NBER WORKING PAPER SERIES

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Working Paper 16409
http://www.nber.org/papers/w16409

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2010

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NBER Working Paper No. 16409
September 2010
JEL No. G23,H7,H72,H74,H83

ABSTRACT

Most state and local retirement plans strive for full funding, at least by actuarial standards. Funding measured at market values fluctuates and often falls short. A common argument for full funding is that pensions are a form of deferred compensation that does not justify a debt. The paper examines public finance, political economy, and financial market issues that bear on optimal funding, broadly and in a series of models.

In a model where most taxpayers hold debt and face intermediation costs, returns on pension assets are less than taxpayers' cost of borrowing. Pension funding is costly and hence zero funding is optimal. The model also implies that unfunded pension promises are properly discounted at a rate strictly greater than the government's borrowing rate. If pension funds serve as collateral, funding can be warranted despite the cost. This is shown in a model with legal ambiguity and default risk. Except in special cases, the optimal funding ratio is less than full funding.

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"Neither a borrower nor a lender be"

(Shakespeare)

1. Introduction

Most state and local retirement plans strive for full funding as measured by actuarial standards. Funds are commonly invested in risky assets. Hence actual funding ratios—the ratios of assets to accrued benefit obligations—fluctuate and are often less than 100%. Press reports about underfunding naturally cause taxpayer anxiety. Recent experience follows this pattern. Though average funding ratios were over 80% before the recent financial crisis, many state and local pension funds are now seriously underfunded (Munnell et al. 2008, 2010). Concerns about funding are reinforced by controversies about actuarial standards. Some economists have argued that officially reported funding ratios are inflated because pension obligations are computed at excessively high discount rates (Novy-Marx and Rauh 2009).

These concerns raise questions about pension funding. What is full funding? Is it optimal? Should it be required at all times or only in expectation? How should government obligations be reported to the public? These questions have important ramifications for financial markets and for state and local governments. Full funding all the time would require overfunding on average and perhaps restrictions on investments. Giving politicians discretion could undermine balanced budget restrictions and thus alter the political economy of state and local governments.

Questions of optimal funding are complex because they are intrinsically linked to a range of challenging economic, financial, and political-economy issues. This includes federal tax laws as motivation for deferred compensation; the political economy of balanced budget rules; the risk sharing implications of defined benefit plans (DB) as compared to defined contributions (DC); the equity premium and its ramifications for investment strategies; and labor issues relating to career employment.

This paper first reviews these issues broadly and then examines a series of models. The main conclusion is that optimal funding depends on taxpayers' cost of funds and on the presence of legal ambiguities and default risks. The analysis is tailored to state and local public finance; the paper is not about federal debt and pensions programs.

The main model implies that *zero* funding of public pensions is optimal. Because empirically over 75% of U.S. families hold debt, the model assumes that most taxpayers/voters are debtors. Intermediation costs on debt create a wedge between taxpayers' discount rates and the rates of return on asset markets, which gives voters an incentive to leave pensions unfunded. The voting equilibrium is nonetheless consistent with a balanced budget rule that prohibits public debt.

In an extension with legal ambiguity and default risk, partial funding can be optimal because it serves as collateral. Funding allows employers to reduce total compensation because unfunded promises create an undiversifiable risk for employees. Optimal pension funding balances intermediation cost against labor cost. This model yields funding ratios near 100% if employees are highly risk averse and if they rely on the pension for most of their retirement income. However, optimal funding is always strictly less than full, except in a behavioral scenario where employees have more "pessimistic" default expectations than their employer.

The analysis has implications for pension accounting and for the tradeoff between DB and DC plans. First, the appropriate discount rate for unfunded DB pensions is the taxpayers' marginal cost of funds. If most taxpayers are borrowers, the appropriate discount rate is a risk-adjusted *borrowing* rate—always a higher rate than the safe interest rate. Second, because funding is costly and full funding is usually suboptimal, regulations that impose full funding would undermine employer incentives to offer DB plans. Most private employers phased out their DB plans after costly funding and insurance requirements were imposed in the 1970s. If current anxieties about public retirement plans lead to excessive funding requirements, the effects may prove similarly destructive.

The paper is organized as follows. Section 2 reviews pension funding in general and comments on funding policies, actuarial standards, and investment strategies of public pension funds. Section 3 sets up a stochastic overlapping generations model with local property taxes to study pension funding. Section 4 examines the impact of legal ambiguities and default risk. Section 5 comments on incomplete financial markets. Section 6 concludes.

2. The Problem of Pension Funding

This section examines several conceptual issues that make pension funding problematic.

A key distinction is between Defined Benefit (DB) and Defined Contribution (DC) plans. DB plans make specific promises about retirement income that are not necessarily funded. Unfunded promises create obligations for the employer just like a bond. However, DB pension obligations tend to be more complicated than bonds because promised payments are often annuitized, contingent on earnings records, and inflation indexed. Default risk is considered low due to strong legal protections (see Brown and Wilcox 2009; Peng 2009). About 80% of state and local retirement plans are DB plans (Munnell et al 2008). In DC plans, employer promises are limited to making contributions. Because DC plans are fully funded by construction, they are not interesting for this paper except for comparison.

2.1. The Ambiguous Meaning of Full Funding

There are at least three conceptually different ways to interpret full funding. I will call them the accounting view, the finance view, and the populist view.

The accounting view considers a pension fully funded if an actuarial measure of fund assets at some date equals an actuarial measure of accrued liabilities. Accounting rules for U.S. state and local governments are set by the Government Accounting Standards Board (GASB). The key rule for pension accounting, GASB 25, gives plan sponsors a choice between several different actuarial methods to compute pension obligations (see Peng 2009). One measure, the accrued benefits obligation (ABO), seeks to determine the present value of benefits earned by current plan participants at the valuation date, assuming no accrual of obligations in the future. All others are projection methods, which involve (1) estimating the lifetime benefits of current participants, including projected future earnings, (2) allocating the cost to past and future service, and (3) treating the past-service component as accrued liability. The most commonly used projection method is Entry-Age Normal (EAN), which allocates cost over time in proportion to earnings (since entry).

Importantly, GASB gives plan sponsors wide discretion in discounting future benefits and allows them to tie the discount rate to the expected return on assets. On the asset side, smoothing methods are common that spread the recognition of a capital gain or loss over several years. This means the actuarial value of assets is a moving average of market values. Measures of funding are the *unfunded actuarial accrued liability* (UAAL), the gap between actuarial liabilities and assets, and the *funding ratio*, which is the ratio of actuarial assets over liabilities. If a plan sponsor makes contributions under GASB rules, the plan should be fully funded on average, but the funding ratio will almost always differ from 100% due to capital gains and losses. GASB rules specify that any positive or negative UAAL should be amortized over time by raising or lowering new contributions relative to the "normal" contribution.

The finance view considers a pension fully funded if the market values of assets equals the present value of promised pensions (e.g., Novy-Marx and Rauh 2009). Distinctive are the rejection of smoothing methods to value assets, the use of state-contingent claims pricing to value future benefits, and the use of economic reasoning to ascertain the scope of obligations (Bulow 1982). Expected asset returns are rejected as discount rates for liabilities. Similar to accounting view, underfunding is measured at a point in time, and degrees of funding are computed as ratios or differences of assets and liabilities.

By populist view, I mean the view reflected in newspaper stories that portray any possible need for future taxpayer support of a public pension fund as scandalous (a "bail out"). This view treats a pension as fully funded only if current pension assets cover all obligations in all future states of nature, even under the most adverse conditions, so that there is absolutely no risk of a shortfall that might require compensating employer contributions.

Zero risk of a shortfall is a much more demanding standard than the others, and perhaps impossible to meet. Full funding in the finance or accounting sense does not rule out underfunding in the future. Indeed, the odds should be about 50:50 with symmetric shocks and fair accounting. There are only two ways to guarantee no future shortfall: a match of current assets and liabilities combined with perfect hedging of all future risks, or an extreme over-collateralization of expected obligations. Perfect hedging is difficult because a DB pension is typically a complicated contingent claims. Collateral is costly, *per se* and because excess funds are difficult to recover by the sponsor.

Uncertainty about future funding ratios can be reduced by a strategy of investing in (relatively) safe assets. Then a modest over-collateralization would rule out future shortfalls with near certainty. Thus the populist view in effect calls for substantial overfunding by actuarial standards combined with a low risk investment strategy.

Fiscally conservative assumptions in the valuation may be a practical way to ensure a funding buffer. This may explain why allegations of underfunding receive so much public attention. If the public is mainly concerned about risk, any study that finds underfunding under pessimistic assumptions is troubling, even if the plan is certified as fully funded by the accountants and perhaps overfunded under a range of more reasonable assumptions.

The multiple meanings of full funding expand the question in the title. The question is not only if pensions should be fully funded, but also in what sense and why.

2.2. Deferred Compensation and Balanced Budget Rules

Pension promises are a form of deferred employee compensation. In exchange for a future pension, employees accept a reduced current salary.

Almost all states and local governments operate under balanced budget rules. The operating budget (general fund) is supposed to be balanced every year. A separate capital budget allows for bond financing, but usually subject to voter approval. Bonds are repaid out of operating funds as the capital asset depreciates. Pension funds are organized as separate entities, funded by contributions from the operating budget and dedicated to paying retirees. If pensions are fully funded, the budget framework ensures that government net worth is always positive.

Underfunded public pensions complicate this fiscal framework. A shortfall due to unexpectedly low investment returns reduces government net worth. A shortfall because of missed contributions means that the operating budget understates employee compensation. Either way, a funding gap encumbers future operating funds just like a bonded debt. If the budget can be balancing by reducing current wage payments in exchange for unfunded pension promises, the balanced budget rule reduces to a prohibition against bond issues. Thus pension funding is intrinsically linked to the political economy of balanced budgets.

This raises two questions. First, what motivates DB retirement plans in the public sector? Tight funding restrictions might be defensible if DB plans had no apparent rationale apart from evading balanced budget rules, but less so if there are efficiency arguments. Second, what justifies balanced budget rules? Without such rules, special purpose pension funds are meaningless. Though citizens might still be interested in measuring pension obligations to estimate future taxes, retirement funds would be interchangeable with other public funds.

2.3. What Motivates Employer Pensions?

The many arguments for employer pensions fall into three main groups—taxes, risk sharing, and human capital/labor issues.

