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ABSTRACT

Implicit tax rates priced in the cross section of municipal bonds are approximately two to three times as high as statutory income tax rates, with implicit tax rates close to 100% using retail trades and above 70% for interdealer trades. These implied tax rates can be identified on the cross section of municipal bonds because a portion of secondary market municipal bond trades involve income taxes. After valuing the tax payments, market discount bonds, which carry income tax liabilities, trade at yields around 25 basis points higher than comparable municipal bonds not subject to any taxes. The high sensitivities of municipal bond prices to tax rates can be traced to individual retail traders dominating dealers and other institutions.

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1 Introduction

The issue of how taxes affect the prices of assets is an important issue in finance, accounting, and economics. In theoretical models examining the effect of taxes on different assets and different agents (for example, Auerbach and King (1983), Dybvig and Ross (1986), and Dammon and Green (1987)), taxes induce clientele effects in the asset holdings of agents and the existence of different tax rates affects relative asset prices. In reality, estimating implicit tax rates on assets is more difficult than theoretical models suggest because of the myriad ways the tax code can be distorted, large investor heterogeneity, and the many different degrees of market frictions faced by different investors. These real-world issues are especially true for investigating if the tax rates faced by individual investors, as opposed to the tax rates faced by corporations, affect asset prices because financial markets are often dominated by financial institutions and dealers. For example, studies using equities find little evidence of implicit tax effects (see, among many others, Boyd and Jagannathan (1994), Fama and French (1998), and Erickson and Maydew (1998)). In Treasury markets, Green and Ødegaard (1993) find that after the 1986 tax reform, the marginal investor in Treasury bonds has a marginal tax rate of zero.

In contrast to government bonds and equities, the municipal bond market is well suited to evaluate how individual tax rates affect asset prices. First, the municipal bond market is large; the Flow of Funds data from the Federal Reserve show that at the end of the first quarter of 2007, there were $2.5 trillion outstanding municipal securities compared to $4.8 trillion Treasuries. Second, municipal bonds are attractive to high net worth individuals. Not surprisingly, individual holdings of municipal bonds dominate the holdings of other corporate entities. At the end of the first quarter of 2007, individuals held 70% of all outstanding municipal bonds. Individuals directly held 36% of all municipal securities outstanding and held 34% through mutual funds, closed-end funds, and other taxable pass-through intermediaries.[1]

Third, individual investors are the marginal pricers in the municipal bond market at an aggregate level. This is evident from the fact that the municipal yield curve trades lower than the Treasury yield curve. Short-maturity municipal yields are equal to the Treasury yield times one minus the income tax rate and the ratio between municipal and Treasury yields decreases with maturity. These stylized facts are matched well by Green's (1993) model, where after-tax yields on municipal and Treasury bonds are equalized by individuals, in contrast to the Treasury

[1] The remainder was held mostly by banks and insurance companies. The proportion of municipal bonds held directly and indirectly by individuals has been well above 70% over our post-1995 sample period, but was around 35% pre-1985. (See Hildreth and Zorn (2005), for a history of developments in the municipal market.)

market where tax-exempt institutions and dealers dominate pricing. Since at an asset class level individual investors set the prices of municipal bonds relative to Treasury securities, we would also expect individual tax rates to affect the cross-sectional pricing of municipal securities.

It would be impossible to study the effect of individual tax rates in the relative prices of municipal bonds if all municipal securities had identical tax treatment with all cashflows exempt from tax.[2] Fortunately, this is not the case. A unique feature of the municipal bond market is at any given time, an individual investor can purchase municipal bonds which are fully tax exempt, where all the bond cashflows are not subject to tax, or municipal bonds subject to income tax or capital gains tax. Municipal bonds bearing income tax liabilities are termed market discount bonds. We exploit this cross-sectional heterogeneity to estimate the effects of individual tax rates on municipal bond prices as well as to characterize how different investor clienteles respond to different tax treatments. Furthermore, the same bond may change its tax treatment over time, changing from say being subject to income tax to becoming fully tax exempt. Thus, we can also identify the effect of taxes from the time series of these bonds as they move across tax boundaries.

The coexistence of fully tax-exempt bonds together with municipal bonds subject to income or capital gains tax arises from how the tax code defines market discount. When a municipal bond is issued, the coupon payments and original issue discount (OID) are exempt from federal income tax. However, the profits from trading municipal bonds in secondary markets are taxable. If market discount exists, which in most situations is defined as a large enough difference between the market price and par value for a bond issued at par, the purchaser of the bond is liable for income tax, otherwise taxes are levied at capital gains rates. These taxes depend on the purchase price of the bond, the bond's original issue yield or price, and original maturity. While most municipal bond trades are not subject to tax, there is an important subset of municipal bond transactions involving bonds subject to income tax. In some years trades involving market discount bonds represent over 30% of all transactions.

Since 1993, market discount is taxed at regular income tax rates. The Internal Revenue Code (IRC) allows small amounts of market discount to be treated as capital gains, which is termed the de minimis exemption. That is, below the de minimis boundary, market discount is taxed as income. Above the de minimis boundary, bonds may be subject to capital gains tax. If a par bond is trading above par all bond cashflows are not subject to tax. Thus, investors face a

[2] A small number of municipal bonds do not have tax-exempt cashflows, such as private activity bonds subject to the AMT. We do not consider these bonds in our analysis.

discontinuous tax treatment from these different tax boundaries. The no tax, capital gains tax, and income tax boundaries provide an excellent venue to compute the implicit tax rates priced by municipal bond market participants.

As expected, we find taxes matter in determining the cross-sectional and time-series prices of tax-exempt bonds. Theoretically if the marginal investor is an individual, the tax burdens of municipal bonds subject to income tax should cause market discount bonds to trade at higher yields to compensate individuals for assuming the tax liabilities attached to these bonds compared to municipal bonds with no tax or only capital gains tax liabilities. This is certainly true empirically. However, the after-tax yields of municipal bonds with the highest tax burdens are higher than can be explained with a present value model of after-tax cashflows constructed using the zero-coupon municipal yield curve. We build the municipal zero curve using only the transactions of interdealer trades of municipal bonds which are fully tax exempt.

Investors purchasing market discount municipal bonds in A-grade credit classes would obtain after-tax yields around 25 basis points higher than yields on comparable securities not subject to tax. The high yields on market discount bonds translate to very high implicit tax rates. We estimate income tax rates of approximately 100% for retail trades and 70% for interdealer trades. These high yields on municipal bonds subject to market discount taxation persist when taking only insured bonds and are even higher, over 45 basis points, for bonds with short 1-2 year maturities. Our results are also robust to considering bonds from the same series trading above or below the de minimis boundary, which makes default risk an unlikely explanation. We also find that several liquidity measures, like trading frequency and the spreads between dealer and customer trades, cannot account for the high yields on below de minimis bonds.

Since the de minimis boundary affects the payment of individual income tax, but not corporate tax, the market discount effect should be concentrated in bonds more likely to be traded by individuals rather than institutions. We confirm this is the case. The high yields on market discount bonds are concentrated in retail trades, which we define as trades of bonds with par value traded less than $100,000. We also show the effect is very small for bank qualified bonds, which are primarily held by institutions, trading below the de minimis boundary. This does not mean that institutions are unaffected by market discount taxation. In fact, institutions purchasing bonds from dealers would obtain yields approximately 20 basis points higher by purchasing market discount bonds, rather than bonds with fully tax-exempt cashflows. We believe dealers and other institutions are unable to take advantage of low market discount bond prices because of the decentralized opacity of the municipal market, the fact that many institutions, especially mutual funds, shun market discount bonds, and the inability to short municipal bonds by dealers to construct hedges.

Our paper falls into a large literature investigating how taxes matter for asset prices (see Poterba (2002) for a summary). We document very large implicit tax rates – much larger than statutory tax rates and also much larger than the previous estimates of implicit tax rates made using equity, Treasury, and corporate bond markets. A unique feature of the municipal market compared to stock and bond markets is that individuals dominate institutions and we trace the high implicit tax rates to transactions likely to be conducted by individual investors. Thus, our results are also consistent with a literature examining how taxes affect the financial decisions of individuals (see Bernheim (2002) for a summary). But, since the yields on market discount bonds are much higher than can be accounted for by valuing the tax liabilities, our results show individuals exhibit extreme sensitivity to these tax payments. There is also a large municipal bond literature, which concentrates on how municipal bonds are priced relative to other assets, like taxable Treasury debt, corporate securities, and equity securities (see, among many others, Auerbach and King (1983), and McDonald (1983)). In contrast, we take advantage of the pricing of municipal bonds relative to each other, rather than relative to other asset classes, to examine tax effects.

Li (2006) is most related to our cross-sectional analysis. Li argues that prices just below the de minimis boundary are dominated because a purchaser of the bond in this region would be better off paying a slightly higher price to avoid market discount taxation. She shows that although trades in this dominated region should not occur in theory, they do occur in practice. In contrast, we track all below de minimis trades, not just trades in a theoretically dominated region, compute yields for buying market discount bonds, estimate implicit tax rates, and examine the effect of retail clienteles. We also track bonds entering or leaving below de minimis territory. These bonds are especially interesting because of their changing tax treatment.

The rest of this paper is organized as follows. Section 2 presents an overview of the tax treatment of gains in municipal bond transactions. We describe the data and the benchmark yield curve in Section 3. Section 4 contains the main results and shows market discount bonds do carry higher yields than other municipal bonds, but these yields are higher than the zero-coupon yield curve justifies. We discuss several implications of our findings in Section 5. Finally, Section 6 concludes.

2 Income and Capital Gains Taxes on Municipal Bonds

We use the term "municipal bonds" to describe all tax-exempt bonds, which includes bonds issued by municipalities, counties, states, other government authorities, and other entities entitled to issue tax-exempt debt. The interest paid on municipal bonds to any investor is not subject to income tax levied by the federal government, but may be subject to state income tax if an investor holds municipal bonds issued by states where the investor is not considered to be a resident. The principle behind the federal tax exemption of municipal debt was that the Supreme Court originally interpreted the U.S. constitution to not allow the Federal government to tax states.[3] When a municipal bond is first issued, OID and interest coupon payments are equivalent because a municipal issuer can change the initial issue price and coupon in opposite directions to produce the same yield. Thus, OID and coupon interest payments on municipal bonds are not taxable. Consistent with the tax exemption of OID, initial issue premiums cannot be deducted.[4]

While the coupons paid by municipal bonds are tax exempt, an investor may pay federal income or capital gains tax when a municipal bond is purchased or sold in the secondary market, just as in the case of a sale of a taxable Treasury or corporate bond. That is, while OID for municipal bonds is non-taxable because the discount arises at issue and is equivalent to interest, discounts from secondary market trades are taxable because the discount arises in a market transaction, rather than from an original issuer. Investors pay income tax when purchasing any municipal bond with market discount. The economic principle behind this is if the bond is held to maturity, the bond price converges to par value. This convergence is deterministic if the yield remains constant and, thus, the increase is considered income. Taxing market discount at income tax rates is also consistent with the classification of OID on a taxable bond as income.

According to current tax law (IRC § 1278(a)(2)(A)), market discount is created when a par

[3] IRC § 103(a) exempts any interest received from municipal bonds for all taxpayers as not counting towards gross investment income that is subject to federal tax. The original constitutional basis for the federal exemption for municipal bonds was affirmed by the Supreme Court in 1895 in the case of *Pollack v. Farmers' Loan and Trust Company*. But, in 1988, the Supreme Court overturned the constitutional basis for the tax exemption in *South Carolina v. Baker*, so the tax exemption of municipal bonds is not protected by the U.S. constitution but now rests with Congress. In May 2008 in *Department of Revenue v. Davis*, the U.S. Supreme Court upheld the practice of states exempting interest on their own bonds from state tax and taxing residents for interest on bonds issued by other states.

[4] A holder must amortize the premium on a municipal bond under IRC § 1.171-1(c)(1) for reporting purposes and to determine the adjusted basis in the bond, but these are not deductible, unlike taxable bonds (see IRS Publication 550).

bond trades for a price less than par, or an OID bond is sold at a discount to the accrued (or compounded accumulated) value of the OID. Bonds with less than one year of maturity are considered to have no market discount. Under the Revenue Reconciliation Act of 1993, market discount is taxed as ordinary income at the time a bond is sold or redeemed (IRC § 1276(a)(1)).[5] To explain the taxation of market discount, we first show how market discount is computed for a par bond. Then, we discuss the cases of a premium and an OID bond.

In our analysis, we do not consider the effect of state or city taxes on municipal bonds, or in effect, we assume the marginal investor in a particular state's bonds is an individual who is a resident of that state (as shown by Cole, Liu and Smith, 1994). Our focus is on the effect of federal income and capital gains taxes on municipal bonds faced by a person purchasing a tax-exempt bond in the secondary market. We also consider only the effect of taxes on bonds held to maturity, following Litzenberger and Rolfo (1984) and Green and Ødegaard (1993). The Internet Appendix discusses the tax treatment of bonds sold prior to maturity.[6]

2.1 Case of a Par Bond

Consider a bond of par value $100 with an original 10-year maturity, paying semi-annual coupons of 10%. We refer to this bond as Bond A. Suppose two years after issue, with eight years to maturity, Bond A trades at a price of $95. The market discount on this bond is $100 - 95 = \$5$. If the bond is held to maturity, the investor owes ordinary income tax on $5, which is paid when the bond matures. This does not imply the present value of the tax is small, as we demonstrate in Section 2.4. For the investor purchasing Bond A, only the final cashflow of the bond is affected because the investor pays no tax on the coupons of the municipal bond.

