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SELF-PROTECTION AND INSURANCE WITH INTERDEPENDENCIES

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ABSTRACT

We study optimal investment in self-protection of insured individuals when they face interdependencies in the form of potential contamination from others. If individuals cannot coordinate their actions, then the positive externality of investing in self-protection implies that, in equilibrium, individuals underinvest in self-protection. Limiting insurance coverage through deductibles can partially internalize this externality and thereby improve individual and social welfare.

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1 Introduction

This paper is concerned with the question as to how much insured consumers should invest in loss reduction measures when they can be contaminated by others due to interdependencies. To motivate the analysis consider the following problem faced by Ms. A, an owner of an apartment in a multi-unit building. Ms. A, who is required to purchase insurance as a condition for her mortgage, needs to determine how much she should invest in protective measures (e.g. a sprinkler system) to reduce the likelihood of a fire occurring in her apartment knowing that there is some chance that one of her neighbors unprotected could experience a fire that could spread to her apartment and cause damage even if she invests in these measures.

More generally, our interest is on examining the equilibrium levels of investment in protective measures when there are interdependencies between individuals and when insurance rates are risk-based. We show that without coordination between those at risk, individuals will, in equilibrium, underinvest in protection relative to the socially optimal decision due to the possibility of being contaminated by others. Restricting the amount of coverage an individual can take by requiring a deductible on insurance policies can encourage investment in protective measures and often improves both individual and social welfare.

To our knowledge no-one has investigated optimal behavior by insureds when they have the opportunity to invest in protective measures and face interdependent risks. Ehrlich and Becker (1972) study the interaction between insurance and self-protection when there are no interdependencies. Schlesinger and Venezian (1986) focus on the joint production of insurance and self-protection in various market settings without interdependencies between insureds. The problem of optimal protection when there are interdependencies between agents has been recently studied by Kunreuther and Heal (2002) and Heal and Kunreuther (2005) when there is no insurance. They developed a game theoretic model for these interdependent security problems where there are two choices facing an agent: don't invest in protection at all or invest in full protection. For the case where there are negative externalities due to the possibility of contagion from others, they show that there can be two Nash equilibria—either everyone invests in protection or no-one invests. The key point is that the incentive that any agent has to invest in risk-reduction measures depends on how she expects the others to behave in this respect. If she thinks that they will **not** invest in protection, then this reduces the incentive for her to do so. On the other hand should she believe that others will invest in risk reducing measures, then it may be best for her to also do so. So there may be an equilibrium where no-one invests in protection, even though all would be better off if they had incurred this cost.

The interdependency problem we are studying raises the question as to the benefits of coordinating individuals' protective decisions so that one can reduce the externalities due to contamination and hence improve both individual and social welfare. In this sense it is related to the study by Shavell (1991) who investigated the optimal decision by individuals to protect their property against theft, acting alone or collectively, when precautions are observable (e.g. iron bars on a window) or unobservable (e.g. use of a safe for storing valuables). Ayres and Levitt (1998) have demonstrated the social benefits of protection when individuals invest in unobservable precautionary measures. They focus on the Lojack car retrieval system that criminals cannot detect. This generates positive externalities that lead to a sub-optimal level of private investment.

The paper is organized as follows. We first consider the case of two identical individuals where there is no possibility of contamination from one individual to another and each individual has an opportunity to invest in mitigation to reduce its losses with premium reductions reflecting the reduced level of risk. We label this base case the **No Contamination** case. We compare this base case with a situation where there can be contamination between the two parties and where the two parties coordinate their actions. This case is labeled **Contamination — First Best**. It will be compared with a situation where the two parties cannot coordinate their actions and thus each party makes a decision independent of the other. This case is labeled **Contamination — Second Best**. We then turn to a situation where there is a required deductible on each insurance policy and show that this can improve welfare if the two parties face the possibility of contamination and cannot coordinate their actions, i.e. the case Contamination — Second Best. The concluding section discusses the policy implications of these findings by highlighting the importance of coordination between agents either voluntarily or through external involvement such as building codes. We also suggest directions for future research.

2 Model

There are two identical agents, i and j, who maximize expected utility with respect to an increasing, concave utility function $u(\cdot)$.[1] Each policyholder has initial wealth w_0 and is exposed to a loss of size L with probability p_0. There is a market for self-protection and a market for insurance. Investing in self-protection reduces the loss probability and investing in insurance transfers wealth from the no-loss to the loss state. The cost of reducing the loss probability to $p_i \leq p_0$ is given by a cost function $\gamma(\Delta p) = \gamma(p_0 - p_i)$ where $\gamma(0) = 0$, $\gamma' > 0$, and $\gamma'' > 0$. The policyholder can purchase insurance coverage I for an actuarially fair premium $P = P(I)$. We assume that there is no moral hazard problem, i.e. the agents' investment in protection is verifiable and contractible by the insurer.

