_t^L}^{\tilde{S}_t(1)}\int_0^\infty \theta dF(\theta|S_t)\,dG(S_t)$ *will be financed in equilibrium.*

Corollary 1 states that the maturity rat race leads to a positive amount of rollover risk when, conditional on the worst signal, rolling over short-term debt fails at date t. This leads to inefficient liquidation with positive probability. Corollary 2 states that this rollover risk in turn can make projects that have positive NPV in absence of early liquidation unprofitable. To illustrate the intuition behind Corollary 2, consider a positive NPV project with expected cash flow $E[\theta] > 1$. When the project is finance entirely through short-term debt, the project will be liquidated at date t for any signal realization $S_t < \tilde{S}_t(1)$, since the uncoordinated rollover decision of the short-term creditors makes continuation of the project infeasible. Given this positive probability of liquidation at time t, the pledgeable worth of the project is given by the expected cash flows minus expected liquidation costs,

$$\underbrace{E[\theta]}_{\text{Expected cash flow}} \quad \underbrace{-(1-\lambda)\int_{S_t^L}^{\tilde{S}_t(1)}\int_0^\infty \theta dF(\theta|S_t)\,dG(S_t)}_{\text{Value destruction from early liquidation}}.$$

In equilibrium creditors will correctly anticipate these liquidation costs, such that in order to receive financing the project's expected cash flows must exceed its setup cost plus the expected liquidation costs. This means that as a result of the maturity rat race and the resulting rollover risk, some positive NPV projects will not be financed in equilibrium.

Both Corollary 1 and 2 become more powerful if we allow for many rollover dates. This is because allowing for more rollover dates leads to a successive unraveling of the maturity structure to the minimum contract length, which increases the equilibrium amount of rollover risk and thus the probability of inefficient liquidation.

Corollaries 1 and 2 raise the question why the financial institution does not internalize the rollover risk and resulting inefficiency of shortening the maturity structure. After all, in equilibrium all creditors just break even, such that ultimately the cost of the inefficient rollover risk is borne by the financial institution. The reason why the financial institution nevertheless has an incentive to shorten its maturity structure is that starting from any conjectured equilibrium with some amount of long-term debt, moving one more creditor to a rollover contract results in a first-order gain, while the increased rollover risk only causes a second-order loss to the financial institution. This can be seen from equation (7). Increasing the fraction of rollover debt α increases the probability that rollover fails by raising $\tilde{S}_t(\alpha)$, the lowest signal for which rollover is possible at the interim date. However, evaluated at the critical signal $\tilde{S}_t(\alpha)$, the payoff to the financial institution is zero. Hence, a small increase in $\tilde{S}_t(\alpha)$ only leads to a second order loss to the financial institution. In contrast increasing the fraction of rollover debt leads to a first order gain to the financial institutions in states where rollover is possible.

3.2 Seniority Restrictions and Covenants

In this subsection we briefly discuss why standard measures to protect long-term debtholders, such as covenants and seniority restrictions, may not be enough to counteract the maturity rat race.

First consider giving seniority to long-term debtholders. This clearly would diminish the short-term debtholders' ability to exploit the long-term debtholders by raising their face value in response to negative information that arrives at rollover dates. This is because if default occurs at time T and long-term debtholders are senior (in contrast to the equal priority assumption we have made throughout the paper), short-term debtholders will not receive a larger share of the liquidation mass, even if they have raised their face values at rollover. However, making long-term debtholders senior will generally *not* eliminate the financial institution's incentive to shorten the maturity structure. This is because, rather than increasing their face value, in the presence of seniority for long-term debt, the rollover creditors may decide to pull out their financing in response to negative news at the rollover date. This again would impose a negative externality on existing long-term creditors, because the short-term creditors may get repaid even in states where the long-term debtholders end up making losses. In other words, even if long-term creditors have *de jure* seniority, they will still not always have *de facto* seniority if short-term creditors can pull out their funding early. Hence seniority will generally not eliminate the financial institution's incentive to shorten the maturity structure.

