NBER WORKING PAPER SERIES

IMPLEMENTING TECHNOLOGY

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Working Paper 12886
http://www.nber.org/papers/w12886

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 2007

We would like to thank Frederic Dufourt, Stefano Eusepi, and Chloe Tergiman for their comments and suggestions. We are grateful to the NSF (Grant # SES-0517910) and the C.V. Starr Center for Applied Economics for their financial support. Part of this research was conducted while Bart Hobijn was a visiting scholar at the Graduate Center of the City University of New York. The views expressed in this paper solely reflect those of the authors and not necessarily those of the National Bureau of Economic Research, Federal Reserve Bank of New York, or those of the Federal Reserve System as a whole. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

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JEL No. O0,O3

ABSTRACT

We introduce a tractable model of endogenous growth in which the returns to innovation are determined by the technology adoption decisions of the users of new technologies. Technology adoption involves an implementation investment that determines the initial productivity of a new technology. After implementation, learning increases the productivity of a technology to its full potential. In this framework, implementation enhances growth, while growth increases obsolescence and reduces implementation. In a calibrated version of our model, the optimal policy involves a subsidy to capital and to implementation and a R&D tax. This policy would lead to a welfare improvement of 7.6 percent. Out of steady-state analysis yields that the transitional dynamics of the detrended variables after a shock to capital are very similar to the dynamics of the neoclassical growth model, but transitory shocks have permanent effects on the level of productivity.

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1 Introduction

Adopting a technology requires an active engagement of the adopter beyond the selection of which technology to adopt. The initial productivity of a newly adopted technology depends on a series of investments undertaken by the adopter. We refer to these investments as the technology implementation process.

There is ample evidence on the importance of technology implementation. Lientz and Rea (1998) recognize that "when implementing a new technology, the current business process is often streamlined and reengineered to take advantage of the technology."[1] Bikson et al. (1987) study the determinants of success in implementing multifunction interactive computer systems[2]. Their findings "indicate that site-to-site variations in the success of implementing new technologies are more fully explained by differences in the implementation process than by differences in the systems or in the organizations." Moreover, Brynjolfsson, Hitt, and Yang (2002) find evidence of substantial organizational investments to accommodate the adoption and enhance the productivity of new IT technologies.

Despite its empirical significance, state of the art models of endogenous innovation and growth ignore the technology implementation process. In this paper, we develop a tractable model of endogenous development and implementation of technologies.

In our model, research and development (R&D) activities determine the *potential* productivity of new technologies. The initial productivity of a new technology and its intensity of adoption are determined, however, by the investments in implementing the technology undertaken by the adopter. Over time, the adopter learns how to use the technology efficiently and the *actual* productivity converges smoothly to the potential. This generates a

[1] Based on the experience of implementing Plato, an educational software, at over 5000 sites, Plato Learning Inc. has written an eight-point technology implementation guide to maximize the educational benefits from the technology. The different stages of implementation involve (*i*) assigning roles prior to adopting technology, (*ii*) deciding on program goals, (*iii*) deciding how technology will be integrated in production process, (*iv*) determining who does what once the technology is adopted, (*v*) planning the deployment of technology, (vi) planning technical support, (*vii*) training professionals and (*viii*) planning the evaluation of the program goals and the implementation process.

[2] They use data covering 530 employees from 55 different departments in 26 private sector organizations

smooth diffusion of new technologies that resembles empirical diffusion curves.

Including the implementation margin in a theory of endogenous innovation and growth has important consequences for the positive and normative implications of the model. On the positive side, better implementation makes new technologies more productive and leads to a faster adoption of technologies. Implementation, therefore, increases the returns to innovation and, consequently, economic growth. Growth, in turn, affects the technology implementation decision. In particular, since higher growth increases the rate of obsolescence of technologies, growth decreases the return to implementing new technologies at a higher productivity level and thus leads to lower implementation investments. It is crucial for this interaction that both technology diffusion as well as obsolescence are endogenously determined in our model.[3]

On the normative side, the consideration of an implementation margin provides an additional growth policy instrument. Besides stimulating growth through R&D, the policy maker can also consider stimulating growth by enhancing the intensity of adoption. As it turns out, these two policy alternatives have very different consequences. R&D subsidies stimulate innovation and growth, but, at the same time, increase endogenous obsolescence and thus reduce implementation. Stimulating implementation increases the rate of diffusion of technologies which in turn increases the return to R&D and thus growth[4].

The implementation decision in our model determines how far below their potential productivity technologies are operated. When aggregating this gap over the technologies available for production, our model yields an endogenous level of TFP, satisfying Prescott's

[3] Endogenous growth and creative destruction do yield an endogenous rate of obsolescence but standard models generally take the rate of diffusion of technologies as exogenous. This is the case, for example, in Aghion and Howitt's (1992) model of creative destruction in which technology diffusion is immediate and in Dinopoulos and Waldo (2005) where diffusion occurs gradually but at an exogenous rate. Comin (2000) and Comin and Gertler (2006) introduce models of both endogenous diffusion and endogenous growth but these models differ from ours in that the intensitity of adoption is not determined by the implementation investments.

[4] Kok (2004) and Sapir (2003) emphasize the importance of policy measures beyond R&D subsidies and tax credits to stimulate growth in the European Union. Many of the alternative policies emphasize the importance of speeding up technology diffusion.

(1997) request.

Ours is not the first model in which the distance from the technological frontier is endogenously determined. There are basically two other mechanisms proposed that create such a distance. The first is knowledge spillovers, which are considered, for example, by Barro and Sala-i-Martin (1997) as well as Eeckhout and Jovanovic (2002). Knowledge spillovers will make certain agents or countries decide to fall behind the frontier and wait for the opportunity to imitate rather than innovate. The second is heterogenous adoption costs, either in the form of entry barriers as in Parente and Prescott (1994), or because newly invented technologies are not appropriate for the skill endowments in the economy, as in Basu and Weil (1998) and Acemoglu and Zilibotti (2001).

The concept of implementation is similar in spirit to the idea of appropriate technology. However, where in the appropriate technology case productivity is exogenously determined by the endowments of its users, the implementation decision that we consider here endogenously determines the productivity of technologies, the intensity with which they are used, and the rate at which they diffuse.

One important virtue of our model is that it is quite tractable and yields a parsimonious representation of the equilibrium dynamics of the relevant aggregates as well as closed form solutions for the underlying diffusion curves that drive the aggregate dynamics.

Our model contains eight preference and technology parameters. Five of them are common to standard business cycles models and only three are new to the calibration literature. We calibrate them exploiting information on the R&D share in the economy, the productivity growth rate and micro estimates of learning by doing. This calibration permits a reasonable quantitative analysis of our model economy.

We take advantage of the tractability of our model by not only analyzing its steady state properties but also its transitional dynamics. Two of our most significant findings on this front are; First, transitory perturbances to the model parameters and/or state variables have permanent effects on the level of productivity. Second, despite having three state variables, the responses of the detrended variables in our model to shocks to the capital stock are very similar to the responses of these variables in the neoclassical growth model.

The steady state welfare cost of the distortions in the economy is 7.6 percent. The policy that restores the first best involves a subsidy to capital and to implementation and a tax on R&D. This is in sharp contrast to standard endogenous growth models that do not include the implementation margin.

The structure of the rest of this paper is as follows. We introduce our model of endogenous growth with technology implementation in Section 2. In Section 3, we present the positive results of our analysis by considering the dynamic properties of the decentralized market equilibrium allocation implied by our model. In Section 4, we introduce the normative part of our analysis. In it, we consider the resource allocation that a social planner would choose, compare it to the decentralized economy, and discuss policies that can support the planner's steady state allocation in the decentralized equilibrium. We conclude in Section 5. We leave the mathematical details behind the main results presented in the text for the Appendix.

2 The model

The model economy that we consider consists of four sectors: a household sector, a final goods sector, an intermediate goods sector, and an R&D sector. In this section, we introduce each of these sectors separately. We consider the equilibrium outcome, when they interact, in the next section.

2.1 Household sector

The representative household in our model economy is endowed with one unit of time that it inelastically supplies at each instant. It also owns the capital stock that it rents to firms at a net rate r_t. The household selects the path of consumption c_t to maximize the present discounted value of the utility flow

$$\frac{\sigma}{\sigma - 1} \int_t^{\infty} e^{-\rho(s-t)} c_s^{\frac{\sigma-1}{\sigma}} ds \tag{1}$$

subject to the standard flow budget constraint.

The resulting optimal consumption path is characterized by the Euler equation[5]

$$\frac{\dot{c}_t}{c_t} = \sigma \left(r_t - \rho \right) \tag{2}$$

2.2 Final goods sector

The final good consumed by households is produced using a set of intermediate goods. Let y_t denote the final good output at time t and let y_{it} be the number of units of the i^{th} intermediate good used in production at time t. A new intermediate good is introduced each instant. Therefore, the range of intermediate goods used in production at time t equals $(-\infty, t]$. The final goods production function is given by

$$y_t = \left(\int_{-\infty}^{t} y_{it}^{\theta} di \right)^{1/\theta} \quad \text{where } 0 < \theta < 1 \tag{3}$$

The market for the final good is perfectly competitive. Throughout, we use the final good as the numeraire good and will normalize its price to unity. Let p_{it} be the price of a unit of the i^{th} intermediate good. Given this price, the demand for intermediate good i at time t is given by

$$y_{it} = \left(\frac{1}{p_{it}} \right)^{\frac{1}{1-\theta}} y_t \tag{4}$$

2.3 Intermediate goods producers

Each intermediate good is provided by a single producer who owns a patent that ensures his monopoly over the production of the good. Intermediate goods suppliers make two types of decisions.

First, at every instant in time, they decide on factor demand decisions, the level of output, and set the price of the intermediate good they supply.

[5] Caselli and Ventura (2000) have shown that, because the growth rate of consumption for a household is the same independent of its level of wealth, the distribution of profits among households does not matter for aggregate household behavior. Therefore, we ignore the issue of distribution of profits across households in our model.

