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EXPLAINING THE FAVORITE-LONGSHOT BIAS:
IS IT RISK-LOVE OR MISPERCEPTIONS?

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ABSTRACT

The favorite-longshot bias describes the longstanding empirical regularity that betting odds provide biased estimates of the probability of a horse winning—longshots are overbet, while favorites are underbet. Neoclassical explanations of this phenomenon focus on rational gamblers who overbet longshots due to risk-love. The competing behavioral explanations emphasize the role of misperceptions of probabilities. We provide novel empirical tests that can discriminate between these competing theories by assessing whether the models that explain gamblers' choices in one part of their choice set (betting to win) can also rationalize decisions over a wider choice set, including compound bets in the exacta, quinella or trifecta pools. Using a new, large-scale dataset ideally suited to implement these tests we find evidence in favor of the view that misperceptions of probability drive the favorite-longshot bias, as suggested by Prospect Theory.

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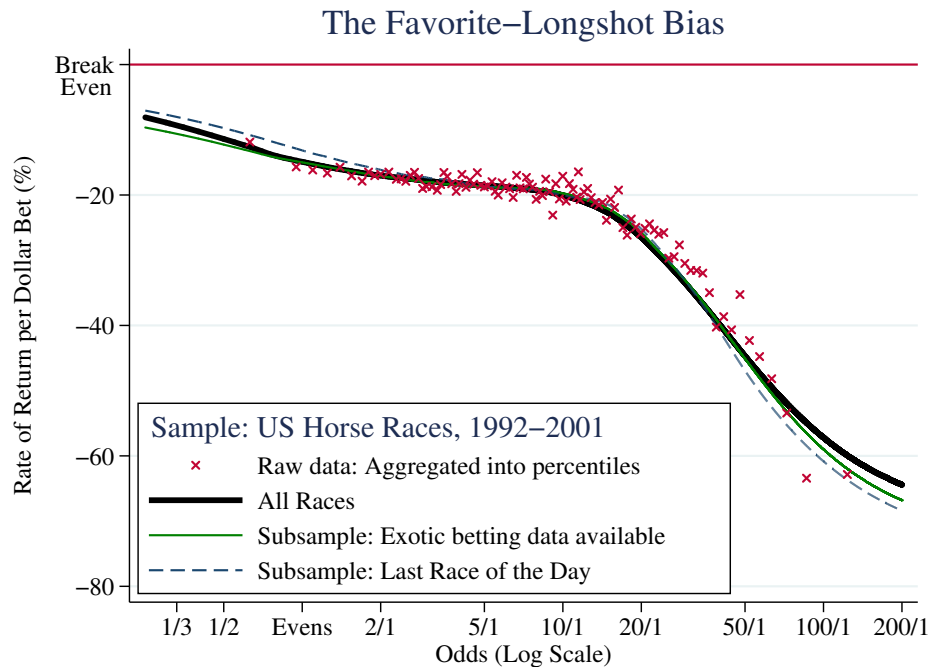
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1 Introduction

The racetrack provides a natural laboratory for economists interested in understanding decision-making under uncertainty. The most discussed empirical regularity in racetrack gambling markets is the *favorite-longshot bias*: equilibrium market prices (betting odds) provide biased estimates of the probability of a horse winning. Specifically, bettors value longshots more than expected given how rarely they win, and they value favorites too little given how often they win. Figure 1 illustrates, showing that the rate of return to betting on horses with odds of 100/1 or greater is about -61% , betting randomly yields average returns of -23% , while betting the favorite in every race yields losses of only 5.5% .

Figure 1: The rate of return on win bets declines as risk increases.



Notes: Sample includes 5,610,580 horse race starts in the United States from 1992–2001. Lines reflect Lowess smoothing (bandwidth=0.4).

Since the favorite-longshot bias was first noted by Griffith in 1949, it has been found in racetrack betting data around the world, with very few exceptions. The literature documenting this bias

is voluminous, and covers both bookmaker- and pari-mutuel markets.¹

Two broad sets of theories have been proposed to explain the favorite-longshot bias. First, neoclassical theory suggests that the prices bettors are willing to pay for various gambles can be used to recover their utility function. While betting at any odds is actuarially unfair, this is particularly acute for longshots—which are also the riskiest investments. Thus, the neoclassical approach can reconcile both gambling and the longshot bias only by positing (at least locally) risk-loving utility functions, as in Friedman and Savage (1948).

Alternatively, behavioral theories suggest that cognitive errors and misperceptions of probabilities play a role in market mis-pricing. These theories incorporate laboratory studies by cognitive psychologists that show people are systematically poor at discerning between small and tiny probabilities, and hence price both similarly. Further, people exhibit a strong preference for certainty over extremely likely outcomes, leading highly probable gambles to be under-priced. These results form an important foundation of Prospect Theory (Kahneman and Tversky, 1979).

Our aim in this paper is to test whether the *risk-love class* of models or the *misperceptions class* of models simultaneously fits data from multiple betting pools. While there exist many specific models of the favorite-longshot bias, we show in Section 3 that each yields implications for the pricing of gambles equivalent to stark models of either a risk-loving representative agent, or a representative agent who bases her decisions on biased perceptions of true probabilities. That is, the favorite-longshot bias can be fully rationalized by a standard rational-expectations expected-utility model, or by appealing to an expected wealth maximizing agent who overweights small probabilities and underweights large probabilities.² Thus, without parametric assumptions, which we are unwilling to make, the two theories are observationally equivalent in win bet data.

We combine new data with a novel econometric identification strategy to discriminate between

¹Thaler and Ziemba (1988), Sauer (1998) and Snowberg and Wolfers (2007) survey the literature. The exceptions are Busche and Hall (1988), which finds that the favorite-longshot bias is not evident in data on 2,653 Hong Kong races, and Busche (1994), which confirms this finding in an additional 2,690 races in Hong Kong, and 1,738 races in Japan.

²Or adopting a behavioral versus neoclassical distinction, we follow Gabriel and Marsden (1990) in asking: “are we observing an inefficient market or simply one in which the tastes and preferences of the market participations lead to the observed results?”

these two classes of theories. Our data include all 6.4 million horse race starts in the United States from 1992 to 2001. These data are an order of magnitude larger than any dataset previously examined, and allow us to be extremely precise in establishing the relevant stylized facts.

Our econometric innovation is to distinguish between these theories by deriving testable predictions about the pricing of compound lotteries (also called *exotic bets* at the racetrack). For example, an *exacta* is a bet on both which horse will come first and which will come second. Essentially, we ask whether the preferences and perceptions that rationalize the favorite-longshot bias (in win bet data) can also explain the pricing of exactas, *quinellas*—a bet on two horses to come first and second in either order—and *trifectas*—a bet on which horse will come in first, which second and which third. By expanding the choice set under consideration (to correspond with the bettor's actual choice set!), we use each theory to derive unique testable predictions. We find that the misperceptions class more accurately predicts the prices of exotic bets, and also their relative prices.

To demonstrate the application of this idea to our data, note that betting on horses with odds between 4/1 and 9/1 has an approximately constant rate of return (at -18% , see Figure 1). Thus, the misperceptions class infers bettors are equally well calibrated over this range, and hence betting on combinations of outcomes among such horses will yield similar rates of return. That is, betting on an exacta with a 4/1 horse to win and a 9/1 horse to come second will yield similar expected returns to betting on the exacta with the reverse ordering (although the odds of the two exactas will differ). In contrast, under the risk-love model, bettors are willing to pay a larger risk penalty for the riskier bet—such as the exacta in which the 9/1 horse wins, and the 4/1 horse runs second—decreasing its rate of return relative to the reverse ordering.