Optimal Insurance Coverage. As insurance is actuarially fair, it is optimal for the risk-averse agent to purchase full insurance, i.e. $I^* = L$, for any level of investment in self-protection. Hence one can investigate the decision on how much self-protection to purchase under conditions of no contamination and contamination independently of the insurance decision. Furthermore since individuals are fully protected by insurance they do not face any risk. They will thus determine their optimal amount of self-protection by maximizing their level of final wealth which, in this case, is equivalent to maximizing their expected utility of wealth. This equivalence does not hold if insurance coverage is restricted and individuals therefore face risk. The optimal level of self-protection is then derived under the maximization of expected utility of final wealth (see Section 3).

[1] One obtains the same qualitative results when considering n rather than two individuals.

2.1 No Contamination

We first review the situation in which one individual cannot be contaminated by the other. As noted above, the optimal amount of self-protection and therefore the optimal loss probability p^* is determined by maximizing the value of final wealth

$$\max_p W(p) = w_0 - \gamma(p_0 - p) - pL.$$

The first and second derivatives with respect to p is $W'(p) = \gamma'(p_0 - p) - L$ and $W''(p) = -\gamma''(p_0 - p) < 0$. The objective function is thus globally concave which implies that we either have a corner solution $p^* = p_0$ if $\gamma'(0) \geq L$ or otherwise the optimal loss probability $p^* < p_0$ is determined by the first order condition

$$\gamma'(p_0 - p^*) = L. \tag{1}$$

The individual thus equates the marginal cost of the loss reduction, $\gamma'(\Delta p)$, with the marginal benefit in premium reduction, L. We now assume that $\gamma'(0) < L < \gamma'(p_0)$ which implies an inner solution $0 < p^* < p_0$. Note that if $\gamma'(p_0) < L$ then $p^* = 0$ because the marginal cost of eliminating the probability of a loss is sufficiently small relative to the magnitude of the loss itself that it is worth investing so there is no exposure to this risk. Similarly if $\gamma'(0) > L$ then the marginal cost of investing in any protection is so high relative to the benefits in reducing the expected loss that it is optimal not to commit any funds to mitigation.

2.2 Contamination

In this section, we introduce the possibility that one agent can be contaminated by the other agent. Denote by $q(p_j)$ the likelihood that agent i is contaminated by the other agent, j, as a function of the other agent's loss probability p_j. Contamination thus introduces an externality between the two agents in the sense that the decision of one policyholder to invest in protection affects the decision of the other policyholder. We assume that contamination is "perfect" in the sense that if a loss is incurred by one policyholder it spreads with probability one to the other policyholder, i.e. $q(p_i) = p_i$ and $q(p_j) = p_j$. The loss and final wealth distribution faced by policyholder i is

event	prob	final wealth
loss	$p_i + (1 - p_i) p_j$	$w_0 - \gamma (p_0 - p_i) - P(I) - L + I$
no loss	$(1 - p_i)(1 - p_j)$	$w_0 - \gamma (p_0 - p_i) - P(I)$

where the actuarially fair premium is given by $P(I) = (p_i + (1 - p_i) p_j) I$.

As above, given that insurance coverage is actuarially fair, it is optimal for the policyholder to purchase full insurance, $I^* = L$, independent of the amount invested in self-protection. Under full coverage, policyholder i's level of final wealth is given by

$$W_i = W(p_i, p_j) = w_0 - \gamma (p_0 - p_i) - (p_i + (1 - p_i) p_j) L.$$

In the following two subsections, we consider the optimal investment in self-protection under the first-best and second-best scenarios in which policyholders can and cannot, respectively, contract on the level of investment in protection.

First-Best. If policyholders can contract on the externalities, i.e. they jointly determine and implement p_i and p_j, the Coase theorem applies and the optimal solution is given by the socially optimal level that maximizes the aggregate level of final wealth

$$W_i + W_j = 2w_0 - \gamma (p_0 - p_i) - \gamma (p_0 - p_j) - 2 (p_i + (1 - p_i) p_j) L.$$

The first and second derivative of the aggregate level of wealth with respect to p_i is given by

$$\begin{aligned}\frac{\partial W_i + W_j}{\partial p_i} &= \gamma'(p_0 - p_i) - 2(1-p_j)L \\ \frac{\partial^2 W_i + W_j}{\partial p_i^2} &= -\gamma''(p_0 - p_i) < 0.\end{aligned}$$

