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COSTS AND BENEFITS OF AN  
ANTI-INFLATIONARY POLICY:  
QUESTIONS AND ISSUES

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ABSTRACT

This paper analyses how the output or unemployment cost of achieving a sustainable reduction in the rate of inflation depends on the structure of the wage-price process and how the "sacrifice ratio" can be minimized. In models where the natural rate is invariant under the anti-inflationary policies, price level inertia is not sufficient for a positive sacrifice ratio. Without sluggishness in the core inflation rate, a zero sacrifice ratio can be achieved simply through intelligent demand management. With sluggish core inflation, the sacrifice ratio is positive unless intelligent demand management is complemented by cost-reducing fiscal measures or effective incomes policy. Letting the exchange rate float does not reduce the sacrifice ratio. If core inflation is partly backward-looking and partly forward-looking, current core inflation may be a function of current and past expectations of future recessions. Conventional sacrifice ratio calculations ignore forward-looking aspects of behaviour and may therefore underestimate the true cost of disinflation. If there is hysteresis in the natural rate (e.g. through a gradual adjustment of the natural rate towards the actual rate) and if there is sluggish core inflation, the sacrifice ratio will become infinite.

Whenever sluggish core inflation is present, credibility of the anti-inflationary (monetary) policy alone cannot obviate a positive sacrifice ratio.

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## I. Introduction

Unlike many contributions to this subject, the present paper has the virtue that at any rate its title (which was chosen by the organizers of the conferences) makes sense. As Hall [1981, p. 432] states "Inflation is an outcome of economic processes, not an exogenous causal influence". At the level of the economy as a whole, it is therefore a nonsense to refer, as is often done, to the costs of inflation or to the benefits from lower inflation. Since inflation is endogenous, the benefits or costs of lower inflation can only be discussed sensibly by specifying the changes in exogenous variables and parameters (policy actions or rules, external events etc.) that bring about and sustain the lower rate of inflation. Partial equilibrium or single-structural-equation "costs of inflation" analyses are void of policy implications.

As regards the benefits from policies to achieve a sustained reduction in the rate of inflation, we shall be brief, because we have nothing much to add to the received wisdom on the topic<sup>1</sup>. First consider a non-stochastic economy. We shall assume that a permanent reduction in the proportional rate of growth of the nominal stock of money is a necessary condition for a permanent reduction in the rate of inflation. Unless debt neutrality prevails or government interest-bearing debt is index-linked, the reduction in the growth rate of the nominal money stock will have to be matched by an equal reduction in the growth rate of the stock of government bonds in order to achieve sustained reduction in the rate of inflation.

It is important to appreciate that it is the same institutional feature of the economy that causes costs to be associated both with policies that cause inflation and with policies to reduce inflation. That feature is the existence of nominal contracts that are not perfectly price level-or inflation-contingent. The reasons for the existence of such contracts have to do with the advantages of using money in the first place -- the benefits from using a common numeraire, standard of deferred payment, medium of exchange and means of payment. They are either too obvious or too deep to be considered in this paper. Bringing down inflation is costly because of the existence of non-contingent money wage and price contracts. In a deterministic setting, imperfect and incomplete indexation of prices and rates of return is the main reason for welfare costs to be associated with policies that cause inflation. The only other cost is the "relabeling cost", i.e. the cost of changing prices more frequently, which can be generalized to the cost of the disruption of well-established methods of transacting business (Carlton, 1982, p. 139).

The best-known cost of non-indexation is the loss of consumer surplus suffered when the demand for real money balances declines as (and to the extent that) the nominal interest rate rises in response to an increase in the (expected) rate of inflation. With the nominal rate of return on money balances assumed fixed (typically at zero) the nominal interest rate represents the opportunity cost of holding money. Partial equilibrium approaches take the real interest rate as given and have the nominal interest rate rising one for one

with an increase in the (expected) inflation rate. This represents a tax on the holding of money balances. If the demand for money is interest-sensitive and money is produced costlessly, this will impose a dead-weight loss of consumer surplus given by the trapezoidal area under the (compensated) money demand function between the low inflation and high inflation quantities of money demanded<sup>2</sup>. General equilibrium approaches treat the real interest rate as endogenous and not necessarily invariant to the policy changes that cause the higher rate of inflation. In a multi-period setting the entire sequence of discounted instantaneous (or single period) trapezoidal utility losses must be considered [Feldstein 1979].

Non-indexation or incomplete indexation extends in practice well beyond the rate of return on high-powered money balances, and affects private contracts as well as tax laws and regulations. Extensive work on the subject has been done by Fischer and Modigliani [1978], Fischer [1981b].

When comparing a situation of low inflation with one of high inflation, the lower level of welfare (presumable) enjoyed in the latter can be attributed either to the policy that brings about the higher inflation (i.e., higher monetary growth) or to the failure to implement policies that permit a partial or complete adaptation to the higher inflation. If indexation is cheap, let alone costless, dead-weight losses, incurred because of a combination of inflation and incomplete indexation, can be attributed with as much justice to the failure to implement policies to adapt to inflation as to the policies that cause

the inflation. This holds for any undesired distributional consequences of policies that cause inflation as much as for the inefficiencies that they entail.

Turning to a stochastic world, a vast literature has sprouted in recent years on the costs of variable or uncertain inflation and on the links between expected inflation and the variability or uncertainty of the price level, the rate of inflation or relative prices<sup>3</sup>. Even in deterministic models it is possible, if not all prices can be adjusted costlessly and continuously, to establish a causal link between the (anticipated) mean inflation rate and the (anticipated) variability of relative prices. The costs of such anticipated variability can be evaluated using conventional deterministic welfare economic tools. In what follows only unanticipated or currently unperceived variability, i.e. inflation or relative price uncertainty, will be considered. Causal chains have been proposed running from high (expected) inflation rates through to relative price uncertainty, suggesting that inflation may raise the noise-to-signal ratio for relative price movements, thus impairing the allocative efficiency of the price mechanism.

Since the expected rate of inflation, its variance and the variance of relative prices are all jointly endogenous in any reasonable macroeconomic model, it is very hard to understand what the "costs of inflation uncertainty" literature is trying to say. It is certainly interesting to study the time series behavior of mean inflation, of inflation variability and uncertainty and of relative price variability and uncertainty (see e.g. Fischer [1981]). By regressing any one of these on any subset of the remaining ones, one cannot hope to extract a structurally invariant relationship. The statement "high expected inflation causes highly uncertain inflation" makes no sense. The statement "certain policies or events that cause the first moment of the distribution function of inflation to increase also tend to raise its second moment" does make sense. E.G. : if higher

mean monetary growth is associated with increasingly unpredictable monetary growth, both the higher mean inflation and the increasing unpredictability of inflation can be attributed to monetary policy.

Perhaps the argument is that there is a structurally invariant relationship between mean monetary growth and the unpredictability of monetary growth for a monetary aggregate with the following two properties. First, it is the relevant causal one in the inflation process and second, the parameters of its distribution function can only be chosen by the authorities subject to some constraints. E.g. the authorities cannot control monetary growth exactly, but they can choose the first two moments of its distribution function, subject to the constraint that mean and variance are positively related. Since it is likely to be welfare-reducing for the authorities to throw more extraneous noise into the system, a further argument for lower mean money growth exists if the variance of the innovation in the money stock process is an increasing function of the expected rate of monetary growth. Unless this is the case, finding a positive pattern of covariation between mean inflation and relative price uncertainty carries no implications for monetary policy.

Note that none of this provides an argument for maximizing the predictability of future monetary growth rates but only for minimizing the variance of the innovation in the money stock process. Consider e.g. the monetary growth rule  $m_{t+1} - m_t = \alpha_t + u_{t+1}^m$ .  $m_t$  is the logarithm of the nominal money stock,  $u_t^m$  is the random, unpredictable, component in the money growth process i.e.  $E_t u_{t+1}^m = 0$ , while  $\alpha_t$  is the predictable component i.e.  $E_t \alpha_t = \alpha_t$ . Let  $\sigma_{m,t}^2 \equiv E_t (u_{t+1}^m)^2$ ; the inflation uncertainty hypothesis can be represented by

$$\frac{\partial \sigma_{m,t}^2}{\partial |\alpha_t|} > 0. \quad \alpha_t \text{ itself could be a non-stochastic known function}$$

of current and past realizations of random variables, i.e. it could be governed by a non-stochastic feedback rule. While such rules tend to make  $m_{t+2} - m_{t+1}$ , say, less predictable, in period  $t$  and earlier, than a non-stochastic open loop path for money growth, such feedback rules may well help diminish the uncertainty that matters. Examples are changes in the information content of observed market prices induced by monetary feedback rules (Turnovsky [1980] Weiss [1980], Buiter [1980, 1981]).

