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ABSTRACT

This paper analyzes the quality of VAR-based procedures for estimating the response of the economy to a shock. We focus on two key issues. First, do VAR-based confidence intervals accurately reflect the actual degree of sampling uncertainty associated with impulse response functions? Second, what is the size of bias relative to confidence intervals, and how do coverage rates of confidence intervals compare with their nominal size? We address these questions using data generated from a series of estimated dynamic, stochastic general equilibrium models. We organize most of our analysis around a particular question that has attracted a great deal of attention in the literature: How do hours worked respond to an identified shock? In all of our examples, as long as the variance in hours worked due to a given shock is above the remarkably low number of 1 percent, structural VARs perform well. This finding is true regardless of whether identification is based on short-run or long-run restrictions. Confidence intervals are wider in the case of long-run restrictions. Even so, long-run identified VARs can be useful for discriminating among competing economic models.

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1. Introduction

Sims's seminal paper *Macroeconomics and Reality* (1980) argued that procedures based on vector autoregression (VAR) would be useful to macroeconomists interested in constructing and evaluating economic models. Given a minimal set of identifying assumptions, structural VARs allow one to estimate the dynamic effects of economic shocks. The estimated impulse response functions provide a natural way to choose the parameters of a structural model and to assess the empirical plausibility of alternative models.[1]

To be useful in practice, VAR-based procedures must have good sampling properties. In particular, they should accurately characterize the amount of information in the data about the effects of a shock to the economy. Also, they should accurately uncover the information that is there.

These considerations lead us to investigate two key issues. First, do VAR-based confidence intervals accurately reflect the actual degree of sampling uncertainty associated with impulse response functions? Second, what is the size of bias relative to confidence intervals, and how do coverage rates of confidence intervals compare with their nominal size?

We address these questions using data generated from a series of estimated dynamic, stochastic general equilibrium (DSGE) models. We consider real business cycle (RBC) models and the model in Altig, Christiano, Eichenbaum, and Linde (2005) (hereafter, ACEL) that embodies real and nominal frictions. We organize most of our analysis around a particular question that has attracted a great deal of attention in the literature: How do hours worked respond to an identified shock? In the case of the RBC model, we consider a neutral shock to technology. In the ACEL model, we consider two types of technology shocks as well as a monetary policy shock.

We focus our analysis on an unavoidable specification error that occurs when the data generating process is a DSGE model and the econometrician uses a VAR. In this case the true VAR is infinite ordered, but the econometrician must use a VAR with a finite number of lags.

We find that as long as the variance in hours worked due to a given shock is above the remarkably low number of 1 percent, VAR-based methods for recovering the response of hours to that shock have good sampling properties. Technology shocks account for a much larger fraction of the variance of hours worked in the ACEL model than in any of our estimated RBC models. Not surprisingly, inference about the effects of a technology shock on hours worked is much sharper when the ACEL model is the data generating mechanism.

Taken as a whole, our results support the view that structural VARs are a useful guide to constructing and evaluating DSGE models. Of course, as with any econometric procedure it is possible to find examples in which VAR-based procedures do not do well. Indeed, we present such an example based on an RBC model in which technology shocks account for less than 1 percent of the variance in hours worked. In this example, VAR-based methods work poorly in the sense that bias exceeds sampling uncertainty. Although instructive, the example is based on a model that fits the data poorly and so is unlikely to be of practical importance.

Having good sampling properties does not mean that structural VARs always deliver small

[1] See for example Sims (1989), Eichenbaum and Evans (1995), Rotemberg and Woodford (1997), Gali (1999), Francis and Ramey (2004), Christiano, Eichenbaum, and Evans (2005), and Del Negro, Schorfheide, Smets, and Wouters (2005).

confidence intervals. Of course, it would be a Pyrrhic victory for structural VARs if the best one could say about them is that sampling uncertainty is always large and the econometrician will always know it. Fortunately, this is not the case. We describe examples in which structural VARs are useful for discriminating between competing economic models.

Researchers use two types of identifying restrictions in structural VARs. Blanchard and Quah (1989), Gali (1999), and others exploit the implications that many models have for the long-run effects of shocks.[2] Other authors exploit short-run restrictions.[3] It is useful to distinguish between these two types of identifying restrictions to summarize our results.

We find that structural VARs perform remarkably well when identification is based on short-run restrictions. For all the specifications that we consider, the sampling properties of impulse response estimators are good and sampling uncertainty is small. This good performance obtains even when technology shocks account for as little as 0.5 percent of the variance in hours. Our results are comforting for the vast literature that has exploited short-run identification schemes to identify the dynamic effects of shocks to the economy. Of course, one can question the particular short-run identifying assumptions used in any given analysis. However, our results strongly support the view that if the relevant short-run assumptions are satisfied in the data generating process, then standard structural VAR procedures reliably uncover and identify the dynamic effects of shocks to the economy.

The main distinction between our short and long-run results is that the sampling uncertainty associated with estimated impulse response functions is substantially larger in the long-run case. In addition, we find some evidence of bias when the fraction of the variance in hours worked that is accounted for by technology shocks is very small. However, this bias is not large relative to sampling uncertainty as long as technology shocks account for at least 1 percent of the variance of hours worked. Still, the reason for this bias is interesting. We document that, when substantial bias exists, it stems from the fact that with long-run restrictions one requires an estimate of the sum of the VAR coefficients. The specification error involved in using a finite-lag VAR is the reason that in some of our examples, the sum of VAR coefficients is difficult to estimate accurately. This difficulty also explains why sampling uncertainty with long-run restrictions tends to be large.

The preceding observations led us to develop an alternative to the standard VAR-based estimator of impulse response functions. The only place the sum of the VAR coefficients appears in the standard strategy is in the computation of the zero-frequency spectral density of the data. Our alternative estimator avoids using the sum of the VAR coefficients by working with a nonparametric estimator of this spectral density. We find that in cases when the standard VAR procedure entails some bias, our adjustment virtually eliminates the bias.

Our results are related to a literature that questions the ability of long-run identified VARs to reliably estimate the dynamic response of macroeconomic variables to structural shocks.

[2] See, for example, Basu, Fernald, and Kimball (2004), Christiano, Eichenbaum, and Vigfusson (2003, 2004), Fisher (2006), Francis and Ramey (2004), King, Plosser, Stock and Watson (1991), Shapiro and Watson (1988) and Vigfusson (2004). Francis, Owyang, and Roush (2005) pursue a related strategy to identify a technology shock as the shock that maximizes the forecast error variance share of labor productivity at a long but finite horizon.

[3] This list is particularly long and includes at least Bernanke (1986), Bernanke and Blinder (1992), Bernanke and Mihov (1998), Blanchard and Perotti (2002), Blanchard and Watson (1986), Christiano and Eichenbaum (1992), Christiano, Eichenbaum and Evans (2005), Cushman and Zha (1997), Eichenbaum and Evans (1995), Hamilton (1997), Rotemberg and Woodford (1992), Sims (1986), and Sims and Zha (2006).

Perhaps the first critique of this sort was provided by Sims (1972). Although his paper was written before the advent of VARs, it articulates why estimates of the sum of regression coefficients may be distorted when there is specification error. Faust and Leeper (1997) and Pagan and Robertson (1998) make an important related critique of identification strategies based on long-run restrictions. More recently Erceg, Guerrieri, and Gust (2005) and Chari, Kehoe, and McGrattan (2005b) (henceforth CKM) also examine the reliability of VAR-based inference using long-run identifying restrictions.[4] Our conclusions regarding the value of identified VARs differ sharply from those recently reached by CKM. One parameterization of the RBC model that we consider is identical to the one considered by CKM. This parameterization is included for pedagogical purposes only, as it is overwhelmingly rejected by the data.

The remainder of the paper is organized as follows. Section 2 presents the versions of the RBC models that we use in our analysis. Section 3 discusses our results for standard VAR-based estimators of impulse response functions. Section 4 analyzes the differences between short and long-run restrictions. Section 5 discusses the relation between our work and the recent critique of VARs offered by CKM. Section 6 summarizes the ACEL model and reports its implications for VARs. Section 7 contains concluding comments.

2. A Simple RBC Model

In this section, we display the RBC model that serves as one of the data generating processes in our analysis. In this model the only shock that affects labor productivity in the long-run is a shock to technology. This property lies at the core of the identification strategy used by King, et al (1991), Gali (1999) and other researchers to identify the effects of a shock to technology. We also consider a variant of the model which rationalizes short run restrictions as a strategy for identifying a technology shock. In this variant, agents choose hours worked before the technology shock is realized. We describe the conventional VAR-based strategies for estimating the dynamic effect on hours worked of a shock to technology. Finally, we discuss parameterizations of the RBC model that we use in our experiments.

2.1. The Model

The representative agent maximizes expected utility over per capita consumption, c_t, and per capita hours worked, l_t :

$$E_0 \sum_{t=0}^{\infty} \left(\beta \left(1+\gamma\right)\right)^t \left[\log c_t + \psi \frac{\left(1-l_t\right)^{1-\sigma}-1}{1-\sigma}\right],$$

subject to the budget constraint:

$$c_t + \left(1+\tau_{x,t}\right) i_t \leq \left(1-\tau_{l,t}\right) w_t l_t + r_t k_t + T_t,$$

where

$$i_t = \left(1+\gamma\right) k_{t+1} - \left(1-\delta\right) k_t.$$

[4]See also Fernandez-Villaverdez, Rubio-Ramirez, and Sargent (2005) who investigate the circumstances in which the economic shocks are recoverable from the VAR disturbances. They provide a simple matrix algebra check to assess recoverability. They identify models in which the conditions are satisfied and other models in which they are not.

Here, k_t denotes the per capita capital stock at the beginning of period t, w_t is the wage rate, r_t is the rental rate on capital, $\tau_{x,t}$ is an investment tax, $\tau_{l,t}$ is the tax rate on labor income, $\delta \in (0,1)$ is the depreciation rate on capital, γ is the growth rate of the population, T_t represents lump-sum taxes and $\sigma > 0$ is a curvature parameter.

The representative competitive firm's production function is:

$$y_t = k_t^{\alpha} \left(Z_t l_t\right)^{1-\alpha},$$

where Z_t is the time t state of technology and $\alpha \in (0,1)$. The stochastic processes for the shocks are:

$$\begin{aligned} \log z_t &= \mu_z + \sigma_z \varepsilon_t^z \\ \tau_{l,t+1} &= \left(1-\rho_l\right)\tau_l + \rho_l \tau_{l,t} + \sigma_l \varepsilon_{t+1}^l \\ \tau_{x,t+1} &= \left(1-\rho_x\right)\tau_x + \rho_x \tau_{x,t} + \sigma_x \varepsilon_{t+1}^x, \end{aligned} \tag{2.1}$$

where $z_t = Z_t/Z_{t-1}$. In addition, ε_t^z, ε_t^l, and ε_t^x are independently and identically distributed (i.i.d.) random variables with mean zero and unit standard deviation. The parameters, σ_z, σ_l, and σ_x are non-negative scalars. The constant, μ_z, is the mean growth rate of technology, τ_l is the mean labor tax rate, and τ_x is the mean tax on capital. We restrict the autoregressive coefficients, ρ_l and ρ_x, to be less than unity in absolute value.

Finally, the resource constraint is:

$$c_t + (1+\gamma) k_{t+1} - (1-\delta) k_t \leq y_t.$$

We consider two versions of the model, differentiated according to timing assumptions. In the *standard* or *nonrecursive version,* all time t decisions are taken after the realization of the time t shocks. This is the conventional assumption in the RBC literature. In the *recursive version* of the model the timing assumptions are as follows. First, $\tau_{l,t}$ is observed, and then labor decisions are made. Second, the other shocks are realized and agents make their investment and consumption decisions.

2.2. Relation of the RBC Model to VARs

We now discuss the relation between the RBC model and a VAR. Specifically, we establish conditions under which the reduced form of the RBC model is a VAR with disturbances that are linear combinations of the economic shocks. Our exposition is a simplified version of the discussion in Fernandez-Villaverde, Rubio-Ramirez, and Sargent (2005) (see especially their section III). We include this discussion because it frames many of the issues that we address. Our discussion applies to both the standard and the recursive versions of the model.

We begin by showing how to put the reduced form of the RBC model into a state-space, observer form. Throughout, we analyze the log-linear approximations to model solutions. Suppose the variables of interest in the RBC model are denoted by X_t. Let s_t denote the vector of exogenous economic shocks and let $\hat{k}_t$ denote the percent deviation from steady state of the capital stock, after scaling by Z_t.[5] The approximate solution for X_t is given by:

$$X_t = a_0 + a_1 \hat{k}_t + a_2 \hat{k}_{t-1} + b_0 s_t + b_1 s_{t-1}, \tag{2.2}$$

[5] Let $\tilde{k}_t = k_t/Z_{t-1}$. Then, $\hat{k}_t = \left(\tilde{k}_t - \tilde{k}\right)/\tilde{k}$, where $\tilde{k}$ denotes the value of $\tilde{k}_t$ in nonstochastic steady state.

where

$$\hat{k}_{t+1} = A\hat{k}_t + Bs_t. \tag{2.3}$$

Also, s_t has the law of motion:

$$s_t = Ps_{t-1} + Q\varepsilon_t, \tag{2.4}$$

where ε_t is a vector of i.i.d. fundamental economic disturbances. The parameters of (2.2)and (2.3) are functions of the structural parameters of the model.

