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SOCIAL SECURITY AND HOUSEHOLD PORTFOLIO ALLOCATION

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## ABSTRACT

The entitlement to social security retirement benefits is a major component of aggregate household wealth. This paper focuses on the impact of social security annuities on household portfolio allocation, extending existing optimizing models of portfolio allocation to explicitly consider the role of social security. The model is implemented using cross-section data. The partial equilibrium impacts of changes in social security benefits on portfolio choice and composition are small but precisely measured. The general equilibrium impacts on asset markets of a social security policy change (focusing on links between social security and dynamic wealth accumulation and between social security benefits and private pension benefits) are generally much larger.

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### SOCIAL SECURITY AND HOUSEHOLD PORTFOLIO ALLOCATION

The entitlement to social security retirement benefits is a major component of aggregate household wealth in the U. S. This paper focuses on the impact of "social security annuities" on household portfolio allocation and exposes the ways in which optimizing agents deal with exogenous changes in government retirement saving policies. Considering the effect of the social security system on portfolio choice and composition is an important step toward understanding the system's impact on savings.

The entitlement to social security benefits in retirement is a type of annuity. Taxes are collected during an individual's working life, and benefits are paid in retirement until death. That this annuity scheme can affect household non-pension saving has been the subject of much research over the past decade. Through its characteristics as an asset, social security can also affect the allocation of non-pension wealth.

Most studies of the impacts of social security on individual wealth accumulation have concentrated on the response of household consumption to changes in social security that generate an increase in lifetime resources (i.e., under the assumption that the present value of anticipated benefits exceeds the present value of contributions).<sup>1</sup> Empirical tests of the impact of social security on individual (or household) saving have been undertaken by Feldstein [11,12], Kotlikoff [21], Leimer and Lesnoy [23], Barro [2], Feldstein and Pellechio [13], Blinder, Grodon, and Wise [3], King and Dicks-Mireaux [20], and Diamond and Hausman [6]. The time-series evidence has been mixed and inconclusive. Microeconomic (cross-section) evidence has generally supported the proposition that social security reduces individual saving, though empirical estimates are again varied. Participation in the social security system is not a choice variable for most individuals. In addition to the changes in the intertemporal consumption decision induced by changes in manadatory social security holdings, there is significant potential for a reallocation of non-pension wealth. Two principal characteristics of the social security annuity describe its influence on household portfolio allocation:

(i) nonmarketability of anticipated benefits<sup>2</sup>, and (ii) integration with private pension benefits.<sup>3</sup>

The paper extends and tests existing optimizing models of portfolio allocation to explicitly consider the role of social security. Section I contains the theoretical framework which serves as the basis for the analysis. Using a cross-section of household data for the U. S., the impact of the social security system on household portfolio allocation is tested in Section II. Section III extends those results by considering the general equilibrium impacts on asset markets of a social security policy change, specifically focusing on links between social security and dynamic wealth accumulation and between social security benefits and private pension benefits. Section IV summarizes the main findings and implications and points toward directions for future research.

## I. SOCIAL SECURITY AND WEALTH ALLOCATION: AN ANALYTICAL FRAMEWORK

To quantify and estimate the impact of social security on the allocation of household wealth, we need a basic approach to modeling portfolio allocation. Beginning with the methodology suggested by Merton [25,26,28] and Samuelson [31] and elaborated in a different context by Roley [30] and Friedman and Roley [15], relative asset shares in total wealth will depend on relative rates of return according to the variancecovariance structure of asset returns.<sup>4</sup> The analysis which follows has its origins in the work of Mayers [24] on the portfolio impacts of nonmarketable assets and in the work of Friedman and Roley [15] on optimal portfolio allocation. The basic modeling approaching of Friedman and Roley is modified to explicitly consider social security.

Abstracting from social security, the general problem of choosing optimal portfolio selection and composition rules can be derived from the maximization of the present discounted value of utility derived from consumption, subject to a wealth constraint. The utility function is assumed to be strictly concave, and the assumption of constant relative risk aversion is maintained throughout. Let

C = consumption

W = total marketable wealth (exclusive of pension annuities) SSW = present value of anticipated social security benefits  $\rho$  = coefficient of constant relative risk aversion  $r_i$  = real net return on the i<sup>th</sup> asset  $\Omega$  = variance-covariance matrix of asset returns  $\delta$  = individual discount rate  $\alpha_i$  = share of W held in the i<sup>th</sup> asset.

Assume that there are no assets provided by the market with risk-free real returns. An examination of real returns over the past decade reveals that this assumption is not so strong as it seems. Even the research on the hedging quality of short-term Treasury bills by Fama and Schwert [8] and by Bodie [5] has shown only that bills are hedges against anticipated inflation. Social security retirement annuities are also not riskless. Prior to the formal indexation (to benefits in 1972 and to some extent today, there are lags in adjusting benefits for inflation. Second, shocks to the economy which affect the growth rate of labor income can alter the real return on social security. Finally, social security benefits may be increased in discrete jumps, increasing the rate of return on taxes paid for some cohorts.

In the absence of a risk-free asset, following Merton [26,28], the problem of choosing optimal portfolio selection and composition rules is formulated in continuous time as

$$\max E[\int_{0}^{t} e^{-\delta t} U[C(t)]dt + B(w(T), T)], \qquad (1)$$

subject to the budget constraint that

$$dW = \alpha' r W dt - Cdt + \alpha' \Omega W.$$
<sup>(2)</sup>

B(W(T),T) is a "bequest valuation function," which is assumed to be concave in W(T). E denotes the expectation operator.

