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ABSTRACT

The value of pension promises already made by US state governments will grow to approximately $7.9 trillion in 15 years. We study investment strategies of state pension plans and estimate the distribution of future funding outcomes. We conservatively predict a 50% chance of aggregate underfunding greater than $750 billion and a 25% chance of at least $1.75 trillion (in 2005 dollars). Adjusting for risk, the true intergenerational transfer is substantially larger. Insuring both taxpayers against funding deficits and plan participants against benefit reductions would cost almost $2 trillion today, even though governments portray state pensions as almost fully funded.

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The true extent of public pension underfunding has been obscured by governmental accounting rules, which allow pension liabilities to be discounted at expected rates of return on pension assets. This paper takes stock of the distribution and magnitude of prospective underfunding in state pension plans given current state pension funding, asset allocation and liability estimates. This analysis demonstrates the large burden that current public pension policy places on future generations. In particular, we show that while the plans appear almost fully funded under government-chosen discount rates, there is a large probability of significant shortfalls in the future. The shortfalls are likely to occur if the economy performs poorly, and so are particularly costly to future generations. The cost of fully insuring future taxpayers and plan participants against these potential shortfalls would approach $2 trillion.

Using a dataset assembled from state government reports, we examine the actual asset allocation of the universe of state-sponsored defined benefit (DB) plans. Given the estimated variance-covariance matrix of state pension investments, we calculate the distribution of intergenerational transfers that will occur under current funding and investment policy. If households form their own financial portfolios taking government pension investment policy into account, then how the government invests pension assets does not matter for intergenerational risk transfers. The extent to which the government taxes current generations to fund pensions, as opposed to waiting to tax future generations, would completely determine the extent of intergenerational risk transfer. However, if households do not undo the government's investment strategy, then investing in risky assets subjects future generations to substantially more risk in the value of their after tax wealth.

If current investment strategies are maintained for 15 years, we estimate (conservatively) that there is a 50% chance of an aggregate underfunding in excess of $750 billion, a 25% chance of an underfunding of at least $1.74 trillion, and a 10% chance of an underfunding in excess of $2.48 trillion (all in 2005 dollars, and under lognormal assumptions regarding the evolution of assets and liabilities). Given that poor returns tend to occur in high marginal utility states, however, these figures understate the pain that would be associated with underfunding. On a risk-adjusted basis, there is a 50% chance of an aggregate underfunding in excess of $2.66 trillion, a 25% chance of an underfunding of at least $3.25 trillion, and a 10% chance of an underfunding in excess of $3.74 trillion (all in 2005 dollars). Furthermore, in the event that public pension funds are in surplus, excess assets are unlikely to be entirely returned to taxpayers, increasing the net expected burden. In addition, we find that the states with more general obligation (GO) debt as a share of revenue also have more underfunding as a share of revenue, although their total pension liabilities are not any larger.

States back pension liabilities with a mix of assets that include stock, bonds, cash, private equity, real estate, and hedge fund exposure. The typical investment strategies in conjunction with accounting rules make the pension funding situation look much better than it actually is. To illustrate this irony, we set forth a "Modest Proposal" in the spirit of Jonathan Swift, whereby state pension funds could be shown to be in substantial surplus if only the state government entities would invest pension assets in a ten-beta levered S&P500 Exchange Traded Fund (ETF). In that case, the "surplus" that would appear to emerge would justify withdrawals from public pension funds sufficient to pay down all outstanding state bonds and pay a $5,000 dividend to every American citizen.

Under any plausible discounting assumptions that reflect the true present value of state pension promises, the underfund in state pension plans is larger than the total magnitude of outstanding state bonds, which was $798 billion as of the end of fiscal year 2005. Under the assumption that state pension promises are riskless, so that accrued benefits cannot be abrogated, the underfunding would amount to $1.9 trillion. If the risk of state pension liabilities is roughly captured by municipal borrowing rates excluding the tax benefit, we find that total state underfunding amounted to $862 billion as of late 2005. This number is smaller because the calculation ascribes a significant probability to a default on pension promises by US states. Given the protections afforded public pension liabilities by state constitutions, this latter measure is almost certainly too conservative.

Furthermore, all of these figures assume that all pension benefits that will be accrued in the future will be fully funded using appropriate discount rates, which we demonstrate has not been the case in the recent past. They also ignore other postretirement employee benefits (OPEBs) which total $380 billion in present value under GASB 45 disclosures. Therefore, our calculations certainly understate the extent of the funding crisis.

The crisis in public pension funds dwarves taxpayers' exposure to underfunded corporate pension plans. As of September 2007, the net financial position of the Pension Benefit Guarantee Corporation (PBGC) was an underfund of $14 billion, with additional "reasonably possible exposure" of $66 billion. The roughly $2 trillion in project benefit obligation (PBO) pension liabilities of US publicly traded corporations were almost fully funded at the point in time we study, though funding may have deteriorated in late 2007 and early 2008. Furthermore, the PBGC is only liable for underfunding on an accumulated benefit obligation basis (ABO), on which basis corporate plans are actually overfunded. The magnitude of the crisis in state pension plans is much larger.

This paper proceeds as follows. Section I calculates the distribution of underfunding of US state pension plans, both relative to state pension liabilities and relative to total state non-pension debt under current reporting standards. Section II analyzes the merits of several possible alternative discount rates and calculates the true distribution of the funding position under these alternatives. Section III describes state DB investment policy. Section IV describes a "Modest Proposal" by which these state pension funds could be shown to have a substantial surplus simply by reallocating pension assets. Section V presents evidence on the distribution of intergenerational transfers inherent in this system. Section VI concludes.

I. The Extent of Underfunding Under Current Reporting

We collected data on the largest DB pension funds sponsored by US state governments. In a typical DB pension plan, an employer pledges an annual pension payment of an amount that is a function of the employee's final salary and years of employment. To assemble the list of plans, we began with data from the US Census of Governments, published by the US Census Department. We listed all plans with more than $1 billion of assets, including only those plans sponsored by state governments. There were 112 such plans at the end of 2005.[1]

While the US corporate sector has moved away from DB plans and towards defined contribution (DC) arrangements such as 401(k) plans, this movement has been very limited in the public sector. A March 2008 Bureau of Labor Statistics (BLS) survey indicates that 80% of state and local government workers are enrolled in a DB plan and under 20% are enrolled in a DC plan (BLS (2008)). The GAO in late 2007 reported that only Alaska and Michigan offered employees a DC plan but not a DB plan, and Indiana and Oregon offered a hybrid plan; all other states offered only DB plans (GAO (2007))[2].

The Census of Governments does not contain any data on DB pension liabilities. We therefore examined the Comprehensive Annual Financial Report (CAFR) for each pension plan and collected total actuarial liabilities for each pension plan, along with the discount rate used by state actuaries to calculate these liabilities. We also collected the total plan assets for these plans as stated in the CAFR. This measure of total plan assets is on an actuarial basis, which allows for smoothing of investment returns over a certain number of years according to actuarial discretion. The US Census Department data are intended to measure assets at their market value.

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[1] As of the time of the study, 2006 data were not yet fully available for the universe of state plans. However, preliminary indications suggest that the 2006 analysis will be very similar to the 2005 analysis.

[2] Nebraska offered a cash balance plan, a type of DB plan in which the value of the plan is presented to employees as a cash balance and the trajectory of benefit accrual with respect to tenure is more concave than in a traditional DB plan.

The statement of liabilities in the CAFRs is an accrued actuarial liability (AAL). In calculating an AAL, state actuaries start with a calculation of the total present value of benefits they expect employees to receive under the plan. To complete this first step, they need to choose a discount rate with which to discount future payments. Furthermore, some of those benefits, have not yet been accrued. Actuaries therefore have to determine the allocation of the present value of future liabilities to past, current, and future years. The flow measure of accruing benefits is called the normal cost, which is the share of the present value of future benefits assigned to a given year. Another way of explaining the AAL is that it is the "portion of the present value of benefits which is not covered by future normal costs" (Civic Federation (2007)).

We begin by taking the AAL calculations at face value, and then consider how different discount rates and normal cost calculations might affect them. We were able to collect liability data on 108 of the 112 plans in the initial list. These plans had an actuarial asset value as of the end of 2005 of $2.164 trillion, compared to $2.175 trillion in the asset measure provided by the Census of Governments. The similarity of the asset measures suggests that the smoothing in actuarial assets is not critical for our cross-sectional analysis. Appendix Table I lists this data.

Adding pension liabilities in the same fashion, we find total liabilities from US state sponsored pension plans of $2.476 trillion as of the end of 2005. The discount rates used by the state plans to calculate these liabilities averaged 7.97% with a standard deviation of 0.39%, and had a median of 8%. The modal discount rate was 8%, with 55 entities using this rate. The minimum rate was 6.5% and the maximum was 8.75%.[3]

Taking the pension liability calculations from the state plans at face value, the state public sector faces an aggregate deficit in pension funding of $312 billion. Similar calculations have also been performed by the Pew Charitable Trust (2007), National Association of Retirement Administrators (2003-2007), and Merrill Lynch Research (2007). Our numbers are similar to those provided in their reports.

The $312 billion pension gap represents an underfunding of 12.6% of state government assets. The top graph of Figure 1 shows the distribution of underfunding relative to pension liabilities across US state-sponsored plans on an unweighted basis and weighted by pension liabilities. The vast majority of plans are underfunded, as is the vast share of total liabilities.

It is instructive to compare this number to total state debt outstanding excluding pensions, which we collected from the US Census Bureau (2005 Survey of State Government Finances). Total state nonpension debt is $798 billion, so that even taking the pension liability calculations from the state plans at face value reveals an underfunding equivalent to 39% of state debt. In

[3] Giertz and Papke (2007) find some evidence that these assumptions are manipulated to reduce funding pressure.

other words, if one treats unfunded pension liabilities as state debt, total state debt was 39% higher at the end of 2005 than if one ignores unfunded pension liabilities as is typically done. The bottom graph of Figure 1 shows the distribution of underfunding relative to total nonpension state debt across US state-sponsored pension plans. For 21 states, adding the pension funding gap at stated levels increases total state debt by at least 60%. For 7 states, adding the pension funding gap at stated levels increases total state debt by more than 100%.

II. Understanding the True Funding Position

A. Possible Discount Rates

As several studies have pointed out previously (see for example Barclays Global Investors (2004)), the 8% average discount rate used by state-sponsored pension plans is almost certainly too high. The discount rate assumptions come from the Government Accounting Standards Board (GASB) ruling 25, and Actuarial Standards of Practice (ASOP) item 27, which together stipulate that the actuarial value of pension liabilities should depend on the assumed return on pension plan assets. Most finance academics and practitioners view this rule as misguided (see Gold (2002) and Bader and Gold (2004)). Financial liabilities should be discounted using discount rates that are specific to the factor (or market) risk inherent in the liabilities.[4] The way the liabilities are funded is irrelevant to their value.

Note that the government treatment differs substantially from the treatment of corporate pension plan liabilities, which under FASB must be discounted using a blended corporate bond yield and for funding purposes must be discounted using Treasury yields (see Rauh (2006) and Bergstresser, Desai, and Rauh (2006) for more information).

States present AAL calculations and disclose the discount rates used to discount the pension payments promised to beneficiaries. They do not disclose the duration of the liabilities. Some practitioner studies (see for example Barclays Global Investors (2004)) assume 14-year durations for pension liabilities based on Waring (2004a, 2004b). Others argue that the true duration is higher because of cost-of-living adjustments (COLAs) in the public plans. For lack of better information, we consider the liabilities of state pension plans to have a 15 year duration and consider what their state will be in 15 years. In effect, we therefore are assuming that states will have to make up shortfalls in pension funding after 15 years. We also examine the robustness of our calculations to 20-year durations. In both cases we assume the convexity of the liabilities to the discount rate is as low as possible given the duration, so we underestimate the true sensitivity

[4] This point has been made in the context of corporate pension plans by Petersen (1996) and Ippolito (2002).

of the liabilities to falling discount rates.[5] That is, we ignore the fact that the duration of the liabilities increases as the discount rate decreases, which makes our estimates conservative.

If we presume that the states accurately forecast liabilities, then we know their expected value in the future. We need only scale up the actuarial liabilities at the stated discount rate. As shown above, total liabilities from US state sponsored pension plans were \$2.476 trillion as of the end of 2005. If the duration is 15 years, the future liability is $(\$2.5 \text{ trillion})*(1.08)^{15} = \$7.9$ trillion in 15 years time. Again, this understates the true future liability, both because the duration we employ is too short, and because we have made the most conservative possible assumption regarding the liabilities' convexity given the duration.

Recall that \$2.5 trillion is the total of state level liabilities in the AAL as of the end of 2005. This liability includes projections for salary growth, but does not consider the projected benefit accruals from additional years of service or new workers. We currently consider only the right discount rate for these liabilities that have been accrued up to the present, i.e. liabilities represented in the AAL calculation that states are required to provide. In other words, we assume that all future accrued liabilities will be fully funded, an assumption that is contradicted by the fact that many states are funding less than 100% of newly accrued liabilities.

The right discount rate for the AAL pension promise depends on the risk structure of the liabilities and its covariance with pricing factors. If state pension promises were truly riskless, meaning that pensioners would get the expected benefit in all states of the world, then one would want to use a risk-free rate.

Our first main candidate for a discount rate is the 15-year strip Treasury rate as of the end of 2005. This serves as a useful if imperfect benchmark for a risk-free rate. The imperfection comes from the fact that long-term Treasury bond yields may contain a risk premium, either because of default risk or in order to entice investors with shorter investment horizons to hold them instead of shorter-maturity government debt, so in fact the actual yield on long-term Treasury bonds is an upper bound on the risk-free rate.

To the extent that there is a risk that retirees will not get the expected benefit, or that the assumptions inherent in projecting the pension payments may be incorrect, then one might want to use a discount rate that is higher than the risk-free rate. However, the risk-free rate would still be the right rate to use if these risks are uncorrelated with pricing factors, which in the context of the Capital Asset Pricing Model (CAPM) means uncorrelated with the market portfolio. In other words, the Treasury rate is approximately the appropriate discount rate if the liabilities are risk-

[5] Essentially, we are assuming the entire liability comes due with a maturity equal to the duration. The single payment "bullet" is less sensitive to falling discount rates than any other distribution of liabilities with the same duration.

free or if their risk is uncorrelated with the market portfolio. We proceed to consider discount rates in the context of the CAPM single-factor pricing model.