The aggregate level of wealth is thus globally concave which implies a unique solution $p_i^*(p_j)$ for each p_j. As the maximization problem is symmetric in i and j, let p_{FB}^* denote the optimal solution which is determined by $p_{FB}^* = p_i^*(p_{FB}^*) = p_j^*(p_{FB}^*)$. If $\gamma'(0) \geq 2(1-p_0)L$, then it is optimal not to invest in protection, i.e. $p_{FB}^* = p_0$. Note that $2(1-p_0)L$ represents the expected joint loss to individuals i and j if neither party invests in protection. In this situation the marginal cost of investing even a penny in protection is greater than the marginal benefit of the joint reduction in losses to individuals i and j from incurring this cost. Note that the smaller p_0 is, the more likely one invests in protection for any given value of $\gamma'(0)$ because the marginal benefits to each individual of the other investing in mitigation is $(1-p_0)L$ which increases as p_0 decreases. Otherwise, the optimal solution is determined by the first-order condition

$$\gamma'(p_0 - p_{FB}^*) = 2(1-p_{FB}^*)L. \tag{2}$$

We can interpret this condition by rearranging it into

$$\gamma'(p_0 - p_{FB}^*) + p_{FB}^* L = L + (1-p_{FB}^*)L. \tag{3}$$

The left hand side of (3) is the marginal cost of investing in protection which is the sum of the marginal dollar cost, $\gamma'(p_0 - p_{FB}^*)$, and the marginal increase in the premium, $p_{FB}^* L$, due to indirectly increasing the likelihood of being contaminated by the other agent. The right hand side of (3) is the marginal benefit of investing in protection which is decomposed into the marginal reduction in premium, L, due to the reduced likelihood of a direct loss and the marginal reduction in premium, $(1-p^*_{FB})\,L$, due to the reduction in the likelihood of contaminating the other agent. The latter marginal benefit represents the benefit from internalizing the positive externality.

Second-Best. In this section, we examine the setting in which the two policyholders cannot contract on the level of investment in self-protection and determine the pure-strategy Nash-equilibria. Policyholder i's best response function $p^*_i(p_j)$ is given by

$$p^*_i(p_j) \in \arg\max_{p_i} W_i(p_i, p_j) = w_0 - \gamma(p_0 - p_i) - (p_i + (1-p_i)\,p_j)\,L.$$

It therefore satisfies the first-order condition

$$\gamma'(p_0 - p^*_i(p_j)) - (1-p_j)\,L = 0.$$

Differentiating with respect to p_j yields

$$-p^{*\prime}_i(p_j)\,\gamma''(p_0 - p^*_i(p_j)) + L = 0$$

i.e.

$$p^{*\prime}_i(p_j) = \frac{L}{\gamma''(p_0 - p^*_i(p_j))} > 0. \tag{4}$$

Policyholder i's strategy is thus a strategic complement to policyholder j's strategy which implies that there are only symmetric pure-strategy Nash-equilibria.

If policyholder j reduces the loss probability to zero, i.e. $p_j = 0$, then there is no contamination to policyholder i and thus $p^*_i(0) = p^*$ which is implicitly determined by (1). Under the assumption $\gamma'(0) < L < \gamma'(p_0)$ we have an inner solution $0 < p^*_i(0) = p^* < p_0$. If policyholder j does not invest in self-protection, i.e. $p_j = p_0$, then policyholder i's best response is determined by

$$\gamma'(p_0 - p^*_i(p_0)) = (1-p_0)\,L.$$

If $\gamma'(0) \geq (1-p_0)L$ then policyholder i's best response is also to not invest in self-protection, i.e. $p_i^*(p_0) = p_0$. Otherwise, if $\gamma'(0) < (1-p_0)L$ then $p_i^*(p_0) < p_0$.

Since $0 < p_i^*(0) = p_j^*(0) < p_0$ and since the best-response functions are increasing, they can only cross the 45 degree line an odd number of times. We thus conclude that if $\gamma'(0) < (1-p_0)L$ then there exists an odd number of pure-strategy Nash-equilibria, $p_{SB}^* = p_i^*(p_{SB}^*) = p_j^*(p_{SB}^*)$, which are all inner solutions and determined by the condition

$$\gamma'(p_0 - p_{SB}^*) = (1 - p_{SB}^*)L. \tag{5}$$

If $\gamma'(0) \geq (1-p_0)L$, then there also exists an odd number of pure-strategy Nash-equilibria with the only difference that the largest equilibrium is at the corner $p_{SB}^* = p_0$, i.e. there is no investment in self-protection in this equilibrium.

In both cases, the smallest and the largest equilibrium are stable with respect to a myopic adjustment process and the other equilibria alternate in terms of stability and instability. The stability condition is characterized by $p_i^{*\prime}(p_{SB}^*) < 1$ which, by equation (4) is equivalent to $\gamma''(p_0 - p_{SB}^*) > L$. If the best-response functions are concave, then there exists a unique pure strategy Nash-equilibrium which is stable with respect to a myopic adjustment process. Figure 1 shows a situation in which there are three Nash-equilibria.

