NBER WORKING PAPER SERIES

EXTENSIVE AND INTENSIVE INVESTMENT OVER THE BUSINESS CYCLE

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Working Paper 14960
http://www.nber.org/papers/w14960

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
May 2009

We thank G. Alessandria, W. Brainard, R. Lucas, G. Violante and L. Wong for comments, and S. Bigio, S. Flynn, H. Tretvoll and V. Tsyrennikov for research assistance, and the Ewing Marion Kauffman Foundation for financial assistance. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

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May 2009, Revised October 2010
JEL No. E22,E32

ABSTRACT

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October 15, 2010

Abstract

Investment of U.S. firms responds asymmetrically to Tobin's Q: investment of established firms – 'intensive' investment – reacts negatively to Q whereas investment of new firms – 'extensive' investment – responds positively and elastically to Q. This asymmetry, we argue, reflects a difference between established and new firms in the cost of adopting new technologies. A fall in the compatibility of new capital with old capital raises measured Q and reduces the incentive of established firms to invest. New firms do not face such compatibility costs and step up their investment in response to the rise in Q. The model fits the data well using aggregates since 1900.

1 Introduction

Extensive and intensive labor supply has been modeled at the individual-worker level (Cogan 1981) and at the aggregate level (Cho and Cooley 1993, Cho and Rogerson 1998). Extensive labor supply – movement in and out of the labor market – is more wage elastic than intensive labor supply – the hours of employed workers.

An even starker contrast exists between the response to aggregate Tobin's Q of extensive investment – capital formation by entering firms and young firms – and the response by intensive investment – capital formation by older firms and listed firms. When aggregate Q rises, new firms and young firms raise their investment, whereas older, publicly listed firms reduce theirs.

To explain the puzzle we use a model in which movements in Q are caused by changes in the cost of capital that affect continuing projects but not new ones. Technological change raises the cost of making new capital compatible with capital already in place. Fitting new wiring or new equipment into an old building originally

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designed for something else is costly. Retraining workers who originally were trained to do something else is also costly. If the arrival of new technology precedes high-Q periods, then a high Q is the result of high costs of adjustment that reflect this incompatibility. Chari and Hopenhayn (1991) have a model in this general spirit, though it is steady-state growth with no variation in Q.

Gilchrist and Williams (2000) and Campbell (1998) estimate vintage-capital models closely related to ours. They too assume a distribution of new investment opportunities each period. Once implemented, the capital that the project embodies cannot be augmented at a later date.[1] Our model relaxes this assumption; it allows firms to invest in continuing projects and the shock to this intensive investment technology is of overriding importance. Investment in capital of the latest vintage (new projects) is estimated to be a small component of the total and is distinguished from investment in capital of older vintages (continuing projects). Furthermore, this intensive investment technology is, at any date, equally efficient regardless of the vintage of the project. Thus, while not a full-blown vintage-capital model, the model we end up with can be solved by hand.

We interpret the shock to the intensive-investment technology as a cost of making new and old capital compatible, or as a cost of adapting old processes to new technology. Yorukoglu (1998) interprets his investment-cost shock similarly. Our distinction between extensive investment (in new projects) and intensive investment (in continuing projects) is also similar to one made by Acemoglu, Gancia and Zilibotti (2010) between innovation and standardization costs. Similarly, Justiniano, Primiceri, and Tambelotti (forthcoming) distinguish shocks to the supply of capital goods from shocks to the investment adjustment cost for installed capital, and find that the latter play a large role in explaining fluctuations. Jovanovic (2009) relates the implementation cost to a depleted stock of investment options. Prusa and Schmitz (1991, 1994) find that firms pursue less radical forms of innovation as they age, presumably because they face such compatibility, adaptability and standardization costs.

The value of capital in place is determined solely by the cost of intensive investment and when that cost rises, we see a substitution towards extensive investment (which is not subject to this shock) and a rise in the stock market and in Q. This is the mechanism that explains the asymmetry. Over the cycle, the mechanism has a smoothing effect on investment because the new-project technology is not affected by the implementation shock that drives aggregate investment.

The model also includes a TFP shock, which is important for explaining other features of the time series but is not involved in generating the asymmetry. Thus as in Greenwood, Hercowicz, and Krusell (1997) and Fisher (2006), we have a TFP shock and a shock to the cost of capital, except that the latter applies only to continuing projects.

[1] Bilbiie et al. (2007) in their Sec. 5 briefly extend to allow variable plant size but do not study the issue of differential elasticity of extensive and intensive investment.

The model assumes two assets. The first is a claim to dividends of the portfolio of all continuing projects run by incumbent firms. The second is a private-equity fund that finances all new projects and sells them to incumbent firms or equivalently as IPOs in the form of new firms. Earlier versions of this paper assumed that new projects were developed by incumbents as spin-outs, as in Franco and Filson (2006), Chatterjee and Rossi-Hansberg (2008) and (implicitly) Prescott and Boyd (1987).[2]

The next section presents the evidence that motivates the paper, and Section 3 presents the model. Section 4 fits the model to the time series of investment – extensive and intensive – and Section 5 contrasts the model to the standard model with just one investment. Section 6 relaxes the assumption that continuing projects are homogeneous. Section 7 concludes.

2 Evidence on Investment and Q

The patterns that we outline above can be seen in both aggregate and sectoral data. Here we summarize that evidence.

Investment of Compustat firms is negatively related to aggregate Q.—Standard and Poor's *Compustat* database consists of public firms that we generally think of as incumbents. The investment rates of these listed firms depend positively on their own Q's but *negatively* on aggregate Q. Figure 1 illustrates this, where the left panel shows a mildly positive relation between firm-level investment and firm-specific Q's in a pooled ordinary least squares regression using data from 1962 through 2006.[3] This is consistent with the standard Q-theory of investment, which suggests that high-Q firms will have higher desired investment rates than low-Q firms. Interestingly, the right panel shows our new fact that investment rates of the same *Compustat* firms respond negatively to aggregate Q, with the observations on the horizontal axis ordered by the unweighted annual averages of the firm-specific Q's. Tables B.1 and B.2 in Appendix B show that the relationships in Figure 1 are robust to the inclusion of fixed effects for years and two-digit standard industry classifications (SICs), and to estimation by GMM to control for potential measurement error in Q.

The same contrast emerges in another set of regressions that use the same *Compustat* data displayed in Figure 1. Firm i's investment in year t as a percentage of its capital stock at the start of the year is denoted by $(X/K)_{i,t}$, aggregate Q is denoted by Q_t and firm i's deviation from it by $Q_{i,t} - Q_t$. Once again estimation is by OLS.

[2]The investment implications do not hinge on the assumed mechanism, but Gompers (2002) shows that IPOs of VC-funded new projects and private sales to firms of such projects outweigh IPOs and private sales of spinouts by a factor of eight or nine.

[3]See Appendix A for descriptions of the sources and methods used to construct all of the data used in our empirics.