Similar to seniority restrictions, covenants (see, for example, Smith and Warner (1979)) may also restrict the ability of short-term creditors to impose externalities on long-term creditors. For example, to reduce the externality that rollover creditors impose on long-term debtholders one may consider a covenant that restricts raising the face value of short-term debt above a certain threshold. This would limit the ability of rollover creditors to dilute long-term debtholders by raising their face values at the rollover date. However, as in the case of seniority restrictions, covenants on the face value of short-term debt may not always prevent the financial institution's incentive to increase short-term financing at the expense of long-term creditors: as before, short-term creditors may choose to pull out their financing at the rollover date. Thus, in order to be effective, both seniority restrictions or covenants would have to be combined with restrictions on short-term creditors withdrawing their funding at interim dates. But if short-term debtholders face restrictions on both the rollover face values they can charge and their ability to withdraw their funding at rollover dates, short-term debt becomes more and more like long-term debt.

In addition, even to the extent that covenants or seniority restrictions can reduce the incentive to shorten the maturity structure, it is not clear that the financial institution will actually use them. In particular, to the extent that financial institutions attach a high value to financial flexibility, they may not be willing to counteract the maturity rat race through covenants or restrictive seniority clauses. Flannery (1994), for example, argues that it is usually hard or even undesirable for financial institutions to use covenants. In fact, this is what we seem to observe in practice: in contrast to corporates, most debt financing used by financial institutions does not contain covenants.

Finally, even in cases where seniority restrictions or covenants can restore an equilibrium in which all financing is long-term, a second, inefficient equilibrium in which all financing is short-term may still continue to exist. The reason is that, given that all other lenders are only providing short-term financing, it is not individually rational for the financial institution to move an individual creditor to long-term financing. This is because to induce an individual creditor to move from a short-term to a long-term contract when everybody else is extending only short-term credit, the financial institution has to promise a high interest rate in order to compensate the long-term creditor for the risk that the remaining short-term creditors may pull out their funding at time t, leaving the long-term creditor stranded. At that interest rate, however, the financial institution is better off under all short-term financing. Through this mechanism, a financial institution with dispersed creditors can get caught in a short-term financing trap, and only a coordinated, simultaneous move by a critical mass of creditors would allow the financial institution to regain access to long-term credit markets.

In summary, thus, neither seniority to long-term debtholders nor covenants are likely to be able to fully resolve the financial institution's incentive to shorten the maturity structure.

4 Conclusion

We provide a model of equilibrium maturity structure for financial institutions that deal with multiple creditors. Our analysis shows that a contractual externality between long-term and short-term debtholders can lead to an inefficient shortening of the maturity structure when the financial institution deals with creditors on a bilateral basis and cannot commit to an aggregate maturity structure. In particular, whenever interim information is mostly about the probability of default, rather than the recovery in default, all short-term financing is the unique equilibrium. This also implies that incentive to shorten the maturity structure is particularly strong during periods of high volatility, such as financial crises, when investors expect substantial default-relevant interim information. The resulting maturity mismatch is inefficient, which stands in contrast to a number of other existing theories of maturity mismatch. Hence, to the extent that maturity mismatch is driven by the forces outlined in this paper, it may be desirable to include limits on maturity mismatch in future financial regulation.

5 Appendix

Proof of Proposition 1: To prove the claim we need to show that starting from any conjectured equilibrium involving any amount of long-term debt, i.e. for all $\alpha \in [0,1)$, in expectation the financial institution is better off by moving an additional creditor to a rollover contract. From (7) we know that this is the case when

$$E\left[(D_{0,T}-D_{t,T})\int_{\bar{D}(S_t)}^{\infty} dF\left(\theta|S_t\right)|S_t \geq \widetilde{S}_t\left(\alpha\right)\right] > 0. \tag{17}$$

Implicit in equation (17) is that as the equityholder, the financial institution only gets paid when rollover succeeds at time t (i.e. $S_t \geq \widetilde{S}_t(\alpha)$) and when the project's cash flow θ exceeds the total value of debt that is to be repaid at time T, $\bar{D}(S_t)$.

Before proving that (17) holds for any $\alpha \in [0,1)$ under the Condition 1, we first establish a lemma that will be useful in the proof.