The tax argument is straightforward. Compensation paid in form of a pension is taxable only when the pension is paid out. Hence deferred compensation means deferred taxes. Assets in funded plans compound without being taxed repeatedly. Not surprisingly, pensions—both private and public—became popular after WWII, at a time of high marginal taxes on regular, non-sheltered savings. Note, however, that the tax argument applies to all deferred compensation and does not provide a rationale for DB plans. A DC plan would provide the same tax shelter with less employer entanglement.

Risk sharing provides a clear distinction between DC and DB plans. By construction, the employer has no obligation to a DC plan beyond making the initial contribution. In a DB plan, in contrast, the employer is responsible for specific benefits.

There are two microeconomic risk-sharing issues that favor DB pensions. One is adverse selection in the market for life annuities. Annuities are less subject to adverse selection when provided to employees as group than if employees tried to buy annuities individually. Such risk pooling is automatic in a DB plan, but difficult to implement in a DC plan. Second, DB investments can be more diversified as they may include “alternative” asset classes not easily held in a DC plan. Both issues are microeconomic in the sense that risks is shared among employees, so the benefit has essentially no cost to the employer.

There are also two sets of “aggregate” risks, one tied to funding strategy, the other tied to the benefit design. The funding-related uncertainty is about real investment returns. In a DB plan, the sponsor in effect owns the risk and returns of the investment portfolio—though under some conditions, the risk is shared with working age plan members (see Bohn 2010). Benefit-related uncertainty is caused by a variety to features that make the real cost of pensions payments stochastic, e.g., an indexation of benefits to future wages, deviations from one-for-one indexing to inflation, and uncertainty about beneficiaries’ collective longevity.

Human capital issues are complex and arguably job and firm specific, but relevant. DB pensions are commonly tied to final salaries times years of service, which means they favor long-reserving employees who earn raises throughout their career. They penalize employees who quit early—with final nominal salary far below the expected end-of-career level—or are unsuccessful. Hence a DB pension can serve as an incentive or retention mechanism.

The specifics depend on the labor market. One polar case is a competitive labor market. Then total compensation at all times equals the marginal product of a worker’s wage. Pension accruals that increase with age would be reflected in a flatter age-salary profile, and restrictions on mobility due to pensions would be inefficient. A second polar case is a unionized work place with seniority system, where young employees are paid less than their marginal product in exchange for an entitlement to earn a premium later. A DB pension can facilitate rewards for seniority. More generally, firm-specific human capital and frictions to mobility may make labor relationships imperfectly competitive. A DB pension can help facilitate implicit contracts that encourage efficient investment in human capital; but it might become an obstacle to job mobility.

It is an open question why the public sector has not followed the private sector trend towards DC plans. DB pensions were common in the private sector in the 1950s to 1980s. Since then, many private firms have terminated their DB plans and shifted to DC or Cash Balance (hybrid) pension schemes.

One line of explanation is that the demise of private sector DB pensions is inefficient and regrettable—a result of bad regulation and credit risk. A mandatory pension guarantee systems is in effect a tax on well-funded plans. ERISA gave creditworthy employers incentives to convert to DC, causing adverse selection and triggering a death spiral of more plan terminations and adverse selection. In this view, DB plans in the public sector are still efficient because they are not subject to the same regulations. The ability to offer DB plans may give public employers a comparative advantage in recruiting.

An opposing view holds that the move from DB to DC in the private sector was an efficient response to increased job mobility, which may reflect a reduced importance of firm-specific human capital. In this view, the public sector's failure to follow is signal of inefficiency, perhaps due to inertia or resistance from unions. Career employment remains prevalent in the public sector. One may argue that this is efficient, e.g., because of specialized training or to avoid conflicts of interest ("revolving doors"); but the argument is not obvious and it may not apply to all public sector jobs.

While these human capital and labor relations issues should be noted, they remain unsettled and are not the focus of this paper. (See Friedberg 2010 for a review).

2.4. Balanced Budget Rules and Full Funding

Balanced budgets are extremely popular according to opinion polls, even at the federal level.[1] Full funding of pensions has a similar popular appeal to fiscal conservatism. A failure to make pension contributions is easily understood as violation of the balanced budget principle.

The popularity of balanced budgets is a challenge to economic theory. Theories of optimal dynamic taxation call for unbalanced budgets essentially all the time, because deficits and surpluses help to stabilize tax rates when there are fluctuations in spending and in the tax base. Hence a positive theory of balanced budget and other fiscal rules requires political economy frictions or some other motive for restricting intertemporal optimization.

The political economy literature is somewhat unsatisfactory with regard to normative analysis at the state and local level. The theoretical literature has focused on national debt and identified a range of circumstances that favor excessive debt (see, e.g., Persson and Tabellini, 2000; Alesina and Perotti, 1995). A constitutional rule imposing balanced budgets is a natural corrective mechanism for such distortions. But because rigid rules are costly, theory suggests that rational voters should demand sophisticated balanced budget rules that are conditional or cyclically adjusted. The empirical literature has focused practical questions about the effects of different rules, taking their existence as given.

Political economy models of national debt are in principle applicable to state and local governments. Notably, government authority is often divided among multiple agents. Hence strategic interactions between them can create common pool problems, and responses to fiscal shocks may be delayed. Uncertainty about reelection is also likely to shorten politicians' planning horizons and invite the use of debt as tool to tie the hands of successor governments.

There may also be simple principal-agent explanations of why voters like balanced budgets at the local level. Voters must monitor politicians who act as their agents. Credible information about local budgets is often unavailable or costly, e.g., requiring time to attend

[1] For example, according to a Nov.2009 CNN poll, 67% of U.S. voters agree that "the government should balance the budget even when the country is in a recession and is at war," suggesting support for balanced budgets even under extreme conditions. This paper is concerned with state and local debt.

meetings. Each voter must monitor are multiple entities—the city, the county, the state. If politicians have no authority incur debt, the potential damage from political favoritism, corruption, or other monitoring failures is bounded by current revenue, whereas damages can be huge if debt is allowed.[2] Simple rules also economize on information cost. A headline saying the balanced budget rule was violated is much easier to understand than a complicated budget document. A full funding requirement for pensions may appeal to voters for similar reasons. Funding requirements are problematic, however, because the calculations require numerous assumptions that can be manipulated and because the management of pension funds poses new monitoring problems, as illustrated by recurrent scandals involving "pay to play" schemes.

One key problem area for funding calculations is the discounting of future benefits. The methods prescribed by accountants are economically unjustified, as noted above. Because pensions are complex contingent claims, the correct discount rates are difficult to determine and controversial even among economic experts. Such ambiguities are especially troubling if reported funding gaps serve as summary statistic to information-constrained voters.

Voters who try to hold politicians accountable for pension funding will generally encounter tradeoffs between type-I versus type-II errors. Pensions that are fully funded under some (reasonable) assumptions can easily be portrayed as underfunded under more unfavorable "conservative" assumptions. Fiscally conservative assumptions may be appropriate if the damage from underfunding is greater than the cost of overfunding, but not otherwise. While weak funding rules undermine the balanced budget rule as net worth

[2] Such information problems do not imply "fiscal illusion"—the notion that voters underestimate the cost of deficit-financed expenditures because they observe the benefits but not the long-run cost. Persistent one-sided errors are difficult to reconcile with basic rationality and seem implausible in this age of cynicism about politicians. The issue here is risk reduction. An appropriate analogy is the question how much signature authority to give to an accountant—control over current accounts or also the power to borrow.

Note that state and local bond issues commonly require voter approval, and are widely considered politically acceptable subject to such approval. If voters are concerned that public officials grant excessive pensions to public employees without properly informing voters, one might consider requiring voter approval for new or significantly expanded public pensions. If DB pensions are efficient, requiring voter approval would be preferable to populist proposals to abolish public sector DB pensions.

constraint, overly restrictive rules might discourage the establishment of pensions plans even when they are efficient.

Recent changes in accounting for retiree health benefits illustrate this tradeoff. GASB 43 treats the present value of projected future employer payments for retiree health as a liability even if these payments are not vested—i.e., even if employee contributions could be raised or benefits canceled at any time. This recent GASB rule has triggered a wave of benefit cuts and premium increases by public employers. These responses suggest that voter information is incomplete and accounting rules matter.

Regarding fund management, the key problem is how to monitor investment managers. If public funds are invested in risky assets, performance evaluation is difficult. Unexpected losses are not necessarily a sign of incompetence or fraud. Fund management problems would be eliminated if pensions were left unfunded.

In summary, there are many information problems that might influence voters, but there is no coherent model. The analysis below will motivate balances budgets differently—as commitment device that avoids costly debt.

One should note that there is a literature on state and local pensions without balanced budgets. D'Arcy et al. (1999) argue that under distortionary taxes, optimal pension funding should be governed by tax smoothing arguments. They conclude that a range of funding levels can be optimal, depending on the growth rates of taxes and expenditures. Lucas and Zeldes (2009) note that pension funding and investment strategy would be irrelevant if Ricardian neutrality applied. They also assume tax smoothing and find that optimally public pension portfolios should hold (at least some) equity. Bader and Gold (2007) examine tax arbitrage opportunities in public pensions and argue that bond investments would help local taxpayers minimize federal taxes. I take a different approach in the analysis below because I consider the popularity of balanced budget rules as a given—a stylized fact that economic theory should explain or at least respect.

2.5. Investment Strategy and Interest Rates

Investment strategy matters for pension funding because return risk is a major source of fluctuations in funding ratios. Even with "conservative" funding, the returns on assets and liabilities invariably differ because DB plans liabilities are contingent on variables like wage growth and survival rates, which are difficult to hedge on financial markets. Often the mismatch between assets and liabilities is increased intentionally by strategies that strive for high asset returns, e.g. via investments in the stock market or in "alternatives" such as hedge funds or private equities.

Finance theory teaches that high returns usually involve risk. This suggests the use of safe interest rates as measure of risk-adjusted returns on *all* financial assts. However, explaining the equity premium is a challenge (Mehra and Prescott 1985). Managers of public pension funds must necessarily take a stand on the explanation. If the premium is due to risk aversion—say, a crash premium a la Barro and Ursua (2009)—the use of safe interest rates may be correct. If the premium is instead due to liquidity effects, investor myopia, or other non-fundamental reasons, an all-equity portfolio may promise abnormal returns. Problems of performance measurement are a complicating factor. Lo (2008) notes that many hedge funds have payoff profiles similar to short positions in far-out-of-the-money put options, i.e. seemingly abnormally high returns most of the time (a high Sharpe ratio), but with a possibility of severe losses in rare states of nature.

