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# SHOOTING THE AUCTIONEER

Roger E. A. Farmer
Andrew Hollenhorst

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Roger E. A. Farmer
UC, Los Angeles
Department of Economics
Box 951477
Los Angeles, CA 90095-1477
and NBER
rfarmer@econ.ucla.edu

Andrew Hollenhorst
Department of Economics UCLA
8283 Bunche Hall: Box 951477
Los Angeles CA 90095-1477
ahollen@ucla.edu

# Shooting the Auctioneer*


Roger E. A. Farmer
rfarmer@econ.ucla.edu

Andrew Hollenhorst
ahollen@ucla.edu


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**Abstract**

Most dynamic stochastic general equilibrium models of the macroeconomy assume that labor is traded in a spot market. Two exceptions by David Andolfatto and Monika Merz combine a two-sided search model with a one-sector real business cycle model. These hybrid models are successful, in some dimensions, but they cannot account for observed volatility in unemployment and vacancies. Following suggestions by Robert Hall and Robert Shimer, this paper shows that a relatively standard DSGE model with sticky wages *can* account for these facts. Using a second-order approximation to the policy function we simulate moments of an artificial economy with and without sticky wages and we document the dependence of unemployment and vacancy volatility on two key parameters; the disutility of effort and the degree of wage stickiness. We compute the welfare costs of the sticky wage equilibrium and find them to be small.


## 1 Introduction

Most dynamic stochastic general equilibrium models (DSGE) of the macroeconomy are built around a spot market for labor. Two exceptions, Andolfatto [2] and Merz [17], combine the two-sided search model of Mortensen


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and Pissarides [21], [20], [22] with a one-sector real business cycle model. These hybrid models are successful in some dimensions at explaining how unemployment and vacancies move over the business cycle but they cannot account for observed volatility in unemployment and vacancies. This paper shows that a DSGE model with rigid wages *can* account for these facts.

Shimer [25] suggests that the problem with search theoretic models is that they are typically closed with a Nash bargaining solution. Nash bargaining, as a wage-setting mechanism, allows too much wage flexibility relative to the data. Hall [10], [11] has explored Shimer's suggestion that models with rigid or partially adjusting wages may be more successful than flexible wage economies at explaining the facts. This paper builds on the Hall-Shimer approach by constructing a fully specified dynamic general equilibrium model and studying the properties of alternative wage determination mechanisms.

We construct a version of a real business cycle model in which we add a two-sided matching technology similar to those studied by Andolfatto and Merz. We use this artificial economy to study the properties of three alternative equilibria. In the first, the wage is chosen to mimic the social planning solution: We call this a flexible wage economy. In the second equilibrium the real wage grows at the rate of underlying technological progress but is unresponsive to current productivity shocks. We call this the rigid wage solution. Finally, we study an economy in which the real wage adjusts 19% of the way towards the efficient solution in every quarter. We call this a sticky wage economy.

In the context of the Mortensen-Pissarides model, Hagedorn and Manovskii [9] have shown that the Hall-Shimer volatility puzzle can be solved by choosing a high value for the outside option of the worker. Although our model of the labor market is embedded into a fully specified DSGE environment we find that a disutility of effort parameter, related to Hagedorn and Manovskii's outside option, plays an important role in influencing unemployment and vacancy volatility. We are not, however, able to resolve the Hall-Shimer puzzle with this parameter alone. Since our model allows for variable search intensity we find that the social planning optimum has counterfactual implications for the Beveridge curve. In order to generate the observed negative correlation between unemployment and vacancies and simultaneously to generate the magnitude of unemployment and vacancy volatility observed in the data we need to choose the disutility of effort and the degree of wage stickiness together.

Since we need sticky wages to explain the data it seems reasonable to ask if the welfare cost of failing to adjust the real wage is large. Using a second order approximation to the utility function we find that when wages adjust 19% of the way to their optimal value in each period that the welfare cost relative to the first best is roughly 40 cents per person per quarter in terms of foregone consumption. It seems likely that a cost of this magnitude could be explained by menu costs.

To summarize; our paper makes three contributions to existing literature. The first is the finding that a sticky wage DSGE economy does a good job of explaining the time series properties of unemployment and vacancies in the U.S. Second, to resolve the Hall-Shimer volatility puzzle in a DSGE model with variable search intensity we need both sticky wages and the ability to pick the reservation wage. Finally we find the welfare costs of the sticky wage equilibrium are small. Hence we are able to show that the main features of the Hall-Shimer sticky wage equilibrium continue to hold in a standard production economy with risk averse consumers and capital accumulation whilst preserving the ability of the standard RBC model to explain other features of the data.

# 2 Related Literature

In the Andolfatto [2] and Merz [17] models, unemployment and vacancies enter differently into the social objective function since vacancies use units of commodities but unemployment uses labor as an input. In the model developed below we have made them symmetric (unemployment and vacancies both impose a time cost) to emphasize a stark implication of the RBC model. In the social planning solution both vacancies and unemployment should be procyclical.[1] Our reason for modifying the Adolfatto-Merz approach is that their models have difficulty in explaining the volatility and cyclical properties of unemployment and vacancies. This problem was documented in U.K. data by Millard, Scott and Sensier [18] and in U.S. data by Shimer [25] who points out that when search models are closed with a Nash bargaining solution, they deliver counterfactual labor market predictions.

[1] In order to generate negatively correlated unemployment and vacancies, both Mertz and Andolfatto study versions of their respective models in which search intensity by workers is fixed. Merz studies a version of her model with variable search intensity in which she finds (Merz [17] Table 3 page 282) that unemployment and vacancies are positively correlated.

We are not the only people currently working on the problem of combining search with an RBC model and closely related papers include those by Blanchard and Galí [4], Costain and Reiter [5], Gertler and Trigari [8], den Haan, Ramey Watson [6], Fujita and Ramey [7], Hall [12] and Veracierto [27]. Although our model is different in some respects from those studied by each of these authors our findings are complementary. Like Blanchard and Galí [4], Gertler and Trigari [8] and Hall [12], we study the effects of closing the model with a sticky wage although we do not attempt to provide a microfoundation for this assumption as in Menzio [16]. Instead, we adopt Moen's [19] approach of competitive search equilibrium in which competitive market makers announce the wage at which firms and workers must agree if search is to take place at their location.

In Moen's work competition between market makers enforces a first best equilibrium. In our view this assumption is strong and the process by which a competitive wage is established is likely to take time. If one identifies the competitive market maker of Moen with the Walrasian auctioneer, our approach is one where the auctioneer has been removed and replaced either by a fixed or slowly adjusting wage; hence the title of our paper 'shooting the auctioneer.'

# 3 The Social Planning Problem

In this section we describe an artificial economy that adapts the standard one-sector real-business-cycle model by adding a search technology for moving labor between leisure and productive activities. We solve for the social planning optimum and show how the model with unemployment and vacancies is related to a standard environment with a spot market for labor.

