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PECUNIARY AND MARKET MEDIATED EXTERNALITIES:  
TOWARDS A GENERAL THEORY OF THE WELFARE  
ECONOMICS OF ECONOMIES WITH IMPERFECT  
INFORMATION AND INCOMPLETE MARKETS

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ABSTRACT

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This paper presents a simple but quite general framework for analyzing the impact of informational externalities. By identifying the traditional pecuniary effect of these externalities which nets out, the paper greatly simplifies the problem of determining when tax interventions can be Pareto improving. **In** some cases it also leads to simple tests, based on readily observable indicators of the efficacy of a particular tax policy. The framework of the paper is used to analyze adverse selection, **signalling**, moral hazard, incomplete contingent claim markets and queue rationing equilibria.

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**Pecuniary and Market Mediated Externalities:**

**Towards a General Theory of the Welfare  
Economics of Economies with Imperfect  
Information and Incomplete Markets**

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A significant portion of the recent literature in public finance has been devoted to the analysis of newly discovered externalities in markets with imperfect information. Signalling (see Spence (1973)), search (see Diamond and Maskin (1979), Mortenson (1979)), and insurance (see Shavell (1979)) dated externalities are all cases in point. In each instance the papers involved attempt first to establish the existence of a "real" externality and then to develop a prescription for welfare improving government interventions. In most of the literature neither of these steps is straightforward. Yet there is a framework, developed in this paper, which readily identifies "real" externalities (i.e. those leading to inefficient market allocations) and provides simple prescriptions, based on observable indicators, for the appropriate direction of government intervention.

Concern with whether information based externalities are "real" seems to be rooted in the long, but now almost dormant, tradition of analyzing the social impact of an individual's actions in terms of pecuniary and non-pecuniary externalities; with the latter being "real" externalities which justify government intervention. The zero net impact of pecuniary externalities (i.e. the consequences of an individual's impact on others through his impact on market prices) is often ascribed, if only

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informally, to the fact that as the number of agents in an economy becomes large, the effect of a single individual on market determined variables, like prices, becomes negligible. Consequently, these effects can be ignored in making welfare calculations. The implicit concern in the literature seems to be that, in atomistically competitive economies, other *market-mediated externalities* may be negligible for similar reasons (hence the care with which the literature examines limiting cases with large numbers of agents). However, as this paper makes clear, pecuniary externalities in the traditional sense are a very special case and market-mediated externalities are in general entirely analogous to traditional externalities (e.g. air pollution).

There are, however, pecuniary effects which arise from any government intervention and these do not net out in welfare calculations for atomistically competitive economies. The complexity of many analyses of informational externalities is often attributable to a failure to identify and eliminate these terms.<sup>1</sup> Doing so within the framework developed in this paper provides an approach which substantially simplifies the analysis of the appropriate nature of government intervention in imperfectly informed market economies.<sup>2</sup>

The paper consists of two sections and a brief conclusion. Section I develops the basic framework and analysis. Section II then uses this formulation to investigate a broad range of welfare situations in which market-mediated externalities play an important role. It analyses the local welfare implications of adverse selection, signalling, moral hazard, incomplete markets and queue-rationing equilibria.

## I. The Basic Model and Results

The agents in the model consist of households, firms, and a government with the following characteristics.

1. See either the literature on search externalities - e.g. Mortenson (1979) or Diamond and Maskin (1979) - or that on incomplete securities markets - e.g. Diamond (1967) or Loong and Zeckhauser (1981). In general, the complexity of these analyses arises because the authors concentrate on comparing relevant marginal rates of substitution; a framework, which makes it difficult to identify the "transfer" or pecuniary externalities which net out.
2. An important aspect of this approach is that it identifies externalities with those who bear the burden rather than those generating the effects. In practice this makes it much easier to identify when an externality is indeed non-pecuniary.

### A. Households

Households maximize a utility function,

$$u^h(x^h, z^h), h = 1, \dots, H,$$

where,

$x^h = (x_1^h, \bar{x}^h)$  = Consumption vector of household  $h$ ,  $x_1^h$  is consumption of the numeraire good,  $\bar{x}^h = (x_2^h, \dots, x_N^h)$  is consumption of the  $N-1$  non-numeraire goods,

$z^h$  = vector of  $N^h$  other variables which affect the utility of household  $h$  (e.g. levels of pollution, average quality of a good consumed).

Households maximize  $u^h$  subject to a budget constraint of the form.

$$x_1^h + q \cdot \bar{x}^h \leq I^h + \sum_F a^{hf} \cdot \pi^f,$$

taking  $q, \pi^f, I^h, a^{hf}$  and  $z^h$  as fixed,

where

$q$  = a vector of prices of the  $N-1$  non-numeraire goods,

$\pi^f$  = profits of firm  $f$ ,

$a^{hf}$  = fractional holding of household  $h$  in firm  $f$ ,  $\sum_H a^{hf} = 1$ ,

$I^h$  = a lump sum government transfer to household  $h$ ,  $I = (I^1, \dots, I^H)$ ,

We will also use,

$E^h(q, z^h, u^h)$  = the expenditure function of household  $h$  which gives the minimum expenditure necessary to obtain a level of utility  $u^h$ , when prices are  $q$  and  $z^h$  is level of "other" variables.

It is well-known that,

$$\begin{aligned} x_k^h(q; z^h, u^h) &= \text{the compensated demand for good } k \text{ given } z^h \text{ and } u^h \text{ fixed} \\ &= \frac{\partial E^h}{\partial q} \Big|_{z^h, u^h} \end{aligned} \quad (1)$$

Finally,

$x^h(q, I) = (x_1^h(q, I), \bar{x}^h(q, I))$  = the demand function (uncompensated) of household  $h$ .<sup>3</sup>

We will assume that this function is differentiable.<sup>4</sup>

### B. Firms

Firms maximize the profit function,

$$\pi^f = y_1^f + p \cdot \bar{y}^f,$$

where,

$y^f = (y_1^f, \bar{y}^f)$  = Production vector<sup>5</sup> of firm  $f$  with  $y_1^f$  and  $\bar{y}^f$  defined analogously to  $x_1^h$  and  $\bar{x}^h$ ,

$p$  = Vector of producers' prices for the  $N - 1$  non-numeraire goods.

Firms maximize profits subject to the constraint that,

$$y_1^f - G^f(\bar{y}^f, z^f) \geq 0,$$

where,

$G^f$  = a production function of the usual sort,

$z^f$  = vector of other  $N^f$  variables affecting firm  $f$  analogously defined to  $z^h$ .

The firm's maximum profit function,

$$\pi_0^f(p, z^f),$$

has the property that,

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3. The household demand function depends on the entire vector of transfers because both  $z^h$  and  $z^1, \dots, z^F$  (which determine  $\pi^1, \dots, \pi^F$  and hence household income) may depend on the consumption choices of other households. In a pure exchange economy.

$x^h(q, I) = x^h(q, I^h; z^h(q, I))$  where  $x^h(q, I^h; z^h)$  is the usual demand function treating  $z^h$  as fixed. Also, for the sake of expositional simplicity, household factor endowments have been arbitrarily set to zero. This has no substantive impact on the analysis.