The 'state' of the system is composed of the variables on the right side of (2.2):

$$\xi_t = \begin{pmatrix} \hat{k}_t \\ \hat{k}_{t-1} \\ s_t \\ s_{t-1} \end{pmatrix}.$$

The law of motion of the state is:

$$\xi_t = F\xi_{t-1} + D\varepsilon_t, \tag{2.5}$$

where F and D are constructed from A, B, Q, P. The econometrician observes the vector of variables, Y_t. We assume Y_t is equal to X_t plus iid measurement error, v_t, which has diagonal variance-covariance, R. Then:

$$Y_t = H\xi_t + v_t. \tag{2.6}$$

Here, H is defined so that $X_t = H\xi_t$, that is, relation (2.2) is satisfied. In (2.6) we abstract from the constant term. Hamilton (1994, section 13.4) shows how the system formed by (2.5) and (2.6) can be used to construct the exact Gaussian density function for a series of observations, $Y_1, ..., Y_T$. We use this approach when we estimate versions of the RBC model.

We now use (2.5) and (2.6) to establish conditions under which the reduced form representation for X_t implied by the RBC model is a VAR with disturbances that are linear combinations of the economic shocks. In this discussion, we set $v_t = 0$, so that $X_t = Y_t$. In addition, we assume that the number of elements in ε_t coincides with the number of elements in Y_t.

We begin by substituting (2.5) into (2.6) to obtain:

$$Y_t = HF\xi_{t-1} + C\varepsilon_t, \; C \equiv HD.$$

Our assumption on the dimensions of Y_t and ε_t implies that the matrix C is square. In addition, we assume C is invertible. Then:

$$\varepsilon_t = C^{-1}Y_t - C^{-1}HF\xi_{t-1}. \tag{2.7}$$

Substituting (2.7) into (2.5), we obtain:

$$\xi_t = M\xi_{t-1} + DC^{-1}Y_t,$$

where

$$M = \left[I - DC^{-1}H\right] F. \tag{2.8}$$

As long as the eigenvalues of M are less than unity in absolute value,

$$\xi_t = DC^{-1}Y_t + MDC^{-1}Y_{t-1} + M^2DC^{-1}Y_{t-2} + \ldots . \tag{2.9}$$

Using (2.9) to substitute out for ξ_{t-1} in (2.7), we obtain:

$$\varepsilon_t = C^{-1}Y_t - C^{-1}HF\left[DC^{-1}Y_{t-1} + MDC^{-1}Y_{t-2} + M^2DC^{-1}Y_{t-3} + ...\right],$$

or, after rearranging:

$$Y_t = B_1Y_{t-1} + B_2Y_{t-2} + ... \ + u_t, \tag{2.10}$$

where

$$u_t = C\varepsilon_t \tag{2.11}$$

$$B_j = HFM^{j-1}DC^{-1}, \ j = 1, 2, ... \tag{2.12}$$

Expression (2.10) is an infinite-order VAR, because u_t is orthogonal to Y_{t-j}, $j \geq 1$.

Proposition 2.1. *(Fernandez-Villaverde, Rubio-Ramirez, and Sargent) If C is invertible and the eigenvalues of M are less than unity in absolute value, then the RBC model implies:*

- *Y_t has the infinite-order VAR representation in (2.10)*
- *The linear one-step-ahead forecast error Y_t given past Y_t's is u_t, which is related to the economic disturbances by (2.11)*
- *The variance-covariance of u_t is CC'*
- *The sum of the VAR lag matrices is given by:*

$$B(1) \equiv \sum_{j=1}^{\infty} B_j = HF[I - M]^{-1} DC^{-1}.$$

We will use the last of these results below.

Relation (2.10) indicates why researchers interested in constructing DSGE models find it useful to analyze VARs. At the same time, this relationship clarifies some of the potential pitfalls in the use of VARs. First, in practice the econometrician must work with finite lags. Second, the assumption that C is square and invertible may not be satisfied. Whether C satisfies these conditions depends on how Y_t is defined. Third, significant measurement errors may exist. Fourth, the matrix, M, may not have eigenvalues inside the unit circle. In this case, the economic shocks are not recoverable from the VAR disturbances.[6] Implicitly, the econometrician who works with VARs assumes that these pitfalls are not quantitatively important.

2.3. VARs in Practice and the RBC Model

We are interested in the use of VARs as a way to estimate the response of X_t to economic shocks, i.e., elements of ε_t. In practice, macroeconomists use a version of (2.10) with finite lags, say q. A researcher can estimate $B_1, ..., B_q$ and $V = Eu_tu_t'$. To obtain the impulse response functions, however, the researcher needs the B_i's and the column of C corresponding to the shock in ε_t that

[6] For an early example, see Hansen and Sargent (1980, footnote 12). Sims and Zha (forthcoming) discuss the possibility that, although a given economic shock may not lie exactly in the space of current and past Y_t, it may nevertheless be 'close'. They discuss methods to detect this case.

is of interest. However, to compute the required column of C requires additional identifying assumptions. In practice, two types of assumptions are used. Short-run assumptions take the form of direct restrictions on the matrix C. Long-run assumptions place indirect restrictions on C that stem from restrictions on the long-run response of X_t to a shock in an element of ε_t. In this section we use our RBC model to discuss these two types of assumptions and how they are imposed on VARs in practice.

2.3.1. The Standard Version of the Model

The log-linearized equilibrium laws of motion for capital and hours in this model can be written as follows:

$$\log \hat{k}_{t+1} = \gamma_0 + \gamma_k \log \hat{k}_t + \gamma_z \log z_t + \gamma_l \tau_{l,t} + \gamma_x \tau_{x,t}, \tag{2.13}$$

and

$$\log l_t = a_0 + a_k \log \hat{k}_t + a_z \log z_t + a_l \tau_{l,t} + a_x \tau_{x,t}. \tag{2.14}$$

From (2.13) and (2.14), it is clear that all shocks have only a temporary effect on l_t and $\hat{k}_t$.[7] The only shock that has a permanent effect on labor productivity, $a_t \equiv y_t/l_t$, is ε_t^z. The other shocks do not have a permanent effect on a_t. Formally, this *exclusion restriction* is:

$$\lim_{j\to\infty} \left[E_t a_{t+j} - E_{t-1} a_{t+j}\right] = f\left(\varepsilon_t^z \text{ only}\right). \tag{2.15}$$

In our linear approximation to the model solution f is a linear function. The model also implies the *sign restriction* that f is an increasing function. In (2.15), E_t is the expectation operator, conditional on the information set $\Omega_t = \left(\log \hat{k}_{t-s}, \log z_{t-s}, \tau_{l,t-s}, \tau_{x,t-s};\ s \geq 0\right)$.

In practice, researchers impose the exclusion and sign restrictions on a VAR to compute ε_t^z and identify its dynamic effects on macroeconomic variables. Consider the $N \times 1$ vector, Y_t. The VAR for Y_t is given by:

$$\begin{aligned} Y_{t+1} &= B(L)\, Y_t + u_{t+1},\ E u_t u_t' = V, \\ B(L) &\equiv B_1 + B_2 L + ... + B_q L^{q-1}, \\ Y_t &= \begin{pmatrix} \Delta \log a_t \\ \log l_t \\ x_t \end{pmatrix}. \end{aligned} \tag{2.16}$$

Here, x_t is an additional vector of variables that may be included in the VAR. Motivated by the type of reasoning discussed in the previous subsection, researchers assume that the fundamental economic shocks are related to u_t as follows:

$$u_t = C\varepsilon_t,\ E\varepsilon_t\varepsilon_t' = I,\ CC' = V. \tag{2.17}$$

Without loss of generality, we assume that the first element in ε_t is ε_t^z. We can easily verify that:

$$\lim_{j\to\infty} \left[\tilde{E}_t a_{t+j} - \tilde{E}_{t-1} a_{t+j}\right] = \tau \left[I - B(1)\right]^{-1} C\varepsilon_t, \tag{2.18}$$

[7] Cooley and Dwyer (1998) argue that in the standard RBC model, if technology shocks have a unit root, then per capita hours worked will be difference stationary. This claim, which plays an important role in their analysis of VARs, is incorrect.

where τ is a row vector with all zeros, but with unity in the first location. Here:

$$B(1) \equiv B_1 + ... + B_q.$$

Also, $\tilde{E}_t$ is the expectation operator, conditional on $\tilde{\Omega}_t = \{Y_t, ..., Y_{t-q+1}\}$. As mentioned above, to compute the dynamic effects of ε_t^z, we require $B_1, ..., B_q$ and C_1, the first column of C.

The symmetric matrix, V, and the B_i's can be computed using ordinary least squares regressions. However, the requirement that $CC' = V$ is not sufficient to determine a unique value of C_1. Adding the exclusion and sign restrictions does uniquely determine C_1. Relation (2.18) implies that these restrictions are:

$$\text{exclusion restriction: } [I - B(1)]^{-1} C = \begin{bmatrix} \text{number} & \underline{0} \\ \text{numbers} & \text{numbers} \end{bmatrix},$$

where $\underline{0}$ is a row vector and

$$\text{sign restriction: } (1,1) \text{ element of } [I - B(1)]^{-1} C \text{ is positive.}$$

There are many matrices, C, that satisfy $CC' = V$ as well as the exclusion and sign restrictions. It is well-known that the first column, C_1, of each of these matrices is the same. We prove this result here, because elements of the proof will be useful to analyze our simulation results. Let

$$D \equiv [I - B(1)]^{-1} C.$$

Let $S_Y(\omega)$ denote the spectral density of Y_t at frequency ω that is implied by the q^{th}-order VAR. Then:

$$DD' = [I - B(1)]^{-1} V [I - B(1)']^{-1} = S_Y(0). \tag{2.19}$$

The exclusion restriction requires that D have a particular pattern of zeros:

$$D = \begin{bmatrix} \underset{1\times1}{d_{11}} & \underset{1\times(N-1)}{0} \\ \underset{(N-1)\times1}{D_{21}} & \underset{(N-1)\times(N-1)}{D_{22}} \end{bmatrix},$$

so that

$$DD' = \begin{bmatrix} d_{11}^2 & d_{11}D_{21}' \\ D_{21}d_{11} & D_{21}D_{21}' + D_{22}D_{22}' \end{bmatrix} = \begin{bmatrix} S_Y^{11}(0) & S_Y^{21}(0)' \\ S_Y^{21}(0) & S_Y^{22}(0) \end{bmatrix},$$

where

$$S_Y(\omega) \equiv \begin{bmatrix} S_Y^{11}(\omega) & S_Y^{21}(\omega)' \\ S_Y^{21}(\omega) & S_Y^{22}(\omega) \end{bmatrix}.$$

The exclusion restriction implies that

$$d_{11}^2 = S_Y^{11}(0), \;\; D_{21} = S_Y^{21}(0) / d_{11}. \tag{2.20}$$

There are two solutions to (2.20). The sign restriction

$$d_{11} > 0 \tag{2.21}$$

selects one of the two solutions to (2.20). So, the first column of D, D_1, is uniquely determined. By our definition of C, we have

$$C_1 = [I - B(1)] D_1. \tag{2.22}$$

We conclude that C_1 is uniquely determined.

2.3.2. The Recursive Version of the Model

In the recursive version of the model, the policy rule for labor involves $\log z_{t-1}$ and $\tau_{x,t-1}$ because these variables help forecast $\log z_t$ and $\tau_{x,t}$:

$$\log l_t = a_0 + a_k \log \hat{k}_t + \tilde{a}_l \tau_{l,t} + \tilde{a}'_z \log z_{t-1} + \tilde{a}'_x \tau_{x,t-1}.$$

Because labor is a state variable at the time the investment decision is made, the equilibrium law of motion for $\hat{k}_{t+1}$ is:

$$\begin{aligned} \log \hat{k}_{t+1} &= \gamma_0 + \gamma_k \log \hat{k}_t + \tilde{\gamma}_z \log z_t + \tilde{\gamma}_l \tau_{l,t} + \tilde{\gamma}_x \tau_{x,t} \\ &\quad + \tilde{\gamma}'_z \log z_{t-1} + \tilde{\gamma}'_x \tau_{x,t-1}. \end{aligned}$$

As in the standard model, the only shock that affects a_t in the long run is a shock to technology. So, the long-run identification strategy discussed in section 2.3.1 applies to the recursive version of the model. However, an alternative procedure for identifying ε_t^z applies to this version of the model. We refer to this alternative procedure as the 'short-run' identification strategy because it involves recovering ε_t^z using only the realized one-step-ahead forecast errors in labor productivity and hours, as well as the second moment properties of those forecast errors.

