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A DYNAMIC SPECIFIC-FACTORS MODEL  
OF INTERNATIONAL TRADE

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ABSTRACT

In a dynamic economy land and capital serve not only as factors of production but as assets which individuals use to transfer income from working periods to retirement. Static models of international trade based on the specific-factors model incorporate only the first of these. Once the second is recognized the supply of capital and evaluation of land can be derived from underlying intertemporal optimization behavior.

Changes in the terms of trade and in the endowments of fixed factors do not necessarily have the same effects on factor prices and the composition of output as they do in the static specific-factors model. Changes in these variables affect both total savings and the amount of savings that is diverted toward investment in land. Results derived from the traditional static model are more likely to emerge when the sector using land as a factor of production has a higher labor share than the sector using capital. In this case the land-using sector dominates factor markets more than asset markets.

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## I. Introduction

A major achievement of the factor-endowments theory of international trade has been to provide simple, intuitive insights into the relationships between commodity prices and factor prices, factor supplies and factor rewards, and factor endowments and the pattern of production and trade. The Stolper-Samuelson (1941), Rybczynski (1955) and Heckscher-Ohlin theorems describe these relationships for the two-factor, two-commodity case.

A number of factor-endowment models of international trade have incorporated land as a third factor of production distinct from labor and capital. Jones (1971) provides a thorough analysis of a model in which one of two factors (land or capital) is used specifically in the production of each of two commodities, while a third factor (labor) is used commonly in both production activities. The formulation has its origins in Ricardo's (1817) theory of rent and capital accumulation, while Viner (1959) has used a similar formulation in a trade context.<sup>1</sup>

The specific-factors model has become popular in trade theory. In particular, it has been used widely as a framework to analyze movements of capital. Unlike the two-factor Heckscher-Ohlin model, the specific-factors model does not imply that trade in commodities, unless it leads to complete specialization, removes any incentive for foreign investment.<sup>2</sup> Despite its origins in Ricardo's dynamic theory of rent, most formulations of the specific-factors model are static. In the tradition of the factor-endowments approach, capital, as well as land and labor, is available in fixed supply.

This paper develops a dynamic, two-sector, three-factor model in which the supply of capital is derived from individual savings behavior. The dynamic specification is a variant of Samuelson's (1958) overlapping

generations (OLG) model. There are an infinite number of generations, only two of which, one young and working and one old and retired, are economically active in any period. Workers supply labor to earn a wage. Some wage income is consumed. What is saved is invested in land or capital. The retired generation consumes the value of their assets and the income they have earned. There are a fixed (and constant) number of workers entering the labor force each period, and there is a fixed supply of land. The supply of capital is determined by the savings and portfolio allocation of workers in the previous period.<sup>3</sup>

Incorporating land and capital together in a dynamic model illustrates the role of land not only as a factor of production but as an asset, unlike capital, in fixed supply.<sup>4</sup> A channel through which changes in exogenous variables affect the economy is through their effect on the value of this asset. The value of land in turn has implications for the supply of capital.<sup>5</sup>

Because of this evaluation effect, many results that emerge from the static specific-factors model about the relationships between commodity prices and factor prices, factor endowments and factor rewards, and factor endowments and the pattern of production, must be amended. For example, a permanent increase in the relative price of one commodity does not necessarily lower the steady-state income of the factor specific to the industry producing the other commodity. An increase in the price of the land-using commodity, by raising the value of land, reduces savings available for investment in capital. The return on capital may consequently rise.

In addition, the effect on overall welfare in steady state of a change in the terms of trade is not solely determined by the pattern of trade. A permanent increase in the relative price of the land-using commodity can lower the steady-state welfare even of a large exporter of that commodity. It does

so by raising the price of land, diverting savings away from investment in capital. The steady state of an economy in which land serves as both a store of value and a productive asset must always have an interest rate in excess of the Golden Rule. Permanent changes that reduce investment act to reduce steady-state welfare.

Finally, an increase in the supply of the non-specific factor (labor) does not necessarily reduce the wage and raise the incomes of specific factors in steady state. An increase in the labor force, because it increases the supply of savings, can raise the steady-state capital stock sufficiently to raise the wage and lower the rate of interest and return on land.

In a dynamic context, then, asset-evaluation effects on the price of land can offset many of the predictions of the static specific-factors model. It turns out, however, that simple conditions on technologies in the two sectors indicate when the asset-evaluation effect dominates the static effect. If the elasticity of substitution between land and labor is one, then whenever the labor share in the land-using sector is larger than in the capital-using sector, the predictions of the static model are maintained. Conversely, if the labor-share is larger in the capital-using sector the asset evaluation effect dominates. In the first case the land-using sector claims a larger share of the labor force than it does of portfolio wealth. Consequently effects operating through the labor market, which are analyzed in the static model, dominate. In the second case the asset-market effects dominate.

The outline of the paper is as follows: Part II presents the basic assumptions of the model. The existence and stability of the steady state are discussed in Part III. Part IV treats the effect of a permanent shift in the terms of trade, both on impact and in steady state. The relationship between the supplies of fixed factors and factor rewards are investigated in Part V.

Part VI considers the role of savings behavior while some concluding remarks are provided in Part VII.

## II. The Model

In each period  $t$  the economy is identical to the three-factor, two-commodity model analyzed in a static context by Jones (1971) and Mussa (1974). One commodity, commodity 1, employs capital and labor in production. Its output, given factor inputs of  $K_1$  of capital and  $L_1$  of labor, is

$$Q_1 = F(K_1, L_1)$$

The second commodity, commodity 2, employs land and labor. With factor inputs of  $T_2$  of land and  $L_2$  of labor an amount

$$Q_2 = G(T_2, L_2)$$

of this commodity is produced. Both  $F$  and  $G$  are continuous, twice differentiable and linear homogeneous. Labor is intersectorally mobile.

Given the period  $t$  price of commodity 2 in terms of commodity 1,  $p_t$ , and the period  $t$  factor endowments  $K_t$ ,  $L_t$  and  $T_t$ , competition in factor markets will yield a wage  $w_t$ , an interest rate  $r_t$ , a land rent  $\pi_t$  and an allocation of labor  $L_{1t}$  that, in the absence of corner solutions, satisfy the equations

$$w_t = F_L(K_t, L_{1t}) \tag{2.1}$$

$$w_t = p_t G_L(T_t, L_t - L_{1t}) \quad (2.2)$$

$$r_t = F_K(K_t, L_{1t}) \quad (2.3)$$

$$\pi_t = p_t G(T_t, L_t - L_{1t}) - w_t(L - L_{1t}) \quad (2.4)$$

Equations (2.1) - (2.4) characterize equilibrium in period  $t$  as a function of factor supplies and the relative commodity price.