The tax code provides a de minimis exception in IRC § 1278(a)(2)(C), which states:

If the market discount is less than $\frac{1}{4}$ of 1 percent of the stated redemption price of

[5] Prior to May 1, 1993, market discount was treated as a capital gain. Market discount of taxable bonds has been taxed as income since July 18, 1984 under the Tax Reform Act of 1984. The Revenue Reconciliation Act of 1993 made tax-exempt bonds purchased at a market discount after April 30, 1993 subject to the same ordinary income classification rule as taxable bonds issued after July 18, 1984.

[6] In the case of an OID bond there is an ex-ante incentive to sell a bond early, but none for a par or a premium bond, but this effect is negligible. The value of a municipal bond may also be affected by other issues not easily captured in simple cashflow discounting methods. Constantinides and Ingersoll (1984) and Strnad (1995), among others, demonstrate the value of a bond should include implicit tax options, which are generated by trading the bonds to time the realizations of capital gains and losses. Chalmers (2000) notes these effects are much less prevalent in the municipal bond market because bond premium amortization cannot be deducted as an expense and it is difficult to short municipal bonds.

the bond at maturity multiplied by the number of complete years to maturity (after the taxpayer acquired the bond), then the market discount shall be considered to be zero.

This de minimis rule imposes a discontinuity between income and capital gains tax rates at the de minimis cut-off. The de minimis boundary is deterministic and is specific to each bond. Bonds trading below the de minimis threshold are market discount bonds, which we also refer to as below de minimis bonds, and we denote the de minimis boundary as DM.

Applied to our example, the de minimis boundary of Bond A is $100-100\times0.0025\times8 = \$98$ at time $t = 2$ as Bond A has eight complete years to maturity. Thus, if Bond A trades for \$98.50, say, two years after issue, this price is above the de minimis boundary and Bond A is has no market discount. If the investor holds Bond A to maturity, the investor would pay capital gains tax on $100 - 98.50 = \$1.50$ when Bond A matures. Again, like the market discount case, only the final cashflow of the bond is affected. Naturally, since the top federal income tax rate is currently 35%, which is higher than the current long-term capital gains rate of 15%, once a bond crosses the de minimis boundary, it becomes subject to the more onerous income tax treatment.

2.2 Premium and OID Bonds

Municipal bond premiums as a result of secondary market transactions are not deductible under IRC § 171(a)(2), but capital gains are subject to tax. This asymmetry in the tax law means a premium bond has the same tax treatment as a par bond. Thus, for a par or premium bond, there are two important tax boundaries. Above par, all the bond cashflows are not subject to tax. If the purchase price falls between the de minimis boundary and par, the difference between par and the purchase price is subject to capital gains tax. Below de minimis bonds purchased in the secondary market have market discounts that are taxed at income tax rates.

OID bonds have the same no tax, capital gains tax, and income tax regions. But, since the bond is not issued at par, the bounds need to be changed to take into account the effect of accreted OID (also called the adjusted issue price), which decreases OID by interest accumulated at the original issue yield. Briefly, instead of using the par value, the de minimis boundary is defined relative to the revised issue price, which is the present value of the remaining cashflows of the bond discounted at the bond's original issue yield.[7] A full treatment of the OID bond

[7] The bond's original issue price is defined as the reoffering price, which is the bond price in the primary

case is given in Appendix A.

2.3 Computing Municipal Bond Prices

To adjust for taxes, we define an "after-tax yield," Y_τ, on municipal bonds, assuming the bonds are held to maturity. We implicitly define Y_τ to solve

$$P = \sum_{n=1}^{N} \frac{100 \times C/2}{(1 + Y_\tau/2)^{n-1+w}} + \frac{100 - \text{tax}}{(1 + Y_\tau/2)^{N-1+w}} - \frac{A}{360} 100 \times C, \tag{1}$$

where P is the price of a municipal bond on \$100 par value, Y is the semi-annual yield-to-maturity of the bond, C is the semi-annual coupon rate implying a six-month coupon rate of $C/2$ every six months, and N is the number of remaining coupon payments occurring at 6-month intervals. The fraction w is defined as:

$$w = \frac{180 - A}{180},$$

where A is the number of accrued days from the beginning of the interest payment period to the settlement date. We follow the 30/360 convention in the municipal bond market to compute A, so we count 30 days for each complete month to make 180 days in each interest rate period and 360 days in one calendar year.

The appropriate tax payment is payable at maturity and is given by

$$\text{tax} = \begin{cases} 0 & \text{if } P \geq RP \\ \tau_C \times (RP - P) & \text{if } DM < P < RP \\ \tau_I \times (RP - P) & \text{if } P \leq DM, \end{cases} \tag{2}$$

for RP the revised price of the bond (which is par value for a par or a premium bond), τ_C the capital gains rate, and τ_I the income tax rate. The de minimis boundary is given by $DM = RP - 100 \times 0.0025 \times \text{floor}(N/2)$. The number of complete years of maturity is given by $\text{floor}(N/2)$, where $\text{floor}(\cdot)$ rounds the number of remaining cashflows downwards to the nearest integer. In equation (1), taxes reduce the final cashflow, and hence increase the after-tax yield, for bonds trading below the revised price. Taxes only affect the last cashflow of the bond as the coupons are exempt from tax.

offering when the bond is sold to the public. Green, Hollifield and Schürhoff (2007b) document that some ultimate individual owners, especially small retail investors, often receive prices much higher than the reoffering price. For a par or premium bond the revised price is simply par value.

The standard definition of yield sets tax $= 0$, which is commonly referred to as the "tax-exempt yield" because it is the yield computed on tax-exempt municipal bonds.[8] We refer to the yield computed under the assumption of tax $= 0$ as the "yield" and denote it as Y.

In computing the after-tax yield, Y_τ, when tax $\neq 0$ we assume the income tax rate and the capital gains rate applied at maturity are the top marginal federal tax rates in the year of the trade. For example, a trade in 2006 would use $\tau_I = 0.35$ and $\tau_C = 0.15$. These rates have changed across our sample and start at $\tau_I = 0.396$ and $\tau_C = 0.28$ in 1995.[9] Municipal bonds do respond to perceptions of future, and actual, changes in tax rates (see, for example, the summary of Fortune (1996)) and agents may anticipate future changes in the tax schedule. We also compute the tax rates implied from secondary market trades using equation (1). These can be identified by bonds trading in different tax boundaries.

2.4 Calibrating the Effects of Taxes on Tax-Exempt Bonds

In this section we gauge the effect of taxes on municipal bond prices and show we should expect to see large tax effects in data. Consider a \$100 face value bond paying semi-annual coupons of rate C with a maturity of $N/2$ years. This bond was originally issued at par. If the current municipal yield curve is flat at the after-tax yield y, then the price of this bond, assuming the bond is held to maturity, is given by:

$$P = \left(1 - \frac{\tau}{(1+y/2)^N}\right)^{-1} \left[\sum_{n=1}^{N} \frac{100 \times C/2}{(1+y/2)^n} + \frac{100 \times (1-\tau)}{(1+y/2)^N}\right], \tag{3}$$

which is derived by rearranging equation (1). The bond price P and the tax rate τ depend on each other and must be solved jointly, with

$$\tau = \begin{cases} 0 & \text{if } P \geq 100 \\ \tau_C & \text{if } DM < P < 100 \\ \tau_I & \text{if } P \leq DM \end{cases}$$

and the de minimis boundary $DM = 100(1 - 0.0025 \times N/2)$. An investor buying this bond at price P would have an IRR of y. However, the quoted yield on this bond in order to produce an

[8] When tax $= 0$, equation (1) simplifies to Rule G-33 of the Municipal Securities Rulemaking Board to compute municipal bond prices. See http://www.msrb.org/msrb1/rules/ruleg33.htm

[9] These rates are available from the IRS. See, for example, http://www.irs.gov/formspubs/article/0,,id=150856,00.html for the 2006 federal tax rate schedule.

IRR of y must be higher than y because of the effect of taxes. An investor buying this bond at P would be quoted a tax-exempt yield $\tilde{y}$ that satisfies the equation:

$$P = \sum_{n=1}^{N} \frac{100 \times C/2}{(1+\tilde{y}/2)^n} + \frac{100}{(1+\tilde{y}/2)^N}.$$

For bonds where $y > C$ the tax reduces the final bond payment and lowers the bond price. Consequently, this raises the bond's yield, $\tilde{y}$, relative to the fully tax-exempt municipal yield, y, to compensate the investor for bearing the tax liability. Thus, we can compute the additional yield required by bonds subject to tax, $\tilde{y} - y > 0$, for these bonds to have the same required return as a newly issued municipal security with yield y. In our empirical work, we construct a proxy for the tax-exempt yield y using only fully tax-exempt bonds and compare it to the after-tax transactions yield $\tilde{y}$ of each bond.

To illustrate the effects of taxes, we choose $C = 3.8\%$, which is the average yield on a 5-year bond over our sample, and maturities of 2, 5, and 10 years. In Figure 1, we graph the additional yield $\tilde{y} - y$ required by this bond to produce an IRR of y, which is the yield on fully non-taxable municipal bonds. We conservatively assume $\tau_I = 0.35$ and $\tau_C = 0.15$, which are the lowest tax rates in our sample. Naturally, as maturity shortens, the effect of taxes rises because the final tax payment at maturity is worth more in present value terms. We vary y from 2% to 7%. The effect of taxes increases with y as there is a larger tax payment on the capital gain as the purchase price of the bond decreases when y increases.

Figure 1 shows the effect of taxes cannot be ignored. Below $y = 3.8\%$, there are no tax effects, so $\tilde{y} - y = 0$. As the yield rises above 3.8%, the price of the bond falls below par and the bond first becomes subject to capital gains taxes. The additional yields required are below 5 basis points in the capital gains region. As y further increases, income taxes now apply and there is a discrete jump in $\tilde{y} - y$. The 10-year bond has the largest region in the capital gains area because its long maturity causes its de minimis boundary to be lowest. For $y = 0.045$, the additional yield required is 21 (33) basis points for a 10-year (2-year) maturity bond. We should expect to see effects of this magnitude in data. As yields reach 7%, the additional yields required are over 150 (80) basis points for a 10-year (2-year) bond, but this is an extreme case. In summary, if individuals set marginal prices in municipal bond markets, we should expect to see significant differences in cross-sectional municipal bond yields for market discount bonds versus municipal bonds carrying no tax liabilities.

3 Data

3.1 MSRB Municipal Bond Transactions

Our data on municipal bonds is the Municipal Securities Rulemaking Board (MSRB) dataset, which contains all transactions of municipal bonds involving municipal bond dealers.[10] The MSRB database lists a price, a trade date, and the par value traded of each transaction. From January 24, 1995 to August 25, 1998, only interdealer transactions are included in the data. After August 25, 1998, all transactions between dealers and customers are recorded with an indicator denoting whether the transaction is a sale or purchase.

Over our sample period from January 1995 to April 2007, the MSRB database contains 70,611,395 individual transactions involving 2,080,291 unique municipal securities, which are identified through a CUSIP number. The MSRB database contains only the coupon, dated date of issue, and maturity date of each security. We obtain other issue characteristics for all the municipal bonds traded in the sample from Bloomberg. Specifically, we collect information on the bond type (callable, putable, or sinkable, etc.); coupon type (floating, fixed, or OID); the issue price and yield; the tax status (federal and/or state tax-exempt, or subject to the Alternative Minimum Tax (AMT); the size of the original issue; the S&P rating; and whether the bond is insured.

We focus on bonds issued in the 50 states that are exempt from federal and state income taxes and which are not subject to the AMT. We first remove all transactions less than $10,000. We take bonds rated by S&P with a rating of A- or higher, which we refer to as the "A-Grade" class of municipal bonds. Over our sample period, there have been zero defaults in A-Grade municipal bonds.[11] We take only straight bonds with maturities one to 10 years because market discount does not apply for bonds with less than one year of maturity and there are relatively few straight bonds with maturities longer than 10 years. Bonds with very long maturities are often issued with call or sinking fund provisions. We also do not take transactions within a month of

[10] At initiation of the database these trades were originally made available with a one-month lag, but trades are now made available with a one-day lag through the Bond Market Association and with a short lag of 15 minutes through data vendors such as Bloomberg and Reuters.

[11] Defaults of investment-grade municipal bonds have been much lower than investment-grade corporate bonds (see studies on municipal defaults by, among others, Litvack and Rizzo (2000), and Woodell, Montrone and Brady (2004)). The famous defaults on the bonds of the Washington Public Power Supply System and Orange County, CA occurred in 1990 and 1994, respectively, before our sample starts in 1995. Likewise, the downgrading of several municipal bond insurers from their AAA ratings and the financial distress of Jefferson County, AL in 2008 is also outside our sample period.

issue because Green, Hollifield and Schürhoff (2007b) document significant aftermarket effects on newly issued bonds. Appendix B contains a detailed description of our data filters.