Our research question is most similar to Jullien and Salanié (2000) and Gandhi (2007) who attempt to differentiate between preference- and perception-based explanations of the favorite-longshot bias using only data on the price of win bets. The results of the former study favor perception-based explanations and the results of the latter favor preference-based explanations. Rosett (1965) conducts a related analysis in that he considers both win bets and combinations of

win bets as present in the bettors' choice set. Ali (1979), Asch and Quandt (1987) and Dolbear (1993) test the efficiency of compound lottery markets. We believe that we are the first to use compound lottery prices to distinguish between competing theories of possible market (in)efficiency. Of course the idea is much older: Friedman and Savage (1948) notes that a hallmark of expected utility theory is "that the reaction of persons to complicated gambles can be inferred from their reaction to simple gambles."

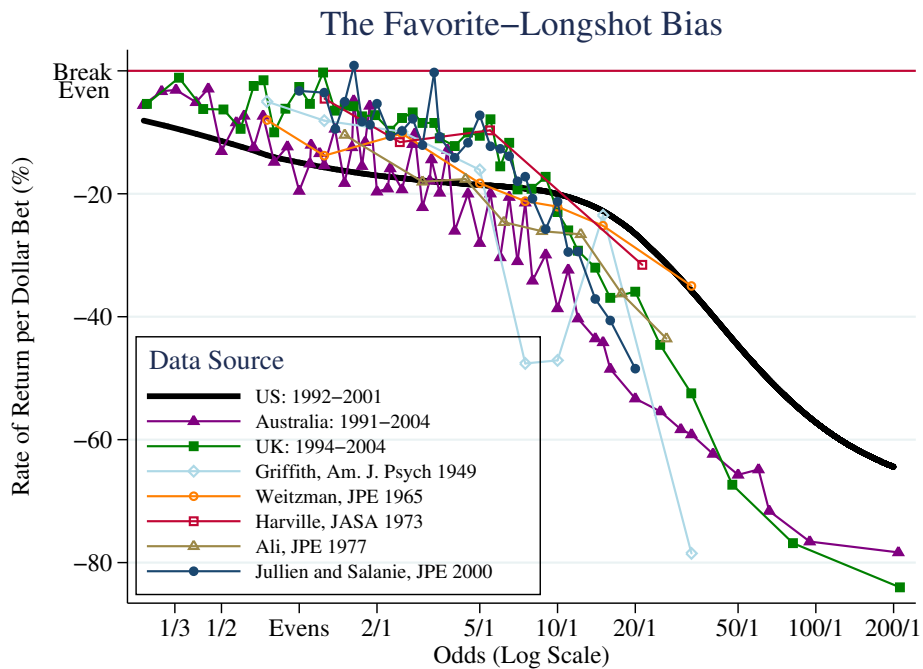
2 Stylized Facts

Our data contains all 6,403,712 horse starts run in the United States between 1992 and 2001. These are official jockey club data; the most precise available. Data of this nature are extremely expensive, which presumably explains why previous studies have used substantially smaller samples. The appendix provides more detail about the data.

We summarize our data in Figure 1. We calculate the rate of return to betting on every horse at each odds, and use Lowess smoothing to take advantage of information from horses with similar odds. Data are graphed on a log-odds scale so as to better show their relevant range. The vastly better returns to betting on favorites rather than longshots is the favorite-longshot bias. Figure 1 also shows the same pattern for the 206,808 races (with 1,485,112 horse starts) for which the jockey club recorded payoffs to exacta, quinella or trifecta bets. Given that much of our analysis will focus on this smaller sample, it is reassuring to see a similar pattern of returns.

Figure 2 shows the same rate of return calculations for several other datasets. We present new data from 2,725,000 starts in Australia from the South Coast Database, and 380,000 starts in Great Britain from flatstats.co.uk. The favorite-longshot bias appears equally evident in these countries, despite the fact that odds are determined by pari-mutuel markets in the United States, bookmakers in the United Kingdom, and bookmakers competing with a state-run pari-mutuel market in Australia. Figure 2 also includes historical estimates of the favorite-longshot bias, showing it has been stable since first noted in Griffith (1949).

Figure 2: The favorite longshot bias has persisted for over 50 years.



The literature suggests two other empirical regularities to explore. First, Thaler and Ziemba (1988) suggest that there are positive rates of return to betting extreme favorites, perhaps suggesting limits to arbitrage. This is not true in any of our datasets, providing a similar finding to Levitt (2004): despite significant anomalies in the pricing of bets, there are no profit opportunities from simple betting strategies.

Second, McGlothlin (1956), Ali (1977) and Asch, Malkiel, and Quandt (1982) argue that the rate of return to betting moderate longshots falls in the last race of the day. These studies have come to be widely cited despite being based on small samples. Kahneman and Tversky (1979) and Thaler and Ziemba (1988) interpret these results as consistent with loss aversion: most bettors are losing at the end of the day, and the last race provides them with a chance to recoup their losses. Thus, bettors underbet the favorite even more than usual, and overbet horses at odds that would eliminate their losses. The dashed line in Figure 1 separates out data from the last race; while the point estimates differ slightly, these differences are not statistically significant. If there was evidence of loss aversion in earlier data, it is no longer evident in recent data, even as the

favorite-longshot bias has persisted.

In the next section we argue that these facts cannot separate risk-love from misperception-based theories. We propose new tests based on the requirement that a theory developed to explain equilibrium odds of horses winning should also be able to explain the equilibrium odds in the exacta, quinella and trifecta markets separately, as well as the equilibrium odds in exacta and quinella markets jointly.

3 Two Models of the Favorite-Longshot Bias

We start with two extremely stark models, each of which has the merit of simplicity. Both are models where all agents have the same preferences and perceptions, but as we suggest below, can be expanded to incorporate heterogeneity. Equilibrium price data cannot separately identify more complex models from these representative agent models.

3.1 The Risk-Love Class

Following Weitzman (1965), we postulate expected utility maximizers with unbiased beliefs and utility $U(\cdot): \mathbb{R} \rightarrow \mathbb{R}$. In equilibrium, bettors must be indifferent between betting on the favorite horse A with probability of winning p_A and odds of $O_A/1$, and betting on a longshot B with probability of winning p_B and odds of $O_B/1$:

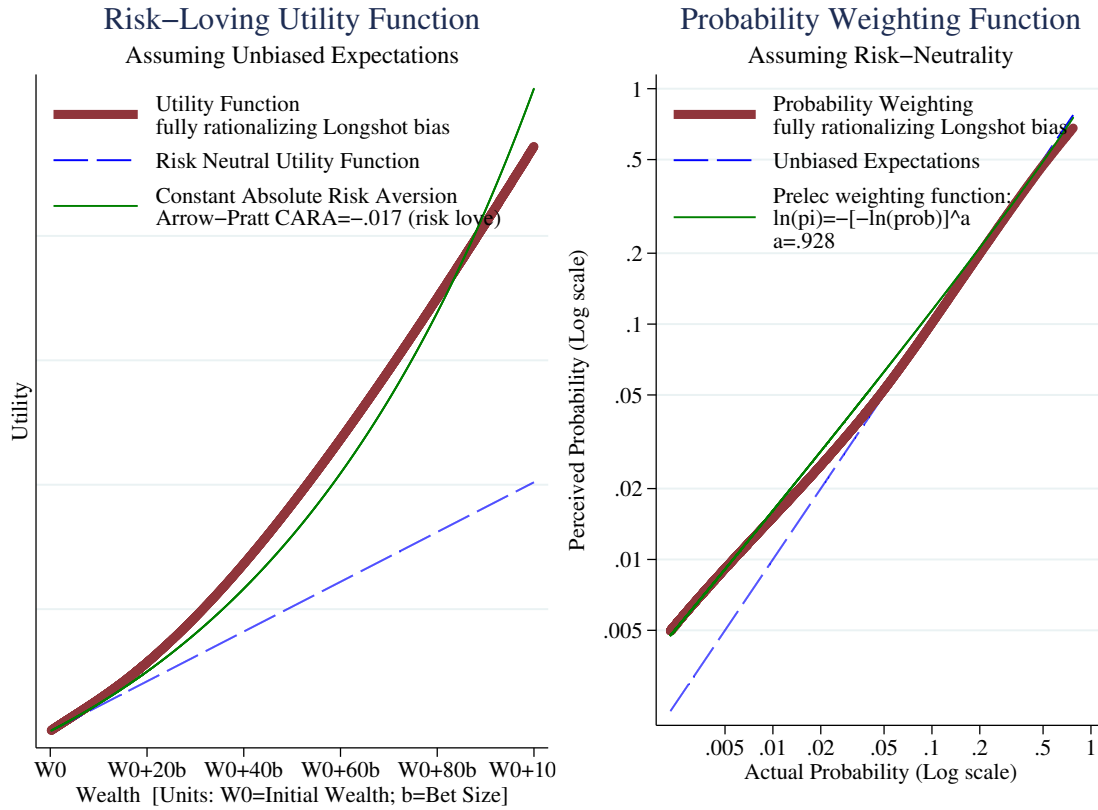
$$p_A U(O_A) = p_B U(O_B) \quad (\text{normalizing utility to zero, if the bet is lost}).^3 \quad (1)$$

The odds (O_A, O_B) and the probabilities (p_A, p_B) of horses winning, which we observe, identify the representative bettor's utility function up to a scaling factor.⁴ To fix the scale we normalize utility to zero if the bet loses, and to one if the bettor chooses not to bet. Thus, if the bettor is indifferent between accepting or rejecting a gamble offering odds of $O/1$ that wins with probability

³We also assume that, consistent with the literature, each bettor chooses to bet on only one horse in a race.

⁴See Weitzman (1965), Ali (1977), Quandt (1986) and Jullien and Salanié (2000) for prior examples.

Figure 3: The win data is completely rationalized by both classes.



p , then $U(O) = \frac{1}{p}$. The left panel of Figure 3 performs precisely this analysis, backing out the utility function required to explain all of the variation in Figure 1.

As can be seen from Figure 3, a risk-loving utility function is required to rationalize the better accepting lower average returns on longshots, even as they are riskier bets. The figure also shows that a CARA utility function fits the data reasonably well.

Several other theories of the favorite-longshot bias yield implications that are observationally equivalent to this risk-loving representative agent model. Some of these theories are clearly equivalent—such as that of Golec and Tamarkin (1998), which argues that bettors prefer skew rather than risk—as they are theories about the shape of the utility function. It can easily be shown that richer theories—such as that of Thaler and Ziemba (1988) where “bragging rights” accrue from winning a bet at long odds, or that of Conlisk (1993) in which the mere purchase of a bet on

a longshot may confer some utility—are also equivalent.⁵

3.2 The Misperceptions Class

Alternatively, the misperceptions class postulates risk-neutral subjective expected utility maximizers, whose subjective beliefs are given by the *probability weighting function* $\pi(p): [0, 1] \rightarrow [0, 1]$. In equilibrium, there are no opportunities for subjectively-expected gain, so bettors must believe that the subjective rates of return to betting on any pair of horses A and B are equal:

$$\pi(p_A)(O_A + 1) = \pi(p_B)(O_B + 1) = 1. \quad (2)$$

Consequently, data on the odds (O_A, O_B) and the probabilities (p_A, p_B) of horses winning reveal the misperceptions of the representative bettor.⁶

Note that the misperceptions class allows more flexibility in the way probabilities enter the representative bettor’s value of a bet, but it is more restrictive than the risk-love class in terms of how payoffs enter that function. More to the point, without restrictive parametric assumptions, each class of models is just-identified, so each yields identical predictions for the pricing of win bets.

The right panel of Figure 3 shows the probability weighting function $\pi(p)$ implied by the data in Figure 1. The low rates of return to betting longshots are rationalized by bettors who bet as though horses with tiny probabilities of winning actually have moderate probabilities of winning. The specific shape of the declining rates of return identifies the probability weighting function at

⁵Formally, these theories suggest agents maximize expected utility, where utility is the sum of the felicity of wealth, $y(\cdot): \mathbb{R} \rightarrow \mathbb{R}$, and the felicity of bragging rights or the thrill of winning, $b(\cdot): \mathbb{R} \rightarrow \mathbb{R}$. Hence the expected utility to a bettor with initial wealth w_0 of a gamble at odds O that wins with probability p can be expressed as: $E(U(O)) = p[y(w_0 + O) + b(O)] + [1 - p]y(w_0 - 1)$. As before, bettors will accept lower returns on riskier wagers (betting on longshots) if $U'' > 0$. This is possible if either the felicity of wealth is sufficiently convex or bragging rights are increasing in the payoff at a sufficiently increasing rate. More to the point, revealed preference data do not allow us to separately identify effects operating through $y(\cdot)$, rather than $b(\cdot)$.

⁶While we term the divergence between $\pi(p)$ and p misperceptions, in non-expected utility theories, $\pi(p)$ can be interpreted as a preference over types of gambles. Under either interpretation our approach is valid, in that we test whether gambles are motivated by nonlinear functions of wealth or probability. In (2) we implicitly assume that $\pi(1) = 1$, although we allow $\lim_{p \rightarrow 1} \pi(p) \leq 1$.

each point.⁷ This function shares some of the features of the decision weights in Prospect Theory (Kahneman and Tversky, 1979), and the figure shows that the one-parameter probability weighting function in Prelec (1998) fits the data quite closely.

While we have presented a very sparse model, a number of richer theories have been proposed that yield similar implications.⁸ For instance, Ottaviani and Sørensen (2009) show that initial information asymmetries between bettors may lead to misperceptions of the true probabilities of horses winning. Moreover, Henery (1985) and Williams and Paton (1997) argue that bettors discount a constant proportion of the gambles in which they bet on a loser, possibly due to a self-serving bias in which losers argue that conditions were atypical. Because longshot bets lose more often, this discounting yields perceptions in which betting on a longshot seems more attractive.

3.3 Implications for Pricing Compound Lotteries

We now show how our two classes of models—while each just-identified based on data from win bets—yield different implications for the prices of exotic bets. As such, our approach responds to Sauer (1998, p.2026), which calls for research that provides “equilibrium pricing functions from well-posed models of the wagering market.”

