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AGGREGATE OUTPUT WITH OPERATING RATES AND INVENTORIES AS BUFFERS BETWEEN VARIABLE FINAL DEMAND AND QUASI-FIXED FACTORS

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ABSTRACT

Empirical evidence has long shown that output varies more in the short-run than do all factor inputs, including employment and hours worked. There is also evidence that all factors, including capital, start adjusting within a few months, suggesting that production models should treat all measured factor inputs as quasi-fixed.

In such a context, long-run equilibrium involves the choice of average factor proportions, including an average operating rate, that minimize total costs of producing the desired level of output. In response to unexpected or temporary changes in demand or cost conditions, optimal temporary equilibrium involves some changes in factor demands coupled with the joint use of pricing and production decisions to make best use of the buffering capacity provided by inventories and operating rates.

Applying this framework to aggregate annual data, this paper concentrates on the econometrics of the production or operating rate decision, since the operating rate is the key adjusting variable in the short-run. The operating rate decision also reveals most clearly the important consequences of quasi-fixity, and shows how our model contrasts with more conventional treatments. Other models of temporary equilibrium of production usually assume either the strict applicability of the underlying production function (requiring the assumption of either completely flexible product prices or at least one fully variable factor if quantity rationing is not to take place) or that current output is determined by aggregate demand without reference to the production function constraint.

The assumed long-run production structure is two-level CES, with the inner function's vintage bundle of capital and energy combining with efficiency units of labour in the outer function. Long-run average cost minimization assumptions are used to derive the parameters of the production function, assuming constant returns to scale and constant growth of labour efficiency. These assumptions about the functional form and properties of the long-run production function are tested against various alternatives in the context of the derived temporary equilibrium output decision.

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1. Empirical and Theoretical Background.

The theoretical and econometric literature on the short-run or temporary determination of aggregate output has long been in an unsettled state. Although Keynes and the classics both argued that labour could be treated as a variable factor that could be immediately (and costlessly) adjusted to keep firms on their production functions, the evidence has persistently failed to support that assumption. The evidence takes the form of the finding of short-run increasing returns to employment and average hours; and of the almost universal result that all factors (including hours) adjust in the short-run by less than the amount required to be consistent with an underlying production function[1]. Okun's Law[2] reports the empirical regularity of an "approximate 3-to-1 link between output and the unemployment rate" (Okun 1970, p. 137). This finding of apparent short-run increasing returns to labour in the United States has been duplicated in many countries, although in countries such as Japan where employment is much more unaffected by short-term changes in output, the Okun's Law ratio reaches such high levels (28 to 1 in Hamada and Kurosaka 1984) as to demand the treatment of labour as a quasi-fixed factor. Many macroeconometric models implicitly accept the quasi-fixity of labour by deriving desired employment and/or desired hours from a production function and by finding significantly less than immediate response of actual employment towards the target value. Since such models typically determine output from the demand side, without explicit reference to the production function, attention is diverted away from the fact that the partial adjustment of the most variable factor implies that all factors are quasi-fixed.

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[1] Fair (1969) provides an extensive summary of the previous literature. See also Solow (1973).
[2] The original 1962 paper is reprinted as an appendix to Okun (1970).

Over the past fifteen years there have been many studies of production based on the translog and other flexible functional forms. The substitution and other parameters are usually estimated from cost share equations based on the assumption of full and immediate adjustment. These production models are usually represented by their dual cost functions, and their primal forms often remain unspecified, so that their maintained hypothesis of constant factor utilization remains untested. Where this assumption has been indirectly tested, in the context of the factor share equations, it has been heavily rejected (e.g. Mohr 1980).

More recent work involves what Berndt, Morrison and Watkins (1981) have described as "third generation" production models wherein flexible functional forms for production are combined with assumed costs of adjustment for one or more quasi-fixed factors to give a dynamic model of factor demands. The adjustment costs for the quasi-fixed factors imply overshooting for at least one of the variable factors. Morrison and Berndt (1981) test for, and find, significant quasi-fixity of capital and non-production workers, following Oi's (1962) suggestion that quasi-fixity of labour is likely to be more prevalent for supervisory and staff employees than for production workers. Our hypothesis is that all types of labour are quasi-fixed, and that it is therefore necessary to take explicit account of the choice of a utilization or operating rate. Berndt and Morrison (1981) suggest that capacity output should be defined, following the notion introduced earlier by Klein and Preston (1967), as that level of output where the short-run and long-run cost functions are tangent. We agree with their suggestion, but note that where all factors are quasi-fixed there is considerable ambiguity (noted earlier by Stigler 1939) in defining the notional short-run cost function, especially where, as argued in this paper, the costs of abnormal utilization rates do not generally show up in current measured costs. This suggests defining normal capacity in terms of the underlying production function at average utilization rates. The temporary equilibrium level of output will then differ from normal output in a manner determined by the utilization or operating rate decision.

As long as there are at least some important quasi-fixed factors, it will in general not be optimal to meet all unexpected changes in final demand by changes in output. Several authors have emphasized that where changes in production are costly and demand is variable it will be optimal to use changes in inventories and in prices, along with changes in production, to meet unforseen or temporary changes in final demand.[3]. Other authors have shown in more detail why there are many prices that are set by producers and not changed unless there arise fundamental or sustained changes in expected demand or cost conditions[4]. More recent work has emphasized the joint optimality for buyers and sellers in "customer markets" (Okun 1981, chap. 4) to maintain relations characterized by relatively stable prices and sustained patterns of supply[5]. Okun argues that the advantages of continuity in customer markets for goods and services are similar to those that bind firms and workers in career labour markets. As Kuh (1965) and others have pointed out in the context of the labour market, the importance of continuity in both labour and product markets means that currently measured prices and quantities will not appear to satisify the conditions for short-term optimality. That does not mean that the strategies followed are not optimal, only that the books are balanced over a longer time span than the normal periods used for econometric estimation.

Another important strand of literature has emphasized that firms facing uncertain demand and cost conditions will tradeoff flexibility against static efficiency, because technologies that can produce at least cost under known demand and cost conditions are less easily adaptable to unexpected changes in those conditions. The optimal tradeoff between flexibility and static efficiency is that which minimizes the present value of current and expected future costs[6]. Quasi-fixity of factor inputs and flexibility of plant design are likely to be mutually

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[3]Blinder (1981, 1982) and Hay (1970) both emphasize the interdependence of output, inventory, and pricing decisions

[4] The early evidence goes back to the 1930s Oxford studies in the price mechanism, e.g. Hall and Hitch (1939).

[5] Gordon (1981) provides a survey of recent theories and evidence of gradual price adjustment.

[6] Insightful early analysis of this trade-off may be found in Stigler (1939) and Hart (1940).

re-enforcing, since flexibility will have a high payoff where quasi-fixity is great, and the benefits of quasi-fixity (whether showing up as smaller total adjustment costs, lower average transactions costs in markets with high continuity, or lower initial costs for no-rush construction) are less costly to obtain if ex ante plant design facilitates flexible ex post changes in operating rates, factor mix, and output characteristics.