From the proposition that governments should not throw extraneous random noise into the economic system by randomizing their monetary policy rules, it also does not follow that minimizing uncertainty about future inflation or future relative prices is sensible policy. E.g. by freezing all relative prices through legislative fiat their predictability is maximized but any shocks to demand or supply will have to be absorbed through rationing and other disequilibrium mechanisms. Random shocks to the system will, in general, be absorbed by unexpected changes in prices, in quantities produced, sold and consumed and by unexpected changes in the values of policy instruments such as the money supply, which are set according to some contingent or conditional rule. By removing the elements of conditionality in the monetary rule to enhance the predictability of the future money stock path, shocks will have to be absorbed in some other way. Only detailed analysis of fully specified models can determine what kind of monetary rule maximizes expected utility.

A final argument for policies consistent with low or zero inflation is that price level stability (zero inflation) is the only stable



equilibrium. Positive inflation inevitably entails rising rates of inflation. (The symmetric argument for negative inflation rates is not made; there have been hyper-inflations but no hyper-deflations).

The political economy of inflationary monetary growth creating pressures for higher and more inflationary rates of monetary growth has never been spelled out satisfactorily. The historical experience of most OECD countries would seem to contradict it. If any inflation carried in itself the seeds of a hyper-inflation (with some non-negligible probability) the case for striving for price level constancy would of course be strengthened considerably.

II. The cost of policies to achieve a permanent reduction in inflation.

In Part II of this paper we shall consider the familiar Okun style output costs of securing a lasting reduction in the rate of inflation. There are well-known objectives to taking the cumulative net output loss (perhaps discounted) associated with an anti-inflationary policy as a measure of its cost. It ignores the benefits of additional output produced at home by the unemployed and not recorded in GNP, as well as the marginal valuation of their leisure and search time. We shall proceed regardless for two reasons. First, it has been argued (see e.g. Gordon [1973]) that if one attempts a conventional triangle approach grounded in applied welfare economics, the answers don't come out all that differently from the crude Okun's gap measure. Second, it is hard to take seriously an approach that cannot differentiate between an employed worker taking a vacation and a worker becoming unemployed. Conventional microeconomic analysis models utility as increasing in leisure, i.e. leisure is a "good thing", and thereby confines labour to the category of "bads". Overwhelming empirical evidence on the importance of work (i.e. of being employed, of having a job) for most people's well-being, happiness and even sanity has not made much of a dent in the "extended holiday" approach to unemployment.

Formally, it is quite easy to combine the "leisure as a good thing" and "work as a good thing" approaches. Consider the state-dependent utility function  $v(\cdot, \cdot)$  given below.  $c$  is a vector of goods and services other than leisure.  $L$  is the endowment of time and  $l$  hours spent working.  $\Theta$  is an indicator variable which takes on values  $\Theta = 1$  if employed and  $\Theta = 0$  if unemployed.

$$U = v(c, L-l, \theta)$$

The utility function is well-behaved in consumption and leisure (strictly increasing and quasi-concave). The benefit from being employed is represented by the assumption that for any given  $c = \bar{c}$  and  $l = \bar{l}$  we have

$$v(\bar{c}, L-\bar{l}, 1) > v(\bar{c}, L-\bar{l}, 0)^4$$

Depending on what further properties one attributes to the utility function<sup>5</sup> and on the budget constraints when employed and unemployed, one could still have people choosing voluntary unemployment.

In this paper only conventional output gap costs will be considered. Even if they have no clear welfare significance, it would still be worthwhile from a positive economics viewpoint to know their magnitude.

Bringing down inflation with a policy-invariant natural rate.

In this section we consider models that have the long-run natural rate property: the same level of real output (rate of unemployment) is consistent with any steady-state and fully anticipated rate of inflation. While output and unemployment can differ from their natural levels outside the steady state, these natural levels themselves are taken to be constant. For this class of models we shall show that price level inertia is not sufficient to generate output costs of policies to bring down inflation. For there to be such costs, inertia should attach to the rate of change of the price level. A number of "Keynesian" models, e.g. Buiter and Miller's [1981, 1982] variant of Dornbusch's [1976] overshooting model, Calvo's [1982 a,b,c] continuous time version of

Taylor's [1980] staggered overlapping money wage contract model,<sup>6</sup> Mussa's [1981] sticky price model and Obstfeld and Rogoff's [1982] version of a price adjustment rule of Barro and Grossman [1976] all have the property that, in principle, inflation can be reduced or eliminated costlessly. Well-designed and credible monetary and fiscal policy can make these economic systems mimic the real behavior of a completely flexible money wage and price economy.

After considering these models we analyze the cost of bringing down inflation in models with sluggish core inflation. These costs, and the properties of desirable anti-inflationary policies turn out to depend crucially on the relative importance of backward-looking ("long term contracts") versus forward-looking ("expectations") determinants of current inflation.

#### Closed economy models.

This subsection deals with closed economy models. Much of the analysis only requires consideration of the wage-price block. Where a complete, if rudimentary, macroeconomic model is required, the following standard log-linear IS-LM model will be used.

$$(1) \quad m - p = -\lambda r + ky \quad \lambda \geq 0; k \geq 0$$

$$(2) \quad y(t) = -\gamma(r(t) - \dot{p}(t,t)) + \varepsilon(m(t) - p(t)) + \eta f(t) \quad \gamma > 0; \varepsilon \geq 0; \eta \geq 0$$

$m$  is the nominal stock of money balances,  $p$  the general price level,  $r$  the short nominal interest rate,  $y$  real output and  $f$  a measure of fiscal impact on aggregate demand. All variables except  $r$  are measured in logarithms. For any variable  $x$ ,  $\dot{x}(t)$  denotes its time derivative, i.e.

$$\dot{x}(t) \equiv \lim_{h \rightarrow 0} \left( \frac{x(t+h) - x(t)}{h} \right)$$

$x(s,t)$  denotes the value of  $x$  expected, at time  $t$ , to prevail at time  $s$ .

I assume  $x(s,t) = x(s)$ ,  $x \leq t$ . The expected instantaneous rate of change of  $x$  is denoted

$$\dot{x}(t,t) \equiv \lim_{\substack{h \rightarrow 0 \\ h > 0}} \left( \frac{x(t+h,t) - x(t,t)}{h} \right) . \text{ The unexpected change in } x \text{ is}$$

$$\text{denoted by } \frac{\partial}{\partial t} x(s,t) = \lim_{\substack{h \rightarrow 0 \\ h > 0}} \left( \frac{x(s,t) - x(s,t-h)}{h} \right) .$$

Costless disinflation with price flexibility.

Let capacity output or the natural level of output be denoted  $\bar{y}$ . This is treated as exogenous and constant. The benchmark case for costless disinflation is represented by the flexible price level model which complements (1) and (2) with the assumption of continuous full capacity utilization.

$$(3) \quad y = \bar{y} .$$

For simplicity it is assumed here and in what follows that the initial position of the economy is one of full stationary equilibrium with a high constant rate of monetary growth  $\bar{\mu}$ . All models considered will have the classical property that the steady state rate of inflation equals the steady state rate of growth of money.<sup>7</sup>

It is easily checked that, regardless of what the initial rate of inflation happens to be, an unanticipated, immediately implemented and permanent fixing of the rate of monetary growth  $\dot{m}(t) \equiv \mu(t)$  at  $\bar{\mu}$  will immediately and permanently set the rate of inflation at  $\bar{\mu}$  if expectations are rational. If the IS curve is vertical ( $\gamma=0$ ) and there is a real balance effect ( $\epsilon \neq 0$ ) this follows trivially from the stronger proposition that real money balances are determined uniquely by  $\bar{y}$  and  $f$ .

