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ABSTRACT

The traditional current account can be an inaccurate measure of the change in the net foreign asset (NFA) position. Using gross asset and liability positions at the country level, a number of 'valuation effects' have been identified which contribute to changes in NFA but do not enter the reported current account. This paper uses new developments in the analysis of portfolio allocation in general equilibrium to investigate valuation effects in a two-country model. The model can be used to analyze both qualitatively and quantitatively the role of valuation effects. Broadly speaking, the valuation effects in the model correspond to those in the data, and have the effect of enhancing cross country risk sharing. But there is a key distinction between "unanticipated" and "anticipated" valuation effects. Unanticipated effects can be large, dominating the movement in NFA, but anticipated effects arise only at higher orders of approximation and are small for reasonable parameterisations. The paper also analyses the determinants of international portfolio positions, and their role in generating valuation effects from asset price and terms of trade changes.

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1 Introduction

Open economy macroeconomic models typically pay close attention to the current account as a measure of the evolution of an economy's net external assets. The growth of current account imbalances, and in particular the US current account deficit, has recently brought this linkage to the forefront of economic policy discussion. Since countries must satisfy intertemporal budget constraints, large and growing current account deficits will reduce net external assets and should require the establishment of future trade surpluses.

This traditional view of the current account has been put into question more recently, however. Recently constructed data suggest that traditional measures of the current account may give an inaccurate measure of the movement of an economy's net external wealth (Lane and Milesi Ferretti 2001, 2006). In these studies, corrected measures of net external assets incorporate changes in asset prices, returns, and currency exchange rates. These adjustments change an economy's net external wealth through separate 'valuation effects' on gross assets and liabilities. Moreover, since the mid 1990's, there have been huge increases in the scale of gross external assets and liabilities, which has led these previously unmeasured valuation effects to increase dramatically relative to the traditional measures of the current account. A number of studies have emphasized the empirical relevance of these valuation effects (Tille 2003, Higgens et al. 2005, Lane and Milesi Ferretti, 2005, Gourinchas, 2007).

By now, economists have recognized the importance of correctly measuring the impact of valuation effects (and more generally, differential assets returns) on net external assets. Until recently however, there has been little impact of these new empirical findings on the traditional modeling of the current account and net external asset movements in open economy macro models. One of the key reasons for this is that it has proven difficult to incorporate classic principles of portfolio choice into the conventional dynamic general equilibrium open economy model. Recent developments in the literature, however, now provide techniques for making progress in combining portfolio choice with general equilibrium open macro models[1]. This paper makes use of these new techniques to provide a qualitative and quantitative analysis of the ability of theoretical models to account for valuation effects in the evolution of net external assets, and to explore the interaction

[1]See, for instance Coeurdacier (2005), Evans and Hnatkovska, (2005), Kollmann (2006), Engel and Matsumoto (2006), Devereux and Sutherland (2006), (2007), Tille and Van Wincoop, (2007).

between valuation effects and traditional measures of the current account.

We start by developing a basic framework within which to analyze valuation effects on the evolution of net foreign assets. This framework is then applied to a simple two-country endowment economy model in which each country faces two sources of risk - one from capital income, which is assumed to be internationally diversifiable through equity sales, and the other from labor income, which cannot be directly diversified. Although the model is simple, it allows us to illustrate in an analytical example the main elements of the dichotomy between the traditionally measured current account and the valuation channel in determining the movement of net external assets. Defining the valuation channel as the gap between the movement of net external assets and the standard measure of the current account, we show that the valuation effect may be broken into anticipated and unanticipated components. The anticipated component of the valuation effect captures expected excess returns on a country's portfolio due to differences in the covariance risk associated with each country's traded equity.[2] Such country risk premia allow, in principle, for permanent imbalances in national current accounts. In addition, there may be time-varying anticipated excess returns that are associated with current account adjustment. The unanticipated component of the valuation effect captures the way in which national portfolios are structured so as to hedge against consumption risk. In this model, a basic property of the unanticipated valuation component is that it should co-vary negatively with the traditional current account. The model also allows for a decomposition of unanticipated valuation effects into those coming from movements in rates of return on assets, and those coming from movements in the portfolio holdings.

Having defined these different components of valuation effects, we go on to provide a quantitative account of the importance of each component in the evolution of net assets. We show that the model indicates that anticipated valuation effects are very small, except for counterfactually high values of risk aversion and differences in country endowment volatilities. But unanticipated valuation components may represent a large fraction of the volatility of net external assets, even when the model is calibrated to realistic sizes of gross national portfolios. Moreover, unanticipated valuation effects in the model behave

[2] In our basic model, asset returns depend on dividend payments and capital gains terms. Since what matters for portfolio choice is total asset returns rather than its components, we focus on expected excess returns coming from both sources. The decomposition of the expected excess returns between dividend payments and changes in asset prices will depend on the process driving the dividend stream.

in quite a similar fashion to those imputed from the net foreign assets (NFA) data - in particular, they are large as compared to traditional macro shocks, they dominate the movements in NFA, they tend to be negatively correlated with the current account, and they are approximately i.i.d.

One aspect of the recent portfolio discussion emphasizes the difference between the effects of shocks to returns for a given portfolio, and the effects of adjustment in the portfolio itself. In our model, both effects form part of the dynamics of NFA. Unanticipated valuation channels involve both shocks to returns, and movements in portfolio holdings. But in our quantitative decomposition of the volatility of net external assets, the latter channel plays at best a small role. The biggest driver of the volatility of net external assets is the unanticipated movement in returns, holding the portfolio constant. Portfolio adjustment and movements in expected returns can also create anticipated valuation effects. But our analysis suggests that these effects arise only at higher orders of approximation and are quantitatively very small.

The main results of the paper are presented in the context of a one-good world economy with stochastic endowments. In a later section, we show how the decomposition of valuation effects extends to a context with differentiated home and foreign goods. In this section, we also emphasize the important role of bonds as well as equities in risk sharing, and both asset prices and terms of trade changes in generating unanticipated valuation effects. In this model, bond trading can achieve substantial risk sharing, even for a very small international exposure to equities, suggesting a possible motive for 'home equity bias'. [3]In this case, valuation effects come from movements in the real exchange rate, as well as movements in asset prices. Moreover, valuation effects in this context are substantially larger than in the model without real exchange rate volatility.

The paper's contribution is also pedagogical. We document how valuation effects enter in the evolution of net foreign assets, and at what order of approximation each effect is important. To this extent, the paper can be seen as a theoretical underpinning for some traditional 'portfolio balance' modeling, which combined goods and asset market modeling in one framework, but based on assumed rules of thumb behaviour with respect to portfolio composition. At the same time, our analysis naturally places a limit on the

[3]These results are similar to those in recent papers by Coeurdacier et al. (2008) and Coeurdacier and Gourinchas (2008).

potential importance of each component of valuation effects. In one sense, our results suggest that in order to support the importance of some key elements of portfolio balance models, it would be necessary to develop models of risk-bearing that differ substantially from those of the standard intertemporal stochastic model that underlies the traditional open economy macro framework used in this paper.

There is a large and growing literature on valuation effects and current account dynamics in general equilibrium models. Notable recent papers are Cavallo and Tille (2006), Ghironi, Lee and Rebucci (2006), and Pavlova and Rigobon (2007). Cavallo and Tille (2006) and Ghironi, Lee and Rebucci (2006) provide a careful quantitative accounting of the impact of valuation effects in models in which the portfolio structure is calibrated to match the data. Pavlova and Rigobon (2008) present a rich continuous time dynamic model in which the portfolio rules can be obtained in closed form, but follow a different line of inquiry from that considered here.

The paper is structured as follows. The next section discusses some properties of the data on the current account and net external assets. We then set out a simple model of the current account in the face of capital and labour income risk. Following this, we discuss the properties of the solution method for portfolio choice. We then explore some analytical results on valuation effects. After this, we present quantitative results on the importance of anticipated and unanticipated valuation effects. The main sections of the paper are based on a simple single-good model with trade in equities. In the last section of the paper we extend the analysis to a two-good model with trade in both equities and bonds.

2 Stylised Facts

Here, we provide a brief description of the evolution of net external assets and their decomposition in terms of the conventional measure of the current account, and those driven by valuation effects[4]. We focus on a subset of OECD countries. Start with a simple decomposition of net external assets into the conventional current account, as measured in balance of payments accounting, and valuation terms. Thus, for country i

[4]Similar discussion is provided in Kollmann (2006), Gourinchas (2007), Lane and Milesi Ferretti (2006) among others.

at time t, we have:

$$NFA_{it} - NFA_{it-1} = CA_{it} + VAL_{it} \tag{1}$$

We compute these using the IMF/Lane-Milesi-Ferretti External Wealth of Nations (EWN) dataset on international investment positions, and from balance of payments data on the current account. As discussed in Lane and Milesi-Ferretti (2006), Tille (2003), and Gourinchas (2007), movements in VAL_{it} are driven by asset price and exchange rate changes which cause revisions in the value of gross external assets and liabilities, but are not incorporated in the income account as returns paid or received on gross external liabilities or assets.

We derive VAL_{it} indirectly, since NFA_{it} is reported in the EWN data-base (and updated using the IMF IIP), and CA_{it} is observable from Balance of Payments data. To make the VAL_{it} variable comparable with our model, we scale by GDP. Thus, we define

$$val_{it} = \frac{(NFA_{it} - NFA_{it-1})}{GDP_{it}} - \frac{CA_{it}}{GDP_{it}} \equiv \Delta nx_{it} - ca_{it}, \tag{2}$$

Since NFA_{it} and CA_{it} are reported in US dollars, we use US dollar GDP_{it} from the OECD database. The variable val_{it} is constructed for a sample of 23 OECD countries for the period 1980-2006. Table 1 describes the characteristics of val_{it}. The first column of the table reports the standard deviation of val_{it} for each country. As noted in Lane and Milesi-Ferretti (2006), the valuation term is highly volatile, with an average standard deviation of 0.07 across countries. The second column illustrates the fraction of the total variation in Δnx_{it} accounted for by valuation effects; $VR_i =$var(val_i)/var(Δnx_i) over the sample. For most countries, this is well above 50 percent. The average value is 0.90, and the US is highest at 1.39. In terms of accounting for the variation in net external assets, for most countries the valuation effects completely dominate the share attributable to the current account in the variation of net external assets.

The valuation term is of course not independent of the current account itself. The third column of Table 1 reports the correlation coefficient between val_{it} and ca_{it} for each country. In the endowment economies explored below, this correlation is negative. The results in the data are mixed. For 14 of the 23 countries in the sample, corr(ca_i, val_i) is negative, with the highest negative correlation being for the US.

Kollmann (2006) finds that NFA is mostly negatively correlated with the current account. He also notes Δnx_{it} is approximately i.i.d. for most countries, while the current account displays substantial persistence. Here, when we impute the valuation effect as the difference between the two, we find that val_{it} inherits the persistence properties of the Δnx_{it} series. The measured val_{it} has no serial correlation for almost all countries. Table 1 reports the results from the AR(1) regression $val_{it} = c_1 + c_2 val_{it-1}$ for each country. The AR(1) coefficient is insignificant for almost all countries. Below, we show that val_{it} as defined in the theoretical model should be *i.i.d.*

The definition of VAL used in the model below includes dividend returns on equity, which may be properly measured in the current account. But even if we define the valuation term using the trade account (which does not contain any asset returns), we get similar results. Take the following decomposition

$$vai_{it} = \Delta nx_{it} - ta_{it}, \tag{3}$$

where ta_{it} is the trade account to GDP ratio. Thus vai_{it} is the sum of the valuation term to GDP ratio, plus the income account to GDP ratio. In practice, vai_{it} and ta_{it} behave very similarly. Table 2 reports the identical results to Table 1 for this decomposition. As before, the variance of the valuation term is very high relative to the variance of net external assets - the average value is again about 0.9. Thus, a large component of nx is driven by portfolio effects, rather than trade balance effects. In addition, we find that the correlation between vai and ta is negative now for most countries. Finally, constructed in this way, vai is transitory - the AR(1) coefficient is again insignificant for most countries.

In the model below, the presence of valuation effects is critically tied to the size of a country's gross asset and liability position. Figures 1 and 2 illustrate the relationship between gross positions and valuation effects. In the figure, gross positions are measured by the average value of assets plus liabilities to GDP over the sample.[5] Figure 1 suggests that there is a positive relationship between the gross position and the standard deviation of $\sigma(VAL)$. Countries with higher gross positions have higher volatility of the valuation residual. But Figure 2 indicates that this is not true for the VR measure. That is, there is no clear relationship between the gross positions and the degree to which net foreign asset changes are accounted for by VAL. For most countries, VR is close to 1.

These stylized facts are 'first-order' in nature. Interpreted this way, as we discuss below, val and vai can thus be thought of as the result of an optimal risk-sharing port-

[5] Clearly this is an imperfect statistic since both measures have been distinctly trending upwards over the sample.

folio, because they can be interpreted as implicit insurance against business cycle shocks. Gourinchas (2007) refers to these as 'unpredictable' valuation terms. But other recent discussion of valuation effects in international financial data stress the presence of 'predictable' valuation effects at the national level, meaning that there are predictable excess returns on some component of a country's gross assets relative to the same component in its gross liabilities. As a rough measure of this, Table 1 computes the average valuation effect over the sample for each country. If valuation changes were just attributable to first-order risk-sharing, then this should be a very small number. In fact, it is negative, and a relatively large share of GDP for many countries. For the US, it is positive and 1.4 percent of GDP.

Gourinchas and Rey (2005) estimate a substantial excess return on US assets relative to liabilities, for all components of its international portfolio. For portfolio equity and debt securities, Curcuru, Dvorak, and Warnock (2008) argue that the actual excess return to the US is quite small. But for FDI, Higgins, Klitgaard and Tille (2006) find a 2-3 percent persistently higher return on US assets abroad than foreign assets held in the US. Lane and Milesi Ferretti (2007) provide an overview of some of the measurement problems inherent in these estimates. Lane and Milesi Ferretti (2005) take a larger sample of countries, and find that average rates of return on assets and liabilities have had significant differences over substantial periods of time for many countries.

Gourinchas and Rey (2007) highlight a somewhat different predictable valuation effect. They find that, conditional on an increase in the US trade balance deficit, the US experiences a predictable persistent increase in the excess return on its international investment portfolio, thereby reducing the required increase in the future trade balance surplus required to achieve overall intertemporal budget balance.

While unpredictable valuation gains or losses are relatively easy to model, in terms of an optimal insurance arrangement, it has proven much more difficult to integrate the findings of predictable excess returns into general equilibrium modeling. This is because these effects are of a 'higher order' nature. In our analysis below, we examine higher order approximations of portfolio choice within a standard general equilibrium framework, and explore the degree to which they give rise to predictable valuation effects on the evolution of net external assets.

3 Definition and Decomposition of Valuation Effects: A Simple Example Model

3.1 The Budget Constraint and the Definition of VAL

We illustrate how the measured current account and valuation effects interact in a simple two-country endowment model with two traded assets, and a single world consumption good. In this section we focus on the definition of valuation effects in the context of a simple endowment economy. We also show how the theoretical valuation effect can be decomposed into unpredictable, predictable, constant and time varying components using approximate solutions for portfolio behaviour. Section 4 solves explicitly for the different components of the valuation effect and presents some numerical calculations which illustrate some of the quantitative predictions of the model.