What are the implications of this theory and evidence for the specification and estimation of aggregate production models? In our view, any model designed to embody explicit production constraints and yet be consistent with the possibly widespread importance of costly and time-consuming factor adjustments, customer markets for goods, and career or long-term (implicit) contracts for labour is likely to need the following features:

1. Explicit minimization of measured short-run costs should be expected to apply on average, and not on a period-to-period basis;
2. Similarly, a production structure based on measured factor inputs should be expected to hold on average, and not during each production period;
3. If quasi-fixity of factors is empirically important, then firms will equip themselves to operate over a range of feasible utilization rates, and will choose their factor quantities, plant designs, and normal operating rates so as to minimize average costs over the expected pattern of operating rates;
4. The long-term commitments implied by the quasi-fixity of factor inputs implies that factor demand decisions be based on expected future demand and cost conditions;
5. Given the expected joint role of inventories, operating rates, and price changes in meeting unexpected or temporary changes in final demand, all three decisions should be specified and estimated consistently, with their key interdependencies made explicit;
6. The treatment of the production decision as an operating rate decision dictates the choice of a production structure that can equally well be represented by its direct form as

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[6](cont'd) The trade-off is clearly stated in terms of modern production theory by Fuss and McFadden (1978).

by its dual cost function.

2. Model Specification

For simplicity of exposition, we shall develop the model in terms of a two-level CES production function[7]. using efficiency units of labour (assuming Harrod-neutral technical progress) and an inner CES bundle of capital-plus-energy to produce q, the aggregate gross output of the energy-using sector. Consistent long-term planning for output and factor inputs must therefore be constrained by the CES relationship between expected profitable future output (q^*) and target inputs of the capital-energy bundle (k_e^*) and labour (ΠN_{ne}^*, where Π is the index of employee efficiency[8] and N_{ne}^* the desired level of employment).

$$q^* = [\mu(\Pi N_{ne}^*)^{(\tau-1)/\tau} + \nu k_e^{*(\tau-1)/\tau}]^{\tau/(\tau-1)} \tag{1}$$

For any given value of desired future output, the first-order conditions for cost minimization can be used to define the desired factor inputs, shown in (2) and (3):

$$k_e^* = [1+(\nu/\mu)^{\tau}(\Pi p_{ke}/W_{ne})^{\tau-1}]^{\tau/(1-\tau)} q^* \tag{2}$$

$$N_{ne}^* = (1/\Pi)[q^{*(\tau-1)/\tau} - \nu k_e^{*(\tau-1)/\tau}/\mu]^{\tau/(\tau-1)} \tag{3}$$

where p_{ke} is the price index for the capital-energy bundle and W_{ne} is the average annual wage in the non-energy sector. Given cost minimization, the factor price frontier (Samuelson 1953-54) or minimum attainable cost index is defined by:

$$c_{ken} = [\mu^{\tau}(W_{ne}/\Pi)^{1-\tau} + \nu^{\tau} p_{ke}^{1-\tau}]^{1/(1-\tau)} \tag{4}$$

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[7] The bundling of capital and energy in a separable subfunction is supported by the results of Berndt and Wood (1979) for the United States and Artus (1983) for the other major OECD countries. The use of a two-level CES function as a way of combining flexibility of parameters with reasonable simplicity of functional form was suggested by Sato (1967).

[8] Since employment is the direct measure of labour input, Π includes the effect of trend changes in average weekly hours.

Under circumstances of uncertainty and quasi-fixity, it may not be expected that actual output will equal desired output, or that desired factor ratios will equal optimal ones, except on average. [9] The main focus of this paper is on the output decision, for given quantities of the quasi-fixed factors. We first define a measure of the quantity of output that would be forthcoming if the actual factor inputs were combined according to the underlying production function:

$$q_{sv} = [\mu(\Pi N_{ne})^{(\tau-1)/\tau} + \nu k_{ev}^{(\tau-1)/\tau}]^{\tau/(\tau-1)} \quad (5)$$

where k_{ev} is the vintage bundle of capital and energy based on the separable CES inner function. If q^* and current relative factor prices had been accurately foreseen, then, in the absence of unforseen or temporary fluctuations in demand, actual and optimal factor inputs would be equal, actual output would equal q_{sv}; actual costs would follow the factor price frontier, and inventory stocks would be at their optimal levels.

Why do changes in cost or demand conditions provide an incentive to produce at some level other than q_{sv}? This question is probably best answered by treating factor utilization, or the operating rate, as a factor of production, and then deriving an exact or an approximate equation for its optimal level. We have already seen that only unexpected or temporary changes in demand or cost conditions can provide an incentive to vary the operating rate, since in the absence of such variations the actual and desired quantities of measured factor inputs will be equal, and the operating rate will be constant at the value that minimizes average costs[10]. When demand or cost conditions fluctuate, firms have, in

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[9]If nominal wage rates are expected to rise at the general rate of inflation plus the rate of increase in the labour efficiency index, as would be required for equilibrium growth, then current prices may be used instead of future prices in equations (2) and (3) in the absence of specific information about future movements in the prices of energy and capital goods relative to the general rate of inflation.

[10] The optimal normal operating rate is naturally a function of the degree of uncertainty; in conditions of lower uncertainty firms would not need to invest so heavily in flexibility, and they would thereby lower average costs, in part by investing in smaller buffer stocks of inventories and excess capacity.

addition to whatever changes they choose to make in their quasi-fixed factors, three interdependent instruments available to them: variations in the operating rate, variations in inventory stocks, and changes in prices. Given the demand and cost conditions, decisions about the values for two of these instruments implies the value for the third. The short-term decision problem for the representative firm can be characterized as minimizing the notional short-term disequilibrium cost function based on the divergences between actual and normal values for the operating rate, inventory stocks, and price increases:

$$c_d = (|q/q_{sv} - 1|, |k^*_{inv}/k_{inv} - 1|, |p_q/c_{ken}|) \qquad (6)$$

subject to the demand function for non-inventory sales,[11]

$$s = s_o p_q^{\alpha} \qquad (7)$$

and the inventory stock identity:

$$k_{inv} = k_{inv-1} + q - s + m_{ne} \qquad (8)$$

where m_{ne} is the level of non-energy imports. For reasons already discussed, it is not possible to obtain direct evidence about the functional form of the cost function (6), since the consequences of abnormal factor utilization, non-optimal inventories, and excessively variable prices will not generally show up in the current period's costs or revenues, but will appear gradually. Fortunately, to obtain an operational model for estimation, all that need be assumed is that there is a symmetrically rising marginal cost of proportionate differences from normal utilization rates, from desired inventories, and price changes not directly linked to changes in the factor price frontier[12]. Optimal short-term response to, e.g., changes in demand conditions requires mutually dependent responses of operating rates, inventories, and prices in order to equalize the marginal costs of using the alternative responses. The optimal temporary

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[11]For the open economy, with imports as an additional source of supply, there is an additional decision variable. In the MACE model (Helliwell et al, 1984), which provides the first macroeconomic application of the production structure described in this paper, this is dealt with by introducing a third level in the nested CES supply structure. In the top level, there is a long-term CES relationship between non-energy imports and the gross output, q, of the domestic energy-using sector in meeting final demands (including exports). This is addressed explicitly later on.