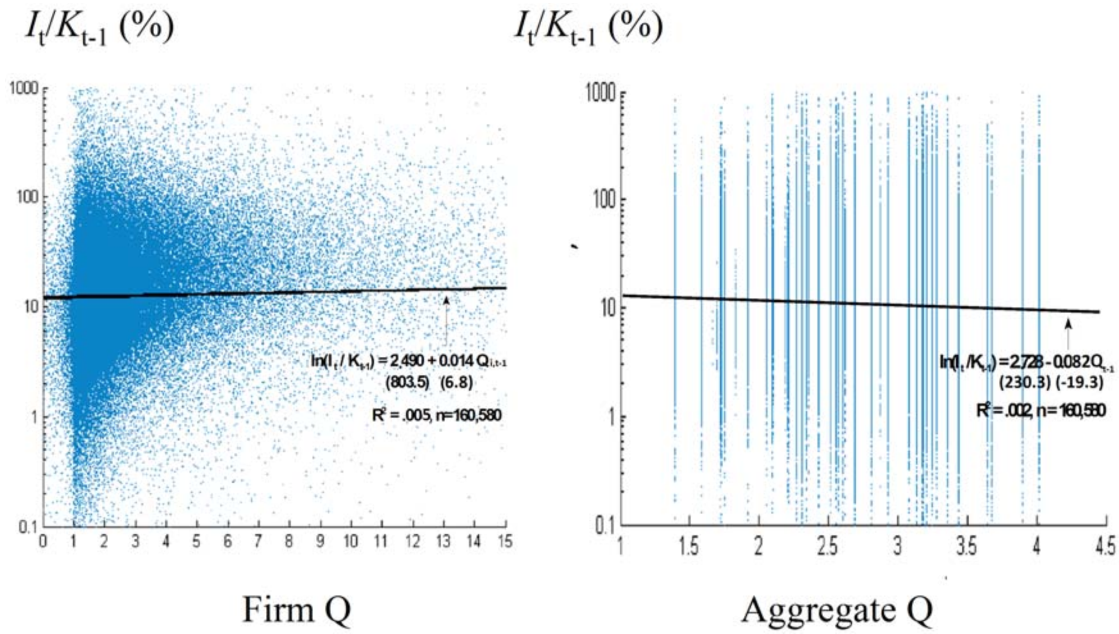

Figure 1: Firm-level investment rates and Q, Compustat sample, 1962-2006.

The estimates and t-statistics are

$$\ln\left(\frac{X}{K}\right)_{i,t} = \underset{(253.8)}{2.751} - \underset{(-20.7)}{0.082}Q_t + \underset{(6.7)}{0.015}\left(Q_{i,t} - Q_t\right),$$

with R^2=.01 and 160,580 firm-level observations. With fixed effects for two-digit SIC industries, the coefficient for Q_t becomes -0.133, suggesting an even stronger negative elasticity of investment to aggregate Q, and the coefficient for $(Q_{i,t} - Q_t)$ falls slightly to 0.012. Both coefficients remain highly significant statistically and the R^2 rises to 0.85. Table 1 shows that this negative aggregate Q-elasticity of investment also holds for firms in five broadly-defined sectors, with the effects of Q_t on investment strongest for business and industrial firms and for biotech and healthcare firms.

Investment by young firms is positively related to aggregate Q.—Although all *Compustat* firms are what we would call incumbents, the firms that listed recently are most likely to exhibit investment behavior similar to that of new firms. To us, this means that the investment rates of recent listings could well be positively related to Q_t even though investment rates of the entire pool of incumbents are negatively related to Q_t. Table 2 shows that this is indeed the case, with columns 1-2 and 5-6 indicating a strong and positive elasticity of investment to aggregate Q for firms that had been listed for two years or less and three years or less, respectively. And as we would expect, the size of the positive coefficient on Q_t is larger for the former (i.e., narrower) definition of "young" firms. At the same time, and again as expected, the complement of firms listed less recently in columns 3-4 and 7-8 have an even more

TABLE 1.—INVESTMENT REGRESSIONS FOR FIVE BROADLY-DEFINED SECTORS, 1962-2006

Dependent variable: log of investment as a percentage of capital stock at start of year

	Business and industrial		Biotech and healthcare		Energy		Communications and electronics		Computers and internet	
$Q_{j, t-1}$	-0.135 (13.91)	-0.149 (16.24)	-0.124 (14.88)	-0.121 (14.58)	-0.032 (2.56)	-0.066 (5.26)	-0.022 (3.71)	-0.040 (6.78)	-0.003 (0.050)	-0.008 (1.24)
$Q_{i, t-1} - Q_{j, t-1}$	0.022 (3.35)	0.020 (3.25)	0.022 (7.27)	0.020 (7.82)	0.027 (2.20)	0.022 (2.11)	0.013 (3.85)	0.011 (3.94)	0.007 (3.17)	0.007 (3.05)
2-digit SIC industries (number of effects)	no	yes (13)	no	yes (4)	No	yes (9)	no	yes (7)	no	yes (5)
R^2	0.015	0.852	0.013	0.834	0.010	0.846	0.006	0.849	0.003	0.848
No. of observations	30,232		17,564		24,327		34,288		22,498	

Notes: Estimation is by OLS with T-statistics based on robust standard errors in parentheses. The independent variable $Q_{j, t-1}$ is the mean Tobin's Q of all *Compustat* firms in broadly-defined sector *j* at the end of the previous year. The independent variable $Q_{i, t-1} - Q_{j, t-1}$ is the difference between the individual firm's Q_i and its broad sectoral average, Q_j. The broadly-defined sectors are the same as those used in Gompers *et al*. (2008). They aggregate the 69 sectors defined by Thomson Venture Economics into nine broader sectors, including the five that we consider here. We then assign *Compustat* firms to these broad sectors using a mapping from three-digit SIC industries provided by Anna Kovner. The mapping is not exclusive, as a given three-digit SIC code can be associated with more than one broadly-defined sector. Thus, some three-digit SIC codes and their associated firms appear in more than one of the sectoral regressions above. Table B.3 in Appendix B provides details of the mapping. Our regressions with industry fixed effects control for variation across the narrower SIC sectors within each broad sector at the two-digit level.

TABLE 2.—INVESTMENT REGRESSIONS FOR YEARS SINCE COMPUSTAT LISTING, 1962-2006

Dependent variable: log of investment as percentage of capital stock at start of year

Years since listing	≤ 2 years		> 2 years		≤ 3 years		> 3 years	
Q_{t-1}	0.163 (15.56)	0.267 (20.23)	-0.116 (28.20)	-0.041 (7.74)	0.128 (14.80)	0.231 (21.25)	-0.128 (30.15)	-0.056 (10.20)
$Q_{i,t-1} - Q_{t-1}$	0.010 (3.31)	0.009 (3.11)	0.013 (5.36)	0.011 (4.90)	0.008 (3.97)	0.007 (3.61)	0.016 (4.73)	0.013 (4.30)
Fixed effects for 2-digit SIC industries	no	yes	No	yes	no	yes	no	yes
R^2	0.010	0.834	0.009	0.843	0.007	0.847	0.012	0.846
No. of observations	31,132		129,448		44,162		116,418	

Notes:

1. Estimation is by OLS with T-statistics based on robust standard errors in parentheses.
2. The independent variable $Q_{, t-1}$ is the mean Tobin's Q of all *Compustat* firms at the end of the previous year.
3. The independent variable $Q_{i, t-1} - Q_{t-1}$ is the difference between the individual firm's Q_i and the aggregate average Q.

deeply negative aggregate Q-elasticity than the entire set of *Compustat* firms.

Venture capital flows into young firms.—A systematic source of micro evidence on entering firms is Thomson's *VentureXpert* sample of venture-backed firms. Venture capitalists (VCs) invest almost exclusively in start-up firms. One cannot compute a firm-specific Q for these firms, but one can use the Q that prevails for listed firms in particular sectors. Gompers *et al.* (2008, Table 4-5, pp. 11-12) show that VC investments have responded elastically to Q for their sector using annual data from 1975 to 1998. In particular, OLS regressions of the annual number of investments in each of 69 *VentureXpert* sectors on one-year lags of their average sectoral Q's (computed as market-to-book ratios from *Compustat*) deliver a coefficient on Q of 0.330 in a specification without year or industry fixed effects and a coefficient of 0.172 with year and industry effects included. They also obtain a coefficient on Q of 0.043 when regressing firm-level investment on sectoral Q's with fixed effects for industries and years. Thus they find a strong positive response by VC-backed firms to sector-specific Qs in same sectors for which our Table 1 documents a significant negative response by the *Compustat* firms.