In order to define the valuation effect in the model it is sufficient to focus on the budget constraint of home households. This is given by

$$\alpha_{1,t} + \alpha_{2,t} = \alpha_{1,t-1} r_{1,t} + \alpha_{2,t-1} r_{2,t} + Y_t - C_t \tag{4}$$

where Y is the endowment received by home agents, C is consumption of home agents, $\alpha_{1,t-1}$ and $\alpha_{2,t-1}$ are the real holdings of the two assets (purchased at the end of period $t-1$ for holding into period t) and $r_{1,t}$ and $r_{2,t}$ are gross real returns. Note that $\alpha_{1,t-1}$ and $\alpha_{2,t-1}$ are external holdings of assets, i.e. home households' claims on (or liabilities to) foreign households. For purposes of exposition, it is easier to develop the results assuming assets are in zero net supply and that domestic households are the default owners of domestic equity. Thus, equity trade takes place through derivative assets α_1 and α_2. This has no bearing on the results. In the discussion below, we show the relationship between α_1 and the total home country holdings of home stocks (which are not in zero net supply). The stochastic process determining endowments and the nature of the assets and the properties of their returns are specified in more detail below.

We define $W_t = \alpha_{1,t} + \alpha_{2,t}$ to be the total net claims of home agents on the foreign country at the end of period t (i.e. the net foreign assets, NFA, of home agents). Defining $r_{x,t} = r_{1,t} - r_{2,t}$ as the "excess return" on asset 1, the budget constraint can then be re-written as

$$W_t = Y_t - C_t + r_{2,t} W_{t-1} + \alpha_{1,t-1} r_{x,t}. \tag{5}$$

Given that $\alpha_{1,t-1}$ and $\alpha_{2,t-1}$ are external holdings of assets, market clearing in asset markets must imply $\alpha_{1,t-1} + \alpha^*_{1,t-1} = 0$ and $\alpha_{2,t-1} + \alpha^*_{2,t-1} = 0$, where asterisks indicate foreign variables. To simplify notation in this example, we can drop the subscript from $\alpha_{1,t}$ and simply refer to α_t. Note that $\alpha_{1,t} = -\alpha^*_{1,t-1} = \alpha_t$, $\alpha_{2,t} = W_t - \alpha_t$ and $\alpha^*_{2,t} = W^*_t + \alpha_t$, where W^* is foreign net external assets, and $W_t + W^*_t = 0$.

Now consider the definition of the current account and the valuation effect. Equation (5) can be rearranged to give a representation for the change in net external wealth as

$$\Delta W_t = Y_t - C_t + (r_{2,t} - 1)W_{t-1} + \alpha_{t-1} r_{x,t} \tag{6}$$

where $\Delta W_t = W_t - W_{t-1}$.

We wish to decompose ΔW_t into current account and valuation terms in a manner analogous to the data. The clearest approach within this model is to take the first term $Y_t - C_t + (r_{2,t} - 1)W_{t-1}$, as a measure of the conventional current account, and the second term $\alpha_{t-1} r_{x,t}$, as a measure of valuation effects which impact on net external assets, but do not directly enter into the current account. We focus on the excess return on the portfolio $\alpha_{t-1} r_{x,t}$ as the principle measure of the valuation term, since the optimal portfolio depends on the properties of total returns rather than its individual components. However, the expression $Y_t - C_t + (r_{2,t} - 1)W_{t-1}$ differs from the measured current account in two ways. First, r_{2t} is the return on a stock, and therefore includes both dividends and capital gains terms. Capital gains are not usually counted as part of the measured current account. Secondly, the current account may include dividend payments paid on domestic and foreign portfolios that are measured in the excess return term $r_{x,t}$. Because we approximate around a symmetric steady state with $\overline{W} = 0$ (see below), the capital gain terms do not actually affect the approximations for the current account that are reported below. [6] In addition, we show below that for highly persistent shocks, most of the variability in $r_{x,t}$ comes from capital gains terms, and not from movements in dividends. Thus, measuring the current account as $Y_t - C_t + (r_{2,t} - 1)W_{t-1}$ and the valuation term as $\alpha_{t-1} r_{x,t}$ accords closely with the balance of payments accounting procedures for highly

[6] We could avoid this feature by including a market in non-contingent commodity bonds, and allowing the bond to be the reference asset. In that case, the return in the $(r_t - 1)W_{t-1}$ term would be that on bonds, and would not contain any capital gains. In a fully symmetric environment, there would be no trade in bonds anyway, and the results would be exactly identical to those reported below. More generally, with highly persistent shocks, the gains from bond trade would be slight.

persistent shocks. More generally, section 2 above shows that in the data, the first-order properties of the valuation term defined using the trade balance as a residual (so that the definition of VAL in the data includes total returns, including both dividend terms and all capital gains) behaves very similarly to that using the current account. Hence, since our measure of the current account in the model is essentially equivalent to the trade balance, there is little discrepancy between the treatment of model and data in this dimension.

We therefore rewrite (6) as follows

$$\Delta W_t = CA_t + VAL_t \tag{7}$$

where

$$CA_t = Y_t - C_t + (r_{2,t} - 1)W_{t-1} \tag{8}$$

$$VAL_t = \alpha_{t-1} r_{x,t} \tag{9}$$

Our analysis focuses on the behaviour of VAL as defined in (9). In Section 4 the budget constraint and the corresponding definition of VAL will be embedded in a simple DSGE model. In common with most DSGE models of portfolio allocation it will be necessary to solve the model by approximation. The particular method we use follows Devereux and Sutherland (2006). In order to analyse VAL it is necessary to solve for the approximate behaviour of α and r_x. In section 3.3 we therefore discuss the nature of approximate solutions for α and r_x within a DSGE model. In particular, we define some terms relating to the true and approximated solutions for α and r_x.

3.2 Other Details of the Example Model

Before considering solutions for α and r_x we complete the description of the model. Agents in the home country have a utility function of the form

$$U_t = E_t \sum_{\tau=t}^{\infty} \theta_\tau u(C_\tau) \tag{10}$$

where C is consumption and $u(C) = (C^{1-\rho})/(1-\rho)$. The discount factor, θ_τ, is determined as follows

$$\theta_{\tau+1} = \theta_\tau \beta C_{A\tau}^{-\eta} / \bar{C}_A^{-\eta}, \ \theta_0 = 1 \tag{11}$$

where $0 < \eta < \rho$, $0 < \beta < 1$, C_A is aggregate home consumption and $\bar{C}_A$ is a constant.[7]

In what follows we assume that $\eta > 0$ is positive, which ensures strict stationarity in the first-order approximated model and a determinate value of net foreign assets, $\bar{W}$. For convenience we choose $\bar{C}_A$ in (11) so that $\bar{W} = 0$. This is achieved by setting $\bar{C}_A = \bar{Y}$ where $\bar{Y}$ is the level of endowment in the non-stochastic steady state (hence $\bar{C}_A$ can be interpreted as the non-stochastic steady state level of home consumption). [8]

The budget constraint for home agents is given by (4). Foreign agents face a similar consumption and portfolio allocation problem with an analogous utility function and budget constraint.

It is assumed that endowments are the sum of 'capital income' components, Y_K and 'labour income' components Y_L, so that

$$Y_t = Y_{K,t} + Y_{L,t}. \tag{12}$$

The two countries may trade assets representing claims on capital income, but labour income is non-diversifiable. Endowments are determined by the following AR(1) stochastic processes

$$\log(Y_{K,t}/\bar{Y}_K) = \mu \log(Y_{K,t-1}/\bar{Y}_K) + \varepsilon_{K,t}, \quad \log(Y_{L,t}/\bar{Y}_L) = \mu \log(Y_{L,t-1}/\bar{Y}_L) + \varepsilon_{L,t} \tag{13}$$

where $0 \leq \mu \leq 1$. $\bar{Y}_K$ and $\bar{Y}_L$ are the steady state levels of capital and labour income and ε_K, ε_L, are zero-mean i.i.d. shocks which are symmetrically distributed over the interval $[-\epsilon, \epsilon]$ with $Var[\varepsilon_K] = \sigma_K^2$, $Var[\varepsilon_L] = \sigma_L^2$. In what follows, we make the further assumption that $\sigma_K^2 = \sigma_L^2$, and define $\zeta = \text{corr}[\varepsilon_K, \varepsilon_L]$ as the correlation between capital and labour income shocks. Foreign income processes are defined analogously, and we assume zero covariance between home and foreign income shocks.

Equilibrium consumption plans must satisfy the resource constraint

$$C_t + C_t^* = Y_t + Y_t^* \tag{14}$$

[7]Following Schmitt Grohe and Uribe (2003), θ_τ is assumed to be taken as exogenous by individual decision makers. The impact of individual consumption on the discount factor is therefore not internalized.

[8]Since the algebraic expressions for optimal portfolios become very unwieldy when $\eta > 0$, to make the exposition easier to interpret, in the explicit expressions developed in this section, we focus on the limiting case where η becomes infinitesimally small. Even with $\eta = 0$, the conditional second moments are still well defined. In order to solve the numerical impulse responses below, η is set equal to 0.001, so that the model is strictly stationary.

where C^* is foreign consumption and Y^* is the foreign endowment.

The two traded assets are equity claims on the home and foreign capital income. The real payoff on a unit of home equity is Y_K and the real price is Z_E. Thus the gross real rate of return is

$$r_{1,t} = \frac{Y_{K,t} + Z_{E,t}}{Z_{E,t-1}} \tag{15}$$

The real return on foreign equity is defined analogously, where Z_E^* is the price of the foreign equity.[9]

At the end of each period agents select the portfolio of assets to hold into the following period. The first-order condition for the choice of $\alpha_{1,t}$ can be written in the following form

$$E_t\left[u'(C_{t+1})r_{1,t+1}\right] = E_t\left[u'(C_{t+1})r_{2,t+1}\right] \tag{16}$$

and the first-order condition for home consumption is

$$u'(C_t) = \beta E_t\left[u'(C_{t+1})r_{2,t+1}\right] \tag{17}$$

Similar conditions arise from the foreign consumption and portfolio choices.

A competitive equilibrium is defined by (5), (16) and its foreign counterpart, (15) and the analogous equation for r_{2t}, (17) and its foreign counterpart, and (14). These implicitly give the solutions for the equilibrium values of C, C^*, r_1, r_2, $Z_{E,t}$, $Z_{E,t}^*$, W_t and α_t.

3.3 Approximate Solutions for α and r_x

We now discuss the nature of approximate solutions for α and r_x. Approximate solutions are defined around a non-stochastic steady state where $\bar{\alpha}$ and $\bar{Y}$ are defined to be the equilibrium values of α and Y. It is simple to see from (16) and (17) that $r_1 = r_2 = 1/\beta$

[9]Recall that the two assets are assumed to be in zero net external supply, i.e. $\alpha_1 + \alpha_1^* = 0$ and $\alpha_2 + \alpha_2^* = 0$. It should be noted that the two assets in this model are paper claims on the endowments streams, Y_K and Y_K^*. It is the paper claims that are in zero net supply, not the underlying supplies of equity. The underlying capitalised value of the endowment streams has a strictly positive net value. Define S and S^* to be the capitalised value (i.e. the discounted present value or equity value) of Y_K and Y_K^* respectively. Given the way the budget constraint is defined, home households are the default recipients of home capital income. They are thus the default owners of S, the capitalised value of Y_K. If they additionally hold α_1 of paper claims on Y_K, they implicitly hold $S + \alpha_1$ of home equity, while foreign households hold α_1^*. So total home and foreign holdings of home equity are $S + \alpha_1 + \alpha_1^*$ which equals S (given the constraint that $\alpha_1 + \alpha_1^* = 0$).

in the non-stochastic steady state. The fact that innovations to the exogenous driving processes are symmetrically distributed over the interval $[-\epsilon, \epsilon]$ ensures that any residual in an equation approximated up to order N can be captured by a term denoted $O\left(\epsilon^{N+1}\right)$.

Consider an approximation of α up to order N

$$\alpha_t = \beta\bar{Y}\left[\tilde{\alpha} + \hat{\alpha}_t^{(1)} + \hat{\alpha}_t^{(2)} + ... + \hat{\alpha}_t^{(N)}\right] + O\left(\epsilon^{N+1}\right) \tag{18}$$

where $\tilde{\alpha} = \bar{\alpha}/(\beta\bar{Y})$ and $\hat{\alpha}_t = (\alpha_t - \bar{\alpha})/(\beta\bar{Y})$. $\hat{\alpha}^{(i)}$ is the *order-i component* of $\hat{\alpha}_t$. In this expression $\beta\bar{Y}$ is a convenient normalising factor which simplifies notation in later derivations. This normalisation also allows $\tilde{\alpha}$ and $\hat{\alpha}_t$ to be interpreted (approximately) in terms of GDP units.[10]

In what follows we confine attention to the first two terms in this approximation, $\tilde{\alpha}$ and $\hat{\alpha}_t^{(1)}$. Notice that, by definition, $\tilde{\alpha}$ is constant and therefore captures the average or steady-state element of portfolio holdings, while the (first-order) time varying element in portfolio holdings is captured by $\hat{\alpha}_t^{(1)}$.

Agents make their portfolio decisions at the end of each period and are free to rearrange their portfolios each period. In a recursive equilibrium the equilibrium asset allocation will be some function of the state of the system in each period - which is summarised by the state variables. We therefore postulate that the true portfolio (i.e. the equilibrium portfolio in the non-approximated model) is a function of state variables, $\alpha_t = \alpha(Z_t)$ where Z is the vector of state variables. We can therefore deduce that $\hat{\alpha}_t^{(1)}$ is a linear function of the first-order deviation of Z from $\bar{Z}$, i.e.

$$\hat{\alpha}_t^{(1)} = \varrho\hat{Z}_t^{(1)}$$

where ϱ is a vector of coefficients.

When analysing a DSGE model up to first-order accuracy, the standard solution approach is to use the non-stochastic steady-state of the model as the approximation point, and to use a first-order approximation of the model's equations to solve for the first-order component of each variable. Neither of these steps can be used to solve for the equilibrium of a portfolio problem. This is because in the non-stochastic equilibrium, the portfolio optimality condition (16) implies that both assets pay the same rate of return. This implies that any value for α is consistent with equilibrium. A similar problem arises in a

[10] Note that α may be negative so log-deviations of α are undefined.

first-order approximation of the model. First-order approximation of (16) implies that the expected returns on the two assets are identical, i.e. $E_t[r_{1,t+1}] = E_t[r_{2,t+1}]$. Again, any value of α is consistent with equilibrium. So neither the non-stochastic steady state nor a first-order approximation of the model provide enough equations to tie down the zero or first-order components of α.

The basic problem is that, in the non-approximated model, agents' preferences across the two assets depend on the differing risk characteristics of assets, but the non-stochastic steady state and the first order approximated model do not capture these differing risk characteristics. By definition, the non-stochastic equilibrium excludes risk, while the first-order approximated model imposes certainty equivalence. It is clear that the risk characteristics of assets only show up in the second-moments of model variables, and it is only by considering higher-order approximations of the model that the effects of second-moments can be captured. In Devereux and Sutherland (2006) we show that a second-order approximation of the portfolio optimality conditions provides a condition which makes it possible to tie down $\tilde{\alpha}$. Having established this starting point, it is relatively straightforward to extend the procedure to higher-order components of α. In Devereux and Sutherland (2007) we show that the solution for the first-order component of α can be derived from third-order approximations of the portfolio optimality conditions.[11] The general solution approach is outlined in the Appendix.

