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DEBT AND TAXES IN THE THEORY OF PUBLIC FINANCE

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ABSTRACT

If a specified amount of government spending must be financed, how should that finance be divided between taxes and government borrowing? In the case of a temporary increase in government spending, it has been argued that debt finance is optimal because the small increments in all future tax rates to finance interest payments involves a smaller excess burden than the single large tax rate increase that would be required to avoid an initial increase in the national debt. This argument ignores the excess burden of debt finance that results if the initial capital stock is smaller than optimal (e.g., because of taxes or capital income).

The first section of the present paper shows how the debt-finance advantage of a small increase in tax rates can be explicitly balanced against the disadvantage of the excess burden that arises from additional debt. The analysis shows that, with plausible parameter values, the excess burden of debt finance is likely to outweigh the advantage of avoiding a large single tax change and therefore that financing a temporary increase in government spending by an immediate tax increase is likely to be preferable to debt financing.

The second section examines the appropriate response to a permanent increase in government spending and shows that such spending cannot be financed by a permanent increase in government debt. Moreover, whenever the golden rule level of capital intensity is an optimality condition independent of the level of government spending, any increase in government spending should be matched by an equal increase in tax revenue.

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Debt and Taxes in the Theory of Public Finance

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This paper investigates what is probably the most basic question in public finance: If a specified amount of government spending must be financed, how should that finance be divided between taxes and government borrowing? Rather surprisingly, this question has been given relatively little analytic attention.

The nineteenth century writers on fiscal theory advocated balanced budgets but did so as a matter of virtue and prudence rather than as a result of an analysis of economic efficiency.[1] Balanced budgets were also preferred as a matter of equity on the "benefit principle" of taxation as a way of forcing the beneficiaries of government spending to pay for those outlays.[2] It was also emphasized by Wicksell (1896) and others that the principle of balanced budgets causes the political process to weight the costs and benefits of government spending more carefully. I shall ignore these issues in the current paper in order to focus directly on the relative economic efficiency of taxation and debt finance.

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[1] See, for example, Bastable (1903) and the discussion in Buchanan and Wagner (1977, Chapter 2).

[2] See Musgrave (1959) for a discussion of this.

There is, however, one common line of reasoning which suggests the contrary conclusion that any temporary increase in government spending should be financed by borrowing with only enough increase in taxes to finance the interest on the increased public debt. This conclusion starts from the observation that the excess burden of taxation depends on the square of the tax rate. It follows from this that it is better to have a large number of small increments in the tax rate over time to finance interest payments than to have a single large increase in the tax rate to finance the initial spending.[3]

This argument ignores the possibility that an increase in the public debt involves an additional excess burden. If the initial capital stock is smaller than optimal (e.g., because of taxes on capital income) and the increase in government borrowing reduces the capital stock further,[4] the debt financing entails a separate excess burden that must be explicitly recognized in the choice between debt and taxes.[5] Equivalently, if the interest rate on the public debt that the government uses to finance the increased spending (or the marginal product of the capital displaced by public borrowing) exceeds the discount rate that is appropriate for intertemporal welfare aggregation, the

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[3]This line of argument can be found in Barro (1979).

[4]There is a large literature, including an important paper by Modigliani (1961), that establishes that an increase in government debt reduces the capital stock. Barro (1974) showed that under certain extreme conditions the public borrowing would completely be offset by an induced equal increase in private saving. I shall ignore this possibility in the current paper unless I explicitly indicate otherwise.

[5]Note that the question of whether there is an excess burden of the public debt is different from the traditional question of whether the debt induces any burden. Several authors, including Buchanan (1958), Diamond (1965), Meade (1958) and Modigliani (1961) have shown that the debt conveys a burden on future generations but did not discuss the implications of this for the choice between debt finance and tax finance.

debt finance involves a first-order excess burden.

The first section of the present paper shows how the debt-finance advantage of a small increase in tax rates can be explicitly balanced against the disadvantage of the excess burden that arises from additional debt. The analysis shows that, with plausible parameter values, the excess burden of debt finance is likely to outweigh the advantage of avoiding a large single tax change and therefore that financing a temporary increase in government spending by an immediate tax increase is likely to be preferable to debt financing.

The second section examines the appropriate response to a permanent increase in government spending and shows that such spending cannot be financed by a permanent increase in government debt. There is a very brief concluding section.