If neither  $\gamma$  nor  $\lambda$  equal zero, the behavior of expected real money balances is governed by

$$\dot{l}(t,t) = (\lambda^{-1} + \gamma^{-1})l(t) + \mu(t) - (\gamma^{-1} + \lambda^{-1}k)\bar{y} + \gamma^{-1} \eta f(t)$$

where

$$l(t) \equiv m(t) - p(t).$$

Since  $\lambda^{-1} + \gamma^{-1}\epsilon$  is positive there is, for a given information set, a unique continuous convergent solution given by

$$l(t) = \left( \frac{\lambda + k\gamma}{\gamma + \lambda\epsilon} \right) \bar{y} - \int_t^{\infty} e^{(\lambda^{-1} + \gamma^{-1}\epsilon)(t-s)} [\gamma^{-1}\eta f(s,t) + \mu(s,t)] ds$$

When both  $f$  and  $\mu$  are expected to remain constant at  $\bar{f}$  and  $\bar{\mu}$  respectively for all future time this simplifies to

$$(4) \quad l(t) = \left( \frac{\lambda + k\gamma}{\gamma + \lambda\epsilon} \right) \bar{y} - \frac{\eta\lambda}{\gamma + \lambda\epsilon} \bar{f} - \frac{\gamma\lambda}{\gamma + \lambda\epsilon} \bar{\mu}.$$

Thus, while the level of the real stock of money balances will, in general, be a function of the rate of monetary growth (even across steady states) through the effect of anticipated money growth on the expected rate of inflation and thus on the nominal interest rate, the adjustment of the real stock of money balances to a new rate of growth of the nominal money stock will be instantaneous. The price level will jump discontinuously if required to satisfy (4). After that the rate of inflation equals the new constant rate of money growth.

The general solution for the expected rate of inflation for constant  $\bar{y}$  and  $\bar{f}$  is:

$$(5) \quad \dot{p}(t,t) = (\lambda^{-1} + \gamma^{-1}\epsilon) \int_t^{\infty} e^{(\lambda^{-1} + \gamma^{-1}\epsilon)(t-s)} \mu(s,t) ds.$$

$$= \bar{\mu} \text{ if } \mu(s,t) = \bar{\mu} \text{ for } s \geq t.$$

The actual rate of inflation -- the sum of anticipated and unanticipated inflation -- is given by

$$(6) \quad \dot{p}(t) = \int_t^{\infty} e^{(\lambda^{-1} + \gamma^{-1}\epsilon)(t-s)} [(\lambda^{-1} + \gamma^{-1}\epsilon)\mu(s,t) + \frac{\partial}{\partial t} \mu(s,t)] ds. \quad 8$$

Note that in this model the credibility of current announcements of future policy is both necessary and sufficient for a sustained reduction in inflation to any level. While, by construction, there never are any output costs of bringing down inflation, a desired reduction in inflation may not be achievable, regardless of the actual past and current path of monetary growth, simply because expectations concerning future monetary growth are sufficiently pessimistic. Inflation, and policies to combat inflation, are exclusively forward-looking. I shall return to this credibility issue when I discuss the fiscal preconditions for a sustained deceleration of monetary growth below.

Costless disinflation with price level inertia.

The following four sluggish price adjustment mechanisms permit, in principle, costless and instantaneous sustained reductions in the rate of inflation.

$$(7) \quad \dot{p}(t) = \Psi(y(t) - \bar{y}) + \mu^+(t) \quad \Psi > 0 \quad (\text{Dornbusch})$$

$$(8) \quad \dot{p}(t) = \Psi(y(t) - \bar{y}) + \dot{\tilde{p}}(t) \quad (\text{Barro - Grossman})$$

$$(9) \quad \dot{p}(t) = \Psi(y(t) - \bar{y}) + \dot{\bar{p}}(t) \quad (\text{Mussa})$$

$$(10a) \quad \dot{p}(t) = \delta(v(t) - p(t)) \quad \delta > 0 \quad (\text{Calvo})$$

$$(10b) \quad \dot{v}(t,t) = \delta(v(t) - p(t) - \Psi(y(t) - \bar{y}))$$

In all four cases the price level,  $p$ , is treated as predetermined: it cannot move discontinuously in response to current changes in expectations about the future. Also in all four cases, the rate of change of the price level is an increasing function of excess demand pressure, measured as the excess of the level of current output (which is viewed as demand-determined) over the natural level of output. In equations (7), (8) and (9) it is only current excess demand which, given "core" inflation, affects current inflation. In Calvo's model both current and anticipated future excess demand affects current inflation. Equations (7), (8) and (9) only differ in the augmentation term, or core inflation -- the rate of inflation when there is no excess demand or supply. In equation (7) core inflation is identified with the right hand side derivative of the money stock path:

$$\mu^+(t) \equiv \lim_{\substack{h \rightarrow 0 \\ h > 0}} \frac{m(t+h) - m(t)}{h}$$

In equation (8) core inflation is given by  $\dot{\bar{p}}$ , which is the right-hand side derivative of the equilibrium price path that would be generated if the price level were fully flexible. This is of course the rate of inflation calculated in equation (5). Mussa's equation (9), as developed by Obstfeld [1982], specifies core inflation as the right-hand side derivative of the price path  $\bar{p}(t)$  which would clear the output market



(set  $y(t) = \bar{y}$ ) given the actual (and in general non-Walrasian equilibrium) values of the other endogenous variables,  $p(t)$ ,  $\dot{p}(t,t)$  and  $r(t)$ .  $\bar{p}(t)$  is therefore defined by:

$$\bar{y} = -\gamma(r(t) - \dot{p}(t,t)) + \varepsilon(m(t) - \bar{p}(t)) + \eta f(t).$$

In the spirit of Taylor, Calvo specifies the current contract price,  $v(t)$ , as a forward-looking moving average with exponentially declining weights of the future expected general price level  $p(t)$  and future expected excess demand:

$$(11a) \quad v(t) = \delta \int_t^{\infty} [p(s,t) + \Psi(y(s,t) - \bar{y})] e^{-\delta(s-t)} ds \\ + e^{\delta t} \lim_{\tau \rightarrow \infty} (e^{-\delta \tau} v(\tau, t))$$

Note that the current contract price  $v(t)$  is non-predetermined. The general price level, which is predetermined, is a backward-looking, exponentially declining moving average of past contract prices:

$$(11b) \quad p(t) = \delta \int_{t_0}^t v(s) e^{-\delta(t-s)} ds + e^{-\delta(t-t_0)} p(t_0)$$

In a full employment, stationary equilibrium  $\dot{v}(t,t) = \dot{v}(t) = \dot{p}(t)$ . A positive (zero, negative) steady state rate of inflation requires a current contract price,  $v(t)$ , above (equal to, below) the current general price level,  $p(t)$ .

It will be apparent that all four mechanisms in equations (7) - (10) have flexible core inflation. It would appear that simply reducing monetary growth to  $\bar{\mu}$ , say, and keeping it there would be sufficient

to reduce inflation in (7) without output costs. In equations (8), (9) and (10a,b) the further proviso of credibility of current announcements of future reductions in monetary growth has to be added.

The reason that this isn't quite correct is that a cet. par. reduction in money growth would in general affect the money market and output market. To effect a permanent reduction in inflation at full employment other policy parameters will have to be changed when monetary growth is reduced. Steady-state output  $y = \bar{y}$  is, from (4), sustained by monetary and fiscal policy as follows:

$$y(t) = \bar{y} = \frac{\eta\lambda}{\lambda + k\gamma} \bar{f} + \frac{\gamma\lambda}{\lambda + k\gamma} \bar{\mu} + \left( \frac{\gamma + \lambda\epsilon}{\lambda + k\gamma} \right) (m(t) - p(t)) .$$

The long-run effect of a reduction in money growth is to reduce inflation one-for-one, reduce the nominal interest rate (but less than one-for-one if there is a real balance effect<sup>9</sup>) and thus raise the demand for real money balances. In the classical flex-price model the real money balances required to effect an instantaneous transition to low inflation at full employment comes about through a discontinuous jump down in the price level. With a predetermined price level, the required increase in real money balances can instead be achieved by a discrete, discontinuous increase in the level of the nominal money stock at the same time,  $t$ , that its rate of change is reduced permanently.

The required money jump is given by:

$$(12a) \quad dm(t) = \frac{-\gamma\lambda}{\gamma + \lambda\epsilon} d\bar{\mu} \quad 10 .$$

Alternatively, expansionary fiscal policy could accompany the reduction in money growth in such a way as to leave the nominal interest rate unchanged. The required fiscal expansion is given by

$$(12b) \quad d\bar{f} = \frac{-Y}{\eta} d\bar{\mu} \quad .$$

This fiscal action relies on the direct aggregate demand effect of increased public spending or lower taxes. We shall show below how cost and price reducing indirect tax cuts could be used as a substitute for an increase in the nominal money stock as a means for achieving an increase in real money balances at a given before-tax price level.