After merging our transactions data with the descriptive data and applying our data filters, we are left with a sample of 6,753,847 transactions on 294,442 unique securities. Thus, each bond trades 23 times, on average, over our 12 year sample. A small fraction (5.03%) of issues trade only once. In the top panel of Figure 2, we plot the total number of trades each month. The large jump in the number of trades in August 1998 is due to the inclusion of all trades between dealers and customers being added to the database at this date.

In the bottom panel of Figure 2, we plot the proportion of bond transactions each month involving bonds trading below their de minimis boundaries. The figure also overlays the 5-year zero-coupon yield (see below). Naturally, as interest rates increase, bond prices decline and the number of transactions involving bonds with prices below de minimis increases. This is clearly seen in the large spike of market discount transactions (over 30%) taking place in 2000. In 1998 and over 2001-2003 as interest rates decreased, the number of market discount transactions decreases. Nevertheless, because of the large amount of transactions in our database, there are still a sizeable number of below de minimis trades in these years. For example, in 2002, there are 12,803 transactions of below de minimis bonds, while there are 31,526 trades of bonds with prices below de minimis in 2003. As interest rates started to increase since 2004, the proportion of below de minimis trades increases to end above 10% at the end of our sample in April 2007.

In Table 1, we report proportions of the 6,753,847 transactions falling into various categories. We note this is a sample based on transactions, not issues, and municipal bonds that are purchased directly at issue but never subsequently traded do not appear. Most of the trades involve bonds issued in California (14%) and New York (13%). Florida, Texas, New Jersey, and Michigan each represent around 5% of all trades. Approximately 45% of all transactions are general obligation bonds and 49% are revenue bonds. Most of the transactions (70%) are AAA rated and 64% of all transactions are insured bonds.

Only a minority (11.6%) of the transactions involve par bonds, unlike corporate and U.S. Treasury issues, which are issued almost exclusively at par. The large cross-section of original issue prices is important for our analysis because the main reason bonds decline in price is through increasing interest rates with the credit risk in our A-grade municipal bonds being negligible.[12] Bonds issued at different prices will decline at different amounts when interest

[12] Most bonds issued at discount are issued slightly below par, but there are a large number of bonds issued at deep discount. For bonds issued at premiums, many bonds (15% of all CUSIPs) carry substantial premiums of at

rates rise. Thus, at a given time when interest rates have risen, the bonds trading below de minimis will not all have been issued at one particular point in time. This substantially reduces, but does not eliminate, the fixed time effects of the dated dates of issue on our analysis.

Finally, municipal bond markets are generally illiquid. Downing and Zhang (2004), Hong and Warga (2004), Harris and Piwowar (2006), and Green, Hollifield and Schürhoff (2007a), among others, find large trading costs, especially for retail customers in the municipal bond market. We purge all transactions with par amounts traded below $10,000 from our sample to minimize these effects. The median par amount traded is $50,000 with par amounts of $820,000 lying at the 95th percentile. Even after removing small trades, liquidity may still be an important determinant in pricing. To partially account for this, we treat interdealer transactions separately from dealer transactions with customers. The proportion of transactions between dealers and customers is 67.5% in our sample. The remainder of trades (32.5%) are interdealer trades.

3.2 Municipal Zero Yield Curves and Model-Implied Yields

To provide a benchmark for all municipal bond trades, we construct a daily municipal zero-coupon yield. In constructing the zero curve, we use only interdealer trades of all fully-exempt municipal bonds in our A-grade sample. Interdealer trades are reported continuously through the sample and dealers, who are taxed symmetrically on capital gains and income, provide liquidity between customers so interdealer trades lie between customer to dealer transactions. Another benefit of not using customer to dealer transactions is that there is sometimes considerable price dispersion in customer to dealer types of trades (see, for example Harris and Piwowar (2006), and Green, Hollifield and Schurhoff (2007b)). We choose not to include the largest customer to dealer trades in creating the zero curve because we later segregate customer trades by different transaction sizes to contrast their different behavior from the interdealer zero curve. Appendix C provides further details on the construction of the zero yield curve.

We refer to bond prices computed using the zero yield curve as "model-implied." We do not take a stand that the model-implied yields represent fundamental value. Our focus is on the relative cross-sectional prices of municipal bonds and the model-implied yield curve serves as a way to express bond prices relative to a common standard. Ultimately, many of our comparisons involve yields of bonds trading above or below de minimis and in these relative yield spreads the effect of specification error or estimation error of the zero coupon yield curve cancels out.

least $5 above par.

To benchmark the yields of bond transactions, we compute model-implied yields which we denote as Y^m. The model-implied yields are the corresponding empirical equivalent of the tax-exempt IRR y in the model of Section 2.4. If a bond trades at par, then the appropriate tax-exempt benchmark yield would be the par yield implied by the zero curve. However, as municipal zero curves are not flat, the timing of coupon payments and maturity affect the calculation of the yield. To create the tax-exempt benchmark yield, Y^m, for a particular bond, we treat the bond as if all its cashflows were tax exempt and value the cashflows using the tax-exempt zero curve. We denote this price as P^m. The model-implied yield, Y^m, is the yield corresponding to P^m. This procedure creates an artificial bond with identical cashflows to the original bond, except the cashflows are fully tax exempt and they are discounted using the tax-exempt zero coupon curve. Thus, Y^m is the theoretical tax-free yield of the municipal bond implied by the zero curve. We refer to the difference between transactions yields and model-implied yields, $Y - Y^m$, as "yield spreads." This is the empirical counterpart to the yield spread $\tilde{y} - y$ in the model of Section 2.4. We expect the yield on a market discount bond to be greater than its model-implied yield, $Y - Y^m > 0$, to compensate the investor for bearing the tax liability in purchasing the market discount bond.

4 Tax Effects in Tax-Exempt Bonds

In Section 4.1 we show after-tax yields on below de minimis bonds are higher than the zero curve predicts. Section 4.2 investigates two obvious explanations, default and liquidity risk, neither of which can account for the effect. In Section 4.3 we show that the high yields are concentrated among small retail trades, which we define as trades below $100,000 par value. Section 4.4 examines below de minimis transactions between dealers and customers. In Section 4.5 we examine the effect of taxes on bonds crossing into, or out of, taxable regions. We estimate implied income tax rates in Section 4.6.

4.1 Tax Effects in the Cross Section

To characterize the effects of taxes on the cross-section of municipal bonds, we partition all trades with transaction price P into one of three bins: (1) transactions not involving any tax liability where bond prices are greater than par for par or premium bonds or revised price (RP) for OID bonds, $P \geq RP$; (2) bonds trading between revised price and the de minimis (DM)

boundary, (DM, RP), which are subject to capital gains tax; and (3) market discount transactions where $P \leq DM$, which are subject to income tax.

We first compare transactions yields, Y, against model-implied yields, Y^m, described in Section 3.2. We compute yield spreads each day for each bin and then average the yield spreads across time. Table 2 reports the results. We report standard errors of the time-series averages in parentheses similar to the method of Fama and MacBeth (1973) for computing standard errors of factor risk premia with time-series estimates. Panel A shows the average yield spreads for bonds trading above revised price is around 5 basis points below the zero curve. Trades in the (DM, RP) bin have a yield spread of 4 basis points. For bonds trading below de minimis, transactions yields are 45 basis points higher than predicted by the zero yield curve. Looking at all trades, the difference in yields between market discount bonds and their fully tax-exempt counterparts is over 50 basis points. Similarly, for interdealer trades, the difference in yield spreads between bonds trading above revised price and below de minimis is 37 basis points. These spreads are economically large and close to our calibration in Section 2.4 predicted. In summary, taxes matter for pricing the cross section of municipal bonds.

This result is perhaps not surprising, given evidence that individual investors react rationally to tax effects in other asset pricing decisions, like allocations to mutual funds or tax-deferred accounts (see, for example, Bergstresser and Poterba (2002), and Bernheim (2002)). Panel A of Table 2 clearly demonstrates market discount bonds have higher yields than fully tax-exempt bonds and this is consistent with investors requiring market discount bonds to have higher yields to compensate them for bearing income tax liabilities associated with these bonds. However, Panel A does not address the question of whether the higher yields are too low or too high relative to the present value of the income taxes.

An investor buying a market discount bond, or a bond with a capital gains tax liability, will receive an after-tax yield of Y_τ, defined in equation (1). If the investor receives fair compensation for bearing the tax burden, the after-tax yield will be the same as the yields on comparable fully non-taxable bonds and we expect, on average, $Y_\tau - Y^m = 0$. To examine if the higher yields on market discount bonds represent compensation for bearing the income taxes, we report average after-tax yield spreads, $Y_\tau - Y^m$, in Panel B of Table 2.

Not surprisingly, Panel B shows the after-tax yield spreads are smaller than the raw yield spreads in Panel A. However, they are not zero on average. For bin 1 the after-tax yield spreads are exactly the same as the tax-exempt yield spreads in Panel A since if $P \geq RP$ there are no tax effects. Bonds in the capital gains region, (DM, RP), are priced approximately at intrinsic value, with an after-tax yield spread of one and zero basis points, respectively, for all trades and interdealer trades. However, for market discount bonds, the after-tax yield spread is 21 basis points for all trades and 12 basis points for interdealer trades. Column 4 thus summarizes that market discount bonds provide yields 26 basis points higher (13 basis points higher for interdealer trades) than bonds trading above revised price. In summary, market discount bonds seem to be trading at prices too low, or yields too high, after valuing their income tax liabilities.

4.2 Default and Liquidity Risk

Two obvious explanations for the results in Table 2 are default and liquidity risk, which we now examine.[13] We focus on bonds above revised price and bonds trading below de minimis as Table 2 shows that bonds with prices trading between (DM, RP) have after-tax yields close to zero.

We first measure the after-tax yield spread for bonds with short maturities between 1-2 years. There are two reasons for considering these short maturity bonds. First, short maturity bonds have the lowest cumulative default risk. Second, investors may be unwilling to purchase market discount bonds with high after-tax yields because they have shorter investment horizons than the typical maturities of these bonds and are less willing to bear price risk. Such investors would focus on short maturity bonds. The first row of Table 3 reports that for bonds with 1-2 year maturities, the difference in the after-tax yield spread for market discount bonds and bonds trading above revised price is 46 basis points, which is larger than the corresponding after-tax yield spread difference of 26 basis points using all bonds in Table 2. This is especially puzzling because short maturity bonds are most likely to be held to maturity and these investments carry the least cumulative default risk.

By construction, our sample is specifically constructed to minimize default risk by taking only A-grade bonds. Nevertheless, as a second default risk control, we take only insured bonds with the highest AAA credit ratings. The second row of Table 3 reports the after-tax yield spreads on these bonds. The after-tax yield spread difference between bins 1 and 3 is 20 basis points, which is a little lower than the raw 26 basis point spread reported in Table 2. This indicates that insurance helps to reduce the after-tax yield spread but insurance per se does not remove the de minimis premium.

The fact that we still observe high yields on bonds with market discount among insured

[13] The higher after-tax yields of below de minimis bonds are not due to these bonds having higher empirical duration than bonds trading above the de minimis boundary. These results are available in the Internet Appendix.

bonds does not rule out a default story if default is a Peso problem and the bond market is pricing in an extremely rare event. To implement a very strict default control, we employ the following strategy. Municipal bonds are usually issued in series, with many bonds of different types and maturities being issued simultaneously by the same issuer. We consider above and below de minimis bonds with different tax treatments issued in the same series. The third row of Table 3 shows the yield differences on bonds with and without market discount are extremely unlikely to be due to default risk. Bonds in the same series with different tax treatments have an after-tax yield spread difference of 21 basis points between bonds above RP and bonds below DM. Thus, it is highly unlikely that default risk is behind the high yields of market discount bonds. This result is similar to the fact that default risk also cannot explain the declining ratio of Treasury to municipal bond yields as maturity increases (see Chalmers (1998)).

In the fourth row we take a simple control for liquidity by taking only New York and California bonds. Municipal bonds from these states tend to be the most liquid, as noted by Biais and Green (2005), because these states have high income tax rates and have many residents with high marginal tax rates for whom in-state municipal bonds are attractive investments.[14] Bonds issued in these states comprise 27% of all trades (see Table 1). For these states, the after-tax yield spread difference between fully tax-exempt and market discount bonds is 24 basis points. Thus, even in these more liquid markets, the high yields of below de minimis bonds persist.

In the final row of Table 3 we compute the premium excluding the years 1995, 1999-2000, and 2006-2007. These periods coincided with a large proportion of de minimis trades. For example, the path of 5-year municipal interest rates in bottom plot of Figure 2 rose from 4% to 5% over 1999 to 2000 and there were a large proportion of trades that involved below de minimis bonds in 2000. There are also large numbers of below de minimis trades in 1995 and 2006-7. Excluding these periods increases the after-tax yield spread difference between bins 1 and 3 to 37 basis points, which is a little larger than the raw yield spread difference of 26 basis points in Table 2.

