NBER WORKING PAPER SERIES

ASSET PRICING TESTS WITH LONG RUN RISKS IN CONSUMPTION GROWTH

George M. Constantinides
Anisha Ghosh

Working Paper 14543
http://www.nber.org/papers/w14543

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 2008

We thank Ravi Bansal, Alan Bester, John Campbell, Raj Chakrabarti, John Cochrane, Eugene Fama, Lars Hansen, Christian Julliard, Dana Kiku, Ralph Koijen, Oliver Linton, Sydney Ludvigson, Alex Michaelides, Lubos Pastor, Annette Vissing-Jorgensen, and Amir Yaron for helpful comments. We remain responsible for errors and omissions. Constantinides acknowledges financial support from the The Center for Research in Security Prices of the University of Chicago Booth School of Business. Ghosh acknowledges financial support from the London School of Economics. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

Asset Pricing Tests with Long Run Risks in Consumption Growth
George M. Constantinides and Anisha Ghosh
NBER Working Paper No. 14543
December 2008, Revised January 2011
JEL No. G12

ABSTRACT

A novel methodology in testing the long-run risks model of Bansal and Yaron (2004) is presented based on the observation that, under the null, the potentially latent state variables, "long-run risk" and the conditional variance of its innovation, are known a¢ ne functions of the observable market-wide price-dividend ratio and risk free rate. In linear forecasting regressions of consumption growth and returns by the price-dividend ratio and risk free rate, the model implies much higher forecastability than what is observed in the data over 1931 –2009. The co-integrated variant of the model by Bansal, Gallant, and Tauchen (2007), also implies much higher forecastability of returns than what is observed in the data. Finally, we reject the models' implications in jointly pricing the cross-section of returns and fitting the unconditional time series moments of consumption and dividend growth. The results suggest that either some important state variable is missing or that the models should be generalized in a way that the lagged price-dividend ratio and risk free enter the regressions in a non-linear fashion.

George M. Constantinides
The University of Chicago
Booth School of Business
5807 South Woodlawn Avenue
Chicago, IL 60637
and NBER
gmc@ChicagoBooth.edu

Anisha Ghosh
Tepper School of Business
Carnegie Mellon University
5000 Forbes Avenue
Pittsburgh PA 15213
anishagh@andrew.cmu.edu

1 Introduction

A burgeoning literature in finance addresses investors' attitudes towards the timing of resolution of uncertainty of future consumption and cash flows through the class of preferences introduced by Epstein and Zin (1989), Kreps and Porteus (1978), and Weil (1989). Models initiated by Bansal, Dittmar, and Lundblad (2005), Bansal and Yaron (2004), and Hansen, Heaton, and Li (2008) have rich implications on prices and show promise in explaining the time series and cross-sectional properties of returns of financial assets. These models pay particular attention to the low frequency properties of the time series of dividends and aggregate consumption—hence their characterization as "long run risks" (LRR) models.[1]

In this paper we revisit the particular LRR model introduced by Bansal and Yaron (2004) (hereafter B-Y) that has received wide attention in the literature. Whereas we formally reject both this model and its co-integrated variant introduced by Bansal, Gallant, and Tauchen (2007), our primary contribution lies in the novelty of our empirical methodology which provides new insights and guides our search for promising models of long run risks.

Our first methodological contribution addresses the feature of the B-Y model (and of related models) that the LRR variable and the conditional variance of its innovation are latent. The filtering of these latent variables potentially introduces observation error and decreases the power of the tests. We argue that these "latent" state variables are, in fact, observable because both the aggregate price-dividend ratio and risk free rate are functions of only these two state variables under the model assumptions. Specifically, in the log-linearized version of the B-Y model, the aggregate log price-dividend ratio and log risk free rate are affine functions of the two state variables, with coefficients that are known functions of the preference parameters and of the parameters of the time-series processes. This observation allows us to invert the affine system and express the two state variables as known affine functions of the observable aggregate log price-dividend ratio and log risk free rate. Whereas this empirical methodology is common in the context of the class of affine term structure models (for example, Dai and Singleton (2000) and Duffee (2002)), it has not previously been employed in testing LRR models.

The second methodological contribution stems from the fact that the log-linearized version of the B-Y model implies that the expected market return, equity premium, dividend growth, and consumption growth are affine functions of the two state variables. Since the state variables themselves are known affine functions of the observable

[1] See also, Alvarez and Jerman (2005), Bansal, Dittmar, and Kiku (2009), Bansal, Gallant, and Tauchen (2007), Bansal, Kiku, and Yaron (2007), Bansal and Shaliastovich (2010), Beeler and Campbell (2009), Bekaert, Engstrom, and Xing (2009), Chen, Favilukis, and Ludvigson (2008), Colacito and Croce (2005), Constantinides and Ghosh (2010), Croce, Lettau, and Ludvigson (2008), Ferson, Nallareddy, and Xie, (2009), Ghosh and Constantinides (2010), Hansen and Scheinkman (2009), Lettau and Ludvigson (2009), Lustig, Van Nieuwerburgh, and Verdelhan (2008), Malloy, Moskowitz, and Vissing-Jorgensen (2008), Parker and Julliard (2005), and Piazzesi and Schneider (2006).

aggregate log price-dividend ratio and log risk free rate, it follows that the expected market return, equity premium, dividend growth, and consumption growth are affine functions of the observable aggregate log price-dividend ratio and log risk free rate. Therefore, the time-series properties of the model are readily testable with in-sample linear forecasting regressions and out-of-sample linear predictive regressions of the market return, equity premium, dividend growth, and consumption growth on the lagged price-dividend ratio and risk free rate. The tests on the predictability of the market return, equity premium, and dividend growth are robust to observation error of consumption flows and the possibility of improper temporal aggregation of consumption flows because consumption data are not used in these tests.

Our third methodological contribution is based on the fact that the log-linearized version of the B-Y model implies that the log pricing kernel is an affine function of the two state variables and their lags. Since the state variables themselves are known affine functions of the observable aggregate log price-dividend ratio and log risk free rate, it follows that the log pricing kernel is an affine function of the aggregate log price-dividend ratio, the log risk free rate, and their lags, in addition to consumption growth. Thus we obtain a set of Euler equations of consumption for the cross-section of returns.

The final methodological contribution lies in simultaneously testing through GMM the Euler equations of consumption *and* the restrictions imposed on the model parameters by the unconditional moments of the aggregate dividend and consumption growth, thereby increasing the power of the test.

Our first set of tests is motivated by the implications of the B-Y model regarding in-sample forecasting and out-of-sample prediction of the market return, equity premium, dividend growth, and consumption growth through linear regressions on the lagged price-dividend ratio and risk free rate.[2] Specifically, we simulate the B-Y model, calibrated at the monthly, quarterly, and annual frequencies, and demonstrate that the model implies much higher predictability of the aggregate consumption growth rate, market return, and equity premium than that observed in the data over 1931 – 2009. The out-of-sample predictability tests produce negative results as well. Furthermore, the model calibrated at the annual frequency implies much higher predictability of the 3-year and 5-year consumption growth rate compared to the historical data. The co-integrated variant of the model by Bansal, Gallant, and Tauchen (2007), that introduces as an additional state variable the consumption-dividend ratio, when calibrated at the annual frequency implies forecastability of consumption and dividend growth

[2] The predictability of the market return, equity premium, dividend growth, and consumption growth through linear regressions on the lagged price-dividend ratio and risk free rate has been the subject of an extensive literature. Examples include Ang and Bekaert (2007), Binsbergen and Koijen (2010), Boudoukh, Richardson, and Whitelaw (2008), Campbell and Shiller (1988), Campbell and Thompson (2008), Cochrane (2008), Fama and French (1988), Kelly and Pruitt (2010), Lettau and Van Nieuwerburgh (2008), and Welch and Goyal (2008)).

consistent with the historical data. However, it implies much higher forecastability of the market return and premium than that observed in the data. Moreover, like the B-Y model, the cointegrated model performs poorly in predicting out-of-sample the growth rates and returns. Overall, these results provide robust time series evidence against the LRR model of B-Y and its co-integrated variant and suggest that either some important state variable is missing or that the model should be generalized in a way that the lagged price-dividend ratio and risk free enter the regressions in a non-linear fashion.

In simultaneously testing through GMM the Euler equations of consumption for the market return and risk free rate and the restrictions imposed on the model parameters by the unconditional moments of the aggregate dividend and consumption growth over 1931 − 2009, we find that the pricing error for the risk free rate is 3.3% and that for the market return is 1.9%. When we extend the asset system to include the "Value", "Growth", "Small" capitalization, and "Large" capitalization portfolios in addition to the market return and risk free rate, we find that the "Small" capitalization portfolio has pricing error 2.6% and the "Growth" portfolio has pricing error −2.9%. The co-integrated variant of the model by Bansal, Gallant, and Tauchen (2007) produces similar results. The overidentifying restrictions test has p-value smaller than 1% for each model specification.

We address the potential problem of temporal aggregation of consumption by repeating our estimation and tests using quarterly data over the post-war period. The results are very similar to those obtained using annual data over the sub-period, suggesting that our findings are unlikely to be driven by problems associated with temporal aggregation.

The paper is organized as follows. In Section 2, we describe the estimation and testing methodology of the LRR model. We discuss the data in Section 3. In Section 4, we present the results of the in-sample forecasting regressions and the out-of-sample predictive regressions. In Section 5, we present the empirical evidence on the cross-section of returns. In Section 6, we consider an extension of the LRR model that introduces, as a third state variable, the co-integrating residual of the logarithms of consumption and aggregate dividend levels. In Section 7, we address the possibility of structural breaks within the period 1931 − 2009 by repeating our tests in the post-war sub-period. We also address the issues related to temporal aggregation of consumption by repeating our tests with quarterly data. Section 8 concludes. The appendix contains derivations and details of the testing methodology.

2 The Model and Its Testable Implications

We describe the LRR model of B-Y and derive its testable implications for the predictability of the market return, equity premium, dividend growth, and consumption growth. Then we derive its testable implications for the equity premium and the cross-section of returns.

2.1 Model

The Bansal and Yaron (2004) LRR model introduces the novel state variable, x_t, and the variance of its innovation, σ_t^2, that jointly drive the conditional mean of the aggregate consumption and dividend growth rates:

$$
\begin{aligned}
x_{t+1} &= \rho_x x_t + {}_x\sigma_t \varepsilon_{x,t+1}, & (1)\\
\sigma_{t+1}^2 &= (1 - \upsilon)\sigma^2 + \upsilon\sigma_t^2 + \sigma_w \varepsilon_{\sigma,t+1}, & (2)\\
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t \varepsilon_{c,t+1}, & (3)\\
\Delta d_{t+1} &= \mu_d + \phi x_t + \varphi\sigma_t \varepsilon_{d,t+1}, & (4)
\end{aligned}
$$

where c_{t+1} is the logarithm of the aggregate consumption level and d_{t+1} is the logarithm of the aggregate stock market dividends. The shocks $\varepsilon_{x,t+1}$, $\varepsilon_{\sigma,t+1}$, $\varepsilon_{c,t+1}$, and $\varepsilon_{d,t+1}$ are assumed to be *i.i.d.* $N(0,1)$ and mutually independent. The time-series specification in equations (1)-(4) introduces nine parameters: μ_c, μ_d, ϕ, φ, ρ_x, ${}_x$, σ, υ, and σ_w. In Appendix A.1, we derive various unconditional moments of consumption and dividend growth rates as functions of the time-series parameters.

The model further assumes that the consumer has the version of Kreps and Porteus (1978) preferences adopted by Epstein and Zin (1989) and Weil (1989). These preferences allow for separation between the coefficient of risk aversion and the elasticity of intertemporal substitution. The utility function is defined recursively as

$$V_t = \left[(1 - \delta)C_t^{\frac{1-}{\theta}} + \delta \left(E_t\left[V_{t+1}^{1-}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-}}, \tag{5}$$

where δ denotes the subjective discount factor, > 0 is the coefficient of risk aversion, > 0 is the elasticity of intertemporal substitution, and $\theta = \frac{1-}{1-\frac{1}{}}$. Note that the sign of θ depends on the relative magnitudes of and . The standard time-separable power utility model is obtained as a special case when $\theta = 1$, i.e. $= \frac{1}{}$.

For this specification of preferences, Epstein and Zin (1989) and Weil (1989) show that, for any asset j, the first-order conditions of the consumer's utility maximization yield the Euler equation,

$$E_t\left[\exp(m_{t+1} + r_{j,t+1})\right] = 1, \tag{6}$$

where

$$m_{t+1} = \theta \log\delta - \frac{\theta}{}\Delta c_{t+1} + (\theta - 1)r_{c,t+1} \tag{7}$$

is the natural logarithm of the intertemporal marginal rate of substitution; $E_t[.]$ denotes expectation conditional on time t information; $r_{j,t+1}$ is the continuously compounded return on asset j; and $r_{c,t+1}$ is the unobservable continuously compounded return on an asset that delivers aggregate consumption as its dividend each period.

We rely on log-linear approximations for the log return on the consumption claim, $r_{c,t+1}$, and on the market portfolio (the return on the aggregate dividend claim), $r_{m,t+1}$, as in Campbell and Shiller (1988):

$$
\begin{aligned}
r_{c,t+1} &= \kappa_0 + \kappa_1 z_{t+1} \square z_t + \Delta c_{t+1}, && (8)\\
r_{m,t+1} &= \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} \square z_{m,t} + \Delta d_{t+1}, && (9)
\end{aligned}
$$

where z_t is the log price-consumption ratio and $z_{m,t}$ the log price-dividend ratio. In equation (8), $\kappa_1 = \frac{e^{\overline{z}}}{1+e^{\overline{z}}}$ and $\kappa_0 = log(1 + e^{\overline{z}}) \square \kappa_1 \overline{z}$ are log-linearization constants, where $\overline{z}$ denotes the long-run mean of the log price-consumption ratio. Similarly, in equation (9), $\kappa_{1,m} = \frac{e^{\overline{z}_m}}{1+e^{\overline{z}_m}}$ and $\kappa_{0,m} = log(1 + e^{\overline{z}_m}) \square \kappa_1 \overline{z}_m$, where $\overline{z}_m$ denotes the long-run mean of the log price-dividend ratio.

B-Y show that z_t and $z_{m,t}$, are affine functions of the state variables, x_t and σ_t^2,

$$
\begin{aligned}
z_t &= A_0 + A_1 x_t + A_2 \sigma_t^2, && (10)\\
z_{m,t} &= A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2. && (11)
\end{aligned}
$$

The coefficients A_0, A_1, A_2, $A_{0,m}$, $A_{1,m}$, and $A_{2,m}$ depend on the parameters of the utility function and those of the stochastic processes for consumption and dividend growth rates (see Appendix A.2.1 for expressions for these coefficients).

For this model specification, the log risk free rate from period t to $t+1$ may also be expressed as an affine function of the state variables (see Appendix A.2.2 for expressions for $A_{0,f}$, $A_{1,f}$, and $A_{2,f}$),

$$
\begin{aligned}
r_{f,t} &= \square \log E_t \left[\exp(m_{t+1})\right], \\
&= A_{0,f} + A_{1,f} x_t + A_{2,f} \sigma_t^2. && (12)
\end{aligned}
$$

Equations (11) and (12) express the observable variables, $z_{m,t}$ and $r_{f,t}$, as affine functions of the latent state variables, x_t and σ_t^2. These equations may be *inverted* to express the unobservable state variables, x_t and σ_t^2, as affine functions of the observables, $z_{m,t}$ and $r_{f,t}$, (see Appendix A.2.3 for details and expressions for α_0, α_1, α_2, β_0, β_1, and β_2),

$$
\begin{aligned}
x_t &= \alpha_0 + \alpha_1 r_{f,t} + \alpha_2 z_{m,t}, && (13)\\
\sigma_t^2 &= \beta_0 + \beta_1 r_{f,t} + \beta_2 z_{m,t}. && (14)
\end{aligned}
$$

2.2 Testable Implications for Predicting Returns and Growth Rates

Equations (9), (11), (4) and (12) imply that the expected market return is given by

$$E_t[r_{m,t+1}] = B_0 + B_1 x_t + B_2 \sigma_t^2, \tag{15}$$

and the expected equity premium is given by

$$E_t[r_{m,t+1} \square r_{f,t}] = E_0 + E_1 x_t + E_2 \sigma_t^2, \tag{16}$$

both affine functions of the state variables, x_t and σ_t^2. The model generates time-varying expected market return and premium. The coefficients $\{B_i, E_i\}_{i=0}^2$ are known functions of the underlying time-series and preference parameters.