It can be checked easily that provided the money stock is raised according to (12a) or a fiscal stimulus is provided according to (12b) when monetary growth is lowered to  $\bar{\mu}$ , the rate of inflation will settle immediately and permanently at  $\bar{\mu}$  while output remains equal to  $\bar{y}$  throughout.<sup>11</sup>

For Calvo's model the existence of costless disinflation policies may not be quite as transparent as for the other three. First note that, treating  $y - \bar{y}$  as exogenous both roots of the dynamic system (10a,b) are zero. Following Buiter [1983c] the convergent solutions for  $v$  and  $p$  are given by:

$$(13a) \quad p(t) = p(t_0) + \delta \int_{t_0}^t K(s) ds + \delta^2 \Psi \int_{t_0}^t \int_s^{\infty} (y(\tau, s) - \bar{y}) d\tau ds$$

$$(13b) \quad v(t) = p(t) + K(t) + \Psi \delta \int_t^{\infty} (y(\tau, t) - \bar{y}) d\tau$$

$K$  is a parameter to be determined from the terminal boundary condition.

A natural transversality condition do determine K (and one consistent with (11a)) is

$$(13c) \quad K(t) = \lim_{\tau \rightarrow \infty} [v(\tau, t) - p(\tau, t)] = \lim_{\tau \rightarrow \infty} \frac{\dot{v}(\tau, t)}{\delta} = \lim_{\tau \rightarrow \infty} \frac{\dot{p}(\tau, t)}{\delta}$$

It is easily seen from equations (1) and (2) that along any (anticipated) full employment path ( $y(\tau, t) = \bar{y}$ ) with a constant expected rate of money growth  $\bar{\mu}$  and a constant expected value of  $f$ , it must be true that  $K(t) = \frac{\bar{\mu}}{\delta}$ . Thus, as  $\mu$  is lowered from  $\bar{\mu}$  to  $\bar{\bar{\mu}}$  and  $m$  is increased once and for all to maintain full employment,  $v$  will drop discontinuously by  $\frac{\bar{\mu} - \bar{\bar{\mu}}}{\delta}$  and inflation is reduced costlessly and permanently. The dependence of the current price level on a "two-sided" moving average of output expectations will re-emerge in a slightly modified form when inflation inertia is added to price level inertia.

If monetary or fiscal policy actions to maintain full employment when the monetary growth rate is reduced are ruled out, the increased demand for real money balances resulting from a successful anti-inflationary policy can only be satisfied by a lowering of the price level path relative to the nominal money stock path. This will involve unemployment and excess capacity in all four sticky price level models. E.g., in the Dornbusch variant,

$$\int_{t_0}^t (y(s) - \bar{y}) ds = \Psi^{-1}(\ell(t_0) - \ell(t))$$

By the time the economy settles down to a new stationary equilibrium and a new, higher, stationary stock of real money balances after a reduction in  $\mu$ , the undiscounted cumulative net output loss will be

$$(13) \quad \int_{t_0}^{\infty} (y(s) - \bar{y}) ds = \Psi^{-1} \frac{\gamma\lambda}{\gamma + \lambda\epsilon} (\bar{\mu} - \bar{\mu})$$

The net output cost of reducing steady state inflation decreases as the short-run Phillips curve steepens and the interest sensitivity of money demand decreases.

These costs of bringing down the price level will be different for the four price adjustment mechanisms considered in this section. They are not considered any further here because of space constraints. The methods of the next section dealing with inflation inertia can be brought to bear on the problem of price level inertia with obvious modifications.

Inflation inertia.

The simplest way of introducing inflation inertia is by postulating a backward-looking adaptive process for core inflation,  $\pi$ , as in (14a,b).

$$(14a) \quad \dot{p} = \Psi(y - \bar{y}) + \pi$$

$$(14b) \quad \dot{\pi} = \xi(\dot{p} - \pi) \quad \xi > 0$$

Both the price level,  $p$ , and core inflation,  $\pi$ , are predetermined. It is now no longer possible to avoid paying the output or unemployment cost of bringing down inflation merely by manipulating aggregate demand as in the previous section. In the model of equations (14a, b) a sustained reduction in inflation requires a reduction in core inflation. Since

$$\pi(t) = \pi(t_0) + \xi\Psi \int_{t_0}^t (y(s) - \bar{y}) ds, \text{ the undiscounted cumulative net}$$

output cost of bringing down inflation permanently is given by

$$(15) \quad \int_{t_0}^{\infty} (y(s) - \bar{y}) ds = \frac{\pi(\infty) - \pi(t_0)}{\xi \Psi}$$

It is a decreasing function of the slope of the short run Phillips curve and an increasing function of the mean lag of the backward-looking core inflation process  $\frac{1}{\xi}$ . This loss measure can not be altered by changing the timing or the intensity of the anti-inflationary package. Given  $\xi$ ,  $\phi$  and the desired reduction in steady state inflation, the net output cost is a constant. (See e.g. Miller [1979], Buiter and Miller [1982, 1983], Miller and O'Donnell [1983]).

Clearly, with discounting, policy makers will not be indifferent between different trajectories with the same net output cost. We shall not address these issues as they are both well-understood (see e.g. Phelps [1972] and Hall [1976]) and comparatively unimportant: characterizing efficient policies (and inefficient policies!) would seem to be more useful than searching for an optimal policy.

The major drawback of the model of core inflation in equation (14b) is the complete irrelevance for current core inflation of current and/or past anticipations of future economic events. To remedy this (14b) is replaced by (15a, b). Core inflation,  $\pi$ , is a backward-looking exponentially declining moving average of past "contract inflation",  $q$ , while  $q$  is a forward-looking moving average of future expected inflation.

$$(15a) \quad \pi(t) = \pi(t_0)e^{-\xi_1(t-t_0)} + \xi_1 \int_{t_0}^t q(s)e^{-\xi_1(t-s)} ds \quad \xi_1 > 0$$

$$(15b) \quad q(t) = \xi_2 \int_t^{\infty} p(\tau, t)e^{-\xi_2(\tau-t)} d\tau \quad 12$$

By differentiating (15a, b) and using (14a) we obtain the two equation system:

$$(16a) \quad \dot{\pi}(t) = -\xi_1(\pi(t) - q(t))$$

$$(16b) \quad \dot{q}(t, t) = -\xi_2(\pi(t) - q(t)) - \xi_2 \Psi(y(t) - \bar{y})$$

Comparing this with Calvo's model in equation (10a, b) two differences stand out. First equations (16a, b) have "slipped a derivative" as compared to (10a, b). Inertia now attaches both to the level and the first derivative of the price level path. Second, the mean lags of the backward-looking core inflation process,  $\xi_1^{-1}$ , and of the forward-looking contract inflation process,  $\xi_2^{-1}$  are permitted to be different from each other.

Treating the output gap  $y - \bar{y}$  as exogenous, the convergent solutions for  $\pi(t)$  and  $q(t)$  are found to be: <sup>13</sup>

$$(17a) \quad \pi(t) = \pi(t_0) + \xi_1 \xi_2 \Psi \int_{t_0}^t \int_s^{\infty} e^{(\xi_2 - \xi_1)(s-\tau)} (y(\tau, s) - \bar{y}) d\tau ds$$

$$(17b) \quad q(t) = \pi(t) + \xi_2 \Psi \int_t^{\infty} e^{(\xi_2 - \xi_1)(t-s)} (y(s, t) - \bar{y}) ds$$

For convergence it is assumed that  $\xi_2 \geq \xi_1$ : current contract inflation adjusts more quickly than core inflation or the mean lag of the contract inflation process is no longer than that of the core inflation process.

Equation (17b) shows that "current contract inflation"  $q(t)$  can be brought down discontinuously by announcing, at  $t$ , a credible path of future recession. Core inflation  $\pi(t)$  is a function of the expectations, formed at each instant in the past, of the entire future

path of the output gap. It can only come down (gradually) in response to credible announcements that policies to generate a recession will be pursued in the future.

To get a better appreciation of the cost of bringing down inflation in this model, we now consider an example of a specific path for expected output.

