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ABSTRACT

We examine the relationship between monetary-policy-induced changes in short interest rates and yields on long-maturity default-free bonds. The volatility of the long end of the term structure and its relationship with monetary policy are puzzling from the perspective of simple structural macroeconomic models. We explore whether richer models of risk premiums, specifically stochastic volatility models combined with Epstein-Zin recursive utility, can account for such patterns. We study the properties of the yield curve when inflation is an exogenous process and compare this to the yield curve when inflation is endogenous and determined through an interest-rate/Taylor rule. When inflation is exogenous, it is difficult to match the shape of the historical average yield curve. Capturing its upward slope is especially difficult as the nominal pricing kernel with exogenous inflation does not exhibit any negative autocorrelation - a necessary condition for an upward sloping yield curve as shown in Backus and Zin (1994). Endogenizing inflation provides a substantially better fit of the historical yield curve as the Taylor rule provides additional flexibility in introducing negative autocorrelation into the nominal pricing kernel. Additionally, endogenous inflation provides for a flatter term structure of yield volatilities which better fits historical bond data.

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1 Introduction

The response of long-term interest rates to changes in short term interest rates is a feature of the economy that often puzzles policy makers. For example, in remarks made on May 27, 1994, Alan Greenspan expressed concern that long rates moved too much in response to an increase in short rates:

> In early February, we thought long-term rates would move a little higher as we tightened. The sharp jump in [long] rates that occurred appeared to reflect the dramatic rise in market expectations of economic growth and associated concerns about possible inflation pressures.[1]

Then in his February 16, 2005, testimony, Chairman Greenspan expressed a completely different concern about long rates:

> Long-term interest rates have trended lower in recent months even as the Federal Reserve has raised the level of the target federal funds rate by 150 basis points. Historically, even distant forward rates have tended to rise in association with monetary policy tightening. ... For the moment, the broadly unanticipated behavior of world bond markets remains a conundrum.[2]

Chairman Greenspan's comments are a reflection of the fact that we do not yet have a satisfactory understanding of how the yield curve is related to the structural features of the macroeconomy such as investors' preferences, the fundamental sources of economic risk, and monetary policy.

Figure 1 plots the nominal yield curve for a variety of maturities from one quarter—which we refer to as the short rate—up to forty quarters for US treasuries starting in the first quarter of 1970 and

[1] Testimony of Chairman Alan Greenspan before the U.S. Senate Committee on Banking, Housing, and Urban Affairs, May 27, 1994. Federal Reserve Bulletin, July 1994.

[2] Testimony of Chairman Alan Greenspan before the U.S. Senate Committee on Banking, Housing, and Urban Affairs, February 16, 2005.
`http://www.federalreserve.gov/boarddocs/hh/2005/february/testimony.htm`

ending in the last quarter of 2005.[3] Figure 2 plots the average yield curve for the entire sample and for two subsamples. Figure 3 plots the standard deviation of yields against their maturities. Two basic patterns of yields are clear from these figures: (1) On average the yield curve is upward sloping, and (2) there is substantial volatility in yields at all maturities. Chairman Greenspan's comments, therefore, must be framed by the fact that long yields are almost as volatile as short rates. The issue, however, is the relationship of the volatility at the long end to the volatility at the short end, and the correlation between changes in short-term interest rates and changes in long-term interest rates.

We can decompose forward interest rates into expectations of future short-term interest rates and interest rate risk premia. Since long-term interest rates are averages of forward rates, long-run interest rates depend on expectations of future short-term interest rates and interest rate risk premiums. A significant component of long rates is the risk premium and there is now a great deal of empirical evidence documenting that the risk premiums are time-varying and stochastic. Movements in long rates can therefore be attributed to movements in expectations of future nominal short rates, movements in risk premiums, or some combination of movements in both.

Moreover, if monetary policy is implemented using a short-term interest rate feedback rule, *e.g.*, a Taylor rule, then inflation rates must adjust so that the bond market clears. The resulting endogenous equilibrium inflation rate will then depend on the same risk factors that drive risk premiums in long rates. Monetary policy itself, therefore, could be a source of fluctuations in the yield curve in equilibrium.

We explore such possibilities in a model of time-varying risk premiums generated by the recursive utility model of Epstein and Zin (1989) combined with stochastic volatility of endowment growth. We show how the model can be easily solved using now standard affine term-structure methods. Affine term-structure models have the convenient property that yields are maturity-dependent linear functions of state variables. We examine some general properties of multi-period default-free bonds in our model assuming first that inflation is an exogenous process, and by allowing inflation

[3] Yields up to 1991 are from McCulloch and Kwon (1993) then Datastream from 1991 to 2005.

to be endogenous and determined by an interest-rate feedback rule. We show that the interest rate feedback rule—the form of monetary policy—can have significant impacts on properties of the term structure of interest rates.

2 The Duffie-Kan Affine Term Structure Model

The Duffie and Kan (1996) class of affine term-structure models, translated into discrete time by Backus et al. (2001), is based on a k-dimensional vector of state variables z that follows a "square-root" model

$$z_{t+1} = (I - \Phi)\theta + \Phi z_t + \Sigma(z_t)^{1/2}\varepsilon_{t+1},$$

where $\{\varepsilon_t\} \sim \text{NID}(0, I)$, $\Sigma(z)$ is a diagonal matrix with a typical element given by $\sigma_i(z) = a_i + b_i'z$ where b_i has nonnegative elements, and Φ is stable with positive diagonal elements. The process for z requires that the volatility functions $\sigma_i(z)$ be positive, which places additional restrictions on the parameters.

The asset pricing implications of the model are given by the pricing kernel, m_{t+1}, a positive random variable that prices all financial assets. That is, if a security has a random payoff h_{t+1} at date-$t+1$, then its date-t price is $E_t[m_{t+1}h_{t+1}]$. The pricing kernel in the affine model takes the form

$$-\log m_{t+1} = \delta + \gamma' z_t + \lambda' \Sigma(z_t)^{1/2}\varepsilon_{t+1},$$

where the $k \times 1$ vector γ is referred to as the "factor loadings" for the pricing kernel, the $k \times 1$ vector λ is referred to as the "price of risk" vector since it controls the size of the conditional correlation of the pricing kernel and the underlying sources of risk, and the $k \times k$ matrix $\Sigma(z_t)$ is the stochastic variance-covariance matrix of the unforecastable shock.

Let $b_t^{(n)}$ be the price at date-t of a default-free pure-discount bond that pays 1 at date $t + n$, with $b_t^{(0)} = 1$. Multi-period default-free discount bond prices are built up using the arbitrage-free pricing restriction

$$b_t^{(n)} = E_t[m_{t+1} b_{t+1}^{(n-1)}]. \tag{1}$$

Bond prices of all maturities are log-linear functions of the state:

$$-\log b_t^{(n)} = A^{(n)} + B^{(n)} z_t,$$

where $A^{(n)}$ is a scalar, and $B^{(n)}$ is a $1 \times k$ row vector.

The intercept and slope parameters, which we often refer to as "yield-factor loadings," of these bond prices can be found recursively according to

$$\begin{aligned} A^{(n+1)} &= A^{(n)} + \delta + B^{(n)}(I - \Phi)\theta - \frac{1}{2}\sum_{j=1}^{k}(\lambda_j + B_j^{(n)})^2 a_j, \\ B^{(n+1)} &= (\gamma' + B^{(n)}\Phi) - \frac{1}{2}\sum_{j=1}^{k}(\lambda_j + B_j^{(n)})^2 b_j', \end{aligned} \tag{2}$$

where $B_j^{(n)}$ is the j-th element of the vector $B^{(n)}$. Since $b^{(0)} = 1$, we can start these recursions using $A^{(0)} = 0$ and $B_j^{(0)} = 0,\ j = 1, 2, ..., k$.

Continuously compounded yields, $y_t^{(n)}$ are defined by $b_t^{(n)} = \exp(-n y_t^{(n)})$, which implies $y_t^{(n)} = -(\log b_t^{(n)})/n$. We refer to the short rate, i_t, as the one-period yield: $i_t \equiv y_t^{(1)}$.

This is an arbitrage-free model of bond pricing since it satisfies equation (1) for a given pricing kernel m_t. It is not yet a structural equilibrium model, since the mapping of the parameters of the pricing model to deeper structural parameters of investors' preferences and opportunities has not yet been specified. The equilibrium structural models we consider will all lie within this general class, hence, can be easily solved using these pricing equations.

3 A 2-Factor Model with Epstein-Zin Preferences

We begin our analysis of structural models of the yield curve by solving for equilibrium real yields in a representative-agent exchange economy. Following Backus and Zin (2006) we consider a representative agent who chooses consumption to maximize the recursive utility function given in Epstein and Zin (1989). Given a sequence of consumption, $\{c_t, c_{t+1}, c_{t+2}, ...\}$, where future consumptions can be random outcomes, the intertemporal utility function U_t is the solution to the recursive equation:

$$U_t = [(1-\beta)c_t^{\rho} + \beta\mu_t(U_{t+1})^{\rho}]^{1/\rho}, \tag{3}$$

where $0 < \beta < 1$ characterizes impatience (the marginal rate of time preference is $1 - 1/\beta$), $\rho \leq 1$ measures the preference for intertemporal substitution (the elasticity of intertemporal substitution for deterministic consumption paths is $1/(1-\rho)$), and the certainty equivalent of random future utility is

$$\mu_t(U_{t+1}) \equiv E_t \left[U_{t+1}^{\alpha}\right]^{\frac{1}{\alpha}}, \tag{4}$$

where $\alpha \leq 1$ measures static risk aversion (the coefficient of relative risk aversion for static gambles is $1-\alpha$). The marginal rate of intertemporal substitution, m_{t+1}, is

$$m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{\rho-1} \left(\frac{U_{t+1}}{\mu_t(U_{t+1})}\right)^{\alpha-\rho},$$

Time-additive expected utility corresponds to the parameter restriction $\rho = \alpha$.

In equilibrium, the representative agent consumes the stochastic endowment, e_t, so that $\log(c_{t+1}/c_t) = \log(e_{t+1}/e_t) = x_{t+1}$, where x_{t+1} is the log of the ratio of endowments in $t+1$ relative to t. The log of the equilibrium marginal rate of substitution, referred to as the real pricing kernel, is therefore given by

$$\log m_{t+1} = \log\beta + (\rho-1)x_{t+1} + (\alpha-\rho)\left[\log W_{t+1} - \log\mu_t(W_{t+1})\right], \tag{5}$$

where W_t is the value of utility in equilibrium.