The time series specification of the model implies that the expected consumption growth rate is given by

$$E_t[\Delta c_{t+1}] = \mu + x_t, \tag{17}$$

and the expected dividend growth rate is given by

$$E_t[\Delta d_{t+1}] = \mu_d + \phi x_t, \tag{18}$$

both affine functions of the state variable x_t.

Since the state variables, x_t and σ_t^2 are affine functions of the observables $z_{m,t}$ and $r_{f,t}$, we may express the expected market return, equity premium, dividend growth, and consumption growth as affine functions of the observables $z_{m,t}$ and $r_{f,t}$ with coefficients known functions of the model parameters.

In Section 4, we test the predictive implications of the model through in-sample linear forecasting regressions and out-of-sample linear predictive regressions of the market return, equity premium, dividend growth, and consumption growth on the lagged price-dividend ratio and risk free rate.

2.3 Testable Implications for the Equity Premium and the Cross-Section of Returns

We substitute the log-affine approximation for $r_{c,t+1}$ in equation (8) into the expression for the pricing kernel (equation (7)), and noting that z_t is given by equation (10), we have,

$$\begin{aligned} m_{t+1} &= (\theta \log \delta + (\theta \square 1)\left[\kappa_0 + (\kappa_1 \square 1)\, A_0\right]) + \left(\square \frac{\theta}{} + (\theta \square 1)\right) \Delta c_{t+1} \\ &\quad + (\theta \square 1)\kappa_1 A_1 x_{t+1} + (\theta \square 1)\kappa_1 A_2 \sigma_{t+1}^2 \square (\theta \square 1) A_1 x_t \square (\theta \square 1) A_2 \sigma_t^2. \end{aligned} \tag{19}$$

Equation (19) for the pricing kernel involves the unobservable (from the point of view of the econometrician) state variables, x_t and σ_t^2, and, hence, is not directly testable on a cross-section of asset returns. Substituting the expressions for x_t and σ_t^2 from equations (13) and (14) into the pricing kernel in equation (19), we have,

$$m_{t+1} = c_1 + c_2 \Delta c_{t+1} + c_3 \left(r_{f,t+1} - \frac{1}{\kappa_1} r_{f,t} \right) + c_4 \left(z_{m,t+1} - \frac{1}{\kappa_1} z_{m,t} \right). \qquad (20)$$

The parameters $c = (c_1, c_2, c_3, c_4)'$ are functions of the parameters of the time-series processes and the preference parameters (see Appendix A.2.4 for details). The above expression for the pricing kernel is entirely in terms of observables. We substitute this expression into the set of Euler equations (6) to obtain a set of moment restrictions that are expressed entirely in terms of observables.

We first examine the empirical plausibility of the model when the set of assets consists of the market portfolio and the risk free rate, thereby focusing on the equity premium and risk free rate puzzles. To the set of their Euler equations we add moment restrictions implied by the time-series specification of the model. We estimate the parameters with GMM and test the specification of the model with the overidentifying restrictions. We then examine the ability of the model to explain the cross-section of returns. The set of assets consists of the "Value", "Growth", "Small" capitalization, and "Large" capitalization stocks, in addition to the market portfolio and the risk free rate. To the set of their Euler equations we add moment restrictions implied by the time-series specification of the model and test with GMM.

3 Data

We use annual, quarterly, and monthly data on prices and dividends from January 1929 through December 2009. We use annual consumption data from January 1929 through December 2009 and quarterly consumption data from January 1947 through December 2009.

The proxy for the market is the Centre for Research in Security Prices (CRSP) value-weighted index of all stocks on the NYSE, AMEX, and NASDAQ. The construction of the size and book-to-market portfolios is as in Fama and French (1993). In particular, for the size sort, all NYSE, AMEX, and NASDAQ stocks are allocated across 10 portfolios each year according to their market capitalization at the end of June of the previous year. NYSE breakpoints are used in the sort. "Small" and "Large" denote the bottom and top market capitalization deciles, respectively. For the book-to-market equity sort, all NYSE, AMEX, and NASDAQ stocks are allocated across 10 portfolios each year according to their book equity (BE) to market equity (ME) ratio at the end of the previous year. NYSE breakpoints are used in the sort. "Growth" and "Value" denote the bottom and top BE/ME deciles, respectively.

The monthly portfolio return is the sum of the portfolio price and dividends at the end of the month, divided by the portfolio price at the beginning of the month. The quarterly portfolio return is the sum of the portfolio price at the end of the quarter and uncompounded dividends over the quarter, divided by the portfolio price at the beginning of the quarter. The annual portfolio return is the sum of the portfolio price at the end of the year and uncompounded dividends over the year, divided by the portfolio price at the beginning of the year.

The proxy for the monthly risk free rate is the arithmetic return on one-month Treasury Bills from Ibbotson Associates. The proxy for the quarterly risk free rate is the compounded arithmetic return on one-month Treasury Bills over the quarter. The proxy for the annual risk free rate is the compounded arithmetic return on one-month Treasury Bills over the year.

The price-dividend ratio of the market is the market price at the end of the period, divided by the sum of dividends over the previous twelve months. The dividend growth rate is the sum of dividends over the period, divided by the sum of dividends over the previous period. Consumption data are obtained from the Bureau of Economic Analysis. The consumption growth rate is the per capita personal consumption expenditure on nondurable goods over the period, divided by the per capita personal consumption expenditure on nondurable goods over the previous period.

All nominal monthly and quarterly log returns and growth rates are converted to real by subtracting the realized log inflation rate over the period. All nominal annual log returns and growth rates are converted to real by subtracting the log inflation rate over the period forecasted using an ARMA$(1; 1)$ model.

Table 1 provides descriptive statistics for the continuously compounded returns on the assets, the market-wide price-dividend ratio, and the aggregate consumption and dividend growth rates for the annual sample over the period $1931 - 2009$. The table illustrates the well documented equity premium and the size and value premia. Over the sample period, the annual equity premium over the risk free rate has mean 5.8% and the volatility of the market return is 19.7%. The annual risk free rate has mean 0.8% and standard deviation 5.0%. The annual mean premium of small over large stocks is 4.7% and of value over growth stocks is 4.5%. Value stocks are much more volatile than growth stocks and small stocks are much more volatile than large stocks.

The annual log price-dividend ratio on the market has a mean of 3.38 and standard error of 0.45 over the sample period. The average annual log dividend growth rate on the market portfolio is 1.4% with volatility 11.9%. Finally, the annual log consumption growth has a mean of 1.4% and standard deviation of 2.6% over the sample period.

4 Forecasting Returns and Growth Rates

We pointed out in Section 2.2 that the LRR model of B-Y implies that the expected rate of return of the market, equity premium, dividend growth, and consumption growth are affine functions of the lagged price-dividend ratio and risk free rate. We test this implication of the model with in-sample linear forecasting regressions and out-of-sample linear predictive regressions. The in-sample forecasting tests are carried out at the monthly, quarterly, and annual frequencies and produces negative results, thereby ruling out the possibility that the model is rejected because it is interpreted at the wrong frequency. The out-of-sample predictability tests are carried out at the annual frequency and produce negative results which reinforce the in-sample results.

In Table 2, we report the results of in-sample linear forecasting regressions of the market return and equity premium at various frequencies over the full sample period 1931 : 1 through 2009 : 12. The regression coefficients are uniformly statistically insignificant at the monthly, quarterly, and annual frequencies and the adjusted R^2 statistics are small as reported in Panels A □ C. In order to interpret the reported adjusted R^2 statistics, we calibrate and simulate the model. At the monthly frequency, we calibrate the model using the parameter values suggested by Bansal, Kiku, and Yaron (2007); at the quarterly and annual frequencies, we calibrate the model using the parameter values estimated by GMM and reported in Tables 4 and 12, respectively. At each frequency, we generate $10,000$ histories, run the in-sample forecasting regressions, and obtain the distributions of the adjusted R^2 statistics under the null hypothesis of the model. The distributions of the adjusted R^2 statistics at the annual frequency are displayed in Panels A and B of Figure 1. In Table 2 and below each reported adjusted R^2 statistic, we report in brackets the p-values that the model is correct. The p-values of the regressions of the market return and premium at all frequencies are lower than 2%, except for the p-value of the forecasting regression of the premium at the monthly frequency which is 8%.[3]

In Table 2, Panels D and E, we also report the results of in-sample linear forecasting regressions of the market return and equity premium at 3- and 5-year horizons, respectively. Consistent with earlier results, we find that the price-dividend ratio and risk free rate perform well at forecasting the market return and equity premium at lower frequencies with the adjusted R^2 statistic varying from 18.9% to 30.2%. In order

[3] Our results in Table 2, Panel C and Figure 1 differ from the results reported in Beeler and Campbell (2009). Beeler and Campbell (2009) report results on in-sample linear forecasting regressions of the equity premium by the market-wide price-dividend ratio. They report a low R^2 that is consistent to that implied by the model as inferred from simulations. Their results seem to contradict ours in Table 2, Panel C and Figure 1. However, a crucial distinction between their forecasting regressions and ours is that we use both the price-dividend ratio and risk free rate as forecasting variables whereas they use only the price-dividend ratio. We do so because our theoretical discussion in Section 2 reveals that under the null, the expected equity return and premium are affine functions of both the price-dividend ratio and risk free rate.

to interpret the reported adjusted R^2 statistics, we calibrate and simulate the model using the parameter values in Table 12. The p-values of the R^2 statistics are high.

In Table 3, Panels A – C, we report the corresponding results for the sub-period 1976 : 1 through 2009 : 12.[4] The adjusted R^2 statistics are low or negative, with the exception of the regression of the market return at the annual frequency which has adjusted R^2 3.2%. Comparison of Table 2, Panel C and Table 3, Panel C demonstrates the instability of the results across subperiods. For the market return, the adjusted R^2 is 1.4% over 1931 : 1 – 2009 : 12 and 3.2% over 1976 : 1 – 2009 : 12. For the equity premium, the adjusted R^2 is 3.8% over 1931 : 1 – 2009 : 12 and negative over 1976 : 1 – 2009 : 12.

Additional evidence against the model comes from the out-of-sample prediction of the market return and equity premium. We evaluate the out-of-sample performance of these forecasts using an out-of-sample R^2 statistic as in Campbell and Thompson (2008) and Welch and Goyal (2008).[5] In Table 3, Panel D, we show that the mean-square-error statistics are negative which means that the predictive regression has higher mean-squared prediction error than the historical average return.

The strongest evidence against the model comes from the forecasting regressions for the aggregate consumption growth rate. We report the results of in-sample linear forecasting regressions of consumption growth for the full sample period only at the annual, 3-year, and 5-year frequencies because quarterly consumption data is only available since 1947 : 1 (Table 2); and at the quarterly and annual frequencies for the sub-period 1976 : 1 through 2009 : 4 (Table 3). Over the full period, the adjusted R^2 statistic of the consumption growth regression at the annual frequency is 8.4% but the p-value is less than 1% (see Figure 1, Panel C) which means that the model of B-Y implies much higher predictability of consumption growth than the observed predictability. Likewise, the model implies much higher predictability of consumption growth at the 3-year, and 5-year frequencies than that observed in the data.Over the post-war sub-period, the adjusted R^2 is 23.6% in-sample but negative out-of-sample.

Finally, we report the results of in-sample linear forecasting regressions of dividend growth at the annual, 3-year, and 5-year frequencies for the full sample period 1931 : 1

[4] Our choice of the start date 1976 : 1 is motivated to coincide with the Welch and Goyal (2008) comprehensive study on forecasting.

[5] We estimate the regression coefficients over 1931 : 1 – 1975 : 4 and predict the market return and equity premium for the following year, 1976 : 1 – 1976 : 4. We then enlarge the estimation window to 1931 : 1 – 1976 : 4 and predict the market return and equity premium for the year 1977 : 1 – 1977 : 4. We repeat this procedure until 2009. We impose two restrictions suggested in Campbell and Thompson (2008): first, we set the regression coefficients to zero whenever they have the wrong sign (different from the theoretically expected sign estimated over the full sample); second, we set the forecast to the zero whenever it is negative. The out-of-sample performance of these forecasts is evaluated using an out-of-sample R^2 statistic: ($R^2_{OOS} = 1 - \frac{MSE_A}{MSE_N}$), where MSE_A denotes the mean-squared prediction error from the predictive regression implied by the model and MSE_N denotes the mean-squared prediction error of the historical average return.

through 2009 : 4 (Table 2); and at the annual frequency over the sub-period 1976 : 1 through 2009 : 4 (Table 3). At the annual frequency, the results are positive. Over the full period, the adjusted R^2 is 7.0% and the p-value of the model is 21.9% (see Figure 1, Panel D). Over the post-war sub-period, the adjusted R^2 is 7.6%. In the out-of-sample linear predictive regression at the annual frequency, however, the adjusted R^2 is negative (Table 3, Panel D). Moreover, at the 3-year, and 5-year frequencies, the model implies much higher predictability of consumption growth at the 3-year, and 5-year frequencies than that observed in the data with p-values 3.1% and 3.5%, respectively.

Overall the results reported in this section provide robust time series evidence against the LRR model of B-Y in both the full period and the sub period and at all frequencies. In the next section, we address the implications of the model on the cross-section of returns.

5 Empirical Evidence on the Equity Premium and the Cross-Section of Returns

5.1 Methodology

First, we estimate the time-series parameters of aggregate consumption and dividend growth without reference to the Euler equations. We choose the nine parameters of the time-series model (1)-(4) to match the following nine sample moments: the unconditional mean, variance, and first-order autocorrelation of consumption and dividend growth rates, the correlation between consumption and dividend growth rates, and the variance of squared consumption and dividend growth rates. These estimates are reported in Section 5.2 and serve as a benchmark for comparison when we subsequently re-estimate them from the joint system of time-series moment restrictions and Euler equations.

In Sections 5.3 and 5.4, we address the equity premium and the cross-section of returns, respectively. The system of equations consists of the Euler equations of consumption on a given set of assets along with the restrictions imposed on the model parameters by the unconditional moments of the aggregate consumption and dividend growth which we described above. We estimate the parameters with GMM using the efficient weighting matrix and test the model with the overidentifying restrictions. We verify the robustness of the tests by replacing the efficient weighting matrix with the identity matrix. The tests are carried out at the annual frequency. Later on, we test the robustness of the tests by repeating them at the quarterly frequency and in subperiods.

In Section 5.3, the set of assets consists of the market portfolio and the risk free rate, thereby focusing on the equity premium and the risk free rate puzzles. We introduce two unconditional Euler equations for the market portfolio and the risk free rate along with four Euler equations for the market portfolio and the risk free rate conditional on the lagged log price-dividend ratio of the market and the lagged log risk free rate. To this set of pricing restrictions we append the nine moment restrictions implied by the time-series specification of the model which we stated earlier. Thus, we have a total of 15 moment conditions. The total number of parameters to be estimated is 12, consisting of nine time-series parameters and three preference parameters. In Section 5.4, the set of assets consists of the "Value", "Growth", "Small" capitalization, and "Large" capitalization stocks, in addition to the market portfolio and the risk free rate. The Euler equations for these six assets along with the nine time-series moment restrictions give 15 moment restrictions in 12 parameters. The numerical search for a global minimum is described in Appendix A.5.

5.2 Time-Series Properties of Aggregate Consumption and Dividend Growth

In Table 4 under the label "Data", we display the sample averages of the nine moments of the consumption and dividend growth rates which we aim to match. The nine parameters of the time-series model (1)-(4) are chosen such that the nine model-generated moments, as displayed under the adjacent column labeled "$Model^{ts}$", exactly match their sample averages. This is feasible because the model is just identified. The point estimates of the nine model parameters, along with the associated standard errors in parentheses, are displayed in the first row of the table.[6] The persistence parameter (ρ_x) of the LRR variable is 0.32 and is statistically significantly positive at conventional levels of significance. This lends support to the major risk channel highlighted in the LRR literature—a predictable component in the aggregate consumption and dividend growth rates.