The initial position at  $t_0$  is one of current and expected future full employment with actual, core and current contract inflation all equal to each other. At  $t_0$  output is unexpectedly lowered to  $y(t_0) < \bar{y}$ . The output gap is then expected to decay exponentially at a rate  $\gamma$  so that

$$(18) \quad y(\tau, t) - \bar{y} = (y(t_0) - \bar{y}) e^{-\gamma(\tau - t_0)}, \quad \tau \geq t_0, \quad \gamma > 0$$

Substituting (18) into (17a, b) yields

$$(19a) \quad \pi(t) = \pi(t_0) + \frac{\xi_1 \xi_2 \Psi}{(\gamma + \xi_2 - \xi_1) \gamma} (y(t_0) - \bar{y}) (1 - e^{-\gamma(t - t_0)})$$

$$(19b) \quad q(t) = \pi(t) + \frac{\xi_2 \Psi}{\gamma + \xi_2 - \xi_1} (y(t_0) - \bar{y}) e^{-\gamma(t - t_0)}$$

The long-run effect on core inflation of this policy is, from (19a)

$$(20a) \quad \pi(\infty) - \pi(t_0) = \frac{\xi_1 \xi_2 \Psi}{(\gamma + \xi_2 - \xi_1)} (y(t_0) - \bar{y}) = \dot{p}(\infty) - \dot{p}(t_0^-) = q(\infty) - q(t_0^-) \quad 14$$

The undiscounted cumulative net output loss incurred for this reduction in inflation is

$$(20b) \quad \int_{t_0}^{\infty} (y(t) - \bar{y}) dt = \frac{y(t_0) - \bar{y}}{\gamma}$$



Consider the special case where  $\gamma = \xi_1$  (the mean lag of the output process equals the mean lag of the core inflation process). In this case the impact of the current and expected future recession is to reduce actual inflation and current contract inflation immediately to their new long-run equilibrium levels:

$$\dot{p}(t) = q(t) = \pi(t_0) + \Psi(y(t_0) - \bar{y}) \quad , \quad t \geq t_0.$$

Core inflation only approaches its long-run equilibrium value gradually according to

$$\pi(t) = \pi(t_0) + \Psi(y(t_0) - \bar{y})(1 - e^{-\gamma(t - t_0)}).$$

In this case actual inflation and current contract inflation have been eliminated immediately and permanently, but the output cost is still to come. This would be obvious to an observer who can measure core inflation, which has yet to be brought down. It will be shown below, that there exist policies that reduce both actual and core inflation to any desired level before any of the output costs have actually been incurred. This raises the problem of time inconsistency and credibility of policy.

#### "Gradualism" versus "Cold-turkey"

It can be seen from (19b) and (20a) that if  $\gamma$  is larger (smaller) than  $\xi_1$ , current contract inflation will on impact fall below (stay above) its new long-run equilibrium level. This might suggest that a short sharp recession would, by changing current contract inflation promptly, be a more effective means of bringing down core inflation than a longer and milder recession. In fact the opposite is the case. From (20a, b) it follows that, holding constant the cumulative net output

loss  $\frac{y(t_0) - \bar{y}}{\gamma}$ , a deeper initial recession (a smaller value of  $y(t_0)$ ) followed by a faster recovery (a larger value of  $\gamma$ ) would produce a smaller reduction in steady-state inflation. The reason is, from (17a), that later expected output gaps, although discounted as regards their effect on  $\pi(t)$ , are also counted again and again (in fact continuously) on the interval  $t_0 - t$ .

In the presence of inflation inertia which is due to contractual or other institutional arrangements rather than to sluggish expectation adjustments, there are "diminishing returns" to cold turkey deflation and gradualism is preferable. Only if a short sharp shock can break down the nominal inertia, i.e. if the adjustment equations (15a,b) or (16a,b) are not structurally invariant to certain dramatic changes. In the policy regime, is there a case for anti-inflationary heroics.

Time inconsistency and the timing of anti-inflationary benefits and output costs.

It has already been shown that it is possible, through credible policy announcements, to bring down actual and current contract inflation immediately -- before the output losses (whose expectation generated this reduction in inflation) have been incurred. It will be apparent from equations (17a,b,c) that it is also possible, if policy announcements are credible, to bring down core inflation gradually (i.e. over any finite time interval) to any desired level before any of the output costs have been incurred. Consider e.g. a policy which, starting at  $t_0$  keeps output at its capacity level  $\bar{y}$  until  $\bar{t} > t_0$ , at  $\bar{y}$  from  $\bar{t}$  till  $\bar{\bar{t}} > \bar{t}$  and again at  $\bar{y}$  after  $\bar{\bar{t}}$ .

$$(21) \quad y(\tau, s) = \begin{cases} \bar{y} & ; t_0 < \tau \leq \bar{t} \\ \bar{\bar{y}} & ; \bar{t} < \tau \leq \bar{\bar{t}} \\ \bar{y} & ; \tau > \bar{\bar{t}} \end{cases}$$

It follows that

$$\pi(\bar{t}) = \pi(t_0) + \frac{\xi_1 \xi_2 \Psi}{(\xi_2 - \xi_1)^2} \left[ 1 - e^{-(\xi_2 - \xi_1)(\bar{t} - t_0)} - e^{-(\xi_2 - \xi_1)(\bar{\bar{t}} - \bar{t})} + e^{-(\xi_2 - \xi_1)(\bar{\bar{t}} - t_0)} \right] (\bar{\bar{y}} - \bar{y})$$

A recession strategy announced (and made credible) at  $t=t_0$  which permits a reduction in core inflation at  $t=\bar{t}$  to  $\bar{\pi}(\bar{t})$ , say, is therefore given by (21) with  $\bar{\bar{y}}$  defined by

$$(22) \quad \bar{\bar{y}} - \bar{y} = \frac{[\bar{\pi}(\bar{t}) - \pi(t_0)](\xi_2 - \xi_1)^2}{\xi_1 \xi_2 \Psi \left( 1 - e^{-(\xi_2 - \xi_1)(\bar{t} - t_0)} - e^{-(\xi_2 - \xi_1)(\bar{\bar{t}} - \bar{t})} + e^{-(\xi_2 - \xi_1)(\bar{\bar{t}} - t_0)} \right)}$$

As expected, the announced future recession is deeper the larger the reduction in core inflation  $\bar{\pi}(\bar{t}) - \pi(t_0)$  that is required.

Having achieved the desired reduction in core inflation at  $t=\bar{t}$  without actually having suffered any output costs as yet, the temptation to renege on the earlier commitment to create a recession between  $\bar{t}$  and  $\bar{\bar{t}}$  would be hard to resist.

The argument that the recession must take place in order to validate and confirm the expectations held between  $t=t_0$  and  $t=\bar{t}$  and thus to preserve or invest in credibility for future policy announcements is unlikely to prove a political winner. Why have a recession when core inflation has already subsided? A policy maker treating bygones as bygones will at  $t=\bar{t}$  and beyond, keep output at its capacity level.

What this suggests is that any optimal policy will be time-inconsistent (Kydland and Prescott [1977], Buiter [1981, 1983b]) if it has the property that some output costs still have to be incurred (if previous expectations are to be validated) after the inflation objectives have been achieved. Unless credible precommitment is possible such policy announcements will not be believed by the private sector.

Time consistent policies must be characterized by a better matching of the time profiles of costs and benefits: if a credible strategy cannot have the costs following the benefits, the recession will have to be brought forward in time. If a speedy reduction in inflation is sought, a deep, short recession will be the only credible strategy. It has already been shown that "short-sharp shocks" of this kind are likely to be inefficient.

The model under consideration not only has implications for policy design but also for policy evaluation. Consider again the general expressions (17a,b,c). Conventional cost measures focus on output or unemployment costs incurred up to the time that a given reduction in core inflation has been achieved. When forward-looking expectations and inflation inertia play a role, as they do in this model, some, most or all of the costs attributable to the reduction in core inflation may be incurred after the anti-inflationary objective has been achieved.

#### Higher order inertia.

It is not difficult to visualize economic systems in which not only the price level and the core rate of inflation but also the rate of change of core inflation adjust sluggishly. Attributing inertia to higher derivatives of the price and wage process is the continuous time analogue

to increasing the number of periods for which nominal contracts hold in discrete time models. E.g. by slipping another derivative in the model of equations (14a and 16a, b) we obtain:

$$\ddot{\pi}(t) = \xi_1(\dot{q}(t) - \dot{\pi}(t))$$

$$\ddot{q}(t,t) = \xi_2(\dot{q}(t) - \dot{\pi}(t) - \Psi(y(t) - \bar{y}))$$

$$\ddot{p}(t) = \Psi(y(t) - \bar{y}) + \dot{\pi}(t)$$

With nominal inertia in the price level, core inflation and the rate of change of core inflation, the output costs of bringing down steady state inflation will be high indeed. <sup>15/</sup>

Some Open Economy Considerations and the Sensible Use of Fiscal Policy in a Disinflationary Programme.