Lemma 1 $E\left[\frac{1}{D_{t,T}(S_t)} - \frac{1}{D_{0,T}}|S_t \geq \widetilde{S}_t(\alpha)\right] = 0.$

Proof. Using (1) and (4), we can write the rollover breakeven constraint as

$$\int_0^{\bar{D}(S_t)} \frac{D_{t,T}(S_t)}{\bar{D}(S_t)} \theta dF(\theta|S_t) + D_{t,T}(S_t) \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t) = K, \tag{18}$$

where we define

$$K = \frac{1 - \int_{S_L}^{\widetilde{S}_t(\alpha)} \lambda E[\theta|S]\, dG(S_t)}{\int_{\widetilde{S}_t(\alpha)}^{S^H} dG(S_t)}. \tag{19}$$

In similar fashion, we can rewrite the long-term breakeven constraint (5) as

$$E\left[\int_0^{\bar{D}(S_t)} \frac{D_{0,T}}{\bar{D}(S_t)} \theta dF(\theta|S_t) + D_{0,T} \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t)\, |S_t \geq \widetilde{S}_t(\alpha)\right] = K. \tag{20}$$

To show that $E\left[\frac{1}{D_{t,T}(S_t)} - \frac{1}{D_{0,T}}|S_t \geq \widetilde{S}_t(\alpha)\right] = 0$, note that from (18) we know that

$$\frac{1}{D_{t,T}(S_t)} = \frac{1}{K}\left[\int_0^{\bar{D}(S_t)} \frac{1}{\bar{D}(S_t)} \theta dF(\theta|S_t) + \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t)\right],$$

and from (20) it follows that

$$\frac{1}{D_{0,T}} = \frac{1}{K} E\left[\int_0^{\bar{D}(S_t)} \frac{1}{\bar{D}(S_t)} \theta dF(\theta|S_t) + \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t)\, |S_t \geq \widetilde{S}_t(\alpha)\right].$$

This implies that

$$\frac{1}{D_{0,T}} = E\left[\frac{1}{D_{t,T}(S_t)}|S_t \geq \widetilde{S}_t(\alpha)\right].$$

Note that by Jensen's inequality this also implies that $E\left[D_{t,T}(S_t)\,|S_t \geq \widetilde{S}_t(\alpha)\right] > D_{0,T}$. ■

We now proceed to prove that for any maturity structure that involves any amount of long-term debt, the financial institution has an incentive to shorten its maturity.

Proof. Assume that Condition 1 holds. In order to prove the assertion, we rewrite (17) as

$$E\left[\left(D_{0,T}-D_{t,T}\left(S_t\right)\right)\int_{\bar{D}(S_t)}^{\infty}dF\left(\theta|S_t\right)|S_t\geq\widetilde{S}_t\left(\alpha\right)\right] \tag{21}$$

$$= E\left[\left(\frac{1}{D_{t,T}\left(S_t\right)}-\frac{1}{D_{0,T}}\right)D_{t,T}\left(S_t\right)D_{0,T}\int_{\bar{D}(S_t)}^{\infty}dF\left(\theta|S_t\right)|S_t\geq\widetilde{S}_t\left(\alpha\right)\right] \tag{22}$$

$$= \underbrace{E\left[\frac{1}{D_{t,T}\left(S_t\right)}-\frac{1}{D_{0,T}}|S_t\geq\widetilde{S}_t\left(\alpha\right)\right]}_{=0}E\left[D_{t,T}\left(S_t\right)D_{0,T}\int_{\bar{D}(S_t)}^{\infty}dF\left(\theta|S_t\right)|S_t\geq\widetilde{S}_t\left(\alpha\right)\right]$$

$$+cov\left(\frac{1}{D_{t,T}\left(S_t\right)}-\frac{1}{D_{0,T}},D_{t,T}\left(S_t\right)D_{0T}\int_{\bar{D}(S_t)}^{\infty}dF\left(\theta|S_t\right)|S_t\geq\widetilde{S}_t\left(\alpha\right)\right) \tag{23}$$

Using Lemma 1 and dividing by the constant term D_{0T}, we see that (17) holds whenever

$$cov\left(\frac{1}{D_{t,T}\left(S_t\right)}-\frac{1}{D_{0,T}},D_{t,T}\left(S_t\right)\int_{\bar{D}(S_t)}^{\infty}dF\left(\theta|S_t\right)|S_t\geq\widetilde{S}_t\left(\alpha\right)\right)>0. \tag{24}$$

We know that for all $S_t \geq \widetilde{S}_t(\alpha)$, $D_{t,T}(S_t)$ is decreasing in S_t. This follows from stochastic dominance. This implies that $\frac{1}{D_{t,T}(S_t)} - \frac{1}{D_{0,T}}$is increasing in S_t. Moreover, from Condition (1) we know that on the interval $S_t \geq \widetilde{S}_t(\alpha)$, $D_{t,T}(S_t)\int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t)$ is weakly increasing in S_t. We also know that $D_{t,T}(S_t)\int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t)$ must be strictly increasing on some interval (when $S_t = \widetilde{S}_t(\alpha)$ the expression is zero, while it is strictly positive for positive realizations of S_t). This implies that the covariance of these two terms is indeed positive, which establishes that (17) indeed holds for any $\alpha \in [0,1]$.

