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OFFSHORING AND UNEMPLOYMENT

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ABSTRACT

In this paper, in order to study the impact of offshoring on sectoral and economywide rates of unemployment, we construct a two sector general equilibrium model in which labor is mobile across the two sectors, and unemployment is caused by search frictions. We find that, contrary to general perception, wage increases and sectoral unemployment decreases due to offshoring. This result can be understood to arise from the productivity enhancing (cost reducing) effect of offshoring. If the search cost is identical in the two sectors, or even if the search cost is higher in the sector which experiences offshoring, the economywide rate of unemployment decreases. We also find multiple equilibrium outcomes in the extent of offshoring and therefore, in the unemployment rate. Furthermore, a firm can increase its domestic employment through offshoring. Also, such a firm's domestic employment can be higher than a firm that chooses to remain fully domestic. When we modify the model to disallow intersectoral labor mobility, the negative relative price effect on the sector in which firms offshore some of their activity becomes stronger. In such a case, it is possible for this effect to offset the positive productivity effect, and result in a rise in unemployment in that sector. In the other sector, offshoring has a much stronger unemployment reducing effect in the absence of intersectoral labor mobility than in the presence of it. Finally, allowing for an endogenous number of varieties provides an additional indirect channel, through which sectoral unemployment goes down due to the entry of new firms brought about by offshoring.

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1 Introduction

"Offshoring" is the sourcing of inputs (goods and services) from foreign countries. When production of these inputs moves to foreign countries, the fear at home is that jobs will be lost and unemployment will rise. In the recent past, this has become an important political issue. The remarks by Greg Mankiw, when he was Head of the President's Council of Economic Advisers, that "outsourcing is just a new way of doing international trade" and is "a good thing" came under sharp attack from prominent politicians from both sides of the aisle. Recent estimates by Forrester Research of job losses due to offshoring equaling a total of 3.3 million white collar jobs by 2015 and the prediction by Deloitte Research of the outsourcing of 2 million financial sector jobs by the year 2009 have drawn a lot of attention from politicians and journalists (Drezner, 2004), even though these job losses are only a small fraction of the total number unemployed, especially when we take into account the fact that these losses will be spread over many years.¹ Furthermore, statements by IT executives have added fuel to this fire. One such statement was made by an IBM executive who said "[Globalization] means shifting a lot of jobs, opening a lot of locations in places we had never dreamt of before, going where there is low-cost labor, low-cost competition, shifting jobs offshore", while another statement was made by Hewlett-Packard CEO Carly Fiorina in her testimony before Congress that "there is no job that is America's God-given right anymore" (Drezner, 2004). The alarming estimates by Bardhan and Kroll (2003) and McKinsey (2005) that 11 percent of our jobs are potentially at risk of being offshored have provided politicians with more ammunition for their position on this issue.

While the relation between offshoring and unemployment has been an important issue for politicians, the media and the public, there has hardly been any careful theoretical analysis of this relationship by economists. In this paper, in order to study the impact of offshoring on sectoral and economywide rates of unemployment, we construct a two sector general equilibrium model in which unemployment is caused by search frictions a la Pissarides (2000). Firms in an imperfectly competitive, differentiated products sector use two inputs to produce varieties of an intermediate good. The production of one of these inputs can be offshored after incurring a fixed cost, while the other input (which we call headquarter services) must be produced domestically. There is a large variety of these intermediate goods used in the production of a homogeneous good produced under perfect competition. There is another sector that produces a homogeneous good under perfect competition and whose production is less sophisticated in that it uses only labor (under constant

¹The average number of gross job losses per week in the US is about 500,000 (Blinder, 2006). Also see Bhagwati, Panagariya and Srinivasan (2004) on the plausibility and magnitudes of available estimates of the unemployment effects of offshoring.

returns to scale). In the absence of offshoring there is a unique equilibrium in the economy. It is shown that when we allow the possibility of offshoring, there exists the possibility of multiple equilibria: (1) an equilibrium with no offshoring, (2) a mixed equilibrium where a fraction of firms offshore while others source their inputs domestically, and finally (3) an equilibrium where all firms offshore their input production. Offshoring reduces the cost of producing intermediate goods, and consequently the cost of the good using these intermediates. When a large number of intermediate firms offshore, the cost of production of the intermediate-using sophisticated good is low and therefore, its price is low. This, in turn, results in a high quantity demanded and hence a large scale of production of this good and a large market for intermediate goods. Thus, a large amount of offshoring is feasible and large scale production at a low cost and selling at a low price remain sustainable. On the other hand when very few intermediate firms offshore, the cost of production and the price of the intermediate-using good are high and the resulting scale of production is small, which in turn can support offshoring only by a few firms.

Looking at the impact of offshoring on unemployment and wages, we find that, contrary to general perception, wage increases and sectoral unemployment decreases due to offshoring. This result can be understood to arise from the productivity enhancing (cost reducing) effect of offshoring.² While the incentive to create vacancies (per worker) in the sector where offshoring takes place increases due to the productivity effect of offshoring, in the other sector this incentive increases due to an improvement in its relative price. Therefore, more jobs are created in both sectors, thereby putting an upward pressure on wages and a downward pressure on unemployment in each sector.³

The impact of offshoring on overall economywide unemployment, however, depends on how the structure or the composition of the economy changes. Even though both sectors have lower unemployment post-offshoring, whether the sector with the lower unemployment or higher unemployment expands will also be a determinant of the overall unemployment rate. If the search cost is identical in the two sectors, implying identical rates of sectoral unemployment, then the economywide rate of unemployment declines unambiguously after offshoring. Alternatively, if the search cost is higher in the sector which experiences

²This is due to the increase in the marginal product of the workers at the headquarters arising from employment of more production input per headquarter worker (since the input is now cheaper) in the offshoring firms.

³Offshoring of production activity in one sector makes the other sector relatively more intensive in the use of domestic labor. At the same time, offshoring raises the relative price of the good whose production is not offshored, i.e., of the good that is domestic labor intensive. Therefore, cost of domestic labor (wage rate and market tightness) goes up. This effect is analogous to the Stolper-Samuelson effect.

offshoring (implying a higher wage as well as higher rate of unemployment in that sector), the economywide rate of unemployment also decreases then because some workers move to the other sector which has a lower unemployment rate.

Masked behind the intersectoral reallocation of labor is intra-sectoral reallocation of labor within the differentiated goods sector in response to offshoring. Output is reallocated from firms that do not offshore to firms that offshore because the latter have lower marginal costs of production and hence charge lower prices. However, the reallocation of employment is not the same as that of output. Firms that offshore a part of their production process increase their employment of workers involved in the production of headquarter services. However, since they reduce their employment in production activities, the net impact is ambiguous. The higher the headquarter intensity of an industry, the more likely it is that offshoring firms increase their employment relative to firms that do not offshore.

Next, we modify our model to disallow labor mobility across the labor forces of the two sectors. This can be considered to be the shorter-run version of the model with labor mobility. It also provides some extra insights that we otherwise would have missed. Under both labor mobility and no labor mobility, there are two effects of offshoring on the sector that uses the offshored input. One is the cost reducing or the productivity enhancing effect, while the other is the reduction in the relative price. The second effect is stronger under no labor mobility than under mobile labor and if this effect is strong enough, the sectoral unemployment rate may go up in this sector. Whether this will be the case or not will depend on the importance of this good in final consumption and on the headquarter intensity in the production of this good. We want to reiterate that the rise in unemployment upon offshoring is only a possibility that can happen under certain conditions. A reduction of unemployment is also possible in this shorter run model. The favorable relative price effect of offshoring on the other sector (in which production is always fully domestic) is stronger under no labor mobility than under mobile labor. Therefore, the reduction in the unemployment rate in this sector (due to offshoring) is greater in the short-run than in the model with intersectoral labor mobility.

We finally perform another extension of the model in which we have an endogenous number of varieties of the intermediate good. Offshoring leads to an increase in the variety of intermediates in equilibrium that leads to a further productivity increase and therefore a further reduction in unemployment. Thus, allowing for an endogenous number varieties in the offshoring sector provides an additional indirect channel through which sectoral unemployment goes down.

Our theoretical results are consistent with the empirical results of Amiti and Wei (2005a, b) for the US and the UK. They find no support for the “anxiety” of “massive job losses” associated with offshore

outsourcing from developed to developing countries.⁴ Using data on 78 sectors in the UK for the period 1992-2001, they find no evidence in support of a negative relationship between employment and outsourcing. In fact, in many of their specifications the relationship is positive. In the US case, they find a very small, negative effect of offshoring on employment if the economy is decomposed into 450 narrowly defined sectors which disappears when one looks at more broadly defined 96 sectors. Alongside this result, they also find a positive relationship between offshoring and productivity. These results are consistent with opposing effects on employment (and unemployment) created by offshoring. In this context, Amiti and Wei (2005a) write: “On the one hand, every job lost is a job lost. On the other hand, firms that have outsourced may become more efficient and expand employment in other lines of work. If firms relocate their relatively inefficient parts of the production process to another country, where they can be produced more cheaply, they can expand their output in production for which they have comparative advantage. These productivity benefits can translate into lower prices generating further demand and hence create more jobs. This job creation effect could in principle offset job losses due to outsourcing.” This intuition is consistent with the channels in our model and the reason for obtaining a result that shows a reduction in sectoral and overall unemployment as a result of offshoring.

Before, we try to relate our work to the existing literature, we would like to say a couple of things in defense of our modeling strategy. Firstly, it is quite easy to imagine that there will be fixed costs associated with offshoring. An analytically tractable way to handle fixed costs here is to depart from perfect competition. This is why the intermediate goods sector which offshores input production is assumed to be imperfectly competitive in this paper. An additional benefit of using the imperfectly competitive framework is the ability to study the case when the number of firms is endogenous and to look at productivity effects of offshoring through changes in input variety. We also introduce cost or productivity heterogeneity, which has been done to make the coexistence of offshoring and non-offshoring firms more likely in the same industry in equilibrium (when offshoring is allowed at a cost).

We next turn to the existing literature. While the relationship between offshoring and unemployment has not been analytically studied before by economists, there is now a vast literature on offshoring and outsourcing.⁵ All the models in that literature, following the tradition in standard trade theory, assume full

⁴The offshoring variable they use, which they call offshoring intensity, is defined as the share of imported inputs (material or service) as a proportion of total nonenergy inputs used by the industry.

⁵See for instance Grossman and Helpman (2002, 2003, 2005), Antras (2003), Antras and Helpman (2004) and Feenstra and Hanson (2005).

employment. In spite of this assumption in the existing literature, it is important to note that our results are similar in spirit to those in an important recent contribution by Grossman and Rossi-Hansberg (2006) where they model offshoring as "trading in tasks" and show that even factors of production whose tasks are offshored can benefit from offshoring due to its productivity enhancing effect. Also closely related to our work is a very recent working paper by Davidson, Matusz and Shevchenko (2006) that uses a model of job search to study the impact of offshoring of high-tech jobs on low and high-skilled workers' wages, and on overall welfare. Another paper looking at the impact of offshoring on the labor market is Karabay and McLaren (2006) who study the effects of free trade and offshore outsourcing on wage volatility and worker welfare in a model where risk sharing takes place through employment relationships. Rodriguez-Clare (2006) analyzes the positive productivity effect and the negative terms of trade effect of offshoring on wages and welfare within a Ricardian framework. Bhagwati, Panagariya and Srinivasan (2004) also analyze in detail the welfare and wage effects of offshoring.

It is also important to note that there does exist a literature on the relationship between trade and unemployment. Previous work on unemployment in an open economy includes minimum wage models (Brecher, 1974a, b and Davis, 1998a, b), implicit contract models (Matusz, 1986), efficiency wage models (Brecher, 1992; Brecher and Choudhri, 1994; Copeland, 1989; Hoon, 2001; and Matusz, 1994), and search models (Davidson and Matusz, 2004 and Şener, 2001, Moore and Ranjan (2005)). None of these models deals with offshoring.

2 A Model of Offshoring and Unemployment

2.1 Preferences

All agents share the identical lifetime utility function

$$\int_{t=0}^{\infty} C_t \exp^{-rt} ds, \tag{1}$$

where C is consumption, r is the discount rate, and t is a time index. Asset markets are complete. There is perfect certainty, aside from one-time, unanticipated shocks. The form of the utility function implies that the risk-free interest rate, in terms of consumption, equals r .

Each worker has one unit of labor to devote to market activities at every instant of time. The total size of the workforce is L . The final consumption good C could be assumed to be produced using two goods Z

and X as inputs follows, or equivalently can be considered to be a composite (basket) of these two goods as follows:

$$C = \frac{Z^\gamma X^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \quad (2)$$

We choose the composite consumption good C as numeraire. Let P_z and P_x be the prices of Z and X , respectively. Since the price of $C = 1$, we get

$$P_z^\gamma P_x^{1-\gamma} = 1 \iff P_x = (P_z)^{\frac{\gamma}{\gamma-1}} \quad (3)$$

Therefore, an increase in P_z implies a decrease in P_x .

Also, (2) implies that the relative demand for Z is given by

$$\left(\frac{Z}{X}\right)^d = \frac{\gamma P_x}{(1-\gamma)P_z} = \frac{\gamma (P_z)^{\frac{1}{\gamma-1}}}{(1-\gamma)} \quad (4)$$

So, the relative demand for Z is decreasing in P_z .

2.2 Production and Profit Maximization in the Z sector

Z is produced using a continuum of differentiated intermediate goods produced by monopolistically competitive firms. The production function for Z is given as follows.

$$Z = \left[\int_{i \in I} z(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}, \sigma > 1 \quad (5)$$

where $z(i)$ is the intermediate good of variety i and I is the set of all existing varieties. Now, the above production function for the final good results in the following demand function for intermediate good of variety i :

$$z(i) = \frac{p(i)^{-\sigma}}{\int_{i \in I} p(i)^{1-\sigma} di} P_z Z \quad (6)$$

where $P_z Z$ denotes the aggregate expenditure on Z , the price of Z , P_z is given by

$$P_z \equiv \left[\int_{i \in I} p(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} \quad (7)$$

So the demand for each intermediate good can be re-written as

$$z(i) = \left(\frac{p(i)}{P_z} \right)^{-\sigma} Z \quad (8)$$

Note from the above equation that the elasticity of demand facing each intermediate good producer is σ , which is also the elasticity of substitution between any two varieties. The productivity of an intermediate good producer is denoted by α , and the distribution of the productivity levels of all intermediate good producers is given by a distribution $G(\alpha)$, where $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. We assume here (for the most part) a fixed mass of firms. In the penultimate section (section 4), we will relax this assumption where each entering firm will have to incur a sunk cost equal to F_E in terms of the numeraire good, after which it will observe its realization of productivity drawn from $G(\alpha)$.

