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James R. Hines, Jr.

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ABSTRACT

Alternatives to the current system of separate tax accounting, such as the proposed Common Consolidated Corporate Tax Base in Europe, would apportion a firm's worldwide profits using formulas based on the location of employment, capital or sales. This paper offers a new method of evaluating the accuracy of these apportionment rules and the ownership distortions they create. Evidence from European company accounts indicates that apportionment formulas significantly misattribute income, since employment and other factors on which they are based do a very poor job of explaining a firm's profits. For example, the magnitude of property, employment and sales explains less than 22 percent of the variation in profits between firms, and the prediction estimates from using such a formula exceed half of predicted profits 64% of the time, and exceed twice predicted income 11% of the time. As a result, the use of formulas rewards or punishes international mergers and divestitures by reallocating taxable income between operations in jurisdictions with differing tax rates. The associated ownership distortion is minimized by choosing factor weights to minimize weighted squared prediction errors, for which, based on the European evidence, labor inputs should play little if any role in allocation formulas. But even a distortion-minimizing formula creates large incentives for inefficient ownership reallocation due to the enormous variation in profitability that is unexplained by formulary factors, implying that significant resource allocation costs would accompany European adoption of formulary apportionment methods.

James R. Hines, Jr.
Department of Economics
University of Michigan
343 Lorch Hall
611 Tappan Street
Ann Arbor, MI 48109-1220
and NBER
jrhines@umich.edu

1. *Introduction*

Governments tax active business income earned within their borders, a practice that is fraught with difficulty under any circumstances, and particularly so when a firm earns income in more than one country. Since tax rates differ between countries, multinational firms usually have incentives to arrange their affairs in ways that reallocate taxable income between countries. An excessively transparent method of doing so would be to sell a paper clip from an affiliate in a low-tax location to an affiliate in a high-tax location, charging a price of $1 million. This transaction would create a tax deduction of $1 million in the high-tax buying country, and taxable income of $1 million in the low-tax selling country, thereby reducing total global tax obligations. Cognizant of these incentives, governments have adopted arm's length pricing rules dictating that for tax purposes the prices used for intercompany transactions must be the same as those that would have been chosen by unrelated parties transacting at arm's length. Clearly, the arm's length pricing standard takes care of the problem of $1 million paper clips, but there is widespread concern that the difficulty of applying the arm's length standard to many ordinary cases, to say nothing of complex transactions involving sophisticated financial instruments or intangible property such as patents and trademarks, leaves ample opportunity for tax avoidance.

In reaction to fears about actual or potential tax avoidance under the arm's length pricing standard, there have been numerous calls for stiffer enforcement of the transfer pricing rules, with some expressing doubt that it is possible to craft intercompany pricing rules that can ever succeed. These advocates suggest abandoning altogether the current system of separately accounting for income earned in distinct jurisdictions, replacing it with a system that uses simple formulas to apportion the worldwide income of multinational firms among the jurisdictions in which they have operations. These formulas typically use some combination of employment, sales, and tangible property as implicit indicators of where firms actually earn their incomes. American states currently use simple formulas to apportion the incomes of multistate businesses within the United States, and, relying on that experience, some (e.g., Martens-Weiner, 2006) suggest that formula apportionment might work well internationally.

In order to adopt a system of formula apportionment it is necessary to specify the weights attached to different factors used to apportion income, and the difficulty of doing so in a sensible way makes vivid at least some of the costs associated with replacing separate accounting with a system of formula apportionment. In the United States, where state governments use formulas to apportion corporate income for tax purposes, states have failed to settle on a common formula. It is far from clear what factors properly enter an apportionment formula – assuming that governments could coordinate on a common formula – if the goal is to allocate income roughly according to where it is earned. Furthermore, as noted by Gordon and Wilson (1986), the use of formulas to apportion taxable income effectively converts an income tax into a multiple rate tax on the use of the productive factors that enter the formula, with associated deadweight loss from this haphazard diversity of tax rates. These problems have a common source, which is that profits are not simple scalar functions of employment, sales or tangible property. Instead, profits are produced by many factors, including managerial inputs, that are difficult to measure or subject to reporting manipulation, and therefore omitted from the formulas that governments use. Put simply, the formulas do not apportion income accurately among the jurisdictions in which it is earned.

There is something distasteful and very possibly inequitable about misattributing income for tax purposes, but the associated problems do not stop there, as income misattribution creates incentives for firms to structure their affairs in new ways. Since the formulas apply to affiliates within consolidated groups, it follows that the use of allocation formulas creates incentives to modify the ownership of companies or operations in order to reduce associated tax burdens. Consider, for example, a profitable German company with income taxed by Germany at a high rate. If European governments required companies for tax purposes to allocate their profits among affiliates using formulas that rely heavily on the location of employment, then the German company would have a strong incentive to acquire an unprofitable Irish company with a large labor force. In joining the German and Irish operations under common ownership, many of the German profits would be attributed to Ireland for tax purposes, where they would be subject to the much lower Irish corporate tax rate. Conversely, if the Irish company had large profits and the German company did not, then the use of formula apportionment might discourage a merger of the two firms even if the merger would otherwise make sense for business reasons.

Formula apportionment is typically defended as a pragmatic compromise, representing an imperfect alternative to the current, arguably flawed, system of separate accounting. The purpose of this paper is to analyze the nature and magnitude of ownership distortions created by allocation formulas, the extent to which formulas misattribute income when firms merge or divest their operations. Evidence can be obtained by considering the consequences of hypothetical mergers between firms that are currently independent and therefore report their incomes, employment, sales and property separately. Using data from a large sample of European companies, it is clear that mergers among them, treating two firms for this purpose as though located in different countries, would result in significant reallocations of taxable income, even in the absence of any effect of the mergers on actual operations or profitability.

The formulas used to attribute income between countries can be thought of as forecasts of what fraction of total firm income is likely to have been earned by affiliates with given shares of employment, sales, and property. The analysis in this paper formalizes this notion, identifying conditions under which the formula that minimizes the efficiency cost of ownership distortions is the same as the formula that minimizes the weighted sum of squared residuals in a regression explaining total pretax income. This framework implies that regressions can be used to compare formulas that assign different weights to employment, sales, and property, as well as to construct alternative formulas.

The evidence implies that existing formulas fare poorly from a prediction standpoint: the absolute value of prediction errors from a formula based equally on employment, sales and property exceeds predicted profits 64% of the time, and exceed twice predicted profits 11% of the time. In unconstrained regressions employment seldom gets a significant coefficient. This is hardly surprising given that employment expenditures are costs and therefore subtractions in calculating pretax income, but employment nonetheless persists in playing significant roles in most contemplated and actual allocation formulas. For example, the European Commission (2004) notes that its Home State Taxation pilot scheme would allocate profits of multijurisdictional firms based entirely on employment (either payroll or numbers of employees), or else on an equal-weighted three-factor formula including payroll, sales and property. While none of these schemes would succeed in accurately representing income, use of the employment formulas would be particularly inaccurate. In view of the sizeable ownership and resource allocation distortions introduced by any of these formulas in Europe, the relevant policy question is whether the alternative is sufficiently bad to warrant such a step.

Section 2 describes current systems of separate entity accounting, evidence of taxpayer responses, and proposed formulary alternatives and their consequences. Section 3 analyzes the distortions associated with misattribution of income, and presents a framework that can be used to estimate the consequences of formulary alternatives. Section 4 describes the available data on European companies, and section 5 presents the empirical estimates of the magnitudes of prediction errors due to the use of formulas. Section 6 is the conclusion.