Finally, it remains to establish that $\alpha = 1$ is indeed an equilibrium. The above analysis establishes that there is no profitable local deviation from $\alpha = 1$, since (17) holds at $\alpha = 1$, which means that moving one creditor from a short-term do a long-term contract is strictly unprofitable. It thus remains to check a global deviation, in which the financial institution deviates from a conjectured equilibrium with $\alpha = 1$ by offering long-term contracts to multiple creditors. In this situation, each creditor, only observing his own contract, will assume that all other creditors' contracts remain unchanged (this follows from the concept of 'passive beliefs,' introduced by McAfee and Schwartz (1994): when observing an out-of-equilibrium contract in a game with unobservable offers, a player assumes that all other offers remain unchanged). This implies that also a global deviation from $\alpha = 1$ cannot be profitable, because the payoff from a global deviation is just equal to the payoff from a local deviation scaled by the mass of creditors moved from short-term to long-term debt contracts. As we saw above, the payoff from the local deviation from $\alpha = 1$ is negative. ■

Proof of Proposition 2: The proof follows the same steps as the proof of Proposition **1.** The only change is that the direction of the inequality in (24) is reversed. Following the same argument, all long-term financing is then the unique equilibrium when $D_{t,T}(S_t) \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t)$ is weakly decreasing in S_t on the interval $S_t \geq \widetilde{S}_t(\alpha)$.

Proof of Proposition 3: Assume that the first date at which all creditors roll over is date $t \leq T$. We want to consider a deviation from a conjectured equilibrium in which all creditors first roll over at time t, and then roll over every period after that until T. Of course, when $t = T$, the project is financed entirely through long-term debt and the proof of Proposition 1 implies that there is an incentive to shorten the maturity structure to $T-1$. When $t < T$, on the other hand, we need to extend the proof of Proposition 1. Intuitively, rather than showing that the deviation raises the expected time T payoff of the financial institution, we now show that it raises the expected time t continuation value.

Let V_t be the time-t continuation value for the financial institution. This continuation value is a function of three state variables. The first is the face value of debt that has to be rolled over at time t. Consistent with our earlier notation, we denote the aggregate face value maturing at time t by $\bar{D}_t$. The aggregate face value that needs to be rolled over at time t is the sum of the face value issued at time 0 and at the potential earlier rollover date $t-1$, i.e. $\bar{D}_t = \alpha D_{t-1,t}(S_{t-1}) + (1-\alpha) D_{0,t}$. The second state variable is the time-t distribution of the final cash flow. A sufficient statistic for this distribution is the time t signal S_t. The third state variable is the remaining time to maturity, $T - t$ (which is also equal to the number of the remaining rollover dates). Together this implies that, conditional on all the information released up to time t, we can write the time t continuation value for the financial institution as

$$V_t\left(\bar{D}_t, S_t, T-t\right). \tag{25}$$

Seen from $t = 0$, the expected continuation value for the entrepreneur at time t is then given by

$$\int_{\tilde{S}_{t-1}}^{\infty}\int_{\tilde{S}_{t}}^{\infty} V_t\left(\bar{D}_t, S_t, T-t\right) dG\left(S_t|S_{t-1}\right) dG\left(S_{t-1}\right), \tag{26}$$

where $\tilde{S}_{t-1}$and $\tilde{S}_t$ are the signals below which the project is liquidated at times t and $t-1$, respectively, because rollover fails. Note that because the face value of the debt that is rolled over at $t-1$ depends on the signal at $t-1$, we have to take an expectation over the S_{t-1} when calculating the expected continuation value at time t.