1. Optimal Financing of a Temporary Spending Increase

Consider an economy that is in full-employment growth equilibrium with the population and labor force growing at rate n.[6] There are N_s people in the labor force in year s and each individual supplies an amount of labor ℓ, making the total labor supply $L_s = \ell N_s$. The wage rate per unit of labor rises because of technical progress at rate λ, making $w_s = (1+\lambda)^{s-1} w_1$ for $t \geq 2$. Taxes are levied at rate τ on wage income, producing total revenue

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[6]The assumption of full-employment explicitly rules out demand effects of fiscal policy on the level of economic activity. The extent to which debt finance may be preferred to tax finance in an economy with unemployed resources depends on the interest sensitivity of money demand among other things.

$T_s = \tau w_s L_s$ at time s. There are no taxes on capital income.[7]

The government initially has a spending plan that calls for outlays of G_s in year s. In general, G_s need not equal T_s. The government then decides to increase spending, in year 1 only, by an amount dG. After year 1, government spending (exclusive of interest payments on the national debt) returns to its initial path. How should the increased spending be financed?

To answer the question, the total burden of financing dG dollars of spending by taxes must be compared with the total burden of financing dG dollars of spending by borrowing at time 1 and subsequently paying interest on the debt. If the burdens per dollar of tax revenue and per dollar of borrowing remain constant for amounts up to dG dollars, the optimal financing will be all taxes or all borrowing. If the burden per dollar varies with the amount of taxation or borrowing, a mixed solution may be possible. I shall assume that dG is small enough to imply constant per dollar burdens.

1.1 The Burden of Tax Finance

Financing the increased spending solely by additional taxation (without any change in the government deficit or surplus) requires increasing total tax revenue in year 1 by an amount equal to the increase in government spending: $dT_1 = dG$. Since $T_1 = \tau w_1 L_1$, this implies

(1) $$dG = w_1 L_1 d\tau + \tau w_1 \frac{dL_1}{d\tau} d\tau$$

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[7]A tax on capital income would complicate the analysis substantially without altering the basic structure of the argument as long as debt finance reduces the capital stock by more than tax finance.

$$= w_1 L_1 d\tau + \tau w_1 \frac{dL_1}{d(1-\tau)w_1} \frac{d(1-\tau)w_1}{d\tau} d\tau .$$

Writing $\eta = [dL_1/d(1-\tau)w_1]\ [(1-\tau)w_1/L_1]$ for the uncompensated elasticity of labor supply with respect to the net wage permits equation 1 to be written as

$$(2) \qquad dG = w_1 L_1 d\tau - \frac{\tau w_1 L_1 \eta}{1-\tau} d\tau$$

or

$$(3) \qquad dG = w_1 L_1 \left[\frac{1-\tau - \tau\eta}{1-\tau}\right] d\tau$$

Equation 3 defines the increase in the tax rate required to finance the additional spending. This higher tax rate imposes a burden on the population that exceeds the amount of the revenue raised by the difference between the value of the increase leisure that results from the distortionary taxation and the pretax wages that might have been earned on that decrement of labor supply.[8] Because of the preexisting tax at rate τ, the first unit of increased leisure involves an excess burden of τw. Since the excess burden on the final unit of increased leisure is therefore $(\tau + d\tau)w_1$, the excess burden is $(\tau + 0.5d\tau)w_1 dL_1$, where dL_1 is the induced change in the labor supply implied by the compensated supply function. Thus $dL_1 = [\varepsilon L_1/(1-\tau)]d\tau$ where ε is the compensated labor supply elasticity. The excess burden of increasing the tax

[8]The change in leisure (and labor supply) is, of course, to be evaluated by the compensated labor supply functions.

rate by $d\tau$ is therefore $[\varepsilon w_1 L_1(\tau+0.5d\tau)/(1-\tau)]d\tau$. Note that in the absence of a preexisting tax ($\tau=0$) this reduces to the familiar excess burden "triangle" formula, $0.5\ \varepsilon w_1 L_1 (d\tau)^2$.