In sum, aggregate data as well as micro evidence from *Compustat* and *VenturXpert* show a clear asymmetry in the response of investment to changes in aggregate Q. Entering and young firms respond positively, while older firms respond negatively to changes in aggregate Q. We now present a model that explains the phenomenon.

3 Model

The output of the final good depends on capital, k, and on a technology shock z:

$$\text{output} = zk.$$

The law of motion for capital is

$$k' = (1 - \delta) k + X + Y, \tag{1}$$

where X represents capital produced in continuing projects, and Y is capital produced in new projects.

(i) *Continuing projects.*—Continuing projects can be enlarged at the gross rate of return of $1/q$, which is the same over all continuing projects. That is, an investment today of q units of the consumption good in an existing project yields one unit of new capital. Then if the capital created via existing projects is X, the total cost is qX.

(ii) *New projects.*—A project uses as inputs a unit of the consumption good and an idea. As output it delivers ε units of capital in the following period. The quality, ε, of the project is known at the start. New projects are born each period, and their quality is distributed with a C.D.F. $G(\varepsilon)$ with density $g(\varepsilon)$. Ideas arrive in proportion to the size of the capital stock. Thus the unnormalized distribution of new ideas is $\lambda k G(\varepsilon)$. Ideas cannot be stored. Idea quality is evaluated privately by venture funds – an agent does not know the ε of his own idea.

We pause to note two differences in how investment in new and old projects is treated.

Contracting between agents and venture funds.—All ideas are submitted to venture funds for evaluation. A fund pays the idea's owner only if it uses the idea to float an IPO or sells the capital privately to an incumbent firm, either way receiving the price of q per unit. There are no long-term contracts, and contracts cannot condition on ε.[4] Up front payments by the venture fund cannot be made because anyone could pretend to have an idea and collect the payment. Ex post, payment cannot be made conditional on ε because the fund would always claim that the idea was of the lowest quality acceptable. Therefore the fund pays the minimum that it takes to get the idea owner to develop the project, and that payment is unity. Since the fund knows it can get $q\varepsilon$ for the idea, it will implement all projects with

$$\varepsilon \geq \frac{1}{q} \equiv \varepsilon_{\mathrm{m}}, \tag{2}$$

so that ε_{m} is the quality of the marginal project. The fund sells the projects to the public at the price of q per unit (this is explained further below) and collects revenue $qY = qyk$, where

$$y(q) = \lambda \int_{1/q}^{\infty} \varepsilon dG(\varepsilon). \tag{3}$$

Private-equity dividends.—The profits of the fund are $r(q)k$, where

$$r(q) = \lambda q \int_{1/q}^{\infty} \left(\varepsilon - \frac{1}{q}\right) dG(\varepsilon), \tag{4}$$

and profits are paid out as dividends to the households that own the funds.

Income identity.—Let $c = C/k$, and $x = X/k$. Output is divided between consumption and the two forms of investment so that the income identity reads

$$c = z - qx - h(y), \tag{5}$$

where, if (2) holds,

$$h(y) \equiv \lambda \left[1 - G\left(\frac{1}{q}\right)\right] \tag{6}$$

is the cost of the new projects and qx is the cost of the continuing projects, with each

[4] This is a bit unrealistic because a firm's founder at IPO often does have a considerable stake in the firm. Nevertheless, the VC does have the majority share and the controlling interest.

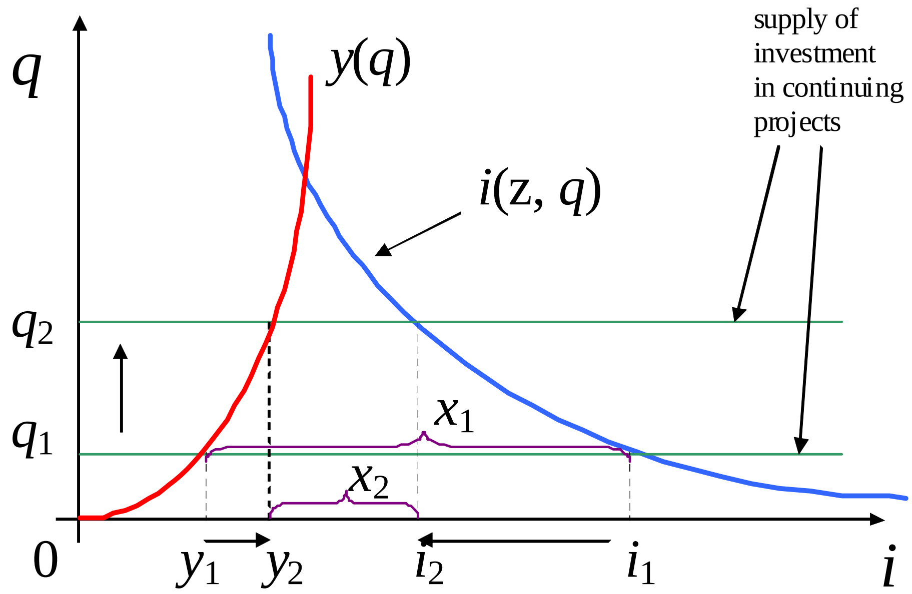

Figure 2: The determination of investment.

term in (5) measured per unit of k. Moreover, (2) and (6) imply that the marginal costs of capital entailed in the two investment margins are equal:[5]

$$h'(y) = q. \tag{7}$$

The determination of investment.—Figure 2 illustrates the effect of a rise in q when z is held constant. Investment, i, equals savings which are determined by households' savings decisions (to be described presently). The households' demand for investment is negatively sloped because a rise in q raises the cost of future consumption and reduces the demand for it (see (11)). The investment rate (i.e., supply) of entering capital is determined by (7) and increasing in q; this is the positively-sloped function in Figure (2). Incumbent investment takes up the slack between the demand for i and the supply of y. As q rises, two things happen: first, savings decline so that i falls from i_1 to i_2; second, the supply of entrants rises from y_1 to y_2. Therefore x falls by more than i.[6] As z rises, on the other hand, the downward-sloping savings curve in Figure 2 shifts to the right (see (11). This causes both x and y to rise.

[5]This is because $h'(y) = -\lambda g(\varepsilon_{\mathrm{m}}) \, d\varepsilon_{\mathrm{m}}/dy$ and $d\varepsilon_{\mathrm{m}}/dy = 1/\lambda\varepsilon_{\mathrm{m}} g(\varepsilon_{\mathrm{m}})$.

[6]These implications remain valid under a more general specification: rewrite (5) as $c = z - qx - h(y)$. Thus q raises the costs of incumbents but not that of entrants. Suppose, instead, that the resource constraint were $c = z - qx - q^{\psi} h(y)$ with $\psi \neq 0$. Then instead of (7) the FOC would be $h'(y) = q^{1-\psi}$ and the conclusions illustrated in Figure 2 would remain qualitatively intact so long as $\psi < 1$. That is, y would be increasing in q and x would be decreasing in q. If, on the other hand, $\psi = 1$, y would

Preferences.—Preferences are $E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t) \right\}$, with

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma}.$$

Let $s \equiv (z, q)$ be the state, assumed i.i.d. with distribution $F(s)$. Contemporaneously, z and q may be correlated. The i.i.d. assumption lets us solve explicitly and discuss results intuitively, but simulations reported in earlier versions of the paper (e.g., Jovanovic and Rousseau 2009) show that the results are largely unchanged if s follows a first-order Markov process.