## 3.1 Setting up the social planning problem

The social planner maximizes the discounted present value of the function

$$J_t = \max \sum_{t=s}^{\infty} \beta^{t-s} E_s \left[ \log \left( C_t \right) - \chi \frac{L_t^{1+\gamma}}{1+\gamma} - b \left( U_t + V_t \right) \right].$$

The first term in the square bracket represents the utility of consumption which we take to be logarithmic. The second term represents the disutility of working in market activity and the third is the utility cost of searching for a job. The cost of search has two components; $U_t$ is time spent searching by a worker for a job, and $V_t$ is time spent by the representative family in its role as an employer searching for workers.

The stock of employment evolves according to the expression

$$L_t = (1 - \delta_L) L_{t-1} + M_t, \tag{1}$$

where we assume that matches separate exogenously at rate $\delta_L$. The term

$$M_t = B (U_t)^{\theta} (V_t)^{1-\theta} \tag{2}$$

is the matching function which we take to be Cobb-Douglas with weight $\theta$.

The problem is constrained by a sequence of capital accumulation constraints,

$$K_{t+1} = K_t (1 - \delta_K) + Y_t - C_t, \qquad t = 1...,$$

and by a production function,

$$Y_t = A_t (K_t)^{\alpha} \left((1+g)^t L_t\right)^{(1-\alpha)}. \tag{3}$$

Output, $Y_t$ is produced using labor $L_t$ and capital $K_t$ which depreciates at rate $\delta_K$. The term $(1+g)^t$ measures exogenous technological progress and $A_t$ is an autocorrelated productivity shock which follows the stationary process

$$A_t = A_{t-1}^{\rho} \exp(\varepsilon_t), \qquad 0 < \rho < 1,$$

$$E_{t-1}(\varepsilon_t) = 0.$$

We assume that $\{\varepsilon_t\}_{t=1}^{\infty}$ is a Markov process with bounded support and we let $\varepsilon^t$ be the history of shocks, defined recursively as;

$$\begin{aligned} \varepsilon^t &= \varepsilon^{t-1} \times \varepsilon_t, \\ \varepsilon^1 &= \varepsilon_1. \end{aligned}$$

The assumption of bounded support is required in Section 5 in which we compute a second order approximation to the policy function.

## 3.2 Solving the social planning problem

The social planner can alter the stock of workers in productive activities by varying the time spent searching for jobs by workers or the time spent searching by firms for workers. Since the stock of labor can only be increased by hiring, the inclusion of employment as a state variable adds an additional propagation mechanism for shocks. Although this mechanism is potentially important, in practice the separation rate from firms is so high that the contribution of this additional component is not large and in our calibrated model most movements in employment at business cycle frequencies are caused by variations in time spent searching by firms or by workers[2].

To model the movements in unemployment and vacancies that would be observed in an efficient allocation we solve the social planning problem. To move labor into and out of productive activity the planner chooses contingent sequences $\{U_t(\varepsilon^t), V_t(\varepsilon^t)\}$. The first-order conditions for the choice of these variables are given by Equations (4) and (5);

$$\theta \frac{M_t}{U_t} \lambda_t = b, \tag{4}$$

$$(1-\theta) \frac{M_t}{V_t} \lambda_t = b, \tag{5}$$

where $\lambda_t$ is the Lagrangian multiplier on the labor accumulation constraint (1). Dividing (4) by (5) implies that the ratio $V_t/U_t$ is constant

$$\frac{V_t}{U_t} = \frac{1-\theta}{\theta}.$$

We define $\varphi_t = V_t/U_t$ to be labor market tightness since when $\varphi_t$ is high there are many firms looking for workers but few workers searching for jobs. From a planning perspective, there is an optimal level of tightness and the most efficient way to increase the labor stock $L_t$ is to increase unemployment and vacancies together. If the data were generated by a social planning solution one would expect to observe that movements in $U_t$ are perfectly correlated with movements in $V_t$.

[2]Shimer [25] cites data from Abowd and Zellner [1] and from the Job Openings and Labor Turnover Survey, to argue that separations occur at a rate of approximately 10% per quarter in the U.S. data. This is a big number - it implies that 40% of the labor force separates from employment in a year.

A consequence of constant labor market tightness is that $\lambda_t$, the shadow price of increasing the stock of labor, is also constant. By the definition of the matching function, Equation (4), and constancy of tightness we have

$$\lambda_t = \frac{b}{\theta} \frac{U_t}{B (U_t)^{\theta} (V_t)^{1-\theta}} = \frac{1}{B} \frac{b}{\theta^{\theta} (1-\theta)^{1-\theta}}. \tag{6}$$

By moving $U_t$ and $V_t$ together the social planner maintains a constant marginal utility cost of creating new matches.

The first order condition for the choice of capital $K_{t+1}$ is given by

$$\frac{1}{C_t} = \beta E_t \left[ \frac{1}{C_{t+1}} \left( 1 - \delta_K + \alpha \frac{Y_{t+1}}{K_{t+1}} \right) \right], \tag{7}$$

and the first order conditions for choosing $\{L_t\}$ is

$$(1-\alpha) \frac{Y_t}{C_t L_t} - \chi L_t^{\gamma} = \lambda_t - \beta (1-\delta_L) E_t [\lambda_{t+1}].$$

Combining Equation (8) with (6) leads to the expression

$$\frac{1}{C_t} \left( (1-\alpha) \frac{Y_t}{L_t} - C_t \chi L_t^{\gamma} \right) = \kappa, \qquad \kappa = (1 - \beta (1-\delta_L)) \frac{1}{B} \left[ \frac{b}{\theta^{\theta} (1-\theta)^{1-\theta}} \right], \tag{8}$$

which is closely related to the static optimizing condition for labor that arises from a standard RBC model. The parameter $\kappa$ drives a wedge between the marginal product of labor and the disutility of employment and as $\kappa \to 0$, Equation (8) converges to the familiar first order condition for labor in the one-sector RBC model. For small values of $\kappa$, the time paths of capital, gdp, consumption and hours are close to the solutions obtained from an RBC economy with a spot market for labor.

The parameter $\chi$ is key to our discussion below of the ability of this model to replicate unemployment dynamics. Although our model is richer than the standard search model studied by Hagedorn and Manovskii [9], $\chi$ plays the same role as the reservation wage in their environment. In a search model with Nash bargaining, $\chi$ would place a lower bound on the wage that the firm could offer. We will return to the role of $\chi$ later in the paper when we discuss its interaction with wage stickiness in helping to generate the Beveridge curve and simultaneously to generate realistic unemployment volatility.

# 4 A Decentralized Model

In this section we study a decentralized version of the model. We assume that a representative worker/firm takes the real wage as given and chooses capital, unemployment and vacancies to maximize expected utility. To close the model we introduce three alternative solution concepts to determine the real wage.

In the first concept we adopt the idea that competition between market makers forces the wage to maximize the expected utility of potential workers, that is, the wage is chosen to implement the social planning optimum. We compare this solution with an alternative, suggested by Hall [10], in which the real wage is unresponsive to current market conditions. We implement this solution by assuming that the real wage is that which would prevail along the non-stochastic balanced growth path. Since the fixed wage solution leads to fluctuations in unemployment and vacancies that are too volatile relative to the data, we also consider a third equilibrium concept in which the real wage adjusts partially each period towards its optimal value.