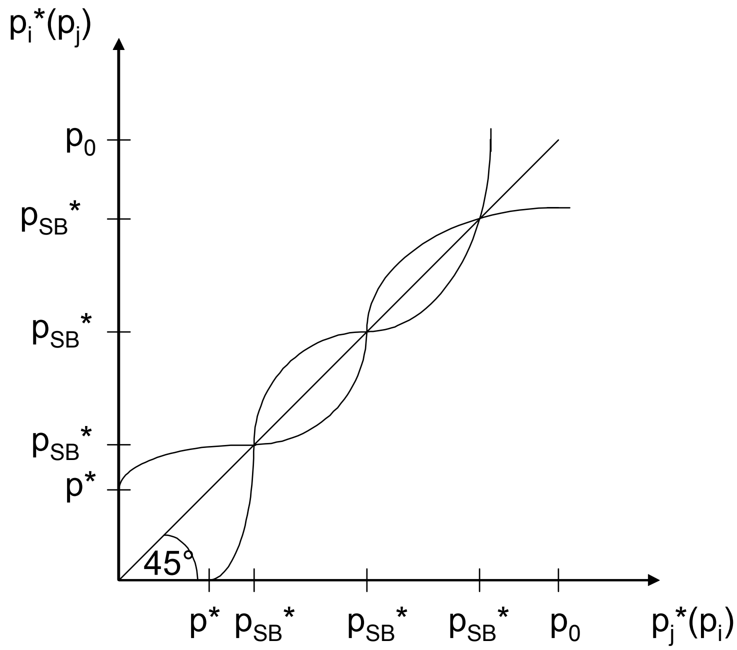

Figure 1

To interpret condition (5), we rearrange it into

$$\gamma'(p_0 - p_{SB}^*) + p_{SB}^* L = L. \tag{6}$$

The left hand side of (6) is the same as under the first-best scenario (3) , i.e. the sum of the marginal dollar cost, $\gamma'(p_0 - p_{SB}^*)$, and the marginal increase in the premium, $p_{SB}^* L$, due to indirectly increasing the likelihood of being contaminated by the other agent. The right hand side of (6), however, differs from the first-best scenario (3). The only marginal benefit of investing in protection is the marginal reduction in premium, L, due to the reduced likelihood of a direct loss. As policyholders cannot contract on the level of investment in self-protection, it is not possible for a policyholder to benefit from the positive externality that his investment poses on the other policyholder as shown in equation (3) for the joint solution.

2.3 Comparison

In the following subsection, we compare the level of investment in any Nash equilibrium with both the one in the first-best scenario and the one if policyholders do not face contamination.

Comparing Second-Best with First-Best. In this section, we compare the optimal level of investment in self-protection in the first-best with the one in the second-best scenario. Suppose it is optimal to not invest in self-protection in the first-best world, i.e. $\gamma'(0) \geq 2(1-p_0)L$. Then it is also not optimal to invest in self-protection in the second-best world, as $\gamma'(0) \geq 2(1-p_0)L > (1-p_0)L$ since an individual does not take into account the positive externalities provided the others when making an investment decision. Now suppose it is optimal to invest in self-protection in the first-best world, i.e. $\gamma'(0) < 2(1-p_0)L$. The optimal solution is then determined by

$$\gamma'(p_0 - p^*_{FB}) = 2(1-p^*_{FB})L.$$

This implies

$$\gamma'(p_0 - p^*_{FB}) > (1-p^*_{FB})L$$

and condition (5) yields $p^*_{SB} > p^*_{FB}$. In any pure-strategy Nash-equilibrium the level of investment in self-protection is thus lower compared to the first-best scenario. The intuition behind this result can be derived from comparing the first-order condition (3) in the first-best scenario under contamination

$$\gamma'(p_0 - p) + pL = L + (1-p)L$$

with the first-order condition (5) in the second-best scenario under contamination

$$\gamma'(p_0 - p) + pL = L.$$

We note that in the second-best scenario it is not possible to internalize the marginal benefit of the policyholder's effect on the other policyholder, $(1-p)\,L$, and he therefore underinvests in self-protection compared to the first-best scenario.