To derive the optimality equations, the problem can be restated in dynamic programming form as

$$J(W(t),t) = \max_{\{C(s),\alpha(s)\}} E(t) \left[ \int^{T} e^{-\delta s} U(C(s)) ds + B(W(T),T) \right],$$
(3)

subject to the same constraints as before. In general, (3) can be expressed as

$$J(W(t_0),t_0) = \max_{\{C(s),\alpha(s)\}} E(t) [\int_{t_0}^{t} e^{-\delta s} U[C(s)] ds + J(W(t),t)].$$
(4)

Letting  $t = t_0 + h$  and performing a Taylor expansion about  $W(t_0)$  yields:

$$J(W(t_{0}), t_{0}) = \max_{\{C,\alpha\}} E(t_{0}) \{e^{-\delta t} U[C(t)] + J(W(t_{0}), t_{0}) + \frac{\partial J(W(t_{0}), t_{0})}{\partial t} + \frac{\partial J(W(t_{0}), t_{0})}{\partial W} (W(t) - W(t_{0})) + \frac{1}{2} \frac{\partial^{2} J(W(t_{0}), t_{0})}{\partial W^{2}} [W(t) - W(t_{0})]^{2}\},$$
(5)

so that

$$0 = \max_{\{C(t),\alpha(t)\}} \{e^{-\delta t} U[C(t)] + \frac{\partial J_t}{\partial t} + \frac{\partial J_t}{\partial W} \alpha' r W(t) - \frac{1}{2} \alpha' \Omega W^2(t)\}.$$
 (6)

Rewriting (6) as

$$\max_{\{C,\alpha\}} \emptyset(\alpha,C,W,t) = 0, \tag{7}$$

we can formulate the maximization subject to the constraint that the asset shares must sum to unity. That is,

$$L = \emptyset + \lambda(1 - \alpha' 1), \tag{8}$$

where  $\lambda$  is the Lagrange multiplier associated with the constraint. The first-order conditions associated with (8) are

$$L_{c}^{*} = 0 = e^{-\delta t} U_{c}^{(C(t))} - J_{W}^{},$$
 (9a)

$$L_{\alpha} = 0 = -\lambda + J_{W} r W + J_{WW} \Omega \alpha' W^{2}, \text{ and}$$
(9b)

$$L_{\lambda} = 0 = 1 - \alpha' 1. \tag{9c}$$

The first condition is familiar; it equates the (discounted) marginal utility of consumption with the marginal utility of wealth. Combining (9b) and (9c) and using the fact that under the assumption of constant relative risk aversion  $\rho = -W(J_{WW}/J_W)$ , we obtain the vector of desired asset shares  $\alpha^*$ :

$$\alpha^{\star} = \frac{1}{\rho} [A]r + B, \qquad (10)$$

where

$$A = \Omega^{-1} - (1'\Omega^{-1} 1)^{-1}\Omega^{-1}11' \Omega^{-1}, \text{ and}$$
(11a)

$$B = (1' \alpha^{-1} 1)^{-1} \alpha^{-1} 1.^{5}$$
(11b)

Given a mandatory nonmarketable position in social security annuities of amount SSW, equation (10) is modified to

$$\alpha^{\star} = \frac{1}{\rho} \left[ \hat{A} \right] \left( r - \rho \; \frac{SSW}{W} \; \hat{\sigma}_{SS} \right) + \hat{B}, \qquad (12)$$

where a caret indicates the inclusion of only the marketable (i.e., non-social security) assets.  $\hat{\sigma}_{SS}$  is the vector of covariances of the returns on the marketable assets with that of social security.

The larger is the ratio of an individual's social security wealth to his non-pension (marketable) wealth, the lower will his demand be for other assets whose returns are positively correlated with that of social security. Thinking of  $(r_k - \rho \frac{SSW}{W} \sigma_{SSk})$  as the "adjusted return" on asset k, the effect of a change in social security on adjusted returns and the composition of non-pension wealth depends on: (i) the individual's risk aversion, (ii) mandatory social security holdings relative to non-pension wealth, and (iii) the correlation of the expected return with that of social security.

The intuition here is clear. Given that participation in the social security annuity system is involuntary and given that the entitlement to benefits is not marketable, we can examine the effect of social security on the allocation of fungible wealth. Expressing required social security holdings SSW as the sum of an optimal amount SSW<sup>\*</sup> (the individual's choice in an unconstrained optimization) plus the excess of required over desired social security, for any asset k,

$$\alpha_{k} = \frac{1}{\rho} \sum_{j} a_{kj} r_{j} - \frac{SSW}{W} \sum_{j} a_{kj} \sigma_{SSj} + \left(\frac{SSW}{W} - \frac{SSW}{W}\right) \sum_{j} a_{kj} \sigma_{SSj} + b_{k}, \quad (13)$$

where  $a_{kj}$  are the elements in the k<sup>th</sup> row of [A] and  $b_k$  is the k<sup>th</sup> element of the vector B. The higher the desired level of social security wealth, the lower will be the relative demand for asset k (as long as  $\sigma_{kSS} > 0$ ). However, to the extent that there is too little (too much) SSW, the relative demand for asset k is increased (decreased) when  $\sigma_{kSS} > 0$ .

The next two sections focus on estimating the model of household portfolio allocation in the presence of pensions and on combining the results with information about the impact of pensions on household saving to simulate the general equilibrium impacts on portfolio allocation of a change in social security benefits relative to non-pension wealth.

- 7 -

# II. ESTIMATING THE IMPACT OF SOCIAL SECURITY ON HOUSEHOLD

## WEALTH ALLOCATION

### A. Background

Much of the previous work on the effects of social security on saving has used aggregate time-series data. "Representative transactor" models are not likely to be good analytical tools for examining the portfolio composition effects. The nonmarketability of anticipated social security benefits makes "high-income" and "low-income" individuals quite different. Because social security benefits are not proportional to preretirement earnings, high-income individuals should have much lower measures of SSW/W than low-income individuals. A macroeconomic-level regression using an aggregate  $\frac{SSW}{W}$  would obscure this difference.<sup>6</sup>

Hence, cross-section data are required to properly implement the model. The household data used in this paper are excerpted from surveys done in 1979 and 1980 under the auspices of the U. S. President's Commission on Pension Policy. Those data represent one of the few attempts since Federal Reserve Board's 1962 survey of consumers (described in Projector and Weiss [29]) to devise an asset data base on the household level. The bulk of the data base comes from two interviews, one year apart. The Wave I interview was conducted in September 1979, and the Wave II interview was conducted in September 1980. Only data from the first wave are used in estimating the model.

### B. The Model and Estimation Results

In keeping with the derivation of the optimal portfolio allocation, the basic model to be estimated is of the form:

- 8 -

$$\alpha_{j} = f(\tau, \frac{SSW}{W}, \frac{PPW}{W}, \psi), \qquad (14)$$

where j indexes assets;  $\tau$  is the unit's marginal tax rate; W, SSW, and PPW represent non-pension wealth and the present values of anticipated social security and private pension benefits, respectively; and  $\psi$  is a vector of other explanatory variables. The logistic transformation of (14) is actually estimated, to reduce heteroscedasticity,<sup>7</sup> i.e.,

$$\ln \left(\frac{\alpha_j}{1-\alpha_j}\right) = \beta_0 + \beta_1 \tau + \beta_2\left(\frac{SSW}{W}\right) + \beta_3 \left(\frac{PPW}{W}\right) + \beta\psi'.$$
(15)

The present values of anticipated social security benefits and private pension benefits are entered separately in (15), so that they are not constrained to have the same impact on portfolio allocation.