2. Separate Accounting and Formulary Methods

The current international practice of using separate accounting to determine taxable income has come under considerable fire from critics who point to the difficulties of enforcing the arm's length pricing standard against the determined behavior of taxpayers. There is direct and indirect evidence that firms currently arrange their affairs in ways that relocate taxable income from high-tax countries to low-tax countries,[1] which is likely to be inefficient. There are several possible solutions to this problem, including stiffer enforcement together with minor modifications of existing rules (Gresik and Osmundsen, 2008), though radical reform is always an option. Whereas separate accounting is generally acknowledged to offer a theoretically satisfying method of measuring income for tax purposes, concern over the practical ability of governments to operate separate accounting underlies much of the appeal of alternative methods of determining the location of business income.

Formulary alternatives to separate accounting can take different forms, relying to differing degrees on sales, property, and employment factors to apportion income among related parties. In the equal-weighted three-factor formula once commonly used by American states, the fraction of a firm's national income taxed by an individual state equals the state's share of the firm's sales, tangible property, and employee compensation, with each of these three factors weighted equally. There is considerable recent interest in possible adoption of a form of formulary apportionment within Europe, the Common Consolidated Corporate Tax Base (Commission of the European Communities, 2001). Gordon and Wilson (1986) analyze some of the distortions introduced by making tax obligations functions of sales, property, and employment rather than the production of income: the tax system thereby discourages sales, investment, and employment. Given these distortions, it is perhaps not surprising that American states have modified their apportionment formulas over time to emphasize the less distortionary sales factors at the expense of property and employment,[2] even though this evolution may or may not be in the interest of states as a group. One of the open questions about formulary methods in the international context is the extent to which the use of separate accounting by the rest of the world increases or reduces the desirability of using formula apportionment within a small federation of countries.[3] The adoption of formulary methods would occasion significant redistributions through changes in the tax obligations of individual firms (Shackelford and Slemrod, 1998) and the tax revenues of individual countries (Devereux and Loretz, 2008; Fuest et al., 2007). Formula apportionment does not require information on location-specific profitability, though this feature may make it more difficult for governments to tailor their tax systems to extract rents from taxpayers in the most efficient possible manner (Gresik, 2008). Even the possible administrative cost and compliance benefits of formulary methods (Mintz, 2004) may be suspect; Roin (2008) notes that many of the methods taxpayers have honed in avoiding income taxes under separate accounting can be deployed to avoid taxes determined by formulary methods, in some cases at greater social cost and to greater effect on tax collections.

One of the costs associated with using formulary apportionment is that these systems create incentives for firms to change their operations through mergers or divestitures. Even in the absence of formula apportionment it is very common for mergers and divestitures to have significant tax effects by triggering the realization of capital gains, changing the ability to offset profits from one operation against losses from another, influencing a firm's ability to claim foreign tax credits, changing asset bases for depreciation purposes, and other tax consequences.[4] There is considerable evidence that tax attributes influence the likelihood and structure of mergers and divestitures, as well as the accompanying transaction prices.[5] Consequently it is

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[1] See, for example, Clausing (2001, 2003), Desai et al. (2003), Hines and Rice (1994), Huizinga and Laeven (2008), and Mintz and Smart (2004); Hines (1999) and Gresik (2001) reviews some of the earlier empirical evidence.
[2] See Goolsbee and Maydew (2000), Anand and Sansing (2000), Edmiston (2002), Wellisch (2004), and, for an alternate viewpoint, Runkel and Schjelderup (2007).
[3] See, for example, Mintz and Weiner (2003) and Riedel and Runkel (2007).
[4] Scholes et al. (2005) offers a nontechnical review of some of the tax considerations in mergers and divestitures.
[5] See, for example, the evidence presented by Auerbach and Reishus (1988), Dhaliwal et al. (2004), Erickson (1998), Erickson and Wang (1999a, 1999b, 2000, 2007), Hayn (1989), Kaplan (1989), Maydew et al. (1999), Schipper and Smith (1991), and Weaver (2000).

reasonable to expect that the ownership incentives created by the adoption of formula apportionment might significantly influence patterns of mergers and divestitures.

3. Distortions

This section analyzes the ownership distortions associated with the use of formulary methods, and in particular, the method by which it is possible to estimate the magnitude of these distortions using data from a cross section of independent firms.

3.1 The extent of income misattribution from using formulas.

It is helpful to consider the tax consequences of a merger of two firms, designated firm 1 and firm 2. Firm 1 operates entirely in country 1, where it faces a profit tax rate of τ_1; firm 2 operates entirely in country 2, where its profits are subject to tax at rate τ_2. Countries 1 and 2 tax multijurisdictional firms using a common formula with weights α_i to each of $i = 1,...,n$ factors such as employment, fixed capital, and sales, with $\sum_{i=1}^{n} \alpha_i = 1$. Firm 1's taxable profits are represented by π_1, where profits equal revenues minus deductible costs including purchases of intermediate goods from other firms, labor and interest expense, but not including the opportunity cost of invested capital. Firm 1's factors are denoted x_{1i} for all $i = 1,...,n$; similarly, firm 2's profits are denoted π_2, and its factors denoted x_{2i}. The aggregate tax obligation for the two firms in the absence of a merger equals:

$$(1) \qquad (\tau_1\pi_1 + \tau_2\pi_2).$$

If the two firms merge in such a way that their factor use and profits are unchanged, then the only difference produced by the merger is that their tax obligations to countries 1 and 2 will be determined by formula, and the total is:

$$(2) \qquad (\pi_1 + \pi_2)\left\{\tau_1\left[\sum_{i=1}^{n} \alpha_i \frac{x_{1i}}{(x_{1i} + x_{21})}\right] + \tau_2\left[\sum_{i=1}^{n} \alpha_i \frac{x_{2i}}{(x_{1i} + x_{2i})}\right]\right\}.$$

The difference between (1) and (2) is $(\tau_1 - \tau_2)\psi$, for which:

(3) $$\psi = \sum_{i=1}^{n} \alpha_i \frac{\pi_2 x_{1i} - \pi_1 x_{2i}}{\left(x_{1i} + x_{2i}\right)}.$$

The difference in total tax obligation induced by formula apportionment is the product of tax rate differences and differences in ratios of profitability to factor use. Clearly, there is no difference if $\tau_1 = \tau_2$ or if $\frac{\pi_2}{\pi_1} = \frac{x_{2i}}{x_{1i}} \forall i$. In the first of these cases the equality of tax rates implies that the taxpayer's total obligation is the same regardless of the jurisdiction to which income is assigned; in the second case, the formulas assign income exactly as it is earned. For most cases, however, neither of these conditions will hold. Introducing a new variable $\left(\frac{\bar{\pi}}{\bar{x}_i}\right)$, the ratio of average profits among all firms $(\bar{\pi})$ and average use of factor i $(\bar{x}_i)$, equation (3) can be rewritten as:

(4) $$\psi = \sum_{i=1}^{n} \alpha_i \left[\pi_2 - x_{2i}\left(\frac{\bar{\pi}}{\bar{x}_i}\right)\right] \frac{x_{1i}}{\left(x_{1i} + x_{2i}\right)} - \sum_{i=1}^{n} \alpha_i \left[\pi_1 - x_{1i}\left(\frac{\bar{\pi}}{\bar{x}_i}\right)\right] \frac{x_{2i}}{\left(x_{1i} + x_{2i}\right)}.$$

In analyzing the implications of (4) it is helpful to define $\bar{s}_1 \equiv \sum_{i=1}^{n} \alpha_i \frac{x_{1i}}{\left(x_{1i} + x_{2i}\right)}$ to be firm one's average share of all factors, weighted as in the common allocation formula, and similarly for firm two, so that $\bar{s}_2 = 1 - \bar{s}_1$. Then (4) can be rewritten as:

(5) $$\psi = \left[\pi_2 - \sum_{i=1}^{n} \alpha_i x_{2i}\left(\frac{\bar{\pi}}{\bar{x}_i}\right)\right]\bar{s}_1 - \left[\pi_1 - \sum_{i=1}^{n} \alpha_i x_{1i}\left(\frac{\bar{\pi}}{\bar{x}_i}\right)\right]\bar{s}_2 - \sum_{i=1}^{n} \alpha_i \left(\frac{\bar{\pi}}{\bar{x}_i}\right)\left[x_{1i} - \bar{s}_1\left(x_{1i} + x_{2i}\right)\right].$$

The right side of equation (5) consists of three terms, the first of which is $\bar{s}_1$ times the difference between π_2 and the weighted average of $x_{2i}\left(\frac{\bar{\pi}}{\bar{x}_i}\right)$, with weights equal to α_i. One can think of this difference as being the residual from a regression equation in which the α_i are regression coefficients used to predict π_2 based on x_{2i}. The mean prediction error of this regression is clearly zero, since the α_i sum to one and firm profits must average the sample mean $\bar{\pi}$, but the sum of squared residuals from this regression depends on the values of the α_i.