Second, at the time they obtain the monopoly right to the supply of the intermediate good, they decide at what initial productivity level to implement the intermediate good's technology. Over time, the producer will learn to produce his intermediate good efficiently and the gap between actual and potential productivity will eventually disappear.

We consider these two choices sequentially. We first solve for the optimal factor demands and pricing rule taking as given the paths for productivity, factor prices, and final goods demand. Then, we solve for the optimal implementation decision.

Factor demands and price setting:

Intermediate goods are produced using capital and labor that are combined using a Cobb-Douglas technology of the form

$$y_{it} = a_{it} k_{it}^{\alpha} l_{it}^{1-\alpha} \quad \text{where } 0 < \alpha < 1 \tag{5}$$

The productivity level with which the i^{th} intermediate good is produced, a_{it}, is time-varying. Labor and capital are homogenous and each intermediate goods producer hires them at the competitive net rates w_t and r_t. The flow profit accrued by the i^{th} intermediate goods producer at time t, π_{it}, is

$$\pi_{it} = p_{it} y_{it} - w_t l_{it} - (r_t + \delta) k_{it} \tag{6}$$

where δ denotes the capital depreciation rate.

The pricing and production decisions are static. Given the production and demand functions, intermediate goods producers, therefore, set p_{it} and demand l_{it} and k_{it} to maximize (6). This yields factor demands that satisfy

$$w_t = \theta (1-\alpha) p_{it} \frac{y_{it}}{l_{it}} \text{ and } r_t + \delta = \theta \alpha p_{it} \frac{y_{it}}{k_{it}} \tag{7}$$

and an optimal price equal to a constant gross markup factor, $1/\theta > 1$, times the marginal cost of production, mc_{it}. That is

$$p_{it} = \frac{1}{\theta} mc_{it} \tag{8}$$

The resulting level of flow profits in each period is given by

$$\pi_{it} = (1-\theta) p_{it} y_{it}, \tag{9}$$

and the value of the firm equals

$$V_{it} = (1-\theta)\int_t^{\infty} e^{-\int_t^s r_j dj} p_{is} y_{is} ds \tag{10}$$

Aggregation over intermediate goods:

As we show in the Appendix, the firms' decisions in the final and intermediate goods sector aggregate to a production function of the form

$$y_t = z_t k_t^{\alpha} l_t^{1-\alpha} = z_t k_t^{\alpha} \tag{11}$$

where the aggregate capital and labor inputs are given by

$$k_t = \int_{-\infty}^{t} k_{it} di \text{ and } l_t = \int_{-\infty}^{t} l_{it} di = 1 \tag{12}$$

Furthermore, the aggregate level of total factor productivity is given by a CES aggregate of the productivity levels of the intermediate goods:

$$z_t = \left[\int_{-\infty}^{t} a_{it}^{\frac{\theta}{1-\theta}} di\right]^{\frac{1-\theta}{\theta}} \tag{13}$$

since this is a power mean of the productivity levels of all intermediates available, we will refer to z_t as *average productivity*.

Firm i's share in aggregate output as well as the inputs is given by

$$\frac{y_{it}}{y_t} = \left(\frac{a_{it}}{z_t}\right)^{\frac{1}{1-\theta}}, \frac{k_{it}}{k_t} = \frac{l_{it}}{l_t} = \left(\frac{a_{it}}{z_t}\right)^{\frac{\theta}{1-\theta}}, \text{ and } \frac{p_{it}}{p_t} = p_{it} = \left(\frac{z_t}{a_{it}}\right) \tag{14}$$

The aggregate factor demands turn out to satisfy the same optimality conditions as the factor demands of the individual firms in the sense that

$$w_t = \theta(1-\alpha)\frac{y_t}{l_t} \text{ and } r_t + \delta = \theta\alpha\frac{y_t}{k_t} \tag{15}$$

This aggregate production function representation allows us to rewrite the value of the firm as

$$V_{it} = (1-\theta)\int_t^{\infty} e^{-\int_t^s r_j dj} \left(\frac{a_{is}}{z_s}\right)^{\frac{\theta}{1-\theta}} y_s ds \tag{16}$$

which depends on the paths of the firm's productivity level a_{it}, aggregate productivity, z_t, the real interest rate, r_t, output, y_t.

Technology implementation:

Prior to starting production, intermediate goods producers decide at which productivity level to implement the technology they use to produce their intermediate good. After that moment, they learn to produce the intermediate good more efficiently and the productivity level a_{it} increases until it reaches the *potential productivity level of intermediate* i, which we denote by $\overline{a}_i$.

Just like in Parente (1994) and Basu and Weil (1998), learning occurs exogenously. We define the *implementation level* of intermediate i at time t as

$$x_{it} = \left(a_{it}/\overline{a}_i\right)^{\frac{\theta}{1-\theta}}, \text{ where } x_{it} \in [0,1] \tag{17}$$

We assume that learning leads x_{it} to evolve according to

$$\dot{x}_{it} = \lambda\left(1 - x_{it}\right) \tag{18}$$

where $\lambda > 0$ is the *learning rate*.

This learning pattern implies that

$$a_{is}^{\frac{\theta}{1-\theta}} = \left(1 - e^{-\lambda(s-t)}\right)\overline{a}_i^{\frac{\theta}{1-\theta}} + e^{-\lambda(s-t)} a_{it}^{\frac{\theta}{1-\theta}} \text{ for } s > t. \tag{19}$$

Given this path of productivity, the value of the firm can be written solely as a function of its current productivity level and the paths of aggregate productivity, the real interest rate, and output. That is, the value function simplifies to

$$\begin{aligned} V_{it}\left(x_{it}\right) &= (1-\theta)\,\overline{a}_i^{\frac{\theta}{1-\theta}} \int_t^{\infty} e^{-\int_t^s r_j dj} \left(\frac{1}{z_s}\right)^{\frac{\theta}{1-\theta}} y_s ds \\ &\quad - (1-\theta)\,\overline{a}_i^{\frac{\theta}{1-\theta}} \left(1 - x_{it}\right) \int_t^{\infty} e^{-\int_t^s r_j dj} e^{-\lambda(s-t)} \left(\frac{1}{z_s}\right)^{\frac{\theta}{1-\theta}} y_s ds \end{aligned} \tag{20}$$

which is linear in the implementation level, x_{it}.

This value consists of two terms. The first term is the present discounted value of the firm's flow profits if it would have implemented its technology at full potential, i.e. $x_{it} = 1$.

The second term is the loss in value due to the firm's productivity being below potential on its learning curve.

Firms only decide where on their learning curve to start. The subsequent path of the implementation level is determined by the exogenous learning process. We assume that new intermediates are adopted at the moment that they are invented. This implies that, at each moment in time, the firm that obtains the monopoly rights to supply intermediate t has to decide on its initial implementation level, x_{tt}. Figure 1 depicts the productivity path, a_{it}, of intermediate i, implemented at time i at level x_{ii} and subsequently converging to $\overline{a}_i$ due to the exogenous learning process.

The implementation cost of good t at time t takes the form

$$C_t^{implement}(x_{tt}) = (1-\theta)\,\xi z_t^{\frac{1}{1-\alpha}} \left(\frac{\overline{a}_t}{z_t}\right)^{\frac{\theta}{1-\theta}} \left[-\ln(1-x_{tt}) - x_{tt}\right] \tag{21}$$

Here, $\xi > 0$ is the implementation cost parameter and x_{tt} is the initial implementation level. In addition, $z_t^{\frac{1}{1-\alpha}}$ is a scaling factor that guarantees balanced growth, while $\left(\frac{\overline{a}_t}{z_t}\right)^{\frac{\theta}{1-\theta}}$ captures the fact that it is more costly to implement technologies whose potential productivity level, $\overline{a}_t$, is further from the average productivity level in the economy, z_t.

The optimal implementation level, x_{tt}, maximizes the difference between the value of the firm and the implementation costs

$$V_{tt}(x_{tt}) - C_t^{implement}(x_{tt}) \tag{22}$$

As we show in the Appendix, x_{tt} is equal to

$$x_{tt} \equiv \left(\frac{a_{tt}}{\overline{a}_t}\right)^{\frac{\theta}{1-\theta}} = \frac{b_{xt}y_t}{\xi z_t^{\frac{1}{1-\alpha}} + b_{xt}y_t} \tag{23}$$

where

$$b_{xt} = \int_t^{\infty} e^{-\int_t^s r_j dj} e^{-\lambda(s-t)} \left(\frac{z_t}{z_s}\right)^{\frac{\theta}{1-\theta}} \frac{y_s}{y_t} ds \tag{24}$$

Intuitively, the marginal benefit of implementation is increasing in the term $b_{xt}y_t$ while the marginal cost is increasing in $\xi z_t^{\frac{1}{1-\alpha}}$. The optimal implementation level is therefore increasing in the former and decreasing in the latter.

The resulting value at t of the firm that produces the last good introduced net of implementation costs, i.e. the value of (22) in the optimal implementation level, equals

$$V_t^* = (1-\theta)\, z_t^{\frac{1}{1-\alpha}} \left(\frac{\overline{a}_t}{z_t}\right)^{\frac{\theta}{1-\theta}} \left[\frac{y_t}{z_t^{\frac{1}{1-\alpha}}}\left(b_{0t}+b_{xt}\right) - \xi \ln\left(1+\frac{y_t}{z_t^{\frac{1}{1-\alpha}}}\frac{b_{xt}}{\xi}\right)\right] \tag{25}$$

where

$$b_{0t} = \int_t^{\infty} e^{-\int_t^s r_j dj}\left(1-e^{-\lambda(s-t)}\right)\left(\frac{z_t}{z_s}\right)^{\frac{\theta}{1-\theta}} \frac{y_s}{y_t} ds \tag{26}$$

2.4 R&D sector

Following Reinganum (1989) and Tirole (1988), we model the R&D process as a patent race. One patent is awarded to the innovator who, within the instant, develops the intermediate good with highest potential productivity.