The production function for an intermediate good producing firm with productivity α is given by

$$z(\alpha) = \frac{1}{\tau^\tau (1-\tau)^{1-\tau}} \alpha m_h(\alpha)^\tau m_p(\alpha)^{1-\tau} \quad (9)$$

where m_h is the labor requirement for certain core activities which have to remain within the home country and m_p is the labor input required for other activities which can potentially be offshored. Denote the marginal cost of a firm with productivity α by $c(\alpha)$. Given the constant elasticity of demand for each firm, the price charged by a firm with productivity α , denoted by $p(\alpha)$ is going to equal $\frac{\sigma}{\sigma-1} c(\alpha)$. If M is the mass of firms in the industry, we can write the price of Z as follows.

$$P_z = M^{\frac{1}{1-\sigma}} \left[\int_{\underline{\alpha}}^{\bar{\alpha}} p(\alpha)^{1-\sigma} dG(\alpha) \right]^{\frac{1}{1-\sigma}} \quad (10)$$

In this section we will assume $M = 1$, that is, there is a unit mass of firms in the industry. As mentioned above, in a later section we endogenize M , where we show that the qualitative results are unchanged. Therefore, for the purposes of this section, the price of Z is simply

$$P_z = \left[\int_{\underline{\alpha}}^{\bar{\alpha}} p(\alpha)^{1-\sigma} dG(\alpha) \right]^{\frac{1}{1-\sigma}} \quad (11)$$

and the demand facing a firm with productivity α is

$$z(\alpha) = \left(\frac{p(\alpha)}{P_z} \right)^{-\sigma} Z \quad (12)$$

If we denote the total amount of labor employed by firm with productivity α by $N(\alpha)$, then we have

$$N(\alpha) = m_h(\alpha) + m_p(\alpha) \quad (13)$$

To produce intermediate goods, a firm needs to open job vacancies and hire workers. The cost of vacancy in terms of the numeraire good is c_z . Denote the number of vacancies posted by a firm by $V(\alpha)$. Any job can be hit with an idiosyncratic shock with probability δ and be destroyed.

Let L_z be the total number of workers who look for a job in sector Z . Define $\theta_z = \frac{v_z}{u_z}$ as the measure of market tightness in sector Z , where $v_z L_z$ is the total number of vacancies in sector Z and $u_z L_z$ is the number of unemployed workers searching for a job in sector Z . The probability of a vacancy filled is $q(\theta_z) = \frac{m(v_z, u_z)}{v_z}$ where $m(v_z, u_z)$ is a constant returns to scale matching function. Since $m(v_z, u_z)$ is constant returns to scale, $q'(\theta_z) < 0$. The probability of an unemployed worker finding a job is $\frac{m(v_z, u_z)}{u_z} = \theta_z q(\theta_z)$ which is increasing in θ_z .

Assuming that each firm employs and hires enough workers to resolve the uncertainty of job inflows and outflows, the dynamics of employment for a firm is

$$\dot{N}(\alpha) = q(\theta_z)V(\alpha) - \delta N(\alpha) \quad (14)$$

The wage for each worker is determined by a process of Nash bargaining with the firm separately which is discussed later. While deciding on how many vacancies to open up the firm correctly anticipates this wage. Therefore, the profit maximization condition for an individual firm can be written as

$$\text{Max}_{V(\alpha), z(\alpha)} \int_t^\infty e^{-r(s-t)} \{p(\alpha)z(\alpha) - w(\alpha)N(\alpha) - c_z V(\alpha)\} ds \quad (15)$$

where $z(\alpha)$ is given in (9) and $N(\alpha)$ is given in (13) and $w(\alpha)$ is taken as given.

Therefore, the firm maximizes (15) subject to (14), (9), and (13). We provide details of the firm's maximization exercise in the appendix. Denoting the Lagrangian multiplier associated with (14) by λ , where λ is the shadow value of an extra job, we get in the steady state

$$\frac{\frac{\sigma-1}{\sigma} \alpha p(\alpha) - w(\alpha)}{(r + \delta)} = \lambda = \frac{c_z}{q(\theta_z)} \quad (16)$$

The expression on the extreme left-hand side is the marginal benefit from a job which equals the present value of the stream of the marginal revenue product net of wage of an extra worker after factoring in the probability of job separation each period. The extreme right-hand side expression is the marginal cost of a job which equals the cost of posting a vacancy, c_z multiplied by the average duration of a vacancy, $\frac{1}{q(\theta_z)}$. Alternatively, $\frac{1}{q(\theta_z)}$ is the average number of vacancies required to be posted to create a job per unit of time. $\frac{c_z}{q(\theta_z)}$ will be the asset value of an extra job for a firm in the wage determination below. An alternative way to write (16) is

$$p(\alpha) = \frac{\sigma}{\sigma-1} \frac{w(\alpha)}{\alpha} + \frac{\sigma(r + \delta)c_z}{(\sigma-1)q(\theta_z)\alpha} \quad (17)$$

This is the mark up equation in the presence of search frictions. So, in addition to the standard wage cost, search cost is added to the marginal cost of producing a unit of output. Note that just as the wage cost is

inversely related to firm productivity, search cost is inversely related as well, because a more productive firm needs a smaller labor force to produce a given level of output.

Further, it is straightforward to see from the first-order conditions that $z(\alpha) = \alpha N(\alpha)$ which when plugged into the dynamic equation of $N(\alpha)$ with the steady state condition $\dot{N}(\alpha) = 0$ imposed gives us $V(\alpha) = \frac{\delta z(\alpha)}{q(\theta_z)\alpha}$. Thus, the number of vacancies that a firm posts is positively related to output and negatively related to firm productivity. The equation above also implies that the ratio of vacancies to employment is the same for all firms. Now, if L_z is the total size of the labor force in the Z sector, then

$$v_z L_z = \int_{\underline{\alpha}}^{\alpha^*} V(\alpha) dG(\alpha) = \frac{\delta}{q(\theta_z)} \int_{\underline{\alpha}}^{\alpha^*} N(\alpha) dG(\alpha) = \frac{\delta}{q(\theta_z)} \int_{\underline{\alpha}}^{\alpha^*} \frac{z(\alpha)}{\alpha} dG(\alpha) \quad (18)$$

Note that, by definition total employment $\int_{\underline{\alpha}}^{\alpha^*} N(\alpha) dG(\alpha)$ equals $(1 - u_z)L_z$, which after minor manipulation gives us

$$u_z = \frac{\delta}{\delta + \theta_z q(\theta_z)} \quad (19)$$

The above is the standard Beveridge curve in Pissarides type search models where the rate of unemployment is negatively related to the degree market tightness θ_z .

2.3 Wage Determination in the Z sector

Wage is determined for each worker through a process of Nash bargaining with his/her employer. The value of an occupied job for a firm, $J(\alpha)$, is the value of λ obtained from firm's maximization problem. It equals $\frac{c_z}{q(\theta_z)}$. Denoting the unemployment benefit in terms of the final good by b and letting U_z denote the income of the unemployed in the Z sector, the asset value equation for the unemployed in this sector is given by

$$rU_z = b + \theta_z q(\theta_z)[E_z - U_z] \quad (20)$$

where E_z is the expected income from becoming employed in the Z sector. As explained in Pissarides (2000), the asset that is valued is an unemployed worker's human capital. The return on this asset is the unemployment benefit b plus the expected capital gain from the possible change in state from unemployed to employed given by $\theta_z q(\theta_z)[E_z - U_z]$.

The asset value equation for an employed worker working for a firm in sector Z with productivity α is given by

$$rE(\alpha) = w(\alpha) + \delta(U_z - E(\alpha)) \Rightarrow E(\alpha) = \frac{w(\alpha)}{r + \delta} + \frac{\delta U_z}{r + \delta} \quad (21)$$

Again the return on being employed equals the wage plus the expected change in the asset value from a change in state from employed to unemployed. The surplus for an unemployed worker from getting a job with a firm is $E(\alpha) - U_z$. The surplus for a firm from an occupied job is $J(\alpha)$. Since the wage is determined using Nash bargaining where the bargaining weights are β and $1 - \beta$, we get the following wage bargaining equation:

$$E(\alpha) - U_z = \beta(J(\alpha) + E(\alpha) - U_z) \quad (22)$$

The equation above is important in delivering our wage equation. Since as seen above $J(\alpha) = \frac{c_z}{q(\theta_z)}$ is the same for all firms, $E(\alpha) - U_z = \frac{\beta}{1-\beta} \frac{c_z}{q(\theta_z)}$, and therefore $E(\alpha) = E_z$ are the same for all α . Plugging this value of $E_z - U_z$ into the asset value equation for the unemployed we have a simplified version of this asset value equation

$$rU_z = b + \frac{\beta}{1-\beta} c_z \theta_z \quad (23)$$

Since we know that $J(\alpha) = \lambda = \frac{\frac{\sigma-1}{\sigma} \alpha p(\alpha) - w(\alpha)}{(r+\delta)}$, this in conjunction with the wage bargaining equation and the asset value equation of an employed worker gives us the wage of a worker in a firm as the weighted average of the return on the asset value of an unemployed person and the marginal revenue product of the worker, the weights being the bargaining power of the firm and the worker respectively. More precisely we have $w(\alpha) = (1 - \beta)rU_z + \beta[\frac{\sigma-1}{\sigma} p(\alpha)\alpha]$, which in conjunction with (23) and the fact that $\frac{\frac{\sigma-1}{\sigma} \alpha p(\alpha) - w(\alpha)}{(r+\delta)} = \frac{c_z}{q(\theta_z)}$ gives us the following simplified wage equation:

$$w_z = b + \frac{\beta c_z}{1-\beta} [\theta_z + \frac{r+\delta}{q(\theta_z)}] \quad (24)$$

Next, we write down the expression for equilibrium profit which is useful in deriving the benefit from offshoring. It is shown in the appendix that the present discounted value of profits of a firm at time t is

$$\Pi_D(\alpha) = \frac{Z P_z^\sigma}{r \sigma^\sigma (\sigma - 1)^{1-\sigma}} \left(w_z + \frac{(r + \delta) c_z}{q(\theta_z)} \right)^{1-\sigma} \alpha^{\sigma-1} + \frac{c_z}{q(\theta_z)} N_t(\alpha) \quad (25)$$

where we have introduced the subscript D to capture the fact that input is produced domestically. The above equation shows that the profit of a firm is increasing in its productivity. To obtain the present discounted value of profit of a firm in steady-state, substitute $N_t(\alpha)$ in the expression above by its steady-state employment. When we allow for free entry of firms to produce differentiated goods, we will assume that each firm enters with $N = 0$, and therefore, its expected present discounted value of profit is obtained by setting $N_t = 0$ in (25).

2.4 Production, Wage Determination and Employment in the X sector

Production of good X is undertaken by perfectly competitive firms. To produce one unit of X a firm needs to hire one unit of labor. In order to hire a worker a firm has to open a vacancy which is costly. The cost of vacancy in terms of the numeraire good is c_x . Define $\theta_x = \frac{v_x}{u_x}$ as the measure of market tightness in the X sector. The probability of a vacancy filled is $q(\theta_x) = \frac{m(v_x, u_x)}{v_x}$ where $m(v_x, u_x)$ is a constant returns to scale function same as in the Z sector. Also, jobs are destroyed with probability δ . The cost of posting a vacancy in sector X is denoted by c_x . Firms in the X sector are perfectly competitive, as opposed to imperfect competition among intermediate goods producers in the Z sector.

Repeating the exercise in the previous section for competitive firms (see Pissarides (2000) for details), we obtain the following three key equations.

$$w_x = (1 - \beta)b + \beta[P_x + c_x\theta_x] \quad (26)$$

$$P_x = w_x + \frac{(r + \delta)c_x}{q(\theta_x)} \quad (27)$$

$$u_x = \frac{\delta}{\delta + \theta_x q(\theta_x)} \quad (28)$$

The above 3 equations determine w_x , θ_x , and u_x , for a given P_x .

Since unemployed workers can search in either sector, the income of the unemployed must be the same from searching in either sector. Imposing (23) which gives us the income of the unemployed searching in Z sector and the corresponding equation for the X sector given by $rU_x = b + \frac{\beta}{1-\beta}c_x\theta_x$ on the labor mobility condition $U_z = U_x$ implies

$$c_z\theta_z = c_x\theta_x \quad (29)$$

That is, the labor market tightness for each sector is inversely proportional to the vacancy cost.

2.5 Solving the Model

Let us define the average productivity of firms as follows.

Definition $\tilde{\alpha} \equiv \left[\int_{\underline{\alpha}}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right]^{\frac{1}{\sigma-1}}$

Next, using (17) write the equation for the price of Z given in (7) as

$$P_z = \frac{\sigma}{\sigma - 1} \left(w_z + \frac{(r + \delta)c_z}{q(\theta_z)} \right) \tilde{\alpha}^{-1} \quad (30)$$

Now, start with a P_x . Determine w_x and θ_x from (26) and (27). Next, θ_z is determined from (29). Then w_z is determined from (24). Since we know θ_z and w_z , we can determine P_z from (30). Therefore, for each P_x there is an associated P_z , determined as described above. It is easy to verify that an increase in P_x implies an increase P_z via the relationship described above. Let us call this *PPP*.

Next, note from (3) that $P_x = (P_z)^{\frac{\gamma}{\gamma-1}}$. That is, an increase in P_z requires a decrease in P_x to keep the price of numeraire at 1. Let us call this *PPN*.

The two relationships between P_x and P_z , *PPP* and *PPN*, uniquely determine the equilibrium values of P_x and P_z . Once we know P_x and P_z we obtain w_x, θ_x , and u_x from (26)-(28), then we obtain θ_z, w_z , and u_z from (29), (24), and (19), respectively.

Notice the Ricardian element in the model. All the prices are determined by the technological variables independent of demand conditions. Diagrammatically, the relative supply of Z is a horizontal curve at the $\frac{P_z}{P_x}$ determined by the intersection of *PPP* and *PPN* curves described above. The relative demand for Z is downward sloping as given by $\frac{Z}{X} = \frac{\gamma(P_z)^{\frac{1}{\gamma-1}}}{(1-\gamma)}$ and is represented by the RD curve in Figure 1. The horizontal RS curve (at the price determined by *PPP* and *PPN* curves) is the relative supply curve. The intersection of the two curves determines the equilibrium $\frac{Z}{X}$.