Values for  $\tau$  are obtained from the TAXSIM model of the National Bureau of Economic Research, given data on earned income, income from assets, and family characteristics.<sup>8</sup> Non-pension wealth W represents the net worth of the unit. The gross asset position is obtained by summing the values of the holdings of the individual assets (financial and nonfinancial). Total liabilities include mortgage and consumer indebtedness. Net worth is just the difference between gross wealth and total liabilities. Details on the construction of household gross social security wealth and private pension wealth variables can be found in the Appendix.

The elements of  $\psi$ , the "other explanatory variables," can be divided into two parts. The first category contains "portfolio scale" variables, the ratio of net worth to permanent income and (the log of) permanent income. Details on the construction of the permanent income variable can be found in the Appendix. These wealth and income variables act as proxies for effects of the size of the total portfolio on portfolio allocation (such as transactions costs). In the second category are relevant individual characteristics, such as self-employment status, labor force status (whether the head of the household is employed), number of children under eighteen years of age, whether the unit is a farm family, and age.<sup>9</sup> There are three brackets for age — under forty (AGE1), between forty and fifty-five (AGE2), and over fifty-five (AGE3).

The following assets were considered: market value of the home, U. S. savings bonds, deposits, non-pension annuities, bonds, equities, and passenger cars.<sup>11</sup> In addition, total liabilities (sum of mortgage and consumer indebtedness) were analyzed in the same framework. The inclusion of liabilities provides another test of the impact of the nonmarketability cnstraints embodied in social security annuities.

Since the model in (15) considers only those observations with positive holdings of the asset, it will be necessary to correct for sample selection bias. Using the procedure suggested by Heckman [18], a first-stage probit model for the probability of owning a given asset was estimated for the full sample. The probit model (for the discrete choice to hold each asset) included as explanatory variables the ratios of non-pension wealth, private pension wealth, and social security wealth to permanent income, (the log of) permanent income, the marginal tax rate, and dummy variables for whether the head of the unit is under age forty, for low current unit earnings (less than \$6000), and for whether the head of the household has at least a high school education. The inverse of Mill's ratio (from the estimated probit equations) was added as an additional regressor in (14) to correct for sample selection bias.<sup>10</sup> Results of the first-stage probit

- 10 -

models for positive asset holdings can be found in Table I. The number of observations in the sample with sufficient information to construct the permanent income and pension wealth variables is 3084.

From Table I, it is clear that the marginal tax rate plays an important role in the discrete choice of which assets to hold. For example, the influence of the marginal tax rate on the decision to buy a house or to buy shares of stock is particularly great. The evidence in Table I makes apparent the need to consider the impact of taxation on portfolio choice as well as on portfolio composition.

For each asset, the probability of ownership rises with the ratio of non-pension wealth to permanent income. It is not obvious <u>a priori</u> what effects on the probability of holding the various assets we should expect of the other two components of wealth. As anticipated social security and private pension benefits are long-term illiquid assets, they might be expected to lessen the chances of holding assets with similar characteristics. However, not all illiquid assets may be perceived as part of saving for retirement (e.g., cars), while liquid assets such as equities might be. The ratios of the present values of private pension benefits and social security benefits to permanent income generally exert a positive impact on the holdings of the marketable assets. Those effects are significant for savings bonds, equities, and passenger cars in the case of private pensions, and for housing annuities, passenger cars, and liabilities in the case of social security. Social security exerted a positive influence in each of those cases except annuities.

- 11 -

TA	BL	E	I

V Constan <b>t</b>	alue of <u>Home</u> -7.478 (0.153)	Savings Bonds -2.874 (0.191)	Deposits 2.393 (0.595)	Bonds -7.351 (2.437)	<u>Equity</u> -8.387 (1.497)	Annuities -3.797 (1.935)	Passenger <u>Cars</u> -3.067 (1.353)	Debt -5.434 (1.162)
PPW/Y <sup>*</sup>	-0.014 (0.009)	0.017 (0.008)	0.009	0.005	0.043	0.002 (0.013)	0.082 (0.036)	-0.001 (0.099)
SSW/Y*	0.079	-0.004	0.012	0.048	-0.006	-0.011	0.054	0.043
	(0.022)	(0.019)	(0.080)	(0.037)	(0.028)	(0.005)	(0.026)	(0.021)
w/Y*	0.031	0.003	-0.109	0.030	0.056	(0.020)	0.014	-0.004
	(0.006)	(0.007)	(0.044)	(0.007)	(0.008)	(0.007)	(0.008)	(0.006)
<b>£n(Y<sup>*</sup>)</b>	0.726	0.078	0.099	0.338	0.566	0.132	0.353	0.587
	(0.118)	(0.121)	(0.100)	(0.245)	(0.152)	(0.197)	(0.138)	(0.119)
Marginal	2.091	3.100	-0.121	3.230	2.663	0.969	1.296	0.013
Tax Rate	(0.482)	(0.470)	(1.515)	(0.927)	(0.554)	(0.459)	(0.609)	(0.470)
Age < 40	-0.636	0.442	-0.088	-0.259	0.306	-0.167	-0.154	0.270
	(0.076)	(0.072)	(0.266)	(0.147)	(0.082)	(0.116)	(0.100)	(0.007)
Earnings	0.636	0.878	1.592	1.784	1.086	0.404	-0.059	-0.914
< 6000	(0.173)	(0.178)	(2.507)	(0.357)	(0.211)	(0.286)	(0.209)	(0.168)
Education	0.115	0.473	0.494	0.632	0.700	0.426	0.424	0.226
	(0.078)	(0.082)	(0.182)	(0.196)	(0.103)	(0.144)	(0.092)	(0.075)
Number Above	1771	797	3060	94	517	134	2745	2417
Number Below 2	1313	2287	24	2990	2467	2950	339	667
$\chi^2(8)$	309.5	68.7	16.1	91.2	309.5	34.2	444.5	230.7

# Probit Model For Positive Asset Holdings

(Standard errors are in parentheses.)

×.