The total burden of financing the increased government spending by taxation is the sum of dG and this excess burden:

(4) $$B_T = dG + [\varepsilon w_1 L_1(\tau+0.5d\tau)/(1-\tau)]dr$$

where B_T is the burden of finance by taxation and $d\tau$ satisfies the financing requirement expressed by equation 3. Thus

(5) $$B_t = dG[1 + \frac{\varepsilon\tau}{1-\tau-\tau\eta} + \frac{1}{2}\varepsilon \frac{dG}{w_1 L_1} \frac{(1-\tau)}{(1-\tau-\tau\eta)^2}]$$

The first term in the bracket represents the resources that are transferred to finance the increased government spending. The second term is the first order welfare loss that occurs because of the preexisting tax. The $\tau\eta$ in the denominator of that term increases this welfare loss, reflecting the fact that the tax increase ($d\tau$) causes a decline in labor supply (if $\eta > 0$) and therefore in the revenue collected at the preexisting tax rate. The final term is the welfare loss caused by the increased tax rate and, unlike the second term, remains even if there is no preexisting tax. Note that this is a second order term in that it is proportional to $(dG)^2/wL$, the square of the increased government spending as a proportion of the total tax base.

1.2 <u>The Burden of Debt Finance</u>

Financing the increased spending without a concurrent tax increase requires increasing the government debt in year 1 by an amount equal to the increased spending, dG. The interest to be paid on that debt is then $r \cdot dG$ in year 2 and all subsequent years.[9] To finance this interest, the government must raise the tax rate by an amount $d\tau_s$ in each future year beginning with year $s = 2$. This increased tax rate will in general not be constant if the tax base $(w_s L_s)$ grows in future years. If the preexisting tax rate remains constant at τ and the labor supply elasticities also remain constant at ε and η, the burden of raising revenue $r \cdot dG$ in year $s \geq 2$ is, by analogy with equation 5,

$$(6) \qquad B_{Ds} = r \cdot dG \left[1 + \frac{\varepsilon - \tau}{1-\tau-\tau\eta} + \frac{1}{2}\varepsilon \frac{dG}{w_1 L_1 (1+n)^{s-1}(1+\lambda)^{s-1}} \frac{(1-\tau)}{(1-\tau-\tau\eta)^2}\right]$$

where the extra term $(1+n)^{s-1}$ reflects the growth of the population from $s = 1$ and the term $(1+\lambda)^{s-1}$ reflects the growth of the wage rate.[10]

A further word about the annual interest cost is appropriate. In the simple model of the economy developed here, it is appropriate to assume

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[9] On the simplifying assumption that the interest rate remains constant at r.

[10] This assumes that labor supply decisions are made myopically as a function of the current net wage. If this is not true, the ε and η of equation 6 would be smaller than the corresponding values of equation 5. Dropping the assumption of myopic labor supply and explicitly recognizing intertemporal labor substitution raises the excess burden of tax finance but does not alter the conclusion (derived below) that the basic excess burden of debt finance is a first order magnitude while the basic excess burden of tax finance is a second order magnitude.

that the interest rate is equal to the marginal product of capital. In a more complete analysis, it would be necessary to recognize that the interest rate on government debt may be less than the marginal product of capital and that the government further reduces the net interest cost by taxing these interest payments. Nevertheless, the government borrowing displaces private capital on the earnings of which a tax would have been collected. On balance, the most appropriate assumption is therefore that the interest rate in equation 6 is equal to the marginal product of capital.

To compare the burden of tax finance and debt finance, the annual debt finance burdens (B_{D_s}) must be discounted back to year s = 1. Since the annual debt finance burdens are valued as the required compensating variations in consumer income, the appropriate discount rate reflects the marginal rate of substitution between consumers' income in adjacent years. Under certain restrictive assumptions this implies that the appropriate discount rate would be the same as the market interest rate, r. More generally, however, the two will not be equal and the appropriate time preference discount rate (δ) will be less than the market interest rate.

An important practical reason for a lower discount rate would be the presence of an individual income tax on interest income. Although I have ignored the existence of such a tax in the derivation of equations 5 and 6 in order to simplify the calculation of the annual excess burden of taxation, the taxation of interest income could hardly be ignored in a more general analysis of the appropriate discount rate.

But even in the absence of a tax on interest income, the appropriate intertemporal discount rate may well be less than the marginal product of capital. Although the market interest rate might be regarded as an appropriate rate for intrageneration aggregations of income, it need not be as an appropriate basis for intergeneration comparisons of income. More specifically, if successive generations are not linked by bequests, there is no rationale for aggregating income by the market interest rate. Even if the generations are linked by bequests, there is no compelling reason to regard the preferences of the bequethers as the normatively appropriate basis for intertemporal comparisons.