## 4.1 Setting up the agent's problem

The decentralized economy is populated by a unit measure of households who optimally choose labor effort $L^S$, search effort $U$, and capital holdings $K$ to maximize utility and operate the production technology for which they choose employment of labor $L^D$, search effort $V$, and the amount of capital to rent from households. We assume markets exist that allow households to perfectly share consumption risk. The solution to this decentralized problem is equivalent to the solution of the problem of a representative agent who acts both as a household and as a firm.

In his role as a household, the agent supplies labor $L_t^S$. His utility function is

$$J_t = \max \sum_{t=s}^{\infty} \beta^{t-s} E_s \left[ \log\left(C_t\right) - \chi \frac{L_t^{1+\gamma}}{1+\gamma} - b\left(U_t + V_t\right) \right]$$

and labor supply in period $t$ is related to search effort $U_t$ and lagged labor supply by the expression

$$L_t^S = \left(1 - \delta_L\right) L_{t-1}^S + U_t \frac{\bar{M}_t}{\bar{U}_t}. \tag{9}$$

$\bar{M}_t/\bar{U}_t$ is the increase in employment when the household increases its search intensity, $U_t$, by one unit. This probability is parametric to the household, but is determined in equilibrium as the ratio of aggregate matches $\bar{M}_t$ to aggregate search intensity $\bar{U}_t$.

The representative worker/firm faces the following sequence of budget constraints;[3]

$$K_{t+1} = K_t\left(1-\delta_K\right) + A_t K_t^{\alpha}\left(\left(1+g\right)^t L_t^D\right)^{1-\alpha} + W_t L_t^S - W_t L_t^D - C_t. \qquad t = 1, \ldots \tag{10}$$

The household can increase its stock of workers, $L_t^D$ by incurring a utility cost $-bV_t$ of search. Every additional unit increase in $V_t$ leads to an increase in the stock of employed workers of $\bar{M}_t/\bar{V}_t$ where $\bar{V}_t$ is aggregate search intensity by all other firms and $\bar{M}_t$ is the aggregate number of matches. This leads to the following expression

$$L_t^D = \left(1-\delta_L\right) L_{t-1}^D + V_t \frac{\bar{M}_t}{\bar{V}_t}, \tag{11}$$

for the accumulation equation faced by the household in its role as a labor demander.

## 4.2 Solving the agent's problem

The representative agent chooses state contingent sequences

$$\left\{K_{t+1}\left(\varepsilon^t\right), U_t\left(\varepsilon^t\right), V_t\left(\varepsilon^t\right)\right\}_{t=1}^{\infty},$$

taking as given the production function (3) and the accumulation constraints (9), (10) and (11). The first order condition for the choice of capital leads to the Euler equation;

$$\frac{1}{C_t} = \beta E_t\left[\frac{1}{C_{t+1}}\left(1-\delta_K+\alpha\frac{Y_{t+1}}{K_{t+1}}\right)\right]. \tag{12}$$

[3] Although we have modeled an economy with a single asset, storable capital, nothing of substance would be added by including a complete set of contingent claims markets. Since this is a representative agent economy, additional markets would serve only to determine the prices for additional assets at which the representative agent would choose not to trade.

The first order conditions for the choice of time spent searching in his capacity as a worker and a firm leads to the following two first-order conditions;

$$\frac{1}{C_t} W_t - \chi \left(L_t^S\right)^{\gamma} = b\left(\frac{\bar{U}_t}{\bar{M}_t} - \beta\left(1-\delta_L\right) E_t\left[\frac{\bar{U}_{t+1}}{\bar{M}_{t+1}}\right]\right), \tag{13}$$

$$\frac{1}{C_t}\left(\left(1-\alpha\right)\frac{Y_t}{L_t^D} - W_t\right) = b\left(\frac{\bar{V}_t}{\bar{M}_t} - \beta\left(1-\delta_L\right) E_t\left[\frac{\bar{V}_{t+1}}{\bar{M}_{t+1}}\right]\right). \tag{14}$$

In a competitive equilibrium, the model is closed by the market equilibrium conditions

$$L_t \equiv L_t^S = L_t^D, \quad \bar{U}_t = U_t, \quad \bar{V}_t = V_t,$$

and by the definition of the aggregate matching function

$$\bar{M}_t = M_t = B U_t^{\theta} V_t^{1-\theta}.$$

Using these market clearing conditions and the definition of tightness, in equilibrium Equations (13) and (14) become

$$\frac{1}{C_t}\left(W_t - \chi L_t^{\gamma} C_t\right) = \frac{b}{B}\left(\varphi_t^{\theta-1} - \beta\left(1-\delta_L\right) E_t\left[\varphi_{t+1}^{\theta-1}\right]\right), \tag{15}$$

$$\frac{1}{C_t}\left(\left(1-\alpha\right)\frac{Y_t}{L_t} - W_t\right) = \frac{b}{B}\left(\varphi_t^{\theta} - \beta\left(1-\delta_L\right) E_t\left[\varphi_{t+1}^{\theta}\right]\right). \tag{16}$$

Since there are two ways of moving labor between leisure and employment, but only one price, the model as it stands is missing an equilibrium condition. Typically, a model of this kind would be closed by adding a Nash bargaining equation to fix the real wage, $W_t$. The Nash bargaining solution, for appropriate choice of bargaining weights, can be shown to implement the social planning solution.[4] Alternatively one might appeal to Moen's idea of

[4] The generalized Nash bargaining solution divides the surplus of a match in proportion to an exogenous bargaining weight. For the case of a Cobb-Douglas matching function, this solution implements the social planning optimum when the bargaining weight is equal to the elasticity parameters $\theta$ of the matching function. This result is a generalization of the Hosios condition [14] to a model with more general utility functions.

competitive market makers to argue that the wage will be chosen to maximize the expected utility of the representative worker. In either case, the imposition of the efficient solution leads to a wage equation of the form

$$W_t = \theta (1-\alpha) \frac{Y_t}{L_t} + (1-\theta) C_t \chi L_t^{\gamma}. \tag{17}$$

Combining Equation (17) with the equilibrium conditions and the first-order conditions of the competitive model, Equations (13) and (14), one arrives at equations for unemployment and vacancies that mimic the first-order conditions of the social planner, Equations (4) and (5).

In the following analysis we will study three different equilibrium concepts. In the first we choose the wage according to Equation (17) to mimic the social planning optimum. We also consider a fixed wage equilibrium in which we allow the real wage to grow at the underlying rate of growth of the economy but we do not allow it to respond to productivity shocks. In the third equilibrium concept we allow the real wage to adjust each period by a fraction $\lambda$ of the way back towards the efficient solution in each period. The equations that define the wage are explained more fully in Section 5.2 after we define a set of stationary variables that allow us to find approximate solutions to our model.

# 5 Computational Issues

This section describes the procedure that we used to compute the properties of equilibria in the artificial economy. We begin by describing the solution algorithm that we used to compute the properties of artificial time series generated by the model. We then describe the alternative wage determination mechanisms that we used to close the model.

## 5.1 The Solution Algorithm

To compute solutions to the model we used a second order approximation to the policy function due to Schmitt-Grohé and Uribe [24]. Their procedure requires that the variables be separated into a set of non predetermined variables $p_t$ and a set of predetermined variables $q_t$. To implement this procedure, one must first find a representation of the model in which all of the variable are stationary.