Let g_t denote the growth rate of potential productivity between the intermediate good developed at $t-dt$ and t. That is:

$$\frac{\dot{\overline{a}}_t}{\overline{a}_t} = g_t \tag{27}$$

The intensity of research and development determines g_t. In particular, the cost of inventing an intermediate good whose potential productivity represents a growth in the technological frontier of g_t is equal to

$$C_t^{R\&D}(g_t) = \begin{cases} 0 & \text{for } g_t < 0 \\ (1-\theta)\,\phi z_t^{\frac{1}{1-\alpha}}\left(\frac{\overline{a}_t}{z_t}\right)^{\frac{\theta}{1-\theta}} g_t & \text{for } g_t \geq 0 \end{cases} \tag{28}$$

where $\phi > 0$ is the R&D cost parameter, $z_t^{\frac{1}{1-\alpha}}$ is a scaling factor that guarantees balanced growth, and $\left(\frac{\overline{a}_t}{z_t}\right)^{\frac{\theta}{1-\theta}}$ reflects the fact that it is more costly to develop technologies whose potential productivity is further from the average productivity in the economy.

The R&D patent race between innovators brings the growth rate of the potential productivity induced by intermediate good t to the point where the value of the intermediate good equals the R&D cost. That is,

$$V_t^* = C_t^{R\&D}(g_t) \tag{29}$$

which can be rewritten as

$$\phi g_t = \frac{y_t}{z_t^{\frac{1}{1-\alpha}}} (b_{0t} + b_{xt}) - \xi \ln\left(1 + \frac{b_{xt}}{\xi} \frac{y_t}{z_t^{\frac{1}{1-\alpha}}}\right) \tag{30}$$

3 Equilibrium

The dynamic equilibrium allocation of resources in this economy can be described by a set of differential equations. In this section, we present these equations, we transform them to allow us to define a balanced growth path and transitional dynamics.

Crucial for the equilibrium path of the economy are three measures of productivity. The first is the *potential productivity level* of the newest intermediate that is introduced, $\overline{a}_t$. The second is the *average potential productivity level* at which all currently available intermediate goods technologies can be operated, which equals

$$\overline{z}_t = \left[\int_{-\infty}^{t} \overline{a}_i^{\frac{\theta}{1-\theta}} di\right]^{\frac{1-\theta}{\theta}} \tag{31}$$

The third is the *average productivity level*, z_t. Note that both the average productivity and average potential productivity levels are aggregates that summarize the potential and actual productivities across intermediate goods.

Definition: The equilibrium of this economy is a path of the variables

$$\{y_t, c_t, i_t, k_t, x_{tt}, b_{xt}, b_{0t}, g_t, z_t, \overline{z}_t, \overline{a}_t\} \tag{32}$$

that satisfies the following 11 equations.

(i) The resource constraint that implies that the final good output is either consumed, saved, used for implementation purposes, or used for R&D

$$y_t = c_t + i_t + (1-\theta) z_t^{\frac{1}{1-\alpha}} \left(\frac{\overline{a}_t}{z_t}\right)^{\frac{\theta}{1-\theta}} (\xi [-\ln(1-x_{tt}) - x_{tt}] + \phi g_t) \tag{33}$$

(ii) The consumption Euler equation[6], (2).

[6] The optimal capital input condition, (15), is what pins down the real interest rate in the Euler equation as a function of the marginal revenue of capital.

(iii) The aggregate production function, (11).

(iv) The capital accumulation equation

$$\dot{k}_t = i_t - \delta k_t \tag{34}$$

(v) The optimal implementation level, x_{tt}, given by (23).

(vi) Equilibrium R&D condition that determines the growth rate g_t, (30).

(vii + $viii$) The expressions for the present discounted value coefficients, b_{xt} and b_{0t}, given by (24) and (26).

(ix) The law of motion for $\overline{a}_t$, (27).

(x) The law of motion for average potential productivity,

$$\dot{\left(\overline{z}_t^{\frac{\theta}{1-\theta}}\right)} = \overline{a}_t^{\frac{\theta}{1-\theta}} \tag{35}$$

(xi) The law of motion for average productivity, derived in the Appendix, which is

$$\dot{\left(z_t^{\frac{\theta}{1-\theta}}\right)} = \lambda\left(\overline{z}_t^{\frac{\theta}{1-\theta}} - z_t^{\frac{\theta}{1-\theta}}\right) + \overline{a}_t^{\frac{\theta}{1-\theta}} x_{tt}^* \tag{36}$$

Since c_t, y_t, k_t, and i_t are not stationary on the equilibrium path, we transform them into stationary variables before exploring the transitional dynamics and steady state. We do so by scaling them by the average productivity trend $z_t^{1/(1-\alpha)}$. The resulting detrended variables are

$$c_t^* = \frac{c_t}{z_t^{\frac{1}{1-\alpha}}},\ y_t^* = \frac{y_t}{z_t^{\frac{1}{1-\alpha}}},\ k_t^* = \frac{k_t}{z_t^{\frac{1}{1-\alpha}}},\ \text{and}\ i_t^* = \frac{i_t}{z_t^{\frac{1}{1-\alpha}}} \tag{37}$$

In addition, we define transformed present discounted value coefficients as

$$b_{xt}^* = b_{xt} \tag{38}$$

$$b_{0t}^* = b_{0t} + b_{xt}^* = \int_t^\infty e^{-\int_t^s r_j dj}\left(\frac{z_t}{z_s}\right)^{\frac{\theta}{1-\theta}} \frac{y_s}{y_t} ds \tag{39}$$

Because our aim is to obtain a stationary representation of the equilibrium path, we detrend our three productivity measures, $\overline{a}_t$, $\overline{z}_t$, and z_t. This yields the following two detrended productivity measures

$$\overline{\chi}_t = \left(\frac{\overline{a}_t}{\overline{z}_t}\right)^{\frac{\theta}{1-\theta}} \text{ and } \chi_t = \left(\frac{\overline{z}_t}{z_t}\right)^{\frac{\theta}{1-\theta}} \tag{40}$$

Here, $\overline{\chi}_t$ represents the gap between the potential productivity of the last intermediate good invented and the average potential productivity level. We therefore refer to it as the *potential productivity gap.* The second measure, χ_t, reflects the gap between average potential productivity and the average actual productivity level at which the current intermediates are implemented. We call this the *implementation gap.*

In the Appendix we define a stationary equilibrium in terms of the 10 transformed variables

$$\{y_t^*, i_t^*, c_t^*, k_t^*, x_{tt}, g_t, b_{0t}^*, b_{xt}^*, \chi_t, \overline{\chi}_t\} \tag{41}$$

The three state variables of the 10 equation system are k_t^*, χ_t, and $\overline{\chi}_t$.

3.1 Steady state

A steady state of this economy is an equilibrium path on which the ten transformed variables are constant. Let the steady state values of these variables be

$$\left\{\widetilde{y}^*, \widetilde{i}^*, \widetilde{c}^*, \widetilde{k}^*, \widetilde{x}, \widetilde{g}, \widetilde{b}_0^*, \widetilde{b}_x^*, \widetilde{\chi}, \widetilde{\overline{\chi}}\right\} \tag{42}$$

As we prove in the Appendix, the steady state exists and is unique whenever

$$\psi = \left[\frac{\theta}{1-\theta} + \left(\frac{1}{\sigma} - 1\right)\frac{1}{1-\alpha}\right] > 0 \tag{43}$$

which is the case under the empirically plausible sufficient condition that $\frac{\theta}{1-\theta} > \frac{1}{1-\alpha}$.[7]

The steady state is determined by the following equilibrium R&D zero profit condition

$$\phi\widetilde{g} = y^*\widetilde{b}_0^* - \xi \ln\left(1 + \frac{y^*\widetilde{b}_x^*}{\xi}\right) \tag{44}$$

[7] We also derive the conditions under which the parameters are such that the household's and intermediate goods producer's objectives are bounded on the balanced growth path.

where the steady state level of output, y^*, is determined by the law of motion of aggregate capital and the aggregate production function and equals

$$\widetilde{y}^* = \left(\frac{\alpha\theta}{\rho + \delta + \frac{\widetilde{g}}{\sigma(1-\alpha)}} \right)^{\frac{\alpha}{1-\alpha}} \tag{45}$$

and the steady state present discounted value terms are

$$\widetilde{b}_0^* = \frac{1}{\rho + \psi\widetilde{g}} \text{ and } \widetilde{b}_x^* = \frac{1}{(\rho + \lambda + \psi\widetilde{g})} \tag{46}$$

The left hand side of (44) reflects the normalized steady state R&D expenditures, while the right hand side corresponds to the normalized present discounted value of the profits net of the costs of implementing a new intermediate goods technology.

Given $\widetilde{g}$, the optimal implementation level satisfies

$$\frac{\widetilde{x}}{1-\widetilde{x}} = \frac{\widetilde{y}^*}{\xi\left(\rho + \lambda + \psi\widetilde{g}\right)}, \text{ and thus } \widetilde{x} = \frac{1}{1 + \frac{\xi(\rho+\lambda+\psi\widetilde{g})}{\widetilde{y}^*}} \tag{47}$$

Note that this expression is decreasing in $\widetilde{g}$. This is because an increase in the growth rate increases the endogenous rate of obsolescence of new technologies. It does so through two channels.

First, increased obsolescence reduces the steady state level of (detrended) capital and thus of output, $\widetilde{y}^*$. This reduces the size of the market and the value of the firm. Second, $\widetilde{g}$ increases the effective discount rate both through a higher interest rate, $\widetilde{r}$, and through a higher replacement rate of demand by future, more productive, intermediate goods.[8] Both of these effects reduce the marginal value of implementing at a higher level and therefore lead to a lower $\widetilde{x}^*$.