Let $u^a_{\Omega,t}$ and $u^l_{\Omega,t}$ denote the population one-step-ahead forecast errors in a_t and $\log l_t$, conditional on the information set, Ω_{t-1}. The recursive version of the model implies that

$$u^a_{\Omega,t} = \alpha_1 \varepsilon_t^z + \alpha_2 \varepsilon_t^l, \; u^l_{\Omega,t} = \gamma \varepsilon_t^l,$$

where $\alpha_1 > 0$, α_2, and γ are functions of the model parameters. The projection of $u^a_{\Omega,t}$ on $u^l_{\Omega,t}$ is given by

$$u^a_{\Omega,t} = \beta u^l_{\Omega,t} + \alpha_1 \varepsilon_t^z, \text{ where } \beta = \frac{cov(u^a_{\Omega,t}, u^l_{\Omega,t})}{var\left(u^l_{\Omega,t}\right)}. \tag{2.23}$$

Because we normalize the standard deviation of ε_t^z to unity, α_1 is given by:

$$\alpha_1 = \sqrt{var\left(u^a_{\Omega,t}\right) - \beta^2 var\left(u^l_{\Omega,t}\right)}.$$

In practice, we implement the previous procedure using the one-step-ahead forecast errors generated from a VAR in which the variables in Y_t are ordered as follows:

$$Y_t = \begin{pmatrix} \log l_t \\ \Delta \log a_t \\ x_t \end{pmatrix}.$$

We write the vector of VAR one-step-ahead forecast errors, u_t, as:

$$u_t = \begin{pmatrix} u_t^l \\ u_t^a \\ u_t^x \end{pmatrix}.$$

We identify the technology shock with the second element in ε_t in (2.17). To compute the dynamic response of the variables in Y_t to the technology shock we need $B_1, ..., B_q$ in (2.16) and the second column, C_2, of the matrix C, in (2.17). We obtain C_2 in two steps. First, we identify the technology shock using:

$$\varepsilon_t^z = \frac{1}{\hat{\alpha}_1}\left(u_t^a - \hat{\beta} u_t^l\right),$$

where

$$\hat{\beta} = \frac{cov(u_t^a, u_t^l)}{var\left(u_t^l\right)},\ \hat{\alpha}_1 = \sqrt{var\left(u_t^a\right) - \hat{\beta}^2 var\left(u_t^l\right)}.$$

The required variances and covariances are obtained from the estimate of V in (2.16). Second, we regress u_t on ε_t^z to obtain:[8]

$$C_2 = \begin{pmatrix} \frac{cov(u^l,\varepsilon^z)}{var(\varepsilon^z)} \\ \frac{cov(u^a,\varepsilon^z)}{var(\varepsilon^z)} \\ \frac{cov(u^x,\varepsilon^z)}{var(\varepsilon^z)} \end{pmatrix} = \begin{pmatrix} 0 \\ \hat{\alpha}_1 \\ \frac{1}{\hat{\alpha}_1}\left(cov(u_t^x, u_t^a) - \hat{\beta} cov\left(u_t^x, u_t^l\right)\right) \end{pmatrix}.$$

2.4. Parameterization of the Model

We consider different specifications of the RBC model that are distinguished by the parameterization of the laws of motion of the exogenous shocks. In all specifications we assume, as in CKM , that:

$$\begin{aligned} \beta &= 0.98^{1/4},\ \theta = 0.33,\ \delta = 1-(1-.06)^{1/4},\ \psi = 2.5,\ \gamma = 1.01^{1/4}-1 \\ \tau_x &= 0.3,\ \tau_l = 0.242,\ \mu_z = 1.016^{1/4}-1,\ \sigma = 1. \end{aligned} \tag{2.24}$$

2.4.1. Our MLE Parameterizations

We estimate two versions of our model. In the *two-shock maximum likelihood estimation (MLE) specification* we assume that $\sigma_x = 0$, so that there are two shocks, $\tau_{l,t}$ and $\log z_t$. We estimate the parameters ρ_l, σ_l, and σ_z, by maximizing the Gaussian likelihood function of the vector, $X_t = (\Delta \log y_t, \log l_t)'$, subject to (2.24).[9] Our results are given by:

$$\begin{aligned} \log z_t &= \mu_z + 0.00953\varepsilon_t^z, \\ \tau_{l,t} &= (1-0.986)\,\bar{\tau}_l + 0.986\tau_{l,t-1} + 0.0056\varepsilon_t^l. \end{aligned}$$

The *three-shock MLE specification* incorporates the investment tax shock, $\tau_{x,t}$, into the model. We estimate the three-shock MLE version of the model by maximizing the Gaussian likelihood function of the vector, $X_t = (\Delta \log y_t, \log l_t, \Delta \log i_t)'$, subject to the parameter values in (2.24). The results are:

$$\begin{aligned} \log z_t &= \mu_z + 0.00968\varepsilon_t^z, \\ \tau_{l,t} &= (1-0.9994)\,\tau_l + 0.9994\tau_{l,t-1} + 0.00631\varepsilon_t^l, \\ \tau_{x,t} &= (1-0.9923)\,\tau_x + 0.9923\tau_{x,t-1} + 0.00963\varepsilon_t^x. \end{aligned}$$

[8] We implement the procedure for estimating C_2 by computing $CC' = V$, where C is the lower triangular Cholesky decomposition of V, and setting C_2 equal to the second column of C.

[9] We use the standard Kalman filter strategy discussed in Hamilton (1994, section 13.4). We remove the sample mean from X_t prior to estimation and set the measurement error in the Kalman filter system to zero, i.e., $R = 0$ in (2.6).

The estimated values of ρ_x and ρ_l are close to unity. This finding is consistent with other research that also reports that shocks in estimated general equilibrium models exhibit high degrees of serial correlation.[10]

2.4.2. CKM Parameterizations

The *two-shock CKM specification* has two shocks, z_t and $\tau_{l,t}$. These shocks have the following time series representations:

$$\begin{aligned} \log z_t &= \mu_z + 0.0131\varepsilon_t^z, \\ \tau_{l,t} &= (1-0.952)\,\tau_l + 0.952\tau_{l,t-1} + 0.0136\varepsilon_t^l. \end{aligned}$$

The *three-shock CKM specification* adds an investment shock, $\tau_{x,t}$, to the model, and has the following law of motion:

$$\tau_{x,t} = (1-0.98)\,\tau_x + 0.98\tau_{x,t-1} + 0.0123\varepsilon_t^x. \tag{2.25}$$

As in our specifications, CKM obtain their parameter estimates using maximum likelihood methods. However, their estimates are very different from ours. For example, the variances of the shocks are larger in the two-shock CKM specification than in our MLE specification. Also, the ratio of σ_l^2 to σ_z^2 is nearly three times larger in the two-shock CKM specification than in our two-shock MLE specification. Section 5 below discusses the reasons for these differences.

2.5. The Importance of Technology Shocks for Hours Worked

Table 1 reports the contribution, V_h, of technology shocks to three different measures of the volatility in the log of hours worked: (i) the variance of the log hours, (ii) the variance of HP-filtered, log hours and (iii) the variance in the one-step-ahead forecast error in log hours.[11] With one exception, we compute the analogous statistics for log output. The exception is (i), for which we compute the contribution of technology shocks to the variance of the growth rate of output.

The key result in this table is that technology shocks account for a very small fraction of the volatility in hours worked. When V_h is measured according to (i), it is always below 4 percent. When V_h is measured using (ii) or (iii) it is always below 8 percent. For both (ii) and (iii), in the CKM specifications, V_h is below 2 percent.[12] Consistent with the RBC literature, the table also shows that technology accounts for a much larger movement in output.

[10] See, for example, Christiano (1988), Christiano, et al. (2004), and Smets and Wouters (2003).

[11] We compute forecast error variances based on a four lag VAR. The variables in the VAR depend on whether the calculations correspond to the two or three shock model. In the case of the two-shock model, the VAR has two variables, output growth and log hours. In the case of the three-shock model, the VAR has three variables: output growth, log hours and the log of the investment to output ratio. Computing V_h requires estimating VARs in artificial data generated with all shocks, as well as in artificial data generated with only the technology shock. In the latter case, the one-step ahead forecast error from the VAR is well defined, even though the VAR coefficients themselves are not well defined due to multicollinearity problems.

[12] When we measure V_h according to (i), V_h drops from 3.73 in the two-shock MLE model to 0.18 in the three-shock MLE model. The analogous drop in V_h is an order of magnitude smaller when V_h is measured using (ii) or (iii). The reason for this difference is that ρ_l goes from 0.986 in the two-shock MLE model to 0.9994 in the three-shock MLE model. In the latter specification there is a near-unit root in $\tau_{l,t}$, which translates into a near-unit root in hours worked. As a result, the variance of hours worked becomes very large at the low frequencies. The near-unit root in τ_{lt} has less of an effect on hours worked at high and business cycle frequencies.

Figure 1 displays visually how unimportant technology shocks are for hours worked. The top panel displays two sets of 180 artificial observations on hours worked, simulated using the standard two-shock MLE specification. The volatile time series shows how log hours worked evolve in the presence of shocks to both z_t and $\tau_{l,t}$. The other time series shows how log hours worked evolve in response to just the technology shock, z_t. The bottom panel is the analog of the top figure when the data are generated using the standard two-shock CKM specification.

3. Results Based on RBC Data Generating Mechanisms

In this section we analyze the properties of conventional VAR-based strategies for identifying the effects of a technology shock on hours worked. We focus on the bias properties of the impulse response estimator, and on standard procedures for estimating sampling uncertainty.

We use the RBC model parameterizations discussed in the previous section as the data generating processes. For each parameterization, we simulate 1,000 data sets of 180 observations each. The shocks ε_t^z, ε_t^l, and possibly ε_t^x, are drawn from *i.i.d.* standard normal distributions. For each artificial data set, we estimate a four-lag VAR. The average, across the 1,000 data sets, of the estimated impulse response functions, allows us to assess bias.

For each data set we also estimate two different confidence intervals: a percentile-based confidence interval and a standard-deviation based confidence interval.[13] We construct the intervals using the following bootstrap procedure. Using random draws from the fitted VAR disturbances, we use the estimated four lag VAR to generate 200 synthetic data sets, each with 180 observations. For each of these 200 synthetic data sets we estimate a new VAR and impulse response function. For each artificial data set the percentile-based confidence interval is defined as the top 2.5 percent and bottom 2.5 percent of the estimated coefficients in the dynamic response functions. The standard-deviation-based confidence interval is defined as the estimated impulse response plus or minus two standard deviations where the standard deviations are calculated across the 200 simulated estimated coefficients in the dynamic response functions.

We assess the accuracy of the confidence interval estimators in two ways. First, we compute the coverage rate for each type of confidence interval. This rate is the fraction of times, across the 1,000 data sets simulated from the economic model, that the confidence interval contains the relevant true coefficient. If the confidence intervals were perfectly accurate, the coverage rate would be 95 percent. Second, we provide an indication of the actual degree of sampling uncertainty in the VAR-based impulse response functions. In particular, we report centered 95 percent probability intervals for each lag in our impulse response function estimators.[14] If the confidence intervals were perfectly accurate, they should on average coincide with the boundary of the 95 percent probability interval.

When we generate data from the two-shock MLE and CKM specifications, we set $Y_t =$

[13] Sims and Zha (1999) refer to what we call the percentile-based confidence interval as the 'other-percentile bootstrap interval'. This procedure has been used in several studies, such as Blanchard and Quah (1989), Christiano, Eichenbaum, and Evans (1999), Francis and Ramey (2004), McGrattan (2006), and Runkle (1987). The standard-deviation based confidence interval has been used by other researchers, such as Christiano Eichenbaum, and Evans (2005), Gali (1999), and Gali and Rabanal (2004).

[14] For each lag starting at the impact period, we ordered the 1,000 estimated impulse responses from smallest to largest. The lower and upper boundaries correspond to the 25^{th} and the 975^{th} impulses in this ordering.

$(\Delta \log a_t, \log l_t)'$. When we generate data from the three-shock MLE and CKM specifications, we set $Y_t = (\Delta \log a_t, \log l_t, \log i_t/y_t)'$.

3.1. Short-Run Identification

Results for the two- and three- Shock MLE Specifications

Figure 2 reports results generated from four different parameterizations of the recursive version of the RBC model. In each panel, the solid line is the average estimated impulse response function for the 1,000 data sets simulated using the indicated economic model. For each model, the starred line is the true impulse response function of hours worked. In each panel, the gray area defines the centered 95 percent probability interval for the estimated impulse response functions. The stars with no line indicate the average percentile-based confidence intervals across the 1,000 data sets. The circles with no line indicate the average standard-deviation-based confidence intervals.