2.6 Equilibria with the possibility of offshoring

Now, suppose firms in the Z sector have the option of procuring input m_p from abroad instead of producing them domestically.⁶ However, they need to incur a fixed cost of rF_V in terms of the numeraire good in each period. Therefore, the present discounted value of the fixed cost of offshoring is going to be F_V . The per unit cost of imported input is w_S in terms of the numeraire good. Now, a firm offshoring its input maximizes $\int_t^\infty e^{-r(s-t)} \{p(\alpha)z(\alpha) - w(\alpha)N(\alpha) - w_s m_p(\alpha) - c_z V(\alpha)\} ds$. The production function now becomes $z(\alpha) = \frac{1}{\tau^\tau(1-\tau)^{1-\tau}} \alpha N(\alpha)^\tau m_p(\alpha)^{1-\tau}$, while the other constraint is the equation of motion of employment that remains the same.

From the first-order conditions of this altered maximization problem, we still have $\lambda = \frac{c_z}{q(\theta_z)}$. With each

⁶We are assuming that under autarky a firm uses one unit of labor to produce every unit of this input. Therefore, under offshoring in place of using m_p units of home labor in production activities the firm just imports m_p units of the imported input from abroad.

firm taking the equilibrium θ_z as given and given that $\dot{\lambda} = 0$ in steady state, we have

$$\frac{N(\alpha)}{m_p(\alpha)} = \frac{\tau w_s}{(1-\tau)(w(\alpha) + (r+\delta)\lambda)} = \frac{\tau w_s}{(1-\tau)\left(w(\alpha) + \frac{(r+\delta)c_z}{q(\theta_z)}\right)} \quad (31)$$

The pricing equation in this case is given by

$$\frac{\sigma-1}{\sigma} \alpha p(\alpha) = w_s^{1-\tau} \left(w(\alpha) + \frac{(r+\delta)c_z}{q(\theta_z)} \right)^\tau \quad (32)$$

Next, we turn our attention to wage bargaining in the post-offshoring equilibrium.

Note that since $\lambda = \frac{c_z}{q(\theta_z)}$, the value of a job is the same for each firm in equilibrium, and therefore, each firm pays the same wage irrespective of whether they are offshoring or not. The wage is given by

$$w_z = b + \frac{\beta c_z}{1-\beta} \left[\theta_z + \frac{r+\delta}{q(\theta_z)} \right] \quad (33)$$

In rest of the section we use the following notational simplification.

Definition $\omega \equiv \frac{w_z + \frac{(r+\delta)c_z}{q(\theta_z)}}{w_s}$

In the above definition ω is the cost of domestic labor relative to foreign labor. As far as the steady state employment and vacancy are concerned, note that in steady-state $\dot{N}(\alpha) = 0$, therefore, $V(\alpha) = \frac{\delta N(\alpha)}{q(\theta_z)}$ as before. The relationship between output and domestic employment for firms that offshore is given by $N(\alpha) = \frac{\tau z(\alpha)}{\alpha} \omega^{\tau-1}$ while for firms that do not offshore, employment is still given by $N(\alpha) = \frac{z(\alpha)}{\alpha}$. Next, write the mark-up equation for an offshoring firm as

$$p(\alpha) = \frac{\sigma}{(\sigma-1)\alpha} \left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \right) \omega^{\tau-1} \quad (34)$$

Starting at the economy's steady state at time t , the present discounted value of profit (gross of fixed offshoring costs) of an offshoring firm (whose employment equals $N_t(\alpha)$ at this starting point), holding the actions of all existing firms taken as given, can be written as⁷

$$\Pi_V(\alpha) = \frac{ZP_z^\sigma}{r\sigma^\sigma(\sigma-1)^{1-\sigma}} \omega^{(\sigma-1)(1-\tau)} \left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \right)^{1-\sigma} \alpha^{\sigma-1} + \frac{c_z N_t(\alpha)}{q(\theta_z)} \quad (35)$$

If a firm keeps using only domestic labor despite offshoring opportunities, the present discounted value of its profit is given by

$$\Pi_D(\alpha) = \frac{ZP_z^\sigma}{r\sigma^\sigma(\sigma-1)^{1-\sigma}} \left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \right)^{1-\sigma} \alpha^{\sigma-1} + \frac{c_z N_t(\alpha)}{q(\theta_z)} \quad (36)$$

⁷The method used is absolutely analogous to the autarky case, that has been spelled out in greater detail above.

Therefore, in order for a firm to offshore its input production, we need $\Pi_V(\alpha) - \Pi_D(\alpha) \geq F_V$. Note that the second term, $\frac{c_z N_t(\alpha)}{q(\theta_z)}$ in both the profit expressions above is the rent derived by an incumbent firm with positive employment relative to a new entrant that starts with zero employment. This comes from the fact that the value of an occupied job is $\frac{c_i}{q(\theta_i)}$ in sector $i = X, Z$.

It is clear from a comparison of the profit expressions above that if any firm with productivity α offshores input production, then any firm with productivity $\alpha' \geq \alpha$ also offshores.

To simplify notation, make the following definitional assumption.

Definition $C_D \equiv \frac{\left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)}\right)^{1-\sigma}}{r\sigma^\sigma(\sigma-1)^{1-\sigma}}; C_V \equiv \omega^{(\sigma-1)(1-\tau)}C_D$

In order for $\Pi_V(\alpha) - \Pi_D(\alpha) \geq F_V$ it must be the case that $C_V > C_D$. Therefore, a necessary condition for a firm to offshore is $\omega > 1$, that is the cost of hiring foreign labor is less than the cost of domestic labor. Denote the productivity of the marginal firm that is indifferent between offshoring and relying on domestic sourcing by α^* , where α^* satisfies

$$\Pi_V(\alpha^*) - \Pi_D(\alpha^*) = F_V \quad (37)$$

It can be easily verified that firms with $\alpha \geq \alpha^*$ offshore, while others rely on domestic sourcing. In the appendix we enumerate 11 equations determining the 11 endogenous variables $P_x, P_z, w_x, w_z, \theta_x, \theta_z, u_x, u_z, L_z, Z, \alpha^*$ in an equilibrium where firms have the option to offshore. There are three possible post-offshoring equilibria in the model: 1) No firm offshores ($\alpha^* \geq \bar{\alpha}$); 2) Some firms offshore ($\alpha^* \in (\underline{\alpha}, \bar{\alpha})$); 3) All firms offshore ($\alpha^* \leq \underline{\alpha}$). Below we provide an intuitive discussion of these equilibria. We first trace out a curve in the $(\frac{P_z}{P_x}, \frac{Z}{X})$ space, called the offshoring curve, such that the fraction of firms offshoring varies from zero to 1 along this curve. The offshoring curve is the locus of mutually consistent pairs of $\frac{P_z}{P_x}$ and Z/X along which the cutoff productivity varies.⁸ The intersection of this curve with the relative demand curve $(\frac{Z}{X})^d = \frac{\gamma P_x}{(1-\gamma)P_z}$ will give us the equilibrium in the post-offshoring case.

2.6.1 Derivation of the Offshoring Curve

Denote the price of Z when the marginal firm offshoring is α^* by P_z^* . The expression for P_z^* is given by

⁸Very loosely speaking, the offshoring curve is the general equilibrium relative supply curve of Z after factoring in the endogenous offshoring decision of firms. For any price, it gives us the relative supply exactly consistent with the number of firms offshoring leading to that price. Note, however, that this relative supply is not a behavioral relationship.

$$P_z^* = \frac{\sigma}{\sigma-1} \left(w_z^* + \frac{(r+\delta)c_z^*}{q(\theta_z^*)} \right) \left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \omega^{*(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right]^{\frac{1}{1-\sigma}} \quad (38)$$

For each $\alpha^* \in [\underline{\alpha}, \bar{\alpha}]$ the equation above along with (3), (26), (27), (29), and (24) determines $P_x, P_z, w_x, w_z, \theta_x, \theta_z$, irrespective of the demand condition, same as in the case of no offshoring. It is easy to verify that an increase in offshoring, which corresponds to a decrease in α^* , implies increases in $P_x, w_x, w_z, \theta_x, \theta_z$ and a decrease in P_z .

It follows from (35) and (36) that in order for a firm with productivity α^* to be indifferent between offshoring and not offshoring, the following condition must be satisfied

$$Z = \frac{F_V \alpha^{*1-\sigma}}{(P_z^*)^\sigma [C_V^* - C_D^*]} \equiv Z(\alpha^*) \quad (39)$$

where $Z(\alpha^*)$ is the Z required for a firm with productivity α^* to be exactly indifferent between offshoring and not offshoring. It is easy to verify that when $Z = Z(\alpha^*)$, $\Pi_V(\alpha) - \Pi_D(\alpha) > F_V$ for $\alpha > \alpha^*$ and $\Pi_V(\alpha) - \Pi_D(\alpha) < F_V$ for $\alpha < \alpha^*$. Denote the $\frac{P_z}{P_x}$ when $\alpha^* > \bar{\alpha}$ (i.e., no firm satisfies the cutoff productivity for offshoring) by p^{NO} and the corresponding price of Z by P_z^{NO} , where the superscript NO is used to capture the no offshoring case. Note that this is the relative price that obtains in autarky equilibrium. Next, note that at a price of P_z^{NO} , even the most productive firm finds it unprofitable to offshore if $\Pi_V(\bar{\alpha}) - \Pi_D(\bar{\alpha}) < F_V$, or using (39), as long as $Z < \underline{Z} = \frac{F_V \bar{\alpha}^{1-\sigma}}{(P_z^{NO})^\sigma [C_V^{NO} - C_D^{NO}]}$. Denote the production of X , when the production of Z equals \underline{Z} , by \underline{X} . Since wages and unemployment rates are unchanged as long as $p = p^{NO}$, lower Z production is associated with higher X production, and hence $Z < \underline{Z}$ implies $X > \underline{X}$. Therefore, for all $Z < \underline{Z}$, $\frac{Z}{X} < \frac{\underline{Z}}{\underline{X}}$, and hence no firm offshores as long as $\frac{Z}{X} < \frac{\underline{Z}}{\underline{X}}$.

Denote the $\frac{P_z}{P_x}$ when $\alpha^* \leq \underline{\alpha}$ (i.e., all firms satisfy the cutoff) by p^{CO} and the corresponding price of Z by P_z^{CO} . This price corresponds to the case of complete offshoring and hence the use of superscript CO . Now in order for all firms to offshore, it must be the case that $\Pi_V(\underline{\alpha}) - \Pi_D(\underline{\alpha}) \geq F_V$, which in turn requires from (39) that $Z \geq \bar{Z} = \frac{F_V \underline{\alpha}^{1-\sigma}}{(P_z^{CO})^\sigma [C_V^{CO} - C_D^{CO}]}$. Denote the production of X , when the production of Z equals \bar{Z} , by \bar{X} . Again, at a relative price of p^{CO} any $Z > \bar{Z}$ implies $X < \bar{X}$. Therefore, at a price of P_z^{CO} all firms offshore as long as $\frac{Z}{X} \geq \frac{\bar{Z}}{\bar{X}}$.

For $\alpha^* \in (\underline{\alpha}, \bar{\alpha})$ it is shown in the appendix that a sufficient condition for $\frac{dZ(\alpha^*)}{d\alpha^*} < 0$ is

$$\omega^{NO} > \left(\frac{1}{\tau} \right)^{\frac{1}{(\sigma-1)(1-\tau)}} \quad (40)$$

where ω^{NO} (the value of ω at $\alpha^* > \bar{\alpha}$) is the cost of domestic labor relative to foreign labor at autarky equilibrium.

Intuitively, for a given Z an increase in the number of firms offshoring has two effects on the net profit from offshoring: an increase in the cost of hiring domestic labor, $w_z + \frac{(r+\delta)c_z}{q(\theta_z)}$, and a decrease in P_z . Whether an increase in the cost of hiring domestic labor increases or reduces the attractiveness of offshoring depends on what happens to $C_V - C_D$. If $C_V - C_D$ decreases with offshoring, then Z must increase for more firms to offshore: $Z'(\alpha^*) < 0$. Note that $C_V - C_D$ is the difference in the profits caused by differing marginal costs in cases of offshoring and no offshoring. Since offshoring firms also use domestic labor to provide headquarter services, more offshoring increases the marginal costs of both domestic firms and offshoring firms. Therefore, due to this effect the profits of both decrease. Since profit is convex in marginal cost ($c^{1-\sigma}$), if both marginal costs (of offshoring and not offshoring) increase proportionately, there would be a greater decline in the profit from offshoring. Moreover, the decline in profit from offshoring is higher the greater the σ . However, since offshoring firms use less domestic labor, there is a smaller increase in the marginal cost of offshoring (the lower the τ the smaller the increase in marginal cost of offshoring). Therefore, the impact of an increase in domestic labor cost on the net profitability from offshoring is ambiguous. If the net profit from offshoring either decreases or not increases enough to offset the effect of decrease in P_z , then Z must increase for more firms to offshore. Condition (40) above is sufficient to ensure that $Z'(\alpha^*) < 0$ which in turn generally implies, as shown in the appendix, that $\frac{Z(\alpha^*)}{X(\alpha^*)}$ is decreasing in α^* in the range $\alpha^* \in (\underline{\alpha}, \bar{\alpha})$.

In the case when $Z'(\alpha^*) > 0$, that is when an increase in the hiring cost of domestic labor raises the net profit from offshoring enough to offset the effect of decrease in P_z , then $\frac{Z(\alpha^*)}{X(\alpha^*)}$ is increasing in α^* in the range $\alpha^* \in (\underline{\alpha}, \bar{\alpha})$.