An explicit utilitarian analysis would instead base the discount rate on the rate of growth of per capita consumption and the elasticity of the marginal utility function. If consumption grows at the same rate as wages (λ) and the elasticity of the marginal utility function is denoted μ, the marginal rate of substitution between income in adjacent years is

$$(7) \qquad 1+\delta = (1+\lambda)^{\mu}$$

and δ is the appropriate discount rate. The long-run growth of real per capita consumption has been less than 2 percent.[11] Although μ is not observable, a value of $\mu = 2$ would be regarded as high since it would imply that the marginal utility of consumption would be reduced to one-half of its previous value by a 41 percent increase in per capita consumption -- the increase that occurred between 1966 and 1982. A value of $\mu = 2$ and $\lambda = .02$ implies δ =

[11]From 1929 through 1982, real per capita consumption in the United States rose an an annual rate of 1.64 percent.

.04. A lower value of μ or λ would imply a lower value of δ. By comparison, the pretax real rate of return on capital has averaged about 12 percent for the postwar period (Feldstein, Poterba and Dicks-Mireaux). An explicit utilitarian calculation thus implies $\delta < r$.

Discounting the annual debt burdens given by equation 6 to time s = 1 by the discount rate δ implies

$$(8) \qquad B_D = \sum_{s=2}^{\infty} B_{Ds}(1-\delta)^{-(s-1)}$$

$$= \frac{rdG}{\delta}[1+ \frac{\varepsilon\tau}{1-\tau-\tau\eta}] + \frac{1}{2}\frac{\varepsilon(1-\tau)}{(1-\tau-\tau\eta)^2}\frac{r^2(dG)^2}{w_1L_1(\delta+\nu+\nu\delta)}$$

where $\nu = n+\lambda+n\lambda$ is the growth rate of total wages.

1.3 Comparing Tax and Debt Finance

We can now compare the burden of tax finance (B_T of equation 5) with the burden of debt finance (B_D of equation 8). The analysis is clearest if we begin by examining the special case in which there is no preexisting tax (τ=0) and in which the tax base remains constant (ν=0). In this special case, $B_D > B_T$ and tax finance is preferable to debt finance if and only if

$$(9) \qquad \frac{r}{\delta}dG[1 + \frac{1}{2}\varepsilon\frac{r\cdot dG}{w_1L_1}] > dG[1 + \frac{1}{2}\varepsilon\frac{dG}{w_1L_1}],$$

or

$$(10) \qquad \frac{r}{\delta} - 1 > \frac{1}{2}\varepsilon\frac{dG}{w_1L_1}(1 - \frac{r^2}{\delta}).$$

Several conclusions (for this special case) are immediately apparent. First, tax finance is preferable only if $r > \delta$. It is the excess of r over δ that implies an excess burden of debt finance that is separate from the excess burden of tax finance. Second, the left hand side (representing the excess burden that is particular to debt finance) is independent of the relative size of the additional government spending while the right hand side (representing the excess burden that is particular to tax finance) varies in proportion to the ratio of the government spending increase to the total tax base. The excess burden of the debt finance is a first order magnitude while the excess burden of tax finance is a second order magnitude. Thus, as dG/wL tends to zero, tax finance tends to be unequivocally better. Third, the excess burden that is particular to debt finance does not depend on the elasticity of labor supply while the excess burden that is particular to tax finance does. Thus, as the compensated elasticity of labor supply tends to zero, tax finance tends to be unequivocally better.

A numerical example will illustrate the likely superiority of tax finance. To make this point, I will overstate the values of those parameters that favor debt finance (ε, dG/wL and δ) and understate the parameter that favors tax finances (r). Let $\varepsilon = 1$, $dG/wL = 0.1$, $\delta = 0.05$ and $r = 0.10$. Then the inequality of (10) is satisfied since $1 > .04$. Indeed, even with $\varepsilon = 1$, $dG/wL = 0.1$ and $r = 0.10$, the inequality is satisfied for any δ less than 0.096. It takes a very small discrepancy between r and δ to dictate tax finance for even a relatively large increase in spending.