The domestic supplies of land and labor are given by natural endowments that are exogenous and, for the purpose of this analysis, constant over time. The size of the labor force is  $L$  and, with no loss of generality, the endowment of land is 1. New investment in capital in period  $t$  takes the form of currently produced units of commodity 1 that are not used for consumption. The supply price of capital in terms of commodity 1 is consequently one. The national supply of capital is determined by individual savings behavior. In the absence of international capital mobility the national and domestic supplies of capital coincide.

Savings is determined as part of a simple life-cycle optimization. Individuals live two periods. In the first period of economic life they provide one unit of labor services in exchange for a wage payment of  $w$  in terms of commodity 1. Using commodity 1 as numeraire, an amount  $c^y$  is spent on current consumption and the remainder invested in land and capital. In the second period the individual consumes the value of his holdings of land and capital, and the income that they generate. This amount is denoted  $c^0$ .

For simplicity the individual's utility function is assumed to be intertemporally additively separable. Lifetime utility as a function of  $c^y$ ,

$c^0$ , and the relative price of commodity 2 each period,  $p^y$  and  $p^0$ , respectively, is therefore given by

$$U = V^y(c^y, p^y) + V^0(c^0, p^0)$$

where  $V^y$  is, in indirect terms, the contribution of the working period to lifetime utility and  $V^0$  that of the retirement period.

The price of land in period  $t$  is  $q_t$ . Denoting the individual's investment in capital as  $k_t$  and in land as  $\lambda_t$ , the budget constraint implies that

$$c_t^y = w_t - k_{t+1} - q_t \lambda_{t+1}$$

$$c_{t+1}^0 = (1 + r_{t+1})k_{t+1} + (q_{t+1} + \pi_{t+1})\lambda_{t+1}$$

If individuals anticipate  $r_{t+1}$ ,  $\pi_{t+1}$  and  $q_{t+1}$  perfectly, then, for strictly positive values of  $k_t$  and  $\lambda_t$  to emerge requires that

$$\frac{q_{t+1} + \pi_{t+1}}{q_t} = 1 + r_{t+1} \quad (2.5)$$

If the left-hand side of expression (2.5) strictly exceeds the right, then  $k_t = 0$  while the opposite inequality implies that  $\lambda_t = 0$ . Assuming positive investment in capital, then, under perfect foresight

$$c_{t+1}^0 = (1 + r_{t+1})(w_t - c_t^y) \quad (2.6)$$



In the working period of life the individual's problem is to choose an expenditure level  $c_t^y$  to maximize

$$V^y(c_t^y, p_t) + V^0[(1 + r_{t+1})(w_t - c_t^y), p_{t+1}]$$

The utility-maximizing level of  $c_t^y$  is denoted by the function  $c^y(w_t, r_{t+1}, p_t, p_{t+1})$ .

In the absence of trade in capital or in claims on land the supplies of domestic and national capital coincide, and all domestic land is nationally owned. Equilibrium in the markets for capital and land then implies:

$$K_{t+1} = L \cdot k_{t+1} \tag{2.6}$$

and

$$l = L \cdot \lambda_{t+1} \tag{2.7}$$

The perfect foresight equilibrium of the economy can be characterized by the period equilibrium conditions (2.1) through (2.4), with  $L_t = L$  and  $T_t = 1$ , given  $K_t$ , and the dynamic equations:

$$K_{t+1} = L[w_t - c^y(w_t, r_{t+1}, p_t, p_{t+1})] - q_t \tag{2.8}$$

and

$$q_t = \frac{\pi_{t+1} + q_{t+1}}{1 + r_{t+1}} \tag{2.9}$$

Since the land price  $q_t$  does not appear in equations (2.1) through (2.4),  $w_t$ ,  $r_t$ ,  $\pi_t$  and  $L_{1t}$  can be expressed as functions of  $K_t$  and  $p_t$  alone, i.e., as  $w(K_t, p_t)$ ,  $r(K_t, p_t)$ ,  $\pi(K_t, p_t)$  and  $L_1(K_t, p_t)$ . It is easy to demonstrate that  $w_K > 0$ ,  $r_K < 0$ ,  $\pi_K < 0$ ,  $L_{1K} > 0$ ,  $w_p > 0$ ,  $r_p < 0$ ,  $\pi_p > 0$ ,  $L_{1p} < 0$ . Here  $x_K = \partial x / \partial K_t$  and  $x_p = \partial x / \partial p_t$ ,  $x \equiv w, r, \pi, L_1$ . See, for example, Jones (1971) and Mussa (1974).

### III. The Steady State

A steady state is characterized by an exogenous, constant relative price  $\bar{p}$  and levels of the capital stock  $\bar{K}$  and value of land  $\bar{q}$  that satisfy

$$\bar{K} = \psi(\bar{K}, \bar{p}) - \bar{q}(\bar{K}, \bar{p}) \quad (3.1)$$

$$\bar{q}(\bar{K}, \bar{p}) = \frac{\pi(\bar{K}, \bar{p})}{r(\bar{K}, \bar{p})} \quad (3.2)$$

Here

$$\psi \equiv L[w(K, \bar{p}) - c^y[w(K, \bar{p}), r(K, \bar{p}), \bar{p}, \bar{p}]]$$

which is aggregate savings given a constant  $K$  and  $\bar{p}$ .

Subject expressions (3.1) and (3.2) to the following restrictions:

Condition 3.1

$$\phi(0, \bar{p}) > q(0, \bar{p})$$

where

$$r(0, \bar{p}) \equiv \lim_{K \rightarrow 0} \left[ \max_{L_1} \frac{F(K, L_1) - \bar{p}G'(L - L_1)L_1}{K} \right]$$

and

$$q(K, \bar{p}) = \frac{\pi(K, \bar{p})}{r(K, \bar{p})}$$

This restriction implies that if the capital stock is zero the capitalized value of land does not satiate aggregate saving. Otherwise a steady state with  $\bar{K} = 0$  would exist, with land serving as the sole store of value. The price of land would be

$$\bar{q} = L\{w(0, \bar{p}) - c^y[w(0, \bar{p}), \tilde{r}, \bar{p}, \bar{p}]\}$$

$$\tilde{r} = \frac{\pi(0, \bar{p})}{\bar{q}}$$

Condition 3.2: There exists a  $\hat{K} > 0$  such that

$$K > \phi(K, \bar{p}) \quad \forall K > \hat{K} \tag{3.4}$$

This condition implies that for sufficiently high levels of  $K$  the savings generated at the consequent factor prices is less than that level of capital.