4. The problem of justifying this kind of differentiability assumption is examined in detail by Starrett (1980) who makes a similar assumption in a slightly different context. The difficulty here is that the usual convexity assumptions of preferences and production functions will not guarantee differentiability. The external effects may create discontinuities. The "excess demand" functions used here include the effect of prices on quantities both directly and indirectly via their impact on externality generating activities (i.e. through their impact of  $z^f$  and  $z^h$ ) which in turn, affect consumption and production choices.

5.  $y_1^f < 0$  represents an input.

$$\frac{\partial \pi_f^k}{\partial p_k} \Big|_{z_f} = y_k^f, k = 1, \dots, N \quad (2)$$

where  $y_k^f$  here denotes the profit maximizing level of the production variable in question.

Finally,

$$y^f(p, I) = (y_1^f(p, I), \bar{y}^f(p, I)) = \text{supply function (uncompensated) of firm } f.$$

We will assume that this function like the demand function is differentiable.

### C. Government

The government produces nothing, collects taxes, distributes the proceeds and receives a net income,

$$R = t \cdot \bar{x} - \sum_H I^h,$$

where,

$$t = (q - p),$$

$$\bar{x} = \sum_H \bar{x}^h \text{ (i.e. the sum of non-numeraire consumption).}$$

An initial equilibrium with no taxes and  $I^h = 0, \forall h$ , will be assumed to exist.<sup>6</sup> At this equilibrium,  $p = q$  and,

$$\bar{x}(q, I) - \sum_F \bar{y}^f(p, I) = 0.$$

A simple test of the Pareto Optimality of this equilibrium is to ask whether there exists a set of taxes, subsidies and lump sum transfer which would (a) leave household utilities unchanged and (b) increase government revenues (assumed to be consumed in numeraire good). This, in turn, implies

6. As described so far the model may not, of course, have an equilibrium price vector. However, having noted that possibility, it is still worth investigating the welfare implications of any equilibria which may exist. The case for this is made fully and compellingly by Starrett (1980). We will also ignore the problem of free goods. Accounting for them would merely complicate the analysis without altering any basic results.

that the problem,

$$\max_{t, I} R = t \cdot \bar{x} - \sum_h I^h \quad (3)$$

subject to

$$I^h + \sum a^{hf} \pi^f = E^h(q, z^h; \bar{u}^h) \quad (4)$$

where,

$\bar{u}^h$  = competitive equilibrium utility levels,

and  $z^h, z^f, \pi^f, p$  end9 are functions of  $t$  and  $I$ , has a solution at  $t = 0$ , if the initial equilibrium is Pareto Optimal. (This is a necessary, but not sufficient, condition for Pareto Optimality.)

To see when this last condition holds, note that, along the constraint of equation (4),

$$\frac{dI^h}{dt} + \sum_{f \in F} a^{hf} \left( \pi_z^f \frac{dz^f}{dt} + \pi_p^f \frac{dp}{dt} \right) = E_q^h \frac{dq}{dt} + E_z^h \cdot \frac{dz^h}{dt}, \quad (5)$$

where,

$\frac{dI^h}{dt}$  = change in lump sum income per unit change in tax.<sup>7</sup>

$\pi_z^f = \left[ \frac{\partial \pi^f}{\partial z^f} \right] = \frac{\partial G^f}{\partial z^f}$ , an  $N^f$ -element vector,

$E_z^h = \left[ \frac{\partial E^h}{\partial z^h} \right] = \left[ \frac{\partial u^h}{\partial z^h} \right]$ , (with  $u^h$  suitably normalized), an  $N^h$ -element vector.

But,  $dq/dt = I_{N-1} + dp/dt$  (here  $I_{N-1}$  is an identity matrix). Therefore, substituting into (4)

and rearranging terms yields,

7. This  $N-1$  element vector is not a conventional derivative, since there is no functional relationship between  $t$  and  $I$ . Also, there is an important simplification which underlies equation (5). The distributions,  $I_1, \dots, I_H$  and, hence, changes in those distributions are assumed to have no impact on  $z^h$  for any household or  $z^f$  for any firm. Including the impact of balancing income distributions merely adds an income induced externality term to the tax induced term derived below. Since this would greatly complicate the notation without altering the basic nature of the results, these terms have been (admittedly somewhat arbitrarily) ignored. Alternatively, all the derivatives of  $z$  (and  $p$  and  $q$ ) with respect to  $t$  can be viewed as total derivatives, taking into account the associated changes in  $I_1, \dots, I_H$  as well as the direct effect of  $t$ .

$$E_q^h + (E_q^h - \sum_f a^{hf} \cdot \pi_p^f) \frac{dp}{dt} = \frac{dI^h}{dt} + \sum_F a^{hf} \pi_z^f \frac{dz^f}{dt} - E_z^h \frac{dz^h}{dt} \quad (6)$$

Next, substitution from equations (1) and (2), summation over all households and use of the fact that  $\sum_h a^{hf} = 1$  yields,

$$\bar{x} + (\bar{x} - \bar{y}) \frac{dp}{dt} = \sum_H \frac{dI^h}{dt} + \left( \sum_F \pi_z^f \frac{dz^f}{dt} - \sum_H E_z^h \frac{dz^h}{dt} \right),$$

where  $\bar{y} = \sum_F y^f = \bar{x}$  in any market equilibrium. Therefore, along the constraint (4),

$$\sum_H \frac{dI^h}{dt} = \bar{x} - \left( \sum_F \pi_z^f \frac{dz^f}{dt} - \sum_H E_z^h \frac{dz^h}{dt} \right), \quad (7)$$

where the "pecuniary" impact of the initial tax change,  $(\bar{x} - \bar{y}) dp/dt$ , has been netted out.

Now, differentiating the objective function (3) with respect to  $t$ , we obtain,

$$\frac{dR}{dt} = \bar{x} + \frac{d\bar{x}}{dt} \cdot t - \sum_H \frac{dI^h}{dt}. \quad (8)$$

Substitution from (7) into (8) yields,

$$\frac{dR}{dt} = \frac{d\bar{x}}{dt} \cdot t + (\Pi^t - B^t), \quad (9)$$

where

$$\Pi^t = \sum_F \pi_z^f \frac{dz^f}{dt}, \quad (10)$$

$$B^t = \sum_H E_z^h \frac{dz^h}{dt}, \quad (11)$$

which is the derivative of  $R$  along directions in which the compensational constraint is satisfied. This can be used as a measure of the net change in welfare.

For the initial equilibrium to be P.O.,  $dR/dt$  must equal zero at  $t=0$ , which implies that,

$$\frac{dR}{dt} = (\Pi' - B') = 0. \quad (12)$$

Thus, Pareto Optimality depends on the absence of any  $z$ 's which change with taxes and affect either profits or household utilities.