2.6.2 Types of Equilibria

Under the sufficient condition (40), the Offshoring Curve(OC) looks like the one depicted in Figure 2a. The equilibrium can be obtained by the intersection of the two curves denoted by RD and OC, since Z and X are not being traded in the world market. Given the Offshoring Curve in Figure 2a, we have the following equilibria:

- a) Unique equilibrium with no offshoring when the RD curve is one labeled I in Figure 2a. Allowing offshoring does not change the equilibrium.
- b) Unique equilibrium with complete offshoring as depicted in Figure 2a when the RD curve is

one labeled IV. The intersection of the relative demand curve with the dotted line shows us the initial equilibrium when offshoring was not allowed. Allowing offshoring moves the economy from no offshoring to complete offshoring.

c) Multiple equilibria as in Figure 2a when the RD curve is the dotted one labeled II. The initial no offshoring equilibrium remains an equilibrium even after offshoring. In other words, there is a coordination problem among firms after offshoring is allowed. A temporary tax break could shift the economy permanently to the offshoring equilibrium

d) Unique, mixed offshoring equilibrium as in Figure 2a when the RD curve is the one labeled III. Mixed equilibrium is one where a fraction of firms offshores while others do not. Allowing for offshoring in this case moves the economy from no offshoring to offshoring by the most productive firms.⁹

Since we get multiple equilibria in some cases, a few words on the stability of equilibria are in order. Note that the Offshoring Curve depicted in Figure 2a is not the standard relative supply curve except in the extreme cases of no offshoring and complete offshoring. Therefore, we cannot use the standard Marshallian or Walrasian notions of stability. For each α^* , the conventional relative supply for Z is a horizontal line. Therefore, we use the following reasonable notion of stability relevant for differentiated goods producing firms. In order for an equilibrium to be stable, any unilateral deviation by a small mass of firms, in terms of their alternative strategies of offshoring or producing their inputs domestically, should result in incentives that alter firms' actions (offshoring versus producing domestically) to push the economy back towards that same (starting) equilibrium outcome. With this definition in mind, look at the interior equilibrium obtained by the intersection of the dotted RD curve with the OC curve in Figure 2a. Starting from this interior equilibrium, if a domestic firm offshores (deviates) P_z falls which results in an increase in the relative demand for Z greater than the required increase in Z/X for an extra firm to offshore. Therefore, more domestic firms have an incentive to offshore, taking us further away from this interior equilibrium. Hence this equilibrium is not stable. Using this concept of stability and instability, it can be seen that all the other equilibria in Figure 2a are stable. In particular the unique, mixed equilibrium obtained by the intersection of the RD curve labeled

⁹For example, with the following parameters, $q(\theta) = k\theta^{\phi-1}$; $k = .25$; $\phi = .5$; $c_x = .05$; $c_z = .05$; $\sigma = 3.8$; $\alpha \in \text{Pareto}[.2, 3.4]$; $\beta = .5$; $b = .25$; $r = .03$; $\delta = .035$; $\tau = .5$; $\gamma = .5$; $L = 1$, we get a unique, mixed equilibrium with $F_V = 1$, $w_s = .25$, a complete offshoring equilibrium with $F_V = .5$, $w_s = .25$, and a no offshoring equilibrium with $F_V = 1$, $w_s = .55$. The rationale for choosing these parameter values is provided in the appendix.

III and the OC curve in Figure 2a is stable.

When $\frac{Z(\alpha^*)}{X(\alpha^*)}$ is increasing in α^* , the Offshoring Curve looks like the one shown in Figure 2b. We cannot get a unique, mixed equilibrium in this case. We get either a no offshoring equilibrium even after allowing for offshoring, if the RD curve is like the one labeled I in Figure 2b, or a complete offshoring equilibrium when the RD curve is like the one labeled III in Figure 2b. If the RD curve is like the one labeled II in Figure 2b, then we get multiple equilibria, however, the argument in the previous paragraph implies that the interior equilibrium is unstable.

Since the types of equilibria in Figure 2b are a subset of the equilibria depicted in Figure 2a, in the comparative static analysis below, we focus on the cases depicted in Figure 2a.

2.7 Impact of offshoring on allocation of labor and unemployment

In this section we do two things. First, we study the impacts of decreases in the fixed cost of offshoring, F_V , and transportation/communication cost (or foreign wage), w_s . Second, we compare the outcome when offshoring was not possible to the two cases of mixed equilibrium and complete offshoring equilibrium when offshoring becomes possible.

2.7.1 Change in the fixed cost of offshoring or in the transportation/communication cost

Suppose there is a decrease in F_V . It is easy to verify from the discussion of the Offshoring Curve that it will shift to the left rendering equilibria with positive amount of offshoring more likely. In Figure 3a the solid curve is the Offshoring Curve for initial level of F_V , while the dashed curve is the Offshoring Curve after F_V has decreased. An implication of the leftward shift of the Offshoring Curve is that even in the case when the no offshoring equilibrium is the unique equilibrium, as in the case with RD curve I in Figure 2a, a decrease in F_V will make equilibria with positive offshoring more likely. If we start from a unique, mixed equilibrium where only a few firms offshore, this reduction in F_V will lead to offshoring by more firms as the equilibrium moves from point A to B in Figure 3a. In both cases the relative price of Z will fall and that of X will rise. P_x rises means w_x , θ_x , w_z , and θ_z also rise. Since θ_x and θ_z go up, both u_x and u_z fall, i.e., the rates of unemployment in both sectors decrease.

A change in transportation, communication cost, or a decrease in Southern wage can be captured by a decrease in w_s in our model. It can be seen from the earlier discussion that \underline{Z} decreases with w_s . As well, a decrease in w_s would imply a lower P_z^{CO} and a higher P_x^{CO} . Therefore, the relative price p^{CO} at which

complete offshoring obtains shifts down. A higher P_x^{CO} implies higher w_z^{CO} and higher θ_z^{CO} as well. The impact on \bar{Z} is as follows. A decrease in w_s has a direct negative effect on \bar{Z} , however, the indirect effects through changes in w_z^{CO} and θ_z^{CO} are ambiguous. The effects can be seen as follows.

$$\bar{Z} = \frac{F_V \alpha^{1-\sigma}}{(P_z^{CO})^\sigma [C_V^{CO} - C_D^{CO}]}$$

It turns out from the above expression that \bar{Z} decreases as w_s decreases. It can also be verified that for each $p(\alpha^*) \in (p^{CO}, p^{NO})$, the required $\frac{Z}{X}$ is smaller the smaller the w_s . Therefore, the Offshoring Curve shifts to the left as shown in Figure 3b. Again, the possibility of equilibria with offshoring increases, as one would expect. Also, if we start from a unique, mixed offshoring equilibrium where only a few firms offshore, this reduction in w_s will lead to offshoring by more firms as the equilibrium moves from point A to B in Figure 3b. In both these cases the relative price of Z will fall and that of X will rise, and through the mechanism outlined above we will get a reduction in sectoral unemployment in each of the two sectors.

Thus we can summarize the comparative static results above in the following proposition:

Proposition 1 *A decrease in the fixed cost F_V of offshoring or in the transportation or communication cost captured by w_s makes an offshoring equilibrium more likely when offshoring is allowed. Also, if we start from a unique, interior (mixed) equilibrium where only a few firms offshore, this reduction in offshoring or transportation or communication costs will lead to offshoring by more firms. In both cases the relative price of Z will fall and that of X will rise, and we will get a reduction in sectoral unemployment in each of the two sectors.*

2.8 Comparing no-offshoring and offshoring equilibria

2.8.1 Sectoral and economywide demand for labor

In an offshoring equilibrium P_x is higher, which means w_x , θ_x , w_z , and θ_z are also higher. Since θ_x and θ_z are higher, both u_x and u_z are lower than in the no-offshoring equilibrium, i.e., the rates of unemployment in both sectors decrease. An increase in the price of good X is able to support higher labor costs in that sector. Since the wage bargaining equation implies that wage and market tightness increase together, we have an increase in both these variables in the X sector. Unemployment goes down as a result. Market tightness in the X and Z sectors go together, and so we get a reduction in Z sector unemployment rate as well. While the reduction in P_z by itself, everything else held constant, should increase unemployment in sector Z , this

is more than offset by the decrease in the cost of production brought about by offshoring. Note that, for firms that offshore, there is now a higher marginal product of headquarter labor, arising from employment of more production input per headquarter worker (since the input is now cheaper) in the offshoring firms. This effect offsets the effect of a decline in the price of Z . An alternative way to look at this is the following. Offshoring of production activity in one sector makes the other sector relatively more intensive in the use of domestic labor. At the same time, offshoring raises the relative price of the good whose production is not offshored, i.e., of the good that is domestic labor intensive. Therefore, cost of domestic labor (wage rate and market tightness) goes up. This effect is analogous to the Stolper-Samuelson effect.

The impact on aggregate unemployment depends on what happens to L_z , the share of labor affiliated with sector Z , and whether c_x is more or less than c_z .

Case I: In the special case of $c_x = c_z$, we have $\theta_x = \theta_z$ and hence $u_x = u_z$. Therefore, aggregate unemployment falls along with the fall in sectoral unemployment due to offshoring.

When $c_x = c_z$, it is easy to show that the size of the labor force in the Z sector post-offshoring is less than in the pre-offshoring equilibrium (See proof in appendix). Even though the result above obtains for $c_x = c_z$, using a continuity argument we can say that it will hold if c_x and c_z are not too different. Numerical simulations confirm that the result on L_z decreasing upon offshoring is valid even when $c_x \neq c_z$. In this case we get the following additional results.

Case II: $c_x < c_z$. In this case, it is easy to verify that $\theta_x > \theta_z$, and hence $u_x < u_z$. That is, Z sector has higher wage as well as unemployment. Now, since offshoring shifts labor from sector Z to sector X , there is going to be an unambiguous decrease in aggregate unemployment. Although the wages of workers in both sectors increase, the number of workers earning higher wage declines.

Case III: $c_x > c_z$. In this case, even though the rate of unemployment decreases in both sectors, since labor moves into the sector with higher unemployment, the impact on aggregate unemployment is ambiguous.

The comparison of the offshoring and no-offshoring equilibria can be summarized as follows:

Proposition 2 *In an offshoring equilibrium, sectoral wages are higher and sectoral unemployment lower than in the pre- or no-offshoring equilibrium. When $c_x \leq c_z$, there is an unambiguous decrease in aggregate unemployment as a result of moving from a no- (or pre-) offshoring equilibrium to an offshoring equilibrium. When $c_x > c_z$, the impact on aggregate unemployment is ambiguous.*

2.8.2 Firm-level demand for labor

In the model, the reallocation of labor, as a result of offshoring, can be summarized as follows. Firstly, there is intersectoral reallocation of labor from sector Z to sector X . Secondly, within sector Z some or all firms move some of their production activities overseas. Thirdly, within that sector demand for labor shifts to headquarter services in firms that end up offshoring their production. Finally, since foreign labor is cheaper, offshoring firms will increase the use of production services and depending on the elasticity of substitution, decrease or increase the use of headquarter services.

To see the impact of offshoring on employment for an individual firm we need to do the following. Let us look at the marginal firm, α^* that is indifferent between offshoring and no offshoring. If this firm doesn't offshore then its employment is given by

$$N^{NO}(\alpha^*) = \frac{z(\alpha^*)}{\alpha^*} = \left(\frac{p(\alpha^*)}{P_z} \right)^{-\sigma} \frac{Z}{\alpha^*} = \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} \alpha^{*\sigma-1} \left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \right)^{-\sigma} Z P_z^\sigma$$

Similarly, the domestic employment of this firm, if it decides to offshore is given by

$$N^O(\alpha^*) = \frac{\tau z(\alpha^*)}{\alpha^*} \omega^{\tau-1} = \left(\frac{p(\alpha^*)}{P_z} \right)^{-\sigma} \frac{\tau Z \omega^{\tau-1}}{\alpha^*} = \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} \alpha^{*\sigma-1} \left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \right)^{-\sigma} \tau Z P_z^\sigma \omega^{(1-\tau)(\sigma-1)}$$

Comparing the above two expressions, note that $N^O(\alpha^*) > N^{NO}(\alpha^*)$, under the sufficient condition $\omega^{NO} > \left(\frac{1}{\tau} \right)^{\frac{1}{(\sigma-1)(1-\tau)}}$ discussed earlier.

Intuitively, a high τ implies greater headquarter intensity, and therefore, employment of offshoring firms can increase because their headquarter activities increase (due to their complementarity with labor used in production). A high ω means a higher relative labor cost in the North relative to the South, which means that offshoring firms steal more business from non-offshoring firms. Therefore, their domestic employment can increase. Finally, a high σ implies greater elasticity of substitution, and therefore, offshoring firms again can steal more business from non-offshoring firms. Thus, if the business stealing effect is sufficiently strong, the domestic employment of offshoring firms can be higher than that of non-offshoring firms.

To compare domestic employment of a firm before and after offshoring we need to compare

$$\tau \omega^{(1-\tau)(\sigma-1)} \left(w_z^O + \frac{(r+\delta)c_z}{q(\theta_z^O)} \right)^{-\sigma} Z^O (P_z^O)^\sigma > \left(w_z^{NO} + \frac{(r+\delta)c_z}{q(\theta_z^{NO})} \right)^{-\sigma} Z^{NO} (P_z^{NO})^\sigma$$

There are several effects. $\omega^{(1-\tau)(\sigma-1)}$ captures the business stealing effect as discussed earlier. However, domestically produced inputs have become costlier due to the higher domestic wage compared to the no offshoring case. This increase in the price of domestic inputs would tend to reduce domestic employment. In

addition, what happens to ZP_z^σ becomes important. An increase in this implies an increase in employment for all firms in the industry. Therefore, the net effect depends on the relative strengths of these effects.

The above discussion on firm-level employment can be summarized in the following proposition:

Proposition 3 *For high enough headquarter intensity τ , a high enough ω , the relative North-South labor cost and for a high enough elasticity of substitution σ , an offshoring firm, controlling for firm-level characteristics, will have higher domestic employment relative to that of a firm which remains fully domestic. Similar variables determine, albeit in a much more complicated manner, whether a firm after offshoring increases its domestic employment or not.*

2.9 Other Comparative Static Exercises

While the focus of this paper is to understand the implications of offshoring for unemployment, we can also use the model to understand how labor market institutions affect offshoring and consequently unemployment. To this end, we study the impact of an increase in the unemployment benefit, b , on offshoring and unemployment. We will also look at the impact of a change in the country size, L .