Condition 3.3: The functions  $\psi(K, \bar{p})$  and  $q(K, \bar{p})$  are, for any finite  $\bar{p}$ , continuous functions of  $K$ .

Condition 3.1 through 3.3 are sufficient to establish:

Theorem 3.1: For any finite, positive relative price  $\bar{p}$  a steady state level of capital,  $\bar{K}$ , and a steady state value of land,  $\bar{q}$ , exist, with  $0 < \bar{K} \leq \hat{K}$ .

The proof is a simple application of the intermediate-value theorem. At  $K = 0$

$$\psi(K, \bar{p}) - q(K, \bar{p}) > K$$

while at  $K \geq \hat{K}$ ,

$$\psi(K, \bar{p}) - q(K, \bar{p}) < K$$

Therefore, there exists a  $\bar{K}$  such that

$$\psi(\bar{K}, \bar{p}) - q(\bar{K}, \bar{p}) = \bar{K}$$

In addition, if  $\psi_K(K, \bar{p}) + dq/dK < 1 \quad \forall K \in (0, \hat{K})$ , the steady state is unique.

Consider now the stability of the model in the neighborhood of steady state. The equations of motion for the capital stock and the price of land, equations (2.8) and (2.9), can be expressed as a linearization around the steady state as follows:

$$x_{t+1} = L(1 - c_w^y)w_K x_t - Lc_r^y r_K x_{t+1} - y_t \quad (3.5)$$

$$y_t = \left[ \frac{\pi_K}{1+r} - \frac{\pi}{(1+r)^2} r_K \right] x_{t+1} + \frac{1}{1+r} y_{t+1} \quad (3.6)$$

where the terms  $c_w^y$ ,  $c_r^y$ ,  $w_K$ ,  $r_K$ ,  $r$ ,  $\pi$  and  $\pi_K$  assume their steady-state values, and  $x_t \equiv K_t - \bar{K}$  and  $y_t \equiv q_t - \bar{q}$ .

These two equations constitute a second-order system of homogeneous linear difference equations which can be expressed as:

$$\begin{bmatrix} x_{t+1} \\ y_{t+1} \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} \quad (3.7)$$

where

$$\Gamma_{11} \equiv \frac{L(1 - c_w^y)w_K}{1 + Lc_r^y r_K}$$

$$\Gamma_{12} \equiv \frac{-1}{1 + Lc_r^y r_K}$$

$$\Gamma_{21} \equiv -\left(\pi_K - \frac{\pi}{r} r_K\right)\Gamma_{11}$$

$$\Gamma_{22} \equiv 1 + r + \Gamma_{12}\Gamma_{21}/\Gamma_{11}$$

The solution to this system is of the form

$$x_t = A_1(\lambda_1)^t + A_2(\lambda_2)^t \quad (3.8)$$

$$y_t = \frac{\lambda_1 - \Gamma_{11}}{\Gamma_{12}} A_1(\lambda_1)^t + \frac{\lambda_2 - \Gamma_{11}}{\Gamma_{12}} A_2(\lambda_2)^t \quad (3.9)$$

where  $\lambda_1$  and  $\lambda_2$  are the two roots of the characteristic equation of (3.7):

$$\lambda^2 - (\Gamma_{11} + \Gamma_{22})\lambda + \Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21} = 0, \quad (3.10)$$

with  $\lambda_1$  and  $\lambda_2$  defined so that  $\lambda_1 \leq \lambda_2$ . The scalars  $A_1$  and  $A_2$  are determined by initial and/or terminal conditions.

At any period  $t$  the capital stock  $K_t$ , and hence  $x_t$ , are predetermined. The price of land  $q_t$ , and hence  $y_t$ , are determined by asset-market equilibrium each period. Considering an arbitrary period,  $t = 0$ ,  $A_1$  must satisfy the initial condition

$$A_1 = x_0 - A_2$$

The nature of the behavior of the capital stock and land price near steady state can be ascertained from  $\lambda_1$  and  $\lambda_2$ . A necessary and sufficient

condition for the existence of a unique stable non-oscillatory convergence path to steady state is that  $0 < \lambda_1 < 1 < \lambda_2$ .

If  $\lambda_1 < 0$  the system exhibits oscillation. The condition  $\Gamma_{11} \geq 0$  is necessary and sufficient to preclude oscillation. Since  $w_K > 0$  and  $r_K < 0$ ,  $\Gamma_{11} \geq 0$  is guaranteed if  $c_w^y \leq 1$  (the marginal propensity to save is nonnegative) and  $c_r^y < -1/Lr_K$ . This second condition requires that an increase in the interest rate not have an overly negative effect on savings. These assumptions are imposed.

If  $\lambda_2 < 1$  and  $\lambda_1 \geq 0$  then for any value of  $A_2$  the system converges to steady state, while if  $\lambda_1 \geq 1$  no path converges. If  $0 \leq \lambda_1 < 1 < \lambda_2$  then  $A_2 = 0$  is a necessary and sufficient condition for convergence. The remainder of the analysis is for the case in which parameter values satisfy the condition that  $0 \leq \lambda_1 < 1 < \lambda_2$ , so that there is a unique stable non-oscillatory equilibrium.

Changes in the values of exogenous variables can now be examined, both in terms of their effect on the steady state itself and characterizing the transition from the initial situation to the new steady state. An important feature of the transition is the degree to which the changes are anticipated. Alternative assumptions about whether or not changes are anticipated are examined. The effect of a change in the terms of trade on factor prices, the price of land and the capital stock is the topic of the next section. Part V considers a change in fixed factor endowments and Part VI a shift in intertemporal preferences.

#### IV. The Terms of Trade

In the static specific-factors model, an increase in the price of the land-using commodity raises the land rent and lowers the return to capital in terms of either commodity. The wage falls relative to the price of the land-using commodity, but rises relative to the price of the other commodity (Jones, 1971).