We discuss the pricing of exactas (picking the first two horses in order) in detail. Prices for these bets are constructed from: the bettors’ utility function, indifference conditions as in (1) or (2), data on the perceived likelihood of the pick for first, A , actually winning (p_A or $\pi(p_A)$, depending on the class), and conditional on A winning, the likelihood of the pick for second, B , coming second ($p_{B|A}$ or $\pi(p_{B|A})$). A bettor will be indifferent between betting on an exacta on horses A then B in that order, paying odds of $E_{AB}/1$, and not betting (which yields no change in wealth, and hence a

⁷ There remains one minor issue: as Figure 1 shows, horses never win as often as suggested by their win odds because of the track-take. Thus we follow the convention in the literature and adjust the odds-implied probabilities by a factor of one minus the track take for that specific race, so that they are on average unbiased; our results are qualitatively similar whether or not we make this adjustment.

⁸While the assumption of risk-neutrality may be too stark, as long as bettors gamble small proportions of their wealth the relevant risk premia are second-order. For instance, assuming log utility, if the bettor is indifferent over betting $x\%$ of their wealth on horse A or B , then: $\pi(p_A) \log(w + wxO_A) + (1 - \pi(p_A)) \log(w - wx) = \pi(p_B) \log(w + wxO_B) + (1 - \pi(p_B)) \log(w - wx)$, which under the standard approximation simplifies to: $\pi(p_A)(O_A + 1) \approx \pi(p_B)(O_B + 1)$, as in (2).

utility of one), if:

Risk-Love Class

(Risk-lover, Unbiased expectations)

$$p_A p_{B|A} U(E_{AB}) = 1$$

Noting $p = \frac{1}{U(O)}$ from (1)

$$E_{AB} = U^{-1}(U(O_A)U(O_{B|A})) \quad (3)$$

Misperceptions Class

(Biased expectations, Risk-neutral)

$$\pi(p_A)\pi(p_{B|A})(E_{AB} + 1) = 1$$

Noting $\pi(p) = \frac{1}{O+1}$ from (2)

$$E_{AB} = (O_A + 1)(O_{B|A} + 1) - 1 \quad (4)$$

Thus, under the misperceptions class, the odds of the exacta E_{AB} are a simple function of the odds of horse A winning, O_A , and conditional on this, on the odds of B coming second, $O_{B|A}$. The preferences class is more demanding, requiring that we estimate the utility function. The utility function is estimated from the pricing of win bets (in Figure 3), and can be inverted to compute unbiased win probabilities from the betting odds.⁹

Our empirical tests simply determine which of (3) or (4) better fit the actual prices of exacta bets. We apply an analogous approach to the pricing of quinella and trifecta bets: the intuition is the same; the mathematical details are described in Appendix B.

Note that both (3) and (4) require $O_{B|A}$, which is not directly observable. In Section 4 we infer the conditional probability $p_{B|A}$ (and hence $\pi(p_{B|A})$ and $O_{B|A}$) from win odds by assuming that bettors believe in conditional independence. That is, we apply the Harville (1973) formula: $\pi(p_{B|A}) = \frac{\pi(p_B)}{1 - \pi(p_A)}$; replacing $\pi(p)$ with p in the risk-love class. This assumption is akin to thinking about the race for second as a “race within the race” (Sauer, 1998). While relying on the Harville formula is standard in the literature—see for instance Asch and Quandt (1987)—we show that our results are robust to dropping this assumption and estimating this conditional probability, $p_{B|A}$, directly from the data.

⁹Our econometric method imposes continuity on the utility and probability weighting functions; the data mandate that both be strictly increasing. Together this is sufficient to ensure that $\pi(\cdot)$ and $U(\cdot)$ are invertible. As in Figure 1, we do not have sufficient data to estimate the utility of winning bets at odds greater than 200/1. This prohibits us from pricing bets whose odds are greater than 200/1, which is most limiting for our analysis of trifecta bets.

3.4 Failure to Reduce Compound Lotteries

As in Prospect Theory, the frame the bettor adopts in trying to assess each gamble is a key issue, particularly for misperceptions-based models. Specifically, (4) assumes that bettors first attempt to assess the likelihood of horse A winning, $\pi(p_A)$, and then assess the likelihood of B coming second given that A is the winner, $\pi(p_{B|A})$. An alternative frame might suggest that bettors directly assesses the likelihood of first-and-second combinations: $\pi(p_A p_{B|A})$.¹⁰

There is a direct analogy to the literature on the assessment of compound lotteries: does the bettor separately assess the likelihood of winning an initial gamble (picking the winning horse) which yields a subsequent gamble as its prize (picking the second-placed horse), or does she consider the equivalent simple lottery (as in Samuelson (1952))? Consistent with (4), the accumulated experimental evidence (Camerer and Ho, 1994) is more in line with subjects failing to reduce compound lotteries into simple lotteries.¹¹

Alternatively, we could choose not to defend either assumption, leaving it as a matter for empirical testing. Interestingly, if gamblers adopt a frame consistent with the reduction of compound lotteries into their equivalent simple lottery form, this yields a pricing rule for the misperceptions class equivalent to that of the risk-love class.¹² Thus, evidence consistent with what we are calling the risk-love class accommodates either risk-love by unbiased bettors, or risk-neutral but biased bettors, whose bias affects their perception of an appropriately reduced compound lottery. By contrast, the competing misperceptions class implies the failure to reduce compound lotteries and posits the specific form of this failure, shown in (4).

This discussion implies that results consistent with our risk-love class are also consistent with a richer set of models emphasizing choices over simple gambles. These include models based on the utility of gambling, information asymmetry or limits to arbitrage, such as Ali (1977), Shin

¹⁰Unless the probability weighting function is a power function ($\pi(p) = p^\alpha$), these different frames yield different implications (Aczél, 1966).

¹¹Additionally, note that (4) satisfies the compound independence axiom of Segal (1990).

¹²To see this, note that identical data (from Figure 1) is used to construct the utility and decision weight functions respectively, so each is constructed to rationalize the same set of choices over simple lotteries. This implies each class also yields the same set of choices over compound lotteries if preferences in both classes obey the reduction of compound lotteries into equivalent simple lotteries.

(1992), Hurley and McDonough (1995), Manski (2006). Any theory that prescribes a specific bias in a market for a simple gamble (win betting) will yield similar implications in a related market for compound gambles if gamblers assess their equivalent simple gamble form. By implication, rejecting the risk-love class substantially narrows the set of plausible theories of the favorite-longshot bias.

4 Results from Exotic Bets

The first panel of Figure 4 shows the difference between the predictions for exactas of the risk-love and misperceptions classes, expressed as a percent of the predictions. This demonstrates the two classes of models yield qualitatively different predictions. Exotic bets have relatively low probabilities of winning, so under the risk-love class a risk penalty results, yielding lower odds. By contrast, the misperceptions model is based on the underlying simple lotteries, many of which suffer smaller perception biases. The risk penalty becomes particularly important as odds get longer; thus the difference in predictions grows along a line from the bottom right to the top left of the first panel of Figure 4.

To focus on shorter-odds bets, in Table 1 we convert the predictions into the price of a contingent contract that pays \$1 if the chosen exacta wins: $Price = \frac{1}{Odds + 1}$. We test the ability of each economic model to predict this price by examining the mean absolute error of the predictions of both models from the actual prices of exotic bets (column 1). We further investigate which of the models produces predictions that are closer, observation-by-observation, to the prices that are actually observed (column 3). The explanatory power of the misperceptions class is substantially greater. The misperceptions class is six percentage points closer to the actual prices of exactas (column 2) an improvement of $6.3/34.3 = 18.4\%$ over the risk-love class.