The defining characteristic of externalities, which (in traditional language) are 'non-pecuniary' and, therefore, justify some form of government intervention, is that they enter utility or profit functions in the form of the  $z$ -variables. The variables involved may be determined by the market interactions of agents (e.g. average product qualities, search times, average levels of unobservable effort  $a$ , with incomplete markets, future prices), and individual agents may have only a negligible impact on their levels. Nevertheless the resulting equilibrium will be Pareto inefficient. As an economy becomes increasingly atomistic, the reduced impact of each individual agent on the level of the "external" variables is just counterbalanced by the increasing number of others affected. This is shown rigorously in Appendix I. Except in the special case (which is unlikely to hold generically) where  $\Pi'$  and  $B'$  exactly cancel each other out, the existence of non-pecuniary externalities will make the initial equilibrium inefficient and guarantee the existence of welfare improving tax measures.

Furthermore, equation (9) provides a simple set of necessary conditions characterizing the optimal level of taxes in the presence of externalities. Since  $dR/dt = 0$  is necessary for optimality, optimal tax levels have the property that,

$$t \cdot \frac{d\bar{x}}{dt} = \Pi' - B' \quad (13)$$

Taxes should be increased until the "constant-rate" loss in tax revenue is exactly equal to the marginal value of the abatement in externality generating activities, where "constant-rate" changes in taxes revenue are the changes in revenue that would have occurred at the existing tax rates.<sup>8</sup>

8. A simple example may help clarify how straightforwardly this rule can be applied in practice. A tax on auto travel that reduces accidents will always be initially beneficial. However, successive tax increases will reduce the revenue yielded by the tax (at constant rates). The tax should be increased until this loss in revenue exactly balances the marginal benefits of reductions in external accident costs.

The remainder of this paper is devoted to applying equation (12) in a variety of familiar welfare situations.

## II. Applications

As noted in the introduction, the most fruitful area for applying equation (12) is in the field of informational externalities. There are several reasons for this. First, informational externalities are almost invariably *market-mediated* in the sense that they affect allocations indirectly through their impact on market equilibria (e.g. by affecting the average quality of a heterogeneous, but observationally identical, commodity which enters the market place) rather than directly through the impact of one agent on another. Therefore, distinguishing market mediated externalities from traditionally pecuniary ones often helps clarify what is at issue. Second, the impact of informational externalities often includes complicated transfers which arise from changes in market equilibria. Ignoring these "pecuniary" effects where they net out greatly simplifies the analysis. Third, the welfare analysis of informational equilibria has been greatly complicated by the variety of informational structures that are assumed. Constrained Pareto optima of many different sorts have been and continue to be defined and discovered. It seems useful, therefore, to ask a simpler question; namely, for a given information structure, are there simple tax interventions which will improve welfare in the compensation sense used here? Focusing on compensated changes in government revenue (i.e.  $dR/dt$ ) allows this kind of question to be answered relatively easily and produces policy prescriptions which can often be based on changes in observable market quantities.

In this section of the paper, simple models of market intervention are developed for adverse selection, signalling, insurance, queue rationing and incomplete risk market equilibria.

### A. Adverse Selection

The simplest case in which the analysis can be applied is to markets with asymmetrically distributed information and heterogeneous quality. The basic model for these situations was developed by Akerlof (1970). To simplify even further we will assume that there is only a single such commodity and that there are no other externalities. Buyers will be assumed to draw randomly

from the market in which the commodity in question is offered for sale, Sellers know the quality of what they are selling. Buyers know only the average quality in the market as a whole. We will assume, in addition, that buyers are perfectly informed about and care only about the average quality of what they buy. Realistically buyers may also care about the range of possible qualities, but taking this into account would change the analysis only in obvious ways and would greatly increase its complexity.<sup>9</sup> Let  $\theta$  denote the quality of each unit of the heterogeneous commodity, and  $\bar{\theta}$  denote the average quality in the market place.

The real situation corresponding most closely to this simple model is a labor market in which firms hire blindly from a pool of workers of heterogeneous quality.

In terms of the model of this paper, the  $z^h$  vectors will consist of a single element which is equal to  $\bar{\theta}$  (although households which do not purchase the commodity may have  $du^h/dz^h=0$ ). Similarly,  $z^f$  for all firms will have a single element equal to  $\bar{\theta}$ . Formally,

$$E^h = E^h(q; \bar{\theta})$$

and

$$\pi^f = \pi^f(p, \bar{\theta}).$$

Under these circumstances, equation (12) for a small tax,  $dt$ , becomes,

$$\frac{dR}{dt} = \left( \sum_F \pi_{\theta}^f - \sum_H E_{\theta}^h \right) \cdot \frac{d\bar{\theta}}{dt} \quad (14)$$

Since  $\pi^f$  increases and  $E^h$  decreases with  $\bar{\theta}$ , this means that any intervention which increases average quality in the market place is beneficial. Thus, any small tax which increases the quality of the heterogeneous commodity is always beneficial.<sup>9a</sup>

9. This simple model applies equally well to a situation in which buyers purchase only a limited number of items and care about the individual qualities of each. In that case ex ante expected utility (the appropriate welfare measure) will depend on the mean and spread of the distribution of "quality" in the market pool.

9a. A question that arises here is whether agents, observing the dependence of quality on price, will behave in the manner described here. The answer, in general, is no. (See Stiglitz-Weiss, (1981), Stiglitz, (1976)). But there are circumstances in which they will, e.g. when labor is engaged at a union hiring hall, in which there are a large number of employers. Then the supply of laborers will be essentially unaffected by any single firm. Hence, a firm will have no incentive to pay a wage in excess of the market wage, and cannot obtain any workers at a lower wage.

What is surprising about this result is its simplicity. The fact that an increase in  $\bar{\theta}$  involves the sale of higher quality inputs by some households suggests the need for a careful balancing of the increased cost of these sales by owner households against the benefits to purchasers. Yet no such calculation is implied by equation (14). The necessary balancing of the costs of selling higher quality items is being done by owner households in the process of maximizing utility. Changes in  $\bar{\theta}$  which result from changes in  $x^h$  by owner households (in response to changes in the market price) are part of the *pecuniary* impact of the original tax change. As such, they net out. This accounts for the simple form of the final policy prescription.

A typical example of tax changes leading to changes in average ability arises where different ability groups have different labor supply elasticities. If high ability workers have greater supply elasticities than low ability workers, a small proportionate wage subsidy will increase average quality.

Finally, it should be noted that there is, at least in principle, an observable basis for judging the effectiveness of government tax policy. Assuming that the average "quality" of labor entering a particular market can be monitored (short of determining the "Quality" of each individual worker) by, for example, taking a statistical sample, any policy of "small" taxes which increase this quality is a beneficial one<sup>10</sup>

### B. Signalling/ Screening

The question of whether a tax has beneficial or harmful effects on a signalling/screening equilibrium<sup>11</sup> can be addressed in a similar way. The principal difference is that signalling

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10. A question that naturally arises at this juncture is whether the compensations required by (4) can actually be carried out given the information available to the government. The answer depends, not unnaturally, on what the government knows and the extent to which lump sum taxes are available. If the government is restricted to commodity taxes and a uniform lump sum tax and knows the characteristics of each of  $M$  classes of consumers (but not the class to which any particular individual belongs) then Pareto improving commodity taxes will in general, exist as long as the number of taxable commodities strictly exceeds  $M$  (i.e.  $N > M$ ). Let the government restrict itself to tax changes which keep each class of consumers, except the first, at a given level of utility. As a rule, this will require  $M-1$  taxes (one for each group except the first). Then let the government change the tax on a further commodity (say good 1), making simultaneous changes in the  $M-1$  other taxes to keep the classes of consumers at all their given level of utility. If the original equilibrium is not P.O., then in general a composite tax change of this kind will exist which raises revenue. This requires  $M$  taxes and, therefore,  $M+1$  commodities.