The first two terms in the marginal rate of substitution are standard expected utility terms: the pure time preference parameter β and a consumption growth term times the inverse of the negative of the intertemporal elasticity of substitution. The third term in the pricing kernel is a new term coming from the Epstein-Zin preferences.

The endowment-growth process evolves stochastically according to

$$x_{t+1} = (1-\phi_x)\theta_x + \phi_x x_t + v_t^{1/2}\varepsilon_{t+1}^x,$$

where

$$v_{t+1} = (1-\phi_v)\theta_v + \phi_v v_t + \sigma_v \varepsilon_{t+1}^v$$

is the process for the conditional volatility of endowment growth. We will refer to v_t as stochastic volatility. The innovations ε_t^x and ε_t^v are distributed NID$(0, I)$.

Note that the state vector in this model conforms with the setup of the Duffie-Kan model above. Define the state vector $z_t \equiv [x_t \;\; v_t]'$, which implies parameters for the Duffie-Kan model:

$$\begin{aligned}
\theta &= [\theta_x \;\; \theta_v]' \\
\Phi &= \text{diag}\{\phi_x,\ \phi_v\} \\
\Sigma(z_t) &= \text{diag}\{a_1 + b_1' z_t,\ a_2 + b_2' z_t\} \\
a_1 &= 0,\ b_1 = [0 \;\; 1]',\ a_2 = \sigma_v^2,\ b_2 = [0 \;\; 0]'.
\end{aligned}$$

Following the analysis in Hansen et al. (2005), we will work with the logarithm of the value function scaled by the endowment:

$$\begin{aligned}
W_t/e_t &= [(1-\beta) + \beta(\mu_t(W_{t+1})/e_t)^\rho]^{1/\rho} \\
&= \left[(1-\beta) + \beta(\mu_t\left(\frac{W_{t+1}}{e_{t+1}} \times \frac{e_{t+1}}{e_t}\right)^\rho\right]^{1/\rho},
\end{aligned} \tag{6}$$

where we have used the linear homogeneity of μ_t (see equation (4)). Take logarithms of (6) to obtain

$$w_t = \rho^{-1} \log[(1-\beta) + \beta \exp(\rho u_t)],$$

where $w_t \equiv \log(W_t/e_t)$ and $u_t \equiv \log(\mu_t(\exp(w_{t+1}+x_{t+1})))$. Consider a linear approximation of the right-hand side of this equation as a function of u_t around the point $\bar{m}$:

$$\begin{aligned} w_t &\approx \rho^{-1} \log[(1-\beta) + \beta \exp(\rho \bar{m})] + \left[\frac{\beta \exp(\rho \bar{m})}{1-\beta+\beta \exp(\rho \bar{m})}\right](u_t - \bar{m}) \\ &\equiv \bar{\kappa} + \kappa u_t, \end{aligned}$$

where $\kappa < 1$. For the special case with $\rho = 0$, *i.e.*, a log time aggregator, the linear approximation is exact with $\bar{\kappa} = 1-\beta$ and $\kappa = \beta$ (see Hansen et al. (2005)). Similarly, approximating around $\bar{m} = 0$, results in $\bar{\kappa} = 0$ and $\kappa = \beta$.

Given the state variables and the log-linear structure of the model, we conjecture a solution for the log value function of the form,

$$w_t = \bar{\omega} + \omega_x x_t + \omega_v v_t,$$

where $\bar{\omega}$, ω_x, and ω_v are constants to be determined. By substituting

$$w_{t+1} + x_{t+1} = \bar{\omega} + (\omega_x + 1)x_{t+1} + \omega_v v_{t+1}.$$

Since x_{t+1} and v_{t+1} are jointly normally distributed, the properties of normal random variables can be used to solve for u_t:

$$\begin{aligned} u_t &\equiv \log(\mu_t(\exp(w_{t+1}+x_{t+1}))) \\ &= \log(E_t[\exp(w_{t+1}+x_{t+1})^{\alpha}]^{\frac{1}{\alpha}})] \\ &= E_t[w_{t+1}+x_{t+1}] + \frac{\alpha}{2}\text{Var}_t[w_{t+1}+x_{t+1}] \\ &= \bar{\omega} + (\omega_x+1)(1-\phi_x)\theta_x + \omega_v(1-\phi_v)\theta_v + (\omega_x+1)\omega_x x_t + \omega_v \phi_v v_t \\ &\quad + \frac{\alpha}{2}(\omega_x+1)^2 v_t + \frac{\alpha}{2}\omega_v^2 \sigma_v^2. \end{aligned}$$

We can use the above expression to solve for the value-function parameters and verify its log-linear solution

$$
\begin{aligned}
\omega_x &= \kappa(\omega_x+1)\phi_x \\
\Rightarrow \omega_x &= \left(\frac{\kappa}{1-\kappa\phi_x}\right)\phi_x \\
\omega_v &= \kappa[\omega_v\phi_v + \frac{\alpha}{2}(\omega_x+1)^2] \\
\Rightarrow \omega_v &= \left(\frac{\kappa}{1-\kappa\phi_v}\right)\left[\frac{\alpha}{2}\left(\frac{1}{1-\kappa\phi_x}\right)^2\right] \\
\bar{\omega} &= \frac{\bar{\kappa}}{1-\kappa} + \frac{1}{1-\kappa}\left[(\omega_x+1)(1-\phi_x)\theta_x + \omega_v(1-\phi_v)\theta_v + \frac{\alpha}{2}\omega_v^2\sigma_v^2\right].
\end{aligned}
$$

The solution allows us to simplify the term $[\log W_{t+1} - \log \mu_t(W_{t+1})]$ in the real pricing kernel in equation (5):

$$
\begin{aligned}
\log W_{t+1} - \log \mu_t(W_{t+1}) &= w_{t+1} + x_{t+1} - \log \mu_t(w_{t+1} + x_{t+1}) \\
&= (\omega_x+1)[x_{t+1} - E_t x_{t+1}] + \omega_v[v_{t+1} - E_t v_{t+1}] \\
&\quad -\frac{\alpha}{2}(\omega_x+1)^2 \mathrm{Var}_t[x_{t+1}] - \frac{\alpha}{2}\omega_v^2 \mathrm{Var}_t[v_{t+1}] \\
&= (\omega_x+1)v_t^{1/2}\varepsilon_{t+1}^x + \omega_v\sigma_v\varepsilon_{t+1}^v - \frac{\alpha}{2}(\omega_x+1)^2 v_t - \frac{\alpha}{2}\omega_v^2\sigma_v^2.
\end{aligned}
$$

The real pricing kernel, therefore, is a member of the Duffie-Kan class with 2-factors and parameters

$$
\begin{aligned}
&\delta = -\log(\beta) + (1-\rho)(1-\phi_x)\theta_x + \frac{\alpha}{2}(\alpha-\rho)\omega_v^2\sigma_v^2 \\
&\gamma = [\gamma_x \qquad \gamma_v]' \\
&\quad = \left[(1-\rho)\phi_x \qquad \frac{\alpha}{2}(\alpha-\rho)\left(\frac{1}{1-\kappa\phi_x}\right)^2\right]' \qquad (7) \\
&\lambda = [\lambda_x \qquad \lambda_v]' \\
&\quad = \left[(1-\alpha) - (\alpha-\rho)\left(\frac{\kappa\phi_x}{1-\kappa\phi_x}\right) \qquad -\left(\frac{\alpha}{2}\right)\left(\frac{\kappa(\alpha-\rho)}{1-\kappa\phi_v}\right)\left(\frac{1}{1-\kappa\phi_x}\right)^2\right]'.
\end{aligned}
$$

We can now use the recursive formulas in equation (2) to solve for real discount bond prices and the real yield curve.

Note how the factor loadings and prices of risk depend on the deeper structural parameters, and the greatly reduced dimensionality of the parameter space relative to the general affine model. Also note that for the time-additive expected utility special case, $\alpha = \rho$, the volatility factor does not enter the conditional mean of the pricing kernel since $\gamma_v = 0$, and also that the price of risk for the volatility factor is zero since $\lambda_v = 0$. Finally, we can see from the expressions for bond prices that the two key preference parameters, ρ and α, provide freedom in controlling both the factor loadings and the prices of risk in the real pricing kernel.

4 Nominal Bond Pricing

To understand the price of nominal bonds, we need a nominal pricing kernel. If we assume that there is a frictionless conversion of money for goods in this economy, the nominal kernel is given by

$$\log(m_{t+1}^{\$}) = \log(m_{t+1}) - p_{t+1}, \tag{8}$$

where p_{t+1} is the log of the money price of goods at time $t+1$ relative to the money price of goods at time t, *i.e.*, the inflation rate between t and $t+1$. Clearly then, the source of inflation, its random properties, and its relationship to the real pricing kernel is of central interest for nominal bond pricing. We next consider two different specifications for equilibrium inflation.

4.1 Exogenous Inflation

If we expand the state space to include an exogenous inflation process, p_t, the state vector becomes $z_t = [x_t \;\; v_t \;\; p_t]'$. The stochastic process for exogenous inflation is

$$p_{t+1} = (1 - \phi_p)\theta_p + \phi_p p_t + \sigma_p \varepsilon_{t+1}^{p},$$

where ε^p_{t+1} is also normally distributed independently of the other two shocks. In this case, the parameters for the affine nominal pricing kernel are

$$\begin{aligned}\delta^{\$} &= \delta + (1-\phi_p)\theta_p \\ \gamma^{\$} &= [\gamma_x \quad \gamma_v \quad \phi_p]' \\ \lambda^{\$} &= [\lambda_x \quad \lambda_v \quad 1]'.\end{aligned}$$

In the exogenous inflation model, the price of inflation risk is always exactly 1, and does not change with the values of any of the other structural parameters in the model. In addition, the factor loadings and prices of risk for output growth and stochastic volatility are the same as in the real pricing kernel. We will refer to this nominal pricing kernel specification as the exogenous inflation economy.

4.2 Monetary Policy and Endogenous Inflation

We begin by assuming that monetary policy follows a simple nominal interest-rate rule. We will abuse conventional terminology and often refer to the interest-rate rule as a Taylor rule. While there are a variety of ways to specify a Taylor rule—see Ang et al. (2004)—we will consider a rule in which the short term interest rate depends on contemporaneous output, inflation and a policy shock:

$$i_t = \bar{\tau} + \tau_x x_t + \tau_p p_t + s_t, \tag{9}$$

where the monetary policy shock satisfies

$$s_t = \phi_s s_{t-1} + \sigma_s \varepsilon^s_t,$$

and where $\varepsilon^s_t \sim \text{NID}(0,1)$ is independent of the other two real shocks.

