

NBER WORKING PAPER SERIES

SOME PLEASANT MONETARIST ARITHMETIC

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Working Paper No. 1295

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
March 1984

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ABSTRACT

Contrary to the conclusion of Sargent and Wallace, it is possible to exogenously and independently vary monetary and fiscal policy and retain steady-state equilibrium in economies like the United States. In particular, the central bank is not forced to monetize increased deficits either now or in the future. This conclusion is based on the fact that the real after-tax yield on government bonds is considerably less than the growth rate of real income except during brief disinflationary periods.

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For the Federal Reserve  
Bank of Minneapolis  
Quarterly Review.

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D.5: February 21, 1984

SOME PLEASANT MONETARIST ARITHMETIC

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Sargent and Wallace (1981) are widely regarded to have demonstrated that monetary policy cannot be manipulated exogenously with a fixed growth path of government expenditures and a fixed tax structure. More succinctly, the central bank can only choose whether to monetize a government deficit now or later. This result can be viewed as a generalization of the Blinder and Solow (1973, 1974), Tobin and Buiter (1976) and Steindl (1974) analyses of the stationary state when it is assumed that (base) money is increased while government spending and the tax rate are fixed so that government borrowing is adjusted passively via open-market operations.<sup>1</sup> The Sargent and Wallace (1981) argument appears persuasive to such authors as King and Plosser (1983), but I believe it is seriously wrong as a guide to understanding monetary policy in the United States. This paper first demonstrates that whether or not money, spending, and taxes can be manipulated independently by the government is an empirical not a theoretical question. Then in Section II, I present evidence that at least the United States government can indeed independently manipulate all three instruments with government debt adjusting in a passive but stable manner.

## I. Steady-State Equilibrium

Miller (1983) has derived a version of the government budget constraint which is useful for studying the long-run growth equilibrium of the economy. Simplifying the notation of his equation (4), we can write this constraint as

$$(1) \quad G - T = \mu M + (\delta - r)D$$

Here,  $G$  and  $T$  are government expenditures and taxes less transfer payments, respectively, both measured as fraction of NNP and exclusive of any government interest payments or the taxes thereon.  $M$  and  $D$  are the money-NNP and debt-NNP ratios, and  $\mu$  and  $\delta$  are the growth rates of nominal money and the real government debt, respectively. Finally,  $r$  is the real after-tax interest rate.<sup>2</sup> That is, the excess of spending over taxes must be financed by either base money creation or by borrowing in excess of the amount needed to pay the real after-tax interest on the government debt.<sup>3</sup>

The question raised by Sargent and Wallace is whether there is only one value of money-creation revenue  $\mu M$  for which debt will be a stable fraction of NNP.<sup>4</sup> This is formalized as asking whether a steady-state equilibrium exists and is stable for alternative values of  $\mu M$ . The steady-state equilibrium debt-NNP ratio  $\bar{D}$  is found from equation (1) to be

$$(2) \quad \bar{D} = \frac{G - T - \mu M}{\gamma - r}$$

where  $\gamma$ , the growth rate of real NNP, must be equal to  $\delta$  if  $D$  is constant. Equation (2) says that if the government is spending more than it takes in explicit taxes and the inflationary tax, there can still be a constant debt-NNP ratio if the growth rate of real NNP exceeds the real after-

tax interest rate. If  $r$  exceeded  $\gamma$ , then any positive excess of  $G$  over  $T + \mu M$  would indeed cause  $D$  to grow without limit. Sargent and Wallace (1981) instead simply assumed that  $r$  exceeds  $\gamma$  and hence inevitably came to the conclusion that the government could not independently choose  $\mu$ ,  $G$ , and  $T$ . I shall argue in Section II below that this assumption is incorrect empirically and proceed here with the analysis on the assumption that  $\gamma$  is greater than  $r$ .

The basic idea is that the government will borrow more than enough to make interest payments if  $D$  is constant and that this net borrowing<sup>5</sup>  $(\gamma - r)D$  increases with  $D$ . So higher deficits  $G - T - \mu M$  will be associated with higher debt-NNP ratios, but they can be financed indefinitely as a matter of arithmetic unless the real interest rate were equal to or greater than the growth rate of real income.