11. Signalling/screening equilibria involve situations in which information is conveyed actively by sellers and/or sought actively by buyers. The traditional model of this kind of equilibrium as developed by Spence (1973), Stiglitz (1975) or Rothschild and Stiglitz (1977) is actually cast in slightly different terms than that presented here. In most familiar versions of the model firms offer price-quantity contracts subject to self-selection constraints. The difficulty with using

equilibria involve, even in the simplest case, two separate quality indices, one for those who acquire a signal and a second for those who do not. Let these be denoted by  $\bar{\theta}_1$  and  $\bar{\theta}_2$  respectively and assume that only firms are affected by their levels. Since signals are costly and wages must, therefore, depend positively on signals, we will assume that  $\bar{\theta}_1 > \bar{\theta}_2$ .

Applying equation (12), the net impact of a small tax  $dt$  is,

$$\frac{dR}{dt} = \sum_i \sum_F \frac{\partial \pi^f}{\partial \bar{\theta}_i} \cdot \frac{d\bar{\theta}_i^f}{dt} \quad (15)$$

Assuming that firms draw at random from the pools of workers with and without signals and that the number of firms is large, this reduces approximately to,

$$\frac{dR}{dt} = \sum_i \frac{\partial \bar{\theta}_i}{\partial t} \left( \sum_F \frac{\partial \pi^f}{\partial \bar{\theta}_i} \right) \quad (16)$$

Since  $\partial \pi^f / \partial \bar{\theta}$  is positive (i.e. higher average worker quality leads to higher profits), it follows immediately that any tax which increases the average quality in both the signalling and non-signalling pools is beneficial. This would be true of a tax which discouraged workers, who are below average in the signalling pool but above the average of the non-signalling pool, from acquiring the signal. Again the simplicity of this result follows from the fact that many complicated pecuniary transfer effects are ignored.

If the value of higher quality to a firm is directly proportional to the number of workers of a particular type that it hires, then equation (16) can be rewritten as,

these models in the present context is that they include two separate departures from the usual full-information Arrow-Debreu model. First, they embody incomplete information and this aspect of the problem is, we hope, captured by the formulation of this paper. Second, the competitive mechanisms and associated definitions of equilibria used in the signalling/screening models differ from the usual Walrasian formulations (these changes are largely responsible for the non-existence problems in most signalling models). Moreover, they differ in ways that make it impossible to think of "small" agents. Nevertheless, it is possible to apply the approach of this paper to signalling/screening equilibria of the usual sort (Non-pecuniary externalities arise because the actions of one firm affect the self-selection constraints of others). However, for the sake of unity of presentation these variations of the signalling/screening model will not be discussed in this paper.

$$\frac{dR}{dt} = n_1 \frac{\partial \bar{\theta}_1}{\partial t} \left[ \frac{\partial \bar{g}}{\partial \theta_1} \right] + n_2 \frac{\partial \theta_2}{\partial t} \left[ \frac{\partial \bar{g}}{\partial \theta_2} \right] \quad (17)$$

where,

$$\sum_F \frac{d\pi^f}{d\theta_i} = n_i \sum_F \left( \frac{n_i^f}{n_i} \right) \left( \frac{\partial g_i^f}{\partial \theta_i} \right) = \frac{n_i \partial \bar{q}}{\partial \theta_i},$$

$n_i^f$  is the number of workers of type  $i$  hired by firm  $f$ ,  $n_i$  is the total number of workers of type  $i$  and,

$$\frac{\partial g_i^f}{\partial \theta_i} = \frac{1}{n_i^f} \frac{\partial \pi^f}{\partial \theta_i}.$$

If we further assume that the overall average quality of the labor force is unaffected by the signal and is fixed, then:

$$n_2 \frac{\partial \bar{\theta}_2}{\partial t} + n_1 \frac{\partial \bar{\theta}_1}{\partial t} + \frac{\partial n_1}{\partial t} (\bar{\theta}_1 - \bar{\theta}_2) = 0$$

Substitution from this expression into (17) yields:

$$\frac{dR}{dt} = \left[ n_1 \frac{\partial \bar{\theta}_1}{\partial t} \right] \left[ \frac{\partial \bar{g}}{\partial \theta_1} - \frac{\partial \bar{g}}{\partial \theta_2} \right] - \frac{\partial n_1}{\partial t} \left[ \frac{\partial \bar{g}}{\partial \theta_2} \right] (\bar{\theta}_1 - \bar{\theta}_2) \quad (18)$$

The first term in (18) captures the "sorting" value of the signal. It is the improvement in quality in the signalling pool (i.e.  $\partial \bar{\theta}_1 / \partial t$ ) multiplied by the differential value of "quality" for workers from the signalling compared to the non-signalling pool. If "quality" is more important for signalling workers then this term will be positive and, therefore, a tax which increases the quality of the signalling pool will tend to be beneficial. If this increase in quality is achieved by reducing the number of workers who signal (i.e.  $\partial n_1 / \partial t < 0$ ), then the second term in (18) will also be positive (since  $\bar{\theta}_1 - \bar{\theta}_2 > 0$  and  $\partial \bar{g} / \partial \theta_2 > 0$ ) and the tax will be unambiguously beneficial (remember that this applies to the case where overall average quality is constant).

Furthermore, if there is no "sorting" (pure hierarchical screening) effect (i.e.

$\partial \bar{g} / \partial \bar{\theta}_1 = \partial \bar{g} / \partial \bar{\theta}_2$ ), then:

$$\frac{dR}{dt} = - \frac{\partial n_1}{\partial t} \left[ \frac{\partial \bar{g}}{\partial \theta_2} \right] (\bar{\theta}_1 - \bar{\theta}_2) \quad (19)$$

and a small tax which reduces the amount of signalling is beneficial.

Finally, if the original equilibrium involves no signalling (i.e.  $n_1 = 0$ ), then (19) again applies.

### C. Moral Hazard and Insurance

In the case of moral hazard arising from insurance contracts, an "externality" arises because the profits of insurance firms (at given levels of policy prices) depend on an unobservable average level of care taken by subscriber households. This could be modelled directly, treating purchasers of insurance as a heterogeneous "pool" (similar to adverse selection) with an average "quality" which in this instance would be the level of care exercised in avoiding "accidents".<sup>12</sup> However, in an insurance situation, a strong case can be made that the technology of providing insurance is characterized by constant returns to scale and, as a result, the necessary derivatives which underlie equation (12) may not exist. Fortunately, with CRTS, firms are an inessential part of the model. The CRTS technology can be subsumed either into the household sector or into the non-CRTS firms which provide the inputs to the CRTS technology. The fundamental results of the paper are not altered. Modelling the insurance problem in this way provides a useful opportunity to show why.