Comparing First-Best with No-Contamination. Let us compare the optimal level of investment without contamination with the one in the first-best scenario with contamination. The first-order condition (1) under no contamination is

$$\gamma'\left(p_0 - p\right) = L$$

and the first-order condition (3) in the first-best scenario under contamination is

$$\gamma'\left(p_0 - p\right) + pL = L + \left(1 - p\right) L.$$

By comparing the marginal costs and benefits of the two scenarios, we see that contamination adds both a marginal cost and a marginal benefit of investing in self-protection. The additional marginal cost, pL, is due to the indirect increase in the likelihood of being contaminated by the other agent while the marginal benefit, $(1-p)\,L$, is due to the internalized positive of effect on the other agent. This implies that investment in self-protection under contamination can be larger or smaller than under no contamination depending on whether the additional marginal benefit is larger or smaller than the additional marginal cost.

Under the condition $p_0 < 1/2$—which seems most relevant for insurance events—the additional marginal benefit is larger than the additional marginal cost of investing in self-protection. It is thus optimal to invest more in self-protection under the first-best scenario with contamination compared to the scenario in which agents cannot be contaminated which yields $p_{FB}^* < p^*$.

To be more precise, suppose it is optimal to not invest in self-protection if there is no contamination, i.e. $\gamma'(0) \geq L$. Then it may still be optimal to invest in self-protection in the first-best world with contamination since the condition for not investing is $\gamma'(0) \geq 2\left(1 - p_0\right) L$ and $2\left(1 - p_0\right) L > L$

for all $p_0 < 1/2$. Now suppose that it is optimal to invest in self-protection without contamination. Then policyholders invest more in self-protection in the first-best world with contamination as the first-order condition $\gamma'(p_0 - p^*) = L$ under no contamination implies

$$\gamma'(p_0 - p^*) < 2(1 - p^*) L$$

for all $p_0 < 1/2$ and condition (3) yields $p^*_{FB} < p^*$.

Comparing Second-Best with No-Contamination. Let us now compare the optimal level of investment without contamination with the one in the second-best scenario with contamination. Suppose it is optimal to not invest in self-protection if there is no contamination, i.e. $\gamma'(0) \geq L$. Then it is also optimal to not invest in self-protection in the second-best world as $\gamma'(0) \geq L$ implies $\gamma'(0) > (1 - p_0) L$. Now suppose that it is optimal to invest in self-protection without contamination. Then policyholders invest less in self-protection in the second-best world with contamination as the first-order condition $\gamma'(p_0 - p^*) = L$ implies

$$\gamma'(p_0 - p^*) > (1 - p^*) L$$

and condition (5) yields $p^*_{SB} > p^*$. In any pure-strategy Nash-equilibrium the level of investment in self-protection is thus lower compared to the scenario in which policyholders do not face possible contamination. The intuition behind this result can again be derived by comparing the first-order conditions (6) in the second-best scenario under contamination

$$\gamma'(p_0 - p) + pL = L$$

with the first-order condition (1) under no contamination is

$$\gamma'(p_0 - p) = L.$$

In the second-best scenario policyholders, by investing in self-protection, face the additional marginal cost of implicitly increasing the likelihood of being contaminated by the other policyholder, pL. The marginal cost equates the marginal benefit of investing in self-protection thus at a lower level of investment.

2.4 Illustrative Example

Suppose that the cost of reducing the probability from p_0 to p_i is given by the function

$$\gamma\left(p_0 - p_i\right) = \frac{c}{2}\left(p_0 - p_i\right)^2.$$

As $\gamma'(0) = 0$, it is optimal under all three scenarios to invest in self-protection. We also assume $\gamma'(p_0) = cp_0 > L$ such that the optimal loss probability is strictly positive. Solving the first-order conditions (1), (2), and (5) yields the following solutions:

$$\begin{aligned} p^* &= p_0 - \frac{L}{c}, \\ p^*_{FB} &= p_0 - \frac{(1-p_0)\,2L}{c-2L}, \\ p^*_{SB} &= p_0 - \frac{(1-p_0)\,L}{c-L}. \end{aligned}$$

We observe that $p^*_{SB} > p^*$ and $p^*_{SB} > p^*_{FB}$. Furthermore, the best response function of individual i in the second-best scenario is given by

$$p^*_i\left(p_j\right) = p_0 - \frac{\left(1-p_j\right)L}{c}.$$

Since it is linear in p_j the solution p^*_{SB} above is the unique pure-strategy Nash-equilibrium.