To check that the economy will in fact move toward the equilibrium debt-income ratio, suppose that actual value of  $D$  differed from its steady-state value  $\bar{D}$ . The growth rate of  $D$  is  $\delta - \gamma$ , the difference in the growth rates of its numerator and denominator. Straightforward manipulations and the assumption of either perfect foresight or indexed government bonds yield the growth rate relation<sup>6</sup>

$$(3) \quad \delta - \gamma = (\gamma - r) \frac{\bar{D} - D}{D} .$$

That is the growth rate of the debt-income ratio will be positive if the actual  $D$  is less than its steady-state value  $\bar{D}$  and negative if  $D$  exceeds  $\bar{D}$ . So  $D$  will gradually converge to  $\bar{D}$  even if the economy were to start from another position as might result from cyclical deficits, wars, short-run monetary or fiscal policy, or changes in the underlying trend values of  $G$ ,  $T$ ,

or  $\mu$  which define the the steady-state equilibrium.

Consider the following simple example:

$$\begin{array}{ll} G = 0.22 & T = 0.18 \\ \gamma = 0.04/\text{year} & r = 0.02/\text{year} \\ \mu = 0.10/\text{year} & M = 0.10 \text{ year} \end{array}$$

Therefore, the steady-state debt-income ratio is

$$\bar{D} = \frac{0.22 - 0.18 - (0.10/\text{year})(0.10 \text{ year})}{(0.04/\text{year}) - (0.02/\text{year})} = \frac{0.03}{0.02/\text{year}}$$

$$\bar{D} = 1.5 \text{ year}$$

Suppose that the Fed decided to increase money growth to  $\mu' = 0.20/\text{year}$  and that this induced  $M$  to fall to  $M' = 0.09 \text{ year}$ . Then the new equilibrium debt-income ratio is

$$\bar{D}' = \frac{0.04 - (0.20/\text{year})(0.09 \text{ year})}{0.02/\text{year}} = 1.1 \text{ year}$$

When this policy is initiated, the growth rate of the debt-income ratio would be

$$\delta - \gamma = \frac{0.02/\text{year}}{1.5 \text{ year}} (1.1 \text{ year} - 1.5 \text{ year})$$

$$\delta - \gamma = -0.0053/\text{year}$$

That is, over the first year of the new policy  $D$  would fall by approximately (1 year)  $(-0.0053/\text{year}) (1.5 \text{ year}) = -0.0080 \text{ year}$  to 1.492 year. The rate of decline would decrease as  $D$  asymptotically approached  $\bar{D}' = 1.1 \text{ year}$ .<sup>7</sup> Thus, the government-budget identity does not pose any problems for the existence or

stability of the steady-state equilibrium as money growth is varied exogenously with fiscal policy fixed. Similarly, government spending or tax rates can be varied exogenously with the other fiscal variable and monetary policy held unchanged. In this way the standard macroeconomic practice of varying fiscal or monetary instruments with government borrowing adjusting passively is shown to be consistent with a stable steady-state equilibrium.

## II. Empirical Issues

Section I -- like Sargent and Wallace (1981) -- is basically an arithmetic exercise. In this section I argue that the empirical evidence favors the relevance of Section I for the United States. There are two substantial differences between the two arithmetic exercises: Sargent and Wallace use before-tax real yields instead of after-tax real yields and they assume that the relevant real yield exceeds the growth rate of real income.

### II.A. Differences about the Relevant Real Yield

The differences over which yield should be compared to real income growth are partially semantic and partially substantive. Sargent and Wallace define an exogenous fiscal policy as a fixed path for the difference between government spending and taxes (exclusive of money or debt creation and interest payments) measured in terms of real goods. I hold the levels of each of these variables (and hence their difference) constant as a fraction of real income. Thus, if decreased money growth reduces real income, it would reduce the level of the future real deficit on my assumption of constant deficits as a fraction of income.