In a dynamic context a change in the relative price of commodities affects asset prices and investment in capital. These create additional effects on factor prices.<sup>6</sup>

##### A. Steady-State Effects

The steady-state effects of a permanent change in the terms of trade on the capital stock and price of land are given by the expressions:

$$\frac{d\bar{K}}{d\bar{p}} = \frac{(-\Gamma_{12})(rS_p - V_p)}{\Delta} = \frac{S_p - \frac{1}{r} V_p}{1 - S_K + \frac{1}{r} V_K} \quad (4.1)$$

$$\frac{d\bar{q}}{d\bar{p}} = \frac{(\Gamma_{12}\Gamma_{21}/\Gamma_{11})S_p + (1 - \Gamma_{11})V_p}{\Delta} = \frac{V_K S_p + (1 - S_K)V_p}{r(1 - S_K + \frac{1}{r} V_K)} \quad (4.2)$$

where

$$S_p \equiv L[(1 - c_w^y)w_p - c_r^y r_p - c_p^y]$$

$$S_K \equiv L[(1 - c_w^y)w_K - c_r^y r_K]$$



$$V_p \equiv \pi_p - (\pi/r)r_p$$

$$V_K \equiv \pi_K - (\pi/r)r_K$$

$$\Delta \equiv -[(1 - \Gamma_{11})(1 - \Gamma_{22}) - \Gamma_{12}\Gamma_{21}]$$

$$c_p^y \equiv \partial c^y / \partial p^y + \partial c^y / \partial p^0 .$$

Here  $S_p$  is the effect of a permanent change in  $\bar{p}$  on total savings, given the capital stock and value of land. Since the effect of interest rate and price level changes on savings is ambiguous, there is no strong presumption about its sign, although the first term is positive if future consumption is non-inferior.  $V_p$  is the effect of a change in  $\bar{p}$  on the price of land, given the capital stock. Since  $\pi_p$  is positive and  $r_p$  is negative this term is unambiguously positive. Finally, the sign of  $\Delta$  is positive if there is a unique, stable path of convergence to steady state.<sup>7</sup>

The effect of a change in the terms of trade can thus be divided into two parts. One is the effect through savings, given the price of land. This is captured by the term  $S_p$ . The second is the effect of the price change on the price of land, given total savings. It is captured by the term  $V_p$ .

### 1. The Savings Effect

To the extent that an increase in  $\bar{p}$  raises savings it acts to raise the stock of capital. The effect of an increase in savings on the land price is ambiguous. It depends upon the sign of  $V_K$ , which reflects the effect of a change in the capital stock on the steady-state land price, given  $\bar{p}$ . There are two conflicting effects. First, an increase in  $K$  reduces profit earned on

land, since the capital-using sector demands more labor, bidding up the wage. This acts to reduce the value of land. Second, an increase in  $K$  reduces the interest rate, so that the discounted value of a given profit stream increases. This acts to raise the price of land. If  $V_K > 0$  the second effect dominates and to the extent that the price change encourages savings ( $S_p > 0$ ) it also acts to increase the price of land. The higher capital stock, by reducing the interest rate, raises land values. If  $V_K < 0$ , meaning that the first effect dominates, to the extent that the price change encourages savings it reduces the value of land, by raising the wage.

The sign of  $V_K$  has implications for other aspects of the behavior of the economy as well. These are discussed below. Its sign is the same as:  $\frac{\pi}{w(L - L_1)} - \frac{rK}{wL_1}$ .<sup>8</sup> Thus if the land-using sector has a lower labor share than the capital-using sector, an increase in the capital stock raises the value of land, and conversely.

The ratio of the labor shares in the two sectors corresponds to their relative importance, in steady state, in the labor market and in portfolios. If the land-using sector has a smaller labor share than the capital-using sector, then, since  $q = \pi/r$ ,  $q/K > (L - L_1)/L_1$ , the land-using sector employs proportionately less of the labor force than the value of land in wealth. In this case the effect a change in the capital stock on the interest rate dominates its effect on the wage in affecting the price of land. If the land-using sector has a larger labor share the converse is true.

## 2. The Land Valuation Effect

The effect of an increase in  $\bar{p}$  through  $V_p$  acts to reduce the capital stock. Given savings, an increase in  $\bar{p}$  raises the price of land. More savings is therefore channeled into investment in land.

The total effect of the increase in  $\bar{p}$  through  $V_p$  on the price of land is, however, ambiguous, depending on the sign of  $1 - S_K$ . If an increase in  $K$ , given  $q$ , raises saving by less than it raises  $K$  itself,  $1 - S_K$  is positive. If  $\Gamma_{21} > 0$ , the condition for a unique stable solution ( $\Delta > 0$ ) insures that  $1 - S_K > 0$ . Otherwise, values of  $S_K > 1$  cannot be excluded on the criterion of a unique stable solution. If, for example,  $S_p = 0$  and  $S_K > 1$ , an increase in  $\bar{p}$  actually lowers the steady-state value of land. The reason is that the drop in the capital stock engenders an even greater drop in savings.

In summary, the criterion for a unique stable solution cannot alone insure that an increase in the price of the land-using commodity raises the price of land and lowers the capital stock in steady state. If the effect of the relative price on savings ( $S_p$ ) is not too strongly positive, then a lower steady-state capital stock is guaranteed. If, in addition, a reduction in the capital stock, given land prices, does not lower savings by an even greater amount, then an increase in the price of the land-using commodity will raise the price of land.

### 3. Commodity Prices and Factor Rewards

The effect of a change in  $\bar{p}$  on factor prices consists of a direct effect, taking factor supplies as given, and an indirect effect through the induced changes in factor supplies. The static specific-factors model, with fixed factor supplies, incorporates only the first. The direct effect of an increase in  $\bar{p}$  is to raise  $\pi$  and to lower  $r$  relative to both commodity prices, and to raise  $w$  but to lower  $w/p$  (Jones, 1971). If in fact an increase in  $\bar{p}$  lowers the steady state capital stock, the effect on the return to land is augmented in steady state, while the effects on the interest rate and wage

are diminished. They can even be reversed: the steady-state interest rate can rise, and the wage can fall in terms of both commodity prices.