The second panel of Figure 4 plots the improvement of the misperceptions class according to the odds of the first and second place horses. When both horses have odds of less than 10/1, which accounts for 70% of our data, the average improvement of the misperceptions class over the

Figure 4: Differences between theories: predictions and improvements.

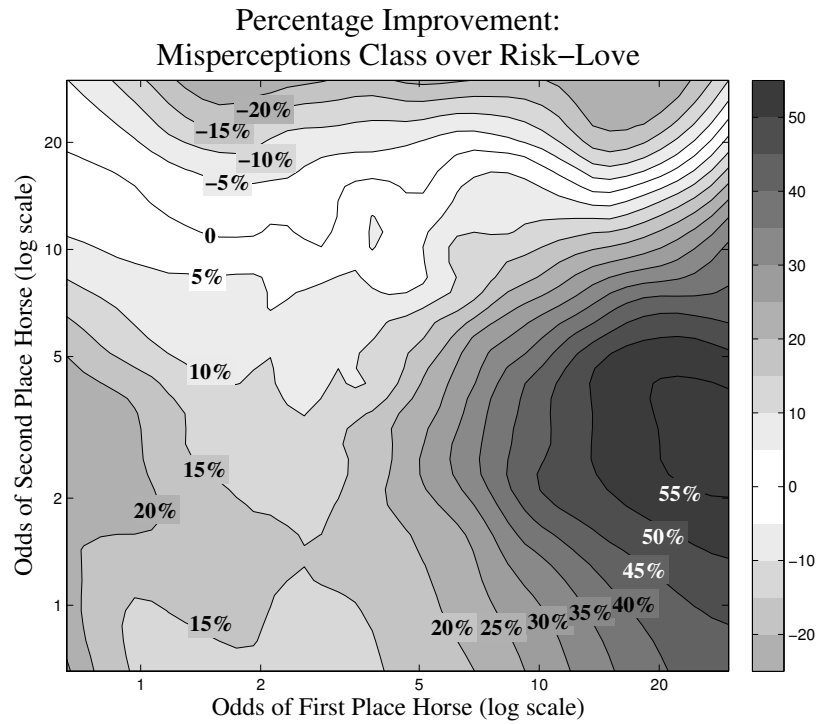
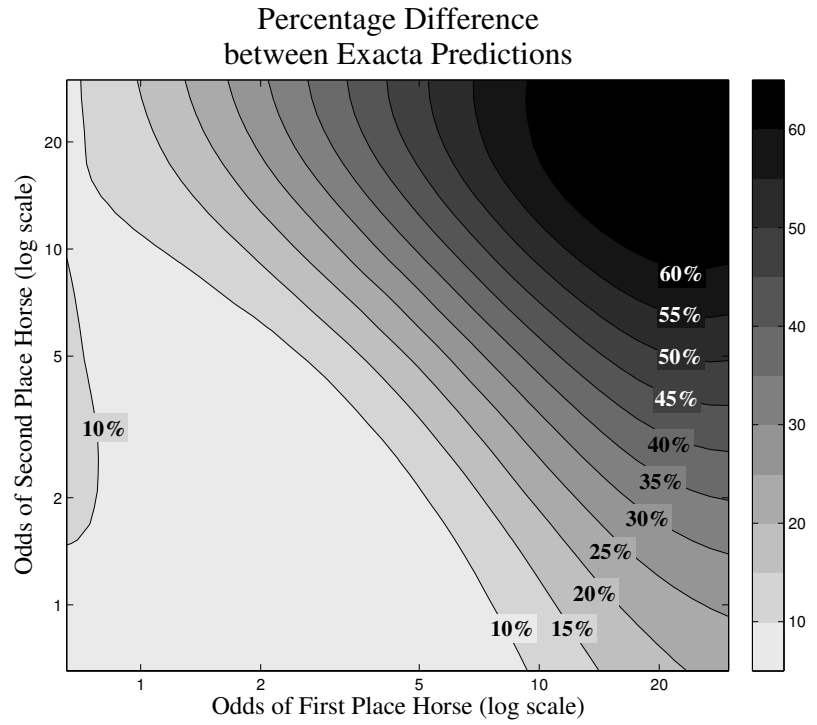


Table 1: Mean Error Based Tests of Risk-Love versus Misperceptions Model

<i>Test:</i>	(1) Absolute Error: Prediction – Actual	(2) Absolute % Error: $\frac{ Prediction - Actual }{Actual}$	(3) Which Prediction is Closer to Actual? (%)
Panel A: Exacta Bets (n=197,551)			
Risk-Love Model	0.0139	34.3%	42.1%
Misperceptions Model	0.0125	28.0%	57.9%
Risk-Love Error – Misperceptions Error	0.00137 (.00002)	6.3% (.1%)	
Panel B: Quinella Bets (n=70,169)			
Risk-Love Model	0.0274	39.0%	46.0%
Misperceptions Model	0.0258	36.3%	54.0%
Risk-Love Error – Misperceptions Error	0.00155 (.00003)	2.7% (.2%)	
Panel C: Trifecta Bets (n=137,756)			
Risk-Love Model	0.00796	100%	28.9%
Misperceptions Model	0.00532	57.4%	71.1%
Risk-Love Error – Misperceptions Error	0.00264 (.00001)	42.9% (.2%)	

Notes: Standard errors in parenthesis. Predictions and actual outcomes are measured in the price of a contract that pays \$1 if the event occurs, zero otherwise.

risk-love class is 16.8%. At long odds (the top and right of the figure) there are clear patterns in the data that are not well explained by either class, leaving room for more nuanced theories of the favorite-longshot bias.

Panels B and C of Table 1 repeat this analysis, but this time extending our test to see which model can better explain the pricing of quinella and trifecta bets. The intuition is similar in all three cases; Appendix B contains further mathematical detail. Each of these tests across all three panels shows that the misperceptions model fits the data better than the risk-love model.¹³

¹³We have also re-run these tests a number of other ways to test for robustness. Our conclusions are unaltered by: whether or not we weight observations by the size of the betting pool, whether we drop observations where the models imply very long odds, whether or not we adjust the models in the manner described in footnote 8; and different

4.1 Relaxing the Assumption of Conditional Independence

Recall that we observe all of the inputs to both pricing models except $O_{B|A}$, the odds of horse B finishing second, conditional on horse A winning. In Section 4 we used the convenient assumption of conditional independence to assess the likely odds of this bet, but there may be good reason to doubt this assumption. For example, if a heavily favored horse does not win a race, this may reflect the fact that it was injured during the race, which would imply that it is very unlikely to come second. That is, the win odds may provide useful guidance on the probability of winning, but may be a poor guide to the race for second. In this section we test the assumption of conditional independence and derive two further tests can distinguish between the risk-love and misperceptions models even if this assumption fails.¹⁴

We test the conditional independence assumption by asking whether we can improve on the predictions of the Harville (1973) formula using other data. As in Section 3.3, the Harville formula is: $p_{B|A} = \frac{p_B}{(1 - p_A)}$, where p_A and p_B reflect the probability that horses A and B , respectively, win the race. We estimate linear probability model where the dependent variable is an indicator variable for whether horse B finished second. A probit specification yields similar results.