Taking only NY and CA bonds in Table 3 is a very crude proxy for liquidity. In Table 4, we examine in more detail the liquidity characteristics of trading frequency and transactions spreads. In Panel A, we first record the number of trades for all bonds in our sample and rank the bonds into quartiles by trading frequency. Bonds in the first quintile trade, on average, 0.5 times per year while bonds in quartile 4 trade, on average, 18.0 times per year. Then, we divide

[14] We also employ other liquidity controls such as matching on the basis of par value traded. These results are available in the Internet Appendix

the transactions of the bonds in each quartile into above revised price and below de minimis bins each day and compute average after-tax yield spreads, $Y_\tau - Y^m$, for each bin. Table 4 reports the average after-tax yield spreads for each quartile and bin. This procedure treats trading frequency as a bond-specific characteristic as the bonds in each quartile are fixed over time. We estimate the trading frequency variable ex-post on the full sample.

In Panel A, we observe the high yields on market discount bonds exist in all four trading frequency quartiles. The average after-tax yield spread difference between bonds trading above revised price and market discount bonds is highest, at 24 basis points, for bonds that are most frequently traded and is lowest, at 13 basis points, for bonds that are least frequently traded. The after-tax yields relative to the zero curve (column 2) are roughly equal, at 15-17 basis points, for all trading frequency quartiles. What drives the spread in column 3 is that the most frequently traded, fully tax-exempt bonds tend to trade slightly below the zero yield curve. Thus, trading frequency as a measure of liquidity points to the largest de minimis premiums occurring in the most liquid bonds.

However, trading frequency is not a complete picture of liquidity because a bond that trades very frequently at disperse prices may be viewed as having less liquidity than a bond that trades infrequently but in large sizes with similar prices for dealer sales to customers and dealer purchases from customers (see comments by Green, Hollifield and Schürhoff (2007a, 2007b)). Large price dispersion can be measured by looking at the average transactions spread, which we define as the yield difference between dealer sales to customers and dealer purchases from customers.[15] We examine this in Panel B of Table 4.

In Panel B, we compute transactions spreads each day for each bond, and then we average the transactions spreads over time. Using each bond's average transactions spread over the sample, we rank the bonds into four quartiles and divide transactions of the bonds in each quartile into the two price bins. We report the average after-tax yield spreads of each bin. On average, transactions spreads range from 3 basis points in quartile 1 to 68 basis points in quartile 4. Panel B does not uncover any monotonic relation between after-tax yield spreads on market discount bonds and transactions spreads. In quartile 4 with the highest transactions spreads, market discount bonds have yields 28 basis points higher than their fully-exempt counterparts. But, in quartile 1 with the lowest transactions spreads, market discount bonds are trading with yields 20 basis points higher than bonds above revised price. Thus, the yield spreads on market

[15] We use the term transactions spread rather than bid-ask spread because the municipal bond OTC market does not have a conventionally defined bid-ask spread corresponding to a traditional centralised exchange.

discount bonds are highest in both quartiles 1 and 4 containing bonds with the lowest and highest transactions spreads, respectively.

In summary, the results of Table 4 suggest that high after-tax yields on market discount bonds are not strongly related to trading frequency and transactions spread characteristics of bonds. In particular, the de minimis premium persists in bonds which are most frequently traded and bonds with the lowest and highest transactions spreads.

4.3 Retail Clienteles and the De Minimis Premium

By definition, market discount taxation is an issue affecting individual investors. Banks, broker/dealers, and corporations are taxed at corporate tax rates. Consequently, banks or corporations trading only with each other may not price a market discount bond at a yield higher than implied by the zero curve.[16] Thus, an alternative hypothesis to default or liquidity for the high yields on market discount bonds is that they are driven, perhaps irrationally, by retail individual investors. The high yields on market discount bonds may also reflect an inconvenience yield to compensate retail investors for handling the taxation payments and computations associated with these bonds. In this section we present evidence consistent with this interpretation.[17]

If retail investors are behind the de minimis premium, then trades of market discount bonds more likely to be made by individual investors should, on average, have higher after-tax yields than trades made by institutional investors. Unfortunately, identities behind municipal bond trades are not available but retail investors are more likely to engage in certain types of transactions, which we examine in Table 5.

Small investors are much more likely to trade in smaller amounts than institutional investors. Even though we eliminate the smallest trades below $10,000 par value in our sample, Table 1 reports over 75% of trades are below $100,000 par value. The top 5% of trades constitute par amounts above $820,000. We define a transaction as a retail trade if the par amount traded is below $100,000 and transactions with par amounts above $100,000 as institutional trades.[18] We define very small trades as those with par amounts traded between $10,000 and $25,000.

Panel A of Table 5 reports after-tax yield spreads of these different trade transaction sizes. The after-tax yield spread between fully tax-exempt and market discount bonds is 25 basis

[16] Market discount taxation may be an issue for mutual funds, which are pass-through taxation vehicles, which we discuss in Section 5.

[17] We thank a referee for suggesting this analysis.

[18] We obtain very similar results if we define institutional trades as those transactions having par amounts traded greater than $1 million.

points for both very small and retail trades, which implies the de minimis premium for very small trades and the vast proportion of retail trades is identical. In contrast, institutional trades uniformly occur below the zero curve. Fully tax-exempt municipal bonds in institutional transactions trade at 12 basis points below the zero curve. Corresponding institutional market discount bonds trade at a premium of only 5 basis points above this level.

Thus, Panel A convincingly demonstrates that the yield premium on market discount bonds occurs for smaller trades mostly likely to be conducted by retail investors. While holdings information on municipal bonds is not available, we can use one particular bond type that should be held largely by institutions as additional evidence. Institutions are much more likely to hold bank qualified (BQ) issues. Under IRC § 265(b)(3)(B), banks can deduct 80% of the carrying cost of a BQ municipal bond but there are no deductions for holding a non-bank qualified municipal bond.[19] Since almost all BQ bonds are held by institutions and are likely to be traded primarily among institutions, the de minimis effect should be close to non-existent for these bonds. Panel B of Table 5 confirms this is the case. For non-BQ issues, the de minimis premium is 27 basis points whereas it is only 5 basis points for BQ issues. In fact, the after-tax yield spread for BQ bonds trading below de minimis is zero.

4.4 Dealer and Customer De Minimis Transactions

The tax treatment of municipal bond dealers is similar to dealers in other securities markets and so dealers treat capital gains the same as ordinary income. Suppose dealers simply facilitate trades between investors, take relatively small speculative positions, and do not arbitrage the different tax regimes facing them and individual investors. Then, the differences in yields between market discount bonds and bonds above revised price should persist in both customer purchases from dealers and customer sales to dealers. We now examine these interactions between dealers and customers. We defer to Section 5 to discuss why dealers are unable, or unwilling, to arbitrage the mispricing of below de minimis bonds.

Table 6 reports transactions spreads (the yield difference between dealer sales to customers and dealer purchases from customers) of below de minimis and above revised price transactions between dealers and customers averaged across the sample. We observe that for retail to dealer transactions, retail investors pay a steep price in terms of the transactions spread, consistent with

[19] Prior to the Tax Reform Act of 1986, institutions could deduct up to 80% of the carrying cost for purchasing any municipal bond. In order for an issue to be classified as BQ, the bonds must be issued by a qualified issuer (no more than $10 million issued in a given year) and the issue must be for public purposes.

the evidence documented by, among others, Downing and Zhang (2004), Harris and Piwowar (2006), and Green, Hollifield Schürhoff (2007a). For fully tax-exempt bonds, the difference in yields between retail sales to dealers and retail purchases from dealers is a large $18.74+18.99 = 38$ basis points. The transactions spread is higher, at $42.03 + 3.26 = 45$ basis points, for below de minimis trades. The transactions spreads in our full sample are close to those estimated by previous authors on smaller sub-samples.

Table 6 demonstrates the de minimis puzzle exists for both retail purchases from dealers as well as retail sales to dealers. The column 3 difference between the after-tax yield spreads of market discount bonds and bonds above revised price is 16 basis points for retail purchases from dealers and is 23 basis points for retail sales to dealers. This difference cannot be arbitraged by a retail investor, as buying a below de minimis bond and selling a bond above revised price to a dealer will net, on average, negative $18.74 + 3.26 = 22$ basis points. The situation where the de minimis premium comes into play is if a retail investor is purchasing a bond in the secondary market, she is much better off buying a market discount bond, which trades at -3 basis points to what the zero curve implies, rather than purchasing a bond trading above revised price for -19 basis points relative to the zero curve. Purchasing a market discount bond rather than a fully tax-exempt bond yields the retail investor an extra 16 basis points. Similarly, if an investor must sell a municipal bond, that investor should avoid selling a market discount bond and instead sell a bond trading above revised price. Doing this saves her from losing 23 basis points.

We examine institutional purchases from and sells to dealers in the next two lines. Institutional transactions spreads are around half of retail transactions spreads, at around 25 basis points. This is consistent with Harris and Piwowar (2006) who show transactions costs decrease with trade size. Interestingly, institutions pay almost the same price as individuals, at -18 basis points relative to the zero curve, when buying a fully tax-exempt bond from a dealer. Institutions receive good prices, with a yield spread of -6 basis points when selling fully tax-exempt bonds to dealers, especially compared to retail investors who lose 19 basis points relative to the zero curve. Again, the de minimis premium is much smaller for institutional trades than for retail trades. There is a difference of -5 basis points between market discount bond yields and fully tax-exempt bond yields for institutional purchases from dealers and 7 basis points for institutional sales to dealers.

In Table 6 institutions buying market discount bonds from a dealer receive an after-tax yield spread of $Y_\tau - Y^m = -23$ basis points relative to the zero curve, which appears to be a much worse deal than retail investors obtain, at -3 basis points. However, banks, insurance companies, and other corporations are not subject to individual income tax, so we examine raw yield spreads, $Y - Y^m$, in columns 4 and 5. Column 4 shows corporations unaffected by personal income tax buying market discount bonds from dealers pay close to the model-implied price, at a very small one basis point above the zero curve and statistically insignificant at the 95% level.

Table 6 shows market discount taxation affects institutions even though market discount taxation only involves individual income taxes. A corporation purchasing a municipal bond in the secondary market would be better off purchasing a market discount bond, where $Y - Y^m =$ one basis point, as opposed to buying a fully tax-exempt bond with a yield spread of -18 basis points. Thus, institutions would obtain an extra 19 basis points of yield, on average, by purchasing market discount bonds.

Similarly, the last row reports institutions potentially lose 34 basis points of yield if they sell a market discount bond to a dealer rather than an above-revised-price bond. It is likely that dealers purchasing market discount bonds from institutions set the prices of these transactions as if they could be sold only to individuals, at an after-tax yield spread of 1 basis point. From row 1 of Table 6, if these large trades are split into small trades, the dealer receives a spread of only $1.48 + 3.26 = 5$ basis points for buying the institutional-size market discount bond. Column 5 shows the institution loses 28 basis points on a raw yield spread basis by selling the market discount bond to a dealer. The institution would save 34 basis points by selling only fully tax-exempt bonds and avoiding selling market discount bonds. Certainly, the pricing of de minimis bonds also adversely affects institutional investors.

4.5 Events When Bonds Cross Taxable Regions

In this section we track individual bonds as they cross into or out of each of the taxable regions. This event-study approach is useful because it gauges the effects of tax on the *same* bond, rather than considering the prices of different bonds in the cross section. We examine events when a bond crosses over the de minimis boundary, DM, in Table 7. We trace the effects of bonds crossing down through DM and up through DM. We consider all such transactions in Panel A and interdealer trades in Panel B. In the first two rows, "Crossing Down" and "Crossing Up," we track trades whose last trade prior to the cross occurred within the last five trading days, but not on the same trading day, as the event trade. In the last two rows, "Crossing Down on the Same Day," and "Crossing Up on the Same Day," we consider trades where the last trade prior to the cross and the trade crossing DM occur on the same trading day.

Not surprisingly, Panel A shows yields of bonds entering (leaving) the income tax region increase (decrease). Bonds entering the below de minimis region increase their yields by 54 basis points when their last trade occurred up to 5 days prior and 39 basis points crossing on the same day. The theoretical changes in these yields, reported in the second last column labeled "ΔY_0^m" for fully tax-exempt bonds are two orders of magnitude smaller than the reactions we see in data.

In the column labeled "$\Delta Y_{0,\tau}$," we report the changes in the after-tax yields between the last trade and the event trade. If we fully account for tax effects using the present value model in equation (1), then these changes in after-tax yields should be close to the changes of the model-implied yields. The changes in after-tax yields are smaller than the yields in the column labeled "ΔY_0," but are clearly not zero. For bonds entering the below de minimis region, the change in after-tax yields is 36 basis points. Similarly, as bonds cross over the DM to the capital gains or no tax regions, the decrease in after-tax yields is 40 basis points for bonds where the last and event trade are not on the same day. Thus, investors give up too much yield when the tax effects are removed.