Now consider equilibrium returns, $r_{1,t}$ and $r_{2,t}$ or, more specifically, the *excess* return, $r_{x,t}$. Consider an approximation of $r_{x,t}$ up to order N

$$r_{x,t} = \bar{r}_x + \frac{1}{\beta}\left[\hat{r}_{x,t}^{(1)} + \hat{r}_{x,t}^{(2)} + ... + \hat{r}_{x,t}^{(N)}\right] + O\left(\epsilon^{N+1}\right) \tag{19}$$

where $\hat{r}_{x,t} = \beta(r_{1t} - r_{2t})$ and $1/\beta$ is the steady state equilibrium value of r_1 and r_2.[12]

What can the solution approach tell us about equilibrium excess returns at different orders of approximation? First, notice that the properties of the non-stochastic steady state tell us immediately that $\bar{r}_x = 0$, i.e. asset returns are equalised when there is no risk. Second, the properties of the first-order approximated model tell us that $E_t[\hat{r}_{x,t+1}^{(1)}] = 0$,

[11] For a related treatment, see also Tille and Van Wincoop (2007).

[12] Note that r_x may be negative so log-deviations of r_x are undefined. Also note that in Devereux and Sutherland (2006, 2007) $\hat{r}_{x,t}$ denotes $\hat{r}_{1t} - \hat{r}_{2t}$ where $\hat{r}_{1t}$ and $\hat{r}_{2t}$ are the log deviations of r_1 and r_2 from their values in the non-stochastic steady state. The definition of $\hat{r}_{x,t}$ used here leads to a considerable simplification of notation when analysing valuation terms.

i.e. certainty equivalence implies that expected asset returns are equalised.

The fact that the first-order expected excess return is zero, i.e. $E_t[\hat{r}^{(1)}_{x,t+1}] = 0$, combined with the nature of the exogenous driving processes ((13) and their foreign counterparts), implies that the realised value of $\hat{r}^{(1)}_{x,t+1}$ will be a linear function of the realised values of the innovations in the exogenous driving processes. Thus, $\hat{r}^{(1)}_{x,t+1}$ will be a linear function of $\varepsilon_{K,t}$ and $\varepsilon_{L,t}$ and their foreign counterparts. In turn this implies that $\hat{r}^{(1)}_{x,t+1}$ will be a zero mean i.i.d. random variable.

It follows from the above discussion that expected excess returns only deviate from zero at orders 2 and higher. In Devereux and Sutherland (2006) we show that $E_t[\hat{r}^{(2)}_{x,t+1}]$ can be solved in conjunction with $\tilde{\alpha}$. Furthermore, we show that $E_t[\hat{r}^{(2)}_{x,t+1}]$ can be written as a linear function of one-period-ahead conditional second moments of first-order realised asset returns and consumption. Depending on the relative size of the covariances between asset returns and consumption in the two countries, $E_t[\hat{r}^{(2)}_{x,t+1}]$ may be greater of less than zero. However, because one-period-ahead conditional second moments are non-time-varying (which in turn is because innovations to exogenous variables are i.i.d.), $E_t[\hat{r}^{(2)}_{x,t+1}]$ will be non-time-varying. In fact $E_t[\hat{r}^{(2)}_{x,t+1}]$ can naturally be thought of as the steady-state equilibrium expected excess return which corresponds to steady-state equilibrium asset holdings, $\tilde{\alpha}$. The Appendix provides a brief demonstration of the link between the solution for $\tilde{\alpha}$ and the solution for $E_t[\hat{r}^{(2)}_{x,t+1}]$.

In a similar way, in Devereux and Sutherland (2007) we show that the third-order component of excess returns, $E_t[\hat{r}^{(3)}_{x,t+1}]$, can be solved in conjunction with the first-order component of asset holdings, $\hat{\alpha}^{(1)}_t$. Again this point is briefly demonstrated in the Appendix. Devereux and Sutherland (2007) further show that $E_t[\hat{r}^{(3)}_{x,t+1}]$ can be written in terms of expected products of first and second-order realised asset returns and consumption. Furthermore, just as $\hat{\alpha}^{(1)}_t$ is time varying, it follows that $E_t[\hat{r}^{(3)}_{x,t+1}]$ is also time varying and it is possible to show that $E_t[\hat{r}^{(3)}_{x,t+1}]$ is a linear function of the first-order component of state variables, i.e.

$$E_t[\hat{r}^{(3)}_{x,t+1}] = \delta \hat{Z}^{(1)}_t$$

where δ is a vector of coefficients which are functions of one-period-ahead conditional second moments. $E_t[\hat{r}^{(3)}_{x,t+1}]$ can naturally be thought of as the time varying element of excess returns that corresponds to the first-order time varying element of portfolio holdings.

The properties of r_x can therefore be summarised as follows: $\bar{r}_x$ is zero; $\hat{r}^{(1)}_{x,t+1}$ is a zero-mean i.i.d. random variable; $E_t[\hat{r}^{(2)}_{x,t+1}]$ is constant and may be non-zero; and $E_t[\hat{r}^{(3)}_{x,t+1}]$ is a linear function of first-order state variables and thus may be time varying.

3.4 Approximate Solutions for VAL

Our analysis of valuation effects is based on an approximate solution for VAL which is constructed using the approximate solutions for α and r_x given in (18) and (19). Using these expressions, together with the fact that $\bar{r}_x = 0$, it is simple to show that an expression for VAL (up to third-order is) is given by

$$VAL_t = \overline{VAL} + \bar{Y}\left[\widehat{VAL}^{(1)}_t + \widehat{VAL}^{(2)}_t + \widehat{VAL}^{(3)}_t\right] + O\left(\epsilon^4\right)$$

where

$$\overline{VAL} = \tilde{\alpha}\bar{r}_x = 0 \tag{20}$$

$$\widehat{VAL}^{(1)}_t = \tilde{\alpha}\hat{r}^{(1)}_{x,t} \tag{21}$$

$$\widehat{VAL}^{(2)}_t = \tilde{\alpha}\hat{r}^{(2)}_{x,t} + \hat{\alpha}^{(1)}_{t-1}\hat{r}^{(1)}_{x,t} \tag{22}$$

$$\widehat{VAL}^{(3)}_t = \tilde{\alpha}\hat{r}^{(3)}_{x,t} + \hat{\alpha}^{(1)}_{t-1}\hat{r}^{(2)}_{x,t} + \hat{\alpha}^{(2)}_{t-1}\hat{r}^{(1)}_{x,t} \tag{23}$$

where $\widehat{VAL}_t = (VAL_t - \overline{VAL})/\bar{Y}$. It proves convenient to normalise by $\bar{Y}$, so $\widehat{VAL}$ can be interpreted in terms of GDP units.[13]

In Section 4 we analyse the first, second and third-order components of VAL, given by (21), (22) and (23), in detail in the context of the simple example model. But, before doing that, we note here that a number of important general properties of $\widehat{VAL}^{(1)}_t$, $\widehat{VAL}^{(2)}_t$ and $\widehat{VAL}^{(3)}_t$ can be established without specific reference to the model. In particular, we show how the unanticipated, anticipated, constant and time-varying components of VAL naturally arise at different orders of approximation. These general properties can be established with reference to the general properties of the approximate solutions for α and r_x given in (18) and (19).

Starting with the first-order component of VAL given in (21), the most obvious feature of $\widehat{VAL}^{(1)}_t$ is that it is a zero mean i.i.d. random variable. This follows from the properties of $\hat{r}^{(1)}_{x,t}$ and $\tilde{\alpha}$. The steady state portfolio, $\tilde{\alpha}$, is a constant while the first-order component

[13]Note that VAL may be negative so log-deviations of VAL are undefined.

of excess returns, $\hat{r}_{x,t}^{(1)}$, has a zero conditional expectation (because of certainty equivalence in the first-order system). Furthermore, the one-period ahead conditional variance of $\hat{r}_{x,t}^{(1)}$ is constant, so $Var_{t-1}[\widehat{VAL}_t^{(1)}]$ is constant.

Now consider the properties of the second-order component of VAL. Equation (22) shows that $\widehat{VAL}_t^{(2)}$ is the sum of two terms. Since $\tilde{\alpha}$ is a constant, the first term in $\widehat{VAL}_t^{(2)}$, $\tilde{\alpha}\hat{r}_{x,t}^{(2)}$, inherits the stochastic properties of $\hat{r}_{x,t}^{(2)}$. It was explained above that $E_{t-1}[\hat{r}_{x,t}^{(2)}]$ can be solved as a function of one-period-ahead conditional second moments. Because these one-period-ahead conditional second moments are non-time-varying, $E_{t-1}[\hat{r}_{x,t}^{(2)}]$ will also be non-time-varying. $E_{t-1}[\hat{r}_{x,t}^{(2)}]$ can naturally be thought of as the steady-state equilibrium expected excess return, or risk premium. It therefore follows that $\tilde{\alpha}\hat{r}_{x,t}^{(2)}$ will have a constant but possibly non-zero conditional mean.

The second term in $\widehat{VAL}_t^{(2)}$, $\hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(1)}$, on the other hand has a zero mean (conditional on time $t-1$ information). The conditional variance of $\hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(1)}$ is however time-varying because $\hat{\alpha}_{t-1}^{(1)}$ is time varying. This term thus captures (one aspect) of the impact of portfolio adjustment on the volatility of VAL. Notice, however, that portfolio adjustment does not give rise to *predictable* time variation in VAL at the second-order level.

The properties of the two components of $\widehat{VAL}_t^{(2)}$ thus imply that $E_{t-1}[\widehat{VAL}_t^{(2)}]$ is constant and may be non-zero while $Var_{t-1}[\widehat{VAL}_t^{(2)}]$ is time varying.

Notice from the discussion so far that $\widehat{VAL}_t^{(1)}$ and $\widehat{VAL}_t^{(2)}$ potentially capture two of the aspects of valuation effects which have been emphasised in the empirical literature. $\widehat{VAL}_t^{(1)}$ potentially captures the large unpredictable swings in valuation effects documented in Table 1, but it has nothing to say about predictable elements of valuation effects. $\widehat{VAL}_t^{(2)}$ on the other hand potentially captures the steady-state mean behaviour of valuation effects. In particular it shows that the steady-state mean valuation effect is associated with steady-state risk premia. The analysis of $\widehat{VAL}_t^{(1)}$ and $\widehat{VAL}_t^{(2)}$ shows however that portfolio adjustment and predictable dynamics in excess returns plays little or no role in valuation effects up to the second-order level. At most, dynamic adjustment of portfolios affects the variance of $\widehat{VAL}_t^{(2)}$.

It is clear therefore that the predictable time-varying valuation effects described by Gourinchas and Rey (2008) do not arise at the level of second-order approximation. We will now demonstrate that it is necessary to go to (at least) the third order level to identify predictable time varying effects. To see this, consider the third-order component of VAL.

In particular focus on the conditional expectation of $\widehat{VAL}_t^{(3)}$, which is given by

$$E_{t-1}[\widehat{VAL}_t^{(3)}] = \underbrace{\tilde{\alpha} E_{t-1}[\hat{r}_{x,t}^{(3)}]}_{\text{time variation in } E[r_x]} + \underbrace{\hat{\alpha}_{t-1}^{(1)} E_{t-1}[\hat{r}_{x,t}^{(2)}]}_{\text{time variation in } \alpha} \tag{24}$$

(which contains just two terms because $E_{t-1}[\hat{\alpha}_{t-1}^{(2)} \hat{r}_{x,t}^{(1)}] = 0$). It is clear from the properties of $\tilde{\alpha}$ (which is constant) and $E_{t-1}[\hat{r}_{x,t}^{(3)}]$ (which is time varying) that the first term, $\tilde{\alpha} E_{t-1}[\hat{r}_{x,t}^{(3)}]$, is time varying. It is also clear from the properties of $\hat{\alpha}_{t-1}^{(1)}$ (which is time varying) and $E_{t-1}[\hat{r}_{x,t}^{(2)}]$ (which is constant) that the second term, $\hat{\alpha}_{t-1}^{(1)} E_{t-1}[\hat{r}_{x,t}^{(2)}]$, is also time varying. In fact, (24) captures and separates the effect of time varying expected returns (i.e $E_{t-1}[\hat{r}_{x,t}^{(3)}]$) on VAL from the effect of time varying portfolio holdings (i.e. $\hat{\alpha}_{t-1}^{(1)}$). $\widehat{VAL}_t^{(3)}$ therefore potentially captures the time-varying predictable valuation effects described by Gourinchas and Rey.[14]

The properties of the different components of VAL can be summarised as follows: $\widehat{VAL}_t^{(1)}$ potentially captures the unpredictable element of VAL; $\widehat{VAL}_t^{(2)}$ potentially captures the constant element of the predictable component of VAL; and $\widehat{VAL}_t^{(3)}$ potentially captures the time-varying element of the predictable component of VAL.

4 Solving for Valuation Effects in the Example Model

We now derive these different components of VAL in the simple example model.

4.1 First-order Valuation Effects

In order to analyse the properties of $\widehat{VAL}_t^{(1)}$ in more detail we first solve for its components $\tilde{\alpha}$ and $\hat{r}_{x,t}^{(1)}$. Following Devereux and Sutherland (2006), it is easy to compute the first-order solutions for consumption, asset prices, and asset returns. Given these, we obtain $\tilde{\alpha}$ as follows

$$\tilde{\alpha} = -\frac{1}{2(1-\beta)} \frac{\phi(\sigma_K^2 + \sigma_K^{2*}) + (1-\phi)(\zeta\sigma_K^2 + \zeta^*\sigma_K^{2*})}{\sigma_K^2 + \sigma_K^{2*}} \tag{25}$$

[14] Equation (18) describes the expected behaviour of $\widehat{VAL}_t^{(3)}$ conditional on information at time $t-1$. More generally, the expected value of $\widehat{VAL}_{t+\tau}^{(3)}$ conditional on information at time t is given by $E_t[\widehat{VAL}_{t+\tau}^{(3)}] = \tilde{\alpha} E_t[\hat{r}_{x,t+\tau}^{(3)}] + E_t[\hat{\alpha}_{t+\tau-1}^{(1)}] E_t[\hat{r}_{x,t+\tau}^{(2)}]$. This follows because the i.i.d. nature of the exogenous innovations ensures that the conditional covariance between $\hat{\alpha}_{t+\tau-1}^{(1)}$ and $\hat{r}_{x,t+\tau}^{(2)}$ is zero.

where $\phi = \bar{Y}_K/\bar{Y}$ is the share of capital income in the total endowment in the non-stochastic steady state. (See the Appendix for an outline of the solution procedure.)

The optimal steady state portfolio, $\tilde{\alpha}$, implies a positive holding of foreign equity, so long as $\phi(\sigma_K^2 + \sigma_K^{2*}) + (1-\phi)(\zeta\sigma_K^2 + \zeta^*\sigma_K^{2*}) > 0$. The total share of domestic equity held by the home country is given by $\frac{\bar{S}+\beta\tilde{\alpha}}{\bar{S}} = \frac{1}{2} - \frac{1}{2}\frac{(1-\phi)(\zeta\sigma_K^2+\zeta^*\sigma_K^{2*})}{\phi(\sigma_K^2+\sigma_K^{2*})}$, where $\bar{S}$ represents the steady state domestic equity to GDP ratio, which is equal to $\beta\phi/(1-\beta)$.[15] A fully diversified portfolio would have each country holding half of the other's equity to GDP ratio, so that $\tilde{\alpha} = -\frac{\phi}{2(1-\beta)}$. From (25), this would obtain if $\zeta = \zeta^* = 0$. The existence and degree of home bias in equity depends on the correlation between capital and labour income in each country. If ζ and ζ^* are less than zero, we have $\tilde{\alpha} > -\frac{\phi}{2(1-\beta)}$, and there is home bias in equity holdings.[16]

The first-order behaviour of the excess return is

$$\hat{r}_{x,t}^{(1)} = \frac{(1-\beta)}{(1-\beta\mu)}(\varepsilon_{k,t} - \varepsilon_{k,t}^*) \tag{26}$$

so the first-order component of the valuation effect, $\widehat{VAL}_t^{(1)}$, is given by

$$\widehat{VAL}_t^{(1)} = \tilde{\alpha}\frac{(1-\beta)}{(1-\beta\mu)}(\varepsilon_{k,t} - \varepsilon_{k,t}^*) \tag{27}$$

We may compare the behaviour of $\widehat{VAL}_t^{(1)}$ to the first-order behaviour of the current account, which is given by[17]

$$\widehat{CA}_t^{(1)} = \frac{\beta}{2}\frac{(1-\mu)}{(1-\beta\mu)}\left[\phi(\hat{Y}_{k,t} - \hat{Y}_{k,t}^*) + (1-\phi)(\hat{Y}_{l,t} - \hat{Y}_{l,t}^*)\right] - \tilde{\alpha}\frac{(1-\beta)^2}{(1-\beta\mu)}(\varepsilon_{k,t} - \varepsilon_{k,t}^*) \tag{28}$$

This expression contains two terms. The first term is the familiar textbook definition of the current account. When there is a rise in home income relative to foreign income, whether capital or labour income, the current account will improve, so long as $\mu < 1$.