Figures 3 and 4 graph the coverage rates for the percentile-based and standard-deviation-based confidence intervals. For each case we graph how often, across the 1,000 data sets simulated from the economic model, the econometrician's confidence interval contains the relevant coefficient of the true impulse response function.

The 1,1 panel in Figure 2 exhibits the properties of the VAR-based estimator of the response of hours to a technology shock when the data are generated by the two-shock MLE specification. The 2,1 panel corresponds to the case when the data generating process is the three-shock MLE specification.

The panels have two striking features. First, there is essentially no evidence of bias in the estimated impulse response functions. In all cases, the solid lines are very close to the starred lines. Second, an econometrician would not be misled in inference by using standard procedures for constructing confidence intervals. The circles and stars are close to the boundaries of the gray area. The 1,1 panels in Figures 3 and 4 indicate that the coverage rates are roughly 90 percent. So, with high probability, VAR-based confidence intervals include the true value of the impulse response coefficients.

Results for the CKM Specification

The second column of Figure 2 reports the results when the data generating process is given by variants of the CKM specification. The 1,2 and 2,1 panels correspond to the two and three-shock CKM specification, respectively.

The second column of Figure 2 contains the same striking features as the first column. There is very little bias in the estimated impulse response functions. In addition, the average value of the econometrician's confidence interval coincides closely with the actual range of variation in the impulse response function (the gray area). Coverage rates, reported in the 1,2 panels of Figures 3 and 4, are roughly 90 percent. These rates are consistent with the view that VAR-based procedures lead to reliable inference.

A comparison of the gray areas across the first and second columns of Figure 2, clearly indicates that more sampling uncertainty occurs when the data are generated from the CKM specifications than when they are generated from the MLE specifications (the gray areas are wider). VAR-based confidence intervals detect this fact.

The sign restrictions are $a, c > 0$. To compute the dynamic response of Y_t to the two technology shocks, we require the first two columns of C. To obtain these, we proceed as follows. Let $D \equiv [I - B(1)]^{-1} C$, so that:

$$DD' = [I - B(1)]^{-1} V [I - B(1)']^{-1} = S_Y(0), \tag{B.1}$$

where, as before, $S_Y(0)$ is the spectral density of Y_t at frequency-zero, as implied by the estimated VAR. The exclusion restrictions require that D have the following structure:

$$D = \begin{bmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{22} & 0 \\ d_{31} & d_{32} & d_{33} \end{bmatrix}.$$

Here, the zero restrictions reflect our exclusion restrictions, and the sign restrictions require $d_{11}, d_{22} \geq 0$. Then,

$$DD' = \begin{bmatrix} d_{11}^2 & d_{11}d_{21} & d_{11}d_{31} \\ d_{21}d_{11} & d_{21}^2 + d_{22}^2 & d_{21}d_{31} + d_{22}d_{32} \\ d_{31}d_{11} & d_{31}d_{21} + d_{32}d_{22} & d_{31}^2 + d_{32}^2 + d_{33}^2 \end{bmatrix} = \begin{bmatrix} S_Y^{11}(0) & S_Y^{21}(0) & S_Y^{31}(0) \\ S_Y^{21}(0) & S_Y^{22}(0) & S_Y^{32}(0) \\ S_Y^{31}(0) & S_Y^{32}(0) & S_Y^{33}(0) \end{bmatrix}$$

and

$$\begin{aligned} d_{11} &= \sqrt{S_Y^{11}(0)},\ d_{21} = S_Y^{21}(0)/d_{11},\ d_{31} = S_Y^{31}(0)/d_{11} \\ d_{22} &= \sqrt{\frac{S_Y^{11}(0) S_Y^{22}(0) - \left(S_Y^{21}(0)\right)^2}{S_Y^{11}(0)}},\ d_{32} = \frac{S_Y^{32}(0) - S_Y^{21}(0) S_Y^{31}(0)/d_{11}^2}{d_{22}}. \end{aligned}$$

The sign restrictions imply that the square roots should be positive. The fact that $S_Y(0)$ is positive definite ensures that the square roots are real numbers. Finally, the first two columns of C are calculated as follows:

$$\left[C_1 \vdots C_2\right] = [I - B(1)] \left[D_1 \vdots D_2\right],$$

where C_i is the i^{th} column of C and D_i is the i^{th} column of D, $i = 1, 2$.

To construct our modified VAR procedure, simply replace $S_Y(0)$ in (B.1) by (4.4).