3 Improving Welfare by Restricting Insurance Coverage

In the section above, we have shown that individuals inefficiently underinvest in self-protection if they cannot coordinate their activities. In this section, we show that restricting insurance coverage, e.g. by requiring a deductible in the insurance policy, can improve individual and social welfare in a second best world with contamination. With a deductible, each individual has to bear part of their own loss and is likely to have more of an incentive to invest in self-protection than if he had full insurance coverage. The additional investment in self-protection creates an extra marginal benefit, $(1-p)\,L$, through the positive externality that exists between individuals. In the following Proposition, we specify conditions under which this benefit outweighs the cost of bearing part of the loss and implies that partial insurance is optimal. It is important, however, that the deductible is enforced by some regulatory entity. In an unregulated environment, an insurer always will deviate by offering full coverage to attract all customers.[2]

Proposition 1 *Suppose that the stability condition $\gamma''(p_0 - p_{SB}^*) > L$ holds where p_{SB}^* is the loss probability in a Nash-equilibrium under full insurance coverage, implicitly defined by* (5)*. Then the optimally enforced deductible is strictly positive if and only if*

$$(1-p_{SB}^*)^2\,L > \left(1-(1-p_{SB}^*)^2\right)\gamma''(p_0 - p_{SB}^*)\,. \tag{7}$$

Proof. See Appendix. ■

Imposing a strictly positive deductible and thereby forcing agents to invest more in self-protection can only be welfare-improving if the marginal benefit of internalizing the externality, i.e. $(1-p)\,L$, is relatively large. This is exactly reflected in the necessary and sufficient condition (7). If p_{SB}^* is relatively small, the marginal benefit of internalizing the externality, $(1-p_{SB}^*)\,L$, is relatively large and (7) is satisfied.

[2] Since individuals face risk under restricted insurance coverage, the proof requires the maximization of expected utility of final wealth.

This result should be contrasted with the case of terrorism insurance considered by Lakdawalla and Zanjani (2005) where protection by one target leads the terrorist to attack a less protected target. Protection thus creates a negative externality and an inefficient overinvestment in self-protection. A governmental subsidy of terrorism insurance can improve social welfare by discouraging investment in protection. In our case, there is a positive externality associated with investment in protection. Social welfare is now improved by limiting insurance through a deductible, thereby encouraging investment in protection.

Remark 2 *In the proof of Proposition 1, we show that small, but strictly positive deductibles induce agents to invest more in self-protection if and only if* $p_{SB}^{*} < 1 - \sqrt{1/2}$. *Note that this condition is implied by the necessary and sufficient condition* (7)*, i.e. enforcing a strictly positive deductible can only be optimal if it induces agents to invest more in self-protection than they would under full insurance coverage.*

Put differently, if the probability of a loss under full insurance coverage is relatively large, i.e. $p_{SB}^{*} > 1 - \sqrt{1/2}$, limiting insurance by enforcing a deductible discourages the investment in protection. This is related to the finding of Ehrlich and Becker (1972) who show that the absence of market insurance can discourage the investment in self-protection if the probability of a loss is relatively large.

3.1 Illustrative Example

We continue our previous illustrative example in Section 2.4 with $\gamma''(p_0 - p_i) = c$. Then

$$p_{SB}^{*} = p_0 - \frac{(1 - p_0) L}{c - L}$$

with $cp_0 > L$ which implies $\gamma''(p_0 - p_i) = c > L$. The necessary and sufficient condition (7) for the optimal enforced deductible to be strictly positive is

$$p_0 < 1 - \sqrt{\frac{(c-L)^2}{c(c+L)}}.$$

4 Implications for Policy

The bundling of protection and insurance has a long history dating back to the factory mutuals founded in the early 19th century in New England (Bainbridge, 1952). These mutual companies offered factories an opportunity to pay a small premium in exchange for protection against potentially large losses from fire while at the same time requiring inspections of the factory both prior to issuing a policy and after one was in force. Poor risks had their policies canceled; premium reductions were given to factories that instituted loss prevention measures. For example, the Boston Manufacturers worked with lantern manufacturers to encourage them to develop safer designs and then advised all policyholders that they had to purchase lanterns from those companies whose products met their specifications. In many cases, insurance would only be provided to companies that adopted specific loss prevention methods. For example one company, the Spinners Mutual, only insured risks where automatic sprinkler systems were installed. The Manufacturers Mutual in Providence, Rhode Island developed specifications for fire hoses and advised mills to buy only from companies that met these standards.

Private insurers today should consider requiring protective measures as a condition for insurance with respect to standard homeowners coverage to reduce the negative externalities due to contagion. However, all insurers would have to find it in their financial interest to follow this strategy because of the contractual arrangements with respect to claims payments. Any insurer who provides protection to individual i is responsible for losses incurred by this policyholder no matter who caused the damage.[3] One reason for this arrangement between insurer and insured is the difficulty in assigning

[3] With respect to fire damage, a classic case is H.R. Moch Co., Inc. v Rensselaer Water Co. 247N.Y.160, 159 N.E. 896 which ruled that "A wrongdoer who by negligence sets fire to a building is liable in damages to the owner where

causality for a particular event. Without protection requirements by other insurers, a competitive insurer will have to charge premiums that reflect the actions of policyholders who are independently deciding how much to invest in protection given that they have some chance of being contaminated by others.