If government pension payments are likely to be higher in a bull market and lower in a bear market, then a higher discount rate than the risk-free rate is warranted. If government pension payments are likely to be higher in the event of a market downturn, then a lower discount rate than the risk-free rate is warranted.

There are two primary risk factors that affect the evolution of the liabilities that have been accrued until now and that may be correlated with market pricing factors: i.) default risk; and ii.) the risk that salary increases will be higher or lower than expected. There are other risk factors in the liabilities, e.g. mortality and retirement assumptions, but these are unlikely to be correlated with the market. There is also the issue of the magnitude of future benefit accruals (due to additional years of service), but again, we begin under the assumption that states will fully fund all future accrued benefits and consider just the liabilities accrued until now.

We start by considering discount rates that assume possible positive covariance of default risk with the market, but zero covariance of salary increases with the market.[6] We then consider the likely extent of salary increase covariance and how it would affect our results. One hint as to the appropriate discount rate that reflects default risk comes from state-specific general obligation credit ratings, to which yields on states' municipal general-obligation bonds are closely related. If government pension liabilities have the same priority as other government debt, then the discount rate should be related to municipal bond yields.

This leads us to consider another discount rate candidate: the yield on state general obligation municipal bonds with a similar duration to the pension liabilities, grossed up to eliminate the tax preference given to borrowing. In other words, for state *i* at time *t*, the tax-corrected yield would be roughly $r_{it} / (1-\tau_b)$, where τ_b is a personal investor's marginal tax rate on interest income (35%). We will refer to this rate as the grossed-up muni rate.

The grossed-up muni rate rate is appropriate if the state is equally likely to default on its pension obligations as it is to default on its other debt, but investors cannot receive tax preference for bond issues in which the proceeds are used to fund pensions. State constitutions often build in protections for government-sponsored pensions (National Conference on Public Employee Retirement Systems (2007)), suggesting that their priority may even be higher than that of general government debt and that the default probabilities of state pension obligations are at least as low

[6] Inkmann and Blake (2007) derive optimal asset allocation as a function of initial funding ratios when the probability of default covaries with the asset allocation. Maurer, Mitchell, and Rogalla (2008) consider optimal asset allocation in the context of the German public sector pension reform.

as those of state general obligation debt. Furthermore, attempts by states to issue Pension Obligation Bonds (POBs), which raise financing at the tax-preferred municipal bond rate in order to fund state pension funds has lead to IRS rulings deeming these instruments "arbitrage bonds" and disallowing tax deductibility (Ellement (2003)).

Despite these rulings, states may still be able to exploit the arbitrage to some extent by increasing their issuance of general obligation debt that they classify as being used to fund operations but which in fact allows them more financial slack. With this financial slack they could increase their pension contributions, perhaps not with an obvious lump sum, but gradually over time. To the extent that states can perform this arbitrage, the grossed-up muni rate is an upper bound on the true risk of the pension liabilities.

To value the liabilities in today's terms, a higher discount rate is required due to the fact that the plans might default. The muni rate grossed up by the tax rate represents the risk that states will default on their general obligation bonds. Assuming the CAPM is the correct model, that rate is higher than the risk-free rate because there is a correlation of default risk on general obligation bonds with the equity market. In the absence of correlation between the magnitude of the pension promise and the stock market, the rate for pensions should be similar.

Now we relax the assumption of zero covariance in salary increases. What if the evolution of pension liabilities is correlated with the market over long horizons? Recall that by examining accrued liabilities, we are not considering variation in the size of the public sector, just about the current promises to current workers. However, a correlation between real wages and the market could generate positive covariance between pension liabilities and the market through variation in salary growth, a point made by Black (1989) and others. This issue has been analyzed in a general context with closed form solutions by Sundaresan and Zapatero (1997). Empirically, Benzoni et al (2006) show that while the correlation between earnings growth and stock returns are negligible on a short horizon, the correlation is higher on a longer positive horizon. Lucas and Zeldes (2006) discuss these effects in the context of corporate pension plans with a model in which the value of human capital and the value of the stock market have positive covariance. In data simulated from their model of corporate outcomes, the one-year correlation between earnings growth and stock returns is zero, the three-year correlation is 0.11, and the five-year correlation is 0.22.

The extent to which the grossed-up muni rate understates the discount rate as a result of this correlation depends on the volatility of wage growth and market returns, as well as their correlation. The loading of liabilities on the market is

$$\beta_L = \frac{\sigma_{L,M}}{\sigma_M^2} = \frac{\rho_{L,M}\sigma_L\sigma_M}{\sigma_M^2}$$

where σ_L is the volatility (standard deviation) of liabilities, σ_M is the volatility (standard deviation) of the market portfolio, $\sigma_{L,M}$ is the covariance between the two, and $\rho_{L,M}$ is the correlation coefficient between the two. The discount rate would have to be adjusted upwards by β*(market premium), where for market premium we use lower and upper bounds of 6.5% and 8% respectively. The volatility of liabilities (σ_L) relevant for this equation is the volatility of currently accrued pension liabilities as the future unfolds. It includes variation due to possible default and salary growth, but again not due to future service accruals.

We follow two procedures for investigating the short- and long-horizon covariance between the market and public pension liabilities. First, we examine the annual standard deviation of the actual liabilities for some samples of state pension funds for which we have a balanced panel of data over a medium horizon. We consider one sample of 70 plans for which we have data for 9 years (1997-2005) and another of 28 plans for which we have data for 13 years (1993-2005). Table I shows the outcome of this analysis. The volatility of liability growth in these samples is 1.56% and 1.52% respectively, and the beta with respect to the market portfolio is 0.008 and -0.011 respectively. At one-year horizons, therefore, there appears to be no appreciable covariance between the evolution of liabilities and returns on the market portfolio.

There is not enough data on liability growth to measure longer-horizon covariance between liabilities and the market portfolio. However, it is possible to obtain long series of salaries of government employees. Our second procedure makes use of salary data and considers the fact that most DB pension plans make the payout a linear function of the average of the final several years of average pay. We use the current population survey (CPS) of the US Census for salary data. Appendix Table II shows the raw data. We use these data to calculate the three-year moving average of full-time government employee average wages during the period 1962-2006.

Table II shows this analysis. The 3-year moving average of real government employee salary growth has a mean of 0.7% and a standard deviation of 2.2%. The covariance of this salary growth series with respect to excess returns on the market portfolio is 0.056, and the beta with respect to excess returns on the market portfolio is 0.007. Collapsing the data into 3-year horizons, we find a correlation of -0.098 and a beta of -0.019. Collapsing the data into 5-year horizons, we find a correlation of -0.234 and a beta of -0.017. While these betas are certainly measured imprecisely, there is no evidence that government salary growth and the stock market are co-integrated at either short or long horizons.

Figure 2 shows the experience of government salaries versus market returns over three 15-year experiences covered by the sample. While 45 years of data is not sufficient to reject a statistical hypothesis that these series are co-integrated over 15-year horizons, the overall pattern appears to be that government wages have kept pace with inflation but little more, while equity markets have increased dramatically during two of these time periods.

Note that even under the completely unrealistic assumption that the liabilities have a volatility of 0.05 and a correlation with the market of 0.25, the implied discount rate would be higher by only 6.5%*0.05*0.25/0.16 = 51 basis points. Even this implausibly high excess discount rate is only one seventh of the 3.5% in excess of the risk-free rate employed by the median plan. Furthermore, although the present value of liabilities is lower under this higher rate, the potential magnitude of the underfunding is much larger, as we show in Section V.

It is perhaps unsurprising that government worker wages covary little with equity markets, given the possibility of state-level fiscal stimulus and that government employment terms are less driven by market forces. We note that this finding says nothing about the corporate sector, where real wage growth might be highly correlated with the stock market (Lucas and Zeldes (2006)).

Table III shows the distribution of state municipal bond ratings and implied discount rates as of 2005. 46% of states have AA ratings from S&P, with 32% above that range and roughly 22% below.

B. The Extent of Underfunding Under Different Discount Rates

Figure 3 repeats the analysis of Figure 1 using tax-corrected municipal yields. Figure 4 repeats the analysis using Treasury yields. Figure 5 summarizes these relations. To calculate the liability under a different discount rate we assume a 15-year duration and use the simple equation:

$$L_{adjusted} = L_{stated} * \left(\frac{1 + r_{stated}}{1 + r_{adjusted}} \right)^{15}$$

The liabilities calculated in this manner represent lower bounds on the liabilities for each discount rate. This is because our analysis underestimates the convexity of the liabilities, and consequently underestimates the extent to which the liabilities rise as discount rates fall.[7]

[7] The stated liability is calculated by discounting each time-*t* ahead liability at the stated discount rate, $L_{stated} = \sum_t \frac{L_t}{(1+r_{stated})^t}$. If each of the liabilities were known, then the "true" adjusted liability could be calculated by discounting using the adjusted discount rate,

$$L_{adjusted}^{true} = \sum_t \frac{L_t}{(1+r_{adjusted})^t} = L_{stated} \times \sum_t \frac{L_t/(1+r_{stated})^t}{L_{stated}} \times \left(\frac{1+r_{stated}}{1+r_{adjusted}} \right)^t = L_{adjusted} \times \sum_t \frac{L_t/(1+r_{stated})^t}{L_{stated}} \times \left(\frac{1+r_{stated}}{1+r_{adjusted}} \right)^{t-15}.$$

In section A above, we showed that if one treats unfunded pension liabilities discounted at stated discount rates as state debt, total debt was 39% higher at the end of 2005 than if one ignores unfunded pension liabilities as is typically done. Using the adjusted muni rate, pension underfunding adds 113% to state GO debt. Total liabilities increase to $3.1 trillion, so that underfunding increases to $901 billion, compared to $798 billion of state GO debt. Using the risk-free Treasury rate, pension underfunding adds 234% to state GO debt. Total liabilities increase to $4.0 trillion, so that underfunding increases to $1.9 trillion.

The total underfunding under the adjusted muni rate ($901 billion) is less than the total underfunding under the Treasury rate ($1.9 trillion) because the adjusted muni rate gives weight to the possibility that the states can default on their promises to public employees. That is, if true pension underfunding is only $901 billion, current public employees are exposed to default risk on their pensions. Insurance against this default would have a fair market cost of $1 trillion.

This analysis raises the question of whether pension and GO debt are substitutes or complements in the cross-section. We therefore also consider the relation between state general obligation (GO) debt and state pension liabilities and funding. We do this in order to understand the concentration of risk in the system. Are the large pension liabilities and large state general obligations concentrated in the same states, or are they present in different states?

Figure 6 shows several versions of this analysis. Each graph in the left column shows the cross-sectional relation between state GO debt scaled by revenues and net state pension funding scaled by revenues. Each graph in the right column shows the cross-sectional relation between state GO debt scaled by revenues and state pension liabilities scaled by revenues. The top graphs measure liabilities at stated discount rates, the middle graphs at the tax-adjusted muni rate, and the bottom graphs at the Treasury rate.

The left column of graphs shows that states with higher GO debt as a share of state revenues clearly also have larger pension underfunding as a share of state revenues. Note that this relation is not simply driven by the size of the state, since both variables are scaled by state revenues. In particular, the left column of graphs shows that each additional dollar of state GO debt is associated with $0.28 more pension underfunding at stated discount rates, $0.39 more pension underfunding at the tax-adjusted muni rate, and $0.44 more pension underfunding at the

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If $r_{adjusted} < r_{stated}$, then $\left(\frac{1+r_{stated}}{1+r_{adjusted}}\right)^{t-15}$ is a convex function of t, so the sum on the right hand side of the previous equation exceeds one, the value of the function evaluated at the weighted-average duration of 15 years. Our "adjusted" liabilities L_{stated} are consequently less than the "true" adjusted liabilities L_{stated}^{true}.

Treasury rate. The right column of graphs shows that states with more GO debt as a share of revenue do not have large pension liabilities as a share of revenue. Indeed, it is pension underfunding, not simply pension plan size, that correlates with the existence of GO debt.

C. Other Considerations: Actuarial Methods and Benefits to Be Accrued

The earning of pension benefits for workers is typically back-loaded in the workers' careers. Given the standard DB contract, the tenure-benefits function is convex. One way to account for this is to use a projected unit credit (PUC) method for calculating normal cost. This method projects compensation growth to retirement but counts only benefits related to years of service up to the present time as part of the AAL. This means that if the DB-covered government workforce gets closer to retirement, the AAL will grow in a convex way. So current AAL and recent AAL growth might mask large increases in the near future.

The more common method of AAL calculation in our sample is the entry age (EA) method which smoothes the earning of benefits over time. Under this method, the AAL is likely to grow over time in a close to linear fashion. Munnell et al (2008) find that 70% of public sector plans use the EA method, with the remainder using PUC or another method.

In the above calculations, we combine liabilities calculated under the PUC and EA methods. We note that the PUC method may disguise large looming increases in liabilities as workers enter the convex portion of their benefit accrual curve. The EA method, on the other hand, would overstate liabilities if we think that governments have some way to stop future benefit accruals, as corporations have by converting to cash balance plans. Given the relative lack of DC plans in the public sector, and the protections of state pension liabilities offered by state constitutions, we would argue that the EA method gives an accurate picture.

Note that neither method assumes that there will be any growth in state workers. Furthermore, the AAL implicitly assumes that all future normal costs will be fully funded. Thus the current extent of pension underfunding must be seen as relative to benefits that have been accrued until now, with some smoothing of benefit accrual possible under the EA method.