Examination of equations 5 and 8 shows that the pre-existing of an initial tax at rate τ has two effects. First, the preexisting tax implies an extra first-order excess burden which is equal to $(dG)\varepsilon\tau/(1-\tau-\tau\eta)$ for tax finance and to $r(dG)\varepsilon\tau/(1-\tau-\tau\eta)$ for debt finance. With $r > \delta$, the increased burden is greater for the debt finance. Second, the preexisting tax implies that the second-order excess burden terms of both B_T and B_D are multiplied by $(1-\tau)/(1-\tau-\tau\eta)^2$; if $\eta > 1$, this term is greater than one and the second order excess burden terms are both increased proportionately. Since the second-order excess burden term is larger in the tax burden than in the debt burden,[12] this increases the tax burden by more than it increases the debt burden. Although these effects point in opposite directions, the net effect is unambiguously to increase the attractiveness of debt finance relative to tax finance. To see this, note that combining equations 5 and 8 and simplifying shows that $B_D > B_T$ if and only if:

$$(11) \qquad \frac{r}{\delta} - 1 > \frac{1}{2}\,\varepsilon\,\frac{dG}{w_1 L_1}\left(1 - \frac{r^2}{\delta+\nu+\nu\delta}\right)\frac{1-\tau}{(1-\tau-\tau\eta)(1-\tau+\tau(\varepsilon-\eta))}.$$

Since $\varepsilon - \eta$ can be expressed as the marginal propensity to consume leisure out of exogenous income[13] and therefore $\varepsilon - \eta < 1$, the final term in square brackets is unambiguously greater than one. The other terms are the same as in inequality 10 (except for the terms involving ν which were previously taken to be zero) so that preexisting tax means that r/δ must be greater than $B_D > B_T$.

Despite the additional terms, it remains true that for plausible

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[12]The second-order term of the debt burden equals the second-order term of the tax burden multiplied by $r^2/(\delta+\nu+\delta\nu) < 1$.

[13]This follows directly from Slutsky decomposition of the uncompensated elasticity: $\eta = \varepsilon-\partial wL/\partial y$ where y is exogenous income.

parameter values $B_D > B_T$ and tax finance if preferable. Since high values of τ, η and ν favor debt finance, I will select $\tau = 0.5$, $\eta = 0.5$, and $\nu = 0.05$. With $\varepsilon = 1$, $r = 0.1$ and $dG/wL = 0.1$, inequality 11 is satisfied for any $\delta <$ .089. With more realistic parameter values, the inequality implies that tax finance is even more preferable to debt finance.

One further issue deserves comment. In assessing the cost of debt finance I have assumed that individuals do not adjust their private saving to offset the effect of government borrowing on the nation's capital stock. It might instead be assumed, following Barro (1974), that individuals would not reduce their consumption in response to a temporary tax increase but would instead reduce their previously accumulated assets (or borrow) in order to spread the burden over the future in the same way that the government could by borrowing, including a reduction in bequests to force future generations to share in the financing of the incremental expenditure dG. This would increase the burden of tax finance and could in principle reverse the preference between debt and tax finance. Although some individuals may of course behave in approximately this way, I believe (and have argued elsewhere at length, e.g., Feldstein 1982) that such a theory of behavior is of very limited overall empirical relevance. A government that uses personal taxation to finance a war or a temporary bulge in domestic spending is likely to find that such taxation reduces consumption by substantially more than if the same spending is instead financed by government borrowing.

2. Optimal Financing of a Permanent Spending Increase

The previous section focused on the optimal choice between debt and taxes to finance a temporary increase in government spending. The analysis was an example of second-best policy for an economy in which some constraint on government behavior prevented eliminating the difference between the marginal product of capital and the rate of time discount. If that difference could be eliminated, an increase in debt would be the preferred way to finance a temporary increase in spending. But in the second-best situation of an inadequate capital stock, an increase in taxation will, for plausible parameter values, be preferable.

The present section examines a very different problem: the optimal financing of a permanent increase in the per capital level of government spending in an economy in steady state growth, with optimality defined as maximizing the steady state utility level of a representative individual. No constraint on tax policies is assumed although it is recognized that the government does not have direct control over the economy's resources.

By analogy with the question posed in section 1, it is now natural to ask under what conditions a permanent increase in government spending should be financed by an equal increase in taxes and under what conditions it should be financed by an increase in government debt. Such a question may seem natural, but it fundamentally misconceives the nature of the long-run financing problem.