The steady state potential productivity gap equals

$$\widetilde{\widetilde{\chi}} = \frac{\theta}{1-\theta}\widetilde{g}, \tag{48}$$

[8] There is a third effect of $\tilde{g}$ on the effective discount rate. Namely, the positive effect that $\tilde{g}$ has on the growth of aggregate demand. Our assumption that $\psi > 0$ implies that this effect is dominated by the previous two and therefore the effective discount rate is increasing in $\tilde{g}$.

while the implementation gap is

$$\widetilde{\chi} = \frac{\lambda + \widetilde{\widetilde{\chi}}}{\lambda + \widetilde{x}^* \widetilde{\widetilde{\chi}}} \tag{49}$$

on the balanced growth path.

For completeness, the scaled steady state levels of capital, investment and consumption are given by the following expressions:

$$\widetilde{k}^* = (\widetilde{y}^*)^{1/\alpha} \tag{50}$$

$$\widetilde{i}^* = \left(\delta + \frac{1}{1-\alpha}\widetilde{g}\right)\widetilde{k}^* \tag{51}$$

$$\widetilde{c}^* = \widetilde{y}^* - \widetilde{i}^* - (1-\theta)\,\widetilde{\widetilde{\chi}}\widetilde{\chi}\left[\xi\left(-\ln\left(1-\widetilde{x}\right) - \widetilde{x}\right) + \phi\widetilde{g}\right] \tag{52}$$

Comparative statics of growth and implementation in steady state

Since we focus on the interaction between innovation and technology implementation, we consider next how the steady state growth rate, $\widetilde{g}$, and implementation level, $\widetilde{x}$, vary as a function of the R&D costs, ϕ, the implementation costs, ξ, as well as the learning rate, λ. Table 1 summarizes the signs of these comparative static exercises. The details underlying these results are presented in the Appendix.

The intuition behind these results is best understood through Figure 2. This figure depicts the steady state R&D free entry condition, (44), as the g-locus and the steady state optimal implementation condition, (47), as the x-locus. The latter is downward sloping because of the negative effect that increased obsolescence has on the implementation level. The former is a vertical line because we have substituted the optimal implementation level into the value of the intermediate good firm that appears in the R&D free entry condition. This figure reduces the comparative statics of $\widetilde{g}$ and $\widetilde{x}$ to shifts in the g- and x-loci.

An increase in the R&D cost parameter, ϕ, reduces the R&D efforts of the innovators, for a given x, causing an inward shift in the g-locus, and has no effect on the x-locus. As a result, an increase in the R&D cost reduces the steady state growth rate, increases the present discounted value of demand faced by an intermediate good producer and leads to an increase in the steady state implementation level.

An increase in the learning rate, λ, makes firms implement less and depend more on their subsequent learning for a given g. Thus, the x-locus shifts inward in response to an increase in the learning rate. On the other hand, λ increases the productivity growth rate for the intermediate good firm thus increasing its value and the return to R&D investments. As a consequence, an increase in λ shifts the g-locus outward. The result is that, in response to an increase in the learning rate, the steady state growth rate increases while the implementation level decreases.

An increase in the implementation cost reduces the return to implementation for a given g causing an inward shift in the x-locus. It also reduces the value of an intermediate good firm and the return to innovation causing an inward shift in the g-locus. This means that our diagram, in principle, does not suffice to determine the sign of the effect of an increase of ξ on $\widetilde{g}$ and $\widetilde{x}$. In the Appendix we show, however, that the downward movement in the x-locus dominates the leftward shift of the g-locus in this case and that both the steady state growth rate as well as the implementation level are decreasing in the implementation cost.

These comparative statics actually provide an interesting insight into the effects of two alternative policies aimed at stimulating long-run growth: (i) subsidizing R&D, by reducing ϕ, and (ii) subsidizing implementation and speeding up diffusion through reducing ξ. Both of these policies will increase the long-run, steady state, growth rate. R&D subsidies, however, come at the cost of reducing implementation and slowing down diffusion because of the obsolescence cost it imposes. This is not the case for a reduction of implementation costs. Such a reduction will also increase the implementation level and diffusion.

Technology diffusion

Underlying the steady state is a continuous process of diffusion of new intermediate goods. Contrary to other models of endogenous growth, the rate of diffusion of (intermediate good) technologies is endogenously determined in the model here. This rate of diffusion can be considered in two ways.

The first way is to consider the market share of an individual intermediate good. We will denote the market share of intermediate i at time t as

$$s_{it} = \frac{p_{it} y_{it}}{y_t} = \begin{cases} 0 & \text{if } t < i \\ \left(\frac{1}{p_{it}}\right)^{\frac{\theta}{1-\theta}} = \left(\frac{a_{it}}{z_t}\right)^{\frac{\theta}{1-\theta}} & \text{otherwise} \end{cases} \tag{53}$$

In the Appendix, we show that in the steady state

$$s_{it} = \left[1 - (1 - \widetilde{x})\, e^{-\lambda(t-i)}\right] e^{-\frac{\theta}{1-\theta}\widetilde{g}(t-i)} \widetilde{\widetilde{\chi}}\widetilde{\chi} \tag{54}$$

As shown in equation (55), two opposite forces drive the dynamics of the market share for i^{th} intermediate good. On the one hand, learning to produce efficiently the intermediate good induces a productivity gain that raises the market share. On the other, the increase in the average productivity due to the development of new (more productive intermediate goods) and to learning makes obsolete the i^{th} intermediate good reducing its market share.

$$\dot{s}_{it} = \overbrace{\lambda e^{-\frac{\theta}{1-\theta}\widetilde{g}(t-i)} \widetilde{\widetilde{\chi}}\widetilde{\chi}}^{\text{Learning}} - \overbrace{\left[\lambda + \frac{\theta}{1-\theta}\widetilde{g}\right] s_{it}}^{\text{Obsolescence}} \tag{55}$$

When the implementation costs are large the market initial share is so small that the learning effect dominates and the market share is initially increasing. As the market share increases, the endogenous obsolescence of the intermediate good dominates and the market share declines to zero.

An initially increasing and subsequently decreasing market share is consistent with evidence on product life cycles documented in Jovanovic and McDonald (1994), Aizcorbe, Corrado, and Doms (2000), and Kotler (2005).

The second way to characterize the rate of diffusion is to consider the share of expenses in intermediate goods that are newer than good i. Given the factor demands (7), this is equivalent to the fraction of workers employed in the production of an intermediate good intermediate good newer than i. This fraction is given by

$$S_{it} = \int_i^t \frac{p_{\iota t} y_{\iota t}}{y_t} d\iota = \int_i^t l_{\iota t} d\iota \tag{56}$$

As we show in the Appendix, on the balanced growth path, this adoption share follows a diffusion curve of the form

$$S_{it} = 1 - e^{-\frac{\theta}{1-\theta}\widetilde{g}(t-i)} \left[\widetilde{\chi} - (\widetilde{\chi} - 1)\, e^{-\lambda(t-i)}\right] \text{ for } t \geq i \tag{57}$$

3.2 Transitional dynamics

The equilibrium dynamics of the model are determined by three state variables. Thus, analytical results about the dynamic properties are beyond the scope of our analysis. We resort, instead, to a numerical approximation of the transitional dynamics for a specific set of calibrated parameter values. The numerical approximation of the transitional dynamics is based on the log-linearization derived in the Appendix.

Calibration:

The parameters that need to be calibrated are listed in Table 2. We calibrate our model such that t is measured in years.

For the preference parameters, i.e. the discount rate, ρ, and the intertemporal elasticity of substitution, σ, as well as for the capital depreciation rate, δ, we use the parameter values from Cooley and Prescott (1995).

Given our model's emphasis on R&D and implementation, it seems particularly appropriate to use the evidence from Corrado, Hulten, and Sichel (2006, CHS in the following) to calibrate most other parameters from our model. CHS provide an analysis of the sources of growth of the U.S. business sector that includes extensive measures of intangible capital, including R&D.

We calibrate the demand elasticity parameter, θ, and the capital elasticity of output, α, to match U.S. income shares reported by CHS. First, labor costs represent 60% of corporate income. Second, returns to intangible capital represent 15% of the price. In our model these are the profits that flow to the implementation and R&D costs.

The remaining parameters are the learning rate, λ, and the implementation and R&D cost parameters, ξ and ϕ. These parameters are chosen to match three observations, the first two of which are based on CHS. First, investment in R&D by the U.S. business sector is approximately 5.7% of corporate income. Second, adjusted for intangible capital, labor productivity in the U.S. grew at an average rate of 1.9% a year over the 1973-2003 period. Finally, we use evidence from Bahk and Gort (1993) on learning by doing in U.S. manufacturing plants. In particular, we choose our parameters to match their empirical result that a 1% increase in a firm's cumulative output leads to a 0.028% increase in its TFP level.

The way in which we specifically match the parameters with these facts is described in Appendix A. The resulting values of the parameters are listed in the last column of Table 2.

Steady state:

The steady state values of the equilibrium variables are given in the 'equilibrium' column of Table 3. The resulting implementation level is about 4.3%, while the implementation gap is 4.6. The relatively low implementation level in steady state induces the market share of new intermediates to increase at first and then to start decreasing after about 10 years. This can be seen from the 'equilibrium' curve in the top panel of Figure 3. That is, 10 years into the life cycle of an intermediate in this economy the endogenous obsolescence starts to dominate the learning effect. The implied diffusion curve is plotted as the 'equilibrium' curve in the bottom panel. In the decentralized equilibrium, 50% of the workers produce intermediates that were invented less than 18 years ago.

Transitional dynamics:

We compare the transitional dynamics of our model with those of the standard Neoclassical growth model, explained for example in Barro and Sala-i-Martin (2004). That is, if growth is exogenous, constant and equal to the steady state growth rate, $\widetilde{g}$, and if implementation costs are zero, such that $\xi = 0$, then our model boils down to the Neoclassical growth model with a markup distortion[9]. In particular, we compare the dynamics in response to a 1% deviation of capital above its steady state detrended level. Figure 4 contains the impulse responses for the model with implementation, the 'implementation' line, and the Neoclassical benchmark, the 'NC benchmark' line.