Permanent income has a significant positive effect on the discrete choice to hold particular assets. Similarly, education exerts a positive impact on asset holdings. Low age exhibits a negative effect except on holdings of savings bonds. Low household earnings, <u>ceteris paribus</u>, have negative influence.

Table II contains the estimated coefficients for the basic asset demand equations. The ratio of net worth (marketable wealth) to permanent income exerts a depressing effect across the menu of assets. The coefficient on (the log of) permanent income is frequently not significantly different from zero. In general, the relative share demand for financial assets has a negative income elasticity and a positive elasticity for nonfinancial assets.

# TABLE II

# Asset Demand Equations

Dependent Variable:  $\ln(\alpha_j/(1 - \alpha_j))$ .

			1 1				-	
	Value	Saving		n 1.		A	Passenge	
	of Home	Bonds	Deposits	<u>Bonds</u>	Equity	Annuities	Cars	Debt
Constant	7.419	-2.111	-2.857	-9.697	4.642	0.810	-0.947	-2.266
Joins Earre		(3.097)		(6.801)	(5.069)	(9.837)	(1.005)	(1.589)
+				. ,	•			
PPW/Y*	-0.0004	0.013	-0.002	-0.163	-0.017	-0.162	0.004	0.014
	(0.004)	(0.028)	(0.002)	(0.298)	(0.072)	(0.101)	(0.002)	(0.002)
SSW/Y <sup>*</sup>	-0.058	0.036	0.000	0.366	-0.203	-0.297	0.022	0.007
22M/1	(0.038)	(0.036)	0.002 (0.001)	(0.116)	(0.072)	(0.149)	(0.022	(0.001)
	(0.020)	(0.017)	(0.001)	(0.110)	(0.072)	(0.14))	(0.000)	(0.001)
W/Y*	-0.099	-0.108	-0.043	-0.044	-0.060	-0.043	-0.018	-0.159
		(0.026)		(0.022	(0.019)	(0.054)	(0.008)	(0.013)
*								
٤n(Y <sup>*</sup> )	-0.628	-0.226	-0.082	-0.872	-0.564	-0.373	-0.033	0.064
	(0.136)	(0.269)	(0.116)	(0.679)	(0.442)	(0.698)	(0.101)	(0.153)
	0 100	0 (77	0.055	0 705	0 500	2 02/	0 110	0 (55
Marginal	0.180	0.677	-0.055	-0.705	0.503 (0.241)	3.934 (2.001)	-0.112 (0.301)	0.655 (0.269)
Tax Rate	(0.060)	(0.974)	(0.363)	(1.861)	(0.241)	(2.001)	(0.301)	(0.209)
Self -	-0.311	-0.421	-0.131	-0.355	-0.195	-0.166	-0.284	0.168
Employed	(0.084)	(0.248)	(0.127)	(0.595)	(0.310)	(0.570)	(0.104)	(0.142)
	(,				. ,			
Farm	0.165	-0.895	-0.059	-2.323	-1.212	0.285	-0.920	-0.415
Family	(0.191)	(0.693)	(0.287)	(1.129)	(0.600)	(1.008)	(0.241)	(0.333)
			0 011	0 700	0.144	0 701	0 200	0 0 0 0
Unem-	-0.078	1.063	0.311	-0.792	0.164 (0.373)	-0.731 (0.641)	-0.399 (0.092)	-0.038 (0.139)
ployed	(0.084)	(0.265)	(0.112)	(0.603)	(0.3/3)	(0.041)	(0.092)	(0.139)
Number of	0.106	-0.194	-0.169	-0.284	-0.093	-0.091	0.055	0.114
Kids < 18		(0.059)	(0.030)	(0.121)	(0.090)	(0.161)	(0.025)	(0.034)
			•	•				
AGE 2	-0.185	-0.014	0.339	-0.623	0.150	0.060	-0.152	-0.759
	(0.102)	(0.159)	(0.089)	(0.500)	(0.061)	(0.515)	(0.069)	(0.103)
	0.100	0 000	0.010	0.017	0 000	0.0(5	0.0(5	1 570
AGE 3						-0.265		-1.573
	(0.123)	(0.085)	(0.120)	(0.040)	(0.133)	(0.605)	(0.09))	(0.149)
Inverse o	f							
Mill's		-0.016 .	-10.304	-0.363	-0.891	0.370	0.409	-0.104
Ratio						(1.618)		
Number wi			-					
Positive		_		<b>.</b> .				
Holdings	1771	797	3060	94	517	134	2745	2417
$\overline{R}^2$	0 00	0 22	0.04	0.94	0 20	0 20	0.25	0 22
ĸ	0.22	0.22	0.24	0.24	0.28	0.20	0.35	0.32
F	24.59	8.78	34.99	2.43	2.75	2.56	48.54	34.66
-	2	0.70		2010		2000		

(Standard errors are in parentheses.)

An increase in the marginal tax rate, <u>ceteris paribus</u>, raises holdings of housing, equities, annuities, and debt relative to net worth; negative effects occur for deposits, bonds, and vehicles. Only the former category of estimated impacts is pronounced and statistically significant, however. Feldstein's [10] finding that, under the special features of the U.S. tax system, higher marginal tax rates raise the relative demand for equities is borne out. The results in Table II are conditional on the household's holding the particular assets in question. Insofar as changes in the marginal tax rate affect the discrete choice of which assets to hold, the estimated coefficients in Table II are underestimates of the impact of personal taxation on portfolio composition.

Coefficient estimates for variables representing individual characteristics are statistically significant at the 95 percent confidence level. Self-employed individuals and farmers, <u>ceteris</u> <u>paribus</u>, hold less of their wealth in financial assets than do the rest of the sample. Larger families hold more of their wealth in housing and cars and are more highly levered. Relative shares of financial assets increase with age (with the exception of U. S. savings bonds), while relative positions in physical assets decline with age. Leverage also declines with age.

The estimated initial impact of the social security variable (ratio of present value of anticipated benefits to net worth) on portfolio composition can be found in the third row of Table II. All of the coefficients are significantly different from zero at the 95-percent confidence level, with negative effects noted for housing, equities, and annuities. Those effects are intuitive, in the first two cases because of the "inflation hedge" properties of the assets and in the last because compulsory purchases of social security annuities are a

- 15 -

substitute for market-provided annuities. Positive effects are observed for savings bonds, deposits, bonds, and vehicles; unlike indexed social security benefits, bonds and deposits represent nominal claims. Also of interest is the positive coefficient on SSW/W in the liabilities regression, indicating that an increase in social security wealth relative to marketable wealth increases leverage, a finding consistent with agents' trying to "undo" the nonmarketability of anticipated future social security benefits.