5.3 Evidence on the Equity Premium

We address the equity premium and risk free rate puzzles by introducing two unconditional Euler equations for the market portfolio and the risk free rate along with four Euler equations conditional on the lagged log price-dividend ratio of the market and the lagged log risk free rate. Combined with the nine time-series moment restrictions, this system of 15 restrictions and 12 parameters (9 time-series parameters plus 3 preference parameters) is overidentified. The results are displayed in Table 4.

The GMM overidentifying restrictions test rejects this model with J-stat 17.7 and

[6] The standard errors are Newey-West corrected using two lags.

asymptotic p-value less than 1%.[7] The annual pricing error for the risk free rate is □3.3% and is economically significant. The annual pricing error for the market return is □1.9%.

The second row of Table 4 displays the point estimates of the time-series and preference parameters when both the pricing restrictions and the time-series restrictions are used in the estimation. The risk aversion estimate is 8 which is reasonable although the standard error is large at 6.59. The point estimate of the IES is 0.6; however the standard error is 1.33 and we cannot reject the hypothesis that the elasticity exceeds the value of one.

The persistence parameter of the LRR variable is much higher at 0.70, compared to the value of 0.32 estimated from the time-series model alone. This suggests that the B-Y model requires much higher predictability of consumption growth to explain the equity premium and risk free rate puzzles than the predictability estimated from the time series of consumption growth alone.

5.4 Evidence on the Cross-Section of Returns

In Table 5, we report the results of tests on the cross-section of returns consisting of the "Value", "Growth", "Small" capitalization, and "Large" capitalization stocks, in addition to the market portfolio and the risk free rate.

The results reinforce the conclusions drawn from the 2-asset system. As in the two-asset system, the GMM test rejects this model with J-stat 13.6 and asymptotic p-value less than 1%. The annual pricing errors for the market return, “Large” portfolio, "Value" portfolio, and risk free rate are small. The annual pricing errors for the “Small” and “Growth” portfolios are larger at 2.6% and □2.9%, respectively. The estimates of the risk aversion and the IES are remarkably similar to the corresponding estimates in the 2-asset system. As in the 2-asset system, the B-Y model requires much higher predictability of consumption growth to explain the cross-section of returns than the predictability estimated from the time series of consumption growth alone.

6 A co-integrated Long Run Risks Model

Bansal, Gallant, and Tauchen (2007) consider a variant of the LRR model of B-Y that imposes a co-integrating restriction between the logarithm of the aggregate stock market dividends and consumption. Bansal, Dittmar, and Kiku (2007) point out that this co-integrating relation measures long run covariance risks in dividends and is important in understanding sources of risk and explaining the equity risk premia across

[7] Note that the J-stat has an asymptotic chi-squared distribution with 3 degrees of freedom under the null.

investment horizons.[8] We consider a log-linearized variant of the Bansal, Gallant, and Tauchen (2007) model that yields closed-form expressions for asset prices. We estimate and test the model using an extension of the methodology introduced in Section 2.

6.1 The Model and Testable Implications

The aggregate consumption growth, the LRR variable, and the variance of its innovation are modeled as in equations (1)-(3). Therefore, the pricing kernel, the log price-consumption ratio, and the risk free rate are functions of the LRR variable and the variance of its innovation, given by equations (19), (10), and (12), respectively.

The point of departure is the imposition of a co-integrating restriction between the logarithm of the aggregate stock market dividends and consumption,

$$d_t - c_t = \mu_{dc} + s_t, \tag{21}$$

where the cointegrating residual, s_t, is an $I(0)$ process with the cointegrating coefficient set at one,[9]

$$s_{t+1} = \lambda_{sx} x_t + \rho_s s_t + {}_s\sigma_t \varepsilon_{s,t+1}. \tag{22}$$

The shocks $\varepsilon_{c,t+1}$, $\varepsilon_{x,t+1}$, $\varepsilon_{\sigma,t+1}$, and $\varepsilon_{s,t+1}$ are assumed to be *i.i.d.* $N(0,1)$ and mutually independent.

From equation (21), we have,

$$\begin{aligned} \Delta d_{t+1} &= \Delta c_{t+1} + \Delta s_{t+1} \\ &= \mu_c + (1 + \lambda_{sx}) x_t + (\rho_s - 1) s_t + \sigma_t \varepsilon_{c,t+1} + {}_s\sigma_t \varepsilon_{s,t+1}, \end{aligned} \tag{23}$$

where the second line follows from equations (3) and (22).

The model has three state variables, the LRR variable, the variance of its innovation, and the co-integrating residual. Note that the B-Y model obtains as a limiting special case when $\rho_s = 1$. We conjecture that the log price-dividend ratio is an affine function of the state variables,

$$z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2 + A_{3,m} s_t. \tag{24}$$

In Appendix $A.3.1$, we verify this conjecture and explicitly solve for the coefficients.

[8] In a different context, Lettau and Ludvigson (2001) and Menzly, Santos, and Veronesi (2004) apply the co-integrating residual between consumption, labor income, and aggregate stock market dividends to explain the cross-section of returns.

[9] Bansal, Gallant, and Tauchen (2007) perform a heteroskedasticity-robust augmented Dickey-Fuller test for a unit root in $d_t - c_t$ and the results provide strong evidence for a cointegrating relationship between the variables with a coefficient equal to unity.

The co-integrating residual is observable as the demeaned difference between the log aggregate dividend and consumption levels (see equation (21)). We invert equations (12) and (24) and express the unobservable state variables, x_t and σ_t^2, in terms of the observables, $z_{m,t}$, $r_{f,t}$, and s_t, (see Appendix $A.3.2$ for details and expressions for α_0, α_1, α_2, α_3, β_0, β_1, β_2 and β_3),

$$\begin{aligned} x_t &= \alpha_0 + \alpha_1 r_{f,t} + \alpha_2 z_{m,t} + \alpha_3 s_t, \quad (25) \\ \sigma_t^2 &= \beta_0 + \beta_1 r_{f,t} + \beta_2 z_{m,t} + \beta_3 s_t. \quad (26) \end{aligned}$$

Equations (9), (24), (25), (26), and (12) imply that the expected market return and equity premium are affine functions of the state variables. Furthermore, it is straightforward to see that the expected consumption and dividend growth rates are affine functions of the state variables. Since the state variables, x_t and σ_t^2 are affine functions of the observables $z_{m,t}$, $r_{f,t}$, and s_t, we may express the expected market return, equity premium, dividend growth, and consumption growth as affine functions of the observables $z_{m,t}$, $r_{f,t}$, and s_t, with coefficients known functions of the model parameters. In the next section, we test the predictive implications of the model through in-sample linear forecasting regressions and out-of-sample linear predictive regressions of the market return, equity premium, dividend growth, and consumption growth on the lagged price-dividend ratio, risk free rate, and the difference between the log dividend and consumption levels.

Finally, we derive the pricing implications of the model. Using equations (7), (8), and (10), we write the pricing kernel as,

$$\begin{aligned} m_{t+1} &= (\theta \log \delta + (\theta - 1)[\kappa_0 + (\kappa_1 - 1) A_0]) + \left(-\frac{\theta}{} + (\theta - 1) \right) \Delta c_{t+1} \\ &\quad + (\theta - 1)\kappa_1 A_1 x_{t+1} + (\theta - 1)\kappa_1 A_2 \sigma_{t+1}^2 \\ &\quad - (\theta - 1) A_1 x_t - (\theta - 1) A_2 \sigma_t^2, \quad (27) \end{aligned}$$

Substituting the expressions for x_t and σ_t^2 from equations (25) and (26) into equation (27), we express the pricing kernel entirely in terms of observables (see Appendix $A.3.2$ for details),

$$m_{t+1} = c_1 + c_2 \Delta c_{t+1} + c_3 \left(r_{f,t+1} - \frac{1}{\kappa_1} r_{f,t} \right) + c_4 \left(z_{m,t+1} - \frac{1}{\kappa_1} z_{m,t} \right) + c_5 \left(s_{t+1} - \frac{1}{\kappa_1} s_t \right). \quad (28)$$

In the next section, we first examine the empirical plausibility of the model when the asset menu consists of the market portfolio and the risk free rate. The lagged log price-dividend ratio of the market and the lagged log risk free rate are used as instruments. The Euler equations for the two assets along with the two chosen instruments give 6 moment restrictions. To this set of pricing restrictions, we add moment restrictions implied by the time-series specification of the model. In particular, we include the following 7 moments of consumption and dividend growth rates: $E(\Delta c_{t+1})$, $Var(\Delta c_{t+1})$, $Corr(\Delta c_{t+1}, \Delta c_{t+2})$, $Corr(\Delta c_{t+1}, \Delta c_{t+3})$, $Var(\Delta d_{t+1})$, $Corr(\Delta d_{t+1}, \Delta d_{t+2})$, and $Corr(\Delta c_{t+1}, \Delta d_{t+1})$ (see Appendix A.4 for expressions for these moments in terms of the time-series parameters). Thus, we have a total of 13 moment conditions. The total number of parameters to be estimated is 12, including 9 time-series parameters and 3 preference parameters. We estimate the parameters with GMM and test the specification of the model using the overidentifying restriction.

We then examine the ability of the model to explain the cross-section of returns. In this case, the asset menu consists of the market portfolio, the risk free rate, and portfolios of "Small" capitalization, "Large" capitalization, "Growth" and "Value" stocks. The Euler equations for the 6 assets give 6 moment restrictions. To this set of pricing restrictions, we add the 7 moment restrictions implied by the time-series specification of the model. This gives, once again, a total of 13 moment conditions in 12 parameters. We estimate the parameters and test the model specification with GMM.

6.2 Empirical Evidence on the Co-integrated Model

In Table 6, we report the results of in-sample linear forecasting regressions of the market return, equity premium, and the consumption and dividend growth rates at the annual frequency for the full sample period 1931 through 2009. We do not report results at the monthly and quarterly frequencies because reliable data on the co-integrating residual, the demeaned difference between the log aggregate dividend and consumption levels, is not available at the monthly frequency and is only available for the postwar subperiod at the quarterly frequency. In Table 7, Panels A and B, we report the corresponding results at the quarterly and annual frequencies, respectively, for the sub-period 1976 : 1 through 2009 : 4.

Tables 6 and 7 reveal that the difference between the log aggregate dividend and consumption levels, s_t, has substantial incremental forecasting power for the aggregate consumption and dividend growth rates over and above that contained in the log price-dividend ratio and risk free rate at the annual and quarterly frequencies. In-sample forecasting regressions for the consumption growth rate have adjusted-R^2 19.8% and p-value 71.3% over the full period (Table 6, Row 1 and Figure 2, Panel C) and adjusted-R^2 38.3% over the subperiod (Table 7, Panel B, Row 1) at the annual frequency. The coefficient of s_t is statistically significant in both cases. Similar results are obtained for the aggregate dividend growth rate for which the adjusted-R^2 is 35.6% and the p-value is 68.4% over the full period (Table 6, Row 2 and Figure 2, Panel D) and the adjusted-R^2 is 15.1% over the subperiod (Table 7, Panel B, Row 2). However, Table 7, Panel C reveals that the state variables of the co-integrated model fail to retain their predictive power for consumption and dividend growth rates out-of-sample. The out-of-sample R^2 for the consumption growth rate is □51.7% and for the dividend growth rate □23.7%.

The state variables do not forecast the market return and equity premium as predicted by the co-integrated model. At the annual frequency and over the full period, the adjusted-R^2 for the market return is 2.0% (Table 6, Row 3) and for the equity premium is 4.4% (Table 6, Row 4). However, the p-values of these regressions are zero, meaning that the co-integrated model implies much higher predictability of the market return and equity premium using the market-wide price-dividend ratio, risk free rate, and the demeaned dividend-consumption ratio than the observed predictability (see Figure 2, Panels A and B).

Furthermore, the observed predictability by the state variables of the co-integrated model is unstable. At the annual frequency over the 1976 □ 2009 sub-period, the adjusted-R^2 for the market return is 0.0% (Table 7, Panel B, Row 3) and for the equity premium is negative (Table 7, Panel B, Row 4). Given the poor in-sample forecasting performance of the model-implied state variables, it is not surprisingly the out-of-sample R^2 for the market return and equity premium are □5.5% and □5.0%, respectively (Table 7, Panel C, Rows 3 and 4). At the quarterly frequency over the 1976 □ 2009 sub-period, the adjusted-R^2 for the market return is negative (Table 7, Panel A, Row 3) and for the equity premium is 1.0% (Table 7, Panel A, Row 4).

The pricing implications of the model for the 2-asset system are displayed in Table 8. The point estimate of the auto-correlation coefficient, ρ_s, of the co-integrating residual, s_t, is 0.96 and the asymptotic standard error is 0.76. Therefore, the data cannot distinguish the co-integrated model from the B-Y model which obtains as a limiting special case when $\rho_s = 1$. This explains why the conclusions drawn from Table 8 are similar to our earlier conclusions from Table 4. Specifically, the persistence parameter of the LRR variable is much higher at 0.94, compared to the value of 0.32 estimated from the time-series model alone in Table 4. Therefore, the co-integrated model, as the B-Y model, requires much higher predictability of consumption growth to explain the equity premium and risk free rate puzzles than the predictability estimated from the time series of consumption growth alone. The GMM overidentifying restrictions test rejects this model with J-stat 9.19 and asymptotic p-value less than 1%. The point estimates of the parameters are similar to those in Table 4. The annual pricing error for the risk free rate is 2.9% and for the market return is 1.5%.

The pricing implications of the model for the 6-asset system are displayed in Table 9. The point estimate of the auto-correlation of the co-integrating residual is 0.90 and the asymptotic standard error is 1.30. As in the 2-asset system, the data cannot distinguish the co-integrated model from the B-Y model and the conclusions drawn from Table 9 are similar to the earlier conclusions drawn from Table 5. Again, the co-integrated model, as the B-Y model, requires much higher predictability of consumption growth to explain the cross-section of returns than the predictability estimated from the time series of consumption growth alone. The GMM overidentifying restrictions test rejects this model with J-stat 9.12 and asymptotic p-value less than 1%. The point estimates of the parameters are very similar to those in Table 5. The annual pricing errors for the "Small" portfolio (4.3%) and "Value" portfolio (3.2%) are economically large.

The co-integrated LLR model generalizes the LRR model of B-Y by introducing the difference between the log dividend and consumption levels as a third state variable. The combined evidence from the out-of-sample predictive regressions and the pricing tests suggest that the problems identified with the model of B-Y remain to be resolved.

7 Robustness Tests

In Section 7.1, we address the robustness of our results to the post-war sub-period. In Section 7.2, we explore the pricing implications of the B-Y model at the quarterly, as opposed to the annual, frequency.

7.1 Robustness to the Post-War Sub-Period

Since the period prior to 1947 was one of great economic uncertainty, including the Great Depression, World War II, and structural breaks in the equity premium, rejection of the LRR models in the full sample may be due to their poor performance in the pre-war period. Pastor and Stambaugh (2001) document evidence of breaks in the equity premium in the early thirties and forties, and in the early and mid-nineties. Lettau, Ludvigson, and Wachter (2008) find evidence of a break in the consumption variance around 1992, followed by a break in the log price-dividend ratio of the market around 1995. Lettau and Van Nieuwerburgh (2008) report evidence of two breaks in the mean of the aggregate price-dividend ratio around 1954 and 1994.

We explore the possibility that rejection of the LRR model of B-Y and its co-integrated extension is due, in part, to failure to account for regime shifts over the period 1931 − 2009. In Section 4, we already provide evidence that the lack of out-of-sample predictability of the market return, equity premium, consumption growth, and dividend growth in linear regressions on the price-dividend ratio and risk free rate over the period 1931 − 2009 (Table 2) persists over the sub-period 1976 − 2009 (Table 3). In Section 6, we add the difference between the log dividend and consumption levels as a third predictive variable, as implied by the LRR model of Bansal, Gallant, and Tauchen (2007), and provide evidence that the lack of out-of-sample predictability over the period 1931 − 2009 (Table 6) persists over the sub-period 1976 − 2009 (Table 7).