The nature of the government's equilibrium budget constraint implies that the correct question is: under what conditions should a permanent increase in government spending be financed by an increase in taxes and under what conditions should it be financed by a <u>decrease</u> in government debt. To see this, consider an economy in steady state growth at rate n with current labor force N. The government has debt B on which it pays interest at rate r which is equal to the marginal product of capital. It levies a tax T and has initial government spending G. The government's total outlays are therefore G + rB. It must finance this by a combination of tax revenue and the increase in outstanding debt, $\dot{B}$. On a steady state growth path, the debt must grow at the same rate as population: $\dot{B}/B = n$ or $\dot{B} = nB$. Thus the government's budget constraint in equilibrium is

(12) $$G + rB = T + nB.$$

Dividing each term by the labor force and using lower case letters for the resulting values (thus $g = G/N$) gives

(13) $$g = t-(r-n)b.$$

If the economy's equilibrium corresponds to the golden rule level of capital intensity, $r = n$ and the taxes are necessarily equal to government spending. More generally, if the capital intensity is less than the golden rule level, $r > n$ and equation 13 implies that $g < t$. Equation 13 also shows that an increase in government spending must be financed either by an increase in taxes with per capita debt unchanged ($dg = dt$) or, if taxes are unchanged, by a <u>decrease</u> in per capita debt ($dg = -(r-n)db$). Of course, decreasing the steady state level of per capital debt requires a period of increased tax revenue. <u>Thus, the real choice open to a government that increases permanently the level of spending is either to increase the level of taxation permanently by an equal amount or to increase the level of taxation tem-</u>

porarily by a greater than equal amount in order to reduce the existing government debt. In either case, an increase in government spending requires an increase in tax revenue.

When is it optimal for the government to respond to an increase in steady-state spending by an equal increase in the steady-state level of taxation, leaving the per capita level of government debt unchanged? Although an interesting general characterization of necessary conditions does not seem possible, it is clear from equation 13 that is optimal to offset any increase in g by an equal increase in t whenever the golden rule level of capital intensity is an optimality requirement that is independent of the level of g.

An important special case will illustrate the existence of such a condition. Assume that each individual lives two periods, works an amount (ℓ) during the first period and is fully retired during the second period. The individual receives a wage rate w per unit of labor, pays a proportional tax on his income at rate $1-\theta$, and therefore earns net labor income of $\theta\omega\ell$. The individual's decision about how must to work and to save is made by maximizing a log-linear utility function[14]

(14) $$u = \alpha \ln c_1 + (1-\alpha) \ln c_2 + \beta \ln (1-\ell)$$

subject to the budget constraint

(15) $$c_2 = (\theta\omega\ell - c_1)(1 + r)$$

The first order conditions imply that the optimal labor supply is $\ell = 1/(1+\beta)$ and that $c_1 = \alpha\theta\omega\ell$. The individual's utility can therefore be

[14]The analysis would not be changed if a function of g was added to the terms in equation 14.

written

$$(16) \qquad u = \phi + \ln \theta\omega + (1-\alpha) \ln (1+r)$$

where ϕ is a constant. The government's problem is to choose θ to maximize u of equation 16, subject of course to the constraints implied by the government's budget, by the capital accumulation process, and by the links between capital intensity and the values of w and r.

If k is the capital stock per unit of labor and f(k) is the output per unit of labor, the marginal productivity conditions imply $r = f'$ and $w = f - kf'$. Instead of specifying a level of government debt, I will write k_g for the amount of government capital. The government's budget constraint is, by analogy with equation 13,

$$(17) \qquad g = (1-\theta)w\ell + (f'-n)\ell k_g.$$

The stock of private capital (K_p) at any time is equal to the previous savings of the current generation of retirees:

$$(18) \qquad K_p = (\theta\omega\ell - c_1)N\ (1+n)^{-1}.$$

In steady state equilibrium, the rate of growth of the private capital stock must equal the rate of growth of the population: $\dot{K}_p/K_p = n$. Equation 18 therefore implies that in steady state equilibrium

$$(19) \qquad \ell k_p = (\theta\omega\ell - c_1)(1+n)^{-1},$$

or, using the fact that $c_1 = \alpha\theta\omega\ell$,

$$(20) \qquad \ell k_p = (1-\alpha)(1+n)^{-1}\theta\omega\ell.$$

In equilibrium, the ratios of k_g and k_p to the total capital stock per unit of labor must remain constant: let $k_g = \gamma k$ and $k_p = (1-\gamma)k$. Using this constancy, equations 17 and 20 can be combined to solve for the capital accumulation constraint:

$$(21) \qquad g - (1-\theta)w\ell = (f'-n)[\ell k-(1-\alpha)(1+n)^{-1}\theta\omega\ell].$$

The right hand side term in square brackets is the government capital stock per worker, i.e., the excess of total capital per worker over the privately provided savings of the current retirees. The entire right hand side is thus equal to the earnings of the government's capital in excess of the amount required to maintain that equilibrium level of government capital per capita. The government's excess capital income can finance the difference between government spending and tax revenue shown on the left hand side of the equation.