Finally, it is worth repeating that the liabilities in the measures above include projected salary increases, but not effects of working another year (future normal cost). States are subject to GASB guidelines that they should at least fund normal cost, although they may report that they chose to fund less than the normal cost. However, the normal cost understates liability if the true liability has a lower discount rate than the assumed discount rate. In other words, working another year is in fact only partly funded under current funding guidelines, because it is assumed that states can just put $\$1/(1+r)^T$ in the pension fund to cover each \$1 of promised benefits, where r is the state-chosen discount rate.[8]

In this paper, we make the assumption that all future accrued benefits will be fully funded. However, according to Munnell et al (2008) a full 43% of state and local governments are not even paying their ARCs based on normal cost calculations at state-chosen discount rates. This suggests that future benefit accruals are unlikely to be fully funded and that the expected funding deficit is even worse than our findings suggest.

III. State DB Pension Investment Policy

Data on asset allocation within the public pension plans in our sample were collected from three sources: the US Census of Governments, the *Pensions and Investments* (P&I) survey, and the annual reports of the entities themselves if necessary. As shown in Table IV, each of these data sources offers a somewhat different breakdown of asset allocation.

We use the information from these sources to allocate assets into the following categories: domestic stocks, domestic government bonds, domestic corporate bonds, international stocks, international government bonds, international corporate bonds, cash and cash equivalents, private equity, real estate equity, mortgages, and other (primarily hedge funds). We use the following procedure to aggregate information from the different sources.

Step 1. If no assets are in the "Other" census category that includes trusts, we begin with the census numbers and then use the P&I survey to decompose stocks into international and domestic stocks, and bonds into international and domestic bonds. We also use the P&I data to determine allocations to private equity and real estate. If no P&I survey data exists, or if the P&I numbers appear to conflict with the census numbers, then we collect data from the annual report to obtain these decompositions.

Step 2. If the "Other" census category that includes trusts is less than 5% in the census data, we categorize these assets based on the P&I survey and/or data from the annual reports.

Step 3. If the "Other" census category that includes trusts is greater than 5% in the census data, we use the P&I survey as the primary source and use the census data and if necessary the annual reports to distinguish domestic from international securities in the stock and bond categories.

The summary statistics in Table IV illustrate the outcome of this categorization. We obtain asset allocation data for 95 of the 108 plans in our sample (covering 99.3% of the assets).

[8] Munnell et al (2008) report that 57% of state and local governments paid their full Annual Required Contributions (ARCs) in 2006, with 16% contributing 80-99%, 13% contributing 60-79%, and 14% even less than 60%. Mitchell and Smith (1994) find wide variations in this funding behavior.

We find the largest allocation is to domestic stock, which has a 45.8% asset-weighted mean. International stock and domestic corporate bonds are the next largest categories at 16.4% and 14.4%, respectively. Domestic government bonds represent 9.7% of the asset-weighted portfolio. The other categories combined make up the remaining 13.7%, with private equity and real estate equity each amounting to about 4%. These figures bear similarity to the tabulations in Rauh (2008) of corporate pension plans, in which total allocations to equity among major pension sponsors as of the end of 2003 were approximately 60%.

IV. A Modest Proposal

Current government accounting standards encourage states to fund pension liabilities in expectation, meaning that the expected future value of pension assets should equal the expected future value of pension liabilities. This ignores the fact that any asset allocation strategy generates a distribution of potential outcomes, with first-order welfare implications. For example, a highly risky asset allocation strategy might allow pension liabilities to be funded in expectation (on average), even though they might be massively underfunded 99% of the time.

In this section, we present an illustration of the problems inherent in focusing only on meeting pension liabilities in expectation. If you accept the logic of GASB 25 and ASOP 27, then you must accept the "modest proposal" we provide. This proposal is in the spirit of Jonathan Swift's 1729 essay, "For Preventing The Children of Poor People in Ireland From Being a Burden to Their Parents or Country, and For Making Them Beneficial to The Public."[9] If you do not accept this modest proposal, then you cannot accept the logic of GASB 25 and ASOP 27.

Suppose public pension fund managers increased their allocation in the stock market from 60% to 70%. Assuming a 6.5% equity premium, a 4.5% risk free rate, and a 15 year duration, plans in aggregate would miraculously be fully funded, as opposed to $300 billion short under current asset allocation. All that GASB 25 and ASOP 27 require for full funding is that in expectation, the plans will have $7.9 trillion in assets in 15 years' time. An investment strategy with a beta of one generates a 9% expected return, and the existing $2.175 trillion in assets invested over 15 years generates $7.9 trillion in assets in expectation in 15 years' time.

Now, by investing in futures contracts on the market or ETFs to continuously maintain a market beta of 10, investment managers could free far more cash for the benefit of the people. In fact, we calculate that such an investment strategy would allow for a windfall distribution in the

[9] The premise of Swift's proposal was that the children could be eaten: "I have been assured by a very knowing American of my acquaintance in London, that a young healthy child well nursed is at a year old a most delicious, nourishing, and wholesome food, whether stewed, roasted, baked, or boiled; and I make no doubt that it will equally serve in a fricassee or a ragout" (Swift (1729)).

amount of $5,000 to every American man, woman and child, all while providing for a state pension and OPEB system that appears fully funded. Under a nearly 100% annual expected return (= 4.5% + Exp[10*Log[1.065]]), one only needs to invest $320 million today to meet (in expectation) a liability of $7.9 trillion. This "frees up" $2.17 trillion of assets today. After paying off the entire $800 billion in state debt, there is $1.37 trillion left, or almost $5,000 for each of 280 million Americans. Of course, while funded in expectation due to a large right tail of outcomes, the system is underfunded at maturity with 99.9% probability. With greater than 99% probability the shortfall in 15 years would exceed $7 trillion.

The "modest proposal" presented above is provided in Swiftian language in the accompanying box.

A Modest Proposal, adapted from Jonathan Swift (1729)
For Preventing the Employees of the States and Municipalities from Being a Burden to their Children or Country

It is a melancholy object to those who peruse the financial statements of this great country's public entities, when they see the pension plans, welfare benefit plans, and other post employment benefits crowded with massive liabilities. These pension and benefit plans, instead of relying on the high returns that can be earned by investing in levered equity positions, private equity, and hedge funds, are forced to employ taxpayer money in simple allocations of balanced stock and bond investing, siphoning off money from the school budgets of their helpless infants: who as they grow up must either turn thieves to pay their tax bills or leave their dear native country to Belize or the Cayman Islands.

We think it is agreed by all parties that this prodigious grasp of the state employee pensions onto the arms and heels and into the pockets of children, and frequently the pockets of their mothers and fathers, is in the present deplorable state of the union a very great additional grievance; and, therefore, whoever could find out a fair, cheap, and easy method of making these employees' pensions sound, useful, and beneficial component entities of the commonwealth, would deserve so well of the public as to have his statue set up for a preserver of the nation.

But our intention is very far from being confined to provide only for the professed beggars who are the employees of the states and municipalities; it is of a much greater extent, and shall in fact provide for all citizens and workers and their children who are so burdened by taxes as to have to demand our charity in the streets.

As to our own part, having turned our thoughts for many months upon this important subject, and maturely weighed the several schemes of other projectors, we have always found them grossly mistaken in the computation. It is true, an employee just beginning to accrue a pension may be supported by the meager returns that present managers acting unconscionably are generating by investing in low-yielding debt instruments. But it is exactly as the financial demands of state and local pensioners grow large that we propose to provide for them in such a manner as instead of being a charge upon the taxpayers and their children, or needing our support for the rest of their lives, they shall on the contrary contribute to the feeding, and partly to the clothing of many households.

There is likewise another great advantage in our scheme, that it will prevent those voluntary terminations of defined benefit pension plan, and that horrid practice of converting them to defined contribution plans, alas! Too frequent among us! Sacrificing the poor innocent public employees more to avoid the funding expense than the simple administrative costs. The fate of the public employee on a cash balance plan would move tears and pity in the most savage and inhuman breast.

The number of dollars in this land owed to state employee pensions alone is usually reckoned at $2.5 trillion at an 8% discount rate and a 15 year duration; this being granted our society needs $7.9 trillion in 15 years' time if our state pensions are to be fully funded.

The question therefore is how this number shall be provided for, which, as we have already said under the present situation of affairs, is utterly impossible by all the methods hitherto proposed. For we can neither tax current employees to the necessary extent, nor cut spending to the necessary extent, nor expect employees to work over the age of 60. We confess that public employees learn the rudiments of retirement much earlier, during which time, they can however be properly looked upon only as pensioners. We have indeed been informed by a principal gentleman in the county of Cook in the state of Illinois, who protested to us that he never knew above one or two instances of public employees over the age of 60, even in a part of the land so renowned for public employment.

We shall now therefore humbly propose our own thoughts, which we hope will not be liable to the least objection. We have been assured by our very knowing acquaintances at our university that a young healthy investment in an exchange traded fund that continuously maintained a market beta loading of 10 would generate 10 times the expected excess market return (which we take conservatively to be a simple annual rate of 6.5%). With continuous rebalancing, and accounting for the risk free rate (which we take conservatively to be 4.5%), this investment strategy would yield an expected annual return of over 90% per annum.

We have reckoned upon the simplest of desktop spreadsheet applications that an investment of $320 million thus made would reach in expectation $7.9 trillion in 15 years' time. Seeing that $2.175 trillion of funds have been set aside in public pension funds being invested in irresponsibly low-yielding assets, in excess of $2.17 trillion could be removed from pension funds and spent on paying down our eight hundred billion of state debt. This would then leave $1.37 trillion to be spent on our nation's schools, highways, and baseball stadiums or distributed as a windfall gain to taxpayers.

We do therefore humbly offer it to public consideration that of the two hundred eighty million citizens of the United States, each could receive a share of this $1.37 trillion amounting to a windfall tax dividend of nearly $5,000.

We have already computed the fee for the managers in such an ETF (in which we include all of the investment bankers, hedgies, capital markets desk traders, accountants, and desktop spreadsheet laborers) to be about 1% per annum. While this raises the amount that must be set aside now from $320 million to $9 billion, we believe that no gentleman would repine to give even $10 billion as such a donation to the capital markets would only reduce the value of his $5,000 net profit by approximately $50. The squire will have $4,950 in net profit and be fiscally fit until the next fiscal catastrophe should ensue.

Those who are more thrifty (as we must confess the times require) may donate some of the proceeds to paying down the federal debt, which ignoring the artificial dressing due to the off-budget position of Medicare and Social Security will make admirable improvements in our nation's fiscal position. As to our financial centers such as New York and Chicago, investment bankers we may be assured will not be wanting in helping state and local governments to establish their derivatives positions.

A very worthy person, a true lover of his country, and whose virtues I highly esteem, was lately pleased in discoursing on this matter to offer a refinement upon our scheme. He said that many gentlemen of this nation, having of late destroyed their retirement savings, he conceived that the want of money in 401(k) plans might be well supplied by the windfall gains from the new investment strategy. But with due deference to so excellent a friend and so deserving a patriot, we cannot be altogether in his sentiments; for as to the individual investors, my American acquaintance assured me, from frequent experience, that their investment performance in their 401(k) plans was so generally poor and the frequency of their trading so disagreeable, so that to fatten their defined contribution accounts now would not answer the charge.

We profess, in the sincerity of our hearts, that we have not the least personal interest in endeavoring to promote this necessary work, having no other motive than the public good of our country, by advancing our trade, providing for elderly and infants alike, relieving the poor, and giving some pleasure to the rich. We ourselves are not participants in public defined benefit pension plans, as we are not employees of the state, and are unlikely to be so unless in the future we become employed by state universities.

V. Intergenerational Transfers in the System

Section IV showed that analyzing average funding outcomes provides an insufficient picture of the welfare implications of current policy. In this section, we examine the distribution of potential outcomes in terms of the magnitude of the funding shortfall or surplus. Future funding surpluses represent transfers from the current generation to future generations, while future funding shortfalls represent transfers from future generations to the current generation.

We also consider the fact that the magnitude of the potential transfers does not adequately characterize the true cost of the uncertainty in the plans' future funding status. Funding shortfalls will occur if the assets perform poorly over relatively long horizons, meaning that the economy as a whole has grown at a rate that has fallen short of expectations. That is, our generation will be foisting a large financial obligation on our children precisely in those states of the world in which our children's standard of living is lower than our own. The worse off the next generation is, the greater the burden they will carry. In order to account for the fact that funding shortfalls coincide with high marginal utility states of the world, we consider here the risk-neutral distribution of the future funding surplus or shortfall.[10]

Characterizing the distribution of the future funding shortfall/surplus is equivalent to characterizing the joint distribution of the plans' future assets and liabilities. The plans' future liabilities may be calculated using the current actuarial values of the liabilities, the discount rates employed by the plans, and the duration of the plans' liabilities. While the actuarial liabilities of the plans and the discount rates they employ are stated annually in the CAFRs, the CAFRs do not provide the duration of the plans' liabilities. Our estimates of the true magnitude of plans' liabilities consequently depend critically on our assumptions regarding the duration of the plans' liabilities.

As noted previously, corporate DB plans have shorter durations than state plans, both because they have fewer young workers, and because they typically do not provide COLAs. We assume that the duration of state DB pension liabilities is 15 years. Figure 7 shows the expected future liabilities of the states' DB pension liabilities as a function of the duration of the plans' liabilities, for durations from 15 to 20 years, in trillions of 2005 dollars.[11,12]

Given the expected magnitude of the plans' liabilities we can calculate the probability of each possible funding outcome, *i.e.,* the distribution of the possible future aggregate surplus, under standard log-normal assumptions regarding the evolution the plans' assets and liabilities. If the liabilities are well forecast (*i.e.,* known), and if plans maintain their current investment strategies, then the probability that the realized value of the assets exceeds the value of the liabilities by at least s is

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[10] The risk-neutral probability of an outcome is the fair market price (in cents) of a dollar, plus interest, received in the event of the outcome.

[11] Real discount rates are calculated assuming each plans' stated discount rate accounts for both the real discount rate and inflation, $(1+\mu^{nominal}) = (1+\mu^{real})(1+i)$, where $\mu^{nominal}$ is the stated discount rate, μ^{real} is the real component, and i is inflation. The component due to expected inflation comes from a similar relation for bonds, $(1+r^{nominal}) = (1+r^{real})(1+i)$, using US Treasury data on yields. At the end of 2005 the real yield curve was basically flat at two percent (2.06% for 15 years, and 2.04% for 20 years), while nominal interest rates on long-dated bonds were roughly four and a half percent (4.50% for 15 years, and 4.61% for 20 years).