Similar patterns are also found for only interdealer trades in Panel B. Bonds crossing down into de minimis territory increase their after-tax yields by 16 basis points compared to a predicted change of less than one basis point. Bonds crossing up through DM gain in price by 21 basis points on an after-tax basis, also compared to predicted changes of less than one basis point.

Patterns like those observed in Table 7 suggest a trading strategy to exploit the mispricing of market discount bonds relative to the yield curve. Investors could identify bonds trading close to, but slightly above, the de minimis boundary. If interest rates rise, these bonds are likely to decrease in price much more than the typical bond giving large negative convexity. While shorting long-dated bonds in municipal markets is generally difficult, if an institution is benchmarked relative to a broad-based index, like the Lehman municipal bond indices, then bonds trading slightly above their de minimis boundaries could be underweighted in increasing interest rate environments. The converse strategy is to buy bonds trading slightly below DM. These bonds decrease in yield, or increase in price, much more than their tax effects justify when they cross the de minimis boundary.

In summary, bonds crossing into taxable thresholds suddenly trade at higher after-tax yields than taxes seem to justify. Similarly, investors seem to be willing to give up after-tax yields when bonds cross in regions with lower tax treatments. Thus, the de minimis boundary gives rise to large negative convexity. We now turn to estimating the implicit tax rates priced by individual investors by below de minimis bonds.

4.6 Implied Income Tax Rates

So far in our analysis we have computed the after-tax yield in equation (1) assuming the tax rates in the year of trade. In this section, we use the prices of market discount bonds to compute an implied tax rate. We denote the transaction price for a trade in the below de minimis region as P. We assume the market discount on a bond trading below de minimis is paid at maturity of the bond. Since the payment of income tax occurs only once at maturity, all the intermediate cashflows of the bond are identical to a fully tax-exempt bond. We use P^m to denote the model-implied price if the bond were not subject to tax, and the difference between P and P^m is the present value of the tax liability:

$$P = P^m - \frac{(RP - P) \times \tau_I}{(1 + r_N/2)^{N-1+w}} \tag{4}$$

using the same notation as equation (1) where τ_I is the income tax rate we wish to estimate.

We estimate implicit tax rates in two ways. First, we obtain direct estimates by inverting τ_I using equation (4). We compute estimates of τ_I using all market discount bonds on each trading day and then average the daily implicit tax rate estimates across days. This procedure is analogous to the average yields reported in the analysis so far. Second, we observe in equation (4), the tax rate appears linearly. Thus, we can treat τ_I as a regression coefficient by placing a rational expectations error on the RHS of equation (4). We run a cross-sectional regression each day using market discount bond prices and then average the OLS coefficients τ_I across time similar to the approach of Fama and MacBeth (1973). This approach has the advantage that we can add other instruments to the regressions as controls. We use fixed effects for each year, dummies for different bond types (general obligation or revenue), dummies for different original issue prices (par or premium), and dummies for the eight most traded states (CA, NY, FL, TX, NJ, MI, OH, and PA). On the other hand, the OLS estimates have a disadvantage since by adding other instruments the implicit tax rates do not correspond to investable yields as in the direct estimation method.

Table 8 reports the results. In the direct estimates in Panel A, the implied income tax rate that equates the transaction price and the price implied by the zero curve is 88% across all trades. Taking only interdealer trades results in an implicit income tax rate estimate of 71%. These are much larger than historical tax rates during the sample. However, since the high yields on below de minimis bonds are concentrated in retail trades, separating retail from institutional trades results in implicit tax rates of 96% for retail trades and 24% for institutional trades.[20] The implicit tax rate on retail trades is over twice as high as the highest personal income tax in the sample at 39.6%, which applies in the pre-2000 period. Our estimate of the institutional tax rate of 24% over the whole sample is close to the implicit tax rates of approximately 20% in the corporate bond market estimated by Liu et al. (2007). Panel B reports OLS estimates of the implicit tax rates. With additional controls, the implied income tax rate across all trades is 70%, but is a very high 129% for retail trades and 65% for institutional trades, respectively.

Over our sample, the statutory top income tax rate dropped from 39.6% to 35%. The largest change occurred from 2002, where the tax rate was 38.6% to 2003, where the tax rate was 35%.[21] Table 8 reports estimates of tax rates pre- and post-2003. Consistent with the fall in statutory tax rates over these subsamples, implicit tax rates also decline from 108% to 51% using all trades in Panel A. Taking only retail trades show implicit tax rates fall from 118% to 54% pre- and post-2003. Interestingly, tax rates taking only institutional trades are quite steady and decline only from 26% to 20%. This is consistent with Sullivan's (2007) estimates of modestly falling effective corporate tax rates over this period. The same pattern of lower implicit tax rate estimates post-2003 is also observed in the OLS estimates in Panel B, where implicit tax rates on retail trades decline from 137% to 89%.

5 Discussion

The reason there are high implicit tax rates priced in municipal market discount bonds is not immediately clear. Our results show the effect is concentrated in retail trades, and the fact that retail investors react to taxes is consistent with how taxes alter the financial decisions of individual investors in many other situations. But, the price reaction to these tax rates is many times greater than they should be according to a present value model. A rational story for the abnormally high yields on market discount bonds is that the high yields represent an inconvenience yield demanded by individual investors to deal with the complexities of computing the

[20] If we take only institutional market discount bond trades for par value traded greater than $1 million, the implied tax rate is 28%.

[21] During the early 2000's, the Economic Growth and Tax Relief Reconciliation Act of 2001 progressively reduced income tax rates from 39.6% in 2000, but The Jobs and Growth Tax Relief Reconciliation Act of 2003 accelerated these and reduced the income tax rate to its present level of 35% from 2003 onwards.

tax liabilities and tracking the de minimis boundary. An alternative story is that the extremely high implicit tax rates are consistent with a behavioral story that individuals have a particular aversion to tax not justified by rational models (see McCaffery (1994)).[22]

However, while individual investors may be acting as marginal pricers, another underlying question is why dealers, corporations, and tax-exempt institutions do not trade and profit from the low prices on market discount bonds. The actions of these institutions should eliminate, or at least mitigate, any tax effect. The high yields on market discount bonds suggest this process has not occurred and individuals dominate dealers and other institutions in the municipal market.

There are three reasons why the de minimis premium may continue to persist. First, some sophistication is required to identify which bonds are trading below de minimis. Many retail investors would defer such computations, especially for the more complicated calculations for OID bonds, to financial professionals, and most would not perform these calculations in a real-time trading environment. Even for professional investors, specialized computer systems are needed to track and compute de minimis boundaries for a very large number of municipal securities. For example, over our sample period there are over 2 million unique CUSIPs. Initial offering terms required to compute the de minimis boundaries are also more difficult to obtain than for corporate issues. While the largest municipal mutual funds and most banks and insurance companies are capable of performing these calculations, our conversations with municipal dealers and large mutual municipal fund managers indicate that most large investors do not routinely perform de minimis calculations.

Second, many mutual fund companies, which control around one third of investments in municipal bonds, deliberately avoid market discount transactions even though purchasing these bonds would be good deals for their investors. Many investors in municipal mutual funds place their money with these funds expecting to receive distributions entirely exempt from tax, or perhaps expecting to pay capital gains tax, which may be evidence of superior bond picking ability by the mutual fund manager. Mutual fund managers picking up cheap market discount bonds would pass through income tax liabilities onto their clients. While some mutual fund managers are permitted to do such trades, the largest municipal bond funds not holding AMT bonds pass through income taxes only once every ten or so years, from our conversations with mutual fund managers.

[22] Fennell and Fennell (2003) comment that studying the effects of tax aversion is difficult in many applications, partly because tax evasion is illegal in the real world. Our findings possibly represent an example of tax aversion in a legal market.

To illustrate the reluctance of many mutual fund managers to engage in transactions involving market discount bonds, we tabulate the 2006 distributions of the largest 20 mutual funds listed in the Bond Buyer 2007 Yearbook. Of the largest 20 funds, 17 funds exclusively distributed only untaxed dividends. The three funds that distributed dividends subject to income tax held high yield municipal bonds or municipal bonds subject to the AMT. The prospectuses of these funds make clear that these funds distributed income dividends.[23] Among the largest 10 funds, nine funds exclusively distributed untaxable dividends. The largest plain-vanilla municipal bond offerings from Franklin, Fidelity, and Vanguard all do not distribute taxable dividends.[24] Thus, a large segment of active mutual fund managers deliberately do not buy market discount bonds. Importantly, because of the underlying retail clientele of these funds, many municipal bond funds are also unlikely to engage in transactions generating income tax liabilities in the future.

The third reason we might expect the de minimis phenomenon to persist is that the municipal bond market is opaque and decentralized, as Green, Hollifield, and Schürhoff (2007a) emphasize. The dealer market is also very concentrated, with the five largest dealers out of over 1,600 dealers accounting for over 50% of all trades.[25] This means that even if an investor is aware of the high yields on market discount bonds and sophisticated enough to perform calculations associated with the de minimis boundary, that investor may not be able to purchase mispriced market discount bonds if dealers never offer that investor any suitable transactions. Only the largest investors with good deal flow would have opportunity to take advantage of the de minimis premium. Many of the large investors able to see a large amount of deal flow are mutual funds who cannot take advantage of market discount deals.

If mutual funds generally avoid below de minimis trades, why do dealers and institutions not actively purchase more market discount bonds? In the Treasury bond market, Green and Ødegaard (1997) show dealers and large tax-exempt institutions dominate because they have lower trading costs and can take unlimited interest expenses and loss deductions. Their estimates of implicit tax rates in the Treasury market are zero. Municipal dealers have the same

[23] These funds were Franklin High Yield Tax-Free Income Fund, Eaton Vance National Municipal Fund, and RiverSource Tax Exempt High Income Fund.

[24] We also search the CRSP mutual fund database with all funds names including the words "tax-free," "tax-exempt," "municipal," or "TE." After removing equity funds and money market funds, we tabulate taxable and untaxable distributions from the largest 100 funds at December 2006. In 2006, 82 of the largest 100 funds did not distribute taxable dividends. For the other 18 funds most of the taxable dividends likely result from the funds receiving interest from private activity bonds rather than from the funds buying market discount bonds.

[25] SEC report http://sec.gov/news/studies/munireport2004.pdf

tax treatment as their Treasury counterparts. What prevents them from eliminating the market discount effect?

Dealers should still view the taxation of market discount bonds by individuals as a profitable trading opportunity. However, a major impediment is that dealers have little ability to hedge the purchase of market discount bonds on their books. Unlike Treasury bonds, shorting municipal bonds is very difficult because only tax-exempt authorities and institutions can pay tax-exempt interest. An investor lending a municipal bond to a dealer would receive a taxable dividend because that dividend is paid by the dealer, not a tax-exempt institution. Even if an active repo municipal market existed, it may be close to impossible to locate a suitable municipal bond as a hedge because of the sheer number of municipal securities. Shorting related interest rate securities, like Treasuries and corporate bonds, opens up potentially large basis risk. Another reason arbitrage may be limited is because the trading costs are much higher than Treasury markets.

6 Conclusion

The municipal bond market is a unique place to study the effects of individual tax rates. Although at issue, coupons and original issue discount of municipal bonds are exempt from federal income tax, in secondary markets an individual investor can purchase a municipal bond that is fully tax exempt, or have income or capital gains tax liabilities. The number of municipal bonds subject to tax is not small; in some years the proportion of municipal bonds transactions subject to income tax on market discount is above 30%. Municipal bonds bearing income tax are termed market discount bonds. Since individuals dominate in municipal bond markets, the cross section of municipal bonds trading with and without tax allows us to investigate how asset prices respond to individual tax rates.

We find taxation plays an important role in determining municipal bond prices. Not surprisingly, yields on market discount bonds are higher than yields on corresponding municipal bonds where all cashflows are exempt from tax to compensate individual investors for bearing the tax when purchasing market discount bonds. These effects are both highly statistically and economically significant. However, we find the yields on municipal bonds subject to tax are higher than can be explained by valuing the after-tax cashflows of the bond using the zero-coupon municipal yield curve. In particular, municipal bonds in A-grade credit classes bearing the highest tax burdens have after-tax yields approximately 25 basis points higher than the tax-exempt yield curve. The yields on short maturity market discount bonds are especially high.

Default risk and liquidity controls such as trading frequency and the spread between dealer and customer transactions do not explain the de minimis premium. We find the high yields on market discount bonds are concentrated in retail trades, which we define as transactions where the par value traded is less than $100,000. Individual retail investors may demand significantly more yield to hold market discount bonds as an inconvenience premium to pay income tax and to track the boundaries between income tax, capital gains, and no tax regions. The implicit tax rates implied by market discount bond prices are close to 100% using only retail trades and are above 70% for interdealer trades. This effect of taxes in the municipal bond market is much larger than previous estimates of implicit taxes, especially estimates of implicit tax rates in equity, Treasury, and corporate bond markets.

It is likely that the high yields on market discount bonds could persist for some time, if individuals continue to demand higher yields to purchase market discount bonds. In municipal bond markets, individuals dominate dealers and other financial institutions. Tax-exempt issuers are prevented by law from arbitraging the effect. Many large mutual funds shun purchasing market discount bonds because it would mean passing on income tax liabilities onto their clients. Finally, the hedging ability of dealers to offset long positions in market discount bonds is difficult because it is difficult to short municipal securities.