One apparent omission in the literature is a consideration of taxpayers' access to financial markets. This is critical for public retirement plans because the ultimate plan sponsors are the state and local taxpayers. Many taxpayers hold debt and face borrowing costs. The next section will examine the ramifications.

3. A Model of Local Government

This section examines a model of local public pensions. The model has quite a number of necessary elements—a local population, a local government, a public work force, a national or world economy that determines outside options, and a national tax system that motivates tax-sheltered savings. Hence I start with an outline.

A main assumption is that all debt is subject to intermediation cost. This may be motivated by costs of monitoring and screening that must be incurred to protect lenders against moral hazard and adverse election; it is introduced as an exogenous wedge between the cost of borrowing and the return to investing. To keep the model tractable, I reduce the life cycle to three periods and abstract from demographic fluctuations. The local government is financed by property taxes on homes. As in Epple and Schipper (1981), public debt and pensions are capitalized in home prices.[3]

To anticipate, intermediation costs will provide an argument against pension funding. The intuition is that contributions to a public pension fund require taxes. Taxpayers who are in debt must borrow to pay taxes. If the borrowing rate exceeds the return on fund assets, taxpayers are better off if pensions are left unfunded and if taxes are deferred until the pension payments are due. While this intuition is essentially deterministic, it is presented in a contingent claims setting to show that intermediation cost and risk premiums have quite different implications.

A model with taxpayers as borrowers faces some challenges. It must explain why most individuals are borrowers, why individuals nonetheless desire pension savings, and why a balanced budget rule has voter support despite taxpayers' incentives to defer taxes. In the model, the coexistence of debt and pension savings is motivated by a federal tax system that imposes lower income taxes in retirement than in prime working age. Voters will support balanced budgets because the cost of public debt would be capitalized in home prices.

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[3] Home ownership is a convenient device to make voters care about the community's long run future without having to model intergenerational altruism. An earlier version of this paper also explored optimal funding in a model with general taxes. Then mobility becomes an important issue: selfish individuals would have stronger incentives to incur debt and to underfund pensions, but debt would also be more dangerous for those who are relatively immobile.

3.1. Assumptions and Benchmark Allocation

The community is populated by overlapping generations of individuals who live for three periods. The three cohorts are interpreted as young, middle, and old age. Time is indexed by t and cohorts by age i (i=1,2,3). The community has an exogenous number of residents (N^i) in each cohort. Three periods is the minimum needed to model essential age-earnings and pension vesting issues, while keeping model analytically tractable. The young supply one unit of labor, the middle-aged supply $e > 1$ units (capturing an age-earnings link), and the old are retired. All individuals have preferences over consumption (c_t^i), local public services (g_t), and housing services.

The outside world determines wages and returns to financial investments. Let w_t be the marginal product of a labor unit. Let financial assets be valued under a pricing kernel $m_{tn} = m(s_{t+n} \mid s_t)$; that is, payoffs $a_{t+n} = a(s_{t+n})$ in state of nature s_{t+n} in period t+n are valued in period t at conditional expectation $E_t[m_{tn}a_{t+n}]$. Let $m_{t+1} = m_{t1}$ be the one-period pricing kernel and let $1 + r_t = 1/E_{t-1}m_t$ define the safe interest rate.[4]

The community has $\overline{N}$ identical houses that can be owned by residents or by commercial owners. Owner-occupied houses provide a consumption value $v(g_t)$ per period for the young and middle-aged, which depends in part on public services; $v(g)$ is increasing and concave. Individual utility is

$$U_t = u(c_t^1 + h_t^1 v(g_t)) + \beta u(c_{t+1}^2 + h_{t+1}^2 v(g_{t+1})) + \beta^2 u(c_{t+2}^3), \tag{1}$$

where $\beta \in (0,1)$ captures time preference and $h_t^i \in [0,1]$ indicates home ownership. (Ownership greater than one would be treated as commercial.) Homeowners pay property taxes T_t.

Commercial owners earn the value $v(g_t)$ as rental income but incur a management cost of $\chi_H > 0$, so net rental income is $v(g_t) - \chi_H - T_t$. Commercial owners capitalize rental income under the pricing kernel *m*, which yields a house value

[4] Though the life-cycle arguments below employ essentially deterministic reasoning, a stochastic setting is important because current disputes about pension accounting are centrally about discounting stochastic claims. Hence risk premiums and intermediation cost should be clearly distinguished. For readers not used to pricing kernels: simply think of *m* as a discount factor like 1/(1+r).

$$H_t = \sum_{n\geq 0} E_t\left[m_{tn}\left(v(g_{t+n}) - \chi_H - T_{t+n}\right)\right] \quad (2)$$

Assume $\overline{N} > N^1 + N^2$, so not all houses can be owner-occupied. Then (2) defines the equilibrium house price. Also assume (for simplicity) χ_H is high enough that all residents prefer to own rather than rent, so $h_t^1 = h_t^2 = 1 \forall t$. That is, the young in period t buy a house at price H_t, they pay taxes T_t and (later) T_{t+1}, and they sell the house at price H_{t+2} when old.

As simple benchmark, abstract from income taxes but allow for national old age transfers *TR* (social security), and assume complete markets. (Section 5 considers incomplete market.) Then individuals maximize utility subject to the budget constraint

$$w_t - c_t^1 - T_t - H_t + E_t\left[m_{t1}(ew_{t+1} - c_{t+1}^2 - T_{t+1})\right] + E_t\left[m_{t2}(TR_{t+2} + H_{t+2} - c_{t+2}^3)\right] = 0 \quad (3)$$

The optimality conditions are

$$\frac{\beta u'(\tilde{c}_{t+1}^2(s_{t+1}))}{u'(\tilde{c}_t^1(s_t))} = m(s_{t+1} \mid s_t) \forall s_{t+1} \text{ and } \frac{\beta^2 u'(c_{t+2}^3(s_{t+2}))}{u'(\tilde{c}_t^1(s_t))} = m(s_{t+2} \mid s_t) \forall s_{t+2}, \quad (4)$$

where $\tilde{c} = c + v(g)$. Thus marginal utilities are aligned with the pricing kernel across periods and states of nature. Because individuals can save on their own, there is no need for pension funds. Under empirically plausible assumptions, net financial assets are negative from youth to middle age and positive from middle to old age.[5] This is assumed in the following. Let $\overline{U}_t$ denote the maximized utility, which is a function of public services, home prices, and property taxes.

The task of local government is to provide public services that maximize homeowner utility $\overline{U}_t$. Assume

$$g_t = G_t / \overline{N} = G(L_t^1, L_t^2) / \overline{N} \quad (5)$$

is produced by public employees, where *G* is increasing, concave, and linearly homogeneous, and L_t^1 and L_t^2 are the number of young and middle-age employees.[6] Assuming the

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[5] That is, houses are costly, earnings peak in middle age, and social security and home sales are not enough to finance retirement. As rough calibration, suppose w=1, e=1.5, a period of 20 years, 3% annual return on savings of 3%, house price of 5 times annual wages, 1% annual property taxes; so H=0.25 and T=0.05 per 20 year period. Assume v=0.2, slightly above the carrying cost of a house of 0.16, TR=0.2, and 3% time preference. Then $\tilde{c}^1 = \tilde{c}^2 = c^3 \approx 1.05$, savings are -0.15 in period 1, +0.60 in period 2, and -0.60 in period 3. So individuals borrow when young and save in middle age.

[6] An interesting extension would be to consider differential productivity of "experienced" employers who worked for the same government in the previous period versus new middle age hires. This would provide a naturally motivation for career employment and a way to address the human capital issues raised in Section 2.

government pays market wages, labor costs are $W_t = w_t L_t^1 + e w_t L_t^2$. To minimize cost, relative productivity must match relative wages, so

$$\frac{\partial G}{\partial L^2}(l, 1-l) / \frac{\partial G}{\partial L^1}(l, 1-l) = e.$$

This defines the optimal share of young workers $l^* = L^1/(L^1 + L^2)$. Because public debt would be neutral (capitalized in house prices), setting $T_t = W_t / \overline{N}$ is without loss of generality. Optimal public employment maximizes the value of services minus their cost, so $\partial(v(g_t) - T_t)/\partial L_t^i = 0$. This implies

$$g_t = g_t^* \equiv (v')^{-1}\left(w_t \frac{l^* + e(1-l^*)}{\overline{N}} \right) \text{ and } T_t = T_t^* \equiv w_t g_t^* \frac{(l^* + e(1-l^*))\overline{N}}{G(l^*, 1-l^*)}.$$

Because $(v')^{-1}$ is decreasing, optimal services are declining in the cost of labor. The implied house price is

$$H_t^* = \sum_{n \geq 0} E_t \left[m_{tn} \left(v(g_t^*) - T_t^* - \chi_H \right) \right]. \tag{6}$$

Assume $v(g_t^*) - T_t^* > \chi_H$, so house prices are positive.

3.2. Income Taxes and Intermediation Cost

Individuals are in reality subject to national income taxes. As simple proxy for progressive taxation, assume marginal tax rates τ^i are constant but age-specific. Because the age-earnings profile peaks in middle age, assume $\tau^2 > \tau^1$ and $\tau^2 > \tau^3$. Taxes apply to wage income minus pension contributions, to returns on regular (taxable) investments, and to pension payouts.

Income taxes provide a strong motivation for pension plans. Because the setup cost of a pension plan is negligible compared to the tax benefits, all employers will offer at least a DC pension plan.[7] With competition between employers, all tax savings accrue to employees, so the wage w_t remains unchanged. Because employer contributions would reduce cash wages one for one, assume (w.l.o.g.) that DC plans have zero employer contributions.

Regular investments, debt, and pensions now require separate accounting. Let $x_t^i \geq 0$ denote retirement contributions, let r_{xt+1}^i be the returns, and let

[7] Individual IRA accounts or large tax exemptions for capital income would deliver similar tax benefits, but U.S. tax laws allow greater contributions in employer plans. IRAs can be subsumed under DC plans. It is beyond the scope of this paper to explain why Congress enacted laws that in effect subsidize pension savings; the focus here is on the implications.

$$X_{t+2} = [x_t^1(1+r_{xt+1}^1) + x_{t+1}^2](1+r_{xt+2}^2). \tag{7}$$

be pension payouts. Payouts in middle age are prohibited. Individuals may also hold regular taxable assets $a_t^i \geq 0$ with returns r_{at+1}^i and borrow amounts $d_t^i \geq 0$.