Now take the derivative of (26) with respect to α. This yields

$$\int_{\tilde{S}_{t-1}}^{\infty}\int_{\tilde{S}_{t}}^{\infty} \frac{\partial V_t}{\partial \bar{D}_t}\frac{d\bar{D}_t}{d\alpha} dG\left(S_t|S_{t-1}\right) dG\left(S_{t-1}\right). \tag{27}$$

To prove that there is a profitable deviation from a conjectured equilibrium in which all creditors roll over for the first time at time t, we need to show that this expression is positive. From the definition $\bar{D}_t = \alpha D_{t-1,t}\left(S_{t-1}\right) + (1-\alpha)D_{0,t}$ we know that $\frac{d\bar{D}_t}{d\alpha} = D_{t-1,t}\left(S_{t-1}\right) - D_{0,t}$. This means that we need to show that

$$\int_{\tilde{S}_{t-1}}^{\infty}\int_{\tilde{S}_{t}}^{\infty} \frac{\partial V_t}{\partial \bar{D}_t}\left[D_{t-1,t}\left(S_{t-1}\right) - D_{0,t}\right] dG\left(S_t|S_{t-1}\right) dG\left(S_{t-1}\right) > 0. \tag{28}$$

Before we proceed with the proof, we now extend Lemma 1 to the multiperiod setting.

Lemma 2 $E\left[\frac{1}{D_{t-1,t}(S_{t-1})} - \frac{1}{D_{0,t}}|S_{t-1} \geq \widetilde{S}_{t-1}\left(\alpha\right)\right] = 0.$

Proof. Proceeding analogously to the steps in the proof of Lemma 1, we can write the rollover breakeven constraint from $t-1$ to t as

$$\int_{S_t^L}^{\widetilde{S}_t(\alpha)} \frac{D_{t-1,t}\left(S_{t-1}\right)}{\bar{D}_t\left(S_{t-1}\right)}\lambda E\left[\theta|S_t\right] dG\left(S_t|S_{t-1}\right) + D_{t-1,t}\left(S_{t-1}\right)\int_{\widetilde{S}_t(\alpha)}^{S_t^H} dG\left(S_t|S_{t-1}\right) = K, \tag{29}$$

where we define

$$K = \frac{1 - \int_{S_{t-1}^L}^{\widetilde{S}_{t-1}(\alpha)} \lambda E\left[\theta|S_{t-1}\right] dG\left(S_{t-1}\right)}{\int_{\widetilde{S}_{t-1}(\alpha)}^{S_{t-1}^H} dG\left(S_{t-1}\right)}. \tag{30}$$

In similar fashion, we can rewrite the breakeven constraint for creditors that lend from 0 to t as

$$E\left[\int_{S_t^L}^{\widetilde{S}_t(\alpha)} \frac{D_{0,t}}{\bar{D}_t\left(S_{t-1}\right)}\theta dG\left(S_t|S_{t-1}\right) + D_{0,t}\int_{\widetilde{S}_t(\alpha)}^{S_t^H} dG\left(S_t|S_{t-1}\right)|S_{t-1} \geq \widetilde{S}_{t-1}\left(\alpha\right)\right] = K. \tag{31}$$

To show that $E\left[\frac{1}{D_{t,T}(S_t)} - \frac{1}{D_{0,T}}|S_t \geq \widetilde{S}_t\left(\alpha\right)\right] = 0$, note that from (29) we know that

$$\frac{1}{D_{t-1,t}\left(S_{t-1}\right)} = \frac{1}{K}\left[\int_{S_t^L}^{\widetilde{S}_t(\alpha)} \frac{1}{\bar{D}_t\left(S_{t-1}\right)}\theta dG\left(S_t|S_{t-1}\right) + \int_{\widetilde{S}_t(\alpha)}^{S_t^H} dG\left(S_t|S_{t-1}\right)\right], \tag{32}$$

and from (31) it follows that

$$\frac{1}{D_{0,t}} = \frac{1}{K}E\left[\int_{S_t^L}^{\widetilde{S}_t(\alpha)} \frac{1}{\bar{D}_t\left(S_{t-1}\right)}\theta dG\left(S_t|S_{t-1}\right) + \int_{\widetilde{S}_t(\alpha)}^{S_t^H} dG\left(S_t|S_{t-1}\right)|S_{t-1} \geq \widetilde{S}_{t-1}\left(\alpha\right)\right]. \tag{33}$$