Figure 1: A Simulated Time Series for Hours

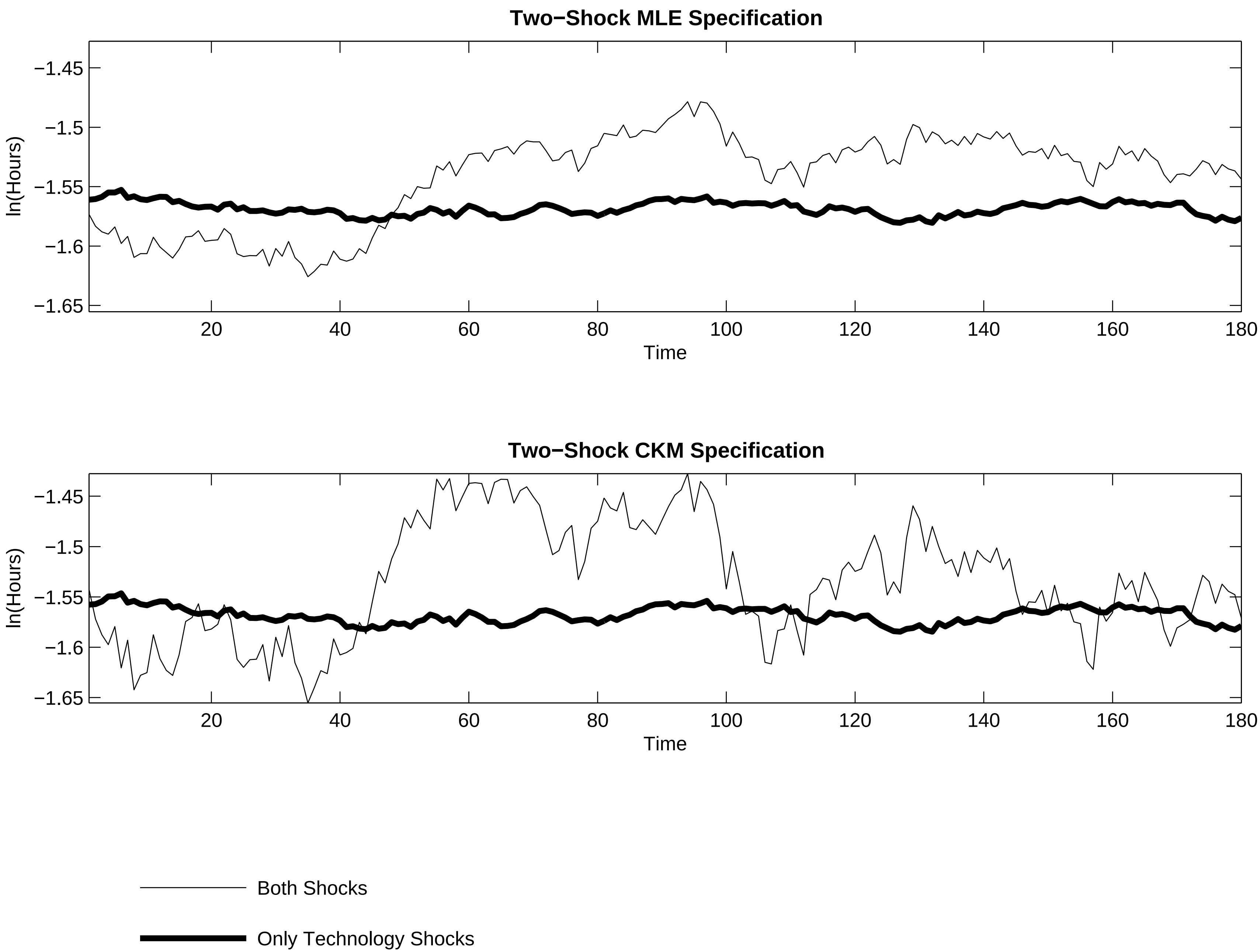

Figure 2: Short−run Identification Results

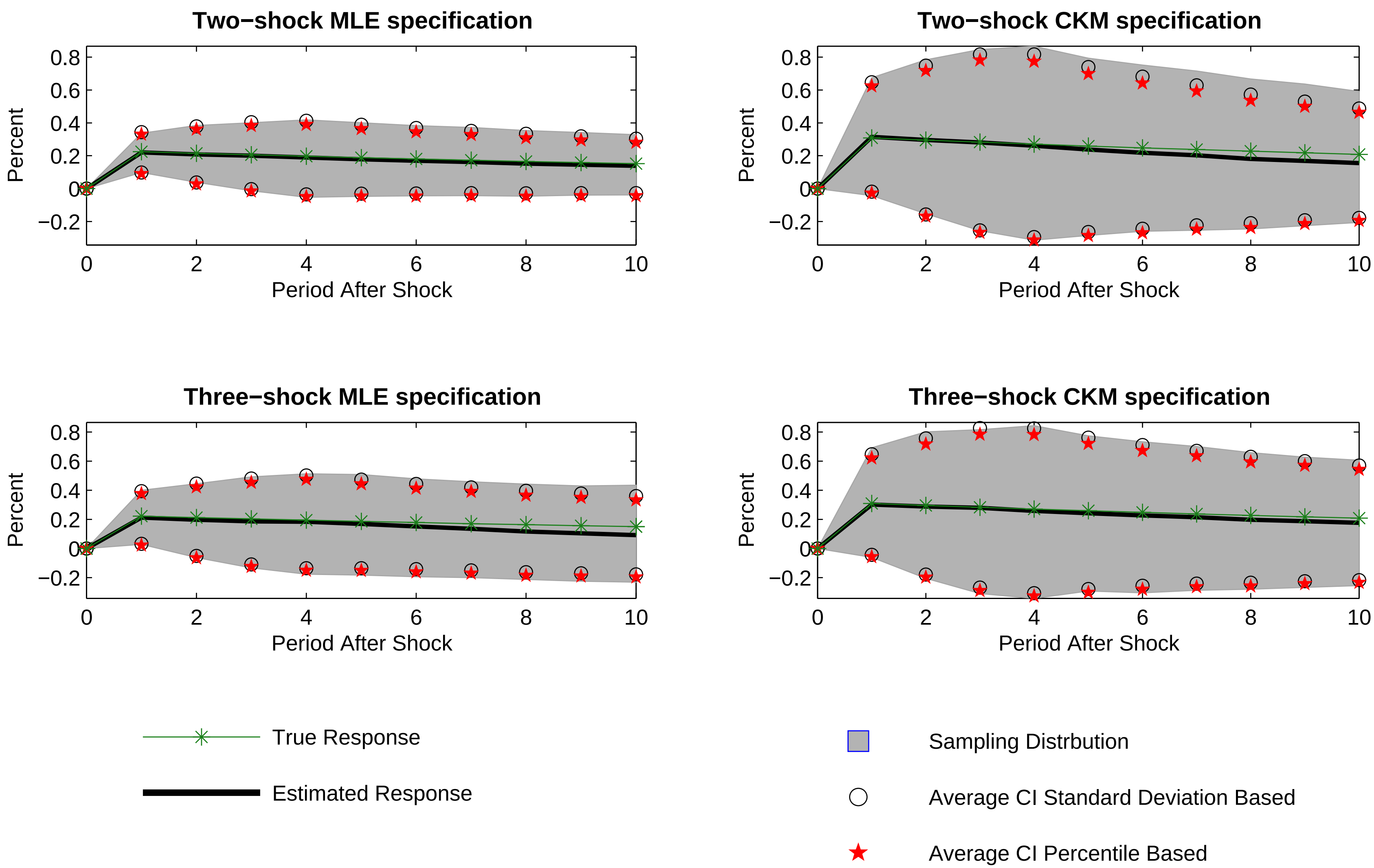

Figure 3: Coverage Rates, Percentile−Based Confidence Intervals

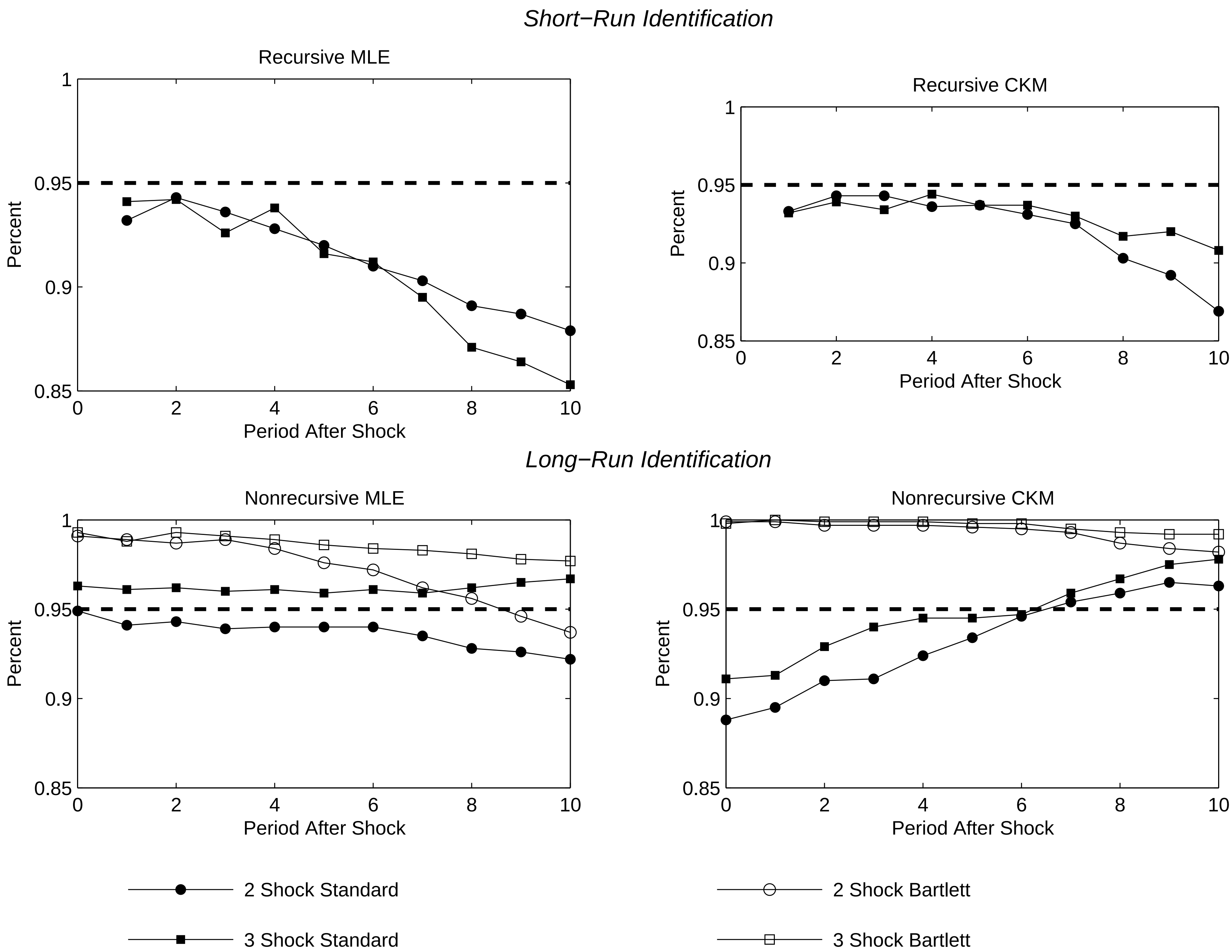

Figure 4: Coverage Rates, Standard Deviation−Based Confidence Intervals

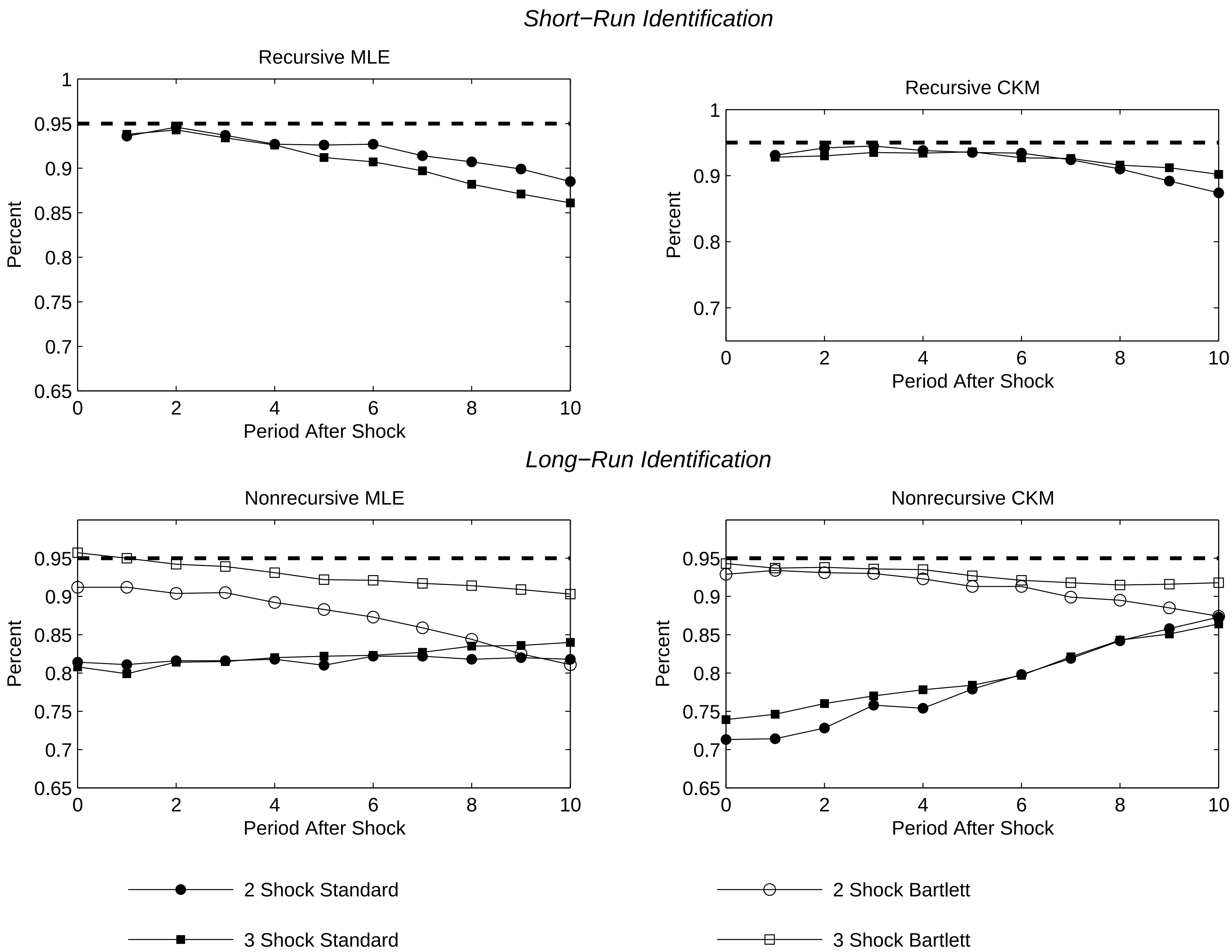

Figure 5: Long−run Identification Results

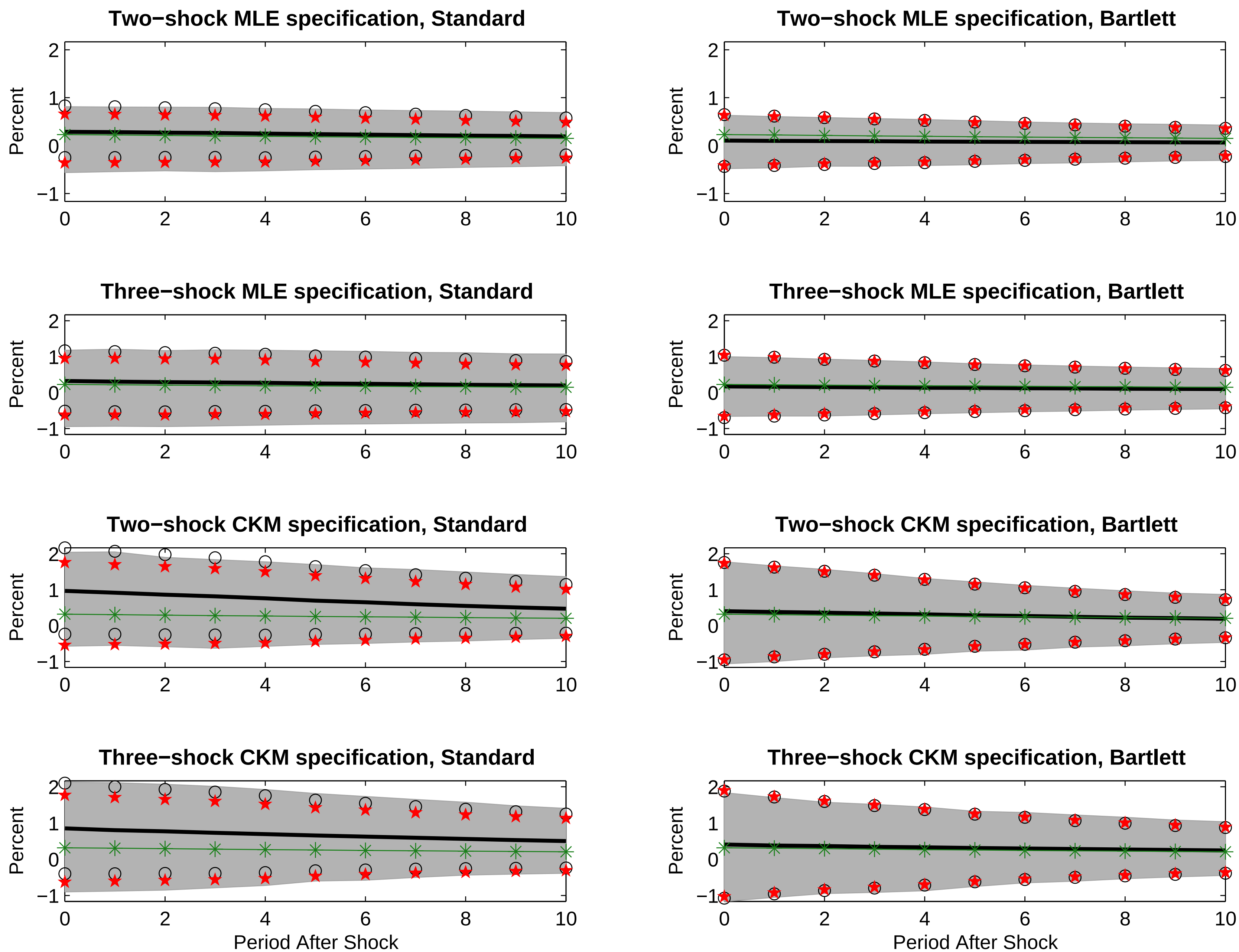

Figure 6: Analyzing Precision in Inference

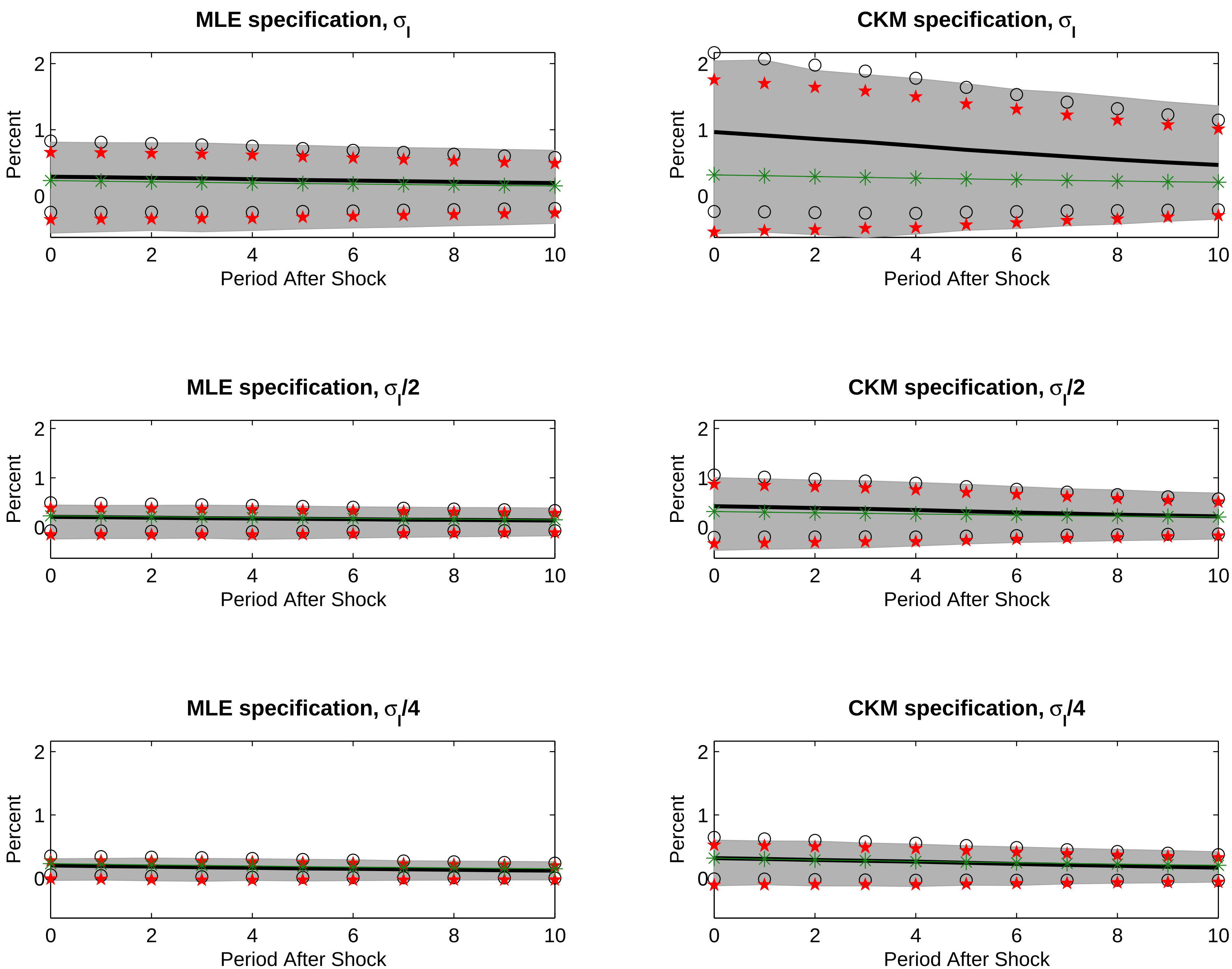

Figure 7: Varying the Labor Elasticity in the Two−shock CKM Specification

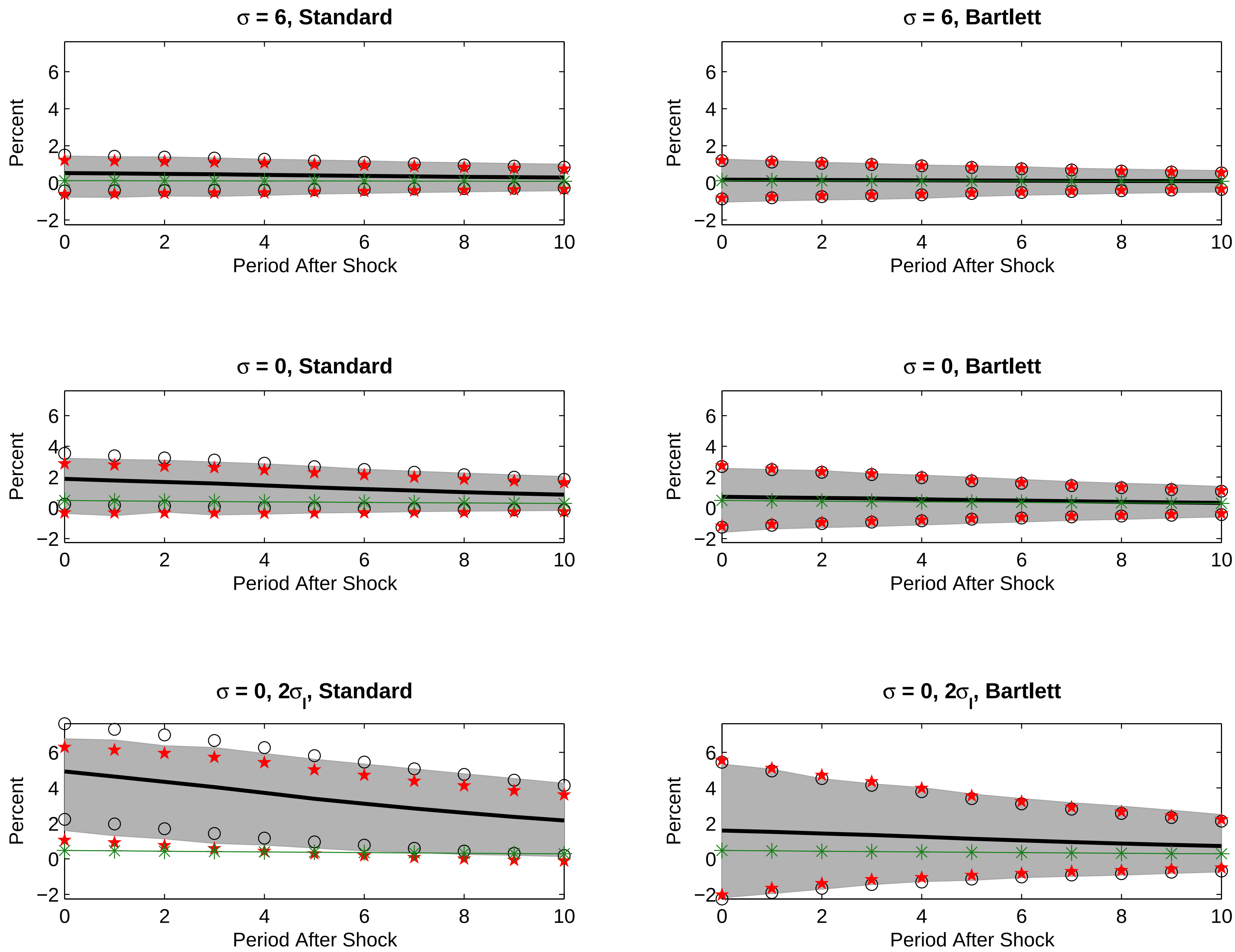

Figure 8: Impact of C_1 on Distortions

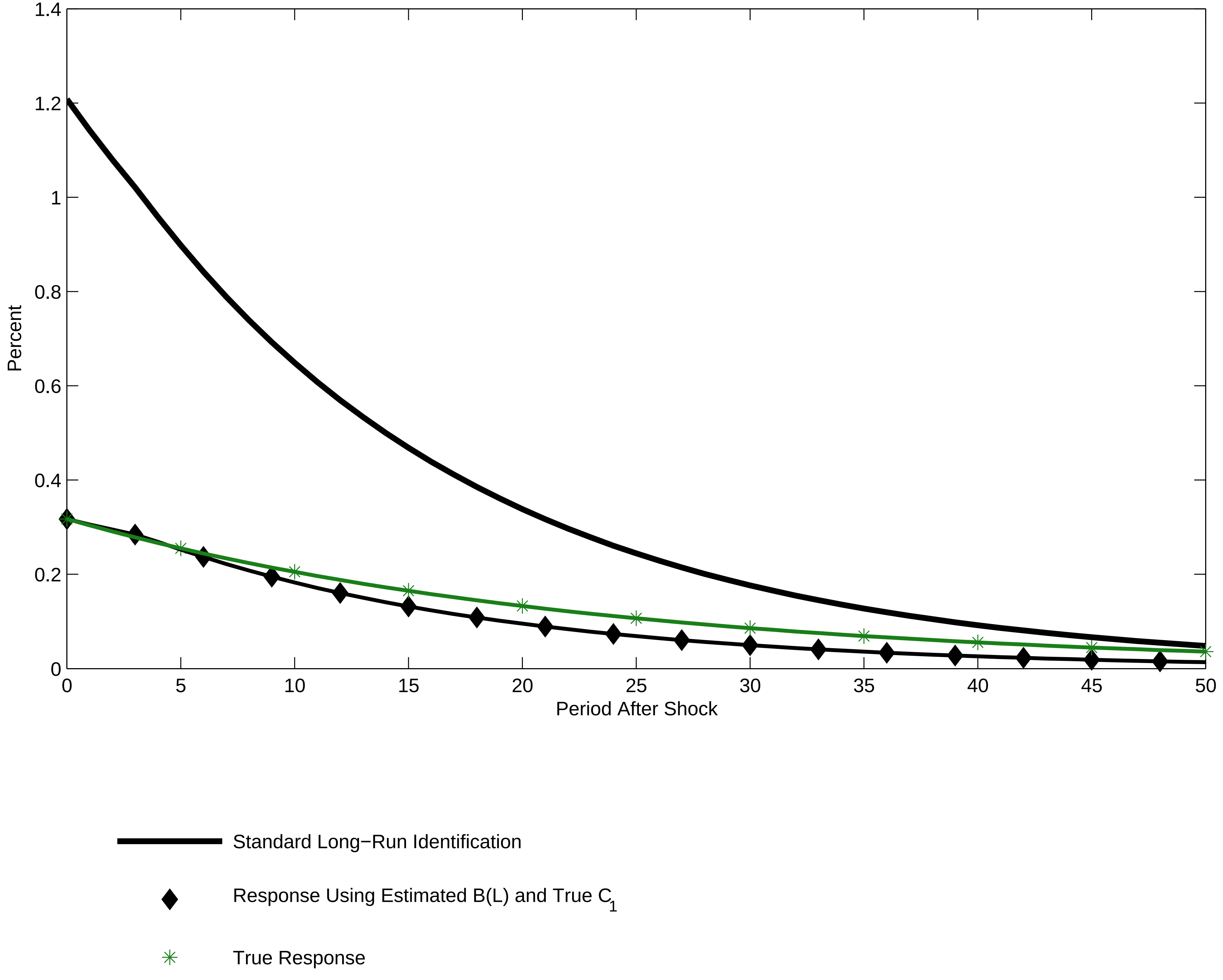

Figure 9: Analysis of Long−run Identification Results

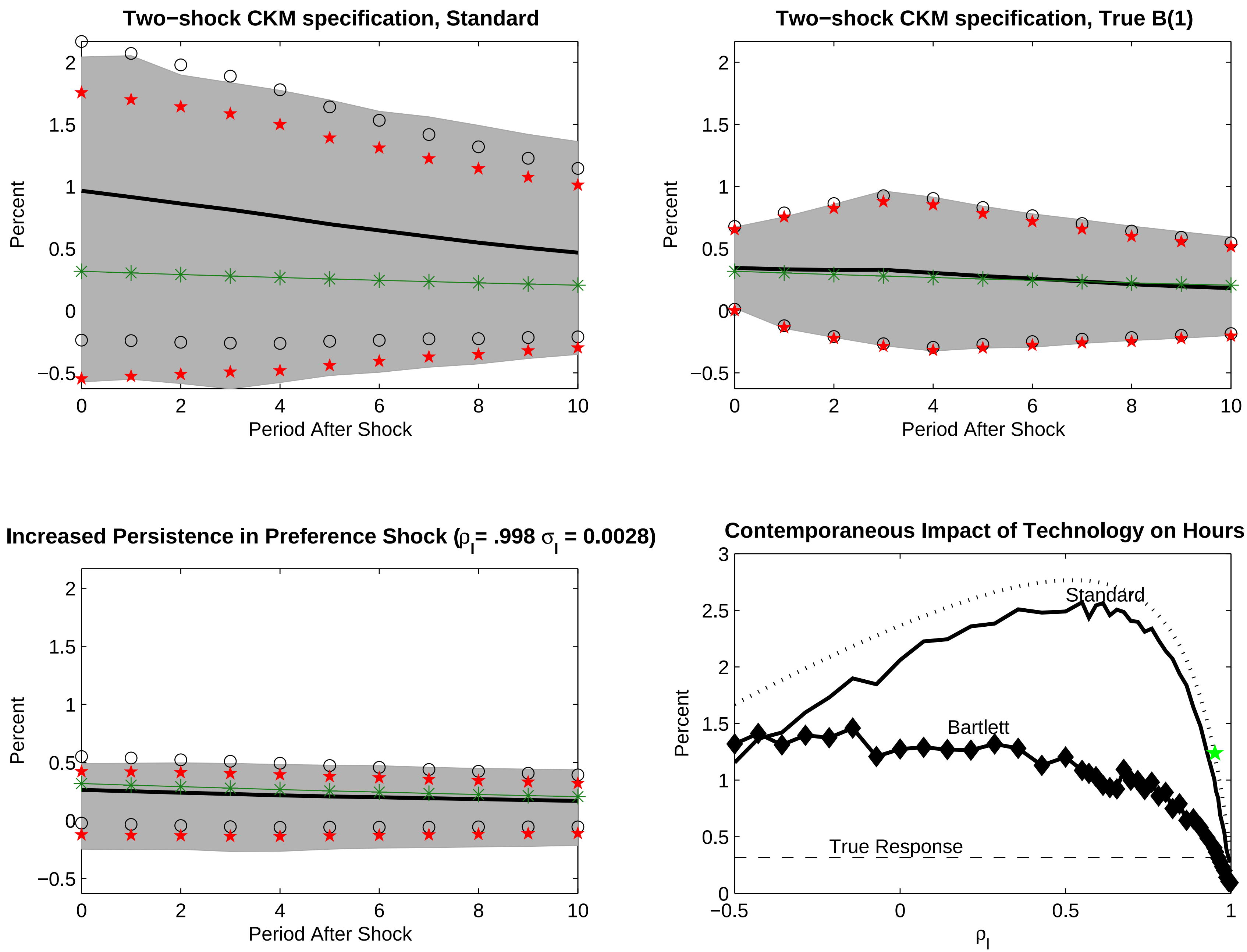

Figure 10: Comparing Long− and Short−Run Identification

Recursive Two−Shock CKM Specification

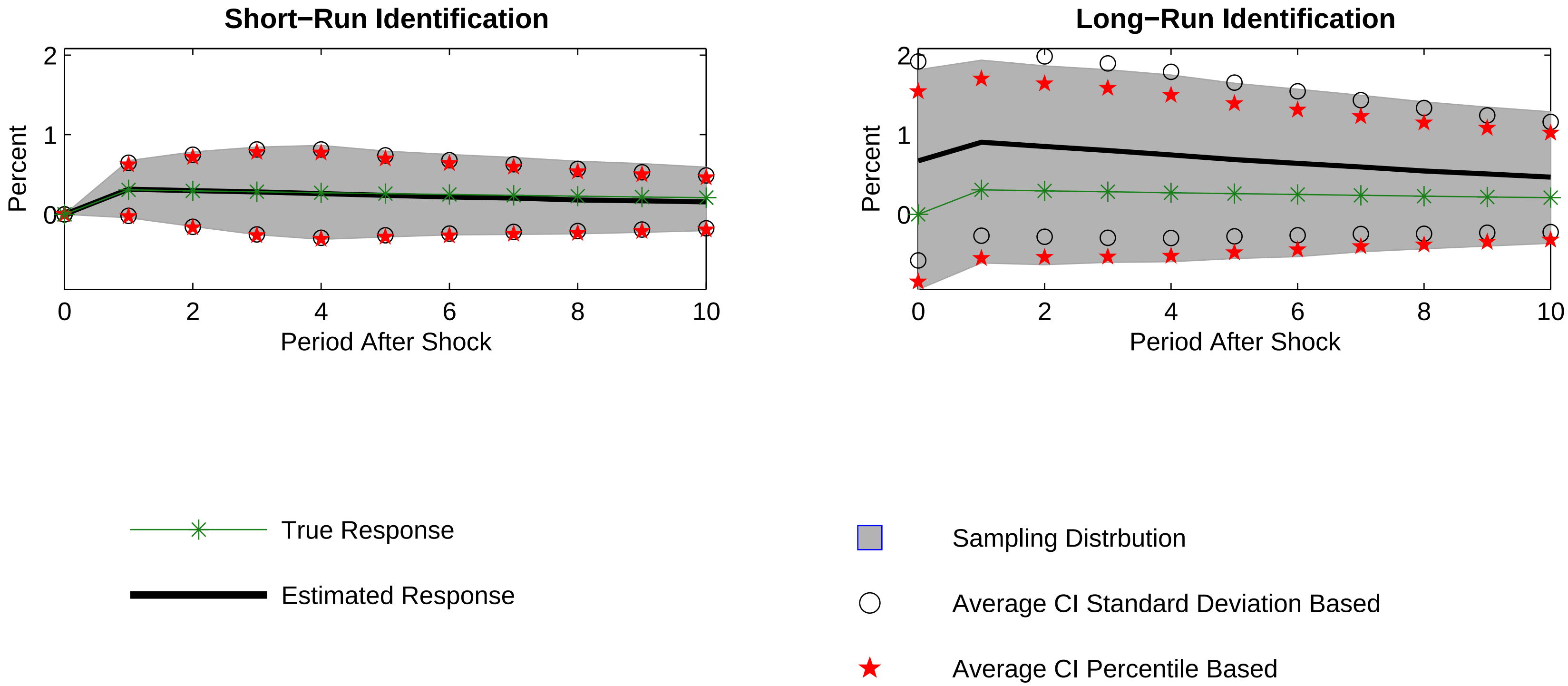

Figure 11: The Treatment of CKM Measurement Error

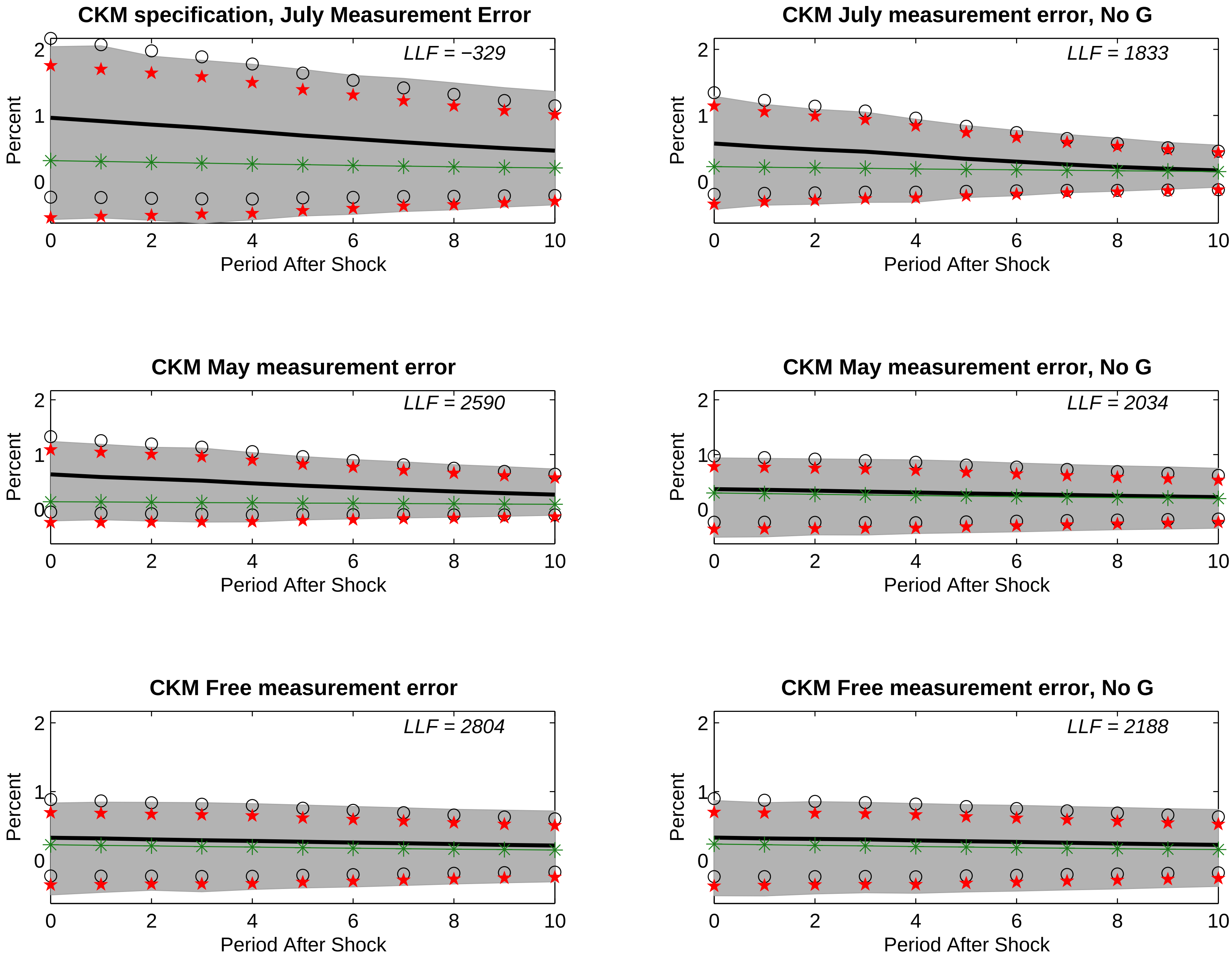

Figure 12: Stochastic Process Uncertainty

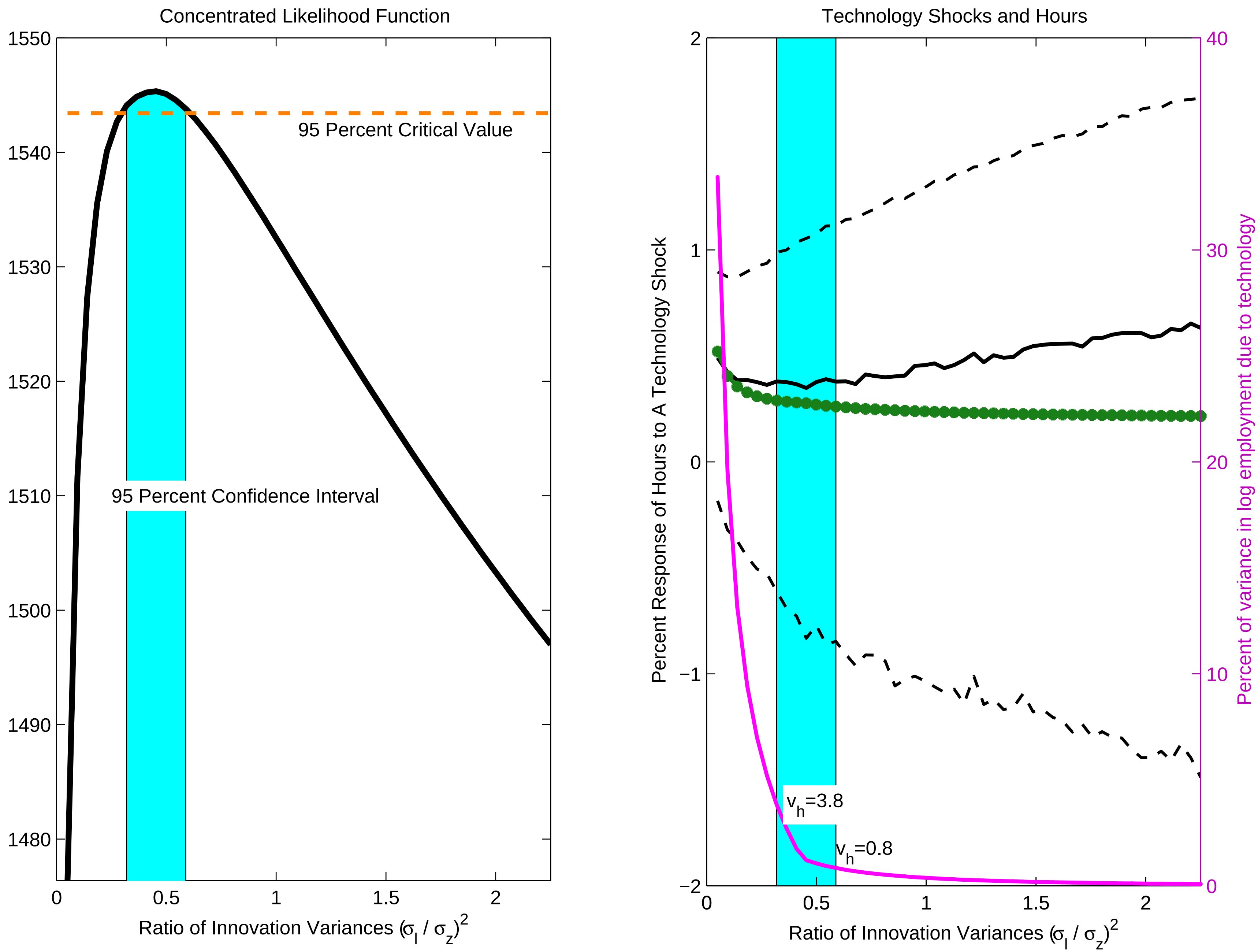

Figure 13: Data Sensitivity and Inference in VARs

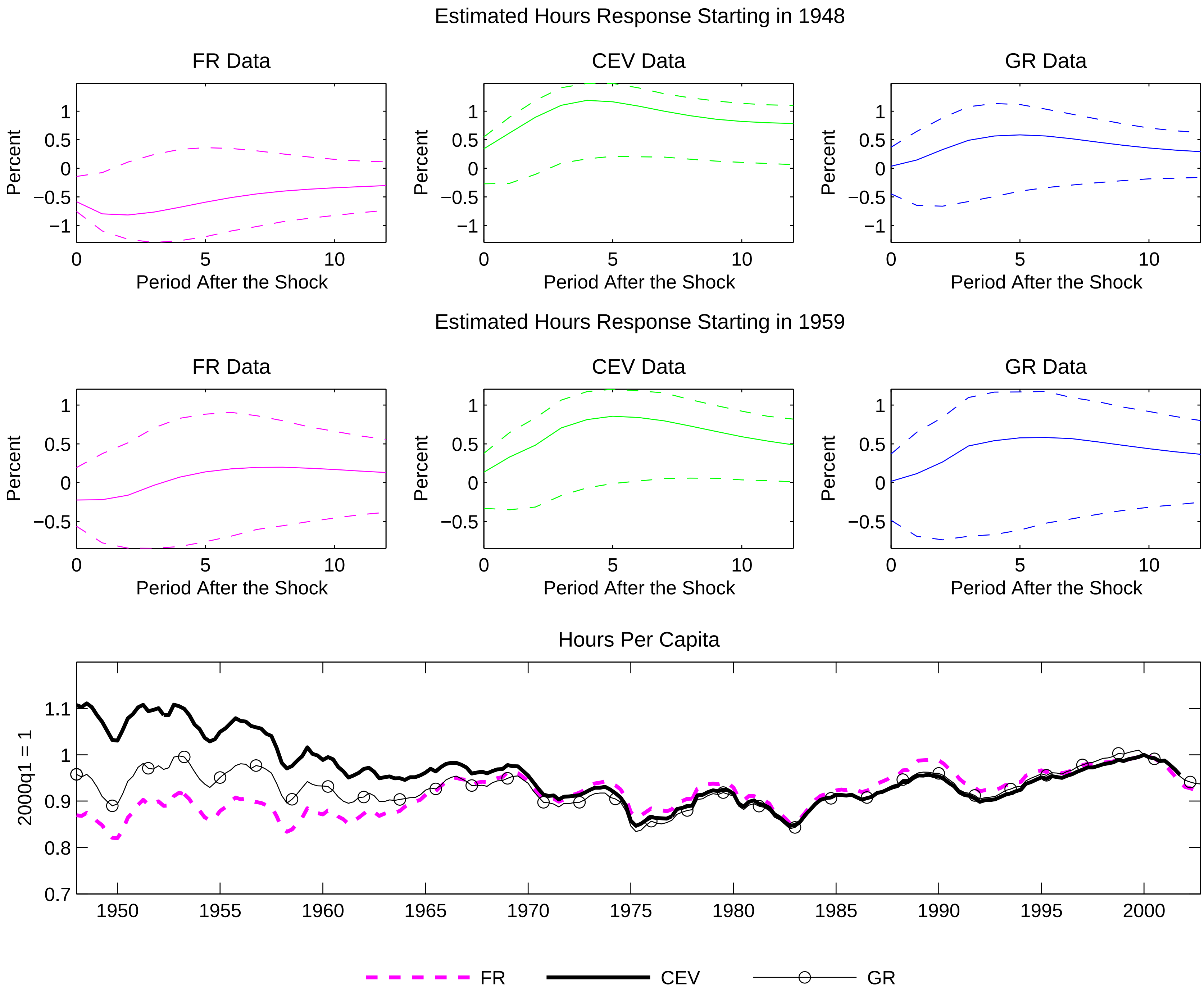

Figure 14: Impulse Response Results when the ACEL Model is the DGP

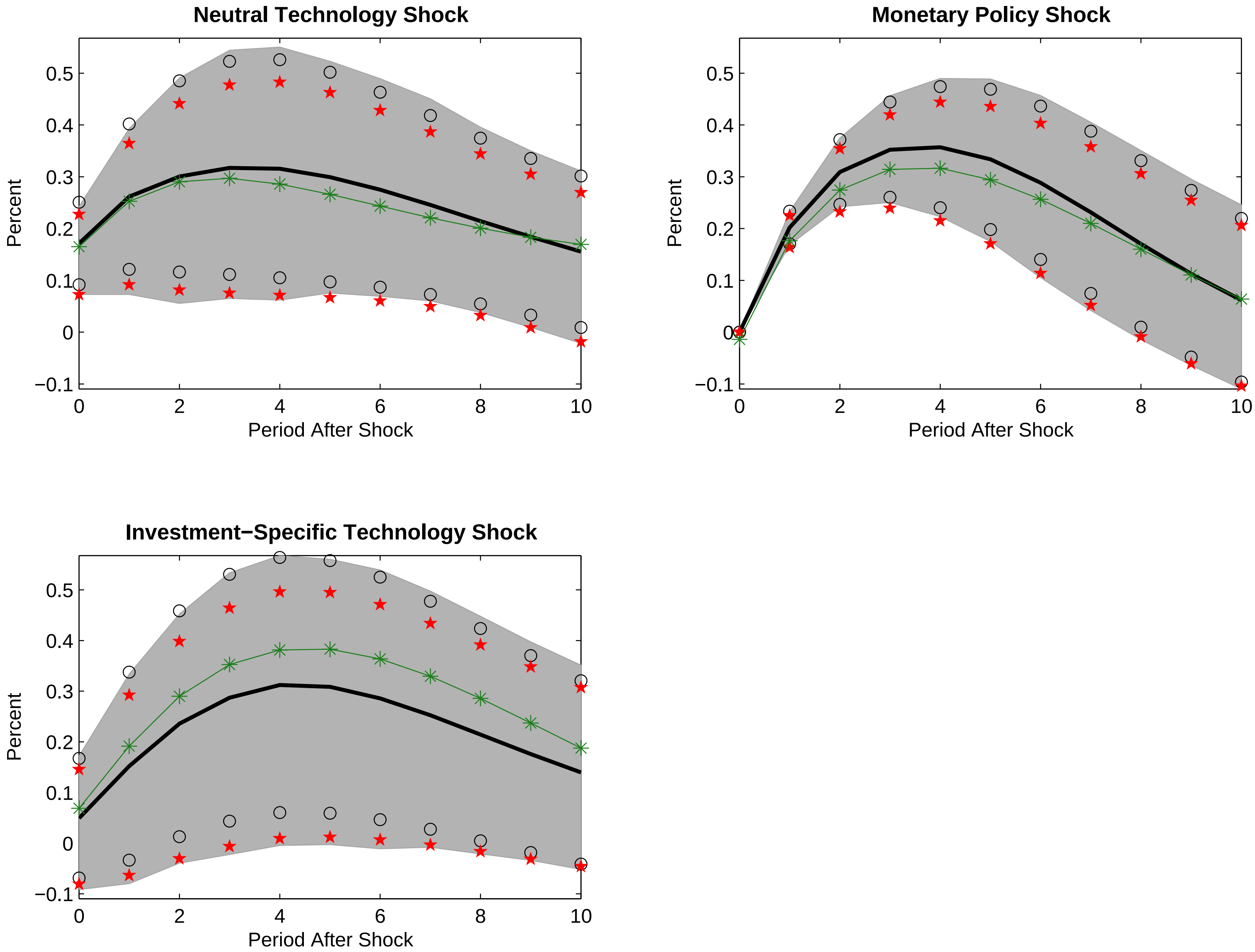

Table 1: Contribution of Technology Shocks to Volatility

Model specification		Unfiltered		HP-filtered		One-step-ahead forecast error	