Assume for simplicity that the universe of insured agents consists of identical households and that a scalar level of effort which reduces the cost of accidents cannot be observed by insurers. Let households maximize,<sup>13</sup>

$$E [U^h(x^h, \mu^h, e^h)],$$

12. The moral hazard issue encompasses far more than insurance problems narrowly construed. However, the "insurance" example captures the fundamental nature of the issues involved in the whole range of moral hazard questions.

13. Accident losses are subsumed in this function. This formulation assumes that the individual commits himself to all non-numeraire expenditures prior to knowing whether there will be an accident. Our results hold for more general formulations.

subject to the constraint that,

$$q \cdot (x^h - w^h) + \gamma(\mu^h, \bar{e}) - I^h - \sum_F a^{hf} \pi^f \leq 0,$$

where  $E$  here denotes an expectation across states of nature,  $\mu^h$  is a vector of insurance payments across states of nature (i.e.  $\mu_1^h$ , the first element of  $\mu^h$ , is the insurance payment made to household  $h$  in state of nature 1),  $\gamma(\mu^h, \bar{e})$  is the premium paid for insurance (which may depend on  $x^h$ , but this situation is described at length below),  $e^h$  is the level of "care" exercised by household  $h$  and  $\bar{e}$  is the "average" level of care exercised by all households, i.e.:

$$\bar{e} = \frac{1}{H} \sum_H e^h$$

and  $w^h$  is the individual endowment vector.

With constant returns to scale in the insurance industry and risk neutral investors, equilibrium in the insurance industry implies:

$$\gamma(\mu^h, \bar{e}) = E(\mu^h | \bar{e})$$

This can be substituted into the household budget constraint so that households choose  $x^h$  and  $\mu^h$  in order to maximize  $E[U^h]$  subject to the constraint:

$$q \cdot (x^h - w^h) + E(\mu^h | \bar{e}) - I^h - \sum_F a^{hf} \pi^f \leq 0$$

where the function  $E[U^h(x^h, \mu^h, e^h)]$  can be treated as a normal utility function.

For a small tax,  $dt$ , the change in maximum expected utility for an individual household is,"

$$\frac{dE[U^h]}{dt} = \lambda^h \left[ \frac{dI^h}{dt} + (w^h - x^h) \frac{dq}{dt} + \sum_F a^{hf} \frac{d\pi^f}{dt} - \frac{dE(\mu^h | \bar{e})}{dt} \cdot \frac{d\bar{e}}{dt} \right]$$

where  $\lambda^h$  is the marginal expected utility of income to household  $h$ . In order to insure that

14. This is the partial derivative of the Lagrangian with respect to  $dt$ , which entails a straight forward application of the envelope theorem.

household  $h$  suffers no net loss in utility,

$$\frac{dI^h}{dt} = -(w^h - x^h) \frac{dq}{dt} - \sum_F a^{hf} \frac{d\pi^f}{dt} + \frac{dE(\mu^h|\bar{e})}{d\bar{e}} \cdot \frac{d\bar{e}}{dt}$$

Summing over all households and applying the same simplifications used in Section I above, the net impact per unit of tax,  $dt$ , is:<sup>15</sup>

$$\frac{dR}{dt} = \sum_H \frac{dE(\mu^h|\bar{e})}{d\bar{e}} \cdot \frac{d\bar{e}}{dt}$$

Since  $dE(\mu^h|\bar{e})/d\bar{e}$  should be negative (more care reduces insurance payments), any small tax which increases household efforts at accident avoidance will improve welfare. Moreover, the net social value of the tax change is just equal to the reduction in expected level of casualty insurance payments. Again this is an observable consequence against which the efficiency of a tax intervention can be measured.

This conclusion runs counter to the conventional idea that competitive insurance markets ought not to be interfered with as long as a competitive equilibrium exists (see for example, Shavell (1979)). However, in most of these models, the insurance industry is constituted so that no set of taxes can alter the level of care. Showing why this occurs reveals a great deal about the appropriate role of government intervention.

If we assume that insurance costs can be made to depend on the complete vector of household consumption, equilibrium in a competitive insurance industry will imply that,

$$\gamma(\mu^h, x^h, \bar{e}) = E(\mu^h|\bar{e}(x^h), x^h)$$

where the  $\bar{e}$  in question is now that of households with consumption vector  $x^h$  since these households constitute a separate insurance class. Under these conditions, the cost of insurance term in the net social impact expression will be,

15. Note that as  $t$  changes, the optimal policy,  $\mu^h$ , will change, but by the envelope theorem, this effect drops out.

$$\frac{dR}{dt} = \sum_H \frac{dE(\mu|\bar{e}(x^h))}{d\bar{e}} \cdot \frac{\bar{e}(x^h)}{dt}$$

where  $x^h$  is being held constant as taxes change. However, if taxes do not affect  $x^h$  then they will not affect  $e^h$  and, thus, will have no impact on  $\bar{e}$ .

Therefore, where insurance premia are conditioned on  $x^h$ , tax interventions will not be able to improve overall consumer welfare.<sup>16</sup> The ultimate policy question is whether insurance firms can monitor individual household consumption levels or whether it is easier for the government to control overall consumption levels via taxes.<sup>17</sup>

#### D. Incomplete Markets

An economy without a full set of Arrow-Debreu contingent commodity markets is one in which many commodities are composites in the same way that a draw from a labor pool of heterogeneous quality is a composite. When changes in demand change market prices, the nature of the composite product will often change. As a result, although the notion of "quality" is no longer unambiguous, small tax interventions will almost invariably exist which can improve an original market allocation. The initial allocation is not, therefore, a Pareto optimum. The recurrent discovery of this kind of inefficiency should not continue to be a surprise.<sup>18</sup>

A simple model of the phenomenon involved is one with two periods. Assume, that, in period two, the state of nature may take on one of  $k$  value. Assume further that there is a single store of value, denoted good zero, whose price in period two depends on the state of nature which materializes at that time.<sup>19</sup> Let an  $n$ -dimensional vector  $s = (s_1, \dots, s_k)$  denote the price of the store-of value good in terms of the numeraire good in each of the period two

16. The original competitive equilibrium may still not be Pareto efficient, but commodity taxes will not help. For a complete characterization of the optimal tax policy see Arnott and Stiglitz (1981).