It is convenient to denote the value of the residual predicting π_2 in this regression, the first bracketed term on the right side of equation (5), as r_2. Likewise the second term on the right side of equation (5) is the product of $\bar{s}_2$ and the analogous residual from the equation predicting π_1 based on x_{1i}; hence this product can be represented as $r_1\bar{s}_2$.

The third term on the right side of (5) is the weighted sum of the difference between x_{1i} and $\bar{s}_1\left(x_{1i}+x_{2i}\right)$, with weights equal to $\alpha_i\left(\frac{\bar{\pi}}{\bar{x}_i}\right)$. This sum is a function not of the ability of factors to predict profitability, but instead of differences in the relative factor intensities of firms 1 and 2, since x_{1i} differs from $\bar{s}_1\left(x_{1i}+x_{2i}\right)$ because $\bar{s}_1$ is an average taken across all input factors, not merely factor *i*. The value of this sum is likely to be of second order importance relative to the prediction errors that constitute the first two terms on the right side of equation (5); for example, it is exactly zero in the case in which a single factor receives unit weight in the allocation formula and other factors have zero weight.[6] Taking this term to be of second order relative to the others, equation (5) can be written:

(6) $$\psi = r_2\bar{s}_1 - r_1\bar{s}_2 .$$

Hence the merger of two firms misallocates their income for tax purposes by an amount equal to the weighted difference of the residuals in the equations explaining their incomes.

3.2 The economic cost of income misattribution.

From the standpoint of efficient resource allocation, the cost of misattributing economic income by using an allocation formula includes the possibility that this misattribution may influence whether firms merge or divest their operations. A merger between firms 1 and 2 is tax favored if $(\tau_1-\tau_2)\psi$ is negative, and tax disfavored if $(\tau_1-\tau_2)\psi$ is positive; the magnitude of $(\tau_1-\tau_2)\psi$ determines the extent of tax incentive. A conglomerate consisting of two affiliates equivalent to firms 1 and 2 similarly faces incentives to divest one affiliate if $(\tau_1-\tau_2)\psi$ is

[6] As noted in the discussion of the empirical evidence in section 5, empirically this third term in equation (5) is of much smaller magnitude than the other terms in the equation.

positive, and to avoid divestiture if $(\tau_1 - \tau_2)\psi$ is negative. These tax incentives say nothing about the business merits of these mergers or divestitures, though as a general matter the existence of such tax incentives can be expected to reduce efficiency by introducing considerations other than pretax profits into ownership decisions.

In order to estimate the magnitude of the ownership distortion due to formula apportionment, it is necessary to understand the extent to which firms are likely to change their ownership of affiliates or other firms in response to these tax incentives. There is apt to be considerable variation, since influencing the decision requires that a firm be sufficiently close to the margin that tax considerations become decisive. Firm size is sure to be correlated with this proclivity: Exxon Mobil is unlikely to attempt to acquire British Petroleum in order to save $100 million in taxes – given the other costs associated with such an acquisition – even though two medium sized firms would find a tax saving of that size an irresistible inducement to merge. Exxon Mobil might, on the other hand, consider acquiring a small oil company in return for a modest tax saving, since relative to the size of the acquisition the tax saving could loom large.

One way to formalize these notions is to specify that the probability that firms 1 and 2 merge is given by:

$$(7) \qquad k + \gamma \frac{(\tau_1 - \tau_2)\psi}{(\tilde{x}_1 + \tilde{x}_2)\tilde{s}_1^2 \tilde{s}_2^2},$$

in which *k* is a function of various non-tax attributes of firms 1 and 2, γ is a constant, $\tilde{x}_1$ is the size of firm 1's assets, and $\tilde{s}_1^2$ is the squared value of firm 1's share of the total assets of firms 1 and 2, $\tilde{s}_1^2 = \frac{\tilde{x}_1^2}{(\tilde{x}_1 + \tilde{x}_2)^2}$. The numerator of the ratio in (7) is the tax consequence of the merger, and the denominator captures that doubling the size of both the target and the acquirer also doubles the required tax saving to have the same effect on merger probabilities. This denominator reflects the cost of merging, and its size effects are not symmetric: whereas increasing the size of the smaller of firms 1 and 2 indeed increases the tax benefit necessary to encourage a merger, increasing the size of the larger of the two firms may reduce the needed tax benefit. Thinking of the much larger firm as the acquirer, as the acquirer grows in size it has greater access to financial and managerial resources that reduce the cost of a merger.

With a probability of merger given by (7), the standard Harberger triangle approximation (Hines, 1999) to the deadweight loss associated with tax incentives, denoted Δ, is given by one-half the product of the tax incentive $\left[(\tau_1 - \tau_2)\psi\right]$ and the induced behavioral change:

(8) $$\Delta = \frac{1}{2}\gamma \frac{(\tau_1 - \tau_2)^2 \psi^2}{(\tilde{x}_1 + \tilde{x}_2)\tilde{s}_1^2 \tilde{s}_2^2}.$$

The value of ψ^2 is given by:

(9) $$\psi^2 = r_2^2 \bar{s}_1^2 + r_1^2 \bar{s}_2^2 - 2 r_1 r_2 \bar{s}_1 \bar{s}_2 .$$

Taking firms 1 and 2 to be randomly matched, it follows from the fact that r_1 and r_2 both have mean zero that the expected value of ψ^2 is:

(10) $$E(\psi^2) = \bar{s}_1^2 E(r_2^2) + \bar{s}_2^2 E(r_1^2).$$

In evaluating (8) it simplifies matters greatly to set $\bar{s}_1^2 = \tilde{s}_1^2$, reflecting that the ratio of two firms' assets roughly tracks the ratios of their factors used in the allocation formulas. Furthermore, the expected value of the squared prediction error is itself a function of firm size, so:

(11) $$\frac{E(r_1^2)}{E(r_2^2)} = \frac{\tilde{s}_1^2}{\tilde{s}_2^2}.$$

Imposing (10) and (11), and the approximation that $\bar{s}_1^2 = \tilde{s}_1^2$, (8) becomes:

(12) $$\Delta = \gamma \frac{(\tau_1 - \tau_2)^2 E(r_1^2)}{(\tilde{x}_1 + \tilde{x}_2)\tilde{s}_1^2} = \gamma \frac{(\tau_1 - \tau_2)^2 E(r_1^2)(\tilde{x}_1 + \tilde{x}_2)}{\tilde{x}_1^2},$$

which in turn implies that:

(13) $$\Delta = \gamma(\tau_1 - \tau_2)^2 \left[\frac{E(r_1^2)}{\tilde{x}_1} + \frac{E(r_2^2)}{\tilde{x}_2} \right].$$

Equation (13) indicates that the deadweight loss from income misattribution by formulary methods is proportional to the product of the square of tax rate differences and sum of squared estimation errors, normalized by asset sizes. Clearly, the formulary system that minimizes the deadweight loss from ownership misallocation associated with income misattribution is one that minimizes the weighted sum of squares of prediction errors, with weights equal to firm assets.