To compute a stationary transformation of the model, we defined the following variables

$$k_t = \log\left(\frac{K_t}{(1+g)^t}\right),\ a_t = \log(A_t),$$

$$y_t = \log\left(\frac{Y_t}{(1+g)^t}\right) \quad c_t = \log\left(\frac{C_t}{(1+g)^t}\right), \quad l_t = \log(L_t),$$

$$u_t = \log(U_t), \qquad v_t = \log(V_t),$$

$$z_t = \log\left(\frac{Y_t}{L_t}\right), \quad w_t = \log\left(\frac{W_t}{(1+g)^t}\right), \quad j_t = \left(\frac{J_t}{(1+g)^{\frac{t}{1-\beta}+\frac{\beta}{(1-\beta)^2}}}\right).$$

The vector $q_t$ consists of the predetermined variables

$$q_t = \{a_t, k_t, l_{t-1}, w_{t-1}\},$$

and the vector of nonpredetermined variables, $p_t$ is given by

$$p_t = \{y_t, c_t, u_t, v_t, z_t, j_t\}.$$

We assume that all uncertainty arises from stochastic productivity shocks that take the form

$$a_{t+1} = \rho a_t + \sigma\varepsilon_{t+1}$$

where $\varepsilon_{t+1} \sim N(0,1)$ and is independent across time and $\sigma$ is the standard deviation of the innovation to the productivity shock. The model is a set of equations

$$E_t f(p_{t+1}, p_t, q_{t+1}, q_t) = 0, \tag{18}$$

where the function $f$ consists of identities, model definitions and first-order conditions. These equations are defined in Appendix A.

## 5.2 Alternative wage determination mechanisms

To compute the decentralized solution under alternative wage determination mechanisms we solved for the time path of $w_t$ in the social planning optimum. This is given by the expression;

$$w_t^{SP} = \log\left((1-\alpha)\theta e^{y_t - l_t} + \chi(1-\theta)e^{c_t - l_t}\right).$$

Next we computed the steady state value $\bar{w}$;

$$\bar{w} = \log\left((1-\alpha)\,\theta e^{y-l} + \chi\,(1-\theta)\,e^{c-l}\right).$$

To compute alternative equilibria we simulated sequences for a set of equations in which the wage is given by the expression

$$w_t = \frac{(1-\lambda)}{(1+g)} w_{t-1} + \lambda w_t^{SP}.$$

By setting $\lambda = 1$ this solution implements the social planning optimum. Alternatively, setting $\lambda = 0$ and choosing an appropriate initial condition fixes the wage equal to its unconditional mean along the balanced growth path. Choosing any other value of $\lambda$ in the interval $(0, 1)$ implements a partial adjustment mechanism in which the logarithm of the real wage adjusts a fraction $\lambda$ of the way towards the social planning optimum in any given period.

The solution to the model, when it exists and is unique, is of the form

$$\begin{aligned}
\underset{6\times1}{p_t} &= g\,(q_t, \sigma)\,, \\
\underset{4\times1}{q_{t+1}} &= h\,(q_t, \sigma) + \eta\sigma\varepsilon_{t+1},
\end{aligned}$$

where $\sigma$ is the standard deviation of the shock $\varepsilon_t$ and $\eta$ is the column vector

$$\eta = [1, 0, 0, 0]'.$$

Schmitt-Grohé and Uribe provide code that generates analytic first and second derivatives of the function $f$ in Equation (18).[5] Evaluating these derivatives at the point

$$\bar{p} = g\,(\bar{q}, 0)\,,\ \bar{q} = h\,(\bar{q}, 0)$$

leads to the second order approximation

$$\underset{6\times1}{\tilde{p}_t} = \underset{6\times1}{\mu_p} + \underset{6\times6}{g_q}\underset{6\times1}{\tilde{q}_t} + \frac{1}{2}\underset{6\times16}{G_{qq}}\underset{16\times1}{\tilde{m}_t}\,, \tag{19}$$

$$\underset{4\times1}{\tilde{q}_{t+1}} = \underset{4\times1}{\mu_q} + \underset{4\times4}{h_q}\underset{4\times1}{\tilde{q}_t} + \frac{1}{2}\underset{4\times16}{H_{qq}}\underset{16\times1}{\tilde{m}_t} + \underset{4\times1}{\eta}\,\sigma\varepsilon_{t+1}. \tag{20}$$

[5] The exact relationship between these expressions and the Schmitt-Grohé-Uribe code is explained in Appendix B.

The terms

$$\tilde{q}_t = (q_t - \bar{q})\,,\ \tilde{p}_t = (p_t - \bar{p})$$

are deviations of $p_t$ and $q_t$ from their non-stochastic steady states and

$$\mu_p = \frac{1}{2} g_{\sigma\sigma}\sigma^2, \quad \mu_q = \frac{1}{2} h_{\sigma\sigma}\sigma^2$$

are bias terms that cause $\tilde{p}_t$ and $\tilde{q}_t$ to differ from zero when the model is nonlinear. The variable

$$\tilde{m}_t = \tilde{q}_t \otimes \tilde{q}_t \equiv vec\left(\tilde{q}_t \tilde{q}_t'\right)$$

is a vector of cross product terms.

The Schmitt-Grohé-Uribe solution has a number of advantages over alternative algorithms. First, it uses the symbolic math feature of Matlab to compute analytic derivatives of a user specified set of functions. This feature mechanizes the process of solving for derivatives by hand and removes a potential source of error. Second, the program computes a second order approximation to the policy function which is essential if one is interested in a welfare comparison of alternative wage determination mechanisms.

# 6 Taking the model to the data

This section describes the procedures we used to pin down key parameters of the model. We begin by describing parameters that are in common with standard RBC models and move on to describe some novel features that arise from our version of a search model of the labor market.

## 6.1 Standard features of the calibration

Table 1a lists the values of six key moments that we used to calibrate parameters.

| Table 1a: | Product Market: Moments from the U.S. data | |
|---|---|---|
| Moment | Description | Value in baseline calibration |
| $g$ | Average quarterly growth rate of per capita gdp | 0.0045 |
| $r$ | Average quarterly real interest rate | 0.0160 |
| $c_y$ | Average ratio of consumption to gdp | 0.75 |
| $l_s$ | Labor's share of gdp | 0.66 |
| $\rho$ | Autocorrelation of TFP | 0.99 |
| $\sigma_\varepsilon$ | Standard deviation of TFP | 0.007 |

Since the artificial economy is based on a Solow growth model, the parameter $g$ which represents the quarterly growth rate of technological progress is equal to the quarterly per capita growth rate of gdp. This was set at 0.0045 which implies an annual per capita growth rate of 1.8%, equal to the U.S. average for the past century. To calibrate the elasticity of capital in production, $\alpha$, we used the assumptions of competitive labor markets and constant returns-to-scale which imply that $\alpha$ is equal to $1 - l_s$, where $l_s$ is labor's share of gdp.

To compute the time series properties of the productivity shock we computed a time series for total factor productivity in the data using the expression

$$TFP = \frac{Y_t}{K_t^\alpha L_t^{1-\alpha}}.$$

We regressed the log of TFP on its own lagged value and computed the first order autocorrelation coefficient and the standard deviation of the residual. This led to values of $\rho = 0.99$ and $\sigma_\varepsilon$ of 0.007.