One way of coordinating protective decisions of individuals is through a monopolistic insurer who can require the adoption of such measures or provide premium incentives for those at risk to adopt them to internalize the externalities due to interdependencies . A competitive insurer may not be able to do this as easily if others in the industry do not take similar actions. von Ungern-Sternberg (1996) provides an empirical study of the pricing and performance of insurance markets in Switzerland and compares the performance of competitive insurers in seven cantons of the country with local state monopolies in the 19 other cantons. The study finds that for very similar products the monopolies charge premiums that are 70 percent lower than for the competitive insurance, they spend substantially more on fire prevention and have much lower damage rates.

Some type of coordinative mechanism may also improve both individual and social welfare without having to rely on the power of the monopolist insurer. One option is for a well-enforced standard or regulation, such as a building code, that requires individuals and firms to adopt cost-effective protective mechanisms when they would not do so voluntarily. One could also turn to the private sector to coordinate decisions through an industry association that stipulates that any member has to follow certain rules and regulations. For example, an apartment owners association could require that all residents in the building adopt certain fire protective measures such as installing a smoke alarm and/or a sprinkler system. The association could then arrange to purchase insurance for all units in the building where the premiums would reflect the required protection that would reduce the chances of a fire occurring.

the fire has its origin, but not to other owners who are injured when it spreads". We are indebted to Victor Goldberg who provided us with this case.

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6 Appendix: Proof of Proposition 1

The economic environment is as above in the second-best scenario in which individuals cannot contract on their investment in self-protection. The only difference is that the insurance policy includes a deductible D, i.e., the insurer pays $L - D$ in case of a loss. The loss and final wealth distribution faced by policyholder i is

event	prob	final wealth
loss	$p_i + (1 - p_i) p_j$	$w_0 - \gamma (p_0 - p_i) - P(D) - D$
no loss	$(1 - p_i)(1 - p_j)$	$w_0 - \gamma (p_0 - p_i) - P(D)$

where the actuarially fair premium is given by $P(D) = (p_i + (1 - p_i) p_j)(L - D)$. Policyholder 1's expected utility of final wealth is given by

$$\begin{aligned} EU_i (p_i, p_j, D) &= (1 - p_i)(1 - p_j) u (w_0 - \gamma (p_0 - p_i) - P(D)) \\ &\quad + (p_i + (1 - p_i) p_j) u (w_0 - \gamma (p_0 - p_i) - P(D) - D). \end{aligned}$$

Policyholder i's best response function $p_i^* (p_j, D)$ is given by

$$p_i^* (p_j, D) \in \arg\max_{p_i} EU_i (p_i, p_j).$$

Let $p_{SB}^* (D)$ denote the symmetric Nash-equilibrium, i.e. $p_{SB}^* (D) = p_i^* (p_{SB}^* (D), D) = p_i^* (p_{SB}^* (D), D)$, which satisfies the first-order condition

$$\begin{aligned} & (1 - p_{SB}^* (D)) (u (w_0 - \gamma (p_0 - p_{SB}^* (D)) - P(D) - D) - u (w_0 - \gamma (p_0 - p_{SB}^* (D)) - P(D))) \\ & + (1 - p_{SB}^* (D))^2 (\gamma' (p_0 - p_{SB}^* (D)) - (1 - p_{SB}^* (D)) (L - D)) u' (w_0 - \gamma (p_0 - p_{SB}^* (D)) - P(D)) \\ & + p_{SB}^* (D) (2 - p_{SB}^* (D)) (\gamma' (p_0 - p_{SB}^* (D)) - (1 - p_{SB}^* (D)) (L - D)) u' (w_0 - \gamma (p_0 - p_{SB}^* (D)) - P(D) - D) \\ = {} & 0. \end{aligned} \tag{8}$$

At $D = 0$ we get the condition

$$\gamma' (p_0 - p_{SB}^* (0)) - (1 - p_{SB}^* (0)) L = 0, \tag{9}$$

where $p_{SB}^* (0) = p_{SB}^*$. For a given deductible D, the level of expected utility is thus given by

$$\begin{aligned} EU_i (p_{SB}^* (D), p_{SB}^* (D), D) &= (1 - p_{SB}^* (D))^2 u (w_0 - \gamma (p_0 - p_{SB}^* (D)) - P(D)) \\ &\quad + (2 - p_{SB}^* (D)) p_{SB}^* (D) u (w_0 - \gamma (p_0 - p_{SB}^* (D)) - P(D) - D). \end{aligned} \tag{10}$$