[12] For each duration the liabilities are conservatively estimated, because we treat them as a single payment due in the future, and consequently underestimate their convexity.

$$P\left[A_T - L_T > s\right] = N\left(d_2^s\right)$$

where N is the cumulative distributions for the standard normal and

$$d_2^s = \frac{\ln\left(\frac{A_0}{s+L_T}\right) + \left(\mu_A - \frac{\sigma_A^2}{2}\right)T}{\sigma_A \sqrt{T}}.$$

This expression is familiar from the Black-Scholes option pricing equation. It says that the probability that the value of the assets exceeds the liability by at least the surplus level depends on the geometric distance, measured in the standard deviation of the log of the asset's future price, between the most probable future asset value ($A_0 \exp\left(\mu_A - \sigma_A^2 / 2\right)T$) and the sum of the future liability and the surplus.

More generally, if the magnitude of the time-T liabilities are uncertain, with the log-liabilities normally distributed around $\ln \overline{L_T} - \frac{\sigma_L^2 T}{2}$ with standard deviation $\sigma_L \sqrt{T}$, where $\overline{L_T}$ is the expected liability at time-T, then the probability that the surplus exceeds s is

$$\int_{z=\ln s}^{\infty} N\left[\frac{\ln\left(e^z - s\right) - \left(\ln \overline{L_T} - \frac{\sigma_L^2 T}{2}\right) - \frac{\rho\sigma_L}{\sigma_A}\left(z - \left(\ln \overline{A_T} - \frac{\sigma_A^2 T}{2}\right)\right)}{\sqrt{1-\rho^2}\,\sigma_L \sqrt{T}}\right] n\left[\frac{z - \left(\ln \overline{A_T} - \frac{\sigma_A^2 T}{2}\right)}{\sigma_A \sqrt{T}}\right] \frac{dz}{\sigma_A \sqrt{T}}$$

where ρ is the correlation between ln L_T and ln A_T, and n is the density for the standard normal. Details are provided in the appendix.

Using the previous equation we can characterize the distribution of the funding surplus in 15 years. This requires estimating the expected growth rate and volatility of the plans' assets, as well as the uncertainty regarding future liabilities, and the correlation of these liabilities with the dedicated pension assets.

Table V presents the volatility of each asset class included in Table IV, and the correlations between the asset classes' returns. For each asset class, with the exception of private equity, we estimate the volatilities and correlations using five years of monthly data, encompassing the calendar years 2001 through 2005. For domestic equity and cash equivalents we use Fama-French factors (MKT and RF). For the bonds, mortgage backed securities, and hedge funds we use Lehman indices (U.S. Treasuries, U.S. Corporate Investment Grade, Global Treasuries ex-U.S., Global Credit Corporate, U.S. Mortgage Backed Securities, and Hedge Fund Index (Asset Weighted)). For foreign equities and real estate we use the MSCI indices (World ex-USA Standard Core and U.S. REIT). For private equity, we use annual data beginning in 1983 from Thomson Financial's Venture Economics database and take a value-weighted average of Venture Capital and Private Equity sector returns (see Kaplan and Rauh (2008) for details). Due to the lack of high-frequency data, we are forced to estimate the volatility of private equity and its correlations with the other asset classes at an annual frequency.

Assuming the plans maintain their current asset allocations, and using the estimated variance-covariance matrix, the volatility of the aggregate asset portfolio is 8.92%. For the volatility of the plans' liabilities we use 1.5%, as this is the number observed directly in the time-series data on plan liabilities. The assets and liabilities also appear uncorrelated in the data, so we present results assuming the correlation between the two is zero. We discuss robustness to these assumptions below.

Figure 8 depicts the distribution of possible plan shortfalls or surplus in 15 years, using the log-normal model for the evolution of the assets and liabilities, and under the conservative assumption that the duration of the states' DB pension plans is only 15 years. The figure depicts the probability density for each surplus, measured in trillions of 2005 dollars, and shows both the objective probabilities (bold curve) and the risk neutral probabilities (thin curve). The objective distribution assumes the plans' assets carry an annual Sharpe ratio of 0.41, which corresponds to roughly a 6.5% market price of risk and a market volatility of 16%.[13] Figure 9 shows the sensitivity of these results to the plans' asset holdings. It shows the cumulative distribution function for the funding surpluses, as a function of the weight on risky assets in the plans' asset portfolios, highlighting the distribution under the states' current allocation.

Table VI, which shows the magnitude of the under or over funding for nine different levels of the cumulative probability distribution, provides an alternative expression of the results of Figure 8. If current investment strategies are maintained for 15 years, and assuming a 6.5% risk-premium, there is a 50% chance of an aggregate underfunding in excess of $750 billion, a 25% chance of an underfunding of at least $1.74 trillion, and a 10% chance of an underfunding in excess of $2.48 trillion (all in 2005 dollars). The probability that the plans are fully funded is barely one third. Even assuming an implausibly high 8% risk premium, the plans are more likely to be underfunded than overfunded in 15 years.

Accounting for state prices, the distribution of funding outcomes looks even worse. The risk-neutral probability that the value of the liabilities will exceed that of the assets in 15 years is 97.3%. With 50 percent probability, the net liability will exceed $2.66 trillion (2005 dollars).

[13] This analysis assumes that public pension plans do not generate negative alpha, i.e. that they do not perform worse than the market benchmark. Coronado, Engen and Knight (2003) provide evidence that public plans earned a significantly lower rate of return than private plans during the period they analyze. Yang and Mitchell (2006) find that certain governance structures can enhance public pension plan investment performance.

We can also calculate the fair market price of insurance against bad funding outcomes. This insurance is essentially an exchange option. It gives the holder the right to trade the assets for the value of the liabilities in *T* years, should the value of the liabilities exceed that of the assets at that time. This exchange option may be priced using Black's formula for pricing options on futures contracts.[14] Under the states' assumptions regarding the future magnitude of their liabilities, the market price of insuring against bad funding outcomes is almost $1.9 trillion, essentially the entire funding shortfall. The cost of this insurance is almost as large as the funding shortfall because the value of the potential over funding is trivial relative to the potential underfunding, only $16 billion.[15] The expected overfunding is roughly $500 billion in 2005 dollars (33% chance of overfunding times $1.5 trillion conditional overfunding), roughly half the magnitude of the expected underfunding, but the surplus comes only in low marginal utility states of the world. The shortfall, on the contrary, comes in high marginal utility states of the world.[16]

In Table VII, we examine the robustness of these results to different assumptions regarding the volatility of liabilities (σ_L) and their real correlation with the market portfolio (ρ). Greater uncertainty regarding the future value of the liabilities (higher σ_L) generates a wider dispersion of outcomes. Higher correlations with the market portfolio reduce the dispersion of the outcomes, because increases in the value of pension liabilities coincide with increases in the value of the assets used to fund them. The overall effect of the uncertainty in the liabilities dominates, so that with $\sigma_L = 0.05$ and $\rho = 0.25$, the outcomes are still more dispersed than with $\sigma_L = 0.015$ and $\rho = 0$. We emphasize, however, that high correlations and high volatility are not consistent with what is seen in the data (see Section I).

VI. Conclusion

Current government accounting rules allow public pension liabilities to be discounted at expected rates of return on pension assets. Under these rules, state pensions appear to be nearly fully funded. This has led some analysts to the conclusion that the public pension system in its current form is not placing any burden on future generations. As we have shown, this view is misguided. Under the government accounting logic, states could always eliminate their

[14] The value of the exchange option is the Black price of the "futures contract" on the liabilities, struck at the futures price of the assets, using the volatility of the liabilities-to-assets ratio,

$$L_0 N\left(d_2 + \sigma_{L/A}\right) - A_0 N\left(d_2\right)$$

where $\sigma_{L/A} = \left(\sigma_A^2 + \sigma_L^2 - 2\rho\sigma_A\sigma_L\right)^{1/2}$, and $d_2 = \left(\ln\left(L_0 / A_0\right) - \frac{\sigma_{L/A}^2 T}{2}\right) / \left(\sigma_{L/A}\sqrt{T}\right)$.

[15] This means that even if the government is likely to squander overfunding (or promise it to beneficiaries as in Bodie (1990)), the value of that option is small taking the state pricing into account.

[16] These numbers assume a 6.5% market price of risk. For numbers assuming an 8% risk premium, see Table VI.

underfunding no matter how large, simply by investing in sufficiently risky assets. In fact, investing in riskier assets may raise expected returns, but it also increases the probability of a severe underfunding. Under current investment strategies and a standard equity premium of 6.5%, state pension plans realize a shortfall with two-thirds probability in 15 years. The expected conditional shortfall is almost $1.5 trillion in 2005 dollars.

Moreover, bad outcomes occur precisely in states for which marginal utility is high, so that the true welfare implications of the potential shortfalls is much larger. The cost of insuring both taxpayers and plan participants against potential underfunding on a 15 year horizon would be almost $2 trillion in 2005 dollars. Investing more in risky assets has no effect on the cost of this insurance.

These results also are relevant for the debate about whether Social Security could be pre-funded with the assets invested in equities to achieve higher expected returns. Even ignoring concerns about the appropriate role of government in allocating capital, our analysis shows that an underfunded system cannot magically reduce the burden on future generations simply by taking more risk.

Perhaps a more stark example on the federal level is the recent decision by the Pension Benefit Guaranty Corporation (PBGC) to increase its asset allocation to equities. The alleged motivation for doing so is to generate higher returns. The logic for taking on risk to get higher returns is even worse for the PBGC than it is for the states. While the states' pension liabilities appear uncorrelated with the market, the PBGC has large unrecognized liabilities, plans it will have to take over as a result of future corporate bankruptcies, which are strongly negatively correlated with the market. This suggests an optimal asset allocation policy that is significantly short, not long, the market.

At a given funding level, increases in public pension investment risk would be irrelevant for taxpayers if taxpayers actually unwind those changes by decreasing their own risk exposures. If taxpayers do not recognize the exposures created by government investment policies, then such policies expose tax-payers to hidden risk.

Appendix: Distribution of the surplus

The probability that the surplus exceeds any given level *s* is

$$P\left[A_T - L_T > s\right] = P\left[\ln A_T > \ln\left(s + L_T\right)\right].$$

Assuming L_T is known, and making the standard log-normal assumptions on A_T, *i.e.,* that $A_T \sim A_0 \exp\left(\left(\mu_A - \frac{\sigma_A^2}{2}\right)T + \sigma_A \sqrt{T}\chi\right)$ where A_0 is the current value of assets, μ_A and σ_A are the asset's expected growth rate and volatility, and χ is a standard normal random variable, then this probability is equal to

$$P\left[\chi > \frac{\ln\left(\frac{s+L_T}{A_0}\right) - \left(\mu_A - \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}}\right] = N\left(d_2^s\right)$$

where $d_2^s = \left(\ln\left(\frac{A_0}{s+L_T}\right) + \left(\mu_A - \frac{\sigma_A^2}{2}\right)T\right) / \left(\sigma_A\sqrt{T}\right)$. That is, the probability of realizing a surplus that exceeds any given level s looks like the Black-Scholes probability of exercising a call option struck at $s + L_T$, using the expected growth rate of assets under the employed measure.

More generally, assuming L_T is unknown,

$$P\left[A_T - L_T > s\right] = \int_{a=\ln s}^{\infty} P\left[a - s > L_T \middle| A_T = a\right] P\left[A_T = a\right] da.$$

If L_T is also log-normally distributed then $L_T \sim L_0 \exp\left(\left(\mu_L - \frac{\sigma_L^2}{2}\right)T + \sigma_L\sqrt{T}\left(\rho\chi + \sqrt{1-\rho^2}\,\chi'\right)\right)$ where χ' is a standard normal random variable uncorrelated with χ and ρ is the correlation of the growth in the assets and liabilities, and we can write the liabilities as

$$L_T = \left(A_T / \alpha\right)^{\rho\sigma_L/\sigma_A} \beta \exp\left(\sqrt{1-\rho^2}\,\sigma_L\sqrt{T}\chi'\right)$$

where $\alpha = A_0 \exp\left(\left(\mu_A - \frac{\sigma_A^2}{2}\right)T\right)$ and $\beta = L_0 \exp\left(\left(\mu_L - \frac{\sigma_L^2}{2}\right)T\right)$. Using this, we can rewrite the right hand side of the previous equation as

$$\int_{a=s}^{\infty} P\left[\ln\left(\frac{a-s}{\beta\left(a/\alpha\right)^{\rho\sigma_L/\sigma_A}}\right) > \sqrt{1-\rho^2}\,\sigma_L\sqrt{T}\chi'\right] P\left[A_T = a\right] da.$$

this may be rewritten, letting $z \equiv \ln a$ and using the fact that χ' and $\ln A_T$ are independently distributed normal random variables, as

$$\int_{z=\ln s}^{\infty} N\left[\frac{\ln\left(\left(e^z - s\right)/\beta\right) - \frac{\rho\sigma_L}{\sigma_A}\left(z - \ln\alpha\right)}{\sqrt{1-\rho^2}\,\sigma_L\sqrt{T}}\right] n\left[\frac{z - \ln\alpha}{\sigma_A\sqrt{T}}\right] \frac{dz}{\sigma_A\sqrt{T}}$$

where n and N are the density and cumulative distributions for the standard normal, respectively.