Equation 21 can be solved for θw as a function of k and this can be substituted into equation 16 (using $\ell = 1/(1+\beta)$) to yield:

$$(22) \qquad u = \phi + \ln \frac{f-nk - g(1+\beta)}{1+(1-\alpha)(f'-n)(1+n)^{-1}} + (1-\alpha) \ln (1+f')$$

The first order condition for maximizing u with respect to k is:[15]

$$(23) \qquad \frac{f'-n}{f-nk-g(1+\beta)} - \frac{(1-\alpha)f''}{1+n} + \frac{(1-\alpha)f''}{1+f'} = 0$$

This is clearly satisfied by $f' = n$, thus establishing the optimality of the golden rule level of capital intensity. It follows immediately from equation

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[15]Although the government does not control k directly, it can achieve the desired level of k by its tax policy.

21 that in this case any increase in g must therefore be financed by an equal increase in tax revenue, $(1-\theta)w$.

Under more general conditions on the nature of the individual's utility function or the structure of the tax system, it may not be optimal for the government to achieve the golden rule level of capital intensity. If, at the initial level of government spending $k_g > 0$ and $f' > n$, the earnings of the government's capital stock help to finance the government spending. An increase in g can then be financed either by an equal increase in taxes ($dt = dg$) or by using a temporary period of even higher taxes to increase k_g $[(r-n)dk_g = d_g]$. In either case, the result is an immediate increase in taxes that is at least equal to the percent rise in g. It is only if the optimal response to a rise in g is an even greater rise in the permanent level of taxes that the immediate response would be a temporary tax decline.

3. Conclusion

An increase in government spending must be financed by an increase in taxes. The optimal choice between tax finance and debt finance is really a choice about the timing of those taxes. A permanent increase in government spending must be matched by at least an equally large permanent increase in taxes unless taxes are increased by even more in the short run. As section 2 showed, there is no way to choose between a permanently higher level of taxes and a permanently higher level of debt.

In the more realistic case in which the government must decide how to finance a temporary rise in spending, the choice is really an empirical question. If the capital stock is initially at an optimal level, it is in general better to finance a temporary rise in spending by an increase in government debt. But when the capital stock is initially below the optimal level, it is likely to be better to finance the spending increase by a concurrent increase in taxes. The analysis of section 1 showed that plausible parameter values imply the optimality of concurrent tax finance.

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Bibliography

Barro, R. (1974), "Are Government Bonds Net Wealth," Journal of Political Economy 82, 6, 1095-1117.

Barro, R. (1979), "On the Determination of the Public Debt," Journal of Political Economy 87, 5, 940-71.

Bastable, C.F. (1903), Public Finance, London: MacMillan Press.

Buchanan, J. and R.E. Wagner (1977), Democracy in Deficit, New York: Academic Press.

Buchanan, J. (1958), Public Principles of Public Debt, Homewood, Illionis: Irwin.

Diamond, P.A. (1965), "National Debt in a Neoclassical Growth Model," American Economic Review 55, 1125-50.

Feldstein, M. (1982), "Government Deficits and Aggregate Demand," Journal of Monetary Economics 9, 1-20.

Feldstein, M., J. Poterba and L. Dicks-Mireaux (1981), "The Effective Tax Rate and the Pretax Rate of Return," National Bureau of Economic Research Working Paper No. 740, forthcoming in the Journal of Public Economics.

Meade, J.E. (1958), "Is the National Debt a Burden?" Oxford Economic Papers, 10, 163-83.

Modigliani, F. (1961), "Long Run Implications of Alternative Fiscal Policies and the Burden of the National Debt," Economic Journal 71, 728-55.

Musgrave, R. (1959), The Theory of Public Finance, New York: McGraw-Hill.

Wicksell, K. (1896), "A New Principle of Just Taxation," in R.A. Musgrave and A. Peacock (eds.), Classics in the Theory of Public Finance (London: MacMillan Press, 1958).