3.3. Interpretation.

Equation (13) implies that the expected economic cost of ownership distortions introduced by misattributing income is proportional to the product of the square of tax rate differences and the expected squared prediction error from using a formula. This comes from the Harberger triangle representation of deadweight loss, which takes a second-order approximation that the distribution of tax benefits is roughly uniform across the affected population, so for those firms whose merger or divestment decisions are influenced by the use of formula allocation, the average economic cost of this distortion equals half of the tax incentive produced by the formula. This, together with the equation determining the likelihood of tax-motivated ownership changes, implies that small tax incentives produce very small expected deadweight losses, whereas large tax benefits for some firms may create significant deadweight losses.

The analysis in section 3.2 considers a potential merger of two firms chosen at random, and it is the randomness of this matching that makes it possible to ignore the third term on the right side equation (9), the expectation of which is zero, and thereby replace equation (9) with equation (10). Clearly, potential merger candidates are not in fact randomly matched, though whether their matching is random relative to potential tax benefits introduced by the use of formula apportionment is another matter. The analysis considers individual firm matches, but once a system of formula apportionment is in place the distribution of firm attributes will change as assets are reallocated in response to tax incentives. As Gordon and Wilson (1986) note, in the absence of merger costs every firm will face the same average tax rate in equilibrium, since any tax differences will be eliminated through the process of tax-motivated mergers and divestments. It is, however, unrealistic to think that asset reallocation can proceed so costlessly or easily. The framework underlying the empirical analysis assumes that potential merger partners are brought together for non-tax business reasons, at which point tax considerations have the potential to influence the outcome by affecting the potential net benefits of a merger. Furthermore, tax-motivated mergers are either sufficiently limited in number that they do not significantly influence the distribution of firm attributes throughout the economy, so these attributes can be taken to be exogenous from the standpoint of the analysis, or else they are quite frequent, in which case it is clear that they are the source of large economic distortions.

Both tax and prediction error terms enter the formula for efficiency cost in Equation (13). Assuming that tax rates are determined by considerations that do not include the details of formula apportionment, and taking firm attributes to be distributed independently of tax rates, it follows that the formula that minimizes the expected squared prediction error also minimizes deadweight loss from ownership distortions. This is the ordinary least squares estimate of the formula components: OLS is the minimum variance unbiased linear estimator, and by construction in this case the mean estimation error is zero and the estimates are linear in the components. Hence in order to implement equation (13) to find the distortion-minimizing formulary apportionment scheme, it is simply necessary to run regressions explaining profitability on the basis of observed factors.

4. Data

In order to evaluate the magnitudes of tax-induced ownership distortions it is necessary to estimate the extent to which apportionment factors explain the variation in firm profitability. The data on European firms come from the Amadeus-Bureau Van Dijk database, which includes firm-level data on over 5 million private and publicly owned non-financial firms in 34 European countries in both eastern and western Europe. The Amadeus database is created by collecting standardized data received from information providers across Europe. The local source for these data is generally company registrar offices, which require all incorporated firms to submit annual filings. The database includes firm-level accounting data in standardized financial format.

The empirical analysis considers only firms with 100 or more employees, since Amadeus coverage of very small firms varies with country- level filing requirements, and tax-motivated international ownership changes are likely to be focused on larger firms. The key variables in this analysis are fixed assets, enterprise value, sales, profit/loss before taxation, number of employees, and labor compensation. Fixed assets are defined as "tangible fixed/long-term assets net of depreciation", which does not include financial assets; fixed assets can be thought of as property plant and equipment (PPE). Enterprise value is the sum of a firm's yearend market capitalization and debt (both long term and short term) minus cash. Excluding firms with missing value for these variables, and using only consolidated statements, the final sample includes 11,103 firms for which there is information on a firm's profit/loss before taxation, and 1,473 firms for which there is information on year-end enterprise value.

Table 1 presents descriptive statistics for these variables;[7] the top panel reports the figures for the sample with information on profits and losses, and the bottom panel for the significantly smaller sample with information on enterprise value. The skewed distribution of firm sizes is evident from this table, in that mean entries for every variable greatly exceed sample medians. In the sample of firms reporting profits and losses, median profit for 2004 was $3.9 million; these firms had median sales of $116.2 million, median property, plant and equipment holdings of $23.6 million, median labor compensation of $21.6 million, and a median of 427 employees. The smaller sample of firms reporting enterprise value had median enterprise values of $160.9 million, and had roughly double the median sales, property plant and equipment, labor compensation and employment of firms not reporting enterprise values.

The empirical work in section 5 uses these data to estimate the extent to which observable factors explain income differences. Since the Amadeus data are financial accounting entries, they need not correspond to taxable incomes, and are potentially subject to their own sources of bias and noise, as executives may manage entries to meet earning targets and otherwise present their firms to financial markets as favorably as possible. Whether these financial data are more or less reliably reported than information presented to tax authorities is an interesting question, but in any case tax data are confidential and therefore unavailable for this analysis, and it is revealing to use the financial data to compare the accuracy of alternative methods of predicting a firm's income.

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[7] Appendix Table 1 presents descriptive statistics for versions of these variables weighted by the square root of firm assets; these are the variables used in the regressions that follow.

The available information covers the operations and incomes of independent companies, whereas the analysis in section 3 concerns separate operations within the same multinational firms. In using data from separate companies to estimate the consequences of allocating income using formulary methods, there is an implicit assumption that the income generating process in a cross section of firms resembles the income generating process among affiliates of the same firm. Certainly this assumption looks sound if mergers are tax-driven marriages of convenience; and even in the case that ownership of divisions or affiliates is determined by pretax profitability, with production, sales, and cost spillovers among components of the firm, it is nonetheless reasonable to expect that to a first order the contribution of an affiliate to total firm profitability is closely related to its profitability as a stand-alone entity.

5. Determinants of Income and Market Valuation

In evaluating the predictive quality of different allocation formulas, it is helpful to start with benchmark formulas whose coefficients are unconstrained by prior choices but instead determined by the data. These benchmark formulas come simply from regressions of income on factors that enter the formulas. These regressions have the potential to reveal the ability of different factors to account for income production, and to help evaluate the extent to which simple formulary rules depart from weights that are indicated by the data.

Table 2 presents regressions of profits and losses, and enterprise values, on measures of sales, tangible property, and employment. The regressions reported in columns 1 and 3 use labor compensation as measures of labor input, whereas the regressions reported in columns 2 and 4 use numbers of employees; as a result, the sample sizes are much larger in the even-numbered columns. In all of the regressions variables are weighted by the square root of firm assets. Observable measures do a creditable job of predicting operating income, in that the R-squareds lie between 0.2 and 0.4, though this largely reflects differences between small firms and large firms. The point estimates in column one imply that, controlling for property and employment, \$100 of additional sales, interacted with the ratio of mean income to mean sales, is associated with \$60.95 of additional operating income. Given the values reported in Appendix Table 1, this implies that \$100 of additional sales, conditional on property and employment, correlates with \$2.80 of additional income. Similarly, controlling for other factors, the regression implies that

$100 of additional property, interacted with the ratio of mean income to mean property, is associated with $25.60 of additional operating income – or that $100 of additional property, conditional on sales and employment, correlates with $7.13 of additional income. The estimated coefficients on sales and tangible property are both significant, though it is notable that the t-statistic on the sales coefficient exceeds that on the property coefficient. The 0.0654 point estimate on labor compensation implies that $100 of additional labor compensation, interacted with the ratio of mean income to mean labor compensation, is associated with $6.54 of additional income, and this magnitude does not differ statistically from zero. The regression reported in column two using employment as a measure of labor input produces similar results.

Columns 3 and 4 report coefficient estimates obtained by repeating these regressions with enterprise value as the dependent variable. Sales and property again significantly affect measured income, and the employment variables perform significantly better in these regressions, with positive and statistically significant estimated coefficients, albeit smaller than those for sales and property.