In the benchmark model the excess capital is used for current and future consumption

[9] The dynamic equilibrium equations of this restricted model are provided in the Appendix.

through intertemporal substitution. The same is true in the model with implementation. However, in the model with implementation there is not just intertemporal substitution of consumption. An above trend capital stock increases the size of the market and thus the present discounted value of the stream of future profits. This raises the benefits from innovation and implementation. As a result, the above steady state level of capital leads to a transitory increase in the growth rate of potential productivity and in the implementation level and to a permanent increase in the level of productivity.[10] This is the main departure of our model from the dynamics of the Neoclassical benchmark.

Relative to the Neoclassical model, the additional implementation and R&D expenditures seem to come mainly at the cost of investment and not of consumption. What is remarkable is that, in terms of the detrended variables, the impulse responses in the model with implementation are very similar to that of the neoclassical growth model.

Our model has two state variables in addition to the capital stock: the implementation gap and the potential productivity gap. Figure 5 plots the impulse responses to a 1% deviation of the these gaps from their steady state level. For comparison purposes, we have also included the impulse response to capital. Increases in these two gaps increase implementation and R&D costs and therefore reduce both implementation and innovation. The increase in the implementation and R&D costs induce a shift in resources from implementation and innovation to investment in physical capital. As a result, these changes induce very significant transitory declines in the implementation level and the growth rate of potential productivity and an important permanent decline in the level of productivity. There seems, however, to be little or no effect on detrended consumption and detrended output.

4 Social planner solution

So far, we have focused on the decentralized equilibrium outcome of our economy. Next, we explore the optimal innovation and implementation decisions from the social planner's perspective.

[10] This may not be obvious from the impulse response functions because output is detrended by $z_t^{\frac{1}{1-\alpha}}$.

In this section we derive the first order necessary conditions for the social planner's problem. We study the steady state implementation level, $\widetilde{x}^{sp}$, and growth rate, $\widetilde{g}^{sp}$, chosen by the planner, and compare them with those resulting from the decentralized equilibrium. Finally, we show how the planner's steady state resource allocation can be supported through taxes and subsidies in the decentralized equilibrium.

The social planner in this economy chooses a path for

$$\{c_s, y_s, i_s, k_s, x_{ss}, g_s, z_s, \overline{a}_s, \overline{z}_s\}_{s=t}^{\infty} \tag{58}$$

to maximize the present discounted value of the representative household's stream of utility, (1), subject to the resource constraint, (33), the final goods production function, (11), the capital accumulation constraint, (34), the law of motion of potential productivity of the newest intermediate, (27), the law of motion of average potential productivity, (35), and the law of motion of average productivity, (36).

The current value Hamiltonian associated with this problem is:

$$\begin{aligned} (59)\quad H_t = {} & \frac{\sigma}{\sigma-1} c_t^{\frac{\sigma-1}{\sigma}} \\ & +\mu_{rt}\left\{ y_t - c_t - i_t - (1-\theta)\, z_t^{\frac{1}{1-\alpha}} \left(\frac{\overline{a}_t}{z_t}\right)^{\frac{\theta}{1-\theta}} \left[\xi\left(-\ln\left(1-x_{tt}^*\right) - x_{tt}^*\right) + \phi g_t\right] \right\} \\ & +\mu_{yt}\left[y_t - \left(z_t^{\frac{\theta}{1-\theta}}\right)^{\frac{1-\theta}{\theta}} k_t^{\alpha} \right] + \mu_{kt}\left[i_t - \delta k_t\right] + \mu_{\overline{a}t}\left[\frac{\theta}{1-\theta}\overline{a}_t^{\frac{\theta}{1-\theta}} g_t\right] \\ & +\mu_{\overline{z}t}\left[\overline{a}_t^{\frac{\theta}{1-\theta}}\right] + \mu_{zt}\left[\lambda\left(\overline{z}_t^{\frac{\theta}{1-\theta}} - z_t^{\frac{\theta}{1-\theta}}\right) + \overline{a}_t^{\frac{\theta}{1-\theta}} x_{tt}^*\right] \end{aligned}$$

where μ_{rt} is the costate variable associated with the resource constraint, μ_{yt} is the costate variable associated with the aggregate production function, $\mu_{\overline{a}t}$ is the costate variable associated with the law of motion of potential productivity of the last intermediate good, $\mu_{\overline{z}t}$ is the costate variable associated with the law of motion of average potential productivity, and μ_{zt} is the costate variable associated with the law of motion of average productivity.

At any instant along the efficient resource allocation path the planner equates the marginal utility cost of implementation to the shadow value of the marginal average productivity that this implementation generates. This is represented by equation (60). The planner also equates the marginal utility cost of a better innovation, i.e. of g_t, to the shadow value of the additional potential productivity this innovation generates through $\overline{a}_t$. Mathematically, this corresponds to equation (61).

$$\xi(1-\theta)\frac{x_{tt}}{1-x_{tt}}c_t^{-\frac{1}{\sigma}} = \mu_{z,t}z_t^{\frac{\theta}{1-\theta}-\frac{1}{1-\alpha}} \tag{60}$$

$$\phi(1-\theta)c_t^{-\frac{1}{\sigma}} = \frac{\theta}{1-\theta}\mu_{\bar{a},t}z_t^{\frac{\theta}{1-\theta}-\frac{1}{1-\alpha}} \tag{61}$$

4.1 Distortions in decentralized equilibrium

We characterize the full dynamics of the planner's optimal resource allocation in the Appendix and focus here on the resulting steady state, also derived in the the Appendix. To distinguish the planner's steady state allocation from that of the decentralized equilibrium, we denote the planner's allocation with a superscript sp. Thus, $\widetilde{x}^{sp}$ and $\widetilde{g}^{sp}$ are the planner's steady state implementation level and growth rate respectively.

The planner's steady state level of detrended output is

$$\widetilde{y}^{*sp} = \left[\frac{\alpha}{\rho+\delta+\frac{\widetilde{g}^{sp}}{(1-\alpha)\sigma}}\right]^{\frac{\alpha}{1-\alpha}} = \left(\frac{1}{\theta}\right)^{\frac{\alpha}{1-\alpha}}\left[\frac{\alpha\theta}{\rho+\delta+\frac{\widetilde{g}^{sp}}{(1-\alpha)\sigma}}\right]^{\frac{\alpha}{1-\alpha}} \tag{62}$$

For a given growth rate, it is higher than the output level in the decentralized equilibrium, because the monopolistic competition between the intermediate goods producers leads to an inefficiently low level of output in the decentralized equilibrium.

The steady state implementation level satisfies

$$\frac{\widetilde{x}^{sp}}{1-\widetilde{x}^{sp}} = \frac{1}{\theta}\left[1+(1-\theta)\left(\frac{\theta}{1-\theta}-\frac{1}{1-\alpha}\right)\frac{\widetilde{\overline{\chi}}\widetilde{\chi}}{\widetilde{y}^{*sp}}\left[\xi\left(-\ln\left(1-\widetilde{x}^{sp}\right)-\widetilde{x}^{sp}\right)+\phi\widetilde{g}^{sp}\right]\right]\frac{\widetilde{y}^{*sp}}{\xi\left(\rho+\lambda+\psi\widetilde{g}^{sp}\right)} \tag{63}$$

We call this the x^{sp}-locus. It is the social planner's counterpart to (47). This implies that, given the growth rate, the planner's implementation level is higher than that in the decentralized equilibrium for three reasons. First, output in the planner's allocation is higher, therefore increasing the marginal benefit of implementation. Second, the monopolistic competition also influences the marginal benefit of implementation because the firms that implement in the decentralized equilibrium equate the marginal revenue product of implementation to its marginal cost. The planner instead equates the marginal product of implementation to its marginal cost. Because of the downward sloping demand curves that the monopolistically competing intermediate goods producers face in the decentralized market, the marginal revenue product is less than the marginal product and, hence, these firms underimplement relative to the social planner. This difference is reflected in the $1/\theta$, term that pre-multiplies the squared bracket in the RHS of equation (63). The final reason that, at a given growth rate, the planner chooses a higher implementation level than is realized in the decentralized equilibrium is that the planner internalizes the effect of the implementation level on the implementation gap. A higher implementation level decreases the implementation gap and, through that, reduces the implementation and R&D costs. This effect is reflected by the second term in the above equation.

The comparison of the steady state output and implementation levels above was done conditional on the growth rate, $\widetilde{g}^{sp}$. Given $\widetilde{x}^{sp}$ and $\widetilde{y}^{*sp}$, the efficient steady state growth rate is determined by the optimal innovation condition

$$\phi = \frac{\theta}{1-\theta} \frac{\overbrace{\left[\xi \left(\frac{\lambda+\rho+\psi\widetilde{g}^{sp}}{\rho+\psi\widetilde{g}^{sp}}\right) \frac{\widetilde{x}^{sp}}{1-\widetilde{x}^{sp}} + \xi \ln\left(1-\widetilde{x}^{sp}\right) - \widetilde{g}^{sp}\phi\right]}^{\text{Mg. Soc. value of } \bar{a}}}{\rho + \frac{1/\sigma-1}{1-\alpha}\widetilde{g}^{sp}} \tag{64}$$

We call this the g^{sp}-locus. The decentralized equilibrium counterpart of this equation, which is a rewritten version of (44), reads

$$0 = \xi \left[\left(\frac{\lambda+\rho+\psi\widetilde{g}}{\rho+\psi\widetilde{g}}\right) \frac{\widetilde{x}}{1-\widetilde{x}} + \ln\left(1-\widetilde{x}\right)\right] - \widetilde{g}\phi \tag{65}$$

For a given implementation level, there are three differences between the optimal innovation condition in the decentralized equilibrium, (65), and that of the social planner, (64).