[15] Note that α_t is interpreted as the home countries external liabilities in the home equity. In the non-stochastic steady state the total capitalised value of home equity is $\overline{Z}_E = \beta\bar{Y}_K/(1-\beta)$ so the equity to GDP ratio is $\overline{Z}_E/\bar{Y} = (\beta\bar{Y}_K/\bar{Y})/(1-\beta) = \beta\phi/(1-\beta)$. Since the home household is the default owner of home equity, it's total holding of home equity (expressed as a ratio to GDP) is $(\overline{Z}_E + \bar{\alpha})/\bar{Y}$.

[16] The potential for home bias in equity holdings arising because capital and labour income co-move negatively has been noted in many previous papers. See for instance Bottazzi et al, Baxter and Jehrmann (1997) (who argue against this explanation) , and Engel and Matsumoto (2008).

[17] Here we approximate the current account around an initial value of NFA equal to zero.

The second term captures the impact of portfolio valuation effects on consumption, and therefore on the current account. The valuation term represents the income gain or loss due to unanticipated changes in the excess return on assets. The sign and size of this will depend on the portfolio position $\tilde{\alpha}$, given in (25).

We measure both the volatility of $\widehat{VAL}_t^{(1)}$ directly, and the volatility relative to the volatility of net foreign assets. Take the case where $\sigma_K^2 = \sigma_K^{2*}$ and $\zeta = \zeta^*$. In addition, to make the discussion simpler, assume that $\zeta > -\frac{\phi}{1-\phi}$. This condition ensures that no country exhibits 'super home-bias', in the sense that it wishes to take a positive external position in domestic equity.[18] Then, from (25) and (27), the standard deviation of $\widehat{VAL}_t^{(1)}$ is

$$\sigma_{t-1}(\widehat{VAL}_t^{(1)}) = |\tilde{\alpha}| \frac{(1-\beta)}{(1-\beta\mu)} \sqrt{2\sigma_K^2} = \frac{\phi + (1-\phi)\zeta}{\sqrt{2}(1-\beta\mu)} \sigma_K \tag{29}$$

The volatility of $\widehat{VAL}_t^{(1)}$ depends positively on the size of the gross asset position. This is consistent with the evidence in Figure 1. In addition $\sigma_{t-1}(\widehat{VAL}_t^{(1)})$ is increasing in the persistence of endowment shocks, and the volatility of shocks. A higher μ has no effect on $\widetilde{\alpha}$, but increases excess returns volatility.

Using (25), (27), and (28), we can define the ratio of the variance of $\widehat{VAL}_t^{(1)}$ to that of the variance in the change in net foreign assets as:

$$VR = \frac{\sigma_{t-1}^2(\widehat{VAL}_t^{(1)})}{\sigma_{t-1}^2(\Delta \hat{W}_t^{(1)})} = \frac{(\phi + (1-\phi)\zeta)^2}{\beta^2[(1-\mu)^2(1-\phi)^2(1-\zeta^2) + \mu^2(\phi + (1-\phi)\zeta)^2]} \tag{30}$$

Theoretically, this can take any value in the range between zero and infinity. When $\phi = 1$ or $\zeta = 1$, there are effectively complete markets, and the right hand side of (30) is $1/\mu^2$, which always exceeds unity. If shocks are quite transitory, then the optimal portfolio keeps net external assets very stable, and the valuation ratio is very high. On the other hand, for low or negative ζ, the optimal portfolio position is small, due to home bias, and the valuation ratio may be very small.

To illustrate how risk sharing works in the model, take a one unit positive home endowment shock. In addition, focus on the special case where $\phi = 1$. In the absence of valuation effects, measured income would rise by 1, while consumption would rise by $\frac{(1-\frac{\beta(1+\mu)}{2})}{(1-\beta\mu)}$, leading to a current account surplus equal to $\frac{0.5\beta(1-\mu)}{(1-\beta\mu)}$, consistent with (28).

[18] For $\phi = 0.36$, (as assumed below), the condition requires only that $\zeta > -0.5625$, which is always satisfied in our computations.

When equities are chosen optimally however, there is a simultaneous negative payoff from the portfolio return, as the excess return on home equity is positive, and the home household holds a negative gross position in external claims on home income. Consumption is adjusted downwards by $(1-\beta)$ times the valuation effect, or $\tilde{\alpha}\frac{(1-\beta)^2}{(1-\beta\mu)}$. Given $\phi = 1$,and $\tilde{\alpha} = 0.5/(1-\beta)$, consumption rises only by $\frac{(1-\frac{\beta(1+\mu)}{2})}{(1-\beta\mu)} - \frac{0.5(1-\beta)}{(1-\beta\mu)} = 0.5$, and the net effect on the measured current account is 0.5. Thus, the home endowment shock is shared equally among home and foreign consumption. The sum of the measured current account and the direct negative valuation term, then leads to a *fall* in net foreign assets for the home country equal to $0.5 - \tilde{\alpha}\frac{(1-\beta)}{(1-\beta\mu)} = -\frac{0.5\beta\mu}{(1-\beta\mu)}$. In the period following the shock, the combination of higher income but lower net foreign assets leads home consumption to rise by 0.5μ, again leading to an optimal sharing across countries. In this way, the initial (unanticipated) valuation effect leads to an expected evolution of net foreign assets following the shock such that the consumption response is equalized across the home and foreign country.

Using the solution for $\tilde{\alpha}$, we may establish that:

$$\text{corr}_{t-1}(\widehat{VAL}_t^{(1)}, \widehat{CA}_t^{(1)}) = -\frac{(\phi + (1-\phi)\zeta)(1-\mu\beta)}{\sqrt{\beta^2(1-\mu)^2(1-\phi)^2(1-\zeta^2) + (1-\beta\mu)^2(\phi+(1-\phi)\zeta)^2}} < 0. \tag{31}$$

Hence, the theoretical correlation in this model is always negative. When either $\zeta = 1$, $\phi = 1$, or $\mu = 1$, this correlation is equal to -1. In the first and second case, this is because equity holdings allow for effectively complete markets, and the excess return on the portfolio acts perfectly to stabilize idiosyncratic domestic income shocks. The effects of any endowment shock on the current account precipitate a movement in the valuation term which is exactly proportional to the shock. In the case $\mu = 1$, the correlation is -1 because the only source of movement in the measured current account (28) is due to movements in the portfolio itself.

4.1.1 Quantitative Implications

We set out there the main parameter values that are used both in this section, and in the extended model evaluation in the next sections. Let the discount factor be $\beta = 0.96$. Again, we look at a symmetric case where the countries have identical volatilities of capital income, and $\zeta = \zeta^*$. The portfolio size $\widetilde{\alpha}$ depends on the value of ζ, the correlation between labour and capital income, ϕ, the share of capital in income, and the discount factor. For ϕ we take the conventional measure for the US economy of $\phi = 0.36$. We set $\mu = 0.9$, with $\sigma_K^2 = 0.02^2$, which is approximately the volatility of annual US GDP growth. Empirical estimates of ζ have varied quite a lot (see Bottazzi et al (1996), and Engel and Matsumoto (2008)). The correct measure of ζ should compare the overall returns to physical capital with those to human capital. Following this procedure, Bottazzi et al (1996) find a range of estimates both negative and positive. In this model, to allow for home bias in equity holdings within this example, it is necessary to have $\zeta < 0$. In section 5 below, we show that in the presence of endogenous terms of trade and bond holdings, we can obtain home equity bias for any value of ζ. Here however, we simply choose a range of values of ζ which give rise to different values for the gross asset and liability positions $\widetilde{\alpha}$. For $\zeta = -.4375$, the home country holds a gross asset and liability position equal to about 100 percent of GDP (approximately the liabilities of the US economy), so $\widetilde{\alpha} = -1$. This implies a high degree of home bias, with 89 percent of domestic equity being held by home consumers.

Table 3 illustrates the values of $\sigma_{t-1}(\widehat{VAL}_t^{(1)})$, VR, and $\text{corr}_{t-1}(\widehat{VAL}_t^{(1)}, \widehat{CA}_t^{(1)})$ for this model. In the Table, we allow for variation in equity holdings and $\widetilde{\alpha}$ by allowing for different values of ζ. Using this calibration, at $\widetilde{\alpha} = -1$, we find $\sigma_{t-1}(\widehat{VAL}_t^{(1)}) = 0.008$, almost a quarter that in the data for the US. To match the US estimate of $\sigma_{t-1}(\widehat{VAL}_t^{(1)}) = 0.03$, we would need $\widetilde{\alpha} = -3.4$, which would reduce the share of domestic equity held by home residents to 62 percent. If shocks were much more persistent, $\sigma_{t-1}(\widehat{VAL}_t^{(1)})$ would be higher. For $\mu = 0.95$, for instance, we have $\sigma_{t-1}(\widehat{VAL}_t^{(1)}) = 0.012$. Table 3 also illustrates the values of VR for various values of equity holdings. At the baseline case with $\widetilde{\alpha} = -1$, VR is 0.82, so the variance of VAL is almost as large as the variance of NFA. As $\widetilde{\alpha}$ rises in absolute value, VR increases, but since shocks are persistent, VR is relatively insensitive to the size of the gross asset position, consistent with the empirical evidence in Figure 2. Intuitively, from (27) and (28), when μ is closer to unity, both $\widehat{VAL}_t^{(1)}$ and $\Delta\hat{W}_t^{(1)}$ are proportional to $\tilde{\alpha}$, so that their ratio is relatively insensitive to the size of gross positions.

This example suggests that in principle, a model of efficient risk-sharing can account for the properties of the valuation shocks, the absence of persistence in these shocks, and their large size relative to overall fluctuations in NFA.[19] Table 3 also shows the correlation

[19] Our results complement those of Ghironi et al. (2006), and Kollmann (2006). Ghironi et al. discuss

between VAL and CA for various equity positions. The baseline correlation is -0.19. This increases substantially (in absolute value) as the equity position becomes more and more diversified, since intuitively, the portfolio position in that case more successfully cushions shocks to the current account.

4.1.2 Alternative Definitions of VAL

How do the above conclusions differ when an alternative definition of the valuation term is used? When we measure valuation effects as coming only from capital gains (associated with fluctuations in equity prices), assuming that dividend payments are included directly in the measured current account, we find that the volatility of VAL is given by

$$\sigma_{t-1}(\widehat{VAL}_Q^{(1)}) = |\tilde{\alpha}| \frac{\mu\beta(1-\beta)}{(1-\mu\beta)} \sqrt{2}\sigma_K \tag{32}$$

With substantial persistence in shocks, this differs only slightly from definition (29) above. Similarly, we may show that $VR_Q = \mu^2 VR$, while

$$\text{corr}_{t-1}(\widehat{VAL}_{Qt}^{(1)}, \widehat{CA}_{Qt}^{(1)}) = \text{corr}_{t-1}(\widehat{VAL}_t^{(1)}, \widehat{CA}_t^{(1)}).$$

Thus, for high persistence in shocks, the valuation term under this alternative definition is similar to the previous definition, while the correlation between the valuation ratio and the current account under this alternative definition is the same as that of the previous definition.

4.2 Second-order Valuation Effects: Anticipated Excess Returns

Now consider the properties of the second-order component of VAL, given by expression (22). It is useful to break the analysis of $\widehat{VAL}_t^{(2)}$ into two stages. First we consider the mean, or expected value of $\widehat{VAL}_t^{(2)}$. Later on, we consider the stochastic behaviour of $\widehat{VAL}_t^{(2)}$. Recall that $E_{t-1}[\widehat{VAL}_t^{(2)}] = \tilde{\alpha} E_{t-1}[\hat{r}_{x,t}^{(2)}]$ which, given the properties of $E_{t-1}[\hat{r}_{x,t}^{(2)}]$, is a constant and may be non-zero.

the way in which financial integration may enhance risk sharing using an alternative approach to portfolio choice. Kollmann focuses on the potential for equity portfolios to facilitate risk sharing in a complete markets environment. His model also implies a negative correlation between NFA and the conventional measure of the current account.

Following Devereux and Sutherland (2006), we obtain the following expression for the second-order component of the expected excess return

$$E_{t-1}[\hat{r}_{x,t}^{(2)}] = \frac{\rho}{2}\frac{(1-\beta)}{(1-\beta\mu)}\left(\phi(\sigma_K^2 - \sigma_K^{2*}) + (1-\phi)(\zeta\sigma_K^2 - \zeta^*\sigma_K^{2*})\right) \tag{33}$$

The expected excess return on the home country asset is negative if the volatility of the foreign capital income shock exceeds that of the home shock, and the covariance of capital and labour income shocks in the foreign country exceed those in the home country. Intuitively, if $\sigma_K^2 < \sigma_K^{2*}$, then the foreign capital income shock is more responsible for world consumption volatility than the home shock. Investors in both countries then must receive a higher expected return on the foreign asset. Even if $\sigma_K^2 = \sigma_K^{2*}$, however, if $\zeta < \zeta^*$, then again world consumption volatility is more correlated with the foreign asset return, and there is a risk premium on the foreign asset.

A risk premium on the foreign asset translates into an expected long run current account imbalance in the following way. Take expectations of a second-order approximation of (7) yields

$$E_{t-1}[\Delta\hat{W}_t^{(1)} + \Delta\hat{W}_t^{(2)}] = E_{t-1}[\widehat{CA}_t^{(1)} + \widehat{CA}_t^{(2)}] + \tilde{\alpha}E_{t-1}[\hat{r}_{x,t}^{(2)}]$$

The first term on the right hand side is the expected current account surplus, evaluated up to second-order, while the second term is the expected excess return on the external portfolio. If a country holds an external portfolio which commands a positive risk premium, so that $\tilde{\alpha}E_{t-1}[\hat{r}_{x,t}^{(2)}] > 0$, then it can sustain a permanent average current account deficit, and yet keep $E_{t-1}[\Delta\hat{W}_t^{(1)} + \Delta\hat{W}_t^{(2)}] = 0$. For instance, if $\phi(\sigma_K^2 - \sigma_K^{2*}) + (1-\phi)(\sigma_{KL} - \sigma_{KL}^*) < 0$, then the home country's asset is less correlated with world consumption risk. Since $\tilde{\alpha} < 0$, we then have $\tilde{\alpha}E_{t-1}[\hat{r}_{x,t}^{(2)}] > 0$, and country 1 can have a permanent current account deficit equal to this. By acting as a 'safe haven', a country with a low volatility of output can on average consume more than its income, if it is willing to hold more risky foreign assets.