Figure 1 plots predicted and actual values of profits and losses, where the prediction is drawn from the model reported in column 1 of Table 3. It is clear from the figure that while predicted values capture the central tendencies of the data, there is just an enormous amount of idiosyncratic variation that no three-factor formula can hope to reflect. Figure 2 performs the same exercise for the equation predicting enterprise value, a dependent variable that is truncated at zero, but that nevertheless exhibits considerable unexplained variation. Of course, constraining the formulas by omitting one or two of the factors, or imposing equality among the coefficients on all three, only reduces the predictive power of the equations.

Tables 3 presents estimated coefficients from regressions that include just two factors, sales and measures of employment. The results indicate that the sales variable does almost all of the work in explaining measured income. The employment coefficients are statistically insignificant in the regressions reported in columns 1 and 2 in which profit is the dependent variable, and the employment variable is insignificant in the regression reported in column 4 in which enterprise value is the dependent variable. Only in column three, in which enterprise value is the dependent variable, does the labor compensation variable have a coefficient that differs significantly from zero, and its t-statistic of 2.06 is quite a bit smaller than the corresponding t-statistic of 8.53 on the sales coefficient.

It may not be surprising that labor cost factors do a very poor job of predicting income, given that labor expenses are deductible in calculating income; whereas capital expenses are typically only partially deductible, and greater sales conditional on factor inputs contribute to income. As Goolsbee and Maydew (2000) and Anand and Sansing (2000) note, American states have moved over time to give greater weight to sales factors in their apportionment formulas, and typically have incentives to do so in order to attract mobile economic activity, quite apart from any improved income attribution (though the destination-based sales definition in the formulas does not correspond exactly to the firm-based sales definition in the Amadeus data). In contrast, the variant of the European Common Consolidated Corporate Tax Base proposal in which income would be allocated purely on the basis of labor inputs would attribute income production particularly poorly.

In practice, formulary methods as used by U.S. states and Canadian provinces, and advocated for use internationally, do not correspond to the unconstrained regressions presented in Tables 2 and 3. Instead these formulas follow fixed rules, such as equal weights on each of the sales, property and employment factors, or double weighting sales in a three-factor formula, or two factor formulas with equal weights on property and employment. Table 4 presents summary statistics from regressions based on these and other formulary alternatives. The table reports R^2 statistics from the underlying regressions in each row. The first row presents data for the unconstrained three-factor regression, and all of the following rows report the extent to which their R^2 statistics exceed that of the unconstrained regression.

The figures in Table 4 illustrate the extent to which constraining the regressions reduces their predictive power. The equal weighted three-factor formula produces an R^2 that is 6.29% smaller than that for the unconstrained formula predicting profits (using labor compensation as a measure of employment), and 4.62% smaller in predicting enterprise value. Use of a three-factor formula with double weight on the sales factor significantly mitigates the loss of R^2, as do the two-factor regressions in which labor compensation does not appear. Even with sales and property constrained each to have 50% weight, these two-factor formulas come within 4.8% of the R^2 of unconstrained three-factor formula predicting profits (using labor compensation as the employment measure) and 1.9% of the R^2 of unconstrained three-factor formula predicting enterprise value.

The two-factor formulas that include labor compensation and are constrained to use equal weights perform very poorly from an R^2 standpoint, the equation in which property and wages predict profits producing an R^2 22% lower than that for the unconstrained three-factor formula. The culprit is clearly the labor compensation variable, as revealed by the one-factor formulas reported in the bottom four rows of Table 4. Whereas use of the sales factor alone to predict profits produces an R^2 value within 7.4% of that produced by an unconstrained three-factor formula, the labor compensation factor generates an R^2 40.2% smaller, and total employment an R^2 58.2% smaller. The property factor alone is similar to the labor factors in generating low R^2 values.

One indicator of the performance of a formula is the distribution of its predictions relative to actual values. Table 5 presents the distribution of the ratio of forecast errors to forecasted incomes for the unconstrained regressions. Columns 1 and 2 indicate that the absolute value of the error in predicting firm profits exceeds half of the predicted value 64% of the time, and the error in predicting enterprise value exceeds half of that predicted value roughly 50% of the time. The table indicates that the absolute value of the error in predicting profits exceeds twice the predicted level of profits 11% of the time, and the error in predicting enterprise value exceeds twice the predicted value 8-9% of the time; clearly, there are significant numbers of cases of large prediction errors.[8]

The constrained regressions perform considerably worse on average than do the unconstrained three-factor formulas. Table 6 presents the distribution of forecast errors for the three-factor formulas in which sales, property and employment are assigned equal weights. The

[8] A similar calculation can be used to measure the absolute magnitude of the third term on the right side of equation (5) relative to predicted firm profits. In order to evaluate this third term it is necessary to specify a merger partner, which is taken to be a firm with mean values of sales, property, and labor compensation. The absolute values of the third term are tiny compared to the prediction errors that are the basis of the calculations presented in Table 5. For example, in the regression reported in the first column, only 12.1% of firms have third terms that exceed (in absolute value) 10% of predicted firm profits, 2.3% of firms have third terms that exceed 25% of predicted profits, fewer than 0.05% of firms have third terms that exceed 50% of predicted profits, and none exceed 75% of predicted profits.

absolute value of the prediction error from the equal-weighted three-factor formula exceeds half of predicted profits 65% of the time, and exceeds twice profits 11-13% of the time. These prediction errors do not differ greatly from those reported for the unconstrained regressions in Table 5, and the differences between the prediction errors of the unconstrained and constrained three-factor regressions are similarly modest for equations predicting enterprise values.

6. Conclusion

Formulary alternatives to separate entity accounting hold the undeniable appeal of reducing certain opportunities for tax-motivated international income reallocation. This comes at a serious cost, which is that the factors that enter the formulas do not accurately correspond to the determinants of business incomes. As a result, the formulas misattribute income, so their use in an international setting would misallocate tax revenue among countries and tax burdens among taxpayers. In so doing, the adoption of formula apportionment creates incentives for new forms of tax avoidance through mergers and divestitures.

Evidence from European companies indicates that commonly proposed formulas predict little more than one fifth of the observed variation in profits, and roughly one third of the variation in market capitalization. In particular, the labor cost factors do a very poor job of predicting income. The analysis in section 3 of the paper shows that the ownership distortions associated with the use of formulary methods are proportional to the mean squared prediction error in a regression explaining firm income, with weights equal to the inverse square root of firm size. Since the estimated coefficients are very unkind to the labor factors, they suggest that governments that must use formulas maximize the accuracy of income attribution and minimize the deadweight loss of ownership distortions by ignoring or significantly downplaying the labor input factors – though since the formulas also distort investment, employment, and other economic decisions in addition to ownership, the total distortion-minimizing formula would incorporate all of these considerations.

Is it sensible to consider formulary alternatives to separate accounting for tax purposes, given the inaccuracy of formulary methods and the incentives they create? Evaluating this question requires a careful comparative examination of all of the unappetizing tax choices that governments face. Hard experience makes problems more evident in the tax systems that governments use than in the alternatives, but it does not follow that tax reform improves matters, since it generally replaces one set of problems with another. It is clear from the evidence that formulas attribute income very imperfectly, so whether the associated costs are acceptable depends on how dire one considers the international tax regime today.

References

Anand, Bharat N. and Richard Sansing, The weighting game: Formula apportionment as an instrument of public policy, *National Tax Journal*, June 2000, 53 (2), 183-200.

Auerbach, Alan J. and David Reishus, The effects of taxation on the merger decision, in Alan J. Auerbach ed., *Corporate takeovers: Causes and consequences* (Chicago: University of Chicago Press, 1988), 157-183.

Clausing, Kimberly A., The impact of transfer pricing on intrafirm trade, in James R. Hines Jr., ed. *International taxation and multinational activity* (Chicago: University of Chicago Press, 2001), 173-194.