The procedure for solving for the steady state levels of output, consumption, capital and the parameters $\beta$ and $\delta_k$ relies only on the steady state resource constraint and capital Euler equation and is identical to the procedure followed in a standard RBC model. This is a consequence of the fact that the friction introduced in the labor market distorts decisions only as the economy transitions from one state to another and is inconsequential in a nonstochastic economy in steady state. To compute the quarterly depreciation rate we solved the steady state equations,

$$(1 + r) = 1 - \delta_K + \frac{\alpha}{k_y}, \tag{21}$$

$$(g + \delta_K)\, k_y = 1 - c_y, \tag{22}$$

for $k_y$, the steady state capital to gdp ratio and $\delta_K$, the depreciation rate as functions of $r$, $\alpha$, $g$ and $c_y$. Equation (21) is a no-arbitrage relationship in the asset market and (22) is the steady state gdp accounting identity.

The unknowns $r$ and $c_y$ were set equal to their historical averages in the data; we set $r = 0.0162$ which represents an annual rate of 6.6% (computed as the average annual yield on the S&P 500) and $c_y = 0.75$, which is the average ratio of consumption to gdp when government consumption is included as part of consumption. The value of $\beta$ was computed from the steady state capital Euler equation

$$\beta \frac{(1 + r)}{(1 + g)} = 1,$$

which gives a value of $\beta = 0.99$. Table 1b lists the values for the parameters $\beta$, $\delta_K$ and $\alpha$, implied by this exercise.

| Table 1b: | Parameter values implied by Moments from Table 1a | |
|---|---|---|
| Parameter | Description | Value in baseline calibration |
| $\beta$ | Quarterly discount factor | 0.99 |
| $\delta_K$ | Quarterly depreciation rate for physical capital | 0.0314 |
| $\alpha$ | Elasticity of capital in production | 0.34 |

## 6.2 Labor market parameters calibrated from the steady state

We now turn to features of our model that differ from standard RBC economies. Table 2a lists some of the moments from data and parameter values that were used to calibrate the labor market portion of the model. We set the separation rate, $\delta_L$, at 10% per quarter based on Shimer's [25] interpretation of the JOLT data, the unemployment rate at 5.8%, and the participation rate to equal 70%.

Table 2a: Labor Market: Parameters chosen to match moments from the U.S. data

| Moment | Description | Value in baseline calibration |
|---|---|---|
| $u$ | Average unemployment rate | 0.058 |
| $p$ | Average participation rate | 0.7 |
| $\gamma$ | Inverse labor supply elasticity | 0 |
| $\theta$ | Elasticity of the matching function | 0.4 |
| $\delta_L$ | Average quarterly separation rate | 0.1 |

To pick the parameter $\gamma$ we used the fact that real business cycle models require high labor elasticity to generate sufficient volatility of hours. In the base-line calibration we picked $\gamma = 0$ which has become standard following Hansen's work [13] on indivisibilities. Finally, we set $\theta$ according to Blanchard and Diamond's [3] estimated value of $\theta = 0.4$.

The steady state values of $U$ and $L$ are related to the unemployment rate $u$, the participation rate $p$, and population size $N$ by the definitions

$$p = \frac{L+U}{N}, \tag{23}$$

$$u = \frac{U}{U+L}. \tag{24}$$

Since the model contains a single representative agent, we normalized the population size to 1 and computed $L$ and $U$ from Equations (23) and (24). This led to a value of $L = 0.66$, and $U = .041$ which implies that the representative agent spends 66% of his time in paid employment and 4.1% searching for a job.

To pin down steady state vacancies, new matches and the parameters $B$ and $b$ we used the labor market first order conditions and the definition of the matching function. In the steady state, the number of new matches must equal the number of jobs destroyed exogenously each period which implies,

$$m = \delta_L L. \tag{25}$$

From the steady state versions of Equations, (15) and (16), and noting that the steady state wage $w$ is given in terms of known quantities by the steady state version of (17) we can solve for $b$ and $V$ as functions of $\chi$

$$b = \frac{w/c - \chi L^{\gamma}}{1-\beta+\beta\delta_L}\frac{m}{U}, \tag{26}$$

$$V = \frac{m}{b\left(1-\beta+\beta\delta_L\right)}\left(\left(1-\alpha\right)\frac{y/c}{L} - w/c\right). \tag{27}$$

Finally, from the definition of the matching function we have

$$B = \frac{m}{U^{\theta}V^{1-\theta}}. \tag{28}$$

In the following table, $b$ and $V$ are calculated for $\chi$ equal to 1.204, a value that we explain in the following section. The steady state values associated with the labor market are collected in table 2b.

Table 2b: Labor market parameter values implied by Tables 1a and 2a

| Moment | Description | Value in baseline calibration |
|---|---|---|
| $U$ | Fraction of time unemployed | .041 |
| $V$ | Fraction of time searching for workers | .051 |
| $L$ | Fraction of time working | 0.66 |
| $b$ | Disutility of search effort | .83 |
| $m$ | Match parameter | 0.066 |
| $B$ | Constant of the matching function | 1.43 |

## 6.3 Labor market parameters associated with the volatility of unemployment and vacancies

There are two important parameters of the model that are not pinned down by the steady state calibration; these are $\chi$, a parameter that governs the disutility of work and $\lambda$, the degree of wage rigidity. The labor market dynamics of the model are highly sensitive to the joint choice of these parameters and they were calibrated to match several stylized facts regarding the cyclical behavior of unemployment and vacancies.

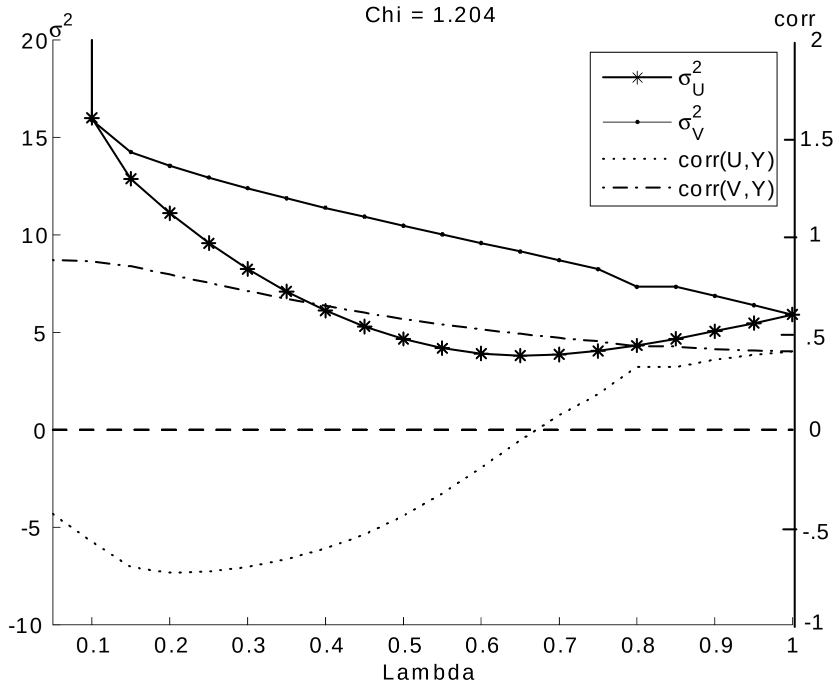


Figure 1: Model statistics for different values of lambda

Figure 1 plots the variances of unemployment and vacancies and the correlations of each with GDP for different values of $\lambda$ holding fixed $\chi$ at 1.204. Figure 2 plots these same variables against $\chi$ holding $\lambda$ fixed at 0.19. To select the values 1.204 and 0.19 we searched over a grid of these parameters and chose values that came close to matching these four moments. This figure illustrates the point that when $\lambda = 1$, (the social planning solution) $U$ and $V$ are both positively correlated with gdp and hence a model like this one with highly elastic search intensity cannot explain the Beveridge curve.