When b goes up, holding $c_z = c_x$, the average cost of employing domestic labor in each sector, given by $\left(w_i + \frac{(r+\delta)c_i}{q(\theta_i)}\right)$, $i = x, z$, remains unchanged for a given P_x . Therefore, holding the number of firms offshoring fixed, from the production side P_z does not change due to this increase in b at a given P_x . It also means that the threshold Z consistent with just making exactly as many firms offshore remains the same. However, an increase in b at given P_x (and therefore, at given P_z) reduces labor market tightness and increases unemployment in both sectors. Therefore, even though the threshold Z remains unchanged we now have a higher threshold Z/X . The offshoring curve shifts right from OC to OC' as a result of the increase in b (Figure 3c). Starting from an initial mixed equilibrium A, we move to B which corresponds to a mixed equilibrium with offshoring by fewer firms and a higher relative price of Z . On top of the direct effect of an increase in b on unemployment, there is a further indirect adverse effect on unemployment through the reduction in P_x . Intuitively, at unchanged P_x , an increase in b increases wage but reduces the market tightness. While the former raises the domestic labor cost, the latter reduces it and offsets the effect of the former. A reduction in P_x reduces domestic labor cost, making offshoring less attractive. Therefore, an increase in b affects offshoring adversely through intersectoral price changes. Effectively, an increase in the unemployment benefit works like a reduction in country size. This in turn leads to fewer firms being able to jump the fixed costs of offshoring.

A comparative static exercise of directly reducing country size, L will give us similar effects in terms of shifting the offshoring curve, the fall in P_x and the consequent rise in unemployment (only the indirect effect).

3 The Case of No Intersectoral Labor Mobility

Since studying the transitional dynamics of the model is very complicated, to study the shorter run implications of offshoring on unemployment, we discuss a case where there is no intersectoral labor mobility. The only connection between the two sectors is through goods prices.

We know that since the composite good C is the numeraire good and its price equals unity, P_x and P_z move in opposite directions. Also, when $\frac{P_z}{P_x}$ rises, it means P_x falls and P_z rises. With no intersectoral labor mobility, i.e., with L_x and L_z held fixed, we need to solve the wage and price equations simultaneously within a sector but separately for the two sectors. Clearly the solution to the equations $w_x = (1 - \beta)b + \beta[P_x + c_x\theta_x]$, $P_x = w_x + \frac{(r+\delta)c_x}{q(\theta_x)}$ will be such that as P_x goes down (as $\frac{P_z}{P_x}$ rises) both w_x and θ_x fall. A fall in θ_x , plugged into the Beveridge curve, implies that sectoral unemployment rate u_x rises. However, as P_z goes up (as $\frac{P_z}{P_x}$ rises), the solution to the simultaneous equations $w_z = b + \frac{\beta c_z}{1-\beta}[\theta_z + \frac{r+\delta}{q(\theta_z)}]$, $P_z = \frac{\sigma}{\sigma-1} \left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \right) \tilde{\alpha}^{-1}$, that hold under autarky, is such that both w_z and θ_z rise. Plugging this rise in θ_z into the Beveridge curve, we get a decline in the sectoral unemployment u_z . Thus, under autarky, for given L_x and L_z , as $\frac{P_z}{P_x}$ rises, $\frac{Z}{X}$ rises. Let us call this relationship, the short-run relative supply curve, and the horizontal relative supply curve we derived earlier, shown in Figure 1, the long-run relative supply curve. At $L_x = L_x^{NO}$ and $L_z = L_z^{NO}$ (where L_i^{NO} represents equilibrium labor force in sector $i = x, z$, in autarky when labor is mobile across sectors) it is easy to see that both the long-run and the short-run curves cut the relative demand curve at exactly the same point A (See Figure 4 where SRS stands for short-run relative supply).

Let us now look at a situation of complete offshoring equilibrium under no intersectoral labor mobility and compare it with the scenario of labor mobility.¹⁰ The condition $P_z = \frac{\sigma}{\sigma-1} \left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \right) \tilde{\alpha}^{-1}$ under autarky now gets replaced with $P_z = \frac{\sigma}{\sigma-1} \left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \right) \omega^{(\tau-1)} \tilde{\alpha}^{-1}$ under complete offshoring where $\omega^{(\tau-1)} < 1$. As in the case of autarky, the short-run supply curve under complete offshoring is again upward sloping. As seen in Figure 4, the long-run complete offshoring equilibrium is also a short-run complete offshoring equilibrium at $L_x = L_x^{CO}$ and $L_z = L_z^{CO}$ (where L_i^{CO} represents equilibrium labor force in sector $i = x, z$, under

¹⁰Note that at this moment complete offshoring is being taken as given. This can be guaranteed for fixed costs of offshoring small enough.

complete offshoring when labor is mobile across sectors).

Holding the economy's labor force constant, increasing L_z shifts the short-run relative supply curve to the right. For small enough fixed costs of offshoring, allowing the possibility of offshoring will give us complete offshoring under both intersectoral labor mobility and no labor mobility. At $L_x = L_x^{NO}$ and $L_z = L_z^{NO}$ (allowing no labor mobility), the short-run relative supply curve under complete offshoring should now be to the right of the one at $L_x = L_x^{CO}$ and $L_z = L_z^{CO}$, since, as shown in the appendix, $L_x^{CO} < L_x^{NO}$. Thus, $\frac{P_z}{P_x}$ under complete offshoring is lower when there is no labor mobility than under labor mobility. Therefore, under complete offshoring Z sector wages are lower and unemployment there higher under no labor mobility than under labor mobility. Under both labor mobility and no labor mobility, there are two effects of offshoring on sector Z . One is the cost-reducing or the productivity-enhancing effect, while the other is the decline in relative price (a fall in $\frac{P_z}{P_x}$). The second effect is stronger under no labor mobility than under mobile labor and if this effect is strong enough, the sectoral unemployment rate may go up in the Z sector. Whether this will be the case or not will depend on parameter γ which represents the importance of Z in the final numeraire good C and headquarter intensity τ . Intuitively, a higher γ implies a higher demand for Z sector output and consequently a higher derived demand for labor in the Z sector, while a higher τ implies higher demand for domestic labor in sector Z . Therefore, with a high γ or τ , a larger amount of labor can be absorbed in the Z sector without a rise in unemployment.

Also the favorable terms-of-trade effect of offshoring for the X sector is stronger under no labor mobility than under mobile labor. Therefore, the reduction in the unemployment rate in the X sector (due to offshoring) is greater in the short-run than in the long run. This means unemployment falls by a considerable amount in the short run and then rises in the long run, with the new long run unemployment rate being lower than the initial long-run unemployment rate.

In the case of incomplete offshoring, when only the most productive firms are able to jump the fixed costs of offshoring, there are several more effects we need to take care of. Firstly, the labor force in the Z sector is larger in size under no labor mobility which shifts, as in the complete offshoring case, the relative supply curve to the right. Second, the equilibrium number of firms offshoring also affects this curve. There are two factors acting in opposite directions on the equilibrium number of firms offshoring. Under no labor mobility (relative to the mobile labor case), wages and labor-market tightness are lower in the Z sector in the North, which reduces the attractiveness of offshoring. However, since the labor force is not allowed to shrink in this sector, the scale effect (captured by $P_z Z$) and hence the attractiveness of offshoring are stronger when labor mobility is not allowed than when it is allowed. The impact of offshoring on sectoral unemployment

rates is qualitatively similar to that in the complete offshoring case. Below we provide numerical examples to confirm that unemployment can go down in the Z sector starting from an initial situation of a mixed offshoring equilibrium.

Table 1

	Labor mobility	Parameters	$\frac{P_z}{P_x}$	α^*	$G(\alpha^*)$	w_x	w_z	ω	u_x	u_z	L_z
I		$F_V = 1, w_s = .25$	2.44	0.24	0.46	.606	.606	2.56	.0521	.05214	.276
II	Yes	$F_V = .75, w_s = .25$	2.39	0.218	0.26	.613	.613	2.59	.0517	.0517	.272
III	NO	$F_V = .75, w_s = .25$	2.36	0.219	0.26	.617	.606	2.56	.0514	.05215	.276
IV	Yes	$F_V = 1, w_s = .2$	2.08	0.225	0.33	.658	.658	3.47	.0488	.0488	.272
V	NO	$F_V = 1, w_s = .2$	2.05	0.225	0.33	.662	.650	3.42	.0485	.0493	.276

The other parameter are as follows. $q(\theta) = k\theta^{\phi-1}$; $k = .25$; $\phi = .5$; $c_x = .05$; $c_z = .05$; $\sigma = 3.8$; $\alpha \in \text{Pareto}[.2, 3.4]$; $\beta = .5$; $b = .25$; $r = .03$; $\delta = .035$; $\tau = .5$; $\gamma = .5$; $L = 1$. We discuss the rationale for choosing these parameter values in the appendix.

Row I presents the results of the initial mixed offshoring equilibrium. Rows II and III present results of a 25% reduction in the fixed cost of offshoring F_V with and without labor mobility, respectively. Comparing rows I and II, we see that reduction in F_V leads to increased offshoring and decreases in unemployment rates in both sectors when there is labor mobility. When the amount of labor in the Z sector is restricted to that in row I, then offshoring reduces unemployment in X sector but increases it slightly in the Z sector. In rows IV and V F_V goes back to 1, but w_s is lowered by 20% compared to row I. Again there is increased offshoring both with and without labor mobility, however, in this case even in the absence of labor mobility, the unemployment rates in both sectors go down as shown in row V, even though the decline is larger in sector X .

Proposition 4 *In the shorter run case where intersectoral labor mobility is not allowed, in an offshoring equilibrium, the reduction in the relative price of Z is greater than what we get with offshoring under intersectoral labor mobility. Thus, the increase in wage and the reduction in sectoral unemployment in sector Z under offshoring are smaller under no labor mobility than under intersectoral labor mobility, with the possibility being there that sectoral unemployment goes up as a result. In the X sector, the increase in wage and the reduction in sectoral unemployment as a result of offshoring are greater.*

4 An Extension: Free Entry

The extension we present here is allowing for free entry of firms in the differentiated inputs sector, so that the mass of firms operating in equilibrium is endogenously determined.¹¹ We are back here to the case of intersectoral labor mobility. As discussed in section 2, before entering, firms incur a sunk cost equal to F_E in terms of the numeraire good. Then they observe their realization of productivity which they draw from a distribution $G(\alpha)$, where $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. The price of Z , P_z , can now be written as

$$P_z = M^{\frac{1}{1-\sigma}} \left[\int_{\underline{\alpha}}^{\bar{\alpha}} p(\alpha)^{1-\sigma} dG(\alpha) \right]^{\frac{1}{1-\sigma}} \quad (41)$$

where M is the mass of active firms. Assuming α^* to be the cutoff productivity above which firms offshore, the key equations determining the equilibrium in the case of endogenous entry are given as follows. The pricing decision of differentiated goods firms is same as before. The equation determining cutoff productivity α^* is again given by (37) which can be written as

$$ZP_z^\sigma (\alpha^*)^{\sigma-1} (C_V - C_D) = F_V \quad (42)$$

The only additional equation is the free entry condition in the differentiated goods sector which can be written as

$$ZP_z^\sigma \left[C_V \int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + C_D \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right] = F_E + (1 - G(\alpha^*))F_V \quad (43)$$

The labor market clearing condition in the Z sector is modified as follows.

$$(1 - u_z)L_z = MZP_z^\sigma \left(\frac{\sigma}{(\sigma-1)} \right)^{-\sigma} \left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \right)^{-\sigma} \left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \tau\omega^{(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right] \quad (44)$$

The product market clearing condition is still given by

$$\frac{Z}{(1 - u_x)(L - L_z)} = \frac{\gamma (P_z)^{\frac{1}{\gamma-1}}}{(1 - \gamma)} \quad (45)$$

Equations (41)-(45) along with (60)-(66) in the appendix determine the following 12 endogenous variables of interest: $P_x, P_z, w_x, w_z, \theta_x, \theta_z, u_x, u_z, L_z, Z, \alpha^*$, and M .

¹¹See Zieseimer (2005) for a completely autarkic, one sector model of unemployment under monopolistic competition.

Next, we show that the results obtained on the impact of offshoring on unemployment continue to hold when the number of varieties is endogenously determined. Establishing this result analytically in the general case is difficult, therefore, we look at the case where $c_x = c_z$. In equilibrium, a potential entrant must be indifferent between entering and not entering an industry. Let $\bar{\pi}$ be the expected annualized per period profit of a potential new entrant in the differentiated intermediate good sector when the economy is in the steady state. This profit is net of both search and production costs, but gross of fixed costs and is given by r times the expression on the left hand side of (43). Since we have $c_x = c_z = c_e$, we have $w_x = w_z = w_e$, $\theta_x = \theta_z = \theta_e$, and $u_x = u_z = u_e$, where the subscript e denotes “economywide”. Let \bar{R} be the annualized per period expected revenue of a potential new entrant. Given the markup equation we derived earlier, we have $\bar{\pi} = \bar{R}/\sigma$. Since $C = \frac{Z^\gamma X^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}$, a constant share γ of total expenditure on C is indirectly on the Z good and consequently on all the intermediate varieties, $z(i)$ combined.

Total expenditure on the numeraire good C is the sum of the wage bill, total profits net of fixed costs, search costs, total fixed costs and export sales that pay for (therefore equal) the import of the offshored production input. As explained earlier, incumbent firms already in the steady state will earn rents relative to a potential new entrant. Such rents will be earned by incumbent firms in both sectors because each occupied job has a value of $\frac{c_e}{q(\theta_e)}$. Let us denote total per-period rents of this type by Γ .¹² We denote the total per period expenditure on the offshored production input by \mathcal{M} . Also, in our framework, when a firm incurs a sunk cost of entry, F_E , right in the beginning, it is equivalent to paying rF_E every period. Recall that rF_V is the per period fixed cost of offshoring. With a total of M firms producing the differentiated intermediate good z we have total revenue in the intermediate goods sector equal to total expenditure, E_z , on Z

$$M\bar{R} = E_z = \gamma\{(1 - u_e) w_e L + M[\bar{\pi} - (1 - G(\alpha^*))rF_V - rF_E] + \Gamma + S + M[(1 - G(\alpha^*))rF_V + rF_E] + \mathcal{M}\} \quad (46)$$

where S is the total search costs incurred by all firms in the economy and is given in steady state by $\frac{\delta c_e (1 - u_e) L}{q(\theta_e)}$. In equilibrium with M endogenously determined, the free entry condition in (43) can be re-written as

$$\bar{\pi} = (1 - G(\alpha^*))rF_V + rF_E \quad (47)$$

using which and using the relation $\bar{\pi} = \bar{R}/\sigma$, we have the equilibrium condition given by

$$\sigma [(1 - G(\alpha^*))rF_V + rF_E] = \frac{1}{M} \gamma \{(1 - u_e) w_e L + \Gamma + S + M[(1 - G(\alpha^*))rF_V + rF_E] + \mathcal{M}\} \quad (48)$$

¹²The total annualized rents per period for the economy as a whole can be written as $\Gamma = \frac{rc_e(1-u_e)L}{q(\theta_e)}$ where as before the subscript e stands for the common, economywide variable.