Appendix

A The De Minimis Boundary for OID Bonds

Computing market discount for an OID bond is more complicated than computing market discount for par or premium bonds. In the case of defining market discount on an OID bond, IRC § 1278(a)(2)(B) replaces the stated redemption price of the bond at maturity by the revised issue price. Thus, for a bond originally issued at discount, market discount is defined as the difference between the purchase price of a bond and the original issue price of the bond plus accreted OID. From issue date to maturity, OID accretes according to the constant yield accrual method (IRC § 1272-1(b)).[26] Since OID is original interest, it is not taxable for a municipal bond, and thus the accretion of OID as the bond matures is also not taxable. Market discount is created when an OID bond trades at a price below the bond's original issue price plus accreted OID. The original issue price plus accreted OID is termed the revised issue price (IRC § 1278(a)(4)) and can be computed as the present value of the remaining cashflows of the bond discounted at the bond's original issue yield.

As an example, consider Bond B, which is an OID bond originally issued with a 10-year maturity paying a 10% semi-annual coupon. Bond B was issued at a price of $88.5301 with a par value of 100. The semi-annual initial yield at issue of this bond is 12%. Figure A-1 illustrates the accreted OID of this bond in the convex solid line. At any point in time, the revised price of the bond is the value of the remaining payments of the bond discounted at its original 12% yield. Suppose that at year 2, an investor buys Bond B at a price of $84. With eight years remaining, the revised issue price of the bond is the discounted value of 16 coupons of $5 received at six-month intervals at a yield of 6% every six months. This revised issue price is $89.8941, which is equivalent to the original issue price of $88.5301 plus $1.3640 in accreted OID. The market discount at $t = 2$ is the difference between the revised issue price and the purchase price, which is $89.8941 - 84.0000 = \$5.8941$. This is shown in Figure A-1 as the solid vertical line at $t = 2$. If the investor holds Bond B to maturity, the market discount of $5.8941 is taxed at income tax rates when the bond matures at $t = 10$.

An alternative way to view the calculation of market discount is as follows. According to the OID schedule of Bond B from $t = 2$ to maturity at $t = 10$, Bond B should increase in price from $89.8941 to $100.0000. This increase of $100.0000 - 89.8941 = \$10.1059$ in the OID schedule is tax exempt. If Bond B is purchased at $t = 2$ for $84 and held to maturity, then a portion of the $100 - 84 = \$16$ gain is tax-free because some of this increase would have happened under the original accrual schedule. Only the gain in excess of the accreted OID is taxable. Thus, the taxable gain, which is considered income, is $100 - 84 - 10.1059 = \$5.8941$, and the income tax on $5.8941 is payable at maturity if the bond is held to maturity.

The de minimis boundary for Bond B is still defined relative to the stated redemption price of the bond. Thus, the de minimis boundary for Bond B at $t = 2$ is $89.8941 - 100 \times 0.0025 \times 8 = \$87.8941$. In Figure A-1, we graph the de minimis boundary in black dots below the accreted OID solid line. Any trade above the de minimis level is considered to have no market discount. Thus, if Bond B trades at $t = 2$ for a price greater than $87.8941, then the gain is considered to be de minimis and there is no market discount, but the gain may be subject to capital gains tax. For example, suppose that Bond B's price at $t = 2$ is $89. The investor would see a gain of $100 - 89 = \$11$ if Bond B is held to maturity, and $10.1059 of this gain is tax-free according to Bond B's accreted OID schedule. Thus, if held to maturity, the investor would pay capital gains tax on $100 - 89 - 10.1059 = \$0.8941$ when Bond B matures.

As a final case, suppose that Bond B is trading above its accreted OID schedule. For example, suppose that at $t = 2$, Bond B trades for $91, which is greater than the revised price of Bond B of $89.8941. An investor buying Bond B and holding it to maturity would see a gain of $100 - 91 = \$9$, but none of this is taxable since under the OID accretion schedule, the investor is entitled to a tax-free gain of $100 - 89.8941 = \$10.1059$ from $t = 2$ to $t = 10$. Thus, the accreted OID acts as a bound below which the OID bond becomes subject to tax, at least at capital gains rates. In addition, if the bond price is below de minimis, the market discount is taxed at income tax rates.

B Data Filters

From January 1995 to April 2007, the original MSRB database contains 70,611,395 individual transactions on 2,080,291 unique municipal bonds. Our final sample consists of 6,753,847 trades on 294,442 unique securities.

1. Tax Status

[26] For bonds issued prior to 27 September, 1985, straight line amortization (or the ratable accrual method) can be used. An investor would never rationally choose straight line amortization because the constant yield method leads to a slower accrual of the market discount.

We consider only bonds that are exempt from federal and state income taxes. Some tax-exempt municipal bonds issued by state and local governments to finance capital projects are classified as private activity bonds and are subject to the AMT. These bonds comprise 3.33% of all CUSIPS and we exclude these bonds from our analysis. We also limit our bond universe to bonds issued in one of the 50 states, and so we exclude bonds issued in Puerto Rico, the Virgin Islands, other territories of the U.S. such as American Samoa, the Canal Zone, and Guam. Bonds issued in these territories constitute less than 0.37% of all bonds.

2. High Credit Ratings

   To focus only on the tax implications of municipal bond trades, we focus on bonds of the highest credit classes. We take only bonds rated by S&P in the AAA, AA+, AA, AA-, A+, A, and A- categories. Many A Grade bonds obtain their credit rating because they are insured by a AAA-rated insurer. Slightly over 60% of all bonds are insured in the MSRB sample. Our S&P rating is collected at two points in time May 2006 for transactions before November 2005 and July 2007 for the remainder. The S&P rating is relevant at the time of maturity for bonds that have expired, or or for current outstanding bonds at those two dates.[27]

3. Straight Bonds

   We further limit our sample to include only bonds paying fixed coupon rates (94.1% of all bonds in the MSRB sample). We also take only straight bonds with no embedded option features, so all our bonds are fixed maturity paying fixed semi-annual coupons. Straight municipal bonds constitute 50.35% of all the bond universe and they generally have shorter maturities than bonds with embedded options. The average maturity at issue of straight bonds is 6.25 years while the average maturity at issue of option-embedded bonds is 15.75 years. The exclusion of option-embedded bonds is to facilitate our computation of yield-to-maturity and market discounts. Including bonds with callable or sinking bond features would entail numerically intensive option-adjusted spread computations involving binomial trees to correctly price the embedded options.

   After these first three requirements, we have transactions on 15,821,095 transactions on 604,118 unique municipal bonds.

4. Avoiding Newly Issued Bonds

   Green, Hollifield and Schürhoff (2007b) document significant underpricing in new municipal bond issues and interesting patterns in the aftermarket trading of these bonds between informed and uninformed customers. To avoid the effect of newly issued bonds, we exclude all the transactions that happened within 30 days of issuance. Transaction of newly issued bonds constitute about 25.9% of the 15.8 million transactions, reflecting the fact that municipal bonds transactions are concentrated during the period right after issuance. However, almost all of these transactions are not trades near de minimis because there is little movement in the yield curve over a 30 day period. We obtain nearly identical results when these trades are included in our sample.

5. Maturities Between One and Ten Years

   Transactions involving straight bonds with maturities longer than 10 years are scarce because most bonds with long maturities are issued with callable or sinking fund provisions. We use only transactions with maturity shorter than 10 years in our analysis. We also take bonds only with maturities greater than one year because long-term capital gains rates apply only to securities held longer than one year and there is no market discount for bonds with a maturity less than one year.

6. Removing Very Small Trades and Outliers

   To avoid the effect of extremely small trades, we exclude all transactions with par amounts traded less than $10,000. Finally, we take only transactions with prices between $80 and $130, and bonds with coupon rates from 1% to 20%.

[27] Unfortunately, neither S&P's Ratings IQuery or Moody's Ratings Delivery Service provide complete historical rating information for public finance issues. Both these companies only provide past ratings changes for bonds that they currently cover. That is, for bonds that have defaulted, matured, or are no longer covered by analysts at S&P or Moody's, past rating information on these individual bonds cannot be directly obtained from these companies. However, S&P and Moody's do publish aggregate historical information on the default experience and transitions between each credit class.

C Municipal Zero-Coupon Bond Curves

We follow the method of Nelson and Siegel (1987), so the zero-coupon yield for maturity n half-years, r_n, is given by:

$$r_n = \beta_0 + (\beta_1 + \beta_2)\frac{1 - \exp(-n/\tau)}{n/\tau} - \beta_2 \exp(-n/\tau), \tag{C-1}$$

which is determined by the parameters $\theta = \{\beta_0, \beta_1, \beta_2, \tau\}$. We estimate these parameters daily by fitting the Nelson-Siegel curve to only interdealer trades traded each business day. We use only interdealer trades as they are the trade type reported continuously over the sample (retail trades only are reported from August 1998) and represent a mid-point between sales to customers or purchases from customers made by dealers. Thus, the interdealer trades avoid any distortions from transaction spreads between dealers and customers. We also take only trades of bonds above revised price as these are fully tax-exempt municipal bonds. We also follow the same data screens as Appendix B, in particular removing any small trades below $10,000.

For each transaction price, we use the zero-coupon rate implied by equation (C-1) to discount the cashflows of the bonds. This gives us a fitted price for each bond, P^m, which we compute by:

$$P^m = \sum_{n=1}^{N} \frac{100 \times C/2}{(1 + r_n/2)^{n-1+w}} + \frac{100}{(1 + r_N/2)^{N-1+w}} - \frac{A}{360} 100 \times C, \tag{C-2}$$

where each cashflow is discounted by the zero-coupon yield. We denote bond prices computed using the fitted zero yield curve as "model-implied" (or "zero-implied") bond prices P^m with corresponding "model-implied" yields Y^m. By definition, Y^m is the yield implied by valuing the bond cashflows using the municipal zero-coupon yield curve on that trading day.

Each trading day, we estimate the parameters θ to minimize the distance between actual transaction prices and the predicted prices using the zero curve:

$$\min_{\theta} \sum_i (P_i^m - P_i)^2, \tag{C-3}$$

where P_i^m is the price of bond i computed using the Nelson-Siegel zero-coupon curve in equation (C-2) and P_i is the transaction price of bond i in data. We take the summation over all bonds traded each day in our sample.[28] We find that the differences between actual yields and zero-implied yields are very small, with the average difference between transaction yields and zero-implied yields over all bonds for our sample being 2 basis points.

[28] In our estimations of the zero curve, there are some bonds that trade more than once per day. For these bonds, there will be more than one trading price per day, but there is only one zero-curve implied model price.

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Table 1: Sample Summary Statistics

States of Issues with Most Transactions	CA 14.13%	NY 12.59%	FL 5.97%	TX 5.92%	NJ 4.47%	MI 3.55%	OH 3.56%	PA 3.53%
Type of Bond	General Obligation 45.22%		Revenue 48.97%		Other 5.81%			
Credit Rating	AAA 70.30%		AA+, AA, AA- 23.66%		A+,A,A- 5.81%			
Insurance	Yes 64.42%	No 35.58%						
Issue Price	Discount 32.23%		Par 11.62%		Premium 56.15%			
Trade Type	Dealer Sales to Customers 43.13%		Dealer Purchases from Customers 24.33%		Interdealer Trades 32.53%			
Coupon Rate	5% 2.25%	25% 4.20%	50% 5.00%	75% 5.25%	95% 6.10%			
Par Amount Traded	5% $10,000	25% $25,000	50% $50,000	75% $100,000	95% $820,000			
Maturity at Trade	1-2 Years 15.00%	2-5 Years 43.16%	5-10 Years 41.84%					

The table lists summary statistics of municipal bond transactions in our sample from the MSRB database where we take only straight bonds with maturities greater than one year and less than ten years at the time of transaction; bonds with a S&P rating of A- or higher; federal and state-exempt bonds not subject to the AMT; bonds issued in one of the 50 states; bonds trading at least 30 days after original issuance; and transactions of at least $10,000. There are 294,442 unique CUSIPs in our sample with a total of 6,753,847 transactions from January 1995 to April 2007. The table reports proportions of the 6,753,847 transactions falling into various categories. For the coupon rate and par amount traded, we report the coupon rates and traded par amounts at various percentiles of the distribution.