		Measure of Variation					
		$\ln l_t$	$\Delta \ln y_t$	$\ln l_t$	$\ln y_t$	$\ln l_t$	$\Delta \ln y_t$
MLE							
Base	Nonrecursive	3.73	67.16	7.30	67.14	7.23	67.24
	Recursive	3.53	58.47	6.93	64.83	0.00	57.08
$\sigma_l/2$	Nonrecursive	13.40	89.13	23.97	89.17	23.77	89.16
	Recursive	12.73	84.93	22.95	88.01	0.00	84.17
$\sigma_l/4$	Nonrecursive	38.12	97.06	55.85	97.10	55.49	97.08
	Recursive	36.67	95.75	54.33	96.68	0.00	95.51
$\sigma = 6$	Nonrecursive	3.26	90.67	6.64	90.70	6.59	90.61
	Recursive	3.07	89.13	6.28	90.10	0.00	88.93
$\sigma = 0$	Nonrecursive	4.11	53.99	7.80	53.97	7.73	54.14
	Recursive	3.90	41.75	7.43	50.90	0.00	38.84
Three	Nonrecursive	0.18	45.67	3.15	45.69	3.10	45.72
	Recursive	0.18	36.96	3.05	43.61	0.00	39.51
CKM							
Base	Nonrecursive	2.76	33.50	1.91	33.53	1.91	33.86
	Recursive	2.61	25.77	1.81	31.41	0.00	24.93
$\sigma_l/2$	Nonrecursive	10.20	66.86	7.24	66.94	7.23	67.16
	Recursive	9.68	58.15	6.88	64.63	0.00	57.00
$\sigma_l/4$	Nonrecursive	31.20	89.00	23.81	89.08	23.76	89.08
	Recursive	29.96	84.76	22.79	87.91	0.00	84.07
$\sigma = 6$	Nonrecursive	0.78	41.41	0.52	41.33	0.52	41.68
	Recursive	0.73	37.44	0.49	40.11	0.00	37.42
$\sigma = 0$	Nonrecursive	2.57	20.37	1.82	20.45	1.82	20.70
	Recursive	2.44	13.53	1.73	18.59	0.00	12.33
$\sigma = 0$ and $2\sigma_l$	Nonrecursive	0.66	6.01	0.46	6.03	0.46	6.12
	Recursive	0.62	3.76	0.44	5.41	0.00	3.40
Three	Nonrecursive	2.23	30.73	1.71	31.11	1.72	31.79
	Recursive	2.31	23.62	1.66	29.67	0.00	25.62