The above can be re-written as

$$[(1 - G(\alpha^*))rF_V + rF_E] = \frac{\gamma}{(\sigma - \gamma)M} \{(1 - u_e)w_eL + \Gamma + S + \mathcal{M}\} \quad (49)$$

which can be further re-written as

$$[(1 - G(\alpha^*))rF_V + rF_E] = \frac{\gamma}{(\sigma - \gamma)M} \left(w_e + \frac{(r + \delta)c_e}{q(\theta_e)} \right) (1 - u_e)L + \frac{\gamma\mathcal{M}}{(\sigma - \gamma)M} \quad (50)$$

Now, for a given α^* , w_e and θ_e are increasing in M , while u_e is decreasing in M . Therefore, $(w_e + \frac{(r+\delta)c_e}{q(\theta_e)})(1 - u_e)L$ on the r.h.s of the above equation is clearly increasing in M . In the case of autarky, which equals the case where $\alpha^* = \bar{\alpha}$ and $\mathcal{M} = 0$, it is shown in the appendix that the proportional change in $(w_e + \frac{(r+\delta)c_e}{q(\theta_e)})$ is less than the proportional change in M , and since the impact on u_e is of second order, $\frac{\gamma}{(\sigma - \gamma)M} (w_e + \frac{(r+\delta)c_e}{q(\theta_e)})(1 - u_e)L$, which we from now on call $\varphi(M)$, would be decreasing in M in autarky. [It is shown in the appendix that under the sufficient condition $(\sigma - 1) > \frac{\gamma}{(1 - \gamma(1 - \tau))}$, $(w_e + \frac{(r+\delta)c_e}{q(\theta_e)})$ under offshoring increases less than proportionally with M for any α^* .] Therefore the r.h.s. of the last equation which equals $\varphi(M)$ under autarky, is represented by a downward sloping EE curve in Figure 5.

When F_V is small, for a given M , allowing for offshoring leads to complete offshoring, i.e., $\alpha^* = \underline{\alpha}$ at which point $G(\alpha^*) = 0$. We now look at the various components of $\varphi(M)$ under complete offshoring and under autarky. For a given M , w_e and θ_e are decreasing in α^* , while u_e is increasing in α^* . Therefore, $(w_e + \frac{(r+\delta)c_e}{q(\theta_e)})(1 - u_e)L$ is higher under offshoring than under autarky. On top of that now $\mathcal{M} > 0$, which means exports are positive and add to the total demand for C and therefore to the expenditure on Z . Thus, in Figure 5, the EE curve under complete offshoring, representing $\varphi(M) + \frac{\gamma\mathcal{M}}{(\sigma - \gamma)M}$, lies completely above the EE curve under no offshoring. The per-firm total costs of entry and offshoring combined are greater under complete offshoring than under no offshoring. If F_V is relatively small, we clearly have equilibrium M , given by the intersection of EE and the per firm fixed cost curve, higher under complete offshoring than under no offshoring.¹³ A higher M under offshoring implies that P_z is lower with endogenous M than with M exogenously fixed at the autarky value, and hence unemployment is lower in a complete offshoring equilibrium with endogenous M than with exogenous M .

Thus, with free entry, we have an increase in the total mass of firms operating in the differentiated intermediate goods sector as a result of offshoring. This, clearly, is an additional channel through which

¹³This is true irrespective of the shape of the EE curve under complete offshoring, since it is completely above the downward sloping EE curve under autarky. This means that the EE curve under offshoring cannot intersect the fixed cost curve to the left of the equilibrium M under autarky (assuming fixed costs of offshoring are relatively small).

offshoring increases the wage rate and reduces unemployment, since this effect increases productivity in the Z sector and thereby increases the relative price of X . While the analysis in this section sharpens our understanding of the free entry case and guides us on how to think about it, it requires making the restriction of relatively small fixed costs of offshoring (that leads to complete offshoring). Free entry in the incomplete offshoring case (the case with larger fixed costs) is analytically intractable, especially since both M and α^* are endogenously determined and they together determine the unemployment rate. Therefore, we construct some numerical examples of incomplete offshoring (mixed equilibrium) to check the robustness of our insights from our above analysis of the complete offshoring case.

Numerical Examples : The movement to greater offshoring here is a result of a reduction in F_V . Offshoring leads to greater decrease in unemployment in the case of endogenous entry. The table below provides numerical examples to show the impact of decreases in F_V and w_s in the cases of no entry and free entry.

Table 2

		Free entry	$\frac{P_z}{P_x}$	α^*	$G(\alpha^*)$	w	ω	u	L_z	M
I	$F_V = 1, w_s = .25$		2.44	0.24	0.46	.606	2.56	.0521	.28	1
II	$F_V = .75, w_s = .25$	No	2.39	0.218	0.26	.613	2.59	.0517	.27	1
III	$F_V = .75, w_s = .25$	Yes	2.37	0.219	0.27	.615	2.6	.0515	.27	1.02
IV	$F_V = 1, w_s = .2$	No	2.08	0.225	0.33	.658	3.47	.0488	.27	1
V	$F_V = 1, w_s = .2$	Yes	2.01	0.229	0.36	.669	3.53	.0482	.27	1.08

The other parameters are same as in the numerical examples presented in Table 1.

The first row gives the results of the initial mixed equilibrium with the entry cost chosen to make $M = 1$. The required entry cost turns out to be $F_E = 3.31$. The second row shows results of lowering F_V with no entry while the third row shows results with free entry. Rows IV and V show the results of lowering w_s from .25 in row I to .2 while keeping F_V at the same level as in row I. The results above make it clear that allowing for entry makes the unemployment reducing effect of offshoring stronger, but the effect is quantitatively small.

5 Discussions and Conclusions

In this paper, in order to study the impact of offshoring on sectoral and economywide rates of unemployment, we construct a two sector general equilibrium model in which unemployment is caused by search frictions.

We find that, contrary to general perception, wage increases and sectoral unemployment decreases due to offshoring. This result can be understood to arise from the productivity enhancing (cost reducing) effect of offshoring. This result is consistent with the recent empirical results of Amiti and Wei (2005a, b) for the US and UK, where, when sectors are defined broadly enough, they find no evidence of a negative effect of offshoring on sectoral employment.

Even though both sectors have lower unemployment post-offshoring, whether the sector with the lower unemployment or higher unemployment expands will also be a determinant of the overall unemployment rate. If the search cost is identical in the two sectors, implying identical rates of unemployment, then the economywide rate of unemployment declines unambiguously after offshoring. Alternatively, even if the search cost is higher in the sector which experiences offshoring (implying a higher wage as well as higher rate of unemployment in that sector), the economywide rate of unemployment decreases because workers move from the higher unemployment sector to the lower unemployment sector. Interestingly, when the possibility of offshoring is allowed for we find the existence of multiple equilibria, resulting in the indeterminacy of sectoral and overall unemployment rates.

Next, the domestic employment of firms that offshore may be higher than those of firms that decide not to offshore. This arises from the complementarity between headquarter services and production that can be offshored, as well as from the business stealing effect. These effects can also increase a firm's domestic employment after offshoring relative to its pre-offshoring level.

When we modify the model to disallow intersectoral labor mobility, the negative relative price effect on the sector in which firms offshore some of their production activity becomes stronger. In such a case, it is possible for this effect to offset the positive productivity effect, and result in a rise in unemployment in that sector. In the other sector, offshoring has a much stronger unemployment reducing effect in the absence of intersectoral labor mobility than in the presence of it.

Finally, allowing for an endogenous number varieties in the offshoring sector provides an additional indirect channel through which sectoral unemployment goes down.

Before ending the paper some remarks on some of our modeling assumptions are in order. Instead of writing a two sector model, we could have written a one sector model to examine the impact of offshoring on unemployment. We have verified that the unemployment reducing effect of offshoring is present even in a one sector model with endogenous entry a la Melitz (2003), but we preferred a two sector model because it provides additional insights about the intersectoral reallocation. We have modeled vacancy cost, c , in terms of the numeraire good which seemed natural given the two sector structure of the model. One could

alternatively model the vacancy cost either in terms of labor or foregone output. In the former case, the vacancy cost would be $c_i w_i$ for sector $i = X, Z$, where w_i is the sectoral wage. In the latter case, it would be $c_i p_i$. Our preliminary investigation suggests that the qualitative results would be unchanged.

The model in the paper can be extended along several dimensions. In our current paper we do not explicitly model outsourcing, as is currently understood to be subject to contracting problems.¹⁴ If firms have the choice to outsource their production activities to a foreign supplier in an incomplete contract framework, whether a firm outsources or produces inputs domestically will depend on the tradeoff between the domestic labor market frictions and contracting costs.

Another possible extension would be to endogenize the bargaining power of workers in the wage bargaining process. In our current model, wages increase with offshoring, however, there is some evidence to suggest that wages of workers have stagnated despite productivity gains coming from globalization and technological progress. If we make workers' bargaining power a decreasing function of the number of firms offshoring, it would be possible to show a decrease in wages resulting from offshoring. This would be similar in spirit to the Mitchell (1985) description of "norm shift" in wage determination. Since our focus in the present paper is on unemployment effects of offshoring, we do not pursue this extension in the present paper.

We can also extend the model to allow for workers with different skills and see whether offshoring affects them differentially. We plan to pursue these extensions in a separate paper, as they seem to be beyond the scope of the current one.

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¹⁴See the very brief literature review in the introduction. Also, see Mitra and Ranjan (2005) for an explicit modeling of incomplete contracts, in the context of external economies of offshoring (but not in the context of unemployment).

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6 Appendix

6.1 Maximization problem of the firm in the no offshoring case

The firm maximizes (15) subject to (14), (9), and (13). Denoting the Lagrangian multiplier associated with (14) by λ , with (9) by ξ , and with (13) by ϕ , the current value Hamiltonian for each firm can be written as

$$H = p(\alpha)z(\alpha) - w(\alpha)N(\alpha) - c_z V(\alpha) + \lambda[q(\theta_z)V(\alpha) - \delta N(\alpha)] \\ + \xi\left[\frac{1}{\tau^\tau(1-\tau)^{1-\tau}}\alpha m_h(\alpha)^\tau m_p(\alpha)^{1-\tau} - z(\alpha)\right] + \phi[N(\alpha) - m_h(\alpha) - m_p(\alpha)]$$

The first order conditions for the above maximization are follows.

$$z(\alpha) : p(\alpha) + z(\alpha)(dp(\alpha)/dz(\alpha)) = \xi \quad (51)$$

$$m_h : \frac{\xi\tau\alpha m_h(\alpha)^{\tau-1}m_p(\alpha)^{1-\tau}}{\tau^\tau(1-\tau)^{1-\tau}} = \phi \quad (52)$$

$$m_p : \frac{\xi(1-\tau)\alpha m_h(\alpha)^\tau m_p(\alpha)^{-\tau}}{\tau^\tau(1-\tau)^{1-\tau}} = \phi \quad (53)$$

$$V(\alpha) : c_z = \lambda q(\theta_z) \quad (54)$$

$$N(\alpha) : w(\alpha) + \lambda\delta - \phi = \dot{\lambda} - r\lambda \quad (55)$$

Now, (52) and (53) imply

$$\frac{m_h(\alpha)}{m_p(\alpha)} = \frac{\tau}{1-\tau} \quad (56)$$

using the above in (52) gives

$$\xi = \frac{\phi}{\alpha} \quad (57)$$

Next, note from (54) that for a given θ_z , λ is constant. Using $\dot{\lambda} = 0$ in (55) we get

$$\phi - w(\alpha) = (r + \delta)\lambda \quad (58)$$

Denoting the elasticity of demand facing each producer by σ , from (51) (57) and (58) we get

$$\frac{\frac{\sigma-1}{\sigma}\alpha p(\alpha) - w(\alpha)}{(r + \delta)} = \lambda = \frac{c_z}{q(\theta_z)} \quad (59)$$

λ is the shadow value of an extra job.

6.2 Derivation of Expression for Present Discounted Value of Profits

The present discounted value of a firm's profit at time t is given by

$$\int_t^\infty e^{-r(s-t)} \{p_s(\alpha)z_s(\alpha) - w_s(\alpha)N_s(\alpha) - c_z V_s(\alpha)\} ds$$

Next, substitute out $V_s(\alpha)$ using $\dot{N}_s(\alpha) = q(\theta_z)V_s(\alpha) - \delta N_s(\alpha)$ to get

$$\int_t^\infty e^{-r(s-t)} \left\{ p_s(\alpha)z_s(\alpha) - \left(w_s(\alpha) + \frac{\delta c_z}{q(\theta_z)} \right) N_s(\alpha) - \frac{c_z}{q(\theta_z)} \dot{N}_s(\alpha) \right\} ds$$

We know that $w(\alpha) = w_z$. Also, if the industry is in steady-state at time t , then w_z, θ_z, P_z , and Z do not change over time and are taken as given by a firm. Since the economy is in steady state and so Z, θ and λ are fixed, and because adjustment costs are linear, a firm with employment N_t at time t will immediately jump to the steady state level of employment and output. Incorporating all this, after integrating the last term of the previous expression by parts, we obtain the present discounted value of a firm's profit at time t as

$$\int_t^\infty e^{-r(s-t)} \left\{ p(\alpha)z(\alpha) - \left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \right) N(\alpha) \right\} ds + \frac{c_z}{q(\theta_z)} N_t(\alpha)$$

where the variables without time subscripts denote steady state values. Next, use $N(\alpha) = \frac{z(\alpha)}{\alpha}$ and the first order condition given in (17) to write the present discounted value of profit of a firm with employment $N_t(\alpha)$ at time t as $\frac{z(\alpha)p(\alpha)}{r\sigma} + \frac{c_z}{q(\theta_z)} N_t(\alpha)$. Finally, using (12) the above can be written as

$$\frac{ZP_z^\sigma}{r\sigma^\sigma(\sigma-1)^{1-\sigma}} \left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \right)^{1-\sigma} \alpha^{\sigma-1} + \frac{c_z}{q(\theta_z)} N_t(\alpha)$$

6.3 Equations determining equilibrium in the offshoring case

$$P_x = (P_z)^{\frac{\gamma}{\gamma-1}} \tag{60}$$

$$w_x = (1-\beta)b + \beta[P_x + c_x\theta_x] \tag{61}$$

$$P_x = w_x + \frac{(r+\delta)c_x}{q(\theta_x)} \tag{62}$$

$$u_x = \frac{\delta}{\delta + \theta_x q(\theta_x)} \tag{63}$$

$$c_z\theta_z = c_x\theta_x \tag{64}$$

$$w_z = b + \frac{\beta c_z}{1-\beta} \left[\theta_z + \frac{r+\delta}{q(\theta_z)} \right] \tag{65}$$

$$u_z = \frac{\delta}{\delta + \theta_z q(\theta_z)} \quad (66)$$

$$P_z = \frac{\sigma}{\sigma - 1} \left(w_z + \frac{(r + \delta)c_z}{q(\theta_z)} \right) \left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \omega^{(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right]^{\frac{1}{1-\sigma}} \quad (67)$$

$$\Pi_V(\alpha^*) - \Pi_D(\alpha^*) = F_V \quad (68)$$

The above 9 equations correspond to equations (3), (26), (27), (28), (29), (24), (19), (37), respectively, in the text.