The first specification of Table 2 shows that the Harville formula is an extremely useful predictor of the probability of a horse finishing second. As a guide for thinking about the explanatory power of the Harville formula, note that the R^2 of specification 1 is about four-fifths the R^2 of the regression of an indicator for whether a horse won the race on its betting odds. Columns 2 and 3 however, provide compelling evidence that we can do better than the Harville formula. Column 2 adds dummy variables representing the odds of the first place horse and the odds of the second placed horse (using 100 odds groupings in each case, each grouping containing 1% of the odds distribution). Column 3 includes a full set of interactions of these fixed effects, estimating the conditional probability non-parametrically from the odds of the first and second place horses; this regression is equivalent to estimating a large table showing the proportion of horses at odds of

functional forms for the price of a bet, including the natural log price of a \$1 claim, the odds, or log-odds.

¹⁴Even if conditional independence fails, it is not immediately obvious that it yields errors that are correlated in such as way as to drive our main results. Even so, this is an issue for empirical testing.

Table 2: Predicting the Conditional Probability of a Second Place Finish

<i>Dependent Variable:</i> Indicator for whether a horse came in second (Conditional on not winning)			
<i>Specification:</i>	(1)	(2)	(3)
Prediction from Conditional Independence (Harville Formula)	0.793 (.0012)	0.908 (.0077)	
Odds of this horse and odds of first horse (100 dummy variables for each)		F=32.3 p=0.00	
Full set of interactions: (this horse * first horse) (10,000 dummy variables)			F=43.5 p=0.00
R^2	0.0782	0.0794	0.0813

Notes: Standard errors in parenthesis. Predictions and actual outcomes are measured in the price of a contract that pays \$1 if the event occurs, zero otherwise.

$O_B/1$ who won the race for second, given the winner was at odds of $O_A/1$. In both columns 2 and 3, F-tests show that these fixed effects are jointly statistically significant.

We now use non-parametrically estimated probabilities as a robustness check of our results in Table 1. That is, rather than inferring $p_{B|A}$ (and hence $\pi(p_{B|A})$ and $O_{B|A}$) from the Harville formula, we simply apply the empirical probabilities estimated using the Lowess procedure of Cleveland, Devlin, and Grosse (1988). We implement this exercise in Table 3, calculating the price of exotic bets under the risk-love and misperception models, but adapting our earlier approach so that $p_{B|A}$ is derived from the data.¹⁵

The results in Table 3 are almost identical to those in Table 1. For exacta, quinella and trifecta bets, the misperceptions model has greater explanatory power than the risk-love model.

¹⁵Because the precision of our estimates of $p_{B|A}$ vary greatly, WLS weighted by the product of the squared standard error of $p_{B|A}$ and p_A might be appropriate. Additionally, we could estimate $p_{B|A}$ directly from column 3 of Table 2. These approaches produce qualitatively identical results.

Table 3: Robustness to Relaxing Conditional Independence Assumption

	(1)	(2)	(3)
<i>Test:</i>	Absolute Error: Prediction – Actual	Absolute % Error: $\frac{ \text{Prediction} - \text{Actual} }{\text{Actual}}$	Which Prediction is Closer to Actual? (%)
Panel A: Exacta Bets (n=197,551)			
Risk-Love Model	0.0117	33.7%	42.9%
Misperceptions Model	0.0109	24.4%	57.1%
Risk-Love Error – Misperceptions Error	0.00082 (.00001)	9.3% (.1%)	
Panel B: Quinella Bets (n=70,169)			
Risk-Love Model	0.0240	37.7%	48.7%
Misperceptions Model	0.0235	33.8%	51.3%
Risk-Love Error – Misperceptions Error	0.00046 (.00002)	3.9% (.2%)	
Panel C: Trifecta Bets (n=137,756)			
Risk-Love Model	0.00650	98.0%	30.4%
Misperceptions Model	0.00464	49.0%	69.6%
Risk-Love Error – Misperceptions Error	0.00186 (.00001)	49.0% (.1%)	

Notes: Standard errors in parenthesis. Predictions and actual outcomes are measured in the price of a contract that pays \$1 if the event occurs, zero otherwise.

5 Simultaneous Pricing of Exactas and Quinellas

Our final test of the two classes relies only on the relative pricing of exacta and quinella bets, and is more stringent as it considers these bets simultaneously.¹⁶ As before, we derive predictions from each class and test which better explains the observed data.

The exacta AB (which represents A winning and B coming second) occurs with probability $p_A * p_{B|A}$; the BA exacta occurs with probability $p_B * p_{A|B}$. By definition, the corresponding quinella

¹⁶Note that these tests are distinct from the work by authors such as Asch and Quandt (1987) and Dolbear (1993), who test whether exacta pricing is arbitrage-linked to win pricing. Instead, we ask whether the same model that explains pricing of win bets can jointly explain the pricing of exacta and quinella bets.

pays off when the winning exacta is either AB or BA and hence occurs with probability $p_A * p_{B|A} + p_B * p_{A|B}$. Each model yields unique implications for the relative prices of the winning exacta and quinella bets, and thus unique predictions for $\frac{p_A p_{B|A}}{p_A p_{B|A} + p_B p_{A|B}}$, which is also the probability horse A wins given that A and B are the top two finishers. Specifically, consider the AB exacta at odds of $E_{AB}/1$, and the corresponding quinella at $Q/1$:

Risk-Love Class

(Risk-lover, Unbiased expectations)

$$\text{Exacta: } p_A p_{B|A} U(E_{AB}) = 1$$

$$p_{B|A} = \frac{U(O_A)}{U(E_{AB})} \quad (5)$$

$$\text{Quinella: } [p_A p_{B|A} + p_B p_{A|B}] U(Q) = 1$$

$$p_{A|B} = U(O_B) \left(\frac{1}{U(Q)} - \frac{1}{U(E_{AB})} \right) \quad (7)$$

Hence from (1), (5) and (7):

$$\frac{p_A p_{B|A}}{p_B p_{A|B} + p_A p_{B|A}} = \frac{U(Q)}{U(E_{AB})} \quad (9)$$

Misperceptions Class

(Biased expectations, Risk-neutral)

$$\text{Exacta: } \pi(p_A) \pi(p_{B|A}) (E_{AB} + 1) = 1$$

$$\pi(p_{B|A}) = \frac{O_A + 1}{E_{AB} + 1} \Rightarrow p_{B|A} = \pi^{-1} \left(\frac{O_A + 1}{E_{AB} + 1} \right) \quad (6)$$

$$\text{Quinella: } [\pi(p_A) \pi(p_{B|A}) + \pi(p_B) \pi(p_{A|B})] (Q + 1) = 1$$

$$\pi(p_{A|B}) = (O_B + 1) \left(\frac{1}{Q+1} - \frac{1}{E_{AB}+1} \right) \Rightarrow p_{A|B} = \pi^{-1} \left(\frac{(O_B+1)(E_{AB}-Q)}{(E_{AB}+1)(Q+1)} \right) \quad (8)$$

Hence from (2), (6) and (8):

$$\frac{p_A p_{B|A}}{p_B p_{A|B} + p_A p_{B|A}} = \frac{\pi^{-1} \left(\frac{1}{O_A+1} \right) \pi^{-1} \left(\frac{O_A+1}{E_{AB}+1} \right)}{\pi^{-1} \left(\frac{1}{O_A+1} \right) \pi^{-1} \left(\frac{O_A+1}{E_{AB}+1} \right) + \pi^{-1} \left(\frac{1}{O_B+1} \right) \pi^{-1} \left(\frac{(O_B+1)(E_{AB}-Q)}{(E_{AB}+1)(Q+1)} \right)} \quad (10)$$

Equations (9) and (10) show that for any pair of horses at win odds $O_A/1$ and $O_B/1$ with quinella odds $Q/1$, each class has different implications for how frequently we expect to observe the AB exacta winning, relative to the BA exacta. That is, each class gives distinct predictions about how often a horse with win odds $O_A/1$ will come in first, given that horses with win odds $O_A/1$ and $O_B/1$ are the top two finishers. As a first test, we regress an indicator for whether the favorite out of horses A and B actually won, given that horses A and B finished first and second, on the predictions of each model.¹⁷ In this simple specification, the misperceptions class yields a robust and significant positive correlation with actual outcomes (coefficient = 0.63; standard error = 0.014, $n = 60,288$), while the risk-love class is negatively correlated with outcomes (coefficient = -0.59 ; standard error = 0.013, $n = 60,288$).