Table 2: Yield Spreads Across Different Price Bins

	(1) $\geq RP$	(2) $(DM\ RP)$	(3) $\leq DM$	(4) (3)–(1)
Panel A: Yield Spreads $Y - Y^m$				
All	-5.32	4.00	44.78	50.10
	(0.07)	(0.40)	(1.04)	(1.01)
# Trades per Day	1946	146	217	
Interdealer	-1.13	2.47	36.10	37.22
	(0.04)	(0.31)	(1.03)	(0.99)
# Trades per Day	610	62	85	
Panel B: After-Tax Yield Spreads $Y_\tau - Y^m$				
All	-5.32	1.46	20.63	25.94
	(0.07)	(0.40)	(1.01)	(0.98)
# Trades per Day	1946	146	217	
Interdealer	-1.13	-0.10	12.07	13.20
	(0.04)	(0.32)	(0.97)	(0.94)
# Trades per Day	610	62	85	

The table reports average yield spreads, $Y - Y^m$, in Panel A and after-tax yield spreads, $Y_\tau - Y^m$, in Panel B expressed in basis points for bonds partitioned into different price bins. The three bins are based on the revised price (RP) and de minimis boundaries (DM). Bin 1, $\geq RP$, contains all transactions with prices above RP where there is no tax liability involved. Bin 2, ($DM\ RP$), includes trades with prices between the de minimis boundary DM and revised price RP, which are subject to capital gains tax. Bin 3, $\leq DM$, contains all the trades with prices below DM, which are subject to income tax. In Panel A, the yield spread $Y - Y^m$ is defined as the actual transaction yield minus the model-implied yield (the yield implied by valuing the bond cashflows using the municipal zero-coupon yield curve on that trading day). In Panel B, the after-tax yield spread, $Y_\tau - Y^m$, is computed as the difference between the after-tax transaction yield and the model-implied yield. All spreads are reported in basis points. In computing averages, we only include days for which at least one trade takes place in all the bins. We also report the average number of trades per day for each bin. We report standard errors of the time-series averages in parentheses computed using the method of Fama and MacBeth (1973).

Table 3: Controls for Default Risk and Liquidity Risk

		(1) $\geq RP$	(2) $\leq DM$	(3) (2)–(1)
1	Short Maturity (1-2 Years) Bonds	-5.63	40.73	46.36
		(0.22)	(1.45)	(1.41)
	# Trades per Day	291	46	
2	Insured Bonds	-6.03	14.10	20.12
		(0.08)	(0.88)	(0.87)
	# Trades per Day	1246	148	
3	Only Bonds from the Same Series	-0.16	21.05	21.21
		(0.43)	(0.89)	(0.95)
	# Series per Day	17	12	
4	Only NY and CA Bonds	-6.88	17.00	23.89
		(0.17)	(1.02)	(1.02)
	# Trades per Day	542	54	
5	Exclude 95, 99-00, 06-07	-5.47	31.43	36.89
		(0.09)	(1.46)	(1.45)
	# Trades per Day	1919	82	

The table reports average after-tax yield spreads (the difference between the after-tax transaction yield, Y_τ, and the model-implied yield, Y^m) in basis points for bonds of different types partitioned into different price bins, representing default or liquidity risk controls. We take only bonds between 1-2 year maturities, only insured bonds, only bonds issued in New York and California, above and below de minimis bonds issued in the same series, and we exclude the years 1995, 1999-2000, and 2006-2007 from the sample. In each case, we partition trades into two bins based on the revised price (RP) and de minimis boundaries (DM). Bin 1, $\geq RP$, contains all transactions with prices above RP where there is no tax liability involved. Bin 2, $\leq DM$, contains all the trades with prices below DM, which are subject to income tax. In computing averages, we only include days for which at least one trade takes place in both bins. We also report the average number of trades per day for each bin. We report standard errors in parentheses computed using the method of Fama and MacBeth (1973).

Table 4: Bond Liquidity Characteristics

		(1) $\geq RP$	(2) $\leq DM$	(3) (2)–(1)
Panel A: Trading Frequency				
Q1	Least Frequently Traded	2.02	15.41	13.39
		(0.16)	(1.06)	(1.05)
	# Trades per Day	61	9	
Q2		-1.62	17.07	18.68
		(0.12)	(1.04)	(1.03)
	# Trades per Day	141	23	
Q3		-4.52	16.09	20.61
		(0.10)	(0.97)	(0.96)
	# Trades per Day	312	52	
Q4	Most Frequently Traded	-6.71	17.51	24.22
		(0.07)	(1.02)	(1.01)
	# Trades per Day	1435	144	
Panel B: Transaction Spreads				
Q1	Lowest Transaction Spreads	-5.83	14.03	19.85
		(0.09)	(0.92)	(0.91)
	# Trades per Day	595	65	
Q2		-7.52	2.39	9.91
		(0.09)	(0.97)	(0.98)
	# Trades per Day	662	38	
Q3		-7.01	6.53	13.54
		(0.08)	(0.86)	(0.86)
	# Trades per Day	486	77	
Q4	Highest Transaction Spreads	-0.09	28.32	28.41
		(0.11)	(1.27)	(1.24)
	# Trades per Day	279	59	

Note to Table 4

The table reports average after-tax yield spreads (the difference between the after-tax transaction yield, Y_τ, and the model-implied yield, Y^m) in basis points for bonds sorted by trading frequency and by transaction spreads. In Panel A, we record the number of trades for all bonds in our sample and rank them into quartiles by trading frequency, defined as the number of trades of each bond per annum. Then, for the bonds in each quartile we divide transactions into above revised price and below de minimis bins and compute after-tax yield spreads, $Y_\tau - Y^m$, for each quartile. The table reports the average after-tax yield spread of each quartile and bin over the sample. Panel B defines the transaction spread as the difference between dealer sales to customers and dealer purchases from customers. We compute transactions spreads for each day for each bond, and then we average the transaction spreads over time. Using each bond's average transaction spread over the sample, we rank the bonds into quartiles and divide the transactions into the two above revised price and below de minimis price bins. In computing averages, we only include days for which at least one trade takes place in both bins. We also report the average number of trades per day for each bin. We report standard errors in parentheses computed using the method of Fama and MacBeth (1973).

Table 5: Transactions Sorted by Clientele Characteristics

	(1) $\geq RP$	(2) $\leq DM$	(3) (2)–(1)
Panel A: Transaction Size			
Very Small Trades	3.00	28.38	25.37
	(0.13)	(1.04)	(1.04)
# Trades per Day	744	102	
Retail Trades	-2.80	22.48	25.28
	(0.09)	(1.00)	(1.00)
# Trades per Day	1485	189	
Institutional Trades	-11.77	-6.53	5.23
	(0.10)	(1.04)	(1.05)
# Trades per Day	476	35	
Panel B: Bank Qualified Issues			
Non-Bank Qualified	-5.29	21.68	26.98
	(0.07)	(1.04)	(1.03)
# Trades per Day	1880	204	
Bank Qualified	-5.58	-0.53	5.06
	(0.16)	(0.79)	(0.79)
# Trades per Day	67	17	

The table reports average after-tax yield spreads (the difference between the after-tax transaction yield, Y_τ, and the model-implied yield, Y^m) in basis points for bond transactions sorted by various clientele characteristics. A transaction is defined as a retail trade if the par amount traded is below $100,000 and defined as a institutional trade otherwise. Very small trades are those with par amount traded less than $25,000. In each case, we partition trades into two bins based on the revised price (RP) and de minimis boundaries (DM). Bin 1, $\geq RP$, contains all transactions with prices above RP where there is no tax liability involved. Bin 2, $\leq DM$, contains all the trades with prices below DM, which are subject to income tax. In computing averages, we only include days for which at least one trade takes place in both bins. We also report the average number of trades per day for each bin. We report standard errors in parentheses computed using the method of Fama and MacBeth (1973).

Table 6: Transactions Spreads

		$Y_\tau - Y^m$		$Y - Y^m$	
	(1) $\geq RP$	(2) $\leq DM$	(3) (2)–(1)	(4) $\leq DM$	(5) (4)-(1)
Retail Investor Buying from Dealer	-18.99	-3.26	15.73		
	(0.12)	(1.51)	(1.49)		
# Trades per Day	1006	98			
Retail Investor Selling to Dealer	18.74	42.03	23.30		
	(0.13)	(1.19)	(1.17)		
# Trades per Day	450	87			
Institution Buying from Dealer	-18.03	-23.11	-5.08	1.41	19.44
	(0.15)	(1.24)	(1.24)	(1.18)	(1.47)
# Trades per Day	240	10		10	
Institution Selling to Dealer	-5.62	1.48	7.09	28.14	33.76
	(0.13)	(1.18)	(1.19)	(1.38)	(1.38)
# Trades per Day	168	16		16	

The table reports the average after-tax yield spread, $Y_\tau - Y^m$, and the raw yield spread, $Y - Y^m$, in basis points for bond transactions between customers and dealers for various price bins. We partition trades into two bins based on the revised price (RP) and de minimis boundaries (DM). The first bin ($\geq RP$) contains all transactions with prices above RP where there is no tax liability involved. The second bin ($\leq DM$) contains all trades with prices below DM, which are subject to income tax. Panel A averages the yield spreads over the whole sample. In computing averages, we only include days for which at least one trade takes place in both bins. We also report the average number of trades per day for each bin. We report standard errors in parentheses computed using the method of Fama and MacBeth (1973).

Table 7: Events when Bonds Cross Out of the Income Tax Region

	ΔY_0	$\Delta Y_{0,\tau}$	ΔY_0^m	# of Trades
Panel A: All Trades				
Crossing Down	53.58	36.15	0.36	11522
	(0.43)	(0.35)	(0.06)	
Crossing Up	-58.86	-40.19	-0.15	57166
	(0.17)	(0.14)	(0.02)	
Crossing Down on the Same Day	39.03	24.49	0.00	28928
	(0.24)	(0.19)	–	
Crossing Up on the Same Day	-45.70	-29.86	0.00	45525
	(0.19)	(0.15)	–	
Panel B: Interdealer Trades				
Crossing Down	28.34	16.27	0.96	1848
	(0.89)	(0.69)	(0.14)	
Crossing Up	-33.84	-20.73	-0.65	6229
	(0.42)	(0.34)	(0.07)	
Crossing Down on the Same Day	29.66	17.10	0.00	7327
	(0.34)	(0.26)	–	
Crossing Up on the Same Day	-28.54	-16.28	0.00	6138
	(0.45)	(0.35)	–	

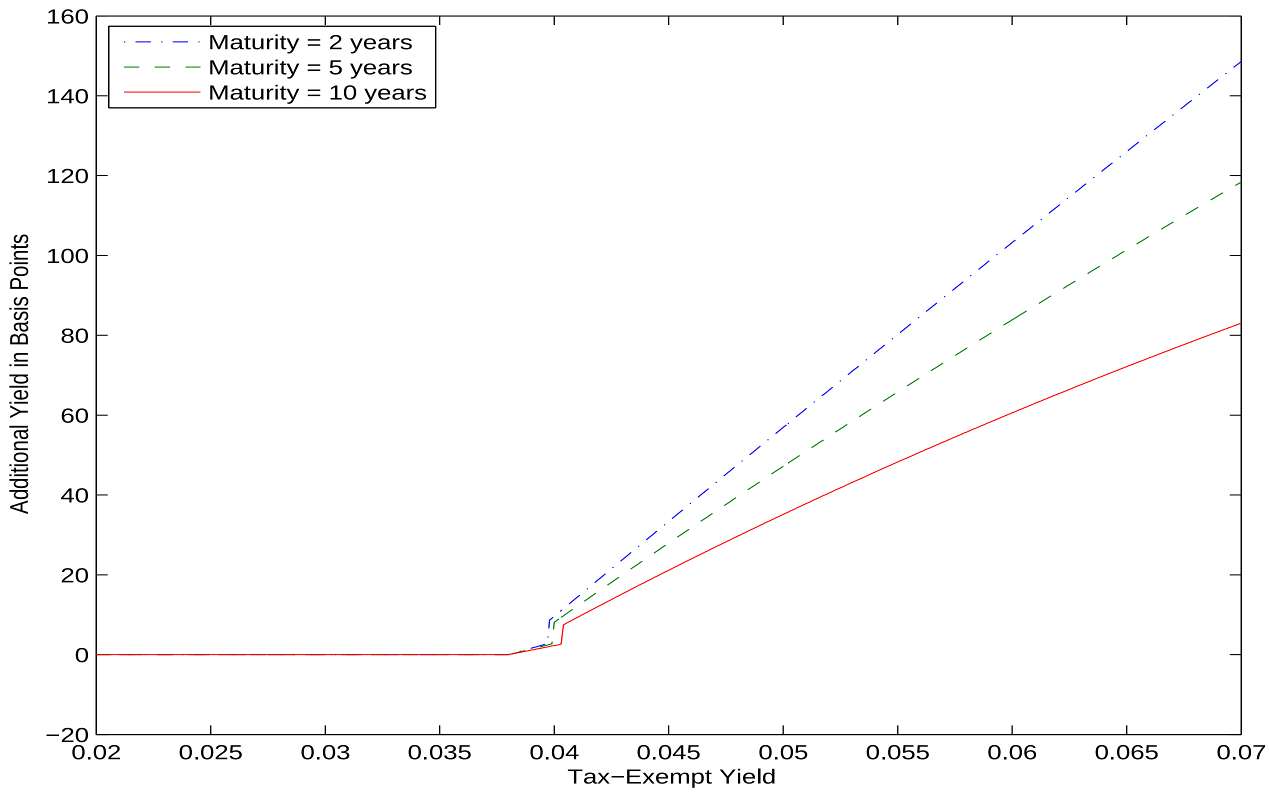
The table lists averages of yields and yield changes for events where bonds cross into and out of the income tax region below the de minimis boundary (DM). We denote the event time of the transaction crossing the DM boundary as time zero. The yield of the prior trade is denoted as Y_{last}. We report the change in yield $\Delta Y_0 = Y_0 - Y_{last}$. The change in the after-tax yields between the event trade and the prior trade is reported as $\Delta Y_{0,\tau} = Y_{0,\tau} - Y_{last,\tau}$. We also report the change in the model-implied yields, $\Delta Y_0^m = Y_0^m - Y_{last}^m$, which is zero for intra-day trades. All the columns with yield changes are expressed in basis points. Panel A includes all trades while Panel B includes only interdealer trades. For the rows labeled "Crossing Down" and "Crossing Up," we track all events where bonds move down or up, respectively, across DM with the last trade happening within the previous five days (but not the same day as the cross). For the rows labeled "Crossing Down on the Same Day" and "Crossing Up on the Same Day," the last trade occurs on the same trading day as the cross. We take only trades where the de minimis boundary does not change across the last trade to the event trade, thus the cross is due to the change in bond prices, not due to a shifting de minimis boundary. We report the number of trades in each category in the last column and report standard errors in parentheses.