[12] In later sections we shall test this assumption indirectly by examining the skewness of the distributions of the ratios of actual to normal operating rates and inventory levels.

equilibrium choice of the three variables can be represented by equations for prices and for either production or inventory change, with the other being determined by the identity linking production and sales. Equations (9) and (10) are log-linear form for the price and production equations, and (11) shows a comparable inventory equation in conventional linear adjustment form. Either (10) or (11) could be used, with the equation (8) used to define the other.

Price adjustment equation

$$p_q/p_{q-1}=(c_{ken}/c_{ken-1})^{\beta_1}\ (k^*_{inv}/k_{inv-1})^{\beta_2}(q/q_{sv})^{\beta_3}\ +\ u_t \qquad (9)$$

Operating rate equation

$$q/q_{sv}=c_q^{\beta_4}(s/s_n)^{\beta_5}(k^*_{inv}/k_{inv-1})^{\beta_6}\ +\ v_t \qquad (10)$$

where c_q is the ratio of current unit costs to the output price and s_n is normal or expected sales.

Inventory adjustment equation (alternative to (10))

$$k_{inv}-k_{inv-1}=\beta_7[s-s_n]+\beta_8[c_q^{\beta_9}k^*_{inv}-k_{inv-1}] \qquad (11)$$

Where the short-term cost variable c_q modifies the normal target stock of inventories to reflect the implications for inventory accumulation of profit-induced changes in the relationship between production and sales.

For the open economy, the short-term supply structure may be more complicated, as imports may provide a short-term buffer as well as a long-term source of supply. If non-energy imports are substitutable with domestic normal output q_{sv} in a long-run CES relationship, then normal or permanent imports will be given by

$$m_{nep}=q_{sv}(p_{mne}/p_q)^{\Phi} \qquad (12)$$

Actual imports may differ from normal imports by lags in the response to relative prices as well as by a potential buffering role played by inventories if there are discrepencies between actual and normal operating rates or inventories. If the production and import buffering responses are symmetric, then we would have the following import equation:

$$m_{ne}/m_{nep}=c_q^{\beta_{10}}(s/s_n)^{\beta_{11}}(k^*_{inv}/k_{inv-1})^{\beta_{12}}+w_t \qquad (13)$$

where m_{nep} is as defined in equation (12). Since m_{nep} is unmeasured, equation (12) must be substituted into equation (13) to obtain equation (14) for estimation:

$$m_{ne}=c_q^{\beta_{10}}(s/s_n)^{\beta_{11}}(k^*_{inv}/k_{inv-1})^{\beta_{12}}\ q_{sv}(p_{mne}/p_q)^{\Phi}+w_t \qquad (14)$$

A finding of significant coefficients for β_{10}, β_{11} or β_{12} would imply a short-term bufering role for imports, and would require that equation (14) and equation (10) be both taken into account to deduce the buffering role played by inventory changes.

In this paper we shall concentrate on the direct estimation of the operating rate equation (10), with some attention to the matching equations for prices and imports, using the inventory stock identity to derive the implications for inventory determination.

Before proceeding to a discussion of estimation and results, there remain some specification issues, one relating to the cost variable c_q and the others to the appropriate definition of normal sales and the desired stock of inventories. The cost variable is actual unit costs relative to the output price, and can be related to the factor price frontier as follows:

$$c_q=TC/qp_q=((TC/q_s)/c_{ken})(c_{ken}/p_q)(q_s/q) \qquad (15)$$

Where TC is actual total costs, using the depreciation rate plus an interest-sensitive rental price of capital[13] to measure the return to capital, and q_s is the level of output that would be forthcoming if the existing quantities of employed factors were used at normal operating rates in the long-term two-level CES production structure, with vintage effects ignored. The first of the three terms of the compound expression measures actual total costs per unit of normal non-vintage output divided by the cost index with cost-minimizing factor proportions. This will always be more than 1.0, as actual factor proportions cannot be better than optimal. Variations in this term show the extent to which the current factor mix is out of line with

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[13]All of the evidence we have assessed shows that the derived cost-minimizing factor proportions treat the real supply price of capital as a constant, while the cost of capital most relevant to the operating rate and inventory decisions is based on a weighted average of the cost of debt financing and the (constant) long-run cost of equity capital. The precise definition is given in Helliwell, MacGregor and Padmore (1984).

current factor prices. The second term is the factor price frontier divided by the output price; variations represent changes in quasi-rents in the output market. On average this term will be less than 1.0, as average revenues over the long haul must be sufficient to cover average costs based on actual rather than currently optimal factor proportions. The third term converts costs per normal unit of output to costs per actual unit, and reemphasizes how unlikely it would be to find actual unit costs rising with increases in the utilization rate: for given levels of the quasi-fixed factors, costs per unit are bound to fall with increases in q/q_s unless the factor input prices, assumed so far to be predetermined, rise as much as proportionately with q/q_s. We shall later test whether the elasticity of the operating rate is, as hypothesized here, equally responsive to the different sources of variation in unit costs relative to the output price. It is possible, for example, that high costs due to, for example, excessively high energy consumption built into existing capital goods, would reduce the temporary equilibrium rate of output differently from changes in profitability caused by, e.g., a worsening in the terms of trade leading to a drop in the market price relative to the factor price frontier.

The definitions of normal sales, s_n, and of desired inventories, k^*_{inv}, need to be settled prior to estimation. It has been traditional for inventory models to equate desired production with expected sales, and to base expected sales on some extrapolation of past sales. However, it is possible to exploit the links between the production, inventory, and factor demand decisions more fully to develop what may be a stronger hypothesis. Changes in sales will induce buffering changes in operating rates or inventories only to the extent that they were not foreseen as being sure enough and permanent enough to justify matching changes in the quantities of quasi-fixed factors. It therefore seems more than natural to use normal output (or normal output plus normal imports in the case of an open economy) to measure the relevant expected sales concept.

For example, if a change in sales is expected, but is thought to be too temporary to justify matching changes in the stocks of quasi-fixed factors, then the difference between sales and (some function of) planned capacity q_{sv} will be the appropriate measure of the gap to be filled by buffering movements of operating rates, inventories, or imports. If normal imports have been roughly constant in relation to normal output, then normal sales can be q_{sv} multiplied by the average ratio of s to q_{sv}. If there have been important price-induced long-term fluctuations in import intensity, then normal sales might be more appropriately defined by adding permanent imports to normal output:

$$s_n = q_{sv} + m_{nep} \qquad (16)$$

Desired inventories could be defined either in relation to normal output or normal sales; since normal output is in any event the main determinant of permanent imports, the simplest definition of long-term desired inventories is q_{sv} mulitiplied by the trend value of the ratio of inventories to q_{sv}.