For investments, assume the distribution of returns is chosen by investors subject to the no-arbitrage conditions $E_t[m_{t+1}(1+r_{xt+1}^i)] = 1 - \chi_{DC}$ and $E_t[m_{t+1}(1+r_{at+1}^i)] = 1$, i=1,2. where $\chi_{DC} \geq 0$ allows for a (small) cost of managing assets. That is, investments can be interpreted as portfolios of Arrow securities that provide payoffs in specific states of nature. This complete markets setting—apart from cost—provides a straightforward accounting for uncertainty and for modeling risk premiums.[8]

For individual debt, assume lenders incur intermediation costs that increase with the level of debt (say, due to screening and monitoring expenses). Borrowers rank sources of debt by borrowing costs, which are intermediation costs minus possible tax benefits (e.g. for mortgages). To express this parsimoniously, let $\chi_d(\hat{d})$ denote the net cost at debt level $\hat{d}$ and $\bar{\chi}_d(d_t^i) = \int_0^{d_t^i} \chi_d(\hat{d})d\hat{d}$ the total cost of debt. For symmetry with assets, assume borrowers choose the return distribution, which can be written as $E_t[m_{t+1}(1+r_{dt+1}(\hat{d})] = 1 + \chi_d(\hat{d})$ on the margin and $E_t[m_{t+1}(1+\bar{r}_{dt+1}(d_t^i)] = 1 + \bar{\chi}_d(d_t^i)$ for total debt. Net cost may be negative for tax-deductible debt. To limit arbitrage, assume marginal costs approach infinity at some (unspecified) maximum debt. To avoid corner solutions, assume $\chi_d(0) \leq 0$.

Given these financing choices, individuals maximize utility subject to the budget equations

$$c_t^1 + h_t^1 T_t = (w_t - x_t^1)(1-\tau^1) + d_t^1 - a_t^1 - h_t^1 H_t \tag{8}$$

$$\begin{aligned} c_{t+1}^2 + h_{t+1}^2 T_{t+1} = {} & (ew_{t+1} - x_{t+1}^2)(1-\tau^2) + d_{t+1}^2 - [1+\bar{r}_{dt+1}(d_t^1)]d_t^1 \\ & +(h_t^1 - h_{t+1}^2)H_{t+1} - a_{t+1}^2 + [1+(1-\tau^2)r_{at+1}^1]a_t^1 \end{aligned} \tag{9}$$

$$c_{t+2}^3 = TR_{t+2} + X_{t+2}(1-\tau^3) + h_{t+1}^2 H_{t+2} - [1+\bar{r}_{dt+1}(d_{t+1}^2)]d_{t+1}^2 + [1+(1-\tau^3)r_{at+2}^2]a_{t+1}^2. \tag{10}$$

[8] Incomplete markets would be a distraction here and are deferred to Section 6. I abstract from management cost on regular savings; they will be return-dominated even without cost.

Proceeding recursively, the first order conditions in middle age for pensions, taxable assets, and debt are

$$\frac{\partial U}{\partial x_{t+1}^2} = -\beta u'(\tilde{c}_{t+1}^2)(1-\tau^2) + E_{t+1}\left[\beta^2 u'(c_{t+2}^3)(1+r_{xt+2}^2)\right](1-\tau^3) + \mu(x_{t+1}^2) = 0, \quad (11)$$

$$\frac{\partial U}{\partial a_{t+1}^2} = -\beta u'(\tilde{c}_{t+1}^2) + E_{t+1}\left[\beta^2 u'(c_{t+2}^3)(1+(1-\tau^3)r_{at+2}^2)\right] + \mu(a_{t+1}^2) = 0 \quad (12)$$

$$\frac{\partial U}{\partial d_{t+1}^2} = \beta u'(\tilde{c}_{t+1}^2) - E_{t+1}\left[\beta^2 u'(c_{t+2}^3)(1+r_{dt+2}^2)\right] + \mu(d_{t+1}^2) = 0 \quad (13)$$

where $\mu(z) \geq 0$ denotes the Kuhn-Tucker multiplier for any constraint $z \geq 0$.

Consider first the case $x_{t+1}^2 > 0$. From (11) and the arbitrage condition for returns, one obtains

$$E_{t+1}\left[\left(\frac{\beta u'(c_{t+2}^3)}{u'(\tilde{c}_{t+1}^2)}\Theta_{x2} - m_{t+2}\right)(1+r_{xt+2}^2)\right] = 0 \text{ with } \Theta_{x2} \equiv \frac{1-\tau^3}{1-\tau^2}(1-\chi_{DC}). \quad (14)$$

Because this condition must hold for all investments, including Arrow securities, the marginal rate of substitution in all states of nature must satisfy

$$\frac{\beta u'(c_{t+2}^3(s_{t+2}))}{u'(\tilde{c}_{t+1}^2(s_{t+1}))} = \frac{1}{\Theta_{x2}} m(s_{t+2} \mid s_{t+1}) \quad \forall s_{t+2} \quad (15)$$

This is similar to (4), but distorted by the tax factor Θ_{x2}. Recall that $(1-\tau^3)/(1-\tau^2) > 1$ and assume χ_{DC} is small. Then $\Theta_{x2} > 1$. This means federal taxes provide a "boost" to pension returns and a strong incentive to contribution in middle age.

If (15) holds, (12) implies $\mu(a_{t+1}^2) > 0$, so $a_{t+1}^2 = 0$. Regular savings are inferior to pensions. In addition, (15) combined with (13) imply

$$\chi_d(d_{t+1}^2) = \Theta_{x2} - 1 > 0. \quad (16)$$

This defines an optimal debt level $d_{t+1}^{2*} > 0$. Intuitively, individuals borrow to fund pensions up to the point where the pension tax incentives in Θ_{x2} are offset by intermediation cost. Hence Θ_{x2} can also be interpreted as borrowing cost: $\Theta_{x2} = 1 + \chi_d(d_{t+1}^2)$.

For completeness, consider the possibility that $x_{t+1}^2 = 0$. Then analogous arguments imply $\frac{\beta u'(c_{t+2}^3(s_{t+2}))}{u'(\tilde{c}_{t+1}^2(s_{t+1}))} \leq m(s_{t+2} \mid s_{t+1})/\Theta_{x2}$, which in turn implies $a_{t+1}^2 = 0$ and $d_{t+1}^2 \geq d_{t+1}^{2*} > 0$. Given $d_{t+1}^2 > 0$, (13) implies $\frac{\beta u'(c_{t+2}^3(s_{t+2}))}{u'(\tilde{c}_{t+1}^2(s_{t+1}))} = m(s_{t+2} \mid s_{t+1})/\Theta(d_{t+1}^2)$, where

$\Theta(d_{t+1}^2)=1/(1+\chi(d_{t+1}^2)\le\Theta_{x2}$. Thus the marginal rate of substitution would be pushed down even more than in the $x_{t+1}^2>0$ case.

To summarize: in all cases, *income taxes imply a downward distortion in the marginal rate of substitution between peak earnings years and retirement*. In the following, to avoid distracting case distinctions, assume the case $x_{t+1}^2 \equiv x_{DC}^2>0$ applies.

For the young, similar first order conditions as in middle age apply for regular assets and debt (not shown). The condition for optimal pension contributions is different because pensions must be held to retirement:

$$\frac{\partial U}{\partial x_t^1}=-u'(\tilde{c}_t^1)(1-\tau^1)+E_t\left[\beta^2 u'(\tilde{c}_{t+2}^3)(1+r_{xt+1}^1)(1+r_{xt+2}^2)\right](1-\tau^3)+\mu(x_t^1)=0. \qquad (17)$$

Using (15) to replace $u'(\tilde{c}_{t+2}^3)$, this implies

$$E_t\left[\left(\frac{\beta u'(\tilde{c}_{t+1}^2)}{u'(\tilde{c}_t^1)}\Theta_{x1}-m_{t+1}\right)(1+r_{xt+1}^1)\right]+\frac{\mu(x_t^1)}{u'(\tilde{c}_t^1)(1-\tau^1)}=0$$

$$\text{where } \Theta_{x1}=\frac{1-\tau^2}{1-\tau^1}(1-\chi_{DC})^2\le\frac{1-\tau^2}{1-\tau^1}<1. \qquad (18)$$

In young age, the tax factor $\Theta_{x1}<1$ discourages retirement contributions.

Life-cycle arguments suggest that the young have strong incentives to borrow. To avoid distracting case distinctions, assume $d_t^1>0$ and $\chi_d(d_t^1)>0$. Then the first-order condition for debt has a zero Kuhn-Tucker multiplier and can be written as

$$E_t\left[\left(\frac{\beta u'(\tilde{c}_{t+1}^2)}{u'(\tilde{c}_t^1)}(1+\chi_d(d_t^1))-m_{t+1}\right)(1+r_{dt+1})\right]=0, \qquad (19)$$

with implies

$$\frac{\beta u'(\tilde{c}_{t+1}^2(s_{t+1}))}{u'(\tilde{c}_t^1(s_t))}=\frac{1}{1+\chi_d(d_t^1)}m(s_{t+1}\,|\,s_t) \quad \forall s_{t+1}. \qquad (20)$$

Optimal debt aligns the marginal rate of substitution with the pricing kernel, now distorted downwards by intermediation cost. Combining (20) and (18), $\mu(x_t^1)>0$ follows from $\Theta_{x1}<1\le 1+\chi_d(d_t^1)$, so $x_t^1=x_{DC}^1=0$. Similar reasoning implies $a_t^1=0$.

The model suggests that the young and middle-aged hold debt. For empirical support, Table 1 shows data on the prevalence of borrowing and on the cost of borrowing. According to the Survey of Consumer Finances 2007 (SCF), more than 80% of families with head of household under 65 hold debt. Mortgages are held by majorities in the 35-64 age brackets, installment debt by majorities in the under-54 age brackets, and credit card debt by majorities in the 35-54 age brackets. Thus the vast majority of U.S. families are debtors, and for most of them (all but prime mortgage debtors who itemize), debt is costly.

The model also implies that young borrowers should not contribute to pension plans. Empirically, many young workers indeed contribute nothing or very little, which is sometimes viewed as puzzling. Indeed, according to SCF, 58% of families under 35 have no retirement accounts at all. Though retirement contributions by the young could be rationalized in model extensions (see below), given the data, zero contributions are a useful benchmark.