In an open economy, appreciation of the exchange rate might seem to offer a mechanism for bringing down inflation more rapidly or at less cost than in a closed economy or an economy on a fixed exchange rate.

This will be considered in an open economy extension of the model with backward-looking core inflation given in (14 a,b). Possible direct cost and price effects of direct and indirect taxes are also introduced here. They can be applied to all closed economy models in an obvious way. The economy is a price taker in the world market for its imports, whose world price is  $p^*$ , and in international financial markets. The country has some market power in the market for its exportable. Perfect capital mobility and international asset substitutability are assumed.  $r^*$  is the foreign nominal interest rate.  $w$  denotes domestic costs, i.e. labour costs per unit of output or even the GDP deflator at factor cost;  $p$  is the consumer price index,  $e$  the price of foreign exchange,  $g$  exhaustive public spending,  $\tau_i$  the rate of indirect taxation and  $\tau_d$  the income tax rate. The model is given in equations (23-29). All coefficients are positive.

$$(23) \quad \frac{d}{dt}(w(t) - \beta\tau_d(t)) = \Psi(y - \bar{y}) + \pi(t) \quad 0 \leq \beta \leq 1$$

$$(24) \quad p(t) = \tau_i(t) + \alpha w(t) + (1 - \alpha)(p^*(t) + e(t)) \quad 0 < \alpha < 1$$

$$(25) \quad \dot{\pi} = \xi(\dot{p} - \pi(t))$$

$$(26) \quad y(t) = -\gamma(r(t) - \dot{p}(t,t)) + \delta(e(t) + p^*(t) - p(t)) + \varepsilon(m(t) - p(t)) + \eta_1 g(t) - \eta_2(\tau_i(t) + \tau_d(t))$$

$$(27) \quad m(t) - p(t) = ky(t) - \lambda r(t)$$

$$(28) \quad \dot{e}(t,t) = r(t) - r^*(t)$$

$$(29a) \quad c \equiv e + p^* - w$$

$$(29b) \quad \ell \equiv m - w$$

The model is similar to the one considered in Buiters and Miller [1982, 83]. Note that taxes have both aggregate demand effects (equation 26) and direct cost effects (equations (23) and (24)). Only a fraction  $\beta$  of an increase in income tax rates is translated into higher wage settlements (equation (23)). One interpretation of (23) is that it is the (adjusted) after-tax money wage  $w - \beta\tau_d$  that is predetermined or sticky rather than the before tax wage  $w$ . Note that the consumer price index can move discontinuously, even if  $w$  is predetermined, through changes in indirect tax rates and in the exchange rate. The latter influences the c.p.i. through the share of imports in final consumption  $1 - \alpha$ .

From equation (23) - (25) and (29b) it follows that

$$(30a) \quad \pi(t) - \pi(t_0) = \xi\psi \int_{t_0}^t (y(s) - \bar{y})ds + \xi(1 - \alpha)(c(t) - c(t_0)) + \xi(\tau_i(t) - \tau_i(t_0)) + \xi\beta(\tau_d(t) - \tau_d(t_0))$$

or

$$(30b) \quad \int_{t_0}^t (y(s) - \bar{y})ds = \frac{\pi(t) - \pi(t_0)}{\xi\psi} - \frac{(1 - \alpha)}{\psi}(c(t) - c(t_0)) - \left[ \frac{(\tau_i(t) - \tau_i(t_0)) + \beta(\tau_d(t) - \tau_d(t_0))}{\psi} \right]$$

The familiar closed economy output costs given in equation (15) are found back as the first term on the right-hand side of (30b). They can be reduced by an appreciation of the real exchange rate,  $c$ , by a cut in indirect taxes and, if  $\beta \neq 0$ , by a cut in direct taxes.

A balanced budget cut in indirect taxes matched by an increase in direct taxes helps reduce the output cost of disinflation only if direct tax increases don't raise before tax wage settlements one-for-one.

Consider the steady state of the model.

$$(31) \quad \dot{\ell} = \tau_i + (1 - \alpha)c - \lambda(r^* - \dot{p}^* + \mu) + k\bar{y}$$

$$(32) \quad \bar{y} = -\gamma(r^* - \dot{p}^*) + (\delta\alpha - \varepsilon(1 - \alpha))c + \varepsilon\ell - \varepsilon\tau_i + \eta_1 g - \eta_2(\tau_i + \tau_d)$$

$$(33) \quad \dot{p} = \pi = \mu = \dot{p}^* + \dot{e}$$

$$(34) \quad r = r^* - \dot{p}^* + \mu$$

Across steady states, a reduction in monetary growth lowers actual and core inflation one-for-one. It will have no effect on competitiveness if there is no real balance effect ( $\varepsilon = 0$ ). If there is a real balance effect, the increased stock of real balances associated with a lower rate of monetary growth will require a loss of competitiveness to maintain equilibrium in the output market if  $(\delta + \varepsilon)\alpha > \varepsilon$ . Assuming that there is no long-run effect on competitiveness from a reduction in the rate of inflation, any favourable short-run or impact effects on the price level, the rate of inflation and the core rate of inflation from an initial appreciation of the exchange rate will not lower the undiscounted cumulative net output cost of securing a sustained reduction in core inflation. Buiter and Miller [1982, 1983] show that, on impact, a reduction in monetary growth will be associated with a discrete, jump appreciation of the nominal exchange rate (a step down in  $e$  and  $c$ ). If  $e$  jumps down, so, from (24), does  $p$  and so, from (25), does  $\pi$ . Core inflation jumps on impact but the apparent reduction in output cost that this entails is nullified by the net depreciation of the real exchange rate that will be required during the remainder of the adjustment process to restore competitiveness. Only if the short, sharp shock of a sudden revaluation breaks down the inertia captured in (23) and (25) will it help reduce the output cost of bringing down inflation.



In principle, by using indirect and, if  $\beta \neq 0$ , direct tax cuts to melt core inflation instantaneously, a costless and immediate transition to a sustained lower rate of inflation is possible. The higher stock of real money balances demanded in a low inflation equilibrium can be provided either by engineering a step increase in the level of the nominal money stock at the same time its growth rate is reduced or by cutting taxes. Indirect tax cuts will do the job and so will direct tax cuts, if  $\beta \neq 0$  and if it is the after-tax money wage rather than the before-tax money wage that is predetermined. If step adjustments in  $m$  are ruled out, immediate attainment of the new long-run equilibrium values of  $m - p$  and  $\pi$  while maintaining full employment will in general require use of all three fiscal instruments,  $g$ ,  $\tau_i$  and  $\tau_d$ .

Finally, in the context of this model, incomes policy can be seen as the ability to "override" the core inflation adjustment equation (25). An extreme version would permit the authorities to pick a new starting value for  $\pi$ . The model clearly isn't rich enough to suggest reasons why such policies have a habit of breaking down.

Hysteresis in the natural rate.

One of the most striking macroeconomic coincidences of the last fifteen years has been the way in which estimates of the natural rate of unemployment have moved up, along with the actual rate of unemployment. In this subsection we consider the implications of the hypothesis that this co-movement represents a causal influence running from current and past actual unemployment to the current natural unemployment rate. Letting  $y$  stand for the actual unemployment rate and  $\bar{y}$  for the natural rate, we postulate that

$$(35) \quad \bar{y}(t) = \bar{y}(t_0)e^{-\theta_2(t-t_0)} + \theta_2 \int_{t_0}^t y(s) e^{-\theta_2(t-s)} ds + \theta_1 \int_{t_0}^t e^{-\theta_2(t-s)} \dot{R}(s) ds$$

or

$$(35') \quad \dot{\bar{y}}(t) = \theta_2(y(t) - \bar{y}(t)) + \theta_1 \dot{R}(t) \quad , \quad \theta_2 \geq 0$$

$R(t)$  stands for whatever structural factors or policies may affect the natural rate (union power, unemployment benefits, minimum wage etc.). The second term on the right-hand side of (35) and the first term on the right-hand side of (35') represent the hypothesis that unemployment destroys human capital by having a negative effect both on attitudes towards working and the aptitude for work. The idea is an old one. Recent formalizations can be found in Buiter and Gersovitz [1981], Hargreaves Heap [1980], Gregory [1982, 1983] and Gregory and Smith [1983]. Clearly, as written in (35) the hypothesis is too strong, since no bounds are set on the natural rate in the long run: by selecting an appropriate path for unemployment, the natural rate can be steered to any level. Such global hysteresis is implausible. Over some finite range of unemployment rates, the hypothesis may, however,

have merit. (35) should be viewed as the log-linear approximation in the relevant range of unemployment rates to a model with local hysteresis. While the long-run or stationary Phillips curve is vertical, it can be made vertical at any point within that range.