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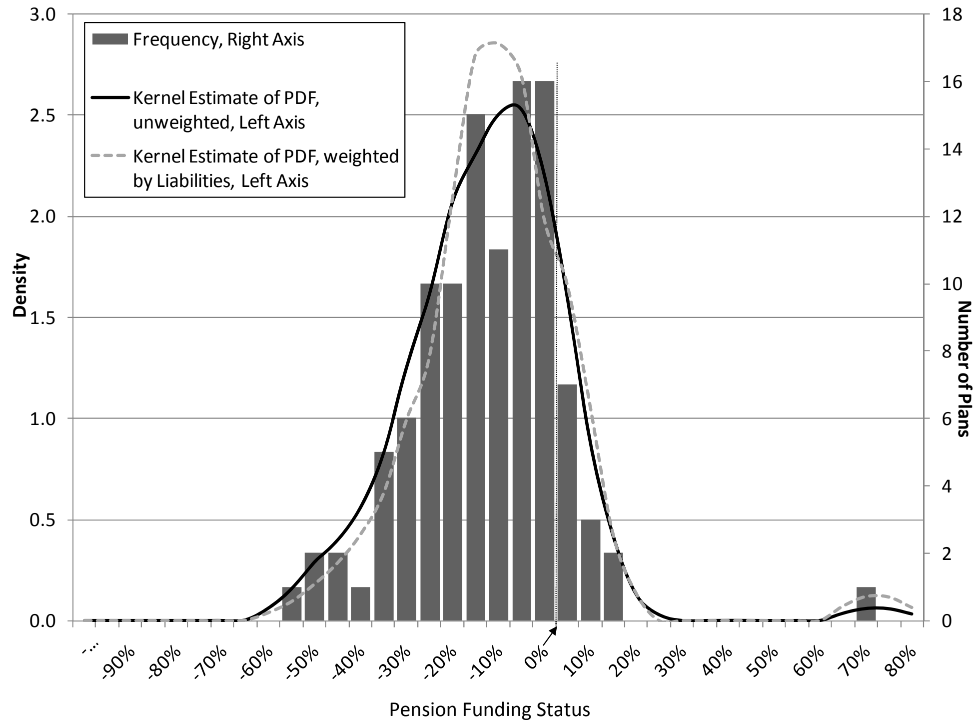


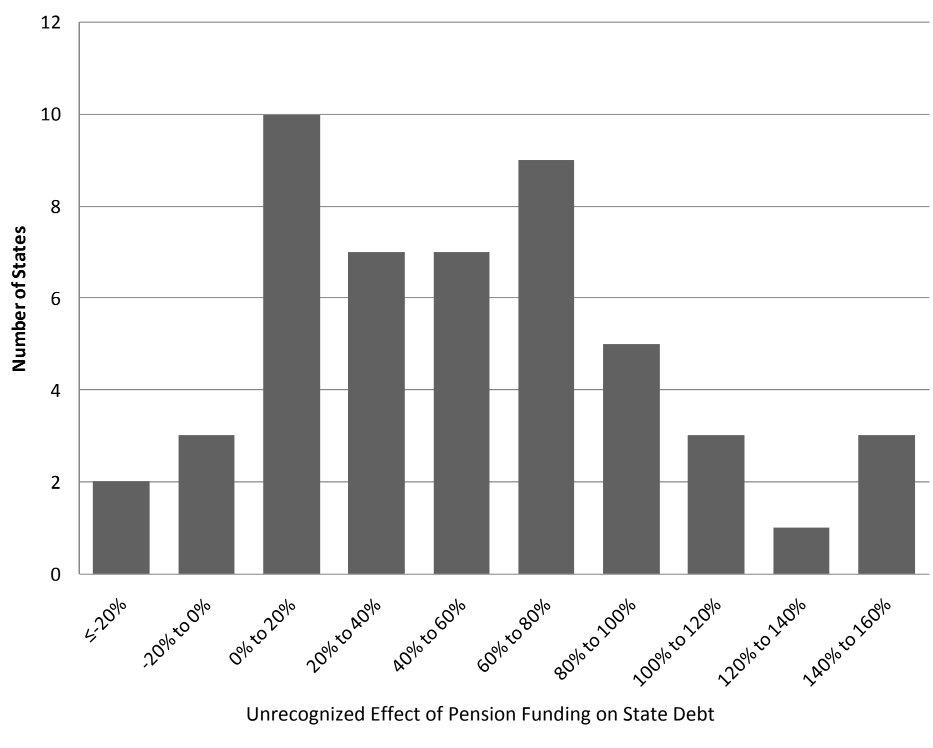


Figure 1: Distribution of Funding Levels in State Pension Plans at Stated Discount Rates
The upper graph shows the distribution of underfunding or overfunding in state pension plans at discount rates provided by the states themselves. The lower graph shows the distribution of the increase in state debt that occurs when adding the pension funding gap calculated at the stated discount rates to state general obligation debt.

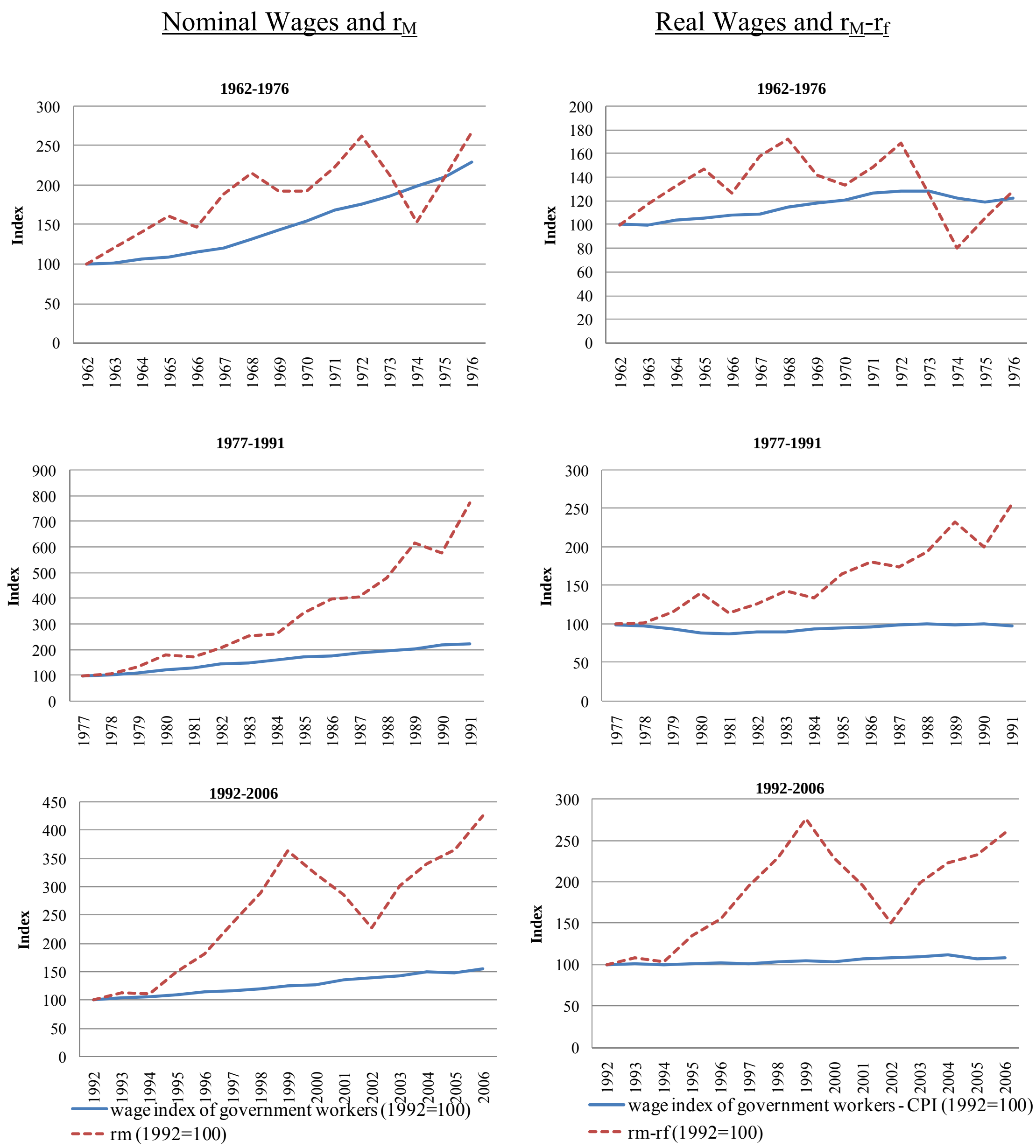
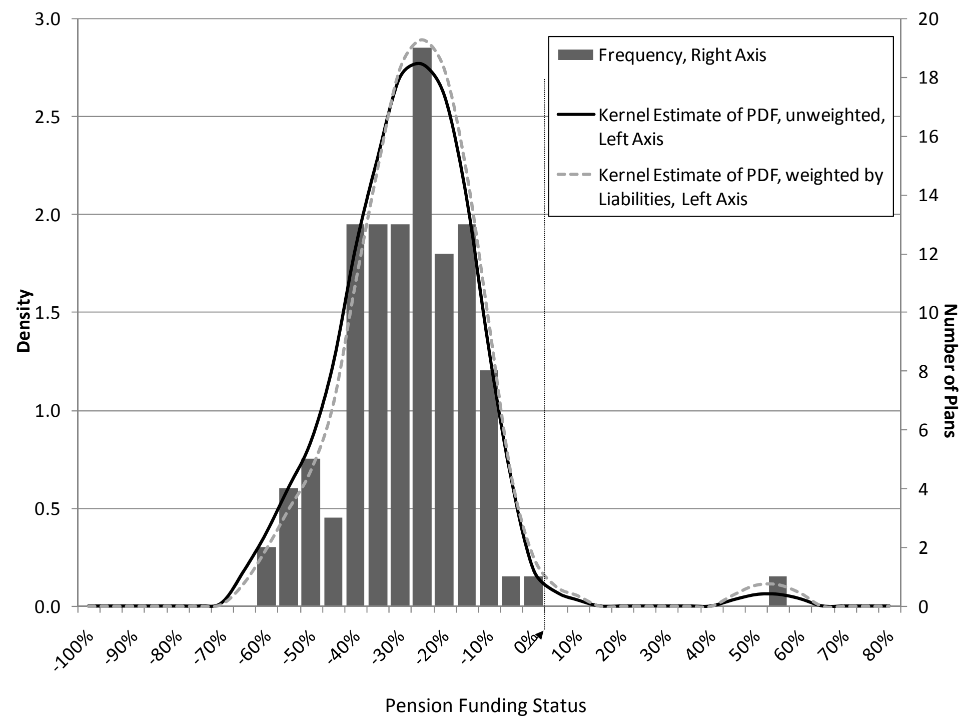


Figure 2: **Government Salary Growth and Market Returns**
This figure shows the extent of co-movement between returns on a wage index of government workers and returns on the market portfolio. The wage index comes from the Current Population Survey (CPS) of the US Census for all government workers between 1962 and 2006. Consumer Price Inflation (CPI) growth is from the Bureau of Labor Statistics (BLS) website. The return on the US stock market and the risk-free rate is extracted from the Ken French data.



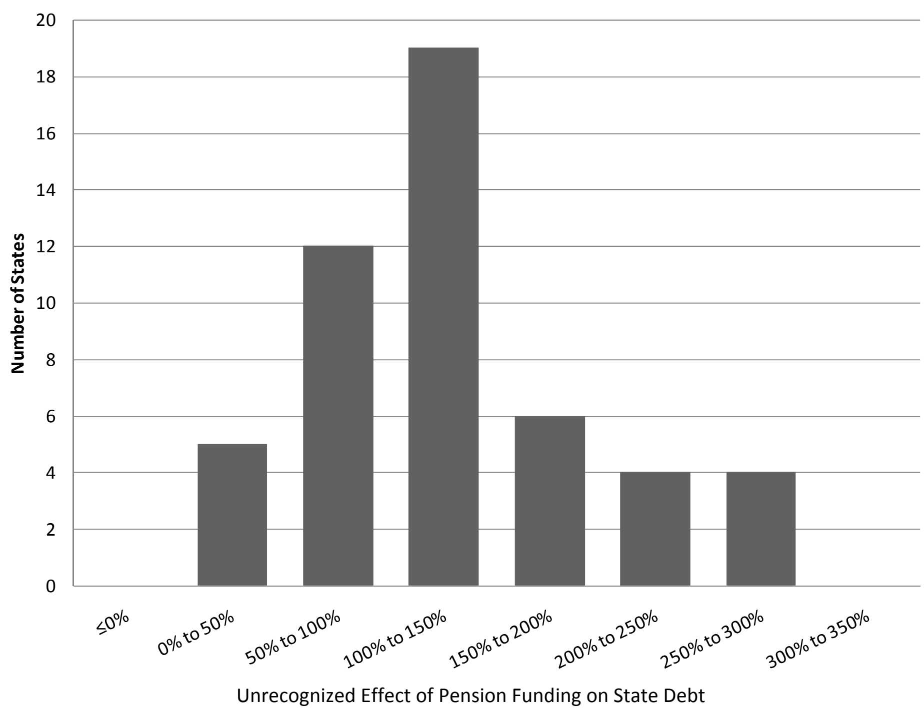


Figure 3. Distribution of Funding Levels in State Pension Plans at the Municipal Bond Rate Excluding the Tax Benefit. Pension liabilities have been adjusted to a discount rate that equals the state's municipal bond discount rate divided by (1-0.35), assuming a 15-year duration. The upper graph shows the distribution of underfunding or overfunding in state pension plans using this rate. The lower graph shows the distribution of the increase in state debt that occurs when adding the pension funding gap calculated at this discount rate to state general obligation debt.