Figure 2 shows how the correlations of $U$ and $V$ with gdp depend on $\chi$. Notably, these correlations switch sign as $\chi$ increases above 1.325 and for values above 1.26 $\lambda$ becomes negative. This singularity when $\chi$ approaches 1.26 is present for all values of $\lambda \in [0, 1]$. These figures make transparent the way in which the imposition of some degree of wage rigidity breaks the one to one correlation between $U$ and $V$ that obtains in the flexible wage economy. We conclude that both the volatilities and correlations with GDP of the key

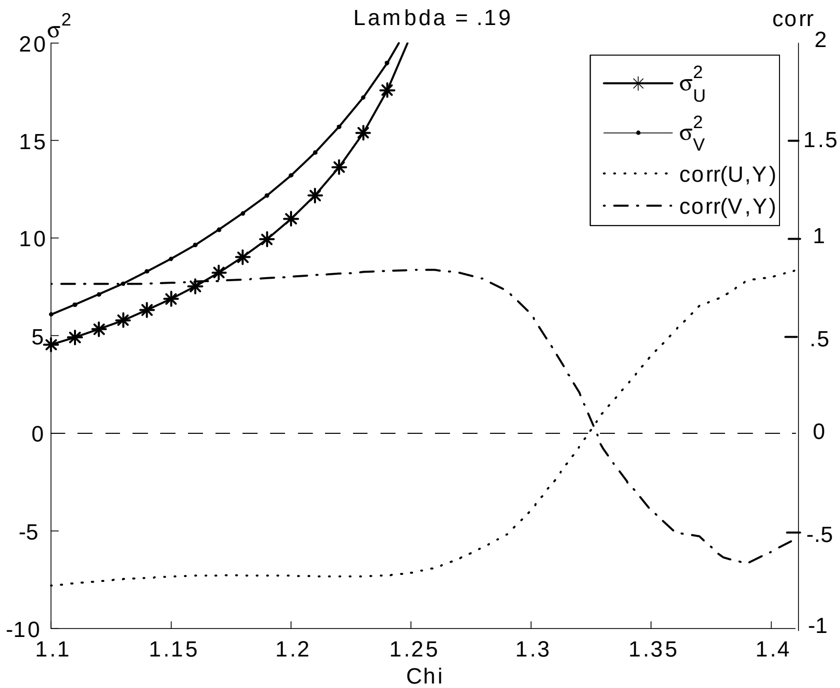


Figure 2: Model statistics for different values of chi

labor market variables, $U$ and $V$, are highly sensitive to the disutility of labor $\chi$ and to the speed of wage adjustment $\lambda$.

# 7 Matching the Data

We now turn to the performance of our model in certain key dimensions by comparing two different calibrated economies with the U.S. data. Table 3 reports the volatilities and correlations with gdp of gdp, consumption, investment, hours worked, labor productivity, the real wage, unemployment and vacancies. The first column reports the moments of the quarterly data from 1955 first quarter, to 2002, fourth quarter. Consumption and investment are both defined as the sum of private plus government components and all variables are in 1996 U.S. dollars and deflated by U.S. resident population. Hours is defined as employment per person multiplied by average hours where employment is total non farm employment from the establishment survey. Productivity is gdp deflated by hours and the real wage is computed as compensation to employees divided by hours and deflated by the 1996 gdp price index. Unemployment is the U.S. unemployment rate for persons over 16 years old and vacancies is an index of help wanted from the St. Louis Federal Reserve data base. All variables have been passed through the HP filter with a smoothing parameter of 1600.

The second two columns of Table 3 report the same moments for two artificial economies. The middle panel is an economy in which the parameter $\lambda$ is set equal to 1 which allows the wage to adjust each period to a value that causes the decentralized solution to mimic the social planning optimum. The third panel reports data for an economy in which the log of the real wage adjusts by a fraction 0.19 towards the social planning optimum wage level in any given period.

Table 3 — Standard deviations in percent (a) and correlations with Gdp (b) for U.S. and artificial economies

| | Quarterly U.S. time series (1955.1-2002.4) | | Flexible wage economy ($\lambda = 1$) | | Sticky wage economy ($\lambda = 0.19$) | |
|---|---|---|---|---|---|---|
| Series | (a) | (b) | (a) | (b) | (a) | (b) |
| Gdp | 1.59 | 1.00 | 1.43 | 1.00 | 1.58 | 1.00 |
| | $NA$ | $NA$ | (0.18) | (0.00) | (0.22) | (0.00) |
| Consumption | 1.04 | 0.85 | 0.71 | 0.95 | 0.75 | 0.94 |
| | $NA$ | $NA$ | (0.10) | (0.007) | (0.11) | (0.011) |
| Investment | 5.45 | 0.93 | 3.75 | 0.98 | 4.27 | 0.98 |
| | $NA$ | $NA$ | (0.46) | (0.003) | (0.61) | (0.004) |
| Hours | 1.80 | 0.92 | 0.79 | 0.73 | 1.05 | 0.71 |
| | $NA$ | $NA$ | (0.10) | (0.062) | (0.18) | (0.064) |
| Productivity | 0.74 | −0.07 | 0.71 | 0.95 | 0.68 | 0.87 |
| | $NA$ | $NA$ | (0.10) | (0.007) | (0.10) | (0.039) |
| Real wage | 0.74 | −0.24 | 0.71 | 0.62 | 0.42 | 0.11 |
| | $NA$ | $NA$ | (0.10) | (0.056) | (0.08) | (0.053) |
| Unemployment | **11.57** | **−0.86** | **5.91** | **0.40** | **11.43** | **−0.72** |
| | $NA$ | $NA$ | (0.36) | (0.038) | (1.50) | (0.067) |
| Vacancies | **13.00** | **0.90** | **5.91** | **0.40** | **13.67** | **0.81** |
| | $NA$ | $NA$ | (0.36) | (0.038) | (1.17) | (0.010) |

Columns 2 and 3 of Table 3 were generated by simulating 100 runs of the model for the baseline parameters setting $\lambda = 1$ for column 2 and $\lambda = 0.19$ for column 3. We refer to the former as a flexible wage economy and to the latter as a sticky wage economy. The disutility of labor $\chi = 1.204$ was chosen together with the wage flexibility parameter $\lambda = 0.19$ to match the four model moments described in the previous section to the corresponding sample moments in the data. For comparison purposes, we set $\chi = 1.204$ in both calibrations. The numbers in parentheses are standard deviations of the reported variable over 100 simulations. In each case, the column labeled (a) reports the standard deviation of a variable and the column labeled (b) is its correlation with gdp. All artificial data has been passed through the HP filter with a smoothing parameter of 1600 in the same way as the real world data.