Note: (a) V_h corresponds to the columns denoted by $\ln(l_t)$.
(b) In each case, the results report the ratio of two variances: the numerator is the variance for the system with only technology shocks and the denominator is the variance for the system with both technology shock and labor tax shocks. All statistics are averages of the ratios, based on 300 simulations of 5000 observations for each model.
(c) 'Base' means the two-shock specification, whether MLE or CKM, as indicated. Three' means the three-shock specification.
(d) For a description of the procedure used to calculate the forecast error variance, see footnote 13.
(e) 'MLE' and 'CKM' refer, respectively, to our and CKM's estimated models.

.

Table 2: Properties of Two-Shock CKM Specification

Panel A: First Six Lag Matrices in Infinite-Order VAR Representation

$$B_1 = \begin{bmatrix} 0.013 & 0.041 \\ 0.0065 & 0.94 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.012 & -0.00 \\ 0.0062 & -0.00 \end{bmatrix},\ B_3 = \begin{bmatrix} 0.012 & -0.00 \\ 0.0059 & -0.00 \end{bmatrix},$$

$$B_4 = \begin{bmatrix} 0.011 & -0.00 \\ 0.0056 & -0.00 \end{bmatrix},\ B_5 = \begin{bmatrix} 0.011 & -0.00 \\ 0.0054 & -0.00 \end{bmatrix},\ B_6 = \begin{bmatrix} 0.010 & -0.00 \\ 0.0051 & -0.00 \end{bmatrix}$$

Panel B: Population Estimate of Four-lag VAR

$$\hat{B}_1 = \begin{bmatrix} 0.017 & 0.043 \\ 0.0087 & 0.94 \end{bmatrix},\ \hat{B}_2 = \begin{bmatrix} 0.017 & -0.00 \\ 0.0085 & -0.00 \end{bmatrix},\ \hat{B}_3 = \begin{bmatrix} 0.012 & -0.00 \\ 0.0059 & -0.00 \end{bmatrix},$$