The state of the economy is (s, k), but since returns are constant and preferences homothetic, k affects neither interest rates nor investment rates.

The equilibrium price of capital.—We shall assume that, as in Figure 2, the size of entering ideas is too small to affect today's market price q of the marginal unit of capital. A price higher than q would draw forth an investment level x that is larger than the supply of savings on the part of households. Moreover, since the quality, ε, of each new project is public knowledge ex ante, the shares of such an entering firm will sell at a price of εq where established firms bid for the capital to the point at which its value equals its cost.

Assets.—The household may own two assets:

(i) *Private equity.*—The price is $p(s)k$ per share in the private-equity fund, and a share pays dividend $r(q)k$. The fund manages only new projects and sells them right away, and benefits through an external effect from the growth of k.

(ii) *Public equity.*—The number of firms is fixed and the size of the representative firm is proportional to k. The purchase of new capital (x created by the consumption-good-conversion technology at the cost of q per unit and y by a purchase from the private-equity funds at the price of q per unit) is financed by a withdrawal of $q(x+y)$ from the firm's earnings. This reduces dividends but leads to exactly offsetting capital gains because the new capital is valued at q per unit as long as $x(s) > 0$ in equilibrium for all s.[7] Thus the dividends of the public-equity fund are zk regardless of its investment activity and the price per share is qk.

The household's budget constraint.—After dividing both sides by k, the constraint is

$$pn' + qN' + c = (r+p)\,n + (z+q)\,N,$$

where n is the number of units of capital of the representative firm that the household owns and $c = C/k$. In equilibrium $n = N = 1$. As in Lucas (1978), we substitute

then be a constant, depending neither on q nor on z, and would solve the equation $h'(y) = 1$. But x would still be decreasing in q.

[7]This ensures that the economy is never at a zero-x corner where q could be less than the reproduction cost as was the case in Sargent (1980).

these conditions into the budget constraint. The resulting public-equity price per unit of k is

$$q = \beta \int \left(\frac{C'}{C}\right)^{-\gamma} (z' + q')\, dF(s'), \tag{8}$$

and the private-equity price is

$$p(s) = (1 - \delta + i(s))\, \beta \int \left(\frac{C'}{C}\right)^{-\gamma} (r(q') + p(s'))\, dF(s'). \tag{9}$$

The multiplicative factor $(1 - \delta + i) = k'/k$ in (9) corrects for the growth of the capital stock to which prices and dividends are proportional. It is absent from (8) because the capital gains it implies are offset exactly by the cost of investment.

Analysis.—Since utility is homothetic and returns are constant, the solution is of the form $C = c(s)\, k$. Moreover, we derive the following solutions in Appendix C:

Proposition 1 ***(homogeneous x)****. Constants $D > 0$ and $\chi > 0$ are defined in (19) and (20), such that in state (z, q)*

$$c = \frac{z + r + (1 - \delta)\, q}{1 + q^{1-1/\gamma} D}, \tag{10}$$

$$i = \frac{(z + r)\, q^{-1/\gamma} D - (1 - \delta)}{1 + q^{1-1/\gamma} D}, \tag{11}$$

and

$$p = (1 - \delta + i)\, \chi q. \tag{12}$$

Recall that the magnitudes given in (10)-(12) are all relative to k. At this point we note the following:

1. Using (11) and (3), we can solve for $x = i - y$.
2. Income is consumed or invested, i.e., $c + qi = z + r$, and the marginal propensity to consume out of income is $\left(1 + q^{1-1/\gamma} D\right)^{-1}$.
3. The marginal propensity to invest out of income is $q^{1-1/\gamma} D^{-1} \left(1 + q^{1-1/\gamma} D\right)^{-1} > 0$.
4. Wealth is $q + p = [1 + (1 - \delta + i[z, q])\, \chi]\, q$, so that it depends positively on q and z, the latter effect working through the rise in capital accumulation that an increase in z causes.
5. Formulas (10)-(12) are valid iff in every state $c + i \leq z + r \iff c + h(y) < z$.

4 Fitting the model to data

We assume the Pareto distribution

$$G(\varepsilon) = 1 - \left(\frac{\varepsilon}{\varepsilon_0}\right)^{-\rho} \tag{13}$$

for $\varepsilon \geq \varepsilon_0 > 0$ and $\rho > 1$. Then for $q < \varepsilon_0^{-1}$, (3) and (4) become

$$y(q) = \frac{\rho \lambda \varepsilon_0^{\rho}}{\rho - 1} q^{\rho - 1} \quad \text{and} \quad r(q) = \frac{\lambda \varepsilon_0^{\rho}}{\rho - 1} q^{\rho}. \tag{14}$$

The series k.—In our model, q is driven by hidden implementation costs. When we come to the data, this will mean that q represents a shock *relative* to the price of capital that the BLS measures and uses to construct its estimated stock of capital. Therefore measured Q equals the model's q. When compatibility problems cause reproduction costs of incumbents to rise, this will not enter their book values (i.e., the denominator of measured Q), and this generates a measurement error that raises measured Q. The model normalizes the measured cost of capital to unity, which means that we implicitly assume that the price index for capital goods is correctly used by the BLS when constructing the capital stock numbers that we shall use here.

The series X.—We compute it as $I-$ IPO$/Q = X$.

The series Y.—IPO values relative to the capital stock are interpreted as qy. This is the value of the composite capital stock brought into the stock market. Division by q yields y.

Calculating z.—Since output is zk, we measure z by the ratio of private output over the course of a given year to private capital at the start of that year. That is, $z_t = GDP_t/k_t$ for all t.

Calculating q.—For q, we use the ratio of market-to-book values of tangible capital in the entire non-financial corporate sector.[8]

Of the six parameters, one is not identified – our data identify only the product $\lambda\varepsilon_0^{\rho}$, and not λ and ε_0 separately, and so we chose $\varepsilon_0 = 1/\max_t q_t$ so that the economy always has a marginal new project, even at the highest sample value of q. The discount factor β was fixed at 0.95. Then the algorithm first chooses (λ, ρ) to fit the y_t series. In view of (14), this does not involve other parameters. With (λ, ρ) thus obtained, the algorithm then chooses (γ, δ) to fit the long-run growth of output (given by $\bar{x} + \bar{y} - \delta$) per head of 1.5 percent, and to explain as much as possible of the variation in x and y.[9] Table 3 reports the parameter values.

[8] See Appendix A for descriptions of the data used in the simulation.

[9] Simulation requires that we solve for the constant D, which can only be done numerically. To obtain estimates of $D = 3.26$ and $\chi = 0.0034$, we fitted a bivariate log-normal distribution of (z, q) to the data and then used this distribution in (19) and (20) to solve for D and χ.

Table 3. Parameter Values Used in Simulation

parameter	value	comment
β	0.95	commonly used discount factor
γ	78.5	$\arg\min_{\gamma,\delta}\left\{(\bar{x}+\bar{y}-\delta-0.015)^2+\sum_t (x_t-\frac{X_t}{K_t})^2\right\}$
δ	0.3	
λ	0.025	$\arg\min_{\lambda,\rho}\sum_t (\frac{\mathrm{IPO}_t}{K_t}-q_t y_t)^2$
ρ	2.1	
ε_0	0.3	equated to $1/\max_t(q_t)$

The resulting fit of the model is in Figure 3, and the time series for (z,q) is given in Figure 4.[10]

The early years of the Great Depression feature an extremely low z and the constraint $i\geq 0$ is violated during the period 1931-34. This period includes extreme events under which (8) holds as an inequality, as in Sargent (1980). But under the estimated $F(s)$, the violation occurs very rarely, namely one-third of a percent of the time. If we did impose non-negativity of i in all dates we believe that the optimal solution (which we then can no longer solve explicitly) would remain very close to that given by (10) and (11). In its desire to avoid excessive variation in i, the curvature parameter γ is estimated to be very large. As $\gamma\to 0$ it becomes impossible for the condition to hold in the face of even moderate variation in q.