The steady-state effect of a rise in the price of the land-using commodity on the interest rate is given by:

$$\begin{aligned} \frac{d\bar{r}}{d\bar{p}} &= r_p + r_k \frac{dK}{d\bar{p}} \\ &= \frac{G_{ww} w_K (L - L_1) L_1}{r K \pi \sigma} \left[ \frac{\sigma \pi}{w (L - L_1)} - \frac{r K}{w L_1} \right] \end{aligned} \quad (4.3)$$

where

$$\sigma \equiv \frac{G_{LT}}{G_L G_T}$$

the elasticity of substitution between land and labor in sector 2.<sup>9</sup> The effect of a change in  $\bar{p}$  on the wage is the negative of that on the interest rate.

From equation (4.3) follows:

Proposition 4.1. An increase in the price of the land-using commodity is more likely to raise the interest rate and lower the wage (i) when the labor share in the land-using industry is smaller than in the capital-using industry and (ii) when the elasticity of substitution between land and labor is large.

If expression (4.3) equals zero, which is the case if industry 2 is Cobb-Douglas and labor shares are equal in the two industries, then the steady-state values of the interest rate and wage do not change at all as a consequence of a commodity price change. In this case steady-state welfare unambiguously falls as a consequence of an increase in  $\bar{p}$ . This is true

regardless of the country's trade position: When expression (4.3) equals zero, changes in the capital stock insure that steady-state factor rewards in terms of the capital-using commodity are independent of the relative price of the land-using commodity. The only effect of a rise in  $\bar{p}$  is a reduction in the real purchasing power of the wage and of interest income.

B. Dynamic Adjustment: An Unanticipated Price Change

Consider now the effect of an unanticipated, permanent increase in the relative price of the land-using commodity in some period  $t$ ,  $t = 0$ . If the economy is initially in steady state, the capital stock cannot immediately assume its new steady state value. From expression (4.1), on impact the capital stock will deviate from its new steady state level by an amount

$$x_0 = - \frac{d\bar{K}}{d\bar{p}} = - \frac{S_p - \frac{1}{r} v_p}{1 - S_K + \frac{1}{r} v_K} \quad (4.4)$$

Imposing this initial condition on the solution (3.8), along with the condition for eventual convergence to the new steady state, implies that  $A_1 = x_0$  and  $A_2 = 0$ . Subsequent values of  $K_t$  are given by

$$K_t = \bar{K} + x_0(\lambda_1)^t \quad (4.5)$$

where  $\bar{K}$  is the new steady state capital stock. As discussed above, the presumption is that  $x_0$  is positive: A rise in  $\bar{p}$  lowers  $\bar{K}$ . Immediately following an unanticipated rise in  $\bar{p}$ , then, the capital stock lies above its new steady state level. It descends toward that value according to (4.5).

From expression (4.2), immediately after the price rise the value of land will deviate from its new, steady state level by

$$y_0 = \frac{\lambda_1 - \Gamma_{11}}{\Gamma_{12}} A_1 \quad (4.6)$$

From (3.10),  $\lambda_1 \gtrless \Gamma_{11}$  as  $\Gamma_{21} \gtrless 0$ . Since  $\Gamma_{12} < 0$ , the coefficient of  $A_1$  in expression (4.6) has the same sign as  $V_K$ . If an increase in the capital stock raises the price of land then, if  $x_0$  is positive so is  $y_0$ . The increase in  $\bar{p}$  causes the price of land to "overshoot" its new steady-state level: as capital decumulates the price of land then descends toward that value. Conversely, if an increase in the capital stock causes the price of land to fall ( $V_K$  negative) then the value of land rises by less than the full steady-state amount on impact, and continues to rise toward its new steady-state value in subsequent periods.

### C. Dynamic Adjustment: An Anticipated Price Change

Consider now what happens if the change in  $\bar{p}$  in period 0 is anticipated as of some previous period, say period  $-s$ ,  $s > 0$ . The steady-state effect of the change is, of course, the same. Assuming that before period  $s$  the economy was in steady state, at the moment of the announcement

$$x_s^a = A_1^a + A_2^a = 0, \quad (4.7)$$

where  $x_t^a$  denotes the derivation of  $K_t$  from its initial steady-state value,

and  $A_1^a$  and  $A_2^a$  are the values of  $A_1$  and  $A_2$  that govern  $x_t^a$  before the actual change in  $\bar{p}$ . Between period  $s$  and period 0

$$x_t^a = A_1^a(\lambda_1)^{t+s} + A_2^a(\lambda_2)^{t+s}$$

Using the fact that  $K_0$  has the same value regardless of whether it is expressed as a deviation from its old or new steady-state level,

$$\begin{aligned} x_0^a &= A_1^a(\lambda_1)^s + A_2^a(\lambda_2)^s \\ &= A_1^p + A_2^p + \frac{dK}{d\bar{p}} = x_0^p + \frac{dK}{d\bar{p}} \end{aligned} \quad (4.8)$$

where  $x_t^p$  is the deviation of  $K_t$  from its new steady-state value and  $A_1^p$  and  $A_2^p$  govern  $x_t^p$  for  $t \geq 0$ . Similarly, for the price of land,

$$\begin{aligned} y_0^a &= \frac{\lambda_1 - \Gamma_{11}}{\Gamma_{12}} A_1^a(\lambda_1)^s + \frac{\lambda_2 - \Gamma_{11}}{\Gamma_{12}} A_2^a(\lambda_2)^s \\ &= \frac{\lambda_1 - \Gamma_{11}}{\Gamma_{12}} A_1^p + \frac{\lambda_2 - \Gamma_{11}}{\Gamma_{12}} A_2^p \end{aligned} \quad (4.9)$$

Convergence to the new steady state continues to require that  $A_2^p = 0$ .

Therefore (4.7), (4.8) and (4.9) together imply that:

$$A_1^a = \frac{\Gamma_{12} \frac{d\bar{q}}{d\bar{p}} + (\Gamma_{11} - \lambda_1) \frac{d\bar{K}}{d\bar{p}}}{(\lambda_2)^s (\lambda_1 - \lambda_2)} \quad (4.10)$$

$$A_2^a = -A_1^a \quad (4.11)$$

$$A_1^p = [(\lambda_1)^s - (\lambda_2)^s] A_1^a - \frac{d\bar{K}}{d\bar{p}} \quad (4.12)$$

Since

$$y_{-s}^a = \frac{\lambda_1 - \Gamma_{11}}{\Gamma_{12}} A_1^a + \frac{\lambda_2 - \Gamma_{11}}{\Gamma_{12}} A_2^a ,$$

then

$$y_t^a = (\lambda_2)^t y_0^a , \quad -s \leq t \leq 0 \quad (4.13)$$

At the moment of the announcement the price of land changes in proportion to the change that will occur when the commodity price change actually happens, discounted by  $(\lambda_2)^{-s}$ .