Since this nominal interest rate rule must also be consistent with equilibrium in the bond market, *i.e.*, it must be consistent with the nominal pricing kernel in equation (8) as well as equation (9), we can use these two equations to find the equilibrium process for inflation. Conjecture a log-linear solution for p_t,

$$p_t = \bar{\pi} + \pi_x x_t + \pi_v v_t + \pi_s s_t, \tag{10}$$

with $\bar{\pi}$, π_x, and π_s constants to be solved.

To solve for a rational expectations solution to the model, we substitute the guess for the inflation rate into both the Taylor rule and the nominal pricing kernel and solve for the parameters $\bar{\pi}$, π_x, π_v, and π_s that equate the pricing-kernel-determined short rate with the Taylor-rule-determined short rate.

From the dynamics of x_{t+1}, v_{t+1}, and s_{t+1}, inflation p_{t+1} is given by

$$\begin{aligned}
p_{t+1} &= \bar{\pi} + \pi_x x_{t+1} + \pi_v v_{t+1} + \pi_s s_{t+1} \\
&= \bar{\pi} + \pi_x(1-\phi_x)\theta_x + \pi_v(1-\phi_v)\theta_v + \pi_x\phi_x x_t + \pi_v\phi_v v_t + \pi_s\phi_s s_t \\
&\quad + \pi_x v_t^{1/2}\varepsilon_{t+1}^x + \pi_v\sigma_v\varepsilon_{t+1}^v + \pi_s\sigma_s\varepsilon_{t+1}^s.
\end{aligned}$$

Substituting into the nominal pricing kernel,

$$\begin{aligned}
-\log(m_{t+1}^{\$}) &= -\log(m_{t+1}) + p_{t+1} \\
&= \delta + \gamma_x x_t + \gamma_v v_t + \gamma_x v_t^{1/2}\varepsilon_{t+1}^x + \gamma_v\sigma_v\varepsilon_{t+1}^v + p_{t+1} \\
&= \delta + \bar{\pi} + \pi_x(1-\phi_x)\theta_x + \pi_v(1-\phi_v)\theta_v \\
&\quad + (\gamma_x + \pi_x\phi_x)\, x_t + (\gamma_v + \pi_v\phi_v)\, v_t + \pi_s\phi_s s_t \\
&\quad + (\lambda_x + \pi_x)\, v_t^{1/2}\varepsilon_{t+1}^x + (\lambda_v + \pi_v)\,\sigma_v\varepsilon_{t+1}^v + \pi_s\sigma_s\varepsilon_{t+1}^s.
\end{aligned}$$

From these dynamics, the nominal one period interest rate $i_t = -log\left(E_t\left[m_{t+1}^{\$}\right]\right)$ is

$$\begin{aligned}
i_t &= \delta + \bar{\pi} + \pi_x(1-\phi_x)\theta_x + \pi_v(1-\phi_v)\theta_v \\
&\quad + (\gamma_x + \pi_x\phi_x)\, x_t + (\gamma_v + \pi_v\phi_v)\, v_t + \pi_s\phi_s s_t \\
&\quad - \frac{1}{2}(\lambda_x + \pi_x)^2 v_t - \frac{1}{2}(\lambda_v + \pi_v)^2\sigma_v^2 - \frac{1}{2}\pi_s^2\sigma_s^2.
\end{aligned}$$

Comparing this to the interest rate rule $i_t = \bar{\tau} + \tau_x x + \tau_p \left(\bar{\pi} + \pi_x x_t + \pi_v v_t + \pi_s s_t\right) + s_t$, gives the parameter restrictions consistent with equilibrium:

$$\begin{aligned}
\pi_x &= \frac{\gamma_x - \tau_x}{\tau_p - \phi_x} \\
\pi_v &= \frac{\gamma_v - \frac{1}{2}(\lambda_x + \pi_x)^2}{\tau_p - \phi_v} \\
\pi_s &= -\frac{1}{\tau_p - \phi_s} \\
\bar{\pi} &= \frac{1}{\tau_p - 1}\left[\delta - \bar{\tau} + \pi_x(1-\phi_x)\theta_x + \pi_v(1-\phi_v)\theta_v - \frac{1}{2}(\lambda_v + \pi_v)^2\sigma_v^2 - \frac{1}{2}\pi_s^2\sigma_s^2\right]. \qquad (11)
\end{aligned}$$

These expressions form a recursive system we use to solve for the equilibrium parameters of the inflation process. See Cochrane (2006) for a more detailed account of this rational expectations solution method.

It is clear from these expressions that the equilibrium inflation process will depend on the preference parameters of the household generally, and attitudes towards risk specifically.

In a similar fashion, we can extend the analysis to any Taylor rule-type that is linear in state variables, including lagged short rates, other contemporaneous yields at any maturity, as well as forward-looking rules, as in Clarida et al. (2000), since in the affine framework, interest rates are all simply linear functions of the current state variables. See Ang et al. (2004) and Gallmeyer et al. (2005) for some concrete examples.

4.3 A Monetary-Policy Consistent Pricing Kernel

Substituting the equilibrium inflation process from equations (10) and (11) into the nominal pricing kernel, an equilibrium 3-factor affine term structure model that is consistent with the nominal-interest rate rule is obtained.

The state space is

$$
\begin{aligned}
z_t &\equiv [x_t \quad v_t \quad s_t]' \\
\Phi &= \mathrm{diag}\{\phi_x,\ \phi_v,\ \phi_s\} \\
\theta &= [\theta_x \qquad \theta_v \qquad 0]' \\
\Sigma(z_t) &= \mathrm{diag}\{a_1 + b_1' z_t,\ a_2 + b_2' z_t,\ a_3 + b_3' z_t\} \\
& \quad a_1 = 0,\ b_1 = [0 \quad 1 \quad 0]' \\
& \quad a_2 = \sigma_v^2,\ b_2 = [0 \quad 0 \quad 0]' \\
& \quad a_3 = \sigma_s^2,\ b_3 = [0 \quad 0 \quad 0]',
\end{aligned}
$$

and the parameters of the pricing kernel are

$$
\begin{aligned}
\delta^{\$} &= \delta + \bar{\pi} + \pi_x(1-\phi_x)\theta_x + \pi_v(1-\phi_v)\theta_v \\
\gamma^{\$} &= [\gamma_x + \phi_x \pi_x \qquad \gamma_v + \phi_v \pi_v \qquad \phi_s \pi_s]' \\
\lambda^{\$} &= [\lambda_x + \pi_x \qquad \lambda_v + \pi_v \qquad \pi_s]'.
\end{aligned}
$$

We will often refer to this nominal pricing kernel specification as the endogenous inflation economy.

The Taylor rule parameters, through their determination of the equilibrium inflation process, affect both the factor loadings on the real factors as well as their prices or risk. Monetary policy through its effects on endogenous inflation, therefore, can result in significantly different risk premiums in the term structure than the exogenous-inflation model. We explore such as possibility through numerical examples.

5 Quantitative Exercises

We calibrate the exogenous processes in our model to quarterly post-war US data as follows:

1. Endowment Growth. $\phi_x = 0.36$, $\theta_x = 0.006$, $\sigma_x = 0.0048(1-\phi_x^2)^{1/2}$;

2. Inflation. $\phi_p = 0.8471$, $\theta_p = 0.0093$, $\sigma_p = 0.0063(1 - \phi_p^2)^{1/2}$;

3. Stochastic volatility. $\phi_v = 0.973$, $\theta_v = 0.0001825$, $\sigma_v = 0.9884 \times 10^{-5}$;

4. Policy Shock. $\phi_s = 0.922$, $\sigma_s = (0.023 \times 10^{-4})^{1/2}$.

The endowment growth process is calibrated to quarterly per capita consumption of durable goods and services, and inflation is calibrated to the nondurables and services deflator, similarly to Piazzesi and Schneider (2006). The volatility process is taken from Bansal and Yaron (2004), who calibrate their model to monthly data. We adjust their parameters to deal with quarterly time-aggregation. We take the parameters for the policy shock from Ang et al. (2004) who estimate a Taylor rule using an affine term-structure model with macroeconomic factors and an unobserved policy shock.

Figures 4 though 7 depict the average yield curves and yield volatilities for different preference parameters for the exogenous and endogenous inflation models. The top plot in each figure depicts the average historical nominal yield curve with stars ($**$), the average real yield curve common across both inflation models with a dashed line ($--$), the average nominal yield curve in the exogenous inflation economy with a dashed-dotted line (- – -), and the average nominal yield curve in the endogenous inflation economy with a solid line (–). The bottom plot depicts yield volatilities for the same cases as the average yield curve plot.

Each figure is computed using a different set of preference parameters. We fix a level of the intertemporal elasticity parameter ρ for each plot, and pick the remaining preference parameters—the risk aversion coefficient α and the rate of time preference β—to minimize the distance between the average nominal yields and yield volatilities in the data and the those implied by the exogenous inflation economy. We pick the Taylor-rule parameters to minimize the distance between the average nominal yields and yield volatilities in the data and the those implied by the endogenous inflation economy. Table 1 reports the factor loadings and the prices of risk for each economy corresponding to the figures. Table 2 reports the coefficients on the equilibrium inflation rate and properties of the equilibrium inflation rate in the endogenous inflation economy.

Figure 4 reports the results with $\rho = -0.5$; here the representative agent has a low intertemporal elasticity of substitution. The remaining preference parameters are $\alpha = -4.83$, $\beta = 0.999$. With this choice of parameters, the average real term structure is slightly downward sloping.

Backus and Zin (1994) show that a necessary condition for the average yield curve to be upward sloping is negative autocorrelation in the pricing kernel.[4] Consider an affine model with independent factors $z_t^1, z_t^2, ..., z_t^k$ with an innovation ϵ_t^j on the j^{th} factor, a factor loading γ_j on the j^{th} factor, and a price of risk λ_j on the j^{th} factor. In such a model, the j^{th} factor contributes

$$\gamma_j^2 \text{Autocov}(z_t^j) + \gamma_j \lambda_j \text{Cov}(z_t^j, \epsilon_t^j), \tag{12}$$

to the autocovariance of the pricing kernel.