17. A similar but slightly more complicated analysis can be applied to the adverse selection case presented earlier.

18. Among the most recent discoverers of this phenomenon are Loong and Zeckhauser (1981).

19. A more conventional approach would be to follow Diamond (1967) and Stiglitz (1972), who assume that the investment good yields a random return. If there are grounds for government intervention in the more restrictive model used here (in which the "real" return to the investment good is fixed at zero), then there are certainly grounds for government intervention in the more general model.

states of nature. The value of this vector will depend upon market conditions in period two which depend, among other things, on prices and the amount of the good zero produced in period 1. If the store-of value good is the only item of capital, then a household's expected utility at the beginning of period 2 depends on its holdings,  $x_0^h$ , of this good at that time and the vector of prices,  $s$ . For each  $x_0^h$  and  $s$ , there is a function  $V^h(x_0^h; s)$  which describes the maximum expected utility of household  $h$  in period 2. For concreteness,  $V^h$  can be written;

$$V^h(x_0^h; s) = \sum_k u_{2k}^h(x_{2k}^h; x_0^h, s_k) b_k$$

where  $b_k$  is the probability that state  $k$  materialized. The vector  $x_{2k}^h$  is the consumption which maximizes the utility of household  $h$  during period 2 in state  $k$ . It is selected to maximize  $u_{2k}^h(x)$  subject to the constraint that

$$p_2 \cdot (x - w_2) \leq x_0 s_k + I_{2k}^h$$

where  $I_{2k}^h$  here denotes the lump sum income of household  $h$  plus net sales of the numeraire good in state  $k$  in period 2.

Looking forward from the beginning of period 1, we will assume that a household's two period expected utility is the sum of its expected utilities in period 1 and period 2 separately. Formally:

$$u_0^h(x_1^h, x_0^h; s) = u_1^h(x_1^h) + V^h(x_0^h; s) \quad (20)$$

where  $x_1^h$  denotes consumption period 1. Households should maximize this two period expected utility subject to the usual budget constraint.

Now consider the impact of a small change in period 1 prices. It will lead to a readjustment in period 1 consumption whose effects will net out. In addition, it will lead to changes in total  $x_0$  purchased and, by this means, to changes in the vector  $s$ . In equation (20), the vector  $s$  enters the overall utility function directly as a kind of externality. Like the quality variable in the adverse selection example it describes the "composition" of a ticket in a lottery. In this instance, the lottery is a subsequent value lottery instead of a quality lottery. Thus, changes in the "prices"  $s$  are non-pecuniary externalities rather than pecuniary externalities in the traditional sense.

Applying equation (12) to this simple model a small change in taxes,  $dt$ , will have a net impact

$$\frac{dR}{dt} = \sum_k \left( \sum_H x_0^h \cdot \lambda_k^h \right) \frac{ds_k}{dt} b_k$$

where  $\lambda_k^h$  is the marginal utility of income to household  $h$  in state  $k$ . Therefore, in general, taxes will exist which will improve overall welfare. Models which conclude otherwise (e.g. Diamond (1967)) typically impose conditions under which  $ds/dt = 0$  or households are indifferent to the pattern of prices which occurs across states of nature (i.e.  $\partial v^h/\partial s = 0$ ). For example, Diamond (1967) achieves this by having only a single good so that  $s_k = 1$  for all  $k$  under all circumstances.<sup>20</sup> The conditions involved are very special ones.

#### E. Queue Rationing

When information is imperfect and search (transactions) are costly, the benefits and costs of entering a market often depend on variables other than price. For instance, the return to a worker entering the labor market depends both on the length of time that he has to search for a job as well as the wage he receives once he is employed. And the length of time that an individual has to search depends on the search activities of other individuals.

Similarly, in product markets, first come first served queues often serve to balance supply and demand. The length of the queue may again depend on the actions of other firms and individuals.<sup>21</sup> In both cases, there is an externality. The question is whether these externalities result in markets being pareto efficient. We now show how these externalities can be analyzed using the framework of this paper.

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20. Circumstances under which  $\partial v^h/\partial s = 0$  have been identified by Stiglitz (1982). He considers a situation in which the store-of-value good yields a random return  $\pi_k$  in state  $k$ . Then, if demand for the store-of-value good is Cobb-Douglas, its total value is independent of the price  $s_k$  and consumers are indifferent to the pattern of  $s_k$  prices across states of nature.
21. Similar externalities arise when firms must bear some part of the hiring and training costs of individuals, and individual quit rates depend on the actions of other firms. Still other search externalities which may be analyzed using our framework are those where the characteristics (quality) of individuals arriving at a firm are affected by the policies of other firms.

In this section, we investigate queue rationing as one example in this category. The reasons for looking at queues are three-fold. First, they have not been investigated as thoroughly as search equilibria.<sup>22</sup> Second, the structure of the models is quite general. And, third queue rationing equilibrium usefully illustrates the set of circumstances in which competitive equilibria are Pareto efficient in ways which most conventional search models do not.

Again, to facilitate the exposition, we will use a very simple model. Let there be a single good, subscript 1, produced by a single factor, subscript 0. Assume that the factor, referred to as labor, is inelastically supplied and is the numeraire. The "good" is supplied in  $N$  separate markets indexed  $i = 1, \dots, N$  in each of which firms guarantee a different waiting time. Let,

$q_i$  = consumer price of the "good" in market  $i = 1, \dots, N, q = (q_1, \dots, q_N)$

$p_i$  = producer price in market  $i = 1, \dots, N,$

$T_i$  = average waiting time for consumers in market  $i = 1, \dots, N, T = (T_1, \dots, T_N).$

Each of the  $i$  markets are assumed to be competitive with both firms and consumers taking prices as given, and consumers taking waiting times as given.

Households will be assumed to divide up their purchases among the several markets. Let,

$x^h = (x_1^h, \dots, x_N^h)$  = vector of purchases by household  $h$

$x = \sum_H x^h.$

Household utility will be assumed to depend on  $x^h$  and, also, implicitly on the waiting time associated with  $x^h$ . Let,

$u^h = u^h(x^h, w - p \cdot x^h, L - T \cdot x^h)$  = utility function of household  $h,$

where

$w$  = total supply of labor = labor income,

<sup>22</sup> An exception is Truman Belwely's unpublished paper, "Equilibrium Theory with Transactions Costs."

$L$  = non-worked hours.<sup>23</sup>

Firms produce the single good subject to the constraint that they meet the specified waiting times for their market segments. Let,

$y_{01}^f$  = labor devoted to production of output by firm  $f = 1, \dots, F$ ,

$y_{00}^f$  = labor devoted to handling queue members,

$y_{01}^f = L^f(y_1^f)$  = Labor requirement function of firm  $f$  producing  $y_1^f$  units of output.

$y_{00}^f = Q^f(T_i, y_1^f)$  = Function defining the service labor required by firm  $f$  to supply an output  $y_1^f$  with a waiting time  $T_i$ .

Assuming, for simplicity, that each firm participates in only one market, profits for firm  $f$  are,

$$\pi^f = p_i y_1^f - (L^f(y_1^f) + Q^f(T_i, y_1^f)).$$

Profits are maximized over a choice of output,  $y_1^f$ , and the market  $i$  (characterized by  $\{p_i, T_i\}$ ) in which the firm chooses to participate.

An equilibrium set of prices equates supply and demand in each of the markets (we ignore, as always, existence problems). The extent to which this equilibrium involves "real" externalities depends on the form of the queue service function,  $Q^f$ .