Results for the "composition effect" of private pension wealth (the PPW/W variable) are not conclusive, with statistically significant coefficients only for passenger cars and for liabilities. Much of the ambiguity probably stems from the fact that, unlike social security, private pension participation is not truly exogenous to individual decisions. Moreover, there is no information in the data on the portfolio composition of pension assets; hence no test can be made of whether households "internalize" the portfolios of their pension funds.

Whether or not to use the same real discount rate for all individuals in computing social security wealth or private pension wealth is a difficult question. Older individuals may be more certain of receiving anticipated pension benefits and may have a lower implicit discount rate. To allow for differences in discount rates, the products of the age brackets and the pension wealth variables were added to the regression model in (14).

Table III reports the regression results when the age-pension interaction terms are included. This attempt to capture age-specific characteristics of the pension wealth variables did not produce significantly different results for the marginal tax rate, the scale variables, the composition variables, or the individual characteristics. In most cases, the coefficients of the interaction

- 16 -

# TABLE III Asset Demand Equations

Dependent Variable: $ln(\alpha_j/(1 - \alpha_j))$ .								
	Value of Home	Savings Bonds	s Deposits	Bonds	Equity	Annuities	Passenge <u>Cars</u>	r Debt
Constant	7.524	-2.054	-2.373	-6.799	5.215	-1.478	-0.990	-1.942
	(1.617)	(3.412)	(1.158)	(9.007)	(5.105)	(9.107)	(1.008)	(1.581)
PPW/W	-0.001 (0.005)	0.011 (0.026)	-0.003 (0.003)	-0.310 (0.342)	0.131 (0.112)	-0.212 (0.114)	0.005	0.012 (0.003)
SSW/W	-0.060	0.034	0.002	0.356	-0.211	0.785	0.020	0.006
	(0.028)	(0.018)	(0.001)	(0.122)	(0.096)	(0.512)	(0.006)	(0.001)
W/Y <sup>*</sup>	-0.100	-0.104	-0.041	-0.041	-0.065	-0.061	-0.018	-0.154
	(0.008)	(0.030)	(0.008)	(0.020)	(0.020)	(0.050)	(0.009)	(0.013)
ln(Y <sup>*</sup> )	-0.636	-0.241	-0.001	0.637	-0.612	-0.355	-0.029	0.031
	(0.136)	(0.270)	(0.116)	(0.804)	(0.443)	(0.636)	(0.101)	(0.152)
Marginal	-0.162	0.649	0.114	-0.845	0.480	4.540	-0.112	0.622
Tax Rate	(0.080)	(0.984)	(0.365)	(2.413)	(0.235)	(1.927)	(0.302)	(0.266)
Self -	-0.312	-0.431	-0.132	-0.567	-0.228	-0.084	-0.285	0.178
Employed	(0.085)	(0.237)	(0.128)	(0.656)	(0.311)	(0.527)	(0.104)	(0.141)
Farm	0.167	-0.912	-0.086	-1.755	-1.165	0.191	-0.920	0.425
Family	(0.191)	(0.702)	(0.288)	(1.959)	(0.607)	(0.905)	(0.241)	(0.330)
Unem-	-0.075	1.059	0.310	0.879	-0.184	-0.276	-0.394	-0.087
ployed	(0.085)	(0.260)	(0.112)	(0.885)	(0.371)	(0.603)	(0.093)	(0.139)
Number of	0.106	-0.189	-0.174	-0.228	-0.089	-0.065	0.055	0.107
Kids < 18	(0.021)	(0.054)	(0.030)	(0.103)	(0.096)	(0.148)	(0.025)	(0.034)
AGE 2	-0.199	-0.010	0.388	0.975	0.161	0.149	-0.150	-0.825
	(0.102)	(0.170)	(0.089)	(0.894)	(0.078)	(0.735)	(0.070)	(0.106)
AGE 3	-0.199	-0.010	0.988	0.009	0.426	1.595	-0.255	-1.688
	(0.126)	(0.097)	(0.116)	(1.145)	(0.205)	(0.880)	(0.096)	(0.150)
AGE 2 x	0.012	0.002	0.002	0.532	0.231	0.005	0.001	0.014
(SSW/W)	(0.010)	(0.004)	(0.003)	(1.370)	(0.195)	(0.592)	(0.004)	(0.005)
AGE 3 x	-0.001	0.014	0.028	0.444	-0.156	-2.001	-0.006	0.045
(SSW/W)	(0.019	(0.007)	(0.007)	(1.513)	(0.164)	(0.631)	(0.007)	(0.011)
AGE 2 x	-0.009	0.002	0.011	0.247	-0.263	0.614	-0.007	0.009
(PPW/W)	(0.009)	(0.005)	(0.007)	(0.693)	(0.146)	(0.543)	(0.007)	(0.009)
AGE 3 x	0.007	0.008	-0.044	1.057	-0.449	0.456	0.008	0.042
(PPW/W)	(0.020)	(0.010)	(0.014)	(0.786)	(0.277)	(0.789)	(0.013)	(0.019)

	TABLE	III
Asset	Demand	Equations

Dependent Variable: $ln(\alpha_j/(1 - \alpha_j))$								
Value Savings					Passenger			
	of Home	Bonds	Deposits	Bonds	Equity	Annuities	Cars	Debt
Inverse	of							
Mill's	-0.753	-0.020 -	-10.256	-0.050	-0.964	0.740	0.415	-0.305
Ratio	(0.322)	(0.450)	(5.309)	(0.661)	(0.450)	(1.480)	(0.170)	(0.416)
R <sup>2</sup>	0.22	0.23	0.24	0.22	0.28	0.35	0.35	0.33
F	20.2	8.49	29.4	2.74	2.88	3.03	40.80	30.88

(Standard errors are in parentheses.)

terms were not significantly different from zero at the 95 percent confidence level. A marked exception is total liabilities, where the impact of social security wealth on leverage increases with age.<sup>12</sup>

To provide a meaningful interpretation of the point estimates of the impact of social security on portfolio allocation, the implied asset (share) demand elasticities with respect to changes in SSW/W are presented in Table IV. While the original coefficient estimates are precisely measured, the elasticities are small absolutely. However, the general equilibrium impact on portfolio allocation of a legislated change in social security wealth depends on the change in desired asset shares, the response of the level of marketable wealth (saving effect), and on any formal integration with private pension systems. Only the first of those effects has been examined thus far.