In our calibration, the exogenous factors in the real economy—output growth and stochastic volatility—all have positive autocovariances and the factor innovations have positive covariances to the factor levels. This implies that $\gamma_j^2 \text{Autocov}(z_t^j)$ and $\text{Cov}(z_t^j, \epsilon_t^j)$ are both positive. For a factor to contribute negatively to the autocorrelation of the pricing kernel requires that the factor loading γ_j and the price of risk λ_j have opposite signs. Additionally, the price of risk λ_j must be large enough relative to the factor loading γ_j to counteract the positive autocovariance term $\gamma_j^2 \text{Autocov}(z_t^j)$.

Output growth has a lower autocorrelation coefficient than stochastic volatility in our calibration, but since output growth has a much higher unconditional volatility, it has a much higher autocovariance than stochastic volatility. The factor loading γ_x in the real economy on the level of output growth is equal to $(1-\rho)\phi_x$, which is nonnegative for all $\rho \leq 1$. Also, the price of risk for output growth λ_x is positive at the parameter values used in Figure 4 since a sufficient condition for it to be positive is $\alpha \leq 0$ and $|\rho| \leq |\alpha|$. From (12), output growth contributes positively to the autocovariance of the pricing kernel.

[4] Piazzesi and Schneider (2006) argue that an upward-sloping nominal yield curve can be generated if inflation is bad news for consumption growth. Such a structure leads to negative autocorrelation in the nominal pricing kernel.

From the real pricing kernel parameters given in (7), the price of risk for volatility is related to the factor loading on the level of volatility by

$$\lambda_v = -\frac{\beta}{1-\beta\phi_v}\gamma_v.$$

Since $1-\beta\phi_v > 0$, the volatility price of risk λ_v and the volatility factor loading γ_v have opposite signs implying that the volatility factor can contribute a negative autocovariance to the pricing kernel. But output growth has the strongest effect on the autocovariance of the pricing kernel, leading to positively autocovariance in the pricing kernel. As a consequence, the average real yield curve is downward sloping. The numerical values for the real pricing kernel's factor loadings and prices of risk from Figure 4 are reported in Panel A of Table 1.

In the exogenous inflation economy, shocks to inflation are uncorrelated to output growth and stochastic volatility—the factor loadings and prices of risk on output growth and stochastic volatility in the nominal pricing kernel are the same as in the real pricing kernel. Average nominal yields in the exogenous inflation economy are equal to the real yields plus expected inflation and inflation volatility with an adjustment for properties of the inflation process. The inflation shocks are positively autocorrelated with a factor loading and a price of risk that are both positive. The average nominal yield curve has approximately the same shape as the real yield curve—it is downward sloping.

In the endogenous inflation economy, inflation is a linear combination of output growth, stochastic volatility, and the monetary policy shock. From Panel A of Table 2, endogenous inflation's loading on output, π_x, is negative. This implies that the nominal pricing kernel's output growth factor loading and price of risk are lower than in the exogenous inflation economy. As a consequence, output growth contributes much less to the autocovariance of the pricing kernel with endogenous inflation. The factor loading and price of risk for stochastic volatility are also lower in the endogenous inflation economy. The policy shocks are positively autocorrelated, but the sign of the loading and the price of risk for the policy shock are of opposite sign. The average nominal yield curve in the endogenous inflation economy is therefore flatter than both the real yield curve and the nominal yield curve with exogenous inflation.

Turning to the volatilities in the bottom plot, the exogenous inflation economy exhibits more volatility in short rates and less volatility in long rates than found in the data. This is a fairly standard finding for term structure models with stationary dynamics (see Backus and Zin (1994)). The volatility of long rates is mainly driven by the loading on the factor with the largest innovation variance and that factor's autocorrelation. The closer that autocorrelation is to zero, the faster that yield volatility decreases in bond maturity. In our calibration, output growth has the largest innovation variance and a fast rate of mean reversion, equal to 0.36. Yield volatility drops quite quickly as bond maturity increases. In general, the lower the loading on output growth, the slower that yield volatility drops as bond maturity increases. Because endogenous inflation is negatively related to output growth, the factor loading on output growth is lower. Yield volatility drops at a slower rate versus maturity in the endogenous inflation economy than in the exogenous inflation economy.

Figure 5, Panel B of Table 1, and Panel B of Table 2 report yield curve properties with a higher intertemporal elasticity of substitution $\rho = 0$, or a log time aggregator. Piazzesi and Schneider (2006) study a model with the same preferences, but without stochastic volatility. The factor loading on output growth in the real economy is higher than in the economy with $\rho = -0.5$ reported in Figure 4 (compare Panel A to Panel B of Table 1.) The average real yield curve and the average nominal yield curve with exogenous inflation are less downward sloping when $\rho = 0$ than when $\rho = -0.5$. Similarly, increasing ρ further to 0.5 (see Figure 6) or 1.0 (see Figure 7) leads to a less downward sloping real yield curve. Since increasing ρ decreases the factor loading on output growth, it also decreases the volatility of real yields: see the bottom plots in Figures 4 to 7.

As ρ increases, the representative agent's intertemporal elasticity of substitution increases implying less demand for smoothing consumption over time. Increasing ρ decreases the representative agent's demand for long term bonds for the purpose of intertemporal consumption smoothing, and leads to lower equilibrium prices and higher yields for real long term bonds. The average real yield curve therefore is less downward sloping as ρ increases. Increasing ρ also reduces the sensitivity of long-term real yields to output growth, leading to less volatile long-term yields: See the bottom plots in Figures 4 to 7.

Nominal yields in the economies with exogenous inflation are approximately equal to the real yields plus a maturity independent constant. But in the economies with endogenous inflation, inflation and output growth have a negative covariance leading to a decrease in the factor loading on output growth with endogenous inflation–see Panels C and D of Tables 1 and 2. For $\rho \geq 0.5$—see Figures 6 and 7, the average nominal yield curve is upward sloping, and the shape of the volatility term structure decays similarly to that in the data.

The final three columns of Table 2 reports unconditional moments of inflation in the economy with endogenous inflation. There are a few notable features. First, the unconditional moments are not particularly sensitive to the intertemporal elasticity of substitution. Second, the unconditional variance of inflation in the calibrated economy is an order of magnitude higher than than in the data: 0.0033 in empirical data and about 0.02 in these economies. Finally, inflation is much more autocorrelated in the data—the AR(1) coefficient is 0.85 in the data and about 0.4 in the model economies.

Figure 8, Figure 9, and Table 3 report results from changing the Taylor rule parameters. We keep the remaining parameters fixed at the values used to generate Figure 7. The top plot in Figure 8 shows that increasing τ_x, the interest rate's responsiveness to output growth shocks, leads to a reduction in average nominal yields, a steepening in the average yield curve, and from the bottom plot an increase in yield volatility.

From Panel A of Table 3, increasing τ_x decreases the constant in the nominal pricing kernel, decreases the factor loading, decreases the price of risk for output growth, and also increases the factor loading for stochastic volatility. The loading on output growth in the pricing kernel drops because the sensitivity of the inflation rate to output growth drops, and the sensitivity of inflation to the to stochastic volatility increases by a large amount—from 6.66 to 8.55.

The top plot in Figure 9 shows that increasing τ_p, the interest rate responsiveness to inflation, leads to a reduction in average nominal yields, a flattening in the average yield curve, and from the bottom plot a decrease in yield volatility.

From Panel B of Table 3, increasing τ_p decreases the constant in the nominal pricing kernel, increases the factor loading on output growth, increases the price of risk for output growth, decreases the factor loading for stochastic volatility, and also drops the factor loading on the monetary policy shock. The constant in the pricing kernel drops because the constant in the inflation rate drops, the factor loading on output growth increases because the sensitivity of the inflation rate to output growth increases, and the sensitivity of inflation to stochastic volatility decreases by a large amount—from 6.66 to 3.58.

Overall, the experiments reported in Figure 8 and Figure 9 show that properties of the term structure depend on the form of the monetary authorities interest rate feedback rule. In particular, the factor loading on stochastic volatility is quite sensitive to the interest rate rule. Since stochastic volatility is driving time-variation in interest rate risk-premiums in this economy, monetary policy can have large impacts on interest rate risk premiums in this economy.

6 Related Research

The model we develop is similar to a version of Bansal and Yaron (2004) that includes stochastic volatility; however, our simple autoregressive state-variable process does not capture their richer ARMA specification. Our work is also related to Piazzesi and Schneider (2006) who emphasize that for a structural model to generate an upward-sloping nominal yield curve requires joint assumptions on preferences and the distribution of fundamentals. Our work highlights how an upward-sloping yield curve can also be generated through the monetary authority's interest rate feedback rule.

Our paper adds to a large and growing literature combining structural macroeconomic models that include Taylor rules with arbitrage-free term structures models. Ang and Piazzesi (2003), following work by Piazzesi (2005), have shown that a factor model of the term structure that imposes no-arbitrage conditions can provide a better empirical model of the term structure than a model based on unobserved factors or latent variables alone. Estrella and Mishkin (1997), Evans and Marshall (1998), Evans and Marshall (2001), Hördahl et al. (2003), Bekaert et al. (2005), and Ravenna and Seppala (2006) also provide evidence of the benefits of building arbitrage-free term-structure models with macroeconomic fundamentals. Rudebusch and Wu (2004) and Ang et al. (2004) investigate the empirical consequences of imposing a Taylor Rule on the performance of arbitrage-free term-structure models.

For an alternative linkage between short-maturity and long-maturity bond yields, see Vayanos and Vila (2006) who show how the shape of the term structure is determined in the presence of risk-averse arbitrageurs, investor clienteles for specific bond maturities, and an exogenous short rate which could be driven by the central bank's monetary policy.

7 Conclusions

We demonstrate that an endogenous monetary policy that involves an interest-rate feedback rule can contribute to the riskiness of multi-period bonds by creating an endogenous inflation process that exhibits significant covariance risk with the pricing kernel. We explore this through a recursive utility model with stochastic volatility which generates sizable average risk premiums. Our results point to a number of additional questions. First, the Taylor rule that we work with is arbitrary. How would the predictions of the model change with alternative specification of the rule? In particular, how would adding monetary non-neutralities along the lines of a New Keynesian Phillips curve as in Clarida et al. (2000) and Gallmeyer et al. (2005) alter the monetary-policy consistent pricing kernel? Second, what Taylor rule would implement an optimal monetary policy in this context? Since preferences have changed relative to the models in the literature, this is a nontrivial theoretical question.

In addition, the simple calibration exercise in this paper is not a very good substitute for a more serious econometric exercise. Further research will explore the tradeoffs between shock specifications, preference parameters, and monetary policy rules for empirical yield curve models that closer match historical evidence.