First, the patent race that determines the R&D intensity in the decentralized economy equalizes the value of the intermediate good producer net of innovation and implementation costs to 0. The social planner, instead, equalizes the marginal value and the marginal cost of innovation. The (scaled) marginal cost of innovation is $\phi > 0$. Hence, the patent race leads to too much innovation in the decentralized equilibrium relative to the planner's.

Second, the planner internalizes the fact that g leads permamently to a higher $\bar{a}^{\frac{\theta}{1-\theta}}$. As a result, the marginal social value of innovating is higher than the market value of the intermediate good porducer, for a given x, for two reasons. On the one hand, a higher g increases potential productivity levels in all future times. Hence the discounting in the RHS of (64). On the other hand, g increases the growth rate of $\bar{a}^{\frac{\theta}{1-\theta}}$ by $\frac{\theta}{1-\theta}\, g$. Hence, the term $\frac{\theta}{1-\theta}$ that pre-multiplies the marginal social value of $\bar{a}$ in the right hand side of equation (64).

Because the x^{sp}-locus is above the x-locus for all growth rates, the planner's steady state can not be such that both the growth rate and the implementation level are below that of the decentralized steady state. Figure 6 adds the planner's g^{sp}- and x^{sp}-loci to Figure 2. It shows the three possible cases of the planner's steady state compared to the decentralized one: (i) $\widetilde{g}^{sp} < \widetilde{g}$ and $\widetilde{x}^{sp} > \widetilde{x}$; (ii) $\widetilde{g}^{sp} > \widetilde{g}$ and $\widetilde{x}^{sp} > \widetilde{x}$; and (iii) $\widetilde{g}^{sp} > \widetilde{g}$ and $\widetilde{x}^{sp} < \widetilde{x}$.

4.2 Supporting the planner's solution

We do not formally characterize these three cases, but instead describe the policies that support the social planner's steady state equilibrium in the decentralized economy. Such a policy analysis serves two main purposes. First, it allows us to consider how private costs and benefits can be corrected to coincide with those in the efficient resource allocation. Second, it helps us understand how optimal R&D policies interact with policies aimed at affecting the implementation and adoption of technologies.

The decentralized and social planner's steady states generally differ in the R&D intensity, in the implementation level, and in the saving rate. To align these three margins in the decentralized equilibrium with those in the planner's allocation, it is necessary to use three instruments.

The instruments we consider to decentralize the social planner's steady state are an R&D subsidy, s_g, a subsidy to implementation, s_x, and a subsidy for capital, s_k. These subsidies can potentially be negative, and thus be a tax. We assume that these subsidies are financed by a lump-sum tax or subsidy that balances the government's budget. Because of the non-distortionary nature of this lump-sum tax, we do not focus on it in our analysis.

We derive the optimality conditions in the decentralized equilibrium under the subsidies in the Appendix. In order to see how the distortionary subsidies can be used to support the planner's solution, we consider each of the optimality conditions that they affect in the decentralized equilibrium sequentially and compare them with those of the planner's steady state.

Conditional on the other subsidies supporting the planner's growth rate and implementation level, the optimal capital input subsidy is such that

$$(66)\qquad \widetilde{y}^{*sp} = \left[\frac{1}{1-s_k}\frac{\alpha\theta}{\rho+\delta+\frac{\widetilde{g}^{sp}}{(1-\alpha)\sigma}}\right]^{\frac{\alpha}{1-\alpha}}$$

Hence, the optimal capital input subsidy is $s_k = 1-\theta$. This corrects for the distortion induced by the monopolistic competition between the intermediate goods suppliers. In the decentralized equilibrium, this distortion leads to an undersupply of output, a less than efficient level of capital, and thus a lower than optimal saving rate. The capital subsidy increases the private marginal return to capital to offset this.

To support the optimal implementation level, conditional on supporting $\widetilde{y}^{*sp}$ and $\widetilde{g}^{sp}$, the implementation subsidy has to satisfy

$$(67)\qquad \frac{\widetilde{x}^{sp}}{1-\widetilde{x}^{sp}} = \frac{1}{(1-s_x)\,\xi\left(\rho+\lambda+\psi\widetilde{g}^{sp}\right)}\widetilde{y}^{*sp}$$

which, combined with (63), yields that the optimal implementation subsidy should solve

$$(68)\qquad \frac{1}{(1-s_x)} = \frac{1}{\theta}\left[1+(1-\theta)\left(\frac{\theta}{1-\theta}-\frac{1}{1-\alpha}\right)\frac{\widetilde{\widetilde{\chi}}\widetilde{\chi}}{\widetilde{y}^{*sp}}\left[\xi\left(-\ln\left(1-\widetilde{x}^{sp}\right)-\widetilde{x}^{sp}\right)+\phi\widetilde{g}^{sp}\right]\right] > 1$$

s_x corrects for the two effects that are not internalized in the decentralized implementation decision: (i) The effect of monopolistic competition on equilibrium profits, reflected by $(1/\theta)$; (ii) the effect of the implementation decision on the implementation gap and thus on the costs of implementing and inventing. Because both of these effects lead to underimplementation in the decentralized equilibrium relative to the efficient allocation, the optimal policy in this context always involves a subsidy to implementation.

To support the planner's growth rate in the decentralized equilibrium, the R&D subsidy has to satisfy

$$(69)\qquad \widetilde{g}^{sp} = \frac{(1-s_x)}{(1-s_g)}\frac{\xi}{\phi}\left[\left(\frac{\rho+\lambda+\psi\widetilde{g}^{sp}}{\rho+\psi\widetilde{g}^{sp}}\right)\frac{\widetilde{x}^{sp}}{1-\widetilde{x}^{sp}}+\ln\left(1-\widetilde{x}^{sp}\right)\right]$$

which, combined with (64), allows us to write

$$(70)\qquad (1-s_g) = (1-s_x)\left(1-s_g'\right),\ \text{where}\ \left(1-s_g'\right) = \frac{1-\theta}{\theta}\left(\frac{\rho}{\widetilde{g}^{sp}}+\psi\right)$$

Thus the optimal R&D subsidy consists of two parts. The first part corrects for the wedge between private and social implementation costs induced by the implementation subsidy s_x. The second part is given by

$$s_g' = \frac{1-\theta}{\theta}\left(\frac{1}{1-\alpha}\left(\frac{\sigma-1}{\sigma}\right) - \frac{\rho}{\widetilde{g}^{sp}}\right) \tag{71}$$

Intuitively, the social benefits from innovation are, in large part, determined by the intertemporal elasticity of substitution of the representative consumer because the intertemporal elasticity of substitution together with the discount rate, ρ, and the growth rate of the economy, is what determines the effective discount rate at which the planner discounts the future gains from current innovation investments.

When the representative consumer has a low intertemporal elasticity of substitution, i.e. $\sigma < 1$, the planner heavily discounts future gains from current R&D investments. Hence, the efficient allocation involves devoting fewer resources to innovation than the decentralized economy. In that case, the optimal R&D subsidy would be lower than the implementation subsidy and potentially involve a tax on innovative activities.

Conversely, when the agent is more willing to intertemporally substitute consumption today for consumption tomorrow, i.e. $\sigma > 1$, the present discounted value of the social payoffs from R&D are larger and the social planner prescribes more growth than in the decentralized economy.

Besides the insights that we obtained on how the distortions in the decentralized equilibrium can be corrected using the optimal choice of three policy instruments, we obtained another important insight. The optimal R&D and implementation policies are inherently intertwined. This implies that any policy discussion about stimulating or reducing the incentives to innovate should also take into account the incentives to implement and adopt the innovations.

4.3 Quantitative evaluation

To illustrate the planner's steady state, we consider it for the calibrated parameter values of Table 2. The 'planner' column of Table 3 shows the efficient allocation of resources for the same parameters as the decentralized equilibrium outcome we discussed in Subsection 3.2. For the calibrated parameter values, the decentralized equilibrium would have too little growth and implementation. At 1.38%, the efficient growth rate is half a tenth of a percentage point higher than that in the market equilibrium. Moreover, the efficient implementation level is 6.75%, which is almost 2.5 percentage points higher than in the decentralized equilibrium.

These distortions in the market implementation level and growth rate means that technologies diffuse at an inefficiently low rate. To see this, compare the efficient 'planner' diffusion curves with the decentralized 'equilibrium' ones in Figure 3. The efficient 50% diffusion time in the bottom panel of the figure is about 16 years, while the 50% diffusion time in the market equilibrium is closer to 18 years.

The misallocation of the resources in the decentralized economy cause a significant welfare loss. Welfare in the steady state of the decentralized equilibrium is 7.6% lower than that resulting from the planner's allocation. At a constant growth rate, this would amount to 0.36% of steady state consumption. This is more than 40 times larger than the welfare cost for log preferences of consumption fluctuations around a linear trend estimated by Lucas (1987).

The optimal policy derived above can offset the resource misallocation caused by the model' distortions in the decentralized equilibrium. For our calibrated parameter values, the optimal policy involves a 15% capital input subsidy, a 35% implementation subsidy, and a 7% R&D tax.

These policies are in sharp contrast with the policy prescriptions of standard endogenous growth models that do not include the implementation margin. These generally yield an optimal R&D subsidy rather than a tax, as in Jones and Williams (1998, 2000).

If, in our economy, the government tried to implement the optimal policy ignoring implementation, i.e. assuming $\xi = 0$, it would choose a capital input subsidy of 15% and an R&D subsidy of 59%.[11] This, however, would lead to a 5% loss in welfare relative to the policies derived above, which represents only a 2% improvement over doing nothing.

[11] The optimal policies for an economy without implmentation are derived in the Appendix.

5 Conclusion

We have introduced the technology implementation decision in a theory of endogenous growth. In our model, the implementation decision determines the initial productivity of the technology to produce a new intermediate good. The gap between the potential and the actual productivity of the new technology is closed over time through exogenous learning. Our model is sufficiently tractable to analyze not only its steady state but also its transitional dynamics. The addition of a technology implementation decision to an, otherwise, standard model of endogenous growth leads to two important insights.