As further evidence of the robustness of our results we present estimation and test results of the Euler equations of consumption over the post-war sub-period 1947 − 2009 on the 2-asset system (Table 10) and the 6-asset system (Table 11). The model is still rejected on both the 2-asset and 6-asset systems with p-values less than 1%. The point estimates of the parameters and the pricing errors are similar in the full period and the post-war sub-period and lend support to the robustness of the empirical methodology.

7.2 Interpretation of the Model at the Quarterly Frequency

We explore the possibility that the LRR model of B-Y applies at the quarterly, as opposed to the annual, frequency. The lack of out-of-sample predictability of the market return, equity premium, dividend growth, and consumption growth reported in Section 4 cannot be attributed to the possibility that the LRR model of B-Y applies at the quarterly, as opposed to the annual, frequency because the tests are carried out at both the quarterly and annual frequencies.

Our rejection of the Euler equations of consumption on the 2-asset and 6-asset systems at the annual frequency is susceptible to the criticism that the LRR model applies at the quarterly frequency and the Euler equations stated at the annual frequency improperly temporally aggregate consumption flows. Therefore, we repeat our estimation and tests of the Euler equations at the quarterly frequency. Since reliable quarterly data are only available over the post-war sub-period, we perform our tests over $1947:2-2009:4$. Tables 12 and 13 display the results for the 2-asset and 6-asset systems, respectively. The annualized pricing errors are very similar to those obtained using annual data over the post-war subperiod.

The results in this section suggest that our findings are unlikely to be driven by the problems associated with temporal aggregation or by the interpetation of the model at the wrong frequency.

8 Concluding Remarks

We present a novel methodology in testing the long-run risks model of Bansal and Yaron (2004) based on the observation that, under the null, the potentially latent state variables, "long-run risk" and the conditional variance of its innovation, are known affine functions of the observable market-wide price-dividend ratio and risk free rate.

Using the methodology, we test the time-series and cross-sectional pricing implications of the model over the sample period $1931-2009$. The results are robust to the interpretation of the model at the annual, quarterly, and monthly frequencies; to the full time period $1931-2009$ versus the post-war sub-period; to the temporal aggregation of consumption flows; and to the co-integration or lack of co-integration of the consumption and dividend levels. Whereas we formally reject the model, we derive valuable insights which should prove useful in guiding future search.

Essentially, the model requires that aggregate consumption growth and returns be much more forecastable by the price-dividend ratio and risk free rate in linear regressions than what we observe in the data. What may be needed is a richer model in which the expected market return, equity premium, consumption growth, and pricing kernel are nonlinear functions of the two state variables. Alternatively, what may be needed is a model that introduces a third state variable that plays a major role in forecasting the expected market return, equity premium, and consumption growth and also plays a major role in explaining the cross-section of returns. The observed differences in the results between the full sample period and the post-war sub-period reinforce the view that the economy experiences structural breaks which may be fruitfully captured by such a state variable.

A Appendix

A.1 Estimation of Time-Series Parameters

The decision interval of the agent is assumed to be annual. We estimate the model at the annual frequency, such that its annual growth rates of consumption and dividends match salient features of observed annual consumption and dividend data. There are 9 parameters to be estimated - μ_c, μ_d, ϕ, φ, ρ_x, ${}_x$, σ, υ, and σ_w.

From the specification of the consumption growth process, we have

$$E\left(\Delta c_{t+1}\right) = \mu_c \tag{29}$$

We also, have

$$\begin{aligned} Var\left(\Delta c_{t+1}\right) &= Var\left(x_t\right) + Var\left(\sigma_t \varepsilon_{c,t+1}\right) + 2Cov(x_t, \sigma_t \varepsilon_{c,t+1}) \\ &= Var\left(x_t\right) + \sigma^2 + 0 \\ &= \frac{{}_x^2 \sigma^2}{1 - \rho_x^2} + \sigma^2 \end{aligned} \tag{30}$$

and,

$$Cov(\Delta c_{t+1}, \Delta c_{t+2}) = \rho_x \frac{{}_x^2 \sigma^2}{1 - \rho_x^2} \tag{31}$$

From the specification of the dividend process, we have

$$E\left(\Delta d_{t+1}\right) = \mu_d \tag{32}$$

$$Var\left(\Delta d_{t+1}\right) = \phi^2 \frac{{}_x^2 \sigma^2}{1 - \rho_x^2} + \sigma^2 \varphi^2 \tag{33}$$

$$Cov(\Delta d_{t+1}, \Delta d_{t+2}) = \phi^2 \rho_x \frac{{}_x^2 \sigma^2}{1 - \rho_x^2} \tag{34}$$

Also, from the consumption and dividend growth processes,

$$Cov(\Delta c_{t+1}, \Delta d_{t+1}) = \phi \frac{{}_x^2 \sigma^2}{1 - \rho_x^2} \tag{35}$$

Finally, we have

$$Var \,^{\square}\!(\Delta c_{t+1})^2\big) = E\left[Var_t \,^{\square}\!(\Delta c_{t+1})^2\big)\right] + Var\left[E_t \,^{\square}\!(\Delta c_{t+1})^2\big)\right] \tag{36}$$

Now,

$$(\Delta c_{t+1})^2 = \mu_c^2 + x_t^2 + \sigma_t^2 \varepsilon_{c,t+1} + 2\mu_c x_t + 2x_t \sigma_t \varepsilon_{c,t+1} + 2\mu_c \sigma_t \varepsilon_{c,t+1} \tag{37}$$

Hence,

$$E_t \,^{\square}\!(\Delta c_{t+1})^2\big) = \mu_c^2 + x_t^2 + \sigma_t^2 + 2\mu_c x_t$$

$$\begin{aligned} Var\left[E_t \,^{\square}\!(\Delta c_{t+1})^2\big)\right] &= Var(x_t^2) + Var(\sigma_t^2) + 4\mu_c^2 Var(x_t) + 4\mu_c Cov(x_t, x_t^2) \\ &\quad + 2Cov(x_t^2, \sigma_t^2) + 4\mu_c Cov(x_t, \sigma_t^2) \end{aligned} \tag{38}$$

Now, $Var(\sigma_t^2) = \frac{\sigma_w^2}{1 \square \upsilon^2}$, $Cov(x_t, \sigma_t^2) = 0$, $Cov(x_t^2, \sigma_t^2) = \frac{{}_x^2 \sigma_w^2 \upsilon}{(1 \square \upsilon^2)(1 \square \upsilon \rho_x^2)}$, $Cov(x_t, x_t^2) = 0$, and

$$Var(x_t^2) = \frac{3 \;{}_x^4 \sigma_w^2 (1 + \upsilon \rho_x^2)}{(1 \square \rho_x^4)(1 \square \upsilon^2)(1 \square \upsilon \rho_x^2)} + \frac{1}{1 \square \rho_x^4}\left[2\sigma^4 + \frac{4\rho_x^2 \;{}_x^4 \sigma^4}{(1 \square \rho_x^2)}\right]$$

Substituting the above expressions into equation (38), we have

$$\begin{aligned} Var\left[E_t \,^{\square}\!(\Delta c_{t+1})^2\big)\right] &= \frac{3 \;{}_x^4 \sigma_w^2 (1 + \upsilon \rho_x^2)}{(1 \square \rho_x^4)(1 \square \upsilon^2)(1 \square \upsilon \rho_x^2)} + \frac{1}{1 \square \rho_x^4}\left[2\sigma^4 + \frac{4\rho_x^2 \;{}_x^4 \sigma^4}{(1 \square \rho_x^2)}\right] \\ &\quad + \frac{\sigma_w^2}{1 \square \upsilon^2} + 4\mu_c^2 \frac{{}_x^2 \sigma^2}{1 \square \rho_x^2} + \frac{2 \;{}_x^2 \sigma_w^2 \upsilon}{(1 \square \upsilon^2)(1 \square \upsilon \rho_x^2)} \end{aligned} \tag{39}$$

Also, from equation (37),

$$Var_t \,^{\square}\!(\Delta c_{t+1})^2\big) = 2\sigma_t^4 + 4x_t^2 \sigma_t^2 + 4\mu_c^2 \sigma_t^2 + 8\mu_c x_t \sigma_t^2$$

Hence,

$$E\left[Var_t \,^{\square}\!(\Delta c_{t+1})^2\big)\right] = 2\frac{\sigma_w^2}{1 \square \upsilon^2} + 2\sigma^4 + \frac{4 \;{}_x^2 \sigma_w^2 \upsilon}{(1 \square \upsilon^2)(1 \square \upsilon \rho_x^2)} + \frac{4 \;{}_x^2 \sigma^4}{1 \square \rho_x^2} + 4\mu_c^2 \sigma^2 \tag{40}$$

Substituting equations (39) and (40) into equation (36), we have

$$\begin{aligned} Var \,^{\square}\!(\Delta c_{t+1})^2\big) &= \frac{3 \;{}_x^4 \sigma_w^2 (1 + \upsilon \rho_x^2)}{(1 \square \rho_x^4)(1 \square \upsilon^2)(1 \square \upsilon \rho_x^2)} + \frac{1}{1 \square \rho_x^4}\left[2\sigma^4 + \frac{4\rho_x^2 \;{}_x^4 \sigma^4}{(1 \square \rho_x^2)}\right] + \frac{3\sigma_w^2}{1 \square \upsilon^2} \\ &\quad + 4\mu_c^2 \frac{{}_x^2 \sigma^2}{1 \square \rho_x^2} + \frac{6 \;{}_x^2 \sigma_w^2 \upsilon}{(1 \square \upsilon^2)(1 \square \upsilon \rho_x^2)} + \frac{4 \;{}_x^2 \sigma^4}{1 \square \rho_x^2} + 2\sigma^4 + 4\mu_c^2 \sigma^2 \end{aligned} \tag{41}$$

Similar calculations yield,

$$
\begin{aligned}
Var\left[E_t\left((\Delta d_{t+1})^2\right)\right] &= \phi^4\left[\frac{3\ {}_x^4\sigma_w^2(1+\upsilon\rho_x^2)}{(1-\rho_x^4)(1-\upsilon^2)(1-\upsilon\rho_x^2)}+\frac{1}{1-\rho_x^4}\left(2\sigma^4+\frac{4\rho_x^2\ {}_x^4\sigma^4}{(1-\rho_x^2)}\right)\right] \\
&\quad +\frac{\sigma_w^2}{1-\upsilon^2}\varphi^4+4\mu_c^2\frac{{}_x^2\sigma^2}{1-\rho_x^2}\phi^2+\frac{2\ {}_x^2\sigma_w^2\upsilon}{(1-\upsilon^2)(1-\upsilon\rho_x^2)}\phi^2\varphi^2
\end{aligned}
$$

$$
\begin{aligned}
E\left[Var_t\left((\Delta d_{t+1})^2\right)\right] &= \left[2\frac{\sigma_w^2}{1-\upsilon^2}+2\sigma^4\right]\varphi^4+\left[\frac{4\ {}_x^2\sigma_w^2\upsilon}{(1-\upsilon^2)(1-\upsilon\rho_x^2)}+\frac{4\ {}_x^2\sigma^4}{1-\rho_x^2}\right]\phi^2\varphi^2 \\
&\quad +4\mu_d^2\varphi^2\sigma^2
\end{aligned}
$$

Hence, we have

$$
\begin{aligned}
Var\left((\Delta d_{t+1})^2\right) &= \phi^4\left[\frac{3\ {}_x^4\sigma_w^2(1+\upsilon\rho_x^2)}{(1-\rho_x^4)(1-\upsilon^2)(1-\upsilon\rho_x^2)}+\frac{1}{1-\rho_x^4}\left(2\sigma^4+\frac{4\rho_x^2\ {}_x^4\sigma^4}{(1-\rho_x^2)}\right)\right]+\frac{3\sigma_w^2}{1-\upsilon^2}\varphi^4 \\
&\quad +4\mu_c^2\frac{{}_x^2\sigma^2}{1-\rho_x^2}\phi^2+\frac{6\ {}_x^2\sigma_w^2\upsilon}{(1-\upsilon^2)(1-\upsilon\rho_x^2)}\phi^2\varphi^2+\frac{4\ {}_x^2\sigma^4}{1-\rho_x^2}\phi^2\varphi^2 \\
&\quad +2\sigma^4\varphi^4+4\mu_d^2\varphi^2\sigma^2 \qquad (42)
\end{aligned}
$$

Equations (29)-(35), (41), and (42) give 9 moments restrictions in the 9 time-series parameters.

A.2 Details of Estimation Methodology

The model is given by the equations

$$
\begin{aligned}
x_{t+1} &= \rho_x x_t + {}_x\sigma_t\varepsilon_{x,t+1}, \\
\sigma_{t+1}^2 &= (1-\upsilon)\sigma^2+\upsilon\sigma_t^2+\sigma_w\varepsilon_{\sigma,t+1}, \\
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t\varepsilon_{c,t+1}, \\
\Delta d_{t+1} &= \mu_d + \phi x_t + \varphi\sigma_t\varepsilon_{d,t+1}.
\end{aligned}
$$

The shocks $\varepsilon_{x,t+1}$, $\varepsilon_{\sigma,t+1}$, $\varepsilon_{c,t+1}$, $\varepsilon_{d,t+1}$ are assumed to be *i.i.d.* $N(0,1)$ and mutually independent.

A.2.1 Expressions for $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_{0,m}$, $\mathbf{A}_{1,m}$, and $\mathbf{A}_{2,m}$

Bansal and Yaron (2004) show that z_t and $z_{m,t}$, are affine functions of the state variables, x_t and σ_t^2,

$$
\begin{aligned}
z_t &= A_0 + A_1 x_t + A_2 \sigma_t^2, \\
z_{m,t} &= A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2,
\end{aligned}
$$

where

$$
\begin{aligned}
A_1 &= \frac{1 \square \frac{1}{}}{1 \square \kappa_1 \rho_x} \\
A_2 &= \frac{0.5\left[\left(\square \frac{\theta}{} + \theta\right)^2 + (\theta \kappa_1 A_1 \quad _x)^2\right]}{\theta\left(1 \square \kappa_1 \upsilon\right)} \\
A_0 &= \frac{log(\delta) + \left(1 \square \frac{1}{}\right)\mu_c + \kappa_0 + \kappa_1 A_2 \sigma^2 (1 \square \upsilon) + 0.5 \theta \kappa_1 A_2 \sigma_w^2}{1 \square \kappa_1}
\end{aligned}
$$

$$
\begin{aligned}
A_{1,m} &= \frac{\phi \square \frac{1}{}}{1 \square \kappa_{1,m} \rho_x} \\
A_{2,m} &= \frac{(1 \square \theta) A_2 \left(1 \square \kappa_1 \upsilon\right) + 0.5\left[\ ^2 + \varphi^2 + \left(\left(\theta \square 1\right)\kappa_1 A_1 + \kappa_{1,m} A_{1,m}\right)^2 \ {}^2_x\right]}{1 \square \kappa_{1,m} \upsilon} \\
A_{0,m} &= \frac{\theta \log(\delta) + \left(\square \frac{\theta}{} + \theta \square 1\right)\mu_c + (\theta \square 1)\kappa_0 + (\theta \square 1)(\kappa_1 \square 1) A_0 + (\theta \square 1)\kappa_1 A_2 \sigma^2 (1 \square \upsilon)}{1 \square \kappa_{1,m}} \\
&\quad + \frac{\kappa_{0,m} + \mu_d + \kappa_{1,m} A_{2,m} \sigma^2 (1 \square \upsilon) + 0.5\left[(\theta \square 1)\kappa_1 A_2 + \kappa_{1,m} A_{2,m}\right]^2 \sigma_w^2}{1 \square \kappa_{1,m}}
\end{aligned}
$$

A.2.2 Risk Free Rate

To derive the expression for the risk free rate, note that

$$
E_t\left[\exp\left(\theta \log \delta \square \frac{\theta}{}\Delta c_{t+1} + (\theta \square 1) r_{c,t+1} + r_{f,t}\right)\right] = 1
$$