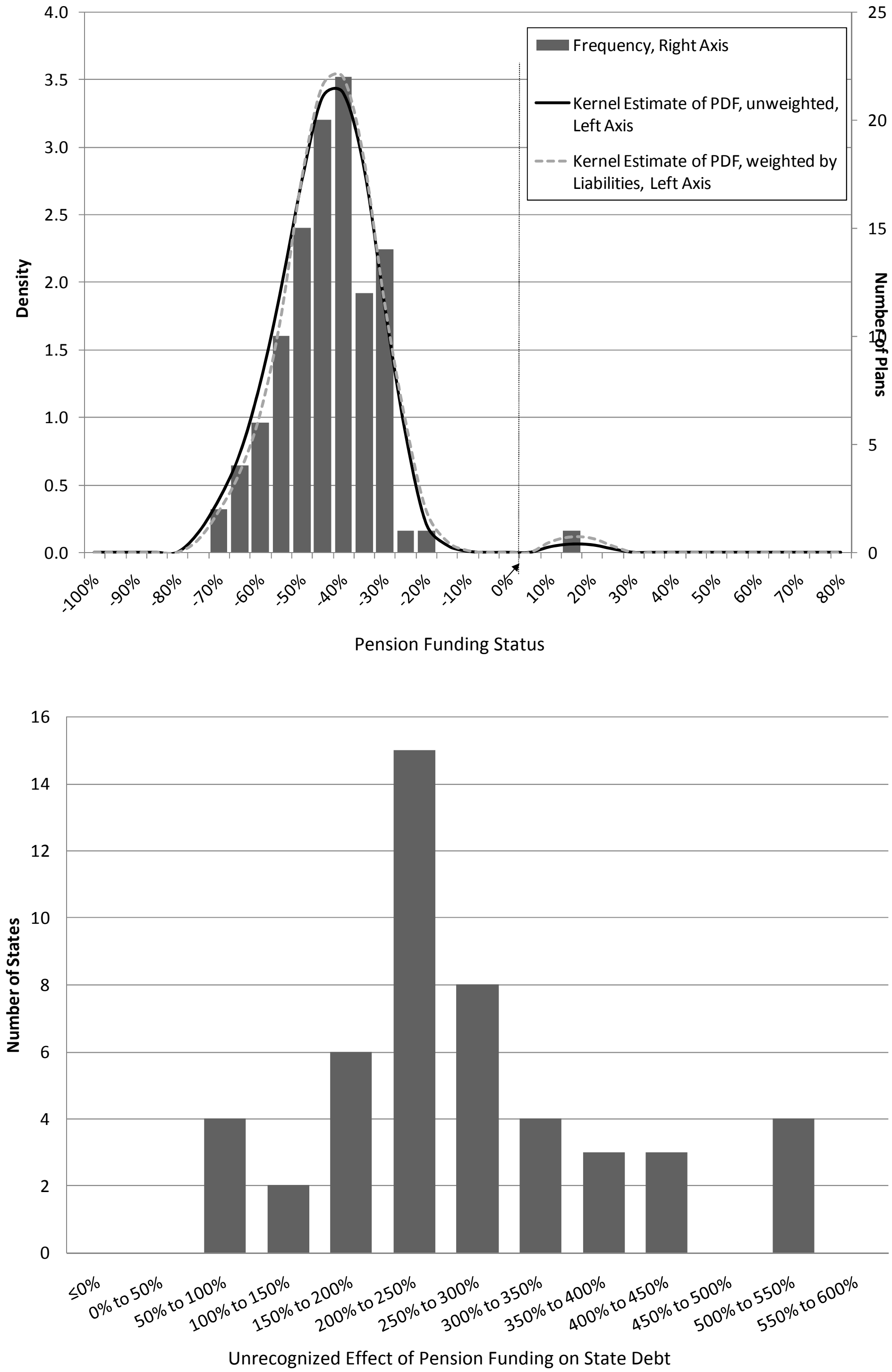
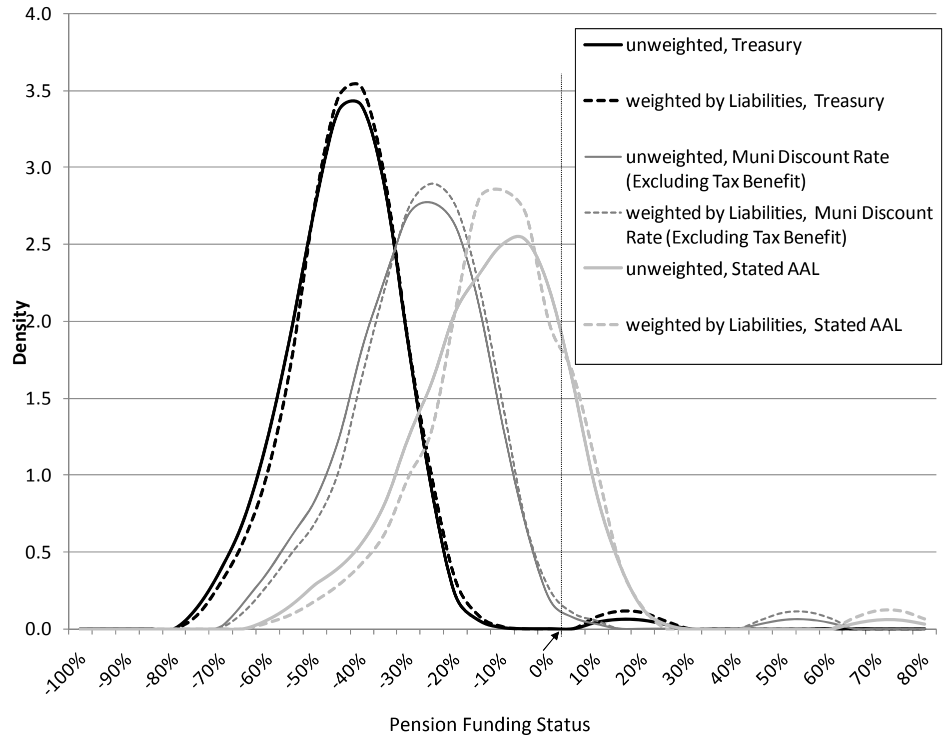


Figure 4. Distribution of Funding Levels in State Pension Plans at the Treasury Rate
Pension liabilities have been adjusted to a discount rate that equals the state's municipal bond discount rate / (1-0.35), assuming a 15-year duration. The upper figure shows the distribution of underfunding or overfunding in state pension plans using this rate. The lower graph shows the distribution of the increase in state general obligation debt that occurs when adding the pension funding gap calculated at this discount rate.



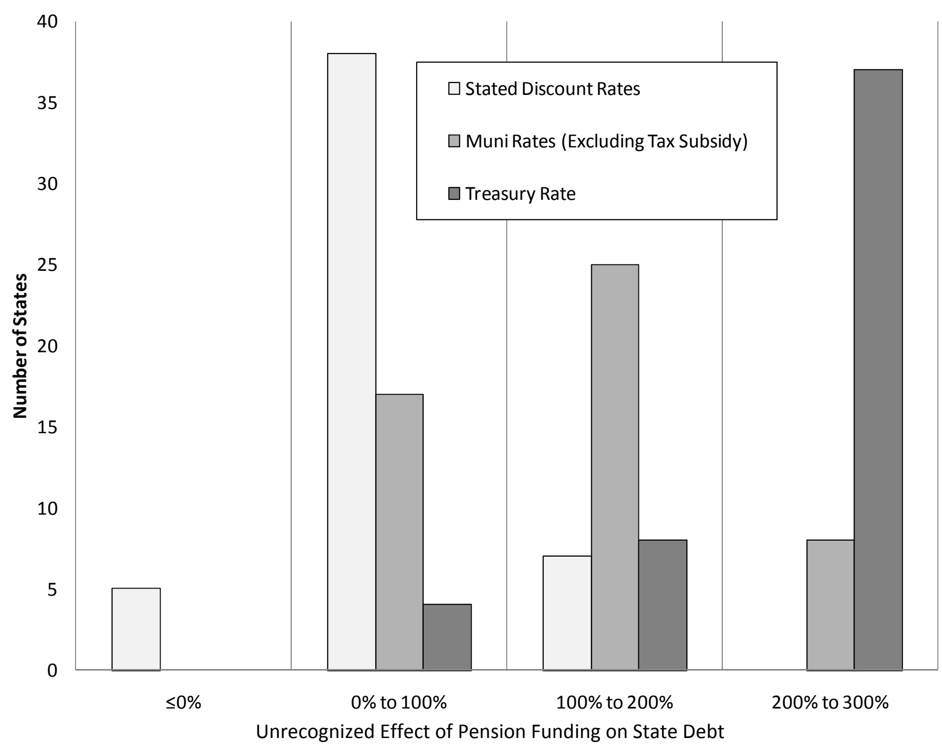


Figure 5. Distribution of Funding Levels in State Pension Plans at Various Rates
The upper graph shows the cross-sectional distribution of state funding statuses as of 2005 under different discount rate assumptions. The lower graph shows the cross-sectional distribution of the impact of pension underfunding on state finances under different discount rate assumptions.

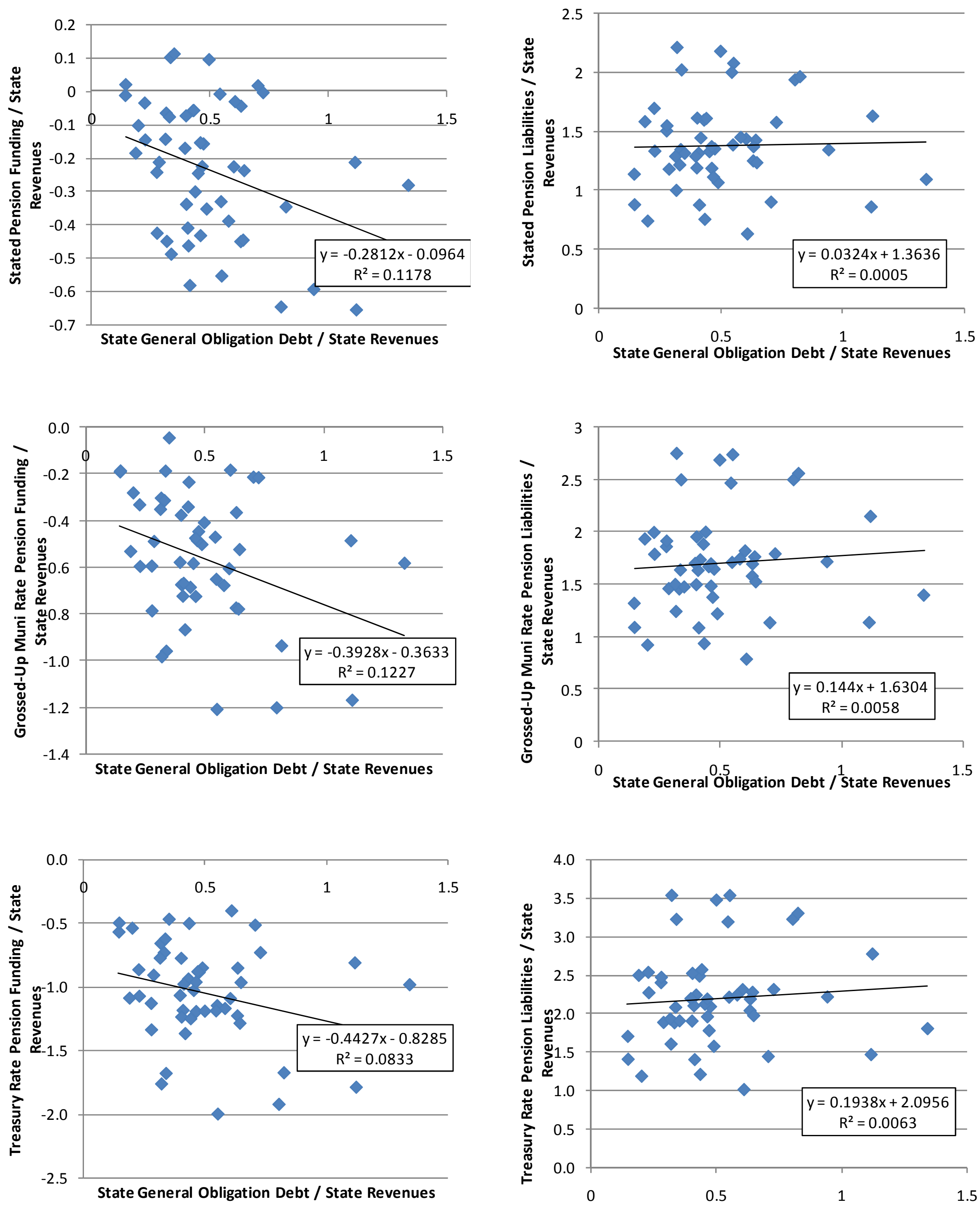


Figure 6: State General Obligation Debt and Pension Funding

The three graphs on the left show the cross-sectional relation between state general obligation (GO) debt scaled by state revenues and pension funding scaled by state revenues for the year 2005. The graphs on the right show the relation between state GO debt scaled by state revenues and state pension liabilities scaled by state revenues. The top graphs measure liabilities at stated discount rates, the middle graphs measure liabilities at the municipal rate grossed up by the statutory tax rate of 35%, and the bottom graphs measure liabilities at the Treasury rate.

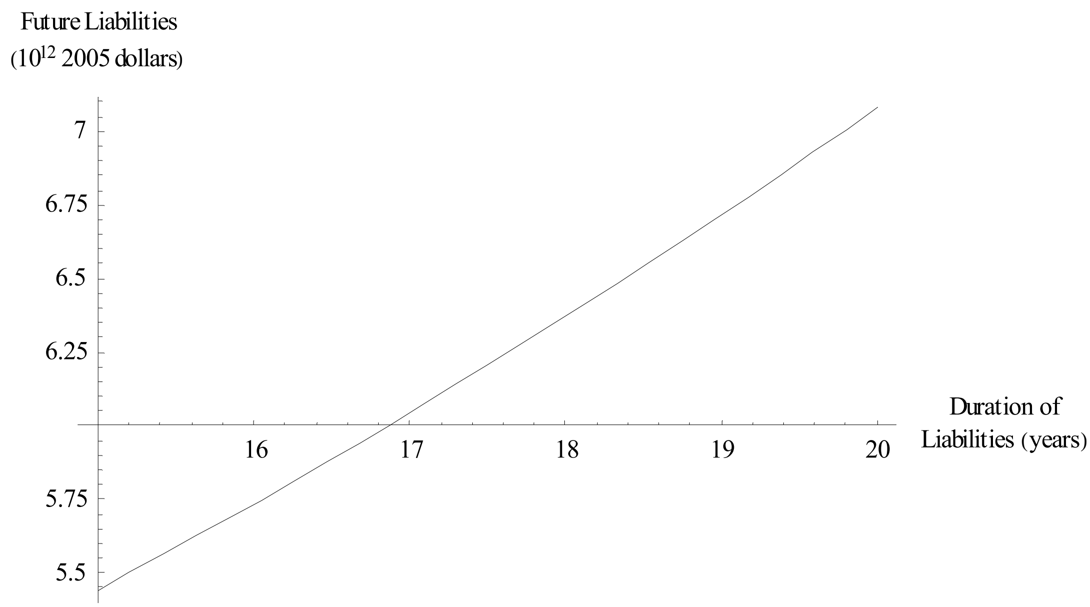


Figure 7: Duration and Future Liabilities
This figure shows the magnitude of the future plan liabilities, in 2005 dollars, as a function of the duration of the plans' current liabilities.