5 Comparison to the one-investment model

In a one-investment model, the income identity would read

$$c+qi=z.$$

In our model it reads

$$c+qi=z+r,$$

where r is given in (4). The parameters determining r are λ and $G(\cdot)$. A neutral rise in the flow of ideas is simply a rise in λ. It raises r in (4) and this is a positive wealth effect on current and future consumption. From (19) it is easy to see that D is decreasing in λ which means in (10) that

$$\frac{\partial c(z,q)}{\partial\lambda}>0 \qquad \text{all } (z,q). \tag{15}$$

The effect on i cannot be signed.

[10]The correlations are $(z_t,q_t)=0.243$, $(z_t,z_{t-1})=0.762$ and $(q_t,q_{t-1})=.861$.

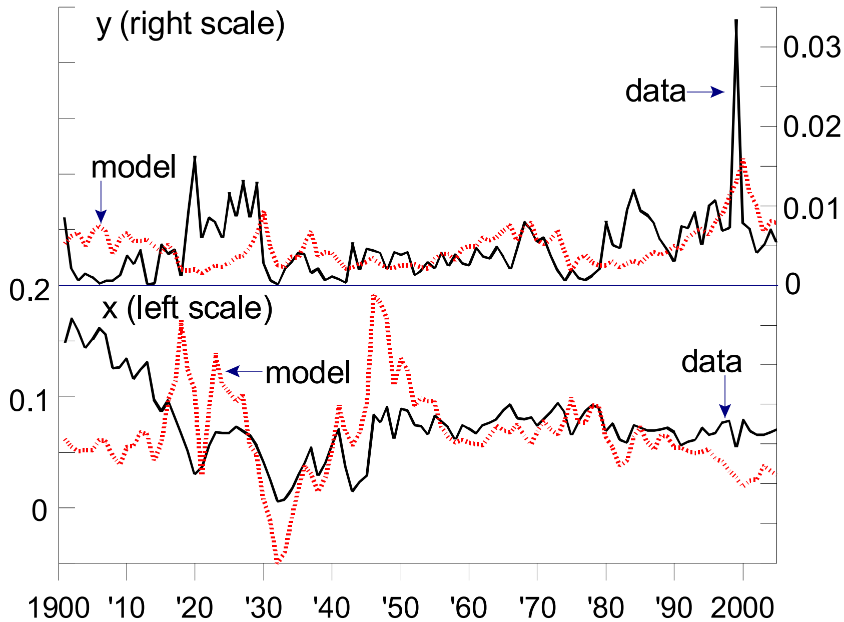

Figure 3: The y and x series and their simulated values, 1901-2005.

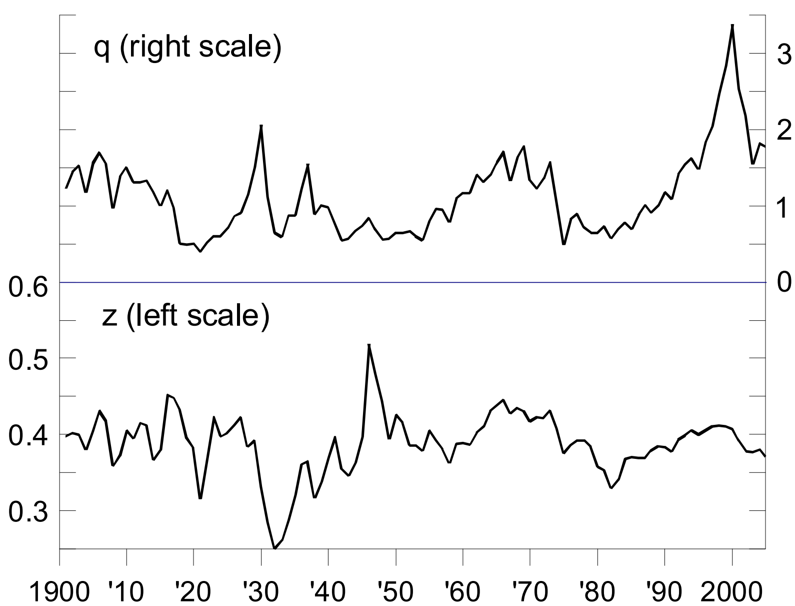

Figure 4: The two shocks, 1901-2005

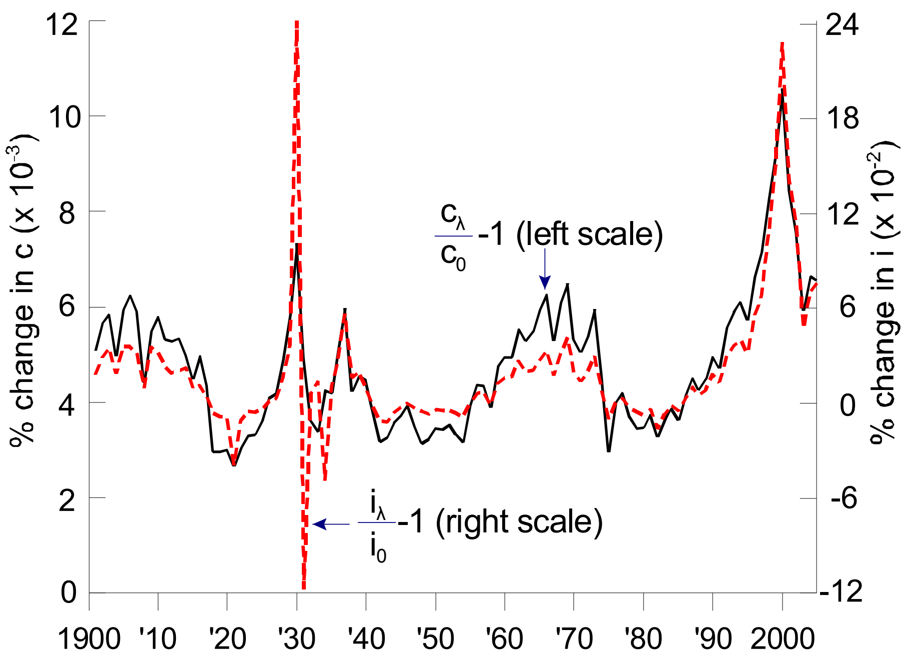

Figure 5: The model economy with λ=0.28 and with λ=0, 1901-2005.

We now simulate c and i for the economy with the parameters as in Table 3, followed by a simulation of c and i for an economy with all its parameters unchanged except that now $\lambda = 0$. Figure 5 reports the percentage increase in c and i that the increase in λ from zero to 0.028 brings about. As (15) implies, consumption rises uniformly, on average by four fifths of one percent. The percentage effect on i is on average larger – about three percent – but it is non-monotonic, being negative in periods when q is low.

In short, the presence of y has a larger cyclical consequence on i than it has on c – the latter rises in all states (z, q). As for i, the presence of y plays a smoothing role. I.e., the new-project margin induces a smaller decline in aggregate investment when q rises, which is not surprising since the new-project sector is immune to the adverse implementation-cost shock. The net result is a smoother investment series than we would see if the y investment were not present.