Hence,

$$
\begin{aligned}
\exp\left(\square r_{f,t}\right) &= E_t\left[\exp\left(\theta\log\delta \square \frac{\theta}{}\Delta c_{t+1} + (\theta \square 1) r_{c,t+1}\right)\right] \\
&= \exp(\theta\log\delta \square \frac{\theta}{}\mu_c \square \frac{\theta}{}x_t + (\theta \square 1)\kappa_0 + (\theta \square 1)\kappa_1 A_0 \\
&\quad +(\theta \square 1)\kappa_1 A_1 \rho_x x_t + (\theta \square 1)\kappa_1 A_2 (1 \square \upsilon)\sigma^2 + (\theta \square 1)\kappa_1 A_2 \upsilon \sigma_t^2 \\
&\quad \square(\theta \square 1) A_0 \square (\theta \square 1) A_1 x_t \square (\theta \square 1) A_2 \sigma_t^2 + (\theta \square 1)\mu_c + (\theta \square 1) x_t \\
&\quad +0.5\left[\left(\square\frac{\theta}{} + \theta \square 1\right)^2 \sigma_t^2 + (\theta \square 1)^2 \kappa_1^2 A_1^2 {}^2_x \sigma_t^2 + (\theta \square 1)^2 \kappa_1^2 A_2^2 \sigma_w^2\right])
\end{aligned}
$$

Therefore, the risk free rate is

$$
\begin{aligned}
r_{f,t} &= \square\theta\log\delta \square \left(\square\frac{\theta}{} + \theta \square 1\right)\mu_c \square (\theta \square 1)\kappa_0 \square (\theta \square 1)(\kappa_1 \square 1) A_0 \square (\theta \square 1)\kappa_1 A_2 (1 \square \upsilon)\sigma^2 \\
&\quad \square 0.5(\theta \square 1)^2 \kappa_1^2 A_2^2 \sigma_w^2 \square \left[(\square\frac{\theta}{} + \theta \square 1) + (\theta \square 1)(\kappa_1 \rho_x \square 1) A_1\right] x_t \\
&\quad \square \left[(\theta \square 1)(\kappa_1 \upsilon \square 1) A_2 + 0.5 \quad \left(\left(\square\frac{\theta}{} + \theta \square 1\right)^2 + (\theta \square 1)^2 \kappa_1^2 A_1^2 {}^2_x\right)\right]\sigma_t^2 \\
&= A_{0,f} + A_{1,f} x_t + A_{2,f}\sigma_t^2
\end{aligned}
$$

where

$$
\begin{aligned}
A_{0,f} &= \square\theta\log\delta \square \left(\square\frac{\theta}{} + \theta \square 1\right)\mu_c \square (\theta \square 1)\kappa_0 \square (\theta \square 1)(\kappa_1 \square 1) A_0 \square (\theta \square 1)\kappa_1 A_2 (1 \square \upsilon)\sigma^2 \\
&\quad \square 0.5(\theta \square 1)^2 \kappa_1^2 A_2^2 \sigma_w^2 \\
A_{1,f} &= \square\left[(\square\frac{\theta}{} + \theta \square 1) + (\theta \square 1)(\kappa_1 \rho_x \square 1) A_1\right] \\
A_{2,f} &= \square\left[(\theta \square 1)(\kappa_1 \upsilon \square 1) A_2 + 0.5 \quad \left(\left(\square\frac{\theta}{} + \theta \square 1\right)^2 + (\theta \square 1)^2 \kappa_1^2 A_1^2 {}^2_x\right)\right]
\end{aligned}
$$

A.2.3 Latent state variables in terms of observable variables

The model implies

$$
\begin{aligned}
z_{m,t} &= A_{0,m} + A_{1,m} x_t + A_{2,m}\sigma_t^2, \\
r_{f,t} &= A_{0,f} + A_{1,f} x_t + A_{2,f}\sigma_t^2.
\end{aligned}
$$

These equations may be inverted to express the state variables in terms of the observables,

$$x_t = \alpha_0 + \alpha_1 r_{f,t+1} + \alpha_2 z_{m,t},$$

where

$$\begin{aligned}
\alpha_0 &= \frac{A_{2,m}A_{0,f} - A_{0,m}A_{2,f}}{A_{1,m}A_{2,f} - A_{2,m}A_{1,f}}, \\
\alpha_1 &= \frac{-A_{2,m}}{A_{1,m}A_{2,f} - A_{2,m}A_{1,f}}, \\
\alpha_2 &= \frac{A_{2,f}}{A_{1,m}A_{2,f} - A_{2,m}A_{1,f}},
\end{aligned}$$

and

$$\sigma_t^2 = \beta_0 + \beta_1 r_{f,t+1} + \beta_2 z_{m,t},$$

where

$$\begin{aligned}
\beta_0 &= \frac{A_{0,m}A_{1,f} - A_{1,m}A_{0,f}}{A_{1,m}A_{2,f} - A_{2,m}A_{1,f}}, \\
\beta_1 &= \frac{A_{1,m}}{A_{1,m}A_{2,f} - A_{2,m}A_{1,f}}, \\
\beta_2 &= \frac{-A_{1,f}}{A_{1,m}A_{2,f} - A_{2,m}A_{1,f}}.
\end{aligned}$$

A.2.4 The pricing kernel in terms of observables

The pricing kernel is given by (19),

$$\begin{aligned}
m_{t+1} &= (\theta \log \delta + (\theta - 1)\left[\kappa_0 + (\kappa_1 - 1)A_0\right]) + \left(-\frac{\theta}{} + (\theta - 1)\right)\Delta c_{t+1} \\
&\quad + (\theta - 1)\kappa_1 A_1 x_{t+1} + (\theta - 1)\kappa_1 A_2 \sigma_{t+1}^2 - (\theta - 1)A_1 x_t - (\theta - 1)A_2 \sigma_t^2
\end{aligned}$$

Substituting the expressions for x_t and σ_t^2 from Section A.1.2 into the pricing kernel, we have

$$m_{t+1} = c_1 + c_2 \Delta c_{t+1} + c_3 \left(r_{f,t+1} - \frac{1}{\kappa_1} r_{f,t}\right) + c_4 \left(z_{m,t+1} - \frac{1}{\kappa_1} z_{m,t}\right)$$

where

$$
\begin{aligned}
c_1 &= \theta \log \delta + (\theta \square 1)[\kappa_0 + (\kappa_1 \square 1)(A_0 + A_1 \alpha_0 + A_2 \beta_0)] \\
c_2 &= \square \frac{\theta}{} + (\theta \square 1) \\
c_3 &= (\theta \square 1)\kappa_1 [A_1 \alpha_1 + A_2 \beta_1] \\
c_4 &= (\theta \square 1)\kappa_1 [A_1 \alpha_2 + A_2 \beta_2]
\end{aligned}
$$

A.3 Estimation Methodology for co-integrated Model

The model is given by the equations

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t \varepsilon_{c,t+1}, \\
x_{t+1} &= \rho_x x_t + {}_x \sigma_t \varepsilon_{x,t+1}, \\
\sigma_{t+1}^2 &= \mu_\sigma + \rho_\sigma \sigma_t^2 + \sigma_w \varepsilon_{\sigma,t+1}, \\
d_t \square c_t &= \mu_{dc} + s_t, \\
s_{t+1} &= \lambda_{sx} x_t + \rho_s s_t + {}_s \sigma_t \varepsilon_{s,t+1}, \\
\Delta d_{t+1} &= \mu_c + (1 + \lambda_{sx}) x_t + (\rho_s \square 1) s_t + \sigma_t \varepsilon_{c,t+1} + {}_s \sigma_t \varepsilon_{s,t+1}.
\end{aligned} \tag{43}
$$

A.3.1 The Dividend Claim

We conjecture that the log price-dividend ratio is an affine function of the state variables, x_t, σ_t^2, and s_t:

$$z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2 + A_{3,m} s_t.$$

The coefficients $A_{0,m}$, $A_{1,m}$, $A_{2,m}$, and $A_{3,m}$ are computed using the method of undetermined coefficients as described below.

The Euler equation for the observable return on the aggregate dividend claim, $r_{m,t+1}$, is,

$$E_t \left[\exp \left(\theta \log \delta \square \frac{\theta}{} \Delta c_{t+1} + (\theta \square 1) r_{c,t+1} + r_{m,t+1} \right) \right] = 1 \tag{44}$$

Substituting the expression for $r_{m,t+1}$ from equation (9) into the above Euler condition, we have

$$
\begin{aligned}
& E_t[\exp(\theta \log \delta \square \frac{\theta}{} \mu_c \square \frac{\theta}{} x_t \square \frac{\theta}{} \sigma_t \varepsilon_{c,t+1} + (\theta \square 1)\kappa_0 + (\theta \square 1)\kappa_1 A_0 \\
& +(\theta \square 1)\kappa_1 A_1 \rho_x x_t + (\theta \square 1)\kappa_1 A_1 \;\; {}_x \sigma_t \varepsilon_{x,t+1} \\
& +(\theta \square 1)\kappa_1 A_2 \mu_\sigma + (\theta \square 1)\kappa_1 A_2 \rho_\sigma \sigma_t^2 + (\theta \square 1)\kappa_1 A_2 \sigma_w \varepsilon_{\sigma,t+1} \\
& +(\theta \square 1)\kappa_1 A_3 \lambda_{sx} x_t + (\theta \square 1)\kappa_1 A_3 \rho_s s_t + (\theta \square 1)\kappa_1 A_3 \;\; {}_s \sigma_t \varepsilon_{s,t+1} \\
& \square(\theta \square 1)A_0 \square (\theta \square 1)A_1 x_t \square (\theta \square 1)A_2 \sigma_t^2 \square (\theta \square 1)A_3 s_t \\
& +(\theta \square 1)\mu_c + (\theta \square 1)x_t + (\theta \square 1)\sigma_t \varepsilon_{c,t+1} \\
& +\kappa_{0,m} + \kappa_{1,m} A_{0,m} + \kappa_{1,m} A_{1,m} \rho_x x_t + \kappa_{1,m} A_{1,m} \;\; {}_x \sigma_t \varepsilon_{x,t+1} + \kappa_{1,m} A_{2,m} \mu_\sigma \\
& +\kappa_{1,m} A_{2,m} \rho_\sigma \sigma_t^2 + \kappa_{1,m} A_{2,m} \sigma_w \varepsilon_{\sigma,t+1} + \kappa_{1,m} A_{3,m} \lambda_{sx} x_t + \kappa_{1,m} A_{3,m} \rho_s s_t \\
& +\kappa_{1,m} A_{3,m} \;\; {}_s \sigma_t \varepsilon_{s,t+1} \square A_{0,m} \square A_{1,m} x_t \square A_{2,m} \sigma_t^2 \square A_{3,m} s_t \\
& +\mu_c + (1 + \lambda_{sx}) x_t + (\rho_s \square 1) s_t + \sigma_t \varepsilon_{c,t+1} + \;\; {}_s \sigma_t \varepsilon_{s,t+1})] \\
= \; & 1
\end{aligned}
$$

Using the assumed conditional log-normality of the stochastic processes, the left-hand-side of the above expression simplifies to

$$
\begin{aligned}
\exp(\quad & \theta \log \delta + \left(\square \frac{\theta}{} + \theta \right) \mu_c + (\theta \square 1)\kappa_0 + (\theta \square 1) \left(\kappa_1 \square 1 \right) A_0 + (\theta \square 1)\kappa_1 A_2 \mu_\sigma \\
& +\kappa_{0,m} + (\kappa_{1,m} \square 1) A_{0,m} + \kappa_{1,m} A_{2,m} \mu_\sigma \\
& + \left[\left(\square \frac{\theta}{} + \theta \square 1 \right) + (\theta \square 1) \left(\kappa_1 \rho_x \square 1 \right) A_1 + (\theta \square 1)\kappa_1 A_3 \lambda_{sx} + \left(\kappa_{1,m} \rho_x \square 1 \right) A_{1,m} + (1 + \lambda_{sx}) \right] x \\
& + \left[\kappa_{1,m} A_{3,m} \lambda_{sx} \right] x_t + \left[(\theta \square 1) \left(\kappa_1 \rho_s \square 1 \right) A_3 + \left(\kappa_{1,m} \rho_s \square 1 \right) A_{3,m} + \rho_s \square 1 \right] s_t \\
& + \left[(\theta \square 1) \left(\kappa_1 \rho_\sigma \square 1 \right) A_2 + \left(\kappa_{1,m} \rho_\sigma \square 1 \right) A_{2,m} \right] \sigma_t^2 \\
& +0.5 \{ \left(\square \frac{\theta}{} + \theta \right)^2 \sigma_t^2 + \left[(\theta \square 1)\kappa_1 A_3 + \kappa_{1,m} A_{3,m} + 1 \right]^2 \;\; {}_s^2 \sigma_t^2 \\
& + \left[(\theta \square 1)\kappa_1 A_1 + \kappa_{1,m} A_{1,m} \right]^2 \;\; {}_x^2 \sigma_t^2 + \left[(\theta \square 1)\kappa_1 A_2 + \kappa_{1,m} A_{2,m} \right]^2 \sigma_w^2 \}) \\
= \; & 1 \qquad (45
\end{aligned}
$$

Since the Euler equation (45) must hold for all values of the state variables, we have

$$
\left[(\theta \square 1) \left(\kappa_1 \rho_s \square 1 \right) A_3 + \left(\kappa_{1,m} \rho_s \square 1 \right) A_{3,m} + \rho_s \square 1 \right] = 0
$$

$$
A_{3,m} = \frac{\rho_s \square 1}{1 \square \kappa_{1,m} \rho_s} \qquad (46)
$$

$$
\left(\square \frac{\theta}{} + \theta \square 1 \right) + (\theta \square 1) \left(\kappa_1 \rho_x \square 1 \right) A_1 + (\theta \square 1)\kappa_1 A_3 \lambda_{sx} + \left(\kappa_{1,m} \rho_x \square 1 \right) A_{1,m} + \kappa_{1,m} A_{3,m} \lambda_{sx} + 1 + \lambda_{sx} = 0
$$

$$A_{1,m} = \frac{1 \square \frac{1}{\ } + \lambda_{sx}(1 + \kappa_{1,m} A_{3,m})}{1 \square \kappa_{1,m}\rho_x} \tag{47}$$

$$\begin{aligned} & (\theta \square 1)\left(\kappa_1 \rho_\sigma \square 1\right) A_2 + \left(\kappa_{1,m}\rho_\sigma \square 1\right) A_{2,m} + 0.5\{\left(\square \frac{\theta}{\ } + \theta\right)^2 \\ & + \left[(\theta \square 1)\kappa_1 A_3 + \kappa_{1,m} A_{3,m} + 1\right]^2 \ {}^{2}_{s} + \left[(\theta \square 1)\kappa_1 A_1 + \kappa_{1,m} A_{1,m}\right]^2 \ {}^{2}_{x}\} \\ = & \ 0 \end{aligned}$$

$$\begin{aligned} A_{2,m} &= \frac{(\theta \square 1)\left(\kappa_1 \rho_\sigma \square 1\right) A_2 + C}{1 \square \kappa_{1,m}\rho_\sigma} \\ C &= 0.5\{\left(\square \frac{\theta}{\ } + \theta\right)^2 + \left[\kappa_{1,m} A_{3,m} + 1\right]^2 \ {}^{2}_{s} \\ & \quad + \left[(\theta \square 1)\kappa_1 A_1 + \kappa_{1,m} A_{1,m}\right]^2 \ {}^{2}_{x}\} \end{aligned} \tag{48}$$

$$\begin{aligned} & \theta \log \delta + \left(\square \frac{\theta}{\ } + \theta\right) \mu_c + (\theta \square 1)\kappa_0 + (\theta \square 1)\left(\kappa_1 \square 1\right) A_0 + (\theta \square 1)\kappa_1 A_2 \mu_\sigma \\ & + \kappa_{0,m} + (\kappa_{1,m} \square 1) A_{0,m} + \kappa_{1,m} A_{2,m}\mu_\sigma + 0.5\left[(\theta \square 1)\kappa_1 A_2 + \kappa_{1,m} A_{2,m}\right]^2 \sigma_w^2 \\ = & \ 0 \end{aligned}$$

$$\begin{aligned} A_{0,m} = & \ \frac{\theta \log \delta + \left(\square \frac{\theta}{\ } + \theta\right) \mu_c + (\theta \square 1)\kappa_0 + (\theta \square 1)\left(\kappa_1 \square 1\right) A_0}{1 \square \kappa_{1,m}} \\ & + \frac{(\theta \square 1)\kappa_1 A_2 \mu_\sigma + \kappa_{0,m} + \kappa_{1,m} A_{2,m}\mu_\sigma + 0.5\left[(\theta \square 1)\kappa_1 A_2 + \kappa_{1,m} A_{2,m}\right]^2 \sigma_w^2}{1 \square \kappa_{1,m}} \end{aligned} \tag{49}$$

A.3.2 Latent State Variables in terms of Observable Variables

We have

$$\begin{aligned} z_{m,t} &= A_{0,m} + A_{1,m} x_t + A_{2,m}\sigma_t^2 + A_{3,m} s_t \\ r_{f,t} &= A_{0,f} + A_{1,f} x_t + A_{2,f}\sigma_t^2 \end{aligned}$$

The above equations may be inverted to express the unobservable state variables, x_t and σ_t^2, in terms of the observables, $z_{m,t}$, $r_{f,t}$, and s_t.