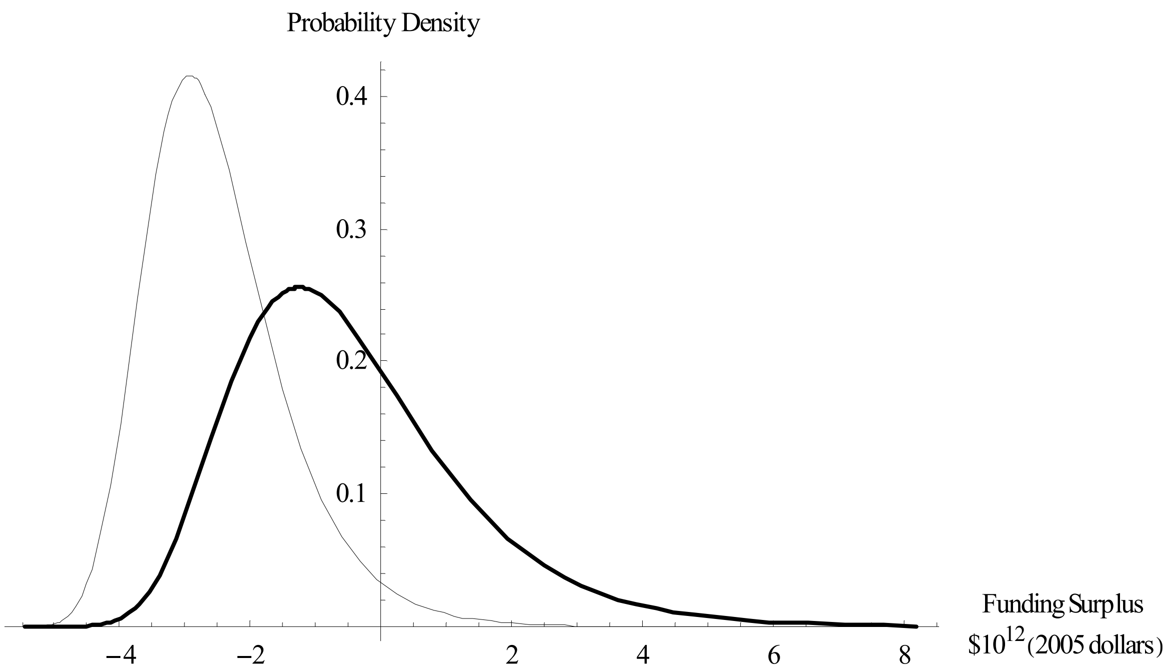
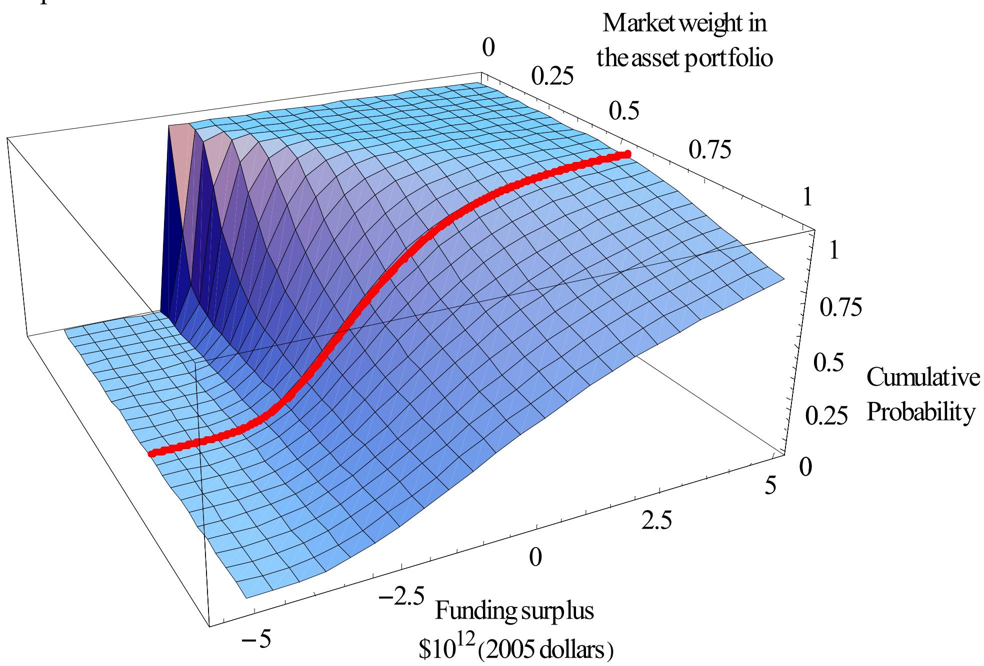


Figure 8: Density of Funding Outcomes Under Current Investment Strategies
This figure shows the probability density for possible outcomes for the funding surplus in 15 years (in trillions of 2005 dollars), assuming plans maintain their current investment strategies. The bold line depicts the objective probability distribution, while the thin line depicts the risk-neutral distribution.

Objective probabilities



Risk-neutral probabilities

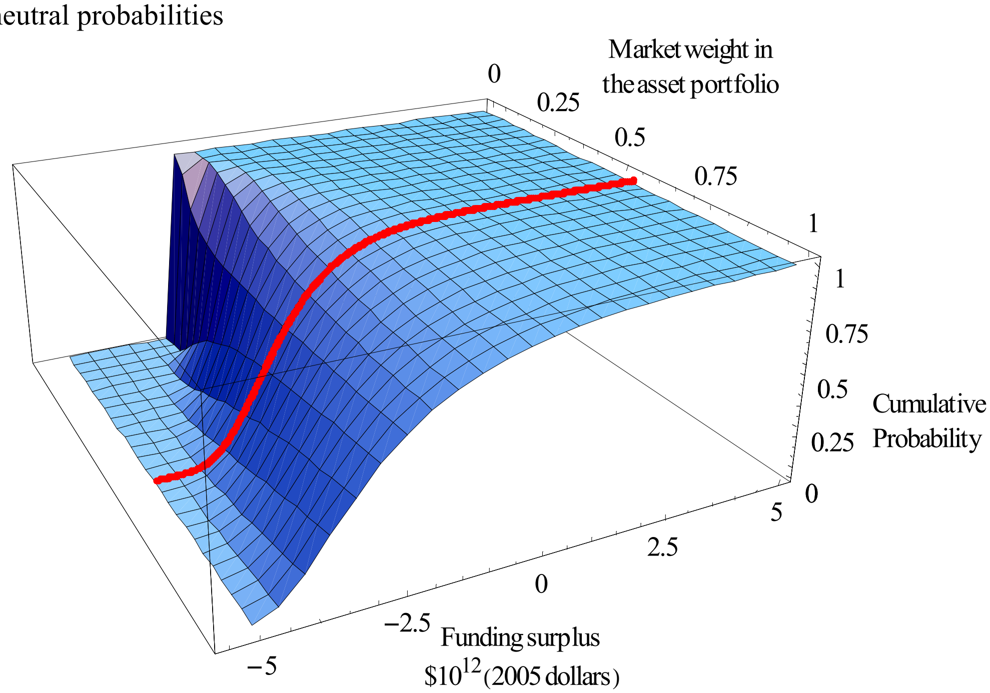


Figure 9: Cumulative Distribution Function of Funding Outcomes
This figure shows the distribution of funding outcomes in 15 years (in trillions of 2005 dollars), as a function of the market weight in the asset portfolio. The figure assumes the actuarial liabilities are calculated using plans' stated discount rates and a 15 year duration. The bold line shows the portfolio with the same volatility as the aggregate state pension assets.

Table I: Growth Rate of Liabilities in Balanced Panels

This table shows the mean and standard deviation of the accrued actuarial liability (AAL) for balanced panels of state government pension plans. All data are from annual reports of state pension plans. The US Census of Governments lists 112 state pension plans with more than $1 billion in assets as of the end of 2005. AAL data are available from annual reports for 108 of those plans as of 2005. For 70 plans we are able to collect a balanced panel back to 1997 and for 28 we are able to collect a balanced panel back to 1993.

	Balanced Panel 1993-2005 Accrued Actuarial Liability (AAL)		Balanced Panel 1997-2005 Accrued Actuarial Liability (AAL)	
	$ billions	Growth Rate	$ billions	Growth Rate
1993	426.4			
1994	465.2	9.10%		
1995	500.4	7.57%		
1996	532.5	6.41%		
1997	562.4	5.61%	1042.5	
1998	602.9	7.21%	1129.8	8.38%
1999	666.3	10.52%	1248.9	10.54%
2000	720.8	8.17%	1348.9	8.01%
2001	780.3	8.26%	1464.3	8.56%
2002	836.2	7.16%	1563.9	6.80%
2003	900.2	7.66%	1674.1	7.05%
2004	948.4	5.35%	1767.4	5.57%
2005	1001.9	5.65%	1878.7	6.30%
Observations Per Year		28		70
Mean		7.39%		7.65%
Standard Deviation		1.53%		1.56%
Beta with respect to (r_m-r_f)		-0.011		0.008

Table II: Covariance Between Government Salary Wage Growth and Stock Market

Government wage growth is calculated from the Current Population Survey (CPS) of the US Census for all government workers between 1962 and 2006. For 3 year moving averages, the data therefore begin in 1965. CPI growth is from the Bureau of Labor Statistics (BLS) website. The return on the US stock market net of the risk-free rate is extracted from the Ken French data.

Horizon (Years)		Growth of 3 year Average Government Salaries – CPI Growth	r_m–r_f	Correlation	Beta
1	Mean	0.7%	5.9%	0.056	0.007
	StDev	2.2%	17.2%		
3	Mean	2.1%	16.3%	-0.098	-0.019
	StDev	5.0%	25.8%		
5	Mean	3.8%	36.5%	-0.234	-0.017
	StDev	4.8%	66.1%		

Table III: Distribution of State Municipal Bond Ratings (S&P) and Implied Yields as of December 2005

This table shows the distribution of S&P municipal bond ratings for each of the 50 US states as of December 31, 2005, as well as the distribution of state pension plans. The yield is the yield on municipal bonds in each ratings category as of December 31, 2005 from Bloomberg. The tax grossed-up yield is the yield divided by (1-0.35), the statutory after-tax rate.

	States	Plans	Yield	Tax Grossed-Up Yield
AAA	9	17	4.08%	6.28%
AA+	7	16	4.14%	6.37%
AA	23	57	4.16%	6.40%
AA-	5	6	4.17%	6.42%
A	2	5	4.27%	6.57%
NR	4	7	4.16%	6.40%
Total	50	108		

Table IV: Asset Allocation in State Pension Funds

The table presents mean asset allocation in state pension funds from two different data sources, and our aggregation from these two sources. We use the following procedure to aggregate information from the different sources. 1.) If no assets are in the "Other" census category that includes trusts, we begin with the census numbers and then use the P&I survey to decompose stocks into international and domestic stocks, and bonds into international and domestic bonds. We also use the P&I data to determine allocations to private equity and real estate. If no P&I survey data exists, or if the P&I numbers appear to conflict with the census numbers, then we collect data from the annual report to obtain these decompositions. 2.) If the "Other" census category that includes trusts is less than 5% in the census data, we categorize these assets based on the P&I survey and/or data from the annual reports. 3.) If the "Other" census category that includes trusts is >5% in the census data, we use the P&I survey as the primary source and use the census data and if necessary the annual reports to distinguish domestic from international securities in the stock and bond categories.

US Census of Governments (N=108)

	Sample Means	
	Unweighted	Weighted by Assets
Corporate Stock	37.5%	39.1%
Government Bonds	7.6%	9.0%
Corporate Bonds	16.1%	14.2%
International	12.5%	14.6%
Cash and Deposits	3.6%	3.1%
Real Property (and Misc Non-Securities)	8.4%	8.6%
Mortgages	0.3%	0.5%
Other Securities (incl Trusts)	13.9%	10.8%

Pensions and Investments (P&I) (N=62)

	Sample Means	
	Unweighted	Weighted by Assets
Domestic stock	45.4%	46.5%
International Stock	16.4%	17.4%
Domestic Fixed Income	26.1%	23.7%
International Fixed Income	0.9%	1.1%
Cash and Equivalents	1.9%	1.5%
Private Equity	4.2%	4.0%
Real Estate Equity	3.7%	3.7%
Mortgages	0.7%	1.4%
Other	0.7%	0.8%

Our Categories (N=95)

	Sample Means	
Our Categories (N=95)	Unweighted	Weighted by Assets
Domestic Stock	44.8%	45.8%
International Stock	15.4%	16.4%
Domestic Government Bonds	9.8%	9.7%
Domestic Corp Bonds	16.2%	14.4%
International Govt Bonds	0.5%	0.4%
International Corp Bonds	0.7%	0.7%
Cash and Cash Equivalents	2.7%	2.2%
Private Equity	3.6%	4.0%
Real Estate Equity	3.7%	3.9%
Mortgages	0.6%	1.4%
Other Including Hedge Funds	2.0%	1.1%

Table V: Variance-Covariance Matrix of Asset Classes

This table presents the volatility of each asset class included in Table 4, and the correlations between the asset classes' returns.