Finally, it would be instructive to compare and contrast the recursive utility model with stochastic volatility to other preference specifications that are capable of generating realistic risk premiums. The leading candidate on this dimension is the external habits models of Campbell and Cochrane (1999). We are currently pursuing an extension of the external habits model in Gallmeyer et al. (2005) to include an endogenous, Taylor-rule driven inflation process.

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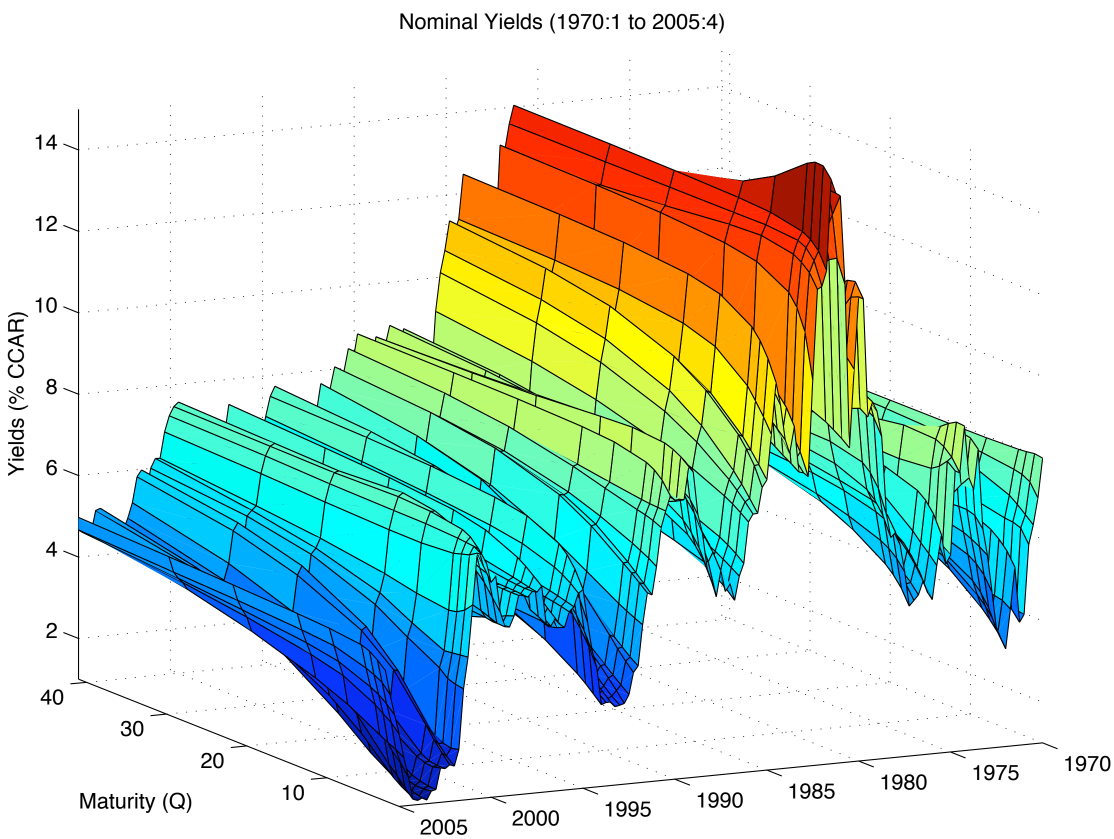


Figure 1: **Time series properties of the yield curve, 1970:1 to 2005:4.**

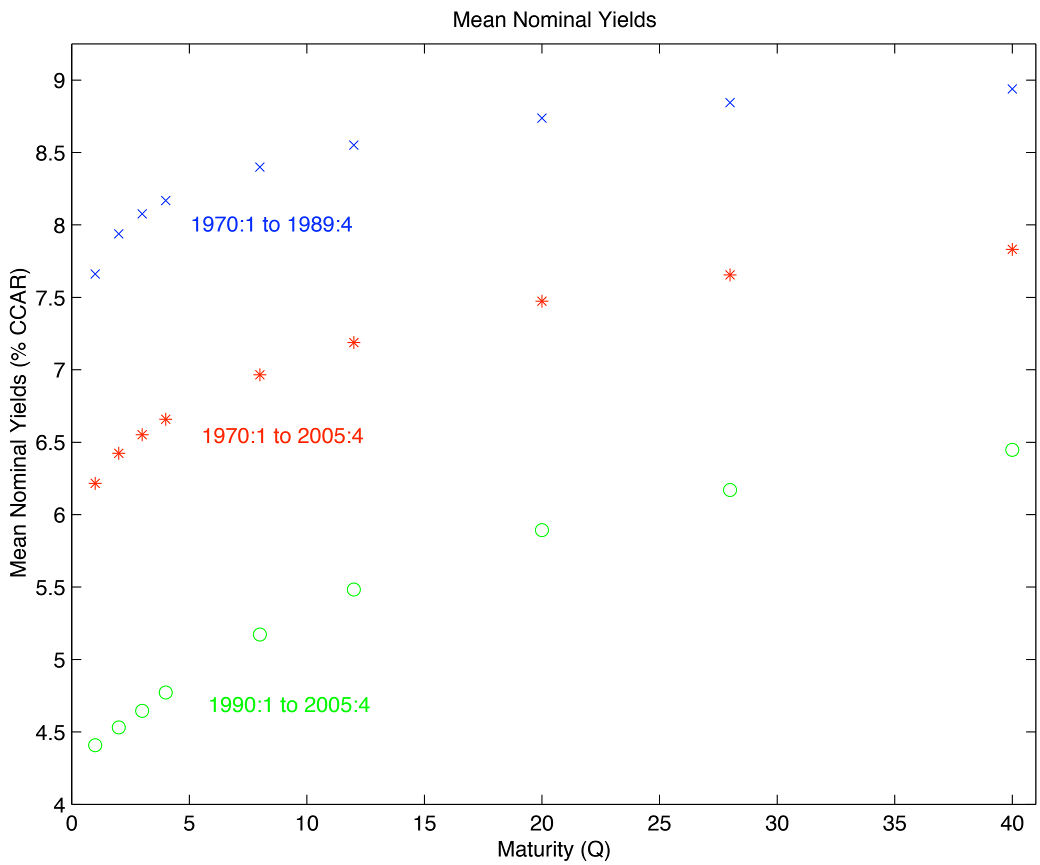


Figure 2: **Average yield curve behavior.**

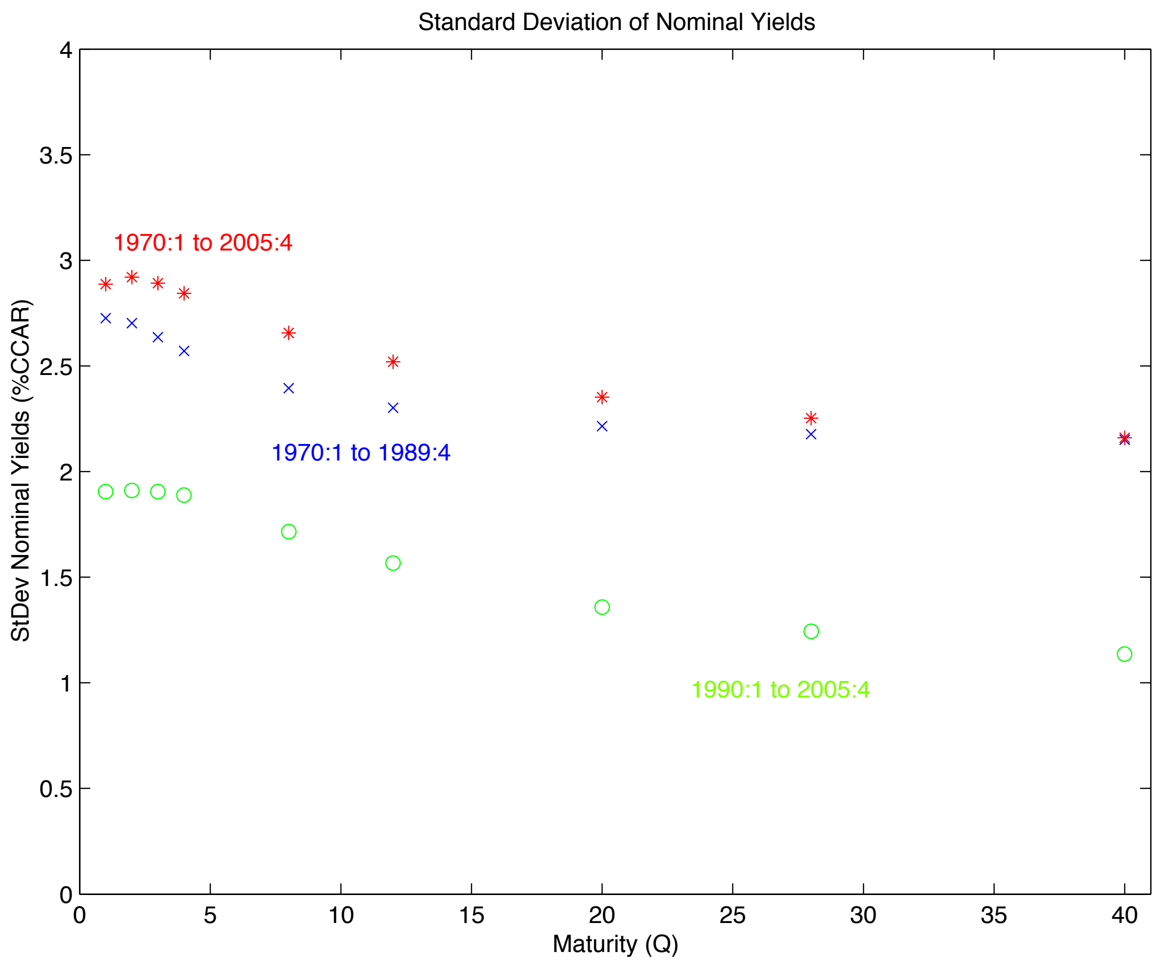
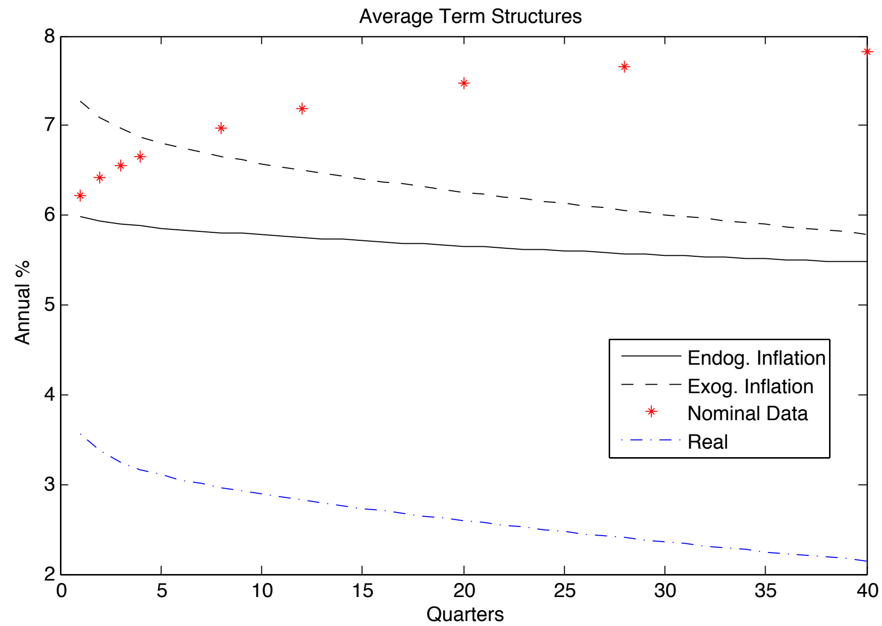