Define,

$$D \equiv A_{1,m}A_{2,f} - A_{1,f}A_{2,m}$$

We have,

$$\begin{aligned}
x_t &= \alpha_0 + \alpha_1 r_{f,t} + \alpha_2 z_{m,t} + \alpha_3 s_t \\
\alpha_0 &= \frac{A_{0,f}A_{2,m} - A_{0,m}A_{2,f}}{D} \\
\alpha_1 &= \frac{-A_{2,m}}{D} \\
\alpha_2 &= \frac{A_{2,f}}{D} \\
\alpha_3 &= \frac{-A_{3,m}A_{2,f}}{D}
\end{aligned}$$

$$\begin{aligned}
\sigma_t^2 &= \beta_0 + \beta_1 r_{f,t} + \beta_2 z_{m,t} + \beta_3 z_{v-g,t} \\
\beta_0 &= \frac{A_{0,m}A_{1,f} - A_{1,m}A_{0,f}}{D} \\
\beta_1 &= \frac{A_{1,m}}{D} \\
\beta_2 &= \frac{-A_{1,f}}{D} \\
\beta_3 &= \frac{A_{1,f}A_{3,m}}{D}
\end{aligned}$$

Now, from equations (7), (8), and (10), the pricing kernel is given by the expression

$$\begin{aligned}
m_{t+1} &= (\theta \log \delta + (\theta - 1)\left[\kappa_0 + (\kappa_1 - 1)A_0\right]) + \left(-\frac{\theta}{} + (\theta - 1)\right)\Delta c_{t+1} \\
&+(\theta - 1)\kappa_1 A_1 x_{t+1} + (\theta - 1)\kappa_1 A_2 \sigma_{t+1}^2 \\
&-(\theta - 1)A_1 x_t - (\theta - 1)A_2 \sigma_t^2
\end{aligned}$$

Substituting the expressions for x_t and σ_t^2 from equations (25) and (26) into the above expression for the pricing kernel, we have

$$m_{t+1} = c_1 + c_2 \Delta c_{t+1} + c_3\left(r_{f,t+1} - \frac{1}{\kappa_1} r_{f,t}\right) + c_4\left(z_{m,t+1} - \frac{1}{\kappa_1} z_{m,t}\right) + c_5\left(s_{t+1} - \frac{1}{\kappa_1} s_t\right)$$

where

$$
\begin{aligned}
c_1 &= \theta \log \delta + (\theta - 1)[\kappa_0 + (\kappa_1 - 1)(A_0 + A_1\alpha_0 + A_2\beta_0)] \\
c_2 &= -\frac{\theta}{} + (\theta - 1) \\
c_3 &= (\theta - 1)\kappa_1[A_1\alpha_1 + A_2\beta_1] \\
c_4 &= (\theta - 1)\kappa_1[A_1\alpha_2 + A_2\beta_2] \\
c_5 &= (\theta - 1)\kappa_1[A_1\alpha_3 + A_2\beta_3]
\end{aligned}
$$

A.4 Estimation of Time-Series Parameters of the co-integrated Model

In this specification, there are 10 parameters to be estimated - μ_c, μ_{dc}, ρ_x, ${}_x$, μ_σ, ρ_σ, σ_w, λ_{sx}, ρ_s, and ${}_s$.

We have

$$E(\Delta c_{t+1}) = \mu_c \tag{50}$$

Define $\sigma^2 = \frac{\mu_\sigma}{1-\rho_\sigma}$. We then have

$$
\begin{aligned}
Var(\Delta c_{t+1}) &= Var(x_t) + Var(\sigma_t \varepsilon_{c,t+1}) + 2Cov(x_t, \sigma_t \varepsilon_{c,t+1}) \\
&= Var(x_t) + \sigma^2 + 0 \\
&= \frac{{}_x^2 \sigma^2}{1 - \rho_x^2} + \sigma^2
\end{aligned} \tag{51}
$$

and,

$$Cov(\Delta c_{t+1}, \Delta c_{t+2}) = \rho_x \frac{{}_x^2 \sigma^2}{1 - \rho_x^2} \tag{52}$$

$$Cov(\Delta c_{t+1}, \Delta c_{t+3}) = \rho_x^2 \frac{{}_x^2 \sigma^2}{1 - \rho_x^2} \tag{53}$$

From the specification of the dividend growth process, we have

$$
\begin{aligned}
Var(\Delta d_{t+1}) &= (1 + \lambda_{sx})^2 Var(x_t) + (\rho_s - 1)^2 Var(s_t) + \\
&\quad \left(1 + {}_s^2\right) \sigma^2 + 2(1 + \lambda_{sx})(\rho_s - 1) Cov(x_t, s_t)
\end{aligned} \tag{54}
$$

where $Var\left(x_t\right) = \frac{{}_x^2\sigma^2}{1 \square \rho_x^2}$, $Cov(x_t, s_t) = \frac{\lambda_{sx}\rho_x}{1 \square \rho_x \rho_s} Var\left(x_t\right)$, and

$$Var\left(s_t\right) = \frac{\lambda_{sx}^2 Var\left(x_t\right) + \ {}_s^2\sigma^2 + \frac{2\lambda_{sx}^2 \rho_x \rho_s Var(x_t)}{1 \square \rho_x \rho_s}}{1 \square \rho_s^2}$$

Also,

$$\begin{aligned} Cov(\Delta d_{t+1}, \Delta d_{t+2}) &= \left(1 + \lambda_{sx}\right)^2 Cov(x_{t+1}, x_t) + \left(\rho_s \square 1\right)^2 Cov(s_{t+1}, s_t) \\ &\quad + \left(1 + \lambda_{sx}\right)\left(\rho_s \square 1\right)\left[Cov(x_{t+1}, s_t) + Cov(x_t, s_{t+1})\right] \\ &\quad + \left(\rho_s \square 1\right) \quad {}_s Cov(s_{t+1}, \sigma_t \varepsilon_{s,t+1}) \end{aligned} \quad (55)$$

where $Cov(x_{t+1}, x_t) = \rho_x Var\left(x_t\right)$, $Cov(s_{t+1}, s_t) = \lambda_{sx} Cov(x_t, s_t) + \rho_s Var\left(s_t\right)$, $Cov(x_t, s_{t+1}) = \lambda_{sx} Var\left(x_t\right) + \rho_s Cov(x_t, s_t)$, $Cov(x_{t+1}, s_t) = \rho_x Cov(x_t, s_t)$, and $Cov(s_{t+1}, \sigma_t \varepsilon_{s,t+1}) = {}_s\sigma^2$.

Finally,

$$Cov(\Delta c_{t+1}, \Delta d_{t+1}) = \left(1 + \lambda_{sx}\right) Var\left(x_t\right) + \left(\rho_s \square 1\right) Cov(x_t, s_t) + \sigma^2 \quad (56)$$

and

$$E(d_t \square c_t) = \mu_{dc} \quad (57)$$

Equations (50)-(57) give 8 moment restrictions in the 8 parameters μ_c, μ_{dc}, ρ_x, ${}_x$, μ_σ, λ_{sx}, ρ_s, ${}_s$

A.5 Optimization Algorithm

Using the initial consistent estimates of the time-series parameters, we update the initial estimates to obtain the final estimates of the time-series parameters and also estimate the preference parameters by performing a 12-dimensional grid search, over the 9 time series parameters and 3 preference parameters, using the full set of 15 moment restrictions. For the persistence parameter of the LRR variable, ρ_x, the grid covers the interval 0.10, 0.15, ..., 0.95. For the persistence parameter of the stochastic variance process, ρ_σ, the grid covers the interval 0.10, 0.11, ..., 0.99. For each of the other time series parameters, the grid consists of 5 evenly spaced points within two standard errors of the initial consistent point estimate. The grid for the risk aversion parameter is $2, 4, ..., 14$, that for the IES is 0.3, 0.6, 0.9, 1.2, 1.5, 1.8, 2.1, 2.4, and that for the rate of time-preference is 0.97, 0.975, ..., 0.995. At each of the grid points, we compute the value of the GMM objective function and report the parameter vector at the grid point that minimizes the criterion function. This choice of grid results in the point estimates of all the parameters being interior points of the grid for both the 2-asset and the 6-asset systems, thereby ensuring that the criterion function has attained its minimum.

References

[1] Alvarez, F., and Jermann, U., (2005), "Using Asset Prices to Measure the Persistence of the Marginal Utility of Wealth," *Econometrica*, 73(6), 1977-2016.

[2] Ang, A. and Bekaert, G., (2007), "Stock Return Predictability: Is it There?" *The Review of Financial Studies*, 20(3), 651-707.

[3] Bansal, R., Dittmar, R.F., and Kiku, D., (2009) "Cointegration and Consumption Risks in Equity Returns," *Review of Financial Studies*, 22, 1343-1375.

[4] Bansal, R., Dittmar, R.F., and Lundblad, C., (2005), "Consumption, Dividends, and the Cross-Section of Equity Returns," *Journal of Finance* 60, 1639-1672.

[5] Bansal, R., Gallant, R., and Tauchen, G., (2007), "Rational Pessimism, Rational Exhuberance, and and Asset Pricing Models," *Review of Economic Studies*, 74, 1005-1033.

[6] Bansal, R., Kiku, D., and Yaron, A., (2007), "Risks For the Long Run: Estimation and Inference," Working Paper.

[7] Bansal, R., and Shaliastovich, I. (2010) "Confidence Risk and Asset Prices" *American Economic Review, Papers and Proceedings*,100, 537-541.

[8] Bansal, R., and Yaron, A., (2004), "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 1481-1509.

[9] Beeler, J. and Campbell, J.Y., (2009), "The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment" Working paper, Harvard University.

[10] Bekaert, G., Engstrom, E., and Xing, Y., (2005), "Risk, Uncertainty, and Asset Prices," Working Paper.

[11] Binsbergen, J., H., van and Koijen, R. S. J. (2010), "Predictive Regressions: A Present-Value Approach," *Journal of Finance*, 65 (4), 1439 - 1471.

[12] Boudoukh, J., Richardson, M., and Whitelaw R. (2008) "The Myth of Long Horizon Predictability," *Review of Financial Studies*, 24, 1577-1605.

[13] Campbell, J.Y., and Shiller, R.J., (1988), "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies*, 1, 195-227.

[14] Campbell, J.Y., and Thompson, S. B., (2008), "Predicting Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average?" *Review of Financial Studies*, 21, 1509-1531.

[15] Chen, X., Favilukis, J., and Ludvigson, S. C, (2010), "An Estimation of Economic Models with Recursive Preferences," *Working Paper*.

[16] Cochrane, J. H., (2008), "The Dog That Did Not Bark: A Defense of Return Predictability," *Review of Financial Studies*, 21(4), 1533-1575.

[17] Constantinides, G.M., and A. Ghosh (2010), "The Cross-Section of Returns with Regime Shifts in Consumption and Dividend Growth," *Working paper*, University of Chicago and Carnegie-Mellon University.

[18] Croce, M. M., Lettau, M., and Ludvigson, S. C., (2009), "Investor Information, Long-Run Risk, and the Duration of Risky Cash Flows," *Working paper*.

[19] Croce, M. M., and Colacito, R., (2010), "Risks for the Long-Run and the Real Exchange Rate," *Working paper*.

[20] Dai, Q., and Singleton, K.J., (2000), "Specification Analysis of Affine Term Structure Models," *Journal of Finance*, 55, 1943-1978.

[21] Duffee, G., (2002), "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance*, 57, 405-443.

[22] Epstein, L.R., and Zin, S.E., (1989), "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica,* 57, 937-969.

[23] Fama, E. F. and French, K. R. (1988), "Dividend Yields and Expected Stock Returns," *Journal of Financial Economics*, 22, 3-25.

[24] Fama, E.F., and French, K.R., (1993), "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics*, 33, 3-56.

[25] Ferson, W., Nallareddy, S, and Xie, B., (2009), "The 'Out-of-Sample' Performance of Long-Run Risk Models," Working paper, University of Southern California.

[26] Ghosh, A., and Constantinides, G. M., (2010), "The Predictability of Returns with Regime Shifts in Consumption and Dividend Growth," Working paper, Carnegie-Mellon University and University of Chicago.

[27] Hansen, L.P., (1982), "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica*, 50, 1029-1054.

[28] Hansen, L.P., Heaton, J., and Li, N., (2008), "Consumption Strikes Back," *Journal of Political Economy*, 116(2), 260-302.

[29] Hansen, L.P., and Scheinkman, J., (2009), "Long-Term Risk : An Operator Approach," *Econometrica*, 77(1), 177-234.

[30] Kelly, B., and Pruitt, S., (2010), "Disaggregate Valuation Ratios and Market Expectations," *Working Paper*, University of Chicago.

[31] Kreps, D.M., and Porteus, E.L., (1978), "Temporal Resolution of Uncertainty and Dynamic Choice Theory," *Econometrica*, 46(1), 185-200.

[32] Lettau, M., and Ludvigson, S., (2001), "Resurrecting the C(CAPM): A Cross-Sectional Test When Risk Premia are Time-Varying," *Journal of Political Economy,* 109: 1238-87.

[33] Lettau, M., and Ludvigson, S., (2009), "Euler Equation Errors," *Review of Economic Dynamics*, 12, 255-283.

[34] Lettau, M., Ludvigson, S., and Wachter, J., (2008), "The Declining Equity Premium: What Role Does Macroeconomic Risk Play?" *Review of Financial Studies*, 21(4), 1653-1687.

[35] Lettau, M., and Van Nieuwerburgh, S., (2008), "Reconciling the Return Predictability Evidence," *Review of Financial Studies*, 21(4), 1607-1652.

[36] Hanno, L., Van Nieuwerburgh, S., and Verdelhan, A., (2008). The Wealth-Consumption Ratio," *Working paper.*

[37] Malloy, C., Moskowitz, T., and Vissing-Jorgensen, A., (2008), "Long-Run Stockholder Consumption Risk and Asset Returns," Forthcoming in *Journal of Finance.*

[38] Menzly, L., Santos, T., and Veronesi, P., (2004), "Understanding Predictability," *Journal of Political Economy*, 112(1), 1-47.

[39] Parker, J.A., and Julliard, C., (2005), "Consumption Risk and the Cross Section of Expected Returns," *Journal of Political Economy*, 113(1), 185-222.

[40] Pastor, L., and Stambaugh, R.F., (2001), "The Equity Premium and Structural Breaks," *Journal of Finance*, 56(4), 1207-1239.

[41] Piazzesi, M., and Schneider, M., (2006), "Equilibrium Yield Curves," in Daron Acemoglu, Kenneth Rogoff, and Michael Woodford, NBER Macroeconomics, Cambridge MA: MIT press, 389-442.

[42] Weil, P., (1989), "The Equity Premium Puzzle and the Risk-Free Rate Puzzle," *Journal of Monetary Economics*, 24(3), 401-421.