In summary, the model yields a simply and empirically plausible life cycle. The young work, buy a house, and borrow. The middle-aged save for retirement and hold debt only for tax reasons. The old consume and liquidate their assets.[9]

Marginal rates of substitution are given by (15) between middle and old age and by (20) between young and middle age. Importantly, both marginal rates of substitution are less than the pricing kernel. The relationship between Θ_{x2} in (15) and $1+\chi_d(d_t^1)$ in (20) is an empirical issue. As a rough calibration, a tax wedge between $\tau^2 = 30\%$ and $\tau^3 = 15\%$ over 20 years would imply an annual tax advantage of about 1% (from $(1-\tau^2)/(1-\tau^3) = 0.85/.70 = (1+0.98\%)^{20}$). A 25bp management cost (say, using index funds) would leave a 75bp annualized gain in Θ_{x2}. Spreads between T-bills and credit card rates are about 14% and spreads on car loans (the most common installment loan) about 5%; see Table 1. Because credit card and car loans have high default rates, the spreads reflect not only intermediation costs but also risk premiums. Hence $\chi_d(d_t^1)$ is difficult to calibrate. As alternative, consider the spread between the Prime Rate and T-bills, which is about 3%. Because Prime borrowers have low credit risk, this spread should largely reflect intermediation cost; half the spread (1.5%) might be a conservative lower bound. Overall, the data suggest that most young families face intermediation cost of several percentage points.

[9] One could easily add a joy-of-giving bequest motive to the model (e.g. for giving the house to the kids), so liquidation should not be taken literally.

This suggests $1+\chi_d(d_t^1) \ge \Theta_{x2} > 1$. Because $\Theta_{x2} = 1+\chi_d(d_t^2)$, the marginal rates of substitution of both cohorts can be written as $m_{t+1}/[1+\chi_d(d)]$ to express their generic dependence on the pricing kernel and on intermediation cost.

Turning to local public finance, pension incentives and costly debt have a key implication: _Most taxpayers face a marginal cost of funds greater than the rate of return on financial assets_.

3.3. Public Pensions and Public Debt

Consider the local government's financing choices with public debt and with DB pensions.

Let D_t denote end-of-period debt. With DB pensions, let w_t^1 and w_t^2 denote period-t "cash" wages paid to young and middle-aged public employees, and let P_t denote employer contributions to the DB plan. Then $\tilde{W}_t = w_t^1 L_t^1 + w_t^2 L_t^2$ is the current wage cost. The period-t budget identity is

$$D_t = (1+r_{Dt})D_{t-1} + \tilde{W}_t + P_t - \overline{N} \cdot T_t \tag{21}$$

The interest r_{Dt} may include an intermediation cost χ_D (constant for simplicity), so $E_{t-1}\left[m_t(1+r_{Dt})\right] = 1+\chi_D \ge 1$. Assuming investors impose the transversality/No Ponzi condition $E_t\left[m_{tn}D_{t+n}\right] \to 0$ as $n \to \infty$, one obtains the intertemporal budget constraint (IBC)

$$D_t^* = \sum_{n \ge 0} E_t\left[m_{tn}\left(\overline{N} \cdot T_{t+n} - \tilde{W}_{t+n} - P_{t+n} - \chi_D D_{t+n}\right)\right] \tag{22}$$

where $D_t^* = (1+r_{Dt})D_{t-1}$ is the start-of-period debt.

Let pension fund assets F_t earns returns r_{Ft}, where $E_{t-1}\left[m_t(1+r_{Ft})\right] = 1-\chi_F$ may be subject to a management cost $\chi_F \ge 0$. All pension benefits B_t are paid from the fund, either from fund assets or pay-as-you go employer contributions. The budget identity is

$$F_t = (1+r_{Ft})F_{t-1} + P_t - B_t. \tag{23}$$

The limit condition $E_t\left[m_{tn}F_{t+n}\right] \to 0$ as $n \to \infty$, implies the IBC

$$F_t^* = \sum_{n \ge 0} E_t\left[m_{tn}\left(B_{t+n} - P_{t+n} + \chi_F F_{t+n}\right)\right] \tag{24}$$

where $F_t^* = (1+r_{Ft})F_{t-1}$ is start-of-period funding position. Combining (22) and (24), the present value of taxes is

$$\overline{N} \cdot \sum_{n\geq 0} E_t\left[m_{tn}T_{t+n}\right] = D_t^* + B_t - F_t^* + \sum_{n\geq 0} E_t\left[m_{tn}\left(W_{t+n} + \chi_F F_{t+n} + \chi_D D_{t+n}\right)\right] \qquad (25)$$

where $W_t = \tilde{W}_t + E_t[m_{t+1}B_{t+1}]$ measures total compensation (current wages and present value of benefits). Thus taxes depend on debt, promised benefits minus funding, plus future compensation and intermediation cost.

Voters care about taxes because they affect house prices. Combining (2) and (25),

$$\begin{aligned} H_t = &\sum_{n\geq 0} E_t\left[m_{tn}\left(v(g_{t+n}) - \chi_H - \tfrac{1}{N}W_{t+n}\right)\right] \\ &-\tfrac{1}{N}\left(D_t^* + B_t - F_t^*\right) - \tfrac{1}{N}\sum_{n\geq 0} E_t\left[m_{tn}\left(\chi_F F_{t+n} + \chi_D D_{t+n}\right)\right] \end{aligned} \qquad (26)$$

The first line in (26) reflects the value of public services and their real cost; the second line reflects financing choices.

To attract employees, an employment package with DB plan must be competitive with DC plans; and with complete markets, the optimal DB plan must essentially replicate DC contributions and benefits. Instead of paying workers w_t and ew_{t+1} in young and middle age, the government pays $w_t^1 = w_t - x_t^1$ and $w_{t+1}^2 = ew_{t+1} - x_{t+1}^2$ and it promises retired workers a pension X_{t+2}, so $B_{t+2} = X_{t+2}L_{t+1}^2$. If $x_{DC}^1 = 0$, as suggested in the previous section, the DB plan simplifies: young employees receive an unreduced salary $w_t^1 = w_t$ and no promise of benefits.[10] Then from (7),

$$W_t = \tilde{W}_t + E_t[m_{t+1}B_{t+1}] = w_t L_t^1 + \left(ew_t - \chi_{DC}x_{DC}^2\right)L_t^2. \qquad (27)$$

This means that to the extent that managing DC investments is costly, a DB plan reduces employment cost. However, if DC costs are small ($\chi_{DC} \approx 0$), the optimal public services, optimal taxes, and optimal public employment are the same as in Section 3.1. Notably,

[10] In practice, legal restrictions may prevent DB plan sponsors from excluding the young. However, vesting requirements could have a similar effect, suggesting an efficiency argument for long vesting periods. The optimality of zero contributions in young age suggests caution in using accounting methods such as entry-age-normal that mechanically allocate projected benefits over an entire career. The interpretation of W_t as total compensation implicitly exploits a single period of contributions; with multi-period accrual, more elaborate notation would be needed.

$g_t \approx g_t^*$, and line 1 in eq. (26) equals H_t^* in (6), so the "real" allocation is essentially unchanged.[11]

Line 2 of (26) shows that initial debt, pension promises, and pension fund assets are fully capitalized. Because of intermediation cost, expected future debt reduces house values. If DB fund management is costly ($\chi_F > 0$), future fund balances also reduce house values.

Recall that most voters—resident homeowners—are debtors. While real estate investors capitalize houses at the pricing kernel *m*, voters discount the future at the marginal rates of substitution (15) and (20), which are strictly greater. Assuming votes takes place every period after homes are traded, middle-aged individuals vote to maximize

$$\eta_t^2 \equiv \left(v(g_t) - T_t\right) + E_t[\frac{m_{t+1}}{1+\chi_d(d_t^2)} H_{t+1}] \tag{28}$$

and the young vote to maximize

$$\eta_t^1 \equiv v(g_t) - T_t + E_t\left[\frac{m_{t+1}}{1+\chi_d(d_t^1)}\left(v(g_{t+1}) - T_{t+1} + \frac{m_{t+2}}{1+\chi_d(d_{t+1}^2)} H_{t+2}\right)\right]. \tag{29}$$

These relations yield several important results.

First, the payoff functions η_t^2 and η_t^1 are strictly decreasing in F_t:

$$\frac{\partial \eta_t^i}{\partial F_t} \equiv -\frac{\partial T_t}{\partial F_t} + E_t\left[\frac{m_{t+1}}{1+\chi_d(d_t^i)} \frac{\partial T_{t+1}}{\partial F_t}\right] = -\frac{1}{\overline{N}} \frac{\chi_F + \chi_d(d_t^i)}{1+\chi_d(d_t^i)} < 0, \text{ for i=1,2.} \tag{30}$$

Thus, the <u>*middle-aged and young voters strictly prefer zero funding*</u>, $F_t = 0$. To see the intuition, suppose funding were positive. Then if funding is reduced, taxes per house can be reduced by $1/\overline{N}$ in period t. Taxes must be increased by $(1 + r_{Ft+1})/\overline{N}$ units in period t+1, which has a present value of $(1-\chi_F)/(1+\chi_d(d_t^i))/\overline{N}$ at taxpayers' marginal rate of substitution. Because $(1-\chi_F)/(1+\chi_d(d_t^i)) < 1$, taxpayers favor reduced funding.

Second, provided $\chi_D < \chi_d(d_t^1)$ and $\chi_D < \chi_d(d_t^2)$, both generations would benefit from a debt-financed tax cut in period t that is reversed in period t+1. This might suggest a bias towards deficit spending. However, note that the $\chi_D D_{t+n}$-terms in (26) enter negatively,

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[11] If DC have high cost, the reduced employment cost $ew_t - \chi_{DC} x_t^2$ of middle-age employees would shift the optimal employment mix ($L_t^1/(L_t^1 + L_t^2) < l^*$) and increase the optimal scale of public services ($g_t > g_t^*$). These real effects would distract from funding issues and are not examined further.

so debt in future periods reduces house values. If voting over public debt is sequential, rational investors must expect positive debt in the future and discount houses values accordingly. This motivates a balanced budget rule—a constitutional provision to outlaw debt once and for all. Voters will approve provided the impact of persistent debt on home prices (the $\chi_D D_{t+n}$-terms) outweighs the short-run cost difference $\chi_d(d) - \chi_D$.[12] Hence the model is consistent with popular support for balanced budgets. In the following, I assume a balanced budget rule applies so $D_t \equiv 0$.