The total employment in the Z sector denoted by N_Z can be written as

$$N_Z = (1 - u_z)L_z = \int_{\underline{\alpha}}^1 N(\alpha) dG(\alpha) = \int_{\underline{\alpha}}^{\alpha^*} \frac{z(\alpha)}{\alpha} dG(\alpha) + \tau \omega^{\tau-1} \int_{\alpha^*}^{\alpha^*} \frac{z(\alpha)}{\alpha} dG(\alpha)$$

Using the demand functions in (6), the above can be written as

$$(1 - u_z)L_z = Z \left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \omega^{(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right]^{\frac{\sigma}{1-\sigma}} \left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \tau \omega^{(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right] \quad (69)$$

Next, $X = (1 - u_x)L_x$, and relative demand $\left(\frac{Z}{X}\right)$ is a function of P_z together imply

$$\begin{aligned} & \frac{(1 - u_z)L_z}{(1 - u_x)(L - L_z)} \\ &= \frac{\gamma (P_z)^{\frac{1}{\gamma-1}}}{(1 - \gamma)} \left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \omega^{(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right]^{\frac{\sigma}{1-\sigma}} \left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \tau \omega^{(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right] \end{aligned} \quad (70)$$

The 11 equations (60)-(68), (69) and (70) determine the following endogenous variables:

$$P_x, P_z, w_x, w_z, \theta_x, \theta_z, u_x, u_z, L_z, Z, \alpha^*.$$

6.4 Shape of Offshoring Curve

As mentioned in the text, for each α^* , $P_x, P_z, w_x, w_z, \theta_x, \theta_z$, are determined irrespective of the demand condition.

Recall from equation (39) in the text that the condition for a firm with productivity α^* to be indifferent between offshoring and not offshoring is

$$Z = \frac{F_V \alpha^{*1-\sigma}}{(P_z^*)^\sigma [C_V^* - C_D^*]} \equiv Z(\alpha^*) \quad (71)$$

Our aim is to find out what happens to the minimum required Z when α^* is lowered. As mentioned in the text, a decrease in α^* implies increases in $P_x, w_x, w_z, \theta_x, \theta_z$ and a decrease in P_z . Therefore, a sufficient, but by no means necessary, condition for $Z'(\alpha^*) < 0$ is that $C_V - C_D$ is increasing in α^* . Note that

$$C_V - C_D \equiv \frac{\left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)}\right)^{1-\sigma} (\omega^{(\sigma-1)(1-\tau)} - 1)}{r\sigma(\sigma-1)^{1-\sigma}}$$

Therefore,

$$\frac{d \log(C_V - C_D)}{d\alpha^*} = (1 - \sigma) \frac{d \log\left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)}\right)}{d\alpha^*} + \frac{d \log(\omega^{(\sigma-1)(1-\tau)} - 1)}{d\alpha^*}$$

which can be re-written as

$$\frac{d \log(C_V - C_D)}{d\alpha^*} = \frac{d \log\left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)}\right) (\sigma - 1)}{d\alpha^*} \left[\frac{1 - \tau \omega^{(\sigma-1)(1-\tau)}}{(\omega^{(\sigma-1)(1-\tau)} - 1)} \right]$$

Since w_z and θ_z are decreasing in α^* , $\frac{d \log\left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)}\right)}{d\alpha^*} < 0$, and hence, $\frac{d \log(C_V - C_D)}{d\alpha^*} > 0$ iff

$$\omega > \left(\frac{1}{\tau}\right)^{\frac{1}{(\sigma-1)(1-\tau)}} \quad (72)$$

It can be seen that the term on the rhs of (72) is decreasing in σ and τ . Therefore, (72) is easily satisfied for high σ and τ . Also, since w_z and θ_z are decreasing in α^* , a sufficient condition for $Z'(\alpha^*) < 0$ is $\omega^{NO} > \left(\frac{1}{\tau}\right)^{\frac{1}{(\sigma-1)(1-\tau)}}$. (Recall that $\omega^* > \omega^{NO} \forall \alpha^* \geq \underline{\alpha}$).

Since we want to plot the offshoring curve in $\left(\frac{Z}{X}, \frac{P_z}{P_x}\right)$ space, we want to establish that $Z'(\alpha^*) < 0$ implies $\frac{Z(\alpha^*)}{X(\alpha^*)}$ is also decreasing in α^* . Define $\psi(\alpha^*) \equiv \frac{Z(\alpha^*)}{X(\alpha^*)}$. Using (69) write the relationship between output and employment in the Z sector as

$$Z(\alpha^*) = A(\alpha^*)(1 - u_z(\alpha^*))L_z(\alpha^*) \quad (73)$$

where

$$A(\alpha^*) = \left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \omega^{(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right]^{\frac{\sigma}{\sigma-1}} \left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \tau \omega^{(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right]^{-1} \quad (74)$$

$Z'(\alpha^*) < 0$ implies $\frac{d \ln A(\alpha^*)}{d\alpha^*} + \frac{d \ln(1 - u_z(\alpha^*))}{d\alpha^*} + \frac{d \ln L_z(\alpha^*)}{d\alpha^*} < 0$. Since $\frac{d \ln(1 - u_z(\alpha^*))}{d\alpha^*}$ is, relative to the other terms, a second-order effect, we take $\frac{d \ln(1 - u_z(\alpha^*))}{d\alpha^*} \approx 0$, and so effectively, $Z'(\alpha^*) < 0$ implies $-\frac{d \ln A(\alpha^*)}{d\alpha^*} > \frac{d \ln L_z}{d\alpha^*}$.

Next, $\psi(\alpha^*) = \frac{A(\alpha^*)L_z(\alpha^*)}{L - L_z(\alpha^*)}$ if $c_x = c_z$. Therefore, we have

$$\frac{d \ln \psi(\alpha^*)}{d\alpha^*} = \frac{d \ln A(\alpha^*)}{d\alpha^*} + \frac{d \ln L_z(\alpha^*)}{d\alpha^*} + \frac{L_z(\alpha^*)}{L - L_z(\alpha^*)} \frac{d \ln L_z(\alpha^*)}{d\alpha^*} = \frac{d \ln A(\alpha^*)}{d\alpha^*} + \frac{1}{l_x} \frac{d \ln L_z(\alpha^*)}{d\alpha^*} \quad (75)$$

where $l_x = (L - L_z(\alpha^*)) / L$. Therefore, $\frac{d \ln \psi(\alpha^*)}{d\alpha^*} < 0$ when $\frac{1}{l_x} \frac{d \ln L_z(\alpha^*)}{d\alpha^*} < -\frac{d \ln A(\alpha^*)}{d\alpha^*}$. We discuss two possible cases.

Case I: $\frac{d \ln L_z(\alpha^*)}{d\alpha^*} < 0$. In this case $\frac{d \ln L_z(\alpha^*)}{d\alpha^*} > \frac{1}{l_x} \frac{d \ln L_z(\alpha^*)}{d\alpha^*}$. Therefore, $Z'(\alpha^*) < 0$ implies $\frac{1}{l_x} \frac{d \ln L_z(\alpha^*)}{d\alpha^*} < -\frac{d \ln A(\alpha^*)}{d\alpha^*}$.

Case II: $\frac{d \ln L_z(\alpha^*)}{d\alpha^*} > 0$. In this case $Z'(\alpha^*) < 0$ implies $-\frac{d \ln A(\alpha^*)}{d\alpha^*} > 0$. So, for $\frac{d \ln \psi(\alpha^*)}{d\alpha^*} < 0$ we need $l_x > \left(\frac{d \ln L_z(\alpha^*)}{d\alpha^*} \right) / \left(-\frac{d \ln A(\alpha^*)}{d\alpha^*} \right)$. [Note that $\frac{d \ln L_z(\alpha^*)}{d\alpha^*} \approx \frac{d \ln Z(\alpha^*)}{d\alpha^*} - \frac{d \ln A(\alpha^*)}{d\alpha^*} < -\frac{d \ln A(\alpha^*)}{d\alpha^*}$ here and so $0 < \left(\frac{d \ln L_z(\alpha^*)}{d\alpha^*} \right) / \left(-\frac{d \ln A(\alpha^*)}{d\alpha^*} \right) < 1$, as is $0 < l_x < 1$.]

We find no indication of the violation of the inequality restriction, $\frac{1}{l_x} \frac{d \ln L_z(\alpha^*)}{d\alpha^*} < -\frac{d \ln A(\alpha^*)}{d\alpha^*}$ from the several numerical examples, constructed with reasonable parameter values, that show that whenever $Z'(\alpha^*) < 0$, we have $\psi'(\alpha^*) < 0$ as well.

6.5 Sectoral reallocation of labor after offshoring

Claim: $c_x = c_z$ implies that the labor force in the Z sector is smaller in an offshoring equilibrium than under a no-offshoring equilibrium.

Proof: $c_x = c_z$ implies $u_x = u_z$ and

$$P_x = w_z + \frac{(r + \delta)c_z}{q(\theta_z)} \quad (76)$$

The relative demand for Z equal to relative supply in an offshoring equilibrium given in (70) above can be re-written as

$$\frac{L_z}{(L - L_z)} = \frac{\gamma(1 - u_x)(P_z)^{\frac{1}{\gamma-1}}}{(1 - u_z)(1 - \gamma)} [A(\alpha^*)]^{-1} \quad (77)$$

where $A(\alpha^*)$ is defined in (74). Let L_z^O denote the size of the labor force in the Z sector in an offshoring equilibrium and L_z^{NO} the size of the labor force in the Z sector in the no-offshoring equilibrium. The expression for L_z^{NO} is obtained by setting $\alpha^* = \bar{\alpha}$ in the above expression. $\alpha^* = \underline{\alpha}$ will capture the complete offshoring case. From equation (77) above we see that $L_z^O < L_z^{NO}$ requires

$$[A(\alpha^*)]^{-1} \tilde{\alpha} < \left(\frac{P_z}{P_z^{NO}} \right)^{\frac{1}{1-\gamma}} \quad (78)$$

Next, using (76) the expression for P_z given in (67) above can be written as

$$P_z = \frac{\sigma}{\sigma-1} P_x \left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \omega^{(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right]^{\frac{1}{1-\sigma}}$$

Also, from (60) $P_x = (P_z)^{\frac{\gamma}{\gamma-1}}$. Therefore,

$$(P_z)^{\frac{1}{1-\gamma}} = \frac{\sigma}{\sigma-1} \left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \omega^{(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right]^{\frac{1}{1-\sigma}} \quad (79)$$

Similarly,

$$(P_z^{NO})^{\frac{1}{1-\gamma}} = \left(\frac{\sigma}{\sigma-1} \tilde{\alpha}^{-1} \right) \quad (80)$$

Using (79) and (80) on the rhs of (78) and cancelling terms we get

$$\tau < 1$$

Therefore, $L_z^O < L_z^{NO}$ for any $\tau < 1$. *QED*

6.6 Free Entry Case: Condition for $\varphi(M)$ to be decreasing in M

Write the expression for P_z as follows.

$$P_z = \frac{\sigma}{\sigma-1} M^{\frac{1}{1-\sigma}} \left(w_z + \frac{(r+\delta)c_z}{q(\theta_z)} \right) \left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \omega^{(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right]^{\frac{1}{1-\sigma}}$$

Note that, for $c_x = c_z$ case $P_x = w_z + \frac{(r+\delta)c_z}{q(\theta_z)}$. Substituting this in the above expression and noting that $P_z = (P_x)^{\frac{\gamma-1}{\gamma}}$, we can write the above as

$$(P_x)^{\frac{-1}{\gamma}} = \frac{\sigma}{\sigma-1} M^{\frac{1}{1-\sigma}} \left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \frac{P_x^{(\sigma-1)(1-\tau)}}{w_s^{(\sigma-1)(1-\tau)}} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right]^{\frac{1}{1-\sigma}}$$

The above implies that

$$\left[1 - \frac{\gamma(1-\tau)\omega^{(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha)}{\left[\int_{\underline{\alpha}}^{\alpha^*} \alpha^{\sigma-1} dG(\alpha) + \omega^{(\sigma-1)(1-\tau)} \int_{\alpha^*}^{\bar{\alpha}} \alpha^{\sigma-1} dG(\alpha) \right]} \right] \frac{d \log P_x}{d \log M} = \frac{\gamma}{\sigma-1}$$

In the case of autarky, $\alpha^* \geq \bar{\alpha}$, therefore, $\frac{d \log P_x}{d \log M} = \frac{\gamma}{\sigma-1} < 1$. In the complete offshoring case, $\alpha^* = \underline{\alpha}$, therefore, the above implies

$$\frac{d \log P_x}{d \log M} = \frac{\gamma}{(\sigma-1)(1-\gamma(1-\tau))}$$

Thus, $\frac{d \log P_x}{d \log M} < 1$ as long as $\frac{\gamma}{(\sigma-1)(1-\gamma(1-\tau))} < 1$ or $(\sigma-1) > \frac{\gamma}{(1-\gamma(1-\tau))}$. Note from the above that for any $\alpha^* \in (\underline{\alpha}, \bar{\alpha})$ the condition $(\sigma-1) > \frac{\gamma}{(1-\gamma(1-\tau))}$ is sufficient for $\frac{d \log P_x}{d \log M} < 1$.