Clausing, Kimberly A., Tax-motivated transfer pricing and U.S. intrafirm trade policies, *Journal of Public Economics*, September 2003, 87 (9-10), 2207-2223.

Commission of the European Communities, *Towards an internal market without tax obstacles: A strategy for providing companies with a consolidated corporate tax base for their EU-wide activities*, 2001, COM 582 final.

Desai, Mihir A., C. Fritz Foley and James R. Hines Jr., Chains of ownership, tax competition, and the location decisions of multinational firms, in Heinz Herrmann and Robert Lipsey, eds. *Foreign direct investment in the real and financial sector of industrial countries* (Berlin: Springer-Verlag, 2003), 61-98.

Devereux, Michael P. and Simon Loretz, The effects of EU formula apportionment on corporate tax revenues, *Fiscal Studies*, 2008, 29 (1), 1-33.

Dhaliwal, Dan S., Merle Erickson and Shane Heitzman, The effect of seller income taxes on acquisition price: Evidence from purchases of taxable and tax-exempt hospitals, *Journal of the American Taxation Association*, 2004, 26 (2), 1-21.

Edmiston, Kelly D., Strategic apportionment of the state corporate income tax: An applied general equilibrium analysis, *National Tax Journal*, June 2002, 55 (2), 239-262.

Erickson, Merle, The effects of taxes on the structure of corporate acquisitions, *Journal of Accounting Research*, Autumn 1998, 36 (2), 279-298.

Erickson, Merle and Shiing-wu Wang, Exploiting and sharing tax benefits: Seagram and DuPont, *Journal of the American Taxation Association*, Fall 1999a, 21 (2), 35-54.

Erickson, Merle and Shiing-wu Wang, Earnings management by acquiring firms in stock for stock mergers, *Journal of Accounting and Economics*, 1999b, 27, 149-176.

Erickson, Merle and Shiing-wu Wang, The effect of transaction structure on price: Evidence from subsidiary sales, *Journal of Accounting and Economics*, 2000, 30, 59-97.

Erickson, Merle and Shiing-wu Wang, Tax benefits as a source of merger premiums in acquisitions of private corporations, *Accounting Review*, March 2007, 82 (2), 359-387.

European Commission, Directorate-General XXI, Taxation and Customs Union, Tax Policy, *Outline of a possible experimental application of Home State Taxation to small and medium-sized enterprises*, TAXUD C.1/DOC (04) 1410, 2004.

Fuest, Clemens, Thomas Hemmelgarn and Fred Ramb, How would the introduction of an EU-wide formula apportionment affect the distribution and size of the corporate tax base? An analysis based on German multinationals, *International Tax and Public Finance*, 2007, 14 (5), 605-626.

Goolsbee, Austan and Edward Maydew, Coveting thy neighbor's manufacturing: The dilemma of state income apportionment, *Journal of Public Economics*, 2000, 75 (1), 125-143.

Gordon, Roger H. and John D. Wilson, An examination of multijurisdictional corporate income taxation under formula apportionment, *Econometrica*, November 1986, 54 (6), 1357-1373.

Gresik, Thomas A., The taxing task of taxing transnationals, *Journal of Economic Literature*, September 2001, 39 (3), 800-838.

Gresik, Thomas A., Optimal separate accounting vs. optimal formula apportionment, Working Paper, University of Notre Dame, 2008.

Gresik, Thomas A. and Petter Osmundsen, Transfer pricing in vertically integrated industries, *International Tax and Public Finance*, June 2008, 15, 231-255.

Hayn, Carla, Tax attributes as determinants of shareholder gains in corporate acquisitions, *Journal of Financial Economics*, April 1989, 23 (1), 121-153.

Hines, James R., Jr., Three sides of Harberger triangles, *Journal of Economic Perspectives*, Spring 1999a, 13 (2), 167-188.

Hines, James R., Jr., Lessons from behavioral responses to international taxation, *National Tax Journal*, June 1999b, 52 (2), 305-322.

Hines, James R., Jr. and Eric M. Rice, Fiscal paradise: Foreign tax havens and American business, *Quarterly Journal of Economics*, February 1994, 109 (1), 149-182.

Huizinga, Harry and Luc Laeven, International profit shifting within multinationals: A multi-country perspective, *Journal of Public Economics*, June 2008, 92 (5-6), 1164-1182.

Kaplan, Steven, The effects of management buyouts on operating performance and value, *Journal of Financial Economics*, 1989, 24, 217-254.

Martens-Weiner, Joann *Company tax reform in the European Union: Guidance from the United States and Canada on implementing formulary apportionment in the EU* (New York: Springer, 2006).

Maydew, Edward L., Katherine Schipper and Linda Vincent, The impact of taxes on the choice of divestiture method, *Journal of Accounting and Economics*, 1999, 28, 117-150.

Mintz, Jack, Corporate tax harmonization in Europe: It's all about compliance, *International Tax and Public Finance*, 2004, 11, 221-234.

Mintz, Jack and Michael Smart, Income shifting, investment, and tax competition: Evidence from provincial taxation in Canada, *Journal of Public Economics*, 2004, 88, 1149-1168.

Mintz, Jack and Joann M. Weiner, Exploring formula allocation for the European Union, *International Tax and Public Finance*, November 2003, 10 (6), 695-711.

Riedel, Nadine and Marco Runkel, Company tax reform with a water's edge, *Journal of Public Economics*, 2007, 91, 1533-1554.

Roin, Julie, Can the income tax be saved? The promise and pitfalls of worldwide formulary apportionment, *Tax Law Review*, 2008, 61, 169.

Runkel, Marco and Guttorm Schjelderup, The choice of apportionment factors under formula apportionment, CESifo Working Paper No. 2072, 2007.

Schipper, Katherine and Abbie Smith, Effects of management buyouts on corporate interest and depreciation tax deductions, *Journal of Law and Economics*, October 1991, 34 (2), 295-341.

Scholes, Myron S., Mark A. Wolfson, Merle Erickson, Edward L. Maydew, and Terry Shevlin, *Taxes and business strategy: A planning approach* (Upper Saddle River, NJ: Prentice Hall, 2005).

Shackelford, Douglas and Joel Slemrod, The revenue consequences of using formula apportionment to calculate U.S. and foreign-source income: A firm-level analysis, *International Tax and Public Finance*, 1998, 5, 41-59.

Weaver, Connie D., Divestiture structure and tax attributes: Evidence from the Omnibus Budget Reconciliation Act of 1993, *Journal of the American Taxation Association*, 2000, 22 (Supplement), 54-71.

Wellisch, Dietmar, Taxation under formula apportionment: Tax competition, tax incidence, and the choice of apportionment factors, *FinanzArchiv*, 2004, 60 (1), 24-41.