Differentiating expected utility with respect to the deductible level and evaluating at $D = 0$ implies

$$\frac{\partial EU_i (p_{SB}^* (D), p_{SB}^* (D), D)}{\partial D} |_{D=0} = -p_{SB}^{*\prime} (0) (1 - p_{SB}^* (0)) L u' (w_0 - \gamma (p_0 - p_{SB}^* (0)) - P(0)). \tag{11}$$

To determine the sign of the first derivative we implicitly differentiate the first-order condition (8) with respect to D and evaluate it at $D = 0$. We derive

$$p_{SB}^{*\prime} (0) (L - \gamma'' (p_0 - p_{SB}^* (0))) = 0.$$

The stability condition $\gamma'' (p_0 - p_{SB}^* (0)) > L$ then implies $p_{SB}^{*\prime} (0) = 0$ and thus

$$\frac{\partial EU_i\left(p_{SB}^*\left(D\right),p_{SB}^*\left(D\right),D\right)}{\partial D}|_{D=0}=0.$$

Next, we determine the sign of the second derivative of expected utility evaluated at $D=0$. It is given by

$$\begin{aligned}
& \frac{\partial^2 EU_i\left(p_{SB}^*\left(D\right),p_{SB}^*\left(D\right),D\right)}{\partial D^2}|_{D=0} \\
= \quad & -p_{SB}^{*\prime\prime}\left(0\right)\left(1-p_{SB}^*\left(0\right)\right)Lu'\left(w_0-\gamma\left(p_0-p_{SB}^*\left(0\right)\right)-P\left(0\right)\right) \\
& +\left(1-p_{SB}^*\left(0\right)\right)^2 p_{SB}^*\left(0\right)\left(2-p_{SB}^*\left(0\right)\right)u''\left(w_0-\gamma\left(p_0-p_{SB}^*\left(0\right)\right)-P\left(0\right)\right). \qquad (12)
\end{aligned}$$

Implicitly differentiating the first-order condition (8) twice with respect to D and evaluating at $D=0$ yields

$$\begin{aligned}
& p_{SB}^{*\prime\prime}\left(0\right)\left(L-\gamma''\left(p_0-p_{SB}^*\left(0\right)\right)\right)u'\left(w_0-\gamma\left(p_0-p_{SB}^*\left(0\right)\right)-P\left(0\right)\right) \\
= \quad & \left(1-p_{SB}^*\left(0\right)\right)\left(1-2\left(1-p_{SB}^*\left(0\right)\right)^2\right)u''\left(w_0-\gamma\left(p_0-p_{SB}^*\left(0\right)\right)-P\left(0\right)\right),
\end{aligned}$$

i.e.

$$p_{SB}^{*\prime\prime}\left(0\right)=\frac{\left(1-p_{SB}^*\left(0\right)\right)\left(1-2\left(1-p_{SB}^*\left(0\right)\right)^2\right)u''\left(w_0-\gamma\left(p_0-p_{SB}^*\left(0\right)\right)-P\left(0\right)\right)}{\left(L-\gamma''\left(p_0-p_{SB}^*\left(0\right)\right)\right)u'\left(w_0-\gamma\left(p_0-p_{SB}^*\left(0\right)\right)-P\left(0\right)\right)}. \qquad (13)$$

If $p_{SB}^*\left(0\right)<1-\sqrt{1/2}$ then $p_{SB}^{*\prime\prime}\left(0\right)<0$ and small deductible levels thus increase the investment in self-protection. Note that this condition is implied by the necessary and sufficient condition (7). Substitution of (13) into the second derivative of expected utility (12) implies

$$\begin{aligned}
& \frac{\partial^2 EU_i\left(p_{SB}^*\left(D\right),p_{SB}^*\left(D\right),D\right)}{\partial D^2}|_{D=0} \\
= \quad & \left(1-\left(1-p_{SB}^*\left(0\right)\right)^2-\frac{\left(1-2\left(1-p_{SB}^*\left(0\right)\right)^2\right)L}{\left(L-\gamma''\left(p_0-p_{SB}^*\left(0\right)\right)\right)}\right)\left(1-p_{SB}^*\left(0\right)\right)^2u''\left(w_0-\gamma\left(p_0-p_{SB}^*\left(0\right)\right)-P\left(0\right)\right).
\end{aligned} \qquad (14)$$

A strictly positive deductible is optimal if and only if $\frac{\partial^2 EU_i\left(p_{SB}^*\left(D\right),p_{SB}^*\left(D\right),D\right)}{\partial D^2}|_{D=0}>0$. Equation (14) implies the condition

$$\left(1-p_{SB}^*\left(0\right)\right)^2L>\left(1-\left(1-p_{SB}^*\left(0\right)\right)^2\right)\gamma''\left(p_0-p_{SB}^*\left(0\right)\right).$$