Figure 3: **Volatility of yields of various maturities.**



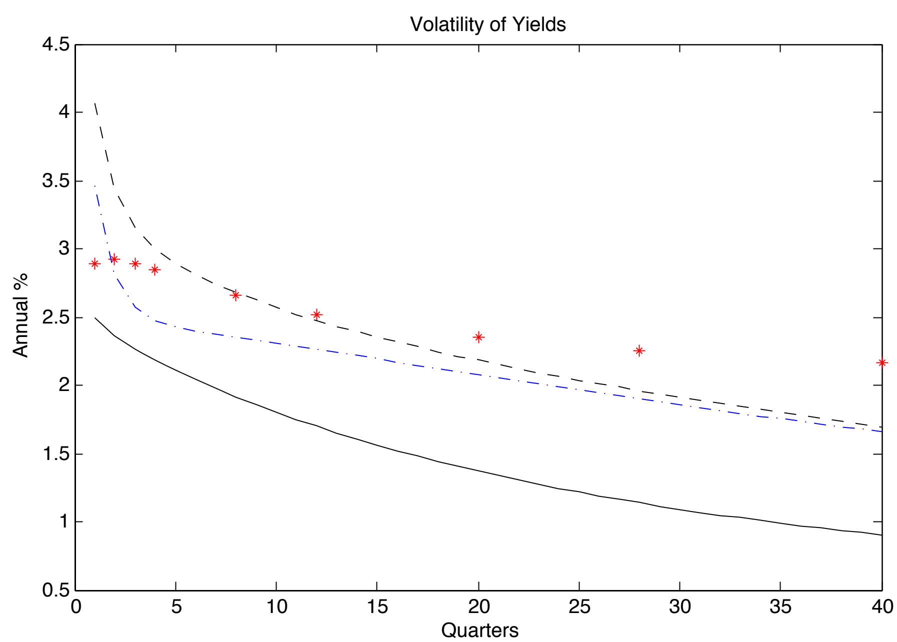
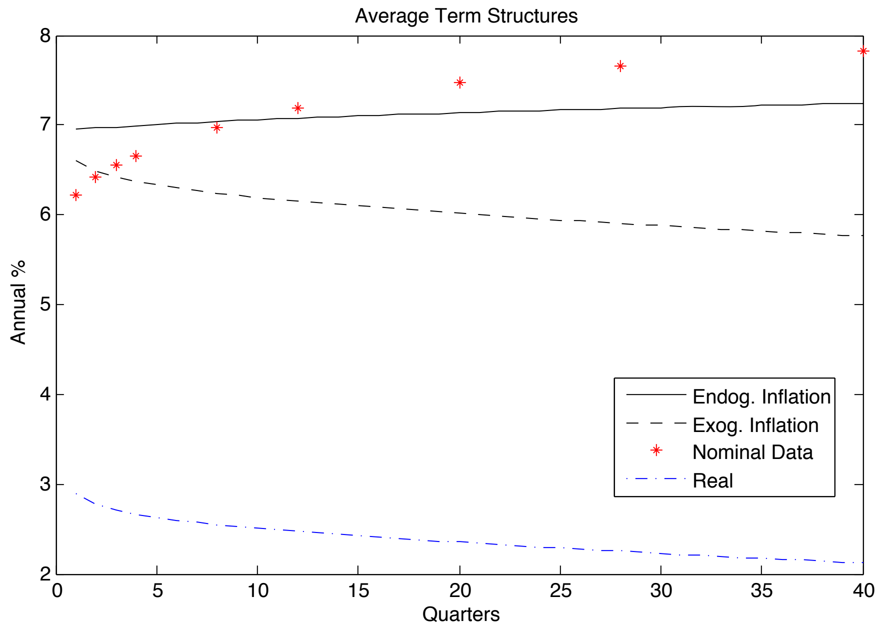


Figure 4: **Average yield curve and volatilities for the Epstein-Zin model with stochastic volatility.** The parameters are $\rho = -0.5$, $\alpha = -4.835$, $\beta = 0.999, \bar{\tau} = 0.003$, $\tau_x = 1.2475 : \tau_p = 1.000$. The top plot is average yields and the bottom plot is yield volatility. The historical moments are plotted with stars ($**$), properties of the real curve are plotted with a dashed line (−−), properties of the yield curve in the exogenous inflation economy are plotted with a dashed-dotted line (- − -), and properties of the yield curve in the economy with endogenous inflation are plotted with a solid line (—).



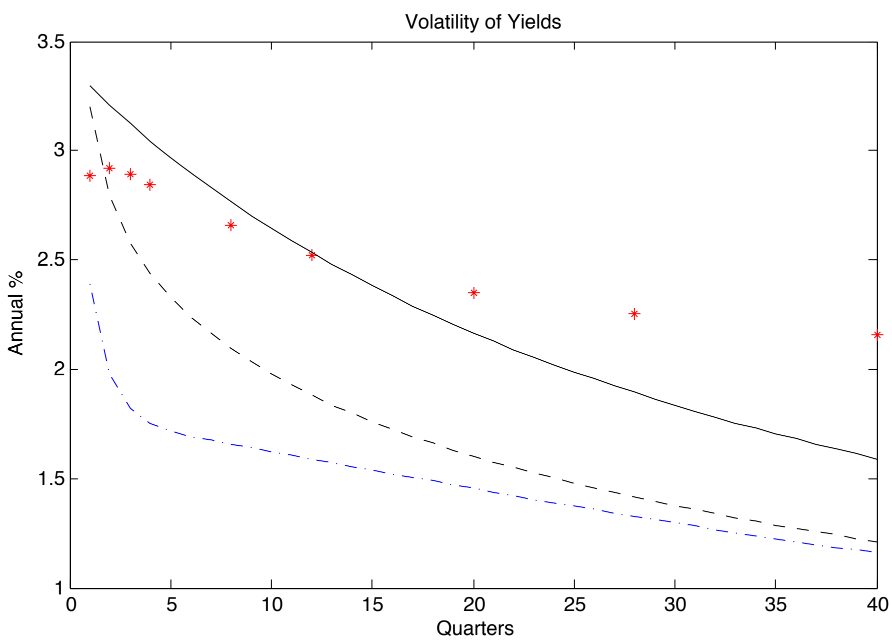
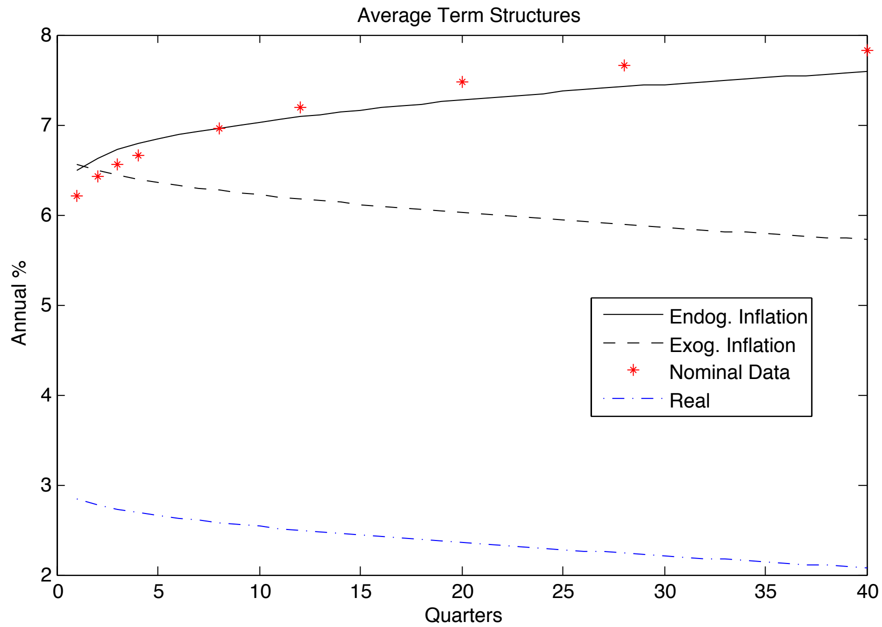


Figure 5: **Average yield curve and volatilities for the Epstein-Zin model with stochastic volatility.** The parameters are $\rho = 0.0$, $\alpha = -4.061$, $\beta = 0.998$, $\bar{\tau} = 0.003$, $\tau_x = 0.973$, $\tau_p = 0.973$. The top plot is average yields and the bottom plot is yield volatility. The historical moments are plotted with stars ($**$), properties of the real curve are plotted with a dashed line ($--$), properties of the yield curve in the exogenous inflation economy are plotted with a dashed-dotted line (- − -), and properties of the yield curve in the economy with endogenous inflation are plotted with a solid line (—).