Figure 1

Errors in Unconstrained Equations Predicting Profits and Losses

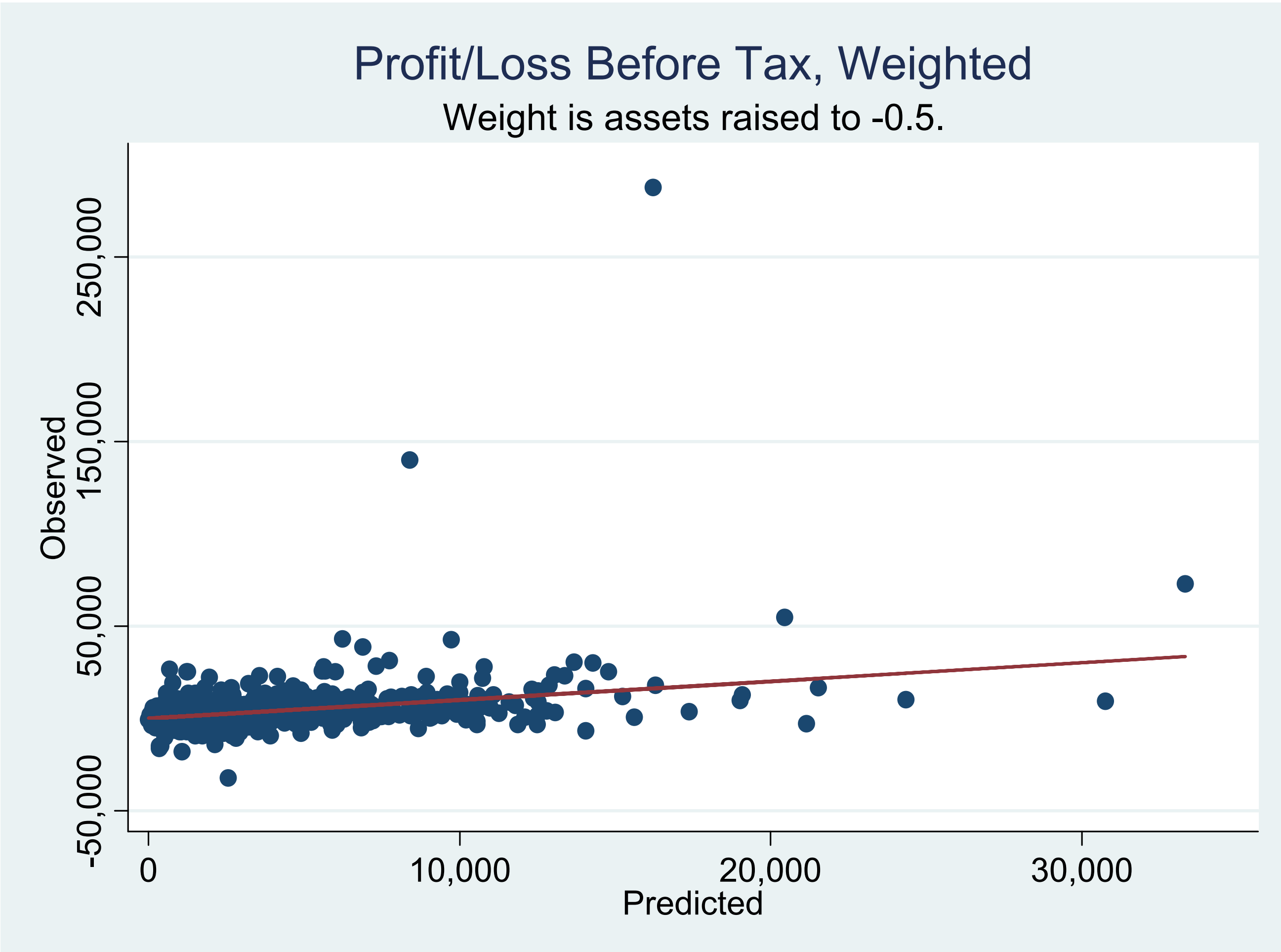

Figure 2

Errors in Unconstrained Equations Predicting Enterprise Market Value

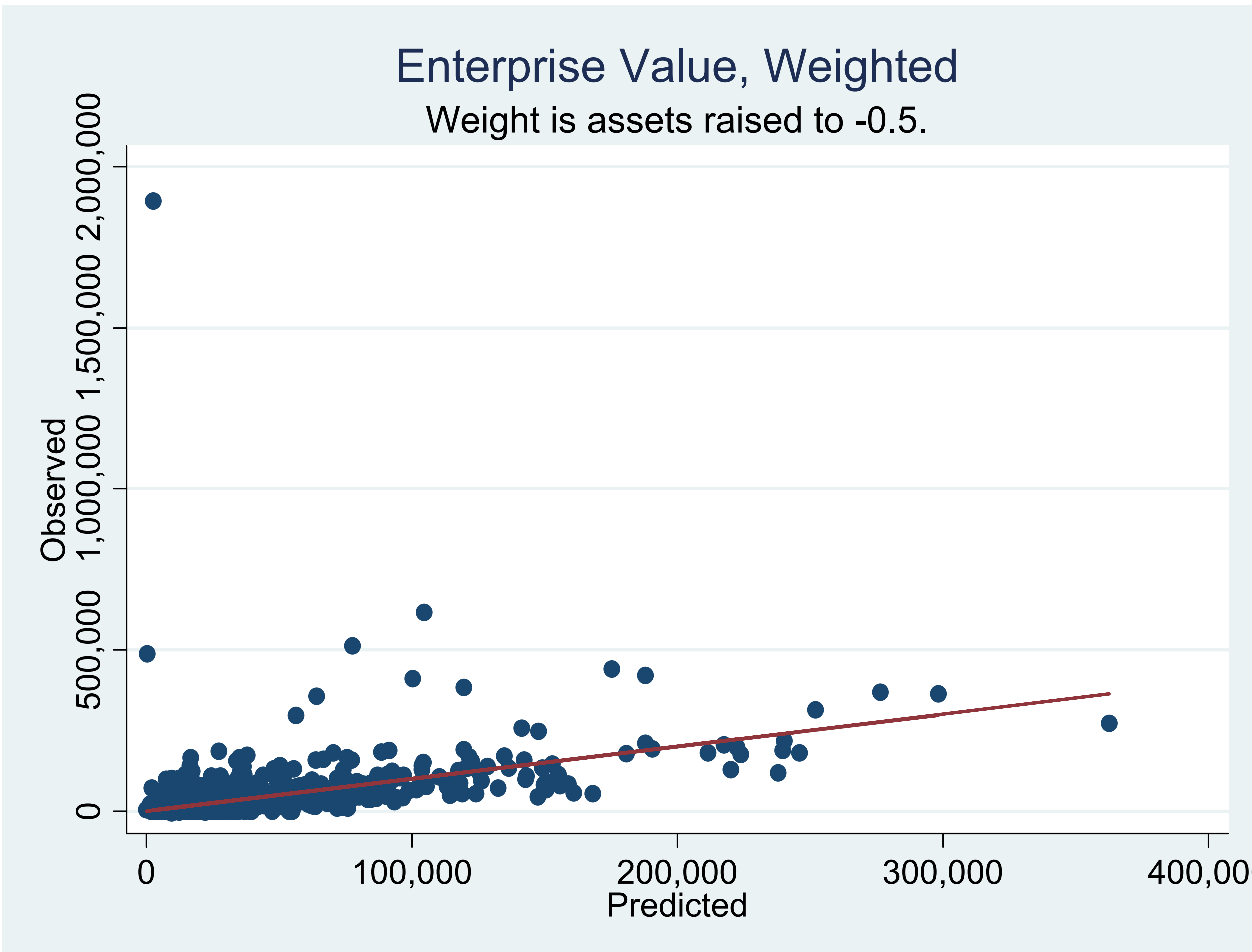

Table 1
Descriptive Statistics - Unweighted

Notes: all variables in 1000s.

Profit/Loss Before Tax Sample	Mean	Median	Standard Deviation	No. Obs.
Profit/Loss Before Tax	50,568	3,887	532,941	11,103
Sales	769,265	116,191	4,996,061	11,103
Property, Plant and Equipment	302,307	23,579	2,508,819	11,103
Labor Compensation	128,030	21,610	776,490	11,103
Employment	2.493	0.427	14.039	11,103

Enterprise Value Sample	Mean	Median	Standard Deviation	No. Obs.
Enterprise Value	2,745,505	160,889	12,200,000	1,473
Sales	2,685,770	252,717	10,500,000	1,473
Property, Plant and Equipment	1,052,864	57,255	4,828,504	1,473
Labor Compensation	470,387	58,523	1,843,462	1,473
Employment	9.024	1.108	33.344	1,473

Table 2
Determinants of Profits, 2004, WLS Results

Notes: The Table presents estimated coefficients from regressions explaining 2004 profits as functions of contemporaneous sales, capital, and labor inputs. All equations are estimated using weighted least squares, with weights equal to one divided by the square root of contemporaneous firm assets. Robust standard errors are presented in parentheses.