# TABLE IV POINT ELASTICITIES OF RELATIVE SHARE DEMAND WITH RESPECT TO SSW/W

Assets	Elasticity
Value of Home	-0.03
U. S. Savings Bonds	-0.04
Deposits	0.01
Bonds	0.17
Equities	-0.13
Annuities	-0.18
Passenger Cars	0.06
Debt	0.01

- 19 -

III. ASSESSING THE IMPACT OF A CHANGE IN SOCIAL SECURITY BENEFITS

The estimation results in the previous section can be used to identify the immediate impact on household portfolio allocation of a change in social security benefits relative to earnings. Those measures are necessary but not sufficient for an analysis of the effects on wealth allocation of a change in social security wealth; changes in social security will affect non-pension wealth accumulation and private pension benefits.

Consider first the wealth accumulation process. Suppose that social security wealth and private pension wealth are among the determinants of an investor's desired wealth; moreover, let them be exogenous to that decision.<sup>13</sup> While individuals are assumed to take private pension wealth as given, the integration of some pension plans with the social security system means that social security wealth will still influence private pension accumulation.

Let the ratio of non-pension wealth to permanent income for individual i in period t be determined according to:

$$\left(\frac{W}{Y^{*}}\right)_{it} = a_{o} + a'D_{it} - a_{s}\left(\frac{SSW}{Y^{*}}\right)_{it} - a_{p}\left(\frac{PPW}{Y^{*}}\right)_{it}, \quad (16)$$

where W, Y<sup>\*</sup>, D, SSW, and PPW denote wealth, permanent income, a vector of individual characteristics, and the present values of social security and private pension benefits, respectively. While the amount of "social security saving" is fixed by law, pension saving is assumed adjust to social security changes, so that

$$\left(\frac{PPW}{Y^*}\right)_{it} = b_0 + b'D_{it} - b_{sp}\left(\frac{SSW}{Y^*}\right)_{it}.$$
 (17)

Combining equations (16) and (17) yields

 $\begin{pmatrix} \frac{W}{Y} \end{pmatrix}_{it} = (a_0 - a_p b_0) + (a' - a_p b') D_{it} - (a_s - a_p b_{sp}) (\frac{SSW}{Y})_{it}.$ (18) Suppose, for example, that Congress legislated a benefit increase sufficient to raise SSW/Y<sup>\*</sup> by Ø percent. From equation (18), the offset to non-pension wealth (assuming that permanent income is unchanged) is just

$$\emptyset \qquad (a_s - a_p b_{sp})$$

increase in social security social security SSW offset to wealth offset to private pension wealth

Hence, ascertaining the general equilibrium impact on asset shares of a change in social security wealth requires knowledge not only of the parameters of the basic model, but also of the offset parameters  $a_s$ ,  $a_p$ , and  $b_{sp}$ .<sup>14</sup>

In another paper using the same data (Hubbard, 1983), I estimated the offsets (evaluated at sample means) to non-pension net worth from an extra dollar of social security wealth and from an extra dollar to private pension wealth to be thirty-three cents and sixteen cents, respectively.<sup>15</sup> Given the information available in the data set, it is not possible to determine the reduction in private pension benefits attendant to a permanent increase in social security benefits. Calculations of the general equilibrium impact on portfolio composition of a permanent change in social security benefits are performed under three values for  $b_{sp}^{}$  - zero, thirty-three cents, and fifty cents. Table V contains the analogues of the elasticities reported in Table IV after adjustments in net worth and private pension benefits have taken place.

Comparing the elasticity measures in Table V with those of Table IV, the importance of considering the linkages among social security, private pension, and fungible net worth is readily apparent. The reductions in net worth and in private pension benefits from an increase in social security benefits alter the ultimate impact of the pension benefit change on portfolio composition. The general equilibrium elasticities are larger than the original impact elasticities. Given the small offset the net worth of private pension wealth, the results are not highly sensitive to the choice of the (assumed) offset to private pension benefits of a change in social security benefits.<sup>16</sup>

The structure of and dynamics of the portfolio <u>adjustment</u> process are also important. This paper has only looked at desired asset shares; the empirical work has implicitly assumed individuals can and do reshuffle their portfolios to quickly balance desired and actual wealth allocation. Much of the recent work on portfolio adjustment processes has focused on the allocation problem in the presence of changes in new investable funds.<sup>17</sup> To be appropriate for the problem at hand, such a model must address the impact of changes in mandated asset holdings (like social security wealth). Adding that dimension represents an important extension of the analysis in the paper.<sup>18</sup> At that stage, more reasonable policy simulations could be designed to evaluate the dynamics of the effects of shfits in the structure of the social security system on household wealth accumulation and allocation.

- 22 -

## TABLE V

# General Equilibrium Elasticities of Relative

Share Demands with Respect to SSW/W

## Asset

# <u>Elasticity</u>

	$b_{sp} = 0.00$	$b_{sp} = 0.33$	$b_{sp} = 0.50$
Value of Home	-0.05	-0.05	-0.04
U. S. Savings Bonds	0.06	0.05	0.05
Deposits	0.02	0.02	0.01
Bonds	0.28	0.26	0.25
Equities	-0.20	-0.19	-0.18
Annuities	-0.28	-0.26	-0.25
Passenger Cars	0.10	0.09	0.08
Debt	0.03	0.03	0.03

## IV. CONCLUSIONS

Concentrating on social security as an annuity, this paper considers the impact of social security on asset markets, through its effects on household wealth allocation. Existing optimizing models of wealth allocation are extended to include the constraint of manadatory participation in the social security system. Results for the impact of social security on portfolio choice and composition depend on household holdings of social security annuities relative to non-pension wealth and on the correlations of the return on social security with those on marketable assets.