To keep the exposition brief, the simple sluggish core inflation model of equations (14a,b) is added to (35). It is, of course, still true that

$$\pi(t) = \pi(t_0) + \xi \Psi \int_{t_0}^t [y(s) - \bar{y}(s)] ds$$

Since the natural rate no longer is invariant under the disinflation process, deviations from the natural rate cease to be a useful measure of cost. In the absence of tax cuts or incomes policy, core inflation can only be lowered by raising the actual unemployment rate above the natural rate. This, however, will, by (35') begin to raise the natural rate, thus reducing gradually the disinflationary effect of any given increase in the actual rate. Formally, since

$$\bar{y}(t) = \bar{y}(t_0) + \theta_1 (R(t) - R(t_0)) + \theta_2 \int_{t_0}^t (y(s) - \bar{y}(s)) ds$$

it follows, using (35), that if structural factors affecting the actual rate remain unchanged (i.e. if  $\dot{R}(t) = 0$ )

$$\int_{t_0}^t [y(s) - \bar{y}(s)] ds = \int_{t_0}^t [y(s) - \bar{y}(t_0)] e^{-\theta_2(t-s)} ds.$$

The disinflationary effect of any given increase in unemployment above its initial value will decay exponentially over time as the natural rate catches up with the actual rate.

In the hysteresis model a permanent increase in the rate of unemployment  $-\Delta y$  will only buy a finite long-run reduction in the rate of inflation  $-\Delta\pi = \frac{\xi\Psi}{\Theta_2} (-\Delta y)$ . With an exogenous natural rate that same steady state reduction in inflation can be achieved by having the same constant increase in unemployment for only a finite period of time.

The simple hysteresis model outlined here has much the same implications for the inflation-unemployment process as does entering the rate of change of output (unemployment) rather than its level as an argument in the Phillips curve. The simplest version of this model is

$$(36) \quad \dot{p} = \phi \dot{y} + \pi$$

$$(14b) \quad \dot{\pi} = \xi(\dot{p} - \pi)$$

The solution for core inflation is :

$$(37a) \quad \pi(t) = \pi(t_0) + \xi\phi(y(t) - y(t_0))$$

while current inflation is given by

$$(37b) \quad \dot{p}(t) = \phi \dot{y}(t) + \phi \xi (y(t) - y(t_0)) + \pi(t_0)$$

The striking implication of this model is that all anti-inflationary gains from a contractionary policy are completely reversed if the economy is permitted to recover. As Tobin [1980, p.61] says "It is possible that there is no NAIRU, no natural rate, except one that floats with actual history.

It is just possible that the direction the economy is moving in is at least as important for acceleration and deceleration as its level. These possibilities should give policy makers pause as they embark on yet another application of the orthodox demand management cure for inflation".

Note that equation (37b) is consistent with some of the early work on Phillips curves, which argued that both the level and the rate of change of unemployment could be significant in inflation equations but ignored endogenous core inflation (e.g. Phillips (1958) and Lipsey (1960)).

This similarity between the hysteresis model and the model of equations (36) and (14b) is especially striking when we consider the effect, in the hysteresis model, of a constant path  $y(s) = \bar{y}$  for  $t_0 < s \leq t$ . This yields the following expression for core inflation

$$\pi(t) = \pi(t_0) + \frac{\xi \phi}{\theta_2} (\bar{y} - \bar{y}(t_0)) [1 - e^{-\theta_2(t-t_0)}]$$

As  $t$  goes to infinity this approaches

$$\lim_{t \rightarrow \infty} \pi(t) = \pi(t_0) + \frac{\xi \phi}{\theta_2} (\bar{y} - \bar{y}(t_0))$$

Asymptotically, the hysteresis model too has complete reversibility of inflationary gains achieved through contractionary policy.

Although the economic mechanisms involved are very different, the output, or unemployment cost of achieving a permanent reduction in inflation are similar for the hysteresis model and the "unaugmented", pre-Phelps

and Friedman, Phillips curve. With the  $\dot{p} = \phi(y - \bar{y}) + \pi$  specification, an exogenous natural rate  $\bar{y}$  and no adjustment, however gradual, of core inflation  $\pi$  towards actual inflation, the "sacrifice ratio" - the cumulative, undiscounted net output or unemployment cost (expressed as a percentage) divided by the steady state reduction in the inflation rate - is infinite. The same specification with an exogenous natural rate but gradual convergence of core inflation to actual inflation, yields a positive but finite sacrifice ratio, whose exact magnitude depends on the details of the core inflation adjustment mechanism. The assumption of instantaneous adjustment of core inflation through rational perception and anticipation of credible policy actions produces a sacrifice ratio of zero. The hysteresis model with gradually adjusting core inflation again yields an infinite sacrifice ratio. Not because, as with the old Phillips curve, core inflation never adjusts but because the natural rate adjusts gradually towards the actual unemployment rate. The case for any policy action(s) that can "override" the core-inflation adjustment equations is therefore even stronger if the hysteresis hypothesis has anything to recommend it.

Credibility and the consistency of monetary and fiscal policy

All models considered in this paper have the property that a sustained reduction in the rate of inflation requires a long-run reduction in the rate of monetary growth. While the exact nature of the relevant monetary aggregate is not apparent from these models, it seems reasonable to assume that a long-run sustained reduction in the rate of growth of any monetary aggregate presupposes corresponding reduction in the growth

rate of the monetary base. If this is the case, a necessary condition for the credibility of a policy to reduce steady state inflation is the consistency of the long-run monetary objectives with the government's fiscal program.

We can get some sense of the "eventual monetization" implied by the government's fiscal program by considering the government's comprehensive balance sheet (see Buiter [1983c] given in (38)).

$$(38) \quad N(t) \equiv p_k(t)K(t) + p_R(t)R(t) + T(t) + \Pi(t) - \frac{M(t)}{p(t)} - \frac{B(t)}{p(t)} - \frac{p_c(t)C_{It}}{p(t)}$$

$N$  is real government net worth,  $K$  the public sector capital stock,  $p_k$  the present value of the future returns to a unit of public sector capital,  $R$  the number of shares of public sector natural resource property rights,  $p_R$  the price of a share in these property rights,  $T$  the present value of future taxes net of transfers,  $\Pi$  the capital value of the government's note issue monopoly,  $M$  the nominal stock of high-powered money,  $B$  the stock of nominally denominated short bonds,  $C$  the number of consols paying a coupon of £1,  $p_c$  the price of a consol and  $p$  the general price level.

Let  $\delta_k$  be the capital rental rate,  $\delta_R$  the return on a share of public sector natural resource property rights,  $g$  government consumption,  $\tau$  current taxes net of transfers,  $i$  the short nominal interest rate

and  $r$  the short real rate. Then, assuming that  $r(t) = i(t) - \frac{\dot{p}(t,t)}{p(t)}$

$$(39a) \quad p_k(t) = \int_t^\infty \delta_k(s,t) e^{-\int_t^s r(u,s) du} ds$$

$$(39b) \quad p_R(t) = \int_t^\infty \delta_R(s,t) e^{-\int_t^s r(u,s) du} ds$$

$$(39c) \quad T(t) = \int_t^\infty \tau(s,t) e^{-\int_t^s r(u,s) du}$$

$$(39d) \quad \Pi(t) = \frac{1}{p(t)} \int_t^\infty i(s,t) M(s,t) e^{-\int_t^s i(u,s) du} ds = \int_t^\infty r(s,t) \frac{M(s,t)}{p(s,t)} e^{-\int_t^s r(u,t) du} ds$$

$$(39e) \quad p_c(t) = \int_t^\infty e^{-\int_t^s i(u,s) du} ds$$

Note that the capital value of the note issue monopoly,  $\Pi$ , is given by the discounted future income derived from the assets that are (and will be) held to "back" the note circulation. Equilization of expected rates of return is assumed.