Importantly, support for a balanced budget framework does *not* imply support for fully funded pensions. For sequential voting over pension funding, this was shown above. For voting on a once-and-for all rule about pension funding, note that $\chi_F F_{t+n}$ enters negatively in (26). Hence voters would support zero funding as a fiscal rule. Future management costs reinforce the optimality of zero funding. The general point is that homeowners strive to avoid all intermediation cost—cost of debt *and* the cost of fund management.[13]

Third, young voters may favor an even more extreme pension policy. If $\chi_d(d_t^1) > \chi_d(d_t^2)$, young voters face higher borrowing cost than middle-aged public employees. Hence they have an incentive to offer a benefits package with reduced wages and higher benefits, so $w_t^2 < ew_t - x_{DC}^2$ and $x_t^2 > x_{DC}^2$. Compared to the private sector, such benefits would appear "bloated." However, bloated benefits would reduce house prices. Hence middle-aged voters will be opposed; and young voters will approve only if the cost

[12] The exact conditions are complicated but seem mild. For example, the middle aged to support a permanent debt reduction provided $\chi_d(d_t^2) - \chi_D \le \chi_D / r_\infty$ where r_∞ is the interest rate on a consol. Say, if r_∞=5%, the condition holds unless $\chi_d(d_t^2)$ is 21-times greater than χ_D. Also, one could show that the model is consistent with borrowing for capital projects that are eligible for tax-exempt financing, provided the tax subsidy covers the intermediation cost. The tax-exempt debt would be capitalized as if $\chi_D < 0$, which means property values would be increased. Note that for $\chi_d(d_t^1) \ge \chi_d(d_t^2)$, any proposal to defer taxes that is favored by the middle-aged is also supported by the young. The median voter in this model is best interpreted as a median-age homeowner.

[13] A main motive for introducing the variable χ_F is to demonstrate the effects of borrowing cost versus investment cost. Zero funding would be optimal even if $\chi_F = 0$. Even if one abstracted from age-dependent tax rates (i.e., consider the special case $\tau^2 = \tau^3$ and $\chi_F = \chi_{DC} = 0$ so $\Theta_{2x} = 1$ and $\chi_d(d_t^2) = 0$), the young would favor zero pension funding. Then middle-age voters would be indifferent, but trivial side payments (say log-rolling) would ensure unanimous support for zero pension funding.

savings $\chi_d(d_t^1) - \chi_d(d_t^2)$ outweigh the negative house price effect. Thus the model can rationalize bloated benefits as a possibility, but they are not a prediction.

Fourth, because funding is costly, mandatory funding—say, imposed by an outside regulator—would reduce taxpayer incentives to offer a DB plan as compared to a DC plan. If differences in management cost are negligible (say, if $\chi_F \approx \chi_{DC} \approx 0$), only underfunded DB plans are preferable to DC. If full funding were required, only a cost advantage $\chi_F < \chi_{DC}$ would justify a DB plan. (This abstracts from all other differences between DB and DC noted in Section 2.)

In summary, one finds that pension funding is costly and therefore suboptimal. While the model omits realistic complications that may warrant funding (see below), it provides a useful baseline and it suggests caution in imposing funding regulations.

3.4. Accounting Implications of Intermediation Costs

Suppose the optimal public employment contract includes a DB plan. How should the government account for the pension obligations?

Specifically, suppose middle-age government workers receive a wage $w_t^2 = ew_t - x_t^2$ in the current period and are promised $X_{t+1} = x_t^2(1 + r_{xt+1}^2)$ in retirements. Total benefits $B_{t+1} = X_{t+1}L_t^2$ may be unfunded or (partially) backed by a fund F_t earning returns r_{Ft+1}. To simplify, assume young employees receive a wage $w_t^1 = w_t$ and no pension entitlement.

Taxpayers and investors are not unanimous on how to value such pension obligations. Outside investors apply the pricing kernel *m* and compute

$$AAL_t^m = E_t\left[m_{t+1}B_{t+1}\right] = E_t\left[m_{t+1}(1 + r_{xt+1})\right]x_t^2 L_t^2 = x_t^2 L_t^2 .$$

Taxpayers use a marginal rate of substitution based on (15) or (20), generically denoted $m_{t+1}/[1 + \chi_d(d)]$. This implies

$$AAL_t = E_t[\tfrac{m_{t+1}}{1+\chi_d(d)} B_{t+1}] = \tfrac{1}{1+\chi_d(d)} x_t^2 L_t^2 < AAL_t^m .$$

Thus discounting by the pricing kernel overstates the opportunity cost of pension promises for taxpayers. Standard finance theory is correct that discounting must reflects the risk characteristics of pension *liabilities* (not assets). However, the discount rates must include an allowance for intermediation cost, i.e., they are properly derived from taxpayers' *borrowing rates*, not from returns available to investors.

As simple adjustment for intermediation cost, one may add a spread to investment-based discount rates. For example, "safe" pension benefits are commonly discounted at duration-matched Treasury rates (Novy-Marx and Rauh 2009). Over the last 20 years, 10-year Treasuries yields have averaged 2.8% in real terms (5.6% nominal, 2.8% inflation). If the median voter faces an intermediation cost of 1.5-3%, one obtains real discount rates of 4%-6%. For pension benefits subject to default risk, one would have to add a risk premium, e.g., the spread between taxable municipal bond and Treasuries (about 1-2%). This suggests real discount rates in the 5-8% range, or nominal 8-11%.[14] Put differently, accountants are correct when they discount pension obligations at rates much greater than Treasury rates—though not for the right reasons. One may even argue that commonly used accounting rates of 7-8% nominal are too low for most taxpayers.

One complication is that because funding is suboptimal, measures of "underfunding" are difficult to interpret. Funding equal to AAL, $F_t / AAL_t = 100\%$, could be labeled "full" funding. However, because fund returns are less than taxpayers' borrowing cost, "full" funding in this sense would lead to expected underfunding in the next period:

$$UAAL_t = E_t\left[\tfrac{m_{t+1}}{1+\chi_d(d)}\left(B_{t+1} - F^*_{t+1}\right)\right] = AAL_t - \tfrac{1-\chi_F}{1+\chi_d(d)} F_t = \left(AAL_t - F_t\right) + \tfrac{\chi_F + \chi_d(d)}{1+\chi_d(d)} F_t \quad (31)$$

which would be strictly positive at $F_t = AAL_t$. A forward looking measure of underfunding must account for current underfunding, $AAL_t - F_t$, plus a term that reflects the inefficiency of carrying funds into the next period.

One way to account for inefficient funding is to discount funded obligations by risk-adjusted asset returns and unfunded obligations at a higher rate that reflects taxpayers' cost of funds. Interestingly, the GASB rule for retiree health plans prescribe exactly the opposite, a higher discount rate on funded liabilities than on unfunded ones (GASB 43); this GASB rule

[14] Because intermediation costs generally differ across individuals (as exemplified by age differences in the model), there is no single "correct" discount rate for everyone. Hence I provide a range of rates; the choice of a particular value would be a matter of setting accounting conventions.

seems unjustified.[15] This application further illustrates that the "intermediation-cost approach" proposed here differs significantly from the finance and accounting approaches.

4. Legal Ambiguity and Default Risk

Legal ambiguity and default risks deserve attention here because they have first order effects on optimal funding.

By legal ambiguity I mean the possibility that the plan sponsor can successfully dispute the scope or existence of pension obligations. By default risk I mean the risk that the plan sponsor will default outright on its liabilities—presumably on debt as well as pensions. A key distinction is that default is usually observable and unambiguous. Hence default should be insurable whereas legal ambiguities are not. Almost by construction, if a plan sponsor finds a legal flaw or otherwise challenges a pension payment, an insurer could raise the same objections. Employees must always worry how the courts might rule, and this uncertainty is uninsurable. (Hence I prefer the term legal *ambiguity* instead of legal *risk*.)

For example, consider a plan that promises a fixed percentage of final salary, which is then indexed to inflation, as is common in DB plans. Final salary is an ambiguous term when the structure of compensation is evolving. A stingy definition—excluding all bonus-type payments—would allow the employer to cut pensions by reducing base salaries and instead paying repeated bonuses. (With inflation, freezing nominal salaries would work similarly.) A generous definition would invite attempts to manipulate the timing of payments to show an abnormally high final salary—a practice known as "pension spiking." Indexing clauses may also cause trouble, e.g., if simple formulas become so "unreasonable" over time that a court may not enforce them.

With legal ambiguity, funding can serve as collateral and provide a floor on benefits. A key question is to what extent pension assets are reserved for pension benefits, or if plan sponsors can reclaim assets under some conditions. Funding is only effective as collateral if it

[15] An additional problem with GASB 43 is that since health benefits are not vested, they are quite risky and must be discounted accordingly.

cannot be diverted. Strong regal restrictions against "raids" by plan sponsors are indeed widespread, even in cases when a plan seems vastly overfunded. Such restrictions are difficult to rationalize without legal ambiguity.[16] Moreover, many retirement funds have a governance structure designed to protect beneficiaries, e.g., through board representation.

To model legal ambiguity, assume each "macro" state of nature (s_{t+1}) has two possible realizations (s_{t+1}^{-}, s_{t+1}^{+}). With probability $\overline{\pi}$, the promised benefits $X(s_{t+1})$ are disputed and the plan sponsor successfully refuses additional contributions. Then beneficiaries share the available funds and actual payments are $X(s_{t+1}^{-}) = \min\{X(s_{t+1}), F^{*}(s_{t+1})/L_t^2\}$. With probability $1-\overline{\pi}$, promised benefits are honored, so $X(s_{t+1}^{+}) = X(s_{t+1})$.[17]

As a behavioral extension, denote employees' perception of legal ambiguity by $\overline{\pi}^e$ and let it be distinct from the employer's view. For simplicity, assume that $\overline{\pi}$ and $\overline{\pi}^e$ are exogenous. Also assume the median voter is middle aged, so the public employer has a well-defined objective of maximizing η_t^2.

The economic implication of legal ambiguity is an uninsurable risk. Retirement consumption is $c_{t+1}^3(s^+) = TR_{t+1} + H_{t+1} + X(s_{t+1})$ if the employer pays as promised. Retirement consumption is $c_{t+1}^3(s^-) = TR_{t+1} + H_{t+1} + F^{*}(s_{t+1})/L_t^2$ if employees share the fund assets. Whenever the funding ratio $f \equiv F^{*}(s_{t+1})/(X(s_{t+1})L_t^2)$ is less than 100%, default implies a discrete downward "jump" in consumption and an upward jump in marginal utility.[18] The latter is increasing in the ratio of promised pension to retirement income, in the degree of underfunding, and in relative risk aversion (denoted γ):

$$\Delta u'(f) \equiv \frac{u'(c_{t+2}^3(s^-)) - u'(c_{t+2}^3(s^+))}{u'(c_{t+2}^3(s^+))} = \left[1-(1-f)\frac{X(s_{t+1})}{c_{t+2}^3(s^+)}\right]^{-\gamma} - 1 \geq 0.$$

The ratio $X(s_{t+1})/c_{t+1}^3(s^+)$ can be called *pension dependence*. It captures to what extent employees rely on a promised pension for their retirement income.