How big can this safe haven effect on the current account be within our simple example? To estimate this, we must combine the solution for $\tilde{\alpha}$ with the expected excess return within the model to obtain:

$$-\frac{\rho}{4}\frac{[\phi(\sigma_K^2 + \sigma_K^{2*}) + (1-\phi)(\zeta\sigma_K^2 + \zeta^*\sigma_K^{2*})]\,[\phi(\sigma_K^2 - \sigma_K^{2*}) + (1-\phi)(\zeta\sigma_K^2 - \zeta^*\sigma_K^{2*})]}{(1-\beta\mu)(\sigma_K^2 + \sigma_K^{2*})}$$

The two key parameters determining the size of this expression are the coefficient of relative risk aversion, and the degree of persistence in endowment shocks. Figure 3 illustrates the excess return and the current account effect. The figure assumes that $\sigma_K^2 = 0.01^2$, and $\sigma_K^{2*} = 0.04^2$, indicating that the foreign country has a much more volatile endowment process. The correlation between capital and labour income in each country is varied in order to allow variation in the value of $\tilde{\alpha}$. We again assume that $\beta = 0.96$, $\phi = 0.36$, $\mu = 0.9$. Clearly, from (33), the excess return will be proportional to the coefficient of relative risk aversion. We allow for $\rho = 1.5$, a conventional value, and a higher value of $\rho = 8$, indicating a high rate of risk aversion, but still well within the range used in asset pricing studies (e.g. Bansal and Yaron, 2004). The gross portfolio position, $-\tilde{\alpha}$, is on the horizontal axis.

For values of $\tilde{\alpha}$ in the range of 0 to -1, the effect of differential risk on the current account is very small. For $\rho = 1.5$, the effect is negligible. But even at the higher value of $\rho = 8$, a 'safe haven' country in this range could expect to have a current account deficit of 0.014 percent of GDP. As total leverage rises, the size of $E_{t-1}[\widehat{VAL}_t^{(2)}]$ rises. For gross asset positions just over 4 times GDP (which is equivalent to a 50 percent holding of total domestic equity), the safe haven effect could finance a current account deficit of 0.25 of a percent of GDP, (with $\rho = 8$). For higher values of μ, this effect is magnified. But even for $\tilde{\alpha} = -5$ and $\mu = .95$, the implied current account deficit is less than 0.5 of a percent of GDP.[20]

4.3 Second-order valuation effects: Portfolio Adjustment

Now let us examine the stochastic properties of $VAL^{(2)}$. Recall that the term $\hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(1)}$ implies that the conditional variance of $VAL^{(2)}$ is time varying. How important is this effect in the determination of the variance of net external assets? That is, how much additional risk-sharing is offered by adjusting the size of the portfolio itself, as opposed to the risk-sharing offered by variable rates of returns for a given portfolio? In order to answer this question, we have to derive a solution for $\hat{\alpha}_t^{(1)}$, the first order component of the portfolio. The resulting expression captures the way in which the portfolio is adjusted in response to movements in the underlying state variables of the economy. Devereux

[20] If we alter preferences to increase the effective rate of risk aversion, the anticipated excess return on the portfolio can increase substantially. For instance, introducing external habit persistence in preferences, such that $U = \frac{1}{1-\sigma}(C_t - \xi\overline{C}_t)^{1-\sigma}$, where $\overline{C}$ is aggregate consumption (where in equilibrium $C = \overline{C}$) increases the *effective* rate of risk aversion. With $\xi = 0.9$, the value of the safe-haven effect is increased by a factor of 10 in the model with habit persistence in preferences.

and Sutherland (2007) show that there is an analytical solution for this, which (for this model) can be written as:

$$\hat{\alpha}_t^{(1)} = \varrho_1 \hat{Y}_{K,t}^{(1)} + \varrho_2 \hat{Y}_{K,t}^{*(1)} + \varrho_3 \hat{Y}_{L,t}^{(1)} + \varrho_4 \hat{Y}_{L,t}^{*(1)} + \varrho_5 \hat{W}_t^{(1)} \tag{34}$$

where the ϱ_i coefficients are complicated functions of parameters and the moments of shocks.[21] These portfolio adjustments will affect the correct measure of valuation, evaluated up to a second-order. In response to movements in the conditional means of consumption and asset returns, agents desire to adjust their portfolio holdings.

How important is the time-variation in the variance of $VAL^{(2)}$? In terms of variance decomposition, Table 4a reports the results of the valuation terms when we solve the model up the second-order. We define the valuation ratios VR_1 and VR_2 respectively as

$$VR_1 = \text{var}(\tilde{\alpha}\hat{r}_{x,t}^{(1)} + \tilde{\alpha}\hat{r}_{x,t}^{(2)})/var(\Delta\hat{W}_t^{(1)} + \Delta\hat{W}_t^{(2)}),$$

and

$$VR_2 = \text{var}(\hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(1)})/var(\Delta\hat{W}_t^{(1)} + \Delta\hat{W}_t^{(2)}).$$

Thus, VR_1 is a measure of the importance of movements in excess returns on the portfolio for the volatility of net foreign assets (up to second-order approximation), holding constant the portfolio holdings. VR_2 is a measure of the volatility in 'portfolio adjustment' as a share of the volatility of net foreign assets. [22] VR_1 is almost the same as the first-order solution VR from Table 3. As gross asset positions rise, the importance of movements in excess returns on the portfolio grows larger. VR_2 was not measured before. For the baseline case, VR_2 is very small relative to VR_1. In Table 4, for a very high degree of home equity bias (and low portfolio diversification), the adjustment of portfolios contributes 3.5 percent of the variation in net external assets. Moreover, as the size of the gross asset

[21] Why does α depend on the shocks and net wealth, as captured in (34)? When (16) is evaluated up to a second-order, a constant $\tilde{\alpha}$ is sufficient to keep the conditional one-step ahead covariance of log consumption and excess returns equal to zero. But when we take a third-order approximation in order to obtain $\hat{\alpha}_t^{(1)}$, the (time-varying) conditional means of consumption and asset returns will affect overall portfolio risk, and agents will have to adjust their portfolio to hedge against this.

[22] Note that this is not the same as 'portfolio rebalancing' (see e.g. Hau and Rey 2008). Since α_t is measured as $Z_{E,t}(\psi_t - 1)$, where $Z_{E,t}$ is the real stock price, and ψ_t is the total share of the home stock held by home agents, changes in ψ_t in response to changes in $Z_{E,t}$ can occur (portfolio rebalancing), even if α_t is held constant.

positions rise, this share falls. Intuitively, as the portfolio position moves towards full risk-sharing, it becomes less necessary to adjust the portfolio in response to shocks.

For more persistent shocks however, portfolio rebalancing can represent a larger fraction of the variability of net foreign assets. Table 4b illustrates the case of $\mu = 0.95$. In that case, we see that the contribution of portfolio adjustment can be as high as 20 percent, when the size of the external gross portfolio is small. Again however, this share diminishes as average portfolios become more diversified.

Despite the small size of the portfolio adjustment term in accounting for the movement in net external assets at the second-order level, it still exhibits the risk-sharing properties of the first-order solution. In Table 3,

$$corr_1 = \text{corr}(\tilde{\alpha}\hat{r}_{x,t}^{(1)} + \tilde{\alpha}\hat{r}_{x,t}^{(2)} + \hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(1)}, \widehat{CA}_t^{(1)} + \widehat{CA}_t^{(2)}),$$

$$corr_2 = \text{corr}(\tilde{\alpha}\hat{r}_{x,t}^{(1)} + \tilde{\alpha}\hat{r}_{x,t}^{(2)}, \widehat{CA}_t^{(1)} + \widehat{CA}_t^{(2)}),$$

$$corr_3 = \text{corr}(\hat{\alpha}_{t-1}^{(1)}\hat{r}_{x,t}^{(1)}, \widehat{CA}_t^{(1)} + \widehat{CA}_t^{(2)}),$$

Thus, the overall second-order valuation term, and the two subcomponents of the valuation term covary negatively with the current account. Portfolio adjustment does play a role as part of the optimal portfolio in the sense that it acts so as to cushion shocks to the current account, as in the case of the first-order valuation effects. But relative to the first-order effect of having an optimally chosen fixed portfolio, this higher-order effect has only a minor impact on the evolution of net external assets.

4.4 Third-order Valuation Effects: Portfolio Adjustment and Time Varying Expected Excess Returns

In the previous sections we examined the terms arising in the first and second-order approximations of VAL. When considering the third order component of VAL it is necessary to analyse the *predictable* time varying behaviour of α and r_x. As discussed in section 3.3 this requires solving for higher-order components of α and r_x. More specifically it is necessary (at least) to solve for the first-order component of α and the *third-order* component of r_x. In the previous section we have already introduced the time-varying solution for $\hat{\alpha}_t^{(1)}$. In conjunction with this first-order solution for $\hat{\alpha}_t^{(1)}$ we may also derive the *expected* third-order component of r_x as a linear function of the state variables as follows

$$E_t[\hat{r}_{x,t+1}^{(3)}] = \delta_1 \hat{Y}_{K,t}^{(1)} + \delta_2 \hat{Y}_{K,t}^{*(1)} + \delta_3 \hat{Y}_{L,t}^{(1)} + \delta_4 \hat{Y}_{L,t}^{*(1)} + \delta_5 \hat{W}_t^{(1)} \tag{35}$$

where again the δ_i coefficients are complicated functions of parameters and the moments of shocks.

Analysis of $E[\widehat{VAL}_t^{(3)}]$ can only be done numerically, via impulse responses. This is reported in the next section.

Note that, while anticipated time variation only enters at a third-order approximation of the VAL, we don't actually need to solve the full model to the third-order to be able to capture the third-order properties of VAL. In fact once we have obtained the solutions of the form (34) and (35), these valuation effects can be evaluated directly from first-order impulse response of the state variables. The next section presents numerical calculations of impulse responses for the example model. These are used to construct numerical calculations for the two terms in (24).

4.5 Impulse Responses and Valuation Effects

To illustrate the role and potential magnitude of the different valuation effects, we consider some impulse responses following an innovation to capital income. These are shown in Figure 4. Again we set $\sigma_K^2 = \sigma_L^2 = \sigma_K^{*2} = \sigma_L^{*2} = .02^2$, $\beta = 0.96$, $\phi = 0.36$, and $\mu = 0.9$ and we choose $\zeta = \zeta^*$ so that home households hold about 90 percent of home equity. Figure 4 shows the impact of a -1% shock to capital income in the home country (Y_K) in period 1. The impact on total income is shown in panel (a). Home country income falls by 0.36% on impact. Panel (b) shows that consumption in the home economy falls by approximately 0.22% in period 1. The impact effect of the shock is therefore to push the home economy into a trade deficit of approximately 0.14% of GDP. This deficit declines to zero as the effects of the shock dissipate.

While the home economy runs a trade and current account deficit following the shock, net foreign assets rise sharply in period 1 and then decline. The sharp rise in NFA in period 1 reflects the first-order unanticipated valuation effect that arises from the effects of the shock on realised equity returns. The shock to home country capital income implies a sharp unanticipated fall in the price of home equity so there is an unanticipated negative excess return on home equity (i.e. $\hat{r}_x$ is negative).[23] Home households have a negative external position in home equity (i.e. $\tilde{\alpha}$ is negative) so the negative excess return in home equity represents a positive valuation effect. The shock to $\hat{r}_x$ is approximately -0.3% so this first-order valuation effect is approximately 0.3% of GDP (i.e. -0.3×-1). This is illustrated in panel (k).

By evaluating the ϱ_i coefficients in (34) we are also able to trace out the dynamic effect of the shock on gross portfolio holdings. These are shown in panel (d) and panel (e). Here $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are home households' holdings of, respectively, home and foreign equity. Panels (d) and (e) show that, for this parameterisation of the model, the movements in gross equity holdings are significantly larger than the movement in NFA. The shock induces home households to increase their gross holdings of home equity by over 1% of GDP while their holdings of foreign equity are reduced by an almost equivalent amount. As discussed in Devereux and Sutherland (2007), the response of gross assets and liabilities following the shock represents a combination of adjustment to overall wealth (the ϱ_5 coefficients) and direct responses to the income shock (the ϱ_1 coefficients).

Evaluation of the δ_i coefficients in (35) allows us also to plot the effects of the shock on the (third-order) expected path of the excess return (i.e. $E[\hat{r}_x^{(3)}]$). This is illustrated in panel (h). The shock leads to a persistent reduction in the expected excess return. The magnitude of this effect is very small however. $E[\hat{r}_x^{(3)}]$ falls by 0.000015% following the shock and gradually returns to zero as the effects of the shock fade.

The dynamic responses of $\hat{\alpha}^{(1)}$ and $E[\hat{r}_x^{(3)}]$ provide us with the information necessary to calculate the two third-order valuation effects in (23). These are illustrated in panel (l). The plot labelled val(α) represents the value of the second term in (23) while the first term in (23) is labelled val(rx). It can be seen from panel (l) that $\hat{\alpha}_{t-1}^{(1)} E_{t-1}[\hat{r}_{x,t}^{(2)}]$ is zero in this parameterisation of the model. This reflects the symmetric nature of the parameterisation, which implies that $E_{t-1}[\hat{r}_{x,t}^{(2)}] = 0$. Dynamic adjustment of $\hat{\alpha}_t^{(1)}$ therefore does not generate any predictable valuation effect. Panel (l) shows however that $\tilde{\alpha} E_{t-1}[\hat{r}_{x,t}^{(3)}]$ is positive following the shock. This reflects the fact that $E[\hat{r}_x^{(3)}]$ is negative (see panel (h)) while $\tilde{\alpha}$

[23] Notice from panel (g) that the prices of both home and foreign equity fall following the shock. The price of foreign equity falls because the expected future rate of return on all equity has to be above its steady state value to be consistent with the rising path of consumption. The price of home equity obviously falls more than the price of foreign equity because the persistent shock to home capital income reduces the income stream to holders of home equity.

is also negative. The persistent negative value of $E[\hat{r}_x^{(3)}]$ therefore represents a positive valuation effect for home households. This effect is, however, minute. At its largest it is only 0.000015 % of GDP! This should be compared to the trade deficit, which is 0.14% of GDP in the period of the shock.

As a further illustration of the size of the third-order valuation effects consider an asymmetric case where $\sigma_K^2 = 0.01^2$ and $\sigma_K^{*2} = .04^2$, i.e. a case where foreign capital income is more volatile than home capital income. This implies a steady-state risk premium in foreign equity of 0.0079% (i.e. $E_{t-1}[\hat{r}_{x,t}^{(2)}] = -0.0079\%$). In this case time variation in $\hat{\alpha}_t^{(1)}$ generates a non-zero valuation effect via the term $\hat{\alpha}_{t-1}^{(1)} E_{t-1}[\hat{r}_{x,t}^{(2)}]$. Impulse responses (not reported) show that this valuation effect is negative (because $E_{t-1}[\hat{r}_{x,t}^{(2)}]$ is negative and $\hat{\alpha}_t^{(1)}$ is positive) and it has a maximum absolute value of 0.0001 following a -1% shock to home capital income. Again this is minute in comparison to the trade deficit created by the shock.

We may therefore conclude that time varying expected returns do act so as to stabilize the impact of a fall in the trade balance in the model. [24]But in practice, this mechanism plays essentially no role at all in the adjustment process in this model. To obtain an economically meaningful pattern of time varying expected valuation effects through movements in excess returns, we would need a model in which risk premia played a much bigger role than they do here, as for instance, in the models of Campbell and Cochrane (1999) or Bansal and Yaron (2004).