[43] Welch, I., and Goyal, A., (2008), "A Comprehensive Look at The Empirical Performance of Equity Premium Prediction," *Review of Financial Studies*, 21, 1455-1508.

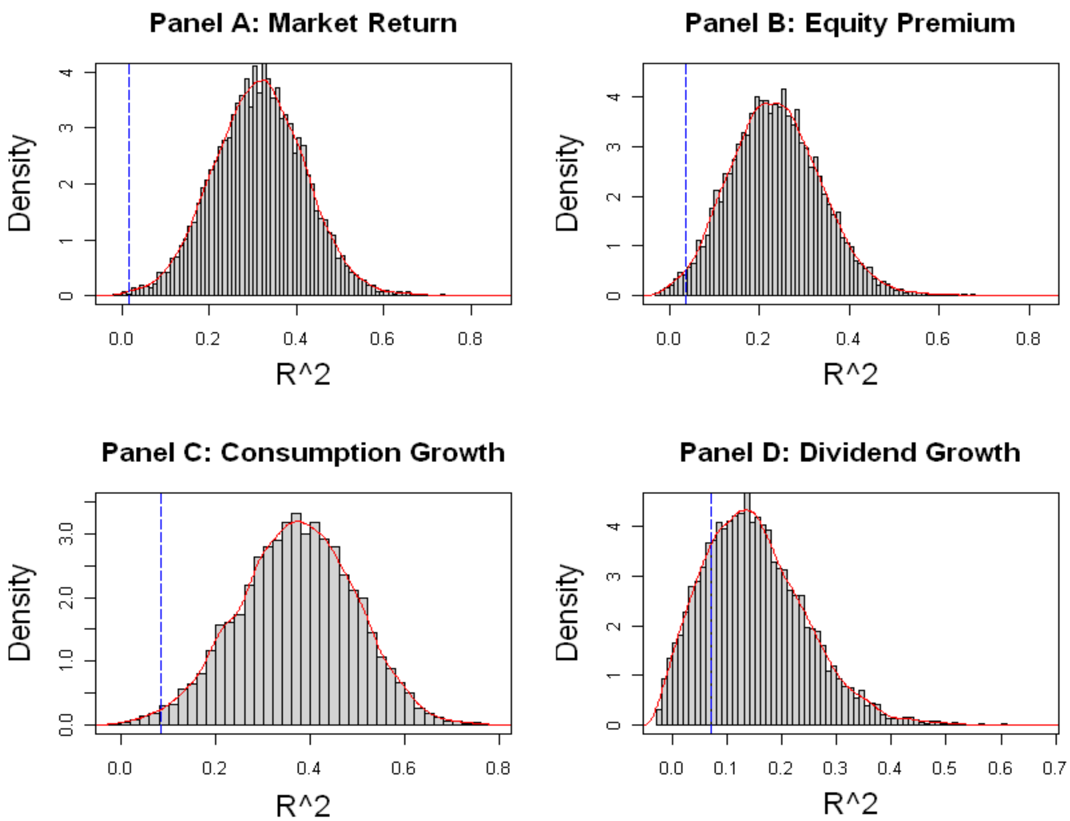

Figure 1: The figure plots the finite-sample distributions of the in-sample $\overline{R}^2$ statistics constructed from 10000 simulations from the B-Y model at the annual frequency. Panels A, B, C, and D show results for the annual market return, equity premium, aggregate consumption growth rate, and aggregate dividend growth rate, respectively. Each panel plots the histogram and the kernel density estimate of the distribution along with the value of the $\overline{R}^2$ statistic obtained in the historical data.

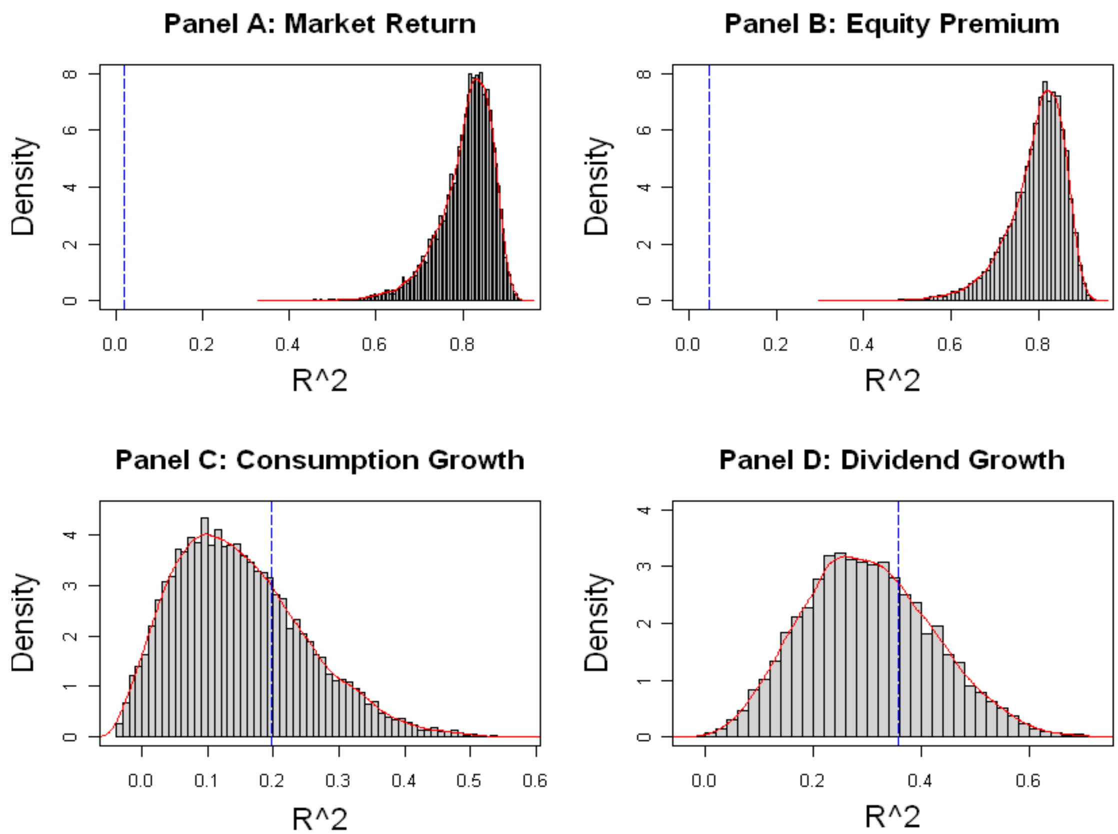

Figure 2: The figure plots the finite-sample distributions of the in-sample $\overline{R}^2$ statistics constructed from 10000 simulations from the cointegrated model at the annual frequency. Panels A, B, C, and D show results for the annual market return, equity premium, aggregate consumption growth rate, and aggregate dividend growth rate, respectively. Each panel plots the histogram and the kernel density estimate of the distribution along with the value of the $\overline{R}^2$ statistic obtained in the historical data.

Table 1: Summary Statistics, 1931-2009

	Mean	*Std. Dev*	*AC(1)*
r_m	0.066	0.197	−0.056
r_f	0.008	0.050	0.645
r_{small}	0.107	0.329	0.074
r_{large}	0.060	0.187	0.017
r_{growth}	0.054	0.210	−0.011
r_{value}	0.099	0.296	−0.124
$log(P/D)$	3.377	0.450	0.877
Δd	0.014	0.119	0.129
Δc	0.014	0.026	0.310

This table reports the descriptive statistics for the annual log returns of the market portfolio, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios, the market-wide log price-dividend ratio and log dividend growth rate, and the aggregate consumption growth rate. The data are annual over the period 1931 – 2009.

Table 2: Model-Implied Forecasting Regressions, 1931-2009

	const.	$log\,(P/D)_t$	$r_{f,t}$	*Adj*-R^2
Panel A: Monthly, In-Sample				
$r_{m,t+1}$	0.033	−0.007	−0.049	0.002
	(0.013)	(0.004)	(0.691)	(0.017)
$r_{m,t+1} - r_{f,t}$	0.033	−0.007	−1.049	0.005
	(0.013)	(0.004)	(0.691)	(0.080)
Panel B: Quarterly, In-Sample				
$r_{m,t+1}$	0.098	−0.022	0.001	0.002
	(0.046)	(0.014)	(0.790)	(0.000)
$r_{m,t+1} - r_{f,t}$	0.098	−0.022	−0.999	0.010
	(0.046)	(0.014)	(0.790)	(0.000)
Panel C: 1-Year, In-Sample				
Δc_{t+1}	−0.043	0.017	−0.086	0.084
	(0.021)	(0.006)	(0.056)	(0.009)
Δd_{t+1}	−0.251	0.078	0.119	0.070
	(0.099)	(0.029)	(0.263)	(0.219)
$r_{m,t+1}$	0.334	−0.081	0.399	0.014
	(0.167)	(0.049)	(0.446)	(0.001)
$r_{m,t+1} - r_{f,t}$	0.334	−0.081	−0.601	0.038
	(0.167)	(0.049)	(0.446)	(0.014)
Panel D: 3-Year, In-Sample				
Δc_{t+1}	−0.014	0.019	−0.123	0.015
	(0.041)	(0.012)	(0.109)	(0.000)
Δd_{t+1}	−0.287	0.104	−0.321	0.036
	(0.162)	(0.048)	(0.434)	(0.031)
$r_{m,t+1}$	1.104	−0.268	1.388	0.189
	(0.222)	(0.066)	(0.594)	(0.558)
$r_{m,t+1} - r_{f,t}$	1.121	−0.275	−0.460	0.191
	(0.219)	(0.065)	(0.585)	(0.780)
Panel E: 5-Year, In-Sample				
Δc_{t+1}	0.063	0.005	0.071	−0.019
	(0.045)	(0.013)	(0.117)	(0.000)
Δd_{t+1}	−0.205	0.093	0.249	0.024
	(0.175)	(0.052)	(0.455)	(0.035)
$r_{m,t+1}$	1.617	−0.381	1.867	0.245
	(0.272)	(0.081)	(0.707)	(0.811)
$r_{m,t+1} - r_{f,t}$	1.772	−0.431	−0.237	0.302
	(0.253)	(0.075)	(0.658)	(0.945)

The table reports results from the Bansal and Yaron (2004) model-implied forecasting regressions for the aggregate consumption growth rate, the aggregate dividend growth rate, the market return, and the equity premium over the period 1931-2009. Panels A, B, C, D, and E report results at the monthly, quarterly, 1-year, 3-year, and 5-year frequencies respectively.

Reliable consumption data is not available at the monthly frequency and is only available for the subperiod at the quarterly frequency, and dividends have a strong seasonal pattern at the monthly and quarterly frequencies. Therefore, we exclude these regressions from the table. The number in parentheses below the adj-R^2 shows the probability of obtaining an adj-R^2 at least as small as that obtained with the historical data if the data were generated by the B-Y model. This probability is computed using 10000 simulations of the same size as the historical data.

Table 3: Model-Implied Forecasting Regressions, 1976-2009

	Panel A: Monthly, In-Sample			
	const.	*log* $(P/D)_t$	$r_{f,t}$	*Adj-*R^2
$r_{m,t+1}$	0.078 (0.027)	−0.017 (0.006)	−1.527 (1.110)	0.012
$r_{m,t+1} - r_{f,t}$	0.078 (0.027)	−0.017 (0.006)	−2.527 (1.110)	0.013
	Panel B: Quarterly, In-Sample			
	const.	*log* $(P/D)_t$	$r_{f,t}$	*Adj-*R^2
Δc_{t+1}	−0.002 (0.005)	0.001 (0.001)	0.064 (0.046)	0.005
$r_{m,t+1}$	0.120 (0.060)	−0.028 (0.016)	0.033 (0.593)	0.008
$r_{m,t+1} - r_{f,t}$	0.120 (0.060)	−0.028 (0.016)	−0.967 (0.593)	0.021
	Panel C: Annual, In-Sample			
	const.	*log* $(P/D)_t$	$r_{f,t}$	*Adj-*R^2
Δc_{t+1}	−0.025 (0.018)	0.009 (0.005)	0.264 (0.081)	0.236
Δd_{t+1}	−0.089 (0.113)	0.025 (0.030)	1.068 (0.502)	0.076
$r_{m,t+1}$	0.354 (0.247)	−0.084 (0.066)	1.062 (1.099)	0.032
$r_{m,t+1} - r_{f,t}$	0.354 (0.247)	−0.084 (0.066)	0.063 (1.099)	−0.010
	Panel D: Annual, Out-of-Sample			
	const.	*log* $(P/D)_t$	$r_{f,t}$	$R^2 = 1 - \frac{MSE_{Model}}{MSE_{Mean}}$
Δc_{t+1}				−1.232
Δd_{t+1}				−0.603
$r_{m,t+1}$	-	-	-	−0.025
$r_{m,t+1} - r_{f,t}$	-	-	-	−0.016

The table reports results from the Bansal and Yaron (2004) model-implied forecasting regressions for the aggregate consumption growth rate, the aggregate dividend growth rate, the market return, and the equity premium over the period 1976-2009. Panels A, B, and C report results at the monthly, quarterly, and annual frequencies respectively. Reliable consumption data is not available at the monthly frequency and dividends have a strong seasonal pattern at the monthly and quarterly frequencies. Therefore, we exclude these regressions from the table. Panel D reports out-of-sample predictive results for the aggregate consumption growth rate, aggregate dividend growth rate, market return, and equity premium at the annual frequency.

Table 4: Tests of the LRR Model on the 2-Asset System over 1931-2009

Parameter	δ			μ_c	μ_d	ϕ	φ	ρ_x	x	σ	υ	σ_w
$Est.^{time□series}$	□	□	□	0.014	0.014	2.90	14.4	0.32	3.70	0.007	0.983	$9.3 \times 10^{□6}$
(*Std. Err.*)				(0.003)	(0.012)	(1.15)	(69.4)	(0.17)	(18.7)	(0.030)	(2.69)	($6.3{\times}10^{□4}$)
$Est.^{full\ model}$	0.98	8	0.6	0.014	0.014	5.20	10.4	0.70	0.70	0.007	0.983	$9.3 \times 10^{□6}$
(*Std. Err.*)	(0.07)	(6.59)	(1.33)	(0.003)	(0.012)	(5.23)	(13.8)	(1.04)	(2.01)	(0.007)	(0.370)	($9.9{\times}10^{□5}$)

Moments	*Data*	$Model^{ts}$	$Model^{full}$	*Pr. Err.*	
$E(\Delta c_{t+1})$	0.014	0.014	0.014	*Mkt*	□0.019 (0.027)
$Std.\ Dev(\Delta c_{t+1})$	0.026	0.026	0.010	*Rf*	□0.033 (0.050)
$\rho(\Delta c_{t+1}, \Delta c_{t+2})$	0.31	0.30	0.34		
$E(\Delta d_{t+1})$	0.014	0.014	0.014	*J-stat*	17.7 (0.001)
$Std.\ Dev(\Delta d_{t+1})$	0.119	0.119	0.081		
$\rho(\Delta d_{t+1}, \Delta d_{t+2})$	0.13	0.12	0.14		
$\rho(\Delta c_{t+1}, \Delta d_{t+1})$	0.61	0.60	0.31		
$Var\left[(\Delta c_{t+1})^2\right]$	$3 \times 10^{□6}$	$3 \times 10^{□6}$	$1 \times 10^{□7}$		
$Var\left[(\Delta d_{t+1})^2\right]$	0.001	0.001	0.0002		

The table reports GMM estimates of the model using annual data over the period 1931-2009 for the efficient weighting matrix. The first row reports the point estimates, along with the associated standard errors in parentheses, of the 9 time-series parameters using the 9 time-series moment restrictions alone. The second row reports estimates of the time-series and preference parameters when both the pricing restrictions and the time-series moment restrictions are used in the estimation. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, and the log risk free rate and its lag. The asset menu consists of the market portfolio and the risk free rate. The lagged price-dividend ratio of the market and the lagged risk free rate are used as instruments. The table shows the values of the time-series moments in historical data as well as the model-implied values of the same. Average pricing errors and their standard errors are presented for each asset. Finally, the table also reports the J-stat for the overidentifying restrictions along with the associated asymptotic p-value in parentheses.