Section II tests the influence of social security wealth on portfolio composition using the model outlined in the text and crosssection data collected under the auspices of the U. S. President's Commission on Pension Policy. Estimation of the model gives interesting results for the effects of marginal tax rates and of social security holdings. Effects of the marginal tax rate generally are particularly strong for housing, equities, annuities, and total liabilities. Changes in social security benefits exhibit significant partial equilibrium impacts on portfolio composition. Negative effects are noted for housing, equities, and annuities; positive effects are observed for saving bonds, deposits, bonds, and vehicles. Also of interest is the positive impact of social security on liabilities, indicating that a permanent increase in social security benefits relative to non-pension wealth increases leverage. The general equilibrium impact on portfolio allocation of a legislated change in social security depends not only on the change in desired asset shares, but also on the response of the level of marketable wealth (saving effect) and on any formal integration with private pension systems.

- 24 -

The third section develops more fully those general equilibrium qualifications to the regression results in section II by considering the response of non-pension wealth accumulation to changes in pension wealth. Specifically, a permanent increase in the present value of anticipated social security benefits of \$1.00 reduces non-pension wealth by thirty-three cents. Using this estimate in conjunction with assumptions about evidence on the impact of the social security system on other forms of retirement saving, the results in section II are augmented to evaluate the general equilibrium effects on portfolio allocation of changes in social security wealth. In many cases, those effects are much large than the partial equilibrium effects.

Two other extensions readily suggest themselves. What are the <u>dynamics</u> of the accumulation and allocation of marketable wealth in response to changes in pension wealth? Second, how will the impacts of the social security system on our variables of interest circumscribe the impacts of changes in policy guiding the use of individual retirement saving plans? While the controversy over the implications of the social security system for household saving is still unresolved, it is also important to examine the program's direct effects on asset markets. That analysis may shed light on the problem of ascertaining household valuations of social security and on the ways in which agents attempt to "undo" constraints on thier asset-holding behavior.

### FOOTNOTES

- \* Assistant Professor of Economics, Northwestern University; and Faculty Research Fellow, National Bureau of Economic Research. I am indebted to Zvi Bodie, Benjamin Friedman, Jerry Hausman, Mervyn King, John Lintner, Clifford Smith, Paul Wachtel, and to workshops at the Board of Governors of the Federal Reserve System, Boston University, Harvard University, University of Maryland, Northwestern University and the University of Pennsylvania for helpful comments and suggestions. The Alfred P. Sloan Foundation and the National Bureau of Economic Research provided financial support for this work.
- <sup>1</sup>In theory, the social security does not have to be unfair in order to reduce household saving. Abel [1] and Hubbard [19] emphasize the impact of social security on saving in the context of individual uncertainty over longevity and find that even a fair social security system can reduce household saving by more than the taxes paid.
- <sup>2</sup>The effects of nonmarketable assets on portfolio structure have been examined by Mayers in the context of the standard Capital Asset Pricing Model of Sharpe [32] and Lintner [22]. Merton [27] concentrates directly ont he nonmarketability of human capital and shows that under certain conditions a tax and transfer system similar to the current social security system can mimic the optimal allocation path, reducing or eliminating the economic ineffciencies stemming from the nonmarketability of human capital.
- <sup>3</sup>The benefits of many private pension plans are formally integrated with the social security system, reducing payments as social security benefits are augmented. Savings models (as in Feldstein [11, 12]) or portfolio allocation models (as in Dicks-Mireaux and King [7]) which include as explanatory variables both private pension and social security annuities should consider not only their offset to non-pension wealth, but also the offset to private pension saving attendant to increases in social security benefits.
- <sup>4</sup>As noted in Merton [26] and in Friedman and Roley [15], those properties (linearity in expected returns and wealth homogeneity) can be derived from a general expected utility maximization in continuous time under the assumptions of (i) constant relative risk aversion and (ii) joint normally distributed expected asset returns. Empirical evidence in support of the first assumption can be found in Friend and Blume [16].
- <sup>5</sup>The matrix A is symmetric with non-negative main diagonal elements. Moreover, the sum of the elements of each individual column of A is zero. For more details, see Roley [30] or Hubbard [19].
- <sup>6</sup>A similar point surfaces in deciding whether to aggregate human wealth and social security wealth. The lack of a proportional relationship between the two major nonmarketable assets suggests that they should be kept separate in any empirical analysis. Above a certain level of (permanent) wage income, increases in human wealth will not be

followed by increases in social security wealth. Earlier tests of the influence on portfolio allocation of nonmarketable assets (such as those of Mayers [24] or Friend and Blume [16]) focused on human capital. Lumping social security wealth together with human wealth may be further questionable because their risk characteristics are not the same. At a minimum, cyclical fluctuations in real wages and the lack of complete price-level indexation of wages make human wealth riskier than social security. (There is also an age heterogeneity effect when considering social security because of the liability represented by future social security tax payments.) Friend and Blume [16, p. 914] made a similar point by concluding that if the return on social security were uncorrelated with the return on the portfolio of marketable risky assets then the inclusion of social security wealth in human wealth would decrease the value of a now more broadly defined "beta coefficient" of human wealth.

- <sup>7</sup>Of course, under this transformation, the adding-up requirements no longer hold.
- <sup>8</sup>See Feenberg [9] for a description of the TAXSIM model. With only one cross-section of units, it is impossible to consider the impacts of variables which are the same across units (such as the return on a particular assets, the rate of inflation, etc.). Marginal tax rates do vary across units. Of course, the problem of endogeneity of the marginal tax rate with respect to portfolio composition still remains.
- <sup>9</sup>Individual characteristics can be important factors in the determination of how wealth is allocated. Families with many children may have larger houses; self-employed individuals may have more of their wealth in business equity, etc.
- <sup>10</sup>Heckman's method [18] does not yield consistent standard errors, and, moreover, as Greene [17] points out, it is impossible to state whether the reported "conventional" standard error is a lower bound or an upper bound of the "true" standard error.
- <sup>11</sup>The category "deposits" includes deposits with financial institutions. Unfortunately, "bonds" in the data set comprise both those whose interest payments are subject to taxation and those whose interest payments are not. "Equity" is the sum of direct holdings and mutual fund shares. All assets are at market value. Other asset categories like business equity, money market funds, and notes and mortgages held had too few observations to ascertain meaningful results.
- <sup>12</sup>This could reflect greater certainty over receipt of anticipated social security benefits. While it is illegal to borrow against social security benefits per se, older individuals may take anticipated benefits into account borrowing against their other assets.