First, the equilibrium effect of growth on implementation is the opposite of the effect of implementation on growth. An increase of the growth rate increases the rate of endogenous obsolescence of technologies. It reduces the present discounted value of profits and therefore the benefit and level of implementation. An increase in implementation leads to a more intensive adoption of new technologies and raises the market value of the firms that produce them. This raises the return to R&D, and thus leads to an increase in long-run growth.

Second, optimal policy in our model does not only involve R&D taxes or subsidies but also requires intervening in the cost of implementation. This suggests that any discussion of policies to stimulate long run growth should not only consider subsidizing the activities of innovators, but, just as importantly, consider subsidizing the implementation, and through it the diffusion, of the technologies that these innovators create.

The model that we analyzed here is basically a stylized model of the world technology frontier and, as such, the normative results in this paper could be interpreted as applying to a 'world growth policy'. In practice, such policies are not decided on at a global level, but, instead, are chosen by national governments. The consideration of optimal national implementation policies versus R&D tax credits becomes even more relevant when it is done in a multi-country context with R&D spillovers. This is the subject of our future research.

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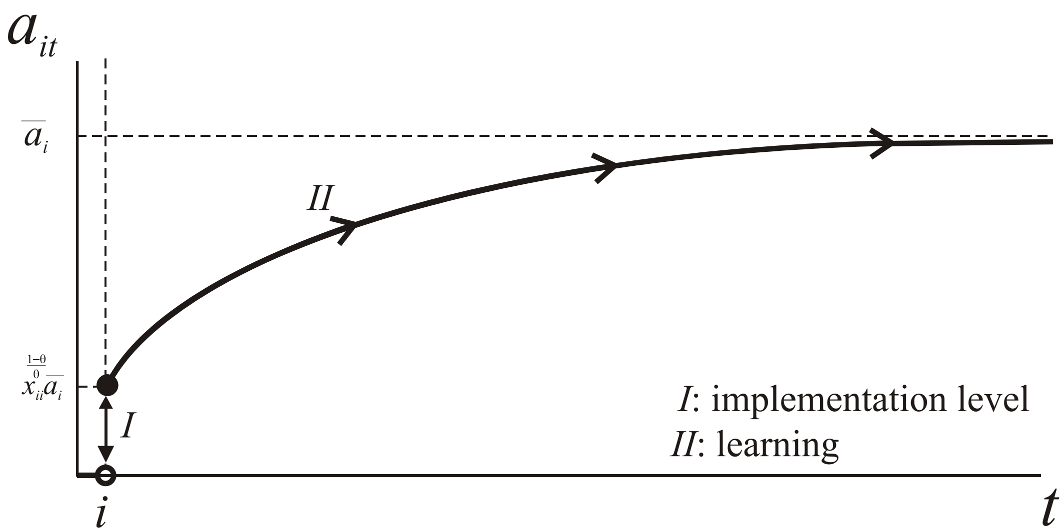

Figure 1: Path of technology specific productivity

Table 1: Comparative statics of decentralized steady state

Sign of partial derivative of...		with respect to...		
		R&D cost	implementation cost	learning rate
		ϕ	ξ	λ
Growth rate	$\widetilde{g}$	-	-	+
Implementation level	$\widetilde{x}$	+	-	-

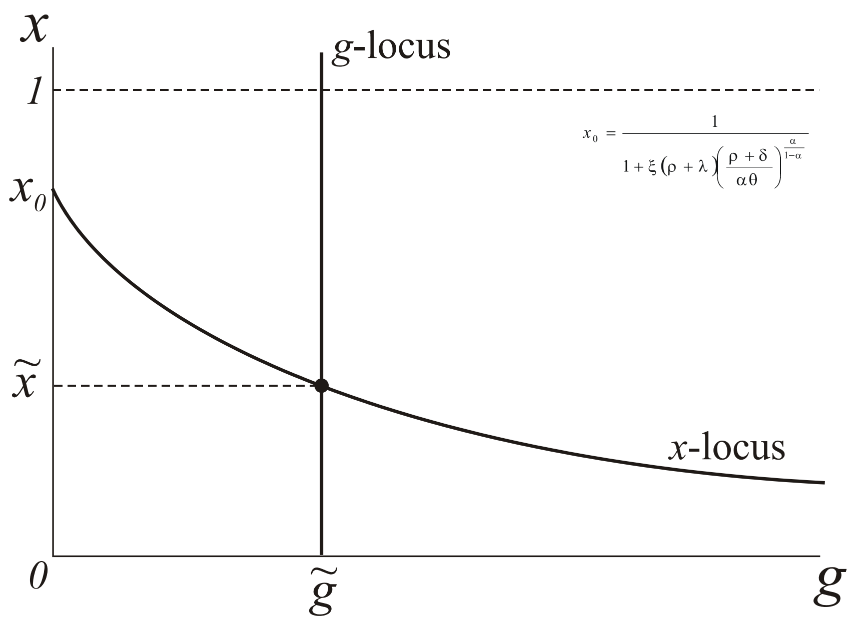

Figure 2: Determination of steady state growth rate and implementation level

Table 2: Model parameters

	parameter	interpretation	value
1.	ρ	discount rate	0.050
2.	σ	intertemporal elasticity of substitution	1.000
3.	θ	reciproce of gross markup factor	0.850
4.	α	capital elasticity of output	0.300
5.	λ	learning rate	0.017
6.	ξ	implementation cost parameter	216
7.	ϕ	R&D cost parameter	115
8.	δ	capital depreciation rate	0.050

Calibration based on t being measured in years.