Table 5: Tests of the LRR Model on the 6-Asset System over 1931-2009

$Parameter$	δ			μ_c	μ_d	ϕ	φ	ρ_x	x	σ	υ	σ_w
$Est.^{time\square series}$	□	□	□	0.014	0.014	2.90	14.4	0.32	3.70	0.007	0.983	$9.3 \times 10^{\square 6}$
$(Std.\ Err.)$				(0.003)	(0.012)	(1.15)	(69.4)	(0.17)	(18.7)	(0.030)	(2.69)	$(6.3\times10^{\square 4})$
$Est.^{full\ model}$	0.98	8.0	0.6	0.014	0.014	2.90	10.4	0.75	0.70	0.007	0.983	$9.3 \times 10^{\square 6}$
$(Std.\ Err.)$	(0.76)	(28.7)	(13.5)	(0.003)	(0.012)	(7.03)	(23.1)	(2.20)	(4.83)	(0.013)	(2.44)	$(6.3\times10^{\square 4})$

$Moments$	$Data$	$Model^{ts}$	$Model^{full}$	$Pr.\ Err.$	
$E(\Delta c_{t+1})$	0.014	0.014	0.014	$Small$	0.026 (0.038)
$Std.\ Dev(\Delta c_{t+1})$	0.026	0.026	0.010	$Large$	□0.018 (0.039)
$\rho(\Delta c_{t+1}, \Delta c_{t+2})$	0.31	0.30	0.40	$Growth$	□0.029 (0.037)
$E(\Delta d_{t+1})$	0.014	0.014	0.014	$Value$	0.015 (0.036)
$Std.\ Dev(\Delta d_{t+1})$	0.119	0.119	0.076	$Market$	□0.016 (0.038)
$\rho(\Delta d_{t+1}, \Delta d_{t+2})$	0.13	0.12	0.06	Rf	□0.011 (0.065)
$\rho(\Delta c_{t+1}, \Delta d_{t+1})$	0.61	0.60	0.21		
$Var\left[(\Delta c_{t+1})^2\right]$	$3 \times 10^{\square 6}$	$3 \times 10^{\square 6}$	$1 \times 10^{\square 7}$	J-$stat$	13.6 (0.003)
$Var\left[(\Delta d_{t+1})^2\right]$	0.001	0.001	0.0002		

The table reports GMM estimates of the model using annual data over the period 1931-2009 for the efficient weighting matrix. The first row reports the point estimates, along with the associated standard errors in parentheses, of the 9 time-series parameters using the 9 time-series moment restrictions alone. The second row reports estimates of the time-series and preference parameters when both the pricing restrictions and the time-series moment restrictions are used in the estimation. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, and the log risk free rate and its lag. The asset menu consists of the market portfolio, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios. The table shows the values of the time-series moments in historical data as well as the model-implied values of the same. Average pricing errors and their standard errors are presented for each asset. Finally, the table also reports the J-stat for the overidentifying restrictions along with the associated asymptotic p-value in parentheses.

Table 6: Co-integrated Model Implied Forecasting Regressions, 1931-2009

	Annual, In-Sample				
	const.	$log\,(P/D)_t$	$r_{f,t}$	s_t	*Adj-*R^2
Δc_{t+1}	□0.076 (0.022)	0.027 (0.006)	0.009 (0.059)	□0.058 (0.017)	0.198 (0.713)
Δd_{t+1}	□0.490 (0.091)	0.147 (0.027)	0.801 (0.247)	□0.414 (0.070)	0.356 (0.684)
$r_{m,t+1}$	0.234 (0.186)	□0.052 (0.055)	0.685 (0.503)	□0.173 (0.143)	0.020 (0.000)
$r_{m,t+1}$ □ $r_{f,t}$	0.234 (0.186)	□0.052 (0.055)	□0.315 (0.503)	□0.173 (0.143)	0.044 (0.000)

The table reports results from model-implied forecasting regressions for the aggregate consumption growth rate, the aggregate dividend growth rate, the market return, and the equity premium over the period 1931-2009 at the annual frequency.

Table 7: Co-integrated Model Implied Forecasting Regressions, 1976-2009

Panel A: Quarterly, In-Sample					
	const.	$log\,(P/D)_t$	$r_{f,t}$	s_t	*Adj-*R^2
Δc_{t+1}	□0.014	0.003	0.049	□0.011	0.056
	(0.007)	(0.001)	(0.046)	(0.004)	
Δd_{t+1}	□0.339	0.068	0.670	□0.284	0.175
	(0.094)	(0.019)	(0.625)	(0.055)	
$r_{m,t+1}$	0.130	□0.022	0.018	□0.017	□0.004
	(0.091)	(0.018)	(0.604)	(0.053)	
$r_{m,t+1}$ □ $r_{f,t}$	0.130	□0.022	□0.982	□0.017	0.010
	(0.091)	(0.018)	(0.604)	(0.053)	

Panel B: Annual, In-Sample					
	const.	$log\,(P/D)_t$	$r_{f,t}$	s_t	*Adj-*R^2
Δc_{t+1}	□0.048	0.017	0.210	□0.046	0.383
	(0.018)	(0.005)	(0.075)	(0.016)	
Δd_{t+1}	□0.188	0.058	0.832	□0.204	0.151
	(0.120)	(0.034)	(0.496)	(0.105)	
$r_{m,t+1}$	0.348	□0.081	1.047	□0.013	0.000
	(0.278)	(0.078)	(1.152)	(0.245)	
$r_{m,t+1}$ □ $r_{f,t}$	0.348	□0.081	0.047	□0.013	□0.043
	(0.278)	(0.078)	(1.152)	(0.245)	

Panel C: Annual, Out-of-Sample					
	const.	$log\,(P/D)_t$	$r_{f,t}$	s_t	$R^2 = 1$ □ $\frac{MSE_{Model}}{MSE_{Mean}}$
Δc_{t+1}					□0.517
Δd_{t+1}					□0.237
$r_{m,t+1}$	-	-	-		□0.055
$r_{m,t+1}$ □ $r_{f,t}$	-	-	-		□0.050

The table reports results from model-implied forecasting regressions for the aggregate consumption growth rate, the aggregate dividend growth rate, the market return, and the equity premium over the period 1976-2009. Panels A and B report results at the quarterly, and annual frequencies respectively. Panel C reports out-of-sample predictive results for the aggregate consumption growth rate, aggregate dividend growth rate, market return, and equity premium at the annual frequency.

Table 8: Tests of the co-integrated LRR Model on the 2-Asset System over 1931-2009

Parameter	δ			μ_c	ρ_x	x	μ_σ	ρ_σ	σ_w	λ_{sx}	ρ_s	s
Estimate	0.99 (0.31)	6.0 (25.3)	0.9 (12.1)	0.011 (0.003)	0.94 (0.79)	0.189 (1.386)	$3.2 \times 10^{\square 6}$ $(3.6 \times 10^{\square 6})$	0.983 (0.021)	$9.3 \times 10^{\square 6}$ $(6.8 \times 10^{\square 4})$	7.305 (26.3)	0.96 (0.76)	4.798 (6.44)
Pr.Err.		*Mean*	*Std.Err.*									
Market		0.015	0.045									
Risk free rate		0.029	0.074									
$J \square stat$	9.19 (0.002)											

The table reports GMM estimates of the model using annual data over the period 1931-2009. Both the pricing restrictions and the time-series restrictions are used in the estimation. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, the log risk free rate and its lag, and the demeaned log dividend-consumption ratio and its lag. The asset menu consists of the market portfolio and the risk free rate. The lagged price-dividend ratio of the market and the lagged risk free rate are used as instruments. The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated asymptotic p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with one degree of freedom.

Table 9: Tests of the co-integrated LRR Model on the 6-Asset System over 1931-2009

Parameter	δ			μ_c	ρ_x	x	μ_σ	ρ_σ	σ_w	λ_{sx}	ρ_s	s
Estimate	0.98 (0.22)	6.0 (18.1)	1.2 (43.3)	0.011 (0.003)	0.96 (0.49)	0.189 (1.235)	$3.2 \times 10^{□6}$ ($1.6\times10^{□6}$)	0.983 (0.019)	$9.3 \times 10^{□6}$ (0.003)	9.305 (23.0)	0.90 (1.30)	4.798 (6.130)
Pr.Err.		*Mean*	*Std.Err.*									
Small		0.043	0.044									
Large		0.0001	0.046									
Growth		□0.011	0.043									
Value		0.032	0.042									
Market		0.002	0.044									
Risk free rate		0.021	0.078									
J □ stat	9.12 (0.003)											

The table reports GMM estimates of the model using annual data over the period 1931-2009. Both the pricing restrictions and the time-series restrictions are used in the estimation. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, the log risk free rate and its lag, and the demeaned log dividend-consumption ratio and its lag. The asset menu consists of the market portfolio, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios.The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated asymptotic p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with one degree of freedom.

Table 10: Tests of the LRR Model on the 2-Asset System over 1947-2009

Parameter	δ			μ_c	μ_d	ϕ	φ	ρ_x	x	σ	ρ_σ	σ_w
$Est.^{time□series}$	□	□	□	0.012	0.023	4.61	5.43	0.68	0.62	0.011	0.988	$1.4 \times 10^{□5}$
(Std. Err.)				(0.002)	(0.010)	(1.43)	(1.11)	(0.17)	(0.31)	(0.001)	(9.32)	(0.006)
$Est.^{full\ model}$	0.98	10.0	0.6	0.012	0.023	4.61	3.21	0.85	0.31	0.009	0.988	$1.4 \times 10^{□5}$
(Std. Err.)	(0.31)	(29.7)	(10.1)	(0.002)	(0.010)	(2.89)	(1.70)	(0.59)	(0.56)	(0.003)	(0.658)	($3.7\times10^{□4}$)

Moments	*Data*	$Model^{ts}$	$Model^{full}$	*Pr. Err.*	
$E(\Delta c_{t+1})$	0.012	0.012	0.012	*Mkt*	□0.013 (0.029)
$Std.\ Dev(\Delta c_{t+1})$	0.016	0.015	0.010	*Rf*	□0.027 (0.054)
$\rho(\Delta c_{t+1}, \Delta c_{t+2})$	0.28	0.28	0.22		
$E(\Delta d_{t+1})$	0.023	0.023	0.023	*J-stat*	17.6 (0.001)
$Std.\ Dev(\Delta d_{t+1})$	0.075	0.075	0.038		
$\rho(\Delta d_{t+1}, \Delta d_{t+2})$	0.23	0.23	0.35		
$\rho(\Delta c_{t+1}, \Delta d_{t+1})$	0.38	0.38	0.33		
$Var[(\Delta c_{t+1})^2]$	$1.8 \times 10^{□7}$	$3 \times 10^{□7}$	$2 \times 10^{□7}$		
$Var[(\Delta d_{t+1})^2]$	0.0002	0.0001	$3 \times 10^{□5}$		

The table reports GMM estimates of the model using annual data over the period 1947-2009. The asset menu consists of the market portfolio and the risk free rate. The lagged price-dividend ratio of the market and the lagged risk free rate are used as instruments. Results are reported for the efficient weighting matrix. The table presents the parameter estimates along with the associated standard errors in parentheses. Both the pricing restrictions and the time-series restrictions are used in the estimation. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated asymptotic p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with three degrees of freedom.

Table 11: Tests of the LRR Model on the 6-Asset System over 1947-2009

Parameter	δ			μ_c	μ_d	ϕ	φ	ρ_x	x	σ	ρ_σ	σ_w
$Est.^{time□series}$ (*Std. Err.*)	□	□	□	0.012 (0.002)	0.023 (0.010)	4.61 (1.43)	5.43 (1.11)	0.68 (0.17)	0.62 (0.31)	0.011 (0.001)	0.988 (9.32)	$1.4 \times 10^{□5}$ (0.006)
$Est.^{full\ model}$ (*Std. Err.*)	0.98 (0.38)	14.0 (71.6)	0.6 (11.8)	0.012 (0.002)	0.023 (0.009)	4.61 (2.64)	4.32 (1.27)	0.75 (0.72)	0.31 (0.45)	0.011 (0.002)	0.988 (0.529)	$1.4 \times 10^{□5}$ ($3.0\times10^{□4}$)
Moments	*Data*	$Model^{ts}$	$Model^{full}$			*Pr. Err.*						
$E(\Delta c_{t+1})$	0.012	0.012	0.012			*Small*	0.008 (0.054)					
$Std.\ Dev(\Delta c_{t+1})$	0.016	0.015	0.012			*Large*	□0.014 (0.054)					
$\rho(\Delta c_{t+1}, \Delta c_{t+2})$	0.28	0.28	0.14			*Growth*	□0.027 (0.050)					
$E(\Delta d_{t+1})$	0.023	0.023	0.023			*Value*	0.021 (0.052)					
$Std.\ Dev(\Delta d_{t+1})$	0.075	0.075	0.053			*Market*	□0.013 (0.053)					
$\rho(\Delta d_{t+1}, \Delta d_{t+2})$	0.23	0.23	0.15			*Rf*	0.003 (0.089)					
$\rho(\Delta c_{t+1}, \Delta d_{t+1})$	0.38	0.38	0.19									
$Var\left[(\Delta c_{t+1})^2\right]$	$1.8 \times 10^{□7}$	$3 \times 10^{□7}$	$2 \times 10^{□7}$			*J-stat*	16.9 (0.001)					
$Var\left[(\Delta d_{t+1})^2\right]$	0.0002	0.0001	$5 \times 10^{□5}$									

The table reports GMM estimates of the model using annual data over the period 1947-2009. Both the pricing restrictions and the time-series restrictions are used in the estimation. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, and the log risk free rate and its lag. The asset menu consists of the market portfolio, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios.The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated asymptotic p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with three degrees of freedom.

Table 12: Tests of the LRR Model on the 2-Asset System using Quarterly Data over 1947:2-2009:4

Parameter	δ			μ_c	μ_d	ϕ	φ	ρ_x	x	σ	ρ_σ	σ_w
$Est.^{full\ model}$	0.98	10.0	0.6	0.002	0.005	5.371	4.286	0.950	0.113	0.022	0.80	$2.0 \times 10^{□6}$
(*Std. Err.*)	(0.19)	(44.1)	(6.59)	(0.001)	(0.005)	(0.814)	(0.436)	(0.062)	(0.073)	(0.001)	(7.758)	(0.065)

	Pr. Err.
Mkt	□0.003
	(0.034)
Rf	□0.003
	(0.035)
J-stat	83.6
	(0.000)

The table reports GMM estimates of the model using quarterly data over the period 1947:2-2009:4. The asset menu consists of the market portfolio and the risk free rate. The lagged price-dividend ratio of the market and the lagged risk free rate are used as instruments. Results are reported for the efficient weighting matrix. The table presents the parameter estimates along with the associated standard errors in parentheses. Both the pricing restrictions and the time-series restrictions are used in the estimation. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated asymptotic p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with three degrees of freedom.

Table 13: Tests of the LRR Model on the 6-Asset System using Quarterly Data over 1947:2-2009:4

Parameter	δ			μ_c	μ_d	ϕ	φ	ρ_x	x	σ	ρ_σ	σ_w
$Est.^{full\ model}$ (*Std. Err.*)	0.98 (0.31)	10.0 (65.6)	0.6 (17.4)	0.002 (0.001)	0.005 (0.005)	5.371 (0.896)	4.286 (0.579)	0.950 (0.071)	0.113 (0.084)	0.022 (0.001)	0.80 (16.5)	$2.0 \times 10^{□6}$ (0.076)

	Pr. Err.
Small	□0.004 (0.033)
Large	□0.002 (0.034)
Growth	□0.006 (0.034)
Value	0.007 (0.035)
Market	□0.003 (0.034)
Rf	□0.003 (0.035)
J-stat	83.6 (0.000)

The table reports GMM estimates of the model using quarterly data over the period 1947:2-2009:4. Both the pricing restrictions and the time-series restrictions are used in the estimation. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, and the log risk free rate and its lag. The asset menu consists of the market portfolio, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios.The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated asymptotic p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with three degrees of freedom.