In the short-term production equation specified in this section, the aggregate demand influences are captured by the separate roles of s and of the output price as part of c_q. As emphasized in earlier models with quasi-fixed but endogenous output and prices (e.g. Hay (1970) and Rotemberg (1982)), exogenous shifts in demand conditions are appropriately measured as variations in s_o rather than in s. Any change in s_o will show up partly through changes in s and partly through changes in p_q. The use of s rather than s_o in the quantity adjustment equations raises no special problems of estimation or interpretation as long as s is appropriately treated as an endogenous variable for estimation purposes, and if the total effects of demand shocks are evaluated using the complete model with endogenous prices and sales.

3. Parameter Estimates and Tests Against Alternative Models

If there are economically important variations in operating rates, and hence if the production function based on measured capital, energy, and labor inputs holds on an average basis, there are implications for the appropriate estimation methods for the parameters of the underlying production structure. Two methods are appropriate, and we have used them both.

The first method is a separable two-stage process, whereby sample averages and trends, along with assumed equality, on average, between actual and cost-minimizing factor proportions, are used to reduce to a minimum the number of parameters requiring direct estimation. As described in the Appendix an iterative maximum liklihood procedure is used, in the context of the equation for the derived demand for energy, to find the retrofitting coefficient (reflecting the extent to which energy use is adjustable ex post) and the long-term elasticity of substitution in the energy-capital bundle. An iterative procedure is also used to define consistently the elasticity of substitution in the outer CES function and the rate of Harrod-neutral technical progress. Given the parameters of the long term technology, equation (5) is then used to define normal output and equation (10) is subsequently estimated to determine the parameters of the operating rate decision, and hence the joint role of operating rates and inventory changes as buffers between variable demand and the quasi-fixed factors represented by q_{sv}.

The second feasible estimation strategy is to use direct estimation of the production equation to jointly determine the longer term technology and the temporary equilibrium production response. This extended strategy can be used as a check on the results obtained from the first estimation strategy, and is necessary if one wishes to increase the complexity of the longer-term structure to such a point that there are too many parameters to be reliably estimated from average optimality and derived factor demand equations. We have used this extended strategy to test alternative models of the pace and nature of technical progress, and especially to test various hypotheses about whether there has or has not been a post-1973 slowdown in the rate of technical progress[14].

In this paper, we shall emphasize the temporary equilibrium determination of the output decision, for given parameters of the underlying production structure, obtained in the

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[14] Results of the tests for the Canadian case, which tend to support the hypothesis that there has been no post-1973 slackening in the underlying rate of technical progress, are reported in Helliwell, MacGregor and Padmore (1984).

manner described in the Appendix. The results for the matching price and import equations are reported in Helliwell, MacGregor, and Padmore (1984). Our example application uses annual Canadian data for a 29–year estimation period running from 1954 through 1982. Two–stage least squares is used for estimation, and all of the right hand variables are treated as jointly endogenous variables. The eligible instrumental variables for the first stage regressions are taken from a causally ordered list of exogenous and pre–determined variables from the macroeconomic model in which the supply structure is embedded. The results for the operating rate equation are as follows:

$$\ln q = \ln q_{sv} - \underset{(11.20)}{.25340} \ln c_q \quad (17)$$

$$+ \underset{(18.84)}{.55404} \ln s/s_n + \underset{(2.93)}{.093749} \ln k^{*}_{inv}/k_{inv}$$

where $s/s_n = [s/q_{sv}]/\langle s/q_{sv}\rangle$

where $\langle s/q_{sv}\rangle$, the sample average of the ratio of sales to normal output, is equal to 1.3396,

and where $k^{*}_{inv} = \langle k_{inv-1}/q_{sv}\rangle q_{sv}$

where $\langle k_{inv-1}/q_{sv}\rangle$, the sample average of the ratio of inventories to normal output, is equal to 0.23365.

2SLS 1954–1982 ; s.e.e.=0.00583; $\bar{R}^2$=.9998; DW=1.21

F–test for constraints on $\ln q_{sv}$ and intercept = 0.38

Coefficient of skewness=−0.1474 with standard deviation of 0.4335

Coefficient of kurtosis=−0.7800 with standard deviation of 0.8452

The parameter estimates show substantial buffering roles for both inventories and operating rates, as implied by the sales coefficient being significantly above zero (which would have indicated no buffering role for operating rates) and 1.0 (which would indicate no buffering role for inventories if imports and production played symmetric buffer roles). In fact, estimation of equation (14) for imports shows significant relative price effects ($\Phi = 1.3$ after three years) but no significant buffering role. Thus production would play a buffering role for unexpected or temporary sales changes unless the sales coefficient in equation (17) were over 1.33, since 1.33 is the average ratio of sales to normal output. There is also a substantial effect from the short-term profitability variable, as shown by the significant negative coefficient on c_q.

The functional form of equation (17) assumes that there are symmetric costs of upward and downward divergences of the operating rate away from its normal value. If this assumption is seriously false, then one would expect to find substantial non-normality of the distribution of residuals in equation (17). The coefficients of skewness and kurtosis are shown below the equation, to provide evidence on this score. Both indicate some non-normality of an expected sort: the negative skewness suggests that costs may rise faster with large positive than with large negative divergences from normal utilization rates, and the platykurtic distribution reflects less than expected frequencies in the tails of the distribution, as one would expect to find if the cost function were flat over a region near the average operating rate, and then more sharply rising with larger divergences. In total, the evidence of non-normality is slight enough (the chi-square of 2.88 is significant only at 10%) that the assumed form for the cost function for divergences of actual from normal output is not likely to be seriously inappropriate.

How confident can we be of these results, and to what extent can they be taken to support our view that there are economically important, and empirically explicable, variations in the operating rates for quasi-fixed factors? A first and obvious question to ask is whether the model is internally consistent with its own assumptions about the definitions of normal sales, desired inventories, and the treatment of the short-term output decision as an operating decision. These assumptions jointly imply a number of restrictions on the value of the coefficient on q_{sv}, which appears in the definitions of s_n and of k_{inv}, as well as in the denominator of the operating rate. Given the definitions of s_n and k_{inv}, the assumptions jointly imply that q_{sv} must have a coefficient of 1.0 in the equation for q. This constraint was imposed during estimation, and is tested by the F-statistic reported below the equation. The restrictions implied by the model are accepted so easily that the standard error of the estimate actually falls when they are imposed, since the saving on the degrees of freedom more than offsets the small reduction in explained variance.