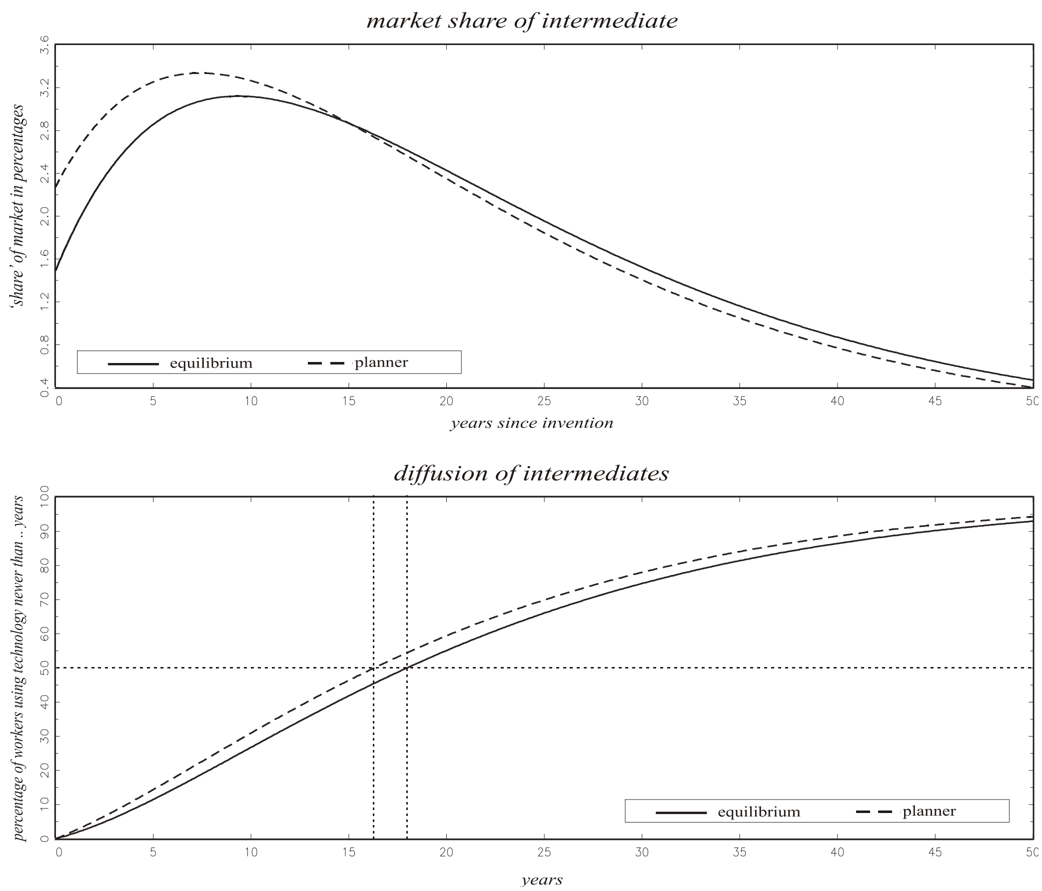

Figure 3: Steady state diffusion curves

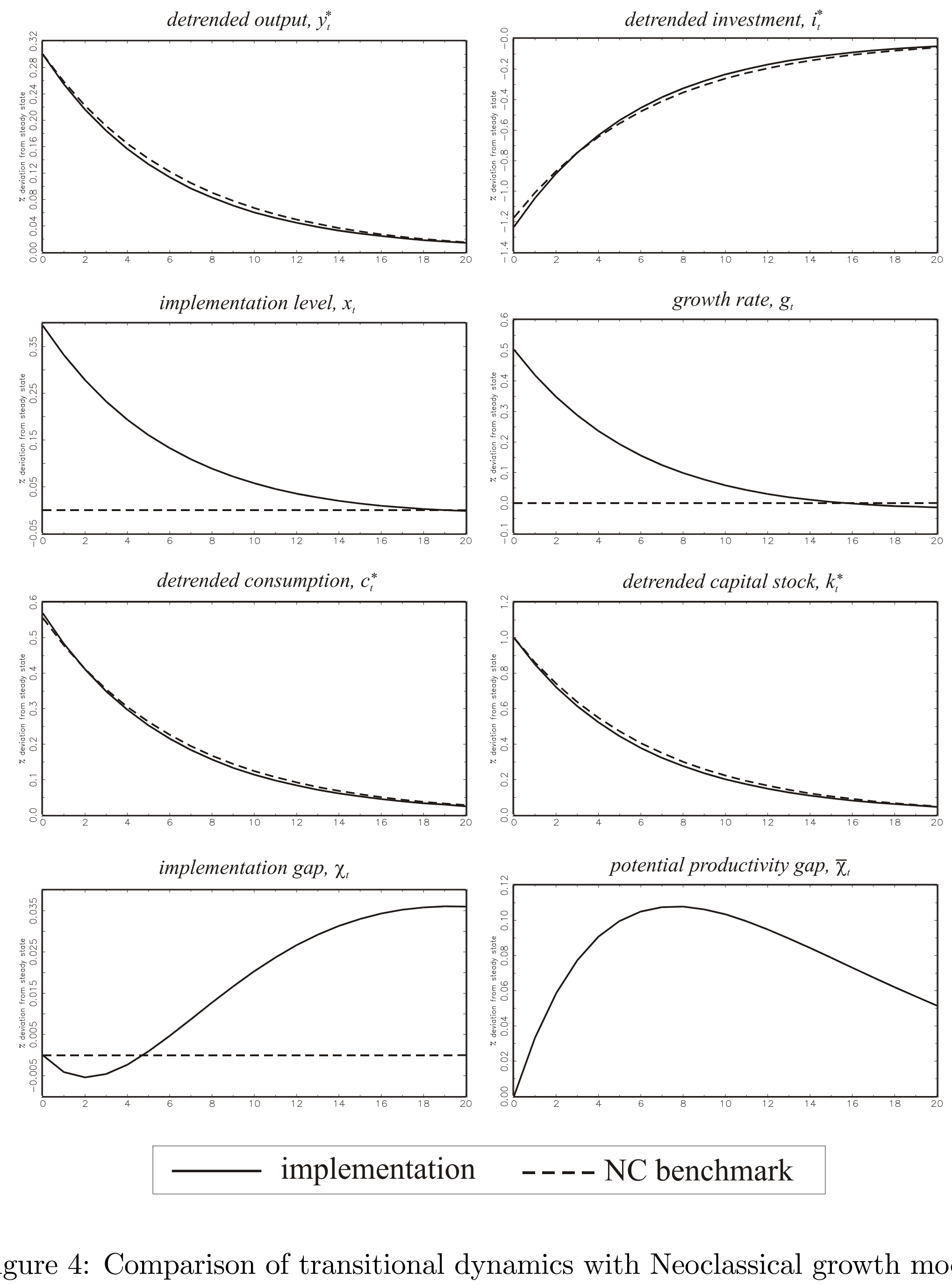

Figure 4: Comparison of transitional dynamics with Neoclassical growth model

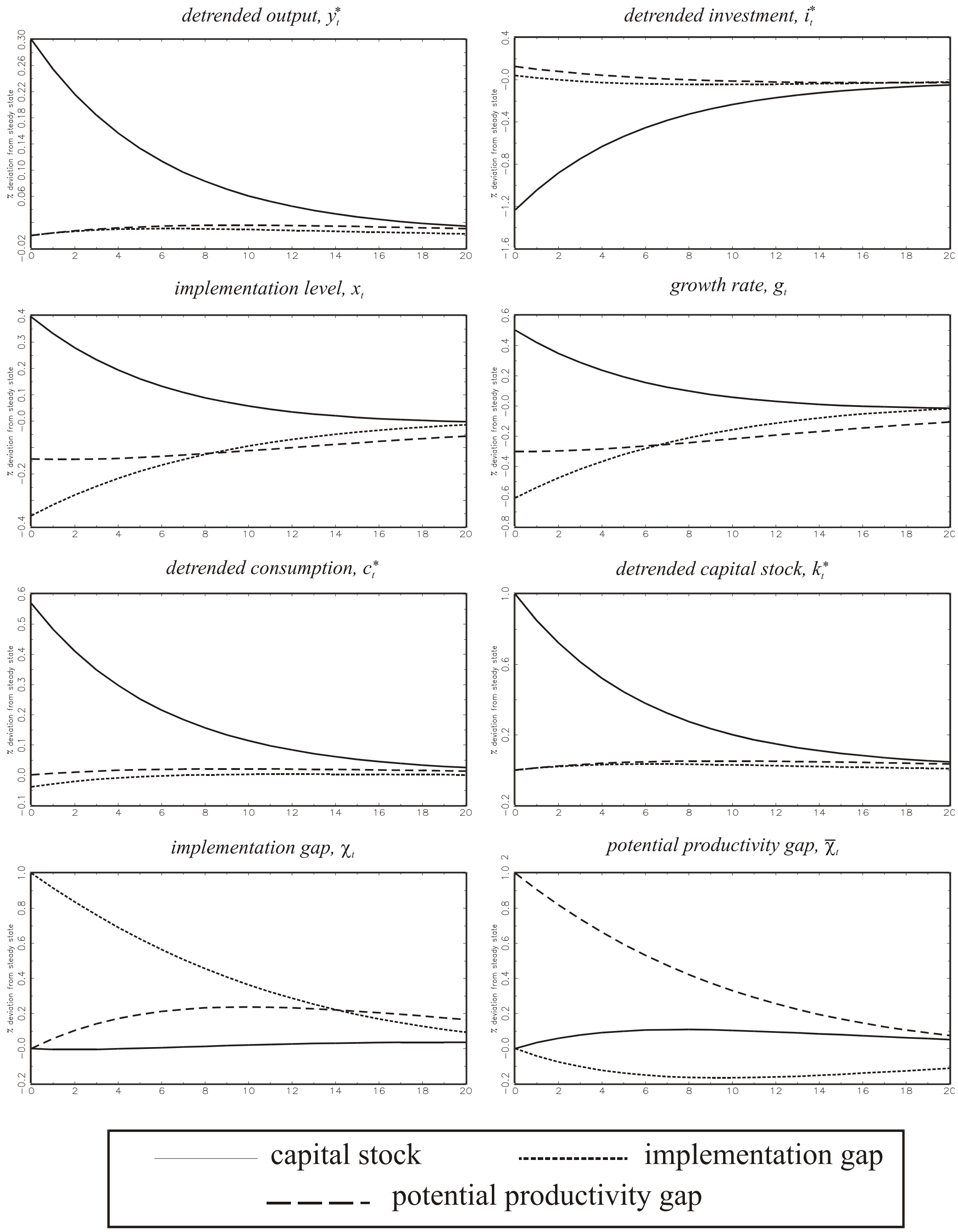

Figure 5: Transitional dynamics in response to shocks in all three state variables

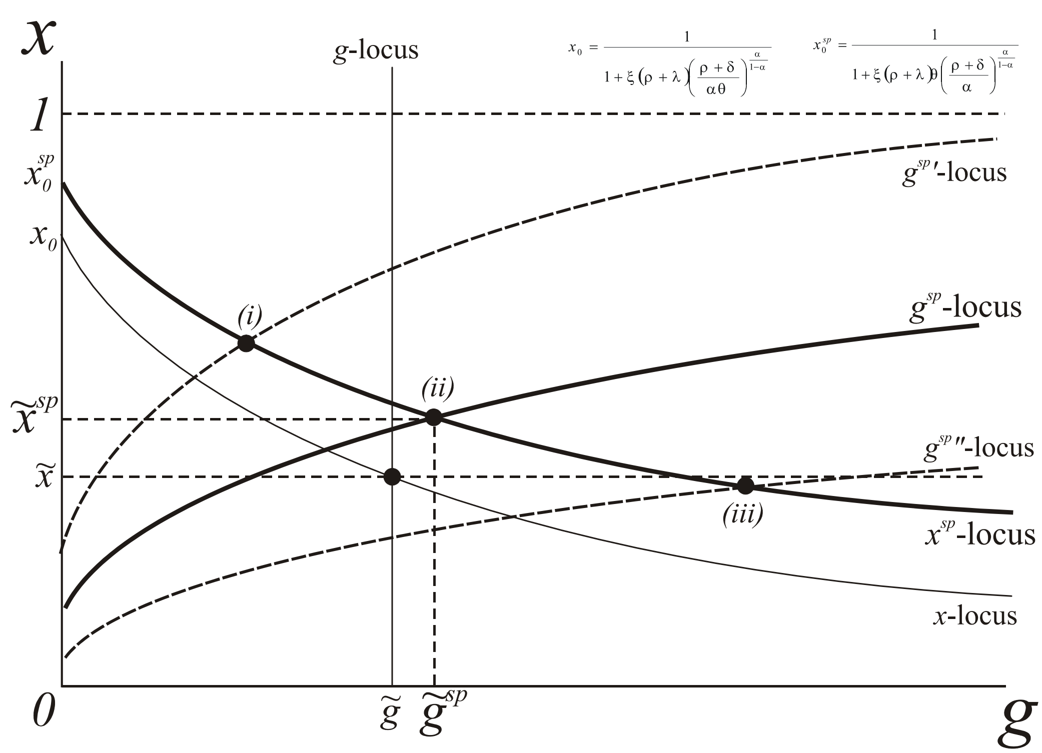

Figure 6: Determination of planner's steady state growth rate and implementation level

Table 3: Steady state values in decentralized equilibrium and social planner's allocations

variable	interpretation	equilibrium	planner
$\widetilde{g}$	growth rate	0.0133	0.0138
$\widetilde{x}$	implementation level	0.0431	0.0675
$\widetilde{\chi}$	implementation gap	4.5857	4.2991
$\widetilde{\widetilde{\chi}}$	potential productivity gap	0.0754	0.0784
$\widetilde{y}^*$	output	1.3863	1.4823
$\widetilde{c}^*$	consumption	1.0916	1.1171
$\widetilde{i}^*$	investment	0.2050	0.2590
$\widetilde{k}^*$	capital	2.9706	3.7132

Welfare in the decentralized equilibrium is 7.6% lower than in the planner's allocation.