This difference is relevant only if lower money growth (and hence a higher debt-income ratio) reduces real output as supposed by Sargent and Wallace. They make a crowding-out argument because in their life-cycle framework more government debt means that less wealth will be held in the form of capital. Thus tax receipts on capital returns go down as tax receipts on government debt goes up.<sup>8</sup>

Suppose instead that individuals are fully rational and care about their children as themselves. In that kind of world, government accounts are consolidated into those of the individuals which it represents so that whether the government finances by taxes or bonds is irrelevant to individual choices about consumption and accumulation of physical capital. Measured saving will equal the unaffected capital accumulation plus however many new government bonds are issued instead of tax receipts. But individuals will not be concerned with how many IOUs they are writing themselves.<sup>9</sup> It should be noted that on this latter view of saving behavior, the real interest rate is unaffected by the level of the debt-income ratio.<sup>10</sup>

In conclusion, there are good reasons to suppose that the difference in the way in which exogenous fiscal policy is defined is not a substantive one. The fall in private capital which Sargent and Wallace associate with higher levels of the debt-income ratio need not occur. Nonetheless, it will be shown in Section II.B that even the real before-tax yield on government securities has been generally well below the growth rate of real income. In that case even in the Sargent and Wallace world exogenous variations in the deficit need not be monetized by the central bank.

## II.B. Real Yields versus Real Growth

As anyone who has ever looked at real before-tax yields on government securities is aware, it is a simple matter to show that secular real before-tax yields have not approached corresponding growth rates of real output. It follows directly that real after-tax yields must be even less. This is not to suggest that the real return to capital in the economy is less than the growth rate of real output; but the real rate of return on government bonds and bills is clearly far below this average social return. Presumably the difference reflects both nonpecuniary services and a very low correlation with the market return, but that really is not at issue in understanding the implications of the government budget constraint.

Ibbotson and Sinquefield (1982) have compiled (before-tax) real rates of return for U.S. Treasury bills and bonds for the years 1926-1981. The arithmetic (geometric) means of the yields for long-term government bonds and Treasury bills are 0.3 and 0.1 (-0.1 and 0.0) percent per annum, respectively. The corresponding nominal yields were 3.1 and 3.1 (3.0 and 3.0). So even if all holders were tax exempt, the experience of the last 55 years suggests that the real after-tax yield on government securities has been nowhere near the 3.0 percent per annum average growth rate of real income over the same years.<sup>11</sup>

It would be possible to increase the estimated real yield somewhat, but I have been unable to find any study that indicates an average real yield on government securities as high as 3 percent even without any allowance for income taxes. Taking account of income taxes would lower these estimates; so there seems to be no doubt empirically that for the U.S. the growth rate of real income exceeds the real after-tax yield on government securities.

### III. A Possible Reconciliation

The point of this paper is a technical one: In the United States, dynamic inconsistencies do not result from treating government expenditures, taxes, and base-money growth as simultaneously exogenous. A current deficit is therefore not per se inflationary in the sense of requiring future increased money growth as claimed by Sargent and Wallace (1981).

This conclusion would not hold for all economies nor need it hold always for the United States. Suppose for example that as the ratio of government debt to income and hence physical capital rises, the yield on government debt rises toward that of physical capital instead of remaining constant as assumed above and by Sargent and Wallace. Then if the equilibrium debt-income ratio were to increase to the point that the real after-tax yield on government securities equalled or exceeded the growth rate of real income, the economy would cross over to the explosive character analyzed by Sargent and Wallace (1981). While this may have occurred for other countries in the past, the United States does not yet seem near that point.

To see this first consider the fiscal 1983 deficit estimated at \$208 billion by the U.S. Council of Economic Advisers (1983, p. 26). If we allow for a cyclical component based on moving from the assumed 10.7 to 6.0 percent unemployment, the "structural" deficit would be about \$117.5 billion less — that is, about \$90 billion. This amount is only \$5 billion more than actual fiscal 1982 interest payments. So even without taking account of the large offsetting state government surpluses, there is no evidence of substantial differences between secular government spending (exclusive of interest) and net taxes. Furthermore, current ratios of government debt to income are far below the 1946 value of 1.1.<sup>12</sup>

In conclusion, the Sargent and Wallace (1981) propositions should not be generally applied in analyses of the United States or similar economies. Where they are applied, they should be justified by evidence that the real after-tax yield on government bonds really does exceed the growth rate of real income or would do so under the circumstances being considered. It is hardly surprising that arithmetic alone cannot give a real answer to a substantive economic question.