Where does the perverse negative correlation between the predictions of the risk-love class and

¹⁷In the rare event when horses A and B had the same odds we coded the indicator as 0.5.

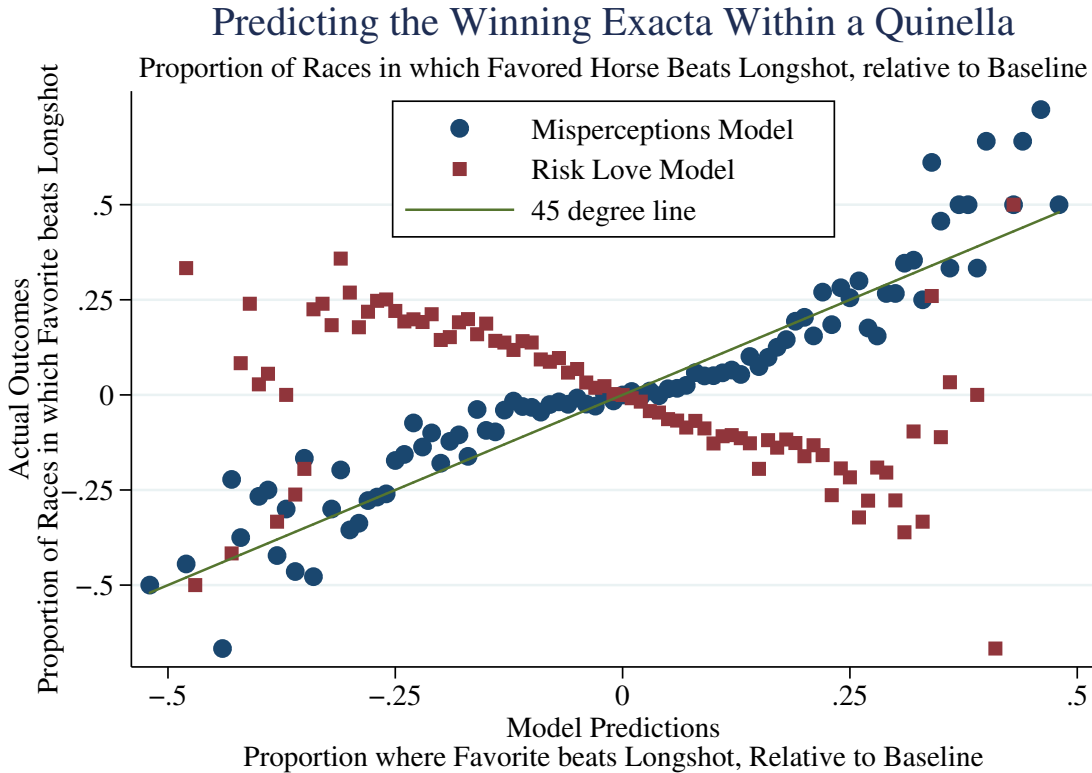
actual outcomes come from? To fix ideas, consider the case where the relative favorite, A , has win odds of $4/1$ and the relative longshot, B , has win odds of $9/1$. The data give average exacta odds $E_{AB} = 39/1$ and $E_{BA} = 44/1$ and average quinella odds $Q = 20/1$. These data agree extremely closely with the predictions of the misperceptions class, so when (10) is applied to data from such a race, it makes accurate predictions about $\frac{P_A P_{B|A}}{P_A P_{B|A} + P_B P_{A|B}}$.

Now examine the predictions of the risk-love class. Under this class, an exacta bet at odds $E_{AB} = 39/1$ is interpreted as having a large risk penalty, as should be clear from Figures 1 and 3. But if, in fact, compound bets are priced according to the misperceptions class—where bets at odds of $4/1$ and $9/1$ do not attract much of a risk (or misperceptions) penalty—then by inferring the existence of such a penalty, the risk-love model implicitly underestimates the probability that exacta AB will occur. This underestimation is severe enough that the risk-love class predicts a less than 50% chance that the relatively favored horse A will finish first out of A and B . But given that A is the favorite of the two horses ($O_A/1 = 4/1 < 9/1 = O_B/1$), it is in fact more likely than not to finish before B .

If, instead, the relative longshot B wins, the exacta $E_{BA} = 44/1$ is observed. Applying the logic above, the risk-love class infers that this price incorporates an even larger risk penalty, and thus assigns a low probability to this exacta. In turn, this means that it yields a low probability of horse B coming in first, given that A and B are the top two finishers. However, the risk-love class will make this prediction *only when* exacta E_{BA} is observed—that is, horse B actually wins the race—which is exactly wrong. Thus, conditional on A and B being the top two finishers, the risk-love class predicts the relative favorite, A , will win with probability less than $\frac{1}{2}$ whenever A wins. Similarly, whenever B wins, the risk-love class predicts that the relative longshot B will win with probability less than $\frac{1}{2}$, and hence predicts that the probability that the relative favorite A wins is more than $\frac{1}{2}$. These two forces lead to the perverse negative correlation found above.

The risk-love class fails here because it insists that the same risk-penalty be priced into all bets of a given risk, regardless of the pool from which the bets are drawn. Yet exotic bets with middling risk—relative to the other bets available in a given pool—do not tend to attract large risk-penalties,

Figure 5: Dropping Conditional Independence



Notes: Chart shows model predictions from (3) and (4) and actual outcomes relative to a fixed-effect region baseline. For the first-two finishing horses the baseline controls for: (a) the odds of the favored horse, (b) the odds of the longshot, (c) the odds of the quinella, and (d) all interactions of (a), (b) and (c). The plot shows model predictions after removing fixed effects, rounded to the nearest percentage point, on the x - axis and actual outcomes, relative to the fixed effects, on the y - axis.

even if those bets would be very risky relative to bets in the win pool (Asch and Quandt, 1987).