$$\hat{B}_4 = \begin{bmatrix} 0.0048 & -0.0088 \\ 0.0025 & -0.0045 \end{bmatrix}$$

Panel C: Actual and Estimated Sum of VAR Coefficients

$$\hat{B}(1) = \begin{bmatrix} 0.055 & 0.032 \\ 0.14 & 0.94 \end{bmatrix},\ B(1) = \begin{bmatrix} 0.28 & 0.022 \\ 0.14 & 0.93 \end{bmatrix},\ \textstyle\sum_{j=1}^{4} B_j = \begin{bmatrix} 0.047 & 0.039 \\ 0.024 & 0.94 \end{bmatrix}$$

Panel D: Actual and Estimated Zero-Frequency Spectral Density

$$S_Y(0) = \begin{bmatrix} 0.00017 & 0.00097 \\ 0.00097 & 0.12 \end{bmatrix},\ \hat{S}_Y(0) = \begin{bmatrix} 0.00012 & 0.0022 \\ 0.0022 & 0.13 \end{bmatrix}.$$

Panel E: Actual and Estimated One-Step-Ahead Forecast Error Variance

$$V = \hat{V} = \begin{bmatrix} 0.00012 & -0.00015 \\ -0.00015 & -0.00053 \end{bmatrix}$$

Panel F: Actual and Estimated Impact Vector

$$C_1 = \begin{pmatrix} 0.00773 \\ 0.00317 \end{pmatrix},\ \hat{C}_1 = \begin{pmatrix} 0.00406 \\ 0.01208 \end{pmatrix}$$

Table 3: Percent Contribution of Shocks in the ACEL model to the Variation in Hours and in Output

	Types of shock		
Statistic	Monetary Policy	Neutral Technology	Capital-Embodied
variance of logged hours	22.2	40.0	38.5
variance of HP filtered logged hours	37.8	17.7	44.5
variance of Δy	29.9	46.7	23.6
variance of HP filtered logged output	31.9	32.3	36.1

Note: Results are average values based on 500 simulations of 3100 observations each.
ACEL: Altig Christiano, Eichenbaum and Linde (2005).