There are two features of Table 3 that are important. Notice first, that gdp, consumption investment and hours have the same statistical properties in the fixed wage and the flexible wage economies. In each case the correlations with gdp and the standard deviations of these series are within one standard deviation of each other. The reasons for the differences of these statistics from the U.S. data are, by now, well understood and the model does not add much that is new in this dimension.[6] The fixed and flexible wage economies differ substantially, however, in their predictions for the behavior of unemployment and vacancies.

In the data the standard deviations of unemployment and vacancies are equal to 11.57 and 13.00. Vacancies are procyclical with a correlation coefficient with gdp of 0.9 whereas unemployment is countercyclical with a correlation coefficient of $-0.86$. In the social planning optimum, in contrast, unemployment and vacancies each have a standard deviation of 5.91, they are perfectly correlated with each other and correlated with gdp with a coefficient of 0.40. Contrast this with the sticky-wage artificial economy. Here, unemployment has a standard deviation of 11.43 and vacancies has a standard deviation of 13.67. As in the U.S. data, these variables are negatively correlated with each other. Vacancies is procyclical with a correlation coefficient with gdp of 0.81 and unemployment is countercyclical with a correlation coefficient with gdp of $-0.72$. We conclude that the sticky wage model does a reasonably good job of matching the labor market facts without com-

[6] For example, investment is too smooth in our simulated environment. This is typically addressed by adding a cost of adjustment to the model.

promising the ability of the model to explain gdp, hours, consumption and investment.

Although our model performs well in some dimensions its behavior with respect to the real wage and productivity is disappointing. By adding an additional shock to the model we might hope to reduce the model implications that productivity and gdp are strongly correlated whereas the correlation in the data is much weaker. The model also has counterfactual implications for the correlation between the real wage and GDP although this may partly depend on the timing of our labor adjustment equation. We assume that new workers can immediately contribute to gdp whereas a more standard assumption is to impose a one-period delay. We plan to explore this issue in future research.

# 8 Welfare Costs of Sticky Wages

If the sticky wage economy does a good job of replicating the real world data, one might ask the question: What is the difference in welfare between the flexible wage equilibrium and the equilibrium with sticky prices? To answer this question, we computed a second order approximation to expected utility for values of $\lambda$ ranging from 0 to 1. When $\lambda = 0$, the wage grows each period at the rate $g$, but is unresponsive to innovations in the technology shock. For our calibrated value $\lambda = 0.19$, the wage adjusts each period by 19% of the difference between its previous value and the optimal wage for the period.

We follow Lucas [15] and calculate the welfare cost of a sticky wage regime parameterized by $\lambda$ as the fraction of consumption an agent would give up each period in return for moving to the wage regime $\lambda = 1$. To formalize this, define the expected utility of an agent in an economy parameterized by $\lambda$ and in the social planning economy ($\lambda = 1$) respectively as

$$V^{\lambda} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log\left(C_t^{\lambda}\right) - \chi \frac{\left(L_t^{\lambda}\right)^{1+\gamma}}{1+\gamma} - b\left(U_t^{\lambda} + V_t^{\lambda}\right) \right], \tag{29}$$

$$V^{SP} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log\left(C_t^{SP}\right) - \chi \frac{\left(L_t^{SP}\right)^{1+\gamma}}{1+\gamma} - b\left(U_t^{SP} + V_t^{SP}\right) \right], \tag{30}$$

where the superscript $\lambda$ refers to the state contingent allocations in the sticky wage economy and superscript $SP$ refers to the social planning allocation.

The welfare cost of living in an economy with wage stickiness, which we denote $\psi(\lambda)$, is implicitly defined by

$$V^{\lambda} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log\left( (1-\psi(\lambda)) C_t^{SP} \right) - \chi \frac{\left(L_t^{SP}\right)^{1+\gamma}}{1+\gamma} - b\left(U_t^{SP} + V_t^{SP}\right) \right].$$

Since the period utility function is additively separable and logarithmic in utility we obtain the following expression for $\psi(\lambda)$ in terms of the difference in expected lifetime utility between the two regimes

$$\psi(\lambda) = 1 - \exp\left[ (1-\beta)\left( V^{\lambda} - V^{SP} \right) \right].$$

We used the Schmitt-Grohé-Uribe algorithm discussed above to obtain an approximate difference equation describing the evolution of lifetime utility $j_t$ over time. That the approximation is second order accurate is especially important in the welfare calculation as in a first order approximation the certainty equivalence property would obtain. Figure 3 plots the welfare cost for different values of $\lambda$ with all other model parameters fixed at the values tabulated in the calibration section.

We are most interested in the welfare cost in the economy where the degree of wage stickiness, $\lambda$, delivers plausible movements of unemployment and vacancies over the business cycle, which we argued is the economy where $\lambda = 0.19$. While there is still a welfare loss in this economy, we found it to be much less than in the fully rigid case. The representative agent would be willing to give up 0.0071% of consumption each period in order to live in the flexible wage economy. This is roughly \$0.40 per person per quarter which is a relatively small number. This suggests that relatively low unmodelled costs of rapid wage adjustment, menu costs for instance, could serve as a plausible explanation for equilibrium wage stickiness. On the other hand, when wages are made completely sticky the welfare cost is 0.37% of consumption or \$20.86 per person per quarter, a number that would require much higher costs to justify.

# 9 Conclusion

We have shown that a relatively simple modification to a standard real business cycle model, of the kind initially studied by Andolfatto and Merz, cannot easily explain the properties of unemployment and vacancies in the U.S. data.

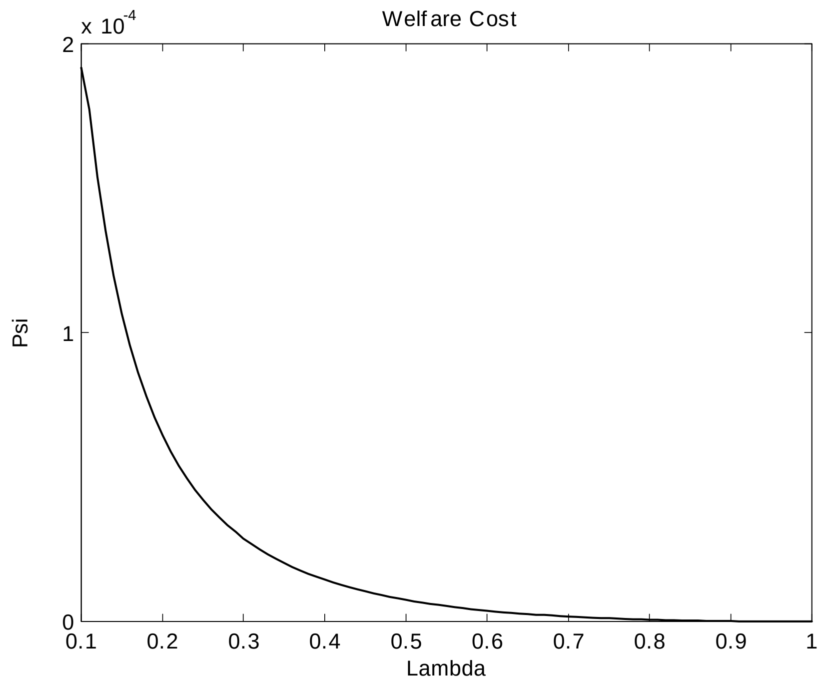


Figure 3: The welfare cost of sticky prices

The problem with this model is the one identified by Shimer: unemployment and vacancies are not volatile enough and they have the wrong correlation with gdp. We modified the model using Hall's suggestion that a model with rigid wages may provide a better representation of the data. As pointed out by Hall, the rigid wage model does not leave firms or workers with an incentive to change their behavior, in effect, because the search model has a missing market.