5 Real Exchange Rate Valuation Effects

Up to now, all the valuation effects we have analysed are due to changes in asset prices (and to a minor extent, dividend payments). But Lane and Milesi Ferretti (2007) emphasize the importance of exchange rate changes in valuation gains and losses. To do full justice to the role of nominal exchange rate variation on the valuation of net external assets is beyond the scope of the present paper. However, we can easily extend the model to incorporate the impact of terms of trade and *real* exchange rate movements in generating

[24] We experimented with a range of parameter values for the persistence of the shocks and relative risk aversion. In all cases, the results were similar, with the effects of time-varying expected returns being very small. Note that a similar result on the stabilization features of expected returns is found in a different context by Pavlova and Rigobon (2008). They also found that the effect is small.

valuation effects. In this section, we do this by allowing for differentiation between home and foreign goods, as well as introducing another shock in the form of a 'demand shock'. We also allow for trade in bonds whose payoffs are defined in terms of units of home or foreign goods. In some cases, this simple extension has quite dramatic implications for the structure of external portfolios, as well as the source of valuation effects.

To save space, we describe the extended model in the Appendix. We assume that agents in both countries have elasticity of substitution θ between the consumption of home and foreign goods. In addition, the share of the home good in the home price index is γ_t. We allow for home bias in consumption, so that $E(\gamma) > 0.5$. In addition, we allow for 'demand shocks', which affects the intensity of preferences for the home good relative to the foreign good. In particular, assume that

$$\gamma_t = \gamma \exp(v_t)$$

where $v_t = \varsigma v_{t-1} + \varepsilon_{v,t}$, where $\varepsilon_{v,t}$, is a zero-mean i.i.d. shock which are symmetrically distributed over the interval $[-\epsilon, \epsilon]$ with $Var[\varepsilon_v] = \sigma_v^2$. The foreign household has preferences with weight $\gamma_t^* = \gamma \exp(-v_t)$ on the *foreign* good. This specification means that a positive v_t shock increases both home *and* foreign demand for the home good.

5.1 Equity Trade Only

First, look at the case of equity trade only. The solution for $\tilde{\alpha}$ is then a complicated function of parameters and shock variance and covariances. In the special case where $\gamma = 0.5$ and $\sigma_K^{*2} = \sigma_K^2$ however, we may express the solution as

$$\begin{aligned}\tilde{\alpha}_e &= -\frac{(\theta-1)}{2(1-\beta)}\frac{(1-\beta\varsigma)^2[1-\theta+(1-\phi)(1-\zeta)(\theta-2\phi)]\sigma_K^2}{(1-\beta\varsigma)^2[(1-\theta)^2+2(1-\phi)(1-\zeta)(\theta-\phi)]\sigma_K^2+2(1-\beta\mu)^2\sigma_v^2} \\ &\quad -\frac{1}{(1-\beta)}\frac{(1-\beta\mu)^2\sigma_v^2}{(1-\beta\varsigma)^2[(1-\theta)^2+2(1-\phi)(1-\zeta)(\theta-\phi)]\sigma_K^2+2(1-\beta\mu)^2\sigma_v^2} \qquad (36)\end{aligned}$$

When $\theta \to \infty$, this recovers (25), the equilibrium equity holding in the one good model, and demand shocks play no role in the demand for equity. On the other hand, when $\theta = 1$, endogenous movements in the terms of trade act so as to fully insure against endowment shocks. The equity position is then designed only to hedge against demand shocks. A positive demand shock will increase home relative to foreign consumption, and also increase the return on home relative to foreign equity (due to a terms of trade improvement). Thus, the home country will diversify out of domestic equity. In addition, when $\phi = 1$, we again have $\tilde{\alpha}_e = -0.5/(1-\beta)$, so there is perfect pooling, as before, since this equity position perfectly hedges against both demand shocks and endowment shocks.

5.2 Bond and Equity Trade

Now extending the asset menu to allow for bond trading, we may solve for the equilibrium vector of portfolio holdings of equities and bonds. First, we may show that in the special case of no demand shocks at all (i.e. $\sigma_v^2 = 0$)

$$\tilde{\alpha}_b = \left[\begin{array}{ccc} 0 & 0 & \frac{(1-\gamma)(2\gamma-1-\rho(2\gamma\theta-1)}{\rho(1-\mu\beta)} \end{array} \right]. \tag{37}$$

The striking feature of (37) is that equilibrium equity holdings are zero. Home agents hold no foreign equity, and take no measures to diversify away their status quo holding of all domestic equity. This means that, when agents can trade in real bonds, there is *complete home bias* in the equity portfolio. This holds independently of the size of shocks, the covariance of capital and labour income shocks ζ, the value of the elasticity θ, or any other parameters of the model.[25] Moreover, we can easily establish that (37) achieves full cross country risk sharing (up to first-order) in the model with endogenous terms of trade movements and $\sigma_v^2 = 0$. That is, an optimal bond portfolio supports complete assets markets, with no need for any foreign equity portfolio[26].

Why do agents not wish to hold equity in portfolios in this example? The reason is that the bond portfolio allows a claim on the terms of trade, and the deviation from full risk-sharing across countries is also proportional to the terms of trade. Up to first-order, the relative rate of return on bonds (i.e. foreign bonds relative to home bonds) is equal to the movement in the terms of trade. Full risk-sharing requires that consumption movements adjusted for real exchange rate movements are equalized across countries. But

[25] To be clear, there is a singularity at the points $\phi = 1$, and/or $\theta = 1$, where perfect pooling of equity would achieve the same allocation as (37).

[26] In a different context, with multiple shocks and capital accumulation, Coeurdacier et al. (2008) show how a combination of bond holdings and equity holdings can support complete risk sharing, with equity holding motives based on variations in income orthogonal to terms of trade movements. Coeurdacier and Gourinchas (2008) show that such a separation in equity and bond portfolios is supported empirically. Finally, Devereux and Saito (2007), in a very different context, also show that bond holding may substitute for international trade in equity.

departures from this are also driven by movements in the terms of trade. Hence, a bond portfolio can ensure full cross country risk sharing.

In general, $\tilde{\alpha}_b = \frac{(1-\gamma)(2\gamma-1-\rho(2\gamma\theta-1)}{\rho(1-\gamma\beta)}$ may be positive or negative. In the case $\gamma = 0.5$, the sign of $\tilde{\alpha}_b$ is determined only by the size of θ. When $\theta > 1$, home agents hold a negative position in foreign denominated bonds. This is because relative home consumption rises as relative home endowments increase, when $\theta > 1$, while simultaneously, the return on the home bond tends to fall, as the terms of trade depreciates, so that home bonds represent a good hedge against consumption risk. [27]

In the more general case where there are both demand and endowment shocks, the optimal portfolio will include both equity and bond holdings. We now explore the importance of valuation effects in this setting.

5.3 Valuation Effects

We may compute the analogous valuation terms that we constructed in the one-good model above. Table 4 reports the results, using the benchmark calibration, for the economy with trade only in equities, and the case where both equities and bonds are traded. We also report the breakdown of valuation effects into dividend payments, asset price movements, and relative price movements. In the baseline calibration, we set all parameters as in Table 3, except $\theta = 1.5$, and $\gamma = 0.6$, which are close to standard consensus values for these parameters in the literature. We assume that endowment shocks have the same volatility and persistence as before. We set ζ equal to the same value as in the baseline case of the previous model. As regards the calibration for demand shocks, there is little empirical evidence to determine σ_v^2 and ς. We follow Couerdacier et al. (2007) in setting $\sigma_v^2 = 0.01^2$, and choose the same persistence for demand shocks as for endowment shocks. Again, we assume all shocks are independent. Finally, we set the intertemporal elasticity of substitution, ρ= 1.5.

Table 5 shows the values for the portfolio positions and valuation effects. In the equity only economy, there is still considerable home-bias, given our calibration. Home investors hold about 70 percent of home equity. In the equity-bond economy, home agents take a positive position in foreign bonds, and a negative external position in domestic equity,

[27] When $\gamma > 0.5$, it is no longer true that the sign of $\tilde{\alpha}_b$ is determined only by the size of θ. But, for reasonable values of parameters, we would anticipate that $\tilde{\alpha}_b < 0$, as before.

backed by a positive holding of foreign equity.[28] Equity holdings display considerable home bias - home agents hold 82 percent of home equity. Bond holdings carry most of the weight in terms of cross-country risk-sharing. Gross bond holdings are about 100 percent of GDP.

Table 5 also illustrates the details of the valuation effects in this extended model. In both the equity-only economy and the equity-bond economy, the volatility of the valuation term is considerably higher than in the previous model, as we now have an additional shock. In both cases, the standard deviation of the valuation term is about 3 percent. In both cases also, the valuation ratio significantly higher than before, at the same value of $\tilde{\alpha}$.

How are the valuation effects decomposed between dividend movements, asset price movements, and terms of trade movements? Table 5 shows that in the model with only equity trade, most of the variance of the valuation effect is accounted for by movements in asset prices. Equity price volatility accounts for about eight times as much in terms of overall valuation movements as does dividend movements. Also, the share of volatility accounted for by movements in the terms of trade is very small.

In the case with both equity and bond trade, the importance of terms of trade in valuation effects increases considerably. Table 5 shows that, in this case, the dividend movements play only a tiny role in valuation effects. Asset price movements remain important, but a significant part of the volatility of the valuation term is accounted for by movements in the terms of trade.

These results indicate that the size and composition of first-order valuation effects may be affected in important ways by the structure of the economy and the availability of assets. Nevertheless, the valuation mechanism still operates in the same way as in the simple model. Valuation effects act so as to enhance risk sharing between countries. In all cases, we see a negative correlation between the current account and the valuation terms.

For brevity, we do not report higher order aspects of valuation effects for the extended model. As we would expect from the results of the previous section, in the current model, expected valuation effects (evaluated up to second-order) and time varying expected valuation effects (evaluated up to third-order) are very small.

[28] A positive holding in foreign bonds ensures that consumption is insured against demand shocks, which, in the absence of portfolio diversification, would both increase consumption and increase the return on home bonds through a terms of trade appreciation.

6 Conclusion

This paper has shown how recent developments in the analysis of portfolio structure in open economy models may be applied to study the role of valuation effects in the movement of net external assets. While in traditional balance of payments theory, the change in net external assets should be equal to the current account, empirical evidence indicates that for most countries, the evolution of net external assets is dominated by valuation gains and losses coming from changes in asset prices and exchange rates, which do not enter into the measured current account. This gives rise to a valuation term, which can be measured as the gap between the change in net external assets and the current account. The paper shows that a simple model of risk-sharing based on optimal portfolio choice can provide a reasonable qualitative and quantitative account of the properties of this valuation term up to the first-order, where valuation effects are 'unanticipated'. The source of these valuation effects, the degree to which they act so as to provide cross country risk sharing, and their decomposition into asset price changes and terms of trade changes, will depend on the structure of international goods markets and the availability of international assets.

Recent literature has also suggested the presence of 'anticipated', or higher-order valuation effects, giving rise to anticipated average excess returns and anticipated time-varying excess returns. We show that these higher order effects do in principle play a role in the movement of net external assets. In practice however, for the benchmark international macro model with standard preferences and realistically calibrated consumption risk, these effects are quantitatively very small.

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Table 1: Valuation term based on Current Account data

	sd(val)	VR	corr(val,ca)	corr(val,gdp)	ar(1)	mean(val)
Australia	0.06	0.88	0.30	-0.03	0.13	0.00
Austria	0.04	1.12	-0.34	0.05	0.20	0.00
Canada	0.04	1.12	-0.39	0.05	0.61**	0.01
Denmark	0.05	0.62	0.22	-0.03	-0.08	-0.01
France	0.04	0.94	-0.06	0.00	-0.19	0.00
Germany	0.03	0.45	0.24	0.08	-0.13	0.00
Iceland	0.06	0.44	0.09	-0.18	-0.17	-0.02
Ireland	0.15	0.92	0.05	-0.23	-0.08	0.00
Italy	0.04	1.05	-0.26	-0.16	-0.01	0.00
Japan	0.03	0.98	-0.13	0.06	-0.39	-0.01
Korea	0.05	0.88	-0.31	0.30	0.07	-0.02
Mexico	0.04	1.28	-0.50	0.22	-0.16	-0.01
Netherlands	0.10	0.99	-0.08	0.04	-0.21	-0.05
New Z.	0.13	0.99	-0.08	-0.08	0.39*	-0.01
Norway	0.05	0.60	-0.35	-0.27	0.15	-0.01
Portugal	0.04	0.42	0.09	0.32	-0.01	0.00
Spain	0.04	0.70	0.11	-0.02	0.01	-0.01
Sweden	0.11	1.14	-0.35	-0.07	0.04	-0.02
Switzerland	0.15	1.14	-0.36	-0.06	0.09	-0.01
Turkey	0.04	0.71	-0.01	-0.19	0.10	-0.02
UK	0.05	0.89	0.01	-0.10	-0.24	0.00
US	0.03	1.40	-0.54	0.11	0.31	0.01

Sd(val) refers to the standard deviation of the VAL term, using the current account as residual, VR refers to the variance of VAL/GDP, corr(VAL,CA) and corr(VAL,GDP) refers to the correlation of VAL with the current account and GDP respectively. AR(1) is the estimated coefficient of VAL on its lagged value. Mean(VAL) is the average VAL over the sample. Sample 1980-2007.

Table 2: Valuation term based on Trade Balance data

	sd(vai)	VR	corr(vai,ca)	corr(vai,gdp)	ar(1)	mean(vai)
australia	0.06	0.88	0.3	-0.02	0.15	-0.03
austria	0.04	1.49	-0.58	0.13	0.38	0.02
canada	0.04	1.07	-0.29	0.06	0.54*	-0.04
denmark	0.05	0.78	0.12	-0.03	0.01	-0.03
france	0.04	0.93	0.01	0.04	-0.2	0
germany	0.03	0.57	0.33	0.21	0.12	-0.02
iceland	0.06	0.55	0.35	-0.26	-0.05	-0.06
ireland	0.18	1.22	-0.45	-0.16	0.14	-0.15
italy	0.07	1.05	-0.27	-0.08	-0.03	-0.01
japan	0.03	1	-0.13	-0.02	0.45*	-0.01
korea	0.05	1.02	-0.37	0.36	0.19	-0.03
Mexico	0.04	1.37	-0.58	0.34	-0.03	-0.03
netherlands	0.1	1.02	-0.14	0.07	-0.27	-0.05
new zealand	0.13	0.99	-0.06	-0.07	0.41*	-0.08
norway	0.05	0.57	-0.19	-0.3	0.12	-0.03
portugal	0.06	0.67	0.09	0.37	0.52*	0.06
Spain	0.06	0.81	0.19	0.02	0.15	0.01
Sweden	0.1	0.96	-0.01	0.01	-0.12	-0.05
Switzerland	0.14	1	-0.1	0	0.04	0.07
Turkey	0.04	0.85	-0.12	-0.28	0.15	0.02
UK	0.05	0.86	0.05	-0.05	-0.26	0.02
US	0.03	1.29	-0.48	0.07	0.26	0.02

Sd(val) refers to the standard deviation of the VAL term using the trade balance as residual, VR refers to the variance of VAL/GDP, corr(VAI,CA) and corr(VAI,GDP) refers to the correlation of VAL with the current account and GDP respectively. AR(1) is the estimated coefficient of VAI on its lagged value. Mean(VAI) is the average VAI over the sample. Sample 1980-2007.

Table 3a

Home Eq.	Stdval	Valrat	Corr(val,CA)
0.89	0.008	0.82	-0.19
0.84	0.012	1	-0.26
0.8	0.015	1.1	-0.32
0.76	0.018	1.17	-0.38
0.71	0.022	1.21	-0.43
0.67	0.025	1.23	-0.48
0.62	0.028	1.26	-0.52
0.58	0.032	1.27	-0.56
0.53	0.035	1.28	-0.6

Table 3b

Home Eq.	Stdval	Valrat	Corr(val,CA)
0.89	0.014	1.08	-0.27
0.84	0.02	1.12	-0.36
0.8	0.026	1.14	-0.44
0.76	0.032	1.15	-0.51
0.71	0.038	1.16	-0.56
0.67	0.043	1.16	-0.61
0.62	0.049	1.17	-0.66
0.58	0.055	1.17	-0.7
0.53	0.061	1.17	-0.73

Table 3a represents the case with shock persistence 0.9. Table 3b has persistence 0.95. Home eq. represents the share of home equity held by home consumers. Stdval is the standard deviation of the valuation term. Valrat is the ratio of the variance of VAL to the variance of the change in NFA. Corr(Val,CA) is the correlation of VAL and the current account.