Another easy test of the model is to compare its explanation of output to that of the underlying production function. If the latter were always binding, then it would explain output with only random residuals, and with a standard error not significantly larger than that of the main model. This hypothesis is nested within our model, and can be tested by constraining equal to 0.0 all of the coefficients other than that on q_{sv}. This raises the standard error of estimate from .0058 to .025, and the F-statistic of 126.1 on the restrictions implies rejection, at a very high level of significance, of the hypothesis that there are no economically important changes in the utilization rates for capital and employment.

Another alternative approach is to adjust labour and capital inputs by separate utilization rates, and then to assume that there are no remaining variations in factor utilization. If this procedure gives more accurate output predictions than equation (17), it could then be implemented by deriving and fitting separate equations for each of the measured utilization rates. But first it will be necessary to see whether the available measures of factor utilization can be combined with the underlying production function to give better explanations of output than does equation (17). If they cannot, then there is no reason to develop models or equations for the separate utilization variables.

Average hours worked are the usual measure of labour utilization, and that is the one we have tested[15]. For capital utilization, there are no generally available direct measures so indirect measures must be used[16]. We shall make use of the capacity utilization series published by the Bank of Canada[17]. We have constructed three alternative models of ouput determination based on these utilization series. Table 1 uses the non-nested hypothesis test suggested by Atkinson (1970) to compare these models with the results of equation (17), as re-estimated over the shorter sample period for which the utilization series is available. The first alternative uses average hours to adjust the labour input and the Bank of Canada capacity utilization series to adjust the capital-plus-energy bundle k_{ev}. The second alternative adjusts both N_{ne} and k_{ev} by the utilization rate series, while the third leaves employment unadjusted and adjusts the capital-energy bundle by means of the utilization rate[18]. In all cases the adjustments are done by multiplying the utilization rate, relative to its trend value, by the relevant factor input[19].

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[15] The labour input is thus in terms of the product of employment and average hours. Several studies have provided evidence that where employment and hours are entered separately in a production function the short-run returns to the latter are higher than to the former, e.g. Feldstein (1967). Some have argued that hours are a truly variable factor, with pecuniary diseconomies (due to the overtime and shift premia) requiring a high marginal product, e.g. Lucas (1970) and Craine (1973), while others (including us) would argue that hours are also quasi-fixed (Lazear 1981), and only appear to have a very high marginal product because they are collinear with unmeasured changes in the rate of utilization of all quasi-fixed factors.

[16] Klein and Preston (1967) suggested a trend trend-through-peaks method that many researchers have used since. More recently, Berndt and Fuss (1982) have suggested using capital asset valuation to adjust for variations in capital utilization. Depending on the extent to which current earnings are capitalized into current share prices, the latter procedure may also capture the effects of any changes in labour utilization, as the rents to all quasi-fixed factors tend to appear as cyclical variations in profits.

[17] As described in Schaefer (1980), the potential capital/output ratio is defined by the trend through troughs of the actual capital/output ratio, and not by any direct evidence from firms. It is subject to substantial historical revisions as new troughs are observed, and is unable to disentangle the effects of changes in relative prices from those of longer-run changes in technical progress.

[18] This third alternative is the one applied to determine production in the Bank of Canada's SAM model of the Canadian economy.

[19] It is necessary to remove the trends from the utilization measures, since the rate of technical progress in q_{sv} is defined so as to ensure that q_{sv} and q have the same average growth rate over the entire sample period. If the downward trend in the average work week, and the likely upward trend in average machine hours (Foss 1981) were included as separate factors, the former would raise the rate of growth of the labour efficiency index, while the latter would reduce it.

The results in Table 1 show that none of the three alternative models contain information that improves the basic model of equation (17), as shown by the insignificant coefficients on the auxiliary variable when H_0 is equation (17). By contrast, when each of the three alternatives is in turn made H_0, the additional information in equation (17) is so great as to reject H_0, as shown by the high t-statistics on the auxiliary variables. Of the alternatives to equation (17), the best is that obtained by adjusting k_{ev} by the Bank of Canada's capacity utilization series, but even in this case the standard error is three times as large as that from equation (17) fitted over the same sample period. These results therefore support our view that the observed operating rate contains much systematic information beyond that provided by conventional measures of utilization of employed factors.

Table 2 shows the results of some tests of different definitions of the cost variable c_q. The first experiment redefines c_q to include only the first two terms of equation (15), with their coefficients constrained to be equal. Unit costs are therefore defined per unit of normal output q_{sv} rather than per unit of actual output. This has the effect of raising all of the coefficients, and also of raising the standard error of estimate from .0058 to .0078. To the extent that the simultaneous equation estimation methods used do not adequately protect against spurious correlation, this increase in standard error may not represent as decisive a preference for the definition of c_q adopted in equation (17). The F-statistic on the constraints is higher than in equation (17), but not significantly so. The increase in the F-statistic may be reflecting a difference in the impacts of disequilibrium costs and of changes in quasi-rents in the output market. To show the extent to which this is so, we re-estimated the equation, still with only the first two terms of c_q, but with their coefficients freely estimated. This raises slightly the coefficient on the ratio of actual costs (per unit of normal output) relative to fully adjusted minimum costs c_{ken}, and reduces the coefficient on the terms of trade term to insignificance. The standard error of estimate is slightly reduced, but remains substantially higher than that of equation (17). To summarize this evidence, it suggests that the disequilibrium costs term has been the most important component of

c_q over the 29-year sample period, without rejecting the maintained hypothesis that c_q is appropriately defined in equation (17).

Finally, Table 3 shows results from some tests of alternative definitions of normal sales. Alternatives to the main model are provided by four different ARIMA models based on previous values of actual sales. Atkinson tests of the resulting output equations show that in each case the ARIMA models are rejected by equation (17), and in no case do the alternative models for normal sales add anything to equation (17).

4. **Summary and Implications**

The tests so far completed with our model of temporary production equilibrium tend to support fairly strongly the main elements of the model: quasi-fixity of measured factor inputs combined with variable factor utilization influenced strongly by unexpected or temporary sales changes, abnormal profitability, and gaps between actual and target stocks of inventories. If the model is acceptable, it provides a potentially important bridge between supply-determined and demand-determined macroeconomic systems, and a means of consistently and coherently integrating supply and demand influences at the aggregate level.

At any level of aggregation, a supply model that combines explicit cost-minimizing factor substitution with short-term departures from normal utilization rates has the potential for providing an enriched and possibly more accurate picture of macroeconomic dynamics. It can also provide a framework for treating short-run disturbances consistently with longer-term substitution and technical progress in the analysis of aggregate productivity movements[20]. Imbedded in a macroeconomic framework, the short-term supply structure outlined in this paper, properly supported by the associated equations for imports, prices and factor demands, would provide the necessary supply constraints in an integrated manner. It also provides a

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[20]The supply model is used to analyze the Canadian productivity experience in Helliwell (1984) and Helliwell, MacGregor and Padmore (1984), and is applied to comparable data for the seven major OECD countries by Helliwell, Sturm and Salou (1984).

framework for using those supply constraints to condition and channel the influence of changes in cost and demand conditions. We think that the results reported in this paper help to confirm the importance and determinants of the buffering roles for operating rates and inventories, and hence to show how important it is to have a supply framework that treats factors as being quasi-fixed and final demand as uncertain.