Although the rigid wage model does better in some dimensions than the flexible wage economy, it overshoots on unemployment volatility and leads to gdp fluctuations that are too small. An intermediate model in which the real wage adjusts by 19% of the way each quarter towards the flexible wage solution, does a much better job. This model performs as well as the standard RBC model at capturing the volatility of hours, gdp, investment and consumption. In addition it captures the observed volatility of unemployment and has close to the correct volatility for vacancies. More important, we find that unemployment is countercyclical and vacancies are procyclical, just as they are in the U.S. data.

We compared the welfare properties of alternative equilibria and found that the rigid wage solution is associated with a welfare cost of roughly \$20.86 per person, a relatively large number. The partial adjustment equilibrium, on the other hand, is associated with a welfare cost of only \$0.40 per quarter. This suggest that there may some small unmodelled cost of wage adjustment that is missing from the model, but which causes the sticky wage equilibrium to dominate, at least for fluctuations of the magnitude that we have observed in the post-war period.

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# Appendix A: The function f

Production Function

$$f_1 = y_t - a_t - \alpha k_t - (1-\alpha) l_t. \tag{A1}$$

Euler equation

$$f_2 = e^{-c_t} - \frac{\beta}{1+g} e^{-c_{t+1}} (1 - \delta_K + \alpha e^{y_{t+1} - k_{t+1}}). \tag{A2}$$

Gdp accounting identity

$$f_3 = (1+g) e^{k_{t+1}} - (1-\delta_K) e^{k_t} - e^{y_t} + e^{c_t}. \tag{A3}$$

Technology shock process

$$f_4 = a_t - \rho a_{t-1}. \tag{A4}$$

Utility function definition

$$f_5 = c_t - b\left(e^{u_t} + e^{v_t}\right) - \frac{\chi}{1+\gamma} (e^{l_t})^{1+\gamma} + \beta e^{j_{t+1}} - e^{j_t}. \tag{A5}$$

Wage equation

$$f_6 = e^{w_t} - \frac{(1-\lambda)}{1+g} e^{w_{t-1}} - \lambda \left(\theta (1-\alpha) e^{y_t - l_t} + (1-\theta) \chi (e^{l_t})^{\gamma} e^{c_t}\right). \tag{A6}$$

Vacancy first order condition

$$f_7 = e^{w_t - c_t} - (1-\alpha)\, e^{y_t - c_t - l_t} + \frac{b}{B} e^{\theta (v_t - u_t)} - \frac{b}{B} \beta \left(1 - \delta_L\right) e^{\theta (v_{t+1} - u_{t+1})}. \tag{A7}$$

Unemployment first order condition

$$f_8 = e^{w_t - c_t} - \chi \left(e^{l_t}\right)^{\gamma} - \frac{b}{B} e^{(1-\theta)(u_t - v_t)} - \frac{b}{B} \beta \left(1 - \delta_L\right) e^{(1-\theta)(u_{t+1} - v_{t+1})}. \tag{A8}$$

Labor accumulation equation

$$f_9 = e^{l_t} - e^{\theta u_t + (1-\theta) v_t} - (1-\delta_L) e^{l_{t-1}}. \tag{A9}$$

Definition of productivity

$$f_{10} = z_t - y_t + l_t. \tag{A10}$$

# Appendix B

The Schmitt-Grohé-Uribe code generates arrays $h_x, h_{xx}, h_{\sigma\sigma}, g_x, g_{xx}$ and $g_{\sigma\sigma}$. The arrays $g_{xx}$ and $h_{xx}$ are three dimensional and may be unpacked into ten 4×4 and four 4×4 matrices respectively. The second order approximation in matrix form can then be written as follows

$$\tilde{p}_t = g_{\sigma\sigma}\sigma^2 + g_q\tilde{q}_t + \begin{bmatrix} \tilde{q}_t' g_{qq}^1 \tilde{q}_t \\ \vdots \\ \tilde{q}_t' g_{qq}^9 \tilde{q}_t \end{bmatrix}, \tag{B1}$$

$$\tilde{q}_{t+1} = h_{\sigma\sigma}\sigma^2 + h_q\tilde{q}_t + \begin{bmatrix} \tilde{q}_t' h_{qq}^1 \tilde{q}_t \\ \vdots \\ \tilde{q}_t' h_{qq}^4 \tilde{q}_t \end{bmatrix}, \tag{B2}$$

where $\tilde{q}_t$ is $4 \times 1$ and $\tilde{p}_t$ is $9 \times 1$. Using Kronecker product notation and the fact (see Hamilton [**?**] page 265) that

$$vec\,(ABC) = (C' \otimes A)\, vec(B), \tag{B3}$$

it follows that

$$vec(\tilde{q}_t' h_{qq}^i \tilde{q}_t) = \tilde{q}_t' \otimes \tilde{q}_t' vec(h_{qq}^i) = vec\left(h_{qq}^i\right)' \tilde{m}_t,\ i = 1, ..4 \tag{B4}$$

$$vec(\tilde{q}_t' g_{qq}^i \tilde{q}_t) = \tilde{q}_t' \otimes \tilde{q}_t' vec(g_{qq}^i) = vec\left(g_{qq}^i\right)' \tilde{m}_t,\ i = 1, ..9 \tag{B5}$$

where

$$\underset{16\times 1}{\tilde{m}_t} = \underset{4\times 1}{\tilde{q}_t} \otimes \underset{4\times 1}{\tilde{q}_t}\,, \tag{B6}$$

is a $16 \times 1$ column vector. Defining

$$G_{qq} = \begin{bmatrix} vec\left(g_{qq}^1\right)' \\ \vdots \\ vec\left(g_{qq}^9\right)' \end{bmatrix}, \tag{B7}$$

$$H_{qq} = \begin{bmatrix} vec\left(h_{qq}^1\right)' \\ \vdots \\ vec\left(h_{qq}^4\right)' \end{bmatrix}, \tag{B8}$$

and $\mu_q = h_{\sigma\sigma}\sigma^2,\ \mu_p = g_{\sigma\sigma}\sigma^2$ leads to the expressions

$$\tilde{p}_t = \mu_p + g_q\tilde{q}_t + G_{qq}\tilde{m}_t, \tag{B9}$$

$$\tilde{q}_{t+1} = \mu_q + h_q\tilde{q}_t + H_{qq}\tilde{m}_t, \tag{B10}$$

which corrspond to equations (19) and (20) in the text.