6 Heterogeneity in continuing-project investments

We now show the results are robust to adding heterogeneity in x. So far we assumed that all continuing projects could augment their capital at a unit cost of q. Suppose, instead that continuing projects are heterogeneous. Suppose that each project delivers a unit of capital tomorrow, but that the cost, ξ, is project-specific, and suppose that

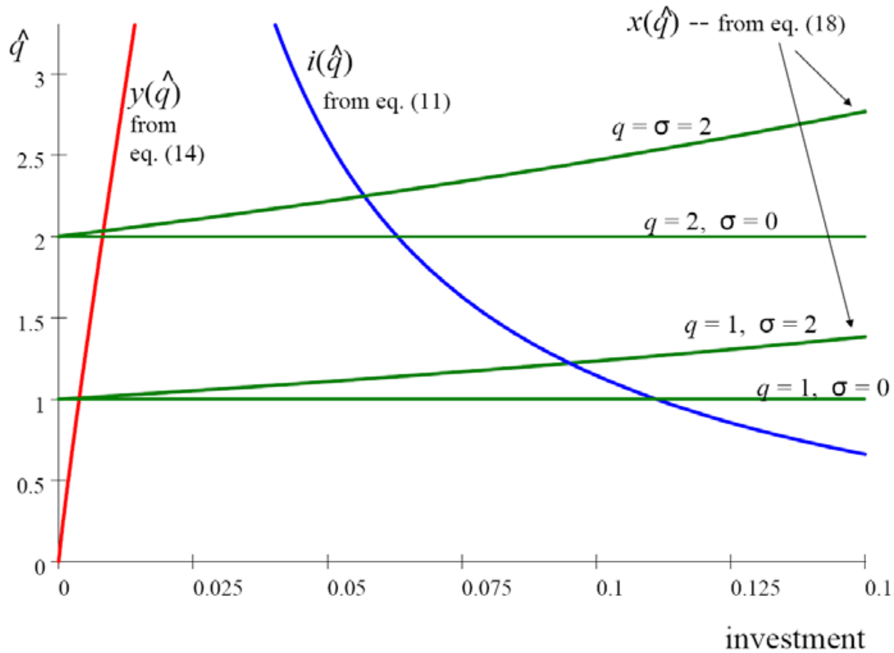

Figure 6: The effect of heterogeneous x projects.

the distribution of ξ is Pareto:

$$\#\text{ of available continuing projects cheaper than } \xi = k\left(1-\left(\frac{\xi}{q}\right)^{-1/\sigma}\right); \qquad \xi \geq q. \tag{16}$$

The parameter σ is an index of heterogeneity of continuing projects. As $\sigma \to 0$ we get the homogeneous case in which on continuing-projects, each unit of capital costs q. But the equilibrium price of shares, call it $\hat{q}$, now depends on q and z and we do not have an analytic solution for it, but the Appendix proves the following result:

Proposition 2 ***(heterogeneous x)****. When (16) holds, the income identity reads*

$$z = c + h(y) + q\left(\frac{1-(1-x)^{1-\sigma}}{1-\sigma}\right) \tag{17}$$

where the function $y(\cdot)$ is defined in (3), and the investment function is

$$x = 1 - \left(\frac{\hat{q}}{q}\right)^{-1/\sigma}. \tag{18}$$

As $\sigma \to 0$, $q\left(\frac{1-(1-x)^{1-\sigma}}{1-\sigma}\right) \to qx$ so that (17) becomes the same as (5). And the price of shares, $\hat{q}$, converges to q for all (z, q).

Figure 6 shows a simulated version of Figure 3, and then add two supply curves for x covering the case $\sigma > 0$, each pertaining to a hypothetical realization of the shock q. When $\sigma > 0$ one must distinguish $\hat{q}$ from q, and so the vertical axis now measures $\hat{q}$, and the curves $y(\hat{q})$ and $i(\hat{q})$ both are represented as functions of this variable, although the functional forms are taken from (10) and (11) so that they are correct only for the case $\sigma = 0$. The curve $i(\hat{q})$ plots (11) evaluated at the sample mean value $z = 0.36$. The supply of new projects $y(\hat{q})$. We draw the supply curve for x for $q \in \{1, 2\}$ and for $\sigma \in \{0, 2\}$. We also plot the supply of new projects $y(q)$ from (14) at the values of ε_0, ρ, and λ given in Table 1.

We must distinguish the price of capital, $\hat{q}$, from the shock q. Note that at $\sigma = 2$, the variance in the distribution of the costs of continuing projects is infinite, whereas the variance of new-project quality is $\frac{\rho}{\rho-2}\left(\frac{\varepsilon_0}{\rho-1}\right)^2 = 1.56$. Thus the supply of continuing projects is more elastic not because their entire population is more homogenous but, rather, because there are far fewer new projects (measuring in total $\lambda = 0.025$) than continuing projects (measuring unity in total), so that among the *implemented* new projects the variance is larger.

7 Conclusion

We have documented an asymmetry in the response of investment to aggregate Q: Small and young firms respond positively to it, while large and old firms respond negatively. We argue that this occurs because a high Q is a signal of low compatibility of old capital with the new and, hence, of high implementation costs specific to incumbents alone. Entrants do not face compatibility problems because they start *de novo*, and raise their investment when the cost is high.

Introducing the second, new-project margin into a one-investment model dampens the time-series variation in investment, raising investment in states when the implementation cost is high and lowering it in states when the cost is low. There seems to be little or no effect on consumption variation.

Finally, variations in the implementation cost are driven by technological change. On this view, aggregate stock-price volatility is caused by technological change, a property that our model shares with other models that feature shocks to the cost of investment.

8 Appendix A. Data and Methods

In this appendix we document the data sources and methods used to construct the series depicted in our figures and included in the empirical analysis.

Figure 1.—The investment rate of an individual firm (i.e., $((X/K)_{i,t})$ is its annual expenditures on property, plant, and equipment (*Compustat* item 30) as a percentage

of its total assets at the start of the year (item 6). We compute the firm-specific $Q_{i,t}$ using year-end data from *Compustat*. The numerator of Q is the value of a firm's common equity at current share prices (the product of *Compustat* items 24 and 25), to which we add the book values of preferred stock (item 130) and of long and short-term debt (items 9 and 34). We use book values of preferred stock and debt in the numerator because prices of preferred stocks are not available from *Compustat* and we do not have information on issue dates for debt from which we might better estimate market value. Book values of these components are reasonable approximations of market values so long as interest rates do not vary excessively. The denominator of Q is computed in the same way except that book value of common equity (*Compustat* item 60) is used in place of market value. Our micro-based Q measures thus focus on the value of a firm's outstanding securities and implicitly assume that the proceeds of these issues are fully applied to the formation of capital, both physical and intangible.

Figure 3.—To construct the y_t series shown in the upper panel of Figure 3, we begin with the value of IPOs as a proxy for the numerator, Y_t. This is measured as the aggregate year-end market value of the common stock of all firms that enter the University of Chicago's Center for Research in Securities Prices (CRSP) files in each year from 1925 through 2005, excluding American Depository Receipts. The CRSP files include listings only from the New York Stock Exchange (NYSE) from 1925 until 1961, with American Stock Exchange and NASDAQ firms joining in 1962 and 1972 respectively. This generates large entry rates in 1962 and 1972 that for the most part do not reflect initial public offerings. Because of this, we linearly interpolate between entry rates in 1961 and 1963 and between 1971 and 1973, and assign these values to the years 1962 and 1972 respectively. For 1901-1924 we obtain market values of firms that list for the first time on the NYSE using our pre-CRSP database of stock prices, par values, and book capitalizations that we collected for all common stocks traded on the NYSE using the *The Commercial and Financial Chronicle, Bradstreet's*, *The New York Times*, and *The Annalist* (see Jovanovic and Rousseau, 2001). We measure the denominator, K_{t-1}, as the previous end-of-year stock of private fixed assets from the BEA (2006, Table 6.1, line 1) for 1925 through 2005. For 1900-1924, we use annual estimates from Goldsmith (1955, Vol. 3, Table W-1, col. 2, pp. 14-15) that include reproducible, tangible assets (i.e., structures, equipment, and inventories), and then subtract government structures (col. 3), public inventories (col. 17), and monetary gold and silver (col. 18). We then join the result with the BEA series and divide by aggregate q_t (described below).