REFERENCES

Artus, P. (1983) "Capital, Energy, and Labour Substitution: The Supply Block in OECD Medium-Term Models" Paris: **OECD Working Papers**, No. 2.

Atkinson, A.C. (1970) "A Method for Discriminating Between Models" **Journal of the Royal Statistical Society Series B** 32:323-253.

Baily, M.N. (1974) "Wages and Employment Under Uncertain Demand" **Review of Economic Studies** 4: 37-50.

Berndt, E.R. and M.A. Fuss (1982) "Productivity Measurement Using Capital Asset Valuation to Adjust for Variations in Utilization" NBER Working Paper No. 895. Cambridge:National Bureau of Economic Research.

Berndt, E.R. and C.J. Morrison (1981) "Capacity Utilization Measures: Underlying Economic Theory and an Alternative Measure" **American Economic Review** 71: 48-57.

Berndt, E.R., C.J. Morrison, and G.C. Watkins (1981) "Dynamic Models of Energy Demand: An Assessment and Comparison" in E.R. Berndt and B.C. Fields, eds., **Modeling and Measuring Natural Resource Substitution** Cambridge Mass.: MIT Press.

Berndt, E.R. and D.O. Wood (1979) "Engineering and Econometric Interpretations of Energy-Capital Complementarity" **American Economic Review** 69: 342-354.

Blinder, A.S. (1981) "Inventories and the Structure of Macro Models" **American Economic Review** 71: 11-16.

Blinder, A.S. (1982) "Inventories and Sticky Prices: More on the Microfoundations of Macroeconomics" **American Economic Review** 72: 334-348.

Brechling, F. and P. O'Brian (1967) "Short-run employment functions in manufacturing industries: An international comparison" **Review of Economics and Statistics** 49: 277-287.

Craine, R. (1973) "On the Service Flow from Labour" **Review of Economic Studies** 40: 39-46.

Davidson, R., and J.G. MacKinnon (1981) "Several Tests for Model Specification in the Presence of Alternative Hypotheses" **Econometrica** 49:781-794.

Davidson, R., and J.G. MacKinnon (1982) "Some Non-Nested Hypothesis Tests and the Relations Among Them" **Review of Economic Studies** 49:551-566.

Fair, R.C. (1969) **The Short-Run Demand for Workers and Hours** Amsterdam: North-Holland.

Feldstein, M.S. (1967) "Specification of the Labour Input in the Aggregate Production Function" **Review of Economic Studies** 34: 375-386.

Foss, M.F. (1981) "Long-run changes in the workweek of fixed capital" **American Economic Review** 71: 58-63.

Fuss, M. and D. McFadden (1978) "Flexibility versus efficiency in ex ante plant design". In M. Fuss and D. McFadden, eds. **Production Economics: A Dual Approach to Theory and Applications** Amsterdam: North-Holland, pp. 311-364.

Gordon, R.J. (1981) "Output Fluctuations and Gradual Price Adjustment" **Journal of Economic Literature** 19: 493-530.

Hall, R.L. and C.J. Hitch (1939) "Price Theory and Business Behaviour" **Oxford Economic Papers** No. 2, May 1939, 12-45.

Hamada, K. and Y. Kurosaka (1984) "The Relationship Between Production and Unemployment in Japan" **European Economic Review** 25: 71-94.

Hart, A.G. (1940) **Anticipations, Uncertainty, and Dynamic Planning** Chicago: Univerity of Chicago Press (second edition, New York: Kelley, 1951).

Hay, G.A. (1970) "Production, Price and Inventory Theory" **American Economic Review** 60: 531-545.

Helliwell, J.F. (1984) "Stagflation and Productivity Decline in Canada, 1974-82" **Canadian Journal of Economics** 17: 191-216.

Helliwell, J.F., P. Sturm and G. Salou (1984) "International Comparison of the Sources of Productivity Slowdown 1973-1982" **NBER Working Paper** No. 1465. Cambridge: National Bureau of Economic Research.

Helliwell, J.F., R.N. McRae, P. Boothe, M. Margolick, T. Padmore, A. Plourde and R. Plummer (1984) "Energy and the National Economy: An Overview of the MACE Model" in A.D. Scott, ed., **Progress in Natural Resource Economics.** Oxford: Oxford University Press.

Helliwell, J.F., M.E. MacGregor and T. Padmore (1984) "Canadian Economic Growth and Productivity 1955-1990" **U.B.C. Department of Economics Discussion Paper 84-30.**

Klein, L.R. and R.S. Preston (1967) "Some New Results in the Measurement of Capacity Utilization" **American Economic Review** 57: 34-58.

Kuh, E. (1965) "Cyclical and secular labor productivity in United States manufacturing" **Review of Economics and Statistics** 47: 1-12.

Lazear, E.P. (1981) "Agency, Earnings Profiles, Productivity, and Hours Restrictions" **American Economic Review** 71: 606-620.

Lucas, R.E. (1970) "Capacity, Overtime, and Empirical Production Functions" **American Economic Review** 60: 23–27

Mohr, M.F. (1980) "The Long-Term Structure of Production, Factor Demand and Factor Productivity in U.S. Manufacturing Industries". In J.W. Kendrick and B.N. Vaccara, eds. **New Developments in Productivity Measurement and Analysis** Chicago: University of Chicago Press (NBER Studies in Income and Wealth, vol. 44) pp. 137–238.

Morrison, C.J. and E.R. Berndt (1981) "Short-run labour productivity in a dynamic model" **Journal of Econometrics** 16: 39–65.

Oi, W.Y. (1962) "Labour as a quasi-fixed factor" **Journal of Political Economy** 70: 538–55 (December 1962).

Okun, A.M. (1970) **The Political Economy of Prosperity** New York: Norton.

Okun, A.M. (1981) **Prices and Quantities: A Macroeconomic Analysis** Washington: Brookings.

Rotemberg, J.J. (1982) "Monopolistic Price Adjustment and Aggregate Output" **Review of Economic Studies** 49: 517–531.

Samuelson, P.A. (1953–54) "Prices of Factors and Goods in General Equilibrium" **Review of Economic Studies** 21: 1–20.

Sato, K. (1967) "A two-level CES production function" **Review of Economic Studies** 34: 201–218.

Schaefer, G. (1980) "Measuring Capacity Utilization: a Technical Note." **Bank of Canada Review** May 1980:3–13.

Solow, R.M. (1973) "Some evidence on the short-run productivity puzzle". In J. Bhagwati and R.S. Eckaus, eds., **Development and Planning: Essays in Honour of Paul Rosenstein-Rodan** 316–325 (London: Allen and Unwin) Chap. 17.