Since the present value of future planned public consumption cannot exceed public sector net worth (a constraint I shall assume to hold with strict equality), we have

$$(40) \quad G(t) = N(t)$$



where

$$(40') \quad G(t) = \int_t^{\infty} g(s,t) e^{-\int_t^s r(u,t) du} ds$$

Let  $S(t) \equiv \Pi(t) - \frac{M(t)}{p(t)}$ . Integrating by parts it is found that  $S(t)$  is the present value of future seigniorage, i.e.

$$(41) \quad S(t) \equiv \frac{1}{p(t)} \int_t^{\infty} \dot{M}(s,t) e^{-\int_t^s i(u,t) du} ds \equiv \int_t^{\infty} \frac{\dot{M}(s,t)}{M(s,t)} \frac{M(s,t)}{p(s,t)} e^{-\int_t^s r(u,t) du} ds$$

Treating  $S(t)$  as the residual item, (40) and (38) tell us the amount of revenue to be raised through seigniorage (in present value terms), given the present value of the government's consumption program and the government's tangible and intangible non-monetary assets and liabilities, i.e.

$$(42) \quad S(t) = G(t) - \left[ p_k(t)U(t) + P_R(t)R(t) + T(t) - \left( \frac{B(t) + p_c(t)C(t)}{p(t)} \right) \right]$$

Let  $y$  denote trend output and  $n$  its rate of growth. A real (index-linked) consol will have a coupon yield  $\bar{R}$  if the instantaneous real rate of return is  $r-n$ , where  $\bar{R}$  is given by

$$\bar{R}(t) = \left[ \int_t^{\infty} e^{-\int_t^s [r(u,t)-n] du} ds \right]^{-1}$$

We can solve (42) for a constant proportional rate of monetary growth  $\frac{\dot{M}}{M}$  and a constant trend income velocity of circulation,  $V \equiv \frac{PY}{M}$  to yield

$$(43) \quad \frac{\dot{M}}{M} = V\bar{R}(t) \left[ \frac{G(t) - T(t)}{y(t)} - \left( \frac{p_k(t)K(t) + p_R(t)R(t)}{y(t)} \right) + \frac{B(t) + p_c(t)C(t)}{p(t)y(t)} \right]$$

If (and only if) the public sector consumption and tax programs together with its other non-monetary assets and liabilities, imply a high value of  $\frac{\dot{M}}{M}$ , then a fiscal correction is a necessary condition for achieving credibility for an anti-inflationary policy. Note that in full steady-state equilibrium (43) becomes the familiar expression

$$\frac{\dot{M}}{M} = V \left( \frac{g - \tau}{y} - (r-n) \left( \frac{p_k K + p_R R}{y} - \frac{(B + p_c C)}{py} \right) \right)$$

Eventual monetary growth is governed in steady state by the trend public sector current account (or consumption account) deficit, with debt service evaluated at the real interest rate net of the natural rate of growth. This deficit measure can differ dramatically from the conventionally measured public sector financial deficit or PSBR, which is often and erroneously taken as a guide to eventual monetization.

### III. Conclusion.

One conclusion that emerges strongly from this paper is the importance of fiscal policy in securing a lasting reduction in inflation at least cost. First, the long-run reduction in monetary growth which is necessary for a sustained reduction in inflation is only credible if it is consistent with the government's spending and tax programs and its outstanding non-monetary assets and liabilities. Second, indirect tax cuts (and under certain conditions direct tax cuts) can be used in ways first suggested by Okun [1978] to secure a painless melting away of core inflation. Tax cuts or a once-and-for all increase in the level of the nominal money stock path must also be used in order to provide the higher stock of real money balances demanded when the inflation rate is lower at a given price level rather than through a further downward shift in the price level path.

A final comment suggested by the analysis of the paper relates to the apparent contrast between the findings of R. J. Gordon who has documented many historical episodes during which bringing down inflation appears to have been costly (Gordon [1982]) and T. S. Sargent who finds that the ends of four hyperinflations in the post-World War I era were achieved without dramatic output losses (Sargent [1982]). These findings can be reconciled by arguing that during hyperinflations inflation inertia (if not price level inertia) disappears. All the advantages of longer term non-contingent nominal contracts are overridden by the need to revise prices almost continuously. During hyperinflations (at any rate

in their final phases) the inflation process is characterized by models like the ones in equations (8), (9) and (10a,b) or even by the purely classical flexible price model.

If there is no inflation inertia but still some price level inertia, optimal anti-inflationary policy has the following features. First, a credible announcement of current and future reductions in monetary growth. This was provided by fiscal reform and currency reform plus the general realization that something had to be done and was going to be done to stop the hyperinflations. Second, a once-and-for all increase in the level of the nominal money stock to raise the stock of real money balances without any need to lower the price level path. The real world counterpart to this were the very large increases in the nominal money stocks in the periods following the sudden ending of the hyperinflations [Sargent, op.cit.]. Such money stock jumps make no sense in a flexible price model but <sup>17</sup> may be called for in models with price level inertia.

Gordon considered episodes of moderate inflation. Long-term non-contingent nominal contracts are adopted because they permit economic agents to economize on frequent, costly renegotiations, on the search and information costs of first identifying all possible relevant contingencies and then monitoring them and on the costs of enforcing complicated conditional contract clauses. Continuously variable and perfectly flexible prices or fully contingent contracts are costly and undesirable when the benefits from changing prices frequently outweigh the costs. In moderate inflations, long-term nominal contracts are still viewed as viable and desirable by private agents. Such changes in the length

of these contracts and in other relevant characteristics as one would expect to occur when the trend rate of inflation changes, are likely to be second-order for the range of inflation rates experienced in most OECD countries since World War II. With the unconditional long-term nominal contract structure intact, even fully credible announcements of future reductions in monetary growth will not remove the need for a period of (expected) output losses and unemployment if inflation is to be brought down.

Footnotes

1. An elegant statement and extension of the traditional theory of the welfare costs of inflation is Fischer [1981b].
2. This assumes that lump-sum taxes are available to the government. If higher monetary growth and the associated higher inflation increase the real value of new money issues (if the elasticity of demand for real money balances with respect to the interest rate is less than unity) the same real public spending program can be financed with lower explicit taxes. If these taxes are distortionary, the usual welfare loss measure overstates the true cost.
3. The most careful and informative mark in this area has been done by Fischer [1981a,b, 1981]. See also Taylor [1981].
4.  $v(\bar{c}, [-\ell, 1]) > v(\bar{c}, L, 0)$  for any  $\ell$  ( $0 < \ell < L$ ) would mean that even without any pecuniary advantage of employment over unemployment, people would choose to be employed. It is a much stronger condition than the one given in the text.
5. Separability, i.e.  $v(c, L-\ell, \theta) = v(u(c, L-\ell), \theta)$  would be convenient analytically.
6. See also Buiter and Jewitt [1981].
7. The natural rate of growth is assumed equal to zero.
8. Actual and expected  $\bar{y}$  and  $\bar{f}$  are again held constant.
9. In the long run,  $r = \left( \frac{-1+\lambda^{-1}k\lambda\varepsilon}{\gamma+\lambda\varepsilon} \right) \bar{y} + \frac{\eta}{\gamma+\lambda\varepsilon} \bar{f} + \frac{\gamma}{\gamma+\lambda\varepsilon} \bar{p}$
10. When both  $\gamma$  and  $\varepsilon$  are equal to zero, a reduction in monetary growth and inflation leaves the nominal interest rate unaffected and costless disinflation is automatic.
11. Note that since  $m$  and  $f$  are manipulated to keep output at its full employment level, the Barro-Grossman equation (8) and the Mussa equation (9) coincide.
12. It is assumed that there is no long-run trend in the rate of inflation.
13. Note that the characteristic roots of the homogeneous system (16a,b) are 0 and  $\xi_2 - \xi_1$ .

14. For any variable  $x$ , let  $x(t^-) \equiv \lim_{\substack{h \rightarrow 0 \\ h > 0}} x(t-h)$ .
15. This is very similar in spirit to John Flemming's suggestion in Flemming [1976].
16. Hysteresis is the property of dynamic systems that the stationary equilibrium is a function of the initial conditions and/or the transition trajectory towards the steady state. In systems of linear differential equations with constant coefficients such as  $Dx = Ax + Bz$ , hysteresis is present when  $A$  has one or more zero eigenvalues.
17. Except for government revenue reasons. We owe this point to Bob Flood.

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