To construct the x_t series shown in the lower panel of Figure 3, we measure the numerator, X_t, as annual gross private domestic investment from the Bureau of Economic Analysis (BEA 2006, Table 5.2.5, line 4) for 1929-2005, to which we join estimates from Kendrick (1961, Table A-IIb, column 5, pp. 296-7) for 1901-1928. We then divide the result by K_{t-1}.

Figure 4.—For z_t, we use private output, defined as GDP less government expenditures on consumption and investment from the BEA (2006) for 1929-2005, to which

we join Kendrick's (1961, Table A-IIb, pp. 296-7, col. 11) estimates of gross national product less government for 1901-2005. We then divide the result by K_{t-1}.

For aggregate q_t, we use fourth quarter observations underlying Hall (2001) for 1950-99, and then join them with estimates underlying Abel and Eberly (2008) for 1999 to 2005. These authors derive aggregate Tobin's Q from the Federal Reserve Board's *Flow of Funds* Accounts as the ratio of market-to-book values for tangible assets in the entire non-financial corporate sector. We then bring the aggregate q_t series back to 1901 by ratio splicing the "equity Q" measure underlying Wright (2004). Hall's measure of Q exceeds Wright's by factor of more than 1.5 in 1950, when the splice occurs, producing q_t's before 1950 that are considerably higher than Wright's published estimates. We also note that these measures of aggregate q_t are generally smaller than the ones that we constructed for 1962 through 2006 as averages of firm-level data for use in our regression analysis. The difference arises for two reasons. First, since *Compustat* and our backward extension only cover firms that are listed on organized stock exchanges, the micro-based sample is focused on larger and more successful firms. Second, our micro-based measure of Q is based on market and book values of a firm's outstanding securities issues (see notes for Figure 1), the proceeds of which are spent on physical capital and intangibles. Since intangibles probably form an important part of the forward-looking component of stock prices, we might expect our micro-based measures of Q to exceed those based upon tangibles alone. At the same time, our micro and macro series are highly correlated, with $\rho = 0.92$ in their period of overlap (i.e., 1962-2005).

9 Appendix B. Robustness of Results in Figure 1

In this appendix we show the robustness of the regressions in Figure 1 to the inclusion of fixed effects for years and two-digit SIC industries, as well as to estimation with GMM using higher-order moments. Table B.1 includes several regressions that are similar to the one depicted in the left panel of Figure 1. We continue to use OLS in the left panel of Table B.1 so that the first column simply repeats the result shown in left panel of Figure 1. In the second column we add fixed effects for years to the specification, and we include fixed effects for two-digit SIC industries in addition to the year effects in the third column. All three regressions show a positive relation between a firm's own Q and its subsequent investment.

Erickson and Whited (2000, 2002) propose a method for estimating investment regressions that yields consistency in the presence of measurement error in Q. The technique uses higher-order moments of the $Q_{i,t-1}$ as instruments for $Q_{i,t-1}$. We estimated these models using the third, fourth, and fifth moments as instruments in the center panel of Table B.1 and using the third through sixth moments in the right panel. In all cases the coefficients on $Q_{i,t-1}$ are even larger than those obtained with OLS and remain positive and statistically significant at the one percent level.

Table B.1. Q-Regressions for Domestic Investment, 1962-2006

	Dependent variable: log of investment as percentage of capital at start of year								
	OLS			GMM5			GMM6		
$Q_{i,t-1}$	0.014 (6.80)	0.022 (7.33)	0.012 (6.42)	0.174 (4.58)	0.140 (4.67)	0.134 (4.32)	0.131 (10.9)	0.112 (10.2)	0.115 (8.85)
2-digit SIC effects	no	no	yes	no	no	yes	no	no	yes
year effects	no	yes	yes	no	yes	yes	no	yes	yes
R^2	.005	.827	.850	.065	.852	.856	.051	.850	.855

Note: T-statistics based on robust standard errors are in parentheses. There are 160,580 observations included in each regression.

Table B.2. Aggregate Q-Regressions for Domestic Investment, 1962-2006

	Dependent variable: log of investment as percentage of capital at start of year					
	OLS		GMM5		GMM6	
Q_{t-1}	-0.082 (20.5)	-0.135 (33.8)	-2.822 (20.2)	-2.330 (21.6)	-2.297 (21.9)	-1.618 (21.3)
2-digit SIC industry effects	no	yes	no	no	no	no
R^2	.002	.844	.027	.859	.031	.854

Note: T-statistics based on robust standard errors are in parentheses. There are 160,580 observations included in each regression.

Table B.2 presents a set of regressions that explore the robustness of the result in the right panel of Figure 1 to alternative specifications and estimation with higher-order moments. The negative relation between and investment and aggregate Q_{t-1} persists in all of these variations, with the estimated Q-elasticity once again stronger when estimated with GMM.

Table B.3 shows the mapping from three-digit SICs into the 69 sectors defined by Thomson, and then to the five broad sectors used for the regressions in Table 1.

Table B.3. SIC Composition of the Five Broadly Defined Sectors in Table 1

Broad Sector	Corresponding SIC 3-Digit Industries
1-Business and industrial	267-Converted paper and paperboard products, except containers and boxes
	273-Books
ThomsonVC Industries	275-Commercial printing
11-Chemicals and materials	282-Plastics materials and synthetic resins, synthetic rubber, cellulo
40-Industrial equipment	287-Agricultural chemicals
41-Industrial products (other)	289-Misc. chemical products
42-Industrial services	308-Misc. plastics products
50-Manufacturing	329-Abrasive, asbestos, and misc. non-metallic mineral products
	331-Steel works, blast furnaces (including coke ovens), and rolling mills
	335-Rolling, drawing, and extruding of nonferrous metals
	353-Construction, mining, and materials handling machinery and equipment
	354-Metal working machinery and equipment
	355-Special industrial machinery, except metalworking machinery
	357-Computer and office equipment
	358-Refrigeration and service industry machinery
	359-Misc. industrial and commercial machinery and equipment
	362-Electrical industrial apparatus
	367-Electronic components and accessories
	371-Motor vehicles and motor vehicle equipment
	382-Lab apparatus and analytical, optical, measuring and controlling devices
	441-Deep sea foreign transportation of freight
	495-Sanitary services
	873-Research, development, and testing services

Broad Sector	Corresponding SIC 3-Digit Industries
2-Biotech and healthcare	281-Industrial inorganic chemicals
	282-Plastics materials and synthetic resins, synthetic rubber, cellulo
ThomsonVC Industries	283-Drugs
03-Biosensors	287-Agricultural chemicals
04-Biotech equipment	382-Lab apparatus and analytical, optical, measuring and controlling devices
05-Biotech other	384-Surgical, medical, and dental instruments and supplies
06-Biotech research	801-Offices and clinics of doctors of medicine
07-Biotech animal	806-Hospitals
08-Biotech human	809-Misc. health and allied services
09-Biotech industrial	873-Research, development and testing services
51-Med/health products	
52-Med/health services	
53-Medical diagnostics	
54-Medical therapeutics	
58-Pharmaceutical	

Broad Sector	Corresponding SIC 3-Digit Industries
3-Energy	131-Crude petroleum and natural gas
	138-Oil and gas field services
ThomsonVC Industries	343-Heating equipment, except electric and warm air; plumbing fixtures
29-Energy, alternative	344-Fabricated structural metal products
30-Energy, coal	353-Construction, mining, and materials handling machinery and equipment
31-Energy, conservation	367-Electronic components and accessories