This implies that

$$\frac{1}{D_{0,t}} = E\left[\frac{1}{D_{t-1,t}\left(S_{t-1}\right)}|S_{t-1} \geq \widetilde{S}_{t-1}\left(\alpha\right)\right]. \tag{34}$$

■

We now proceed analogously to Proposition 1 to rewrite (28) in as a covariance. Following the same steps as in the proof of Proposition 1 and applying Lemma 2, we find that the deviation is profitable when

$$cov\left(\frac{1}{D_{t-1,t}\left(S_{t-1}\right)} - \frac{1}{D_{0,t}}, D_{t-1,t}\left(S_{t-1}\right)\int_{\tilde{S}_t}^{\infty} -\frac{\partial V_t}{\partial \bar{D}_t}dG\left(S_t|S_{t-1}\right)|S_{t-1} \geq \widetilde{S}_{t-1}\left(\alpha\right)\right) > 0. \tag{35}$$

This condition corresponds to equation (24) in the proof of Proposition 1.

As before, we know that $\frac{1}{D_{t-1,t}(S_{t-1})} - \frac{1}{D_{0,t}}$ is increasing in S_{t-1}. Hence, a sufficient condition for the deviation to be profitable is that

$$D_{t-1,t}\left(S_{t-1}\right)\int_{\tilde{S}_t}^{\infty} -\frac{\partial V_t}{\partial \bar{D}_t}dG\left(S_t|S_{t-1}\right) \tag{36}$$

is increasing in S_{t-1} when $S_{t-1} \geq \widetilde{S}_{t-1}$. Recall that from Condition 2 we know that

$$D_{t-1,t}\left(S_{t-1}\right)\int_{\tilde{S}_t}^{\infty} dG\left(S_t|S_{t-1}\right) \tag{37}$$

is increasing in S_{t-1} when $S_{t-1} \geq \widetilde{S}_{t-1}$. We now show that if Condition 2 holds, then is has to be the case that (36) is increasing in S_{t-1} such that (35) holds.

Proof. To build intuition, consider first what happens if $-\frac{\partial V_t}{\partial \bar{D}_t}$ were independent of S_{t-1} and S_t. If this were the case, (36) would be equal to (37) multiplied by a constant, such that (37) would immediately imply (36). Of course, $-\frac{\partial V_t}{\partial \bar{D}_t}$ is not a constant and depends both on S_{t-1} and S_t. However, we now show that this dependence works in favor of the proof. In other words, if (37) implies (36) when $-\frac{\partial V_t}{\partial \bar{D}_t}$ is a constant, it also implies (36) when we allow for $-\frac{\partial V_t}{\partial \bar{D}_t}$ to depend on S_t and S_{t-1}. In order to see this, it is useful to think of the continuation value V_t as an option on the final payoff, and use the result that the value of an option is convex in its moneyness. When the signal S_{t-1}, is higher, the amount to be rolled over at date t, $\bar{D}_t$, is lower. But when $\bar{D}_t$ is lower, this means that for any realization of S_t, the financial institution's option on the final payoff is further in the money. When the option is further in the money, the 'option delta,' $-\frac{\partial V_t}{\partial \bar{D}_t}$, is larger, because the value of the convexity of the option value. Hence, $-\frac{\partial V_t}{\partial \bar{D}_t}$ is increasing in S_{t-1}. Similarly, when S_t is high, the probability that the option will be in the money. Again, this increases the delta of the option value. However, this means that if we know that $D_{t-1,t}(S_{t-1}) \int_{\tilde{S}_t}^{\infty} dG(S_t|S_{t-1})$ is increasing in S_{t-1}, we know *a fortiori* that $D_{t-1,t}(S_{t-1}) \int_{\tilde{S}_t}^{\infty} -\frac{\partial V_t}{\partial \bar{D}_t} dG(S_t|S_{t-1})$ is increasing in S_{t-1}, which completes the proof. ■

Proof of Corollary 1: Since early liquidation is always inefficient in this model, the socially optimal level of rollover risk is zero. Any positive probability of liquidation means that there is excessive rollover risk. The unraveling of the maturity structure to all short-term financing leads to positive rollover risk when conditional on the worst interim signal the expected cash flow is less than 1, i.e.

$$\int_0^{\infty} \theta dF\left(\theta | S_t^L\right) < 1. \tag{38}$$

Proof of Corollary 2: Proof follows directly from the discussion in the main text.

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