When  $V_K = 0$  the dynamic adjustment path is especially simple:

$\lambda_1 = \Gamma_{11}$ ,  $\lambda_2 = 1 + r$ , and  $y_0^a = \frac{d\bar{q}}{d\bar{p}}$ . Just at the moment  $\bar{p}$  rises the price of land assumes its new steady-state value. Between the period when the increase is first anticipated and it actually occurs the land price moves toward the new steady-state value in proportion to the interest rate. As  $r \rightarrow 0$   $q_{-s}$  jumps



to its new steady-state value at the moment the change in  $\bar{p}$  is first expected. As  $r \rightarrow \infty$  no change occurs until period 0.

Since  $\lambda_2 > \lambda_1$ , the behavior of  $x_t^a$  during the announcement period is dominated by  $A_2^a$ . When  $\Gamma_{21} = 0$ , a permanent increase in  $\bar{p}$  necessarily raises the steady-state value of land, ( $\frac{d\bar{q}}{d\bar{p}} > 0$ ), insuring that  $A_1^a > 0$  and  $A_2^a < 0$ . The anticipation of a rise in  $\bar{p}$  at period 0 therefore leads to capital decumulation during the announcement period: the reason is that the anticipation of a rise in  $\bar{p}$  causes the land price to rise continually, diverting savings away from investment in capital. The capital stock begins to fall as soon as the rise in  $\bar{p}$  is anticipated. The rate of return on capital rises, the wage falls, and the profit on land rises continually during the anticipation period.

Capital decumulation during the announcement period is unlikely to proceed to the extent that, at period 0, the capital stock is below its new steady-state level. Even if  $s \rightarrow \infty$ , so that the rise in  $\bar{p}$  was always expected,  $x_0^p$  has the same sign as  $[(\lambda_1 - \lambda_2)S_p + (1 + r)V_p]$ , which can be negative only if  $S_p$  is very large; i.e., if an increase in  $\bar{p}$  causes a large increase in savings.

## V. Factor Endowments

In the static specific-factors model, factor rewards respond to factor supplies. An increase in the supply of any factor, given commodity prices, lowers its own reward. If the factor is specific to a sector, an increase in its supply lowers the reward of the other specific factor as well, but raises the reward of the nonspecific factor. An increase in the supply of the

nonspecific factor raises the rewards of the two specific factors.

The static model also makes strong predictions about the effect of factor endowments on production patterns. An increase in the supply of either specific factor raises the output of the industry in which it is used and lowers output of the other industry. An increase in the supply of the nonspecific factor raises outputs of both industries.

If preferences are similar and homothetic across countries, trade patterns can then be inferred from relative endowments of the specific factors. Countries tend to export the commodity which uses the specific factor relatively more abundant within their borders.<sup>10</sup>

In a dynamic model in which the capital stock is determined by savings behavior, only the supplies of land and labor are exogenous. Since production is at constant returns to scale, for given commodity prices only the relative endowments of land and labor are relevant for determining factor prices and relative outputs. This section considers the effect of a permanent increase in the labor force on factor prices and the pattern of production.

#### A. Steady-State Effects

The steady-state effects of a permanent increase in the labor force on the capital stock and land values are, respectively:

$$\frac{d\bar{K}}{dL} = \frac{-\Gamma_{12}(rS_L - V_L)}{\Delta} = \frac{S_L - \frac{1}{r} V_L}{1 - S_K + \frac{1}{r} V_K} \quad (5.1)$$

and

$$\frac{d\bar{q}}{dL} = \frac{-(\Gamma_{12}\Gamma_{21}/\Gamma_{11})S_L + (1 - \Gamma_{11})V_L}{\Delta} = \frac{S_L + (1 - S_K)V_L}{r(1 - S_K + \frac{1}{r} V_K)} \quad (5.2)$$

where

$$S_L \equiv [w - c^y + L(1 - c_w^y)w_L - Lc_r^y r_L]$$

$$V_L \equiv [\pi_L - (\pi/r)r_L]$$

Here  $S_L$  is the effect of a change in the labor force on total savings given the capital stock and value of land. The increase in the labor force raises the number of savers but since  $w_L < 0$ , it lowers the wage that each saver receives. Consequently there is no strong presumption about the sign of  $S_L$ .  $V_L$  is the effect of a change in the labor force on the price of land given the capital stock. Since

$$\pi_L = -w_L(L - L_1) \tag{5.3}$$

and

$$r_L = -(L_1/K)w_L \tag{5.4}$$

then

$$V_L = -(K/L_1)V_K \tag{5.5}$$

$V_L$  has the opposite sign of  $V_K$ : If an increase in the capital stock raises the price of land (given savings) an increase in the labor force lowers it, and conversely.

Using (5.3) through (5.5), expressions (5.1) and (5.2) may be written as

$$\frac{dK}{dL} = \frac{\frac{K+q}{L} - (S_K - \frac{1}{r} V_K) \frac{K}{L_1}}{1 - S_K + \frac{1}{r} V_K} \quad (5.1')$$

$$\frac{d\bar{q}}{dL} = \frac{V_K \left( \frac{K+q}{L} - \frac{K}{L_1} \right)}{1 - S_K + \frac{1}{r} V_K} \quad (5.2')$$

### 1. The Value of Capital and Factor Rewards

From (5.1'), if the labor share in the capital-using sector is very small relative to the land-using sector, the aggregate capital stock can actually fall when the labor force rises. If the shares in the two sectors are the same, however, then  $\frac{K}{L_1} = \frac{K+q}{L}$ , and the increase in the capital stock is proportional to the capital-labor ratio in industry 1. In this case an increase in the labor force generates an increase in the capital stock just large enough to absorb the new labor in industry 1 at the initial wage and interest rate. In this case an increase in the labor force has no effect on the wage, rent on land or interest rate in steady state. If the land-using sector has a larger labor share than the capital-using sector, then  $\frac{K+q}{L} < \frac{K}{L_1}$ . In this case an increase in the labor force raises the capital stock in a smaller proportion. The qualitative effect of an increase in the labor force on factor prices is the same as in the static version of the model: the real wage falls and the rent on land and the interest rate rise. If the land-using sector has a smaller labor share, however,  $\frac{K+q}{L} > \frac{K}{L_1}$ . An

increase in the labor force raises the real wage and lowers the rent on land and the interest rate.