Table 4a

Home eq.	VR1	VR2	Corr1	Corr2	Corr3
0.97	0.18	0.035	-0.08	-0.06	-0.05
0.93	0.5	0.021	-0.13	-0.12	-0.04
0.9	0.75	0.01	-0.18	-0.18	-0.04
0.87	0.92	0.007	-0.23	-0.23	-0.04
0.83	1.03	0.004	-0.28	-0.28	-0.03
0.8	1.1	0.003	-0.33	-0.32	-0.03
0.71	1.21	0.001	-0.43	-0.43	-0.02
0.58	1.27	0.001	-0.57	-0.56	-0.02
0.53	1.28	0.001	-0.6	-0.6	-0.01

Table 4b

Home eq.	VR1	VR2	Corr1	Corr2	Corr3
0.97	0.43	0.209	-0.12	-0.08	-0.09
0.93	0.82	0.09	-0.18	-0.16	-0.08
0.9	0.99	0.04	-0.24	-0.23	-0.08
0.87	1.08	0.022	-0.3	-0.3	-0.07
0.83	1.12	0.013	-0.36	-0.35	-0.06
0.8	1.14	0.008	-0.41	-0.41	-0.06
0.71	1.17	0.003	-0.53	-0.53	-0.04
0.57	1.19	0.001	-0.67	-0.67	-0.03
0.53	1.19	0.001	-0.7	-0.7	-0.03

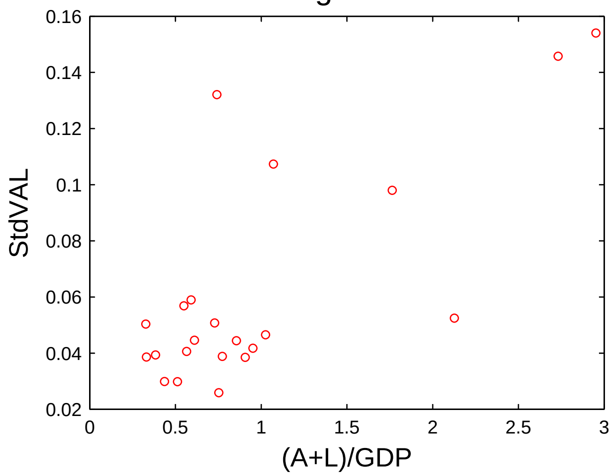
Table 4a represents the case with shock persistence 0.9. Table 4b has persistence 0.95. Home eq. represents the share of home equity held by home consumers. VAR1 and VAR2 represent the share of valuation changes due to excess returns (up to 2nd order), and portfolio re-balancing in total net foreign asset changes. Corr1, Corr2, and Corr3 represent correlations of valuation effects with the current account, as defined in the text.

Table 5: Valuation effects in the Two Good Model

Equity Only	sd(val)	VR	corr(val,CA)
	0.031	1.12	-0.67
Home Eq. 0.7	sd(val_div)	sd(val_DQ)	sd(val_DP)
	0.003	0.022	0.002
Equity and Bond Trade	sd(val)	VR	corr(val,CA)
Home eq. 0.82	0.03	1.14	-0.7
Home Bond 0.97	sd(val_div)	sd(val_DQ)	sd(val_DP)
	0.002	0.011	0.012

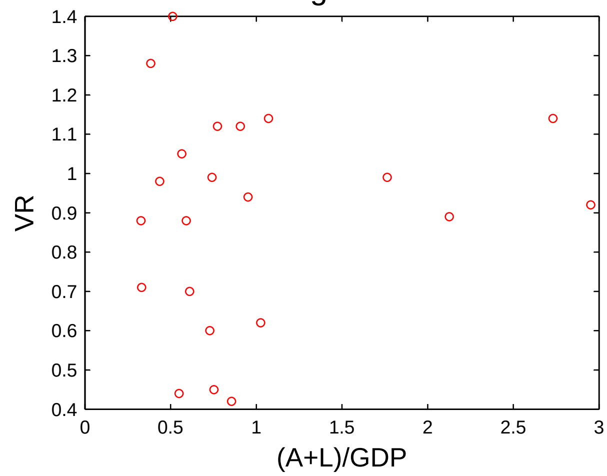
sd(val), VR, and corr(val,CA) refers to the standard deviation of the VAL term, the variance of the ratio of VAL to the CA, and the correlation of VAL with the CA. sd(val_div), sd(val_DQ), and sd(val_DP) are the components of total valuation volatlity coming from dividend movements, asset price movements, and terms of trade movements, respectively.

Figure 1



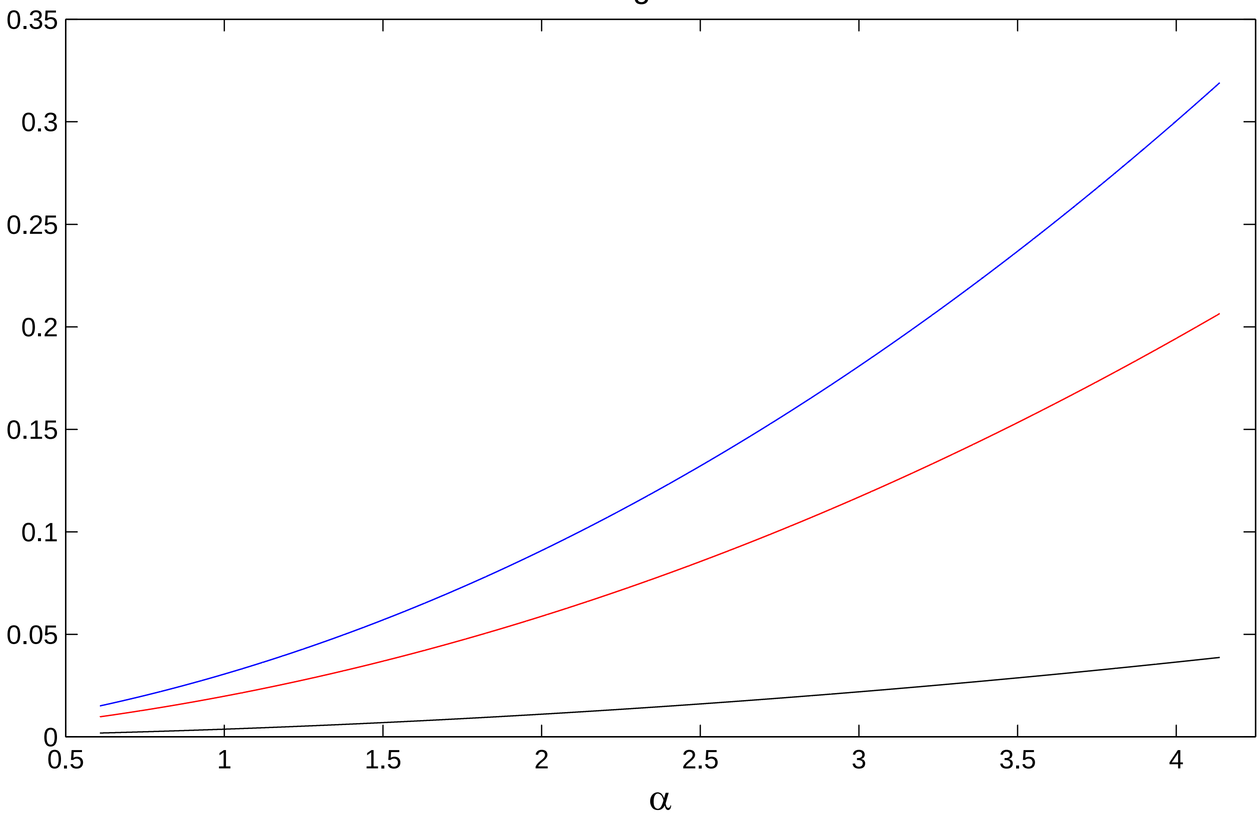
Total gross assets+liabilities vs. standard deviation of VAL

Figure 2



Total gross assets+liabilities vs. valuation ratio

Figure 3



The figure shows the expected value of the valuation term evaluated up to 2nd order, for various parameterisations

Figure 4: Impulse responses

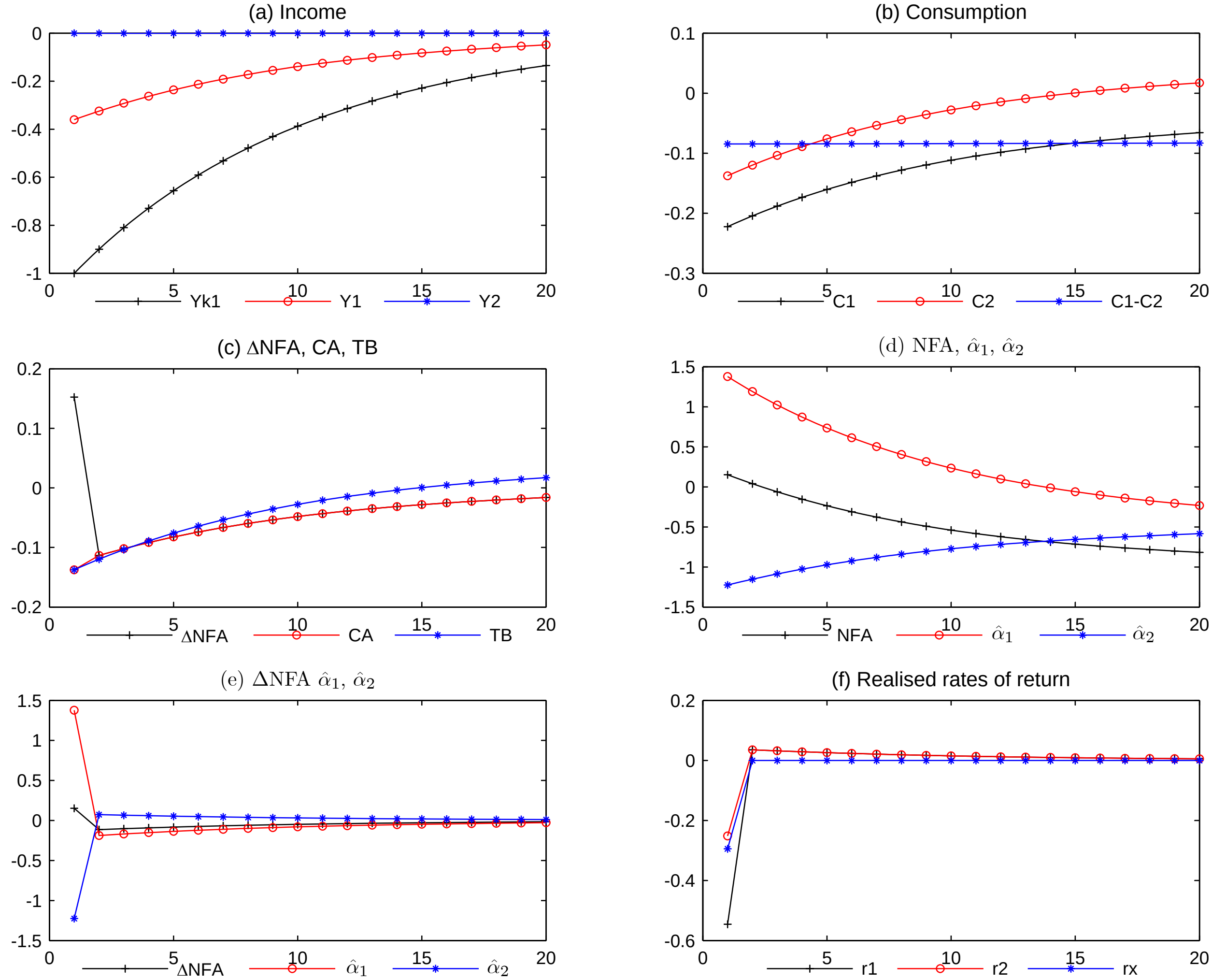
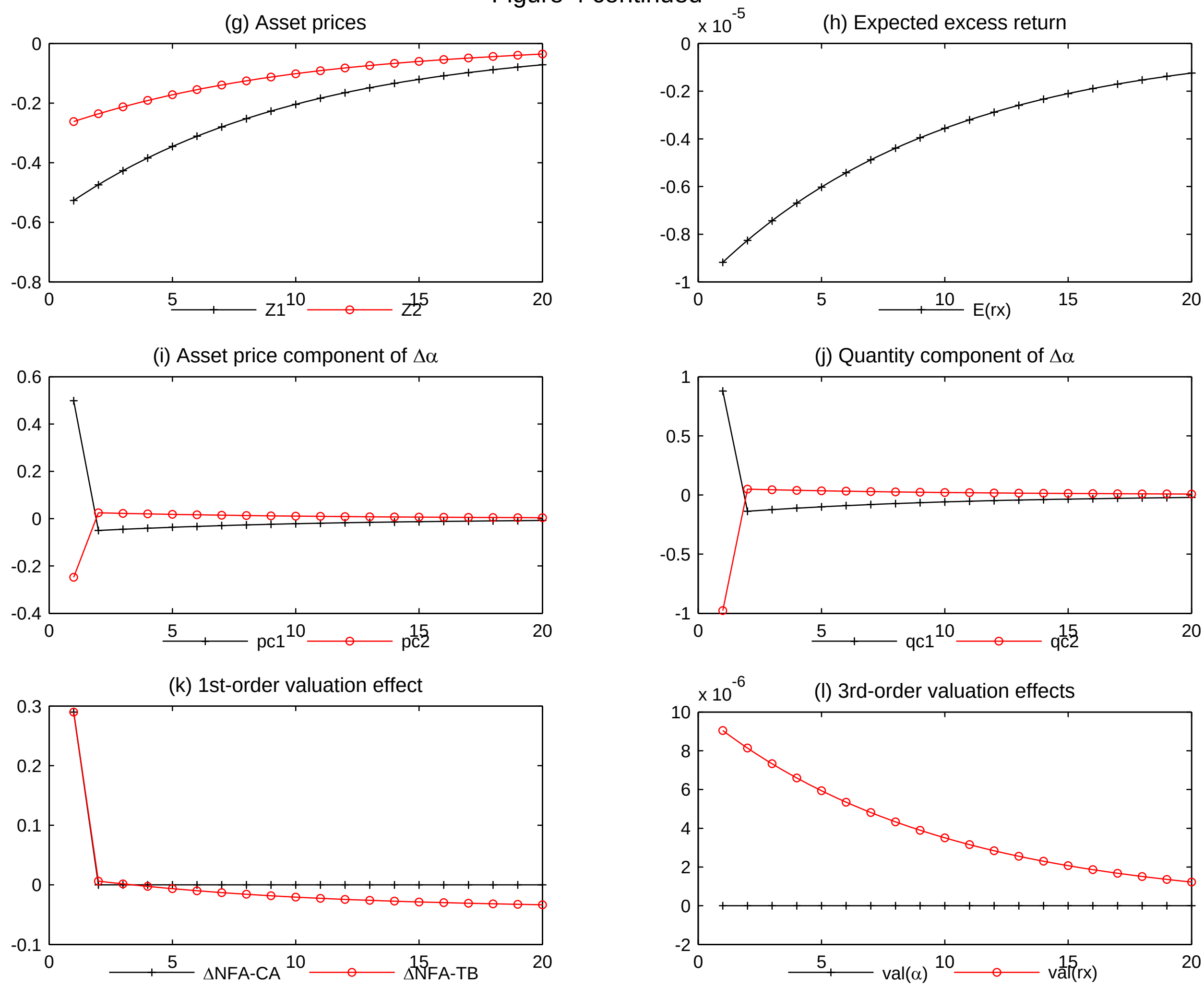


Figure 4 continued



Valuation Effects and the Dynamics of Net External Assets
Michael B Devereux and Alan Sutherland
Technical Appendices (Not for Publication)

Appendix 1: Model solution

This appendix briefly outlines the solutions approach described in Devereux and Sutherland (2006, 2007).