The simplest alternative is to assume that each firm makes available  $y_1^f$  units of output at the beginning of each market period. Customers line up to purchase a single standard unit size each. Thus, letting the standard size be 1 unit of  $y_1$ , firm  $f$  can expect  $y_1^f$  customers. These must be serviced in  $2T_i$  units of time which requires a service rate,

$$\alpha^f = \text{service rate for firm } f = y_1^f / 2T_i.$$

This service rate of customers per unit time will determine the needed service labor. Let,

23. Waiting times  $T$  will be assumed to be rates per units consumed which implies that there is a standard order quantity. Also since  $T$  represents "average" waiting times, we will assume that consumers are risk neutral with respect to variations in waiting time around this average.

$\alpha^f(y_{00}^f)$  = the service rate attained by  $y_{00}^f$  units of labor.

Then

$$Q^f(y_1^f, T_1) = \alpha^{f^{-1}}(y_1^f/2T_1).$$

A queue service function of this kind contains no  $x$ -variables and hence no "real" externalities. Thus, the competitive allocation will be immune to Pareto improving tax interventions.<sup>24</sup> Queue times an observable quality variables and by themselves do not lead to inefficient market allocations. Because firms can control their queue lengths by controlling output there are no non-pecuniary externalities.

However, other models which are characterized by such externalities can easily be described. Assume, for example, that levels of output,  $y_1^f$ , and prices are determined before the beginning of each market period. Before going to a market, individuals know the price and the expected queue length; only after they go to a market do they learn the queue length associated with each particular firm within the market. Assume, that households appear randomly throughout the market period and join the shortest queue within the market belonging to a firm whose unsold output exceeds the demand of customers already in line. Queue length for any particular vendor will depend on the queues facing other vendors. As a result, service rates required to meet the standard expected waiting time,  $T_1$ , will depend on the queue lengths of other firms which depend, in turn, on their service rates. In a market equilibrium with repeated periods and actual service costs equalling expected cost, the profit function of one firm can be written,

$$\pi^f = p_1 y_1^f - (L^f(y_1^f) + Q^f(T_1, y_1^f, \bar{y}_{00}^f)),$$

where,

24. The firm is not, of course, really restricted to choosing only those prices and service levels (queue length) offered by other firms. An implicit assumption of our analysis is that there does not exist any other  $(p, T)$  combination which would be purchased by individuals (given the values of  $p$  and  $T$  in other markets) and such that, at the quantities demanded, the firm makes a profit.

$\vec{y}_{f0}^s$  = vector of the service labor inputs of other firms (not including firm  $f$ ).

In this formulation, there are obviously "real" externalities and, using equation (12), the net social impact of a tax,  $dt$ , is

$$\frac{dR}{dt} = \sum_f \left[ \sum_f \frac{\partial Q^f}{\partial y_{f0}^s} \cdot \frac{\partial y_{f0}^s}{\partial t} \right]$$

Moreover, the appropriate direction of government policy can readily be determined by examining the impact of taxes on the service burdens of firms.<sup>25</sup>

The range of possible models, including those in which consumers bear externality related waiting costs, goes far beyond the simple variations described here. This brief discussion is not in any way intended to be definitive. What should be clear, however, is the value of the underlying framework for assessing rapidly the welfare implications of different queuing or search formulations.

### III. Conclusion

This paper developed a general methodology within which a wide range of market mediated externalities can be analyzed. In concluding, four aspects of our analysis should be stressed.

First, the reasons that pecuniary externalities can be ignored in Arrow-Debreu economies go far beyond the condition that each individual has a negligible impact on prices. Second, competitive market economies with imperfect information (whether this gives rise to adverse selection, signalling and screening, or moral hazard problems) and incomplete markets will be characterized by what we have identified as market-mediated externalities, of a kind which result in the market equilibrium being pareto inefficient. On the other hand, there are some circumstances in which the presence of queues does not, in itself, imply that markets are inefficient. Third, the pecuniary effects can be identified and eliminated in many welfare calculations by the rigorous and repeated application of the envelope theorem. Finally, doing this, and approaching many informational externalities (as

25. It should be noted that this is the change in service cost at constant output not the observed change in service costs.

well as externalities arising out of costly transactions, queues, etc.) like standard non-pecuniary externalities enables one to identify both the appropriate direction of policy intervention and observable measures of their successful application.

### Appendix I

In order to investigate the nature of pecuniary externalities in the traditional sense, the natural starting point is to examine the impact of a small "balanced budget" shift in excess demand.<sup>26</sup> Let,

$$d\bar{s}_0 = (ds_1^0, ds^0),$$

where,

$ds_1^0 = q \cdot ds^0$  = shift in demand for the numeraire good,

$ds^0 = (N-1)$  vector of shifts in demand for the  $N-1$  non-numeraire goods.

The shift,  $d\bar{s}_0$ , may be ascribed either to a shift in the demand of a single-household or to entry of a new household. An analogous shift with  $ds_1^0 = -p \cdot ds^0$  could be defined and ascribed to a change in behavior by the universe of firms.

Assuming taxes are unchanged, the resulting change in market prices is,

$$dp = dq = J^{-1} \cdot ds^0,$$

where,

$$J = \left[ \frac{dx_j}{dp_k} - \frac{dy_j}{dq_k} \right], j, k = 2, \dots, N = \text{Jacobian}^{27} \text{ of the vector of non-numeraire excess demands.}$$

We assume that the excess demand functions are differentiable and that  $J$  is non-singular at the initial equilibrium.

The change in income necessary to maintain the utility level of household  $h$  in the face of a change in price  $dp = dq$  is,

26. Only balanced budget shifts make sense if we are considering changes in equilibrium allocations. An unbalanced shift in excess demand would preclude the existence of a new equilibrium.

27. Since  $dp_k = dq_k$  in the present instance, it makes sense to talk about this "Jacobian" without treating the  $p$  and  $q$  vectors separately.

$$\frac{dI^h}{dp} = E_q^h + E_z^h \left( \frac{dz^h}{dp} + \frac{dz^h}{dq} \right) - \sum_F a^{hf} \left( \pi_p^f + \pi_z^f \left( \frac{dz^f}{dp} + \frac{dz^f}{dq} \right) \right).$$

Summing over all households and recognizing that  $\pi_p^f = y^f$ ,  $E_q^h = x^h$ ,  $\sum_H a^{hf} = 1$  and

$\sum_F y^f = \sum_H x^h$  yields a total net change in government income compensation,

$$\sum_H \frac{dI^h}{dp} = \sum_H E_z^h \left( \frac{dz^h}{dp} + \frac{dz^h}{dq} \right) - \sum_F \pi_z^f \left( \frac{dz^f}{dp} + \frac{dz^f}{dq} \right).$$

The total change in the government surplus (once these compensations are paid) is,

$$dR/dp = t \cdot \frac{dx}{dp} - \sum_H E_z^h \left( \frac{dz^h}{dp} + \frac{dz^h}{dq} \right) + \sum_F \pi_z^f \left( \frac{dz^f}{dp} + \frac{dz^f}{dq} \right).$$