Expression (5.1') implies the following proposition about the relationship between factor endowments and factor prices among trading countries:

Proposition 5.1. If two countries have identical technologies and if the land-using and capital-using industries have the same labor shares, then, in steady state, the real wage, the rent on land, and the rate of interest (and consequently welfare) are the same in the two countries. If the land-using industry has a higher labor share, then the land-abundant country has a lower real wage and a higher land rent and interest rate in steady state. Conversely, if the land-using industry has a lower labor share, then the land-abundant country has a higher real wage and lower land rent and interest rate.

## 2. The Value of Land

If the land-using sector has a lower labor share than the capital-using sector, then  $V_K > 0$  and  $\frac{K+q}{L} > \frac{K}{L_1}$ . Expression (5.2') is then positive. But if the labor share in the land-using sector is lower, both inequalities are reversed. Either way expression (5.2') is positive. This implies:

Proposition 5.2. An increase in the labor force raises the steady-state price of land.

## B. Dynamic Adjustment

As is the case with a change in the terms of trade, if  $V_K = 0$  upon any change in the population the price of land immediately assumes its new steady

state value. If  $V_K < 0$  and  $\frac{dK}{dL} > 0$ , an unanticipated increase in  $L$  causes, on impact, the value of land to overshoot its new steady-state level from the initial steady state. As capital is accumulated, the land value descends toward the new steady-state level. If  $V_K > 0$  the land value undershoots, on impact, and continues to rise in subsequent periods.

If the change in  $L$  is anticipated then the value of land rises before the actual increase. This diverts savings away from capital investment. The wage falls and the interest rate rises from their initial steady-state levels until the increase in  $L$  actually occurs.

## VI. Savings Shift

A comparative static exercise frequent in the trade literature is to consider the effect of changes in the capital endowment on factor prices and the composition of output. In the static specific-factors model an increase in the capital stock lowers the rate of interest on capital and return to land and raises the wage. In a dynamic optimization model the supply of capital is an endogenous variable. To consider the effect of a change in its supply is consequently not a well-defined exercise. A closely related issue is the effect of a change in preferences that affect the intertemporal allocation of resources. Consider then the effect of a shift in preferences toward current consumption. This shift can be introduced by entering a parameter  $\phi$  into the indirect utility function introduced in section 2,

$$U = V^Y(c^Y, p^Y, \phi) + V^0(c^0, p^0) ; \quad V_{c\phi}^Y > 0$$

From the budget constraint and first-order condition for a maximum

$$\frac{\partial c_t^y}{\partial \phi} = \frac{-V_{c\phi}}{V_{cc}^y + (1 + r_{t+1})^2 V_{cc}^o} > 0$$

The utility-maximizing level of  $c_t^y$  can now be expressed as a function  $c^y(w_t, r_{t+1}, p_t, p_{t+1}, \phi)$ , with  $c_\phi^y > 0$ . The effect of a reduced savings propensity can be represented by an upward shift in  $\phi$ .

#### A. Steady-State Effects

The steady-state effects of a permanent increase in  $\phi$  on the capital stock and price of land are, respectively,

$$\frac{\bar{dK}}{d\phi} = \frac{r\Gamma_{12}c_\phi^y}{\Delta} = \frac{-c_\phi^y}{1 - s_K + \frac{1}{r}V_K} \quad (6.1)$$

and

$$\frac{\bar{dq}}{d\phi} = \frac{\Gamma_{12}\Gamma_{21}}{\Gamma_{11}\Delta} = \frac{-c_\phi^y V_K}{r(1 - s_K + \frac{1}{r}V_K)} \quad (6.2)$$

Expression (6.1) is unambiguously negative: a reduced savings propensity reduces the steady-state capital stock. The interest rate and return on land are consequently higher, and the wage lower.

From (6.2), the effect of a change in the savings propensity on the land price, like other magnitudes, depends upon the sign of  $V_K$ . If  $V_K > 0$  the value of land falls as a consequence of lower savings: the effect of reduced capital in raising the interest rate dominates the effect of lowering the wage. If  $V_K < 0$  the opposite is true. There is no presumption, then, that a reduced savings propensity will lower the price of assets in fixed supply. The capital stock bears the primary impact of shifts in savings behavior.

From expression (6.1) and (6.2) follows:

Proposition 6.1. If two countries have identical technologies and endowments of land and labor, the country with the lower savings propensity will have a lower capital stock, a lower wage, and a higher interest rate and return on land. That country will produce less of the commodity produced in the capital-using sector and more of the commodity produced by the land-using sector. Land will be more or less expensive in the low-savings country depending on whether the land-using sector has a higher or lower labor share than the capital-using sector.

#### B. Dynamic Adjustment

On impact, a permanent, unanticipated increase in  $\phi$  causes the initial steady-state level of capital to exceed the new steady-state level. Over time the capital stock declines to its new steady-state level. As before, upon the increase in  $\phi$ ,  $q_t$  is above or below its new steady-state value as  $V_K \gtrless 0$ .

If the decline in savings is anticipated, then the price of land begins to move toward its new steady-state level before  $\phi$  changes. If  $V_K > 0$ ,  $q$  begins to fall. Capital is actually accumulated during the anticipation



period, the interest rate and return on land fall and the wage rises. If  $V_K < 0$  the opposite happens.

## VII. Conclusion

Land plays a role in a national economy both as a factor of production and as an asset in fixed supply. Static models of international trade that incorporate land typically recognize only the first of these. Changes in international prices and in domestic preferences, technology and population affect the value of land as an asset, which in turn affects the supply of savings allocated toward investment in capital. When the steady-state interest rate exceeds the population growth rate, as is always the case where land is both productive and is valued as an asset, changes that reduce the interest rate raise welfare in steady state, and conversely. Changes in exogenous variables that raise the price of land have a negative effect on steady-state welfare because they divert savings away from investment in capital.

This paper has focussed on land and capital as assets and factors of production, land being fixed in supply while the supply of capital is perfectly elastic. The value of other assets in the economy, not incorporated in this analyses, may respond to changes in international prices or domestic conditions in ways that are important for capital accumulation. Access to monopoly rents or the rents associated with the reputations of particular firms are examples. An interesting problem for international trade theorists is the incorporation of the response of these asset prices to exogenous variables.