[16] Otherwise it would be economically efficient to give the plan sponsor (as residual claimant) the right to recover overfunding. But with legal uncertainty, "overfunding" is not well defined because the scope of obligations is part of the dispute. Hence a right to recover funds would undermine the collateral function.

[17] The same pattern of payoffs would occur in case of default; hence the model applies analogously to default risk I focus on legal ambiguity in part for brevity and in part because default raises the question why fund managers or beneficiaries are not taking out insurance against a default of the plan sponsor (e.g. via credit default swaps).

[18] Note that the relevant funding ratio *f* is the ratio of assets to obligations in the *retirement* period. This is consistent with the definition of unfunded liabilities in Section 3.4 in the sense that *f=1* implies UAAL=0.

Optimal pension design is then a choice of funded benefits, unfunded benefits, and a wage w_t^2 that maximizes η_t^2 subject to the constraint that the employment package gives workers at least the same utility as a private job with DC pension. It is straightforward to show that an interior solution ($0 < f < 1$) requires that

$$1+\frac{\chi_F+\chi_d(d_t^2)}{1-\chi_F}=\frac{1-\overline{\pi}}{1-\overline{\pi}^e}\left[1+\overline{\pi}^e\cdot\Delta u'(f)\right] \tag{32}$$

Whenever the l.h.s. is greater, zero funding is optimal. If the r.h.s. were greater, full funding would be optimal. The l.h.s. is greater than one and interpretable as the cost to taxpayers of a funded as compared to an unfunded pension.[19] The r.h.s. has two parts. The ratio $(1-\overline{\pi})/(1-\overline{\pi}^e)$ is a misperceptions term that exceeds one if employees' confidence in the plan is less than the employer's. The bracketed term $1+\overline{\pi}^e\Delta u'(f)$ exceeds one if employees perceive a probability of default ($\overline{\pi}^e$>0) and if this risk is uninsured ($\Delta u'(f)$>0).

Specific numbers may help build an intuition. For example, consider a 50bp annual funding cost and a 1.5% borrowing cost. Then for a period of 20 years, $\chi_F \approx 10\%$, $\chi_d(d_t^2)\approx 35\%$, and $\frac{\chi_F+\chi_d(d_t^2)}{1-\chi_F}\approx 0.5$. Assume symmetric expectations ($\overline{\pi}=\overline{\pi}^e$) and $\gamma = 4$. Suppose the employer pension is half of planned retirement income. Then an unfunded pension would imply $u'(c_{t+2}^3(s^-))/u'(c_{t+2}^3(s^+))\approx 0.5^{-\gamma}=16$, so $\Delta u'(0)\approx 15$. Equality in (31) would require a risk of $\overline{\pi}=0.5/15\approx 3.4\%$. If $\overline{\pi}\le 3.4\%$, zero funding is optimal. If $\overline{\pi}>3.4\%$ partial funding is optimal. For example, if $\overline{\pi}=25\%$, (31) prescribes $\Delta u'(f)=\frac{\chi_F+\chi_d(d_t^2)}{1-\chi_F}/\overline{\pi}=0.5/0.25=2$, which implies $\frac{c_{t+2}^3(s^-)}{c_{t+2}^3(s^+)}=(1+\overline{\pi}\Delta u'(f))^{-1/\gamma}\approx 0.76$. With 50% pension dependence, one obtains an optimal funding ratio $f = 52\%$.

Table 2 shows results of similar calculations for a range of parameters. Generally, high pension dependence, high risk aversion, and a high degree of legal ambiguity (or default risk) favor funding, whereas intermediation costs discourage funding. Panel (a) shows cutoff probabilities for zero funding. The cutoff probabilities are over 10% for all scenarios with

[19] Because $1/(1+\frac{\chi_F+\chi_d(d_t^2)}{1-\chi_F})=1-\frac{\chi_F+\chi_d(d_t^2)}{1+\chi_d(d_t^2)}$, the cost term in (32) is a rescaled version of the terms in (30) and (31).

25% pensions dependence and for many of the 50%-dependence scenarios with low risk aversion and/or high cost. Thus zero funding remains optimal unless legal risk is substantial.

Panel (b) shows optimal funding ratios. To focus on scenarios with positive funding, all columns assume low intermediation cost and some rows assume very high risk-aversion (up to 10). Many entries are nonetheless under 50%. Funding ratios greater than 80% are optimal only with low cost *and* high risk aversion (γ=10) *and* relatively high pension dependence (≥50%).

Because many state and local government employees are not participating in social security, scenarios with high pension dependence are empirically relevant. They could rationalize empirically observed funding ratios that are near 100%.

Three general results should be noted: First, *full funding is never optimal under symmetric beliefs*. This is because $\Delta u'(f) \to 0$ as the funding ratio approaches 100%. The optimal funding ratio is always bounded away from one, but not from zero. Specifically, zero funding is optimal under symmetric beliefs whenever

$$\bar{\pi}\Delta u' \leq \frac{\chi_F + \chi_d(d_t^2)}{1-\chi_F}. \tag{34}$$

Second, even with asymmetric beliefs, *full funding is suboptimal unless employees are much more pessimistic than the employer*. Specifically, full funding is optimal if and only if

$$\frac{\bar{\pi}^e - \bar{\pi}}{1-\bar{\pi}^e} \geq \frac{\chi_F + \chi_d(d_t^2)}{1-\chi_F} \tag{35}$$

For example, if $\frac{\chi_F + \chi_d(d_t^2)}{1-\chi_F} \approx 0.5$ and the true default probability is zero, the perceived risk must be at least 33%. Otherwise optimal funding is at most partial.

Third, *legal ambiguity is always costly for the plan sponsor*. Even if the actual or perceived risks are low enough that zero funding remains optimal, doubtful promises are discounted. An employer offering a DB plan must give workers the same utility as employers offering DC plans. Because ambiguity about defined benefits imposes idiosyncratic risk on employees, the employer must compensate with a higher wage. Funding can reduce these concerns by serving as collateral, but it has a cost.

Finally, note that legal ambiguity raises some deeper questions about the meaning of promises. If $\bar{\pi}$ were near one, a DB plan would provide an investment-backed retirement income just like a DC plan, except with centralized asset management and with a slight upside chance that the employer might (almost voluntarily) pay extra benefits. This case would raise questions beyond the scope of this paper, e.g., about the meaning of "defined" plans when legal ambiguity is so pervasive that legal definitions are virtually meaningless.

5. Incomplete Markets and Risk Sharing

The models above assumed complete markets. In reality most retirement savers invest in standard financial instruments such as stocks and bonds. Many DC retirement plans restrict investment options further, sometimes down a small menu of mutual funds. DB plans provide a much more sophisticated set of benefits, which includes microeconomic insurance features that savers would find costly or impossible to purchase on their own (e.g., annuities and survivor benefits). Managers of DB plans may also have access to a wider range of investment options, through at a cost. This raises questions about robustness of the previous results. Three issues deserve comment.

(a) Most importantly, incomplete markets do not overturn the principal arguments against funding. The intuition is simple. DC plans are constrained by missing contingent claims markets because retirement incomes must be backed by publicly available financial assets. DB plans are not constrained by missing markets because they are private contracts between employer and employees. Market constraints apply only to the extent that funding is mandatory or desirable, e.g., due to regulations or as collateral. Thus market incompleteness tends to (i) magnify the advantages of DB over DC plans and (ii) increase the opportunity cost of DB funding.

To model incomplete markets, assume only stocks or bonds are traded on liquid financial markets. Stocks are claims on capital and pay a state-contingent return $R^k(s_{t+1})$. Bonds are viewed as safe claims that pay $R^b(s_{t+1} \mid s_t) = \bar{R}^b(s_t) \equiv 1/E_t[m_{t+1}]$ in all states s_{t+1}.

In addition, assume Arrow securities are available at an intermediation cost $\chi_z \geq 0$. Let $z^+(s_{t+1}) \geq 0$ and $z^-(s_{t+1}) \geq 0$ denote long and short positions in Arrow securities on state s_{t+1}, and assume the period-t prices are $\xi^+(s_{t+1}) = \pi(s_{t+1})m(s_{t+1})(1+\chi_z)$, and $\xi^-(s_{t+1}) = \pi(s_{t+1})m(s_{t+1})/(1+\chi_z)$. (Note that symbols +/- are used differently than in Section 4.) The cost $\chi_z \geq 0$ can be interpreted as "markup" over the pricing kernel imposed by insurance companies or investment banks. A setting without Arrow securities is included as limiting case $\chi_z \to \infty$. Assume $m(s_{t+1})$ is not spanned by stocks and bonds.

A retirement portfolio can then be written as

$$X(s_{t+1}) = R^k(s_{t+1})x_t^k + R^b(s_{t+1})x_t^b + z^+(s_{t+1}) - z^-(s_{t+1}) \text{ for all } s_{t+1} \tag{35}$$

where x_t^k and x_t^b are holdings of "standard" stocks and bonds (purchased at unit price). The period-t cost of a portfolio of such a portfolio is

$$x_t = x_t^k + x_t^b + \sum_{s_{t+1}} \xi^+(s_{t+1})z^+(s_{t+1}) - \sum_{s_{t+1}} \xi^-(s_{t+1})z^-(s_{t+1}) \tag{36}$$

The cost of a given portfolio is strictly increasing in χ_z unless the $X(s_{t+1})$ lies in the space spanned by $R^k(s_{t+1})$ and $R^b(s_{t+1})$.