At an initial tax level of zero this becomes,

$$\frac{dR}{dp} = - \sum_H E_z^h \left( \frac{dz^h}{dp} + \frac{dz^h}{dq} \right) + \sum_F \pi_z^f \left( \frac{dz^f}{dp} + \frac{dz^f}{dq} \right).$$

As a function of the initial change in excess demand, the net change in the government surplus<sup>28</sup> is,

$$\frac{dR}{ds^0} = \frac{dR}{dp} \cdot \frac{dp}{ds^0} = (\Pi^P - B^P) \cdot J^{-1}, \quad (A-1)$$

where,

$$\Pi^P = \sum_F \pi_z^f \left( \frac{dz^f}{dp} + \frac{dz^f}{dq} \right),$$

$$B^P = \sum_H E_z^h \left( \frac{dz^h}{dp} + \frac{dz^h}{dq} \right).$$

This represents the net social impact of the initial change in price and, thus, the "pecuniary"

28. The expression in equation (A-1) below ignores the externalities generated by changes in consumption that result from compensating ~~income~~ income transfers. This is not done because the changes in question are negligible. They are not negligible. Rather it is done to avoid keeping track of transfer related externalities which add greatly to the notational burden without affecting the basic substance of the analysis. For rigor we could assume that (1) consumption and production of the numeraire good generates no externalities and (2) households are constrained to consume their compensating allotments of the numeraire good. Also we assume that the original  $ds^0$  shift affect no  $z$ -variables.

externality<sup>29</sup> associated with original change in demand  $ds^0$ .

It only remains to be shown that  $dR/ds^0$  does not vanish as the number of households becomes large. To do this let,

$\mu_m$  = fraction of households of type  $m = 1, \dots, M$  (i.e.  $\mu_m H$  = number of households of type  $m$ )

$\mu_l$  = number of firms of type  $l = 1, \dots, L$  per household (i.e.  $\mu_l H$  = number of firms of type  $l$ )

Since  $dz^f/dp$  and  $dz^f/dq$  ought not to be influenced by the number of households,

$$\Pi^P = \sum_L \mu_l H \cdot \pi_z^l \left( \frac{dz^l}{dp} + \frac{dz^l}{dq} \right) = H \cdot \hat{\Pi}^P,$$

where,

$$\hat{\Pi}^P = \sum_L \mu_l \pi_z^l \left( \frac{dz^l}{dp} + \frac{dz^l}{dq} \right)$$

$$\pi_z^l = \pi_z^f \text{ for firms of type } l$$

$$\frac{dz^l}{dp} = \frac{dz^f}{dp}, \quad \frac{dz^l}{dq} = \frac{dz^f}{dq} \text{ for firms of type } l.$$

The matrix  $\hat{\Pi}^P$  will not change with the number of households  $H$ . Similarly,

$$B^P = H \cdot \hat{B}^P,$$

where,

$$\hat{B}^P = \sum_M E_z^M \left( \frac{dz^m}{dp} + \frac{dz^m}{dq} \right),$$

and  $\hat{B}^P$  should be invariant to changes in  $H$ .

29. This effect differs from the tax effects of the body of the paper in that the  $dz/dp$ ,  $dz/dq$  terms differ from the  $dz/dt$  terms of the body of the paper. However, in both cases, externalities will not matter when either,  $\pi_z^f$  and  $E_z^h$  are zero for all households and firms or when the  $z$ 's are not affected by changes in market prices (other cases are fortuitous). If  $dz^f/dt$  and  $dz^h/dt$  are non-zero, then as a rule  $(dz^f/dp + dz^f/dq)$  and  $(dz^h/dp + dz^h/dq)$  will be non-zero. Thus (again in general), the conditions under which taxes can lead to Pareto improving allocations are precisely ~~the same~~ under which "pecuniary" externalities do not net out.



## BIBLIOGRAPHY

- Akerlof (1970), G., "The Market for Lemons: Qualitative Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, LXXXIV, 84, 288-300.
- Arnott, R. and Stiglitz (1981), J., "Moral Hazard," Unpublished Manuscript.
- Bewley, Truman, "Equilibrium Theory with Transactions Costs," forthcoming manuscript.
- Diamond (1967), P., "The Role of a Stock Market in a General Equilibrium Model With Technological Uncertainty," *American Economic Review*, 57, 759-776.
- Diamond (1979), P. and E Maskin, "An Equilibrium Analysis of Search and Breach of Contract, I: Steady State," *Bell Journal of Economics*, Vd. 10, No. 1, pp. 282-318.
- Diamond, P. and McFadden (1974), D. L., "Some Uses of the Expenditure Function in Public Finance," *Journal of Public Economics*, 3, 321.
- Dixit (1979), A., "Rice Changes and Optimum Taxation in a Many Consumer Economy," *Journal of Public Economics*, 11, 143-157.
- Harberger (1964), A., "Measurement of Waste," *American Economic Review*, 54, 58-76.
- Harberger (1971), A., "Three Postulates for Applied Welfare Economics: An Interpretative Essay" *Journal of Economic Literature*, Vd. 9, No. 3, 785-797.
- Lipsey, R and Lancaster (1956), K., "The General Theory of Second Best," *Review of Economic Studies*, 24, 11-32.
- Loong, L and Zeckhauser (1981), R., "Pecuniary Externalities Matter When Contingent Claims Markets Are Incomplete," Unpublished Paper, Harvard University.
- Mortensen, (1979), D., "The Matching Process as a Non-Co-operative Game", Northwestern Math Center Discussion Paper, No. 284.
- Newbery, B. and Stiglitz (1982), J., "The Choice of Technique and the Optimality of Market Equilibrium With Rational Expectations" *Journal of Political Economy*, Vd. 90, No. 2, 223-246.
- Radner, R and Stiglitz (1976), J., "A Non-Concavity in the Value of Information," Dept. of Economics, University of California, Berkeley.
- Rothchild, M. and Stiglitz (1977), J., "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, XC, 90, 629-650.
- Shavell (1979), S., "On Moral Hazard and Insurance," *Quarterly Journal of Economics*, XCIII, 93, 541-562.
- Spence (1973). E., "Job Market Signalling," *Quarterly Journal of Economics*, LXXXVII, 87, 355-374.
- Starrett (1980), D. A., "Measuring Externalities and Second Best Distortions in the Theory of Local

Public Goods," *Econometrica*, Vd. 48, No. 3, 627-642.

Stiglitz (1972), J. E., "On the Optimality of the Stock Market Allocation of Investment," *Quarterly Journal of Economics*, LXXXVI, 86, No. 1, 55-72.

Stiglitz (1975), J. E., "The Theory of Screening Education and the Distribution of Income," *American Economic Review*, Vd. 65, No. 3, 283-300.

Stiglitz (1982), J. E., "The Inefficiency of the Stock Market Equilibrium," *Review of Economic Studies*, XLIX, 241-261.

Stiglitz, (1976), J.E., "Prices and Queues as Screening Devices in Competitive Markets," IMSSS tech. report no. 212, Stanford Univ.

Stiglitz, J.E. and Weiss, A (1981) "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, Vd. 71, no. 3, 393-410.