Note that (9) and (10) also yield distinct predictions of the winning exacta even *within* any set of apparently similar races (those whose first two finishers are at $O_A/1$ and $O_B/1$ with the quinella paying $Q/1$). Thus, we can include a full set of fixed effects for O_A , O_B , Q and *their interactions* in our statistical tests of the predictions of each class.¹⁸ The residual after differencing out these fixed effects is the predicted likelihood that A beats B , relative to the average for all races in which a horses at odds of $O_A/1$ and $O_B/1$ fill the quinella at odds $Q/1$. That is, for all races we compute the predictions of the likelihood that exacta with the relative favorite winning (AB) occurs, and

¹⁸Because the odds O_A , O_B and Q are actually continuous variables, we include fixed effects for each percentile of the distribution of each variable (and a full set of interactions of these fixed effects).

subtract the baseline $O_A * O_B * Q$ cell mean to yield the predictions for each class of model, relative to the fixed effects. The results, summarized in Figure 5, are remarkably robust to the inclusion of these multiple fixed effects (and interactions): the coefficient on the misperceptions class declines slightly, and insignificantly, while the risk-love class maintains a significant but perversely negative correlation with outcomes. It should be clear that this test, by focusing only on the relative rankings of the first two horses, entirely eliminates parametric assumptions about the race for second place.

These tests imply that a model from the risk-love class that accounts for the pricing of win bets yields inaccurate implications for the relative pricing of exacta and quinella bets. By contrast, the misperceptions class is consistent with the pricing of exacta, quinella and trifecta bets, and, as this section shows, also consistent with the relative pricing of exacta and quinella bets. These results are robust to a range of different approaches to testing the theories.

6 Conclusion

Employing a new dataset which is much larger than those in the existing literature, we document stylized facts about the rates of return to betting on horses. As with other authors, we note a substantial favorite-longshot bias. The term bias is somewhat misleading here. That the rate of return to betting on horses at long odds is much lower than the return to betting on favorites simply falsifies a model in which bettors maximize a function that is linear in probabilities and linear in payoffs. Thus, the pricing of win bets can be reconciled by a representative bettor with either a concave utility function, or a subjective utility function employing non-linear probability weights that violate the reduction of compound lotteries. For compactness, we referred to the former as explaining the data with risk-love, while we refer to the latter as explaining the data with misperceptions. Neither label is particularly accurate as each category includes a wider range of competing theories.

We show that these classes of models can be separately identified using aggregate data by requiring that they explain both choices over betting on different horses to win and choices over

compound bets: exactas, quinellas and trifectas. Because the underlying risk or set of beliefs, depending on the theory, is traded in both the win and compound betting markets, we can derive unique testable implications from both sets of theories. Our results are more consistent with the favorite-longshot bias being driven by misperceptions rather than risk-love. Indeed, while each class is individually quite useful for pricing compound lotteries, the misperceptions class strongly dominates the risk-love class. This result is robust to a range of alternative approaches to distinguishing between the theories.

This bias likely persists in equilibrium because misperceptions are not large enough to generate profit opportunities for unbiased bettors. That said, the cost of this bias is also very large, and debiasing an individual bettor could reduce their cost of gambling substantially.

Appendix Data

Our dataset consists of all horse races run in North America from 1992 to 2001. The data was generously provided to us by Axcis Inc., a subsidiary of the jockey club. The data record performance of every horse in each of its starts, and contains the universe of officially recorded variables having to do with the horses themselves, the tracks and race conditions.

Our concern is with the pricing of bets. Thus, our primary sample consists of the 6,403,712 observations in 865,934 races for which win odds and finishing positions are recorded. We use these data, subject to the data cleaning restrictions below, to generate Figures 1–3 and 5. We are also interested in pricing exacta, quinella and trifectas bets and have data on the winning payoffs in 314,977, 116,307 and 282,576 races respectively. The prices of non-winning combinations are not recorded.

Due to the size of our dataset, whenever observations were problematic, we simply dropped the entire race from our dataset. Specifically, if a race has more than one horse owned by the same owner, rather than deal with coupled runners, we simply dropped the race. Additionally, if a race had a dead heat for first, second or third place the exacta, quinella and trifecta payouts may not be accurately recorded and so we dropped these races. When the odds of any horse were reported as zero we dropped the race. Further if the odds across all runners implied that the track take was less than 15% or more than 22%, we dropped the race. After these steps, we are left with 5,606,336 valid observations on win bets from 678,729 races and 1,485,112 observations from 206,808 races include both valid win odds and payoffs for the winning exotic bets.

Appendix Pricing of Compound Lotteries using Conditional Independence

In the text we derived the pricing formulae for exacta bets explicitly; this appendix extends that analysis to also include the pricing of quinella and trifecta bets. The derivations of these pricing formulae depend on the following two formulae originally derived in Section 3:

Risk-Love Model

(Risk-lover, Unbiased expectations)

$$U(O) = \frac{1}{p} \quad (11)$$

Misperceptions Model

(Biased expectations, Risk-neutral)

$$\pi(p) = \frac{1}{O+1} \quad (12)$$

As in the text, we derive pricing formulae by imposing that the expected utility of all bets is equal. Consider a horse race which includes horses A , B and C . An exacta requires the bettor to correctly specify the first two horses, in order. A quinella is a bet on two horses to finish first and second, but the bettor need not specify their order. A quinella bet on horses A and B gives odds Q_{AB} . A trifecta is a bet on three horses to finish first, second and third, and the bettor must correctly specify their order. A trifecta bet on horses A , B and C , in that order, gives odds T_{ABC} . Thus the quinella and trifecta analogues to equations (3) and (4) in the main text are:

Risk-Love Model

(Risk-lover, Unbiased expectations)

Quinella:

$$[p_A p_{B|A} + p_B p_{A|B}]U(Q_{AB}) = 1$$

$$Q_{AB} = U^{-1} \left(\frac{U(O_A)U(O_{B|A})U(O_B)U(O_{A|B})}{U(O_A)U(O_{B|A}) + U(O_B)U(O_{A|B})} \right) \quad (3q)$$

Trifecta:

$$p_A p_{B|A} p_{C|A,B} U(T_{ABC}) = 1$$

$$T_{ABC} = U^{-1} (U(O_A)U(O_{B|A})U(O_{C|A,B})) \quad (3t)$$

Risk-Love Model

(Risk-lover, Unbiased expectations)

Quinella:

$$[\pi(p_A)\pi(p_{B|A}) + \pi(p_B)\pi(p_{A|B})](Q_{AB} + 1) = 1$$

$$Q_{AB} = \frac{(O_A+1)(O_{B|A})(O_B+1)(O_{A|B})}{(O_A+1)(O_{B|A}) + (O_B+1)(O_{A|B})} \quad (4q)$$

Trifecta:

$$\pi(p_A)\pi(p_{B|A})\pi(p_{C|A,B})(T_{ABC} + 1) = 1$$

$$T_{ABC} = (O_A + 1)(O_{B|A} + 1)(O_{C|A,B} + 1) - 1 \quad (4t)$$

The odds data, O_A , O_B and O_C are directly observable. The utility $U(\cdot)$ and probability weighting $\pi(\cdot)$ functions that we use are shown in Figure 3. In order to price these compound bets we also need the conditional probabilities $O_{B|A}$, $O_{A|B}$ and $O_{C|A,B}$.

As noted in Section 3.3, we provide two approaches to recovering these unobservables. First, we assume conditional independence, as in Harville (1973). Thus, $p_{B|A} = p_B / (1 - p_A)$, $p_{A|B} = p_A / (1 - p_B)$ and $p_{C|A,B} = p_C / (1 - p_A - p_B)$. Our second approach directly estimates $p_{B|A}$, $p_{A|B}$, and $p_{C|A,B}$ using Lowess smoothing as described in Cleveland, Devlin, and Grosse (1988). Under both the Harville and Lowess approach these probability estimates and (11) and (12) are used to recover the relevant odds $O_{B|A}$, $O_{A|B}$ and $O_{C|A,B}$.

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