Stigler, G. (1939) "Production and Distribution in the Short Run" **Journal of Political Economy** 47: 305–327.

APPENDIX

This appendix derives the parameters of the two-level CES production function and describes the estimation procedure. The inner function is discussed first, followed by the outer function.

I. The Inner CES Function:

The vintage inner function which bundles energy and capital has the following form:

$$k_{ev} = (1-\delta_1-\delta_2)k_{ev-1} + i_{new}\,k^*/k^*_e \qquad (1)$$

where:

$k^*/k^*_e = \{\beta+\gamma[(\gamma p_k)/(\beta p_e)]^{\sigma-1}\}^{\sigma/(\sigma-1)}$

and where $i_{new} = i_{ne} + \delta_1 k_{ne-1}$ is re-investment with energy use malleable in the current year.

Each year, as the relative prices of energy and capital change, the optimum energy to capital ratio implied by the basic CES factor bundle, discussed below, can be calculated. A fraction δ_1 of the capital energy bundle is assumed to be retrofitted to this energy intensity. New capital installed in the current year is also assumed to use energy at the optimum rate. This provides i_{new} which is the amount of capital stock installed or retrofitted. Equation (1), the vintage capital energy bundle, enters the outer CES production function.

In equation (1) the business fixed capital stock (exluding energy) (k_{ne}), energy expenditure (e), business fixed investment (excluding energy investment) (i_{ne}), the energy price (p_e) and the scrapping rate (δ_2) are observed variables. The user cost of capital p_k is:

$$p_k = (<\delta_2>+.01\rho_r)p_a$$

where p_a is the observed implicit price of absorption and ρ_r (the long term supply price of capital) is defined as a constant, with a value such that on average total factor earnings exhaust total output over the sample period.

The basic CES inner bundle and the derivation of the optimum ratio of capital to the capital-energy bundle k^*/k^*_e in equation (1) will now be discussed. The optimal factor to bundle ratio is based on the following CES bundle which is denoted by k_e:

$$k_e = (\beta k_{ne}^{(\sigma-1)/\sigma} + \gamma e^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)} \qquad (2)$$

To derive the optimum factor ratio, the partial derivatives of (2) with respect to capital and energy are first calculated and set equal to the prices p_k and p_e. This gives the optimal ratio:

$$(e^*/k^*_{ne}) = [(\gamma/\beta)(p_k/p_e)]^\sigma \qquad (3)$$

where σ is the elasticity of substitution between capital and energy. Next the sample average of (3) is taken and the equation is solved for the ratio (γ/β). This is done by assuming that the actual and the cost-minimizing (based on current relative prices) energy/capital ratios have the same mean values over the sample period:

$$(\gamma/\beta) = (<e/\overline{k}_{ne}>/<p_k/p_e>^\sigma)^{1/\sigma} \qquad (4)$$

where $<x>$ denotes the sample average of x.

The optimum factor to bundle ratio is obtained by substituting $e=k_{ne}[(\gamma/\beta)(p_k/p_e)]^\sigma$ from (3) into (2), and by solving for k^*/k^*_e:

$$k^*/k^*_e = \beta^{\sigma/(1-\sigma)}(1+(\gamma/\beta)^\sigma(p_e/p_k)^{1-\sigma})^{\sigma/(1-\sigma)} \qquad (5)$$

The parameter β is solved from (5) by taking sample averages of both sides. It is assumed that the ratio of the optimal capital stock to the bundle of capital and energy services is equal to one, on average i.e., $<k^*/k_e^*>=1$. The expression for β is therefore:

$$\beta = <[1+(\gamma/\beta)^{\sigma}(p_e/p_k)^{1-\sigma}]^{\sigma/(1-\sigma)}>^{(\sigma-1)/\sigma} \qquad (6)$$

Estimation of σ and δ_1

The elasticity of substitution between capital and energy σ and the retrofitting parameter (δ_1) are determined by estimating the energy demand function:

$$\ln(e) = \ln(e_v) \qquad (7)$$

where e_v is the vintage energy requirement needed to operate the capital stock k_{ne} subject to the prevailing relative energy prices p_e/p_k. e_v is defined by the recursive equation:

$$e_v = (1-\delta_1-\delta_2)e_{v-1} + \{(\gamma p_k)/(\beta p_e)\}^{\sigma} i_{new} \qquad (8)$$

To obtain a starting value, e_v is set equal to e at the beginning of the sample period, on the assumption that no large and surprising changes in energy prices occurred over the preceding few years (the kick-off values start in 1952). The parameter pair (σ,δ_1) which maximizes the likelihood function of the above energy demand regression is chosen as the preferred parameter combination (so a double grid search is required).

II. The Outer CES function.

The outer function which defines normal output q_{sv} is:

$$q_{sv} = [\mu(\Pi N_{ne})^{(\tau-1)/\tau} + \nu k_{ev}^{(\tau-1)/\tau}]^{\tau/(\tau-1)} \qquad (9)$$

The following will first discuss the procedure used to derive expressions for ν,μ and Π. The final values of these parameters depend on the value of τ, the elasticity of substitution between labour and the capital/energy bundle, which is determined iteratively. The iteration method used to calculate τ will be examined last.

Equation (9) can be rewritten by setting $q=q_{sv}$ and by isolating the following expression for Π:

$$\Pi = [(q^{(\tau-1)/\tau} - \nu k_{ev}^{(\tau-1)/\tau})/(\mu N_{ne}^{(\tau-1)/\tau}]^{(\tau-1)/\tau} \qquad (10)$$

Equation (10) is used to obtain an expression for the parameter ν. First the optimum factor ratio is derived in the same way as the inner function i.e., the relative prices are obtained and solved for the factor ratio. Assuming the factor ratio is optimal provides the following ratio:

$$\Pi N_{ne}^*/k_{ev}^* = (p_{ke}\Pi/w_{ne})^{\tau}(\mu/\nu)^{\tau} \qquad (11)$$

where the price of the capital-energy bundle is:

$$p_{ke} = (\beta^{\sigma}p_k^{1-\sigma} + \gamma^{\sigma}p_e^{1-\sigma})^{1/(1-\sigma)}$$

p_{ke} is obtained by the cost-minimization problem using the inner CES function (2).

Equation (10) is substituted into equation (11). The parameter μ drops out and can be determined empirically when Π is normalized, as shown below. The parameter ν is isolated in the substituted equation and sample averages are taken to provide the following expression:

$$\nu = <(p_{ke}/W_{ne})(q/N_{ne})^{(\tau-1)/\tau}>/[<(N_{ne}/k_{ev})^{1/\tau}> + <(p_{ke}/W_{ne})(k_{ev}/N_{ne})^{(\tau-1)/\tau}>] \qquad (12)$$

Note that we cannot get as simple an interpretation as before because of the exponent on the starred variables. Instead we are normalizing so that the sample average of the ratio of the factors raised to the $1/\tau$ power is equal to the average for optimum proportions.