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## FOOTNOTES

\*The author acknowledges helpful conversations with John Haltiwanger and Tom Sargent. The research reported here was supported in part by the National Science Foundation (grant number SES-8207336) and by the NBER's Project on Productivity and Industrial Change in the World Economy and is part of the NBER's research program in International Studies. Any opinions expressed are those of the author and not those of the NBER or the NSF. This is not a report of the National Bureau of Economic Research.

<sup>1</sup>This paper does not attempt to comment on the relevance of the balanced-budget condition within the stationary state. See, however, Fischer (1976) and Auerbach and Rutner (1977) on this point.

<sup>2</sup>See Darby (1975). See Section II for further discussion of the use of the real after-tax yield.

<sup>3</sup>The standard national income accounting definition of the deficit counts as government borrowing and private saving that portion of after-tax nominal interest which represents an adjustment for decline in the real value of the nominal debt. In those terms, we would include in equation (1) the growth rate of the nominal debt  $\delta + \pi$  and the nominal after-tax interest rate  $r + \pi$ , where the inflation rate  $\pi$  cancels. See Jump (1980) and Darby and Lothian (1983). Miller's equation (4) substitutes the steady-state condition that the growth rates of real NNP and real debt are equal, but we leave the equation in this form to analyze behavior out of full steady-state equilibrium.

<sup>4</sup>The fraction of NNP which people desire to hold as money is a decreasing function of the nominal interest rate and hence  $\mu$ . In the relevant range  $\mu$  increases with increases in  $\mu$ , but not proportionately so.

<sup>5</sup>An alternative term for net borrowing  $(\gamma - r)D$  would be "negative debt service."

<sup>6</sup>These manipulations begin with the identity  $\delta - \gamma \equiv \frac{1}{D} [(r + \pi)D + G - T - \mu M - (\gamma + \pi)D]$  which was obtained by taking the time derivative of the natural logarithm of  $D$  where the perfect foresight or indexing assumption allows us to express the nominal after-tax interest rate as the sum of the corresponding real rate and actual rather than expected inflation. (In the steady state there is no need to distinguish actual from expected  $\pi$ .) Then, we have  $\delta - \gamma = \frac{\gamma - r}{D} \left[ \frac{G - T - \mu M}{\gamma - r} - D \right]$  from which equation (3) follows by substitution of equation (2).

<sup>7</sup>Note however that absent perfect foresight or a prior refunding into indexed bonds of long-term bonds — see Darby and Lothian (1983) — this adjustment will be much faster as the real value of the existing bonds and debt service drops.

<sup>8</sup>Tobin (1965) proposed a different mechanism by which inflation might reduce the private capital stock. In either case, improvements in the aggregate production function as firms devote less inputs to conserving cash balances would tend to offset, eliminate, or dominate this capital stock effect so that the effect of money growth on real output is theoretically ambiguous. I have assumed elsewhere (1979a) that the production function effect dominates so that lower inflation rates on net increase real output.

<sup>9</sup>White (1978), Darby (1979b), and Kotlikoff and Summers (1981) all report evidence that "bequest assets" dominate "life-cycle assets" in total U.S. wealth, and this finding supports the assumption of concern about ones children's welfare. Barro (1974, 1978), Kochin (1974), and David and Scadding (1974) all present evidence in support of the "ultrarational" or "Ricardian" view. Note that saving increases if the government finances a tax cut with

increased borrowing not in anticipation of future increased taxes but in anticipation that otherwise total NNP would fall.

<sup>10</sup>Plosser (1982), for example, finds that asset prices are unaffected by the extent to which a given level of government expenditures is financed by borrowing instead of taxes.

<sup>11</sup>Computed from real GNP data in Darby (1984, Table A-20) and Federal Reserve Bulletin, February 1983, p. A52.

<sup>12</sup>High ex post real interest rates experienced during 1981-1982 appear to be a result of slowing inflation (compare 1929-1933) and not a matter of a "regime change" to unprecedentedly high deficits.