Asset Class	Dom. stocks	Dom. gvt. bonds	Dom. corp. bonds	For. stock	For. gvt. bonds	For. corp. bonds	cash	private equity	real estate	MBS	Hedge funds
Asset Class Volatility	15.4%	5.3%	5.2%	15.3%	8.5%	6.6%	0.4%	26.2%	14.3%	2.7%	3.5%
Correlation Matrix											
Domestic stock	1.00										
Domestic gvt. Bonds	-0.04	1.00									
Domestic corp. Bonds	0.01	0.60	1.00								
Internat'l stock	0.28	-0.01	0.04	1.00							
Internat'l gvt. Bonds	0.08	0.32	0.38	0.18	1.00						
Internat'l corp. Bonds	0.14	0.32	0.41	0.24	0.58	1.00					
cash	-0.08	0.04	-0.01	-0.07	-0.11	-0.16	1.00				
private equity	0.51	-0.34	-0.33	0.29	-0.25	-0.52	0.05	1.00			
real estate	0.22	0.16	0.19	0.21	0.28	0.29	-0.08	-0.35	1.00		
MBS	-0.06	0.53	0.56	-0.01	0.31	0.31	0.04	-0.27	0.17	1.00	
hedge funds	0.32	0.08	0.17	0.42	0.32	0.40	-0.04	0.50	0.26	0.07	1.00

Table VI: Cumulative Distribution of Funding Outcomes

Panel A shows the surplus (assets minus liabilities) realized in fifteen or twenty years, in trillions of 2005 dollars, for different levels of the cumulative distribution of outcomes. The table reports the funding surplus at nine levels of the objective probability distribution, assuming market risk-premia of 6.5% and 8%, and also the risk-neutral probabilities. In all columns, we assume that the volatility of liabilities (σ_L) is 1.5% and the real correlation between the liabilities and the market portfolio (ρ) is 0. Panel B shows the probabilities of under and over fundings for each scenario, and the expected magnitude of the realized under or overfunding (again in trillions of 2005 dollars).

Duration	15 years			20 years		
	Market price of risk			Market price of risk		
	6.5%	8%	risk-neutral	6.5%	8%	risk-neutral
Panel A: Magnitude of the under/over funding, at different points in the distribution of outcomes						
1%	-3.48	-3.21	-4.42	-4.91	-4.48	-6.30
5%	-2.86	-2.52	-3.99	-4.05	-3.51	-5.74
10%	-2.48	-2.10	-3.74	-3.52	-2.90	-5.42
25%	-1.74	-1.28	-3.25	-2.47	-1.69	-4.83
50%	-0.75	-0.17	-2.66	-1.01	0.01	-4.05
75%	0.50	1.23	-1.91	0.87	2.20	-3.08
90%	1.88	2.78	-1.08	3.05	4.75	-1.99
95%	2.86	3.88	-0.50	4.63	6.59	-1.21
99%	5.06	6.35	0.80	8.27	10.84	0.60
Panel B: Probability of underfunding and overfunding, and conditional magnitudes of the under/over funding.						
P[s < 0]	66.4%	53.6%	97.3%	64.9%	49.9%	98.3%
E[s|s < 0]	-1.48	-1.30	-2.60	-2.12	-1.83	-3.92
P[s > 0]	33.6%	46.4%	2.7%	35.1%	50.1%	1.7%
E[s|s > 0]	1.53	1.81	0.81	2.41	2.96	1.15

Table VII: Cumulative Distributions of Funding Outcomes Under Different Volatility and Correlation Assumptions

This table shows the cumulative distribution functions of funding outcomes under different assumptions for the volatility of liabilities (σ_L) and the real correlation between the liabilities and the market portfolio (ρ). All values are in trillions of 2005 dollars.

Panel A: Market Price of Risk = 6.5%

σ_L	0	0.015	0.015	0.050	0.050
ρ	0	0.00	0.25	0.00	0.25
1%	-3.34	-3.48	-3.29	-4.70	-4.16
5%	-2.79	-2.86	-2.73	-3.51	-3.13
10%	-2.43	-2.48	-2.38	-2.89	-2.59
25%	-1.73	-1.74	-1.69	-1.86	-1.70
50%	-0.76	-0.75	-0.75	-0.65	-0.65
75%	0.47	0.50	0.44	0.72	0.56
90%	1.85	1.88	1.78	2.17	1.87
95%	2.83	2.86	2.73	3.17	2.78
99%	5.02	5.06	4.87	5.38	4.82

Panel B: Market Price of Risk = 8%

σ_L	0.00	0.015	0.015	0.050	0.050
ρ	0.00	0.00	0.25	0.00	0.25
1%	-3.08	-3.21	-3.02	-4.34	-3.77
5%	-2.46	-2.52	-2.39	-3.12	-2.72
10%	-2.06	-2.10	-2.00	-2.48	-2.17
25%	-1.27	-1.28	-1.23	-1.38	-1.22
50%	-0.18	-0.17	-0.17	-0.07	-0.07
75%	1.20	1.23	1.17	1.43	1.27
90%	2.76	2.78	2.68	3.04	2.73
95%	3.85	3.88	3.75	4.16	3.76
99%	6.32	6.35	6.16	6.65	6.07

Panel C: Risk Neutral

σ_L	0.00	0.015	0.015	0.050	0.050
ρ	0.00	0.00	0.25	0.00	0.25
1%	-4.20	-4.42	-4.24	-6.00	-5.58
5%	-3.87	-3.99	-3.86	-4.89	-4.59
10%	-3.66	-3.74	-3.64	-4.34	-4.11
25%	-3.25	-3.27	-3.22	-3.47	-3.34
50%	-2.68	-2.66	-2.66	-2.53	-2.52
75%	-1.95	-1.91	-1.96	-1.58	-1.70
90%	-1.13	-1.08	-1.18	-0.65	-0.90
95%	-0.56	-0.50	-0.63	-0.03	-0.36
99%	0.74	0.80	0.61	1.30	0.81

Appendix Table I: State Pension Data for 2005, in $ billions

	State Pension Assets	Stated State Pension Liabilities	State General Obligation Debt	State Revenues	State GDP	Number of Plans in Sample (number)	Average Discount Rate (%)
Alabama	28.4	34.0	6.3	22.5	151.3	3	8.00
Alaska	7.3	11.4	5.8	9.1	39.4	2	8.13
Arizona	23.7	25.4	8.0	25.3	212.3	2	8.25
Arkansas	14.5	17.7	4.3	14.9	87.0	3	8.00
California	383.3	398.3	107.4	249.1	1,616.4	3	7.75
Colorado	34.3	46.8	12.4	22.5	214.3	1	8.50
Connecticut	20.0	33.5	23.0	20.6	193.5	3	8.50
Delaware	5.7	5.6	4.4	6.2	56.7	1	8.00
Florida	111.5	103.9	25.9	77.1	666.6	1	7.75
Georgia	60.0	61.3	8.2	36.1	358.4	2	7.50
Hawaii	8.9	13.0	5.8	9.1	54.8	1	8.00
Idaho	8.2	8.8	2.4	7.2	45.9	1	7.75
Illinois	77.6	116.5	48.3	60.1	555.6	4	8.25
Indiana	19.5	29.2	13.3	27.3	236.4	3	7.33
Iowa	18.0	20.2	4.9	15.7	117.6	1	7.50
Kansas	11.3	16.5	5.1	12.5	105.2	1	8.00
Kentucky	27.1	34.3	8.6	21.2	138.6	3	8.00
Louisiana	18.8	29.5	11.5	24.8	180.3	2	8.25
Maine	8.9	11.7	4.6	8.4	44.9	1	8.00
Maryland	34.5	39.1	13.7	28.9	244.4	1	7.75
Massachusetts	33.9	45.7	56.0	41.7	320.1	2	8.25
Michigan	49.5	62.1	26.2	55.7	372.1	4	8.00
Minnesota	37.7	42.4	7.3	31.7	231.4	3	8.50
Mississippi	17.4	24.1	4.3	15.5	79.8	2	8.00
Missouri	32.5	38.6	16.2	26.8	215.1	3	8.00
Montana	5.7	7.0	3.7	5.7	29.9	2	8.00
Nebraska	5.6	6.5	1.7	8.7	72.2	2	8.00
Nevada	17.7	23.4	3.9	11.5	110.2	1	8.00
New Hampshire	4.0	5.3	6.9	6.2	54.1	1	8.50
New Jersey	83.1	101.0	42.3	51.3	427.7	4	8.38
New Mexico	17.5	21.5	5.9	13.3	69.7	2	8.00
New York	220.5	221.4	102.0	140.2	961.4	2	7.50
North Carolina	64.1	59.1	15.8	44.9	350.7	2	7.25
North Dakota	2.7	2.9	1.7	3.9	24.9	2	8.00
Ohio	127.3	159.9	23.1	72.2	442.2	5	8.05
Oklahoma	15.4	25.9	7.5	17.9	121.6	4	7.63
Oregon	51.4	49.3	11.3	22.6	141.8	1	8.00
Pennsylvania	78.0	90.1	27.7	69.8	486.1	2	8.50
Rhode Island	5.4	9.8	6.8	7.3	43.6	1	8.25
South Carolina	24.4	33.4	13.4	23.0	140.1	2	7.75
South Dakota	5.4	5.6	2.6	4.1	30.5	1	8.00
Tennessee	27.8	28.1	3.6	24.7	225.0	1	7.50
Texas	134.4	152.4	18.2	96.1	989.3	4	7.75
Utah	14.7	15.7	5.3	13.1	88.4	3	8.00
Vermont	2.8	2.9	2.8	4.6	23.1	3	8.00
Virginia	40.4	49.6	16.9	37.3	350.7	1	8.00
Washington	44.8	50.6	17.0	36.8	271.4	4	8.00
West Virginia	5.0	10.7	5.0	12.2	53.1	1	8.00
Wisconsin	68.6	69.0	18.8	34.4	217.0	1	8.00
Wyoming	5.3	5.2	0.9	5.9	27.2	3	8.00
TOTAL($billions)	2,164	2,476	798	1,638	12,290		

Appendix Table II: Government Employee Wage Data from the Current Population Survey

CPS Year	Mean Wage Income	Standard Deviation of Wage Income	Growth in Mean Wage Income	CPI Inflation	Government Wage Growth Minus CPI Inflation	rf	rm	rm-rf
1962	5000	2724				2.7%	-10.3%	-13.1%
1963	5033	2446	0.7%	1.3%	-0.7%	3.1%	20.9%	17.8%
1964	5327	2881	5.8%	1.3%	4.5%	3.5%	16.3%	12.8%
1965	5471	2632	2.7%	1.6%	1.1%	3.9%	14.4%	10.5%
1966	5791	3159	5.9%	2.9%	3.0%	4.8%	-8.7%	-13.4%
1967	6023	2874	4.0%	3.1%	0.9%	4.2%	28.6%	24.4%
1968	6598	3699	9.6%	4.2%	5.4%	5.2%	14.2%	9.0%
1969	7148	4188	8.3%	5.5%	2.9%	6.6%	-10.8%	-17.4%
1970	7734	3686	8.2%	5.7%	2.5%	6.5%	0.1%	-6.4%
1971	8446	4279	9.2%	4.4%	4.8%	4.4%	16.2%	11.8%
1972	8814	5184	4.4%	3.2%	1.2%	3.8%	17.3%	13.5%
1973	9357	4777	6.2%	6.2%	-0.1%	6.9%	-18.8%	-25.7%
1974	9977	5379	6.6%	11.0%	-4.4%	8.0%	-27.9%	-36.0%
1975	10556	5762	5.8%	9.1%	-3.3%	5.8%	37.4%	31.6%
1976	11499	5646	8.9%	5.8%	3.2%	5.1%	26.8%	21.7%
1977	12248	5944	6.5%	6.5%	0.0%	5.1%	-3.0%	-8.1%
1978	12924	6336	5.5%	7.6%	-2.1%	7.2%	8.6%	1.4%
1979	13829	7108	7.0%	11.3%	-4.3%	10.4%	24.4%	14.0%
1980	14879	8008	7.6%	13.5%	-5.9%	11.3%	33.2%	22.0%
1981	16236	7193	9.1%	10.3%	-1.2%	14.7%	-4.0%	-18.7%
1982	17813	7941	9.7%	6.2%	3.6%	10.5%	20.4%	9.9%
1983	18333	9481	2.9%	3.2%	-0.3%	8.8%	22.7%	13.9%
1984	19926	10540	8.7%	4.3%	4.4%	9.8%	3.2%	-6.7%
1985	21048	11155	5.6%	3.6%	2.1%	7.7%	31.4%	23.7%
1986	21657	11127	2.9%	1.9%	1.0%	6.2%	15.6%	9.4%
1987	23038	11824	6.4%	3.6%	2.7%	5.5%	1.8%	-3.7%
1988	24311	12388	5.5%	4.1%	1.4%	6.4%	17.6%	11.2%
1989	25143	12972	3.4%	4.8%	-1.4%	8.4%	28.4%	20.1%
1990	26749	13631	6.4%	5.4%	1.0%	7.8%	-6.1%	-13.9%
1991	27400	13880	2.4%	4.2%	-1.8%	5.6%	33.6%	28.0%
1992	29008	14717	5.9%	3.0%	2.9%	3.5%	9.1%	5.6%
1993	30067	15131	3.7%	3.0%	0.7%	2.9%	11.6%	8.7%
1994	30719	15738	2.2%	2.6%	-0.4%	3.9%	-0.8%	-4.7%
1995	31699	16922	3.2%	2.8%	0.4%	5.6%	35.7%	30.1%
1996	33301	24617	5.1%	3.0%	2.1%	5.2%	21.2%	16.0%
1997	33627	22699	1.0%	2.3%	-1.3%	5.3%	30.3%	25.1%
1998	34860	21563	3.7%	1.6%	2.1%	4.9%	22.3%	17.4%
1999	36204	21813	3.9%	2.2%	1.6%	4.7%	25.3%	20.6%
2000	37003	21555	2.2%	3.4%	-1.2%	5.9%	-11.1%	-16.9%
2001	39358	24954	6.4%	2.8%	3.5%	3.9%	-11.3%	-15.1%
2002	40366	25560	2.6%	1.6%	1.0%	1.6%	-20.8%	-22.5%
2003	41402	28042	2.6%	2.3%	0.3%	1.0%	33.1%	32.1%
2004	43600	30343	5.3%	2.7%	2.6%	1.2%	13.0%	11.8%
2005	43281	27485	-0.7%	3.4%	-4.1%	3.0%	7.3%	4.4%
2006	45182	33045	4.4%	3.2%	1.2%	4.8%	16.2%	11.4%
Mean			5.2%	4.4%	0.7%	5.8%	12.2%	6.4%
Volatility			2.6%	2.9%	2.6%	2.7%	16.8%	17.0%