## Appendix

### The Effect of a Change in the Terms of Trade on Factor Prices in Steady State

This appendix derives expression (4.3) in the text, indicating the effect of an increase in the relative price of the land-using commodity,  $\bar{p}$ , on the interest rate,  $\bar{r}$ . From duality

$$r_p = -(L_1/K)w_p \quad (\text{A.1})$$

$$r_K = -(L_1/K)w_K \quad (\text{A.2})$$

Since

$$w = F_L \quad (\text{A.3})$$

$$w_p = F_{LL} \frac{dL_1}{dp} \quad (\text{A.4})$$

and from the first-order conditions (2.2) and (2.3)

$$\frac{dL_1}{dp} = \frac{G_L}{F_{LL} + pG_{LL}} \quad (\text{A.5})$$

and

$$\frac{dL_1}{dp} = \frac{-F_{KL}}{F_{LL} + pG_{LL}} \quad (\text{A.6})$$

Thus 
$$\frac{dL_1}{dp} = - \frac{G_L}{F_{KL}} \frac{dL_1}{dK} \quad (\text{A.7})$$

Similarly, since also

$$w = pG_L(L - L_1) \quad (\text{A.8})$$

$$w_K = -pG_{LL} \frac{dL_1}{dK} \quad (\text{A.9})$$

Combining (A.1), (A.2), (A.4), (A.7) and (A.9)

$$r_p = \frac{G_L F_{LL}}{pG_{LL} F_{KL}} r_K \quad (\text{A.10})$$

Therefore, using (4.1)

$$r_p + r_K \frac{dK}{dp} = r_K \left[ \frac{G_L F_{LL}}{pG_{LL} F_{KL}} + \frac{S_p - \frac{1}{r} v_p}{1 - S_K + \frac{1}{r} v_K} \right] \quad (\text{A.11})$$

But, using (A.1), (A.2) and (A.10),

$$S_p = \frac{G_L F_{LL}}{pG_{LL} F_{KL}} S_K \quad (\text{A.12})$$

$$V_p = G + V_K \frac{G_L F_{LL}}{p G_{LL} F_{KL}} \quad (\text{A.13})$$

Substituting (A.12) and (A.13) into (A.11) gives

$$\frac{d\bar{r}}{d\bar{p}} = \frac{r_K \left[ \frac{G_L F_{LL}}{p G_{LL} F_{KL}} - \frac{1}{r} G \right]}{1 - S_K + \frac{1}{r} V_K} \quad (\text{A.14})$$

From duality,

$$F_{LL}/F_{KL} = -K/L_1 \quad (\text{A.15})$$

$$G_{LL} = -G_{TL}/(L - L_1) \quad (\text{A.16})$$

which upon substitution into (A.14), using (A.2) once again, gives expression (4.3).

## Footnotes

1. Kenen (1965) develops a more complicated version of the model incorporating both natural resources and human capital as well as labor and physical capital.
2. Examples of papers that use the specific-factors model to analyze capital flows are by Brecher and Findlay (1983) and Srinivasan (1983).
3. Other trade models have derived the capital stock from underlying savings behavior. This is done in a two-sector infinite-horizon context by Stiglitz (1970) and Findlay (1978) and in an OLG framework by Buiter (1981). The last is a one-sector model which Buiter uses to investigate the implications of capital mobility. None of these papers incorporates land as a separate asset and factor of production.
4. Kareken and Wallace (1977) and Fried (1980) develop two-sector OLG models in which land is the only asset. There is no capital accumulation, and labor is the other productive factor aside from land.
5. The implications for international trade of the role of land as an asset as well as a factor of production are largely unexplored. A literature does exist on the public finance implications for a closed economy of land's use as an asset. Feldstein (1977) shows how the evaluation effect of a tax on land destroys the presumed neutrality of a tax on a fixed factor. Feldstein's analysis has been extended by Calvo, Kotlikoff, and Rodriguez (1979) and Chamley and Wright (1983), in particular.

6. For the analysis in this section the assumption that investment in capital uses only the commodity produced by the capital-using sector involves some loss of generality. A more complete analysis would introduce a price index for investment. Examination of the polar opposite case, an investment good produced by the land-using sector, did not yield conclusions upsetting those discussed here. For starkness, and because it seems empirically more interesting, the analysis sticks with the case of an investment good produced by the capital-using sector.

7. This can be demonstrated by defining the function

$$\psi(\rho) \equiv (\rho - \Gamma_{11})(\rho - \Gamma_{22}) - \Gamma_{12}\Gamma_{21}$$

The characteristic roots of the system (3.7),  $\lambda_1$  and  $\lambda_2$ , are obtained by setting  $\psi(\rho) = 0$ . For  $\rho \in (\lambda_1, \lambda_2)$ ,  $\psi(\rho) < 0$ . Since  $\Delta = -\psi(1)$ ,  $\Delta$  is positive if and only if  $\lambda_1 < 1 < \lambda_2$ . The conditions  $\Delta > 0$  and  $c_r^y < -1/Lr_K$  imply the condition

$$1 - S_K + \frac{1}{r} V_K > 0 .$$

This states that, ceteris paribus, an increase in the capital stock must not increase investment in capital by an even greater amount.

8. From the zero-degree homogeneity of  $F_K$ ,  $r_K K + w_K L_1 = 0$ , so that  $r_K = -\frac{w_K L_1}{K}$ . Therefore

$$V_K = \pi_K - \frac{\pi}{r} r_K = w_K \left[ -(L - L_1) + \frac{\pi L_1}{rK} \right]$$

$$= w_K \frac{wL_1(L - L_1)}{rK} \left[ \frac{\pi}{w(L - L_1)} - \frac{rK}{wL_1} \right]$$

Since  $w_K$  is positive the relative shares determine the sign of  $\pi_K - \frac{\pi}{r} r_K$ .

9. The derivation of this expression is provided in the appendix.

10. If the elasticities of substitution between the specific and non-specific factors differ across sectors, the relative endowments of specific to non-specific factors also matter. Countries with relatively more of the nonspecific factor will, given relative supplies of the specific factors, produce and export more of the commodity produced by the sector with a greater elasticity of substitution. See Mussa (1974).

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