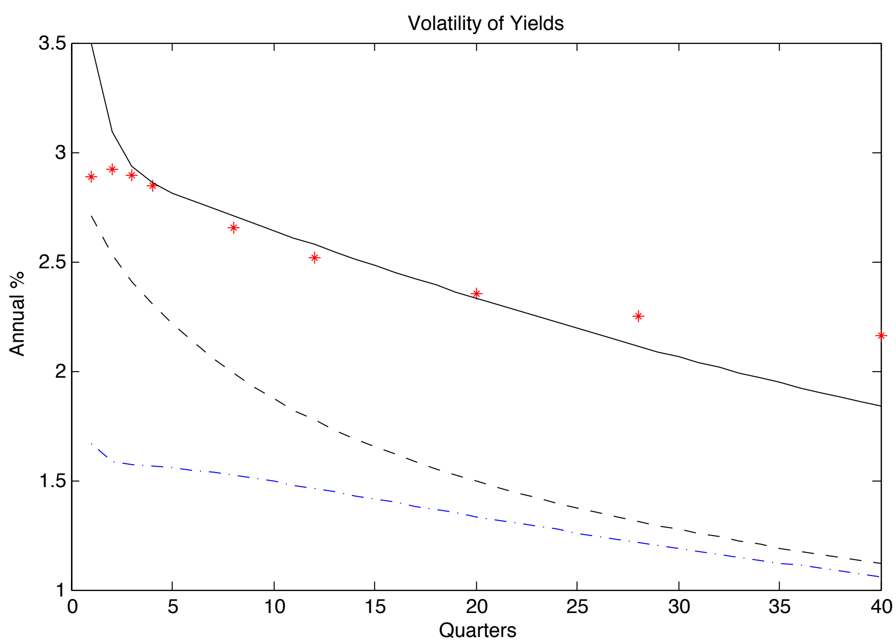


Figure 6: **Average yield curve and volatilities for the Epstein-Zin model with stochastic volatility.** The parameters are $\rho = 0.5$, $\alpha = -4.911$, $\beta = 0.994$, $\bar{\tau} = -0.015$, $\tau_x = 3.064 : \tau_p = 2.006$. The top plot is average yields and the bottom plot is yield volatility. The historical moments are plotted with stars ($**$), properties of the real curve are plotted with a dashed line ($--$), properties of the yield curve in the exogenous inflation economy are plotted with a dashed-dotted line (- – -), and properties of the yield curve in the economy with endogenous inflation are plotted with a solid line (—).

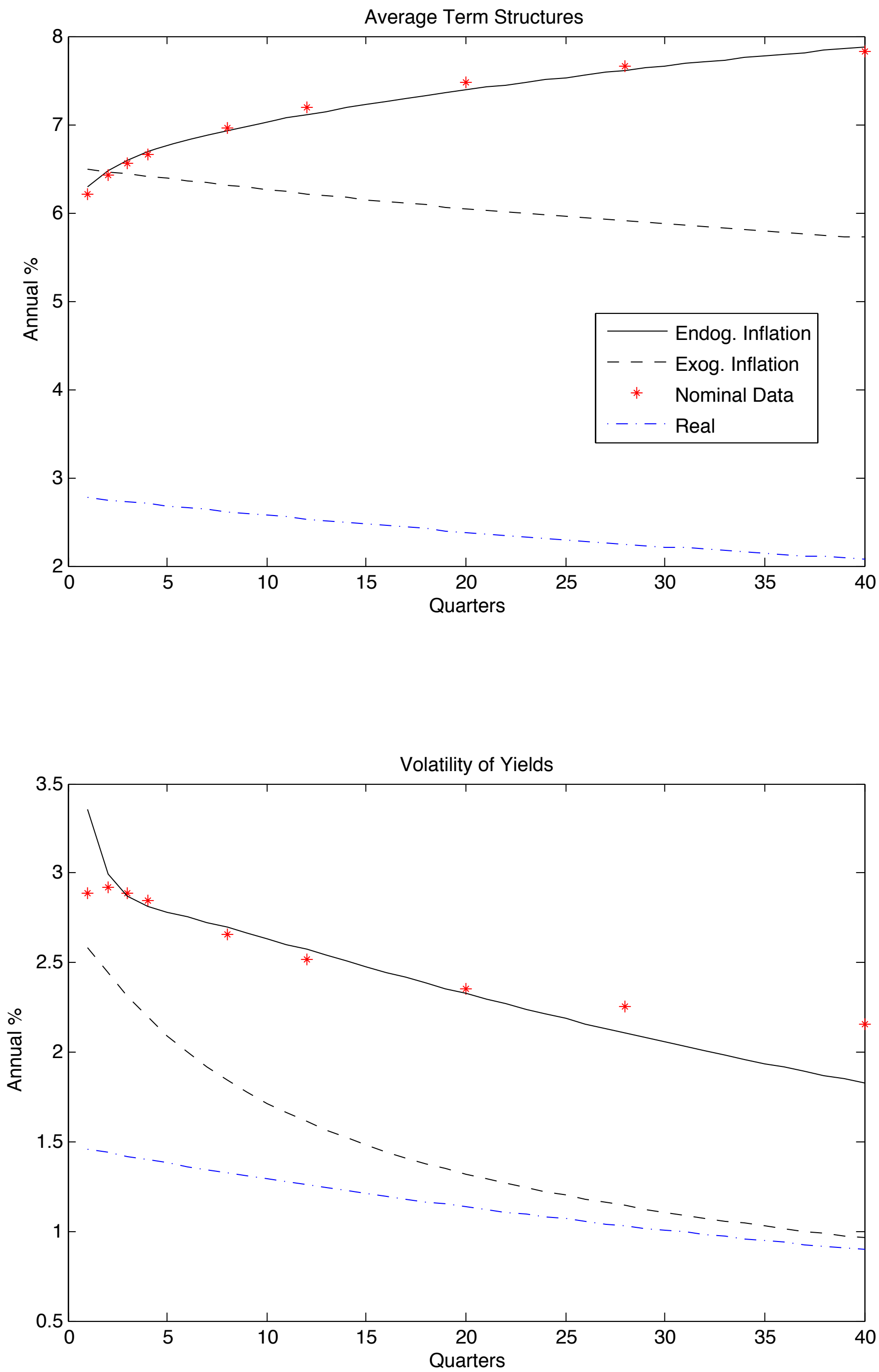


Figure 7: **Average yield curve and volatilities for the Epstein-Zin model with stochastic volatility.** The parameters are $\rho = 1.0$, $\alpha = -6.079$, $\beta = 0.990$, $\bar{\tau} = -0.004$, $\tau_x = 1.534$, $\tau_p = 1.607$. The top plot is average yields and the bottom plot is yield volatility. The historical moments are plotted with stars ($**$), properties of the real curve are plotted with a dashed line ($--$), properties of the yield curve in the exogenous inflation economy are plotted with a dashed-dotted line (- – -), and properties of the yield curve in the economy with endogenous inflation are plotted with a solid line (—).

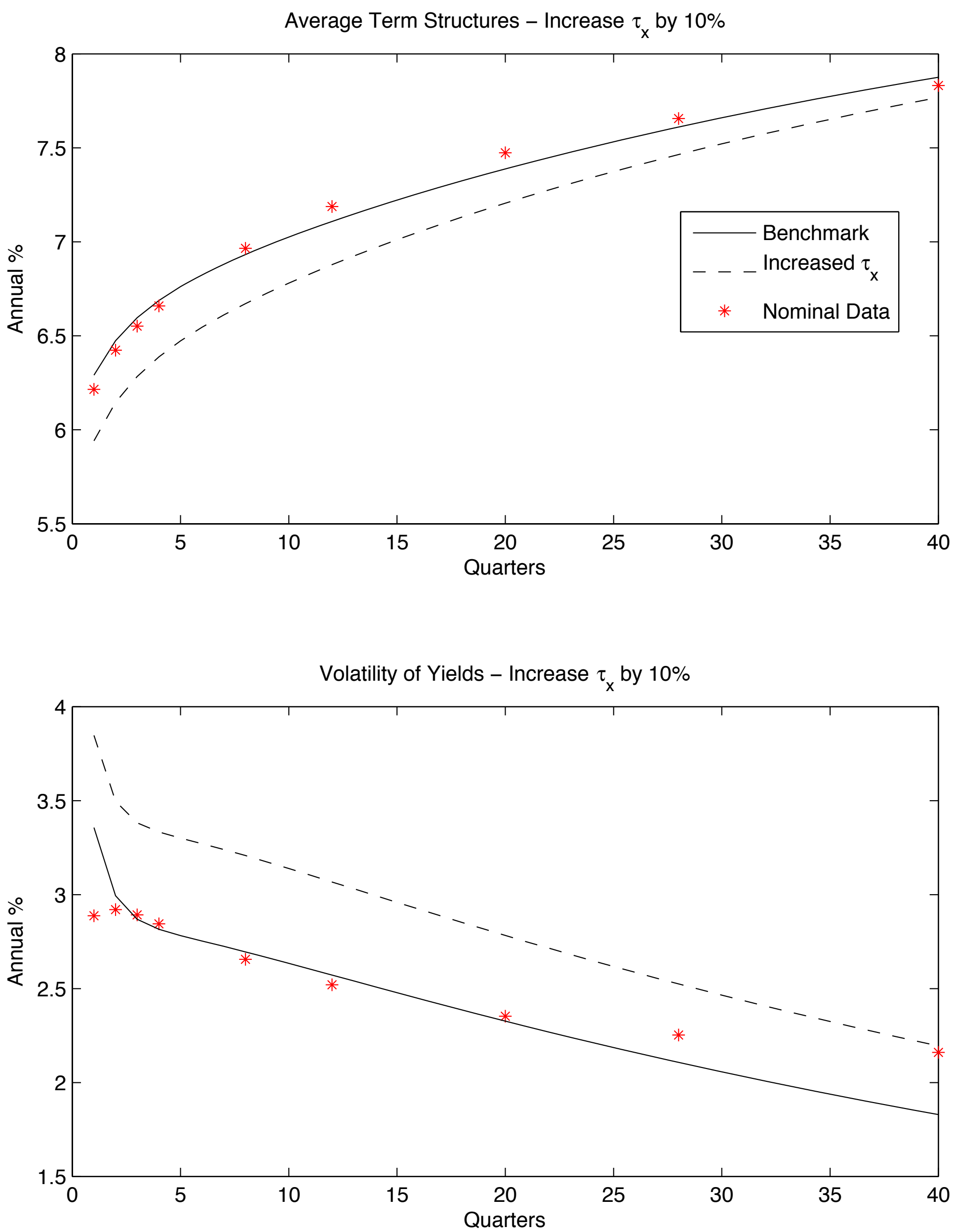


Figure 8: **The effects of increasing τ_x.** The baseline parameters are $\rho = 1.0$, $\alpha = -6.079$, $\beta = 0.990$, $\bar{\tau} = -0.004$, $\tau_x = 1.534$, $\tau_p = 1.607$. Historical data is plotted with stars (∗∗), results with the baseline parameters are plotted with a solid line (—), and results when the feedback from output growth to short-term interest rates is increased by 10% are plotted with a dashed line (- -). The top plot is average yields. The bottom plot is yield volatility.