Let $U^{DC}(\chi_z, \chi_{DC})$ denote the utility of an individual who invests optimally in a DC pension plan constrained by (35) and (36); U^{DC} is strictly decreasing in χ_z. Similarly, let $U^{DB}(\chi_z, \chi_F; w_t^1, w_{t+1}^2)$ denote an employee's utility working under a DB plan that offers first and second period wages (w_t^1, w_{t+1}^2) and invests optimally. Provided $\chi_F \leq \chi_{DC}$, a DB plan can replicate the optimal DC plan and use the same funding and investment policy. Hence

$$U^{DB}(\chi_z, \chi_F; w_t - x_t^1, ew_{t+1} - x_{t+1}^2) \geq U^{DC}(\chi_z, \chi_{DC}) \tag{37}$$

holds for any χ_z. However, the DB plan is superior in two ways. First, a DB plan has a comparative advantage over DC because it can settle non-traded claims internally—across employee groups or with taxpayer—without incurring transaction cost. The relevance of internal settlement depends in part on risk sharing opportunities (discussed below) and funding. Second, reduced funding raises taxpayer utility by the same reasoning as in Section 3, so zero funding is still optimal. Zero funding is even more advantageous because it avoids the cost χ_z. If (37) holds with strict inequality, a plan that maximizes η_t^1 or η_t^2 subject to

$U^{DB}(\chi_z, \chi_F; w_t^1, w_{t+1}^2) \geq U^{DC}(\chi_z, \chi_{DC})$ will have lower cost (i.e., $w_t^1 < w_t - x_t^1$ and/or $w_{t+1}^2 < ew_{t+1} - x_{t+1}^2$). Reduced cost would justify an expansion of public services and it would increase property values.

(b) The optimality of zero contributions by the young should be flagged as a more fragile result. It relies on several implicit assumptions: (i) Debt was assumed state contingent, which allows the young to hedge risks without holding assets. If one assumed "inflexible" debt instead, meaning a debt with exogenous return distribution, it is straightforward to show that the young will make pension contributions—either in a DC plan or via their employer in an optimal DB plan—provided debt returns differ sufficiently from the optimal return distribution.[20] (ii) Pensions contributions in middle age were assumed unconstrained. If one assumed a binding upper bound on x_{t+1}^2, contributions in young age would be a way to increase pension assets that benefit from low taxes in retirement.[21] However, there is no incentive to shift taxable income from young to old age unless $(1-\tau^3)/(1-\tau^1) > 1$; it is questionable if condition holds empirically. (iii) The model abstracts from career employment issue. Unless career employment promises productivity gains, pension benefits for the young are difficult to justify.

(c) Restrictions on the space of financial contracts matter only to the extent that the missing markets are needed to implement the unconstrained optimal allocation. Finance theory implies that the aggregate pricing kernel is the marginal rate of substitution of an investor who bears a representative share of aggregate risks. If output is produced with labor and capital, the fundamental risks are shocks to wages, shocks to the return to capital (including houses). This suggests that m_{t+1} is positively correlated with next period's wage, returns to capital, and house prices.

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[20] Note that inflexible debt has ramifications for optimal policy. Because inflexible debt breaks the alignment between pricing kernel and marginal rates of substitution in (20), there is a potential for fiscal policy to improve risk sharing between younger and older generations, as in Bohn (2009). A detailed examination is left for future research because tax instruments are limited in the property tax model and because intergenerational risk sharing is more promising at the national level.

[21] Formally, the shadow value of the upper bound on x_{t+1}^2 would reduce the marginal rate of substitution between middle and old age consumption and enter (17) in a way that reduces $\mu(x_t^1)$, potentially down to zero.

From budget equation (9), the income of the middle aged is naturally correlated with w_{t+1}. Optimal risk sharing calls for $\tilde{c}^2_{t+1}$ to be perfectly correlated with m_{t+1}. Assuming inflexible debt so the young contribute to pensions, optimal pension accruals from young to middle age should provide equity exposure and hedging against wage risk. From (10), the risk exposure of retirees is largely determined by pensions. Hence optimal pension payments to retirees should provide positive equity exposure and positive exposure to wage risk. (Because both cohorts own houses, optimal hedging against housing risk is unclear.)

These arguments suggest that missing markets for wage-indexed claims are a serious constraint on DC plans.[22] DB plans, in contrast, can provide internal risk sharing because the hedging needs younger and older cohorts go in opposite directions. This requires DB plans to promise wage-linked pensions, i.e., pensions that cannot be matched by funding. Final salary plans can be interpreted as practical implementation, because of all possible salary indexation schemes, the final salary is closest to the retirement period. Thus the arguments in (a) above apply to wage indexing and final salary plans.

Finally, one destructive scenario should be noted. Consider a DB plan that has made promises—optimally—that are not fully funded and cannot be backed by traded assets; say, a wage-linked pension. Suppose regulators suddenly demand that the plan be fully funded in all states of nature. This mandate can be satisfied only by buying Arrow securities at marked-up prices or by over-collateralizing the plan with stocks and bonds. Both options are expensive, perhaps more costly than offering a DC plan. Hence uncertainty about funding rules gives plan sponsors an incentive to shift from DB plans to DC (or hybrid) plans that are more easily backed by standard financial assets—perhaps even preemptively. To conclude, the analysis of incomplete markets suggests that taxpayer gains from underfunding are no less than in the complete market model and arguably greater.

[22] The seriousness of the problem and the ramifications for portfolio choice depend in part on the correlation between wages are equity returns. Baxter-Jermann (1997) and Bohn (1999) estimate a high correlation. Constantinides et al. (2002) argue for a low correlation.

6. Concluding comments

This paper examines state and local pension funding in an overlapping generations model with intermediation cost. There are three main lessons.

First, funding is difficult to justify when taxpayers hold debt and face intermediation cost. Then taxpayers' cost of funding public pensions is greater than the (risk-adjusted) rate of return available on fund investments. Put simply: Why should taxpayers vote to accumulate assets in a public retirement plans that buys Treasury notes yielding, say, 2% when they are paying 15% interest on their credit cards and 7% on car loans? Though part of the spread between consumer loans and Treasury rates reflects default risk, another part reflects intermediation cost. This cost could be avoided by underfunding public pensions and thereby deferring taxes. Hence in the model, zero funding is optimal.

Similar arguments apply when pension funds hold equities and other risky assets. For any asset class, if returns on pension assets and on taxpayer debt are adjusted for risk, there is a gap that equals the cost of borrowing plus the cost of fund management. Hence Shakespeare's advice not to borrow and lend makes sense—unless taxpayers are gullible enough to believe that pension managers can earn abnormal returns that justify the costs.

Second, legal ambiguity—the risk that unfunded pension promises may not be enforceable—and default risk are plausible countervailing forces. Pension funds that are irrevocably dedicated to the benefit of employees can serve as collateral. Collateral is valued by employees and allows employers to reduce total compensation. Hence funding can be optimal despite intermediation cost. However, collateral arguments justify at most partial funding. Full funding remains suboptimal except in a scenario where employees are irrationally pessimistic about unfunded pensions. Thus the model is broadly consistent with the fact that most public retirement plans are funded, but assets usually fall short of full funding (Munnell et al 2008).

Third, mandatory funding would increase the cost of DB plans, in absolute terms and relative to DC plans. Regulations that require excessive funding are therefore costly and damaging to DB plans. In the private sector, DB plans have largely vanished in the post-ERISA period of increasingly stringent funding rules. One must wonder if public sector DB plans await the same fate.[23] The demise of DB pensions would be socially costly because DB pensions have important efficiency advantages over defined contribution plans.

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[23] Recently, pension fund losses triggered by the financial crisis have led to political demands that public pensions be operated so taxpayers face absolutely no risk of future shortfalls. The required funding would be far in excess of expected obligations and could be prohibitively costly.

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Table 1: What Percentages of U.S. Families are Borrowers?

Generation (in Model)	Age Bracket	Percentage of Families who hold			
		Any Debt	Type of Debt:		
			Mortgage	Installment Loan	Credit card
		(1)	(2)	(3)	(4)
Young	<35	**83.5%**	*37.3%*	**65.2%**	*48.5%*
	35-44	**86.2%**	**59.5%**	**56.2%**	**51.7%**
Middle	45-54	**86.8%**	**65.5%**	**51.9%**	**53.6%**
	55-64	**81.8%**	**55.3%**	*44.6%*	*49.9%*
Old	65-74	**65.5%**	*42.9%*	*26.1%*	*37.0%*
	≥75	*31.4%*	*13.9%*	*7.0%*	*18.8%*
Memo: Interest Rate Spreads over Treasuries					
		Prime vs. TB 3mo	Fixed 30y vs Tr.10yr	Car loan vs Tr.3yr	Av. Card vs TB 3mo
Current (July 2010)		3.1%	1.7%	4.9%	14.2%
Average 1990-2009		3.2%	1.7%	NA	NA

Legend: **Bold** = majority. *Italics* = minority.

Sources: Percentage of families' data from Survey of Consumer Finances 2007. Mortgage refers to mortgages secured by primary residence. Rate spread averages are own calculations based on FRB release H.15. Current spreads from the Wall Street Journal for July 9, 2010. Prime vs. TB 3mo = Prime rate minus 3-month Treasury bill rate. Fixed 30y vs Tr.10yr = 30-year fixed rate mortgage rate minus 10-year Treasury rate. Car loan vs Tr.3yr = Average rate on 36 month car loans minus 3-year Treasury rate (Per SCF, car loans are the most common installment loans). Av. Card vs TB 3mo = Average rate on credit cards minus 3-month Treasury bill rate.

Table 2: Funding as collateral against legal ambiguity or default

(a) Maximum probability of enforcement problems so ZERO funding is optimal

Funding cost		Annual	0.25%	0.5%	1.0%
Borrowing cost		Annual	0.75%	1.5%	3.0%
Cost factor		20-year	**0.224**	**0.505**	**1.316**
Pension Dependence	Risk aversion	**Risk factor** $\Delta u'$	(1)	(2)	(3)
25%	2	**0.8**	28.8%	64.9%	(never)
25%	3	**1.4**	16.3%	36.8%	96.0%
25%	4	**2.2**	10.4%	23.4%	60.9%
50%	2	**3.0**	7.5%	16.8%	43.9%
50%	3	**7.0**	3.2%	7.2%	18.8%
50%	4	**15.0**	1.5%	3.4%	8.8%
75%	2	**15.0**	1.5%	3.4%	8.8%
75%	3	**63.0**	0.4%	0.8%	2.1%
75%	4	**255.0**	0.1%	0.2%	0.5%

(b) Optimal funding ratios for given probabilities of enforcement problems

Default Probability:		**5%**	**10%**	**25%**
Funding cost		0.25%	0.25%	0.25%
Borrowing cost		0.75%	0.75%	0.75%
Optimal $\Delta u'$ = Cost/Default		4.477	2.239	0.895
Pension Dependence	Risk aversion	(1)	(2)	(3)
25%	2	0	0	0
25%	4	0	0	41%
25%	10	37%	56%	75%
50%	2	0	11%	45%
50%	4	31%	49%	70%
50%	10	69%	78%	88%
75%	2	24%	41%	64%
75%	4	54%	66%	80%
75%	10	79%	85%	92%