Dependent Variable:	Profit/Loss Before Tax		Enterprise Value	
	(1)		(2)	
Sales	0.6095	0.5908	0.2480	0.2061
	(0.1462)	(0.1099)	(0.0552)	(0.0927)
Property, Plant and Equipment	0.2560	0.2599	0.4754	0.4991
	(0.0832)	(0.0814)	(0.0368)	(0.0407)
Labor Compensation	0.0654		0.1532	
	(0.0655)		(0.0478)	
Employment		0.0784		0.1627
		(0.0721)		(0.0701)
No. of Obs.	11,103	11,103	1,473	1,473
R-Squared	0.2109	0.2135	0.3404	0.3629

Table 3
Determinants of Profits, Two-Factor, WLS Results

Notes: The Table presents estimated coefficients from regressions explaining 2004 profits as functions of contemporaneous sales and labor inputs. All equations are estimated using weighted least squares, with weights equal to one divided by the square root of contemporaneous firm assets. Robust standard errors are presented in parentheses.

Dependent Variable:	Profit/Loss Before Tax		Enterprise Value	
	(1)		(2)	
Sales	0.7158	0.7227	0.5987	0.5688
	(0.1331)	(0.0912)	(0.0702)	(0.0931)
Labor Compensation	0.1000		0.1200	
	(0.0657)		(0.0582)	
Employment		0.0836		0.1366
		(0.0716)		(0.0771)
No. of Obs.	11,103	11,103	1,473	1,473
R-Squared	0.1996	0.2017	0.2787	0.2953

Table 4
Income Prediction Accuracy among Formulas

Notes: The Table presents R^2 statistics from regressions explaining 2004 profits as functions of contemporaneous sales, capital, labor costs, and number of employees. All equations are estimated using weighted least squares, with weights equal to one divided by the square root of contemporaneous firm assets.

Dependent Variable:	Profit/Loss Before Tax				Enterprise Value			
	Obs.	R^2	% loss in RSS	% loss in RSS	Obs.	R^2	% loss in RSS	% loss in RSS
3-factor, unconstrained (wages)	11,103	0.2109	0.00%		1,473	0.3404	0.00%	
3-factor, unconstrained (employees)	11,103	0.2135		0.00%	1,473	0.3629		0.00%
3-factor, equal weights (wages)	11,103	0.1976	-6.29%		1,473	0.3247	-4.62%	
3-factor, equal weights (employees)	11,103	0.1866		-12.62%	1,473	0.3346		-7.80%
3-factor, double sales weights (wages)	11,103	0.2059	-2.35%		1,473	0.3187	-6.38%	
3-factor, double sales weights (employees)	11,103	0.2017		-5.52%	1,473	0.3352		-7.64%
2-factor (sales and PPE), unconstrained	11,103	0.2101	-0.35%	-1.59%	1,473	0.3344	-1.78%	-7.85%
2-factor (sales and PPE), constrained	11,103	0.2007	-4.81%	-5.99%	1,473	0.3340	-1.88%	-7.95%
2-factor (sales and wages), unconstrained	11,103	0.1996	-5.36%		1,473	0.2787	-18.14%	
2-factor (sales and employees), unconstrained	11,103	0.2017		-5.55%	1,473	0.2953		-18.64%
2-factor (sales and wages), constrained	11,103	0.1822	-13.58%		1,473	0.2648	-22.23%	
2-factor (sales and employees), constrained	11,103	0.1594		-25.36%	1,473	0.2625		-27.66%
2-factor (PPE and wages), unconstrained	11,103	0.1643	-22.11%		1,473	0.3320	-2.48%	
2-factor (PPE and employees), unconstrained	11,103	0.1564		-26.76%	1,473	0.3540		-2.44%
2-factor (PPE and wages), constrained	11,103	0.1641	-22.19%		1,473	0.3203	-5.92%	
2-factor (PPE and employees), constrained	11,103	0.1434		-32.87%	1,473	0.3101		-14.54%
1-factor (sales)	11,103	0.1978	-6.20%	-7.37%	1,473	0.2750	-19.23%	-24.23%
1-factor (PPE)	11,103	0.1196	-43.29%	-43.99%	1,473	0.2886	-15.22%	-20.46%
1-factor (wages)	11,103	0.1261	-40.23%		1,473	0.2109	-38.04%	
1-factor (employees)	11,103	0.0892		-58.24%	1,473	0.1820		-49.85%

Table 5: Prediction Errors in the Unconstrained - Three Factor Model

	Profit/Loss Before Tax		***Enterprise Value***	
Model	***1***	***2***	***1***	***2***
Number of Firms	11,103	11,103	1,473	1,473
mean[|π-hat(π)| / hat(π)]	1.09	1.08	2.43	1.00
median[|π-hat(π)| / hat(π)]	0.69	0.69	0.50	0.51
Percent of firms for which:				
|π-hat(π)| > 0.1 * hat(π)	93.11%	92.88%	90.77%	90.97%
|π-hat(π)| > 0.25 * hat(π)	82.28%	82.37%	75.56%	76.37%
|π-hat(π)| > 0.5 * hat(π)	63.69%	63.95%	49.63%	51.60%
|π-hat(π)| > 1 * hat(π)	30.42%	30.38%	14.80%	16.23%
|π-hat(π)| > 2 * hat(π)	11.02%	10.83%	8.35%	9.23%
|π-hat(π)| > 5 * hat(π)	2.04%	1.95%	2.58%	3.26%

Notes:

Model *1* is the following specification: dependent variable regressed on sales, net PPE, and labor costs. *Model 2* is identical to *Model 1* except that number of employees is substituted for labor costs.

A firm's actual profit level or enterprise value is denoted π, whereas the predicted value is denoted hat(π).

Table 6: Prediction Errors in the Constrained - Three Factor Model

	Profit/Loss Before Tax		***Enterprise Value***	
Model	***1***	***2***	***1***	***2***
Number of Firms	11,103	11,103	1,473	1,473
mean[|π-hat(π)| / hat(π)]	1.07	1.19	2.53	0.96
median[|π-hat(π)| / hat(π)]	0.66	0.73	0.53	0.55
Percent of firms for which:				
|π-hat(π)| > 0.1 * hat(π)	93.00%	93.46%	92.12%	92.12%
|π-hat(π)| > 0.25 * hat(π)	82.18%	82.69%	78.82%	78.62%
|π-hat(π)| > 0.5 * hat(π)	64.23%	65.45%	53.16%	53.84%
|π-hat(π)| > 1 * hat(π)	30.50%	33.31%	13.37%	16.09%
|π-hat(π)| > 2 * hat(π)	10.77%	13.10%	6.86%	8.28%
|π-hat(π)| > 5 * hat(π)	1.86%	2.65%	1.83%	2.65%

Notes:

Model 1 is a constrained estimation using the following independent variables: sales, net PPE, and labor costs, each constrained to have identical coefficients. Model 2 is identical to Model 1 except that number of employees is substituted for labor costs.

A firm's actual profit level or enterprise value is denoted π, whereas the predicted value is denoted hat(π).

Appendix Table 1
Descriptive Statistics - Weighted

Notes: The weight is one divided by the square root of contemporaneous firm assets.

Profit/Loss Before Tax Sample	Mean	Median	Standard Deviation	No. Obs.
Profit/Loss Before Tax	900	455	3,855	11,103
Sales	19,541	12,009	27,330	11,103
Property, Plant and Equipment	5,442	2,517	10,813	11,103
Labor Compensation	3,726	2,311	4,947	11,103
Employment	0.081	0.046	0.155	11,103

Enterprise Value Sample	Mean	Median	Standard Deviation	No. Obs.
Enterprise Value	28,830	11,005	70,097	1,473
Sales	31,781	16,403	44,394	1,473
Property, Plant and Equipment	9,806	3,694	18,506	1,473
Labor Compensation	6,689	3,897	8,968	1,473
Employment	0.142	0.073	0.290	1,473