- <sup>13</sup>Exogeneity is more obvious for social security wealth than for private pension wealth. Some individuals (certainly those with defined contribution plans) have control over their employee pension saving.
- <sup>14</sup>To see this, note that

$$d(\frac{SSW}{W}) = \frac{dSSW}{W}(1 + (a_s - a_p b_{sp}) (\frac{SSW}{Y^*}) \text{ and}$$
$$d(\frac{PPW}{W}) = \frac{dSSW}{W}(-b_{sp} + (a_s - a_p b_{sp})((\frac{PPW}{Y^*})).$$

Only if  $b_{sp} = a_s/a_p$  will dSSW/W be equal to d(SSW/W).

- <sup>15</sup>See Hubbard [19] for the background of the wealth accumulation model and for the estimation procedure. More specifically, the offsets were estimated from the ratios of the pension wealth variables to permanent income to the ratio of net worth to permanent income.
- <sup>16</sup>A possible extension would be to obtain a more informed estimate based on an examination of a cross-section of private pension plans.
- <sup>17</sup>See, for example, the survey in Friedman [14] and his development of the "optimal marginal adjustment model," which distinguishes between the allocation of new investable funds and the reallocation of existing wealth.
- <sup>18</sup>That extension was not pursued here because of the dubious quality of the second wave of data in the survey.

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### APPENDIX

### Constructing a Proxy for Permanent Income

The concept of permanent income (or normal annual earnings) is important both for wealth accumulation (because of the age profile of the ratio of net worth to permanent income implied by the life-cycle model) and for portfolio allocation (because of its connection with the normal level of transactions). Permanent income is not an observable variable, so that some sort of estimation procedure is needed to obtain a proxy based on (observable) current earnings.

Let the model for determining permanent income be such that

(A1) 
$$\ln (Y_i^*) = X_i \beta + S_i$$
,

Where  $X_i$  is a vector of observable variables for the i individual,  $\beta$  is the associated parameter vector, and  $S_i$  measures luck or skill, where  $S_i$  has mean zero and variance  $\sigma_S^2$ . In a given year, earnings will differ from permanent income because of one's position in the ageearnings profile and because of transitory earnings. Letting  $Z_{it}$  denote earnings in period t by the i individual,

(A2) 
$$\ln (Z_{it}) - \ln (Y_i^*) + f(A_{it}) + \varepsilon_{it}$$

where A represents age and  $\varepsilon_{it}$  represents the transitory portion of earnings (with mean zero and variance  $\sigma_{\varepsilon}^2$ ). By assumption,  $S_i$  and  $\varepsilon_{it}$  are uncorrelated. The combination of (Al) and (A2) yields the following earnings function:

(A3) 
$$\ln (Z_{it}) = X_i \beta + f(A_{it}) + S_i + \varepsilon_{it}.$$

An estimate of permanent income for each individual in the sample can be constructed from the estimate  $\hat{\beta}$ , if we can inpute a value of  $S_i$ . Given the residual from the earnings equation  $(S_i + \varepsilon_{it})$ , the minimum-variance estimator  $S_i$  (given  $S_i + \varepsilon_{it}$ ) it just:

(A4) 
$$\hat{S}_{i} = \frac{\sigma_{S}^{2}}{\sigma_{S}^{2} + \sigma_{\varepsilon}^{2}}$$
  $(S_{i} + \varepsilon_{it})$ .

From (A1), (A2), and (A4), the estimate of permanent income is

(A5) 
$$\ln (Y_i^*) = X_i \beta + f(A_{it}) + \hat{S}_i^*$$

Separate earnings equations were estimated for male heads of households and for wives; households headed by a woman were deleted from the sample. A significant portion of the individual (almost twenty-five percent) had current earnings of less than \$2500, probably in part because of part-time work. The model is designed to predict earnings of "full-time" employees, so the sample was truncated at earnings of \$2500. The resulting bias from selecting on the dependent variable was corrected for using Heckman's (1979) two-stage procedure.<sup>\*\*</sup>

<sup>\*</sup>Because only a single cross section of data is used, it is not possible to obtain estimates of  $\sigma_s^2$  and  $\sigma_s^2$  in addition to  $\beta$ . It is necessary to assume a value for  $\sigma_s^2/(\sigma_s^2 + \sigma_\epsilon^2)$ based on a study of longitudinal earnings data. Following the survey of such studies in King and Dicks-Mireaux (1982), I assumed that  $\sigma_s^2/(\sigma_s^2 + \sigma_\epsilon^2) = 0.5$ .

<sup>&</sup>quot;The drill is the same as in the asset demand case in the text. In the first stage, a probit analysis of the full sample yields the parameters of the probability that an individual will be in the truncated sample. The second stage is to estimate the earnings function by ordinary least squares with the addition of the universe of Mill's ratio (computed for each observation in the truncated sample) as an explanatory variable.

The probit model for men includes as explanatory variables youth, old age, low education, marriage, part-time work, and unemployment. The women's probit model was identical to that for men with addition of number of children under two, the number of children between two and five, and the number of children between five and twelve as explanatory variables. Regressions included as independent variables occupational dummies, a cubic polynomial in variables. Regressions included as independent variables occupational dummies, a cubic polynomal in variables. Regressions included as independent variables occupational dummies, a cubic polynomial in age, race (white versus nonwhite), education levels, self-employment status, martial status, and the inverse of the Mill's ratio from the probit analysis.

Details of the maximum-likelihood estimates of the probit model for earnings of less than \$2500 or of the second stage estimates of the earnings equation are available on request. Total household permanent income is the same as the estimates for husbands and for wives.

#### Computation of the Pension Wealth Variables

Gross social security wealth is computed according to the following procedures. The Social Security Administration (SSA) was able to match 5516 repondents with social security records. On the basis of an assumption of two percent future growth of real wages and given the law in 1979, statisticians of the SSA calculated the projected Primary Insurance Amount (PIA) for age 65 for the non-retired. For those retired, the actual PIA was directly available. In computing the present values, a real discount rate of three percent was used, with average inflation projected to be four percent. Standard mortality assumptions were used in the present value calculation. The household's social security wealth is the sum of the husband's and the wife's social security wealth. Results were not very sensitive to changes in assumptions.

Present values were also computed for private pension benefits. Attempts were made to calculate the present value of pension benefits for all respondents who currently receive a pension, who currently participate in a plan, or who are terminated with a vested benefit. When information was available, the present value of all pension benefits accrued to date was calculated from the respondent's questionnaire, the EBS-1 forms, and the employer questionnaire. In the calculations used in the paper, a real discount rate of three percent was used in conjunction with a four percent inflation rate. Again, standard mortality assumptions were used.

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- A4 -