The solution for the steady state portfolio, $\tilde{\alpha}$ is based on a second-order approximation of the portfolio optimality conditions. These are given by

$$E_t\left[\hat{r}^{(2)}_{x,t+1} - \rho\hat{C}^{(1)}_{t+1}\hat{r}^{(1)}_{x,t+1}\right] = 0 \tag{38}$$

$$E_t\left[\hat{r}^{(2)}_{x,t+1} - \rho\hat{C}^{*(1)}_{t+1}\hat{r}^{(1)}_{x,t+1}\right] = 0 \tag{39}$$

Equations (38) and (39) can be combined to show

$$E_t\left[(\hat{C}^{(1)}_{t+1} - \hat{C}^{*(1)}_{t+1})\hat{r}^{(1)}_{x,t+1}\right] = 0 \tag{40}$$

and

$$E_t\left[\hat{r}^{(2)}_{x,t+1}\right] = \frac{\rho}{2}E_t\left[(\hat{C}^{(1)}_{t+1} + \hat{C}^{*(1)}_{t+1})\hat{r}^{(1)}_{x,t+1}\right] \tag{41}$$

These two equations express the portfolio optimality conditions in a form which is particularly convenient for deriving equilibrium portfolio holdings and excess returns. Equation (40) provides an equation which must be satisfied by equilibrium portfolio holdings, $\tilde{\alpha}$. And equation (12) shows the corresponding equilibrium expected excess return.

In order to evaluate the left hand side of equation (40) it is sufficient to derive expressions for the first-order behaviour of consumption and excess returns. This requires a first-order accurate solution for the non-portfolio parts of the model. Portfolio decisions affect the first-order solution of the non-portfolio parts of the model in a particularly simple way. This is for three reasons. First, portfolio decisions only enter the non-portfolio parts of the model via budget constraints.[29] Second, the only aspect of the portfolio

[29] In fact, this property is not critical for the implementation of the solution method. It is straightforward to generalise the method to handle cases where portfolio decisions affect equations other than the budget constraint.

decision that enters a first-order approximation of the budget constraints is $\tilde{\alpha}$, the steady-state portfolio. And third, to a first-order approximation, the portfolio excess return is a zero mean i.i.d. random variable. The fact that only the steady-state portfolio enters the first-order model can be illustrated by considering a first-order approximation of the home budget constraint. For period $t+1$ this is given by

$$\hat{W}_{t+1}^{(1)} = \frac{1}{\beta}\hat{W}_{t}^{(1)} + \hat{Y}_{t+1}^{(1)} - \hat{C}_{t+1}^{(1)} + \tilde{\alpha}\hat{r}_{x,t+1}^{(1)} \tag{42}$$

where $\hat{W}_t = (W_t - \bar{W})/\bar{Y}$. Notice that the deviation of α from its steady-state value does not enter this equation because excess returns are zero in the non-stochastic steady state, i.e. $\bar{r}_x = 0$.

The three properties just listed imply that it is possible to combine (42) with the other non-portfolio equations of a model to obtain expressions for $\hat{C}_{t+1}^{(1)}$, $\hat{C}_{t+1}^{*(1)}$ and $\hat{r}_{x,t+1}^{(1)}$ conditional on $\tilde{\alpha}$. These expressions can then be substituted into (40) and the resulting equation can be solved to yield the equilibrium value of $\tilde{\alpha}$.

Having solved for $\tilde{\alpha}$ it is simple to derive the reduced form solutions for $\hat{C}_{t+1}^{(1)}$, $\hat{C}_{t+1}^{*(1)}$ and $\hat{r}_{x,t+1}^{(1)}$ which can be substituted into (41) to obtain a solution for $E_t[\hat{r}_{x,t+1}^{(2)}]$. It is in this sense that the steady state behaviour of α can be solved in conjunction with the expected second-order behaviour of the excess return. Notice that the resulting solution for $E_t[\hat{r}_{x,t+1}^{(2)}]$ will be a function of one-period ahead conditional second moments.

The first-order behaviour of α can be obtained using a similar procedure applied to third-order approximations of the portfolio optimality conditions and second-order approximations to the other parts of the model. Taking a third-order approximation of the home and foreign country portfolio first-order conditions yields

$$E_t\left[\begin{array}{c} \hat{r}_{x,t+1}^{(2)} + \hat{r}_{x,t+1}^{(3)} \\ -\rho\hat{C}_{t+1}^{(1)}\hat{r}_{x,t+1}^{(1)} - \rho\hat{C}_{t+1}^{(1)}\hat{r}_{x,t+1}^{(2)} - \rho\hat{C}_{t+1}^{(2)}\hat{r}_{x,t+1}^{(1)} + \frac{\rho^2}{2}\hat{C}_{t+1}^{(1)2}\hat{r}_{x,t+1}^{(1)} \end{array}\right] = 0 \tag{43}$$

$$E_t\left[\begin{array}{c} \hat{r}_{x,t+1}^{(2)} + \hat{r}_{x,t+1}^{(3)} \\ -\rho\hat{C}_{t+1}^{*(1)}\hat{r}_{x,t+1}^{(1)} - \rho\hat{C}_{t+1}^{*(1)}\hat{r}_{x,t+1}^{(2)} - \rho\hat{C}_{t+1}^{*(2)}\hat{r}_{x,t+1}^{(1)} + \frac{\rho^2}{2}\hat{C}_{t+1}^{*(1)2}\hat{r}_{x,t+1}^{(1)} \end{array}\right] = 0 \tag{44}$$

Combining these two conditions implies that portfolio holdings must ensure that the following holds

$$E_t\left[\begin{array}{c} -\rho(\hat{C}_{t+1}^{(1)} - \hat{C}_{t+1}^{*(1)})\hat{r}_{x,t+1}^{(2)} - \rho(\hat{C}_{t+1}^{(2)} - \hat{C}_{t+1}^{*(2)})\hat{r}_{x,t+1}^{(1)} \\ +\frac{\rho^2}{2}(\hat{C}_{t+1}^{(1)2} - \hat{C}_{t+1}^{*(1)2})\hat{r}_{x,t+1}^{(1)} \end{array}\right] = 0 \tag{45}$$

while expected returns are given by

$$E_t[\hat{r}^{(3)}_{x,t+1}] = E_t\left[\begin{array}{c} \frac{\rho}{2}(\hat{C}^{(1)}_{t+1} + \hat{C}^{*(1)}_{t+1})\hat{r}^{(2)}_{x,t+1} + \frac{\rho}{2}(\hat{C}^{(2)}_{t+1} + \hat{C}^{*(2)}_{t+1})\hat{r}^{(1)}_{x,t+1} \\ -\frac{\rho^2}{4}(\hat{C}^{(1)2}_{t+1} + \hat{C}^{*(1)2}_{t+1})\hat{r}^{(1)}_{x,t+1} \end{array}\right] \tag{46}$$

These are the third-order equivalents of (40) and (41).

Notice that the left hand side of (45) can be evaluated using first and second order expressions for consumption and excess returns. It is thus sufficient to solve a second-order approximation to the non-portfolio parts of the model. As before, portfolio decisions affect the second-order solution of the non-portfolio parts of the model in a particularly simple way. This is again for three reasons. First, portfolio decisions only enter the non-portfolio parts of the model via budget constraints. Second, the only aspects of the portfolio decision that enters a second-order approximation of the budget constraints are $\tilde{\alpha}$ and $\hat{\alpha}^{(1)}_t$. And third, to a first-order approximation, the portfolio excess return at the first and second orders is a zero mean i.i.d. random variable. The fact that only $\tilde{\alpha}$ and $\hat{\alpha}^{(1)}_t$ enter the second-order model can be illustrated by considering a second-order approximation of the home budget constraint. For period $t+1$ this is given by

$$\begin{aligned} \hat{W}^{(1+2)}_{t+1} &= \frac{1}{\beta}\hat{W}^{(1+2)}_t + \hat{Y}^{(1+2)}_{t+1} - \hat{C}^{(1+2)}_{t+1} + \tilde{\alpha}\hat{r}^{(1+2)}_{x,t+1} + \frac{1}{2}\hat{Y}^{(1)2}_{t+1} \\ &\quad -\frac{1}{2}\hat{C}^{(1)2}_{t+1} + \hat{\alpha}^{(1)}_t\hat{r}^{(1)}_{x,t+1} + \frac{1}{\beta}\hat{W}^{(1)}_t\hat{r}^{(1)}_{2,t+1} \end{aligned} \tag{47}$$

where, to simplify notation, for variable X the sum of the first and second-order components is denoted $\hat{X}^{(1+2)}$, i.e. $\hat{X}^{(1+2)} = \hat{X}^{(1)} + \hat{X}^{(2)}$.

Equation (47) can be used in conjunction with the other non-portfolio equations of the model to obtain solutions for $\hat{C}^{(2)}_{t+1}$, $\hat{C}^{*(2)}_{t+1}$ and $\hat{r}^{(2)}_{x,t+1}$. These can be substituted into (45) and the resulting equation can be solved to yield a solution for $\hat{\alpha}^{(1)}_t$.

Having solved for $\hat{\alpha}^{(1)}_t$ it is simple to derive the reduced form solutions for $\hat{C}^{(2)}_{t+1}$, $\hat{C}^{*(2)}_{t+1}$ and $\hat{r}^{(2)}_{x,t+1}$ which can be substituted into (46) to obtain a solution for $E_t[\hat{r}^{(3)}_{x,t+1}]$. It is in this sense that the first-order dynamics of α can be solved in conjunction with the expected third-order behaviour of the excess return.

Note that all the approximations here are fundamentally based on Samuelson's (1970) theorem on portfolio approximations. In this sense, they apply strictly for only very small shocks around a non-stochastic steady state. Thus, the model is of limited applicability for dealing with global shocks, in much the same way as is the solution of DSGE models up to first or second order.

Appendix 2: The extended model

Here we spell out the details of the model with endogenous terms of trade, preference shocks, and trade in bonds and equity. Again, agents in the home country have utility functions of the form given by (10). Now however, C is a consumption index defined across all home and foreign goods, given by:

$$C_t = \left[\gamma_t^{\frac{1}{\theta}} C_{Ht}^{\frac{\theta-1}{\theta}} + (1-\gamma_t)^{\frac{1}{\theta}} C_{Ft}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{48}$$

where C_H and C_F are aggregators over individual home and foreign produced goods. The parameter θ in (48) is the Armington elasticity of substitution between home and foreign goods. The parameter γ_t measures the importance of consumption of the home good in preferences. For $\gamma_t > 0.5$, we have 'home bias' in preferences. We assume that γ_t is affected by a stochastic 'demand' shock, which affects the intensity of preferences for the home good relative to the foreign good. In particular, assume that

$$\gamma_t = \gamma \exp(v_t)$$

where $v_t = \varsigma v_{t-1} + \varepsilon_{v,t}$, where $\varepsilon_{v,t}$, is a zero-mean i.i.d. shock which are symmetrically distributed over the interval $[-\epsilon, \epsilon]$ with $Var[\varepsilon_v] = \sigma_v^2$. The foreign consumption aggregator analogous to (48) is defined as

$$C_t^* = \left[\gamma_t^{*\frac{1}{\theta}} C_{Ft}^{*\frac{\theta-1}{\theta}} + (1-\gamma_t^*)^{\frac{1}{\theta}} C_{Ht}^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{49}$$

where $\gamma_t^* = \gamma \exp(-v_t)$. Thus, when $\gamma > 0.5$, there is on average home bias towards the domestically produced good in both home and foreign preferences. But a positive shock to v_t will shift both home *and* foreign demand towards the home produced good, and away from the foreign produced good.

Given this specification, the aggregate CPIs for home and foreign agents are therefore

$$P_t = \left[\gamma_t P_{Ht}^{1-\theta} + (1-\gamma_t) P_{Ft}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{50}$$

$$P_t^* = \left[(1-\gamma_t^*) P_{Ht}^{1-\theta} + \gamma_t^* P_{Ft}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{51}$$

where P_H and P_F are the aggregate price indices for home and foreign goods.

The budget constraint of the home country agent is then

$$P_t C_t + P_t W_{t+1} = P_{Ht} Y_t + P_t \sum_{k=1}^{N} \alpha_{k,t-1} r_{kt} \tag{52}$$

where W_t denotes home country net external assets in terms of the home consumption basket. The final term represents the total return on the home country portfolio. We now allow for trade in up to $N = 4$ assets; home and foreign equity, as well as home and foreign bonds. We compare an equilibrium where agents trade only in equities to one where there is trade in both equities and bonds.

Equity prices and returns are as described above, except that home equity is now a claim on the capital income return on the home good, Y_{Kt}, and similarly for foreign equity. Thus, the return on the home equity, in terms of the home CPI, is given by

$$r_{et} = \frac{P_{Ht}Y_{Kt}/P_t + Z_{Et}}{Z_{Et-1}} \tag{53}$$

where Z_E is now the price of the home equity in terms of the home consumption basket. The return on a home country bond is written as

$$r_{bt} = \frac{P_{Ht}/P_t}{Z_{Bt-1}} \tag{54}$$

where Z_B is the price of the home bond in terms of the home consumption basket. The foreign equity and foreign good-denominated bond are defined analogously.

From (52), we define the evolution of net foreign assets, evaluated up to the first-order, as

$$\hat{W}_t - \hat{W}_{t-1} = \frac{1-\beta}{\beta}\hat{W}_{t-1} + \widehat{P}_{Ht} - \widehat{P}_t + \hat{Y}_t - \hat{C}_t + \tilde{\alpha}'\hat{r}_{x,t} + O\left(\epsilon^2\right) \tag{55}$$

where $\tilde{\alpha}'$ represents the vector of portfolio holdings. In the case of equity trade alone, as before, $\tilde{\alpha}$ is the real holding of home equity, and $\hat{r}_{x,t}$ is the excess return on home equity relative to foreign equity. When both equities and bonds can be traded, defining the home bond as the residual asset, $\tilde{\alpha}'$ is the vector of real holdings of home equity, foreign equity, and foreign bonds, given by:

$$\tilde{\alpha}' = \left[\begin{array}{ccc} \tilde{\alpha}_e & \tilde{\alpha}_e^* & \tilde{\alpha}_b^* \end{array}\right]$$

The excess return is then defined as:

$$\hat{r}'_{x,t} = \left[\begin{array}{ccc} (\hat{r}_{e,t} - \hat{r}_{b,t}) & \left(\hat{r}^*_{e,t} - \hat{r}_{b,t}\right) & \left(\hat{r}^*_{b,t} - \hat{r}_{b,t}\right) \end{array}\right] = \left[\begin{array}{ccc} \hat{r}_{x,1,t} & \hat{r}_{x,2,t} & \hat{r}_{x,3,t} \end{array}\right].$$

The zero-order optimal portfolio may be constructed using the same procedure as before. In equilibrium, households choose a portfolio of home and foreign equity, and home and foreign bonds so as to satisfy a portfolio selection equation coming from a second-order approximation of a condition akin to (16) above.