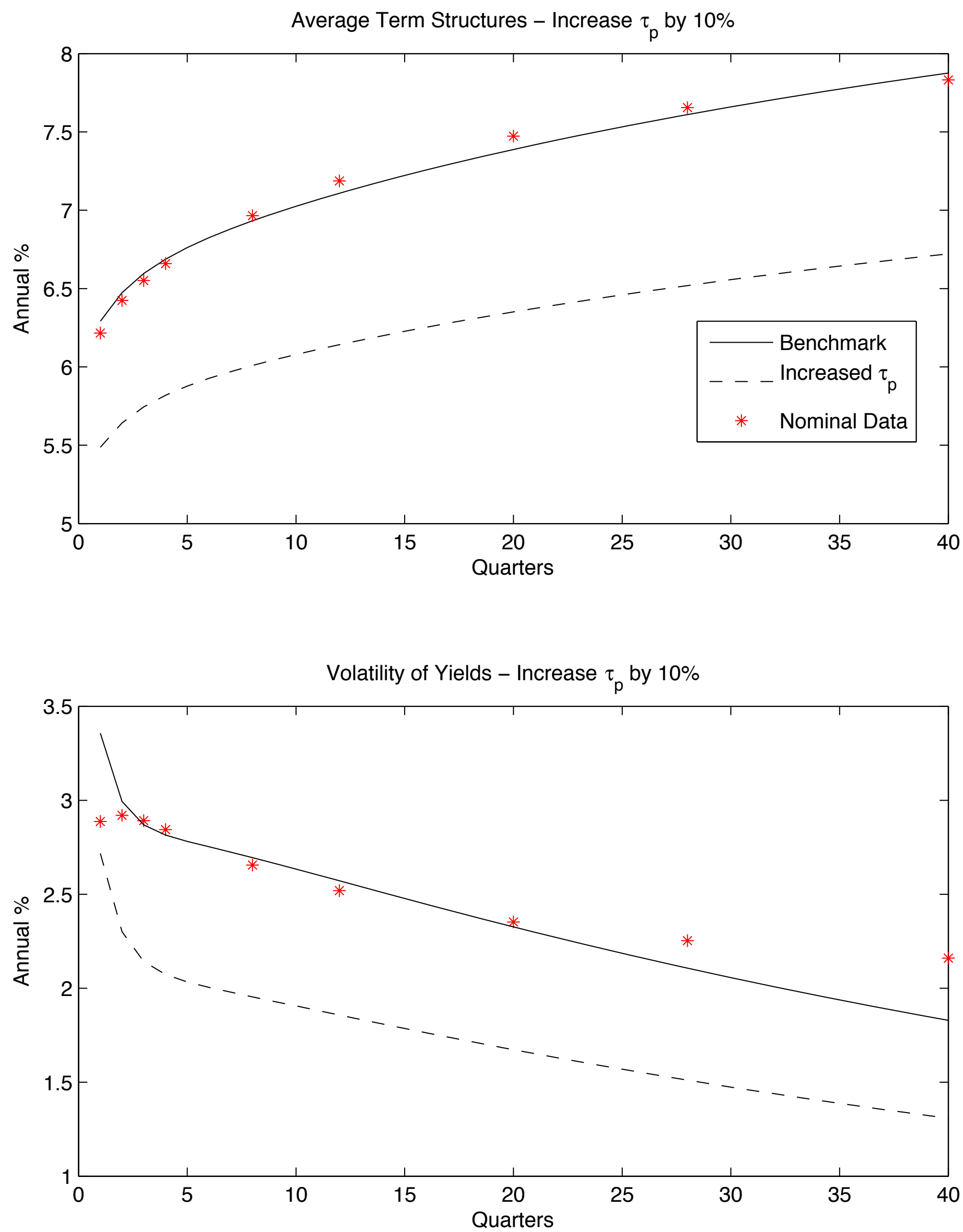


Figure 9: **The effects of increasing τ_p.** The baseline parameters are $\rho = 1.0$, $\alpha = -6.079$, $\beta = 0.990$, $\bar{\tau} = -0.004$, $\tau_x = 1.534$, $\tau_p = 1.607$. Historical data is plotted with stars ($**$), results with the baseline parameters are plotted with a solid line (—), and results when the feedback from inflation to short-term interest rates is increased by 10% are plotted with a dashed line (- -). The top plot is average yields. The bottom plot is yield volatility.

Table 1: **Factor loadings and prices of risk**

	Constant	Factor loadings (γ's) on				Prices of risk (λ's) on			
		x_t	v_t	p_t	s_t	ϵ^x_{t+1}	ϵ^v_{t+1}	ϵ^p_{t+1}	ϵ^s_{t+1}
Panel A: $\rho = -0.5,\ \alpha = -4.835,\ \beta = 0.999,\ \bar{\tau} = 0.003,\ \tau_x = 1.2475,\ \tau_p = 1.000$									
Real kernel	0.01	0.54	25.45	–	–	8.25	-902.98	–	–
Exogenous inflation	0.01	0.54	25.45	0.85	–	8.25	-902.98	1.00	–
Endogenous inflation	0.02	0.14	21.63	–	-1.44	7.15	-906.90	–	-1.56
Panel B: $\rho = 0.0,\ \alpha = -4.061,\ \beta = 0.998,\ \bar{\tau} = 0.003,\ \tau_x = 0.973,\ \tau_p = 0.973$									
Real kernel	0.01	0.36	20.07	–	–	7.34	-677.11	–	–
Exogenous inflation	0.01	0.36	20.07	0.85		7.34	-677.11	1.00	
Endogenous inflation	0.02	0.00	33.56	–	-1.51	6.34	-663.24	–	-1.63
Panel C: $\rho = 0.5,\ \alpha = -4.911,\ \beta = 0.994,\ \bar{\tau} = -0.015,\ \tau_x = 3.064,\ \tau_p = 2.006$									
Real kernel	0.01	0.18	32.23	–	–	8.93	-972.61	–	–
Exogenous inflation	0.01	0.18	32.23	0.85		8.93	-972.61	1.00	
Endogenous inflation	0.02	-0.45	38.34	–	-0.56	7.18	-966.33		-0.61
Panel D: $\rho = 1.0,\ \alpha = -6.079,\ \beta = 0.990,\ \bar{\tau} = -0.004,\ \tau_x = 1.534,\ \tau_p = 1.607$									
Real kernel	0.02	0.00	51.82	–	–	10.99	-1398.00	–	–
Exogenous inflation	0.02	0.00	51.82	0.85	–	10.99	-1398.00	1.00	–
Endogenous inflation	0.03	-0.44	58.30	–	-0.74	9.76	-1391.30		-0.80

The table reports the affine term structure parameters for the real term structure, the nominal term structure in the exogenous inflation economy, and the nominal term structure in the economy with endogenous inflation. The parameters in each panel are computed using a different set of preference parameters. We fix a level of the intertemporal elasticity parameter ρ and choose the remaining preference parameters—the risk aversion coefficient α and the rate of time preference β to minimize the distance between the average nominal yields and yield volatilities in the data and the those implied by the economy with an exogenous inflation rate. We pick the Taylor-rule parameters to minimize the distance between the average nominal yields and yield volatilities in the data and the those implied by the economy with an endogenous inflation rate.

Table 2: **Properties of Endogenous Inflation**

	$\bar{\pi}$	π_x	π_v	π_s	$E(p_t)$	$\sigma(p_t)$	AR(1)
Panel A: $\rho = -0.5,\ \alpha = -4.84,\ \beta = 0.999,\ \bar{\tau} = 0.003,\ \tau_x = 1.25,\ \tau_p = 1.00$							
	0.01	-1.11	-3.92	-1.56	0.01	0.02	0.37
Panel B: $\rho = 0.0,\ \alpha = -4.06,\ \beta = 0.998,\ \bar{\tau} = 0.003,\ \tau_x = 0.97,\ \tau_p = 0.97$							
	0.01	-1.00	13.87	-1.63	0.01	0.02	0.44
Panel C: $\rho = 0.5,\ \alpha = -4.91,\ \beta = 0.994,\ \bar{\tau} = -0.012,\ \tau_x = 3.06,\ \tau_x = 2.01$							
	0.02	-1.75	6.28	-0.61	0.01	0.03	0.37
Panel D: $\rho = 1.0,\ \alpha = -6.08,\ \beta = 0.990,\ \bar{\tau} = -0.004,\ \tau_x = 1.53,\ \tau_p = 1.61$							
	0.01	-1.23	6.66	-0.80	0.01	0.02	0.37

The table reports properties of p_t in the economy with endogenous inflation. The equilibrium inflation rate coefficients on output, stochastic volatility, and the monetary policy shock are reported. Additionally, the unconditional mean, the unconditional standard deviation, and the first-order autocorrelation of inflation are reported.

Table 3: **Comparative Statics for the Taylor Rule Parameters**

	Constant	Nominal pricing kernel Factor loadings (γ's)			Prices of risk (λ's)			Equilibrium inflation loadings			
		x_t	v_t	s_t	ϵ^x_{t+1}	ϵ^v_{t+1}	ϵ^s_{t+1}	$\bar{\pi}$	π_x	π_v	π_s
Panel A: τ_x increased by 10% from 1.53 to 1.69											
Baseline	0.03	-0.44	58.30	-0.74	9.76	-1391.30	-0.80	0.01	-1.23	6.66	-0.80
Increased τ_x	0.02	-0.49	60.13	-0.74	9.63	-1389.40	-0.80	0.01	-1.35	8.55	-0.80
Panel B: τ_p increased by 10% from 1.61 to 1.77											
Baseline	0.03	-0.44	58.30	-0.74	9.76	-1391.30	-0.80	0.01	-1.23	6.66	-0.80
Increased τ_p	0.02	-0.39	55.30	-0.66	9.90	-1394.40	-0.71	0.01	-1.09	3.58	-0.71

The table reports the effect of changing the Taylor rule parameter τ_x or τ_p on the affine term structure parameters as well as properties of p_t in the endogenous inflation economy. The equilibrium inflation rate coefficients on output, stochastic volatility, and the monetary policy shock are reported. The baseline parameters are $\rho = 1.0$, $\alpha = -6.08$, $\beta = 0.990$, $\bar{\tau} = -0.004$, $\tau_x = 1.53$, $\tau_p = 1.61$.