Table 8: Implicit Income Tax Rates

	Full Sample		Pre-2003		Post-2003	
Trade Type	Income Tax Rate	T-stat	Income Tax Rate	T-stat	Income Tax Rate	T-stat
Panel A: Direct Estimates						
All	0.88	39.7	1.08	38.1	0.51	15.8
Interdealer	0.71	32.2	0.87	33.6	0.40	10.5
Retail	0.96	43.0	1.18	42.0	0.54	16.7
Institutional	0.24	12.5	0.26	11.2	0.20	6.15
Panel B: OLS Estimation						
All	0.70	9.25	0.79	7.05	0.53	9.20
Interdealer	0.80	9.07	0.96	6.97	0.52	18.8
Retail	1.29	7.74	1.37	1.45	0.89	6.04
Institutional	0.65	30.4	0.74	22.8	0.53	21.8

The table reports estimates of implicit tax rates from the prices of market discount bonds. In Panel A, we invert the estimated tax rate τ_I directly from equation (4). We compute estimates of τ_I using all market discount bonds on each trading day and then report the mean and t-statistics of the daily time-series estimates. In Panel B, we treat τ_I as an OLS coefficient and add fixed year effects and other controls to the regression implied from equation (4). We use fixed effects for each year, dummies for different bond types (general obligation or revenue), dummies for different original issue prices (par or premium), and dummies for the eight most traded states (CA, NY, FL, TX, NJ, MI, OH, and PA). Panel B reports time-series estimates of the OLS coefficient τ_I following Fama and MacBeth (1973). Retail trades refer to trades with par value smaller than $100,000.

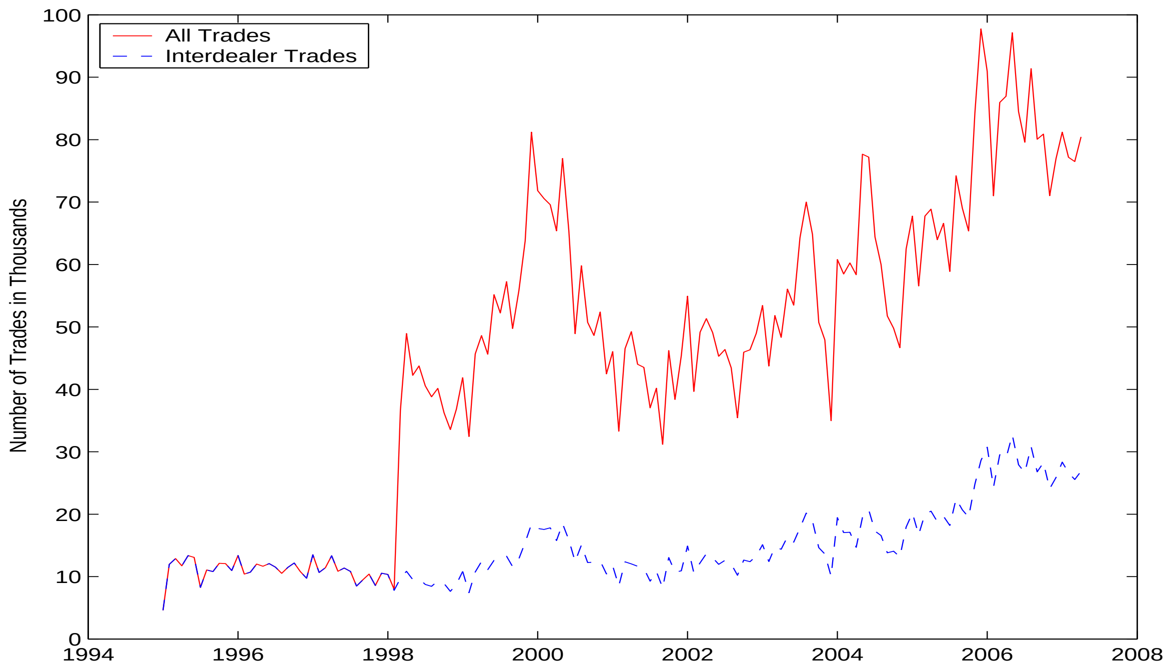
Figure 1: Additional Yields Required by Below De Minimis Bonds



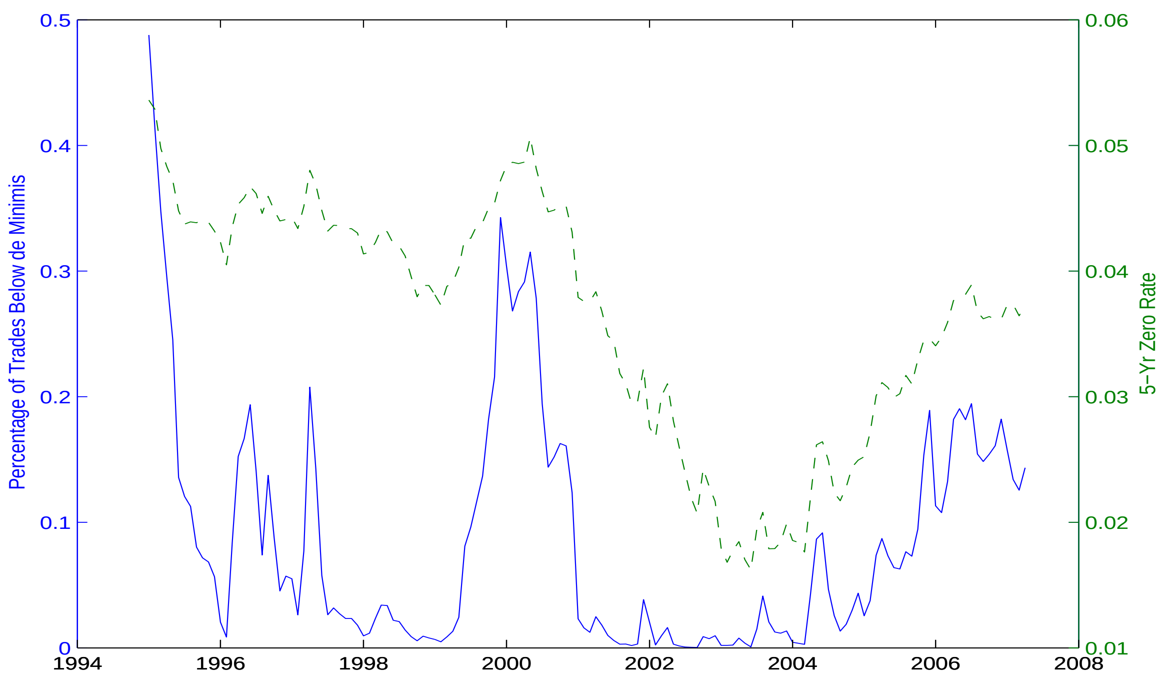
We consider par bonds of different maturities paying semi-annual coupons of 3.8%. For a given tax-exempt yield y (on the x-axis), we compute the additional yield $\tilde{y} - y$ (in basis points on the y-axis) above the tax-exempt yield required to obtain the same IRR y on the after-tax cashflows as the tax-exempt yield y. We assume that the income and capital gains tax rates are $\tau_I = 0.35$ and $\tau_C = 0.15$, respectively.

Figure 2: Trades of Municipal Bonds

Number of Trades Each Month

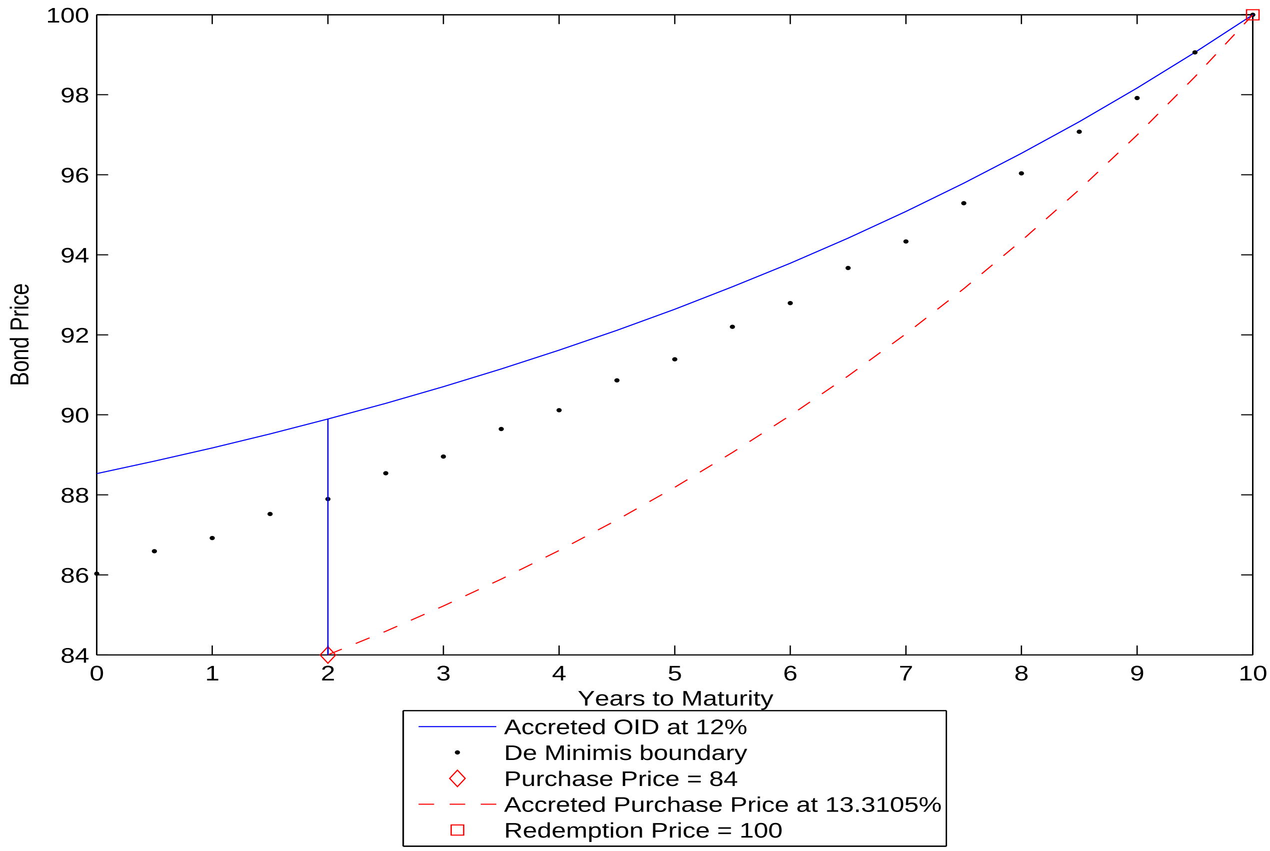


Proportion of Bonds Trading Below De Minimis



In the top panel, we plot the number of trades each month in our sample, totalling 6,753,847 over January 1995 to April 2007. The bottom panel plots the proportion of trades below the de minimis boundary each month, as a fraction of the total amount of trades in that month, in the solid line. In the dashed line, we plot the 5-year zero-coupon municipal bond yield, which is computed using the method detailed in Appendix C.

Figure A-1: Illustration of Market Discount for an OID Bond



Consider an OID bond originally issued with a 10-year maturity paying a 10% semi-annual coupon. At $t = 0$, this bond is issued at a price of $88.5301 with a par value of 100. The semi-annual initial yield at issue of this bond is 12%. The solid line plots the accreted OID of this bond, also called the revised price of the bond, which is the value of the remaining payments of the bond discounted at its original 12% yield. At time $t = 2$, an investor purchases this bond in the secondary market at a price of $84. At $t = 2$, the revised issue price of the bond is $89.8941, which is equivalent to the original issue price of $88.5301 plus $1.3640 in accreted OID. The market discount at $t = 2$ is the difference between the revised issue price and the purchase price, which is $89.8941 - 84.0000 = \$5.8941$ and graphed as a solid vertical line at $t = 2$. The plot also shows the accreted purchase price of $84 from $t = 2$ to the redemption value of $100 at $t = 10$, representing accretion at a yield of 13.3105%, in the dashed line and the de minimis boundary in black dots.