32-Energy, enhanced recovery	382-Lab apparatus and analytical, optical, measuring and controlling devices
33-Energy, other	491-Electrical services
55-Oil and gas exploration	492-Gas production and distribution
59-Pollution and recycling	495-Sanitary services
68-Utilities	753-Automotive repair shops
	871-Engineering, architectural, and surveying services
	873-Research, development, and testing services

Broad Sector	Corresponding SIC 3-Digit Industries
4-Communications and electronics	335-Rolling, drawing, and extruding of nonferrous metals
	355-Special industry machinery, except metalworking machinery
	357-Computer and office equipment
Thomson VC Industries	359-Misc. industrial and commercial machinery and equipment
02-Batteries	362-Electrical industrial apparatus
12-Communications, other	366-Communications equipment
24-Data communications	367-Electronic components and accessories
27-Electronics equipment	369-Misc. electrical machinery, equipment, and supplies
28-Electronics, other	381-Search, detection, navigation, guidance, aeronautical, and nautical
35-Facsimile transmission	382-Lab apparatus and analytical, optical, measuring, and controlling devices
36-Fiber optics	386-Photographic equipment and supplies
39-Industrial automation	481-Telephone communications
56-Optoelectronics	737-Computer programming, data processing, and other computer-related
60-Power supplies	871-Engineering, architectural, and surveying services
62-Satellite communication	
64-Semiconductors, other electronics	
65-Telephone related	
69-Wireless communications	

Broad Sector	Corresponding SIC 3-Digit Industries
5-Internet and computers	357-Computer and office equipment
	367-Electronic components and accessories
Thomson VC Industries	393-Musical instruments
14-Computer, other	481-Telephone communications
15-Computer peripherals	731-Advertising
16-Computer programming	737-Computer programming, data processing, and other computer related
17-Computer services	738-Misc. business services
18-Computer software	
19-Computer hardware	
25-Digital imaging and computer graphics	
26-E-commerce technology	
43-Internet communications	
44-Internet content	
45-Internet E-commerce	
46-Internet programming	
47-Internet services	
48-Internet software	
67-Turnkey integrated systems and solutions	

10 Appendix C: Proofs

Proof of Proposition 1: We define two constants for which no explicit solution exists but conditional on which we fully characterize consumption and the two investments in Proposition 1. The first, D, satisfies the equation

$$D = \left(\beta \int \left(\frac{1+q^{1-1/\gamma}D}{z+r+(1-\delta)q}\right)^{\gamma} [z+(1-\delta)q]\, dF(s)\right)^{1/\gamma}. \tag{19}$$

and the second is defined in terms of D where c is also known up to the parameter D as stated in (10)

$$\chi = \left(1 - \frac{\int (c')^{-\gamma} q' (1-\delta+i[s'])\, dF(s')}{\int^{-\gamma} c^{-\gamma} (z'+[1-\delta]q')\, dF(s')}\right)^{-1} \frac{\int (c')^{-\gamma} r' dF(s')}{\int c^{-\gamma} (z'+[1-\delta]q')\, dF(s')} \tag{20}$$

Since $\frac{C'}{C} = \frac{c'}{c}\frac{k'}{k}$, (8) implies

$$qc^{-\gamma}\left(\frac{k}{k'}\right)^{-\gamma} = \beta \int (c')^{-\gamma} [z'+(1-\delta)q']\, dF(s')$$

Therefore

$$c\frac{k}{k'} = \left(\frac{\beta}{q}\int (c')^{-\gamma}[z'+(1-\delta)q']\, dF(s')\right)^{-1/\gamma}$$

Now

$$c\frac{k}{k'} = \frac{c}{1-\delta+\frac{1}{q}(z+r-c)} = \frac{1}{\frac{1}{c}\left(1-\delta+\frac{1}{q}z\right)-\frac{1}{q}}$$

Therefore

$$\frac{1}{\frac{1}{c}\left(1-\delta+\frac{1}{q}[z+r]\right)-\frac{1}{q}} = \left(\frac{\beta}{q}\int (c')^{-\gamma}[z'+(1-\delta)q']\, dF(s')\right)^{-1/\gamma}$$

i.e.,

$$\frac{1}{c}\left(1-\delta+\frac{1}{q}(z+r)\right) = \frac{1}{q} + \left(\frac{\beta}{q}\int (c')^{-\gamma}[z'+(1-\delta)q']\, dF(s')\right)^{1/\gamma}$$

i.e.,

$$c = \frac{1-\delta+\frac{1}{q}(z+r)}{\frac{1}{q}+\left(\frac{\beta}{q}\int (c')^{-\gamma}[z'+(1-\delta)q']\, dF(s')\right)^{1/\gamma}} = \frac{z+r+(1-\delta)q}{1+q^{1-1/\gamma}\left(\beta\int (c')^{-\gamma}[z'+(1-\delta)q']\, dF(s')\right)^{1/\gamma}},$$

i.e., (10) Then $i = \frac{1}{q}(z+r-c) = \frac{1}{q}\left(z+r-\frac{z+r+(1-\delta)q}{1+q^{1-1/\gamma}D}\right)$, i.e., (11).

In (9), we substitute $[1-\delta+i(s')]\,q'$ for $p(s)$, which leads to

$$\frac{p(s)}{(1-\delta+i[s])\,q}=\frac{\int (c')^{-\gamma}\left(r'+\chi\left[1-\delta+i(s')\right]q'\right)dF(s')}{\int c^{-\gamma}\left(z'+[1-\delta]\,q'\right)dF(s')}\equiv\chi,$$

and thence to

$$\chi=\left(1-\frac{\int (c')^{-\gamma}\,q'\left(1-\delta+i[s']\right)dF(s')}{\int c^{-\gamma}\left(z'+[1-\delta]\,q'\right)dF(s')}\right)^{-1}\frac{\int (c')^{-\gamma}\,r'dF(s')}{\int c^{-\gamma}\left(z'+[1-\delta]\,q'\right)dF(s')}$$

which, together with (10), leads to (20).

Proof of Proposition 2: When projects with $\xi\in[0,\xi_{\max}]$ are implemented, the number of new machines (per unit of k) built is

$$x=1-\left(\frac{\xi_{\max}}{q}\right)^{-1/\sigma}. \tag{21}$$

Now, when the marginal project just breaks even, $\xi_{\max}=\hat{q}$, and we have (18). Solving (21) for the marginal project yields

$$\xi_{\max}=q\,(1-x)^{-\sigma}. \tag{22}$$

Letting $kP(\xi)$ denote the unnormalized C.D.F. defined in (16), the cost of x, per unit of k, is

$$\text{cost}=\int_q^{\xi_{\max}}\xi dP(\xi)=\frac{1}{\sigma}\int_q^{\xi_{\max}}\left(\frac{\xi}{q}\right)^{-1/\sigma}d\xi,$$

changing variables to $u=\xi/q$,

$$\begin{aligned}\text{cost}=\quad&\frac{q}{\sigma}\int_1^{\xi_{\max}/q}u^{-1/\sigma}du=\frac{q}{\sigma}\left.\frac{u^{1-1/\sigma}}{1-1/\sigma}\right|_1^{\xi_{\max}/q}=\frac{q}{1-\sigma}\left(1-\left[\frac{\xi_{\max}}{q}\right]^{1-1/\sigma}\right)\\ =\quad&q\left(\frac{1-(1-x)^{1-\sigma}}{1-\sigma}\right)\quad\text{(using (22))}\end{aligned}$$

Since the cost is in units of today's goods, this implies (17).

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