The value of Π, the labour productivity index for Harrod-neutral technical progress, is derived by the following procedure. Output attributable to labour is is defined by rewriting equation (10):

$$\mu\Pi^{(\tau-1)/\tau} = (q^{(\tau-1)/\tau} - \nu k_{ev}{}^{(\tau-1)/\tau})/N_{ne}{}^{(\tau-1)/\tau} \qquad (13)$$

The technical progress index is modelled to grow at a constant rate. It is estimated by ordinary least squares by regressing the log of the value provided by equation (13) on an annual time index. Given the final value of τ, the fitted values of log $(\mu\Pi^{(\tau-1)/\tau})$ can be estimated for each year. The value of μ is calculated by setting $\Pi=1.0$ in 1971. The technical progress factor for 1971 is therefore log μ. The labour efficiency index is defined simply as the exponent of log $(\mu\Pi^{(\tau-1)/\tau})$ since the parameter μ remains a constant throughout the sample period. The labour efficiency index is calculated to grow at an annual rate of 1.7%.

Finally an estimate of τ is needed to derive final values of the above parameters. The iterative procedure uses the expression for the optimum factor ratio, equation (11). The log of this equation provides the following form that can be estimated:

$$\ln\ (\Pi N^*_{ne}/k^*_{ev}) = \tau \ln(\mu/\nu) + \tau\ \ln(p_{ke}\Pi/W_{ne}) \qquad (14)$$

τ is the coefficient of the inverse price ratio. An arbitrary value of τ is used to define μ, ν, and Π. Equation (14) is then estimated by ordinary least squares and the estimated coefficient provides a new value of τ, which is used to redefine the other parameters in the next round. The process is repeated until the value of τ in equation (14) converges. This value is used to obtain the final values of μ, ν, Π and normal output q_{sv}.

The following are the estimated values of the parameters for the nested production function:
$\beta=.74175$; $\gamma=.15943$; $\mu=.0813723$; $\nu=.655263$; $\sigma=.8700$; $\tau=.53$.

The version of the model used in this paper assumes a constant annual growth rate for Π. Estimated following the procedures described above, it has the value 1.0 in 1971, and has an annual growth rate of 1.70%. In Helliwell, MacGregor and Padmore (1984), the assumption of constant underlying technical progress is tested against, and found to be superior to, a number of alternative models involving some form of slowdown in the rate of technical progress.

q_s, which is defined in the text as the level of output forthcoming if the existing quantities of employed factors were used at normal operating rates with vintage effects ignored, is obtained by using k_e (2) instead of k_{ev} (1) in the outer function (9). All parameters are kept at the estimated values given above.

Table 1

Atkinson Tests of Alternative Models of Output Determination

2SLS 1957-1982	Atkinson test	
H_0:Eq. (17); H_1:Case 1	.449	
H_0:Case 1; H_1:Eq. (17)	18.270*	
H_0:Eq. (17); H_1:Case 2	0.065	
H_0:Case 2; H_1:Eq. (17)	43.747*	
H_0:Eq. (17); H_1:Case 3	0.104	
H_0:Case 3; H_1:Eq. (17)	12.886*	
Model	s.e.e.	$\bar{R}^2$
Equation (17)	.005498	.9998
Case 1	.023716	.9955
Case 2	.052447	.9779
Case 3	.017286	.9976

Note: H_0 denotes the maintained null hypothesis and it is tested against the alternative hypothesis H_1. The Atkinson test first requires the residuals from regressing the estimated maintained hypothesis against the estimated alternative hypothesis. These residuals are included as an independent variable in the regression of the maintained hypothesis. The above table reports the t-statistic for the variable. If it is significant it indicates that H_1 adds significant explanatory power to H_0 and it implies the rejection of the null hypothesis against H_1. The J-test proposed by Davidson and MacKinnon (1981,1982) gives the same accept-or-reject advice for each of our comparisons. The J-test results are therefore not included in the table.

* in the above table denotes rejection of the null hypothesis H_0 against the alternative hypothesis H_1 at the 99% confidence level.

Table 2

A Comparison of the Operating Rate Equation
Under Alternate Definitions of the Cost Variable, c_q.

Equation	Equation (17)	Model 1	Model 2
ln q			
ln q_{sv}	1.0	1.0	1.0
ln c_q	-.25340 (11.20)	-	
ln $(TC/q_s)/c_{ken}$	-	-.32562 (7.93)	-.40270 (8.13)
ln c_{ken}/p_q	-	-0.32562 (7.93)	-.014008 (0.10)
ln s/s_n	.55404 (18.84)	.73766 (14.20)	.65568 (11.24)
ln k^*_{inv}/k_{inv}	.093749 (2.93)	.11127 (2.47)	.14668 (3.42)
s.e.e.	.00583	.00778	.00694
R^2	.9998	.9996	.9997
D-W	1.21	1.24	1.31
F-test on constraints on q_{sv} and intercept	0.38	1.86	.038
2SLS 1954-82			

Table 3

Atkinson Tests of Alternative Models of Normal Sales

2SLS 1956-1982	Atkinson test	
H_0:Eq.(17);H_1:Case 1	.412	
H_0:Case 1;H_1:Eq.(17)	12.304*	
H_0:Eq.(17);H_1:Case 2	.798	
H_0:Case 2;H_1:Eq.(17)	12.365*	
H_0:Eq.(17);H_1:Case 3	.489	
H_0:Case 3;H_1:Eq.(17)	12.469*	
H_0:Eq.(17);H_1:Case 4	.306	
H_0:Case 4;H_1:Eq.(17)	12.350*	
Model	s.e.e.	$\bar{R}^2$
Equation (17)	.005333	.9998
Case 1	.015890	.9982
Case 2	.016079	.9982
Case 3	.016168	.9982
Case 4	.016047	.9982

* in the above table denotes rejection of the null hypothesis H_0 against the alternative hypothesis H_1 at the 99% confidence level.

The following models, which all employ either a first or second order auto-regressive moving average process, were used to provide estimates of normal sales. These values were tested in the operating rate equation against the base model, equation (17). The models are:

Case 1 $\quad s_{nt}=\beta_1 s_{t-1}+\rho\epsilon_{t-1}+\epsilon_t$

Case 2 $\quad s_{nt}=\beta_1 s_{t-1}+\dots\beta_5 s_{t-5}+\rho\epsilon_{t-1}+\epsilon_t$

Case 3 $\quad s_{nt}=\beta_1 s_{t-1}+\rho\epsilon_{t-1}+\theta\epsilon_{t-2}+\epsilon_t$

Case 4 $\quad s_{nt}=\beta_1 s_{t-1}+\beta_2 s_{t-2}+\rho\epsilon_{t-1}+\theta\epsilon_{t-2}+\epsilon_t$

The first data sample used is 1947-55, since sales data are available since 1947. The forecasting models are reestimated each year, so that the parameters depend only on information available at that time. The procedure is used to obtain estimates of normal sales for each year from 1956 to 1982.