6.7 Parameters for numerical examples

$\sigma = 3.8$ and $\alpha \in \text{Pareto}[.2, 3.4]$ are taken from Bernard, Eaton, Jensen and Kortum (2003). $\delta = .035$ corresponds to the monthly average job destruction rate in the United States. The time preference parameter, $r = .03$, is in the range of values commonly used in Macro literature. $m = kv^\phi u^{1-\phi}$ is the standard matching function used in the literature and our choice of $\phi = .5$ is in the range of the estimate of this matching parameter in Blanchard and Diamond (1989) where it ranges from .43 to .75. $k = .25$ was chosen to get reasonable values of unemployment rates. The Nash bargaining parameter $\beta = .5$ is the most commonly used in the literature. In the absence of any clear guidance for other parameters we tried many different values and found the results to be robust to their alternative values. We report the results obtained using some specific values.

Figure 1: Equilibrium without offshoring

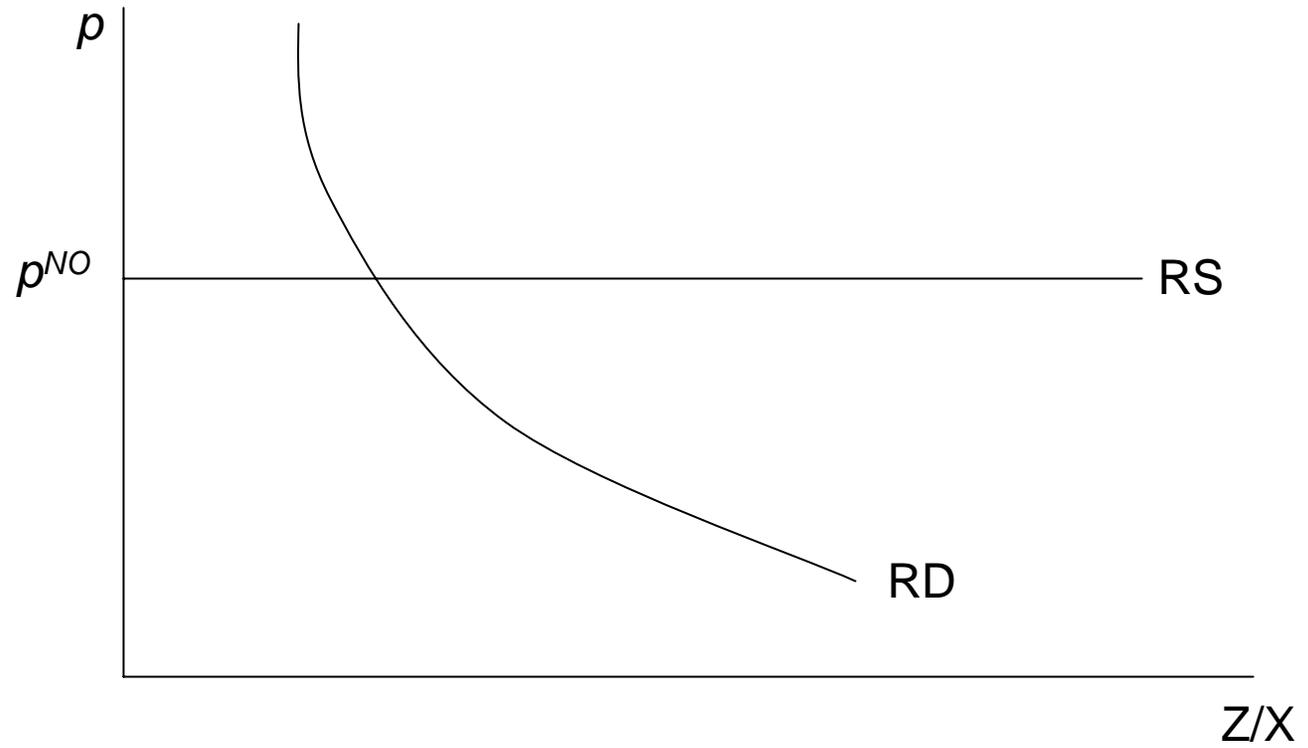


Figure 2: Equilibria with the possibility of Offshoring

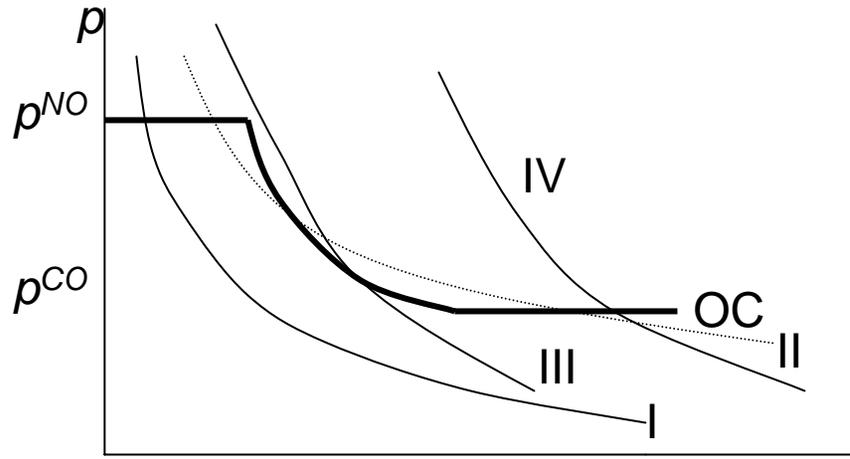


Figure 2a

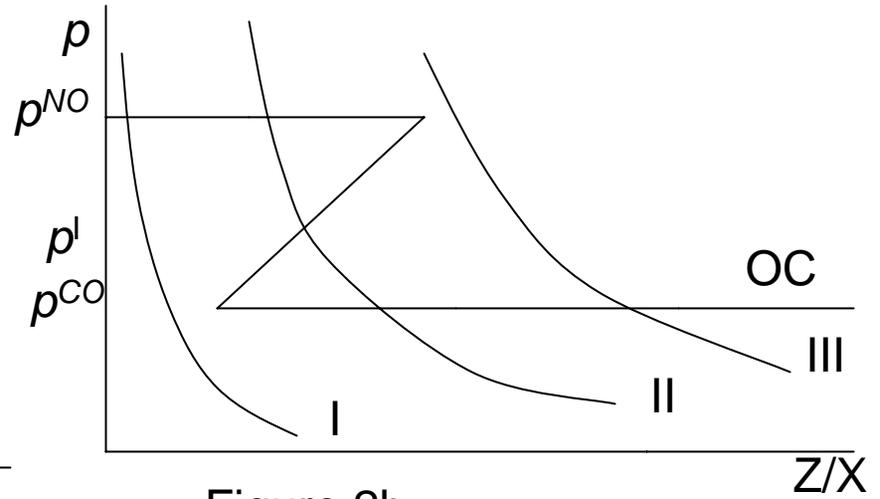


Figure 2b

I, II, III, and IV are the 4 possible positions of the RD curve

Figure 3: Comparative Statics

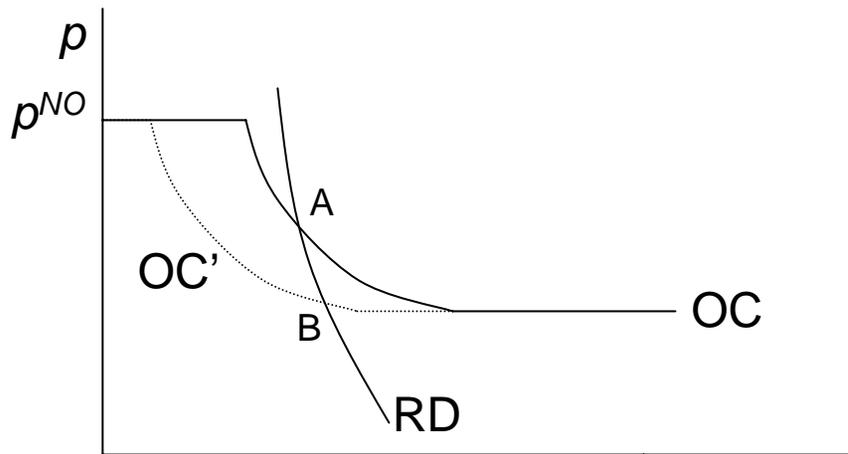


Figure 3a: Decrease in $F_V^{Z/X}$

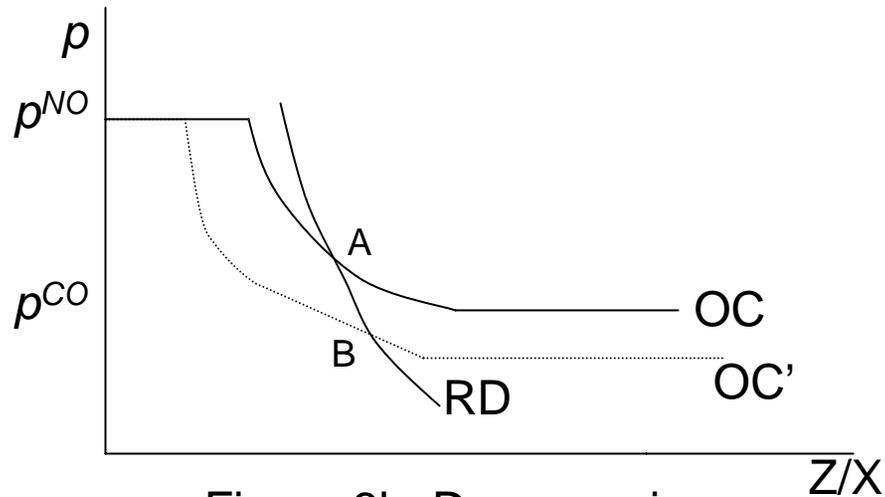


Figure 3b: Decrease in w_s

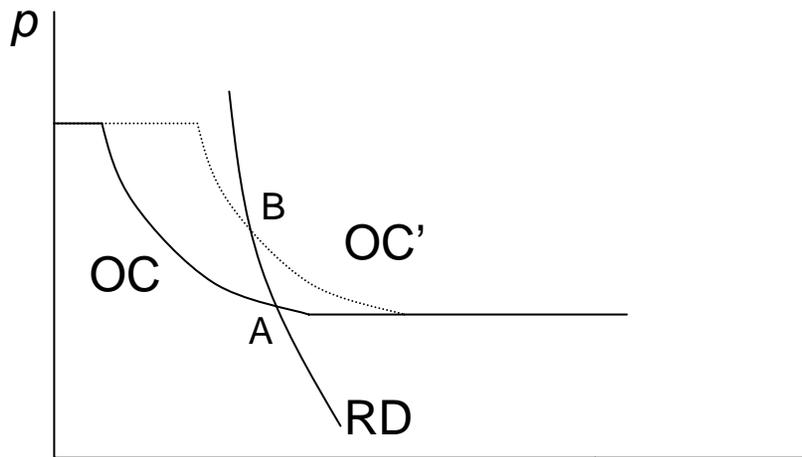


Figure 3c: Increase in b

Figure 4: Offshoring Equilibrium With No Intersectoral Labor Mobility

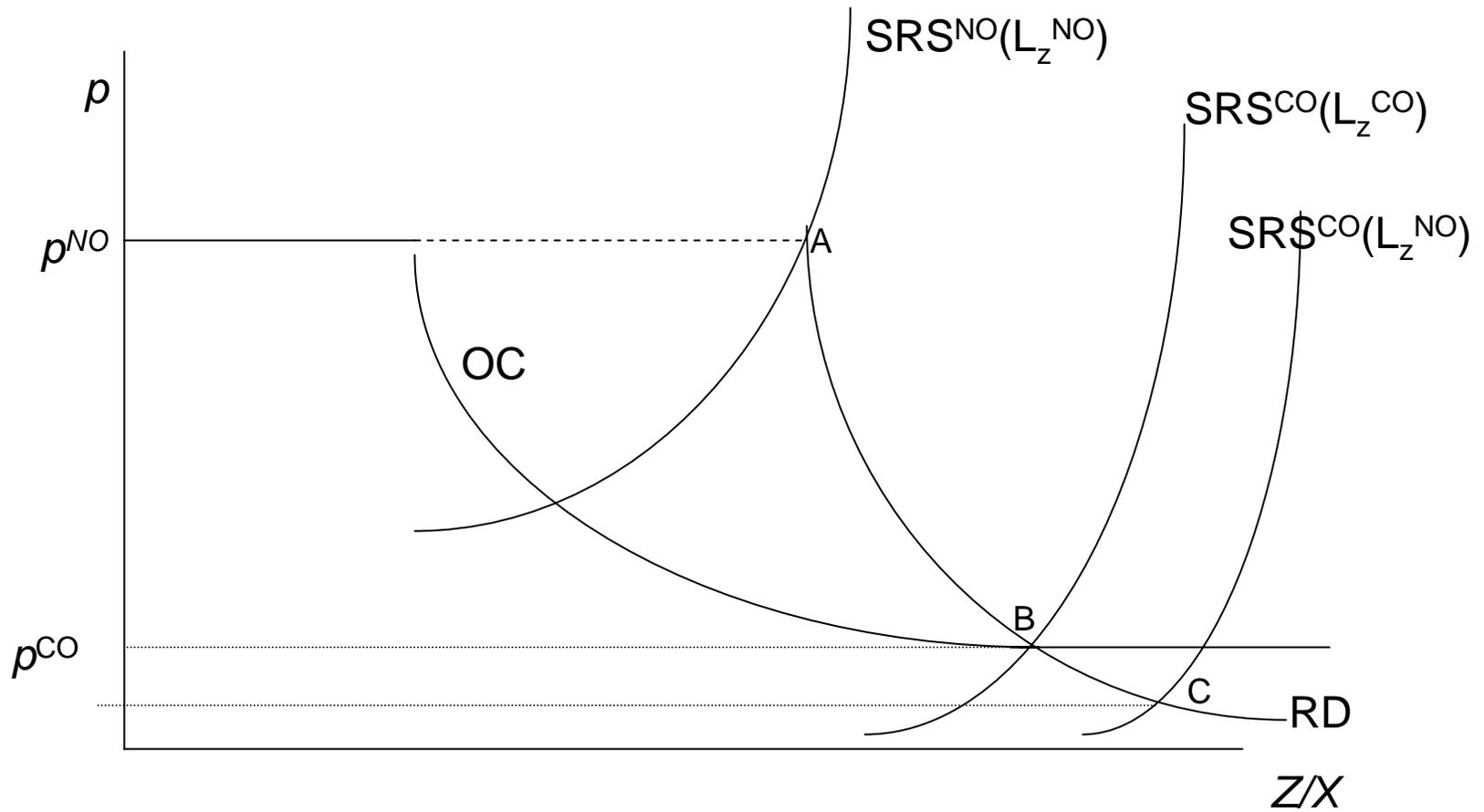


Figure 5: Equilibrium number of firms

